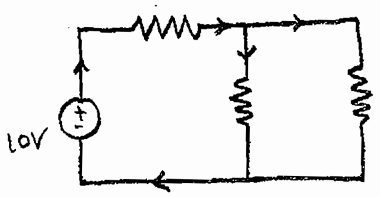
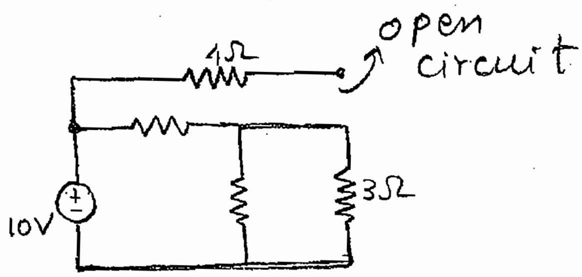
13/8/2013

# FUNDAMENTALS

| CIRCUITS | NETWORKS |
|---|---|
| → Current is intended to flow through all elements. This closed path concept is circuit.   | → Current does not necessarily flow through all the elements.   |

→ Circuits or networks are interconnection of various components to act together.

→ Most of our practical sys. are big inter-connector n/ws but we do circuit analysis to some of its parts where electrical energy flows.

So circuits are building blocks of networks.

- ## NETWORK COMPONENTS / ELEMENTS

All our applications are our components, but when these components are modelled as a circuit or n/ws, we use fundamental ~~co~~ n/w component to model them like,

V, I, R, L, C, etc

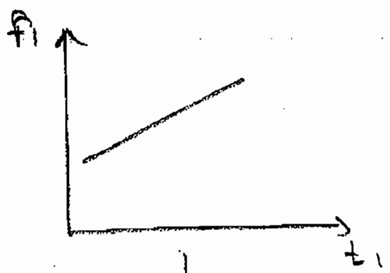
i/p $\rightarrow$ [n/w] $\rightarrow$ ?

Analysis
(unique solution)

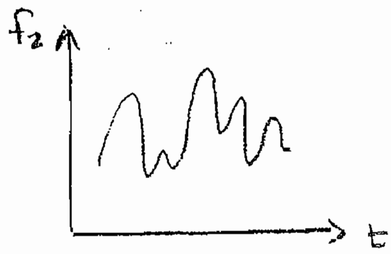
i/p $\rightarrow$ [??] $\rightarrow$ o/p

Synthesis
(various procedures can be performed)

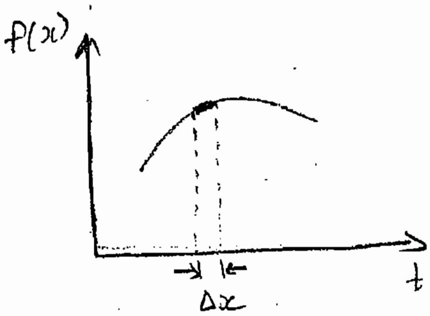
- **LINEAR COMPONENT / CIRCUIT**



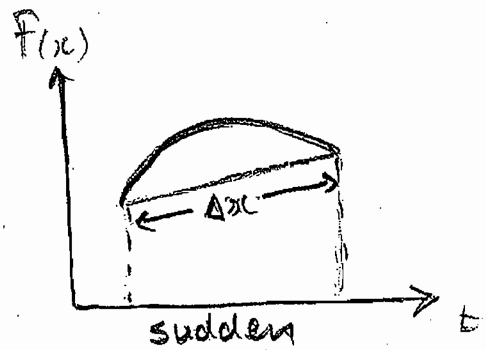

can be modeled mathematically
$y = mx + c$




can not be modeled mathematically




small incremental changes

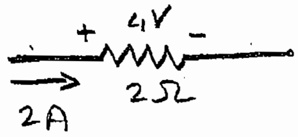



sudden big changes

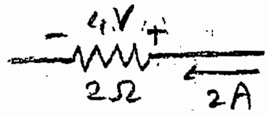
→ Most of our practical components / n/w are non linear in nature, but any non-linear sys. can be linearised for small incremental changes in time.
But the same sys. under sudden big changes undergo non-linear mode of operation.
We design all our n/w practically for specified

ratings & as long as they obey Ohm's Law, Kirchoff's Law, Superposition principle, etc. they are said to be linear.

- **BILATERAL ELEMENTS :-**




$P_{lost} = 8\,W$

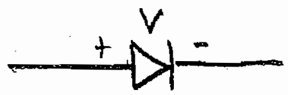



$P_{lost} = 8\,W$

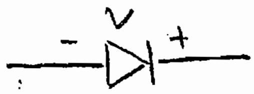
Property independent to
Voltage → Polarity
Current → dirⁿ

eg:- R, L, C

- **UNILATERAL ELEMENTS :-**




Frwd bias
Can conduct
$R_{ON} = \underline{0}\ \Omega$

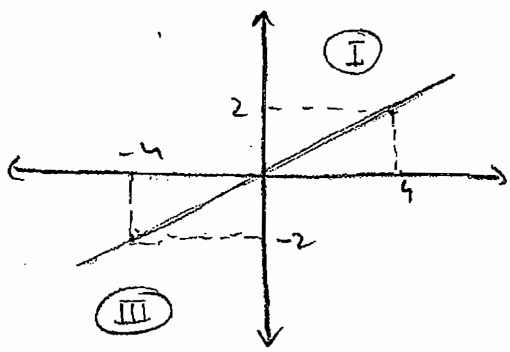



Reverse bias
Cannot conduct
$R_{ON} = \underline{\infty}\ \Omega$

Property depends on
Voltage → Polarity
Current → dirⁿ

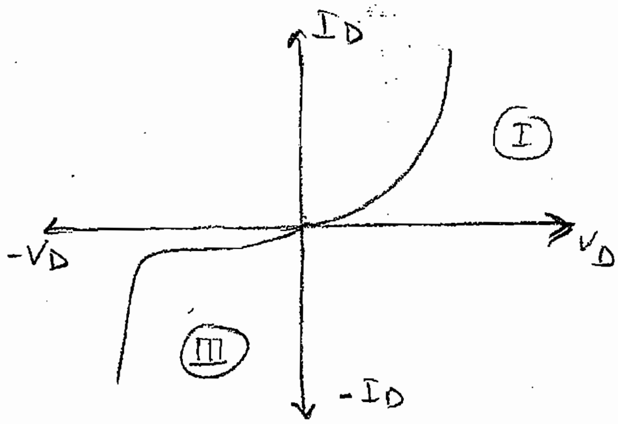
eg:- BJT, FET, diode, etc.

**Based on V-I characteristics :-**




I = III

Bilateral



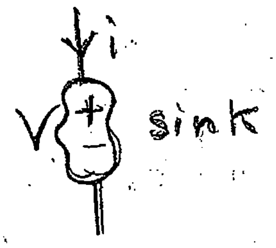

I ≠ III

Unilateral

- **PASSIVE ELEMENTS (sinks)**

- absorbs
- dissipate
- waste
- convert
- store

} electrical energy




$P_{absorbed} \Rightarrow +ve$

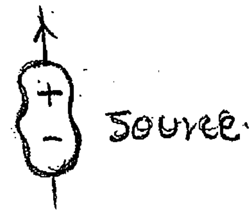
$P_{delivered} \Rightarrow -ve$

eg:- R, L, C.

- **ACTIVE ELEMENTS (sources)**

- Energize
- Drive externally
- deliver
- give out

} electrical energy

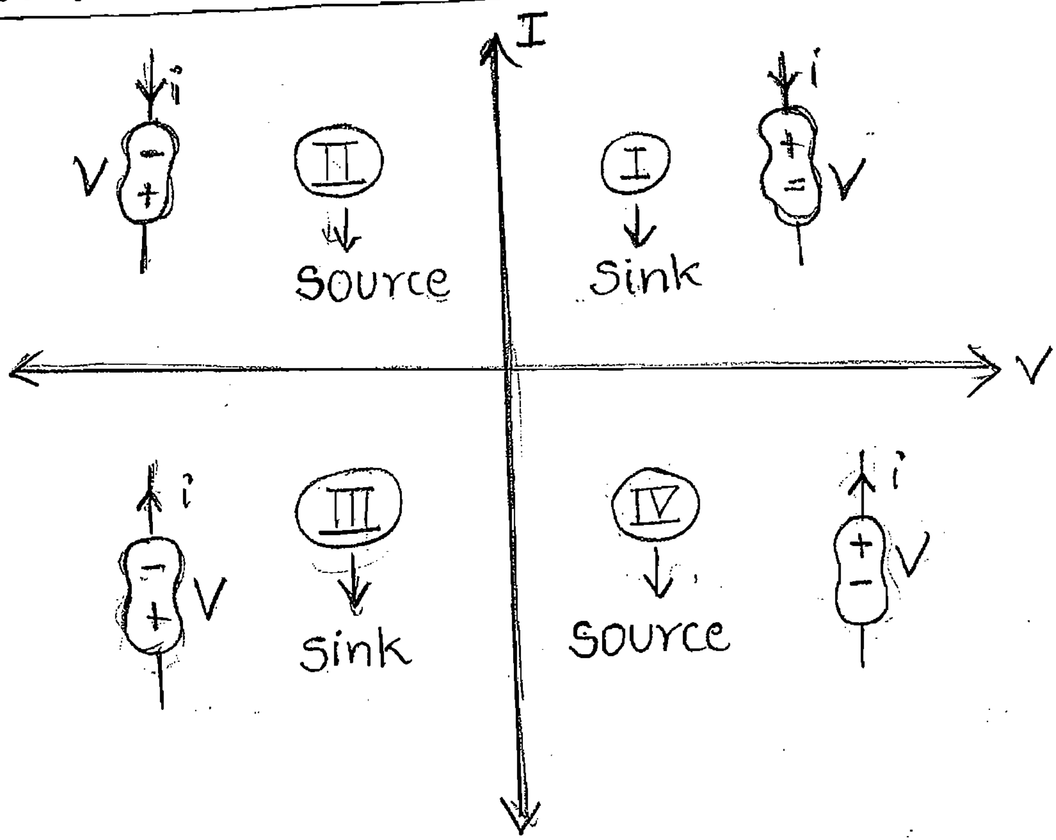



$P_{absorbed} \Rightarrow -ve$

$P_{delivered} \Rightarrow +ve$

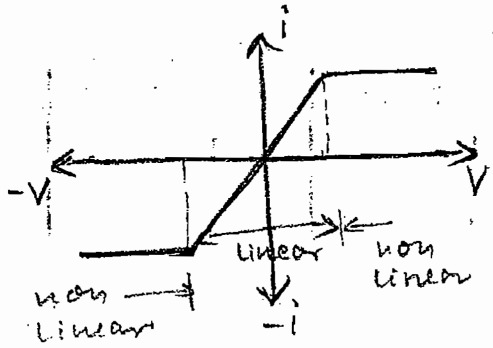
eg:- V, I

Based on V-I characteristics :-



Eg:-

1) The static V-I charac. of component is shown below, then component is ______.

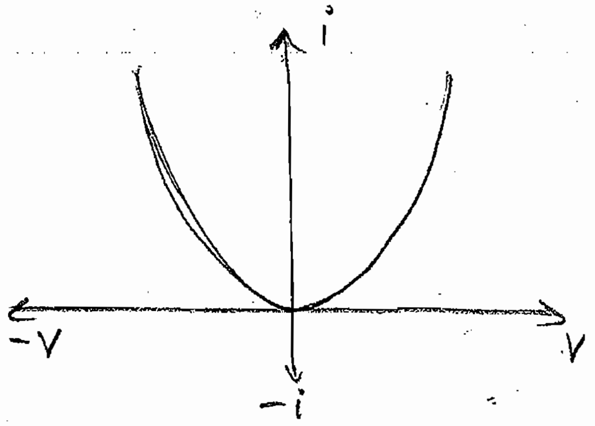



① Linear, active, bilateral

② Linear, passive, bilateral

③ Non-linear, active, unilateral

✓④ Non-linear, passive, bilateral

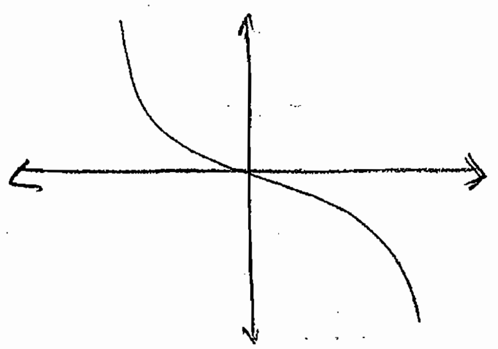



→ Non-linear

→ Unilateral

→ Both active & passive

overall → Active



→ Non linear

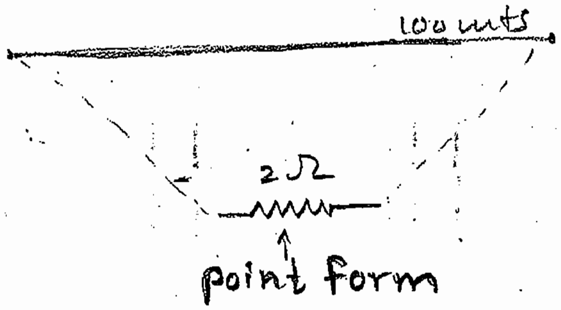
→ Active

→ Bilateral.

Active elements can act as passive elements, but passive elements can't act as active

eg:- Capacitor always acts as a sink; either it charges or discharges.

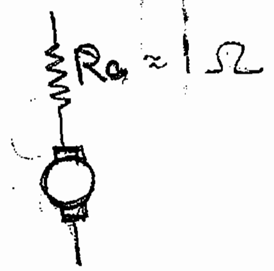
- **LUMPED PARAMETERS**




Eg :-

→ PCB component

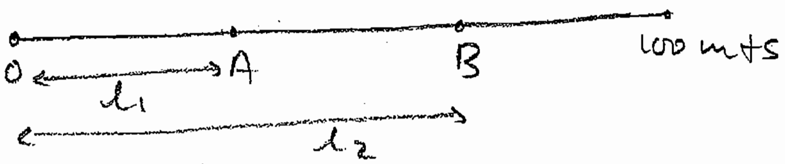
→




Properties :-

- Simple
- Linear Algebraic equation
- Solutions are fast
- Approximated values.

- **DISTRIBUTED PARAMETERS**




$$R_{actual} = \frac{\partial R}{\partial l}$$

Eg :-

- Long Tx line
- Emf concepts
- Antenna (waveguide)

Properties :-

- Complex
- Linear Differential Equations
- Solutions are Tedious
- Very accurate values.

→ Excat modelling

2) The relations like $v = n\lambda$ holds good for ____.

IES (a) Lumped ✓(b) Distributed

(c) Lumped & Dist. (d) None

- NODE (n) :

A node is a point of interconnection or junction b/w 2 or more components.

- BRANCH (b) :

A branch is an elemental connection between two nodes.

- Degree of a Node ($\delta$) :

No. of branches incident or connected at any node represents its degree.

If $\delta = 2$ → simple node ($n_s$)

$\delta > 2$ → principle node ($n_p$)

NOTE :-

For any ckt or n/w

$$\boxed{\sum_{i=1}^{n} \delta_i = 2 \times b}$$

- MESH (m) :

Mesh is a closed path of ckt or n/w which should not have further closed path in it.

- Loops ($l$) :

Loops are all possible closed path of n/w

NOTE :-

→ For any ckt or n/w : $\boxed{m = b - n + 1}$

→ The mini. no. of eq$^n$ to solve any ckt or n/w is :

$$m = b - n + 1$$

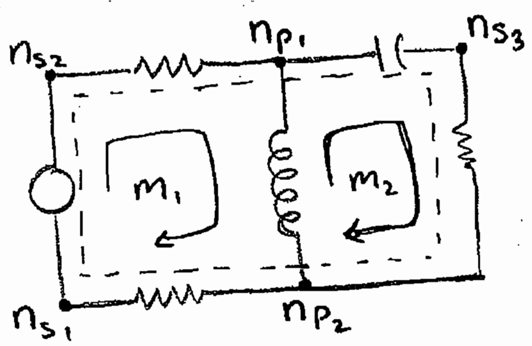
→ Meshes are specifically called as independent loops.

→ All meshes are by default loops but all loops are not meshes.

→ In nodal analysis we may neglect simple node & one of the principle node is considered as reference.

Eg:-

3)




$n = 5$

$b = 6$

$m = 2$

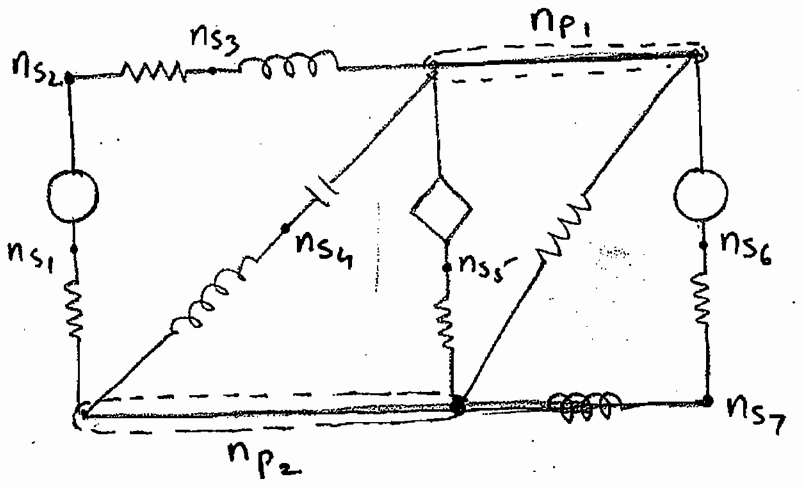
$l = 2 + 1 = 3$

$\Sigma S_i = 2 + 2 + 2 + 3 + 3$

$= 12$

$b \times 2 = 6 \times 2 = 12$

4)




$n = \cancel{8}\,9$

$b = \cancel{8}\,12$

$m = 4$

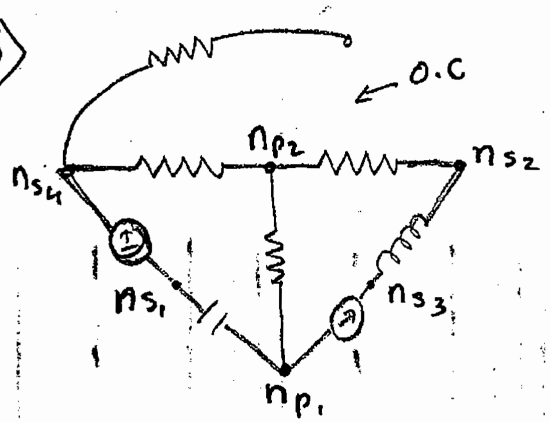
$l = 4 + 6 = 10$

$\Sigma \delta_i = (7 \times 2) + 5 + 5$

$= 24$

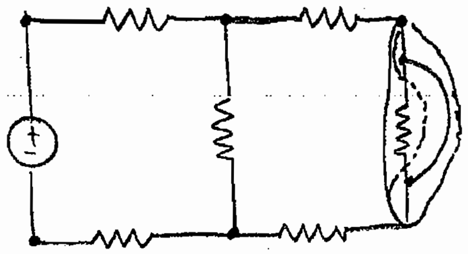
$= 2 \times 12$

5)



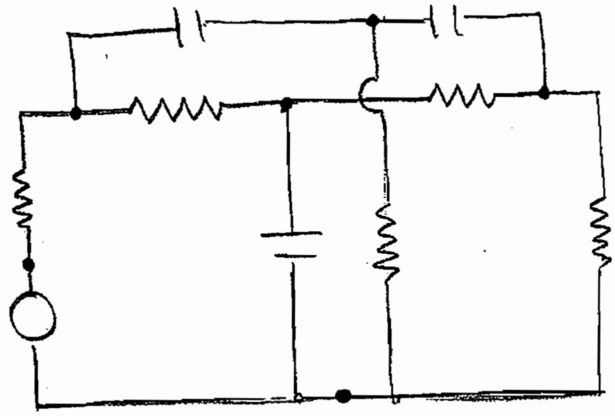

$n = 6$
$b = 7$
$m = 2$

6)

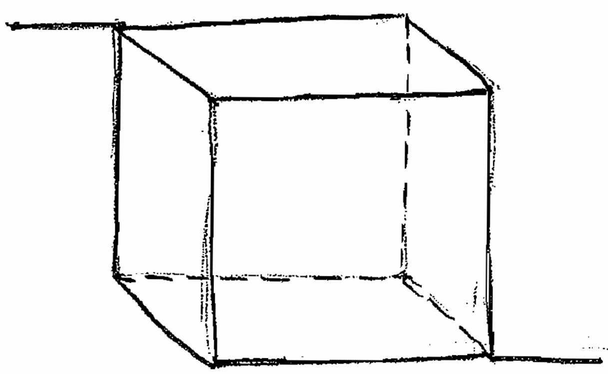


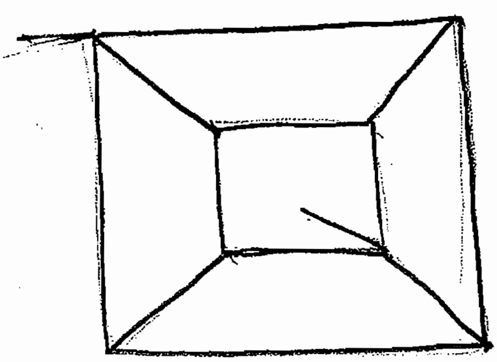
$n = 5$
$b = 6$
$m = 2$

7) The mini. no. of eq$^n$ required to solve
GATE the ckt bellow is $\underline{\quad 4 \quad}$.



$$m = b - n + 1$$
$$= 9 - 6 + 1$$
$$= 4$$





Non-planar $\longrightarrow$ Planar

# * OHM'S LAW

→ L.T.I

→ Temperature is const.

→ Uniform cross section of material

$$\boxed{J = \sigma E}$$ (1st Form)

Current density ← $J$; Conductivity ← $\sigma$; Electric field intensity ← $E$

$$\frac{I}{a} = \sigma \frac{V}{l}$$

$$V = \left[\frac{l}{\sigma a}\right] I$$

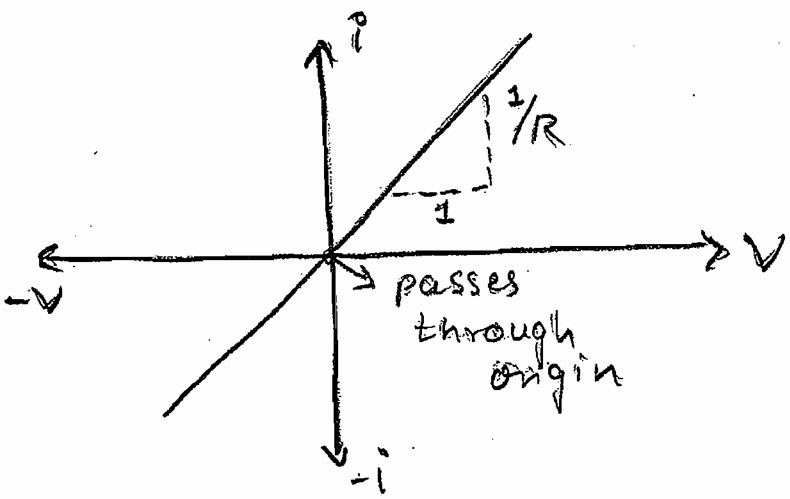
but $\frac{1}{\sigma} = \rho$ (resistivity)

$$V = \left[\frac{\rho l}{a}\right] I$$

$$\boxed{V = IR}$$ (2nd Form) Circuital form of Ohm's Law in 'R'

R → Resistance ($\Omega$)

graphically, $I = \left(\frac{1}{R}\right) V$



$\boxed{L} \rightarrow$ Electromagnetic

$$\boxed{\psi = Li} \quad (3^{rd} \text{ Form})$$

$$\psi = N\phi$$

↳ Flux linkage (Wb-T)

$$\boxed{N\phi = Li}$$

$$N\frac{d\phi}{dt} + \phi\frac{dN}{dt} = L\frac{di}{dt} + i\frac{dL}{dt}$$

$$\therefore \boxed{V = L\frac{di}{dt}} \quad (4^{th} \text{ Form})$$

Critical form of Ohm's Law in 'L'

$\boxed{C} \rightarrow$ Electro-static

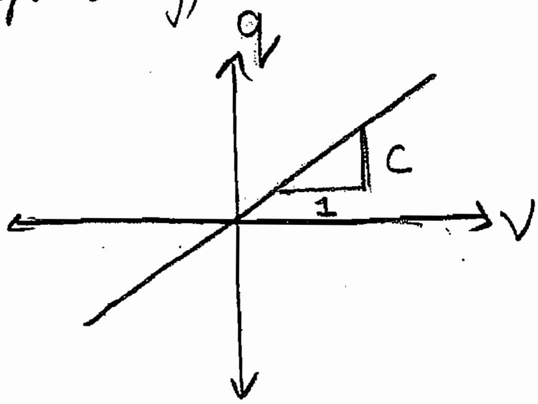
$$\boxed{q = CV} \quad (5^{th} \text{ Form})$$

$$\frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt} \rightarrow \text{If } \varepsilon = 0 \Rightarrow \frac{dC}{dt} = 0$$

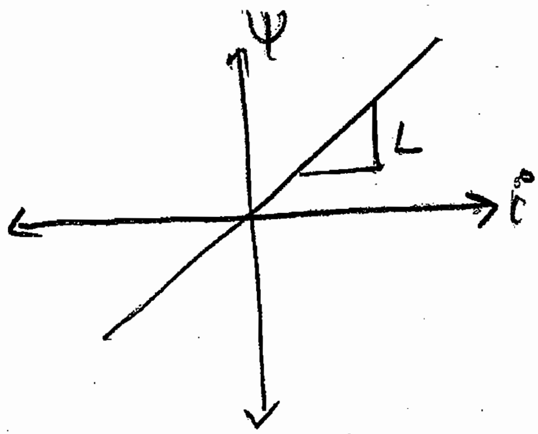
$$\boxed{i = C\frac{dV}{dt}} \quad (6^{th} \text{ Form})$$

Critical form of Ohm's Law in 'C'

Graphically,




Slope = C




Slope = L

# * DC CIRCUIT ANALYSIS

## • Properties of DC Supply :-

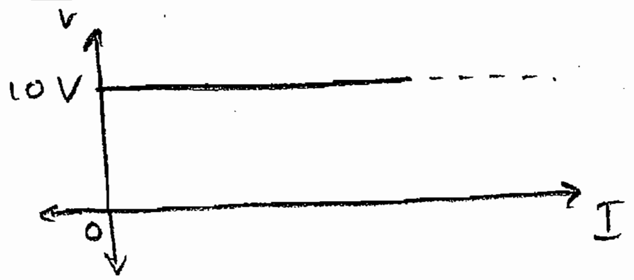
→ Unipolar
→ Unidirection
→ No change in phase / polarity
→ Power freq. = 0 Hz.

They are used in small, independent isolated power supply systems, where electrical energy can be stored in small capacities.

