FLUID MECHANICS AND HYDRAULIC MACHINES

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Compiled by

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Content

Properties of Fluids

Skip to Questions (IAS, IES, GATE)

Highlights

Definition of fluid
A fluid is a substance which deforms continuously when subjected to external shearing forces.

Characteristics of fluid
1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.
3. It is interesting to note that a solid suffers strain when subjected to shear forces whereas a fluid suffers Rate of Strain i.e. it flows under similar circumstances.

Ideal and Real Fluids
1. Ideal Fluid

An ideal fluid is one which has

no viscosity
no surface tension
and incompressible

2. Real Fluid

An Real fluid is one which has

viscosity
surface tension
and compressible

Naturally available all fluids are real fluid.

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Viscosity

Definition: Viscosity is the property of a fluid which determines its resistance to shearing stresses.

Cause of Viscosity: It is due to cohesion and molecular momentum exchange between fluid layers.

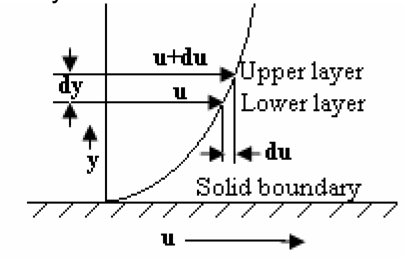
Newton's Law of Viscosity: It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

The constant of proportionality is called the co-efficient of viscosity.

When two layers of fluid, at a distance 'dy' apart, move one over the other at different velocities, say u and u+du

Velocity gradient = $\frac{du}{dy}$

According to Newton's law $\tau \propto \frac{du}{dy}$ or $\tau = \mu \frac{du}{dy}$



Velocity Variation near a solid boundary

Where = constant of proportionality and is known as co-efficient of Dynamic viscosity or only Viscosity

As $\mu = \frac{\tau}{\left[\frac{du}{dy}\right]}$ Thus viscosity may also be defined as the shear stress required producing unit *rate of shear strain*

Units of Viscosity

S.I. Units: Pa.s or N.s/m^2
C.G.S Unit of viscosity is Poise= dune-sec/cm^2
One Poise= 0.1 Pa.s
1/100 Poise is called centipoises.
Dynamic viscosity of water at 200C is approx= 1 cP

Kinematic Viscosity

It is the ratio between the dynamic viscosity and density of fluid and denoted by

Mathematically $\nu = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{\rho}$

Units of Kinematic Viscosity

S.I units: m^2/s
C.G.S units: stoke = cm^2/sec
One stoke = 10-4 m^2/s
Thermal diffusivity and molecular diffusivity have same dimension, therefore, by analogy, the kinematic viscosity is also referred to as the *momentum diffusivity* of the fluid, i.e. the ability of the fluid to transport momentum.

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Effect of Temperature on Viscosity

With increase in temperature
Viscosity of liquids decrease
Viscosity of gasses increase
Note: 1. Temperature response are neglected in case of Mercury
2. The lowest viscosity is reached at the critical temperature.

Effect of Pressure on Viscosity

Pressure has very little effect on viscosity.
But if pressure increases intermolecular gap decreases then cohesion increases so viscosity would be increase.

Classification of fluids

1. Newtonian Fluids

These fluids follow Newton's viscosity equation.
For such fluids viscosity does not change with *rate of deformation*

2. Non- Newtonian fluids

This fluid does not follow Newton's viscosity equation.
Such fluids are relatively uncommon e.g. Printer ink, blood, mud, slurries, polymer solutions.

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Non-Newtonian Fluids $(\tau \neq \mu \frac{du}{dy})$		
Purely Viscous Fluids		**Visco-elastic Fluids**
Time - Independent	**Time - Dependent**	
1. Pseudo plastic Fluids $\tau = \mu\left(\frac{du}{dy}\right)^n ; n < 1$ Example: Blood, milk **2. Dilatant Fluids** $\tau = \mu\left(\frac{du}{dy}\right)^n ; n > 1$ Example: Butter **3. Bingham or Ideal Plastic Fluid** $\tau = \tau_o + \mu\left(\frac{du}{dy}\right)^n$ Example: Water suspensions of clay and flyash	**1.Thixotropic Fluids** $\tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$ *f(t)is decreasing* Example: Printer ink; crude oil **2. Rheopectic Fluids** $\tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$ *f(t)is increasing* Example: Rare liquid solid suspension	**Visco- elastic Fluids** $\tau = \mu \frac{du}{dy} + \alpha E$ Example: Liquid-solid combinations in pipe flow.

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Surface tension

Surface tension is due to cohesion between particles at the surface.

Capillarity action is due to both cohesion and adhesion.

Surface tension

The tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

Pressure inside a curved surface

For a general curved surface with radii of curvature r_1 and r_2 at a point of interest $\Delta p = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

a. Pressure inside a water droplet, $\Delta p = \frac{4\sigma}{d}$

b. Pressure inside a soap bubble, $\Delta p = \frac{8\sigma}{d}$

c. Liquid jet. $\Delta p = \frac{2\sigma}{d}$

Capillarity

A general term for phenomena observed in liquids due to inter-molecular attraction at the liquid boundary, e.g. the rise or depression of liquids in narrow tubes. We use this term for capillary action.

Capillarity rise and depression phenomena depends upon the surface tension of the liquid as well as the material of the tube.

1. General formula, $h = \dfrac{4\sigma\cos\theta}{\rho g d}$
2. For water and glass θ = 0°, $h = \dfrac{4\sigma}{\rho g d}$
3. For mercury and glass θ = 138°, $h = -\dfrac{4\sigma\cos 42}{\rho g d}$

(h is negative indicates capillary depression)
Note: If adhesion is more than cohesion, the wetting tendency is more and the angle of contact is smaller.

Questions (IAS, IES, GATE)

Fluid

1. The drag force exerted by a fluid on a body immersed in the fluid is due to
(a) pressure and viscous forces (b) pressure and gravity forces
(c) pressure and surface tension Forces (d) viscous and gravity forces **[IES-2002]**

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2. Which one of the following sets of conditions clearly apply to an ideal fluid?
(a) Viscous and compressible (b) Nonviscous and incompressible
(c) Nonviscous and compressible (d) Viscous and incompressible
[IAS-1994]

Viscosity

3. Newton's law of viscosity depends upon the **[IES-1998]**
(a) stress and strain in a fluid (b) shear stress, pressure and velocity
(c) shear stress and rate of strain (d) viscosity and shear stress

4. The shear stress developed in lubricating oil, of viscosity 9.81 poise, filled between two parallel plates 1 cm apart and moving with relative velocity of 2 m/s is **[IES-2001]**
(a) 20 N/m^2 (b) 19.62 N/m^2 (c) 29.62 N/m^2 (d) 40 N/m^2

5. The SI unit of kinematic viscosity (υ) is
(a) m^2/s (b) kg/m-s (c) m/s^2 (d) m^3/s^2 **[GATE-2001]**

6. What are the dimensions of kinematic viscosity of a fluid?
(a) LT^{-2} (b) L^2T^{-1} (c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$ **[IES-2007]**

7. An oil of specific gravity 0.9 has viscosity of 0.28 Strokes at 38^{0}C. What will be its viscosity in Ns/m^2 ?
(a) 0.2520 (b) 0.0311 (c) 0.0252 (d) 0.0206 **[IES-2005]**

8. Kinematic viscosity of air at 20^{0}C is given to be 1.6×10^{-5}m^2/s. Its kinematic viscosity at 70^{0}C will be vary approximately **[GATE-1999]**
(a) 2.2×10^{-5}m^2/s (b) 1.6×10^{-5}m^2/s (c) 1.2×10^{-5}m^2/s (d) 3.2×10^{-5}m^2/s

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9. When a flat plate of 0.1 m^2 area is pulled at a constant velocity of 30 cm/sec parallel to another stationary plate located at a distance 0.01 cm from it and the space in between is filled with a fluid of dynamic viscosity = 0.001 Ns/m^2, the force required to be applied is
(a) 0.3 N (b) 3 N (c) 10 N (d)16N **[IAS-2004]**

Newtonian fluid

10. For a Newtonian fluid **[GATE-2006; 1995]**
(a) Shear stress is proportional to shear strain
(b) Rate of shear stress is proportional to shear strain
(c) Shear stress is proportional to rate of shear strain
(d) Rate of shear stress is proportional to rate of shear strain

11. In a Newtonian fluid, laminar flow between two parallel plates, the ratio (τ) between the shear stress and rate of shear strain is given by **[IAS-1995]**

(a) $\mu \frac{d^2\mu}{dy^2}$ (b) $\mu \frac{du}{dy}$ (c) $\mu\left(\frac{du}{dy}\right)^2$ (d) $\mu\left(\frac{du}{dy}\right)^{1/2}$

12. Consider the following statements: **[IAS-2000]**
1. Gases are considered incompressible when Mach number is less than 0.2
2. A Newtonian fluid is incompressible and non- viscous
3. An ideal fluid has negligible surface tension
Which of these statements is /are correct?
(a) 2 and 3 (b) 2 alone (c) 1 alone (d) 1 and 3

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Non-Newtonian fluid

13. If the Relationship between the shear stress τ and the rate of shear strain $\frac{du}{dy}$ is expressed as

$\tau = \mu\left(\frac{du}{dy}\right)^n$ then the fluid with exponent n>1 is known as which one of the following?

[IES-2007]

(a) Bingham Plastic (b) Dilatant Fluid (c) Newtonian Fluid (d) Pseudo plastic Fluid

14. The relations between shear stress (τ) and velocity gradient for ideal fluids, Newtonian fluids and non-Newtonian fluids are given below. Select the correct combination.

[IAS-2002]

(a) $\tau = 0;\ \tau = \mu.(\frac{du}{dy})^2;\ \tau = \mu.(\frac{du}{dy})^3$ (b) $\tau = 0;\ \tau = \mu.(\frac{du}{dy});\ \tau = \mu.(\frac{du}{dy})^2$

(c) $\tau = \mu.(\frac{du}{dy});\ \tau = \mu.(\frac{du}{dy})^2;\ \tau = \mu.(\frac{du}{dy})^3$ (d) $\tau = \mu.(\frac{du}{dy});\ \tau = \mu.(\frac{du}{dy})^2;\ \tau = 0$

15. Fluids that require a gradually increasing shear stress to maintain a constant strain rate are known as
[IAS-1997]
(a) rhedopectic fluids (b) thixotropic fluids (c) pseudoplastic fluids (d) Newtonian fluids

16. Match List 1 (Type of fluid) with List II (Variation of shear stress) and select the correct answer:

List I	List II
A. Ideal fluid	1. Shear stress varies linearly with the rate of strain
B. Newtonian fluid	2. Shear stress does not vary linearly with the rate of strain
C. Non-Newtonian fluid	3. Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain
D. Bingham plastic	4. Shear stress is zero

[IES-2001]

	A	B	C	D		A	B	C	D
(a)	3	1	2	4	(b)	4	2	1	3
(c)	3	2	1	4	(d)	4	1	2	3

17. Match List I(Rheological Equation) with List II(Types of Fluids) and select the correct the answer:

List I	List II
A. $\tau = \mu(du/dy)^n$,n=1	1. Bingham plastic
B. $\tau = \mu(du/dy)^n$,n<1	2. Dilatant fluid
C. $\tau = \mu(du/dy)^n$, n>1	3. Newtonian fluid
D. $\tau = \tau_0 + \mu$ (du/dy)^n, n=1	4. Pseudo-plastic fluid

[IES-2003]

	A	B	C	D		A	B	C	D
(a)	3	2	4	1	(b)	4	1	2	3
(c)	3	4	2	1	(d)	4	2	1	3

18. Assertion (A): Blood is a Newtonian fluid. **[IES-2007]**
Assertion(R): The rate of strain varies non-linearly with shear stress for blood.

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Surface tension

19. Surface tension is due to **[IES-1997]**
(a) viscous forces (b) cohesion (c) adhesion
(d) the difference between adhesive and cohesive forces

20. The dimension of surface tension is
(a) ML^{-1} (b) L^2T^{-1} (c) $ML^{-1}T^1$ (d) MT^{-2} **[GATE-1996]**

21. The dimensions of surface tension is
(a) N/m^2 (b) J/m (c) J/m^2 (d)W/m **[GATE-1995]**

22. If the surface tension of water-air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter will be
(a) 0.146N/m^2 (b) 73N/m^2 (c) 146N/m^2 (d) 292 N/m^2 **[IES-1999]**

Capillarity

23. The capillary rise at 20^{0}C in clean glass tube of 1 mm diameter containing water is approximately **[IES-2001]**
(a) 15 mm (b) 50 mm (c) 20 mm (d) 30 mm

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Compressibility and Bulk Modulus

24. Which one of the following is the bulk modulus K of a fluid? (Symbols have the usual meaning)

(a) $\rho \frac{dp}{d\rho}$ (b) $\frac{dp}{\rho d\rho}$ (c) $\frac{\rho d\rho}{dp}$ (d) $\frac{d\rho}{\rho dp}$ **[IES-1997]**

25. When the pressure on a given mass of liquid is increased from 3.0 MPa to 3.5 MPa, the density of the liquid increases from 500 kg/m^3 to 501 kg/m^3.What is the average value of bulk modulus of the liquid over the given pressure range? **[IES-2006]**

(a) 700 MPa (b) 600MPa (c) 500MPa (d) 250MPa

Vapour Pressure

26. Which Property of mercury is the main reason for use in barometers?

(a) High Density (b) Negligible Capillary effect
(c) Very Low vapour Pressure (d) Low compressibility **[IES-2007]**

27. Consider the following properties of a fluid:

1. Viscosity 2. Surface tension 3. Capillarity 4. Vapour pressure

Which of the above properties can be attributed to the flow of jet of oil in an unbroken stream? **[ESE-2005]**

(a) 1 only (b) 2 only (c) 1 and 3 (d) 2 and 4

28. In case of liquids, what is the binary diffusion coefficient proportional to? **[IES-2006]**

(a) Pressure only (b) Temperature only (c) Volume only (d) All the above

29. Match List I (Physical properties of fluid) with List II (Dimensions/Definitions) and select the correct answer: **[IAS-2000]**

List I	List II
A. Absolute viscosity	1. du/dy is constant
B. Kinematic viscosity	2. Newton per meter
C. Newtonian fluid	3. Poise
D. Surface tension	4. Stress/Strain is constant
	5. Stokes

	A	B	C	D		A	B	C	D
(a)	5	3	1	2	(b)	3	5	2	4
(c)	5	3	4	2	(d)	3	5	1	2

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Answers with Explanation

1. Ans. (d)
2. Ans. (b)
3. Ans. (c)
4. Ans. (c) du=2 m/s; dy= 1cm = 0.01 m; μ = 9.81 poise = 0.981 Pa.s

Therefore (τ) = $\mu \dfrac{du}{dy}$ = 0.981 x $\dfrac{2}{0.01}$ = 19.62 N/m^2

5. Ans. (a)
6. Ans. (b)
7. Ans. (c) specific Gravity=0.9 therefore Density = 0.9 x 1000 =900 Kg/m^3
One Stoke = 10^{-4} m^2/s
Viscosity (μ) = $\rho\nu$ = 900 x 0.28 x 10^{-4} = 0.0252 Ns/m^2

8. Ans. (a) Viscosity of gas increases with increasing temperature.
9. Ans. (a) Given, μ = 0.001 Ns/m^2 and du = (V – 0) = 30 cm/sec = 0.3 m/s and distance (dy) = 0.01 cm = 0.0001 m

Therefore, Shear stress (τ) = $\mu \dfrac{du}{dy} = \left(0.001\dfrac{\text{Ns}}{\text{m}^2}\right)\times\dfrac{(0.3\text{m/s})}{(0.0001\text{m})}$ =3N/m^2

Force required (F) = τ x A = 3 x 0.1 = 0.3 N
10. Ans. (c)
11. Ans. (b)
12. Ans. (d)
13. Ans. (b)
14. Ans. (b)
15. Ans. (a) $\tau = \mu\left(\dfrac{du}{dy}\right)^n + f(t)$ where f(t) is increasing

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16. Ans. (d)
17. Ans. (c)
18. Ans. (d) A is false but R is true
19. Ans. (b)
20. Ans. (b)
21. Ans. (c)
22. Ans. (d) P=$\dfrac{4\sigma}{d} = \dfrac{4\times 0.073}{0.001} = 292\,N/m^2$
23. Ans. (d) $h = \dfrac{4\sigma}{\rho g d} = \dfrac{4\times 0.073}{1000\times 9.81\times 0.001} \approx 30\,mm$
24. Ans. (a)
25. Ans. (d) $\dfrac{500\times(3.5-3.0)}{(501-500)} = 250\,MPa$
26. Ans. (c)
27. Ans. (d)
28. Ans. (d)
29. Ans. (d)

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Problem

1. A circular disc of diameter D is slowly in a liquid of a large viscosity (μ) at a small distance (h) from a fixed surface. Derive an expression of torque(T) necessary to maintain an angular velocity (ω)

Ans. T= $\frac{\pi\mu\omega D^4}{32h}$

2. A metal plate 1.25 m x 1.25 m x 6 mm thick and weighting 90 N is placed midway in the 24 mm gap between the two vertical plane surfaces as shown in the Fig. The Gap is filled with an oil of specific gravity 0.85 and dynamic viscosity 3.0N.s/m^2. Determine the force required to lift the plate with a constant velocity of 0.15 m/s.
Ans. 168.08N

3. A 400 mm diameter shaft is rotating at 200 rpm in a bearing of length 120 mm. If the thickness of oil film is 1.5 mm and the dynamic viscosity of the oil is 0.7 Ns/m^2 determine:
(i) Torque required overcoming friction in bearing;
(ii) Power utilization in overcoming viscous resistance;
Ans. (i) 58.97 Nm (ii) 1.235 kW

4. In order to form a stream of bubbles, air is introduced through a nozzle into a tank of water at 20^{0}C. If the process requires 3.0mm diameter bubbles to be formed, by how much the air pressure at the nozzle must exceed that of the surrounding water? What would be the absolute pressure inside the bubble if the surrounding water is at 100.3 kN/m^2? (σ = 0.0735 N/m)
Ans. P_{abs}= 100.398 kN/m^2 (Hint. Bubble of air but surface tension of water)

5. A U-tube is made up of two capillaries of diameters 1.0 mm and 1.5 mm respectively. The U tube is kept vertically and partially filled with water of surface tension 0.0075kg/m and zero contact angles. Calculate the difference in the level of the menisci caused by the capillarity.
Ans. 10 mm

6. If a liquid surface (density ρ) supports another fluid of density, ρ_b above the meniscus, then a balance of forces would result in capillary rise h= $\frac{4\sigma coc\theta}{(\rho-\rho_b)gd}$

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Pressure and its Measurements

Skip to Questions (IAS, IES, GATE)

Highlights

1. The force (P) per unit area (A) is called pressure (P) Mathematically, $p = \frac{P}{A}$

- If compressive normal stress 'σ' then p = - σ
- Normal stress at a point may be different in different directions then [but presence of shear stress]

$$p = -\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

- Fluid at rest or in motion in the absence of shear stress

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} \text{ and } p = -\sigma_{xx} = -\sigma_{yy} = -\sigma_{zz}$$

- The stagnation pressure at a point in a fluid flow is the total pressure which would result if the fluid were brought to rest isentropically.

Stagnation pressure (p_o) = static pressure (p) + dynamic pressure $\left(\rho \frac{v^2}{2}\right)$

2. Pressure head of a liquid, $h = \frac{p}{w}$ $[\because p = \rho g h = wh]$

Where w is the specific weight of the liquid.

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3. Pascal's law states as follows:
"The intensity of pressure at any point in a liquid at rest is the same in all directions".

4. The atmospheric pressure at sea level (above absolute zero) is called standard atmospheric pressure.
(i) Absolute pressure = atmospheric pressure + gauge pressure

$$P_{abs.} = P_{atm.} + P_{gauge}$$

(ii) Vacuum pressure = atmospheric pressure - absolute pressure (Vacuum pressure is defined as the pressure below the atmospheric pressure)

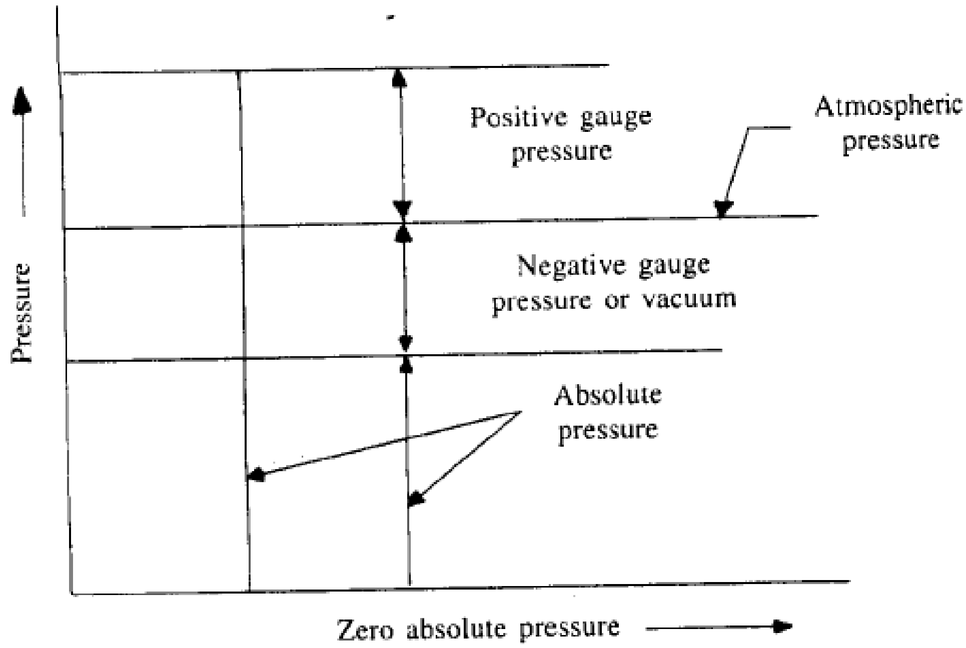

Fig. 2·6. Relationship between pressures.

5. Manometers are defined as the devices used for measuring the pressure at a point in fluid by balancing the column of fluid by the same or another column of liquid.

6. Mechanical gauges are the devices in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Some commonly used mechanical gauges are:
(i) Bourdon tube pressure gauge, (ii) Diaphragm pressure gauge,
(iii) Bellow pressure gauge and (iv) Dead-weight pressure gauge.

7. The pressure at a height Z in a static compressible fluid (gas) undergoing isothermal compression ($\frac{p}{\rho}$ = const);

$$p = p_o e^{-gz/RT}$$

Where P_o = Absolute pressure at sea-level or at ground level
z = height from sea or ground level
R = Gas constant
T = Absolute temperature.

8. The pressure and temperature at a height z in a static compressible fluid (gas) undergoing adiabatic compression ($p/\rho^{\gamma} = const.$)

$$p = p_0\left[1-\frac{\gamma-1}{\gamma}gZ\frac{\rho o}{po}\right]^{\frac{\gamma}{\gamma-1}} = p_0\left[1-\frac{\gamma-1}{\gamma}\frac{gZ}{RT_o}\right]^{\frac{\gamma}{\gamma-1}}$$

$$and\ temperature, T = T_o\left[1-\frac{\gamma-1}{\gamma}\frac{gZ}{RT}\right]$$

Where Po, To are pressure and temperature at sea-level; γ = 1.4 for air.

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9. The rate at which the temperature changes with elevation is known as Temperature Lapse-Rate. It is given by

$$L = \frac{-g}{R}\left(\frac{\gamma - 1}{\gamma}\right)$$

if (i) γ = I, temperature is zero. (ii) γ > I, temperature decreases with the increase of height

Questions (IAS, IES, GATE)

Pressure of a Fluid

1. A beaker of water is falling freely under the influence of gravity. Point B is on the surface and point C is vertically below B near the bottom of the beaker. If PB is the pressure at point B and Pc the pressure at point C, then which one of the following is correct? **[IES-2006]**

(a) $P_B=P_c$ (b) $P_B<P_c$ (c) $P_B>P_c$ (d) Insufficient data.

2. The standard sea level atmospheric pressure is equivalent to
(a) 10.2 m of fresh water of ρ = 998 kg/m^3 (b) 10.1 m of salt water of ρ = 1025 kg/m^3
(c) 12.5 m of kerosene of ρ = 800 kg/m^3
(d) 6.4 m of carbon tetrachloride of ρ = 1590 kg/m^3 **[IAS-2000]**

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Hydrostatic law and Aerostatic law

3. Hydrostatic law of pressure is given as **[IES 2002; IAS-2000]**

(a) $\frac{\partial p}{\partial z} = \rho g$ (b) $\frac{\partial p}{\partial z} = 0$ (c) $\frac{\partial p}{\partial z} = z$ (d) $\frac{\partial p}{\partial z} = const.$

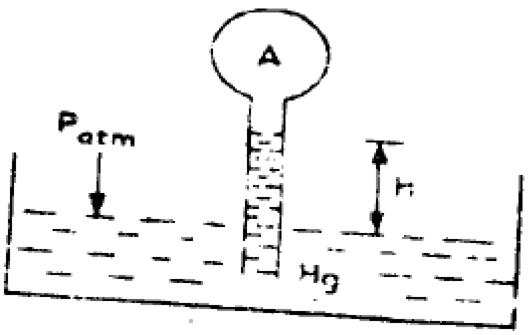
Absolute and Gauge Pressures

4. The reading of the pressure gauge fitted on a vessel is 25 bar. The atmospheric pressure is 1.03 bar and the value of g is 9.81m/s^2. The absolute pressure in the vessel is
(a) 23.97 bar (b) 25.00 bar (c) 26.03 bar (d) 34.84 bar **[IAS-1994]**

5. The standard atmospheric pressure is 762 mm of Hg. At a specific location, the barometer reads 700 mm of Hg. At this place, what does an absolute pressure of 380 mm of Hg correspond to? **[IES-2006]**

(a) 320 mm of Hg vacuum (b) 382 of Hg vacuum
(c) 62 mm of Hg vacuum (d) 62 mm of Hg gauge

6. In given figure, if the pressure of gas in bulb A is 50 cm Hg vacuum and P_{atm}=76 cm Hg, then height of column H is equal to
(a) 26 cm (b) 50 cm
(c) 76 cm (d) 126 cm



[GATE-2000]

Manometers

7. The pressure difference of two very light gasses in two rigid vessels is being measured by a vertical U-tube water filled manometer. The reading is found to be 10 cm. what is the pressure difference?
[IES 2007]
(a) 9.81 kPa (b) 0.0981 bar (c) 98.1 Pa (d) 981 N/m^2

8. A manometer is made of a tube of uniform bore of 0.5 cm^2 cross-sectional area, with one limb vertical and the other limb inclined at 30^0 to the horizontal. Both of its limbs are open to atmosphere and, initially, it is partly filled with a manometer liquid of specific gravity 1.25.If then an additional volume of 7.5 cm^3 of water is poured in the inclined tube, what is the rise of the meniscus in the vertical tube?
[IES-2006]
(a) 4 cm (b) 7.5 cm (c) 12 cm (d) 15 cm

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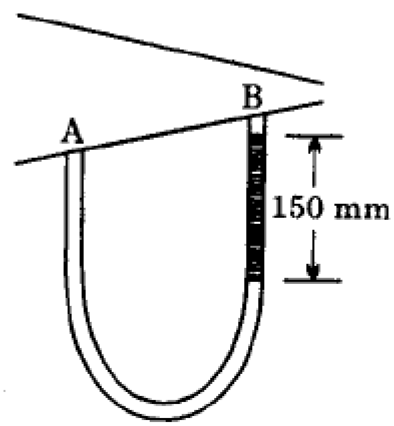
9. A U-tube manometer with a small quantity of mercury is used to measure the static pressure difference between two locations A and B in a conical section through which an incompressible fluid flows. At a particular flow rate, the mercury column appears as shown in the figure. The density of mercury is 13600 Kg/m^3 and g = $9.81 m/s^2$. Which of the following is correct?
(a) Flow Direction is A to B and P_A-P_B = 20 KPa
(b) Flow Direction is B to A and P_A-P_B = 1.4 KPa
(c) Flow Direction is A to B and P_B-P_A = 20 KPa
(d) Flow Direction is B to A and P_B-P_A = 1.4 KPa

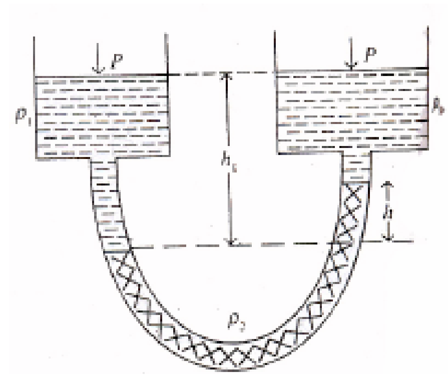


[GATE-2005]

10. The balancing column shown in the diagram contains 3 liquids of different densities ρ_1, ρ_2 and ρ_3. The liquid level of one limb is h_1 below the top level and there is a difference of h relative to that in the other limb.
What will be the expression for h?

(a) $\frac{\rho_1 - \rho_2}{\rho_1 - \rho_3} h_1$ (b) $\frac{\rho_2 - \rho_2}{\rho_1 - \rho_3} h_1$

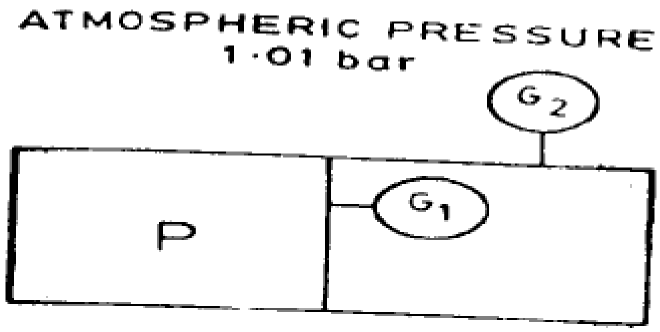
(c) $\frac{\rho_1 - \rho_3}{\rho_2 - \rho_3} h_1$ (d) $\frac{\rho_1 - \rho_2}{\rho_2 - \rho_3} h_1$



[IES-2004]

11. A mercury-water manometer has a gauge difference of 500 mm (difference in elevation of menisci). What will be the difference in pressure?
(a) 0.5 m (b) 6.3 m (c) 6.8 m (d) 7.3 m **[IES2004]**

12. The pressure gauges G_1 and G_2 installed on the system show pressures of P_{G1} = 5.00bar and P_{G2} = 1.00 bar. The value of unknown pressure P is? (Atmospheric pressure 1.01 bars)
(a) 1.01 bar (b) 2.01 bar
(c) 5.00 bar (d) 7.01 bar
[GATE-2004]



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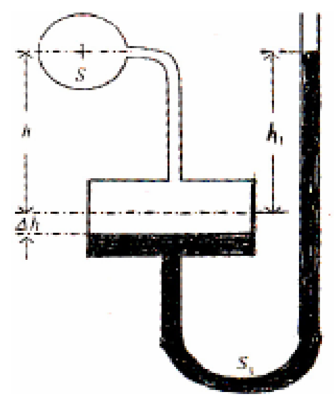
13. To measure the pressure head of the fluid of specific gravity S flowing through a pipeline, a simple micro-manometer containing a fluid of specific gravity S_1 is connected to it. The readings are as indicated as the diagram. The pressure head in the pipeline is
(a) $h_1S_1 - hS - \Delta h(S_1 - S)$ (b) $h_1S_1 - hS_1 + \Delta h(S_1 - S)$
(c) $hS - h_1S_1 - \Delta h(S_1 - S)$ (d) $hS - h_1S_1 + \Delta h(S_1 - S)$
[IES-2003]

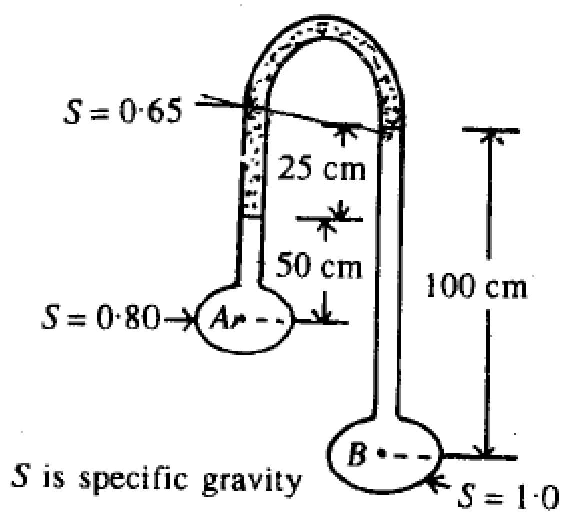


14. Pressure drop of flowing through a pipe (density 1000 kg/m^3) between two points is measured by using a vertical U-tube manometer. Manometer uses a liquid with density 2000 kg/m^3. The difference in height of manometric liquid in the two limbs of the manometer is observed to be 10 cm. The pressure drop between the two points is:
(a) 98.1 N/m^2 (b) 981 N/m^2 (c) 1962 N/m^2 (d) 19620 N/m^2 **[IES 2002]**

15. The pressure difference between point B and A (as shown in the above figure) in centimeters of water is

(a) -44 (b) 44

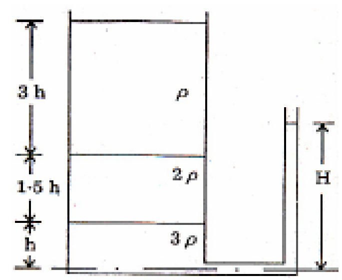
(c) -76 (d) 76



[IAS-2002]

16. There immiscible liquids of specific densities ρ , 2 ρ and 3 ρ are kept in a jar. The height of the liquids in the jar and at the piezometer fitted to the bottom of the jar is as shown in the given figure. The ratio H/h is

(a) 4 (b) 3.5 (c) 3 (d) 2.5



[IES-2001]

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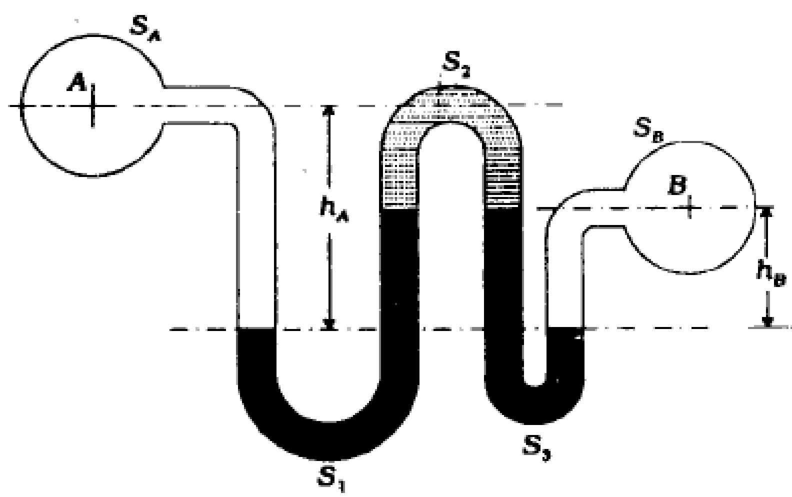
17. Differential pressure head measured by mercury oil differential manometer (specific gravity of oil is 0.9) equivalent to a 600 mm difference of mercury levels will nearly be

(a) 7.62 m of oil (b) 76.2 m of oil (c) 7.34 m of oil (d) 8.47 m of oil **[IES-2001]**

18. A double U-tube manometer is connected to two liquid lines A and B. Relevant heights and specific gravities of the fluids are shown in the given figure. The pressure difference, in head of water, between fluids at A and B is



(a) $S_Ah_A + S_1h_B - S_3h_B + S_Bh_B$ (b) $S_Ah_A - S_1h_B - S_2(h_A - h_B) + S_3h_B - S_Bh_B$

(c) $S_A h_A + S_1 h_B + S_2(h_A - h_B) - S_3 h_B + S_B h_B$ (d) $h_A S_A - (h_A - h_B)(S_1 - S_3) - h_B S_B$

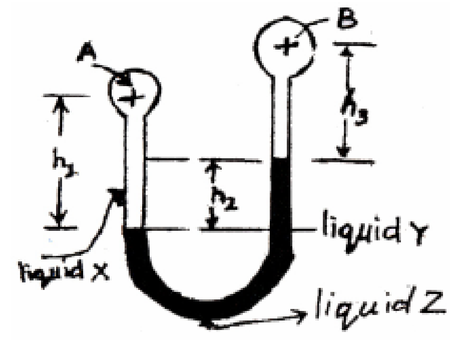
[IAS-2001]

19. A differential manometer is used to measure the difference in pressure at points A and B in terms of specific weight of water, W. The specific gravities of the liquids X, Y and Z are respectively s_1, s_2 and s_3.

The correct difference is given by : $\left(\frac{P_A}{W} - \frac{P_B}{W}\right)$

[a]. $h_3 s_2 - h_1 s_1 + h_2 s_3$ [b]. $h_1 s_1 + h_2 s_3 - h_3 s_2$

[c]. $h_3 s_1 - h_1 s_2 + h_2 s_3$ [d]. $h_1 s_1 + h_2 s_2 - h_3 s_3$

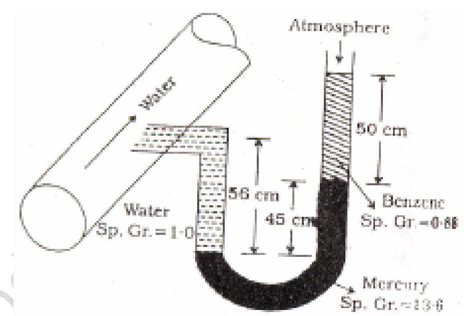


[IES-1997]

20. A U-tube manometer is connected to a pipeline conveying water as shown in the Figure. The pressure head of water in the pipeline is

[a]. 7.12 m [b]. 6.56 m

[c]. 6.0 m [d]. 5.12 m

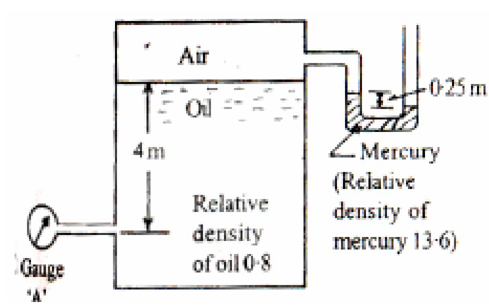


[IES-2000]

21. The reading of gauge 'A' shown in the given figure is

(a) -31.392 kPa (b) -1.962 kPa

(c) 31.392 kPa (d) 19.62 kPa

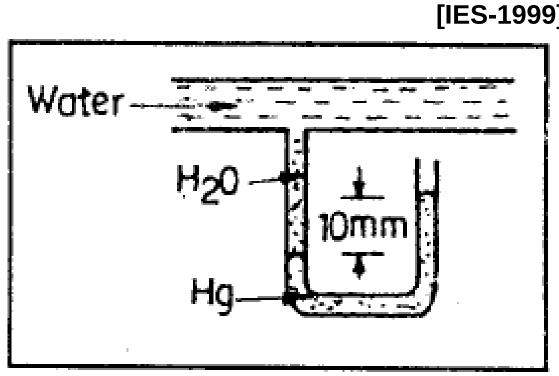


[IES-1999]

22. A mercury manometer is used to measure the static pressure at a point in a water pipe as shown in Figure. The level difference of mercury in the two limbs is 10 mm. The gauge pressure at that point is

(a) 1236 Pa (b) 1333 Pa

(c) Zero (d) 98 Pa



[GATE-1996]

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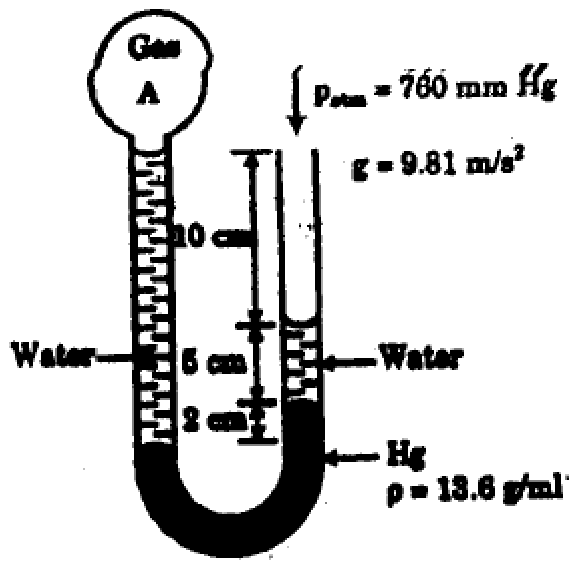
23. Refer to Figure, the absolute pressure of gas A in the bulb is

(a) 771.2 mm Hg (b) 752.65 mm Hg

(c) 767.35 mm Hg (d) 748.8 mm Hg



[GATE-1997]

24. The pressure gauge reading in meter of water column shown in the given figure will be

(a) 3.20 m (b) 2.72 m

(c) 2.52 m (d) 1.52 m

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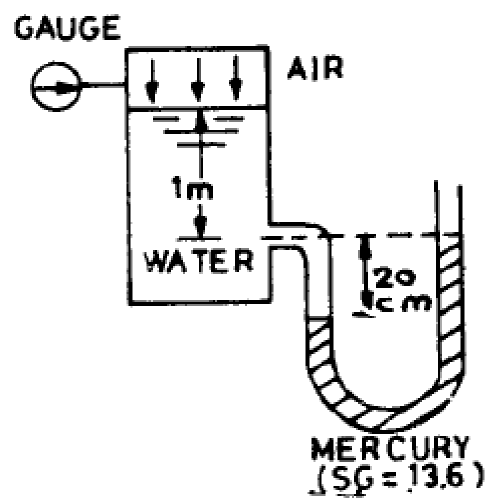


[IAS-1995]

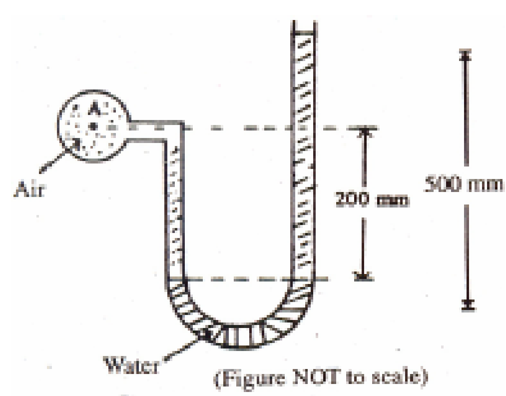
25. In the figure shown below air is contained in the pipe and water is the manometer liquid. The pressure at 'A' is approximately:

[a]. 10.14 m of water absolute

[b]. 0.2 m of water

[c]. 0.2 m of water vacuum

[d]. 4901 pa.



(Figure NOT to scale)

[IES-1998]

Piezometer

26. A vertical clean glass tube of uniform bore is used as a piezometer to measure the pressure of liquid at a point. The liquid has a specific weight of 15 kN/m^3 and a surface tension of 0.06 N/m in contact with air. If for the liquid, the angle of contact with glass is zero and the capillary rise in the tube is not to exceed 2 mm, what is the required minimum diameter of the tube? **[IES-2006]**

(a) 6 mm (b) 8 mm (c) 10 mm (d) 12 mm

27. When can a piezometer be not used for pressure measurement in pipes? **[IES-2005]**

(a) The pressure difference is low
(b) The velocity is high
(c) The fluid in the pipe is a gas
(d) The fluid in the pipe is highly viscous.

Mechanical gauges

28. Match List I with List II and select the correct answer using the codes given below the lists:

List I (Device)	List II (Use)
A. Barometer	1. Gauge pressure
B. Hydrometer	2. Local atmospheric pressure
C. U-tube manometer	3. Relative density
D. Bourdon gauge	4. Pressure differential

Codes:

	A	B	C	D		A	B	C	D
(a)	2	3	1	4	(b)	3	2	1	4
(c)	3	2	4	1	(d)	2	3	4	1

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29. In a pipe-flow, pressure is to be measured at a particular cross-section using the most appropriate instrument. Match List I (Expected pressure range) with List II (Appropriate measuring device) and select the correct answer: **[IES-2002]**

List I	**List II**
A. Steady flow with small position gauge pressure	1. Bourdon pressure gauge
B. Steady flow with small negative and positive gauge pressure.	2. Pressure transducer
C. Steady flow with high gauge pressure	3. Simple piezometer.
D. Unsteady flow with fluctuating pressure	4. U-tube manometer

Codes:

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
[a].	3	2	1	4	[b].	1	4	3	2
[c].	3	4	1	2	[d].	1	2	3	4

30. A siphon draws water from a reservoir and discharges it out at atmospheric pressure. Assuming ideal fluid and the reservoir is large, the velocity at point P in the siphon tube is

(a) $\sqrt{2gh_1}$ (b) $\sqrt{2gh_2}$ (c) $\sqrt{2g(h_2-h_1)}$ (d) $\sqrt{2g(h_2+h_1)}$ **[GATE-2006]**

Answers with Explanations

1. Ans. (a) For free falling body relative acceleration due to gravity is zero ∴ P= ρ gh if g=0 then p=0 (but it is only hydrostatic pr.) these will be atmospheric pressure through out the liquid.

2. Ans. (b) ρgh must be equal to 1.01325 bar = 101325 N/m^2

For (a) $998\times9.81\times10.2=99862\text{ N/m}^2$

(b) $1025\times9.81\times10.1=101558\text{ N/m}^2$

(c) $800\times9.81\times12.5=98100\text{ N/m}^2$

(d) $1590\times9.81\times6.4=99826\text{ N/m}^2$

3. Ans. (a)

4. Ans. (c) Absolute pressure = Atmospheric pressure + Gauge Pressure = 25+1.03 = 26.03 bar

5. Ans. (a)

6. Ans. (d) for 50 cm Hg vacuum add 50 cm column. Therefore H = 76 +50 = 126 cm

7. Ans. (d) $\Delta p=\Delta h\times\rho\times g=0.1\times1000\times9.81$ N/m^2 = 981 N/m^2

8. Ans. (a) Let 'x' cm will be rise of the meniscus in the vertical tube. So for this 'x' cm rise quantity of 1.25 s.g. liquid will come from inclined limb. So we have to lower our reference line = x sin30^o = x/2.
Then Pressure balance gives us

$$\left(x+\frac{x}{2}\right)\times1250\times9.81=\left(\frac{7.5}{0.5}\right)\sin30^o\times1000\times9.81$$

$$\text{or } x=4$$

9. Ans. (a) $P_B+150\,mm-Hg=P_A\ Or\ P_A-P_B=0.150\times13600\times9.81\approx20\,kPa$ and as P_A is greater than P_B therefore flow direction is A to B.

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10. Ans. (d) $h_1\rho_1=h\rho_2+(h_1-h)\rho_3$

11. Ans. (b) $h=y\left(\frac{s_h}{s_l}-1\right)\text{m of light fluid or h} = 0.5\left(\frac{13.6}{1}-1\right)=6.3\text{m of water.}$

12. Ans. (d) Pressure in the right cell = P_{G_2} +Atmospheric pressure = 1.01 +1.0 = 2.01 bar

Therefore P = P_{G_1} + Pressure on the right cell = 5 + 2.01 = 7.01 bar

13. Ans. (a) Use 'hs' rules; The pressure head inthe pipeline(H_p)

$$H_p+hs+\Delta hs-\Delta hs_1-h_1s_1=0\ or\ H_p=h_1s_1-\ hs-\Delta h(s_1-s)$$

14. Ans. (b) $h=y\left(\frac{s_h}{s_l}-1\right)\text{m of light fluid or h} = 0.1\left(\frac{2}{1}-1\right)=0.1\text{m of light fluid}$

The pressure dropbetween the two points is $=h\rho g=0.1\times9.81\times1000=981\text{ N/m}^2$

15. Ans. (b) Use 'hs' formula

$h_A - 50\times 0.8 - 25\times 0.65 + 100\times 1 = h_B \ or \ h_B - h_A = 43.75\,\text{cm of water column}$

16. Ans. (c) Use 'hs' formula
$3h\times\rho + 1.5h\times 2\rho + h\times 3\rho - H\times 3\rho = 0$ Or H/h = 3

17. Ans. (d) $h = y\left(\frac{s_h}{s_l} - 1\right)\text{m of light fluid or h} = 0.600\left(\frac{13.6}{0.9} - 1\right) = 8.47\,\text{m of oil}$

18. Ans. (d) Use 'hs' formula
$H_A + h_A S_A - (h_A - h_B)S_1 + (h_A - h_B)S_3 - h_B S_B = H_B$
$Or\ H_B - H_A = h_A S_A - (h_A - h_B)(S_1 - S_3) - h_B S_B$

19. Ans (a) Use 'hs' formula
$\frac{P_A}{w} + \text{h}_1\text{s}_1 - \text{h}_2\text{s}_3 - h_3 s_2 = \frac{P_B}{w}\ Or\ \frac{P_A}{w} - \frac{P_B}{w} = h_3 s_2 - \text{h}_1\text{s}_1 + \text{h}_2\text{s}_3$

20. Ans. (c) Use 'hs' formula; $H + 0.56\times 1 - 0.45\times 13.6 - 0.5\times 0.88 = 0$

21. Ans. (b) Use 'hs' formula;
$H_A - 4\times 0.8 + 0.25\times 13.6 = 0\ Or\ H_A = -0.2\,\text{m of water column}$
$= -0.2\times 9.81\times 1000\ \text{N/m}^2 = -1.962\,kPa$

22. Ans. (a)
$h = y\left(\frac{s_h}{s_l} - 1\right)\text{m of light fluid or h} = 0.010\left(\frac{13.6}{1} - 1\right) = 0.126\,\text{m of water column}$
$\text{Or P} = \text{h}\rho\text{g} = 0.126\times 1000\times 9.81 = 1236\ \text{N/m}^2 = 1236\,Pa$

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23. Ans. (a) Use 'hs' formula;
$H_A + 170\times 1 - 20\times 13.6 - 50\times 1 = h_{atm.}(760\times 13.6)$ [All mm of water]
$\text{Or H}_A = 10488/13.6\,\text{mm of Hg} = 771.2\,\text{mm of Hg}(Abs.)$

24. Ans. (d) Use 'hs' formula;
$H_G + 1\times 1 + 0.2\times 1 - 0.2\times 13.6 = 0\ or\ H_G = 1.52\,\text{m of water column}$

25. Ans. (d) Use 'hs' formula;
$H_{air} + 0.2\times S_{air}(1.3/1000) - 0.5\times 1 = 0\ or\ H_{air} = 0.49974\,\text{m of water column} = 0.49974\times 9.81\times 1000\,Pa$

26. Ans. (b) $h = \frac{4\sigma\cos\theta}{\rho g d} \le 0.002\ or\ d \ge \frac{4\times 0.06\times \cos 0^o}{15000\times 0.002} = 8\,\text{mm}$

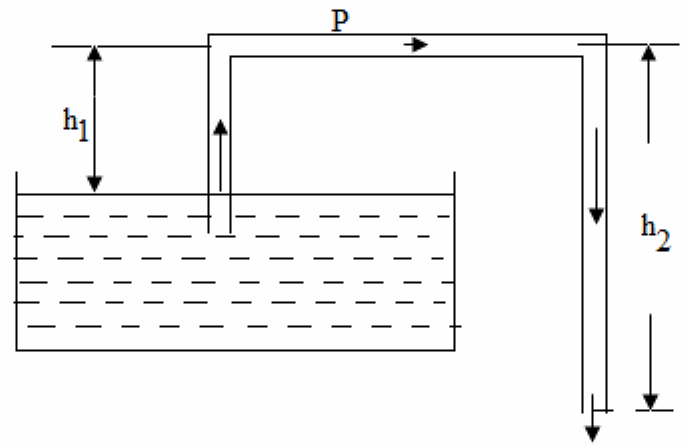
27. Ans. (c)
28. Ans. (d)
29. Ans. (c)

30. Ans. (c) By energy conservation, velocity at point Q

$= \sqrt{2g(h_2 - h_1)}$

As there is a continuous and uniform flow, so velocity of liquid at point Q and P is same.

$V_p = \sqrt{2g(h_2 - h_1)}$



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Hydrostatic Forces on Surfaces

Skip to Questions (IAS, IES, GATE)

Highlights

1. The term hydrostatics means the study of pressure, exerted by a fluid at rest.

2. Total pressure (P) is the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface.

For vertically immersed surface, P = $w\overline{Ax}$

For inclined immersed surface, P = $w\overline{Ax}$

where A = area of immersed surface, and

$\overline{x}$ = depth of centre of gravity of immersed surface from the free liquid surface.

3. Centre of pressure $\left(\overline{h}\right)$ is the point through which the resultant pressure acts and is always expressed in terms of depth from the liquid surface.

For vertically immersed surface, $\overline{h} = \frac{IG}{A\overline{x}} + \overline{x}$

For inclined immersed surface, $\overline{h} = \frac{IG\sin^2\theta}{A\overline{x}} + \overline{x}$

Where I_G stands for moment of inertia of figure about horizontal axis through its centre of gravity.