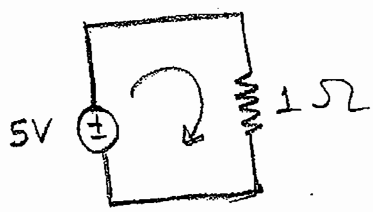
Eg:-
- Machine Tools
- Medical Instruments
- Cell phones, Toys
- Defence Applications

Precision
Accuracy
Automation

## • Standard DC Waveform :-




$T \to \infty$
$f \to 0$



# * AC Circuit Analysis

## • Properties of AC supply :-

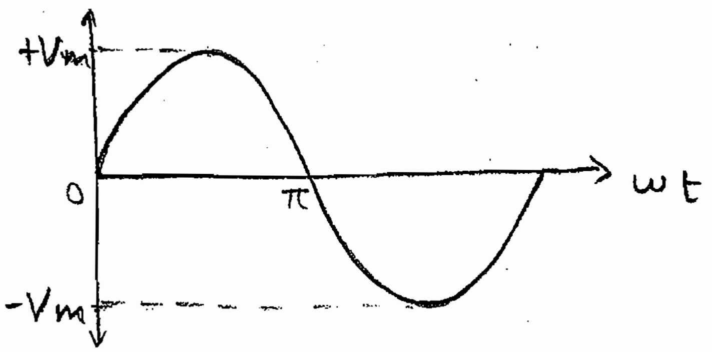
→ Bipolar
→ Bidirection
→ Definite charge in phase / polarity.
→ Power freq. exist (India = 50 Hz)

They are used in Large, Bulk, Continuous power supply systems, where electrical energy cannot be stored.

Eg:-
- Domestic
- Industrial Applications.

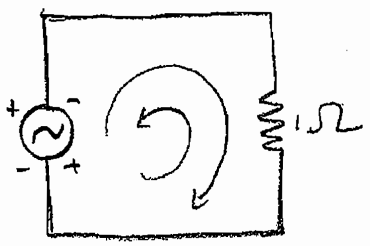
} Robust, Powerful

## • Standard AC Waveform :-




Sinusoid → sine, → cosine

$V = V_m \sin \omega t$



# VOLTAGE :

$$V = \frac{dW}{dq}$$

→ It is the force (Emf), which can drive charges.

→ Units : volts or J/C.

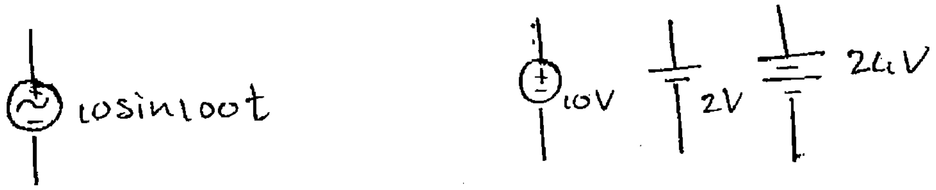
→ Range : kV, mV, V, MV, $\mu$V.

→ Symbols : $v$, $\bar{V}$, $v(t)$

→ Circuit Symbols :

AC

DC




→ Examples :

DC → Cell, Battery, Fuel cells.
P.V solar panels
Rectified power sources
Conventional DC convertor

AC → UPS
Inverter
Alternator

# CURRENT :

$$i = \frac{dq}{dt} \quad \text{or} \quad I = \frac{\Delta Q}{\Delta t} = \frac{Q}{t}$$

→ It is rate of flow of charge

→ Units : Ampere (A) or C/sec

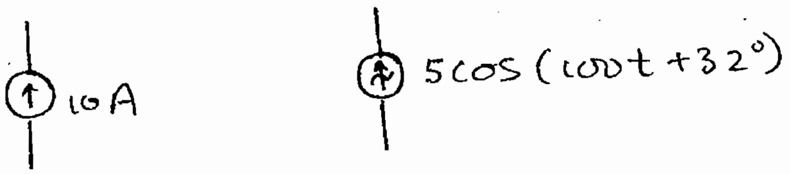
→ Range : μA, mA, A, kA

→ Symbols : $i$ , $I$ , $\bar{I}$ , $i(t)$

→ Circuit Symbols :

DC

AC




⇒ Examples :

DC → A DC series generator can be modeled as DC current source

→ A BJT can be modeled as a DC dependent current source.

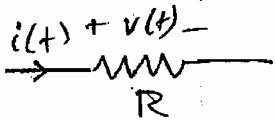
AC → A Feeder can be modeled as a AC current source.

(Feeder → const. current density conductors.)

# RESISTANCE :

→ It is electrical property of matter.

→ "Resistor" is a component to model it.




↳ it is classified based on the material.

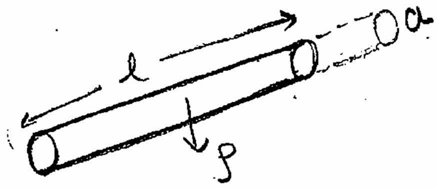
- Carbon
- Tungsten
- Ceramic

→ Units : ohms ($\Omega$), $\frac{V}{A}$

→ Range : $\mu\Omega$, $m\Omega$, $\Omega$, $k\Omega$, $M\Omega$, $G\Omega$

$$V = IR \Rightarrow I = \frac{V}{R}$$

## Basic Formula :




$$R = \frac{\rho l}{a} \ \Omega$$

$l$ → length of material

$a$ → cross section area

$\rho$ → specific resistance (or) resistivity of material

→ Resistance depends upon temperature

$$\boxed{R_t = R_0[1 + \alpha t]}$$

$\alpha$ → temperature co-eff. of resistance

$\alpha$ is +ve → conductors

$\alpha$ is −ve → semiconductors.

Examples :-

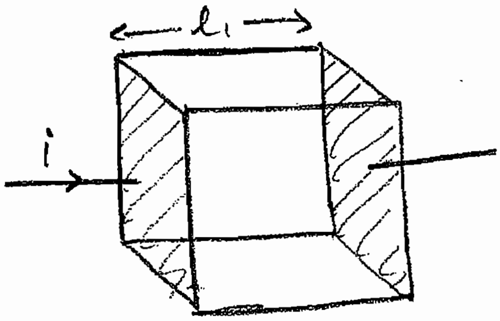
→ All industrial & Domestic wiring.

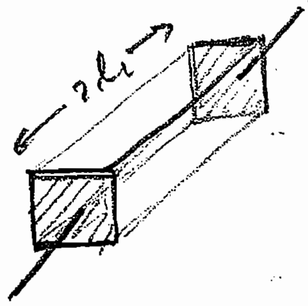
→ Communication & Tx. lines

→ PCB components.

Eg:-

1) A cube shaped material has a resistance of 2 Ω between any of its opposite faces. Now if this material is stretched in one direction by applying a linear force to double its original length, then the resistance between the two opposite stretched faces is ______ .




$$R_1 = \frac{\rho_1 l_1}{a_1} = 2$$

$\rho_2 = \rho_1$

$l_2 = 2 l_1$

$V_1 = V_2$

$l_1 a_1 = l_2 a_2$

$\therefore \quad a_1 = 2 a_2$

$\therefore \quad a_2 = \frac{a_1}{2}$

$$R_2 = \frac{\rho_2 l_2}{a_2} = \frac{\rho_1 (2 l_1)}{a_1/2}$$

$$= 4 \left[ \frac{\rho_1 l_1}{a_1} \right]$$

$$= 4 (2)$$

$$= 8\ \Omega$$

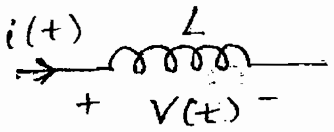
# INDUCTANCE

Electromagnetic property matter

→ "Inductor" is a component to model it.

↳ It is classified based on CORE material

- Iron
- Ferrite
- air




Units : Henry (H), $\frac{V\text{-sec}}{A}$,

Range : μH, mH, H

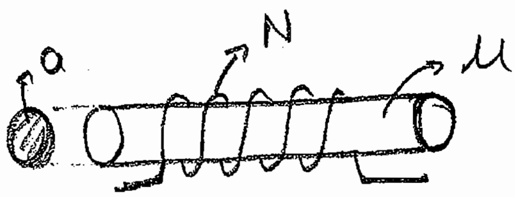
$$V = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int v\,dt$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$= \underbrace{\frac{1}{L}\int_{-\infty}^{0} v\,dt}_{\text{initial current}} + \frac{1}{L}\int_{0}^{t} v\,dt$$

$$i = I_0 + \frac{1}{L}\int_{0}^{t} v\,dt$$

## Basic Formula :-




$$L = \frac{\mu N^2 a}{l} \quad H$$

$\mu = \mu_0 \mu_r$ → permeability of CORE

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$\mu_r = 1$ (air)

$\mu_r > 1000$ (iron)

N → No. of turns of coil

a → cross-sectional area of core ($m^2$)

l → effective length of magnetic flux path (m)

Examples :-

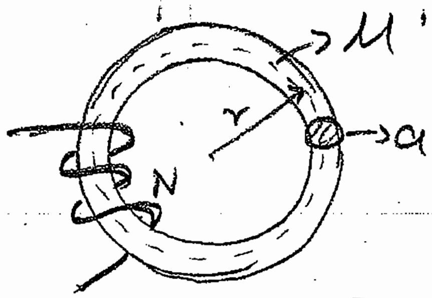
→ Filters

→ Choke coils

→ M/C windings

→ Tx lines.

mH/ph/km




$\boxed{l = 2\pi r}$

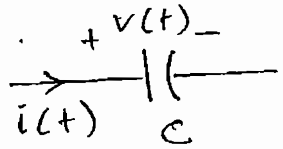
↳ mean circumference.

# CAPACITANCE :

→ Electrostatic property of matter

→ "Capacitor" is a component to model it.

↳ It is classified based on Dielectric element/material.




→ Electrolytic

→ Ceramic

→ Polyster

Units : Faraday (F), $\frac{A\text{-sec}}{V}$

Range : pF, nF, μF, mF

$$i = C\frac{dV}{dt} \quad ; \quad V = \frac{1}{C}\int i\,dt$$

$$V = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \underbrace{\frac{1}{C}\int_{-\infty}^{0} i\,dt}_{\text{initial voltage}} + \frac{1}{C}\int_{0}^{t} i\,dt$$

$$\therefore \quad V = V_0 + \frac{1}{C}\int_0^t i\,dt$$

## Basic Formula :-

$$C = \frac{\varepsilon A}{d} \; F$$

$\varepsilon = \varepsilon_0 \varepsilon_r \rightarrow$ permitivity of dielectric

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\varepsilon_r = 1$ (air)

$\varepsilon_r = 6$ (polyster)

$\varepsilon_r = 700$ (ceramic)

$d \rightarrow$ dist. between electrodes (m)

$A \rightarrow$ common cross-sectional area b/w electrodes ($m^2$)

## Examples :-

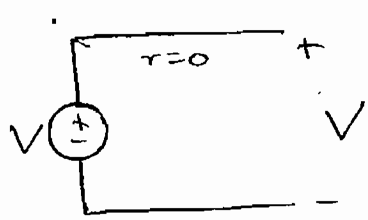
→ Filters

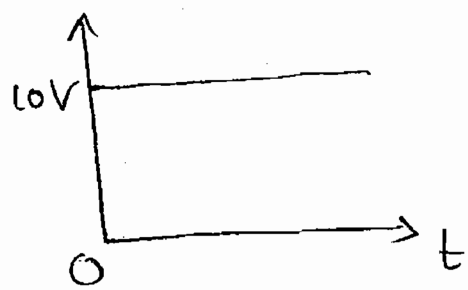
→ Power system

→ PDC circuits

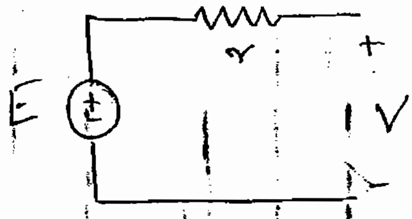
→ Tx lines $\mu F$/ph/km

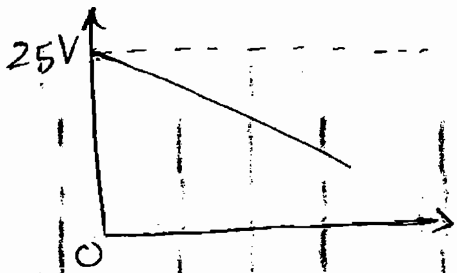
## IDEAL VOLTAGE SOURCE :-



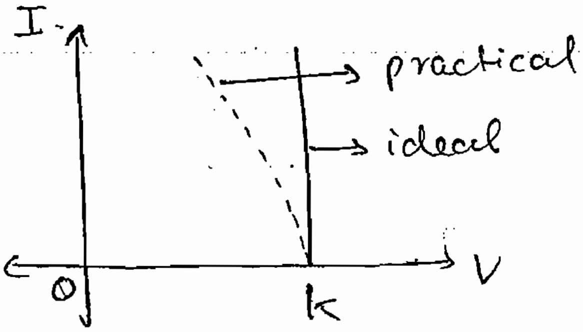



## PRACTICAL VOLTAGE SOURCE :-

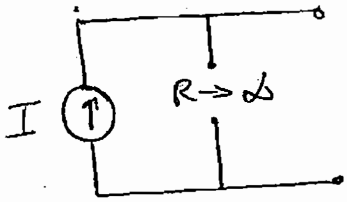


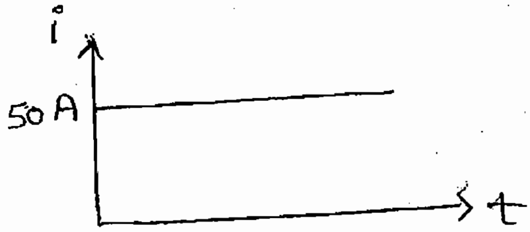




## V-I Characteristics :-

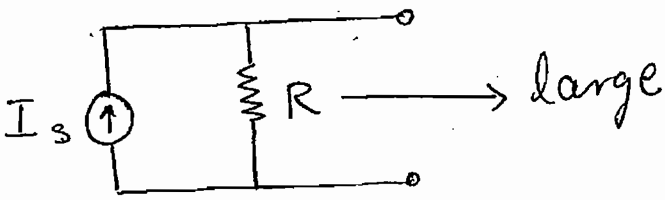


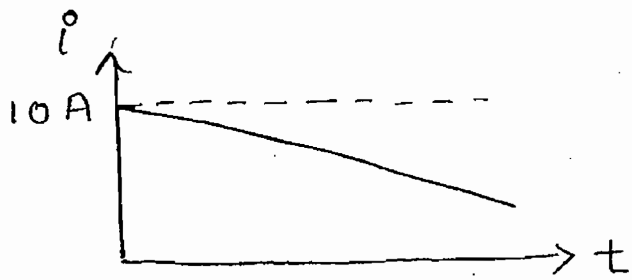

## IDEAL CURRENT SOURCE



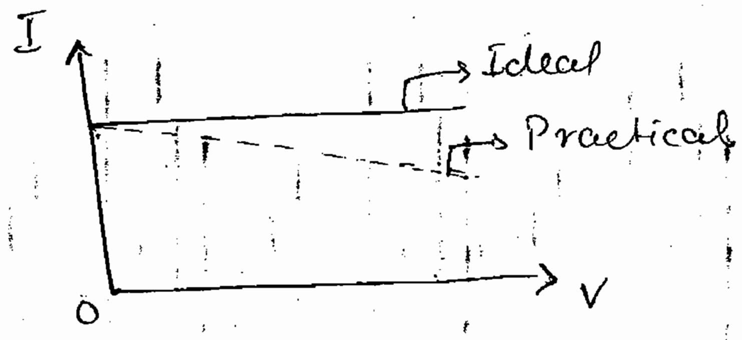




## PRACTICAL CURRENT SOURCE



## V-I Characteristics :-



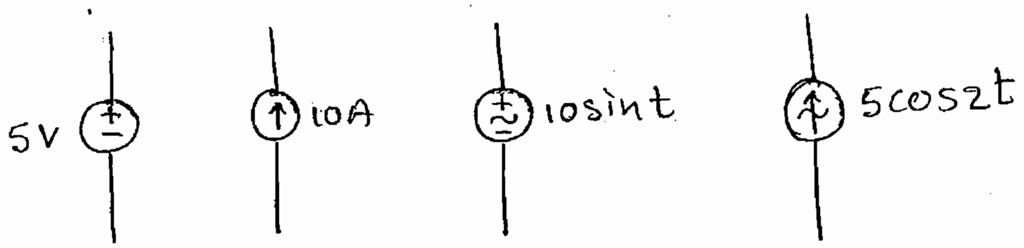

## INDEPENDENT SOURCES :-

- Properties
- Characteristics
- Values [Mag, Freq., etc]

These are independent to any other parameter within or outside the circuit.

Examples :-

→ Ideal sources



# DEPENDENT SOURCE :-

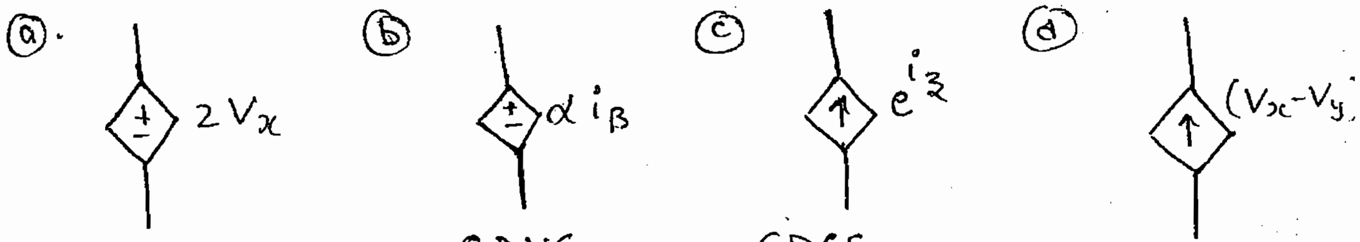
- Properties
- Characteristics
- Values [Mag, Freq, etc]

} depend upon any other parameter within or outside the ckt

## Examples :-

→ Practical sources
→ BJT
→ Solar cell

→ Standards ④ Types :-




| ⓐ | ⓑ | ⓒ | ⓓ |
|---|---|---|---|
| VDVS<br>VCVS | CDVS<br>CCVS | CDCS<br>CCCS | VDCS<br>VCCS |

→ Unlike independent sources dependent sources can not be suppressed in terms of resistance; as these models by themselves represent complex circuits

# POWER :

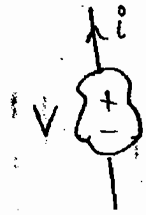
Rate of change in energy.

$$P = \frac{dW}{dt} = \frac{dE}{dt} \quad ; \quad P = \frac{E}{t}$$

Units : Watts or J/sec

Range : mW, W, kW, MW, GW

$$\boxed{1\,hp = 746\ W}$$




→ Source

$$P_{delivered} = +Vi \quad W$$
$$= \frac{dW}{dq} \cdot \frac{dq}{dt} = \frac{dW}{dt}$$

L.T.I

$\boxed{R}$

$$P_R = V_R i_R = i_R^2 R = \frac{V_R^2}{R}$$

$\boxed{L}$ → $\boxed{P_{avg} = 0}$

→ instantaneous power

$$P_L = V_L \cdot i_L = L i \frac{di}{dt} \quad W$$

$\boxed{C}$ → $\boxed{P_{avg} = 0}$

→ instantaneous power

$$P_c = i_c V_c = C V \frac{dV}{dt} \quad W$$

ENERGY :-

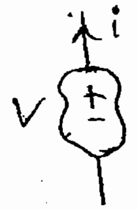
Capacity to do work (electrical)

$\boxed{E = \int P\,dt}$ $\quad$ $\boxed{E = P \times t}$

Units : Joules (or) W-sec

1 unit of E.E. = 1 kWH

$$1\,kWH = 1000\ W \times 1\ hr$$
$$= 500\ W \times 2\ hr$$
$$= 100\ W \times 10\ hr = 2000\ W \times \frac{1}{2}\ hr$$



$$E_{delivered} = +\,V \cdot i \cdot t \quad J$$

$$\boxed{1\ \text{KWH} = 36 \times 10^5\ J}$$

$\boxed{R}$

$$E_R = \int P_R\,dt = \int V_R\, i_R\, dt = \int i_R^2 R\, dt = \int \frac{V_R^2}{R} dt$$

→ But for L.T.I.

$$E_R = V_R \cdot i_R \cdot t = i_R^2 R \cdot t = \frac{V_R^2}{R} \cdot t \quad J$$

$\boxed{L}$

$$E_L = \int P_L\, dt = \int L i \frac{di}{dt} \cdot dt$$

→ Now for L.T.I

$$E_L = \int L i \frac{di}{dt} \cdot dt = \frac{1}{2} L i^2$$

But $\psi = L i$

$$\therefore \boxed{E_L = \frac{1}{2} L i^2 = \frac{1}{2} \psi i = \frac{\psi^2}{2L} \quad J}$$

$\boxed{C}$

$$E_C = \int P_C\, dt = \int C V \frac{dV}{dt} \cdot dt$$

→ But for L.T.I

$$E_C = \int C V \frac{dV}{dt}\, dt = \frac{1}{2} C V^2$$

But $q = CV$

$$\boxed{E_C = \frac{1}{2} C V^2 = \frac{1}{2} q V = \frac{q^2}{2C} \quad J}$$

# ENERGY STORAGE CAPACITY in a Battery :—

→ Ampere-Hours (Ahr)

Eg: Pencil cell (AA) → 1.5 V , 500 m Ahr
Cell phones → 3.7 V (1500 – 1800) m Ahr
Cell batteries → 12 V, 40 Ahr

Eg:-
40 Ahr → 40 A × 1 hr
→ 20 A × 2 hr
→ 1 A × 40 hr
→ 80 A × $\frac{1}{2}$ hr

# KIRCHOFF'S LAWS :—

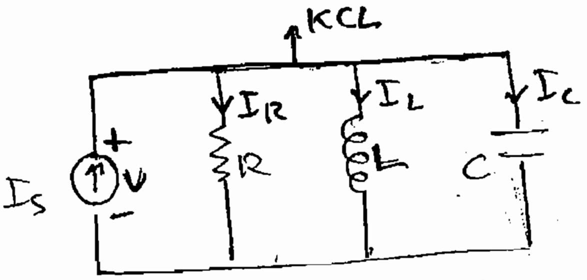
⟨1⟩ K 1st Law → KCL → Node

$$\sum i \Big|_{node} = 0$$

$i$ → $\frac{V}{R}$ , $\frac{1}{L}\int V dt$ , $C\frac{dV}{dt}$

Based on law of conservation of charge.

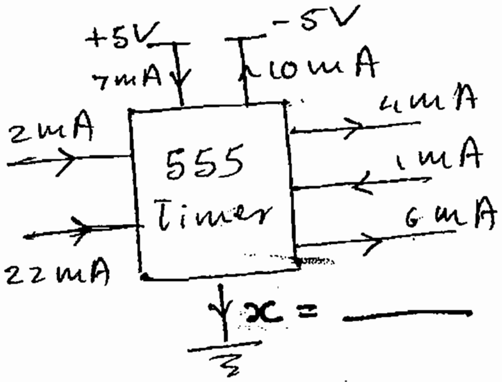
Eg:




$$-I_s + I_R + I_L + I_c = 0$$

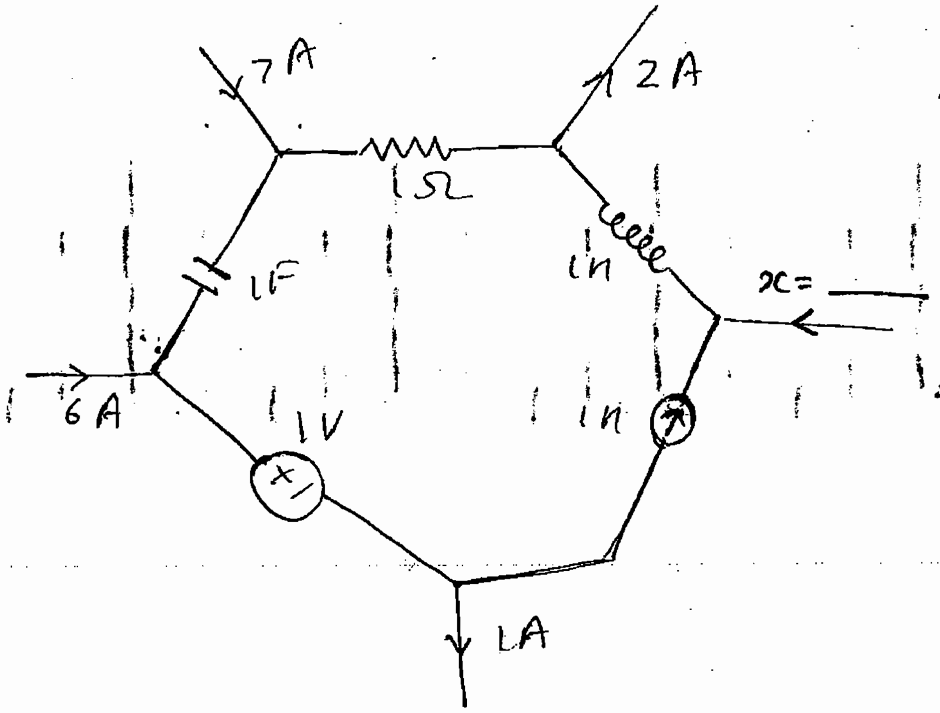
$$I_s = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dV}{dt}$$

Eg:-




$$2 + 22 + 7 + 1 = x + 6 + 4 + 10$$

$$\therefore \boxed{x = 12\ mA}$$




$$7+6+x = 1+2$$
$$13+x = 3$$
$$\boxed{x = -10}$$

∴ Actually the current is coming out of the node.

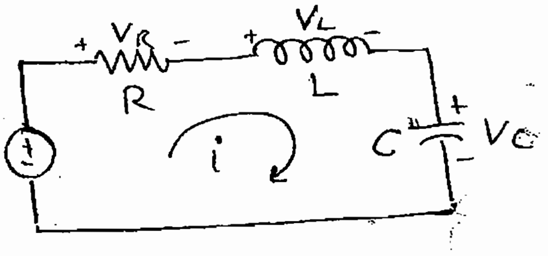
## <2> K 2nd Law → KVL → Mesh

→ Based on law of conservation of energy

$$\sum V \Big|_{Mesh} = 0$$

$$iR \qquad L\frac{di}{dt} \qquad \frac{1}{C}\int i\,dt$$

Eg:

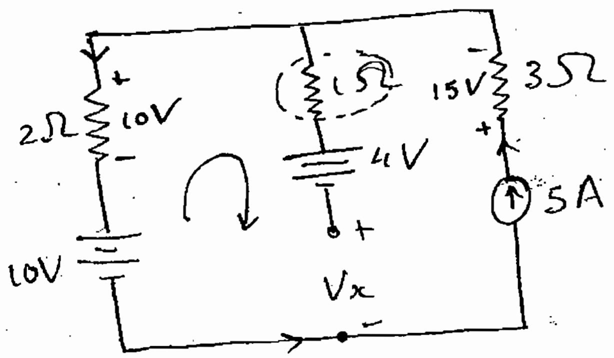



KVL

$$-V_S + V_R + V_L + V_C = 0$$

$$V_S = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

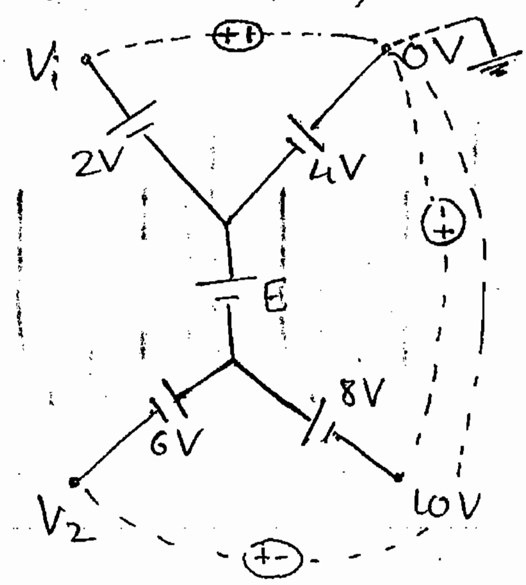
Eg:- Find $V_x$ = ______




$$+10+10-4 = V_x$$

∴ $\boxed{V_x = 16\ V}$

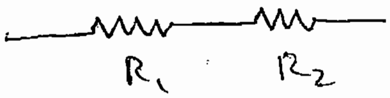
Eg: Find $E, V_1, V_2$.




$$0 - 4 - E - 8 = 10$$
$$\boxed{E = -22\,V}$$
$$V_2 + 6 - 8 = 10$$
$$V_2 = 12\,V$$
$$V_1 - 2 + 4 = 0$$
$$\boxed{V_1 = -2\,V}$$
$$V_2 + 6 + E + 4 = 0$$
$$\boxed{V_2 = 12\,V}$$

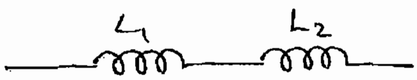
- Series Connection of Elements :-

**R**



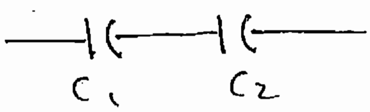

$$R_s = R_1 + R_2$$

**L**



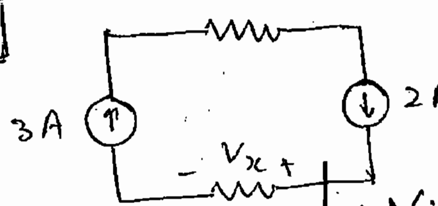

$$L_s = L_1 + L_2$$

**C**



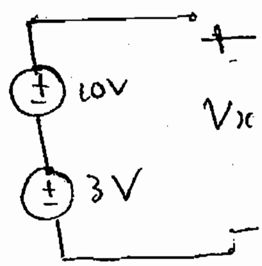

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

**I**




→ Violating KCL

**V**



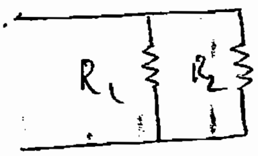

→ Current sources of different values can never exist in series. They violate KCL

→ If 2 current sources are in series, they must be equal both in magnitude & direction.

Voltage sources of any value can be series.

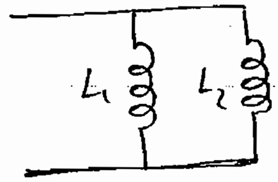
- **Parallel Connection of Elements:**

R



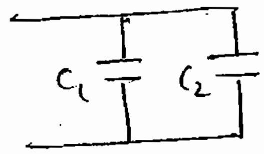

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

L



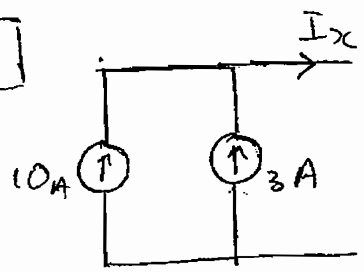

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$$

C

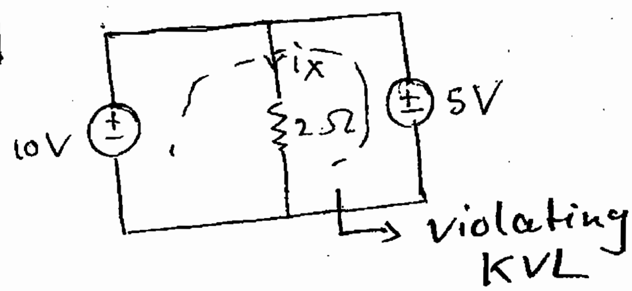



$$C_p = C_1 + C_2$$

I




V




→ Never 2 ideal voltage sources of diff values can exist in ||. They violate KVL

→ If 2 vltg sources exist in ||, they must be equal both in magnitude and polarity.

→ Practical vltg sources can always exist in parallel.

→ Current sources of any value can be in parallel.

# Open Circuit / o.c

→ In an o.c, $i = 0$ for any voltage

$R_{o.c} = \frac{V}{0} = \infty \, \Omega$

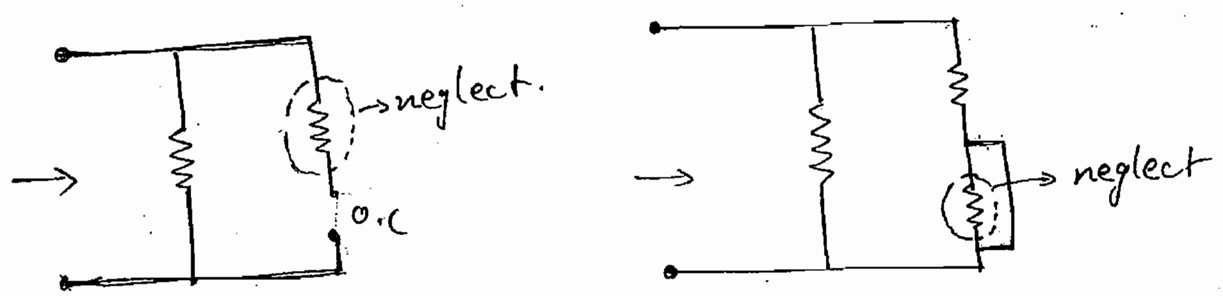
→ Any passive element in series to o.c can be neglected.

# Short Circuit / s.c

→ In a s.c, $V = 0$ for any (i)

$R_{s.c} = \frac{0}{i} = 0 \, \Omega$

→ Any passive element in parallel to s.c. can be neglected.

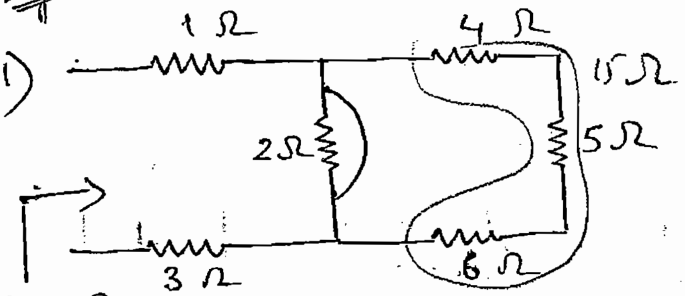



In high vltg engineering o.c. live wire faults are more dangerous than s.c., to human beings.
However s.c always have protection, both at high & low level. vltg by designing ~~same~~ correct rated fuses.

→ Resistance is offered by the path where current can flow as seen from target terminal.

Eg:-

1)

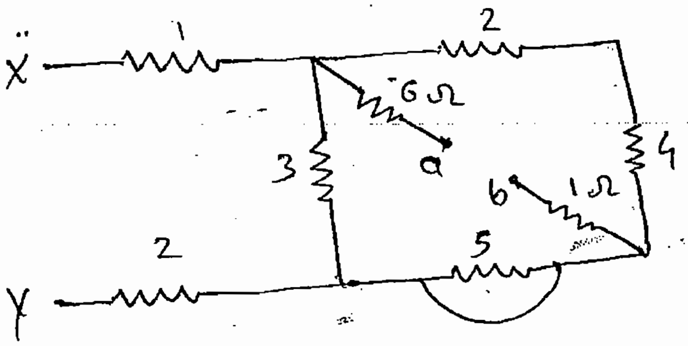



Req = ?

~~4+5+6=15~~

$R_{eq.} = 1 + 3 = 4\ \Omega$

2)



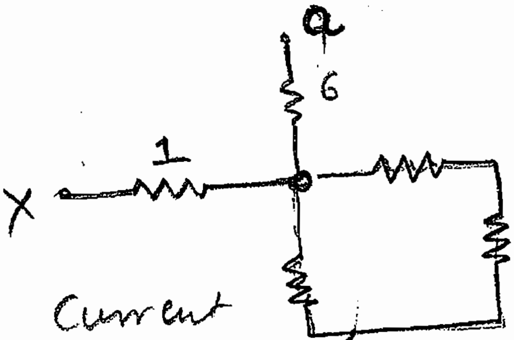

$R_{xy}$, $R_{ab}$, $R_{xa}$
$R_{yb}$.

$\frac{3 \times 6}{9} = 2$ ; $R_{xy} = 1 + 2 + 2 = 5\ \Omega$

$R_{ab} = 1 + 6 + 2 = 9\ \Omega$

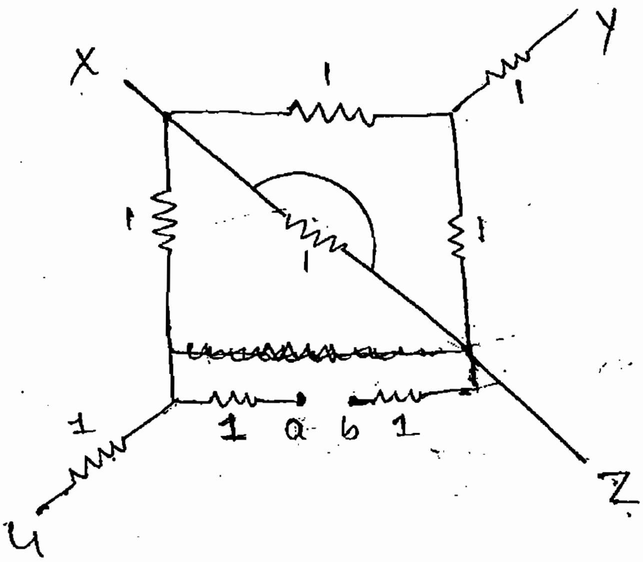
$R_{xa} = 1 + 6 = 7\ \Omega$

$R_{yb} = 1 + 2 = 3\ \Omega$




Current will not flow here.

3)




$R_{xy} = \frac{1}{2} + 1 = \frac{3}{2}\ \Omega$

$R_{xz} = 0\ \Omega$

$R_{xu} = 2\ \Omega$

$R_{yz} = \frac{1}{2} + 1 = \frac{3}{2}\ \Omega$

$R_{yu} = \frac{1}{2} + 3 = \frac{7}{2}\ \Omega$

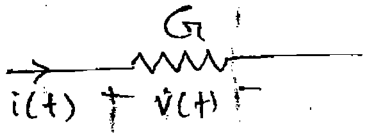
$R_{ay} = 3 + \frac{1}{2} = \frac{7}{2}\ \Omega$

$R_{zu} = 2\ \Omega$ $\quad R_{bx} = 1\ \Omega$

$R_{ab} = 3\ \Omega$ $\quad R_{by} = 2 + \frac{1}{2} = \frac{5}{2}\ \Omega$ $\quad R_{az} = 2\ \Omega$

# CONDUCTANCE :

→ It is the ability to conduct electrically
→ It is used to further classify conductor (metals)




Units :- mho (℧) or seimens (S) or $\frac{A}{V}$

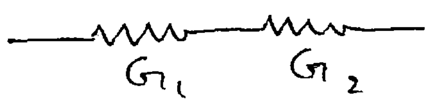
$G = \frac{1}{R}$ ; $G = \frac{a}{\rho l}$

$\boxed{G = \frac{\sigma a}{l}}$ ; $\because \frac{1}{\rho} = \sigma$ (conductivity)

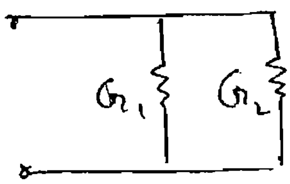
Units for '$\sigma$' → $(\Omega\text{-}m)^{-1}$
→ ℧/m or S/m

$V = \frac{I}{G}$ ; $I = V \cdot G$

$P_G = V_G \cdot i_G = \frac{i_G^2}{G} = V_G^2 \cdot G$ W




$\frac{1}{G_s} = \frac{1}{G_1} + \frac{1}{G_2}$




$G_p = G_1 + G_2$

## Based on Conductivity :—

Rank → $\boxed{1}$ Silver

$\boxed{2}$ Copper → used in high current density compact sys.

→ Domestic, Industrial, M/c, PCB

[3] Gold

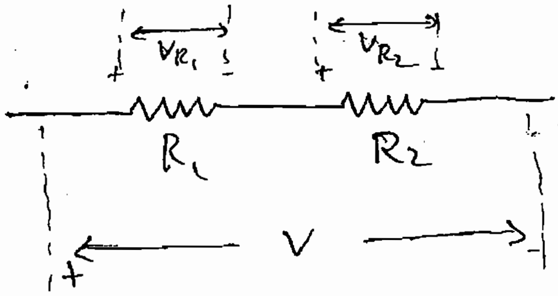
[4] Alluminium. → used in external Tx & distribution lines.

- → cheap
- → Light weight

# Voltage Division Rule

→ series connected elements only

[R]




[L]

$$V_{L_1} = V\left[\frac{L_1}{L_1+L_2}\right]$$

$$V_{L_2} = V\left[\frac{L_2}{L_1+L_2}\right]$$

[C]

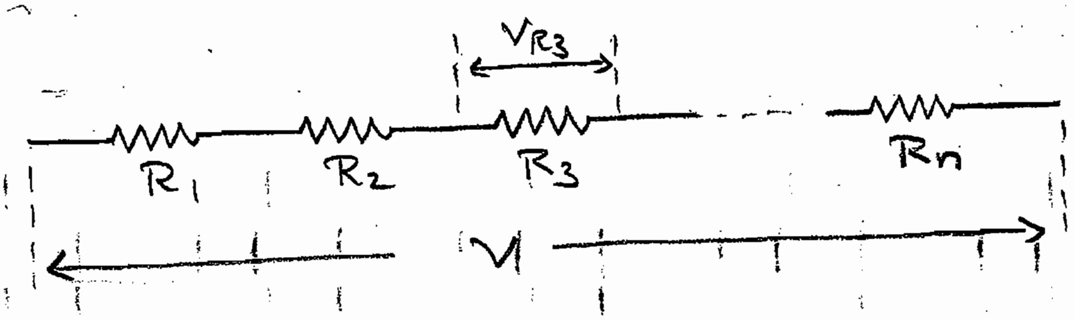
$$V_{C_1} = V\left[\frac{C_2}{C_1+C_2}\right]$$

$$V_{C_2} = V\left[\frac{C_1}{C_1+C_2}\right]$$

[G]

$$V_{G_1} = V\left[\frac{G_2}{G_1+G_2}\right]$$

$$V_{G_2} = V\left[\frac{G_1}{G_1+G_2}\right]$$




$$V_{R_3} = V\left[\frac{R_3}{\sum_{i=1}^{n} R_i}\right]$$

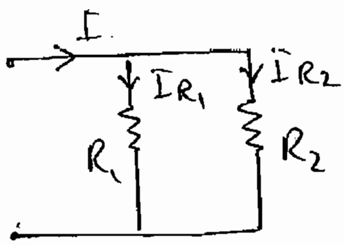
But suppose these are capacitors:

$$V_{C_3} = V\left[\frac{1/C_3}{\sum_{i=1}^{n} 1/C_i}\right]$$

# Current Division Rule :-

Parallel connected elements only.

$\boxed{R}$



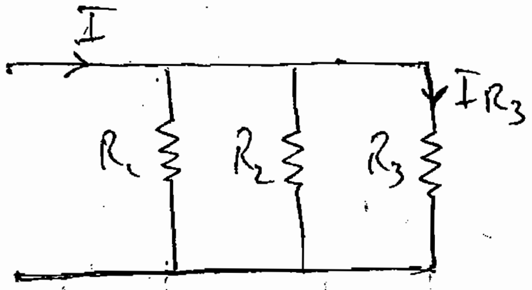

$$I_{R_1} = I\left[\frac{R_2}{R_1+R_2}\right]$$

$$I_{R_2} = I\left[\frac{R_1}{R_1+R_2}\right]$$

$\boxed{L}$ $I_{L_1} = I\left[\frac{L_2}{L_1+L_2}\right]$ ; $I_{L_2} = I\left[\frac{L_1}{L_1+L_2}\right]$

$\boxed{C}$ $I_{C_1} = I\left[\frac{C_1}{C_1+C_2}\right]$ ; $I_{C_2} = I\left[\frac{C_2}{C_1+C_2}\right]$

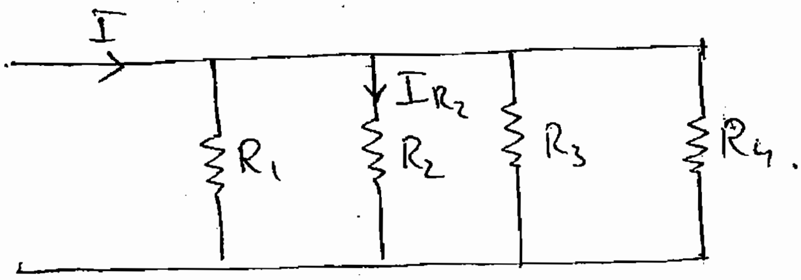
$\boxed{G}$ $I_{G_1} = I\left[\frac{G_1}{G_1+G_2}\right]$ ; $I_{G_2} = I\left[\frac{G_2}{G_1+G_2}\right]$



$$I_{R_3} = I\left[\frac{R_1 R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

→ But suppose capacitors are there

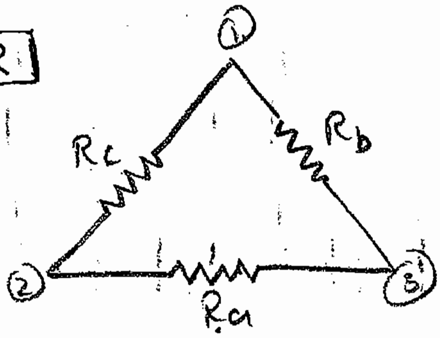
$$I_{C_3} = I\left[\frac{1/C_1 \cdot 1/C_2}{\frac{1}{C_1}\cdot\frac{1}{C_2} + \frac{1}{C_2}\cdot\frac{1}{C_3} + \frac{1}{C_3}\cdot\frac{1}{C_1}}\right]$$

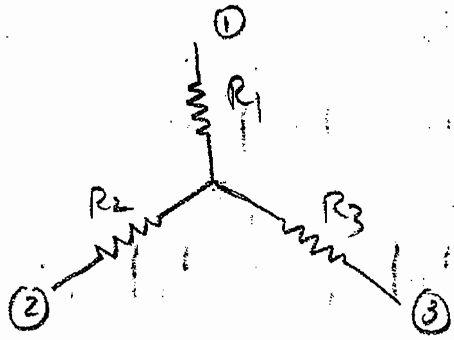


$$I_{R_2} = I\left[\frac{R_1 R_3 R_4}{R_1R_2R_3 + R_2R_3R_4 + R_3R_4R_1 + R_4R_1R_2}\right]$$

# DELTA - TO - STAR

$\boxed{R}$




$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_3 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$\boxed{L}$

$$L_1 = \frac{L_a L_c}{L_a + L_b + L_c}$$

$$L_2 = \frac{L_a L_b}{L_a + L_b + L_c}$$

$\boxed{C}$

$$\frac{1}{C_1} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

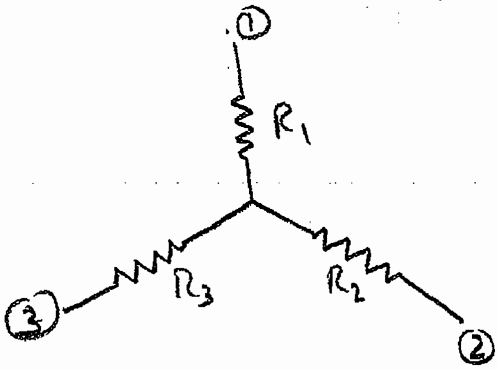
$$\frac{1}{C_2} = \frac{\frac{1}{C_a} \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$\boxed{G}$

$$\frac{1}{G_1} = \frac{\frac{1}{G_a} \cdot \frac{1}{G_c}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

$$\frac{1}{G_2} = \frac{\frac{1}{G_a} \cdot \frac{1}{G_b}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

# STAR - TO - DELTA




$\boxed{R}$

$$R_a = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_b = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_c = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$\boxed{L}$

$$L_a = L_1 + L_2 + \frac{L_1 L_2}{L_3}$$

$\boxed{C}$

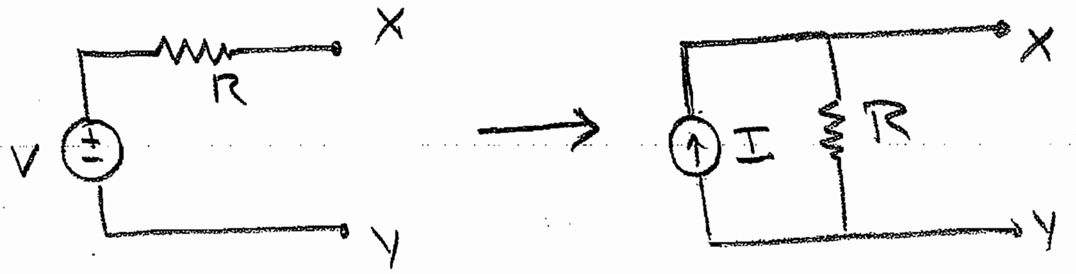
$$\cancel{R_{ca}}\ \frac{1}{C_a} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{\frac{1}{C_1} \cdot \frac{1}{C_2}}{\frac{1}{C_3}}$$

$\boxed{G}$

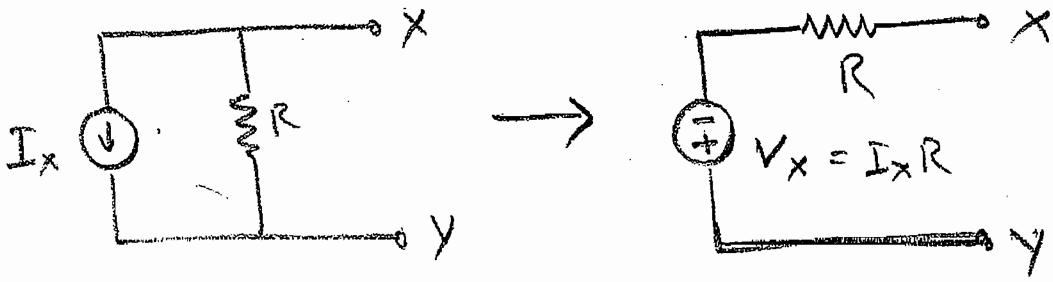
$$\frac{1}{G_a} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{\frac{1}{G_1} \cdot \frac{1}{G_2}}{\frac{1}{G_3}}$$

# SOURCE TRANSFORMATION

→ An ideal voltg source in series with resis. can be converted into ideal current source in $\parallel^{el}$ with same resis. across the same terminals & vice-versa.




$$I = \frac{V}{R}$$

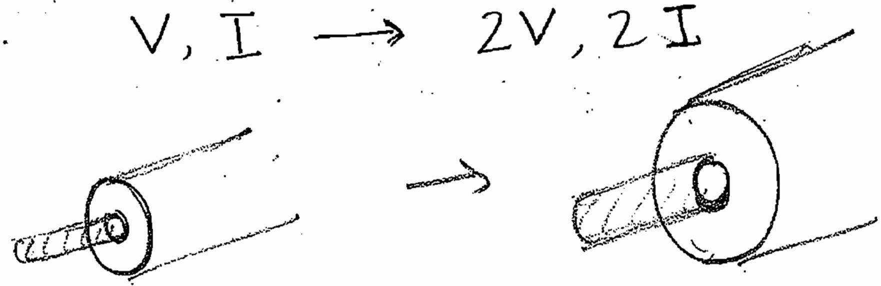



## RATINGS / Specifications :

→ They represent the maxi. permissible or allowable safe values for continuous operation of an electrical device.

→ Most of our electrical or electronic components or ntws will have vltg, current, power, freq. ratings etc.

$V, I \longrightarrow 2V, 2I$



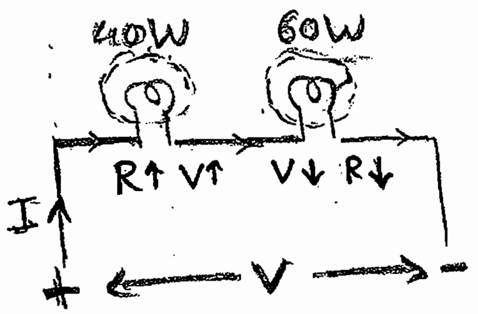
I ↑ → conductor cross-sectional area ↑

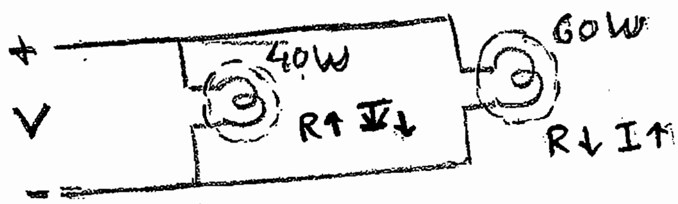
V ↑ → insulation withstanding capacity ↑

→ Most of our Electrical or Electronic equipment are designed for constant Voltage.

→ But their current carrying capacity depends upon Loding level.

| Low Wattage | High Wattage |
| --- | --- |
| V ↑ | V ↓ |
| I ↓ | I ↑ |
| a ↓ | a ↑ |
| R ↑ | R ↓ |



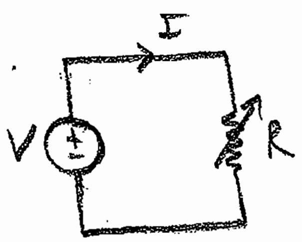




which glows brighter?

= 40W

which glows brighter?

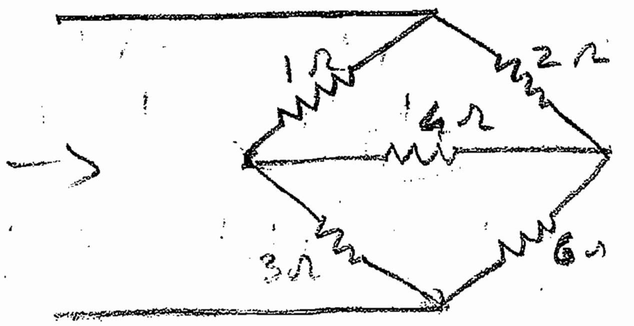
= 60W




If load ↑ ⇒ more power is drawn
⇒ more current is drawn as vltg remain same.