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4. The total force on a curved surface is given by

P= $\sqrt{P^2H + P^2V}$

where P_H = horizontal force on curved surface

= total pressure force on the projected area of the curved surface on the vertical plane = $w\overline{Ax}$

Pv= vertical force on submerged curved surface

= weight of liquid actually or imaginary supported by curved surface.

The direction of the resultant force P with the horizontal is given by

$$\tan\theta = \frac{P_V}{P_H} \quad or \quad \theta \tan^{-1}\frac{P_V}{P_H}$$

5. Resultant force on a sluice gate P = $P_1 - P_2$

Where P_1 = pressure force on the upstream side of the sluice gate, and

P_2 = pressure force on the downstream side of the sluice gate.

6. For a lock gate, the reaction between two gates is equal to the reaction at the hinge,

i.e. N=R

Also reaction between the two gates, N= $\frac{P}{2\sin\alpha}$

Where P = resultant water pressure on the lock gate = $P_1 - P_2$, and

χ = inclination of the gate to normal of side of lock.

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Questions (IAS, IES, GATE)

1. Which one of the following statements is correct?
The pressure centre is:
(a) the cycloid of the pressure prism
(b) a point on the line of action of the resultant force
(c) at the centroid of the submerged area.
(d) always above the centroid of the area **[IES-2005]**

2. A semi – circular plane area of diameter 1 m, is subjected to a uniform gas pressure of 420 kN/m^2. What is the moment of thrust (approximately) on the area about its straight edge? **[IES-2006]**
(a) 35 kNm (b) 41 kNm (c) 55 kNm (d) 82 kNm

3. A horizontal oil tank is in the shape of a cylinder with hemispherical ends. If it is exactly half full, what is the ratio of magnitude of the vertical component of resultant hydraulic thrust on one hemispherical end to that of the horizontal component?
(a) $2/\pi$ (b) $\pi/2$ (c) $4/(3\pi)$ (d) $3\pi/4$ **[IES-2006]**

4. A circular plate 1.5 m diameter is submerged in water with its greatest and least depths below the surface being 2 m and 0.75 m respectively. What is the total pressure (approximately) on one face of the plate? **[IES-2007, IAS-2004]**
(a) 12kN (b) 16kN (c) 24kN (d) None of the above

5. A tank with four equal vertical faces of width l and depth h is filled up with a liquid. If the force on any vertical side is equal to the force at the bottom, then the value of h/l will be **[IAS-2000; IES-2001]**
(a) 2 (b) $\sqrt{2}$ (c) 1 (d) 1/2

6. The vertical component of the hydrostatic force on a submerged curved surface is the **[IAS-1998, 1995, IES-2003]**
(a) mass of liquid vertically above it
(b) weight of the liquid vertically above it
(c) force on a vertical projection of the surface
(d) product of pressure at the centroid and the surface area.

7. Consider the following statements regarding a plane area submerged in a liquid:
1. The total force is the product of specific weight of the liquid, the area and the depth of its centroid.
2. The total force is the product of the area and the pressure at its centroid.
Of these correct statements are:
(a) 1 alone (b) 2 alone (c) both 1 and 2 false (d) both 1 and 2 **[IAS-1995]**

8. A vertical dock gate 2 meter wide remains in position due to horizontal force of water on one side. The gate weights 800 Kg and just starts sliding down when the depth of water upto the bottom of the gate decreases to 4 meters. Then the coefficient of friction between dock gate and dock wall will be **[IAS-1995]**
(a) 0.5 (b) 0.2 (c) 0.05 (d) 0.02

9. A circular disc of radius 'r' is submerged vertically in a static fluid up to a depth 'h' from the free surface. If h > r, then the position of centre of pressure will

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(a) be directly proportional to *h* *(b)* be inversely proportional of *h*
(c) be directly proportional to r *(d)* not be a function of *h* or r. **[IAS-1994]**

10. A circular annular plate bounded by two concentric circles of diameter 1.2m and 0.8 m is immersed in water with its plane making an angle of 45^o with the horizontal. The centre of the circles is 1.625m below the free surface. What will be the total pressure force on the face of the plate? **[IES-2004]**

(a) 7.07 kN (b) 10.00 kN (c) 14.14 kN (d) 18.00kN

11. A plate of rectangular shape having the dimensions of 0.4m x 0.6m is immersed in water with its longer side vertical. The total hydrostatic thrust on one side of the plate is estimated as 18.3 kN. All other conditions remaining the same, the plate is turned through 90^o such that its longer side remains vertical. What would be the total force on one of the plate?

(a) 9.15 kN (b) 18.3 kN (c) 36.6 kN (d) 12.2 kN **[IES-2004]**

12. Consider the following statements about hydrostatic force on a submerged surface:

1. It remains the same even when the surface is turned.
2. It acts vertically even when the surface is turned.

Which of these is/are correct? **[IES-2003]**

(a) Only 1 (b) Only 2 (c) Both 1 and 2 (d) Neither 1 nor 2

13. The depth of centre of pressure for a rectangular lamina immersed vertically in water up to height 'h' is given by **[IES-2003]**

(a) h/2 (b) h/4 (c)2h/3 (d) 3h/2

14. The point of application of a horizontal force on a curved surface submerged in liquid is

(a) $\frac{I_G}{A\bar{h}} - \bar{h}$ (b) $\frac{I_G + A\bar{h}^2}{A\bar{h}}$ (c) $\frac{A\bar{h}}{I_G} + \bar{h}$ (d) $\frac{I_G}{\bar{h}} + A\bar{h}$

Where A = area of the immersed surface

$\bar{h}$ =depth of centre of surface immersed

I_G=Moment of inertia about centre of gravity

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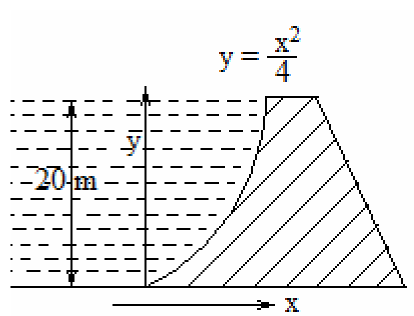
15. A dam is having a curved surface as shown in the figure. The height of the water retained by the dam is 20m; density of water is 1000kg/m^3. Assuming g as 9.81 m/s^2, the horizontal force acting on the dam per unit length is.

(a) 1.962×10^2 N (b) 2×10^5N
(c) 1.962×10^6 N (d) 3.924×10^6 N

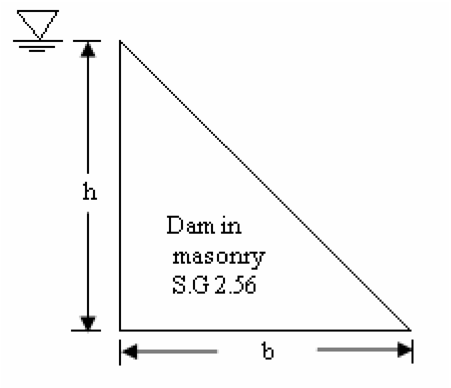


[IES-2002]

16. A triangular dam of height h and base width b is filled to its top with water as shown in the given figure. The condition of stability

(a) b = h (b) b = 2.6 h

(c) b = $\sqrt{3h}$ (d) b = 0.625 h



[IES-1999]

17. A vertical sluice gate, 2.5 m wide and weighting 500 kg is held in position due to horizontal force of water on one side and associated friction force. When the water level drops down to 2 m above the bottom of the gate, the gate just starts sliding down. The co efficient of friction between the gate and the supporting structure is **[IES-1999]**

(a) 0.20 (b) 0.10 (c) 0.05 (d) 0.02

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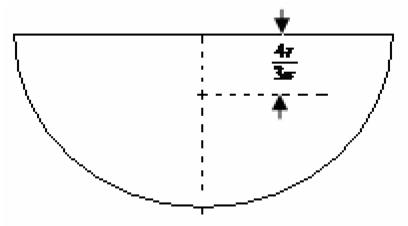
Answers with Explanation

1. (b)

2. (a) Force (P) = p.A = $420\times\frac{\pi.1^2}{4\times 2}$

Moment (M) = $P\times\bar{h}$

$$=420\times\frac{\pi\times 1^2}{4\times 2}\times\frac{4\times(1/2)}{3\times\pi}=35\,kNm$$

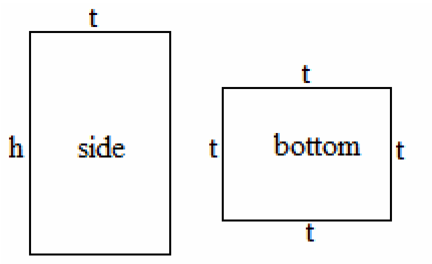


3. (b) $P_H=\rho g A\bar{x}=\rho g\left(\frac{\pi.r^2}{4\times 2}\right).\frac{4r}{3\pi}=\frac{2}{3}\rho g r^3$

$$P_V=\rho g\forall=\rho g.\frac{1}{4}.\left(\frac{4}{3}\pi r^3\right)\;\therefore\frac{P_V}{P_H}=\frac{\pi}{2}$$

4. (C) $P=\rho g A\bar{x}=\rho g\left(\frac{\pi\times 1.5^2}{4}\right)\times\left(\frac{0.75+2}{2}\right)=24\,kN$

5. (a) $P_{bottom}=P_{side}$ or $h\rho g.t.t=\rho g t h.(h/2)\,or\,\frac{h}{t}=2$



6. (b)

7. (d)

8. (c) $\mu P = W$ or $\mu \rho g(4\times 2).(4/2) = 800\times g$ or $\mu = 0.05$

9. (a)

10. (b) $\rho g A\bar{x}$ **=** $1000\times 9.81\times \frac{\pi}{4}(1.2^2 - 0.8^2)\times 1.625 \approx 10kN$

11. (b)

12. (a)

13. (c)

14. (b)

15. (c) $P_H = \rho g A\bar{x} = 1000\times 9.81\times (20\times 1)\times (20/2) = 1.962\times 10^6\ N$

16. (b)

17. (b) $\mu P = W$ or $\mu \rho g A\bar{x} = mg$ or $\mu = \frac{m}{\rho A\bar{x}} = \frac{500}{1000\times (2\times 2.5)\times (2/2)} = 0.1$

S. K. Mondal

Buoyancy and Flotation

Skip to Questions (IAS, IES, GATE)

Highlights

1. The tendency for an immersed body to be lifted up in the fluid, due to an upward force opposite to action of gravity is known as buoyancy.

2. The floating bodies may have the following types of equilibrium:
(i) Stable equilibrium.
(ii) Unstable equilibrium, and
(iii) Neutral equilibrium.

3. The metacenter is defined as a point of intersection of the axis of body passing through e.g. (G) and original centre of buoyancy (B), and a vertical line passing through the centre of buoyancy (B_1) the titled position of the body.

4. The distance between the centre of gravity (G) of a floating body and the metacenter (M) is called **metacentric height.**

5. The metacentric height (GM) by experimental method is given by:

(i) GM = BM - BG when G is higher than B
= BM + BG when G is lower than B

& $BM = \frac{I}{V}$

$$GM = \frac{W_1 z.l}{W.d}\left(= \frac{W_1 z}{W \tan\theta}\right)M$$

Where W_1= known weight.
z = distance through which W_1 is shifted across the axis of the tilt,
l = length of the plumb bob, and
d = displacement of the plumb bob.
θ = angle of tilt ($\tan\theta = \frac{d}{l}$)

6. Time of rolling, $T = 2\pi\sqrt{\frac{k^2}{GM.g}}$ **(VIMP)**

Where k = radius of gyration about e.g. (G), and
GM = metacentric height of the body.

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Questions (IAS, IES, GATE)

1. Assertion (A): The buoyant force for a floating body passes through the centroid of the displaced volume.
Reason (R): The force of buoyancy is a vertical force & equal to the weight of fluid displaced.
[IES-2005]

2. Which one of the following is the condition for stable equilibrium for a floating body?
(a) The metacenter coincides with the centre of gravity
(b) The metacenter is below the center of gravity
(c) The metacenter is above the center of gravity
(d) The centre of buoyancy is below the center of gravity **[IES-2005]**

3. Resultant pressure of the liquid in case of an immersed body acts through which one of the following? **[IES-2007]**
(a) Centre of gravity (b) Centre of pressure (c) Metacenter (d) Centre of buoyancy

4. A hydrometer weighs 0.03 N and has a stem at the upper end which is cylindrical and 3 mm in diameter. It will float deeper in oil of specific gravity 0.75, than in alcohol of specific gravity 0.8 by how much amount? **[IES-2007]**
(a) 10.7 mm (b) 43.3 mm (c) 33 mm (d) 36 mm

5. A wooden rectangular block of length l is made to float in water with its axis vertical. The centre of gravity of the floating body is $0.15l$ above the centre of buoyancy. What is the specific gravity of the wooden block? **[IES-2007]**
(a) 0.6 (b) 0.65 (c) 0.7 (d) 0.75

6. If B is the centre of buoyancy, G is the centre of gravity and M is the Metacentre of a floating body, the body will be in stable equilibrium if **[IES-2007]**
(a) MG=0 (b) M is below G (c) BG=0 (d) M is above G

7. The metacentric height of a passenger ship is kept lower than that of a naval or a cargo ship because **[IES-2007]**
(a) Apparent weight will increase
(b) Otherwise it will be in neutral equilibrium
(c) It will decrease the frequency of rolling
(d) Otherwise it will sink and be totally immersed

8. A metallic piece weighs 80 N air and 60 N in water. The relative density of the metallic piece is about **[IAS-2002]**
(a) 8 (b) 6 (c) 4 (d) 2

9. Match List I (Nature of equilibrium of floating body) with List II (Conditions for equilibrium) and select the correct answer using the codes given below the Lists:

List I (Nature of equilibrium of floating body)	List II (Conditions for equilibrium)
A. Unstable equilibrium	1. MG=0
B. Neutral equilibrium	2. M is above G
C. Stable equilibrium	3. M is below G
	4. BG=0

(Where M,G and B are metacenter, centre of gravity and centre of gravity and centre of buoyancy respectively.)
Codes:

	A	B	C		A	B	C
(a)	1	3	2	(b)	3	1	2
(c)	1	3	4	(d)	4	2	3

[IAS-2002]

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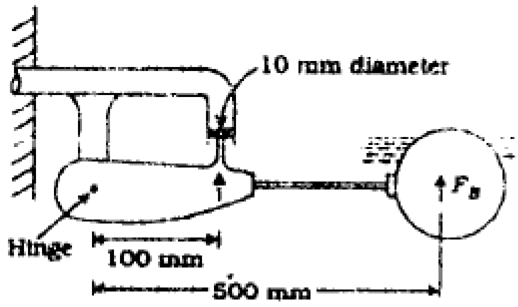
10. A float valve of the 'ball-clock' type is required to close an opening of a supply pipe feeding a cistern as shown in the given figure.

The buoyant force F_B required to be exerted by the float to keep the valve closed against a pressure of 0.28 N/mm is

(a) 4.4 N (b) 5.6N
(c) 7.5 N (d) 9.2 N



[IAS-2000]

11. Assertion (A): A body with rectangular cross section provides a highly stable shape in floatation. **IAS-1999]**

Reason (R): The centre of buoyancy shifts towards the tipped end considerably to provide a righting couple.

12. A weight of 10 tonne is moved over a distance of 6m across the deck of a vessel of 1000 tonne floating in water. This makes a pendulum of length 2.5m swing through a distance of 12.5cm horizontally. The metacentric height of the vessel is **[IAS-1997]**

(a) 0.8m (b) 1.0m (c) 1.2m (d) 1.4m

13. The fraction of the volume of a solid piece of metal of relative density 8.25 floating above the surface of a container of mercury of relative density 13.6 is **[IAS-1997]**

(a) 1.648 (b) 0.607 (c) 0.393 (d) 0.352

14. Consider the following statements regarding stability of floating bodies:

1. If oscillation is small, the position of Metacentre of a floating body will not alter whatever be the axis of rotation
2. For a floating vessel containing liquid cargo, the stability is reduced due to movements of gravity and centre of buoyancy.
3. In warships and racing boats, the metacentric height will have to be small to reduce rolling

Of these statements:

(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 alone is correct (d) 3 alone is correct. **[IAS-1997]**

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15. If a cylindrical wooden pole, 20 cm in diameter, and 1m in height is placed in a pool of water in a vertical position (the gravity of wood is 0.6), then it will

(a) float in stable equilibrium (b) float in unstable equilibrium
(c) float in neutral equilibrium (d) start moving horizontally. **[IAS-1994]**

16. An open tank contains water to depth of 2m and oil over it to a depth of 1m. If the specific gravity of oil in 0.8, then the pressure intensity at the interface of the two fluid layers will be **[IAS-1994]**

(a) 7848 N/m^2 (b) 8720 N/m^2 (c) 9747 N/m^2 (d) 9750 N/m^2

17. Consider the following statements

For a body totally immersed in a fluid.

I. the weight acts through the centre of gravity of the body.
II. the up thrust acts through the centroid of the body.

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Of these statements: **[IAS-1994]**

(a) both I and II are true (b) I is true but II is false

(c) I is false but II is true (d) neither I nor II is true

18. Assertion (A): A circular plate is immersed in a liquid with its periphery touching the free surface and the plane makes an angle θ with the free surface with different values of θ, the position of centre of pressure will be different. **[IES-2004]**

Reason (R): Since the centre of pressure is dependent on second moment of area, with different values of θ, second moment of area for the circular plate will change.

19. An open rectangular box of base 2m X 2m contains a liquid of specific gravity 0.80 up to a height of 2.5m. If the box is imparted a vertically upward acceleration of 4.9 m/s^2, what will the pressure on the base of the tank? **[IES-2004]**

(a) 9.81 kPa (b) 19.62 kPa (c) 36.80 kPa (d) 29.40 kPa

20. Assertion (A): For a vertically immersed surface, the depth of the centre of pressure is independent of the density of the liquid. **[IES-2003]**

Reason (R): Centre of pressure lies above the centre of area of the immersed surface.

21. Match List I with List II and select the correct answer:

List-I(Stability)	List-II(Conditions)
A. Stable equilibrium of a floating body	1. Centre of buoyancy below the centre of gravity
B. Stable equilibrium of a submerged body	2. Metacentre above the centre of gravity
C. Unstable equilibrium of a floating body	3. Centre of buoyancy above the centre of gravity
D. Unstable equilibrium of a submerged body	4. Metacentre below the centre of gravity

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	2	3	4	1
(c)	4	1	2	3	(d)	2	1	4	3

[IES-2002]

22. A barge 30m long and 10m wide has a draft of 3m when flowing with its sides in vertical position. If its centre of gravity is 2.5m above the bottom, the nearest value of metacentric height is **[IES-2001]**

(a) 3.28m (b) 2.78m (c) 1.78m (d) zero

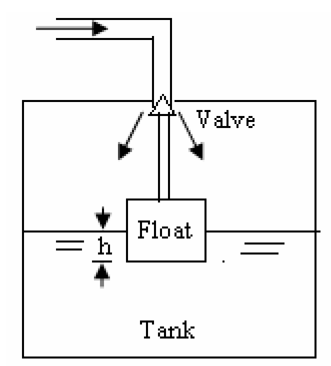
23. A block of aluminum having mass of 12 kg is suspended by a wire and lowered until submerged into a tank containing oil of relative density 0.8. Taking the relative density of aluminum as 2.4, the tension in the wire will be (take g=10 m/s^2) **[IES-2001]**

(a) 12000N (b) 800 N (c) 120 N (d) 80N

24. A float of cubical shape has sides of 10 cm. The float valve just touches the valve seat to have a flow area of 0.5 cm^2 as shown in the given figure. If the pressure of water in the pipeline is 1 bar, the rise of water level h in the tank to just stop the water flow will be

(a) 7.5 cm (b) 5.0 cm

(c) 2.5 cm (d) 0.5 cm



[IES-2000]

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25. Stability of a freely floating object is assured if its centre of
(a) Buoyancy lies below its centre of gravity **[IES-1999]**
(b) Gravity coincides with its centre of buoyancy
(c) Gravity lies below its metacenter
(d) Buoyancy lies below its metacenter.

26. Match List I with List II regarding a body partly submerged in a liquid and select answer using the codes given below: **[IES-1999]**

List-I	List-II
A. Centre of pressure	1. Points of application of the weight of displace liquid.
B. Centre of gravity	2. Point about which the body starts oscillating when tilted by a small angle
C. Centre of buoyancy	3. Point of application of hydrostatic pressure force
D. Matacentre	4. Point of application of the weight of the body

	A	B	C	D		A	B	C	D
(a)	4	3	1	2	(b)	4	3	2	1
(c)	3	4	1	2	(d)	3	4	2	1

27. If a piece of metal having a specific gravity of 13.6 is placed in mercury of specific gravity 13.6, then **[IES-1999]**
(a) the metal piece will sink to the bottom
(b) the metal piece simply float over the mercury with no immersion
(c) the metal piece will be immersed in mercury by half
(d) The whole of the metal piece will be immersed with its top surface just at mercury level.

28. A bucket of water hangs with a spring balance. if an iron piece is suspended into water from another support without touching the sides of the bucket, the spring balance will show **[IES-1999]**
(a) An increased reading (b) A decreased reading (c) no change in reading
(d) Increased or decreased reading depending on the depth of immersion.

29. The least radius of gyration of a ship is 9m and the metacentric height is 750 mm. The time period of oscillation of the ship is **[IES-1999]**
(a) 42.41 s (b) 75.4 s (c) 20.85 s (d) 85 s

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Answers with Explanations

1. Ans. (a)
2. Ans. (c)
3. Ans. (d)
4. Ans. (d) $V_{oil}=\frac{W}{\rho_{oil}g}$ and $V_{al}=\frac{W}{\rho_{al}g}$ Now $V_{oil}-V_{al}=\frac{W}{g}\left(\frac{1}{\rho_{oil}}-\frac{1}{\rho_{al}}\right)=\frac{\pi.(0.003)^2h}{4}$
5. Ans. (c)
6. Ans. (d)
7. Ans. (c)
8. Ans. (c)
9. Ans. (b)
10. Ans. (a)

Pressure force on valve $(F_V)=pressure\times area=0.28\times\frac{\pi\times10^2}{4}N=22N$

Taking moment about hinge, $F_V\times100=F_B\times500\ or\ F_B=4.4\,N$

11. Ans. (a)
12. Ans. (c)
13. Ans. (c)
14. Ans. (c)
15. Ans. (b)
16. Ans. (a)
17. Ans. (b)
18. Ans. (c)
19. Ans. (d) $p=h\rho(g+a)$
20. Ans. (c)
21. Ans. (d)
22. Ans. (b)
23. Ans. (d) $T=mg-v\rho g$
24. Ans. (c)
25. Ans. (c)
26. Ans. (c)
27. Ans. (d)
28. Ans. (c)
29. Ans. (c) $T=2\pi\sqrt{\frac{k^2}{GM.g}}=2\pi\sqrt{\frac{9_2}{0.750\times9.81}}$ =20.85 s

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FLUID KINEMATICS

Skip to Questions (IAS, IES, GATE)

Highlights

1. A fluid motion may be analyzed by following one of the two alternative approaches:

1. **Lagrangian approach:** In this approach the observer concentrates on the movement of a single particle. The path taken by the particle and the changes in its velocity and acceleration are studied.
2. **Eulerian Approach:** In this approach the observer concentrates on a point in the fluid system.
   Velocity, acceleration and other characteristics of the fluid at that particular point are studied.

2. One dimensional flow: When the dependent variables are functions of only one space co-ordinate say x.
The number of dependent variable does not matter.

- The axis of passage does not have to be a straight line.
- One dimensional flow may takes place in a curved passage

3. Two dimensional flow:

Flow over a long circular cylinder is two-dimensional flow.

4. Axi-symmetric flow: Velocity profile is symmetrical about the axis of symmetry.

- Flow is invariant in the circumferential i.e. θ-direction
- Mathematically, $\frac{\partial}{\partial \theta}=0$
- It is two dimensional flow, because the only independent co-ordinates are x and y or r and z.

5. Steady flow: The dependent fluid variables at point in the flow do not change with time. i.e. $\frac{\partial}{\partial t}$ {dependent fluid variables} $\equiv 0$

i.e. $\frac{\partial u}{\partial t}=\frac{\partial \upsilon}{\partial t}=\frac{\partial w}{\partial t}=0=\frac{\partial p}{\partial t}=\frac{\partial \rho}{\partial t}$ etc.

- A flow is said to be steady when conditions do not change with time at any point.
- In a converging steady flow, there is only convective acceleration.
- Local acceleration is zero in steady flow.
- The flow of a liquid at const. rate in a conically tapered pipe is classified as steady, non-uniform flow.
- In a steady flow streamline, path line and streak line are coincident.

Uniform flow: A flow is said to be uniform at an instant of time if the velocity, in magnitude, direction and sense, is identical throughout the flow field.

i.e. $\frac{\partial}{\partial s}$ {dependent fluid variables} $\equiv 0$

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i.e. $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} = 0 = \frac{\partial \upsilon}{\partial x} = \frac{\partial \upsilon}{\partial y}$ -----etc

- Uniform flow occurs when the (spatial) rate of change of velocity is zero.
- Uniform flow can take place in a conical passage.
- In uniform flow constant velocity vector occur.

7. Acceleration in fluid flow:

Total acceleration= Convective acceleration+ Local acceleration

$a_x = u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$a_y = u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} + w\frac{\partial \upsilon}{\partial z} + \frac{\partial \upsilon}{\partial t}$

$a_z = u\frac{\partial w}{\partial x} + \upsilon\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

and $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

- In a natural co-ordinate system the acceleration a_n in a normal direction when local and convective terms are present is given by $a_n = \frac{\partial \upsilon_n}{\partial t} + \frac{\upsilon^2}{r}$

8. Streamline: A streamline in a fluid flow is a line tangent to which at any point is in the direction of velocity at that point at that instant.

- Since the component of velocity normal to a streamline is zero, there can be no flow across a streamline.
- A streamline cannot intersect itself nor can any streamline intersect another streamline.

Equation $\frac{\partial x}{u} = \frac{\partial y}{\upsilon} = \frac{\partial z}{w}$

Above equation is valid for flow, steady or unsteady, uniform or non-uniform viscous or non-viscous, compressible or incompressible.

- ***For a steady flow streamline, path line and streak lines are coincides.***
- A streamline is defined in terms of stream function (ψ) i.e. ψ =const.
- A flow has diverging straight streamlines. If the flow is steady, the flow has convective tangential acceleration.
- A flow has parallel curved streamlines and is steady this flow as normal convective acceleration.
- Streamline and velocity potential line must constitute orthogonal network.

Pathline: A pathline is the trace made by a single particle over a period of time.
i.e. It is the path followed by a fluid particle in motion.

Equation $x=\int u dt;\ y=\int \upsilon dt;\ z = \int w dt$

Streakline: It is a curve which gives an instantaneous picture of the location of the fluid particles which have passed through a given point.

CONTINUITY EQUATION

$\rho AV =$ const. In case of compressible fluid.

AV = const. In case of incompressible fluid.

Differential form of continuity equation in Cartesian co-ordinates system.

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$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}+\frac{\partial w}{\partial z}=0$, Vector form $\nabla.\vec{V}=0$, for incompressible flow

General form

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho \upsilon)+\frac{\partial}{\partial z}(\rho w)+\frac{\partial \rho}{\partial t}=0$$

Vector form $\nabla.(\rho\vec{V})+\frac{\partial \rho}{\partial t}=0$

General form valid for

Viscous or Inviscid; steady or unsteady; uniform or non-uniform; compressible or incompressible.

Integral form: $\int_s \rho\vec{V}$ .dA+ $\frac{\partial}{\partial t}\int \rho dv=0$

Differential form of continuity equation in cylindrical co-ordinate system

$$\frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$, for incompressible flow.

- The equation of continuity in fluid mechanics is an embodiment of the law of conservation of mass.
- Existence of stream function resulting continuity of flow.
- For a *possible* case of fluid flow must satisfy continuity eq^n.

Stream function: u = $\frac{\partial \psi}{\partial y}$ and $\upsilon=-\frac{\partial \psi}{\partial x}$;

In cylindrical co-ordinate system, $u_r=\frac{1}{r}\frac{\partial \psi}{\partial \theta}$ and $u_\theta=-\frac{\partial \psi}{\partial r}$

- A stream function is defined when the flow is *continuous*.
- Dimensions of ψ is [L^2T^{-1}]
- Existence of stream function implies that continuity of flow.

If $\psi_2 and \psi_1$ is the values of stream function at point 2 and 1 respectively. The volume rate of flow per unit depth across an element Δs connecting 2 and 1 is given by $\Delta\,\psi$.

- If a stream function ψ exists it implies that the function ψ represents a possible flow field.
- If ϕ is the laplacian then ψ must exists.
- ψ = const. in the streamline.

Strain component

$\varepsilon_{xx}=\frac{\partial u}{\partial x}$ $\varepsilon_{xy}=\frac{1}{2}\{\frac{\partial \upsilon}{\partial x}+\frac{\partial u}{\partial y}\}$

$\varepsilon_{yy}=\frac{\partial \upsilon}{\partial y}$ $\varepsilon_{yz}=\frac{1}{2}\{\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\}$

$\varepsilon_{zz}=\frac{\partial w}{\partial z}$ $\varepsilon_{xz}=\frac{1}{2}\{\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\}$

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Velocity potential function (ϕ)

Or $u = -\frac{\partial \phi}{\partial x}, \upsilon = -\frac{\partial \phi}{\partial y}, w = -\frac{\partial \phi}{\partial z}$

Dimensions of ϕ is $[L^2T^{-1}]$

- If velocity potential (ϕ) exists, the flow should be irrotational.
- If $\phi_1 and \phi_2$ are solution of Laplace equation then, $\phi_1 - \phi_2$ is also a solution of Laplace eq$^{n.}$
- The lines of constant ϕ are normal to the streamlines.

Cauchy- Riemann equation

$$\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \; and \; \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

If ψ is a Laplacian ϕ ***must*** exists.

Rotational or irrotational

Rotational components

$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial \upsilon}{\partial z}\right]$; $\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$; $\omega_z = \frac{1}{2}\left[\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right]$

$$\omega = \frac{1}{2}(\nabla \times \vec{V}) = \frac{1}{2} curl \vec{V} = \frac{\Omega}{2}$$

Ω or curl$\vec{V}$ is called vorticity

Rotation = $\frac{1}{2}$ vorticity.

- A flow is said to be rotational when every fluid element has finite angular velocity about its mass centre.
- A two dimensional flow in x-y plane is rotational if $\frac{\partial \upsilon}{\partial x} = \frac{\partial u}{\partial y}$
- Irrotational flow implies zero vorticity

Circulation in a flow:

Along a closed contour in a flow field

$$\Gamma = \oint \vec{V}.d\vec{s} = \oint (udx + \upsilon dy + wdz)$$

$$\frac{\delta \Gamma}{\delta A} = 2\omega_z = \Omega_z, the \text{ vorticity.}$$

Circulation per unit area equals the Vorticity in flow.

- Irrotational flow is such that circulation is zero.
- Circulation must be zero along a closed contour in an irrotational flow.

Flow net:

Streamline ψ = const.

Velocity potential line ϕ = const.

- The streamlines and velocity potential lines form an orthogonal net work in a fluid flow.
- Observation of a flow net enables us to estimate the velocity variation.
- Streamline and velocity potential lines must constitute orthogonal net work.

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Questions (IES, IAS, GATE)

Acceleration

1. The convective acceleration of fluid in the x-direction is given by:

[a]. $u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}+\omega\frac{\partial \omega}{\partial z}$ [b]. $\frac{\partial u}{\partial t}+\frac{\partial v}{\partial t}+\frac{\partial \omega}{\partial t}$ **[IES-2001]**

[c]. $u\frac{\partial u}{\partial x}+u\frac{\partial v}{\partial y}+u\frac{\partial \omega}{\partial z}$ [d]. $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\omega\frac{\partial u}{\partial z}$

2. In a two-dimensional velocity field with velocities u and v along the x and y directions respectively, the convective acceleration along the x-direction is given by **[GATE-2006]**

(a) $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$ (b) $u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}$ (c) $u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial y}$ (d) $v\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial y}$

3. For a steady two-dimensional flow, the scalar components of the velocity field are V_x = - 2x, V_y=2y, V_z = 0. What are the components of acceleration?

(a) a_x = 0 , a_y = 0 (b) a_x = 4x, a_y =0
(c) a_x = 0, a_y= 4y (d) a_x = 4x, a_y = 4y **[IES-2006]**

4. For a fluid flow through a divergent pipe of length L having inlet and outlet radii of R_1 and R_2 respectively and a constant flow rate of Q, assuming the velocity to be axial and uniform at any cross-section, the acceleration at the exit is **[GATE-2004]**

(a) $\frac{2Q(R_1-R_2)}{\pi L R_2^3}$ (b) $\frac{2Q^2(R_1-R_2)}{\pi L R_2^3}$ (c) $\frac{2Q^2(R_1-R_2)}{\pi^2 L R_2^5}$ (d) $\frac{2Q^2(R_2-R_1)}{\pi^2 L R_2^5}$

5. The area of a 2m long tapered duct decreases as A = (0.5 – 0.2x) where 'x' is the distance in meters. At a given instant a discharge of 0.5 m^3/s is flowing in the duct and is found to increase at a rate of 0.2 m^3/s. The local acceleration (in m^2/s) at x = 0 will be:

[a]. 1.4 [b]. 1.0 [c]. 0.4 [d]. 0.667 **[IES-2007]**

6. The components of velocity in a two dimensional frictionless incompressible flow are u = t^2 + 3y and v = 3t + 3x. What is the approximate resultant total acceleration at the point (3, 2) and t = 2? **[IES-2004]**

[a]. 5 [b]. 49 [c]. 59 [d]. 54

7. Match List I (Pipe flow) with List II (Type of acceleration) and select the correct answer:

List I	List II
A. Flow at constant rate passing through a bend	1. Zero acceleration
B. Flow at constant rate passing through a straight uniform diameter pipe.	2. Local and convective acceleration
C. Gradually changing flow through a bend.	3. Convective acceleration
D. Gradually changing flow through a straight pipe.	4. Local acceleration.

Codes: **[IES-1999]**

	A	B	C	D		A	B	C	D
(a)	3	1	2	4	(b)	3	1	4	2
(c)	1	3	2	4	(d)	1	3	4	2

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Tangential and Normal Acceleration

8. Which one of the following statements is correct?
A steady flow of diverging straight stream lines
(a) is a uniform flow with local acceleration
(b) has convective normal acceleration
(c) has convective tangential acceleration
(d) has both convective normal and tangential accelerations. **[IAS-2004]**

9. For a fluid element in a two dimensional flow field (x-y plane), if it will undergo
(a) translation only (b) translation and rotation **[GATE-1994]**
(c) translation and deformation (d) deformation only

Types of Flow

10. Match List I (Flows Over or Inside the Systems) with List II (Type of Flow) and select the correct answer:

List I	**List II**
A. Flow over a sphere	1. Two dimensional flow
B. Flow over a long circular cylinder	2. One dimensional flow
C. Flow in a pipe bend	3. Axisymmetric flow
D. Fully developed flow in a pipe at constant flow rate	4. Three dimensional flow.

Codes : **[IES-2003]**

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
[a].	3	1	2	4	[b].	1	4	3	2
[c].	3	1	4	2	[d].	1	4	2	3

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11. Match List I (Types of flow) with List II (Basic ideal flows) and select the correct answer: **[IES-2001]**

	List I	**List II**
A.	Flow over a stationary cylinder	1. source + sink + uniform flow
B.	Flow over a half Rankine body	2. doublet + uniform flow
C.	Flow over a rotating body	3. source + uniform flow
D.	Flow over a Rankine oval.	4. doublet + free vortex + uniform flow.

Codes :

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
[a].	1	4	3	2	[b].	2	4	3	1
[c].	1	3	4	2					

12. Match List I with List II and select the correct answer using the code given below the lists: **[IES-2007]**

List I (Condition)	**List II** (Regulating Fact)
A. Existence of stream function	1. Irrotationality of flow
B. Existence of velocity potential	2. Continuity of flow
C. Absence of temporal Variations	3. Uniform flow
D. Constant velocity vector	4. Steady flow

Code :

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	3	2	1	(b)	2	1	4	3
(c)	4	1	2	3	(d)	2	3	4	1

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13. Irrotational flow occurs when: **[IES-1997]**

[a]. flow takes place in a duct of uniform cross-section at constant mass flow rate.
[b]. streamlines are curved.
[c]. there is no net rotation of the fluid element about its mass center.
[d]. fluid element does not undergo any change in size or shape.

14. Which one of the following statements is correct?
Irrotational flow is characterized as the one in which
[a]. the fluid flows along a straight line
[b]. the fluid does not rotate as it moves along
[c]. the net rotation of fluid particles about their mass centres remains zero.
[d]. the streamlines of flow are curved and closely spaced. **[IES-2004]**

Stream Line

15. A streamline is a line:
[a]. which is along the path of the particle **[IES-2003]**
[b]. which is always parallel to the main direction of flow
[c]. along which there is no flow
[d]. on which tangent drawn at any point given the direction of velocity

16. Assertion (A) : Stream lines are drawn in the flow field such that at a given instant of time there perpendicular to the direction of flow at every point in the flow field.
Reason (R) : Equation for a stream line in a two dimensional flow is given by $V_x\,dy - V_y\,dx = 0$.
[IES-2002]

17. Assertion (A): Streamlines can cross one another if the fluid has higher velocity.
Reason (R): At sufficiently high velocity, the Reynolds number is high and at sufficiently high Reynolds numbers, the structure of the flow is turbulent type.
[IES-2003]

18. In a two-dimensional flow, where u is the x-component and v is the y-component of velocity, the equation of streamline is given by
(a) udx-vdy=0 (b) vdx-udy=0 (c) uvdx+dy=0 (d) udx+vdy=0 **[IAS-1998]**

19. A two-dimensional flow field has velocities along the x and y directions given by $u=x^2t$ and $v=-2xyt$ respectively, where t is time. The equation of streamlines is
(a) x^2y=constant (b) xy^2=constant
(c) xy=constant (d) not possible to determine **[GATE-2006]**

$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$ or $\frac{dx}{x^2t}=\frac{dy}{-2xyt}$ integrating both side $\int\frac{dx}{x}=-\frac{1}{2}\int\frac{dy}{y}$ or $\ln(x^2y)=0$

Path Line

20. Consider the following statements regarding a path line in fluid flow:
1. A path line is a line traced by a single particle over a time interval.
2. A path line shows the positions of the same particle at successive time instants.
3. A path line shows the instantaneous positions of a number of a particle, passing through a common point, at some previous time instants.

Which of the statements given above are correctly?
(a) Only 1 and 3 (b) only 1 and 2
(c) Only 2 and 3 (d) 1, 2 and 3 **[IES-2006]**

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Streak Line

21. Consider the following statements:

1. Streak line indicates instantaneous position of particles of fluid passing through a point.
2. Streamlines are paths traced by a fluid particle with constant velocity.
3. Fluid particles cannot cross streamlines irrespective of the type of flow.
4. Streamlines converge as the fluid is accelerated, and diverge when retarded.

Which of these statements are correct? **[IAS-2001]**

(a) 1 and 4 (b) 1, 3 and 4 (c) 1,2 and 4 (d) 2 and 3

22. Which one of the following is the correct statement? **[IES-2007]**
Streamline, path line and streak line are identical when the
(a) flow is steady (b) flow is uniform
(c) flow velocities do not change steadily with time (d) flow is neither steady nor uniform

23. Streamlines, path lines and streak lines are virtually identical for
(a) Uniform flow (b) Flow of ideal fluids
(c) Steady flow (d) Non uniform flow **[GATE-1994]**

Continuity Equitation

24. Which one of the following is the continuity equation in differential from? (The symbols have usual meanings) **[IAS-2004; IAS2003]**

(a) $\frac{dA}{A}+\frac{dV}{V}+\frac{d\rho}{\rho}=const.$ (b) $\frac{dA}{A}+\frac{dV}{V}+\frac{d\rho}{\rho}=0$

(c) $\frac{A}{dA}+\frac{V}{dV}+\frac{\rho}{d\rho}=const.$ (d) AdA+VdV+ρ d ρ =0

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25. Which one of the following equations represents the continuity equation for steady compressible fluid flow? **[IAS-2000]**

(a) $\Delta.\rho V+\frac{\partial\rho}{\partial t}=0$ (b) $\Delta.\rho V+\frac{\partial\rho}{\partial t}=0$ (c) $\Delta.V=0$ (d) $\Delta.\rho\vec{V}=0$

26. The continuity equation for 3-dimenstional flow $\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}+\frac{\delta w}{\delta z}$ =0 is applicable to

(a) steady flow (b) uniform flow **[IAS-1999; IAS 1998]**
(c) ideal fluid flow (d) ideal as well as viscous flow

27. The velocity components in the x and y directions of a two dimensional potential flow are u and v, respectively. Then $\frac{\partial u}{\partial x}$ is equal to

(a) $\frac{\partial v}{\partial x}$ (b) $\frac{\partial v}{\partial x}$ (c) $\frac{\partial v}{\partial y}$ (d) $-\frac{\partial v}{\partial y}$ **[GATE-2005]**

28. In a two-dimensional incompressible steady flow, the velocity component u = Ae^{X} is obtained. What is the other component v of velocity? **[IES-2006]**

$(a) v = Ae^{xy}$ (b) $v = Ae^{y}$
$(c) v = -Ae^{x}y + f(x)$ (d) $v = -Ae^{y}x + f(y)$

29. In a steady, incompressible, two dimensional flow, one velocity component in the X-direction is given by $u=cx^2/y^2$
The velocity component in the y-direction will be
(a) V= -c(x+y) (b) v= -cx/y (c) v= -xy (d) v= -cy/x **[IAS-1997]**

30. The velocity components in the x and y directions are given by **[GATE-1995]**

$\mu = \lambda xy^3 - x^2 2, \;\; v = xy^2 - \frac{3}{4}y^4$

The value of λ for a possible flow field involving an incompressible fluid is
(a) -3/4 (b) -4/3 (c) 4/3 (d) 3

31. Which one of the following stream functions is a possible irrotational flow field?
[a]. $\psi = x^3 y$ [b]. $\psi = 2xy$ [c]. $\psi = Ax^2 y^2$ [d]. $\psi = Ax + By^2$ **[IES-2003]**

32. The components of velocity u and v along x- and y- direction in a 2-D flow problem of an incompressible fluid are **[IAS-1994]**
1. $u= x^2\cos y$; $v= -2x \sin y$
2. u=x+2 ;v=1-y
3. u=xyt ; $v=x^3-y^2t/2$
4. u=lnx+y ;v=xy-y/x

Those which would satisfy the continuity equation would include
(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 3 and 4 (d) 1 and 2

33. The continuity equation in the form $\Delta.\vec{V} = 0$ always represents an incompressible flow regardless of whether the flow is steady or unsteady. **[GATE-1994]**

34. If $\vec{V}$ is velocity vector of fluid, then $\nabla.\vec{V}$ =0 is strictly true for which of the following?
(a) Steady and incompressible flow
(b) Steady and irrotational flow
(c) Inviscid flow irrespective flow irrespective of steadiness
(d) Incompressible flow irrespective of steadiness **[IAS-2007]**

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Circulation and Vorticity

35. Which one of the following is the expression of the rotational component for a two- dimensional fluid element in x-y plane?

(a) $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ (b) $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

(c) $\omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)$ (d) $\omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ **[IAS-2004;IAS-2003]**

36. Which of the following relations must hold for an irrotational two-dimensional flow in the x-y plane? **[IAS-2003; IAS-2004]**

(a) $\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = 0$ (b) $\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$

(c) $\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0$ (d) $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

37. Circulation is defined as line integral of tangential component of velocity about a .... **[GATE-1994]**

Velocity Potential Function

38. The velocity potential function in a two dimensional flow fluid is given by
$\phi = x^2 - y^2$.The magnitude of velocity at the point(1,1)is

(a) 2 (b) 4 (c) $2\sqrt{2}$ (d) $4\sqrt{2}$ **[IAS-2002]**

39. The relation $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}$ = 0 for an irrotational flow is known as which one of the following? **[IES-2007]**

(a) Navier - Stokes equation (b) Laplace equation
(c) Reynolds equation (d) Euler's equation

40. Existence of velocity potential implies that **[GATE-1994]**
(a) Fluid is in continuum (b) Fluid is irrotational
(c) Fluid is ideal (d) Fluid is compressible

41. Which of the following functions represent the velocity potential in a two-dimensional flow of an ideal fluid ? **[IES-2004]**
1. 2x + 3y 2. $4x^2 - 3y^2$ 3. cos (x – y) 4. $\tan^{-1}$ (x/y)
Select the correct answer using the codes given below :
[a]. 1 and 3 [b]. 1 and 4 [c]. 2 and 3 [d]. 2 and 4

Stream Function

42. If for a flow, a stream function exists and satisfies the Laplace equation, then which one of the following is the correct statement? **[IES-2005]**
[a]. The continuity equation is satisfied and the flow is irrotational.
[b]. The continuity equation is satisfied and the flow is rotational.
[c]. The flow is irrotational but does not satisfy the continuity equation.
[d]. The flow is rotational.

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43. For a stream function to exist, which of the following conditions should hold? **[IAS-1997]**
1. The flow should always be irrotational.
2. Equation of continuity should be satisfied.
3. The fluid should be incompressible.
4. Equation of continuity and momentum should be satisfied.
Select the correct answer using the codes given below:
Codes: (a) 1, 2, 3 and 4 (b) 1, 3 and 4 (c) 2 and 3 (d) 2 alone

44. The velocity potential of a velocity field is given by = $x^2 - y^2$ + const. Its stream function will be given by:
[a]. – 2xy + constant [b]. + 2xy + constant
[c]. – 2xy + f(x) [d]. – 2xy + f(y) **[IES-2002]**

45. The stream function in a 2- dimensional flow field is given by = xy.
The potential function is:

[a]. $\frac{(x^2+y^2)}{2}$ [b]. $\frac{(x^2-y^2)}{2}$ [c]. xy [d]. $x^2y + y^2x$ **[IES-2001**

46. A stream function is given by $(x^2 - y^2)$. The potential function of the flow will be:

[a]. 2xy + f(x) [b]. 2xy + constant [c]. $2(x^2 - y^2)$ [d]. 2xy + f(y) **[IES-2000]**

47. The stream function = $x^3 - y^3$ is observed for a two dimensional flow field. What is the magnitude of the velocity at point (1, –1)? **[IES-2004; IES-1998]**

[a]. 4.24 [b]. 2.83 [c]. 0 [d]. – 2.83

48. Which one of the following stream functions is a possible irrotational flow field ?

(a) $\psi = y^2 - x^2$ (b) $\psi = A \sin(xy)$ (c) $\psi = A x^2 y^2$ (d) $\psi = Ax + By^2$ **[IES-2007]**

49. Match List I with List II and select the correct answer using the code given below the lists: **[IES-2007]**

List I (Condition)	**List II** (Regulating Fact)
A. Existence of stream function	1. Irrotationality of flow
B. Existence of velocity potential	2. Continuity of flow
C. Absence of temporal variations	3. Uniform flow
D. Constant velocity vector	4. Steady flow

Code:

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	2	1	4	3
(c)	4	1	2	3	(d)	2	3	4	1

50. For irrotational and incompressible flow, the velocity potential and steam function are given by , respectively. Which one of the following sets is correct ?

(a) $\nabla^2\varphi = 0, \nabla^2\psi = 0$ (b) $\nabla^2\varphi \neq 0, \nabla^2\psi = 0$ (c) $\nabla^2\varphi = 0, \nabla^2\psi \neq 0$ (d) $\nabla^2\varphi \neq 0, \nabla^2\psi \neq 0$ **[IES-2006]**

51. The 2-D flow with, velocity υ =(x+2y+2)i+(4-y)j is

(a) compressible and irrotational (b) compressible and not irrotational

(c) incompressible and irrotational (d) incompressible and not irrotational

[GATE-2001]

52. Consider the following statements:

1. For stream function to exit, the flow should be irrotational.
2. Potential functions are possible even though continuity is not satisfied.
3. Streamlines diverge where the flow is accelerated. **[IAS-2002]**
4. Bernoulli's equation will be satisfied for flow across a cross-section.

Which of the above statements is/are correct?

(a) 1, 2, 3 and 4 (b) 1, 3 and 4 (c) 3 and 4 (d) 2 only

Flow Net

53. Consider the following statements for a two dimensional potential flow:

1. Laplace equation for stream function must be satisfied. **[IAS-2002]**
2. Laplace equation for velocity potential must be satisfied.
3. Streamlines and equipotential lines are mutually perpendicular.
4. Streamlines can interest each other in very high speed flows.

Which of the above statements are correct?

(a) 1 and 4 (b) 2 and 4 (c) 1, 2 and 3 (d) 2, 3 and 4

54. For an irrotational flow, the velocity potential lines and the streamlines are always.

[a]. parallel to each other [b]. Coplanar **[IES-1997]**

[c]. orthogonal to each other [d]. Inclined to the horizontal.

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55. In a flow field, the streamlines and equipotential lines **[GATE-1994]**

(a) are Parallel (b) are orthogonal everywhere in the flow field

(c) cut at any angle (d) cut orthogonally except at the stagnation points

Answer with Explanations

1. Ans. (d)

2. Ans. (a)

3. Ans. (d) $a_x = u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$ Given $u=V_x=-2x$; $v= V_y=2y$ and $w= V_z = 0$

$$a_y = u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} + w\frac{\partial \upsilon}{\partial z} + \frac{\partial \upsilon}{\partial t}$$

4. Ans. (c) at a distance x from the inlet radius $(R_x) = \left(R_1 + \frac{R_2 - R_1}{L}x\right)$ $\therefore$ area $A_x = \pi R_x^2$

$$\therefore u = \frac{Q}{A_x} = \frac{Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^2}$$

Total acceleration $a_x = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$ for constant flow rate i.e. steady flow $\frac{\partial u}{\partial t} = 0$

$$\therefore \quad a_x = u\frac{\partial u}{\partial x} = \frac{Q}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^2} \times \frac{-2Q\frac{R_2 - R_1}{L}}{\pi\left(R_1 + \frac{R_2 - R_1}{L}x\right)^3}$$ at x=L it gives $\frac{2Q^2(R_1 - R_2)}{\pi^2 L R_2^5}$

5. Ans. (c) $\therefore u = \frac{Q}{A_x} = \frac{Q}{(0.5 - 0.2x)}$ local acceleration $\frac{\partial u}{\partial t} = \frac{1}{(0.5-0.2x)} \times \frac{\partial Q}{\partial t}$ at x = 0

$$\frac{\partial u}{\partial t} = \frac{1}{(0.5)} \times 0.2 = 0.4$$

6. Ans. (c) $a_x = u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$ and $a_y = u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} + \frac{\partial \upsilon}{\partial t}$

or $a_x = (t^2+3y).(0)+(3t+3x).(3)+2t$ and $a_y=(t^2+3y).(3)+(3t+3x).(0)+3$

at x=3, y=2 and t=2

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{49^2 + 33^2} = 59.08$

7. Ans. (a)

8. Ans. (c)

9. Ans. (b)

10. Ans. (c)

11. Ans. (b)

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12. Ans. (b)
13. Ans. (c)
14. Ans. (c)
15. Ans. (d)
16. Ans. (b) both are correct but R is not correct explanation of A
17. Ans. (d)
18. Ans. (b) $\frac{dx}{u} = \frac{dy}{\upsilon}$ or $\upsilon dx - u dy = 0$
19. Ans. (a)
20. Ans. (b) 3 is wrong because it defines Streak line.
21. Ans. (b) 2 is wrong.
22. Ans. (a)
23. Ans. (c)
24. Ans. (b) $\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0$

$\therefore$ Integrating, we get log A+ log V+log P= log C
or log(ρ AV)= log C
$\therefore$ ρ AV=C
which is the continuity equation

25. Ans. (d) General continuity equation $\nabla.\rho\vec{V} + \frac{\partial\rho}{\partial t} = 0$

for steady flow $\frac{\partial\rho}{\partial t} = 0$, and for compressible fluid the equation $\nabla.\rho\vec{V} = 0$

for steady, incompressible flow $\frac{\partial\rho}{\partial t} = 0$ and ρ =const. So the equation $\nabla.\vec{V} = 0$

26. Ans. (d)

27. Ans. (d) from continuity eq. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ =o or $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$

28. Ans. (c) From continuity eq. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ =o or $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -Ae^x$ or $v = -Ae^x y + f(x)$

29. Ans. (b)

30. Ans. (d) Just use continuity eq. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ =o

31. Ans. (b) use continuity equation

32. Ans. (a) Checking $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ =o for all cases.

33. Ans. True General continuity equation $\nabla.\rho\vec{V} + \frac{\partial\rho}{\partial t} = 0$ if $\rho = const.$ $\Delta.\vec{V} = 0$

34. Ans. (d) $\nabla.\vec{V} = 0$ Or $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

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35. Ans. (a)

36. Ans. (d) i.e. $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$

37. Ans. closed contour (path) in a fluid flow

38. Ans. (c) u$= -\frac{\partial \phi}{\partial x} = -2x, \quad \upsilon = -\frac{\partial \phi}{\partial y} = +2y$

V$= \sqrt{u^2 + \upsilon^2} = \sqrt{(2x)^2 + (2y)^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ unit

39. Ans. (b)

40. Ans. (b)

41. Ans. (a) Checking $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$ for all the above.

42. Ans. (a) if a stream function ψ exists means a possible case of flow.
if it satisfies the Laplace equation then flow is irrotational.

43. Ans. (d)

44. Ans. (a) Use Cauchy- Riemann equation

$u = -\frac{\partial \phi}{\partial x} = -2x = \frac{\partial \psi}{\partial y}$ And $v = -\frac{\partial \phi}{\partial y} = 2y = -\frac{\partial \psi}{\partial x}$ therefore $d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$

45. Ans. (b) $u = \frac{\partial \psi}{\partial y} = x = -\frac{\partial \phi}{\partial x}$ And $v = -\frac{\partial \psi}{\partial x} = -y = -\frac{\partial \phi}{\partial y}$ therefore $d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$

46. Ans. (b)

47. Ans. (a) $u = \frac{\partial \psi}{\partial y} = -3y^2 = -3$ And $v = -\frac{\partial \psi}{\partial x} = -3x^2$=-3 $\therefore \sqrt{(-3)^2 + (-3)^2}$ =4.24

48 Ans. (a) satisfy Laplace Equation.

49 . Ans. (b)

50. Ans. (a)

51. Ans. (d) continuity equation satisfied but $\omega_z \neq 0$

52. Ans. (c)

1. Stream function is exist for possible case of fluid flow i.e. if continuity is satisfied but flow may be rotational or irrotational, 1 is wrong.
2. Potential function will exist for possible and irrotational flow so both continuity and irrotational must be satisfied, 2 is wrong.

53. Ans. (c) Streamlines never intersects each other.

54. Ans. (c)

55. Ans. (c)

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FLUID DYNAMICS

Skip to Questions (IAS, IES, GATE)

Highlights

1. Reynolds Transport Theorem

N= $\int n\rho dV = N(G,t)$

$$\frac{dN}{dt} = \int_{cs} n(\rho U.dA) + \frac{\partial}{\partial t}\int_{cv} n\rho dV$$

2. Euler's momentum equation for ((i) Three dimensional, (ii) inviscid, (iii) steady flow)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = B_x - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = B_y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = B_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

Equation for two-dimensional, steady flow of an inviscid fluid in a vertical plane

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

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Euler's momentum equation along streamline or irrigational flow

$$\frac{dp}{\rho g} + \frac{1}{g}d\left(\frac{v^2}{2}\right) + dz = 0$$

Euler's equation in the stream wise direction

$$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + u\frac{du}{ds} = 0$$

Euler's equation of motion

$$\frac{dp}{\rho} + V.dV + g.dz = 0$$

(i) Euler's equation of motion is a statement of conservation of momentum for the flow of an inviscid fluid.
(ii) Euler's equation of motion for fluid flow refers to motion with acceleration in general
(iii) Euler's equation of motion is not applicable for viscous flow.
(iv) Euler's equation of motion is a consequence of law of motion

3. Bernoulli's Equation

Generalized equation: $\int \frac{dp}{\rho g} + \frac{V^2}{2} + Z = const.$

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The assumptions made for Bernoulli's equation **[VIMP]**

(i) The liquid is ideal (viscosity, surface tension is zero and incompressible)
(ii) The flow is steady and continuous
(iii) The flow is along the streamline (it is one-dimensional)
(iv) The velocity is uniform over the section and is equal to the mean velocity.
(v) The only forces acting on the fluid are the gravity force and the pressure force

The assumptions **NOT** made for Bernoulli's equation **[VIMP]**

(i) The flow is uniform
(ii) The flow is irrotational

For incompressible fluid flow: $\frac{p}{\rho g}+\frac{V^2}{2g}+Z=const.$

For compressible undergoing adiabatic process: $\frac{\gamma}{\gamma-1}\frac{p}{\rho g}+\frac{V^2}{2g}+Z=const.$

Bernoulli's equation in the stream wise direction: $\int\frac{dp}{\rho g}+\frac{V^2}{2g}+Z=const.$

For forced Vortex flow: $\frac{1}{2}\rho r^2\omega^2+\rho gz+p=const.$

For real fluid: $\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L(loss\ of\ energy\ in\ between)$

For an inviscid flow, **not irrotational**, the Bernoulli's constants $\frac{p}{\rho g}+\frac{V^2}{2g}+z=C$

C_1, C_2, C_3 have different values along different streamlines, whereas for irrotational flow the Bernoulli's constant C is same in the entire flow field.

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4. Navier-Stokes Equation: (General momentum equation)

$$\rho\frac{du}{dt}=\rho B_x+\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}$$

$$\rho\frac{dv}{dt}=\rho B_y+\frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{yx}}{\partial z}$$

$$\rho\frac{dw}{dt}=\rho B_z+\frac{\partial\tau_{zx}}{\partial x}+\frac{\partial\tau_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}$$

And

$$\sigma_{xx}=-p+2\mu\frac{\partial u}{\partial x};\quad \sigma_{yy}=-p+2\mu\frac{\partial v}{\partial y};\quad \sigma_{zz}=-p+2\mu\frac{\partial w}{\partial z};\quad (p=-\frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3})$$

$$\tau_{xy}=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right);\quad \tau_{zy}=\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right);\quad \tau_{zx}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$$

5. Navier-stokes momentum equation for a three dimensional incompressible fluid flow

$$\rho\frac{du}{dt} = \rho B_x - \frac{\partial p}{\partial x} + \mu\nabla^2 u$$
$$\rho\frac{dv}{dt} = \rho B_y - \frac{\partial p}{\partial y} + \mu\nabla^2 v$$
$$\rho\frac{dw}{dt} = \rho B_z - \frac{\partial p}{\partial z} + \mu\nabla^2 w$$

6. Integral Momentum Equation

Total force = surface force + Body force

$$= \int_s \vec{U}(\rho\vec{U}.d\vec{A}) + \frac{\partial}{\partial t}\int_V U\rho dV$$

7. Angular Momentum equation

$$M = \int_s \{(\vec{r}\times\vec{U})(\rho\vec{U}.d\vec{A})\} + \frac{\partial}{\partial t}\int_v (\vec{r}\times\vec{U})\rho dV$$

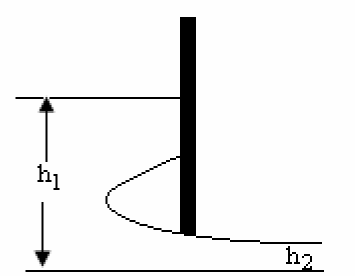
8. Continuity Equation

$$\nabla.(\rho\vec{U}) + \frac{\partial\rho}{\partial t} = 0$$
$$\int_{cs}\rho U.dA + \frac{d}{dt}\int_{cv}\rho dV = 0$$

9. Force on rectangular sluice gate/per unit width of the Gate.

$$F = \frac{\rho g}{2}(h_1^2 - h_2^2) + \rho q(U_1 - U_2)$$

Where: q = Volumetric flow per unit width of the gate.



10. Steady Flow Energy equation

$$\left|\frac{U^2}{2} + gZ + h\right|_1^2 = q - \dot{w}_s$$ Where h = enthalpy/kg; q = heat added/kg; $\dot{w}_s$ = shaft work/kg

S.F.E.E must be satisfied for any fluid flow.

11. Unit of Bernoulli's equation

Unit of each term is *energy per unit weight* =J/kg of liquid= $\frac{N.m}{N}$ =m **[VIMP]**

12. Correction for non Uniform flow

a) Average Velocity

$$U_{avg} = \frac{1}{A}\int udA$$

For laminar flow through round pipe: $\frac{U_{\max}}{U_{avg}} = 2.0$ **[VIMP]**

For plane Poiseuille flow (i.e. Laminar flow between two stationary plates:

$$\frac{U_{\max}}{U_{avg}} = 1.5$$

For velocity distribution $\frac{U}{U_{\max}} = \left(1 - \frac{r}{r_o}\right)^n$; $\frac{U_{\max}}{U_{avg}} = \frac{(1+n)(2+n)}{2}$

b) Kinetic energy correction factor (α)

$$\alpha = \frac{1}{A}\int\left(\frac{U}{U_{avg}}\right)^3 .dA$$

For α =1.0 the flow is *uniform*

For α >1.0 the flow is non uniform.

For laminar flow through round pipe α **=2.0** **[VIMP]**

For turbulent flow through round pipe α **=1.05**

It is conventional to use a value α =1.0 for turbulent flow.

For velocity distribution $\frac{U}{U_{\max}} = \left(1 - \frac{r}{r_o}\right)^n$; $\alpha = \frac{(1+n)^3(2+n)^3}{4(1+3n)(2+3n)}$

b) Momentum correction factor (β)

$$\beta = \frac{1}{A}\int\left(\frac{U}{U_{avg}}\right)^2 dA$$

For β =1.0 the flow is *uniform*

For β >1.0 the flow is non uniform.

For laminar flow through round pipe β **=1.33** **[VIMP]**

It is conventional to use a value β =1.0 for turbulent flow.

For velocity distribution $\frac{U}{U_{\max}} = \left(1 - \frac{r}{r_o}\right)^n$; $\beta = \frac{(1+n)(2+n)^2}{4(1+2n)}$

13. Piezometer head = $\left(\frac{p}{\rho g} + z\right)$

14. For a real fluid moving with uniform velocity the pressure is independent of both depth and orientation.

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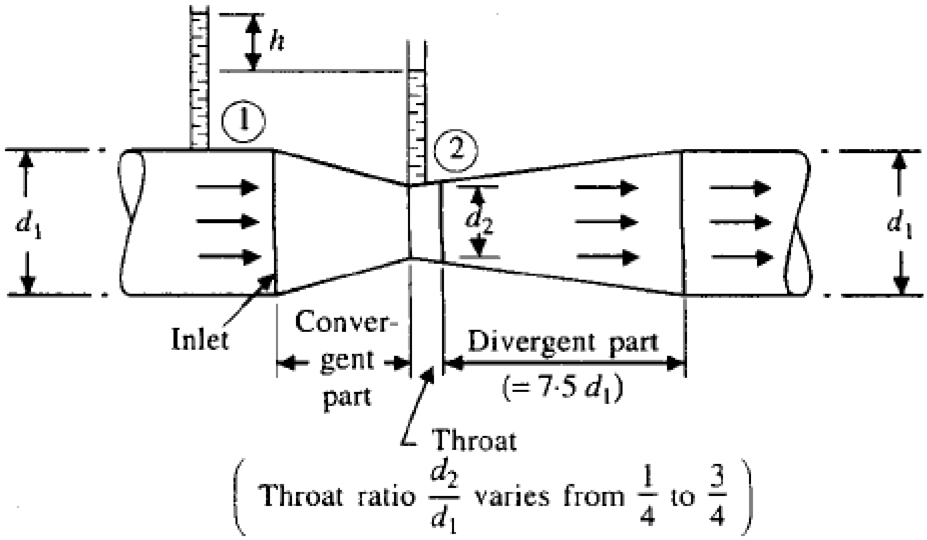
15. Venturimeter



$$Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

(Where $A_2 < A_1$) **[VIMP]**

(i) Co-efficient of discharge of Venturimeter (C_d) varies between 0.96 to 0.98

(ii) Venturimeter is not suitable for *very low velocities* due to variation of C_d

For any Venturimeter (Vertical, inclined, etc) if differential manometer is there we directly get 'h' i.e.

$h = y\left(\frac{s_h}{s_l} - 1\right)$ no correction need for its orientation.

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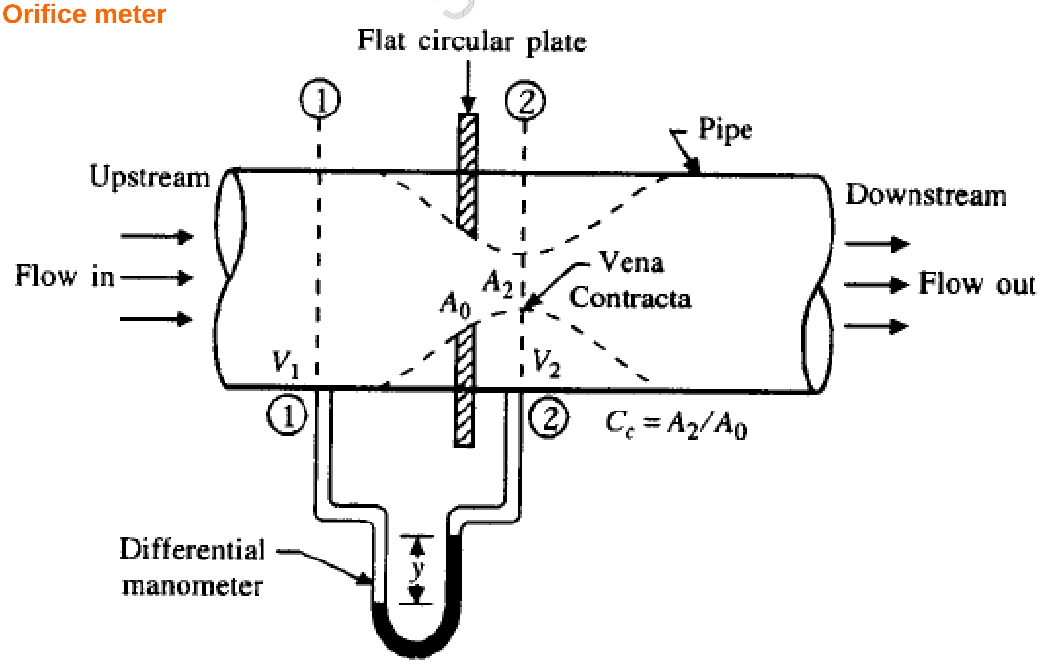
16. Orifice meter



$$Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$ [but here $C_d = C_c \times \frac{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}{\sqrt{1-C_c^2 \times \left(\frac{A_2}{A_1}\right)^2}}$]

Therefore $C_d = f\left(\frac{d}{D}\right)$

17. $C_d = C_c \times C_v$

18. Pitot tube

Pitot tube is a device to measure the *velocity of flow.*

19.What are Rotameter?

The Rotameter is an industrial flowmeter used to measure the flow rate of liquids and gases. The Rotameter consists of a tube and float. The float response to flow rate changes is linear, and a 10-to-1 flow range or turndown is standard. The Rotameter is popular because it has a linear scale, a relatively long measurement range, and low pressure drop. It is simple to install and maintain.

Principle of Operation

The Rota meter's operation is based on the variable area principle: fluid flow raises a float in a tapered tube, increasing the area for passage of the fluid. The greater the flow, the higher the float is raised. The height of the float is directly proportional to the flow rate. With liquids, the float is raised by a combination of the buoyancy of the liquid and the velocity head of the fluid. With gases, buoyancy is negligible, and the float responds to the velocity head alone.

The float moves up or down in the tube in proportion to the fluid flow rate and the annular area between the float and the tube wall. The float reaches a stable position in the tube when the upward force exerted by the flowing fluid equals the downward gravitational force exerted by the weight of the float. A change in flow rate upsets this balance of forces. The float then moves up or down, changing the annular area until it again reaches a position where the forces are in equilibrium. To satisfy the force equation, the Rotameter float assumes a distinct position for every constant flow rate. However, it is important to note that because the float position is gravity dependent, Rotameter must be vertically oriented and mounted.

20. Vortex motion:

The pressure variation along the radial direction for vortex flow along a horizontal plane,

$$\frac{\partial p}{\partial r} = \frac{\rho \upsilon^2}{r}$$

and pressure variation in the vertical plane,

$$\frac{\partial p}{\partial z} = -\rho g$$

Forced vortex flow:

--- Forced vortex flow is one in which the fluid mass is made to rotate by means of some external agency.

$$\upsilon = \omega \times r$$

$$z = \frac{\upsilon^2}{2g} = \frac{\omega^2 r^2}{2g} = \frac{\omega^2 R^2}{2g}$$

where z= height of the parabolo id formed , and

$\omega =$ angular velocity.

--- For a forced vortex flow in an open tank:

Fall of liquid level at centre= rise of liquid level at the ends

--- In case of a closed cylinder,

Volume of air before rotation= volume of air after rotation.

--- If a closed cylindrical vessel completely filled with water is rotated about its vertical axis, the total pressure force acting on the top and bottom are:

$$F_{top} = \frac{\rho}{4}\omega^2 \pi R^4$$

and $F_{bottom} = F_{top}$+weight of water in cylinder

$= F_{top} + w \times \pi R^2 \times H$

Where $\omega = angular$ velocity

R= radius of the vessel,

H=height of the vessel, and

ρ = density of fluid $\left(= \frac{w}{g} \right)$

Free vortex flow:

When no external torque is required to rotate the fluid mass, that type of flow is called free vortex flow. In case of free vortex flow:

$\upsilon \times r$ = constant

$$\frac{p_1}{\rho g} + \frac{\upsilon^2_1}{2g} + z_1 = \frac{p_2}{\rho_g} + \frac{\upsilon^2_2}{2g} + z_2$$

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21. Note: at a distance r, slope of the paraboloid

$$\frac{\partial z}{\partial r} = \tan\theta = \frac{2rw^2}{2g} = \frac{rw^2}{g}$$

22. Note: (i) Volume of a cylinder = $\pi R^2 H$

(ii) Volume of a paraboloid = $\frac{1}{2}.\pi R^2 H$

(iii) volume of a cone = $\frac{1}{3}.\ \pi R^2 H$

Questions (IES, IAS and GATE)

Bernoulli's equation

1. The Bernoulli's equation refers to conservation of **[IAS-2003]**
(a) Mass (b) linear momentum (c) angular momentum (d) energy

2. Bernoulli's equation $\frac{P}{\rho}+\frac{V^2}{2}+gh$ = constant, is applicable for
(a) steady, frictionless and incompressible flow along a streamline
(b) uniform and frictionless flow along a streamline when ρ is a function of p
(c) steady and frictionless flow along a streamline when ρ is a function of p
(d) steady, uniform and incompressible flow along a streamline

3. Bernoulli's theorem $\frac{P}{\rho g}+\frac{V^2}{2g}$+Z= constant is valid
(a) along different streamlines in rotational flow **[IAS-1996]**
(b) along different streamlines in irrotational flow
(c) only in the case of flow of gas
(d) only in the case of flow of liquid

4. Bernoulli's equation can be applied between any two points on a streamline for a rotational flow field. **[GATE-1994]**

5. Which of the following assumptions are made for deriving Bernoulli's equation?
1. Flow is steady and incompressible **[IES-2002]**
2. Flow is unsteady and compressible
3. Effect of friction is neglected and flow is along a stream line.
4. Effect of friction is taken into consideration and flow is along a stream line.
Select the correct answer using the codes given below:
[a]. 1 and 3 [b]. 2 and 3 [c]. 1 and 4 [d]. 2 and 4

6. The expression: **[IES-2003]**

$$\frac{\partial\phi}{\partial t}+\int\frac{\partial p}{\rho}+\frac{1}{2}|\Delta\phi|^2+gz=\text{constant}$$

represents :
[a]. Steady flow energy equation
[b]. Unsteady irrotational Bernoulli's equation
[c]. Steady rotational Bernoulli's equation
[d]. Unsteady rotational Bernoulli's equation

7. Which one of the following statements is correct? While using boundary layer equations, Bernoulli's equation **[IES-2006]**
(a) can be used anywhere
(b) can be used only outside the boundary layer
(c) can be used only inside the boundary layer
(d) cannot be used either inside or outside the boundary layer

8. Assertion (A): Bernoulli's equation is an energy equation. **[IES-1997]**
Reason (R): Starting from Euler's equation, one can arrive at Bernoulli's equation.

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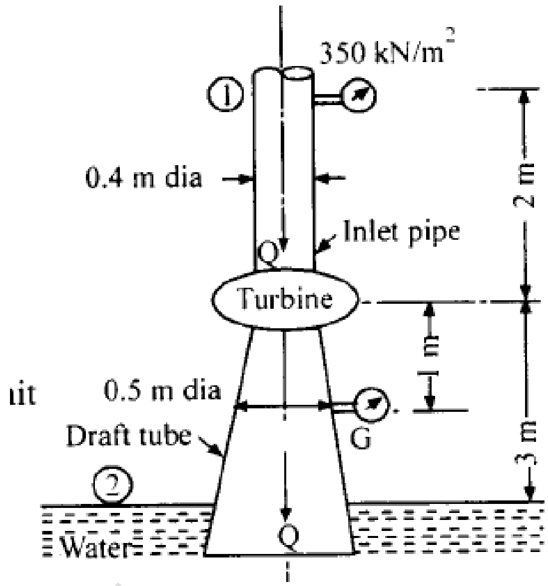
9. Assertion (A) : After the fluid has re-established its flow pattern downstream of an orifice plate, it will return to same pressure that it had upstream of the orifice plate.

Reason (R) : Bernoulli's equation when applied between two points having the same elevation and same velocity gives the same pressure at these points. **[IES-2003]**

10. In the Fig. is shown a turbine with inlet pipe and a draft tube. If the efficiency of turbine is 80 per cent and discharge is 1000 litres/s. find:
(a) The power developed by the turbine, and
(b) The reading of the gauge G.

10. Ans. (a) 344.6 kW
(b) -32.57 kN/m^2



Euler's equation

11. Consider the following assumptions: **[IES-1998]**

1. The fluid is compressible
2. The fluid is inviscid.
3. The fluid is incompressible and homogeneous.
4. The fluid is viscous and compressible.

The Euler's equation of motion requires assumptions indicated in :

[a]. 1 and 2 [b]. 2 and 3 [c]. 1 and 4 [d]. 3 and 4

12. The Euler's equation of motion is a statement of **[IES-2005]**

[a]. Energy balance
[b]. Conservation of momentum for an inviscid fluid
[c].Conservation of momentum for an incompressible flow.
[d]. Conservation of momentum for real fluid.

13. Navier Stoke's equation represents the conservation of
(a) energy (b) mass (c) pressure (d) momentum **[GATE-2000]**

Venturimeter

14. Fluid flow rate Q, can be measured easily with the help of a venturi tube, in which the difference of two pressures, ΔP , measured at an upstream point and at the smallest cross-section and at the smallest cross-section of the tube, is used. If a relation $\Delta P \propto Q^n$ exists, then n is equal to **[IAS-2001]**

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(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

15. Two venturimeters of different area rations are connected at different locations of a pipeline to measure discharge. Similar manometers are used across the two venturimeters to register the head differences. The first venturimeters of area ratio 2 registers a head difference 'h', while the second venturimeters registers '5h'.The area ratio for the second venturimeters is,

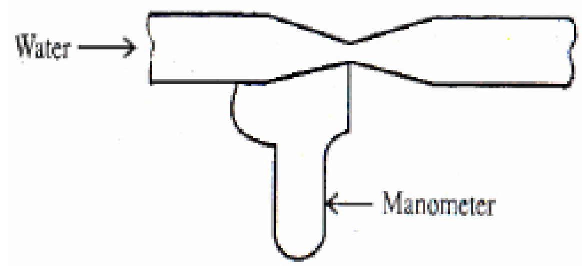
[IAS1999]

(a) 3 (b) 4 (c) 5 (d) 6

16. A horizontal pipe of cross-sectional area 5 cm^2 is connected to a venturimeter of throat area 3 cm^2 as shown in the below figure. The manometer reading is equivalent to 5 cm of water. The discharge in cm^3/s is nearly:

[a]. 0.45 [b]. 5.5

[c]. 21.0 [d]. 370



[IES-1998]

17. An orifice meter with C_d = 0.61 is substituted y Venturimeter with C_d = 0.98 in a pipeline carrying crude oil, having the same throat diameter as that of the orifice. For the same flow rate, the ratio of the pressure drops for the Venturimeter and the orifice meter is:

[IES-2003]

[a]. 0.61 / 0.98 [b]. $(0.61)^2 / (0.98)^2$ [c]. 0.98 / 0.61 [d]. $(0.98)^2 / (0.61)^2$

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18. A Venturimeter in an oil (sp. gr. 0.8) pipe is connected to a differential manometer in which the gauge liquid is mercury (sp.gr.13.6). For a flow rate of 0.16 m^3/s, the manometer registers a gauge differential of 20 cm. The oil-mercury manometer being unavailable, an air-oil differential manometer is connected to the same venturimeter. Neglecting variation of discharge coefficient for the venturimeter, what is the new gauge differential for a flow rate of 0.08 m^3/s?

[IES-2006]

(a) 64 cm (b) 68 cm (c) 80 cm (d) 85 cm

19. A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is

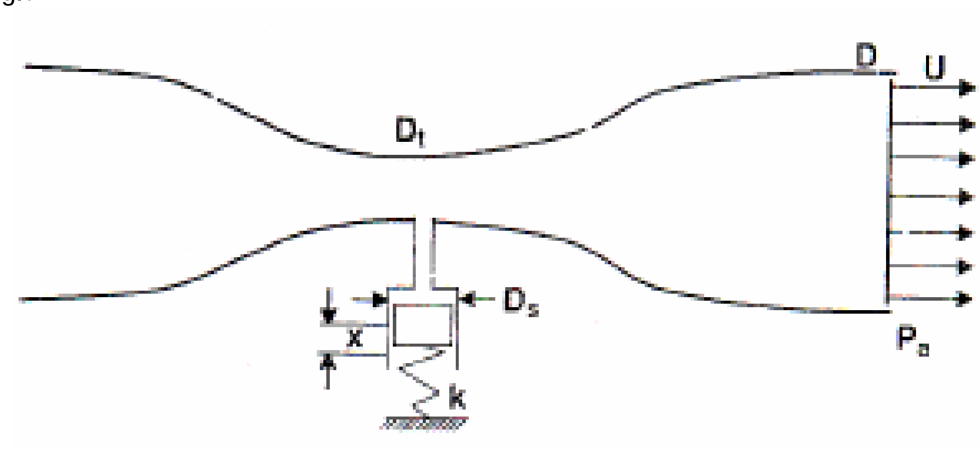
(a) 0.2 m/s (b) 1.0 m/s (c) 1.4 m/s (d) 2.0 m/s **[GATE-2005]**

20. Air flows through a venture and into atmosphere. Air density is ρ; atmospheric pressure P_a; throat diameter is D_t; exit diameter is D and exit velocity is U. The throat is connected to a cylinder containing a frictionless piston attached to a spring. The spring constant is k. The bottom surface of the piston is exposed to atmosphere. Due to the flow, the piston moves by distance x. assuming incompressible frictionless flow, x is

(a) $(\rho U^2/2k)\pi D_s^2$ (b) $(\rho U^2/8k)\left(\frac{D^2}{D^2 t}-1\right)\pi D^2_s$

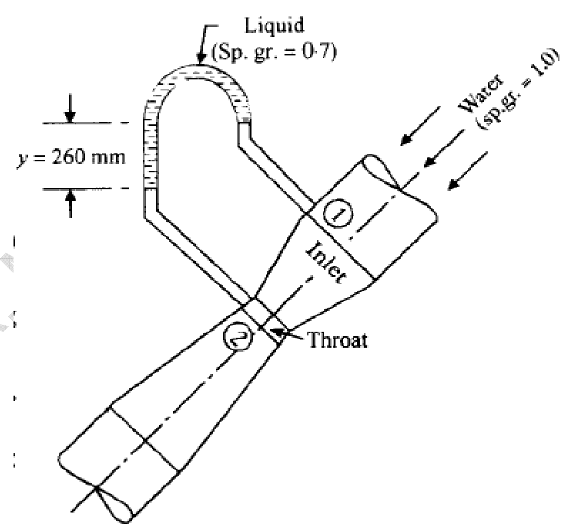
(c) $(\rho U^2 / 2k\left(\frac{D^2}{D^2_t} - 1\right)\pi D^2_s$ (d) $(\rho U^2 / 8k)\left(\frac{D^4}{D^4_t} - 1\right)\pi D^2_s$

Fig.7



[GATE-2003]

21. Determine the rate of flow of water through a pipe 300 mm diameter placed in an inclined position where a Venturimeter is inserted having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of sp. gravity 0·7 in an inverted V-tube which gives a reading of 260 mm. The loss of head between the main and throat is 0·3 times the kinetic head of the pipe.



21. Ans. 0.0222 m^3/s

Orifice meter

22. An orifice meter, having an orifice of diameter d is fitted in a pipe of diameter D. For this orifice meter, what is the coefficient of discharge C_d? **[IES-2007]**
(a) A function of Reynolds number only (b) A function of d/D only
(c) A function of d/D and Reynolds number
(d) Independent of d/D and Reynolds number

23. If a fluid jet discharging from a 50 mm diameter orifice has a 40 mm diameter at its vena contracta, then its coefficient of contraction will be
(a) 0.32 (b) 0.64 (c) 0.96 (d) 1.64 **[IAS-1996]**

24. What is the percentage error in the estimation of the discharge due to an error of 2% in the measurement of the reading of a differential manometer connected to an orifice meter?
(a) 4 (b) 3 (c) 2 (d) 1 **[IAS-2004]**

25. A tank containing water ha two orifices of the same size at depths of 40 cm and 90 cm below the free surface of water. The ratio of discharges through these orifices is:
[a]. 1 : 1 [b]. 2: 3 [c]. 4: 9 [d]. 16: 81 **[IES-2000]**

26. How is the velocity coefficient C_V , the discharge coefficient C_d, and the contraction coefficient C_c of an orifice related? **[IES-2006]**
(a) $C_V = C_cC_d$ (b) $C_c = C_VC_d$ (c) $C_d = C_cC_V$ (d) $C_cC_VC_d = 1$

Pitot tube

27. The velocity of a water stream is being measured by a L-shaped Pilot-tube and the reading is 20 cm.Then what is the approximate value of velocity? **[IES-2007]**
(a) 19.6m/s (b) 2.0 m/s (c) 9.8 m/s (d) 20 cm/s

28. A simple Pitot tube can be used to measure which of the following quantities?
1. Static head 2.Datum head 3.Dynamic head
4.Friction head 5.Total head **[IAS-1994]**
Select the correct answer using the codes given below
Codes: (a) 1,2 and 4 (b) 1,3 and 5 (c) 2,3 and 4 (d) 2,3 and 5

29. Match List I (Measuring Devices) with List II (Measured Parameter) and select the correct answer using the codes given below: **[IES-2004]**

List I	**List II**
A. Pitot tube	1. Flow static pressure
B. Micro-manometer	2. Rate of flow (indirect)
C. Pipe band meter	3. Differential pressure
D. Wall pressure tap	4. Flow stagnation pressure.

Codes:

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
[a].	1	3	2	4	[b].	4	3	2	1
[c].	1	2	3	4	[d].	4	2	3	1

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30. The instrument preferred in the measurement of highly fluctuating velocities in air flow is: **[IES-2003]**
[a]. Pitot-static tube [b]. Propeller type anemometer
[c]. Three cup anemometer [d]. Hot wire anemometer.

31. An instrument which offers no obstruction to the flow, offers no additional loss and is suitable for flow rate measurement is **[IAS-1997]**
(a) Venturimeter (b) Rotameter (c) Magnetic flow meter (d) Bend meter

32. The following instruments are used in the measurement of discharge through a pipe:
1. Orifice meter 2. Flow nozzle 3. Venturimeter **[IAS-1996]**
(a) 1, 3, 2 (b) 1, 2, 3 (c) 3, 2, 1 (d) 2, 3, 1

33. Match List I with List II and select the correct answer:

List I	List II
A. Orifice meter	1. Measurement of flow in a channel
B. Broad crested weir	2. Measurement of velocity in a pipe/ channel
C. Pitot tube	3. Measurement of flow in a pipe of any inclination
D. Rotameter	4. Measurement of upward flow in a vertical pipe

	A	B	C	D		A	B	C	D
(a)	3	1	4	2	(b)	1	3	2	4
(c)	3	1	2	4	(d)	1	3	4	2

[IAS-2000]

34. Assertion (A): In a rotameter the fluid flows from the bottom of the conical rotameter tube with divergence in the upward direction and the position of the metering float indicated the discharge. **[IAS-1996]**

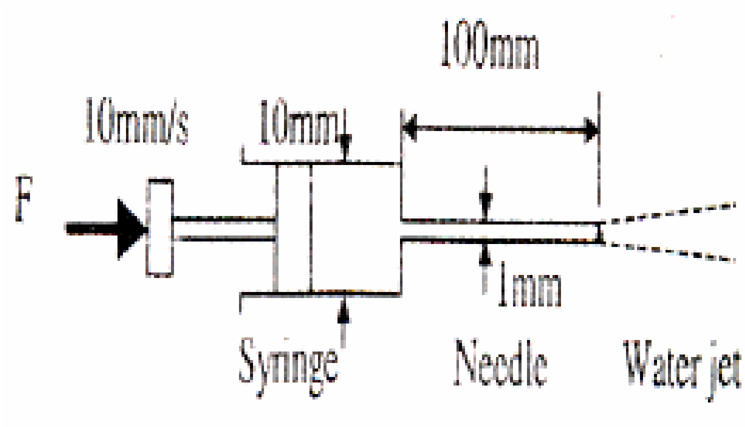
Reason (R): Rotameter float indicates the discharge in terms of its rotation.

Free liquid jet

35. A liquid jet issues from a nozzle inclined at an angle of 60^o to the horizontal and is directed upwards. If the velocity of the jet at the nozzle is 18m/s, what shall approximately be the maximum vertical distance attained by the jet from the point of exit of the nozzle? **[IAS-2004]**

(a) 4.2 m (b) 12.4 m (c) 14.3m (d) 16.5m

Data for Q. 36-37 are given below. Solve the problems and choose correct answers.
A syringe with a frictionless plunger contains water and has at its end a 100 mm long needle of 1 mm diameter. The internal diameter of the syringe is 10 mm. Water density is 1000 kg/m^3. The plunger is pushed in at 10 mm/s and the water comes out as a jet **[GATE-2003]**
Fig. 8



36. Assuming ideal flow, the force F in Newton required on the plunger to push out the water is

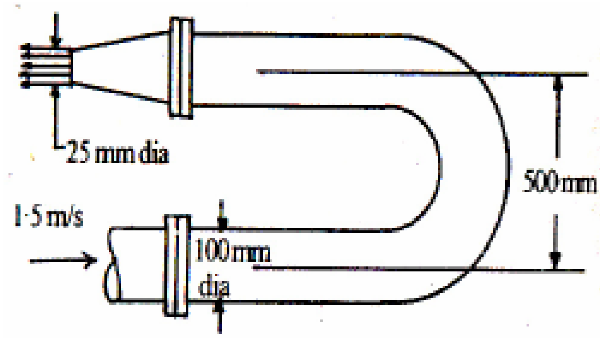
(a) 0 (b) 0.04 (c) 0.13 (d) 1.15 **[GATE-2003]**

38. A constant-head water tank has, on one of its vertical sides tow identical small orifices issuing two horizontal jets in the same vertical plane. The vertical distance between the centres of orifices is 1.5 m and the jet trajectories intersect at a point 0.5 m below the lower orifice. What is the approximate height of water level in the tank above the point o intersection of trajectories? **[IES-2004]**

[a]. 1.0 m [b]. 2.5 m [c]. 0.5 m [d]. 2.0 m

39. The elbow nozzle assembly shown in the given figure is in a horizontal plane. The velocity of jet issuing from the nozzle is:

[a]. 4 m/s [b]. 16 m/s
[c]. 24 m/s [d]. 30 m/s



[IES-1999]

Impulse momentum equation

40. Which one of the following conditions will linearize the Navier-Stokes equations to make it amenable for analytical solutions? **[IES-2007]**

(a) Low Reynolds number (Re<<1) (b) High Reynolds number (Re>>1)
(b) Low Mach number (M<<1) (d) High Mach number (M>>1)

Forced vortex

41. Assertion (A) : A cylinder, partly filled with a liquid is rotated about its vertical axis. The rise of liquid level at the ends is equal to the fall of liquid level at the axis.

Reason (R) : Rotation creates forced vortex motion. **[IES-1999]**

42. Which combination of the following statements about steady incompressible forced vortex flow is correct? **[GATE-2007]**

P: Shear stress is zero at all points in the flow.
Q: Vorticity is zero at all points in the flow
R: Velocity is directly proportional to the radius from the centre of the vortex. S: Total mechanical energy per unit mass is constant in the entire flow field.

(a) P and Q (b) R and S (c) P and R (d) P and S

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43. An open circular cylinder 1.2 m high is filled with a liquid to its top. The liquid is given a rigid body rotation about the axis of the cylinder and the pressure at the centre line at the bottom surface is found to be 0.6 m of liquid. What is the ratio of Volume of liquid spilled out of the cylinder to the original volume?

(a) 1/4 (b) 3/8 (c) 1/2 (d) ¾ **[IES-2007]**

44. A closed cylinder having a radius R and height H is filled with oil of density ρ. If the cylinder is rotated about its axis at an angular velocity of ω, then thrust at the bottom of the cylinder is **[GATE-2004]**

(a) $\pi R^2 \rho g H$ (b) $\pi R^2 \frac{\rho\omega^2 R^2}{4}$ (c) $\pi R^2(\rho\omega^2 R^2 + \rho g H)$ (d) $\pi R^2\left(\frac{\rho\omega^2 R^2}{4} + \rho g H\right)$

Free vortex

45. In a cylindrical vortex motion about a vertical axis, r_1 and r_2 are the radial distances of two points on the horizontal plane (r_2>r_1).If for a given tangential fluid velocity at r_1,the pressure difference between the points in free vortex is one-half of that when the vortex is a forced one, then what is the value of the ratio (r_2/r_1)?

(a) $\sqrt{3/2}$ (b) $\sqrt{2}$ (c) 3/2 (d) $\sqrt{3}$ **[IES-2007]**

46. An inviscid, irrotational flow field of free vortex motion has a circulation constant . The tangential velocity at any point in the flow field is given by /r, where, r is the redial distance form the centre. At the centre, there is a mathematical singularity which can be physically substituted by a forced vortex. At the interface of the free and force vortex motion (r = r_C), the angular velocity is given by:

[a]. $\Omega/(r_C)^2$ [b]. Ω/r_C [c]. Ωr_C [d]. Ωr_C^2 **[IES-1997]**

Answers with Explanations

1. Ans. (d)
2. Ans. (a)
3. Ans. (b)
4. Ans. True
5. Ans. (a)
6. Ans. (b)
7. Ans. (b)
8. Ans. (B)
9. Ans. (d)
10. Ans. (a) 344.6 kW
(b) -32.57 kN/m^2
11. Ans. (b)
12. Ans. (b)
13. Ans. (d)

14. Ans. (d) Q= $\dfrac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \therefore Q^2 \alpha \Delta h$ or $Q^2 \alpha \Delta \rho$

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15. Ans. (b) Q= $\dfrac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^1}} = \dfrac{C_d A_1 A_2' \sqrt{2g5h}}{\sqrt{A_1^2 - A_2'^2}}$ $A_1=2A_2$ and $A_2=(A_1/2)$

That gives $\dfrac{A_1}{A_2^{'}} = 4$

16. Ans. (a)
17. Ans. (b)
18. Ans. (c)
19. Ans. (d) We know, $A_1V_1=A_1V_2$

$\Rightarrow V_2 = \dfrac{D_1^2}{D_2^2} V_1 = \dfrac{16}{4} V_1$

$\therefore \quad V_2 = 4V_1$

Applying Bernoulli's Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

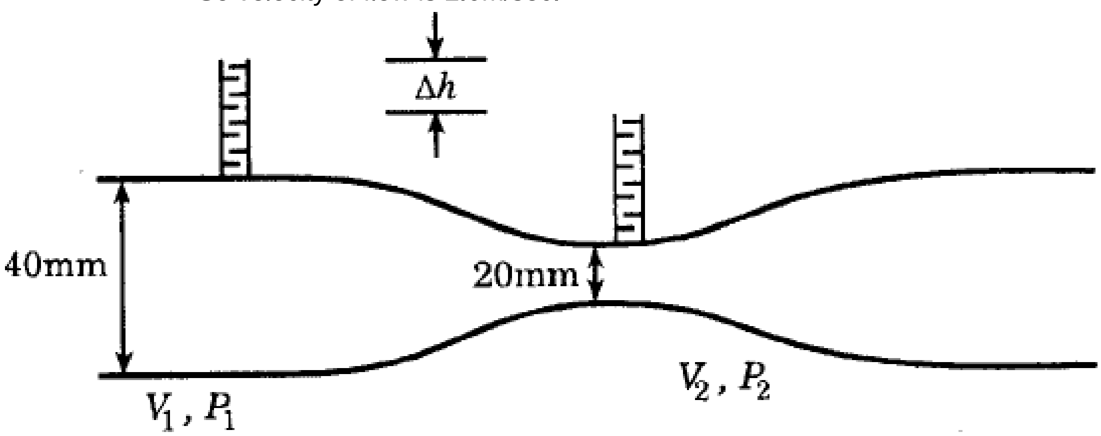
$$\frac{P_1 - P_2}{eg} = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow \frac{15V_1^2}{2} = \frac{30\times10^3}{1000}$$

$\Rightarrow V_1^2 = 4$

$\Rightarrow V_1 = 2.0$m/s

So velocity of flow is 2.0m/sec.



20. Ans. (d)

Applying Bernoulli's equation at points (1) and (2), we have

$$\frac{P_1}{\rho g} + \frac{\upsilon_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\upsilon_2^2}{2g} + z_2$$

Since venturi is horizontal $z_1 = z_2$

Now $\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) = \frac{\upsilon_2^2}{2g} - \frac{\upsilon_1^2}{2g}$

$\Rightarrow$ $(P_1 - P_2) = \frac{\rho g}{2g}(\upsilon_2^2 - \upsilon_1^2) = \frac{\rho}{2}(\upsilon_2^2 - \upsilon_1^2)$

Since $P_2 = P_a$ = atmospheric pressure

$\therefore$ $(P_1 - P_a) = \frac{\rho}{2}(\upsilon_2^2 - \upsilon_1^2)$ ------- (i)

Applying continuity equation at points (i) and (ii), we have

$$A_1\upsilon_1 = A_2\upsilon_2$$

$\Rightarrow$ $\upsilon_1 = \left(\frac{A_2}{A_1}\right)\upsilon_2$ since $V_2 = U$

$$\upsilon_1 = \left(\frac{\frac{\pi}{4}D^2}{\frac{\pi}{4}D_t^2}\right)U$$

$\Rightarrow$ $\upsilon_1 = \left(\frac{D}{D_t}\right)^2 U$

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From equation (i),. $P_1 - P_a = \frac{\rho}{2}\left[\upsilon^2 - \left(\frac{D}{D_t}\right)^2 U^2\right]$

$= \frac{\rho}{2} U^2 \left[1 - \frac{D^4}{D_t^4}\right]$

At point P Spring force = pressure force due air

$-kx = \frac{\pi}{4} D_s^2 \times \frac{\rho U^4}{2}\left[1 - \frac{D^4}{D_t^4}\right]$

$\Rightarrow$ $x = \frac{\pi}{8}\frac{D_s^2 \rho U^2}{k}\left[1 - \frac{D^4}{D_t^4}\right]$

21. Ans. 0.0222 m^3/s

22. Ans. (b) $C_d = C_c \times \frac{\sqrt{1 - \left(\frac{A_0}{A_1}\right)^2}}{1 - C_c^2 \times \left(\frac{A_0}{A_1}\right)^2}$ or, $C_A = f\left(\frac{A_0}{A_1}\right) = F\left(\frac{d}{D}\right)$

23. Ans. (b)

24. Ans. (d) $Q = \frac{C_d A_2 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} = const. \times \sqrt{h}$

or In Q=In(const.)+$\frac{1}{2} In$ h

or $\frac{dQ}{Q} = \frac{1}{2\frac{dh}{h} = \frac{1}{2} \times 2 = 1.\int.}$

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25. Ans. (b)
26. Ans. (c)

27. Ans. (b) $\frac{V^2}{2g}$ =h or, V= $\sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.2}$ = 1.981 m/s

28. Ans. (b)
29. Ans. (b)
30. Ans. (b)
31. Ans. (d)
32. Ans. (c)
33. Ans. (c)
34. Ans. (c)

35. Ans. (b) H= usin $\theta \times t - \frac{1}{2} g t^2$

$$\frac{dH}{dt} = u\sin\theta - gt \quad \text{or} \quad t = \frac{u\sin\theta}{g}$$

$$\therefore H\max = u\sin\theta \times \frac{u\sin\theta}{g} - \frac{1}{2}g \times \left(\frac{u^2\sin^2\theta}{2g}\right) = \frac{18^2\sin^2 60}{2\times 9.8} = 12.4m$$

36. Ans. (b) ρ_{water} =1000kg/m^3

Velocity at points 1=velocity of plunger=10mm/s=0.01m/s

Applying Bernoulli's equation at points 1 and 2, we have

$$\frac{P_1}{\rho q} + \frac{\upsilon_1^2}{2g} + z_1 = \frac{P_2}{\rho q} + \frac{\upsilon_2^2}{2g} + z_2$$

Since z_1-z_2 and P_2=0

$$\frac{P_1}{\rho q} = \frac{\upsilon_2^2}{2g} - \frac{\upsilon_1^2}{2g}$$

$$P_1 = \frac{\rho}{2}(\upsilon_2^2 - \upsilon_1^2) \qquad \text{----- (i)}$$

Applying continuity equation at points (i) and (ii), we have

$$A_1\upsilon_1 = A_2\upsilon_2$$

$$\Rightarrow \quad \upsilon_2 = \left(\frac{A_1}{A_1}\right)\upsilon_1$$

$$\Rightarrow \quad = \frac{\frac{\pi}{4}\times(0.01)^2}{\frac{\pi}{4}\times(0.001)^2}\upsilon_1$$

$= 100\,\upsilon_1$

$=100\times0.01=1$m/s

Now from equation (i),

$$P_1 = \frac{\rho}{2}[\upsilon_2^2 - \upsilon_1^2]$$

$$= \frac{1000}{2}[(1)^2 - (0.01)^2]$$

= 499.95N/m^2

Force required on plunger=$P_1 \times \upsilon_1$

$$= 499.95 \times \frac{11}{4} \times (0.01)^2 = 0.04\text{N}.$$

38. Ans. (b)

39. Ans. (c)

40. Ans. (a)

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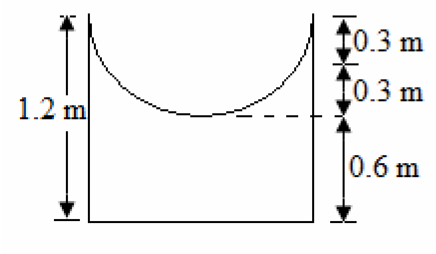


41. Ans. (b)
42. Ans. (B)
43. Ans. (a) $\frac{Volume\ of\ paraboloid}{Total\ volume} = \frac{(1/2)\times A\times 0.6}{A\times 1.2} = 1/4$

44. Ans. (d) We know that

$$\frac{\partial P}{\partial r} = \frac{\rho \upsilon^2}{r} = \frac{\rho.\omega^2 r}{r} = \rho\omega^2 r. \ [\because \upsilon = \omega\times r]$$

$$\therefore \quad \int_0^p \partial p = \int_0^r \rho\omega^2 r dr \qquad [p=\frac{\rho}{2}\omega^2 r^2]$$

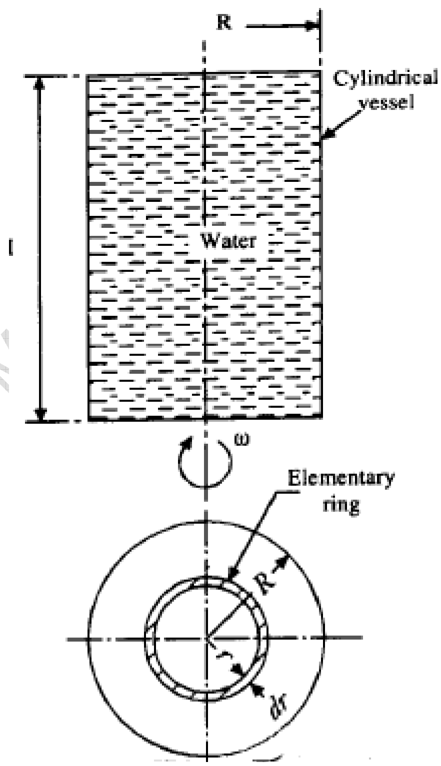
Area of circular ring= $2\pi r dr$
Force on elementary ring
=Intensity of pressure × Area of ring

$$=\frac{\rho}{2}\omega^2 r^2 2\pi r dr$$

$\therefore$ Total force on the top of the cylinder

$$=\int_0^R \frac{\rho}{2}\omega^2 r^2 2\pi r dr \qquad = \frac{\rho}{2}\omega^2 2r\int_0^R r^3 dr$$

$$= \frac{\rho}{2}.\omega^2 2\pi \frac{R^4}{4} = \frac{\rho}{4}\omega^2\times\pi R^4$$



Thrust at the bottom of the cylinder
=Weight of water in cylinder+ Total force on the top of cylinder

$$= \rho g\times\pi R^2\times H + \frac{\rho}{4}\omega^2\times\pi R^4$$

$$= \pi R^2\left[\frac{\rho\omega^2 R^2}{4} + \rho g h\right]$$

45. Ans. (b) For free vortex, $\omega r_1 = const.(k)$

For forced vortex, $V_1 = const.(k) = \frac{c}{r_1}\ Or\ c = \omega r_1^2$

$$(\Delta P)_{forced} = \frac{\rho\omega^2}{2}[r_2^2 - r_1^2], \quad (\Delta P)_{free} = \frac{\rho c^2}{2}\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right] \quad \because c = \omega r_1^2$$

$$2(\Delta P)_{free} = (\Delta P)_{forced} \quad Or \ \ \frac{r_2}{r_1} = \sqrt{2}$$

46. Ans. (a)

DIMENSIONAL AND MODEL ANALYSIS

Skip to Questions (IAS, IES, GATE)

Highlights

1. It may be noted that dimensionally homogeneous equations may not be the correct equation. For example, a term may be missing or a sign may be wrong. On the other hand, a dimensionally incorrect equation must be wrong. The dimensional check is therefore negative check and not a positive check on the correctness of an equation.

2. Rayleigh's Method
This method gives a special form of relationship among the dimensionless group, and has the inherent drawback that it does not provide any information regarding the number of dimensionless groups to be obtained as a result of dimensional analysis. Due to this reason this method has become obsolete and is not favoured for use.

:: Variables outside the 'Show that' are 'least important' and Generally Variables in the numerator are 'most important'

3. The Buckingham's π -theorem states as follows:
"If there are n variables (dependent and independent variables) in a dimensionally homogeneous equation and if these variables contain m fundamental dimensions (such, as; M, L, T, etc.), then the variables are arranged into (n-m) dimensionless terms. These dimensionless terms are called π-terms."

Each dimensionless π-term is formed by combining m variables out of the total n variables, with one of the remaining (n-m) variables i.e. each π-terms contains (m+ 1) variables. These m variables which appear repeatedly in each of π-terms are consequently called repeating variables and are chosen from among the variables such that they together involve all the fundamental dimensions and they themselves do not form a dimensionless parameter.

Selection of repeating variables:
The following points should be kept in view while selecting m repeating variables:

1. m repeating variables must contain jointly all the dimensions involved in the phenomenon. Usually the fundamental dimensions are M, L and T. However, if only two dimensions are involved, there will be 2 repeating variables and they must contain together the two dimensions involved.
2. The repeating variables must not form the non-dimensional parameters among themselves.
3. As far as possible, the dependent variable should not be selected as repeating variable.
4. No two repeating variables should have the same dimensions.
5. The repeating variables should be chosen in such a way that one variable contains geometric property (e.g. length, l; diameter, d; height, H etc.), other variable contains flow property (e.g. velocity, V; acceleration, a etc.) and third variable contains fluid property (e.g. mass density, p; weight density, w, dynamic viscosity, μ etc.).

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Note: Repeating variables are those which are 'least Important' in Rayleigh's Method

Limitations of Dimensional Analysis

Following are the *limitations* of dimensional analysis:

1. Dimensional analysis does not give any clue regarding the selection of variables. If the variables are wrongly taken, the resulting functional relationship is erroneous. It provides the information about the grouping of variables. In order to decide whether selected variables are pertinent or superfluous experiments have to be performed.

2. The complete information is not provided by dimensional analysis; it only indicates that there is some relationship between parameters. It does not give the values of co-efficient in the functional relationship. The values of co-efficient and hence the nature of functions can be obtained only from experiments or from mathematical analysis.