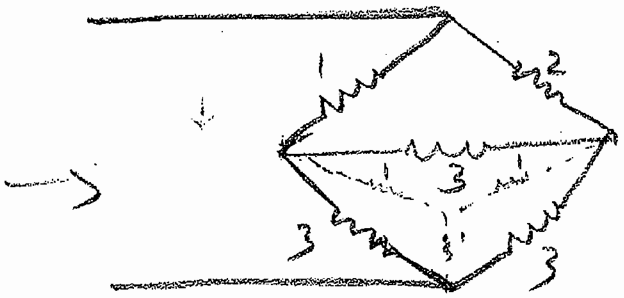
→ The load always decides the power drawing capability or current capability of the system.

# RESISTOR REDUCTION TECHNIQUE :

① 

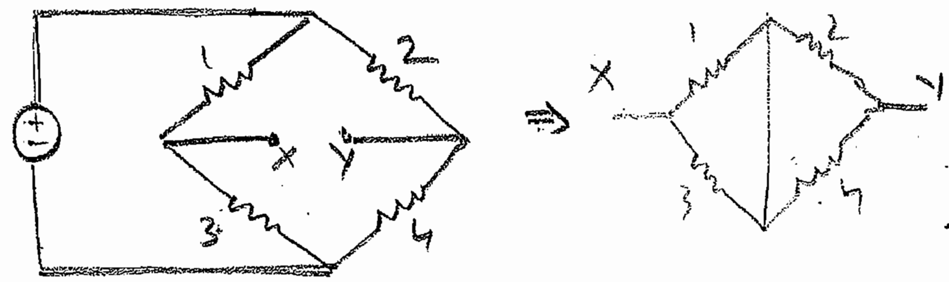

$4 \parallel 8$

$\therefore \dfrac{4\times 8}{12} = \dfrac{8}{3}\,\Omega$

② 


$1 + [2 \parallel 3]$

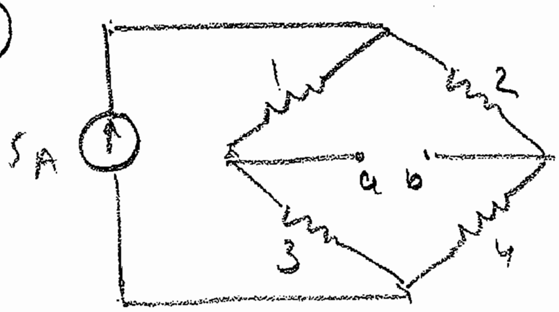
$1 + \dfrac{6}{5} = \dfrac{11}{5}\,\Omega$

③ 

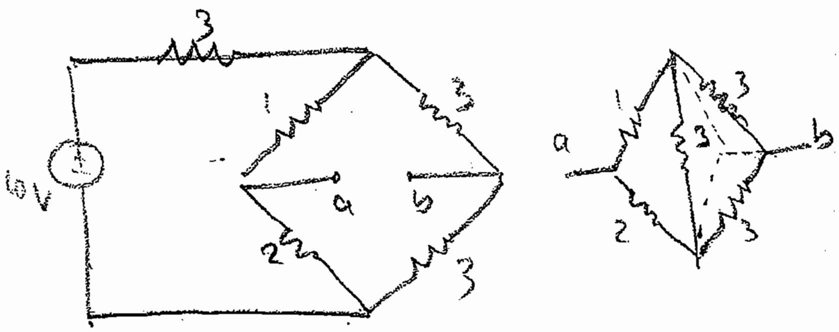

$[1 \parallel 3] + [2 \parallel 4]$

$\dfrac{3}{4} + \dfrac{4}{3}$

$= \dfrac{25}{12}\,\Omega$

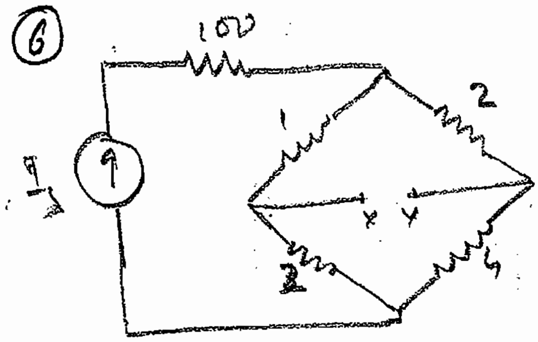
④ 


$\dfrac{3\times 7}{10} = \dfrac{21}{10}$

⑤ 


same as 2nd.

(6)

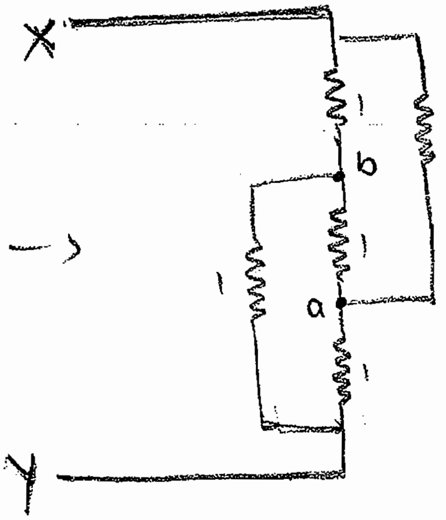



→ Open circuit the current source

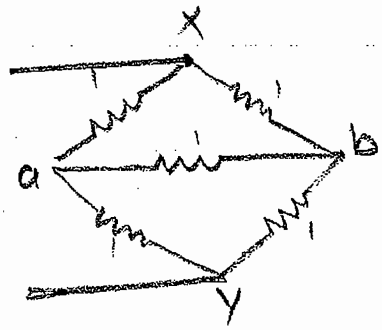
$$R_{xy} = 3 \parallel 6 = \frac{3\times 6}{9}$$

$$= 2\ \Omega$$

(7)

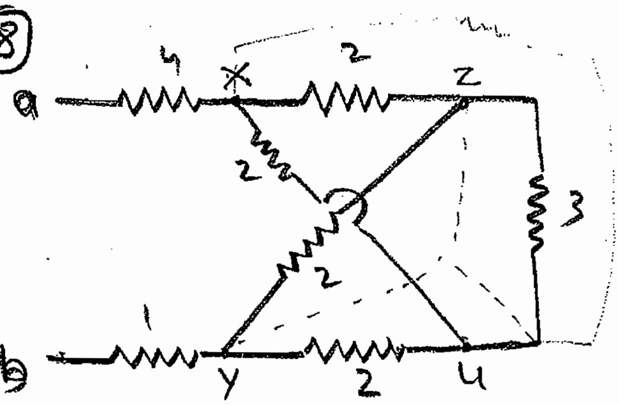


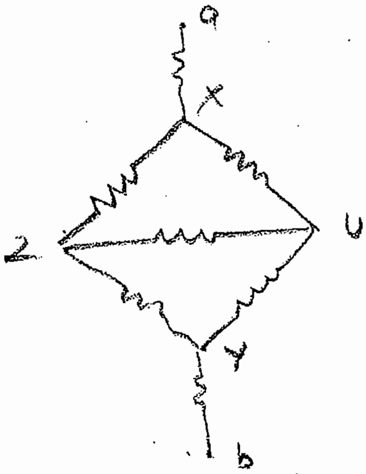

**Node shifting tech.**




$\therefore R_{xy} = 2 \parallel 2 = 1$

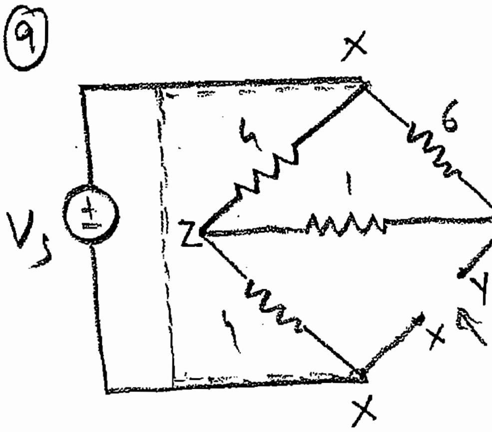
(8)



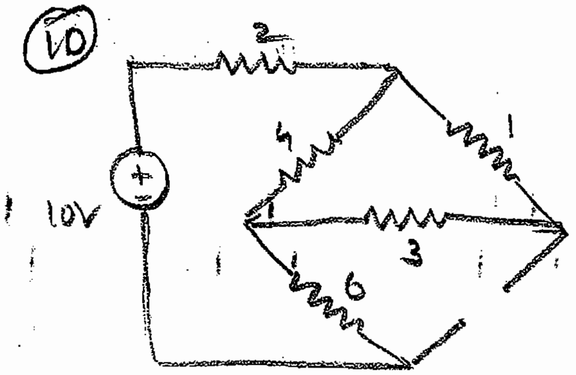

$7\ \Omega$

(9)



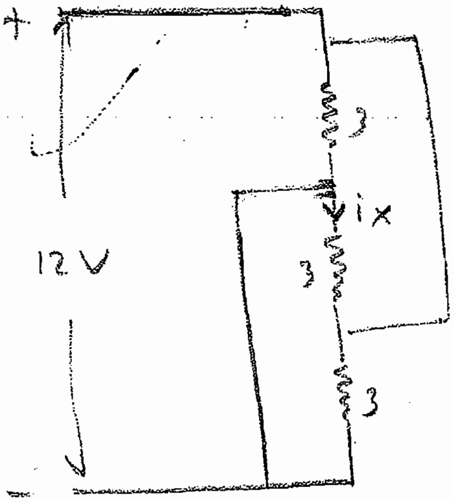

$2\ \Omega$

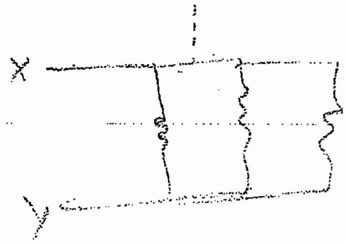
(10)




$9/4\ \Omega$

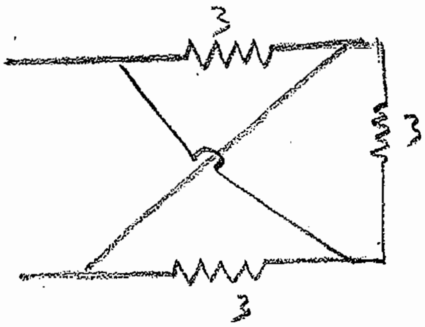
(11)



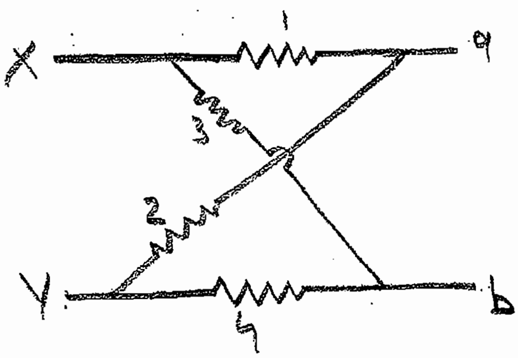




$-4\ A$

(12)




(13)




$R_{xy}$ =  $R_{yb}$ =

$R_{ab}$ =  $R_{xb}$ =

$R_{xa}$ =  $R_{ya}$ =

$R_{xy} = (1+2) \,||\, (3+4) = \frac{3\times 7}{10} = \frac{21}{10}$

$R_{ab} = (1+3) \,||\, (2+4) = \frac{4\times 6}{10} = \frac{12}{5}$

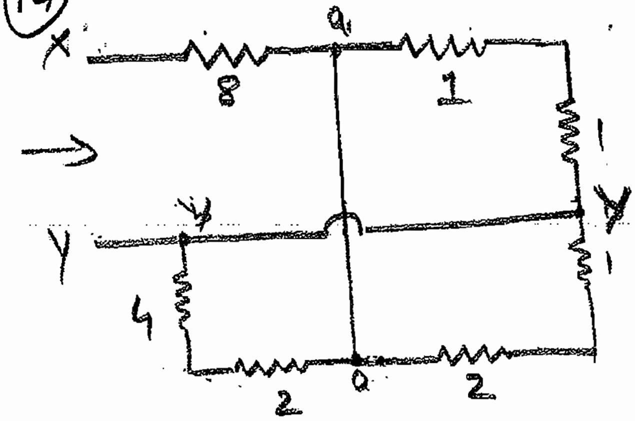
$R_{xa} = 1 \,||\, (3+2+4) = \frac{1\times 9}{10} = \frac{9}{10}$

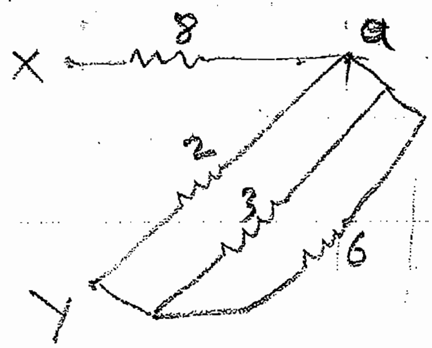
$R_{y_b} = 4 \parallel (1+3+2)$
$= \frac{4\times 6}{10}$
$= \frac{12}{5}$

$R_{xy} = 3 \parallel (1+2+4)$
$=$

$R_{ya} = 2 \parallel (1+3+4)$
$=$

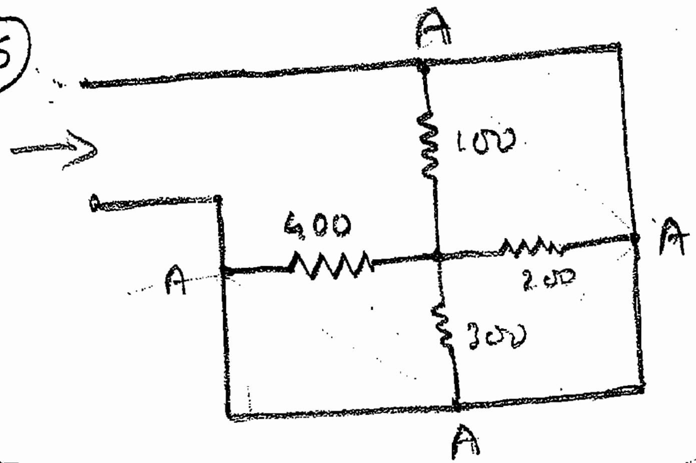
(14)



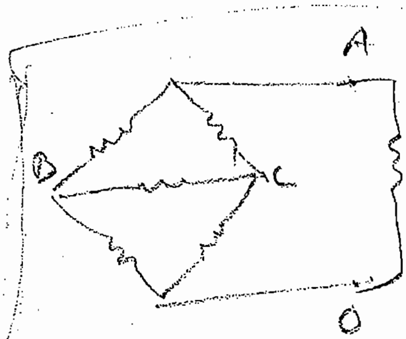




$R_{xy} = 8 + (2 \parallel 3 \parallel 6)$
$= 8 + 1$
$= 9$

(15)

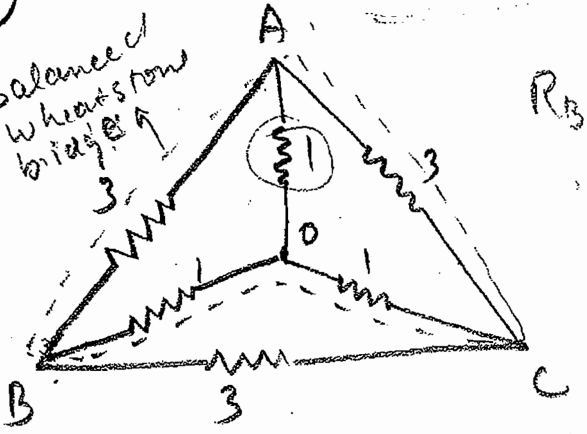



$= 0$




(16)

Balanced wheatstone bridge ?

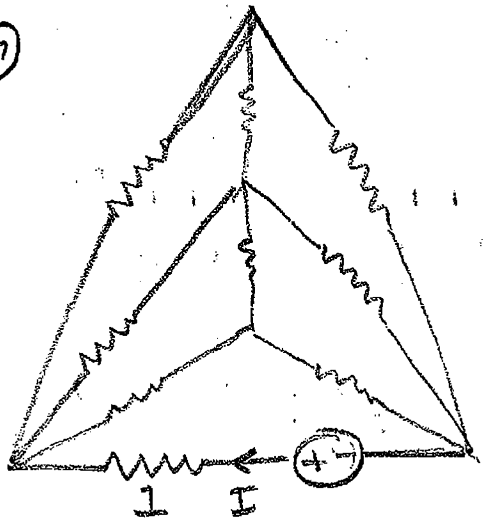



$R_{BC} = 3 \parallel 6 \parallel 2, = 1$

Always convert internal star → delta.
∵ Converting delta to star centre point may not be same.

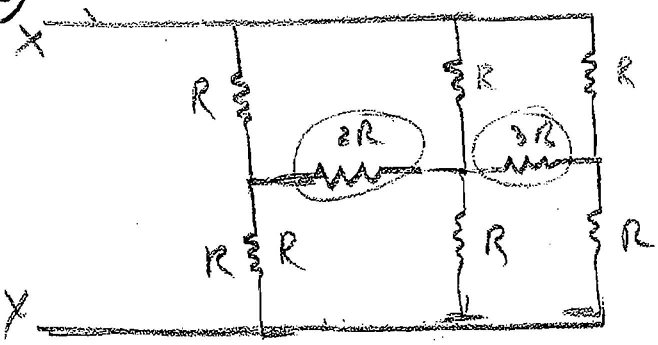
$R_{AO} = ?\ \frac{2}{3}$ $\quad R_{BC} = ?\ 1$

(17)



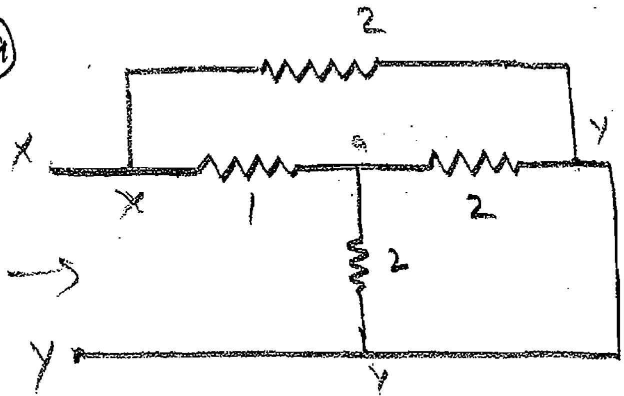

$I = \frac{10}{2} = 5\,A$

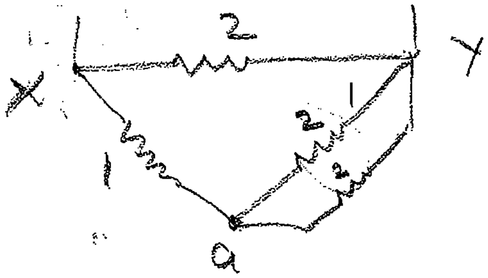
(18)




$\Rightarrow \frac{2}{3}R$

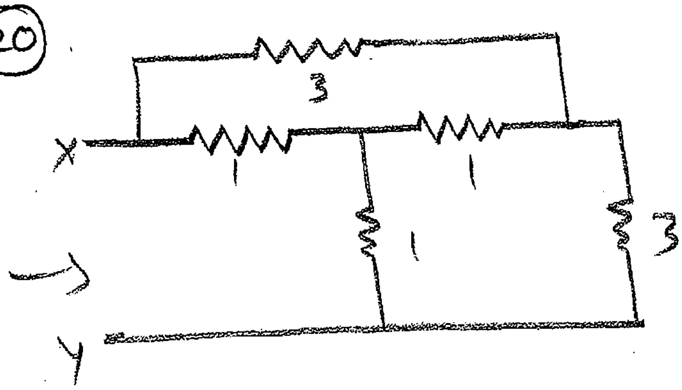
(19)



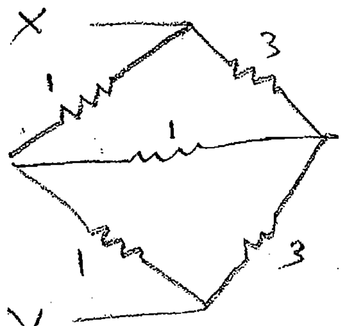




$R_{XY} = 2 \parallel 2 = 1\,\Omega$

(20)

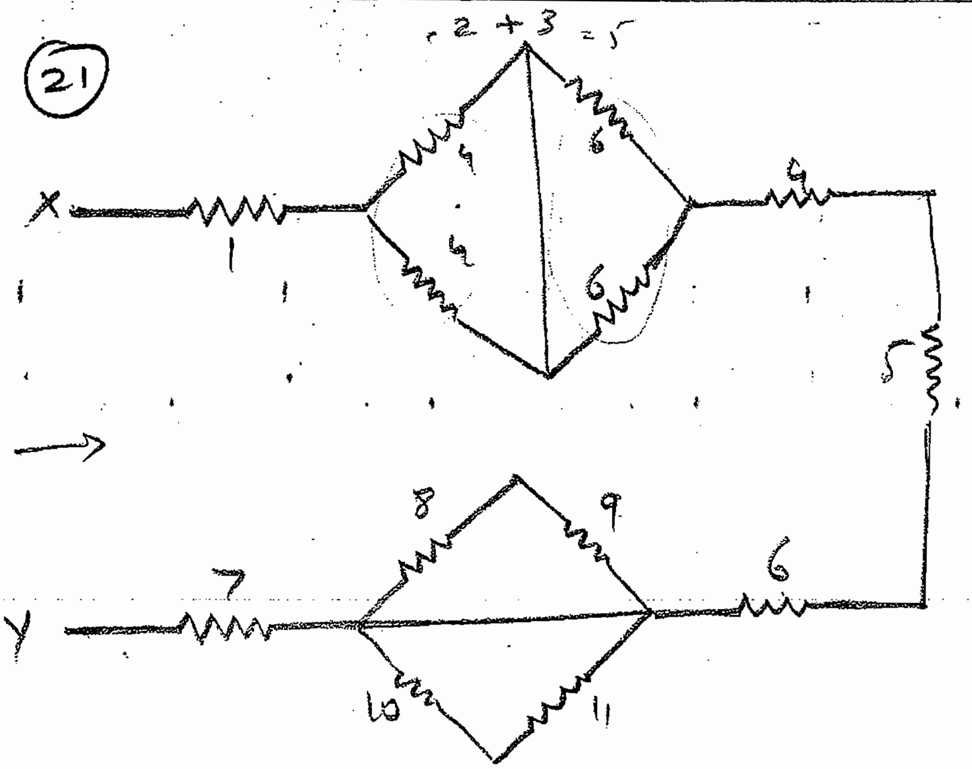



$\Rightarrow$




$R_{XY} = 2 \parallel 6$
$= \frac{3}{2}\,\Omega$

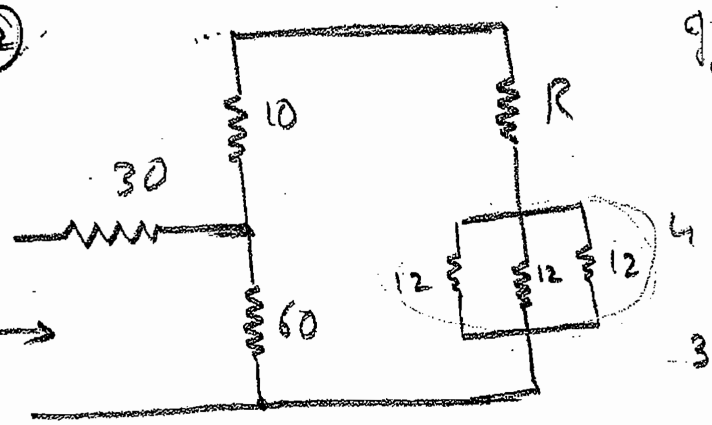
(21)




$R_{xy} = 1+5+4+5+6+7$

$= 28\,\Omega$

(22)




If $R_{eq} = 50$, then

$R =$ ______

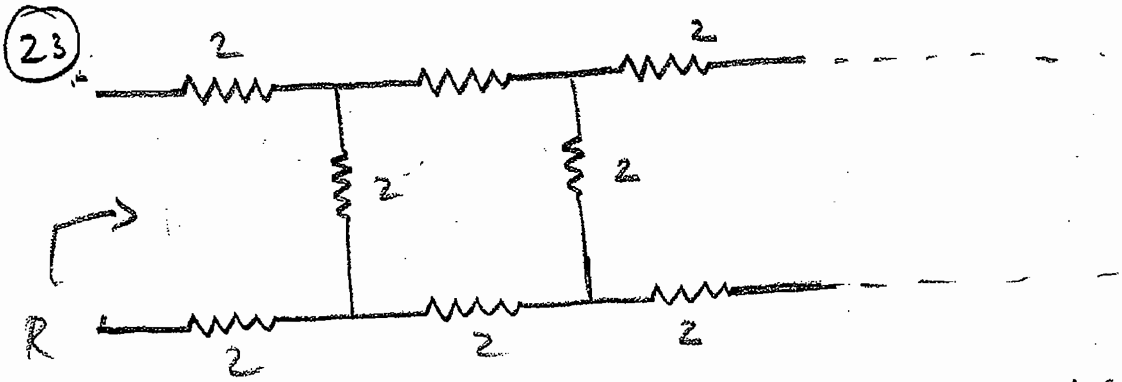
$30 + [60 \,\|\, (14+R)] = 50$

$\therefore \dfrac{60\times(14+R)}{R+74} = 20 \Rightarrow \cancel{60}^{3}(14+R) = \cancel{20}(R+74)$

$\therefore 42 + 3R = R + 74$

$\boxed{R = 16\,\Omega}$

(23)