Similitude

In order that results obtained in the model studies represent the behaviour of prototype, the following three similarities must be ensured between the model and the prototype.

1. Geometric similarity;
2. Kinematic similarity, and
3. Dynamic similarity.

Forces

(i) Inertia Force (Fi) = ρAV^2 and A = L^2

(ii) Viscous Force (F_v) = $\mu \frac{V}{L} A$

(iii) Gravity Force (F_g) = ρALg

(iv) Pressure Force (F_p) = PA

(v) Surface Tension Force (F_s) = σA

(vi) Elastic Force (F_e) = KA

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Table : Dimensionless groups/numbers

Sl. No.	Dimensionless numbers	Aspects			
		Symbol	*Group of variables*	*Significance*	*Field of application*
1.	*Reynolds number*	*Re*	$\frac{\rho VL}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$	Laminar viscous flow in confined passages (where *viscous effects are significant*)
2.	*Froude's number*	*Fr*	$\frac{V}{\sqrt{Lg}}$	$\frac{\text{Inertia force}}{\text{Gravity force}}$	Free surface flows (where *gravity effects are important*)
3.	*Euler's number*	*Eu*	$\frac{V}{\sqrt{p/\rho}}$	$\frac{\text{Inertia force}}{\text{Pressure force}}$	Conduit flow (where *pressure variations are significant*)
4.	*Weber's number*	*We*	$\frac{V}{\sqrt{\sigma/\rho L}}$	$\frac{\text{Inertia force}}{\text{Surface tension}}$	Small surface waves, capillary and sheet flow (where *surface tension is important*)
5.	*Mach's number*	*M*	$\frac{V}{\sqrt{K/\rho}}$	$\frac{\text{Inertia force}}{\text{Elastic Force}}$	High speed flow (where *compressibility effects* are *significant*).

(a) Reynolds Model Law

(i) Motion of air planes,
(ii) Flow of incompressible fluid in closed pipes,
(iii) Motion of submarines completely under water, and
(iv) Flow around structures and other bodies immersed completely under moving fluids.

(b) Froude Model Law:

(i) Free surface flows such as flow over spillways, sluices etc.
(ii) Flow of jet from an orifice or nozzle.
(iii) Where waves are likely to be formed on the surface
(iv) Where fluids of different mass densities flow over one another.

$V_r = T_r = \sqrt{L_r}$ And $Q_r = L_r^{2.5}$; $F_r = L_r^3$

(c) Weber Model Law:

Weber model law is applied in the following flow situations:
(i) Flow over weirs involving very low heads;
(ii) Very thin sheet of liquid flowing over a surface;
(iii) Capillary waves in channels;
(iv) Capillary rise in narrow passages;
(v) Capillary movement of water in soil.

(d) Mach Model Law:

The similitude based on Mach model law finds application in the following:
(i) Aerodynamic testing;
(ii) Phenomena involving velocities exceeding the speed of sound;
(iii) Hydraulic model testing for the cases of unsteady flow, especially water hammer problems.
(iv) Under-water testing of torpedoes.

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(e) Euler Model Law:

(i) Enclosed fluid system where the turbulence is fully developed so that viscous forces are negligible and also the forces of gravity and surface tensions are entirely absent;

(ii) Where the phenomenon of cavitation occurs.

Types of Models

1. Undistorted models;
2. Distorted models.

Undistorted models

An undistorted model is one which is geometrically similar to its prototype.

Distorted models

A distorted model is one which is **not** geometrically similar to its prototype. In such a model different scale ratios for the linear dimensions are adopted. For example in case of a wide and shallow river it is not possible to obtain the same horizontal and vertical scale ratios, however, if these ratios are taken to be same then because of the small depth of flow the vertical dimensions of the model will become too less in comparison to its horizontal length. Thus in distorted models the plan form is geometrically similar to that of prototype but the cross-section is distorted.

A distorted model may have the following distortions:

(i) Geometrical distortion.

(ii) Material distortion.

(iii) Distortion of hydraulic quantities.

Typical examples for which distorted models are required to be prepared are:

(i) Rivers,

(ii) Dams across very wide rivers,

(iii) Harbours, and

(iv) Estuaries etc.

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Reasons for adopting distorted models:

The distorted models are adopted for:

- Maintaining accuracy in vertical dimensions;
- Maintaining turbulent flow;
- Accommodating the available facilities (such as money, water supply, space etc.);
- Obtaining suitable roughness condition;
- Obtaining suitable bed material and its adequate movement.

Merits and Demerits of Distorted Models:

Merits:

1. Due to increase in the depth of fluid or height of waves accurate measurements are made possible.
2. The surface tension can be reduced to minimum.
3. Model size can be sufficiently reduced, thereby its operation is simplified and also the cost is lowered considerably.
4. Sufficient tractive force can be developed to move the bed material of the model.
5. The Reynolds number of flow in a model can be increased that will yield better results.

Demerits:

1. The pressure and velocity distributions are not truly reproduced. .
2. A model wave may differ in type and possibly in action from that of the prototype.
3. Slopes of river bends, earth cuts and dikes cannot be truly reproduced.
4. It is difficult to extrapolate and interpolate results obtained from distorted models.
5. The observer experiences an unfavorable psychological effect.

Scale Effect in Models

By model testing it is not possible to predict the exact behaviour of the prototype. The behaviour of the prototype as predicted by two models with different scale ratios is generally not the same. Such a discrepancy or difference in the prediction of behaviour of the prototype is termed as "scale effect". The magnitude of the scale effect is affected by the type of the problem and the scale ratio used for the performance of experiments on models. The scale effect can be positive and negative and when applied to the results accordingly, the corrected results then hold good for prototype.
Since it is impossible to have complete similitude satisfying all the requirements, therefore, the discrepancy due to scale effect creeps in. During investigation of models only two or three forces which are predominant are considered and the effect of the rest of the forces which are not significant is neglected. These forces which are not so important cause small but varying effect on the model depending upon the scale of the model, due to which scale effect creeps in. Sometimes the imperfect simulation in different models causes the discrepancy due to scale effect.

In ship models both viscous and gravity forces have to be considered, however it is not possible to satisfy Reynolds and Froude's numbers simultaneously. Usually the models are tested satisfying only Froude's law, then the results so obtained is corrected by applying the scale effect due to viscosity.

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In the models of weirs and orifices with very small scale ratio the scale effect is due to surface tension forces. The surface tension forces which are insignificant in prototype become quite important in small scale models with head less than 15 mm.
Scale effect can be known by testing a number of models using different scale ratios, and the exact behaviour of the prototype can then be predicted.

Limitations of Hydraulic Similitude

Model investigation, although very important and valuable, may not provide ready solution to all problems. It has the following limitations:

1. The model results, in general, are qualitative but not quantitative.
2. As compared to the cost of analytical work, models are usually expensive.
3. Transferring results to the prototype requires some judgment (the scale effect should be allowed for).
4. The selection of size of a model is a matter of experience.

Question (IES, IAS and GATE)

Dimensions

1. The dimensionless group formed by wavelength , density of fluid , acceleration due to gravity g and surface tension , is: **[IES-2000]**

[a]. □ /□^{2} g □ [b]. □ /□ g^2 □ [c]. □ g /□^{2} □ [d]. □ /□^{2} g □

2. Match List I (Fluid parameters) with List II (Basic dimensions) and select the correct answer: **[IES-2002]**

	List I		**List II**
A.	Dynamic viscosity	1.	M / t^2
B.	Chezy's roughness coefficient	2.	$M / L t^2$
C.	Bulk moduls of elasticity	3.	$M / L t$
D.	Surface tension ( )	4.	$\sqrt{L}$ /t

Codes:

	A	B	C	D		A	B	C	D
[a].	3	2	4	1	[b].	1	4	2	3
[c].	3	4	2	1	[d].	1	2	4	3

3. In M-L-T system. What is the dimension of specific speed for a rotodynamic pump?

(a) $L^{\frac{-3}{4}}T^{\frac{3}{2}}$ (b) $M^{\frac{1}{2}}L^{\frac{1}{4}}T^{-\frac{5}{2}}$ (c) $L^{\frac{3}{4}}T^{-\frac{3}{2}}$ (d) $L^{\frac{3}{4}}T^{\frac{3}{2}}$ **[IES-2006]**

Rayleigh's method

4. Given power 'P' of a pump, the head 'H' and the discharge 'Q' and the specific weight 'w' of the liquid, dimensional analysis would lead to the result that 'P' is proportional to

[a]. $H^{1/2} Q^2 W$ [b]. $H^{1/2} Q W$ [c]. $H Q^{1/2} W$ [d]. HQW **[IES-1998]**

5. Volumetric flow rate Q, acceleration due to gravity g and head H form a dimensionless group, which is given by: **[IES-2002]**

[a]. $\frac{\sqrt{gH^5}}{Q}$ [b]. $\frac{Q}{\sqrt{gH}}$ [c]. $\frac{Q}{\sqrt{g^3}\,H}$ [d]. $\frac{Q}{\sqrt{g^2}\,H}$

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Buckingham's π -method/theorem

6. If the number of fundamental dimensions equals 'm', then the repeating variables shall be equal to: **[IES-1999, IES 1998, GATE-2002]**

[a]. m and none of the repeating variables shall represent the dependent variable.
[b]. m + 1 and one of the repeating variables shall represent the dependent variable
[c]. m + 1 and none of the repeating variables shall represent the dependent variable.
[d]. m and one of the repeating variables shall represent the dependent variable.

7. In a fluid machine, the relevant parameters are volume flow rate, density, viscosity, bulk modulus, pressure difference, power consumption, rotational speed and characteristic dimension. Using the Buckingham pi (π) theorem, what would be the number of independent non-dimensional groups? **[IES-2007]**

(a) 3 (b) 4 (c) 5 (d) None of the above

8. The variable controlling the motion of a floating vessel through water are the drag force F, the speed v, the length l, the density . dynamic viscosity µ of water and gravitational constant g. If the non-dimensional group are Reynolds number (Re), Weber number (We), Parandtl number (Pr) and Froude number (Fr), the expression for F is given by: **[IES-1997]**

[a]. $\frac{F}{\rho\, v^2/l^2} = f(Re)$ [b]. $\frac{F}{\rho\, v^2/l^2} = f(Re, Pr)$ [c]. $\frac{F}{\rho\, v^2/l^2} = f(Re, We)$ [d]. $\frac{F}{\rho\, v^2/l^2} = f(Re, Fr)$

9. Consider the following statements: **[IES-2003]**

1. Dimensional analysis is used to determine the number of variables involved in a certain phenomenon
2. The group of repeating variables in dimensional analysis should include all the fundamental units.
3. Buckingham's theorem stipulates the number of dimensionless groups for a given phenomenon.
4. The coefficient in Chezy's equation has no dimension.

Which of these are correct?

[a]. 1, 2, 3 and 4 [b]. 2, 3 and 4 [c]. 1 and 4 [d]. 2 and 3

Similitude

10. The drag force D on a certain object in a certain flow is a function of the coefficient of viscosity μ, the flow speed v and the body dimension L(for geometrically similar objects); then D is proportional to

(a) L μ V (b) $\frac{\mu^2 V^2}{L^2}$ (c) $\mu^2 v^2 L^2$ (d) $\frac{\mu L}{V}$ **[IAS-2001]**

11. For a 1: m scale model of a hydraulic turbine, the specific speed of the model N_{sm} is related to the prototype specific speed N_{sp} as

(a) $N_{sm}=N_{sp}/m$ (b) $N_{sm}=mN_{sp}$
(c) $N_{sm}=(N_{sp})^{1/m}$ (d) $N_{sm}=N_{sp}$ **[IAS-1997]**

Froude number *(Fr)*

12. The square root of the ratio of inertia force to gravity force is called **[IAS-2003]**
(a) Reynolds number (b) Froude number (c) Mach number (d) Euler number

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Euler number *(Eu)*

13. Euler number is defined as the ratio of inertia force to: **[IES-1997]**
[a]. viscous force [b]. elastic force [c]. pressure force [d]. gravity force.

Mach number (M)

14. An aeroplane is cruising at a speed of 800 kmph at altitude, where the air temperature is 0^0 C. The flight Mach number at this speed is nearly **[GATE-1999]**
(a) 1.5 (b) 0.254 (c) 0.67 (d) 2.04

15. Match List I (Dimensionless numbers) with List II (Definition as the ratio of) and select the correct answer : **[IES-2001]**

List I		List II	
A.	Reynolds number	1.	Inertial force and elastic force
B.	Froude number	2.	Inertia force and surface tension force
C.	Weber number	3.	Inertia force and gravity force.
D.	Mach number	4.	Inertia force and viscous force.

Codes :

	A	B	C	D		A	B	C	D
[a].	1	2	3	4	[b].	4	3	2	1
[c].	1	3	2	4	[d].	4	2	3	1

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16. It is observed in a flow problem that pressure, inertia and gravity forces are important. Then, similarly requires that **[IES-2006]**
(a) Reynolds and Weber numbers be equal
(b) Mach and Froude numbers be equal
(c) Euler and Froude numbers be equal
(d) Reynolds and Mach numbers be equal

17. Match List I (Flow/Wave) with List II (Dimensionless Number) and select the correct answer: **[IES-2003]**

	List I		**List II**
A.	Capillary waves in channel	1.	Reynolds number
B.	Testing of aerofoil	2.	Froude number
C.	Flow around bridge piers.	3.	Weber number
D.	Turbulent flow through pipes.	4.	Euler number
		5.	Mach number

Codes:

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
[a].	5	4	3	2	[b].	3	5	4	1
[c].	5	4	2	1	[d].	3	5	2	1

Model (or Similarity) Laws

18. Consider the following statements: **[IES-2005]**
1. For achieving dynamic similarity in model studies on ships, Froude numbers are equated.
2. Reynolds number should be equated for studies on aerofoil for dynamic similarity.
3. In model studies on a spillway, the ratio of width to height is equated for kinematic similarity.

What of the statements given above are correct?
[a]. 1, 2 and 3 [b]. 1 and 2 [c]. 2 and 3 [d]. 1 and 3

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Reynolds Model Law

19. Assertion (A): Reynolds number must be same for the model and prototype immersed in subsonic flows. **[IES-2003]**
Reason (R): Equality of Reynolds number for the model and prototype satisfies the dynamic similarity criteria.

20. A model test is to be conducted in a water tunnel using a 1: 20 model of a submarine, which is to travel at a speed of 12 km/h deep under sea surface. The water temperature in the tunnel is maintained, so that is kinematic viscosity is half that of sea water. At what speed is the model test to be conducted to produce useful data for the prototype? **[IES-2002]**
[a]. 12 km/h [b]. 240 km/h [c]. 24 km/h [d]. 120 km/h

Froude Model Law

21. A $\frac{1}{25}$ model of a ship is to be tested for estimating the wave drag. If the speed of the ship is 1 m/s, then the speed at which the model must be tested is **[IAS-2002]**
(a) 0.04 m/s (b) 0.2 m/s (c) 5.0 m/s (d) 25.0 m/s

22. A 1: 20 model of a spillway dissipates 0.25 hp. The corresponding prototype horsepower dissipated will be: **[IES-1998]**
[a]. 0.25 [b]. 5.00 [c]. 447.20 [d]. 8944.30

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23. A ship with hull length of 100 m is to run with a speed of 10 m/s. For dynamic similarity, the velocity for a 1: 25 model of the ship in a towing tank should be :

[a]. 2 m/s [b]. 10 m/s [c]. 20 m/s [d]. 25 m/s **[IES-2001]**

24. A ship's model, with scale 1: 100, has a wave resistance of 10 N at its design speed. What is the corresponding prototype wave resistance in kN? **[IES-2007]**

(a) 100 (b) 1000 (c) 10000

(d) Cannot be determined because of insufficient data

25. A model test is to be conducted for an under water structure which each likely to be exposed for an under water structure, which is likely to be exposed to strong water currents. The significant forces are known to the dependent on structure geometry, fluid velocity, fluid density and viscosity, fluid depth and acceleration due to gravity. Choose from the codes given below, which of the following numbers must match for the model with that of the prototype: **[IES-2002]**

1. Mach number 2. Weber number 3. Froude number 4. Reynolds number.

[a]. 3 alone [b]. 1,2, 3 and 4 [c]. 1 and 2 [d]. 3 and 4

Types of Models (Undistorted models, distorted models)

26. Consider the following statements: **[IES-2003]**

1. Complete similarity between model and prototype envisages geometric and dynamic similarities only.
2. Distorted models are necessary where geometric similarity is not possible due to practical reasons.
3. In testing of model of a ship, the surface tension forces are generally neglected.
4. The scale effect takes care of the effect of dissimilarity between model and prototype.

Which of these statements are correct?

[a]. 1 and 3 [b]. 1, 2 and 4 [c]. 2 and 3 [d] 2 and 4

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Answers with Explanations

1. Ans. (a)
2. Ans. (c)
3. Ans. (c)
4. Ans. (d)
5. Ans. (a)
6. Ans. (c)
7. Ans. (c) No of variable=8

no of independent dimension(m)=3

$\therefore$ no of π term= n-m=8-3=5

8. Ans. (d)
9. Ans. (d) 1 and 4 are wrong, coefficient in Chezy's equation has dimension $[L^{1/2}T^{-1}]$
10. Ans. (a)
11. Ans. (d)
12. Ans. (b)
13. Ans. (c)

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14. Ans. (c)
15. Ans. (b)
16. Ans. (c)
17. Ans. (d)
18. Ans. (d) Mach number should be equated for studies on aerofoil for dynamic similarity.
19. Ans. (b)
20. Ans. (d) Apply Reynolds Model law.

21. Ans. (b) Apply Froude Model law $(F_r)_m=(F_r)_p$ or $\frac{V_m}{\sqrt{gL_m}}=\frac{V_p}{\sqrt{g.L_p}}$

or $\frac{V_m}{V_p}=\sqrt{\frac{L_m}{L_p}}=\sqrt{\frac{1}{25}}=\frac{1}{5}$ or $V_m=\frac{1}{5}=0.2$ m/s.

22. Ans. (d) $P_r=L_r^{3.5}=20^{3.5}$ Therefore $P_p=0.25\times 20^{3.5}=8944$ hp

23. Ans. (a) Use $V_r=\sqrt{L_r}$

24. Ans. (c) We know that $F_r=L_r^3$

or, $\frac{F_p}{F_m}=\left(\frac{L_p}{L_m}\right)^3$ *or* $F_p=F_m\times\left(\frac{L_p}{L_m}\right)^3$ = $10\times(100)^3$ N =10000kN

25. Ans. (d)

26. Ans. (c)1 is wrong. Complete similarity between model and prototype envisages geometric, kinematic and dynamic similarities only.

4 is also wrong. The scale effect takes care of the effect of ~~dissimilarity~~ (Size difference) between model and prototype.

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Boundary Layer Theory

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. When a viscous fluid (Real fluid) flows past an immersed body, a thin boundary layer is formed in the immediate neighborhood of solid surface. In the boundary layer the velocity gradient $\left(\frac{\partial u}{\partial y}\right)$ is very high.

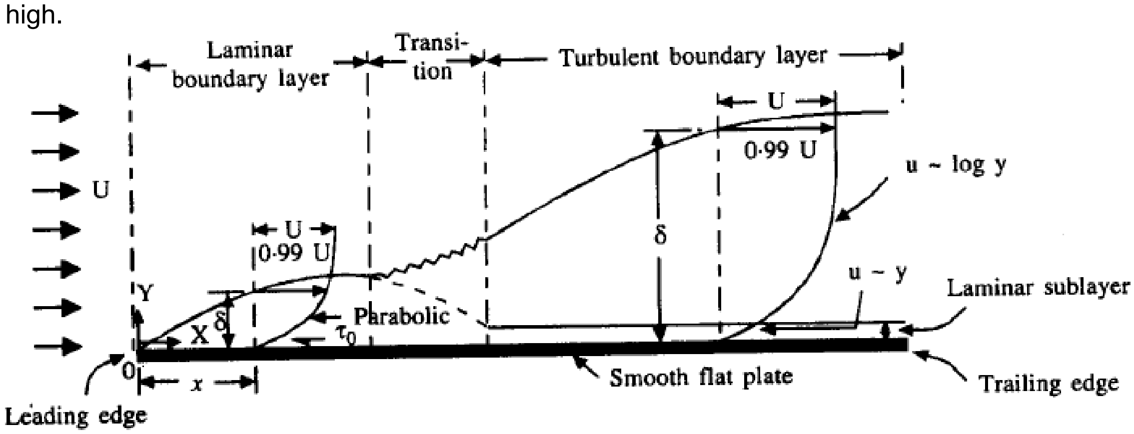

Fig. Boundary layer on a flat plate

2. The resistance due to viscosity is confined only in the boundary layer. The fluid outside the boundary layer may be considered as ideal.

3. Near the leading edge of a flat plate, the boundary layer is wholly laminar. For a boundary layer velocity distribution is parabolic. The thickness of the boundary layer (δ) increases with distance the leading edge, as more and more fluid is slowed down by the viscous boundary, becomes u and breaks into turbulent boundary layer over a transition region.

4. For a turbulent boundary layer, if the boundary is smooth, the roughness projections are covered a very thin layer which remains laminar, called laminar sublayer. The velocity distribution in turbulent boundary layer is given by Log law or Prandtl's one-seventh power law.

As compared to laminar boundary layers, the turbulent boundary layers are thicker.

5. For a flow, when **Re = $\frac{Ux}{\nu}$ < 5 x 10^5 ... boundary layer is laminar, and**

When **Re = $\frac{Ux}{\nu}$ > 5 x 10^5 ... boundary layer is called turbulent.**

Where U = free stream velocity, x = distance from the leading edge, and ν = kinematic viscosity of fluid.

6. The thickness of the boundary layer is arbitrarily defined as that distance from the boundary in which the velocity reaches 99 percent of the velocity of the free stream. It is denoted by the symbol δ

7. Displacement thickness, $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

It is the distance, measured perpendicular to the boundary, by which the main/free stream is displaced on account of formation of boundary layer.

or

It is an additional "wall thickness" that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation

8. Momentum thickness, $\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$

Momentum thickness is defined as the distance through which the total loss of momentum per second is equal to if it were passing a stationary plate.

9. Energy thickness, $\delta_e = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right)dy$

Energy thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in K.E. of the flowing fluid on account of boundary layer formation.

10. Shape Factor = $\frac{\text{Displacement thickness}}{\text{Momentum thickness}} = \frac{\delta^*}{\theta}$

For linear distribution Shape Factor = 3.0

11. Von Kaman momentum integral equation is given as

$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx}$ Where $\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$, and τ_o = shear stress at surface.

This equation is applicable to laminar, transition and turbulent boundary layer flows.

12. Total drag on the plate of length L one side, $F_D = \int_0^L \tau_o B dx$

Local co-efficient of drag or co-efficient of skin friction, $C_D^* \ Or \ f = \frac{\tau_0}{\frac{1}{2}\rho U^2}$

Average co-efficient of drag, $C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$

13. As per Blasius results: (Re < 3×10^5)

The thickness of laminar boundary layer, $\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$ **[VIMP]**

(Where Re_x= Reynolds number)

Average co-efficient of drag, $C_D = \frac{1.328}{\sqrt{\mathrm{Re}_x}}$

Local co-efficient of drag or co-efficient of skin friction, $C_D^* \ Or \ f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$

14. For turbulent boundary layer, the velocity profile is given as: $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$

This equation is not valid very near the boundary where laminar sub-layer exists.

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15. For turbulent boundary layer over a flat plate, the shear stress at the boundary is given as

$$\tau_o = 0.0225 \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$$

16. In case of a turbulent boundary layer:

For $5 \times 10^5 < Re < 10^7$

$$\delta = \frac{0.371x}{(\mathrm{Re}_x)^{1/5}}, \qquad C_D = \frac{0.072}{(\mathrm{Re}_L)^{1/5}}, \quad \text{and } \tau_o = \frac{\rho U^2}{2} \times \frac{0.0576}{(\mathrm{Re}_x)^{1/5}}$$

For $10^7 < Re < 10^9$

$$C_D = \frac{0.455}{(\log_{10} \mathrm{Re}_L)^{2.58}}$$ …… Prandtl- Schlichting empirical equation

17. Total drag on a flat plate due to laminar and turbulent layers:

$$F_{total} = \left[\frac{0.455}{(\log_{10} \mathrm{Re}_L)^{2.58}} - \frac{1700}{\mathrm{Re}_x}\right] \frac{LB\rho U^2}{2}$$

Average co-efficient of drag, $C_D = \left[\frac{0.455}{(\log_{10} \mathrm{Re}_L)^{2.58}} - \frac{1700}{\mathrm{Re}_x}\right]$

18. Compare turbulent boundary layer with laminar boundary layer:

1. Turbulent boundary layers are thicker than laminar boundary layer
2. Velocity in turbulent boundary layers is more uniform
3. In case of a laminar boundary layer, the thickness of the boundary layer increases more rapidly as the distance from the leading edge increases.
4. For the same local Reynolds number. Shear stress at the boundary is less in the case of turbulent boundary layer.

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19. Algorithm for conventional problem

Velocity distribution is given and **calculates**

(i) Boundary layer thickness, (δ)

(ii) Shear stress (τ_o)

(iii) Local co-efficient of drag (C_D^*)

and (iv) Co-efficient of drag (C_D)

Step-I: First calculate θ

Step-II: Use Von Kaman equation $\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx}$ or $\tau_o = \rho U^2 \frac{d\theta}{dx}$ and solve until $\tau_o = something \times \frac{d\delta}{dx}$ come.

Step-III: (a) For laminar boundary layer use Newton's Law of viscosity

$\tau_o = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$ and solve until $\tau_0 = a\ function\ of\ \delta$ comes.

(b) For turbulent boundary layer use $\tau_o = 0.0225 \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$

Step-IV: Equate above τ_o from Step-II and Step-III and find δ is a function of $\dfrac{something \times x}{(\mathrm{Re}_x)^{something}}$

Step-V: Put this value of δ in the τ_o which is calculated in Step-III and rearrange it to a function of Re_x

Step-VI: Calculate, $C_D^* = \dfrac{\tau_o}{\frac{1}{2}\rho U^2}$

Step-VII: Calculate, $C_D = \dfrac{1}{L}\int_0^L C_D^* dx$

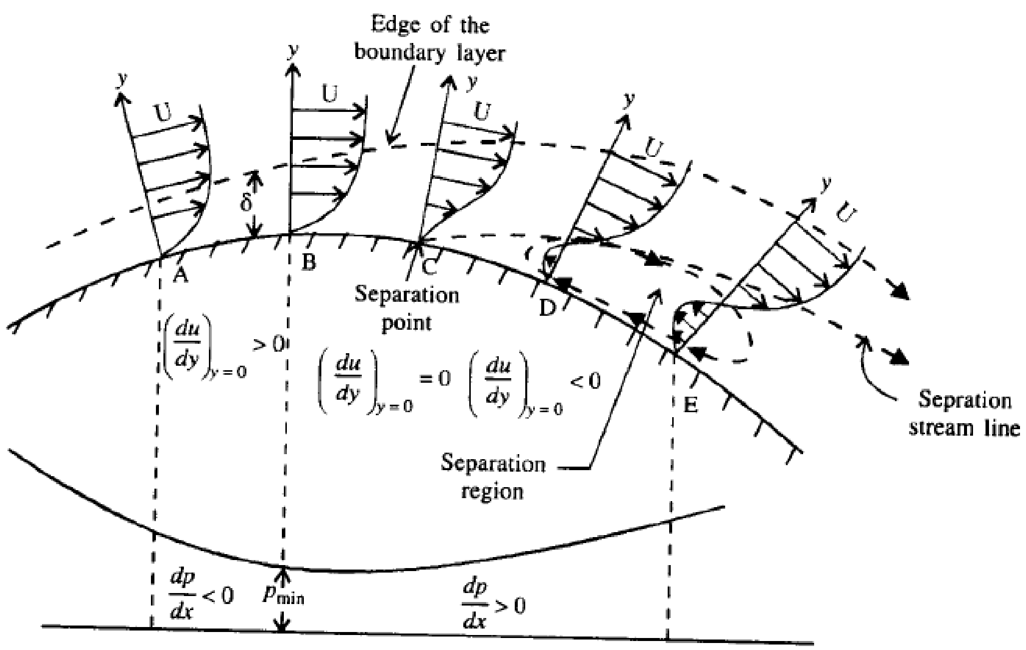
20. Boundary Layer Separation

The velocity gradient, for a given velocity profile, exhibits the following characteristic for the flow to remain attached, get detached or be on the verge of separation.

(i) $\left(\dfrac{\partial u}{\partial y}\right)_{y=0}$ $is + ive$ ….. Attached flow (The flow will not separate)

(ii) $\left(\dfrac{\partial u}{\partial y}\right)_{y=0}$ $is\ zero$ …… The flow is on the verge of separation

(iii) $\left(\dfrac{\partial u}{\partial y}\right)_{y=0}$ $is - ive$ …… Separated flow.



Separation of boundary layer

21. Methods of preventing the separation of boundary Layer:

Following are some of the methods generally adopted to retard separation:

1. Streamlining the body shape.

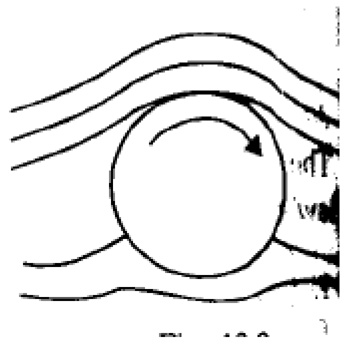
2. Tripping the boundary layer from laminar to turbulent by provision roughness.
3. Sucking the retarded flow.
4. Injecting high velocity fluid in the boundary layer.
5. Providing slots near the leading edge.
6. Guidance of flow in a confined passage.
7. Providing a rotating cylinder near the leading edge.
8. Energizing the flow by introducing optimum amount of swirl in the incoming flow.

22. Describe with sketches the methods to control separation. (U.P.S.C 1997)

Methods to control separation:

1. Motion of solid boundary:

By rotating a circular cylinder lying in a stream of fluid, so that the upper side of cylinder where the fluid as well as the cylinder moves in the same direction, the boundary layer does not form. However on the lower side of cylinder where the fluid motion is opposite to that of cylinder separation would occur.



2. Acceleration of fluid in the boundary layer:

This method of controlling separation consists of supplying additional energy to particles of fluid which are being retarded in the boundary layer. This may be achieved either by injecting the fluid into the region of boundary layer from the interior of the body with the help of some available device as shown in Fig.-1 or by diverting a portion of fluid of the main stream from the region of high pressure to the retarded region of boundary layer through, a slot provided in the body. (Fig-2)

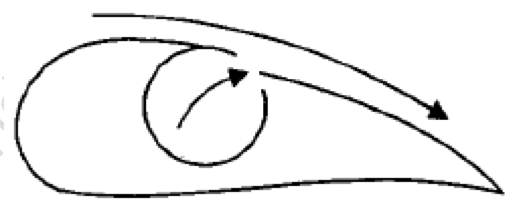


Fig-1:Injecting fluid into boundary layer

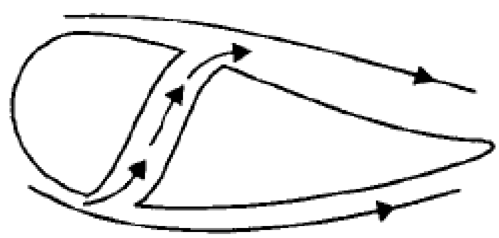
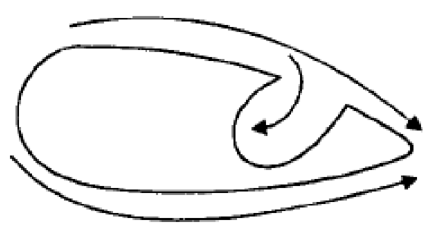


Fig-2: Slotting wing

3. Suction of fluid from the boundary layer:

In this method, the slow moving fluid in the boundary layer is removed by suction through slots or through a porous surface as shown in the Fig.

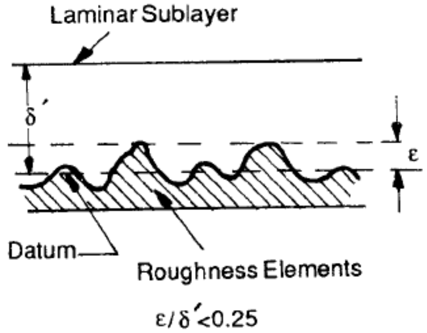


4. Streamlining of body shapes: By the use of suitably shaped bodies the point of transition of the boundary layer from laminar to turbulent can be moved downstream which results in the reduction of the skin friction drag. Furthermore by streamlining of body shapes, the separation may be eliminated.

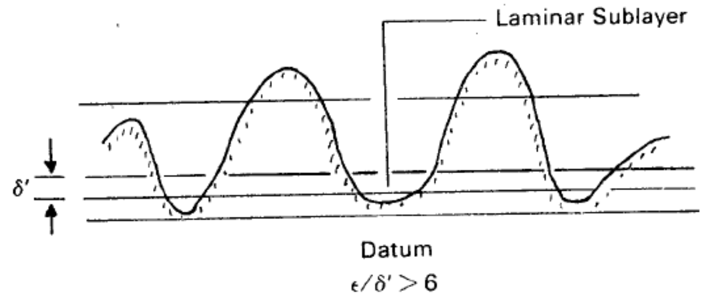
23. Laminar sublayer: The laminar sublayer is usually very thin and its thickness δ' is found by experiments to be $\delta' = 11.6\frac{\nu}{u_*}$

Where $u_* = \sqrt{\tau_o/\rho}$ =shear velocity.

If the roughness magnitude of a surface e is very small compared to δ', i.e. $\varepsilon << \delta$', then such a surface is said to be hydrodynamically smooth. Roughness does not have any influence in such flows while the viscous effects predominate. Usually ε / δ'< 0.25 is taken as the criterion for hydrodynamically smooth surface (Fig.)



(a) Smooth Boundary



(b) Rough Boundary

In the laminar sublayer thickness δ' is very small compared to roughness height ε, i.e. $\varepsilon << \delta$'), in such flows viscous effects are not important and the boundary is said to be hydrodynamically rough. Usually ε / δ'> 6 is taken as the criterion for hydrodynamically rough boundaries.

In the region 0.25 < ε / δ'< 6, the boundary is in the transition regime and both viscosity and roughness control the flow.

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Questions (IAS, IES, GATE)

Boundary layer Definitions and Characteristics

1. In the boundary layer, the flow is **[IES-2006]**
(a) Viscous and rotational (b) Inviscid and irrotational
(c) Inviscid and rotational (d) Viscous and irrotational

2. The critical value of Reynolds number for transition from laminar to turbulent boundary layer in external flows is taken as: **[IES-2002]**
[a]. 2300 [b]. 4000 [c]. 5×10^5 [d]. 3×10^6

3. The development of boundary layer zones labeled P, Q, R and S over a flat plate is shown in the given figure.
Based on this figure, match List I (Boundary layer zones) with List II (Type of boundary layer) and select the correct answer :

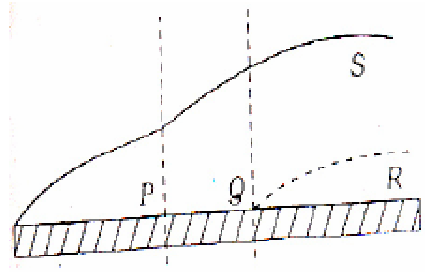

Figure. [IES-2000]

List I		**List II**
A.	P	1. Transitional
B.	Q	2. Laminar viscous sub-layer
C.	R	3. Laminar
D.	S	4. Turbulent

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
[a].	3	1	2	4	[b].	3	2	1	4
[c].	4	2	1	3	[d].	4	1	2	3

4. Velocity defect in boundary layer theory is defined as **[IAS-2003]**
(a) The error in the measurement of velocity at any point in the boundary layer
(b) The difference between the velocity at a point within the boundary layer and the free stream velocity
(c) The difference between the velocity at any point within the boundary layer and the velocity nearer the boundary
(d) The ratio between the velocity at a point in the boundary layer and the free stream velocity

5. (i) Assertion (A): In an ideal fluid, separation from a continuous surface would not occur with a positive pressure gradient. **[IAS-2000]**
Reason (R): Boundary layer does not exist in ideal fluid.

5.(ii) Assertion (A): The thickness of boundary layer cannot be exactly defined.
Reason (R): The Velocity within the boundary layer approaches the inviscid velocity asymptotically. **[IAS-1996]**

Boundary layer thickness (δ)

6. Assertion (A): The thickness of boundary layer is an ever increasing one as its distance from the leading edge of the plate increases. **[IES-1999]**
Reason (R): In practice, 99% of the depth of the boundary layer is attained within a short distance of the leading edge.

7. For the velocity profile u / u_∞ = □, the momentum thickness of a laminar boundary layer on a flat plate at a distance of 1 m from leading edge for air (kinematic viscosity = 2×10^{-5} m^2/s) flowing at a free stream velocity of 2 m/s is given by : **[IES-2001]**
[a]. 3.16 mm [b]. 2.1 mm [c]. 3.16 m [d]. 2.1 m

8. A flat plate, 2m × 0.4m is set parallel to a uniform stream of air (density $1.2 kg/m^3$ and viscosity 16 centistokes) with its shorter edges along the flow. The air velocity is 30 km/h. What is the approximate estimated thickness of boundary layer at the downstream end of the plate? **[IES-2004]**
[a]. 1.96 mm [b]. 4.38 mm [c]. 13.12 mm [d]. 9.51 mm

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Displacement thickness (δ^*)

9. How is the displacement thickness in boundary layer analysis defined? **[IAS-2007]**
(a) The layer in which the loss of energy is maximum
(b) The thickness up to which the velocity approaches 99% of the free stream velocity.
(c) The distance measured perpendicular to the boundary by which the free stream is displaced on account of formation of boundary layer.
(d) The layer which represents reduction in momentum caused by the boundary layer.

10. The displacement thickness at a section, for an air stream (ρ = 1.2 kg/m^3) moving with a velocity of 10 m/s over flat plate is 0.5mm. What is the loss mass rate of flow of air due to boundary layer formation in kg per meter width of plate per second?
(a) 6x10^{-3} (b) 6x10^{-5} (c) 3x10^{-3} (d) 2x10^{-3} **[IAS-2004]**

11. If the velocity distribution in a turbulent boundary layer is given by $\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9}$ then the ratio of displacement thickness to nominal layer thickness will be
(a) 1.0 (b) 0.6 (c) 0.3 (d) 0.1 **[IAS-1998; IES-2006]**

12. The velocity distribution in the boundary over the face of a high spillway found to have the following from:

$$\frac{u}{u_a} = \left(\frac{y}{\delta}\right)^{0.25}$$ **[IAS-1996]**

An a certain section, the free stream velocity u$_\alpha$ was found to be 20m/s and the boundary layer thickness was estimated to be 5cm.The displacement thickness is
(a) 1.0 cm (b) 2.0 cm (c) 4.0 cm (d) 5.0 cm

13. For linear distribution of velocity in the boundary layer on a flat plate, what is the ratio of displacement thickness (□*) to the boundary layer thickness (□)?
[a]. $\frac{1}{4}$ [b]. $\frac{1}{3}$ [c]. $\frac{1}{2}$ [d]. $\frac{1}{5}$ **[IES-2005]**

Momentum thickness (θ)

14. If U∞ = free stream velocity, u = velocity at y and □ = boundary layer thickness, then in a boundary layer flow, the momentum thickness □ is given by:
[a]. $\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$ [b]. $\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right)dy$ **[IES-1997; IAS-2004]**
[c]. $\theta = \int_0^\delta \frac{u^2}{U_\infty^2}\left(1 - \frac{u}{U_\infty}\right)dy$ [d]. $\theta = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right)dy$

15. Given that **[IES-1997]**
δ = boundary layer thickness, δ^* =displacement thickness
δ_e = energy thickness θ = momentum thickness
The shape factor H of a boundary layer is given by
(a) $H = \frac{\delta_e}{\delta}$ (b) $H = \frac{\delta^*}{\theta}$ (c) $H = \frac{\delta}{\theta}$ (d) $H = \frac{\delta}{\delta^*}$

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16. The velocity distribution in the boundary layer is given as u / u_S = y / □, where u is the velocity at a distance y from the boundary, u_S is the free stream velocity and □is the boundary layer thickness at a certain distance from the leading edge of a plate. The ratio of displacement to momentum thicknesses is: **[IES-2001; 2004]**

[a]. 5 [b]. 4 [c]. 3 [d]. 2

Energy thickness (δ e)

17. Which one of the following is the correct relationship between the boundary layer thickness δ, displacement thickness δ^* and the momentum thickness θ?

(a) $\delta > \delta^* > \theta$ (b) $\delta^* > \theta > \delta$ (c) $\theta > \delta > \delta^*$ (d) $\theta > \delta^* > \delta$

[IAS-2004; IES-1999]

Momentum Equation for Boundary Layer by Von-karman

18. For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed respectively, as

$$\frac{U}{U_\alpha} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3; \qquad \delta = \frac{4.64x}{\sqrt{\text{Re}_x}}$$ **[GATE-2004]**

If the free stream velocity is 2 m/s, and air has kinematic viscosity of 1.5×10^{-5}m^2/s and density of 1.23kg/m^3,then wall shear stress at x = 1 m, is

(a) 2.36×10^{2}N/m^2 (b) 43.6×10^{-3}N/m^2 (c) 4.36×10^{-3}N/m^2 (d) 2.18×10^{-3}N/m^2

19. According to Blasius law, the local skin friction coefficient in the boundary-layer over a flat plate is given by: **[IES-2001]**

[a]. $0.332/\sqrt{R_e}$ [b]. $0.664/\sqrt{R_e}$ [c]. $0.647/\sqrt{R_e}$ [d]. $1.328/\sqrt{R_e}$

20. Match List I (Variables in Laminar Boundary layer Flow over a Flat Plate Set Parallel to the Stream) with List II (Related Expression with usual notations) and select the correct answer using the codes given below: **[IES-2004]**

	List I		**List II**
A.	Boundary layer thickness	1.	$1.729/\sqrt{Ux/v}$
B.	Average skin-friction coefficient	2.	$0.332\rho U^2/\sqrt{Ux/v}$
C.	Shear stress at boundary	3.	$5\sqrt{vx/U}$
D.	Displacement thickness.	4.	$0.664\sqrt{v/Ux}$
		5.	$1.328/\sqrt{UL/v}$

Codes:

	A	B	C	D		A	B	C	D
[a].	3	5	4	2	[b].	2	4	1	3
[c].	3	5	2	1	[d].	5	4	1	2

Laminar Boundary Layer

21. The thickness of laminar boundary layer at a distance 'X' from the leading edge over a flat varies as **[IAS-1999; GATE-2002]**

(a) X (b) $X^{\frac{1}{2}}$ (c) $X^{\frac{1}{5}}$ (d) $X^{\frac{4}{5}}$

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22. The laminar boundary layer thickness, □ at any point x for flow over a flat plate is given by: □/ x = **[IES-2002]**

[a]. $\frac{0.664}{\sqrt{Re_x}}$ [b]. $\frac{1.328}{\sqrt{Re_x}}$ [c]. $\frac{1.75}{\sqrt{Re_x}}$ [d]. $\frac{5.0}{\sqrt{Re_x}}$

Turbulent Boundary Layer

23. The velocity profile for turbulent layer over a flat plate is: **[IES-2003]**

[a]. $\frac{u}{U}=\sin\left(\frac{\pi}{2}-\frac{y}{\delta}\right)$ [b]. $\frac{u}{U}=\left(\frac{y}{\delta}\right)^{1/7}$ [c]. $\frac{u}{U}=2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2$ [d]. $\frac{u}{U}=\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$

24. The thickness of turbulent boundary layer at a distance x from the leading edge over a flat plate varies as **[IAS-2003; 2004; 2007; IES-1997; 2000]**

(a) $x^{4/5}$ (b) $x^{1/2}$ (c) $x^{1/5}$ (d) $x^{3/5}$

25. For turbulent boundary layer low, the thickness of laminar sublayer '□' is given by :

[a]. v / u* [b]. 5 v / u* [c]. 575 log v / u* [d]. 2300 v / u* **[IES-1999]**

26. Consider the following statements comparing turbulent boundary layer with laminar boundary layer:

1. Turbulent boundary layers are thicker than laminar boundary layer
2. Velocity in turbulent boundary layers is more uniform
3. In case of a laminar boundary layer, the thickness of the boundary layer increases more rapidly as the distance from the leading edge increases.
4. For the same local Reynolds number. Shear stress at the boundary is less in the case of turbulent boundary layer.

Of these statements:

(a) 1.2.3 and 4 are correct (b) 1 and 3 are correct
(c) 3 and 4 are correct (d) 1 and 2 are correct **[IAS-1997]**

Total Drag Due to Laminar and Turbulent Layers

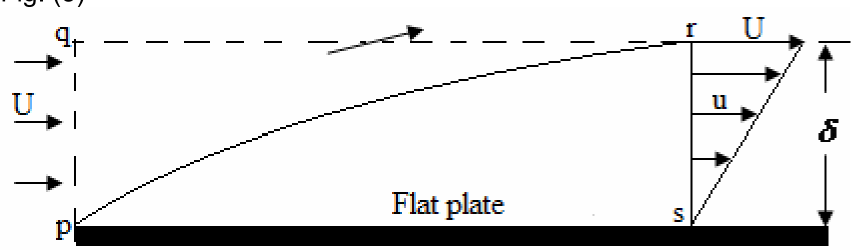
27. Consider an incompressible laminar boundary layer flow over a flat plate of length L, aligned with the direction of an oncoming uniform free stream. If F the ratio of the drag force on the front half of the plate to the drag force on the rear half, then

(a) F<1/2 (b) F = ½ (c) F = 1 (d) F > 1 **[GATE-2007]**

Statement for Linked Answer Questions 28& 29:

A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at U= m/s

Fig. (3)



The thickness of the boundary layer at section r-s is 10 mm, the breadth of the plate is 1 m (into the paper) and the destiny of the gas ρ = 1.0kg/m^3.Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution, μ = U(y/δ),at the section r-s, where y is the height from plate.

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28. The mass flow rate (in kg/s) across the section q-r is
(a) zero (b) 0.05 (c) 0.10 (d) 0.15 **[GATE-2006]**

29. The integrated drag force (in N) on the plate, between p-s, is **[GATE-2006]**
(a) 0.67 (b) 0.33 (c) 0.17 (d) zero

30. In a laminar boundary layer over a flat plate, what would be the ratio of wall shear stresses τ_1 and τ_2 at the two sections which lie at distances x_1=30 cm and x_2=90 cm from the leading edge of the plate? **[IAS-2004]**

(a) $\frac{\tau_1}{\tau_2} = 3.0$ (b) $\frac{\tau_1}{\tau_2} = \frac{1}{3}$ (c) $\frac{\tau_1}{\tau_2} = (3.0)^{1/2}$ (d) $\frac{\tau_1}{\tau_2} = (3.0)^{1/3}$

Boundary Layer Separation and its Control

31. In a boundary layer developed along the flow, the pressure decreases in the downstream direction. The boundary layer thickness would: **[IES-1998]**
[a]. tend to decrease [b]. remain constant [c]. increase rapidly [d].increase gradually.

32. Flow separation is caused by: **[IAS-1996; IES-1997;2000; GATE-2002]**
[a]. reduction of pressure to local vapour pressure [b]. a negative pressure gradient
[c]. a positive pressure gradient [d]. thinning of boundary layer thickness to zero.

33. Flow separation is caused by
(a) thinning of boundary layer thickness to zero (b) a negative pressure gradient
(c) a positive pressure gradient (d) reduction of pressure to local vapour pressure
[IAS-2002]

34. Boundary layer separation takes place when

(a) $\left(\frac{du}{dy}\right)_{y=0}$ = +ve value (b) $\left(\frac{du}{dy}\right)_{y=0}$ = -ve Value

(c) $\left(\frac{du}{dy}\right)_{y=\delta}$ = 0 (d) $\left(\frac{du}{dy}\right)_{y=0}$ =0 **[IAS-2007]**

35. The necessary and sufficient condition which brings about separation of boundary layer is $\frac{dp}{dx} > 0.$
[GATE-1994]

36. Flow separation is likely to take place when the pressure gradient in the direction of flow is
[IAS-1998]
(a) zero (b) adverse (c) slightly favourable (d) strongly favourable

37. Consider the following statements pertaining to boundary layer: **[IES-2003]**
1. Boundary layer is a thin layer adjacent to the boundary where maximum viscous energy dissipation takes place.
2. Boundary layer thickness is a thickness by which the ideal flow is shifted.
3. Separation of boundary layer is caused by presence of adverse pressure gradient.
Which of these statements are correct?
[a]. 1, 2 and 3 [b]. 1 and 2 [c]. 1 and 3 [d]. 2 and 3

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Thermal Boundary Layer

38. For a fluid having Prandtl number equal to unity, how are the hydrodynamic boundary layer thickness δ and the thermal boundary layer thickness δ_t related?

(a) $\delta = \delta_t$ (b) $\delta > \delta_t$ (c) $\delta < \delta_t$ (d) $\delta_t = \delta^{1/3}$

39. Consider a laminar boundary layer over a heated flat plate. The free stream velocity is $U\infty$.At some distance x from the leading edge the velocity boundary layer thickness is δ_t.If the Prandtl number is greater than 1, then

(a) $\delta_v > \delta_T$ (b) $\delta_T > \delta_V$ (c) δ_V **[GATE-2003]**

40. Water (Prandtl number 6) flows over a flat plate which is heated over the entire length. Which one of the following relationship between the hydrodynamic boundary layer thickness (δ) and the thermal boundary layer thickness (δ_t) is true?

(a) $\delta_t > \delta$ (b) $\delta_t < \delta$ (c) $\delta_t = \delta$ (d) Can not be predicted

41. For air near atmosphere conditions flowing over a flat plate, the laminar thermal boundary layer is thicker than the hydrodynamic boundary layer. **[GATE-1994]**

Answers with Explanations

1. Ans. (d)
2. Ans. (c)
3. Ans. (a)
4 Ans. (b)
5 (i) Ans. (a) In Ideal fluid viscosity is zero so no boundary layer is formed.
5.(ii) Ans. (a)
6. Ans. (a)

7. Ans. (b) Thickness of Boundary layer, $\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}} = \frac{5x}{\sqrt{Ux/\nu}} = \frac{5\times 1}{\sqrt{2\times 2/2\times 10^{-5}}}$ = 0.01118 m and

for such velocity distribution $\theta = \frac{\delta}{6} = 1.863\,mm$ nearest ans. (b)

8. Ans. (b) Thickness of Boundary layer, $\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}} = \frac{5L}{\sqrt{\frac{UL}{\nu}}} = \frac{5\times 0.4}{\sqrt{\frac{30\times(5/18)\times 0.4}{16\times 10^{-6}}}}$ = 4.38 mm

9. Ans. (c)

10. Ans. (a) Q (loss per meter)= $\rho\times\delta^*\times$velocity= $1.2\times\left(\frac{0.5}{1000}\right)\times 10$ kg/ms

= 6×10^{-3} kg/ms

11. Ans. (d) displacement thickness (δ^*) = $\delta\int_0^1 (1-z^{1/9})dz = 0.1\delta$

12. Ans. (a) displacement thickness (δ^*) = $\delta\int_0^1 (1-z^{0.25})dz = 0.2\delta = 0.2\times 5 = 1.0cm$

13. Ans. (c) remember it.
14. Ans. (a)
15. Ans. (b)
16. Ans. (c) remember it.
17. Ans. (a) $\delta > \delta^* > \theta > \delta^{**}$

18. Ans. (c) Given: ρ =1.23kg/m^3, v= $\frac{\mu}{\rho} = 1.5\times10^{-5} m^2/s$ u= 2m/s, x=L=1m.

$$R_{ex} = \frac{\rho u L}{\mu} = \frac{2\times1}{\left(\frac{\mu}{\rho}\right)} = \frac{2\times1}{1.5\times10.5} = 1.34\times10^5$$

Now, shear stress, $\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$

Where, $\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3}$ or $\frac{du}{dy} = U\left[\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right]$

Hence $\left(\frac{du}{dy}\right)_{y=0} = \frac{3U}{2\delta}$

Given: $\delta = \frac{4.64x}{\sqrt{\mathrm{Re}_x}} = \frac{4.64\times x}{\sqrt{\frac{\rho U x}{\mu}}}$

Putting x=1, $(\delta)_{x=1} = \frac{4.64}{\sqrt{\frac{2\times1}{1.5\times10^{-5}}}} = 0.0127$

$\therefore$ $\tau_0 = \mu.\frac{du}{dy} = \frac{3}{2}\frac{\mu U}{\delta}$

$$= \frac{3}{2}\times\frac{(1.5\times10^{-5}\times1.23)\times2}{0.0127} = 4.355\times10^{-3}.N/M^2$$

19. Ans. (b)
20. Ans. (c)
21. Ans. (b) $\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}\,x}}$ or $\delta\alpha\frac{5x}{\sqrt{\frac{\rho v x}{\mu}}} or\ \delta\alpha\sqrt{x}$

22. Ans. (d)
23. Ans. (b)
24. Ans. (a) $\frac{\delta}{x} = \frac{0.371}{(\mathrm{Re}\,x)^{1/5}} 0r, \delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}}\times x^{4/5}$

or, $\delta \propto x^{4/5}$

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25. Ans. (b)
26. Ans. (a)

27. Ans. (d) $F_D = some\,Const \times \int_{l_1}^{l_2} x^{-1/2}dx$ Therefore ratio = $\frac{\sqrt{L/2}-0}{L-\sqrt{L/2}} = \frac{1}{\sqrt{2}-1} > 1$

28. Ans. (b) Mass entering from side q-p= Mass leaving from side q-r + Mass leaving the side r-s.
29. Ans. (c) By momentum equation, we can find drag force.

30. Ans. (c) $\tau_o = 0.323\frac{\mu u}{x} \times \sqrt{\mathrm{Re}\,x}$ i.e. $\tau_o \,\alpha\, \frac{1}{\sqrt{x}}$

$$\therefore \frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}} = \sqrt{\frac{90}{30}} = (3)^{1/2}$$

31. Ans. (d)
32. Ans. (c) i.e. an adverse pressure gradient

33. Ans. (c) Separation takes place where $\frac{dp}{dx} > 0$ and $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

34. Ans. (d) but $\frac{\partial p}{\partial x} > 0$

35. Ans. False because Separation takes place where $\frac{dp}{dx} > 0$ and $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

36. Ans. (b)
37. Ans. (c) 2 is wrong it defines displacement thickness.

38.Ans. (a) $\frac{\delta}{\delta_t} = (\mathrm{Pr})^{1/3}$

39. Ans. (a) Prandtl number= $\frac{Molecular\ diffusivity\ of\ mom}{Molecular\ diffusivity\ of\ heat}$

From question, since Prandtl number>1

$\therefore$ Velocity boundary thickness ($\delta\upsilon$) >1 thermal boundary thickness

40. Ans.(b)
41. Ans. False

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Question from conventional paper

To solve the problems below use **"**Algorithm" from **'Highlight'**

1. Explain displacement and momentum boundary layer thickness. Assume that the shear stress varies linearly in a laminar boundary layer such that $\tau = \tau_o\left[1-\frac{y}{\delta}\right]$ **[IES-1998]**

Calculate the displacement and momentum thickness in terms of δ.

2. Derive the integral momentum equation for the boundary layer over a flat plate and determine the boundary layer thickness, δ, at a distance x from the leading edge assuming linear velocity profile (u/U) = y/δ where u is the velocity at the location at a distance y from the plate, and U is the free stream velocity. **[IAS-1998]**

3. When a fluid flows over a flat plate, the velocity profile within the boundary layer may be assumed to be V_x = U $\left[\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]$ for $y \le \delta$ **[IES-1995]**

Where U is a constant and the boundary layer thickness δ is a function of x given by

$\delta = 5\left(\frac{\mu x}{\rho U}\right)^{1/2}$ Here μ and ρ denote the viscosity and density of the fluid respectively. Derive an expression for the variation of V_y across the boundary layer.
i.e. calculate displacement thickness.

4. The velocity profile for laminar flow in the boundary layer of a flat plate is given by
$\frac{u}{U} = \sin\left(\frac{\pi}{2} - \frac{y}{\delta}\right)$ Where u is the velocity of fluid in the boundary layer at a vertical distance y from the plate surface and *U* is the free stream velocity. Prove that the boundary layer thickness δ may be given by the expression, $\delta = \frac{4.795x}{\sqrt{\text{Re}_x}}$ **[IES-1992]**

5. Explain briefly the Boundary Layer Theory as propounded by Prandtl. Obtain an expression for the thickness of the boundary layer for laminar flow assuming the velocity distribution law as

$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ **[IAS-1990]**

Where U= approach velocity of the stream, u= velocity of the stream in the boundary layer at a distance y from the boundary and δ = thickness of the boundary layer.

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LAMINAR FLOW

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. Reynolds number, *Re* < 2000 Laminar flow
   Reynolds number, *Re* > 4000 Turbulent flow

2. In case of laminar flow: The loss of head $\propto$ V, where *V* is the velocity of flow.
   In case of turbulent flow : The loss of head $\propto V^2$ (approx).
   $\propto V^n$ (more exactly), where *n* varies from 1·75 to 2.0

3. The Navier-stokes equation of motion is the general momentum equation for compressible or incompressible, viscous or inviscid flows.
   The Navier-stokes equation must be satisfied for any fluid flow.
   The Navier-stokes equation is a consequence of the law of flow.

4. For a inviscid fluid, $\mu = \nu = 0$, Navier-stokes equation reduces to the Euler's form. The Euler's equation is, therefore, a special case of the Navier-stokes equation.

5. Relationship between shear stress and pressure gradient

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

This equation indicates that the pressure gradient in the direction of flow is equal to the shear gradient in the direction normal to the direction of flow.
This equation holds good for all types of flow and all types of boundary geometry.

6. In case of viscous flow through circular pipe, we have:

   (i) Shear stress, $\tau = -\frac{\partial p}{\partial x}.\frac{r}{2}$

   (ii) Velocity, $u = -\frac{1}{4\mu}.\frac{\partial p}{\partial x}(R^2 - r^2) = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$

   Max. Velocity, $u_{max} = -\frac{1}{4\mu}.\frac{\partial p}{\partial x}R^2$; Average Velocity, $\bar{u} = \frac{u_{max}}{2}$ **[VIMP]**

7. Loss of pressure head, $h_f = \frac{32\mu \bar{u} L}{\rho g D^2}$; Average Velocity, $\bar{u} = \frac{Q}{\pi R^2}$

   Or $\Delta P = \frac{128\mu QL}{\pi D^4}$

8. For the viscous flow the co-efficient of friction is given by, $f = \frac{16}{\text{Re}}$ **[VIMP]**

$$h_f = \frac{32\mu \bar{u} L}{\rho g D^2} = \frac{16 \times 4L\bar{u}^2}{(\rho \bar{u} D/\mu)2Dg} = \frac{16}{\text{Re}} \times \frac{4L\bar{u}^2}{2Dg} \text{ therefore } f = \frac{16}{\text{Re}}$$

But Remember Friction Factor, $f = 4f = \frac{64}{Re}$ (Be careful) **[VIMP]**

9. Flow of viscous fluid between two parallel plates

(a) If one plate is moving and other at rest this flow is known as – **Couette Flow.**

(b) Both plates at rest – Poiseuille flow $\Delta P = \frac{12\mu \bar{u} L}{b^2}$

Question (IES, IAS, GATE)

1. In flow through a pipe, the transition from laminar to turbulent flow does not depend on **[GATE-1996]**

(a) Velocity of the fluid (b) density of the fluid (c) Diameter of the pipe (d) length of the pipe

2. The lower critical Reynolds number for a pipe flow is **[IAS-1995]**

(a) different for different fluids
(b) the Reynolds number at which the laminar flow changes to turbulent flow
(c) more than 2000
(d) the least Reynolds number ever obtained for laminar flow

Relationship Between Shear Stress and Pressure Gradient

3. Which one of the following is the characteristic of a fully developed laminar flow?

(a) The pressure drop in the flow direction is zero
(b) The velocity profile changes uniformly in the flow direction
(c) The velocity profile does not change in the flow direction
(d) The Reynolds number for the flow is critical **[IAS-2004]**

4. The velocity distribution in laminar flow through a circular pipe follows the

(a) linear law (b) parabolic (c) cubic power law (d) logarithmic law **[IAS-1996]**

5. For flow through a horizontal pipe, the pressure gradient dp/dx in the flow direction is

(a) +ve (b) 1 (c) zero (d) –ve **[IAS-1995]**

6. In a steady flow of an oil in the fully developed laminar regime, the shear stress is: **[IES-2003]**

[a]. Constant across the pipe
[b]. Maximum at the centre an decreases parabolically towards the pipe wall boundary
[c]. Zero at the boundary and increases linearly towards the centre.
[d]. Zero at the centre and increases towards the pipe wall.

7. A 40 mm diameter 2m long straight uniform pipe carries a steady flow of water (viscosity 1.02 centipoises) at the rate of 3.0 liters per minute. What is the approximate value of the shear stress on the internal wall of the pipe? **[IES-2004]**

[a]. 0.0166 dyne/cm^2 [b]. 0.0812 dyne/cm^2 [c]. 8.12 dyne/cm^2 [d].0.9932 dyne/cm^2

8. The pressure drop for a relatively low Reynolds number flow in a 600 mm, 30m long pipeline is 70 kPa. What is the wall shear stress? **[IES-2004]**

[a]. 0 Pa [b]. 350 Pa [c]. 700 Pa [d]. 1400 Pa

Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law

9. Laminar developed flow at an average velocity of 5 m/s occurs in a pipe of 10 cm radius. The velocity at 5 cm radius is: **[IES-2001]**

[a]. 7.5 m/s [b]. 10 m/s [c]. 2.5 m/s [d]. 5 m/s

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10. The velocity profile in fully developed laminar flow in a pipe of diameter D is given by $u=u_0(1-4r^2/D^2)$, where is the radial distance from the centre. If the viscosity of the fluid is μ ,the pressure drop across a length L of the pipe is **[GATE-2006]**

(a) $\frac{\mu u_0 L}{D^2}$ (b) $\frac{4\mu u_0 L}{D^2}$ (c) $\frac{8\mu u_0 L}{D^2}$ (d) $\frac{16\mu u_0 L}{D^2}$

11. What is the discharge for laminar flow through a pipe of diameter 40mm having center-line velocity of 1.5 m/s? **[IAS-2004]**

(a) $\frac{3\pi}{59}$ m^3/s (b) $\frac{3\pi}{2500}$ m^3/s (c) $\frac{3\pi}{5000}$ m^3/s (d) $\frac{3\pi}{10000}$ m^3/s

12. Velocity for flow through a pipe, measured at the centre is found to be 2 m/s. Reynolds number is around 800.What is the average velocity in the pipe?

(a) 2 m/s (b) 1.7 m/s (c) 1 m/s (d) 0.5 m/s

[IES-2007]

13. For laminar flow through a long pipe, the pressure drop per unit length increases.

(a) in linear proportion to the cross-sectional area **[GATE-1996]**
(b) in proportion to the diameter of the pipe
(c) in inverse proportion to the cross-sectional area
(d) in inverse proportion to the square of cross-sectional area

14. In fully developed laminar flow in a circular pipe, the head loss due to friction is directly proportional to....... (Mean velocity /square of the mean velocity). **[GATE-1995]**

15. The MINIMUM value of friction factor 'f' that can occur in laminar flow through a circular pipe is: **[IAS-1997]**

(a) 0.064 (b) 0.032 (c) 0.016 (d) 0.008

Flow of viscous fluid between two parallel plates

16. The shear stress developed in lubricating oil, of viscosity 9.81 poise, filled between two parallel plates 10 cm apart and moving with relative velocity of 2 m/s is: **[IES-2001]**

[a]. 20 N/m^2 [b]. 19.62 N/m^2 [c]. 29.62 N/m^2 [d]. 40 N/m^2

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Answers with Explanations

1. Ans. (d) it is totally depends on Reynolds number = $\frac{\rho VD}{\mu}$

2. Ans. (a). The lower critical Reynolds number for a pipe flow is different for different fluids.

3. Ans. (c)

4. Ans. (b) Velocity, $u=-\frac{1}{4\mu}.\frac{\partial p}{\partial x}(R^2-r^2) = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$

5. Ans. (d). For flow through a horizontal pipe, the pressure gradient dp/dx in the flow direction is –ve.

$$\tau=-\frac{\partial p}{\partial x}.\frac{r}{2}$$

6. Ans. (d) $\tau = -\frac{\partial p}{\partial x}.\frac{r}{2}$

7. Ans. (b)

8. Ans. (b) $-\frac{\partial p}{\partial x} = \frac{\Delta P}{L} = \frac{70\times10^3}{30} = 2333;\ \tau_o = -\frac{\partial p}{\partial x}.\frac{R}{2} = 2333\times\frac{0.6}{4} = 350\,Pa$

9. Ans. (a) Velocity, u = $u_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]$ and $\bar{u} = \frac{u_{\max}}{2}$

10. Ans. (d) By Hagen-Poiseuille law, for steady laminar flow in circular pipes

$$\tau = -\mu\frac{\partial u}{\partial r}$$

$$\tau = \frac{-\partial P}{\partial x}.\frac{r}{2}$$

$$\mu\frac{\partial u}{\partial r} = \frac{\partial P}{\partial x}.\frac{r}{2}$$

$$\mu u_0\left(\frac{-8r}{D^2}\right) = \frac{P}{L}.\frac{r}{2} \qquad \text{---------}\left[\therefore u = u_0\left(1-\frac{4r^2}{D^2}\right)\right]$$

P= $\frac{-16\mu L u_0}{D_2}$ [(-) sign is due to drop]

11. Ans. (d) Centre-line velocity= U_{max}=1.5m/s

therefore average velocity ($\bar{U}$) = $\frac{U\max}{2} = \frac{1.5}{2} m/s$

Discharge (Q) =Area$\times$Area $\times average$ velocity= $\frac{\pi}{4}\times\left(\frac{40}{1000}\right)^2\times\frac{1.5}{2}m^3/s$

= $\frac{3\pi}{10,000}m^3/s$

12. Ans. (c) Re = 800 i.e. < 2000 so it is laminar flow and for laminar flow through pipe

$$\frac{U_{\max}}{U_{avg}} = 2 \quad Or \quad U_{avg} = \frac{U_{\max}}{2} = \frac{2}{2} = 1m/s$$

13. Ans. (d) $\frac{\Delta P}{L} = \frac{128\mu Q}{\pi D^4} \infty \frac{1}{D^4} i.e. \infty \frac{1}{A^2}$

14. Ans. True, $h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$

15. Ans. (b) Friction Factor, f = $4f = \frac{64}{\text{Re}}$ Where Max. Re=2000

16. Ans. (b) $\tau = \mu\frac{du}{dy} = \frac{9.81}{10}\times\frac{2}{0.1}$ =19.62 N/m^2

S. K. Mondal

TURBULENT FLOW

Skip to Questions (IAS, IES, GATE)

Highlight

1. co-efficient of friction in terms of shear stress, $f = \dfrac{2\tau_o}{\rho V^2}$ **[VIMP]**

2. The shear in turbulent flow is mainly due to momentum transfer.

3.

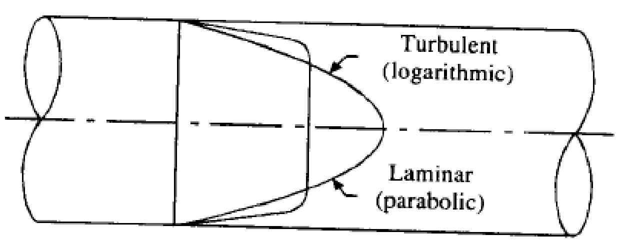


Fig. Velocity Distribution curves for laminar and turbulent flow

4. Characteristics of turbulent flow

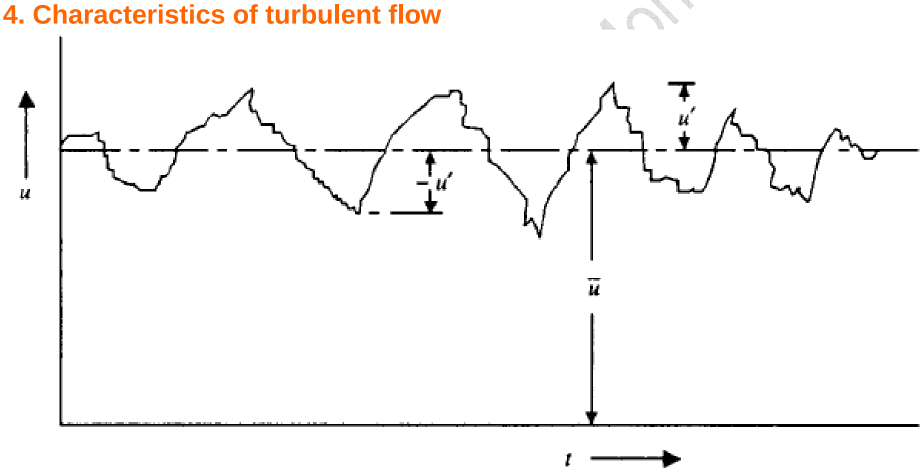


Fig. Variation of u with time

Instantaneous velocity (u) = Average Velocity ($\bar{u}$) + Velocity Fluctuation (u`)

5. Prandtl's mixing length (l) is defined as the average lateral distance through which a small mass of fluid particles would move from one layer to the other adjacent layers before acquiring the velocity of the new layer.

Total shear stress $(\tau) = \tau_{laminar} + \tau_{turbulence} = \mu \dfrac{du}{dy} + \eta \dfrac{du}{dy}$ Where η = eddy viscosity which is not a fluid property but depends upon turbulence conditions of the flow.

The turbulence shear stress $(\tau_t) = \rho l^2 \left(\dfrac{du}{dy}\right)^2$ **[VIMP]**

Where mixing length (l) = k y and k= Karman's Coefficient =0.4

6. Universal velocity distribution equation

$$u = u_{max} + 2.5 u_f \log_e\left(\frac{y}{R}\right) \quad \text{i.e.} \quad u = u_{max} + 5.75 u_f \log_{10}\left(\frac{y}{R}\right)$$ **[VIMP]**

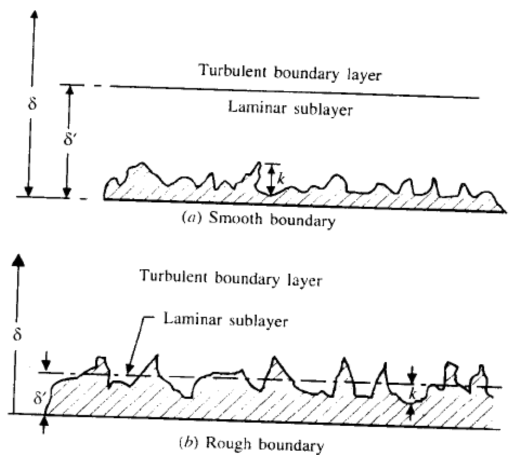
Where Shear Velocity $(u_f) = \sqrt{\frac{\tau_o}{\rho}}$

7. Hydro dynamically smooth and rough boundary

Roughness Reynolds Number, $\frac{u_f k}{\nu}$

For Smooth boundary $\frac{u_f k}{\nu} < 4$

For rough boundary $\frac{u_f k}{\nu} > 100$



8. Co-efficient of friction **[VIMP]**

Type of flow	Pipe	Reynolds number	co-efficient of friction
Laminar	Smooth or Rough	<2000	$f = \frac{16}{\text{Re}}$
Turbulent	Smooth	4000 to 1 lac	$f = \frac{0.0791}{(\text{Re})^{1/4}}$
		1 lac to 4 crore	$f = \frac{0.05525}{(\text{Re})^{0.237}} + 0.0008$
	Rough	>2000	$\frac{1}{\sqrt{4f}} = 2\log_{10}(R/K) + 1.74$

S K Mondal

Questions (IES, IAS, GATE)

Characteristics of Turbulent Flow

1. In a turbulent flow, $\bar{u}, \bar{v}$ and $\bar{w}$ are time average velocity components? The fluctuating components are u', v' and respectively. The turbulence is said to be isotropic if:

[a]. $\bar{u} = \bar{v} = \bar{w}$ [b]. $\bar{u} = \bar{v} = \bar{w}$ **[IES-1997]**

[c]. $(\overline{u'})^2 + (\overline{v'})^2 = (\overline{w'})^2$ [d]. None of the above situations prevails.

2. While water passes through a given pipe at mean velocity V the flow is found to change from laminar to turbulent. If another fluid of specific gravity 0.8 and coefficient of viscosity 20% of that

of water, is passed through the same pipe, the transition of flow from laminar to turbulent is expected if the flow velocity is
(a) 2V (b) V (c) V/2 (d) V/4 **[IAS-1998]**

3. In fully-developed turbulent pipe flow, assuming 1/7th power law, the ratio of time mean velocity at the centre of the pipe to that average velocity of the flow is:
[a]. 2.0 [b]. 1.5 [c]. 1.22[d]. 0.817 **[IES-2001]**

Shear Stresses in Turbulent Flow

4. Shear stress in a turbulent flow is due to: **[IES-1997]**
[a]. the viscous property of the fluid.
[b]. the fluid
[c]. fluctuation of velocity in the direction of flow
[d]. fluctuation of velocity in the direction of flow as well as transverse to it.

5. The shear stress in turbulent flow is
(a) linearly proportional to the velocity gradient
(b) proportional to the square of the velocity gradient
(c) dependent on the mean velocity of flow
(d) due to the exchange of energy between the molecules. **[IAS-1994]**

6. The pressure drop in a 100 mm diameter horizontal pipe is 50 kPa over a length of 10m. The shear stress at the pipe wall is: **[IES-2001]**
[a]. 0.25 kPa [b]. 0.125 kPa [c]. 0.50 kPa [d]. 25.0 kPa

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Prandtl's mixing length theory

7. In a turbulent flow, 'I' is the Prandtl' is mixing length and $\partial \bar{u}/\partial y$ is the gradient of the average velocity in the direction normal to flow. The final expression for the turbulent viscosity υ_t is given by; **[IES-1997]**

[a] $\upsilon_t = I\left(\frac{\partial \bar{u}}{\partial y}\right)$. [b]. $\upsilon_t = I\left|\frac{\partial \bar{u}}{\partial y}\right|$ [c]. $\upsilon_t = I^2\left(\frac{\partial \bar{u}}{\partial y}\right)$ [d]. $\upsilon_t = I^2\left|\frac{\partial \bar{u}}{\partial y}\right|$

8. Prandtl's mixing length in turbulent flow signifies **[GATE-1994]**
(a) the average distance perpendicular to the mean flow covered by the mixing particles.
(b) the ratio of mean free path to characteristic length of the flow field.
(c) the wavelength corresponding to the lowest frequency present in the flow field
(d) the magnitude of turbulent kinetic energy.

Resistance to Flow of Fluid in Smooth and Rough Pipes

9. Flow takes place and Reynolds Number of 1500 in two different pipes with relative roughness of 0.001 and 0.002.The friction factor **[IES-2000]**
(a) will be higher in the case of pipe with relative roughness of 0.001.
(b) will be higher in the case of pipe having relative roughness of 0.002.
(c) will be the same in both the pipes.
(d) in the two pipes cannot be compared on the basis of data given

10. In a fully turbulent flow through a rough pipe, the friction factor 'f' is (Re is the Reynolds number and ξ_S / D is relative roughness) **[IES-1998; IES-2003]**
[a]. a function of Re [b]. a function of Re and ξ_S / D
[c]. a function of ξ_S / D [d]. independent of Re and ξ_S / D

Answers with Explanations

1. Ans. (c)

2. Ans. (d) Rew= $\frac{\rho_w V_w D_w}{\mu_w} = \frac{0.8\rho_f \times V_f \times D_f}{0.2\mu_t} = 4R_{fw}$

$V_f = \frac{V_w}{4} = \frac{V}{4}$

3. Ans. (d) $U_{avg} = \frac{1}{A}\int udA = \frac{1}{\pi R^2}\int_0^R u_{max}\left(\frac{r}{R}\right)^{1/7} 2\pi r dr = \frac{14}{15}u_{max}$

4. Ans. (d)

5. Ans.(b) $f = \frac{2\tau_o}{\rho V^2}$ or $\tau_o = \frac{f\rho V^2}{2}$

6. Ans. (b) $(p_1 - p_2)\frac{\pi D^2}{4} = \tau_o \pi DL$ Or $\tau_o = \frac{\Delta P \times D}{4L}$

7. Ans. (d) $\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

8. Ans. (a)

9. Ans. (c) The flow is laminar (friction factor, $f = \frac{64}{\text{Re}}$) it is not depends on roughness but for turbulent flow it will be higher for higher relative roughness.

10. Ans. (c) $\frac{1}{\sqrt{4f}} = 2\log_{10}(R/K) + 1.74$; f is independent of Reynolds number and depends only on relative roughness (k/D)

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FLOW THROUGH PIPES

Skip to Questions (IAS, IES, GATE)

Highlight

1. Major Energy losses

(a) Darcy-Weisbach Formula:

$h_f = \frac{4flV^2}{2gD}$ Here 'f' is co-efficient of friction **[VIMP]**

$Or\ \frac{flV^2}{2gD}$ Here 'f' is friction factor

Co-efficient of friction

Type of flow	Pipe	Reynolds number	co-efficient of friction
Laminar	Smooth or Rough	<2000	$f = \frac{16}{\text{Re}}$
Turbulent	Smooth	4000 to l lac	$f = \frac{0.0791}{(\text{Re})^{1/4}}$
		1 lac to 4 crore	$f = \frac{0.05525}{(\text{Re})^{0.237}} + 0.0008$
	Rough	>2000	$\frac{1}{\sqrt{4f}} = 2\log_{10}(R/K) + 1.74$

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(b) Chezy's formula:

Mean Velocity, V= $C\sqrt{mi}$

Where *hydraulic mean depth* or *hydraulic radius*, $m = \frac{A}{P} = \frac{Area\ of\ flow}{Wetted\ perimeter}$, dimension [L]

Loss of head per unit length, i = $\frac{h_f}{L}$ it is dimensionless

So dimension of Chezy,s Constant , C is [$L^{1/2}$ T^{-1}]

Note: Darcy-Weisbach formula is used for flow through pipe.
Chezy's formula is used for flow through open channel.

2. Minor Energy Losses

Loss of head due to sudden enlargement, $\frac{(V_1 - V_2)^2}{2g}$ **[VIMP]**

Loss of head due to sudden contraction, k $\frac{V_2^2}{2g} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$ **[VIMP]**

Where k is dynamic loss coefficient = 0.375 for C_c=0.62

Loss of head due to obstruction in pipe, $\left[\dfrac{A}{C_c(A-a)}\right]^2 \dfrac{V^2}{2g}$

Where A= area of pipe, a= area of obstruction

Loss of head at the entrance to pipe, $0.5\dfrac{V^2}{2g}$

Loss of head at the exit of a pipe, $\dfrac{V^2}{2g}$

Loss of head due to bend in the pipe, k$\dfrac{V^2}{2g}$

Where k depends on pipe diameter, radius of curvature and angle of bend

Loss of head in various pipe fittings, k$\dfrac{V^2}{2g}$

Where k depends on type pipe fittings

3. Equivalent pipe: It is defined as the pipe of uniform diameter having **loss of head and discharge equal** to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. To determine the size of the equivalent pipe Dupit's equation is used.

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + ...$$ **[VIMP]**

4. In case of parallel pipes:

(i) Rate of discharge in the main line = sum of the discharges in each of the parallel pipes.

$$Q=Q_1+Q_2+Q_3$$

(ii) The loss of head in each pipe is same.

5. Power transmitted through pipes will be maximum when, $h_f = \dfrac{H}{3}$ **[VIMP]**

Therefore $\eta_{\max} = \dfrac{H - H/3}{H} = 66.7\%$

Diameter of nozzle for maximum power transmission, $d = \left(\dfrac{D^5}{8\,fL}\right)^{1/4}$

6. (a) Energy Gradient Line (EGL), $EGL = \dfrac{P}{\rho g} + \dfrac{V^2}{2g} + z$

(b) Hydraulic Gradient Line (HGL), $HGL = \dfrac{P}{\rho g} + z$

7. Pipe network

An interconnected system of pipes is called a pipe network. The flow to an outlet may come from different pipes. Figure below shows a typical network. In a network:

(1) Flow into each junction must be equal to flow out of each junction.

(2) Algebraic sum of head losses round each loop must be zero.

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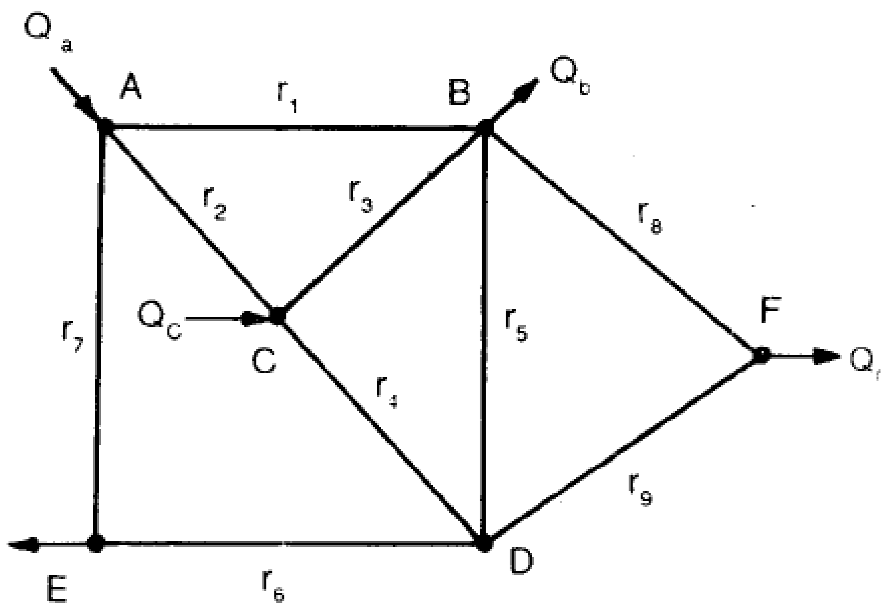

Typical Pipe Network

The head loss in each pipe is expressed as $h_f = rQ^n$. The coefficient r depends upon pipe length, diameter and friction factor. For turbulent flow n is of the order of 2.

8. Water hammer in pipes.

The phenomenon of sudden rise in pressure in a pipe when water flowing in it is suddenly brought to rest by closing the valve is known as water hammer or hammer blow.

Valve closure is gradual when $t > \frac{2L}{C}$

Valve closure is sudden when $t < \frac{2L}{C}$

Where $C = \sqrt{\frac{K}{\rho}}$, C being the velocity of pressure wave produced due to water hammer.

9. The intensity of pressure rise due to water hammer

If t = thickness of pipe wall, L = length of pipe, V = velocity of flow, K = bulk modulus of water, and E = modulus of elasticity for pipe material

$p = V\rho\frac{L}{t}$ For gradual closing

$p = V\rho C$ For Rigid pipe and instantaneous closing

$p = V \times \sqrt{\frac{\rho}{\frac{1}{K} + \frac{D}{Et}}}$ For Elastic pipe and instantaneous closing

Questions (IES, IAS, GATE)

Loss of Energy (or Head) in Pipes

1. Which one of the following statements is true of fully developed flow through pipes?

[a]. The flow is parallel, has no inertia effects, the pressure gradient is of constant value and the pressure force is balanced by the viscous force.

[b]. The flow is parallel, the pressure gradient is proportional to the inertia force and there is no viscous effect

[c]. The flow is parallel, the pressure gradient is negligible and inertia force si balanced by the viscous force.

[d]. The flow is not parallel, the core region accelerates and the viscous drag is far too less than the inertia force. **[IES-1997]**

Darcy-Weisbach formula

2. The head loss in turbulent flow in pipe varies

(a) Directly as the velocity (b) Inversely as the square of the velocity

(c) Inversely as the square of the diameter (d) approximately as the square of the velocity **[IES-2007; IAS-2007]**

3. Two identical pipes of length 'L', diameter 'd' and friction factor 'f' are connected in parallel between two points. For the same total volume flow rate with pipe of same diameter'd' and same friction factor 'f', the single length of the pipe will be

(a) $\frac{L}{2}$ (b) $\frac{L}{\sqrt{2}}$ (c) $\sqrt{2}$ L (d) $\frac{L}{4}$ **[IAS-1999]**

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4. The value of friction factor is misjudged by + 25% in using Darcy-Weisbach equation. The resulting error in the discharge will be: **[IES-1999]**

[a]. + 25% [b]. – 18.25% [c]. – 12.5 % [d]. +12.5%

5. A pipeline connecting two reservoirs has its diameter reduced by 20% due to deposition of chemicals. For a given head difference in the reservoirs with unaltered friction factor, this would cause a reduction in discharge of: **[IES-2000]**

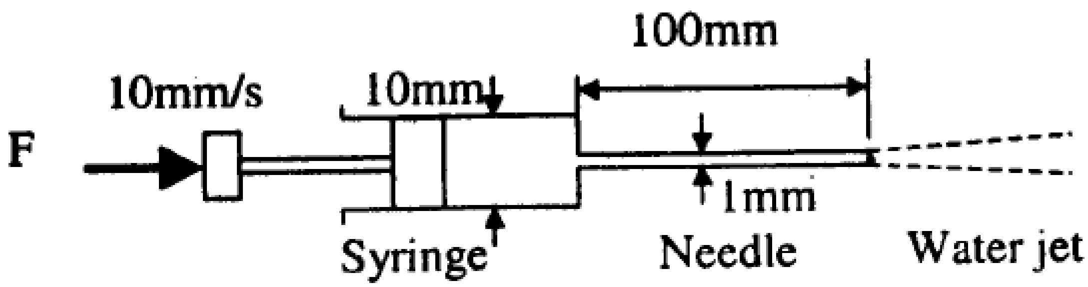
[a]. 42.8% [b]. 20% [c]. 17.8% [d]. 10.6%

6. The loss of head in a pipe of certain length carrying a rate of flow of Q is found to be H. If a pipe of twice the diameter but of the same length is to carry a flow rate of 2Q, then the head loss will be

(a) H (b) H/2 (c) H/4 (d) H/8 **[IAS-1997]**

Data for Q. 7 - 8 are given below. Solve the problems and choose correct answers.

A syringe with a frictionless plunger contains water and has at its end a 100 mm long needle of 1 mm diameter. The internal diameter of the syringe is 10 rom. Water density is 1000 kg/m^3 The plunger is pushed in at 10 mm/s and the water comes out as a jet



7. Assuming ideal flow, the force F in Newton required on the plunger to push out the water is **[GATE-2003]**

(a) 0 (b) 0.04 (c) 0.13 (d) 1.15

8. Neglect losses in the cylinder and assume fully developed laminar viscous flow throughout the needle; the Darcy friction factor is 64/Re. Where Re is the Reynolds number. Given that the viscosity of water is 1.0 x 10-3 kg/s m, the force F in Newton required on the plunger is **[GATE-2003]**

(a) 0.13 (b) 0.16 (c) 0.3 (d) 4.4

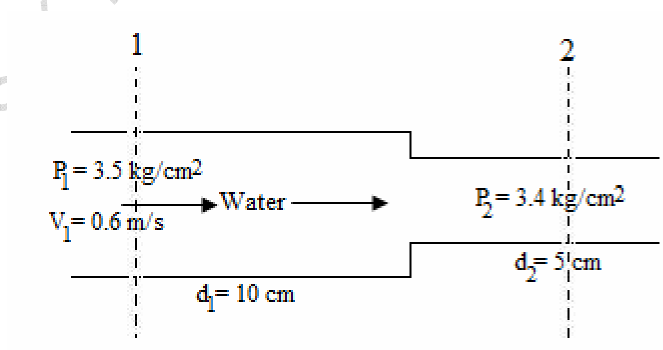
9. The coefficient of friction 'f' in terms of shear stress 'τ_0' is given by

(a) f=$\frac{\rho v^2}{2\tau_0}$ (b) f=$\frac{\tau_0}{\rho v^2}$ (c) f=$\frac{2\tau_0}{\rho v^2}$ (d) f=$\frac{2\rho v^2}{\tau_0}$ **[IAS-2003]**

10. Fluid is flowing with an average velocity of V through a pipe of diameter d. Over a length of L, the "head" loss is given by $h_f = \frac{fLV^2}{2g \times D}$. The friction factor, f, for laminar flow in terms of Reynolds number (Re) is........ **[GATE-1994]**

11. The energy loss between sections (1) and (2) of the pipe shown in the given figure is

(a) 1.276 Kg-m (b) 1.00 Kg-m
(c) 0.724 Kg-m (d) 0.15 Kg-m.
[IAS-1995]



12. Water flows through a 0.6 m diameter, 1000 m long pipe from a 30 m overhead tank to a village. Find the discharge (in liters) at the village (at ground level), assuming a Fanning friction factor f= 0.04 and ignoring minor losses due to bends etc. **[GATE-2001]**

13. A laminar flow is taking place in a pipe. Match List I (Term) with List II (Expression) and select the correct answer using the codes given below the Lists: **[IAS-2002]**

List I (Term)	List II (Expression)
A. Discharge, Q	1. $\frac{16\mu}{\rho VD}$

B. Pressure drop, $\frac{\Delta P}{L}$ 2. $\frac{\pi d^2 \Delta p}{128 \mu L}$

C. Friction factor, f 3. $\frac{32 \mu V}{D^2}$

4. $\frac{\pi d^4 \Delta p}{128 \mu L}$

Codes:

	A	B	C		A	B	C
(a)	2	3	4	(b)	4	3	1
(c)	4	1	3	(d)	1	4	2

14. From a reservoir, water is drained through two pipes of 10 cm and 20 cm diameter respectively. If frictional head loss in both the pipes is same, then the ratio of discharge through the larger pipe to that through the smaller pipe will be

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 4 (d) $4\sqrt{2}$ **[IAS-1998]**

15. The pressure drop in a pipe flow is directly proportional to the mean velocity. It can be deduced that the **[IES-2006]**

(a) Flow is laminar (b) Flow is turbulent (c) Pipe is smooth (d) Pipe is rough

Chezy's formula for loss of head due to friction

16. The hydraulic means depth (where A = area and P = wetted perimeter) is given by:

[a]. P / A [b]. P^2 / A [c]. A / P [d]. $\sqrt{A / P}$ **[IES-2002]**

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17. Which one of the following expresses the hydraulic diameter for a rectangular pipe of width b and height a ?

(a) $\frac{ab}{2(a+b)}$ (b) $\frac{ab}{(a+b)}$ (c) $\frac{2ab}{(a+b)}$ (d) $\frac{a+b}{2ab}$ **[IAS-2007]**

18. Which one of the following is the correct expression for the area of flow for a circular channel? (Where □ = half the angle subtended by water surface at the center and R = radius of the circular channel) **[IES-2004]**

[a]. $R^2\left(2\theta - \frac{\sin 2\theta}{2}\right)$ [b]. $R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$ [c]. $R^2(2\theta - \sin 2\theta)$ [d]. $2R^2(\theta - \sin 2\theta)$

19. For a circular channel, the wetted parameter (where R = radius of circular channel, □ = half the angle subtended by the water surface at the centre) is given by:

[a]. R□/ 2 [b]. 3R□ [c]. 2R□ [d]. R□ **[IES-2003]**

Minor Energy Losses

Loss of head due to sudden contraction

20. Assertion (A): Head loss for sudden expansion is more than the head loss for a sudden contraction for the same diameter ratio.

Reason(R): Head loss varies as the square of the upstream and downstream velocities in the pipe fitted with sudden expansion or sudden contraction. **[IAS-2003]**

21. If coefficient of contraction at the vena contract is equal to 0.62, then what will be the dynamic loss coefficient in sudden contraction in air-conditioning duct? **[IES-2004]**

[a]. 0.25 [b]. 0.375 [c]. 0.55 [d]. 0.65

Loss of head in various pipe fittings

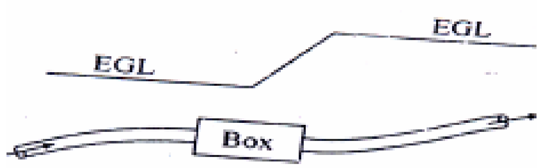
22. A liquid flows downward through at tapped vertical portion of a pipe. At the entrance and exit of the pipe, the static pressures are equal. If for a vertical height 'h' the velocity becomes four times, then the ratio of 'h' to the velocity head at entrance will be:
[a]. 3 [b]. 8 [c]. 15 [d]. 24 **[IES-1998]**

Hydraulic Gradient and Total Energy Lines

23. Which one of the following statements is correct? **[IES-2000]**
[a]. Hydraulic grade line and energy grade line are the same in fluid problems
[b]. Energy grade line lies above the hydraulic grade line and is always parallel to it.
[c]. Energy grade line lies above the hydraulic grade line and they are separated from each other by a vertical distance equal to the velocity head.
[d]. The hydraulic grade line slopes upwards meeting the energy grade at the exit of flow.

24. Point A of head 'H_A' is at a higher elevation than point B of head 'H_B'. The head loss between these points is H_L. The flow will take place. **[IES-1999]**
[a]. always form A to B [b]. from A to B if $H_A + H_L = H_B$
[c]. from B to A if $H_A + H_L = H_B$ [d]. form B to A if $H_B + H_L = H_A$

25.



The energy grade line (EGL) for steady flow in a uniform diameter pipe is shown above. Which of the following items is contained in the box? **[IES-2006]**
(a) A pump (b) A turbine (c) A partially closed valve (d) An abrupt expansion

26. A 12 cm diameter straight pipe is laid at a uniform downgrade and flow rate is maintained such that velocity head in the pipe is 0.5 m. If the pressure in the pipe is observed to be uniform along the length when the down slope of the pipe is 1 in 10, what is the friction factor for the pipe? **[IES-2006]**
(a) 0.012 (b) 0.024 (c) 0.042 (d) 0.050

Pipes in Series or Compound Pipes

27. Two pipelines of equal length and with diameters of 15 cm and 10 cm are in parallel and connect two reservoirs. The difference in water levels in the reservoirs is 3 m. If the friction is assumed to be equal, the ratio of the discharges due to the larger dia pipe to that of the smaller diameter pipe is nearly, **[IES-2001]**
[a]. 3.375 [b]. 2.756 [c]. 2.25 [d]. 1.5

28. A pipe is connected in series to another pipe whose diameter is twice and length is 32 times that of the first pipe. The ratio of frictional head losses for the first pipe to those for the second pipe is (both the pipes have the same frictional constant):
[a]. 8 [b]. 4 [c]. 2 [d]. 1 **[IES-2000]**

29. Two pipelines of equal lengths are connected in series. The diameter of the second pipe is two times that of the first pipe. The ratio of frictional head losses between the first pipe and the second pipe is **[IAS-1996]**

(a) 1:32 (b) 1:16 (c) 1:8 (d) 1:4

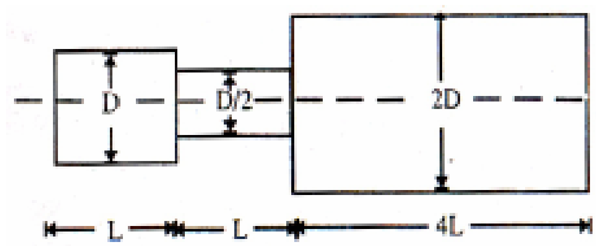
Equivalent Pipe

30. A pipeline is said to be equivalent to another, if in both
- (a) Length and discharge are the same
- (b) Velocity and discharge are the same
- (c) Discharge and frictional head loss are the same
- (d) Length and diameter are the same **[IAS-2007]**

31. The equivalent length of stepped pipeline shown in the below figure, can be expressed in terms of the diameter 'D' as :

[a]. 5.25 L [b]. 9.5 L

[c]. $33\frac{1}{32}$ L [d]. $33\frac{1}{8}$ L



[IES-1998]

32. A stepped pipelines with four different cross-sections discharges water at the rate of 2 litres per second. Match List I (Areas of pipe in sq cm) with List II (Velocities of water in cm/s) and select the correct answer using the codes given below the Lists: **[IAS-2001]**

List I	List II
A. 500	1. 4
B. 100	2. 5
C. 400	3. 10
D. 200	4. 15
	5. 20

Codes:

	A	B	C	D		A	B	C	D
(a)	5	1	2	3	(b)	1	5	2	3
(c)	1	5	3	4	(d)	3	2	5	1

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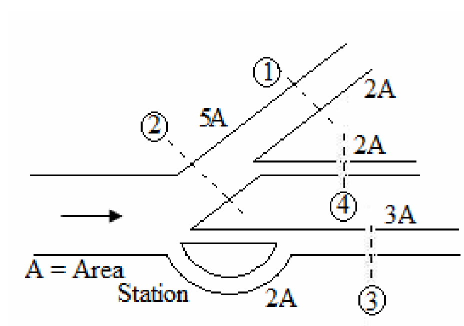
33. A branched pipeline carries water as shown in the given figure. The cross-sectional areas of the pipelines have also been indicated in the figure. The correct sequence of the decreasing order of the magnitude of discharge for the four stations is

(a) 2, 3, 1, 4 (b) 3, 2, 1, 4

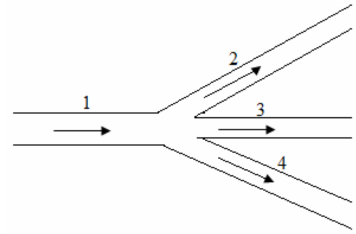
(c) 3, 2, 4, 1 (d) 2, 3, 4,1

[IAS-1996]



34. Pipe '1' branches to three pipes as shown in the given figure. The areas and corresponding velocities are as given in the following table.

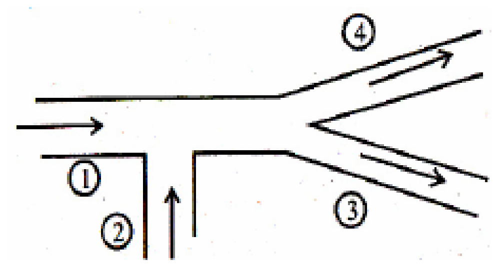
Pipe	Velocity (cm per second)	Area (sq cm)
1.	50	20
2.	V_2	10
3.	30	15
4.	20	10



The value of V_2 in cm per second will be

(a) 15 (b) 20 (c) 30 (d) 35 **[IAS-1995]**

35. A pipe flow system with flow direction is shown in the below figure. The following table gives the velocities and the corresponding areas: **[IES-1998]**



pipe No.	***Area (cm^2)***	***Velocity (cm/s)***
1	50	10
2	50	V_2
3	80	5
4	70	5

The value of V_2 is:

[a]. 2.5 cm/s [b]. 5.0 cm/s [c]. 7.5 cm/s [d]. 10.0 cm/s

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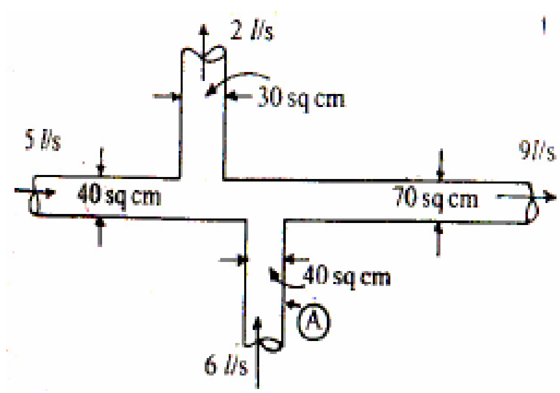
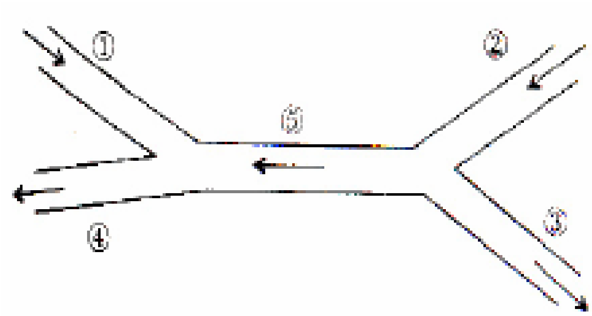
36. The pipe cross-sections and fluid flow rates are shown in the given figure. The velocity in the pipe labeled as A is:

[a]. 1.5 m/s [b]. 3 m/s
[c]. 15 m/s [d]. 30 m/s
[IES-1999]



37. The velocities and corresponding flow areas of the branches labeled ①, ②, ③, ④ and ⑤ for a pipe system shown in the given figure are given in the following table :

[IES-2000]



Pipe Label 1	**Velocity**	**Area**		**Velocity**	**Area**
①	5 cm/s	4 sq cm	②	6 cm/s	5 sq cm
③	V_3 cm/s	2 sq cm	④	4 cm/s	10 sq cm
⑤	V_5 cm/s	8 sq cm			

The velocity V_5 would be:

[a]. 2.5 cm/s [b]. 5 cm/s [c]. 7.5 cm/s [d]. 10 cm/s

38. A compound pipeline consists of two pieces of identical pipes. The equivalent length of same diameter and same friction factor, for the compound pipeline is L_1 when pipes are connected in series, and is L_2 when connected in parallel. What is the ratio of equivalent lengths L_1/L_2? **[IES-2006]**

(a) 32 : 1 (b) 8 : 1 (c) 2 : 1 (d) $\sqrt{2}:1$

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Power Transmission through Pipes

39. For maximum transmission of power through a pipe line with total head H, the head lost due to friction h_f is given by: **[IAS-2007; IES-2001]**

[a]. 0.1 H [b]. H/3 [c]. H/2 [d]. 2H/3

40. Assertion (A): The power transmitted through a pipe is maximum when the loss of head due to friction is equal to one-third of total head at the inlet.

Assertion(R): Velocity is maximum when the friction loss is one-third of the total head at the inlet. **[IES-2007]**

41. What will be the maximum efficiency of the pipeline if one-third of the available head in flow through the pipeline is consumed by friction? **[IAS-2004]**

(a) 33.33% (b) 50.00% (c) 66.66% (d) 75.00%

42. In a pipe flow, the head lost due to friction is 6 m. If the power transmitted through the pipe has to be the maximum then the total head at the inlet of the pipe will have to be maintained at **[IAS-1995]**

(a) 36 m (b) 30 m (c) 24m (d) 18m

Diameter of the nozzle for transmitting maximum power

43. A 20 cm diameter 500 m long water pipe with friction factor u_f = 0.025, leads from a constant-head reservoir and terminates at the delivery end into a nozzle discharging into air. (Neglect all energy losses other than those due to pipe friction). What is the approximate diameter of the jet for maximum power? **[IES-2004]**

[a]. 6.67 mm [b]. 5.98 mm [c]. 66.7 mm [d]. 59.8 mm

Water Hammer in Pipes

44. Velocity of pressure waves due to pressure disturbances imposed in a liquid is equal to: **[IES-2003]**

[a]. (E / □) 1 / 2 [b]. (E □)1 / 2 [c].(□ / E) 1 / 2 [d]. (1/ □ E) 1 / 2

45. Which phenomenon will occur when the value at the discharge end of a pipe connected to a reservoir is suddenly closed? **[IES-2005]**

[a]. Cavitation [b]. Erosion [c]. Hammering [d]. Surging.

Answer with Explanation

1. Ans. (a)

2. Ans. (d) $h_f = \dfrac{4fLV^2}{D\times 2g}$

3. Ans. (d) $h_f = \dfrac{4fLV^2}{2g\times D}$ for same dia. Velocity, V will be (V/2) ΔP Will be $\dfrac{1}{4}$ times

4. Ans. (c) Correct method $h_f = \dfrac{4fLV^2}{2gD} Where V = \dfrac{Q}{A} or\ V^2 = \dfrac{16Q^2}{\pi^2 D^4}$

$$or\ h_f = \frac{64fLQ^2}{2g\pi^2 D^5}\ Or\ Q\infty\frac{1}{\sqrt{f}} \text{ or } \frac{Q'-Q}{Q} = \sqrt{\frac{f}{f'}} - 1 = \sqrt{\frac{1}{1.25}} - 1 = -10.55\%$$

Nearest answer is (c)

But Paper setter calculates it in the way given below.

$$\ln(Q) = -\frac{1}{2}\ln(f)\ Or \frac{dQ}{Q} = -\frac{1}{2}\frac{df}{f} = -\frac{1}{2}\times 25 = -12.5\%$$

Note: This method is used only for small fluctuation and 25% is not small that so why this result is not correct.