The v-i relation in a component is $V = i^2$

$R_{dynamic} =$ ______ $R_{static}$ $\left\{ R_{static} = \dfrac{V}{I} \right.$

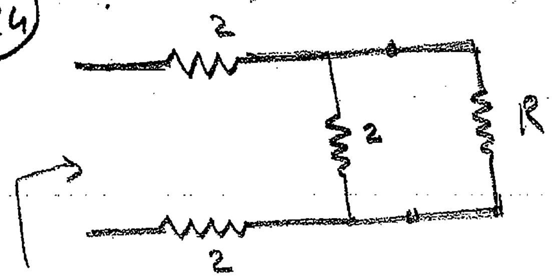
$$V = i^2$$

$$\frac{dV}{dt} = 2\,\frac{di}{dt}$$

$$\boxed{R_{dynamic} = \left[\frac{\frac{dV}{dt}}{\frac{di}{dt}}\right] = 2}$$

$\therefore$ $R_{dynamic} = \underline{2}\; R_{static}$

(24)



$$R = 4 + [2 \,||\, R]$$

$$= 4 + \frac{2R}{2+R}$$

$$\therefore\; 2R + R^2 = 8 + 2R + 4R$$

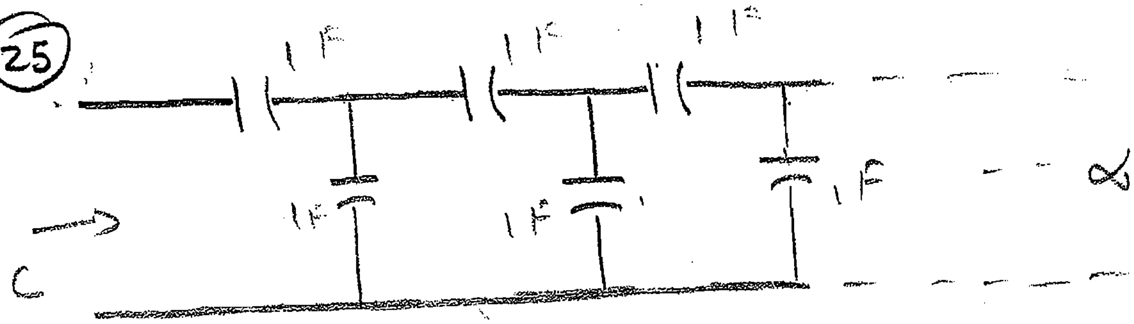
$$\therefore\; R^2 - 4R - 8 = 0$$

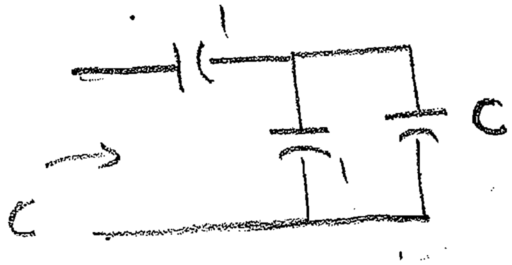
$$R = \frac{4 \pm \sqrt{16 - 4(-8)}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$$R = (2 \pm 2\sqrt{3})$$

$\therefore$ $\boxed{R = (2 + 2\sqrt{3})\,\Omega}$

(25)





$$C = 1 \; S \; (1 + C)$$

$$\therefore\; C = \frac{1+C}{2+C}$$

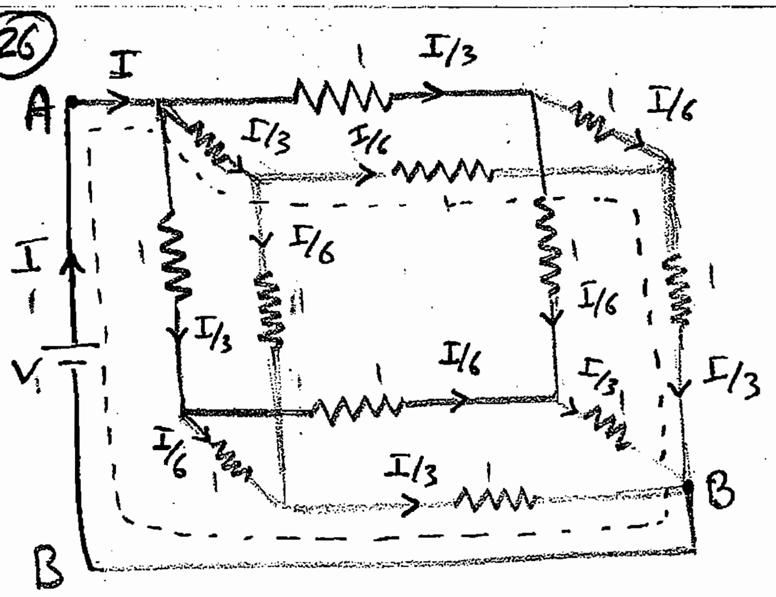
$$2C + C^2 = 1 + C$$

$$C^2 + C - 1 = 0$$

$$C = \frac{-1 \pm \sqrt{5}}{2}$$

$\therefore$ $\boxed{C = \frac{\sqrt{5}-1}{2}\ F}$

(26)




Symmetrical
→ Resis. are equal value
→ Travelling from source to destin.
resis. remains same

Applying KVL in the above loop

$$-V + \frac{I}{3}(1) + \frac{I}{6}(1) + \frac{I}{3}(1) = 0$$

$$V = I\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right] = I\left[\frac{5}{6}\right]$$

$$R_{AB} = \frac{V}{I} = \frac{5}{6}\,\Omega$$

$$\boxed{R_{AB} = \frac{5}{6}r}\ \Omega$$

$$L_{AB} = \frac{5}{6}l$$

$$C_{AB} = \frac{6}{5}C$$

$$G_{AB} = \frac{6}{5}g$$

KVL → for capacitors :-

$$-V + \frac{1}{C}\int \frac{I}{3}dt + \frac{1}{C}\int \frac{I}{6}dt + \frac{1}{C}\int \frac{I}{3}dt = 0$$

$$\therefore V = \frac{1}{C}\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right]\int I\,dt$$

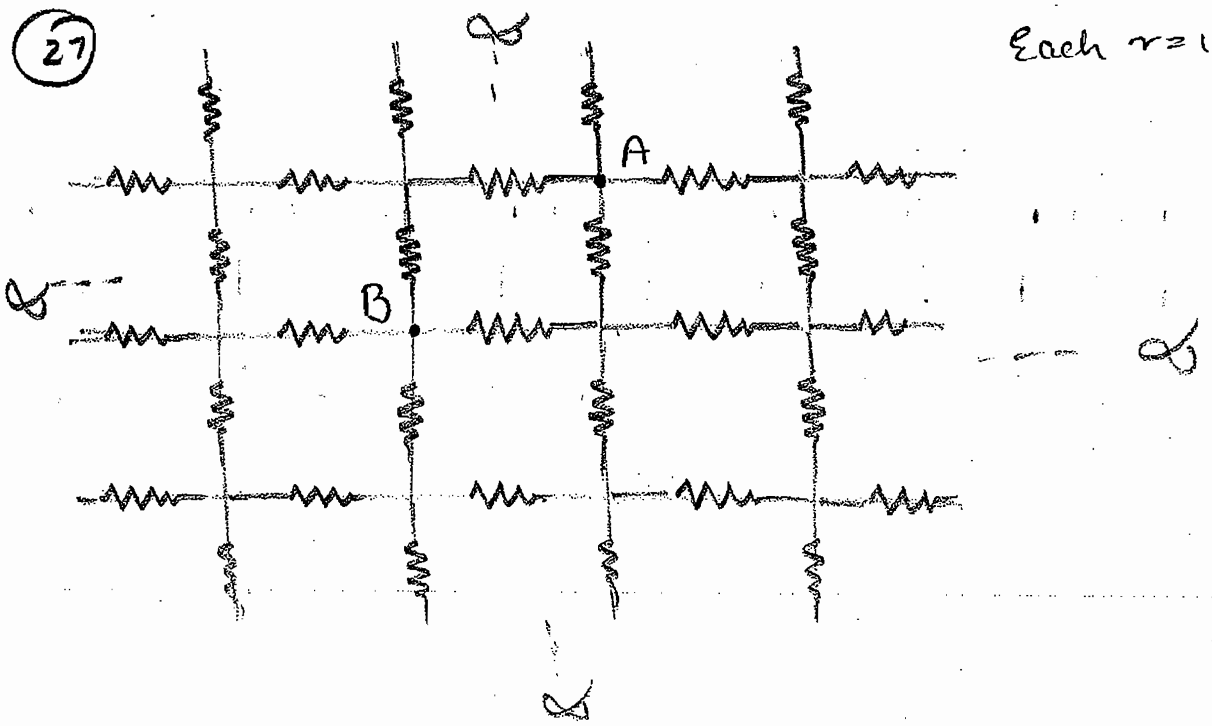
$$= \frac{1}{C}\left[\frac{5}{6}\right]\int I\,dt$$

$$\frac{1}{C_{AB}} = \frac{1}{C}\left[\frac{5}{6}\right]$$

$$\therefore C_{AB} = \frac{6C}{5}$$

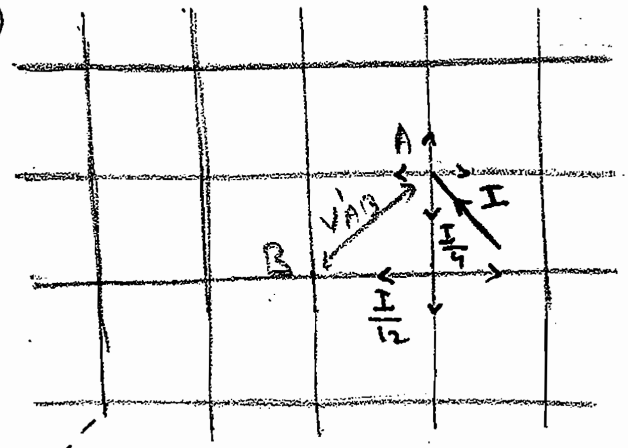
(27)

Each $r = 1\,\Omega$

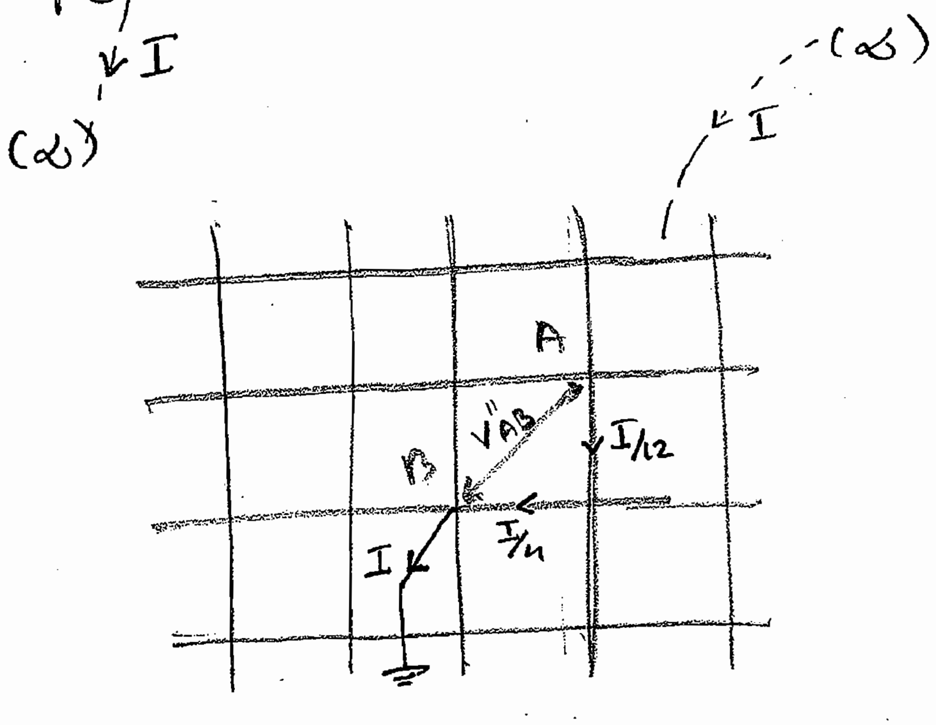


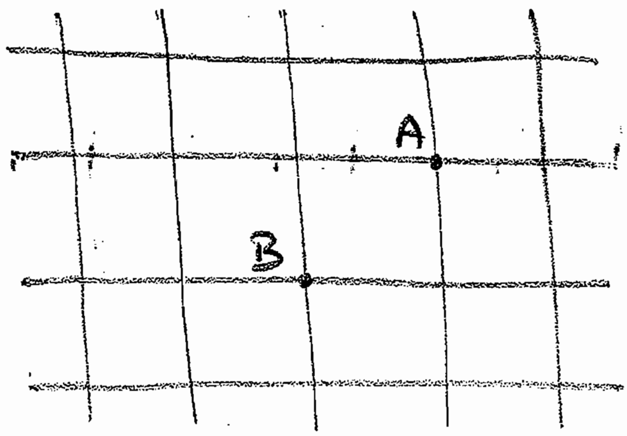

Use Super-position principle:

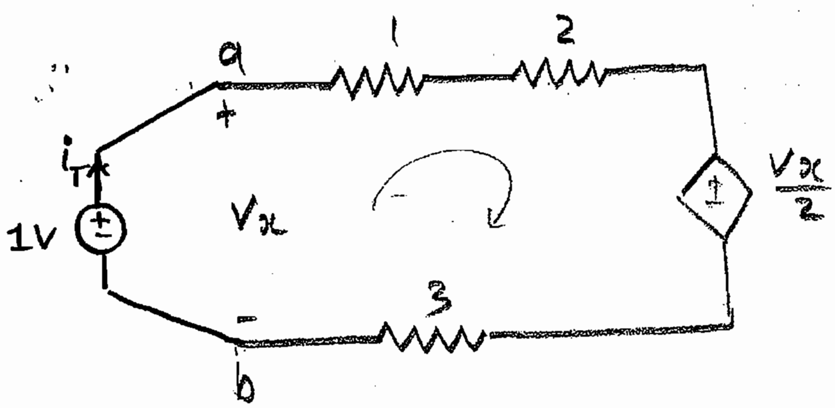
Step (1)




~~Step (2)~~






$R_{ab} = \frac{1}{i_T}$

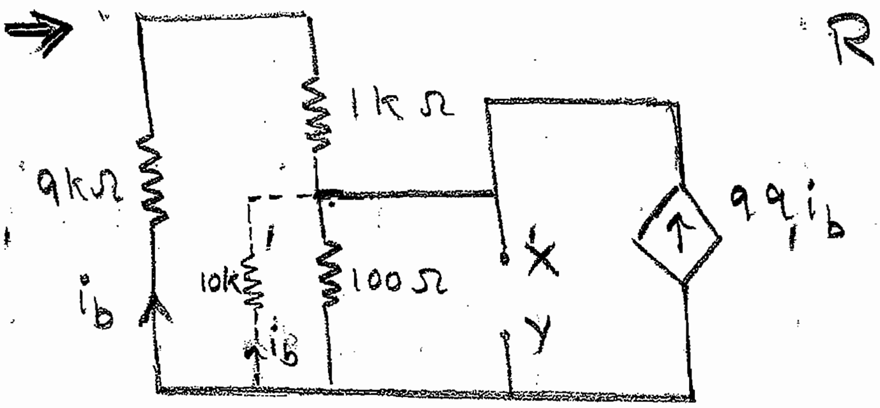
KVL

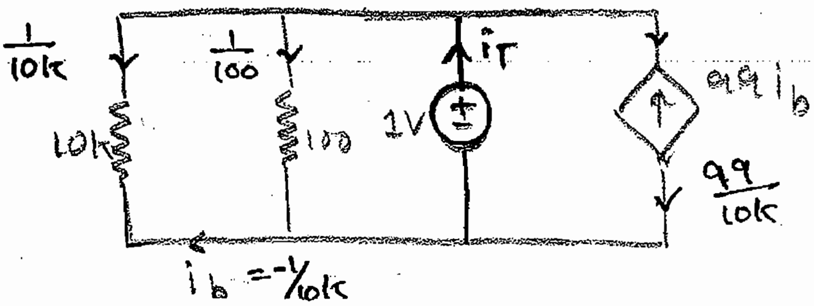
$$-1 + i_T + 2 i_T + \frac{1}{2} + 3 i_T = 0$$

$$6 i_T = \frac{1}{2} \quad \Rightarrow \quad i_T = \frac{1}{12}$$

$$R_{ab} = \frac{1}{i_T} = \frac{1}{1/12} = \boxed{12\,\Omega}$$

$R_{xy} = \underline{\ ?\ }$



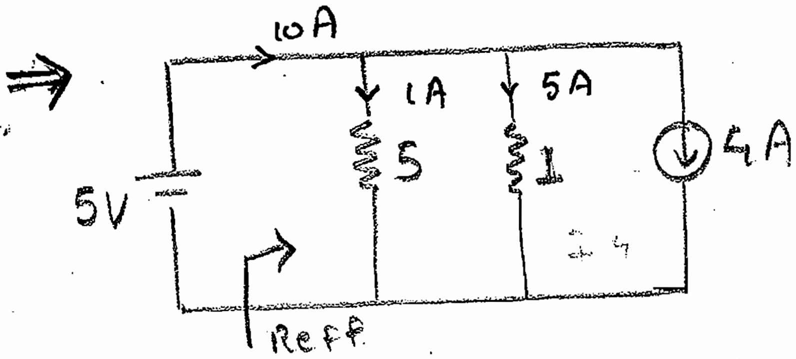

$$R_{xy} = \frac{1}{i_T}$$

$$i_T = \frac{1}{100} + \frac{1}{10k} + \frac{99}{10k}$$

$$i_T = \frac{1}{100} + \frac{1}{10k}[100]$$

$$= \frac{1}{100} + \frac{1}{100} = \frac{2}{100} = \frac{1}{50}$$

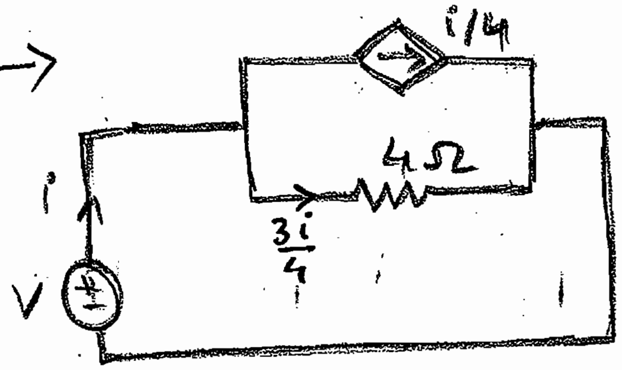
$$\therefore \boxed{R_{xy} = 50\,\Omega}$$




What is the effective resis. seen by the vltg source?

→ Effective resistance is the resistance offered by the circuit under working condition.

$$\therefore R_{eff} = \frac{5}{i_T} = \frac{5}{10} = 0.5\,\Omega$$



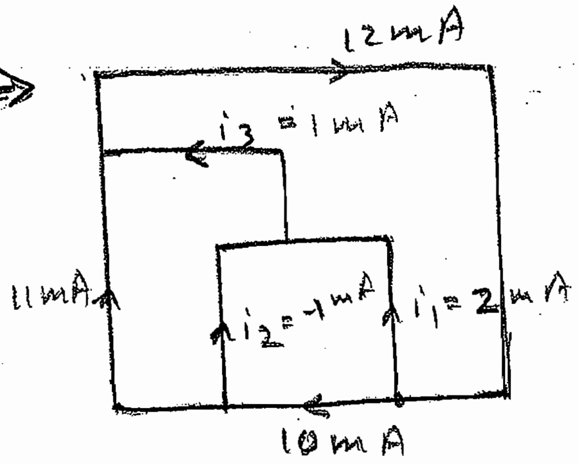

What is $R_{eff}$ seen by the vltg source?

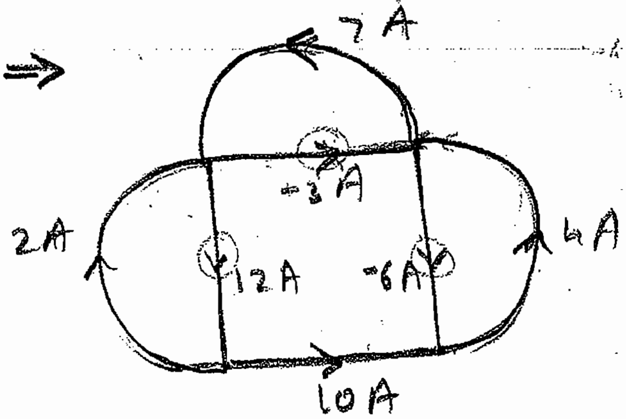
KVL

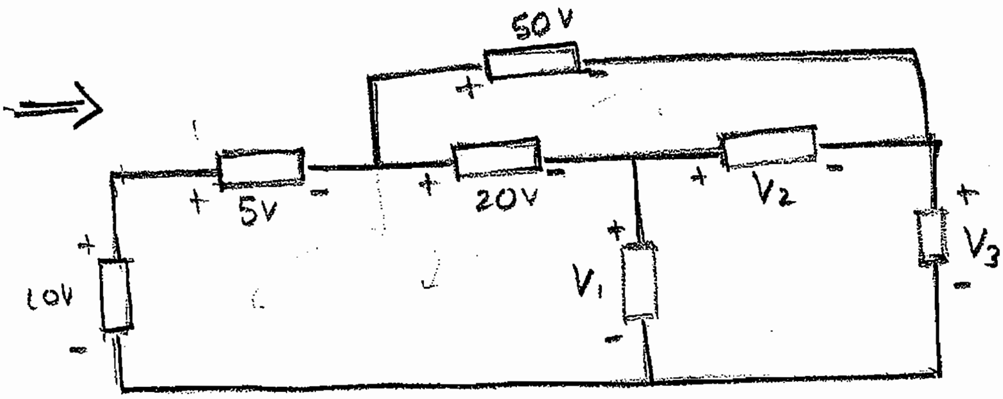
$$-V + \frac{3i}{4} \times 4 = 0$$

$$\therefore V = 3i$$

$$R_{eff} = \frac{V}{i} = 3\,\Omega$$




$10 - 25 = V_1$

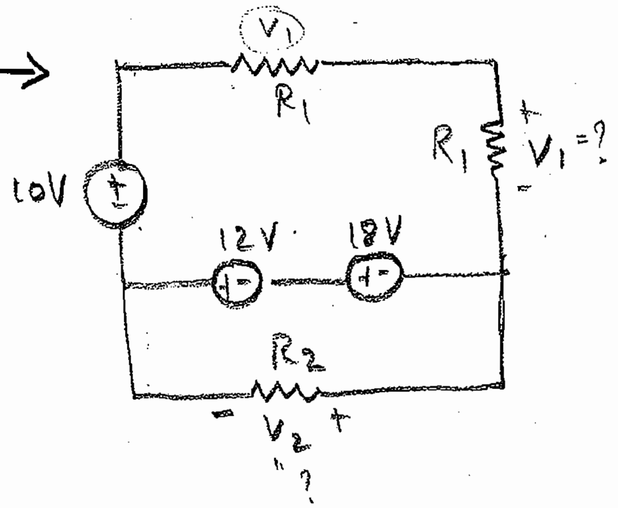
$V_1 = -15$ V

$20 - 50 + V_2 = 0$

$V_2 = 30$ V

$-15 - 30 = V_3$

$V_3 = -45$ V

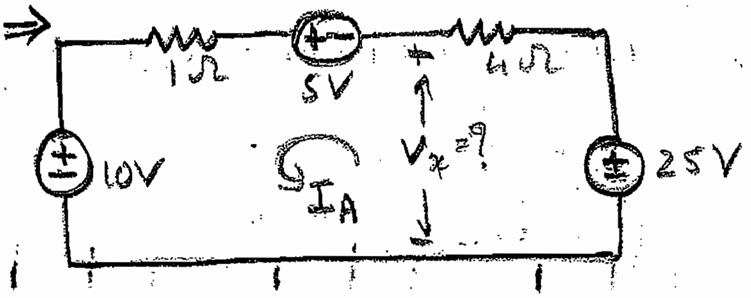



$V_2 = -30$ V

$-10 + 2V_1 - 30 = 0$

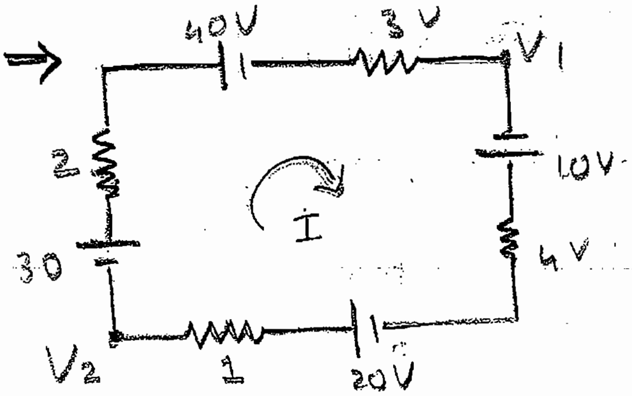
$2V_1 = 40$

$V_1 = 20$



$$I_A = \frac{20}{5} = 4\ A$$

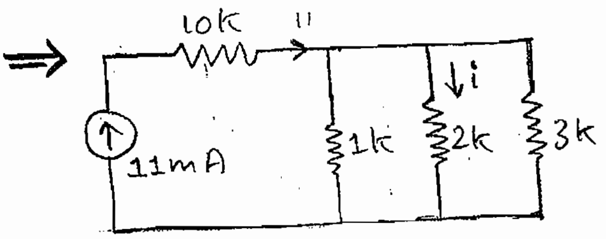
$$V_x = 25 - 4I_A$$
$$= 25 - 16$$
$$= 9\ V$$

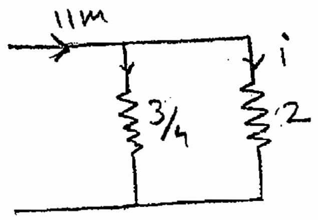


$$I = \frac{20}{10} = 2\ A$$

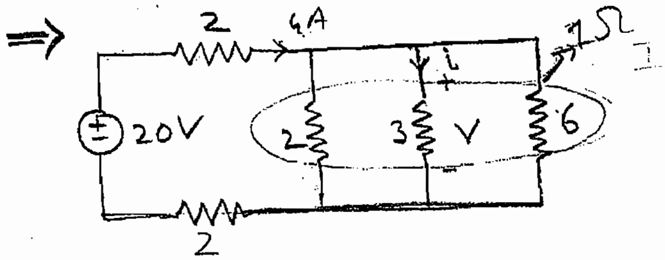
$$V_2 = -30 + 5I + 40 + V_1$$

$$\therefore \boxed{V_2 = V_1 + 20}$$

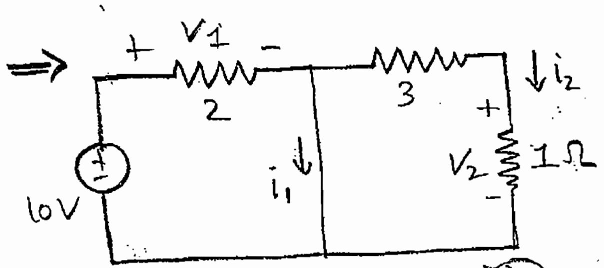




$$i = \frac{\frac{3}{4} \times 11}{\frac{3}{4} + 2}$$
$$= \frac{33}{11} = 3\ A$$

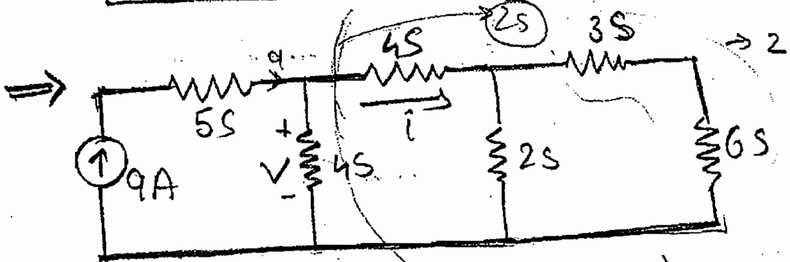


$$V = \frac{1 \times 20}{4 + 1} = \frac{20}{5}$$
$$= 4\ V$$



$$i_1 = \frac{10}{2} = 5\ A \qquad i_2 = 0$$

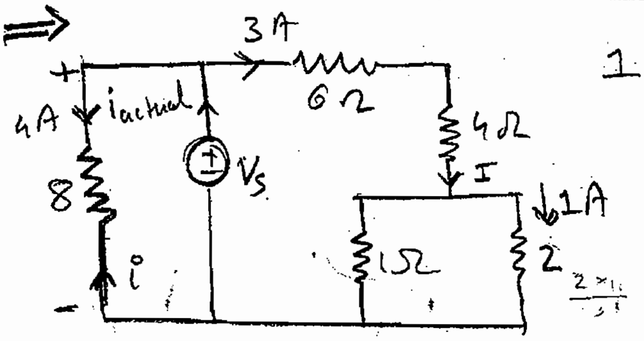
$$V_1 = 2 \times 5 = 10\ V \qquad V_2 = 0$$



$$i = \frac{9 \times 4}{4 + 2} = \frac{\overset{3}{\cancel{9}} \times \overset{2}{\cancel{4}}}{\cancel{6}}$$
$$= 6\ A$$