5. Ans. (a) $h_f = \dfrac{4fLV^2}{2gD} Where V = \dfrac{Q}{A} or\ V^2 = \dfrac{16Q^2}{\pi^2 D^4} or\ h_f = \dfrac{64fLQ^2}{2g\pi^2 D^5}\ Or\ Q\infty D^{5/2}$

$$or\left(1-\frac{Q_2}{Q_1}\right) = 1-(0.8)^{2.5} = 42.75\% \text{ (Reduction)}$$

6. Ans. (d)

$$H = \frac{4fLV^2}{2gD} Where V = \frac{Q}{A} or\ V^2 = \frac{16Q^2}{\pi^2 D^4} or\ H = \frac{64fLQ^2}{2g\pi^2 D^5}\ Or\ H_2 = \frac{64fL(2Q)^2}{2g\pi^2(2D)^5} = \frac{2^2}{2^5}H = \frac{H}{8}$$

7. Ans (b)

8. Ans. (c) Given, υ = viscosity of water =10×10^{-3} kg/sm

Now Re= $\dfrac{\rho\upsilon_2 d}{\upsilon} = \dfrac{1000\times 1\times 0.001}{1\times 10^{-3}}$ =Re=1000---- since= $\upsilon_2 = 1$

Darcy's friction factor, 4f= $\dfrac{64}{Re} = \dfrac{64}{1000} = 0.064$

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So head loss in needle=$h_t = \frac{fl\upsilon_2^2}{2gD} = \frac{0.064\times 0.1\times (1)^2}{2\times 9.8\times 0.001} = 0.3265$ m

Applying Bernoulli's equation at points 1 and 2, we have

$$\frac{P_1}{\rho g} + \frac{\upsilon_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\upsilon_2^2}{2g} + z_2 + h_1$$

$$\therefore \frac{P_1}{\rho g} = \frac{\upsilon_2^2 - \upsilon_1^2}{2g} + h_1$$ [Since z_1=z_2 and P_2=0]

P_1=$\frac{\rho}{2}(\upsilon_2^2 - \upsilon_1^2) + \rho g h_1 = \frac{1000}{2}[(1)^2 - (0.01)^2] + 1000\times 9.8\times 0.3265$

Now force required on plunger=$P_1 \times A_1$ =3699.65$\times \frac{\pi}{4}\times (0.01)^2$ =0.3N

9. Ans. (c)

10. Ans. $\frac{64}{\text{Re}}$

11. Ans. (c). Energy loss between sections 1 and 2

$= \frac{p_1 - p_2}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$ Also $V_1 \times A_1 = V_2 \times A_2$

Or $0.6\times \frac{\pi}{4}\times (0.1)^2 = V_2 \times \frac{\pi}{4}(0.05)^2$ or V_2=0.6×4=2.4 m/s

Energy loss = $\frac{(3.5-3.4)\times 10000\times 9.81}{9.81\times 1000} + \frac{0.6^2}{2g}(1-16) = 0.1\times 10 - \frac{15\times 0.36}{2\times 9.81} = 1 - 0.266 = 0.724$

12. Ans (0.834 m^3/s) $h_f = \frac{fLV^2}{2gD} = \frac{0.04\times 1000\times V^2}{2\times 9.81\times 0.6}$ *Therefore* $\Delta H = H - h_f = 30 - h_f$

$$V = \sqrt{2g\Delta H} \quad Or\, \Delta H = \frac{V^2}{2g} = 30 - h_f = 30 - \frac{0.04\times 1000\times V^2}{2\times 9.81\times 0.6} \Rightarrow V = 2.95\, m/s$$

$$Q = VA = V\times \frac{\pi D^2}{4} = 2.95\times \frac{\pi\times (0.6)^2}{4} = 0.834\, m^3/s$$

13. Ans. (b) here 'C' is wrong. Friction factor=$\frac{64}{\text{Re}}$ and

Coefficient of friction= $\frac{16}{\text{Re}}$ so 'C' would be co-efficient of friction.

14. Ans. (d) h_f = $\frac{4fLV^2}{D\times 2g}$ $\frac{V_R^2}{D_R} = \frac{V_S^2}{D_s}$ or $\frac{V_l}{V_S} = \sqrt{\frac{D_l}{D_s}} = \sqrt{\frac{20}{10}} = \sqrt{2}$

$$\frac{Ql}{Qs} = \frac{A_l V_l}{A_s V_s} = \frac{D_l^2}{D_s^2}\times \frac{V_l}{V_s} = \frac{D_l^2}{D_s^2}\times \sqrt{\frac{D_l}{D_s}} = 4\sqrt{2}$$

15. Ans. (a)

16. Ans. (c)

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17. Ans. (c) Hydraulic diameter= $\frac{4A}{P} = \frac{2ab}{2(a+b)}$

Note: Hydraulic mean depth= $\frac{Ac}{P}$

Hydraulic equivalent diameter= $\frac{4Ac}{P}$

18. Ans. (b)
19. Ans. (c)
20. Ans. (c)
21. Ans. (b)
22. Ans. (c) Apply Bernoulli's Equation

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = z_1 - z_2 = h \qquad \Rightarrow \frac{(4V_1)^2}{2g} - \frac{V_1^2}{2g} = h \qquad \Rightarrow 15\frac{V_1^2}{2g} = h$$

23. Ans. (c)
24. Ans. (c) if flow is from point 1 to point 2 then
Total head at point 1 = Total head at point 2 + loss of head between 1 and 2
25. Ans. (a) Energy increased so box must add some hydraulic energy to the pipeline. It must be a pump that converts Electrical energy to Hydraulic energy.

26. Ans. (b) $h_f = \frac{fL}{D}.\frac{V^2}{2g} \quad Or\, 1 = \frac{f \times 10}{0.12} \times 0.5 \Rightarrow f = 0.024$

27. Ans. (b) Loss of head in larger dia. pipe = Loss of head in smaller dia. pipe

$$h_f = \frac{4\,fLV^2}{2gD} Where V = \frac{Q}{A} or\ V^2 = \frac{16Q^2}{\pi^2 D^4} or\, h_f = \frac{64\,fLQ^2}{2g\pi^2 D^5}\ Or\ Q \infty D^{5/2}$$

$$\frac{Q_1}{Q_2} = \left(\frac{15}{10}\right)^{5/2} = 2.756$$

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28. Ans. (d)

$$h_f = \frac{4\,fLV^2}{2gD} Where V = \frac{Q}{A} or\ V^2 = \frac{16Q^2}{\pi^2 D^4} or\, h_f = \frac{64\,fLQ^2}{2g\pi^2 D^5}$$

$$\frac{h_{f1}}{h_{f2}} = \left(\frac{L_1}{L_2}\right) / \left(\frac{D_1}{D_2}\right)^5 = 32/32 = 1$$

29. Ans. (a) $h_f = \frac{4\,fLV^2}{2gD} Where V = \frac{Q}{A} or\ V^2 = \frac{16Q^2}{\pi^2 D^4} or\, h_f = \frac{64\,fLQ^2}{2g\pi^2 D^5}\ Or\ h_f \infty \frac{1}{D^5}$

$$\frac{h_{f1}}{h_{f2}} = \left(\frac{D_2}{D_1}\right)^2 = (2)^5 = 32$$

30. Ans. (c) $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + ...$

31 Ans. (d) $\frac{Le}{D^5} = \frac{L}{D^5} + \frac{L}{(D/2)^5} + \frac{4L}{(2D)^5} = 33\frac{1}{8}L$

32. Ans. (b) Volume flow rate= A.V= 2000 cm^3/sec

$A_1V_1=A_2V_2=A_3V_3=A_AV_A=2000$

33. Ans. (d) don't confuse with section 1 and section 4 both has area ='2A' as it is vertically up so discharge will be less.

34. Ans. (d) $Q_1=Q_2+Q_3+Q_4$

$50\times 20=V_2\times 10+30\times 15+20\times 10;$

or $1000=10V_2+450+200$

$10V_2=1000-650=350$ and $V_2=35$ cm/sec

35. Ans. (b) $Q_1+Q_2 = Q_3+Q_4$

$50\times 10 + 50\times V_2 = 80\times 5+70\times 5;$

$V_2=5$ cm/sec

36. Ans. (a) $V=\dfrac{Q}{A}=\dfrac{6000}{40}=150cm/s=1.5m/s$

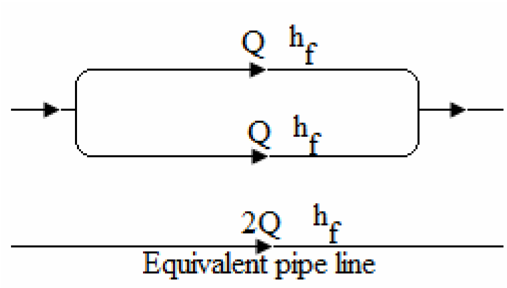
37. Ans. (a) $Q_1 + Q_5 = Q_4$ or $5x4 + V_5x8 = 4x10$ or $V_5 = 2.5$ cm/s

38. Ans. (b) Pipes connected in series, $\dfrac{L_1}{D^5}=\dfrac{L}{D^5}+\dfrac{L}{D^5}$ or $L_1=2L$

Pipes connected in parallel,

$$h_f=\frac{4fLV^2}{2gD} Where V=\frac{Q}{A} or$$

$$V^2=\frac{16Q^2}{\pi^2D^4} or\ h_f=\frac{64fLQ^2}{2g\pi^2D^5}=\frac{64fL_2(2Q)^2}{2g\pi^2D^5}$$



$$L_2=\frac{L}{4}\quad \therefore \frac{L_1}{L_2}=\frac{2L}{L/4}=8$$

39. Ans. (b)

40. Ans. (a)

41. Ans. (c) $h_f=\dfrac{H}{3}\therefore \eta=\dfrac{H-h_f}{H}\times 100=\dfrac{2}{3}\times 100=66.66\%$

42. Ans. (d). Head lost due to friction is 6 m. Power transmitted is maximum when friction head is 1/3 of the supply head. ∴ Supply head should be 18 m.

43. Ans. (d) $d=\left(\dfrac{D^5}{2fL}\right)^{1/4}=\left(\dfrac{0.20^5}{2\times 0.025\times 500}\right)^{1/4}=0.0598m=59.8mm$, Here f is friction factor

$d=\left(\dfrac{D^5}{8fL}\right)^{1/4}$ here f is co-efficient of friction.

44. Ans. (a)

45. Ans. (c)

FLOW THROUGH ORIFICES AND MOUTHPIECES

Skip to Questions (IAS, IES, GATE)

Highlights

1. An orifice is an opening in a fluid container.

2. A mouthpiece is a short tube fitted in place of an orifice. The length of the tube usually 2 to 3 times the diameter of the orifice. It is used to increase the amount of discharge.

3. Coefficient of contraction, $C_c = \dfrac{Area\ of\ jet\ at\ vena\ contracta}{Area\ of\ orifice\ opening}$; $generally = 0.64$

4. Coefficient of velocity, $C_v = \dfrac{Actual\ velocity\ at\ vena\ contracta}{Ideal\ velocity\ of\ the\ jet}$; $generally = 0.98$

5. Coefficient of Discharge, $C_d = \dfrac{Actual\ discharge}{Ideal\ discharge}$; $generally = 0.60$

6. $C_d = C_c \times C_v$ [VIMP]

7. Head lost in orifice = Head of theoretical velocity-Head of actual velocity

$$= \frac{(V_1)_{th}^2}{2g} - \frac{V_1^2}{2g} = \left(\frac{V_1}{C_v}\right)^2 \frac{1}{2g} - \frac{V_1^2}{2g} = \frac{V_1^2}{2g}\left(\frac{1}{C_v^2} - 1\right)$$

8. Discharge through a large rectangular orifice, $Q = \dfrac{2}{3} C_d b \sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right]$

9. Time of emptying a tank through an orifice at its bottom, $T = \dfrac{2A(\sqrt{H_1} - \sqrt{H_2})}{C_d . a . \sqrt{2g}}$

10. Internal Mouthpiece is also known as Borda's Mouthpiece

11. Co efficient of discharge of internal mouthpiece (C_d)

(a) External mouthpiece, C_d= 0.855

(b) Internal mouthpiece, Running full, C_d= 0.707

Running free, C_d= 0.50

(c) Convergent-divergent mouthpiece, C_d= 1.0

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Questions (IES, IAS, GATE)

Flow through an Orifice

1. Match List I with List II and select the correct answer using the codes given below the Lists: **[IES-1997]**

List I (Measuring device)		**List II (Parameter measured)**
A.	Anemometer	1. Flow rate
B.	Piezometer	2. Velocity
C.	Pitot tube	3. Static pressure
D.	Orifice	4. Difference between static and stagnation pressure.

Codes:

	A	B	C	D		A	B	C	D
[a].	1	3	4	2	[b].	1	2	3	4
[c].	2	3	4	1	[d].	2	4	3	1

Co-efficient of discharge *(C_d)*

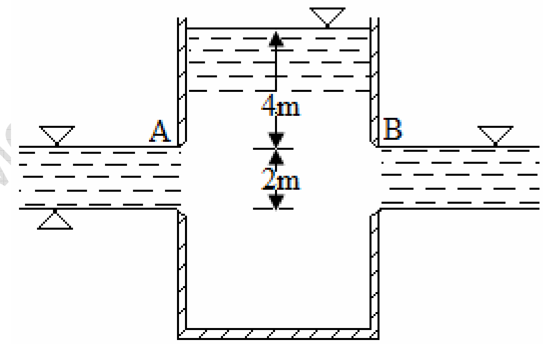
2. A fluid jet is discharging from a 100 mm nozzle and the vena contracta formed has a diameter of 90 mm. If the coefficient of velocity is 0.95, then the coefficient discharge for the nozzle is

(a) 0.855 (b) 0.81 (c) 0.9025 (d) 0.7695 **[IAS-1994]**

Discharge through a Large Rectangular Orifice

3. Water discharges from a two-dimensional rectangular opening into air as indicated at A in the given figure. At B water discharge from under a gate onto the floor. The ratio of velocities V_A to V_B is

$(a)\frac{\sqrt{5}}{2}$ $(b)\frac{1}{2}$ $(c)2$ $(d)\frac{1}{\sqrt{2}}$ **[IAS-1996]**



Discharge through an External Mouthpiece.

4. Given,

H = height of liquid, b = width of notch, a = cross-sectional area, a_1 = area at inlet,

a_2 = area at the throat and C_d = coefficient of drag, **[IES-1997]**

Match List I with List II and select the correct answer using the codes given below the Lists:

List I		**List II**
A.	Discharge through Venturimeter	1. $\frac{2}{3} C_d\ b\ \sqrt{2g}\ H^{3/2}$
B.	Discharge through an external mouthpiece.	2. $\frac{8}{15} C_d\ b\ \sqrt{2g}\ H^{5/2}$
C.	Discharge over a rectangular notch	3. $\frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH}$
D.	Discharge over right angled notch.	4. $0.855a\ \sqrt{2gH}$

Code:

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	3	4	1	2
(c)	2	1	3	4	(d)	2	3	1	4

Answers with Explanation

1. Ans. (a)

2. Ans. (d) $C_c \frac{A_v}{A} = \frac{\frac{\pi}{4}(90)^2}{\frac{\pi}{4}(100)^2} = 0.81$, $C_v = 0.95$ $\therefore C_d = C_c \times C_d = 0.81 \times 0.95 = 0.7695$

3. Ans. (a)

It is to be noted that the side of the reservoir having rectangular opening into air (as denoted by two triangles) should have a average theoretical velocity of water given by,

$$V_A = \sqrt{2gH} \quad \text{where } H = \frac{H_1 + H_2}{2}$$

Theoretical velocity of water from under a gate onto the floor (see the figure) is given by

$$V_B = \sqrt{2gH_1}$$

$$\text{Velocity Ratio, } \frac{V_A}{V_B} = \sqrt{\frac{H_1 + H_2}{2H_1}}$$

4. Ans. (b)

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FLOW OVER NOTCHES AND WEIRS

Questions (IAS, IES, GATE)

Discharge over a Rectangular Notch or Weir

1. Which of the following is/are related to measure the discharge by a rectangular notch? **[IES-2002]**

1. $2/3\ C_d.\ b\ \sqrt{2g}\ H^2$
2. $2/3\ C_d.\ b\ \sqrt{2g}\ H^{3/2}$
3. $2/3\ C_d.\ b\ \sqrt{2g}\ H^{5/2}$
4. $2/3\ C_d.\ b\ \sqrt{2g}\ H^{1/2}$

Select the correct answer using the codes given below:

[a]. 1 and 3 [b]. 2 and 3 [c]. 2 alone [d]. 4 alone

Discharge over a Triangular Notch or Weir

2. A triangular notch is more accurate measuring device than the rectangular notch for measuring which one of the following? **[IAS-2007]**

(a) Low flow rates (b) Medium flow rate (c) High flow rates (d) All flow rates

3. Match List I with List II and select the correct answer using the code given below the lists: **[IES-2007]**

List I (Measuring Instrument)	List II (Variable to be measured)
A. Hot-wire anemometer	1. Discharge
B. Pitot-tube	2. Rotational speed
C. V-notch weir	3. Velocity fluctuations
D. Tachometer	4. Stagnation pressure

Code:

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	4	2	1
(c)	4	3	1	2	(d)	3	4	1	2

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4. A standard 90° V-notch weir is used to measure discharge. The discharge is Q_1 for heights H_1 above the sill and Q_2 is the discharge for a height H_2. If H_2 / H_1 is 4, then Q_2 / Q_1 is: **[IES-2001]**

[a]. 32 [b]. $16\sqrt{2}$ [c]. 16 [d]. 8

Answers

1. Ans. (c)
2. Ans. (a)
3. Ans. (d)
4. Ans. (d)

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FLOW AROUND SUBMERGED BODIES-DRAG AND LIFT

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. A body wholly immersed in a real fluid may be subjected to the following forces:
Drag force (F_D): It is the force exerted by fluid in the direction of flow (free stream).
Lift force (F_L): It is the force exerted by fluid at the right angles to the direction of flow.

2. The mathematical expressions for F_D and F_L are:

$$F_D = C_D \times \frac{\rho U^2}{2} \times A \quad and \quad F_L = C_L \times \frac{\rho U^2}{2} \times A$$

Where C_D = co-efficient of drag.
C_L = co-efficient of lift.
U = free stream velocity of fluid.
ρ = density of fluid. and
A= projected area of the body.

Resultant force, F = $\sqrt{F_D^2 + F_L^2}$

3. Total drag on a body = pressure drag + friction drag

4. A body whose surface coincides with the streamlines when placed in a flow is called a streamlined body. If the surface of the body does not coincide with the streamlines, the body in called **bluff body·**

5. Stokes found out that for R_e < 0·2 the total drag on a sphere is given by

$F_D = 3\pi\mu DU$; and of the total drag

Skin Friction drag = $\frac{2}{3} \times 3\pi\mu DU = 2\pi\mu DU$,and

Pressure drag = $\frac{1}{3} \times 3\pi\mu DU = \pi\mu DU$

6. For sphere, the values of C_D for different Reynolds number are:

Reynolds number (Re)	C_D
(i) Less than 0.2	$\frac{24}{\text{Re}}$
(ii) Between 0.2 and 5.0	$\frac{24}{Re}\left(1+\frac{3}{16Re}\right)$
(iii) Between 5 and 1000	0.4
(iv) Between 1000 and 10^5	0.5
(v) Greater than 10^5	0.2

7. The terminal velocity is the maximum velocity attained by a falling body. The terminal velocity of a body falling through a liquid at rest is calculated from the following relation: $W = F_D + F_B$
Where F_D and F_B are the drag force and buoyant force respectively, acting vertically upward.

8. The velocity of ideal fluid at any point on the surface of the cylinder is given by

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u_θ = 2U sin θ

Where u_θ = tangential velocity on the surface of the cylinder,

U = uniform velocity (or free stream velocity),

θ = the angular distance of the point from the forward stagnation point.

9. The peripheral velocity on the surface of the cylinder due to circulation (u_c) is given by:

$$u_c = \frac{\Gamma}{2\pi R}$$

where Γ = circulation, and R = radius of the cylinder.

10. The resultant velocity on a circular cylinder which is rotated at a constant speed in a uniform flow field is given by,

$$u = u_e + u_c = 2U \sin\theta + \frac{\Gamma}{2\pi R}$$

11. The position of stagnation points is given by

$$\sin\theta = -\frac{\Gamma}{4\pi RU}$$

For a single stagnation point, the condition is $\Gamma = 4\pi UR$ ---in terms of circulation

$u_C = 2U$ ...in terms of tangential velocity.

12. The pressure at any point on the cylinder surface (p) is given by

$$p = p_o + \frac{1}{2}\rho U^2\left[1 - \left(2\sin\theta + \frac{\Gamma}{2\pi UR}\right)^2\right]$$

Where p_o = the pressure in the uniform flow at some distance ahead of cylinder.

13. When a circular cylinder is rotated in a uniform flow, a lift force (F_L) is produced on the cylinder. the magnitude of which is given by

$$F_L = \rho LU\Gamma$$

This equation is known as Kutta-Joukowski equation.

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14. The expression for lift co-efficient for a rotating cylinder in a uniform flow is given by $C_L = \frac{\Gamma}{UR}$ ...in terms of circulation

$C_L = 2\pi \frac{u_c}{U}$ ...in terms of tangential velocity.

15. The generation of lift by spinning cylinder in a fluid stream is called Magnus effect.

16. Circulation developed on the airfoil is given by

$$\Gamma = \pi c U \sin\alpha$$

Where c = chord length. α = angle of attack.

17. The expression for co-efficient of lift for an airfoil is given by

$$C_L = 2\pi \sin\alpha$$

18. When an aeroplane is in stready-state.

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(i) The weight of aeroplane (W) = the lift force $\left(C_L \times \frac{\rho U^2}{2} \times A\right)$

(ii) The thrust developed by the engine = the drag force.

QUESTIONS (IES, IAS, GATE)

Force Exerted by a Flowing Fluid on a Body

1. Whenever a plate is submerged at an angle with the direction of flow of liquid, it is subjected to some pressure. What is the component of this pressure in the direction of flow of liquid, known as? **[IES-2007]**

(a) Stagnation pressure (b) Lift (c) Drag (d) Bulk modulus

Expressions for Drag and Lift

2. Assertion (A): A body with large curvature causes a larger pressure drag and, therefore, larger resistance to motion.
Reason(R): Large curvature diverges the streamlines, decreases the velocity resulting in the increase in pressure and development of adverse pressure gradient leading to reverse flow near the boundary. **[IAS-2002]**

3. The drag force exerted by a fluid on a body immersed in the fluid is due to: **[IES-2002]**

[a]. pressure and viscous forces [b]. pressure and gravity forces

[c]. pressure and surface tension forces [d]. viscous and gravity forces.

4. Assertion (A): In flow over immersed bodies. **[IAS-1995]**
Reason(R): drag can be created without life.
life cannot be created without drag

5. An automobile moving at a velocity of 40 km/hr is experiencing a wide resistance of 2 kN. If the automobile is moving at a velocity of 50 km/hr, the power required to overcome the wind resistance is **[IES-2000]**

[a]. 43.4 kW [b]. 3.125 kW [c]. 2.5 kW [d]. 27.776 kW

6. Which one of the following causes lift on an immersed body in a fluid stream? **[IES-2005]**

[a]. Buoyant forces.
[b]. Resultant fluid force on the body.
[c]. dynamic fluid force component exerted on the body parallel to the approach velocity.
[d]. Dynamic fluid force component exerted on the body perpendicular to the approach velocity.

Stream-lined and Bluff Bodies

7. Improved streaming produces 25% reduction in the drag coefficient of a torpedo. When it is travelling fully submerged and assuming the driving power to remain the same, the crease in speed will be: **[IES-2000]**

[a]. 10% [b]. 20% [c]. 25% [d]. 30%

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8. Match List I with List II and select the correct answer: **[IES-2001]**

	List I		List II
A.	Stokes' law	1.	Strouhal number
B.	Bluff body	2.	Creeping motion
C.	Streamline body	3.	Pressure drag
D.	Karman Vortex Street	4.	Skin friction drag.

Codes:

	A	B	C	D		A	B	C	D
[a].	2	3	1	4	[b].	3	2	4	1
[c].	2	3	4	1	[d].	3	2	1	4

Terminal velocity of a body

9. A parachutist has a mass of 90 kg and a projected frontal area of 0.30 m^2 in free fall. The drag coefficient based on frontal area is found to be 0.75. If the air density is 1.28 kg/m^3, the terminal velocity of the parachutist will be: **[IES-1999]**

[a]. 104.4 m/s [b]. 78.3 m/s [c]. 25 m/s [d]. 18.5 m/s

Circulation and Lift on a Circular Cylinder

10. The parameters for ideal fluid flow around a rotating circular cylinder can be obtained by superposition of some elementary flows. Which one of the following sets would describe the flow around a rotating circular cylinder? **[IES-1997]**

[a]. Doublet, vortex and uniform flow. [b]. Source, vortex and uniform flow.
[c]. Sink, vortex and uniform flow [d]. Vortex and uniform flow.

11. When a cylinder is placed in an ideal fluid and the flow is uniform, the pressure coefficient C_p is equal to: **[IES-2000]**

[a]. $1 - \sin^2$ [b]. $1 - 2\sin^2$ c]. $1 - 4\sin^2$ [d]. $1 - 8\sin^2$

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12. Match List I (Types of flow) with List II (Basic ideal flows) and select the correct answer: **[IES-2001, IAS-2003]**

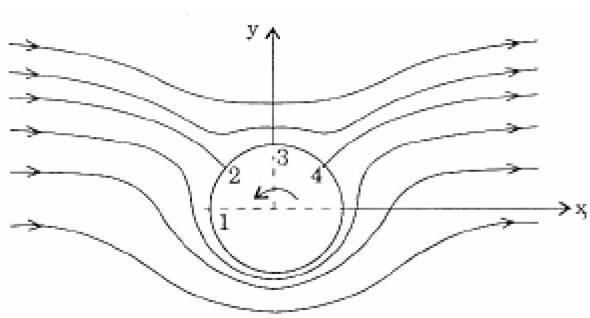
	List I	List II
A.	Flow over a stationary cylinder	1. source + sink + uniform flow
B.	Flow over a half Rankine body	2. doublet + uniform flow
C.	Flow over a rotating body	3. source + uniform flow
D.	Flow over a Rankine oval	4. doublet + free vortex + uniform flow.

Codes :

	A	B	C	D		A	B	C	D
[a].	1	4	3	2	[b].	2	4	3	1
[c].	1	3	4	2	[d].	2	3	4	1

Position of stagnation points

13. A cylindrical object is rotated with constant angular velocity about its symmetry axis in a uniform flow field of an ideal fluid producing streamlines as shown in the figure given above. At which point(s), is the pressure on the cylinder surface maximum? **[IES-2007]**



(a) Only at point 3 (b) Only at point 2 (c) At points 1 and 3 (d) At points 2 and 4

14. A circular cylinder of 400 mm diameter is rotated about its axis in a stream of water having a uniform velocity of 4 m/s. When both the stagnation points coincide, the lift force experienced by the cylinder is: **[IES-2000]**

[a]. 160 kN/m [b]. 10.05 kN/m [c]. 80 kN/m [d]. 40.2 kN/m

Expression for lift co-efficient for rotating cylinder

15. Which one of the following sets of standard flows is superimposed to represent the flow around a rotating cylinder? **[IES-2000]**

[a]. Doublet, vortex and uniform flow [b]. Source, vortex and uniform flow.

[c]. Sink, vortex and uniform flow [d]. Vortex and uniform flow.

Magnus effect

16. The Magnus effect is defined as **[IAS-2002]**

(a) the generation of lift per unit drag force
(b) the circulation induced in an aircraft wing
(c) the separation of boundary layer near the trailing edge of a slender body
(d) the generation of lift on a rotating cylinder in a uniform flow

17. Consider the following statements:

1. The phenomenon of lift produced by imposing circulation over a doublet in a uniform flow is known as Magnus effect.
2. The path-deviation of a cricket ball from its original trajectory is due to the Magnus effect.

Which of the statement given above is/are correct? **[IES-2007]**

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) neither 1 nor 2

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Lift on an Airfoil

18. Consider the following statements: **[IES-1999]**

1. The cause of stalling of an aerofoil is the boundary layer separation and formation of increased zone of wake.
2. An aerofoil should have a rounded nose in supersonic flow to prevent formation of bow shock.
3. When an aerofoil operates at an angle of incidence greater than that of stalling, the lift decreases and drag increase.
4. A rough ball when at certain speeds can attain longer range due to reduction of lift as the roughness induces early separation.

Which of these statements are correct?

[a]. 3 and 4 [b]. 1 and 2 [c]. 2 and 4 [d]. 1 and 3.

19. Which one of the following is true of flow around a submerged body? **[IES-1998]**

[a]. For subsonic, non-viscous flow, the drag is zero

[b]. For supersonic flow, the drag coefficient is dependent equally on Mach number and Reynolds number

[c]. the lift and drag coefficients of an aerofoil is independent of Reynolds number

[d]. for incompressible flow around an aerofoil, the profile drag is the sum of from drag and skin friction drag.

20. When pressure drag over a body is large as compared to the friction drag, then the shape of the body is that of: **[IES-2000]**

[a]. an aerofoil [b]. a streamlined body

[c]. a two-dimensional body [d]. a bluff body.

21. Assertion (A): Aircraft wings are slotted to control separation of boundary layer especially at large angles of attack. **[IES-2003]**

Reason (R): This helps to increase the lift and the aircraft can take off from, and land on, short runways.

Answers with Explanation

1. Ans. (c)
2. Ans. (a)
3. Ans. (a)
4. Ans. (b). Both the statements of A and R are true, but R is not necessarily the explanation for A.

5. Ans. (a) Power, $P = F_D \times V = C_D \times \frac{\rho V^2}{2} \times A \times V \;\; Or \;\; P \infty V^3$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^3 \;\; or\; P_2 = (F_{D1} \times V_1) \times \left(\frac{V_2}{V_1}\right)^3 = \left(2 \times 40 \times \frac{5}{18}\right) \times \left(\frac{50}{40}\right)^3 = 43.4\,kW$$

6. Ans. (d)

7. Ans. (a) $C_{D1} \times V_1^3 = C_{D2} \times V_2^3 \;\; or\; \frac{V_2}{V_1} = \sqrt[3]{\frac{C_{D1}}{C_{D2}}} = \sqrt[3]{\frac{100}{75}} = 1.10$

8. Ans. (c)

9. Ans. (b) $Total\ Drag(F_D) = Weight(W) \;\; or\; C_D \times \frac{\rho V^2}{2} \times A = mg$

$$or V = \sqrt{\frac{2mg}{C_D \times \rho \times A}} = \sqrt{\frac{2 \times 90 \times 9.81}{0.75 \times 1.28 \times 0.3}} = 78.3\,m/s$$

10. Ans. (a)
11. Ans. (*)
12. Ans. (b)
13. Ans. (d)
14. Ans. (d)

For single stagnation point, Circulation $(\Gamma) = 4\pi VR = 4\pi \times 4 \times \frac{0.400}{2} = 10.05\,m^2/s$

And Lift force $(F_L) = \rho L V \Gamma = 1000 \times L \times 4 \times 10.05\,N \Rightarrow \frac{F_L}{L} = 40.2\,kN/m$

15. Ans. (a)
16. Ans. (d)
17. Ans. (c)
18. Ans. (d)
19. Ans. (b)
20. Ans. (a)
21. Ans. (b)

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COMPRESSIBLE FLOW

Skip to Questions (IAS, IES, GATE)

Highlight

1. A fluid is said to be incompressible if its density does not change, or changes very little, with a change in pressure.
If Mach number (M) < 0.3 the flow is incompressible flow.
A compressible flow is that flow in which the density of the fluid changes during flow.
For Bernoulli's equation we may use absolute or gauge pressure because the atmospheric effect cancel out from both sides but **for compressible flow always use absolute pressure.**
PV= mRT, P must be absolute pressure.
In Pressure head, $\frac{P}{\rho g}$, P must be absolute pressure.

2. **Basic Thermodynamic Relations**
Specific heat at constant pressure, $C_p = \frac{\gamma R}{\gamma - 1} = \left(\frac{\partial h}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p$ = a + kT
Specific heat at constant volume, $C_v = \frac{R}{\gamma - 1} = \left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v$ = b + kT
$$C_p - C_v = R \quad and \quad R = \frac{R_{univ}}{M}$$

3. **Adiabatic index** $(\gamma) = \frac{C_p}{C_v}$, if $T\uparrow \quad then\, \gamma \downarrow$
$\gamma = 1 + \frac{2}{n}$, Where n in d.o.f of molecule
$= \frac{5}{3} = 1.67$, for monoatomic gas (n = 3)
$= \frac{7}{5} = 1.4$, for diatomic gas (n = 5)
$= \frac{4}{3} = 1.33$, for tri-atomic gas (n = 6)

For steam γ = 1.3 for superheated steam
γ = 1.135 for dry saturated steam
γ = 1.035 + 0.1 x for wet steam, where 'x' is dryness fraction

4. **Sonic velocity or velocity of sound,** C $= \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} = \sqrt{\gamma RT}$

5. **Mach number,** M $= \frac{Velocity\ of\ fluid}{Velocity\ of\ sound} = \frac{V}{C} = \sqrt{\frac{inertia\ force}{elastic\ force}}$

6. **Propagation of disturbance in compressible fluid**
Case I: When M <1 (i.e. V < C). In this case since V < C the projectile lags behind the disturbance/pressure wave and hence as shown in Fig.(a) the projectile at point B lies

inside the sphere of radius Ct and also inside other spheres formed by the disturbances/ waves started at intermediate points.

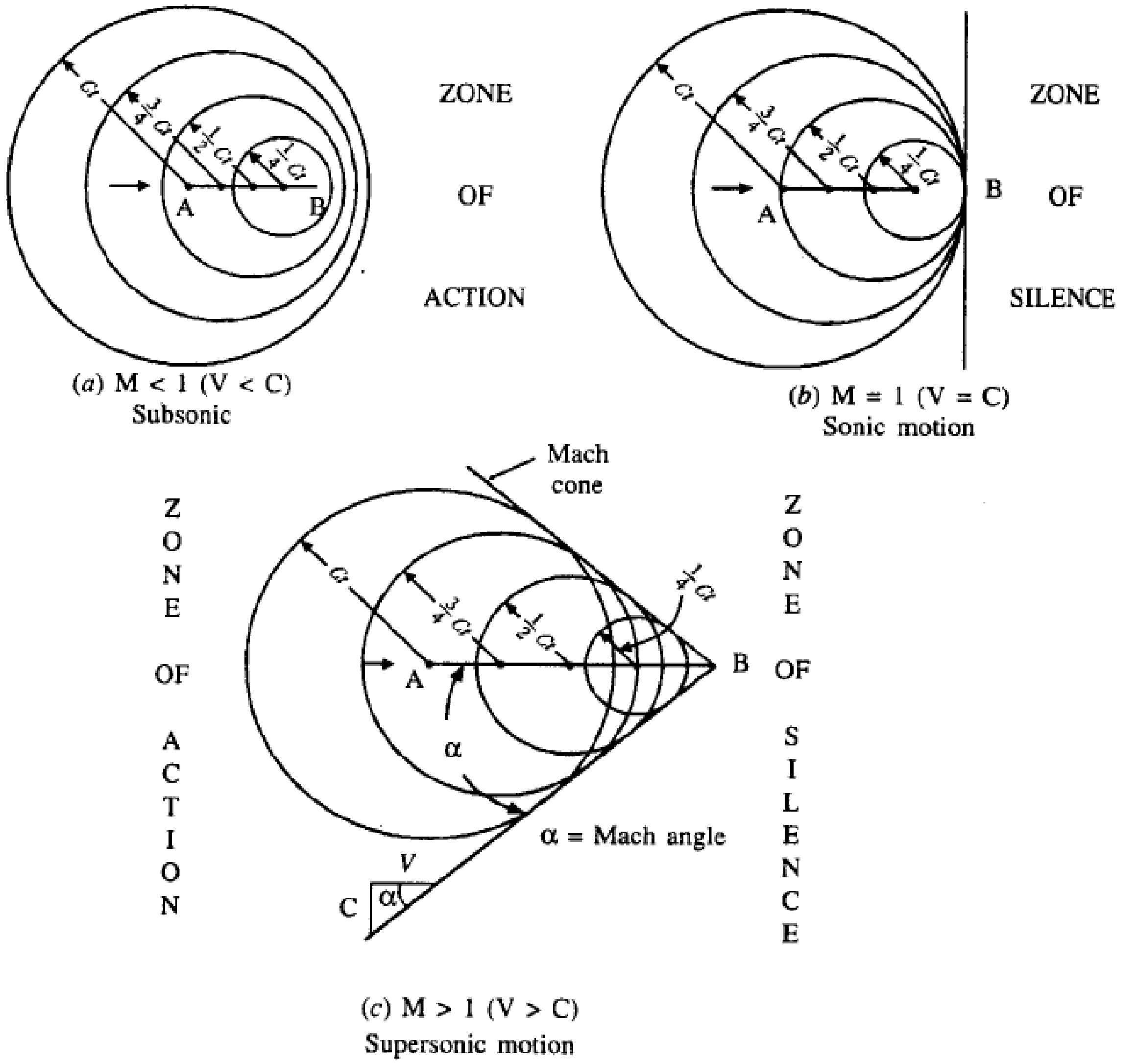


Fig. Nature of propagation of disturbances in compressible flow

Case II: When M = 1 (i.e. V = C). In this case, the disturbance always travels with the projectile as shown in Fig.(b). The circle drawn with centre A will pass through B.

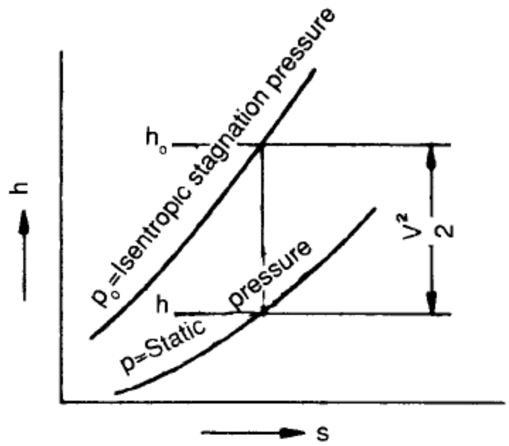
Case III: When M > 1 (i.e. V > C). In this case the projectile travels faster than the disturbance. Thus the distance AB (which the projectile has travelled) is more than Ct, and hence the projectile at point 'B' is outside the spheres formed due to formation and growth of disturbance at t = 0 and at the intermediate points (Fig. (c) If the tangents are drawn (from the point B) to the circles, the spherical pressure waves form a cone with its vertex at B. It is known as Mach cone. The semi-vertex angle a of the cone is known as Mach angle which is given by,

$$\sin\alpha = \frac{Ct}{Vt} = \frac{C}{V} = \frac{1}{M}$$

In such a case (M>1), the effect of the disturbance is felt only in region inside the Mach cone, this region is called **zone of action**. The region outside the Mach cone is called **zone of silence.**
It has been observed that when an aeroplane is moving with supersonic speed, its noise is heard only after the plane has already passed over us.

7. Stagnation Properties

The isentropic stagnation state is defined as the state a fluid in motion would reach if it were brought to rest *isentropically* in a *steady flow*, *adiabatic*, *zero work output* devices.



$$h_o = h + \frac{V^2}{2}$$

$$C_p T_o = C_p T + \frac{V^2}{2}$$

$$or\ \frac{T_o}{T} = 1 + \frac{V^2}{2C_p T} \quad Where\ C_p = \frac{\gamma R}{\gamma - 1}$$

$$Or\ \frac{T_o}{T} = 1 + \frac{V^2(\gamma - 1)}{2\gamma RT}$$

$$Or\ \frac{T_o}{T} = 1 + \frac{(\gamma - 1)}{2} M^2$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)} = \left[1 + \frac{(\gamma-1)}{2} M^2\right]^{\gamma/(\gamma-1)}$$

$$\frac{\rho_o}{\rho} = \left[1 + \frac{(\gamma-1)}{2} M^2\right]^{1/(\gamma-1)}$$

$p_o = p + \frac{\rho V^2}{2}$, When compressibility effects are neglected

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8. Area-Velocity Relationship and Effect of Variation of Area for Subsonic, Sonic and Supersonic Flows

$$\frac{dA}{A} = \frac{dV}{V}\left(M^2 - 1\right)$$

$$Or\ \frac{dA}{A} = \frac{dp}{\rho V^2}\left(1 - M^2\right)$$

$$\frac{dM}{M} = \frac{dV}{V}\left[1 + \frac{\gamma - 1}{2} M^2\right]$$

$$\frac{dA}{A} = \frac{(M^2 - 1)}{\left(1 + \frac{\gamma - 1}{2} M^2\right)} \frac{dM}{M}$$

dA = 0 or A = constant. So M = 1 occurs only at the throat and nowhere else, and this happens only when the discharge is the maximum.
If the convergent – divergent duct acts as a nozzle, in the divergent part also, the pressure will fall continuously to yield a continuous rise in velocity. The velocity of the gas is subsonic before the throat, becomes sonic at the throat, and then becomes supersonic till its exit in isentropic flow, provided the exhaust pressure is low enough.

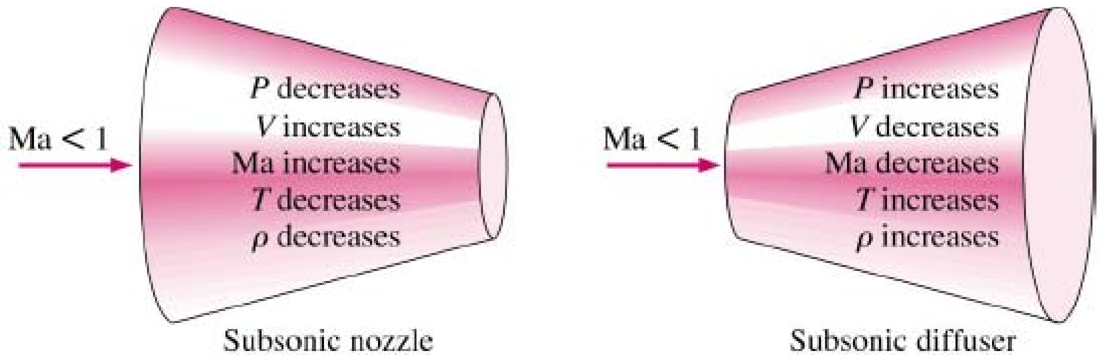

Fig. Subsonic flow (M < 1)

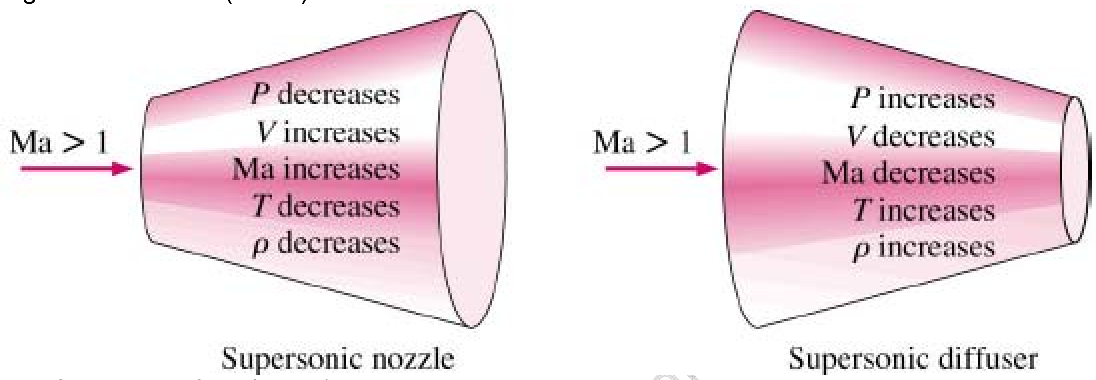

Fig. Supersonic flow (M > 1)

9. Flow of Compressible Fluid Through a Convergent Nozzle

$$Exit\,Velocity\,(V_2) = \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

10. Critical value of pressure ratio, $\left(\frac{p_2}{p_1}\right)^* = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ [VVIMP]

For air, γ=1.4 and $\left(\frac{p_2}{p_1}\right)^* = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}}$ = **0. 528**

For superheated steam γ=1.3 and $\left(\frac{p_2}{p_1}\right)^* = \left(\frac{2}{1.3+1}\right)^{\frac{1.3}{1.3-1}} = 0.546$

For dry saturated steam γ=1.135 and $\left(\frac{p_2}{p_1}\right)^* = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{1.135-1}} = 0.577$

11. Normal shock wave

Whenever a supersonic flow (compressible) abruptly changes to subsonic flow a shock wave (analogous to hydraulic jump in an open channel) is produced, resulting in a sudden rise in pressure, density, temperature and entropy.

A shock wave takes place in the diverging section of a nozzle, in a diffuser, throat of a supersonic wind tunnel, in front of sharp-nosed bodies.

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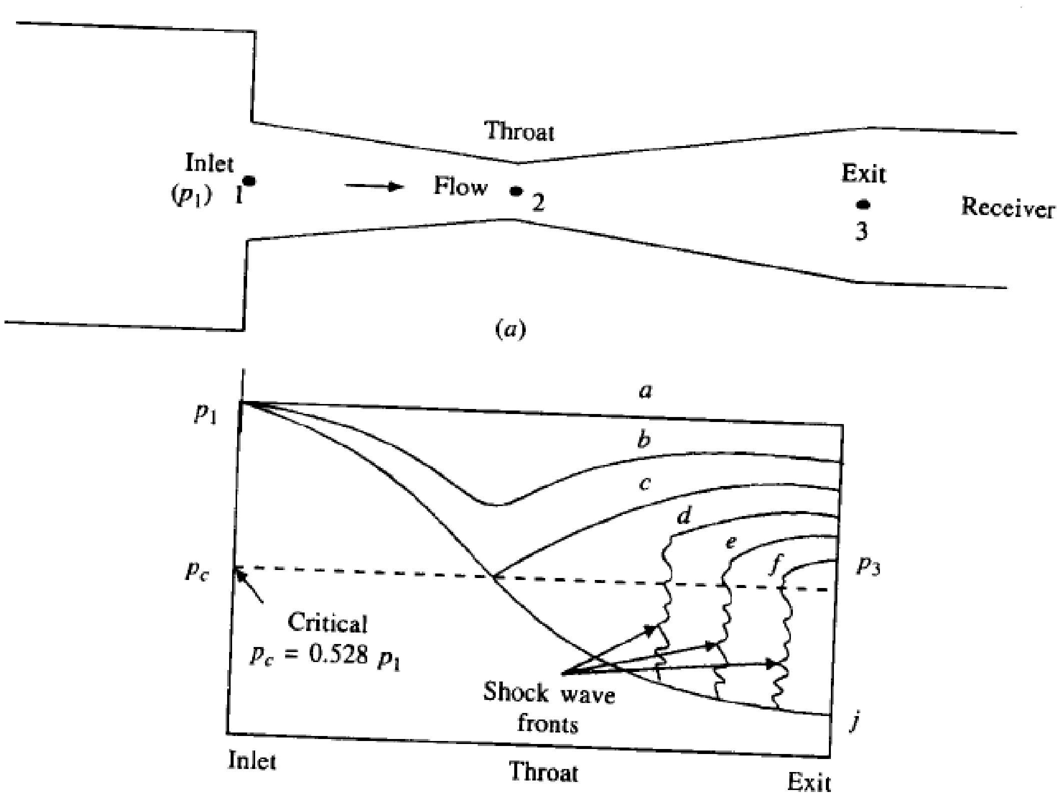


12. Characteristics of a normal shock:

- Shock occurs only when the flow is supersonic, and after the shock the flow becomes subsonic, when the rest of the diverging portion acts as a diffuser. **[VIMP]**
- The stagnation temperature remains the same across the normal shock and hence all over the flow. **[VIMP]**
- Stagnation pressure and stagnation density decreases with M_1 in the same ratio, $\frac{p_{o2}}{p_{o1}} = \frac{\rho_{o2}}{\rho_{o1}}$
- Entropy increases across a shock with consequent decrease in stagnation pressure and stagnation density across the shock.
- Mach number relation , $M_2^2 = \frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}$ **[IMP]**

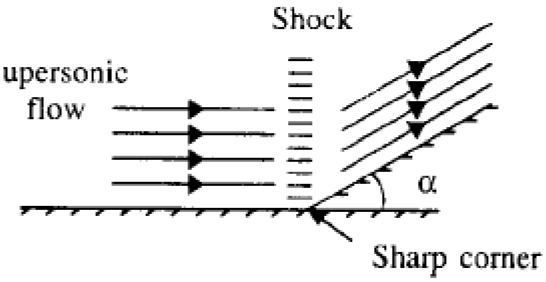
13. Shock strength

The strength of shock is defined as the ratio of pressure rise across the shock to the upstream pressure.

Strength of shock $= \frac{p_2 - p_1}{p_1} = \frac{2\gamma}{\gamma+1}(M_1^2-1)$

14. Oblique shock wave

When a supersonic flow undergoes a sudden turn through a small angle α (positive), an oblique wave is established at the corner.



15. Equations in Normal shock

Continuity equation, G = $\rho_1 V_1 = \rho_2 V_2$ Where G is mass velocity kg/m^{2}s

Momentum equation, F = $p_1 A_1 + \rho_1 A_1 V_1^2 = p_2 A_2 + \rho_2 A_2 V_2^2$ Where $A_1 = A_2$

Energy equation: Stagnation enthalpy, $h_o = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

16. Fanno line

Combining Continuity and Energy equation, $h = h_o - \frac{G^2}{2\rho^2}$

The line resenting the locus of points with the same mass velocity and stagnation enthalpy is called a Fanno line. The end states of the normal shock must lie on the Fanno line.

Adiabatic flow in a **constant area duct *with friction***, in a one dimensional model, has both constant G and constant h_o and hence must follow a Fanno line.

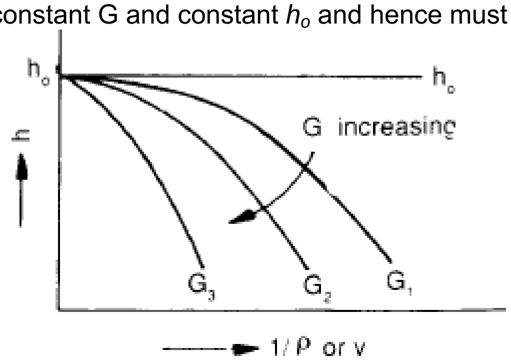


Fig. Fanno line on h – 1/ρ coordinate

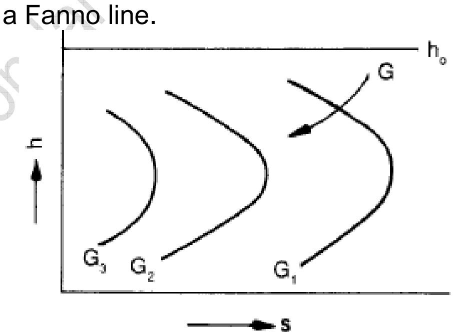


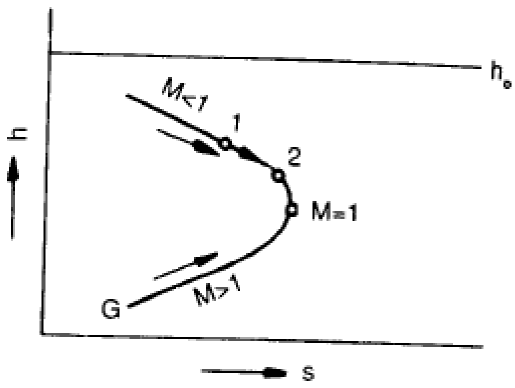
Fig. Fanno line on h-s diagram

17. Reyleigh line

Combining continuity and Momentum equation, Impulse pressure, $I = \frac{F}{A} = p + \frac{G^2}{\rho}$

The line resenting the locus of points with the same impulse pressure and mass velocity is called a Rayleigh line. The end states of the normal shock must lie on the Rayleigh line.

The Rayleigh line is a model for flow in a **constant area duct *with heat transfer***, but **without friction**.



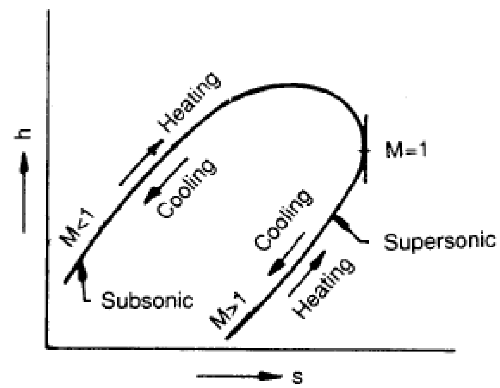
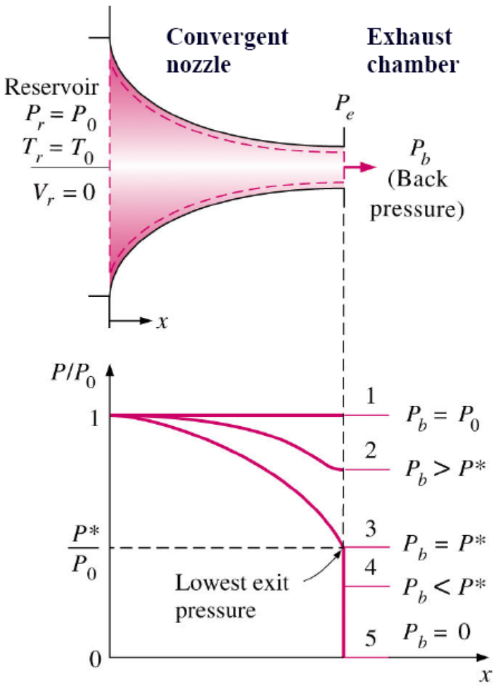


Fig. Fig. Fanno line on h-s plot   Fig. Rayleigh line on h-s plot

From the above diagram it is clear when entropy is maximum Mach number is unity.