S → Seimens (i.e conductance)

⟹



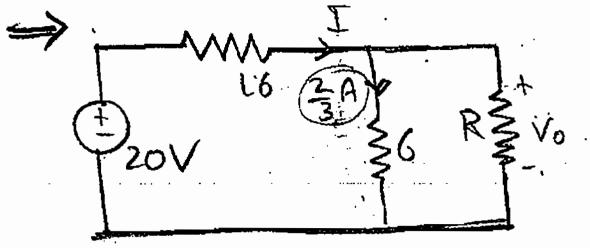

$1 = \frac{1}{3} \times I \Rightarrow I = 3\ A$

$V_s = 3 \times \left(10 + \frac{2}{3}\right)$

$= 32\ V$

$i = -\frac{32}{8} = -4\ A$

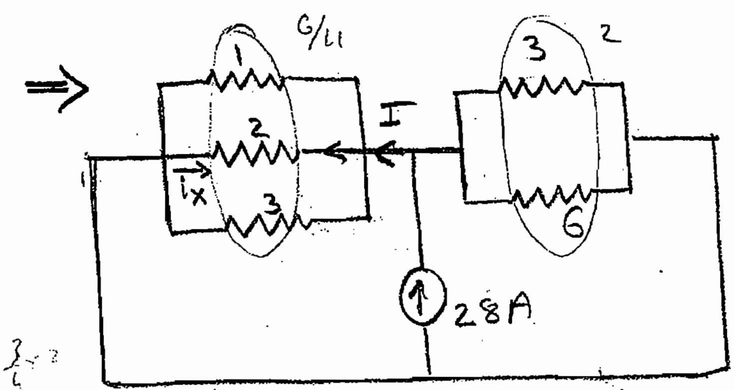
⟹




i] $V_o = 4V$, $R =$ ___9___ 12

$I = \frac{16}{16} = 1\ A$

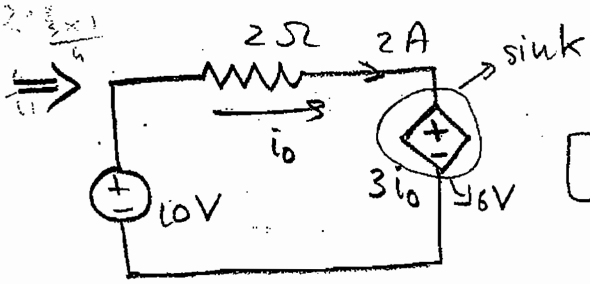
⟹




$I = \frac{2 \times 28}{[2 + 6/11]} =$

$-i_x = \frac{\frac{2\times28\times4}{28} \times \frac{3}{4}}{4(2) + \frac{3}{4}} = 6$

⟹

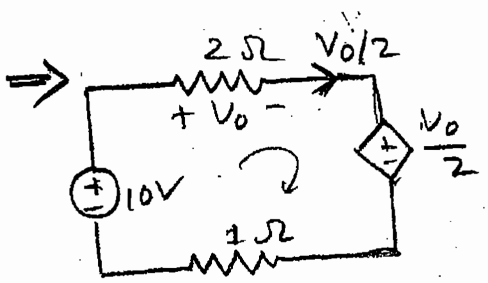


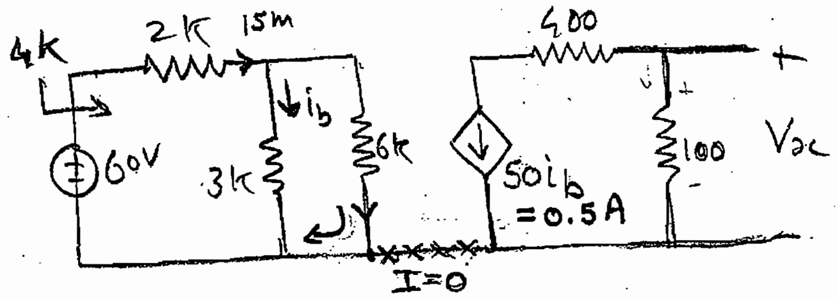

$\frac{10 - 3i_o}{2} = i_o \Rightarrow 5i_o = 10$

$\boxed{i_o = 2}$

$P = -6 \times 2 = -12W$

⟹



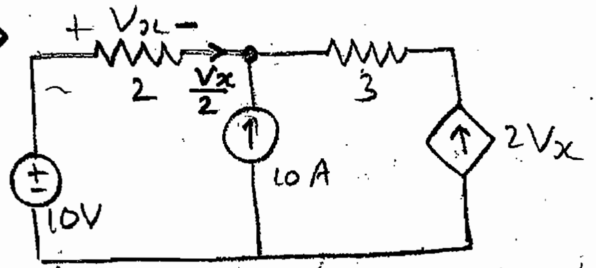

$+10 - V_o - \frac{V_o}{2} - \frac{V_o}{2} = 0$

$\therefore V_o = \frac{10}{2} = 5V$

$i_b = \frac{6 \times 15}{9} = 10\ mA$

$V_x = -100(0.5)$

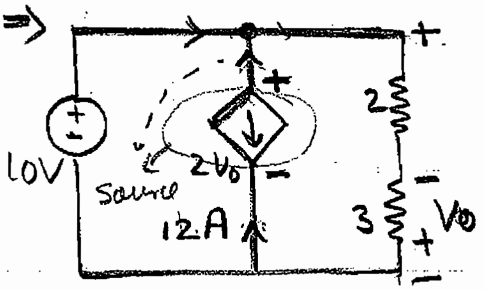
$= -50\ V$




$$\frac{V_x}{2} + 10 + 2V_x = 0$$

$$\therefore\ 5V_x = -20$$

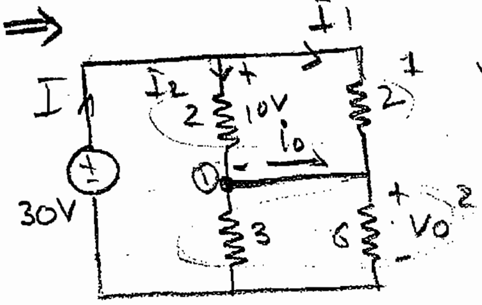
$$\therefore\ \boxed{V_x = -4V}$$




Power delievered by dependent source?

$$V_0 = -\left(\frac{3\times 10}{3+2}\right) = -6V$$

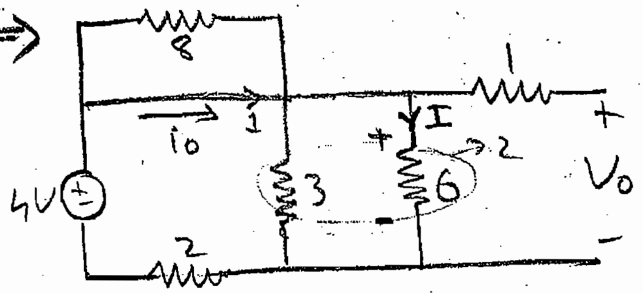
$$P = +10\times 12 = \boxed{120W}$$




$$V_0 = \frac{30\times 2}{3} = 20V$$

KCL ① $\frac{10}{2} = i_0 + \frac{20}{3} \Rightarrow i_0 = \frac{30-40}{6}$

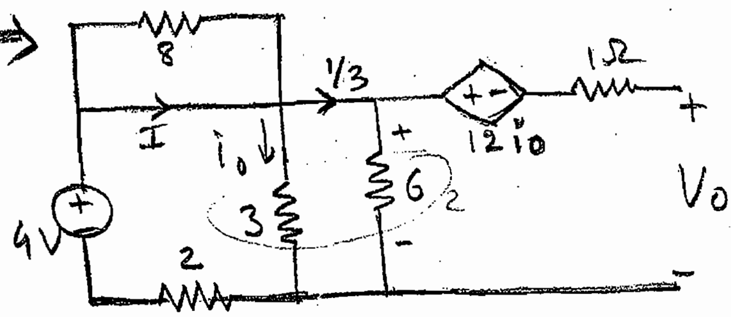
$$i_0 = \frac{-10}{6} = \boxed{-\frac{5}{3}A}$$




$$i_0 = \frac{4}{4} = 1A$$

$$I = \frac{3}{9} = \frac{1}{3}$$

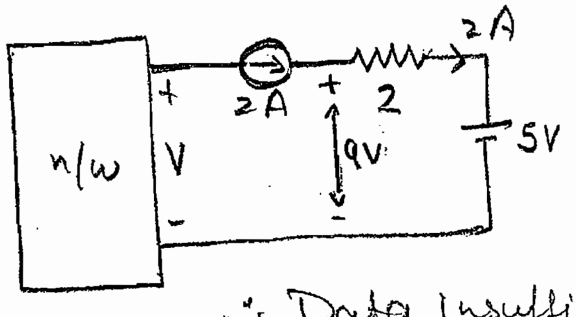
$$V_0 = 6\times\frac{1}{3} = 2V$$




$$I = \frac{4}{4} = 1A$$

$$i_0 = \frac{6}{9} = \frac{2}{3}A$$

$$V_0 = -12\left(\frac{2}{3}\right) + 6\left(\frac{1}{3}\right) = -6V$$

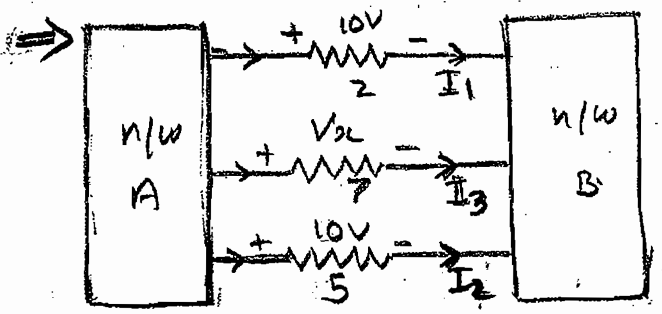



$$\frac{V-5}{2} = 2$$

$$\boxed{V = 9V}\ \times$$

∵ We missed the vltg across the current source

∴ Data Insufficient ✓



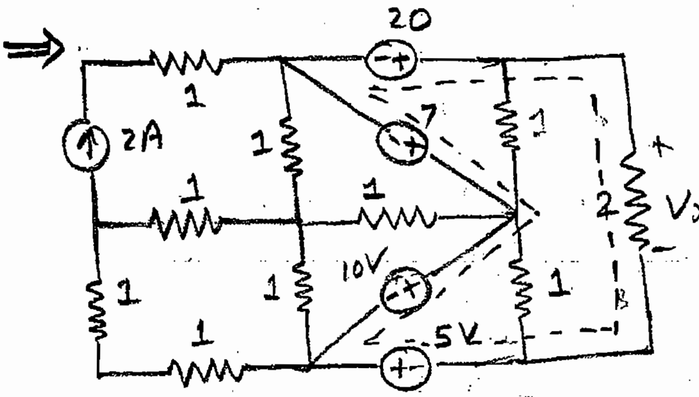

$V_x = ?$

$I_1 = 5A \quad I_2 = 2A$

$\therefore I_3 = I_1 + I_2$ (KCL)

$= -7 A$

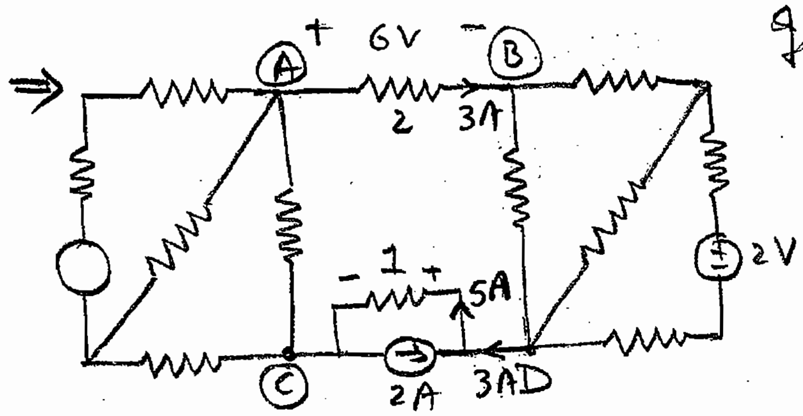
$\boxed{V_x = -49 V}$




$-5 - 10 + 7 - 20 + V_x = 0$

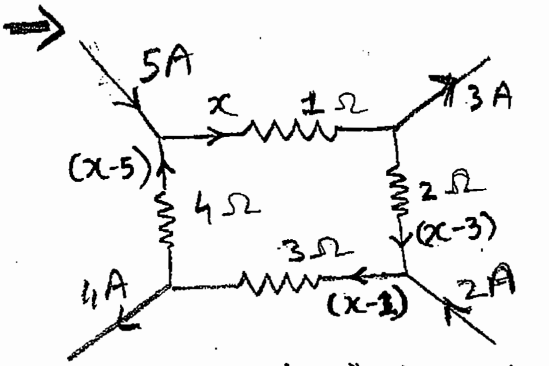
$V_x = 35 - 7$

$\boxed{V_x = 28 V}$




If $V_A - V_B = 6V$, $V_C - V_D = ?$

$\therefore V_C - V_D = -5 V$




Find power lost in n/w

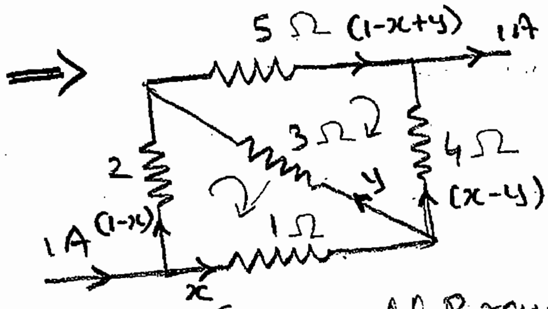
KVL

$x + 2(x-3) + 3(x-1) + 4(x-5) = 0$

$\therefore x = 2.9 A$

$P_T = (2.9)^2 + (0.1)^2(2) + (1.9)^2(3) + (2.1)^2(4)$

$= 36.9 W$




Find all Branch current

KVL ① $2(1-x) - 3y - x = 0$

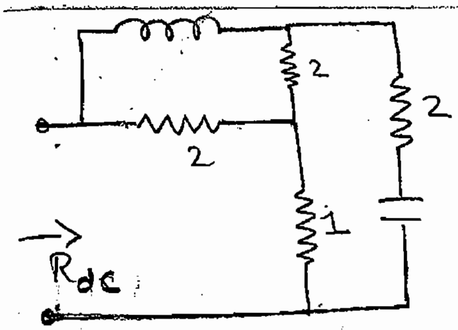
$3x + 3y = 2$ — ①

② $3y + 5(1-x+y) - 4(x-y) = 0$

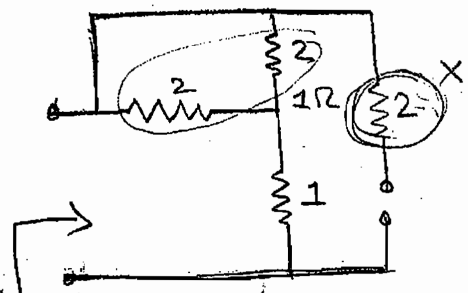
$9x - 12y = 5$ — ②

From ① & ②

$x = 0.619 \quad y = 0.047$




⟹




$R_{dc} = 2\,\Omega$

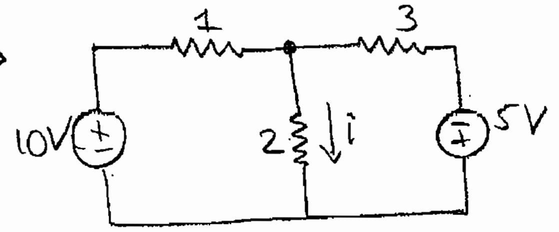
# Methods of Analysis :

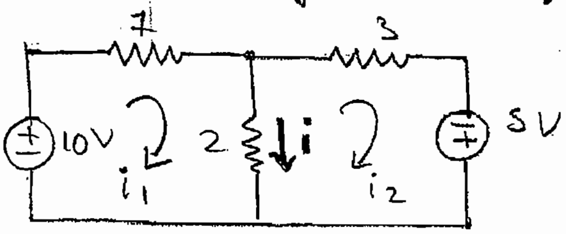
→ Mesh Analysis = KVL + Ohms' Law
  [I↑ V↓]

→ Nodal Analysis = KCL + Ohms' Law
  [V↑ I↓]

In nodal analysis, we can eliminate the use of simple nodes, if not required.

1)




Find 'i' using mesh & nodal analysis

**Mesh Analysis**

① $10 - 3i_1 + 2i_2 = 0$

$\therefore\ 3i_1 - 2i_2 = 10$

② $5 - 5i_2 + 2i_1 = 0$

$\therefore\ 2i_1 - 5i_2 = -5$

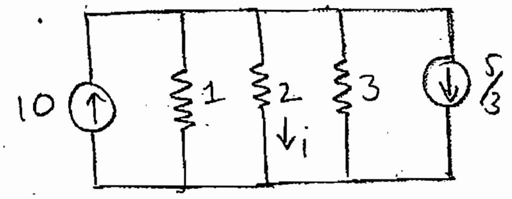
From ① & ② -

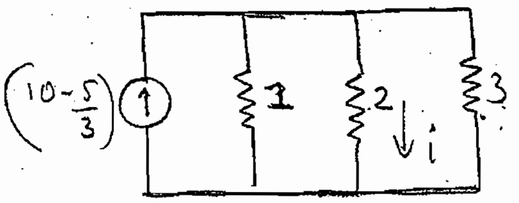
$i_1 = \frac{60}{11} \qquad i_2 = \frac{35}{11}$

Now.

$i = i_1 - i_2$

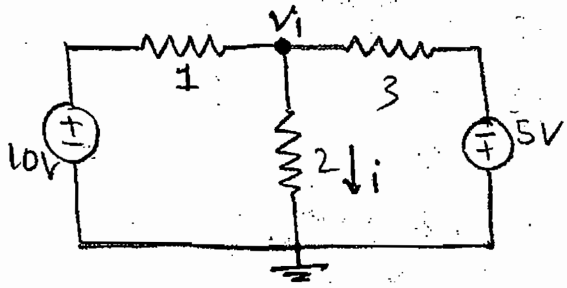
$= \frac{25}{11}$ A




$i = \left(10 - \frac{5}{3}\right)\left[\frac{3}{2+3+6}\right]$

$= \frac{25}{3} \times \frac{3}{11} = \frac{25}{11}$ A




Nodal Analysis

$$\frac{V_1-10}{1}+\frac{V_1}{2}+\frac{V_1-5}{3}=0$$

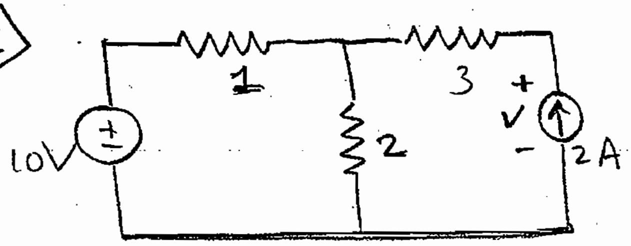
$$\therefore \; 6V_1-60+3V_1+2V_1-10=0$$

$$V_1=\frac{50}{11}\text{ V}$$

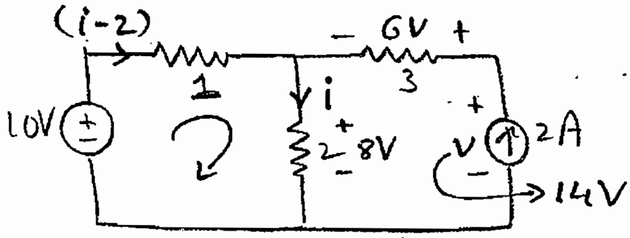
Now,

$$i=\frac{V_1}{2}=\frac{25}{11}\text{ A.}$$

2)



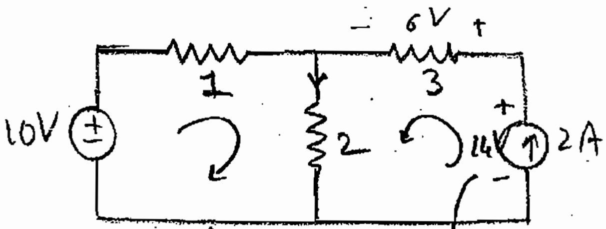

Find power delivered by current source using mesh & nodal.




$$-10+i-2+2i=0$$

$$i=4\text{ A}$$

$$\therefore P_{deliv.}=+\;2\times 14=28\text{ W}$$



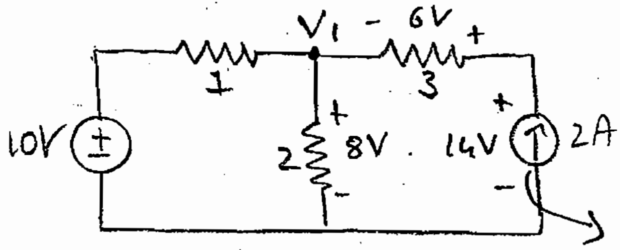

$P_{del}=28W$

$$-10+i_1+2(i_1-i_2)=0$$

$$3i_1-2i_2=10 \quad \text{— (1)}$$

By comparision : $i_2=-2$ — (2)

$$\therefore \; 3i_1=6 \;\Rightarrow\; i_1=2$$

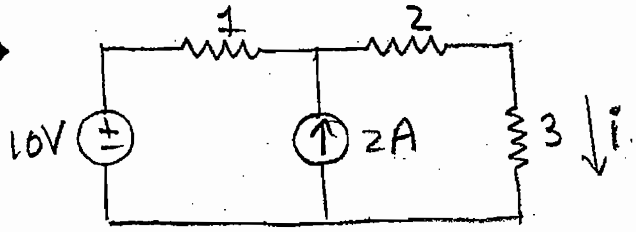



$$\frac{V_1-10}{1}+\frac{V_1}{2}-2=0$$

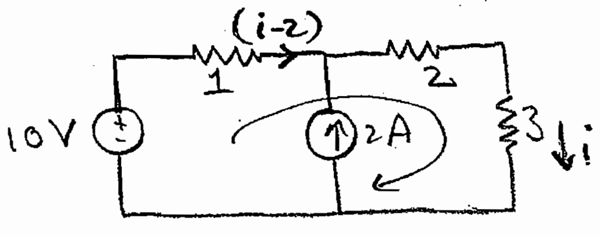
$$\therefore V_1=8\text{ V}$$

$P_{del}=28W$

3)

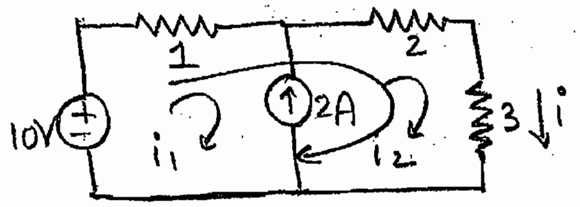



Find i using mesh & nodal analysis




$$-10-i-2+5i=0$$

$$i=2$$



Here $i_2 = i$

$10 - i_1 - 5i = 0$

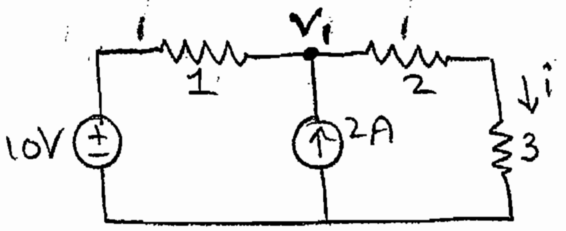
$i_1 + 5i = 10$

Now,

$i - i_1 = 2$

$i_1 = i - 2$

$\therefore\ 6i = 12 \Rightarrow \boxed{i = 2\,A}$

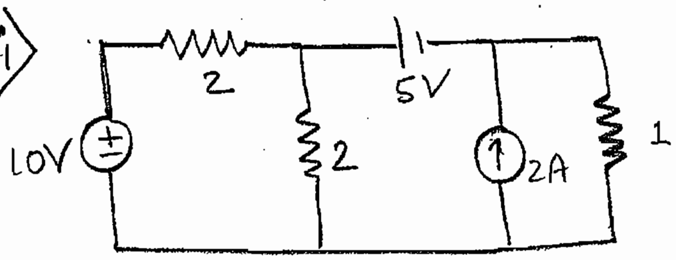


$$\frac{V_1 - 10}{1} - 2 + \frac{V_1}{5} = 0$$

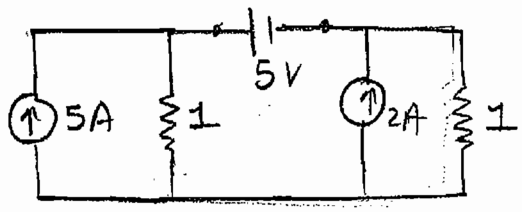
$6V_1 = 60 \Rightarrow V_1 = 10\,V$

$$i = \frac{V_1}{5} = \frac{10}{5} = 2\,A$$

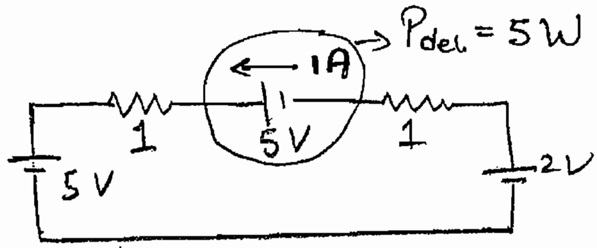
4)

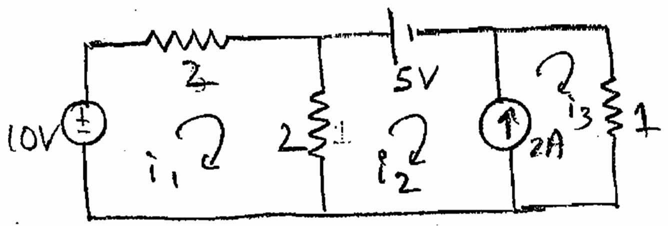


Find the power delivered by 5 V source using mesh and nodal analysis.