19. Isentropic flow through nozzles-Converging Nozzles



- State 1: $P_b = P_0$, there is no flow, and pressure is constant.
- State 2: $P_b < P_0$, pressure along nozzle decreases . Also $P_b > P^*$ (critical).
- State 3: $P_b = P^*$, flow at exit is sonic (M=1), creating maximum flow rate called **choked flow.**

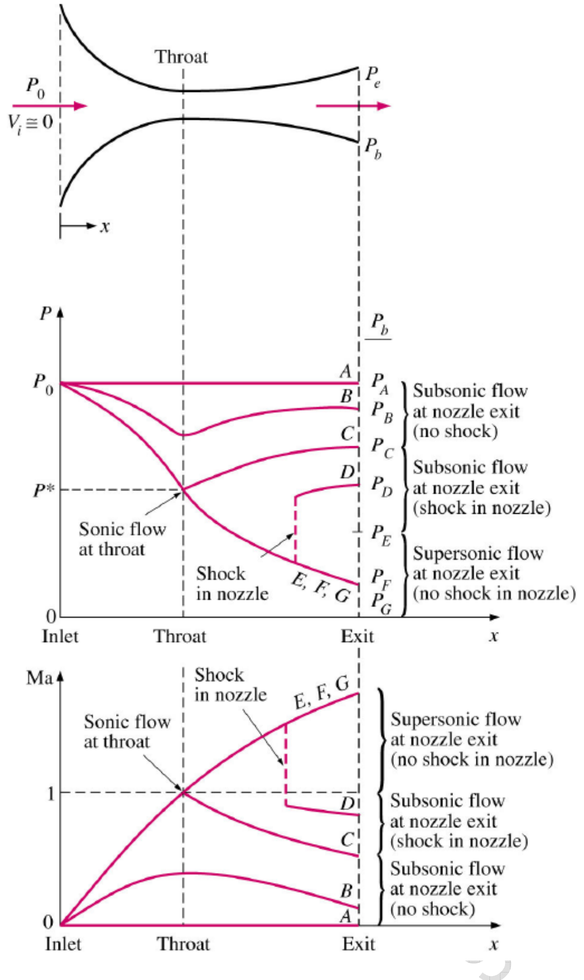
For the above states, *nozzle exit pr is same as exhaust chamber pressure*

- State 4: $P_b < P^*$, there is no change in flow or pressure distribution in comparison to state 3

Here *nozzle exit pressure P_e does not decrease even when P_b is further reduced below critical. This change of pressure from P_e to P_b takes place outside the nozzle exit through a expansion wave*

- State 5: $P_b = 0$, same as state 4.

20. **Isentropic flow through nozzles-Converging-Diverging Nozzles**



1. $P_0 > P_b > P_C$
   - Flow remains subsonic, and mass flow is less than for choked flow. Diverging section acts as diffuser
2. $P_b = P_C$
   - Sonic flow achieved at throat. Diverging section acts as diffuser. Subsonic flow at exit. Further decrease in Pb has no effect on flow in converging portion of nozzle
3. $P_C > P_b > P_E$
   - Fluid is accelerated to supersonic velocities in the diverging section as the pressure decreases. However, acceleration stops at location of **normal shock.** Fluid decelerates and is subsonic at outlet. As P_b is decreased, shock approaches nozzle exit.
4. $P_E > P_b > 0$
   - Flow in diverging section is supersonic with no shock forming in the nozzle. Without shock, flow in nozzle can be treated as isentropic.

Questions (IES, IAS, GATE)

Compressible flow

1. Net force on a control volume due to uniform normal pressure alone **[GATE-1994]**
(a) depends upon the shape of the control volume. (b) translation and rotation
(c) translation and deformation (d) deformation only

Basic Thermodynamic Relations

2. Match List I and List II for questions below. No credit will be given for partial matching in each equation. Write your answers using only the letters A to D and numbers 1 to 6. **[GATE]**

List I	List II
(a) Steam nozzle	1. Mach Number
(b)Compressible flow	2. Reaction Turbine
(c) Surface tension	3. Biot Number
(d) Heat conduction	4. Nusselt Number
	5. Supersaturation
	6. Weber Number

Sonic velocity

3. For a compressible fluid, sonic velocity is **[GATE-2000]**
(a) A property of the fluid
(b) Always given by $(\gamma RT)^{1/2}$ where γ, R and T are respectively the ratio of specific heats, gas constant and temperature in K
(c) Always given by $(\partial p/\partial p)s^{1/2}$. Where p, ρ and s are respectively pressure, density and entropy.
(d) Always greater than the velocity of fluid at any location.

Mach number

4. If a bullet is fired in standard air at 15°C at the Mach angle of 30°, the velocity of the bullet would be: **[IES-2000]**
[a]. 513.5 m/s [b]. 585.5 m / s [c]. 645.5 m / s [d]. 680.5 m / s

5. The stagnation temperature of an isentropic flow of air (k = 1.4) is 400 K. If the temperature is 200K at a section, then the Mach number of the flow will be:
[a]. 1.046 [b]. 1.264 [c]. 2.236 [d]. 3.211 **[IES-1998]**

6. An aero plane travels at 400 km/hr at sea level where the temperature is 15°C. The velocity of the aero plane at the same Mach number at an altitude where a temperature of – 25°C prevailing, would be: **[IES-2000]**
[a]. 126.78 km/hr [b]. 130.6 km/hr [c]. 371.2 km/hr [d]. 400.10 km/hr

Propagation of Disturbance in Compressible Fluid

7. An aircraft flying at an altitude where the pressure was 35 kPa and temperature -38^{0}C, stagnation pressure measured was 65.4 kPa. Calculate the speed of the aircraft. Take molecular weight of air as 28. **[IES-1998]**

8. The eye of a tornado has a radius of 40 m. If the maximum wind velocity is 50 m/s, the velocity at a distance of 80 m radius is: **[IES-2000]**
[a]. 100 m /s [b]. 2500 m /s [c]. 31.25 m /s [d]. 25 m /s

Stagnation Properties

9. In adiabatic flow with friction, the stagnation temperature along a streamline................. (increases/decreases/remains constant) **[GATE-1995]**

10. While measuring the velocity of air (□ = 1.2 kg/m^3), the difference in the stagnation and static pressures of a Pitot-static tube was found to be 380 Pa. The velocity at that location in m/s is: [a]. 24.03 [b]. 4.02 [c]. 17.8 [d]. 25.17 **[IES-2002]**

11. Match List I (Property ratios at the critical and stagnation conditions) with List II (values of ratios) and select the correct answer using the codes given below the Lists:

List I	List II
A. $\frac{T^*}{T_0}$	1. $\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$
B. $\frac{\rho^*}{\rho_0}$	2. $\frac{2}{\gamma+1}$

[IES-1997]

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C. $\frac{p*}{p_0}$ 3. 1

D. $\frac{S*}{S_0}$ 4. $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

Codes:

	A	B	C	D		A	B	C	D
[a].	2	1	4	3	[b].	1	2	3	4
[c].	2	1	3	4	[d].	1	2	4	3

12. In isentropic flow between two points, the stagnation: **[IES-1998]**

[a]. pressure and stagnation temperature may vary

[b]. pressure would decrease in the direction of the flow.

[c]. pressure and stagnation temperature would decrease with an increase in velocity

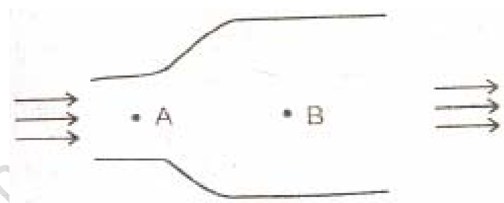
[d]. pressure, stagnation temperature and stagnation density would remain constant throughout the flow.

Area-Velocity Relationship and Effect of Variation of Area for Subsonic, Sonic and Supersonic Flows

13. A compressible fluid flows through a passage as shown in the above diagram. The velocity of the fluid at the point A is 400 m/s.

Which one of the following is correct?

At the point B, the fluid experiences



[IES-2004]

[a]. an increase in velocity and decrease in pressure

[b]. a decrease in velocity and increase in pressure

[c]. a decrease in velocity and pressure

[d]. an increase in velocity and pressure.

14. During subsonic, adiabatic flow of gases in pipes with friction, the flow properties go through particular mode of changes. Match List I (Flow properties) with List II (Mode of changes) and select the correct answer: **[IES-2002]**

	List I	**List II**
A.	Pressure.	1. Increase in flow direction
B.	Density	2. Decreases with flow direction
C.	Temperature.	
D.	Velocity	

Codes:

	A	B	C	D		A	B	C	D
[a].	1	1	2	2	[b].	2	2	2	1
[c].	2	2	1	2	[d].	2	1	1	2

Flow of Compressible Fluid through a Convergent Nozzle

15. Which one of the following is the correct expression for the critical pressure ratio of a nozzle?

[a]. $\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}$ [b]. $\left(\frac{1}{n+1}\right)^{\frac{n}{n-1}}$ [c]. $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$ [d]. $\left(\frac{1}{n+1}\right)^{\frac{1}{n-1}}$ **[IES-2004]**

16. What is the critical pressure ratio for isentropic nozzle flow with ratio of specific heats as 1.5?
[a]. $(0.8)^3$ [b]. $(0.8)^{0.6}$ [c]. $(1.25)^{0.33}$ [d]. $(1.25)^3$ **[IES-2004]**

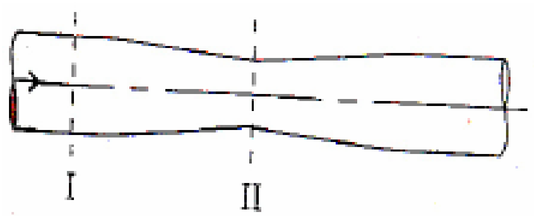
17. If the cross-section of a nozzle is increasing in the direction of flow in supersonic flow, then in the downstream direction. **[IES-2005]**
[a]. Both pressure and velocity will increase.
[b]. Both pressure and velocity will decrease.
[c]. Pressure will increase but velocity will decrease.
[d]. Pressure will decrease but velocity will increase.

18. In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1 + 3x/L)$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is **[GATE-2007]**

(a) $\frac{L}{u_0}$ (b) $\frac{L}{3u_0} In4$ (c) $\frac{L}{4u_0}$ (d) $\frac{L}{2.5u_0}$

Flow through Laval Nozzle (Convergent-Divergent Nozzle)

19. At location-I of a horizontal line, the fluid pressure head is 32 cm and velocity head is 4 cm. The reduction in area at location **II** is such that the pressure head drops down to zero. The ratio of velocities at location -II to that at location-I is: **[IES-2001]**



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[a]. 3 [b]. 2.5 [c]. 2 [d]. 1.5

Normal shock wave

20. Across a normal shock wave in a converging-diverging nozzle for adiabatic flow, which of the following relations are valid?
(a) Continuity and energy equations, equation of state, isentropic relation
(b) Energy and momentum equations, equation of state, isentropic relation
(c) Continuity, energy and momentum equations, equation of state
(d) Equation of state, isentropic relation, momentum equation, mass-conservation Principle **[IES 2007]**

21. In a normal shock wave in one-dimensional flow
(a) pressure, density and temperature increase
(b) velocity, temperature and density increase
(c) pressure, density and temperature decrease
(d) velocity, pressure and density decrease **[IAS-2003]**

22. In a normal shock in a gas, the: **[IES-1998; 2006]**
[a]. upstream shock is supersonic [b]. upstream flow is subsonic
[c]. downstream flow is sonic
[d]. both downstream flow and upstream flow are supersonic.

23. If the upstream Mach number of a normal shock occurring in air (k = 1.4) is 1.68, then the Mach number after the shock is: **[IES-2000]**
[a]. 0.84 [b]. 0.646 [c]. 0.336 [d]. 0.546

24. In a normal shock in a gas: **[IES-2002]**
[a]. the stagnation pressure remains the same on both sides of the shock
[b]. the stagnation density remains the same on both sides of the shock.
[c]. the stagnation temperature remains the same on both sides of the shock
[d]. the Mach number remains the same on both sides of the shock.

25. A normal shock: **[IES-2002]**
[a]. causes a disruption and reversal of flow pattern
[b]. may occur only in a diverging passage
[c]. is more severe than an oblique shock
[d]. moves with a velocity equal to the sonic velocity

26. The fluid property that remains unchanged across a normal shock wave is:
[a]. Stagnation enthalpy [b]. Stagnation pressure.
[c]. Static pressure. [d]. Mass density **[IES-2003]**

27. Consider the following statements:
In the case of convergent nozzle for compressible flow, **[IES-2005]**
1. no shock wave can occur at any pressure ratio.
2. no expansion wave can occur below a certain pressure ratio.
3. expansion wave can occur below a certain pressure ratio
4. shock wave can occur above a certain pressure ratio.
Which of the following statements given above are correct ?
[a]. 1 and 2 [b]. 3 and 4 [c]. 1 and 3 [d]. 2 and 4

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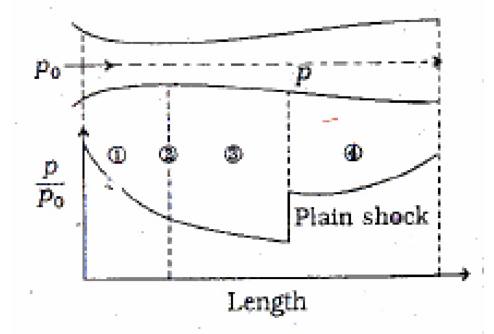
28. The plot for the pressure ratio along the length of convergent-divergent nozzle is shown in the given figure. The sequence of the flow condition labeled ①,②,③ and ④ in the figure is respectively. **[IES-2000]**
[a]. supersonic, sonic, subsonic and supersonic
[b]. sonic, supersonic, subsonic and supersonic
[c]. subsonic, supersonic, sonic and subsonic
[d]. subsonic, sonic, supersonic and subsonic



29. Consider the following statements pertaining to one-dimensional isentropic flow in a convergent-divergent passage: **[IES-2003]**
1. A convergent-divergent passage may function as a supersonic nozzle or a venturi depending on the back pressure.
2. At the throat, sonic conditions exits for subsonic or supersonic flow at the outlet.
3. A supersonic nozzle discharges fluid at constant rate even if the exit pressure is lower than the design pressure.
4. A normal shock appears in the diverging section of the nozzle if the back pressure is above the design pressure but below a certain minimum pressure for venturi operation.

Which of these statements are correct?
[a]. 1, 2, 3 and 4 [b]. 1, 3 and 4 [c]. 2, 3 and 4 [d]. 1 and 2

30. Match List I (Phenomena) with List II (Causes) and select the correct answer: **[IES-2003]**

	List I		List II
A.	Shock wave	1.	Surface tension
B.	Flow separation	2.	Vapour pressure
C.	Capillary rise.	3.	Compressibility
D.	Cavitation	4.	Adverse pressure gradient.

Codes:

	A	B	C	D		A	B	C	D
[a].	3	1	2	4	[b].	4	2	1	3
[c].	3	4	1	2	[d].	4	1	2	3

Oblique shock wave

31. For oblique shock, the downstream Mach number **[IES-1997]**
[a]. is always more than unity [b]. is always less than unity
[c]. may be less or more than unity [d]. can never be unity.

Fanno line

32. Assertion (A): In the case of Fanno line flow, in the subsonic region friction causes irreversible acceleration. **[IES-1997]**
Reason (R): In the case of Fanno line, flow, decrease in entropy is not possible either for supersonic or subsonic flows.

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33. The prime parameter causing change of state in a Fanno flow is: **[IES-1998]**
[a]. heat transfer [b]. Area change [c]. Friction [d]. Buoyancy.

34. Fanno line low is a flow in a constant area duct:
[a]. with friction and heat transfer but in the absence of work.
[b]. with friction and heat transfer and accompanied by work
[c]. with friction but in the absence of heat transfer or work.
[d]. without friction but accompanied by heat transfer and work. **[IES-1997]**

35. Which one of the following statements is correct about the Fanno flow?
(a) For an initially subsonic flow, the effect of friction is to decrease the Mach number towards unity
(b) For an initially supersonic flow, the effect of friction is to increase the Mach number towards unity
(c) At the point of maximum entropy, the Mach number is unity
(d) Stagnation pressure always increases along the Fanno line **[IES 2007]**

Rayleigh line

36. Rayleigh line flow is a flow in a constant area duct: **[IES-1997]**
[a]. with friction but without heat transfer [b]. without friction but with heat transfer
[c]. with both friction and heat transfer [d]. without either friction or heat transfer

37. Which of the following assumptions/conditions are true in the case of Rayleigh flow?
1. Perfect gas. 2. Constant area duct. **[IES-2005]**

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3. Steady one-dimensional real flow. 4. Heat transfer during the flow.

Select the correct answer using the code given below:

[a]. 1, 2 and 3 [b]. 2, 3 and 4 [c]. 1, 3 and 4 [d]. 1, 2 and 4

38. Air at 2 bar and 60°C enters a constant area pipe of 60 mm diameter with a velocity of 40 m/s. During the flow through the pipe, heat is added to the air stream. Frictional effects are negligible and the values of C_p and C_v are that of standard air. The Mach number of the flow corresponding to the maximum entropy will be: **[IES-1999]**

[a]. 0.845 [b]. 1 [c]. 0.1212 [d]. 1.183

Answers with Explanations

1. Ans. (c)

2. Ans. (a)-5,(B)-1,(C)-6,(D)-3

3 Ans. (a) $(\gamma RT)^{1/2}$ only when the process is adiabatic and $(RT)^{1/2}$ when the process is isothermal.

4. Ans. (d) for Mach angle α, $\sin\alpha = \frac{Ct}{Vt} = \frac{C}{V} = \frac{1}{M}$

Where C = $\sqrt{\gamma RT} = \sqrt{1.4\times 287\times(273+15)} = 340 m/s$

$$\therefore V = \frac{C}{\sin\alpha} = \frac{340}{\sin 30} = 680\, m/s$$

5. Ans. (c) $\frac{T_o}{T} = 1+\frac{(\gamma-1)}{2}M^2$ Or $\frac{400}{200} = 1+\frac{(1.4-1)}{2}M^2$ or M = $\sqrt{5} = 2.236$

6. Ans. (c) for same Mach number

$$\frac{V_1}{C_1} = \frac{V_2}{C_2} \Rightarrow V_2 = V_1\times\frac{C_2}{C_1} = V_1\times\sqrt{\frac{T_2}{T_1}} = 400\times\sqrt{\frac{(273-25)}{(273+15)}} = 371.2\, km/hr$$

7. Ans.(349 m/s) Here γ is not given so compressibility is neglected

$p_s = p+\frac{\rho V^2}{2}$ Where, $\rho = \frac{m}{V} = \frac{pM}{RT} = \frac{35\times 28}{8.314\times(273-38)} = 0.5\, kg/m^3$

Therefore $V = \sqrt{\frac{2(p_s-p)}{\rho}} = \sqrt{\frac{2(65.4-35)\times 10^3}{0.5}} = 349\, m/s$

8. Ans. (a)

9. Ans. remains constant.

10. Ans. (d) $p_o = p+\frac{\rho V^2}{2}$, when compressibility effects are neglected

$$V = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2\times 380}{1.2}} = 25.17\, m/s$$

11. Ans. (a)

12. Ans. (b) stagnation temperature cannot vary.

13. Ans. (a) Velocity at A is very high we may say it is supersonic so above diagram is a divergent nozzle.

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14. Ans. (d) Due to friction temperature increase, and pressure decrease in flow direction. Frictional resistance decreases velocity and for same mass flow rate density must increase.

15. Ans. (c)

16. Ans. (a) just use $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

17. Ans. (d)

18. Ans. (b) $dt = \frac{dx}{V} \quad or\, T = \int dt = \int_0^L \frac{dx}{u_o\left(1+\frac{3x}{L}\right)} = \frac{L}{3u_o}\ln 4$

19. Ans. (a) $32 + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \quad or\, \frac{V_2}{V_1} = \sqrt{\frac{32}{V_1^2/2g}+1} = \sqrt{8+1} = 3$

20. Ans. (d)

21. Ans. (a)

22. Ans. (a)

23. Ans. (b) $M_2^2 = \frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}$

$= \frac{(1.4-1)\times 1.68^2+2}{2\times 1.4\times 1.68^2-(1.4-1)} = 0.417 \quad Or\, M_2 = \sqrt{0.417} = 0.646$

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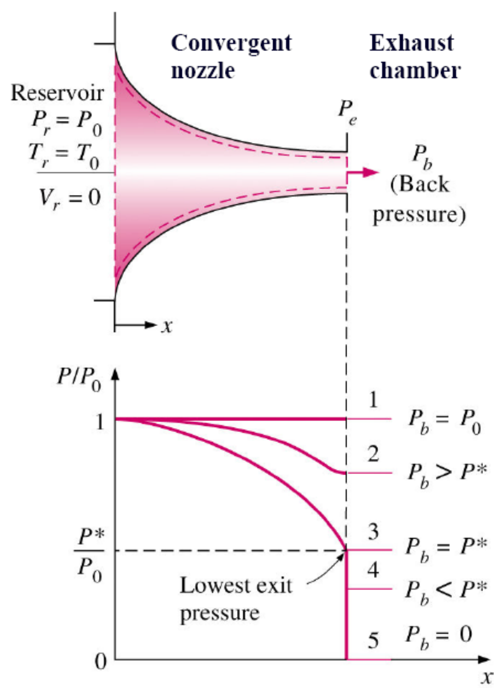
24. Ans. (c)

25. Ans. (b)

26. Ans. (a)

27. Ans. (a)



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- State 2: $P_b < P_0$, pressure along nozzle decreases . Also $P_b > P^*$ (critical).
- State 3: $P_b = P^*$, flow at exit is sonic (M=1), creating maximum flow rate called **choked flow.**

For the above states, *nozzle exit pr is same as exhaust chamber pressure*

- State 4: $P_b < P^*$, there is no change in flow or pressure distribution in comparison to state 3

Here *nozzle exit pressure* P_e *does not decrease even when* P_b *is further reduced below critical. This change of pressure from* P_e *to* P_b *takes place outside the nozzle exit through a expansion wave*

- State 5: $P_b = 0$, same as state 4.

28. Ans. (d)

29. Ans. (b) At the throat, sonic conditions not exits for subsonic flow when it is venturi.

30. Ans. (c)
31. Ans. (c)
32. Ans. (c)
33. Ans. (c)
34. Ans. (c)
35. Ans. (c)
36. Ans. (b)
37. Ans. (b)
38. Ans. (b)

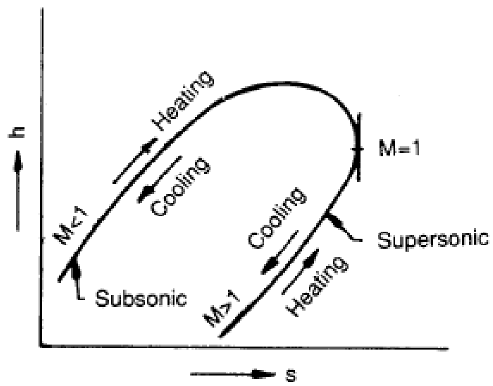

Fig. Rayleigh line on h-s plot

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Flow in open channel

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. An open channel may be defined as a passage in which liquid flows with its upper surface exposed to atmosphere.

2. Flow in a channel is said to be ***uniform**,* if the depth, slope, cross-section and velocity remain constant over a given length of the channel. Flow in a channel is said to be *non-uniform* (or varied) when the channel depth *varies* continuously from one section to another.

3. The flow in the open channel may be characterized as laminar or turbulent depending upon the value of Reynold's number:

when *Re* < 500 ...flow is laminar; when *Re* > 2000 ...flow is turbulent.

when 500<Re<2000---flow is transitional.

4. If Froude number *(Fr)* is less than 1·0, the flow is subcritical or streaming. If *Fr* is equal to 1·0, the flow is critical. If *Fr* is greater than 1·0, the flow is super-critical or shooting.

5. Velocity by Chezy's formula is given by

$V=C\sqrt{mi}$ where C = Chezy's constant,

m = hydraulic radius (or hydraulic mean depth) = $\dfrac{A(area)}{P(wetted\ permeter)}$

i = slope of the bed.

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6. Empirical relations for the Chezy's constant, C

(i) $C=\dfrac{157.6}{1.81+\dfrac{K}{\sqrt{m}}}---Bazin's\ formula$

Where *K* = *Bazin' s constant,*
m = hydraulic radius (or hydraulic mean depth)

(ii) $C=\dfrac{23+\dfrac{0.00155}{S}+\dfrac{1}{N}}{1+\left(23+\dfrac{0.00155}{S}\right)\dfrac{N}{\sqrt{m}}}$ ----Kutter's formula

Where *N* = Kutter's constant, and S = bed slope.

7. The *most economical section* (also called the best section or most efficient section) is one which gives the maximum discharge for a given amount of excavation.

8. Conditions for maximum discharge through different channel sections:
(*a*) Rectangular section

(i) *b = 2y;* (ii) *R* = $\dfrac{y}{2}$

(b) Trapezoidal section
(i) Half top width = sloping side

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or $$\frac{b+2ny}{2} = y\sqrt{n^2+1}$$

(ii) $m = \frac{y}{2}$

(iii) A semi-circle drawn from the mid-point of the top width with radius equal to depth of flow will touch the three sides of the channel. *Best side slope* for most economical trapezoidal section is

$$\theta = 60^0 \; or \; n = \frac{1}{\sqrt{3}} = \frac{1}{\tan\theta}$$

(c) Triangular section

(i) Each sloping side makes an angle of 45° with the vertical.

(ii) Hydraulic radius, m = $\frac{y}{2\sqrt{2}}$

(d) Circular section.

(i) Condition for maximum discharge:

Depth of flow, y = 0·95 diameter of circular channel

Hydraulic radius, R = 0·29 times channel diameter

(ii) Condition for maximum velocity:

Depth of flow, y = 0·81 diameter of circular channel,

Hydraulic radius, R=0.305 diameter

9. For a circular channel,

Area of flow, A=$r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$ $\quad \therefore hydraulic\; mean\; depth = \frac{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2r\theta}$

Wetted perimeter, P=2rθ

Where r=radius of circular channel,

θ=half the angle subtended by the water surface at the centre.

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10. Channel sections of constant velocity are designed particularly in the case of large sewers in which the discharge ranges from a certain minimum value that flows daily to a very large value during rainy season.

11. The total energy of flow per unit weight of liquid is given by

Total energy=z+y+$\frac{V^2}{2g}$

12. Specific energy of a flowing liquid per unit weight,

$$E = y + \frac{V^2}{2g}$$

13. The depth of flow at which specific energy is minimum is called critical depth, which is given by

$y_c = \left(\frac{q^2}{g}\right)^{1/3}$, where q=discharge per unit width. q=$\frac{Q}{b} m^2/s$

14. The velocity of flow at critical depth is known as critical velocity, which is given by

$$V_c = \sqrt{g \times y_c}$$

15. Minimum specific energy is given by

$F_{min} = \frac{3}{2} y_c$ =critical depth

16. (i) A flow corresponding to critical depth(or when Froude number, Fr=1)is known as ***critical flow.***

(ii) When the depth of flow in a channel is greater than critical depth (when Fr<1) the flow is said to be ***sub-critical*** or streaming flow.

(iii) The flow is ***supercritical*** (or shooting or torrential) when the depth of flow in a channel is less than critical depth (when Fr>1).

17. The condition for maximum discharge for given value of specific energy is that the depth of flow should be critical.

18. Hydraulic jump: In an open channel when rapidly flowing stream abruptly changes to slowly flowing stream, a distinct rise or jump in the elevation of liquid surface takes place, this phenomenon is known as hydraulic jump. The hydraulic jump is also known as 'standing wave'.
The depth of flow after the jump is given by

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{2gy_1}} \quad \text{----- (in terms of q)}$$

$$= -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2V_1^2 y_1}{g}} \quad \text{----- (in terms of } V_1\text{)}$$

$$= \frac{y_1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right) \quad \text{----- (in terms of Fr1)}$$

(Where y1=depth of flow of water before the jump)
Height of hydraulic jump, $H_j = y_2 - y_1$
Length of hydraulic jump, L_j =5 to 7 H_j

Loss of energy due to hydraulic jump, $E_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

19. Gradually varied flow (G.V.F.) is one in which the depth changes gradually over a long distance.
Equation of gradually varied flow is given by

$$\frac{du}{dx} = \frac{S_b - S_e}{\left(1 - \frac{V^2}{gy}\right)} \quad \text{--------- (in terms of V)}$$

$$= \frac{S_b - S_e}{\left(1 - Fr^2\right)} \quad \text{----- (in terms of Fr)}$$

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where $\frac{du}{dx}$ = slope of free water surface,
S_b= slope of the channel bed,
S_e=slope of the energy line, and
V=velocity of flow.

20. Afflux is the increase in water level due to some obstruction across the flowing liquid; the curved surface of the liquid with its concavity upwards, is known as 'back water curve'.

Length of back water curve, $l = \frac{E_2 - E_1}{S_b - S_e}$

where $E_1\left(= y_1 + \frac{V_1^2}{2g}\right)$ and $E_2\left(= y_2 + \frac{V_2^2}{2g}\right)$ represent the specific energies at the beginning and end of back water curve.

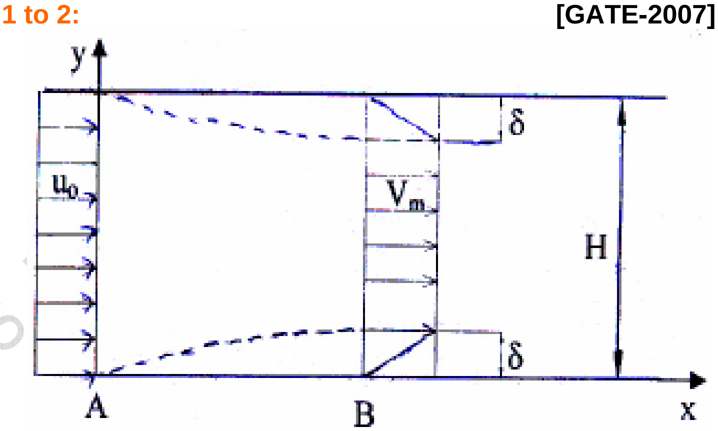
Question (IAS, IES, GATE)

Laminar flow and turbulent flow

Statement for linked answer Question: 1 to 2: **[GATE-2007]**

Consider a steady incompressible flow through a channel as shown below:
The velocity profile is uniform with a value of u_0 at the inlet section A. The velocity profile at section B downstream is

$$u = \begin{cases} V_m \frac{y}{\delta}, & 0 \le y \le \delta \\ V_m, & \delta \le y \le H-\delta \\ V_m \frac{H-y}{\delta}, & H-\delta \le y \le H \end{cases}$$



1. The ratio V_m/u_0 is

(a) $\frac{1}{1-2(\delta/H)}$ (b) 1 (c) $\frac{1}{1-(\delta/H)}$ (d) $\frac{1}{1+(\delta/H)}$

2. The ratio $\frac{p_A - p_B}{\frac{1}{2}\rho u_0^2}$ (where p_A and p_B are the pressures at section A and B, respectively, and is the density of the fluid) is

(a) $\frac{1}{(1-(\delta/H))^2} - 1$ (b) $\frac{1}{[1-(\delta/H)]^2}$ (C) $\frac{1}{(1-(2\delta/H))^2} - 1$ (d) $\frac{1}{1+(2\delta/H)}$

Sub-critical flow, critical flow and supercritical flow

3. Match List I (Flow Depth) with List II (Basic Hydraulic condition Associated there with) and select the correct answer: **[IES-2004]**

List I		List II	
A.	Conjugate depth	1.	Uniform flow

B. Critical depth — 2. Same specific energy
C. Alternate depth — 3. Minimum specific energy
D. Normal depth — 4. Same specific force
5. Same bed slope

Codes:

	A	B	C	D		A	B	C	D
[a].	3	5	4	2	[b].	2	4	1	3
[c].	4	3	2	1	[d].	5	4	1	3

4. An open channel of symmetric right-angled triangular cross-section is conveying a discharge Q. Taking g as the acceleration due to gravity, what is the critical depth?

$(a)\left(\frac{Q^2}{g}\right)^{\frac{1}{3}}$ $(b)\left(\frac{2Q^2}{g}\right)^{\frac{1}{3}}$ $(c)\left(\frac{Q^2}{g}\right)^{\frac{1}{5}}$ $(d)\left(\frac{2Q^2}{g}\right)^{\frac{1}{5}}$ **[IES-2006]**

5. The critical depth of a rectangular channel of width 4.0 m for a discharge of 12 m^3/s is , nearly,
[a]. 300 mm [d]. 30 mm [c]. 0.972 m [d]. 0.674 m **[IES-2001]**

Most Economical Section of Channel

6. How is the best hydraulic channel cross-section defined? **[IES-2005]**
[a]. The section with minimum roughness coefficient.
[b]. The section that has a maximum area of a given flow.
[c]. The section that has a minimum wetted perimeter
[d]. The section that has a maximum wetted area.

Most economical trapezoidal channel section

7. Assertion (A): To have maximum hydraulic efficiency, the trapezoidal section of an open channel should be a half-hexagon. **[IES-1999]**
Reason (R): For any cross-section, a hexagon has the lest-hexagon.

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Hydraulic Jump or Standing Wave

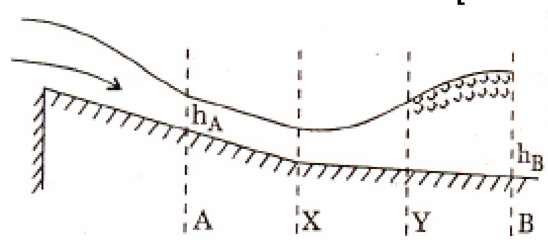
8. A hydraulic jump is formed in a 5.0 m wide rectangular channel with sequent depths of 0.2 m and 0.8m.The discharge in the channel, in m^3/s, is
(a) 2.43 (b) 3.45 (c) 4.43 (d) 5.00 **[IAS-1998]**

9. A hydraulic jump occurs in a channel
(a) Whenever the flow is supercritical **[IES-1997]**
(b) if the flow is controlled by a sluice gate
(c) if the bed slope changes from mild to steep
(d) if the bed slope changes from steep to mild

10. Consider the following statements regarding a hydraulic jump: **[IES-1999]**
1. There occurs a transformation of super critical flow to sub-critical flow.
2. The flow is uniform and pressure distribution is due to hydrostatic force before and after the jump.
3. There occurs a loss of energy due to eddy formation and turbulence.
Which of these statements are correct?
[a]. 1, 2 and 3 [b]. 1 and 2 [c]. 2 and 3 [d]. 1 and 3

11. An open channel flow encounters a hydraulic jump as shown in the figure. The following fluid flow conditions are observed between A and B: **[IES-2001]**

1. Critical depth
2. Steady non-uniform flow
3. Unsteady non-uniform flow
4. Steady uniform flow.



The correct sequence of the flow conditions in the direction of flow is:
[a]. 1, 2, 3, 4 [b]. 1, 4, 2, 3 c]. 2, 1, 4, 3 d]. 4, 2, 3, 1

12. Consider the following statements: **[IES-2003]**
A hydraulic jump occurs in an open channel
1. When the Froude number is equal to or less than one. 2. at the toe of a spillway.
3. downstream of a sluice gate in a canal. 4. When the bed slope suddenly changes.
Which of these are correct?
[a]. 1, 2, 3 and 4 [b]. 1, 2 and 3 [c]. 2, 3 and 4 [d]. 1 and 4

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Answers with Explanations

1. Ans. (c) Continuity equation gives, $u_o \times b \times H = V_m \times b \times (H - 2\delta) + \frac{1}{2} \times b \times 2\delta$

$$\text{or } \frac{V_m}{u_o} = \frac{H}{H-\delta} = \frac{1}{1-\delta/H}$$

2. Ans. (a)
3. Ans. (c) only one matching (B with 3) will give us ans. (c) The depth of flow at which specific energy is minimum is called critical depth.
4. Ans. (a) Note: here Q = discharge per unit width (m^2/s) and **not** m^3/s

5. Ans. (c) Discharge per unit width, $q = \frac{Q}{b} = \frac{12}{4} = 3.0\, m^2/s$

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3^2}{9.81}\right)^{1/3} = 0.972\, m$

6. Ans. (c)
7. Ans. (b)

8. Ans. (c) q = $\sqrt{g y_1 y_2 \frac{(y_2 + y_1)}{2}} = 0.8854$

Q = qL= 0.8854×5=4.43 m^3/s

9. Ans. (b) If the flow changes from supercritical to subcritical
10. Ans. (a)
11. Ans. (b)
12. Ans. (c) only 1 is wrong so (a), (b) and (d) out.

Force Exerted on Surfaces

Skip to Questions (IAS, IES, GATE)

HIGHLIGHT

1. A fluid jet is a stream of fluid issuing from a nozzle with a high velocity and hence a high kinetic energy.

2. The force exerted by a jet of water on a **stationary plate** (F_x):

$F_x = \rho a V^2$ ...for a vertical plate,

$= \rho a V^2 \sin^2\theta$ ...for an inclined plate,

$= \rho a V^2 (1+\cos\theta)$ ...for a curved plate and jet strikes at the centre,

$= 2\rho a V^2 \cos\theta$ ...for a curved plate and jet strikes at one of the tips of the jet.

Where, V = velocity of the jet,
θ = angle between the jet and the plate for inclined plate, and
= angle made by the jet with the direction of motion for curved plates

3. The force exerted by a jet of water on a **moving plate** in the direction of the motion of the plate (F_x):

$F_x = \rho a (V-u)^2$ ...for a moving vertical plate,

$= \rho a (V-u)^2 \sin^2\theta$ ...for an inclined moving plate,

$= \rho a (V-u)^2 (1+\cos\theta)$ ...when jet strikes the curved plate at the centre.

4. When a jet of water strikes a curved moving vane at one of its tips and comes curved out at the other tip, the force exerted and work done is given by (from inlet and outlet velocity triangles):

Force exerted, $F_x = \rho a V_{r1}(V_{w1} \pm V_{w2})$

Work done per second = $\rho a V_{r1}(V_{w1} \pm V_{w2})$ x u

+ ive sign is taken when β < 90° (i.e. 13 is an acute angle)
- ive sign is taken when β > 90° (i.e. 13 is an obtuse angle)
V_{w2} = 0 when β = 90°

Work done per second per N of fluid = $\frac{1}{g}(V_{w1} + V_{w2}) \times u$

For series of vanes:

Force exerted, $F_x = \rho a V_1 (V_{w1} \pm V_{w2})$

Work done per second = $\rho a V_1 (V_{w1} \pm V_{w2})$ x u

Work done per sec. per N of fluid = $\frac{1}{g}(V_{w1} + V_{w2}) \times u$

Where V_1 = absolute velocity of jet at inlet,
V_{wl} = velocity of whirl at inlet,
V_{w2} = velocity of whirl at outlet, and
u = velocity of the vane.

5. For series of radial curved vanes:

Work done per second on the wheel = $\rho a V_1 (V_{w1} \times u_1 \pm V_{w2} \times u_2)$

Efficiency of the radial curved vane, $\eta_{vane} = \dfrac{\rho a V_1 (V_{w1}u_1 \pm V_{w2}u_2)}{\frac{1}{2}(\rho a V_1)\times V_1^2} = \dfrac{2(V_{w1}u_1 \pm V_{w2}u_2)}{V_1^2}$

Where u_1 = tangential velocity of vane at inlet, and
u_2 = tangential velocity of vane at outlet

6. Jet propulsion of ships:

Case I. When the inlet orifices are at right angles to the direction of motion of the ships

Efficiency of propulsion, $\eta = \dfrac{2Vu}{(V+u)^2}$

Conditions for maximum efficiency, $\dfrac{d\eta}{du}$ = 0, i.e. u = V

η_{max} = 50 % (neglecting loss of head due to friction etc. in the intake and ejecting pipes)

Case II. When the inlet orifices face the direction of motion of the ship

Efficiency of propulsion, $\eta = \dfrac{2u}{V+2u}$

Questions (IES, IAS, GATE)

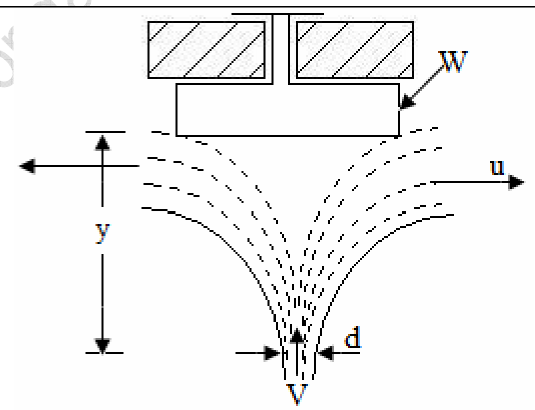
Force Exerted on a Stationary Flat Plate Held Normal to the Jet

1. A vertical jet of water 'd' cm in diameter leaving the nozzle with a velocity of V m/s strikes a disc weighing 'W' kgf as shown in the given figure. The jet is then deflected horizontally. The disc will be held in equilibrium at a distance 'y' where the fluid velocity is 'u', when 'y' is equal to

(a) $(V^2 - u^2)/2g$ (b) $V^2/2g$

(c) W/V^2 (d) W/u^2

[IAS-1996]



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2. A jet of water issues from a nozzle with a velocity of 20m/s and it impinges normally on a flat plate moving away from it at 10m/s.If the cross-sectional area of the jet is 0.02m² and the density of water is taken as 1000 kg/m³, then the force developed on the plate will be **[IAS-1994]**
(a) 10 N (b) 100N (c) 1000N (d) 2000N

Force Exerted on a Curved Vane when the Vane is moving in the Direction of Jet

3. The force of impingement of a jet on a vane increases if: **[IES-2002]**
[a]. the vane angle is increased [b]. the vane angle is decreased
[c]. the pressure is reduced [d]. the vane is moved against the jet.

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Answers

1. Ans. (a)
2. Ans. (d)
3. Ans. (d)

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HYDRAULIC TURBINE

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. Advantages of Hydro Power:

i. Water source is perennially available. No fuel is required to be burnt to generate electricity. It is aptly termed as 'the white coal'. Water passes through turbines to produce work and downstream its utility remains undiminished for irrigation of farms and quenching the thirst of people in the vicinity.
ii. The running costs of hydropower installations are very low as compared to thermal or nuclear power stations. In thermal stations, besides the cost of fuel, one has to take into account the transportation cost of the fuel also.
iii. There is no problem with regards to the disposal of ash as in a thermal station. The problem of emission of polluting gases and particulates to the atmosphere also does not exist; Hydropower does not produce any greenhouse effect, cause the pernicious acid rain and emit obnoxious NO.
iv. The hydraulic turbine can be switched on and off in a very short time. In a thermal or nuclear power plant the steam turbine is put on turning gear for about two days during start-up and shut-down.
v. The hydraulic power plant is relatively simple in concept and self contained in operation. Its system reliability is much greater than that of other power plants.
vi. Modem hydropower equipment has a greater life expectancy and can easily last 50 years or more. This can be compared with the effective life of about 30 years of a thermal or nuclear station.
vii. Due to its great ease of taking up and throwing off the load, the hydropower can be used as the ideal spinning reserve in a system mix of thermal, hydro and nuclear power stations.
viii. Modern hydro-generators give high efficiency over a considerable range of load. This helps in improving the system efficiency.
ix. Hydro-plants provide ancillary benefits like irrigation, flood control, afforestation, navigation and aqua-culture.
x. Being simple in design and operation, the hydro-plants do not require highly skilled workers. Manpower requirement is also low.

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2. Disadvantages of Hydro Power:

i. Hydro-power projects are capital-intensive with a low rate of return. The annual interest of this capital cost is a large part of the annual cost of hydropower installations.
ii. The gestation period of hydro projects is quite large. The gap between the foundation and completion of a project may extend from ten to fifteen years.
iii. Power generation is dependent on the quantity of water available, which may vary from season to season and year to year. If the rainfall is in time and adequate, then only the satisfactory operation of the plant can be expected.
iv. Such plants are often far way from the load centre and require long transmission lines to deliver power. Thus the cost of transmission lines and losses in them are more.
v. Large hydro-plants disturb the ecology of the area, by way of deforestation, destroying vegetation and uprooting people. Strong public opinion against erection of such plants is a deterrent factor. The emphasis is now more on small, mini and micro hydel stations.
vi. Silt content in Indian River is too high, and that creates lot of problems to hydro station.
vii. Some site is so remote there is no access road, for that we have to make road first.

3. Selection of site for a Hydro Project:

The following factors should be considered while selecting the site for hydroelectric power plant.

I. Availability of water
II. Water storage capacity

III. Available water head
IV. Accessibility of the site
V. Distance from the load centre
VI. Type of land of site

I. Availability of water The design and capacity of the hydro-plant greatly depends on the amount of water available at the site. The run-off data along with precipitation at the proposed site with maximum and minimum quantity of water available in a year should be made available to

(a) decide the capacity of the plant,
(b) set up the peak load plant such as steam, diesel or gas turbine plant,
(c) provide adequate spillways or gate relief during flood period.

II. Water storage capacity Since there is a wide variation in rainfall all round the year, it is always necessary to store the water for continuous generation of power. The storage capacity can be estimated with the help of mass curve.

III. Available water head In order to generate the desired quantity of power it is necessary that a large quantity of water at a sufficient head should be available. An increase in effective head, for a given output, reduces the quantity of water required to be supplied to the turbines.

IV. Accessibility of the site The site should be easily accessible by rail and road. An inaccessible terrain will jeopardize the movement of men and material.

V. Distance from the load centre If the site is close to the load centre, the cost of transmission lines and the transmission losses will be reduced.

VI. Type of the land of the site The land of the site should be cheap and rocky. The darn constructed at the site should have large catchment area to store water at high head. The foundation rocks of the masonry dam should be strong enough to withstand the stresses in the structure and the thrust of water when the reservoir is full.

4. A hydraulic turbine is a prime mover that uses the energy of flowing water and converts it into the mechanical energy (in the form of rotation of the runner)

5. In an impulse turbine the pressure energy of water is converted into kinetic energy when passed through the nozzle and form the high velocity jet of water. The formed water jet is used for driving the wheel.

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The Pelton wheel or Pelton turbine is a tangential flow impulse turbine and is used for high head.

6. Some important formulae relating Pelton wheel are:
Work done and efficiencies:

(i) The work done by the jet on runner per second = $\rho a V_1 (V_{w1} \pm V_{w2})$

(ii) The work done per second per unit weight of water striking = $\frac{1}{g}(V_{w1} + V_{w2}) \times u$

(iii) Hydraulic efficiency, $\eta_h = \frac{2(V_{w1} \pm V_{w2})u}{V_1^2}$

$$\eta_h = \frac{power\ developed\ by\ the\ runner}{power\ supplied\ at\ the\ inlet\ of\ turbine}$$

η_h is maximum when u = $\frac{V_1}{2}$, and

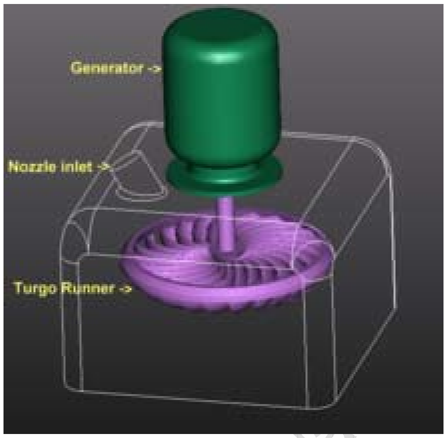
$(\eta_h)_{max} = \frac{1+\cos\phi}{2}$ Assuming No friction (i.e. K = 1)

(iv) Mechanical efficiency, $\eta_m = \dfrac{shaft\ power}{bucket\ power}$

(v) Volumetric efficiency, $\eta_v = \dfrac{Volume\ of\ water\ actually\ striking\ the\ runner}{total\ water\ supplied\ by\ the\ jet\ to\ the\ turbine}$

(vi) Overall efficiency, $\eta_o = \dfrac{shaft\ power}{water\ power} = \dfrac{P}{\rho g Q H}$

7. Turgo turbine



Turgo turbine and generator

The **Turgo turbine** is an ***impulse water turbine*** designed for ***medium head*** applications. Operational Turgo Turbines achieve efficiencies of about 87%. In factory and lab tests Turgo Turbines perform with efficiencies of up to 90%.

Developed in 1919 by Giles as a modification of the Pelton wheel, the Turgo has some advantages over Francis and Pelton designs for certain applications.

First, the runner is less expensive to make than a Pelton wheel. Second, it doesn't need an airtight housing like the Francis. Third, it has higher specific speed and can handle a greater flow than the same diameter Pelton wheel, leading to reduced generator and installation cost.

Turgos operate in a head range where the Francis and Pelton overlap. While many large Turgo installations exist, they are also popular for ***small hydro*** where low cost is very important.

Like all turbines with nozzles, blockage by debris must be prevented for effective operation.

8. Design aspects:

(i) Velocity of jet, $V_1 = C_v\sqrt{2gH}$ $\quad Or \left(C_v = \dfrac{V_1}{\sqrt{2gH}} \right)$

(C_v: 0·98 or 0·99)

(ii) Velocity of wheel, u= ($u_1 = u_2$) = $= K_u\sqrt{2gH}$ $\quad Or \left(K_u = \dfrac{u}{\sqrt{2gH}} \right)$

(K_u, the speed ratio varies from 0·43 to 0·48)

Number of buckets on a runner Z= 15 + $\dfrac{D}{2d}$ = 15 + 0·5 m

Where m (jet ratio) = $\frac{D}{d}$; D and d being the pitch diameters of Pelton wheel and the jet diameter respectively. $\frac{D}{d}$ lies between 11 to 16 for maximum hydraulic efficiency; normally jet ratio is adopted as 12 in practice.

3. In a reaction turbine the runner utilizes both potential and kinetic energies.

9. Formulae for various reaction turbines are as follows:
(a) Francis turbine:
(i) Francis turbine is an inward radial flow reaction turbine having discharge radial at outlet which means the angle made by absolute velocity at outlet is 90° i.e. β= 90° Then V_{w2}= 0 and work done by water on the runner per second per unit weight of water is

$$\frac{1}{g}V_{w1}u_1$$

(ii) Flow ratio, $K_f = \frac{V_{f1}}{\sqrt{2gH}}$; K_f varies from 0·15 to 0·30

(iii) Speed ratio, $K_u = \frac{u_1}{\sqrt{2gH}}$; K_u ranges from 0·6 to 0·9

(iv) The ratio of width (B_1) to the diameter of the wheel (D_l), n = $\frac{B_1}{D_1}$; n varies from 0·10 to 0·45

(v) Discharge, Q = $K_{t1}\pi D_1 B_1 V_{f1} = K_{t2}\pi D_2 B_2 V_{f2}$
Where K_t is known as vane thickness factor/co-efficient; its value is usually of the order of 0·95 or so (always less than unity)

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(b) Kaplan & Propeller Turbine:
It is an axial flow turbine in which the vanes on the hub are adjustable. It is used for low heads where large volumes of water are available. In this turbine a high efficiency is maintained even at part load.

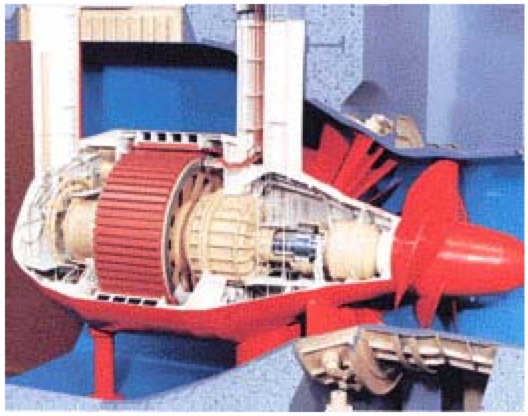
The peripheral velocities at inlet and outlet are equal, i.e. $u_l = u_2$

Discharge, Q = $\frac{\pi}{4}(D_o^2 - D_b^2)\times V_f$

(iii) Speed ratio, $K_u = \frac{u}{\sqrt{2gH}}$; K_u ranges from 1.40 to 2·0

Where, D_o= outside diameter of the runner, and
D_b= diameter of boss (or hub).
V_f = velocity of flow; ($V_{f1} = V_{f2} = V_f$)

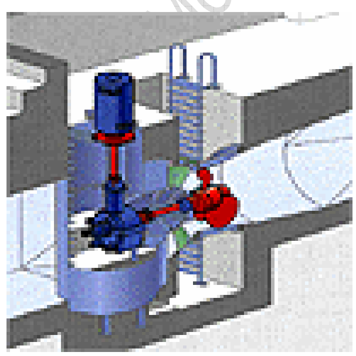
There are several different types of propeller turbines:
Bulb turbine: The turbine and generator is a sealed unit placed directly in the water stream.



Bulb hydropower turbine

Straflo: **The** generator is attached directly to the perimeter of the turbine.