$\Rightarrow$



$-10 + 2i_1 + 2(i_1 - i_2) = 0$

$2i_1 - i_2 = 5$ —— (1)

$2(i_2 - i_1) + 5 + i_3 = 0$

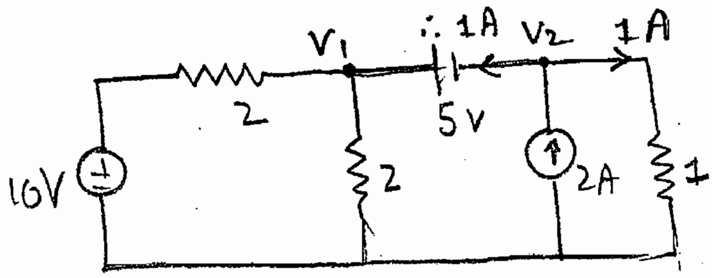
$-2i_1 + 2i_2 + i_3 = 5$ —— (2)

$-i_2 + i_3 = 2$ —— (3)

$i_2 + i_3 = 0$ ~~(4)~~

$2i_2 = -2$

$i_2 = -1 \Rightarrow \boxed{P_{del.} = 5W}$



$$\frac{V_1 - 10}{2} + \frac{V_1}{2} - 2 + \frac{V_2}{1} = 0$$

$V_1 + V_2 = 7$ —— (1)

$V_1 - V_2 = 5$ —— (2)

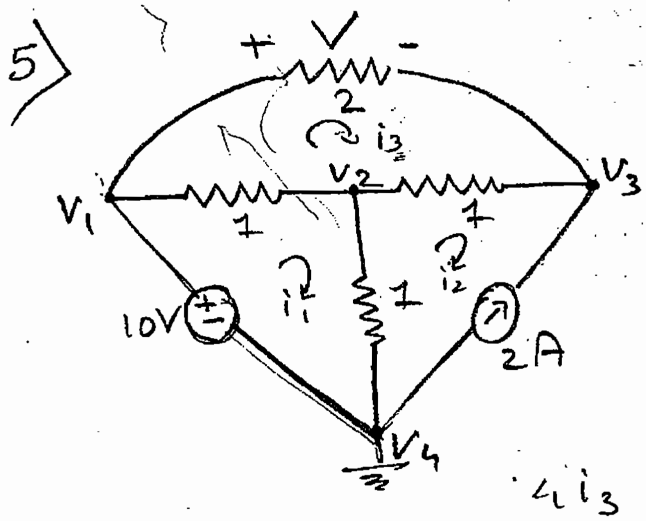
$\therefore\ V_1 = 6\ V$

$V_2 = 1\ V$

$\therefore P_{deliv.} = 5 \times 1$

$= 5\,W.$

5)




Find V using mesh & nodal analysis.

MESH :-

$10 - 2i_1 + i_3 + i_2 = 0$ —(1)

$i_2 = -2$ —(2)

$4i_3 - i_1 - i_2 = 0$ —(3)

$\therefore -2i_1 + i_3 + 8 = 0$

& $-i_1 + 4i_3 + 2 = 0$

$\Rightarrow$

$$\begin{aligned} i_3 - 2i_1 &= -8 \\ -8i_3 + 2i_1 &= +4 \\ \hline -7i_3 &= -4 \end{aligned}$$

$i_3 = \frac{4}{7}$

$\therefore V = 2\left(\frac{4}{7}\right) = \boxed{\frac{8}{7}\,V}$

NODAL :-

$V_1 = 10\,V$ —(1)

$\frac{V_2 - 10}{1} + \frac{V_2 - V_3}{1} + \frac{V_2}{1} = 0$

$\therefore 3V_2 - V_3 = 10$ —(2)

$\frac{V_3 - V_2}{1} + \frac{V_3 - 10}{2} - 2 = 0$

$\therefore 3V_3 - 2V_2 = 14$ —(3)

From (2) & (3);

$$\begin{aligned} 6V_2 - 2V_3 &= 20 \\ -6V_2 + 9V_3 &= 42 \\ \hline 7V_3 &= 62 \end{aligned}$$

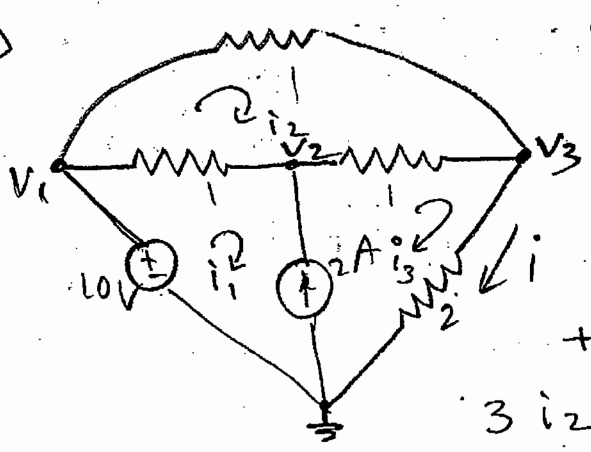
$V_3 = \frac{62}{7}$

$\therefore V = V_1 - V_3$

$= 10 - \frac{62}{7}$

$\boxed{V = \frac{8}{7}\,V}$

6)




Find 'i' using mesh & nodal.

Mesh :-

$\cancel{+10i_1}$ $i_3 - i_1 = 2$ — (1)

$3i_2 - i_1 - i_3 = 0$ — (2)

$10 - i_1 - 3i_3 + 2i_2 = 0$ — (3)

Now, from (2) & (1); $3i_2 - i_3 + 2 - i_3 = 0$

$3i_2 - 2i_3 = -2$ — (4)

Now, from (1) & (3); $-i_3 + 2 - 3i_3 + 2i_2 = 10$

$\therefore\ 2i_2 - 4i_3 = +8$ — (5)

$6i_2 - 4i_3 = -4$

$\underline{\overset{-}{}6i_2 \underset{+}{-} 12i_3 = \overset{-}{}24}$

$8i_3 = -28$

NODAL :- $V_1 = 10\ V$ — (1)

$\frac{V_2 - 10}{1} + \frac{V_2 - V_3}{1} - 2 = 0$

$2V_2 - V_3 = 12$ — (2)

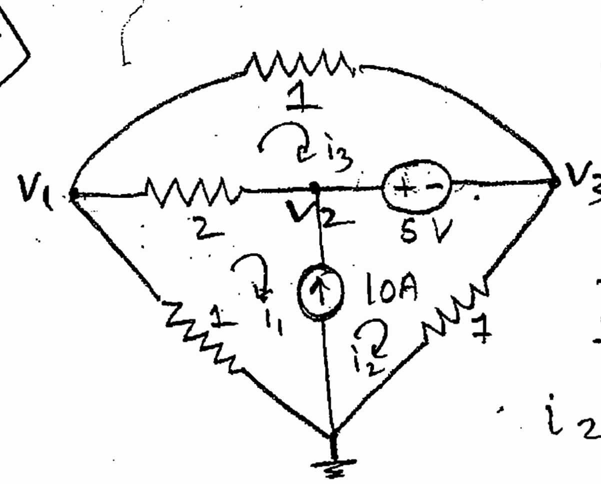
$\frac{V_3 - 10}{1} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$

$\therefore\ -2V_2 + 5V_3 = 20$ — (3)

$\therefore\ 4V_3 = 32$

$V_3 = 8\ V \Rightarrow i = \frac{8}{2} = \boxed{4A}$

7)




What is the power delivered by the vltg source?

Mesh :-

$i_2 - i_1 = 10$ —— (1)

$3i_1 + \cancel{2}i_2 - 2i_3 = -5$

$\therefore 3(i_2 - 10) + \cancel{2}i_2 - 2i_3 = -5$

$3i_2 - 30 + \cancel{2}i_2 - 2i_3 = -5$

$\therefore 4i_2 - 2i_3 = 25$ —— (2)

3×

$3i_3 - 2i_1 = 5$

$3i_3 - 2(i_2 - 10) = 5$

$\therefore -2i_2 + 3i_3 = -15$ —— (3)

2×

$$12i_2 - 6i_3 = 75$$
$$-4i_2 + 6i_3 = -30$$
$$8i_2 = 45$$

$\therefore \boxed{i_2 = \frac{45}{8}}$

$i_3 = \frac{4\left(\frac{45}{8}\right) - 25}{2}$

$= \frac{45 - 50}{4} = -\frac{5}{4}$

$P_{del.} = -5 \times \left(\frac{45}{8} + \frac{5}{4}\right)$

$= \boxed{-\frac{275}{8} \text{ W}}$

NODAL :—

$$\frac{V_1}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{1} = 0$$

$5V_1 - V_2 - 2V_3 = 0$ —— (1)

$(V_2 - V_3) = 5$ —— (2)

$$\frac{V_2 - V_1}{2} - 10 + \frac{V_3}{1} + \frac{V_3 - V_1}{1} = 0$$

$$\therefore -3V_1 + V_2 + 4V_3 = 20 \quad \text{——— (3)}$$

From (1) & (2) ; $7V_1 - V_2 = 20$

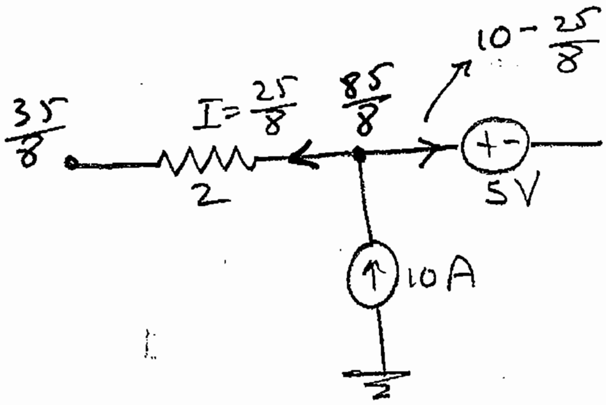
From (2) & (3) ; $5V_1 - 3V_2 = -10$

$$16V_1 = 70$$

$$\therefore \boxed{V_1 = \frac{35}{8}}$$

$$V_2 = 7\left(\frac{35}{8}\right) - 20$$

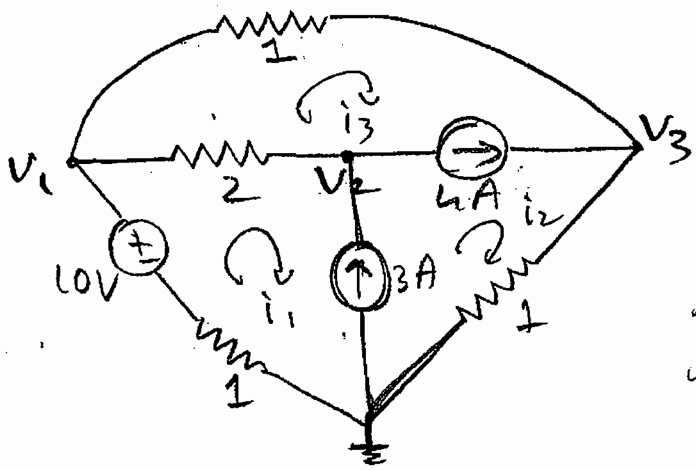
$$\therefore \boxed{V_2 = \frac{85}{8}}$$




$$\therefore P_{delivr} = -5\left(10 - \frac{25}{8}\right)$$

$$= -\frac{275}{8} \text{ W}$$

8)




Find power delivered by voltg source using mesh & nodal.

MESH :-

$$i_1 + i_2 + i_3 = 10 \quad \text{——— (1)}$$

$$i_2 - i_1 = 3 \quad \text{——— (2)}$$

$$i_2 = 3 + i_1$$

$$i_2 - i_3 = 4 \quad \text{——— (3)}$$

$$3 + i_1 - i_3 = 4$$

$$\therefore i_3 = i_1 - 1$$

$\therefore \; i_1 + 3 + i_1 + i_1 - 1 = 10$

$\therefore \; 3i_1 = 8 \;\Rightarrow\; i_1 = \frac{8}{3}$

$\therefore \; P_{deliv.} = 10 \times \frac{8}{3} = \boxed{\frac{80}{3}\,W}$

## NODAL :—

$$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{1} = 0$$

$\therefore \; 5V_1 - V_2 - 2V_3 = 20$ —— (1)

$$\frac{V_2 - V_1}{2} + 4 - 3 = 0$$

$\therefore \; V_2 - V_1 = -2$ —— (2)

$$\frac{V_3}{1} - 4 + \frac{V_3 - V_1}{1} = 0$$

$\therefore \; -V_1 + 2V_3 = 4$ —— (3)

From (1) & (2); $5V_1 - V_1 + 2 - 2V_3 = 20$

$\therefore \; 4V_1 - 2V_3 = 18$ —— (4)

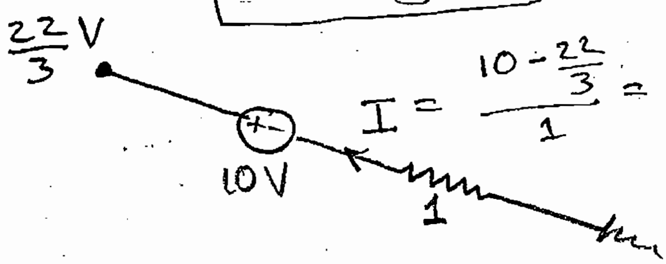
$-V_1 + 2V_3 = 4$

$3V_1 = 22$

$\therefore \; \boxed{V_1 = \frac{22}{3}\,V}$

$\therefore \; V_3 = \frac{4 + \frac{22}{3}}{2}$

$\therefore \; V_3 = \frac{34}{6}\,V$

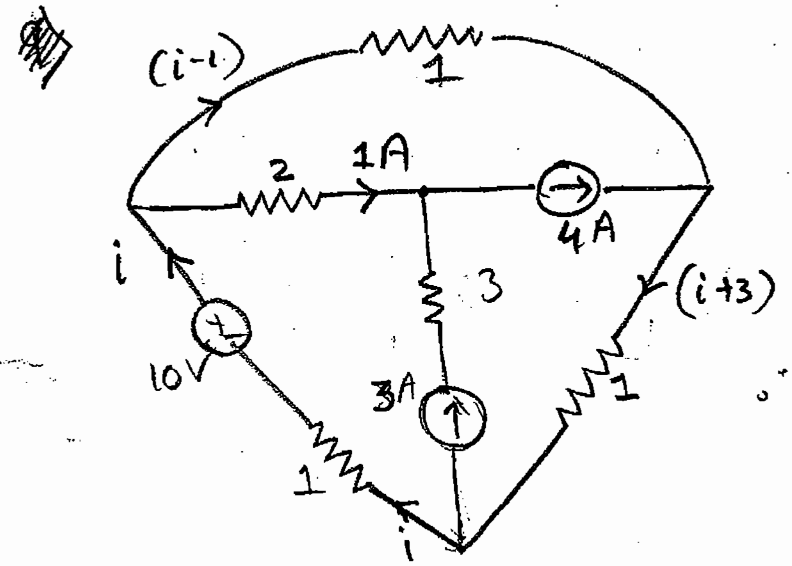



$I = \frac{10 - \frac{22}{3}}{1} = \frac{8}{3}\,A$

now,

$P_{deliv.} = 10 \times \frac{8}{3}$

$= \frac{80}{3}\,W$




$$i - 10 + i - 1 + i + 3 = 0$$

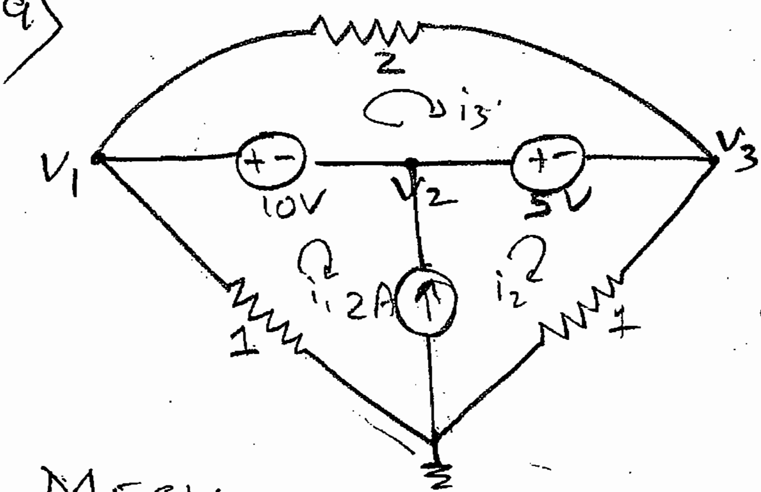
$$3i = 8$$

$$i = \frac{8}{3}$$

$$\therefore P_{deliv.} = 10 \times \frac{8}{3}$$

$$= \frac{80}{3}\ W$$

9)




Find the power delivered by the current source using mesh & nodal.

MESH :-

$$i_1 + 1i_2 + 1i_3 = 0 \quad \text{—— (1)} \times$$

$$2i_3 = 15 \Rightarrow \boxed{i_3 = \frac{15}{2}} \quad \text{—— (2)}$$

$$i_1 + i_2 = -15 \quad \text{—— (3)}$$

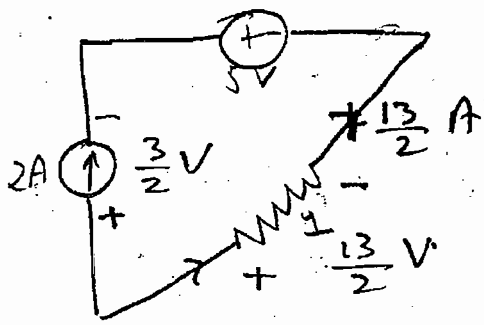
$$-i_1 + i_2 = 2 \quad \text{—— (4)}$$

$$\boxed{i_2 = \frac{-13}{2}}$$

$$i_1 = \frac{-17}{2}$$

$$\therefore P_{deliverd} = -2 \times \frac{3}{2}$$

$$\boxed{= -3W}$$



# NODAL :—

$$\frac{V_1}{1} + \frac{V_1 - V_3}{2} - 2 + \frac{V_3}{1} + \frac{V_3 - V_1}{2} = 0$$

$\therefore \cancel{3V_1 - V_3 = 0}$

$\therefore \quad V_1 + V_3 = 2$ —— (1)

$V_1 - V_2 = 10$ —— (2)

$V_2 - \overset{(2-V_1)}{V_3} = 5$ —— (3)

$\Rightarrow V_2 + V_1 = 7.$

$\therefore \quad 2V_1 = 17$

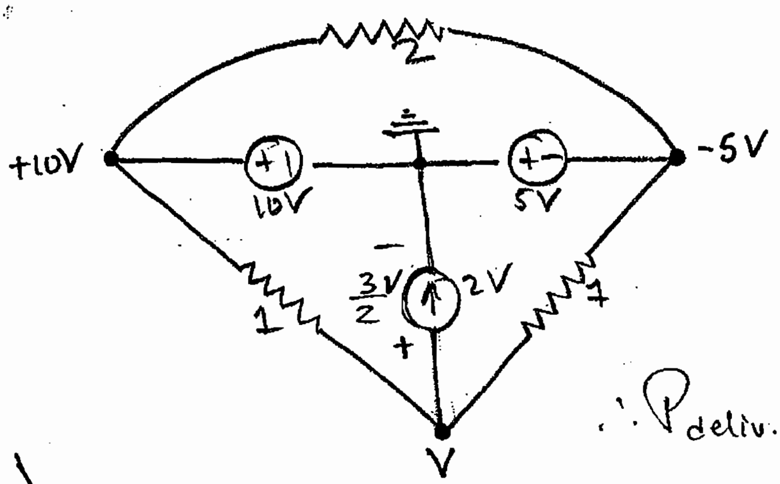
$V_1 = \frac{17}{2}$

$V_2 = \frac{17}{2} - 10$

$\boxed{V_2 = -\frac{3}{2} V}$

$P_{delivered} = 2 \times \left(-\frac{3}{2}\right)$

$= -3 W.$



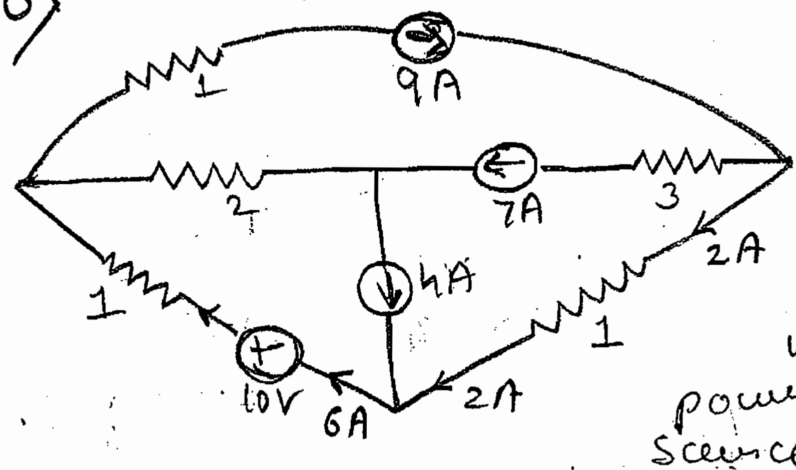

$$\frac{V-10}{1} + \frac{V+5}{1} + 2 = 0$$

$\therefore \quad 2V = 3$

$V = \frac{3}{2} V$

$\therefore P_{deliv.} = -\frac{3}{2} \times 2 = -3W$

10)




Just write mesh & nodal eqn governing the n/w & determine power delivered by vltg source by inspection.

MESH :—

$i_1 - i_2 = 4$ —— (1)     $i_3 = 9\,A$ —— (2)

$i_3 - i_2 = 7$ —— (3) $\Rightarrow$ $i_2 = 2\,A$

$\therefore i_1 = 6\,A$

NODAL :—

$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{2} + 9 = 0$ —— (1)

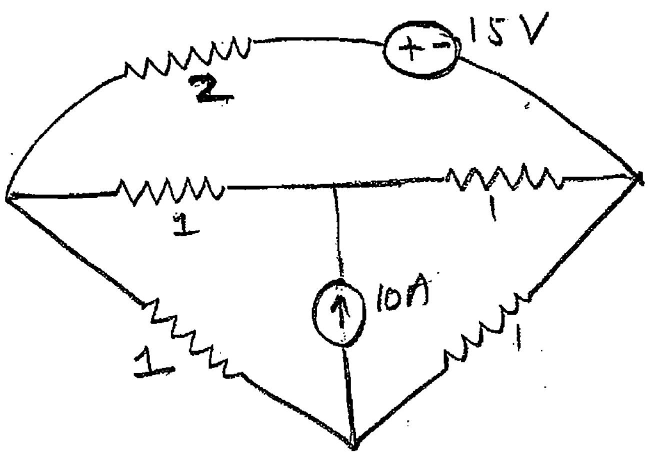
$\therefore \frac{V_2 - V_1}{2} + 4 - 7 = 0$ —— (2)

$\frac{V_3}{1} + \cancel{V_3}\,7 - 9 = 0$ —— (3)

Now,

$P_{delvird} = 10 \times 6 = \boxed{60\,W}$

11)




What is the power delivered by vltg source using mesh & nodal.

MESH :—

$i_2 - i_1 = 10$ —— (1)

$3 i_3 - i_1 - i_2 = -15$ —— (2) $\Rightarrow$ $i_1 + i_2 = 3 i_3 + 15$

$i_1 + i_2 + 2 i_3 = -15$ —— (3) $\Rightarrow$ $3 i_3 + 15 + 2 i_3 = -15$

$P_{delive} = +15 \times \frac{10}{2} = +\frac{075}{1}\,W$     $\therefore i_3 = -5A$

NODAL:-

$$\frac{V_1}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - 15 - V_3}{2} = 0$$

$$\therefore \quad 5V_1 - 2V_1 - V_3 = 15 \quad \text{——} \; (1)$$

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} - 10 = 0$$

$$\therefore \quad -V_1 + 2V_2 - V_3 = 10 \quad \text{——} \; (2)$$

$$\frac{V_3}{1} + \frac{V_3 - V_2}{1} + \frac{V_3 + 15 - V_1}{2} = 0$$

$$\therefore \quad -V_1 - 2V_2 + 5V_3 = -15 \quad \text{——} \; (3)$$

From (1) & (2); $\quad 4V_1 - 2V_3 = 25$

From (2) & (3); $\quad -2V_1 + 4V_3 = -5$

$\quad\quad\quad -4V_1 + 8V_3 = -10$

$\quad\quad\quad 6V_3 = 15$

$$V_3 = \frac{15}{6}\ V = \frac{5}{2}\ V$$

$$V_1 = \frac{25 + \frac{15}{3}}{4}$$

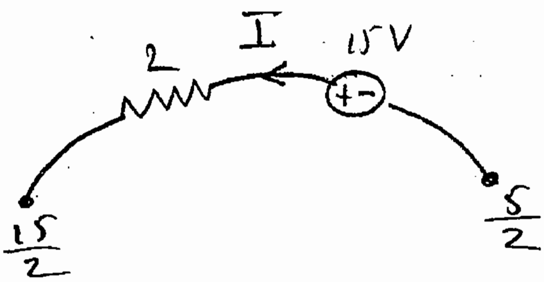
$$= \frac{90}{12} = \frac{15}{2}\ V$$

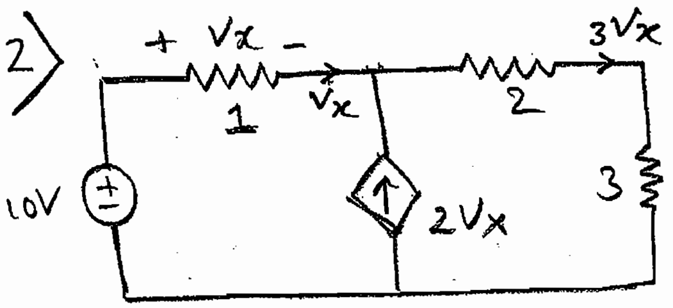
$$\therefore \quad I = \frac{V_3 + 15 - V_1}{2} = \frac{\frac{5}{2} + 15 - \frac{15}{2}}{2}$$

$$= \frac{20}{4} = 5\ A$$

$$\therefore \quad P_{deliver} = 15 \times 5$$

$$\boxed{= 75\ W}$$



12) 


Find $V_x$ using mesh & nodal

KVL

$-10 + V_x + 15V_x = 0$

$\therefore \boxed{V_x = \frac{5}{8} V}$

Link equation:-

$V_x = i_1$ —③

MESH :—

$i_1 + 5i_2 = 10$ —①

$-i_1 + i_2 = 2V_x$ —②

$\therefore -3i_1 + i_2 = 0$ —④

From ① & ④;

$$\begin{array}{r} 3i_1 + 15i_2 = 30 \\ -3i_1 + i_2 = 0 \\ \hline 16i_2 = 30 \end{array}$$

$\therefore$ ~~$i_1 = 10$ V~~

$i_1 = \frac{5}{8}$

$i_2 = 2$ A.