Tube turbine: The penstock bends just before or after the runner, allowing a straight line connection to the generator.



10. Deriaz turbine. It is also known as diagonal turbine. Its runner is so shaped that it can be used both as a turbine as well as a pump and hence it may be classified as a reversible type turbine. As such these turbines are amply suitable for pumped storage hydropower plants.

11. Runaway speed is the maximum speed, governor being disengaged, at which a turbine would run when there is no external load but operating under design head and discharge.

12. A draft tube is a pipe of gradually increasing area used for discharging water from the exit of a reaction turbine. It is an integral part of mixed and axial flow turbines. The efficiency of a draft tube (η_d) is given by

$$\eta_d = \frac{net\ gain\ in\ pressure\ head}{Velocity\ head\ at\ entrance\ of\ draft\ tube} = \frac{\left(\frac{V_2^2 - V_3^2}{2g} - h_f\right)}{\frac{V_2^2}{2g}}$$

Where V_2 = velocity of water at inlet of the draft tube, and
V_3 = velocity of water at outlet of the draft tube

$$\left[or\ h_f = \frac{V_2^2 - V_3^2}{2g} - \eta_d \times \frac{V_2^2}{2g}\right]$$

The draft tube serves the following two purposes:
(i) By diffusion it converts the kinetic energy to pressure energy
(ii) It allows the turbine to be set above tail-water level, without loss of head, to facilitate inspection and maintenance.

13. Specific speed (N_s): of a turbine is defined as the speed of a geometrically turbine which would develop unit power when working under a unit head. It is given by the relation, $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

Where P = shaft power, and H = net head on the turbine
Specific speed plays an important role in the selection of the type of turbine.
Or, The suitability of a turbine for a particular depends on (a) head of water (b) rotational speed (c) power developed, which together fix a parameter called 'specific speed'.

14. Unit quantities are the quantities which are obtained when the head on the turbine is unity. They are given as

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Unit speed, $N_u = \frac{N}{\sqrt{H}}$

Unit discharge, $Q_u = \frac{Q}{\sqrt{H}}$

Unit power, $P_u = \frac{P}{H^{3/2}}$

15. The important characteristic curves of a turbine are
(i) Main or constant head characteristic curves.
(ii) Operating or constant speed characteristic curves.
(iii) Constant efficiency or iso-efficiency or Muschel curves.

16. Cavitation: The formation, growth and collapse of vapour filled cavities or a bubble in a flowing liquid due to local fall in fluid pressure is called cavitation. The critical value of cavitation factor (σ_c) is given by

$$\sigma_c = \frac{(H_a - H_v - H_s)}{H}$$

Where H_a = atmospheric pressure head in meters of water,
H_v = vapour pressure in meters of water corresponding to the water temperature.
H = working head of turbine (difference between head race and tail race levels in meters)
H_s = suction pressure head (or height of turbine inlet above tail race level) in meters.
The value of critical factor depends upon specific speed of the turbine.
If the value of σ is greater than σ_c then cavitation will not occurred in the turbine or pump.

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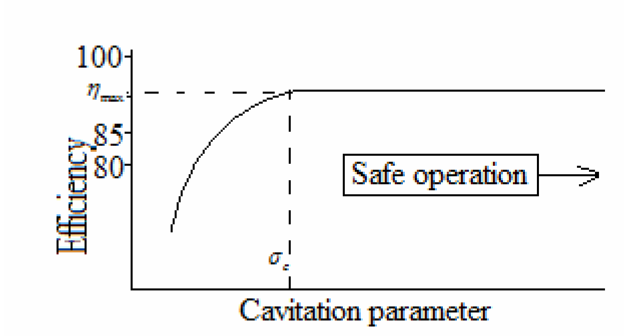
Effect of cavitation:

(i) Roughening of the surface by pitting
(ii) Increase vibration due to irregular collapse of cavities.
(iii) The actual volume of liquid flowing through the machine is reduced.
(iv) Reduce output power
(v) Reduce efficiency

Method to avoid cavitation:

(i) Runner/turbine may be kept under water
(ii) Design cavitation free runner
(iii) Selecting proper material, use stainless steel, alloy steel
(iv) Blades coated with harder material
(v) Selecting a runner of a proper specific speed

Efficiency vs. Cavitation parameter plot



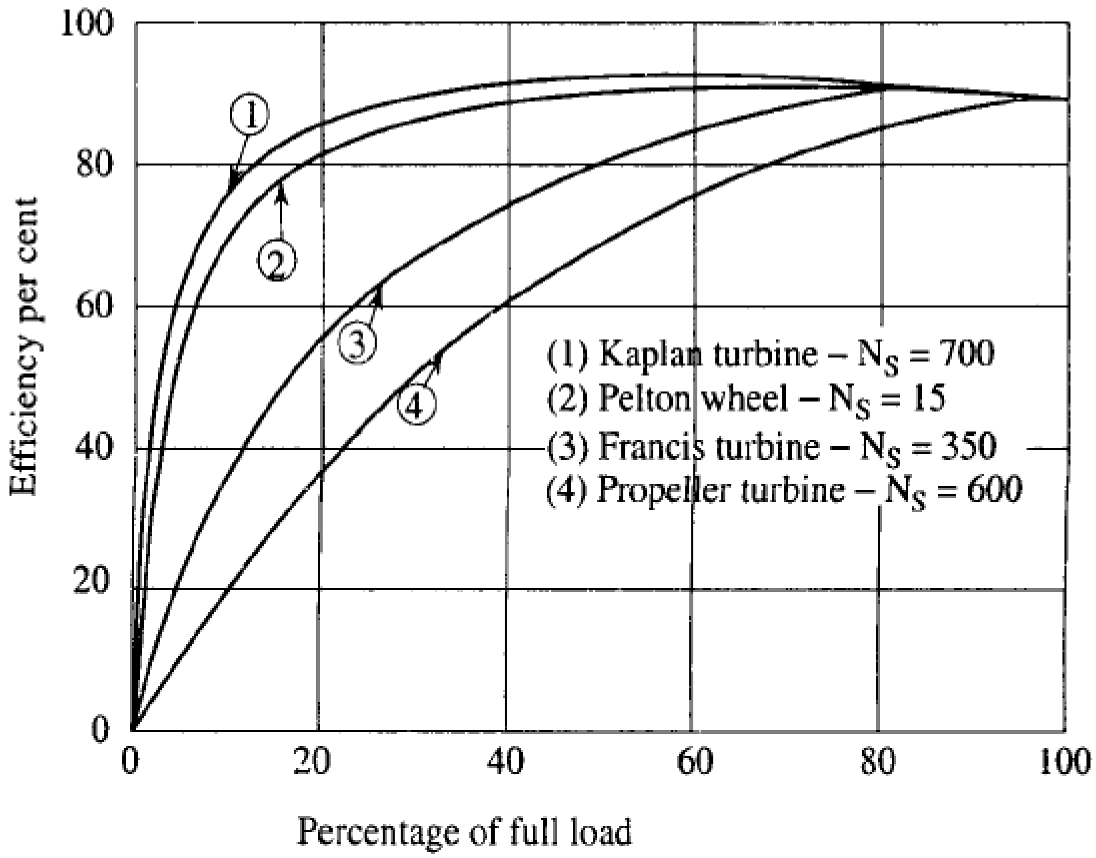
17. A 'surge tank' is a small reservoir or tank in which the water level rises or falls to reduce the pressure swings so that they are not transmitted in full to a closed circuit.
The purpose of a surge tank in high head hydroelectric plants is to prevent water hammer due to sudden load changes.

18. Relation between 'speed' and 'pole' and 'frequency'

N = $\frac{120f}{p}$ N = Rotational speed, rpm ; f = frequency (Hz)

p = no. of pole in the generator

19. Overall efficiency variation with load for various turbines:



20. For any plant the number of turbine should not be less than two so that at least one unit is always available for service in the case of a unit breakdown.

21. Unit quantities: $N_u = \frac{N}{\sqrt{H}}$; $Q_u = \frac{Q}{\sqrt{H}}$; $P_u = \frac{P}{H^{3/2}}$

22. **For model relationship** use: $\frac{H}{N^2D^2} = const.$ $\frac{Q}{ND^3} = const.$ $\frac{P}{N^3D^5} = const.$

and specific speed **[VIMP]**

23. Francis versus Kaplan Turbine

For projects with a head between 30 and 75 m the choice of turbine type stands between Francis and Kaplan .The most important advantages for each type that may influence upon the final choice are mentioned below:

Advantages of Francis Turbine
1 Cavitation characteristic permits considerably higher setting
2. Normally better peak efficiency. At low heads, however, the Kaplan turbines may have as good peak efficiency as the Francis turbines
3. Usually somewhat smaller overall dimensions. This is the fact even though the Francis runner discharge diameter is some what larger that the Kaplan runner.
4. Considerably lower runaway speed and runaway discharge in relation to normal speed and discharge
Advantage of Kaplan Turbine

1. Wider load range with better part load efficiency. The recommended I load range for a Kaplan unit is normally from 100% to 20%.
2. Higher synchronous speed.
3. Hydraulically somewhat better suited for plants with large head variations.

A comparative study of results for head of 50 m and power output of 60 MW of Kaplan versus Francis , from a manufacturer's publication is given below:

The main data for each alternative is as follow:

	Francis	Kaplan
Head (m)	50	50
Specific speed m. kW	250	370
Diameter (m)	3.9	4.2
Speed (rpm)	136.4	200
Setting –Hs	-1 m	-13 m
Spacing	14 m	15.5 m
Output (MW)	60.0	60.0

The final choice of turbine may be made concerning difference in total cost due to submergence, difference in generator cost and evaluation of efficiency.

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Questions (IES, IAS, GATE)

Introduction

1. In a hydroelectric power plant, forebay refers to the **[IAS-1997]**
(a) beginning of the open channel at the dam (b) end of penstock at the valve house
(c) level where penstock begins (d) tail race level at the turbine exit

Classification of Hydraulic Turbines

2. Assertion (A): In many cases, the peak load hydroelectric plants supply power during average load as also during peak load, whenever require. **[IAS-1996]**
Reason(R): Hydroelectric plants can generate a very wide range of electric power, and it is a simple exercise to restart power generation and connecting to the power grid.

Impulse Turbines - Pelton Wheel

3. In the case of Pelton turbine installed in a hydraulic power plant, the gross head available is the vertical distance between **[IAS-1994]**
(a) forebay and tail race (b) reservoir level and turbine inlet
(c) forebay and turbine inlet (d) reservoir level and tail race.

Work done and efficiency of a Pelton wheel

4. Euler equation of turbine giving energy transfer per unit mass E_0 (where U, V_w, V_r and V represents the peripheral, whirl, relative and absolute velocities respectively. Suffix 1 and 2 refer to the turbine inlet and outlet respectively) is given by: **[IES-2003]**

[a]. $E_0 = U_1 V_{w1} - U_2 V_{w2}$ [b]. $E_0 = U_1 V_{r1} - U_2 V_{r2}$

[c]. $E_0 = U_1 V_1 - U_2 V_2$ [d]. $E_0 = V_1 V_{w1} - V_2 V_{w2}$

5. In a Pelton wheel, the bucket peripheral speed is 10 m/s, the water jet velocity is 25m/s and volumetric flow rate of the jet is $0.1 m^3/s$.If the jet deflection angle is120^0 and the flow is ideal, the power developed is **[GATE-2006]**

(a) 7.5kW (b) 15.0 kW (c) 22.5kW (d) 37.5kW

6. In a simple impulse turbine, the nozzle angle at the entrance is 30^0. What is the blade-speed ratio (u/V) for maximum diagram efficiency? **[IAS-2004]**

(a) 0.25 (b) 0.5 (c) 0.433 (d) 0.866

7. For an impulse turbine with exit angle 'ϕ', the maximum hydraulic efficiency is

(a) $\left(1-\frac{\cos\phi}{2}\right)$ (b) $\left(\frac{1}{2}+\cos\phi\right)$ (c) $\left(\frac{1+\cos\phi}{2}\right)$ (d) $\left(\frac{1-\cos\phi}{2}\right)$ **[IAS-1999]**

Definitions of heads and efficiencies

8. The overall efficiency of a Pelton turbine is 70%. If the mechanical efficiency is 85%, what is its hydraulic efficiency? **[IES-2007]**

(a) 82.4% (b) 59.5% (c) 72.3% (d) 81.5%

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Design aspects of Pelton wheel

9. Assertion (A): For high head and low discharge hydraulic power plant, Pelton wheel is used as prime mover. **[IAS-2004]**

Reason(R): The non-dimensional specific speed of Pelton wheel at designed speed is high.

Reaction Turbine

10. Which one of the following is an example of a pure (100%) reaction machine?

(a) Pelton wheel (b) Francis turbine (c) Modern gas turbine (d) Lawn sprinkler

[IAS-1998]

Design of a Francis turbine runner

11. In the case of Francis turbine, velocity ratio is defined as $\frac{V_3}{\sqrt{2gH}}$ where H is the available head and V_3 is the **[IAS-1997]**

(a) absolute velocity at the draft tube inlet (b) mean velocity of flow in the turbine

(c) absolute velocity at the guide vane inlet (d) flow velocity at the rotor inlet

Propeller turbine

12. In which of the following hydraulic turbines, the efficiency would be affected most when the flow rate is changed from its design value? **[IAS-2007]**

(a) Pelton wheel (b) Kaplan turbine (c) Francis turbine (d) Propeller turbine

Kaplan turbine

13. Kaplan turbine is **[GATE-1997]**
(a) a high head mixed flow turbine (b) a low axial flow turbine
(c) an outward flow reaction turbine (d) an impulse inward flow turbine

14. Which one of the following is **not** correct regarding both Kaplan and propeller turbines? **[IAS-1998]**
(a) The runner is axial (b) The blades are wing type
(c) There are four to eight blades (d) The blades can be adjusted

15. Based on the direction of flow, which one of the following turbines is different from the other three? **[IAS-1998]**
(a) Pelton turbine (b) Kaplan turbine (c) De laval turbine (d) Parson's turbine

Draft Tube

16. The use of a draft tube in a reaction type water turbine helps to
(a) Prevent air from entering (b) Increase the flow rate
(c) Convert the kinetic energy to pressure energy **[IES-2007]**
(d) Eliminate eddies in the downstream

17. The function of the draft tube in a reaction turbine is
(a) to enable the shaft of the turbine to be vertical **[IAS-2002]**
(b) to transform a large part of pressure energy at turbine outlet into kinetic energy
(c) to avoid whirl losses at the exit of the turbine
(d) to transform a large part of kinetic energy at the turbine outlet into pressure energy

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18. Assertion (A): A draft tube is used along with high head hydraulic turbines to connect the water reservoir to the turbine inlet.
Reason(R): A draft tube is used to increase both the output and the efficiency of the turbine. **[IAS-2002]**

19. Assertion (A): Pelton turbine is provided with a draft tube.
Reason(R): Draft tube enables the turbine to be set at a convenient height above the tail race without loss of head. **[IAS-2001]**

Specific Speed

20. The specific speed (N_s) of a water turbine is expressed by which one of the following equations? **[IES-2007; IAS-1996]**

(a) $N_s = \frac{N\sqrt{P}}{H^{5/4}}$ (b) $N_s = \frac{N\sqrt{P}}{H^{3/4}}$ (c) $N_s = \frac{N\sqrt{Q}}{H^{5/4}}$ (d) $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$

21. Match List I with II and select the correct answer using the codes given below the lists

List I (Turbines)	List II (Specific speeds in MKS units)
A. Kaplan turbine	1. 10 to 35
B. Francis turbine	2. 35 to 60
C.Pelton wheel with single jet	3. 60 to 300
D. Pelton wheel with two or more jets	4. 300 to 1000

Codes:

	A	B	C	D		A	B	C	D
(a)	4	3	1	2	(b)	3	4	2	1
(c)	3	4	1	2	(d)	4	3	2	1

22. Consider the following statements with regard to the specific speeds of different types of turbine: **[IAS-2004]**

1. High specific speed implies that it is a Pelton wheel
2. Medium specific speed implies that it is an axial flow turbine
3. Low specific speed implies that it is a Francis turbine

Which of these statements given above is/are correct?
(a) 1 only (b) 2 only (c) 3 only (d) none

23. At a hydro electric power plant site, available head and flow rate are 24.5 m and 10.1 m^3/s respectively. If the turbine to be installed is required to run at 4.0 revolution per second (rps) with an overall efficiency of 90%, then suitable type of turbine for this site is
(a) Francis (b) Kaplan (c) Pelton (d) Propeller **[GATE-2004]**

24. In a hydroelectric station, water is available at the rate of 175 m^3/s under a head of 18m. The turbines run at speed of 150 rpm with overall efficiency of 82%. Find the number of turbines required if they have the maximum specific speed of 460………………… 2 (two)
[GATE-1996]

25. The specific speed of a hydraulic turbine is 40. What is the type of that turbine?
(a) Single jet Pelton turbine (b) Multiple Pelton turbine
(c) Francis turbine (d) Kaplan turbine **[IAS-2007]**

26. Specific speed of a Kaplan turbine ranges between **[GATE-1993]**
(a) 30 and 60 (b) 60 and 300 (c) 300 and 600 (d) 600 and 1000

Model Relationship

27. A large hydraulic turbine is to generate 300 kW at 1000 rpm under a head of 40 m. For initial testing, a 1: 4 scale model of the turbine operates under a head of 10 m. The power generated by the model (in KW) will be **[GATE-2006; 1992]**
(a) 2.34 (b) 4.68 (c) 9.38 (d) 18.75

28. If the full-scale turbine is required to work under a head of 30 m and to run at 428 r.p.m., then a quarter-scale turbine model tested under a head of 10 m must run at:
[a]. 143 r.p.m. [b]. 341 r.p.m. [c]. 428 r.p.m. [d]. 988 r.p.m. **[IES-2000]**

Cavitation

29. Cavitation in a hydraulic turbine is most likely to occur at the turbine **[GATE-1993]**
(a) entry (b) exit (c) stator exit (d) rotor exit

30. Cavitation damage in the turbine runner occurs near the **[IAS-2001]**
(a) inlet on the concave side of the blades (b) outlet on the concave side of the blades
(c) outlet on the convex side of the blades (d) inlet on the convex side of the blades

Surge Tanks

31. What is the purpose of a surge tank in high head hydroelectric plants?
(a) To act as a temporary storage during load changes
(b) To improve the hydraulic efficiency
(c) To prevent surges in generator shaft speed
(d) To prevent water hammer due to sudden load changes **[IAS-2007]**

32. Which one of the following is the purpose of a surge tank in a Pelton Turbine station?
(a) It acts as a temporary storage during load change (b) It prevents hydraulic jump
(c) It prevents surges at the transformer

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(d) It prevents water hammer due to sudden reduction in load. **[IAS-2004]**

33. In hydraulic power-generation systems, surge tanks are provided to prevent immediate damage to
(a) draft tube (b) turbine (c) tail race (d) penstocks

34. Match List I with List II and select the correct answer using the codes given below the lists:

List I (Water Turbines)	List II (Application)
A. Pelton	1. High head and low discharge
B. Francis	2. High head and high discharge
C. Kaplan	3. Medium head and medium
	4. Low head and high discharge

Codes:

	A	B	C		A	B	C
(a)	1	3	2	(c)	2	4	3
(b)	1	3	4	(d)	3	2	4

35. Match List I with List II and select the correct answer using the codes given below the lists **[IAS-1994]**

List I	List II
A. Propeller turbine	1. Impulse turbine
B. Tangential turbine	2. Kaplan turbine
C. Reaction is zero	3. Gas turbine
D. Reaction turbine	4. Pelton turbine

Codes:

	A	B	C	D		A	B	C	D
(a)	3	2	1	4	(b)	2	1	4	3
(c)	2	4	1	3	(d)	3	4	2	1

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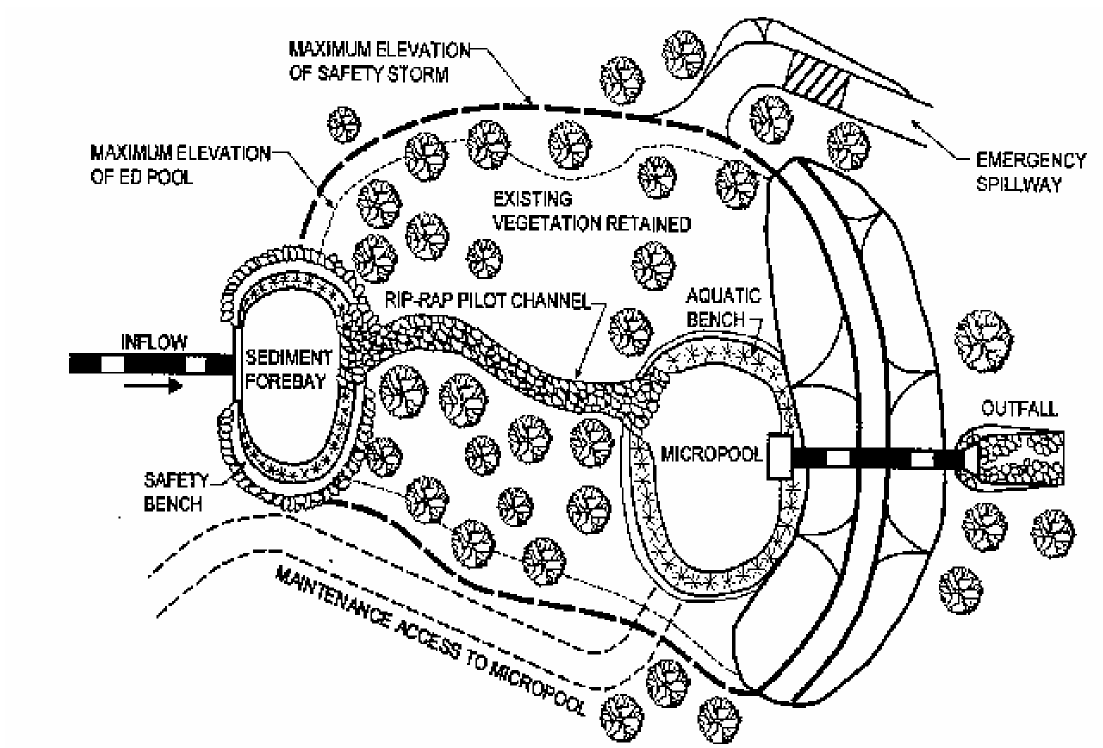
Answer with Explanation

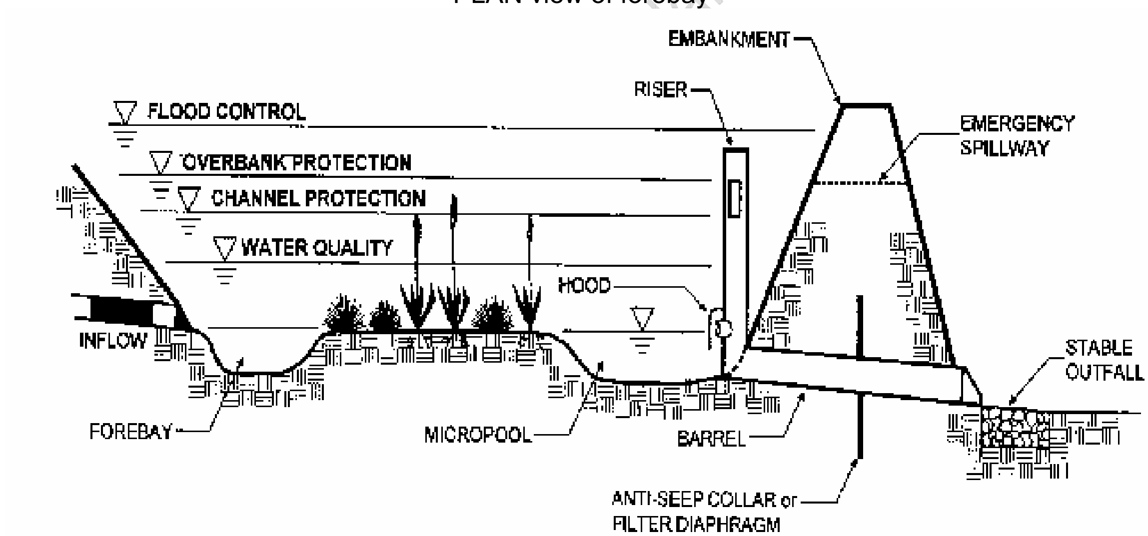
1. Ans. (c) What is a sediment forebay: A sediment forebay is a small pool located near the inlet of a storm basin or other stormwater management facility. These devices are designed as initial storage areas to trap and settle out sediment and heavy pollutants before they reach the main basin. Installing an earth **beam**, **gabion wall**, or other barrier near the inlet to cause stormwater to pool temporarily can form the pool area. Sediment forebays act as a pretreatment feature on a stormwater pond and can greatly reduce the overall pond maintenance requirements.

Why consider a sediment forebay:

These small, relatively simple devices add a water quality benefit beyond what is accomplished by the basin itself. Forebays also make basin maintenance easier and less costly by trapping sediment in one small area where it is easily removed, and preventing sediment buildup in the rest of the facility.



PLAN view of forebay



Profile of forebay

2. Ans. (a)
3. Ans. (b)
4. Ans. (a)

5. Ans. (c) From velocity triangle, Power developed= $\int Q(V_{w_1}+V_{w_2})\times u$=22.5 KW

6. Ans. (c) $\frac{u}{V} = \frac{\cos\alpha}{2} = \frac{\cos 30}{2} = 0.433$

7. Ans. (c)

8. Ans. (a) $\eta_o = \eta_m \times \eta_h \;\; Or \;\; \eta_h = \frac{\eta_o}{\eta_m} = \frac{0.70}{0.85} = 0.8235$

9. Ans. (c) The non-dimensional specific speed of Pelton wheel at designed speed is low.
10. Ans. (d)
11. Ans. (d)
12. Ans. (d)
13. Ans. (b)
14. Ans. (d)
15. Ans. (d)
16. Ans. (c)
17. Ans. (d)
18. Ans. (d) *A is false*. A penstock is used in hydraulic turbine to connect reservoir to the turbine inlet.
19. Ans. (d) For Pelton turbine **no** draft tube needed.
20. Ans. (a)
21. Ans. (a)
22. Ans. (d) 1 is wrong. Low specific speed implies that it is a Pelton wheel
2 is wrong, High specific speed implies that it is an axial flow turbine
3 is wrong, Medium specific speed implies that it is a Francis turbine
23. Ans. (a) Given: H=24.5m, Q=10.1m^3/s ; N=4 rev/s=4×60=240r.p.m.

η_0 =0.90 ∴ Power generated= ρ gQH×0.9
=1000×9.81×10.1×24.5×0.9=2184.7 kW

Again, $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{240\sqrt{2184.7}}{(24.5)^{5/4}}$ =205.80; 51<N_s<255, hence turbine is Francis.

24. Total Power generated = ρ gQH×0.9 = 1000×9.81×175×18×0.82=25313 kW

Again, $N_s = \frac{N\sqrt{P}}{H^{5/4}} = 460 = \frac{150\sqrt{P}}{(18)^{5/4}} \; or \; P = 12927\,kW\,;\text{So no of Turbine} = \frac{25313}{12927} \approx 2$

25. Ans. (b) Specific speed of Pelton Turbine: Single Jet 10-30
Multi Jet 30-60

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26. Ans. (d)
27. Ans. (a)

$$\frac{\mathrm{H}}{\mathrm{N^2D^2}} = \text{const. and} \frac{\mathrm{P}}{\mathrm{N^3D^5}} = \text{const. gives} \frac{P}{H^{\frac{3}{2}}D^2} = \text{const. so,} \left(\frac{P}{H^{\frac{3}{2}}D^2}\right)_m = \left(\frac{P}{H^{\frac{3}{2}}D^2}\right)_p$$

$$or\; P_m = P_p\left(\frac{H_m}{H_p}\right)^{3/2}\left(\frac{D_m}{D_p}\right)^2 = 300\times\left(\frac{10}{40}\right)^{3/2}\times\left(\frac{1}{4}\right)^2 = 2.34$$

28. Ans. (d) $\frac{H}{N^2D^2} = const.\; or \left(\frac{H}{N^2D^2}\right)_m = \left(\frac{H}{N^2D^2}\right)_p \;\; or\; N_m = N_p\sqrt{\left(\frac{H_m}{H_p}\right)}\times\left(\frac{D_p}{D_m}\right)$

$$N_m = 428\sqrt{\left(\frac{10}{30}\right)}\times\left(\frac{4}{1}\right) = 988rpm$$

29. Ans. (d)
30. Ans. (c)
31. Ans. (d)

32. Ans. (d)
33. Ans. (d)
34. Ans. (b) There is no any turbine for High head and high discharge.
35. Ans. (c)

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CENTRIFUGAL PUMP

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. A pump is a contrivance which provides energy to a fluid in a fluid system; it assists to increase the pressure energy or kinetic energy, or both of the fluid by converting the mechanical energy.

2. (a) work done per second per unit weight of liquid

$$= \frac{V_{w2}u_2}{g}$$, assuming flow at inlet to be radial

(b) If the flow is not radial, the expression for work done may be written as:

Work done per second per unit weight of liquid = $\frac{1}{g}(V_{w2}u_2 - V_{w1}u_1)$

Above equation is known as the Euler momentum equation for centrifugal pumps.

The term $\frac{1}{g}(V_{w2}u_2 - V_{w1}u_1)$ is referred to as Euler head (H_e)

(c) Work done per second per unit weight of liquid (or H_e)

$$= \frac{V_2^2 - V_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{V_{r_1}^2 - V_{r_2}^2}{2g}$$

This equation is sometimes called the fundamental equation of a centrifugal pump

3. Suction head (h_s) It is the vertical height of the centerline of pump shaft above the liquid surface in the sump from which the liquid is being raised

Delivery head (h_d) It is the vertical height of the liquid surface in the tank/reservoir to which the liquid is delivered above the centreline of the pump shaft

The sum of suction head and delivery head is known as static head ($H_{stat.}$)

4. Manometric head (H_{mano}) The head against which a centrifugal pump has to work is known as manometric head. It is given as

(i) $H_{mano} = \frac{V_{w2}u_2}{g} - loss\ of\ head\ in\ the\ pump\ (i.e.\ impeller\ and\ casing)$

(ii) $H_{mano} = H_{stat} + loss\ in\ pipes + \frac{V_d^2}{2g}$

$$= (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g}$$

(iii) H_{mano} = total head at outlet of the pump - total head at inlet of the pump

$$= \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) - \left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right)$$

5. The various efficiencies of the pump are

(i) Manometric efficiency, $\eta_{mano} = \frac{gH_{mano}}{V_{w2}u_2}$

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(ii) Volumetric efficiency, $\eta_v = \dfrac{Q}{Q+q}$

Where Q = actual liquid discharge at the pump outlet per second, and
q = leakage of liquid per second from the impeller (through the clearances between the impeller and casing)

(iii) Mechanical efficiency, $\eta_m = \dfrac{\rho g(Q+q)(V_{w2}u_2/g)}{P} = \dfrac{P - P_{mech.loss}}{P}$

Where, P=shaft Power

(iv) Overall efficiency, $\eta_o = \dfrac{\rho g Q H_{mano}}{P}$ or $\eta_o = \eta_{mano} \times \eta_v \times \eta_m$

6. The minimum speed for starting a centrifugal pump is given by

$$N_{\min} = \frac{120 \times \eta_{mano} \times V_{w2} \times D_2}{\pi (D_2^2 - D_1^2)}$$

7. A multi-stage pump is one which has two or more identical impellers (mounted on the same shaft or on different shafts); to produce a high head the impellers are connected in series while to discharge a large quantity of liquid, the impellers are connected in parallel.

8. The specific speed (N_s) of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver unit quantity (one cubic metre of liquid per second) against a unit head (one metre). Thus

$$N_s = \frac{N\sqrt{Q}}{(H_{mano})^{3/4}}$$

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9. For complete similarity between the model and prototype/actual centrifugal pump the following conditions should be satisfied

(i) $$\left[\frac{N\sqrt{Q}}{(H_{mano})^{3/4}}\right]_m = \left[\frac{N\sqrt{Q}}{(H_{mano})^{3/4}}\right]_p$$

(ii) $$\left(\frac{\sqrt{H_{mano}}}{DN}\right)_m = \left(\frac{\sqrt{H_{mano}}}{DN}\right)_p = \frac{H}{D^2N^2}$$

(iii) $$\left(\frac{Q}{D^3N}\right)_m = \left(\frac{Q}{D^3N}\right)_p$$

(iv) $$\left(\frac{P}{D^5N^3}\right)_m = \left(\frac{P}{D^5N^3}\right)_p$$

10. The characteristics curves are used for predicting the behaviour and performance of a pump when it is working under different heads, speeds and rates of flow.

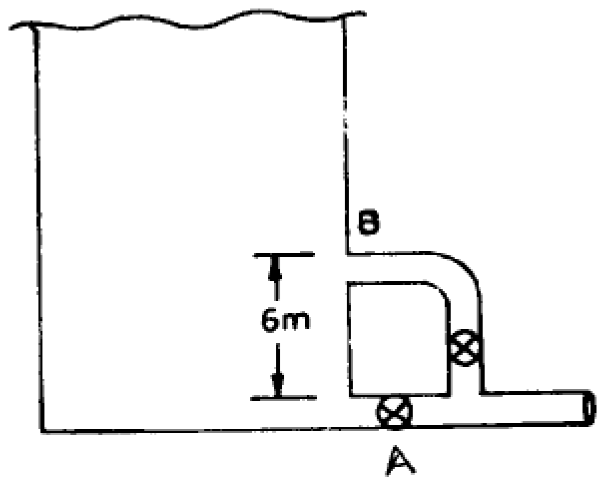
11. The net positive suction head (NPSH) may be defined as "The difference between the net inlet head and the head corresponding to the vapour pressure of the liquid"

12. Cavitation begins to appear in centrifugal pumps when the pressure at the suction falls below the vapour pressure of the liquid. It can be noted by sudden drop in efficiency head and power requirement.

Questions (IES, IAS, GATE)

Working of a Centrifugal Pump

1. The water level in an empty vertical cylindrical tank with top open is to be raised by 6m from a nearby reservoir. The ratio of the cost of pumping through pipes A and B (see given figure) is
(a) 1:6 (b) 2:3
(c) 1:2 (d) 3:5



[IAS-1996]

Work done by the Impeller (or Centrifugal Pump) on Liquid

2. When the speed of a centrifugal pump is doubled, the power required to drive the pump will
(a) increase 8 times (b) increase 4 times (c) double (d) remain the same **[GATE-2000]**

3. The power absorbed by a hydraulic pump is directly proportional to which one of the following?
(a) N (b) N^2 (c) N^3 (d) N^4 **[IES-2007]**
(Where N is the rotational speed of the pump)

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Heads of a Pump

• Common Data Question No. 4 & 5.

A centrifugal pump has an efficiency of 80% . The specifications of the pump are: Discharge = 70 m^3 /hr, head = 7 m, speed = 1450 rpm and diameter = 2000 mm. If the speed of this pump is increased to 1750 rpm.

4. Discharge and head developed are given respectively: **[GATE-2002]**
[a] 84.48n m^3 / Hr and 10.2 m [b] 48.8 m^3 / Hr and 20 m
[c] 48.8 m^3 /Hr and 10.2 m [d] 58.4 m^3 / Hr and 12 m

5. Power input required is given by:
[a] 1.066 kW [b] 1.066 kW [c] 2.12 kW [d] 20 kW

Losses in centrifugal pump

6. A centrifugal pump is required to pump water to an open water tank situated 4 km away from the location of the pump through a pipe of diameter 0.2 m having Darcy's friction factor of 0.01.The average speed of water in the pipe is 2m/s.If it is to maintain a constant head of 5 m in the tank, neglecting other minor losses, then absolute discharge pressure at the pump exit is
(a) 0.449 bar (b) 5.503 bar (c) 44.911 bar (d) 55.203 bar **[GATE-2004]**

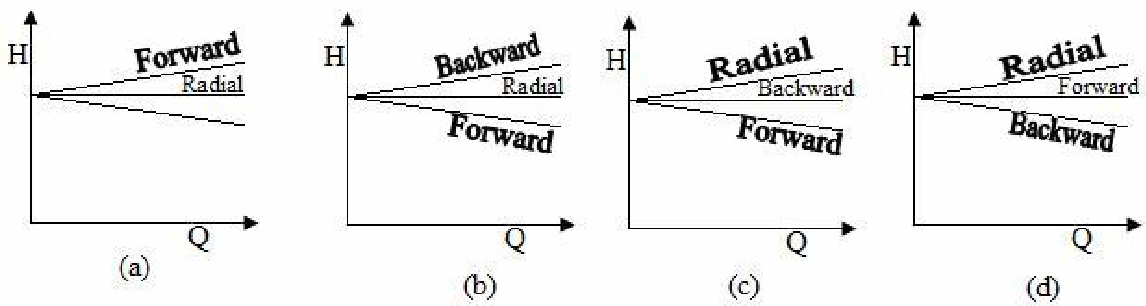
Efficiencies of a centrifugal pump

7. Manometric efficiency of a centrifugal pump is defined as the ratio of

(a) Suction head to the head imparted by the impeller to water
(b) head imparted by the impeller to water to the suction head **[IAS-1996]**
(c) manometric head to the head imparted by the impeller to water
(d) head imparted by the impeller to water to the manometric head

Effect of outlet vane angle on manometric efficiency

8. Which one of the following figures represents theoretical head versus discharge curves for a centrifugal pump with forward radial and backward curved vanes?



[IAS-1999]

9. The vanes of a centrifugal pump are generally **[IES-2007]**
(a) Radial (b) Curved backward (c) Curved forward (d) Twisted

Pumps in parallel

10. Consider the following statements in respect of centrifugal pumps:
1. Heat developed is proportional to the square of the speed of rotation
2. Backward curved bladed impellers are generally used in centrifugal pumps
3. These pumps generally do not require priming
4. Multistage pumps would give higher discharge proportional to the number of stages.

Which of these statements are correct? **[IAS-2003]**
(a) 1 and 2 (b) 2 and 3 (c) 3 and 4 (d) 1 and 4

Specific Speed

11. In terms of speed of rotation of the impeller (N), discharge (Q) and change in total head through the machine, the specific speed for a pump is........... **[GATE-1994]**

12. For discharge 'Q', the specific speed of a pump is 'Ns'.For half discharge with the same head the specific speed will be

(a) N_s (b) $\frac{N_s}{\sqrt{2}}$ (c) $\sqrt{2}$ N_s (d) $2N_s$ **[IAS-1999]**

13. If, in a pump, the discharge is halved, then, assuming that the speed remains unchanged, what would be the ratio of the heads H_1/H_2? **[IES-2007]**
(a) $\sqrt{1/3}$ (b) $\sqrt{2/3}$ (c) $\sqrt[3]{0.25}$ (d) $\sqrt[3]{0.5}$

Model Testing and Geometrically Similar Pumps

14. In utilizing scaled models in the designing of turbo-machines, which of the following relationship must be satisfied? **[IES-2002]**

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[a]. $\frac{H}{ND^3}$ = cosntant ; $\frac{Q}{N^2D^2}$ = constant [b]. $\frac{Q}{D^2\sqrt{H}}$ = cosntant ; $\frac{Q}{N^3D}$ = constant

[c]. $\frac{P}{QH}$ = cosntant ; $\frac{H}{N^2D^2}$ = constant [d]. $\frac{NQ^{1/2}}{H^{3/2}}$ = cosntant ; $\frac{NP^{1/2}}{N^{3/4}}$ = constant

15. A centrifugal pump having an impeller of 10 cm diameter discharges 40 liter/ second when turning at 1000rpm.The corresponding speed of a geometrically similar pump having an impeller of 40cm diameter and 0.8m^3/s discharge will be

(a) 276.4rpm (b) 298.3rpm (c) 312.5rpm (d) 358.2rpm **[IAS-1997]**

16. A centrifugal pump running at 500 rpm and at its maximum efficiency is delivering a head of 30 m at a flow rate of 60 litres per minute. If the rpm is changed to 1000, then the head H in metres and flow rate Q in litres per minute at maximum efficiency are estimated to be

(a) H = 60 , Q = 120 (b) H = 120 , Q = 120 [**GATE-2003]**

(c) H = 60 , Q = 480 (d) H = 120 , Q = 30

17. Which one of the following correctly expresses the specific speed of a turbine and a pump, respectively? **[IAS-2004]**

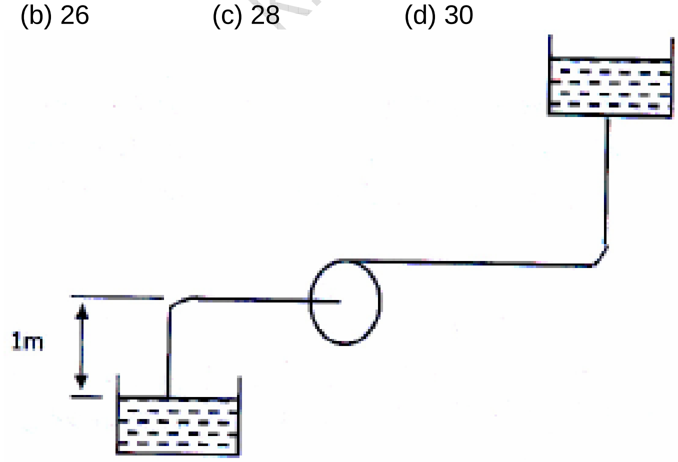
(a) $\frac{N\sqrt{Q}}{H^{3/4}}, \frac{N\sqrt{P}}{H^{5/4}}$ (b) $\frac{N\sqrt{P}}{H^{3/4}}, \frac{N\sqrt{Q}}{H^{5/4}}$ (c) $\frac{N\sqrt{P}}{H^{5/4}}, \frac{N\sqrt{Q}}{H^{3/4}}$ (d) $\frac{N\sqrt{P}}{H^{7/4}}, \frac{N\sqrt{Q}}{H^{3/4}}$

Characteristics of Centrifugal Pumps

Net Positive Suction Head (NPSH)

18. A horizontal-shaft centrifugal pump lifts water at 65^{0}c. The suction nozzle is one meter below pump centerline. The pressure at this point equals 200 kPa gauge and velocity is 3m/s.Stream tables show saturation pressure at 65^{0}C is 25 kPa, and specific volume of the saturated liquid is 0.001020 $m^{3/}$kg.The pump Net Positive Suction Head (NPSH) in meters is **[GATE-2006]**

(a) 24 (b) 26 (c) 28 (d) 30



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Cavitation in Centrifugal Pumps

19. In the case of a centrifugal pump, cavitation will occur if

(a) it operates above the minimum net positive suction head

(b) it operates below the minimum net positive suction head

(c) the pressure at the inlet of the pump is above the atmospheric pressure

(d) the pressure at the inlet of the pump is equal to the atmospheric pressure.

20. Which one of the following helps in avoiding cavitation in centrifugal pumps?
(a) Low suction pressure
(b) High delivery pressure
(c) Low delivery pressure
(d) High suction pressure **[IAS-2004]**

21. Cavitation in a centrifugal pump is likely to occur at the
(a) impeller exit (b) impeller inlet (c) diffuser exit (d) involute casing **[IAS-1996]**

Priming of a Centrifugal Pump

22. Match the items in columns I and II **[GATE-2007]**

Column I	Column II
P: Centrifugal compressor	1. Axial flow
Q: Centrifugal pump	2. Surging
R: Pelton wheel	3. Priming
S: Kaplan turbine	4. Pure impulse

	P	Q	R	S		P	Q	R	S
(a)	2	3	4	1	(b)	2	3	1	4
(c)	3	4	1	2	(d)	1	2	3	4

Operational Difficulties in Centrifugal Pumps

23. Consider the following statements for specific speed:
1. The optimum efficiency of a hydraulic machine depends on its specific speed.
2. For the same power, a turbo machine running at higher specific speed will be smaller in size.
3. Width-diameter ratio of a centrifugal pump increases with the increase in specific speed.
Which of the statements given above is/are correct?
(a) 1 only (b) 1 and 2 only (c) 2 and 3 only (d) 1, 2 and 3 **[IAS-2007]**

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Answers with Explanations

1. Ans. (c)
2. Ans. (b)
3. Ans. (c)
4. Ans. (a)
5. Ans. (a)
6. Ans. (b) Given: d=0.2m, L=4000m
f=0.01, υ=2m/s
Head loss due to friction,

$$h_f = \frac{fL\upsilon^2}{2gd} = \frac{0.01\times 4000\times (2)^2}{2\times 9.81\times 0.2} = 40.77 \text{ m}$$

Pressure corresponding to this head= $\rho g(h_f + h + h_{atm})$
$= 1000\times 9.81(40.77+5+10.3)$
$= 5.50\times 10^5 \text{N/m}^2 = 5.50$ bar

7. Ans. (c)
8. Ans. (a)

9. Ans. (b)
10. Ans. (a)

11. Ans. $\dfrac{N\sqrt{Q}}{H^{3/4}}$

12. Ans. (b) $N_s = \dfrac{N\sqrt{Q}}{H^{3/4}}$ or $N_s \; \alpha \sqrt{Q}$ or $\dfrac{N_s'}{N_s} = \sqrt{\dfrac{Q'}{Q}} = \sqrt{\dfrac{1}{2}}$

or $N_s' = \dfrac{N_s}{\sqrt{2}}$

13. Ans. (c) $N_s = \dfrac{N\sqrt{Q}}{H^{3/4}} = const.\ Or\ H \infty Q^{2/3}\ \dfrac{H_1}{H_2} = \left(\dfrac{Q_1^{\,2}}{Q_2^{\,2}}\right)^{1/3} = 4^{1/3}$

14. Ans. (a)
15. Ans. (c)
16. Ans. (b) N_1=500rpm, H_1=30m Q_1= 60 ι /minute
N_2=1000rpm, H_2=? Q_2=?

Since $\dfrac{\sqrt{H_1}}{DN_1} = \dfrac{\sqrt{H_2}}{DN_2}$

$\therefore$ $H_2 = \left(\dfrac{N_2}{N_1}\right)$

$H_1 = \left(\dfrac{1000}{500}\right)^2 \times 30 = 120m$

$\dfrac{Q_1}{D^3N_1} = \dfrac{Q_2}{D^3N_2}$

$\Rightarrow$ $Q_2 = \left(\dfrac{N_2}{N^1}\right)$

$Q_1 = \left(\dfrac{1000}{500}\right) \times 60 = 120\iota / \min ute$

17. Ans. (c)
18. Ans. (a)

19. Ans. (b)
20. Ans.(a)
21. Ans. (b)
22. Ans. (a)
23. Ans. (d)

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RECIPROCATING PUMPS

HIGHLIGHT

1. The reciprocating pump is a *positive displacement* pump and consists of a cylinder, a piston a suction valve, a delivery valve, a suction pipe, a delivery pipe and crank and connecting rod mechanism operated by a power source e.g. steam engine, I.C. engine or an electric motor.

2. Discharge through a pump per second is given as

$Q = \frac{ALN}{60}$ for a single-acting pump

$Q = \frac{2ALN}{60}$ for a double-acting pump

3. Work done by reciprocating pump per second is given as

$\frac{\rho g A L N}{60}(h_s + h_d)$ for a single-acting pump

$\frac{2\rho g A L N}{60}(h_s + h_d)$ for a double-acting pump

Power required driving the pump

$\frac{\rho g A L N}{60 \times 1000}(h_s + h_d)\,kW$ for a single-acting pump

$\frac{2\rho g A L N}{60 \times 1000}(h_s + h_d)\,kW$ for a double-acting pump

(Where ρg = weight density of liquid in N/m^3)

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4. The difference between the theoretical discharge and actual discharge is called the 'slip' of the pump.

5. Pressure head due to acceleration (ha) in the suction and delivery pipes is given as

$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta$ for suction pipe

$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r \cos\theta$ for delivery pipe.

6. The indicator diagram of a reciprocating pump is the diagram which shows the pressure head of the liquid in the pump cylinder corresponding to any position during the suction and delivery strokes. It is a graph between pressure head and stroke length of the piston for one complete revolution.

7. Work done by the pump is proportional to the area of the indicator diagram.

8. Work done by the pump per second due to acceleration and friction in suction and delivery pipes

$\frac{\rho g A L N}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right)$ for a single-acting pump

$$\frac{2\rho gALN}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right)$$ for a double-acting pump

9. An air vessel is a closed chamber containing compressed air in the upper part and liquid being pumped in the lower part. The air vessels are used:
(i) To get continuous supply of liquid at a uniform rate,
(ii) To save the power required to drive the pump and
(iii) To run the pump at a much higher speed without any danger of separation.

Questions (IES, IAS, GATE)

Classification of Reciprocating Pumps

1. For pumping molasses, it is preferable to employ
(a) reciprocating pump (b) centrifugal pump with double shrouds
(c) open impeller pump (d) multistage centrifugal pump

Air Vessels

2. **[GATE-1998]**

List I	List II
(a) High head, low flow rate	1. Streamlined body
(b) Low head, high flow rate	2. Boundary layer
(c) Heat transfer	3. Orifice meter
(d) Low drag	4. Centrifugal pump
	5. Axial flow pump
	6. Nusselt number

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Answers

1. Ans. (c)
2. Ans. A-4, B-5, C-6, D-1

MISCELLANEOUS HYDRAULIC MACHINES

Skip to Questions (IAS, IES, GATE)

HIGHLIGHTS

1. The hydraulic accumulator is a device used to store the energy of fluid under pressure and make this energy available to hydraulic machines such as presses, lifts and cranes. Its action is similar to that of an electrical storage battery.

Capacity of hydraulic accumulator = p x A x L

[Where p = liquid pressure supplied by pump, A = area of the sliding ram, and
L = stroke or lift of the ram.]

2. A differential accumulator is a special type of accumulator that is used for storing energy at high pressure by comparatively small load on the ram.

3. Hydraulic intensifier is a device which increases the intensity of pressure of a given liquid with the help of low pressure liquid of large quantity

4. Hydraulic press is a device used for lifting heavy loads by the application of much smaller force. It is based on Pascal's law.

5. Hydraulic crane is a device which is used for lifting heavy loads (upto 25 MN).

6. Hydraulic lift is a device used for carrying persons and loads from one floor to another.

7. Hydraulic ram is a device with which small quantities of water can be pumped to higher levels from the available large quantity of water of low head, The efficiency of hydraulic ram is expressed in two ways:

(i) D' Aubuisson's efficiency = $\dfrac{qH}{Qh}$

(ii) Rankine's efficiency = $\dfrac{q(H-h)}{(Q-q)h}$

Where Q = discharge from supply tank to the valve box,
q = discharge from the valve box to delivery tank,
h = height of water in the supply tank above the valve box, and
H = height of water in the delivery tank above the valve box.

8. Hydraulic (or fluid) coupling is a device which is employed for transmission of power from one shaft to another through a liquid medium.

Efficiency of hydraulic coupling, $\eta = \dfrac{\omega_t}{\omega_p}$

(Where ω_t and ω_p are the angular speeds of the turbine shaft and pump shaft respectively)

The magnitudes of input and output torque are equal.

9. Hydraulic torque converter is device used for transmitting increased or decreased torque from one shaft to another.

Efficiency of torque converter, $\eta = \dfrac{\omega_t}{\omega_p}\left(1+\dfrac{T_v}{T_p}\right)$

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(Where T_v = variation of torque caused by fixed guide vanes; T_p = torque of pump impeller).

10. Air lift pump is a device used to lift water from a deep well or sump by utilizing the compressed air.

Questions (IES, IAS, GATE)

Hydraulic Press

1. If a hydraulic press has a ram of 12.5 cm diameter and plunger of 1.25 cm diameter, what force would be required on the plunger to raise a mass of 1 tonne on the ram?
(a) 981N (b) 98.1N (c) 9.81N (d) 0.98N **[IAS-1998]**

Hydraulic Coupling

2. A hydraulic coupling belongs to the category of
(a) power absorbing machines (b) power developing machines
(c) energy generating machines (d) energy transfer machines

3. In a hydraulic coupling, what is the ratio of speed of the turbine runner to that of the pump impeller to maintain circulatory motion of oil?
(a) <1 (b) =1 (c) >1 (d) Can be any value
[IES-2007]

Hydraulic Torque Converter

4. Fluid flow machines are using the principle of either (i) supplying energy to the fluid, or (ii) extracting energy from the fluid. Some fluid flow machines are a combination of both (i) and (ii). They are classified as : **[IES-2002]**
[a]. compressors [b]. hydraulic turbines [c]. torque converters [d]. wind mills

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Answers with Explanations

1. Ans. (b) Pressure on the ram = pressure on the plunger

or $$\left(\frac{F}{A}\right)_R = \left(\frac{F}{A}\right)_p$$

or $$F_R = F_P \times \frac{A_R}{A_p} = 1000 \times 9.81 \times \left(\frac{1.25}{12.5}\right)^2 N = 98.1\text{N}$$

2. Ans. (d)
3. Ans. (b)
4. Ans. (C)

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