$\therefore V_x = \frac{5}{8}$ V

NODAL :—

$\frac{V_1 - 10}{1} - 2V_x + \frac{V_1}{5} = 0$

$5V_1 - 50 - 10V_x + V_1 = 0$

$6V_1 - 10V_x = 50$

$3V_1 - 5V_x = 25$ —①

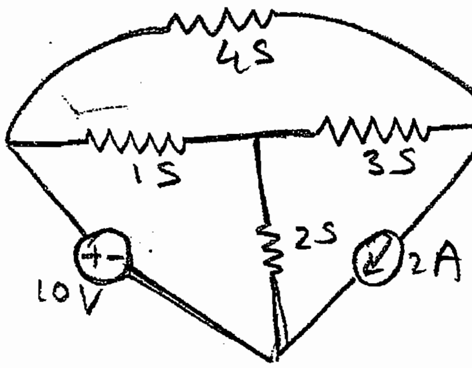
$V_x = 10 - V_1$ —②

$3(10 - V_x) - 5V_x = 25$

$8V_x = 5$

$V_x = \frac{5}{8}$ V

13)




MESH

$$-10 + \frac{(i_1 - i_3)}{1} + \frac{(i_1 - i_2)}{2} = 0 \quad — (1)$$

$$\therefore \quad i_2 = 2 \quad — (2)$$

$$\frac{i_3}{4} + \frac{(i_3 - i_2)}{2} + \frac{(i_3 - i_1)}{1} = 0 \quad — (3)$$

$$P_{3S} = \frac{|i_2 - i_3|^2}{3} = \_\_\_\_\_ \ W.$$

NODAL :—

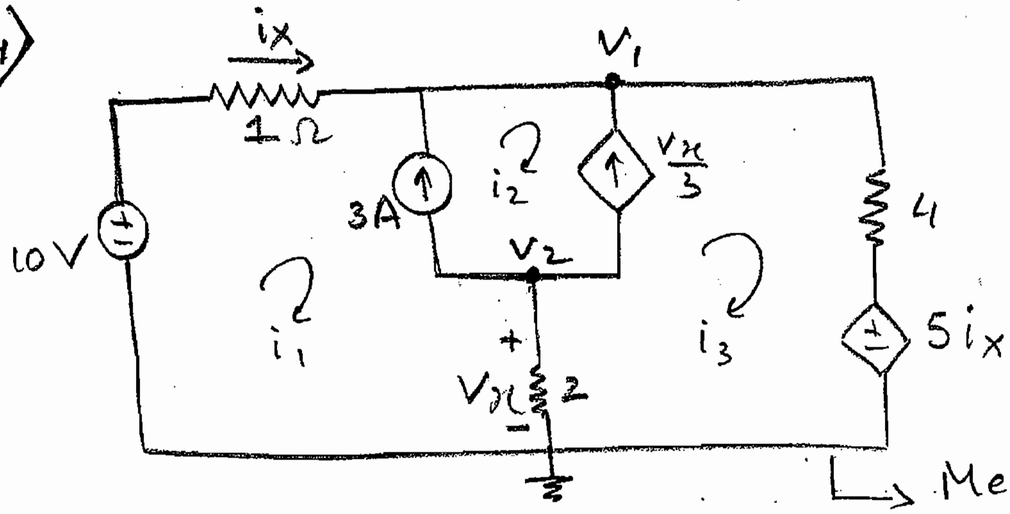
$$V_1 = 10 \quad — (1)$$

$$1[V_2 - V_1] + 2V_2 + 3[V_2 - V_3] = 0 \quad — (2)$$

$$+2 + 3(V_3 - V_2) + 4(V_3 - V_1) = 0 \quad — (3)$$

$$P_{3S} = [V_2 - V_3]^2 \cdot 3 = \_\_\_\_\_ \ W.$$

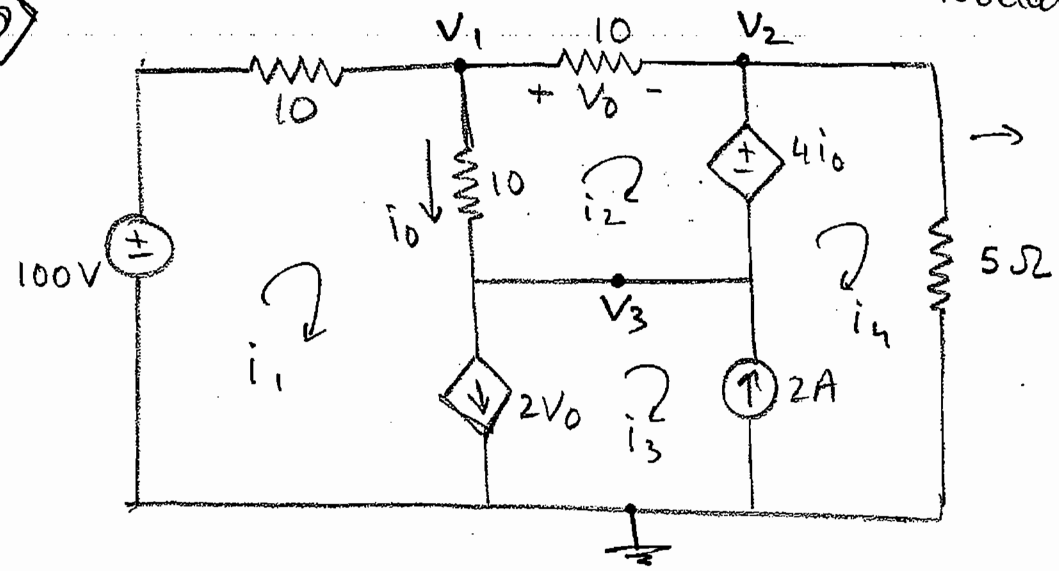
14)




Write mesh & nodal equation governing the circuit.

↳ Mesh = 3 + 2 = 5
Nodal = 2 + 2 = 4

15)




→ Mesh = 4+2
= 6
Nodal
= 3 + 2
= 5

14) MESH :-

$10 - i_1 - 4i_3 - 5i_x = 0$ —— (1)

$-i_1 + i_2 = 3$ —— (2)

$-i_2 + i_3 = \frac{V_x}{2}$ —— (3)

$i_x = i_1$ —— (4)

$V_x = (i_1 - i_3)2$ —— (5)

NODAL :-

$\frac{V_1 - 10}{1} + \frac{V_1 - 5i_x}{4} - 3 - \frac{V_x}{3} = 0$ —— (1)

$\frac{V_2}{2} + 3 + \frac{V_x}{3} = 0$ —— (2)

$V_x = V_2$ —— (3)

$i_x = 10 - V_1$ —— (4)

15) MESH :—

$$100 - 20i_1 + 10i_2 + 4i_0 - 5i_4 = 0 \quad \text{—— (1)}$$

$$-10i_1 + 20i_2 + 4i_0 = 0 \quad \text{—— (2)}$$

$$i_4 - i_3 = 2 \quad \text{—— (3)} \qquad i_1 - i_2 = i_0 \quad \text{—— (5)}$$

$$i_1 - i_3 = 2V_0 \quad \text{—— (4)} \qquad V_0 = 10i_2 \quad \text{—— (6)}$$

NODAL :—

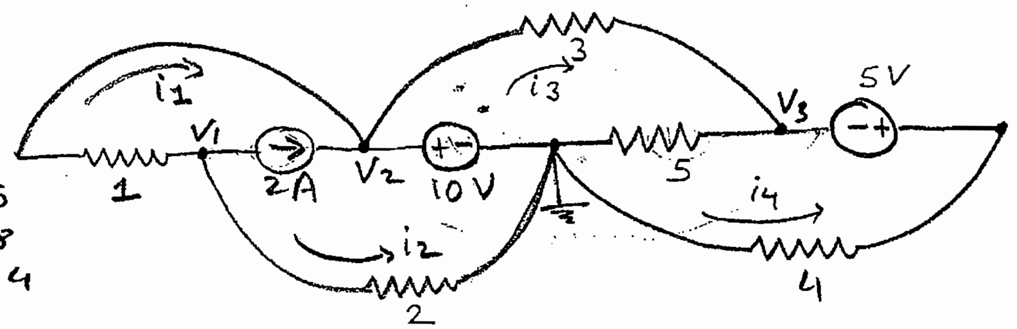
$$\frac{V_1 - 100}{10} + \frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{10} = 0 \quad \text{—— (1)}$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + 2V_0 - 2 + \frac{V_3 - V_1}{10} \quad \text{—— (2)}$$

$$V_2 - V_3 = 4i_0 \quad \text{—— (3)}$$

$$V_0 = V_1 - V_2 \quad \text{—— (4)} \qquad i_0 = \frac{V_1 - V_3}{10} \quad \text{—— (5)}$$

16)




n = 5
b = 8
m = 4

MESH :—

$$i_1 + 10 - 2i_2 = 0 \quad \text{—— (1)} \qquad +i_1 + i_2 = -2 \quad \text{—— (2)}$$

$$8i_3 + 5i_4 = 10 \quad \text{—— (3)} \qquad 5i_3 + 9i_4 + 5 = 0 \quad \text{—— (4)}$$

NODAL :—

$$\frac{V_1 - V_2}{1} + 2 + \frac{V_1}{2} = 0 \quad \text{—— (1)} \qquad V_2 = 10 \quad \text{—— (2)}$$

$$\frac{V_3 - V_2}{3} + \frac{V_3}{5} + \frac{V_3 + 5}{4} = 0 \quad \text{—— (3)}$$

## MESH :-

$$i_1 - 2i_2 = -10$$
$$\underline{-i_1 \pm i_2 = \mp 2}$$
$$-3i_2 = -8$$

$$\boxed{i_2 = \frac{8}{3}\ A}$$

$$i_1 = -10 + 2\left(\frac{8}{3}\right)$$

$$\boxed{i_1 = \frac{-14}{3}\ A}$$

$$40i_3 + 25i_4 = 50$$
$$\underline{-40i_3 \pm 72 i_4 = \mp 40}$$
$$-48 i_4 = 90$$

$$\boxed{i_4 = \frac{-90}{47}\ A} ; \quad \boxed{i_3 = \frac{115}{47}\ A}$$

## NODAL :-

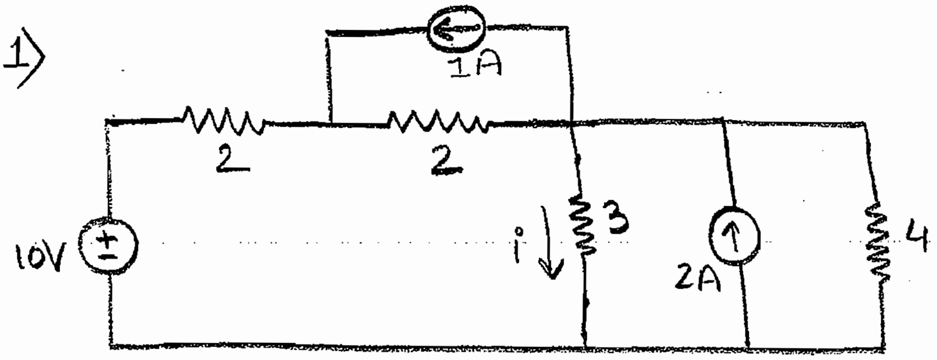
$$V_1 - 10 + 2 + \frac{V_1}{2} = 0$$

# NETWORK THEOREMS IN DC

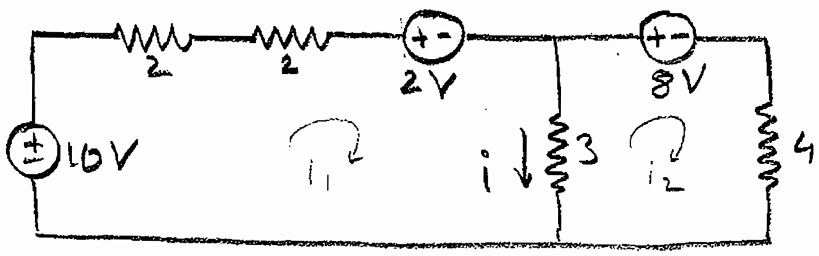
## Theorem 1 :-

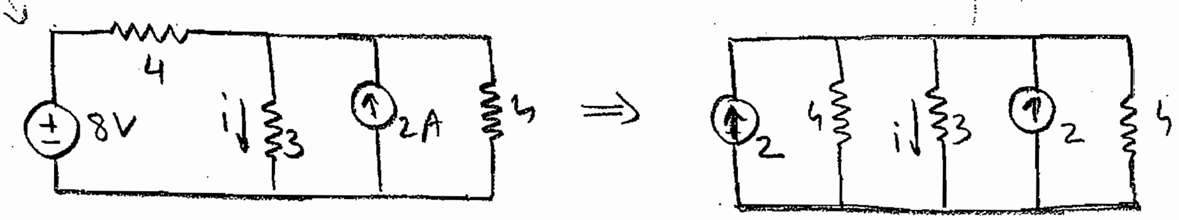
## Source Transformation Technique :-

1)



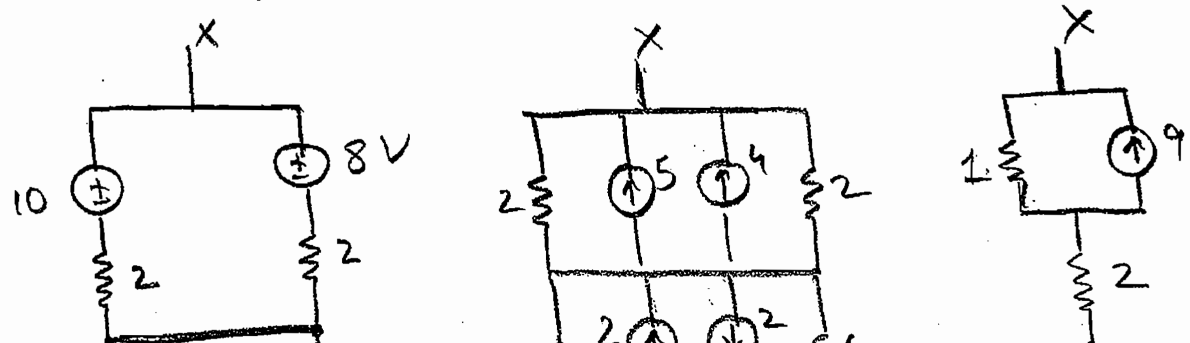

Find 'i' using S.T.T

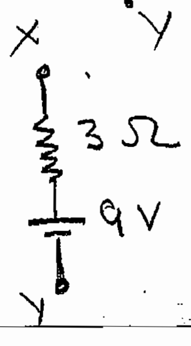


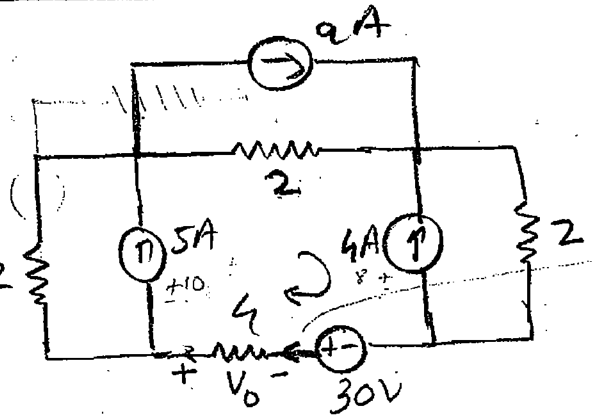

$$\therefore\ i = 4\left(\frac{16}{12+12+16}\right) = 4\left(\frac{16}{40}\right) = \frac{8}{5}\ A$$

2)



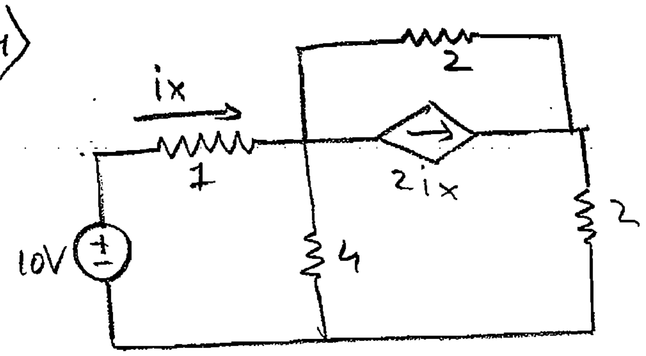
3) Find $V_0$ in one step

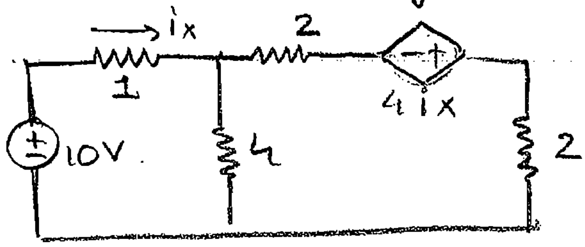


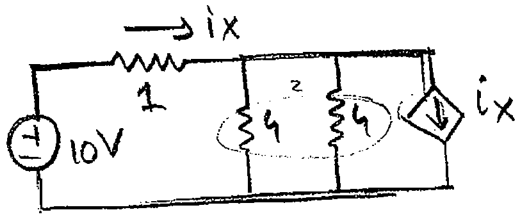

$$V_0 = -50\left(\frac{4}{4+6}\right)$$

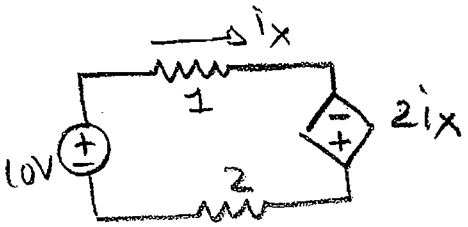
$$= -20\ V$$

4) Find $i_x$ using S.T.T.



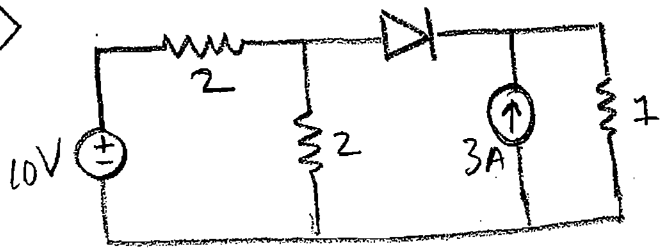



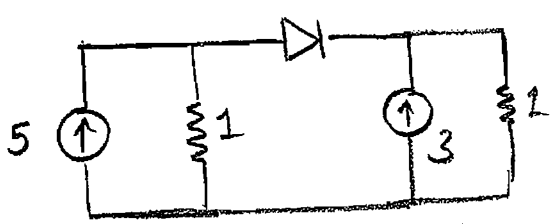



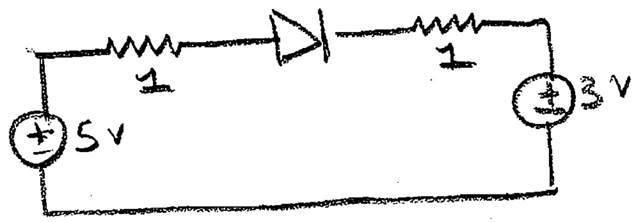



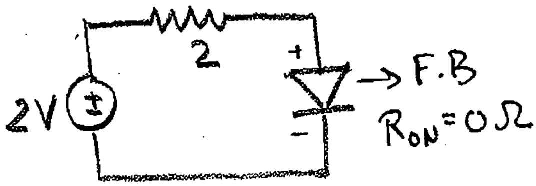

$$i_x = \frac{10 + 2i_x}{3} \Rightarrow \boxed{i_x = 10\ A}$$

5) If diode is ideal one, find current through it.




$$\therefore I_D = \frac{2}{2} = 1\ A$$

Theorem 2 :—

## Superposition Theorem :—

In any linear, active, bilateral n/w, consisting of no. of energy sources, resistances, etc. ; the effect produced in any element when all sources act at a time is equal to sum of effect in same element when each source is considered independently.

1) The no. of sub-circuits to be solved by applying S.P.T. is ______

⇒ Sum of Independent sources only.

2) Which of the foll. electrical parameter cannot be directly evaluated by using S.P.T ?

(a) Voltage (b) Current

(c) Power (d) Charge.

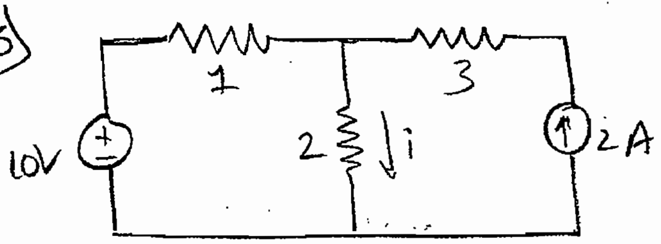
↳ non-linear electrical parameter

→ While applying S.PT, we consider only 1 independent source in every sub-circuit where other vltg sources are replaced by short circuit & ideal current sources are replaced by open circuit.

However dependent sources cannot be suppressed.

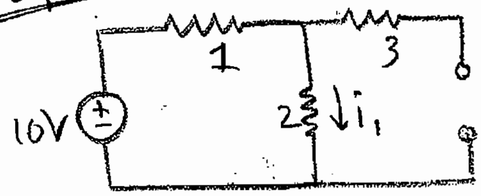
→ Voltages have unique polarities & Current have unique dir^n & they must be ~~as~~ respected ~~by~~ while applying S.P.T.

3)



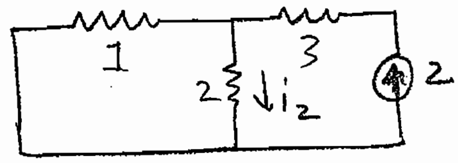

Find 'i' using S.P.T.

Step 1




$i_1 = \frac{10}{3}$ A

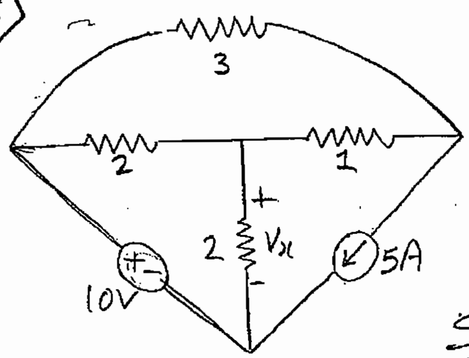
Step 2




$i_2 = 2\left(\frac{1}{3}\right) = \frac{2}{3}$ A

∴ By SPT :- $i = i_1 + i_2 = \frac{10}{3} + \frac{2}{3} = \frac{12}{3} = 4$ A

4)



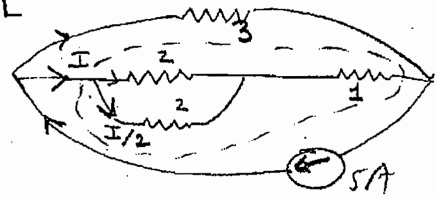

Find $V_x$ with help of S.P.T.

Step 1 (10V only)

$V_x' = 10\left(\frac{2}{2+4/3}\right) = 6$ V

Step 2 (5A only)

$V_x'' = -2\left[5\left(\frac{3}{3+2}\right)\times\frac{1}{2}\right] = -3$ V




Using SPT :-

$V_x = V_x' + V_x'' = 6 - 3 = 3$ V

Check (Nodal) :-

$V_1 = 10$ —— ①

$\frac{V_2 - 10}{2} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$

$4V_2 - 2V_3 = 10$ —— ②

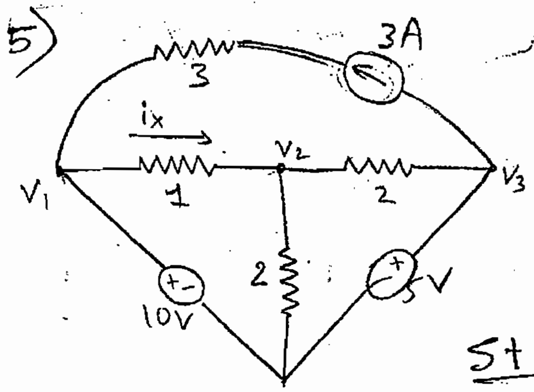
$5 + \frac{V_3 - V_2}{1} + \frac{V_3 - 10}{3} = 0$

$-3V_2 + 4V_3 = -5$ —— ③

$5V_2 = 15$

$V_2 = 3$, So $\boxed{V_x = 3}$

5)




Find $i_x$ using S.P.T.

Step 1 (10 V only)

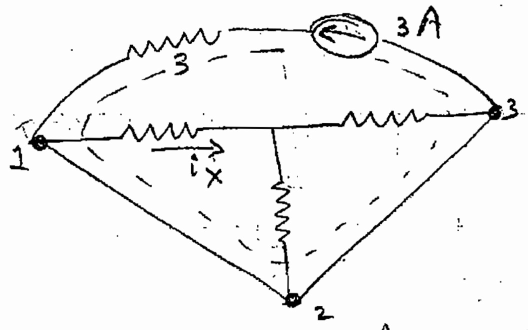
$$i_x' = 10 \times \frac{1}{1+1} = 5\ A.$$

Step 2 (5 V only)

$$i_x'' = -5\left(\frac{2/3}{2/3+2}\right) = -5\left(\frac{2}{8}\right) = -\frac{5}{4}\ A$$

Step 3 (3 A only)

$$i_x''' = 0\ A$$




1, 2, 3 are at same potential.

~~Check (Not~~ By SPT

$$i_x = 5 + 0 - \frac{5}{4} = \boxed{\frac{15}{4}\ A}$$

Check (Nodal)

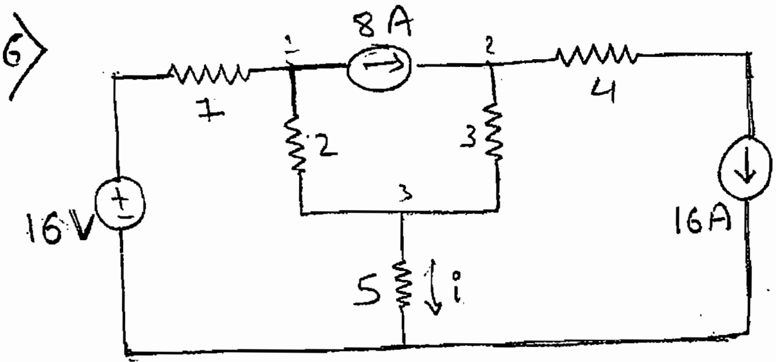
$$\frac{V_2 - 10}{1} + \frac{V_2}{2} + \frac{V_2 - 5}{2} = 0 \Rightarrow 4V_2 = 25$$

$$V_2 = \frac{25}{4}$$

Now,

$$i_x = \frac{V_1 - V_2}{1} = 10 - \frac{25}{4} = \frac{15}{4}\ A$$

6)




What is the power lost in 5 Ω resistor using S.P.T.

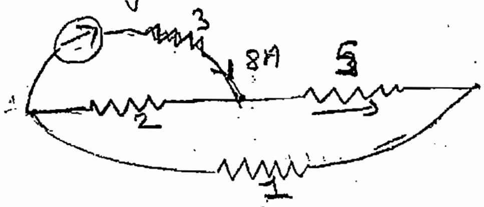
Step 1 :- (16 V only)

$$i' = \frac{16}{8} = \cancel{\frac{4}{4}}\ \cancel{A}\ 2\ A$$

Step 2 :- (8 A only)

$$i'' = 8 \times \frac{2}{8} = 2\ A$$

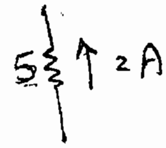
Power is non-linear parameter & hence cannot be calculated directly using S.P.T.



Step 3 :- (16 A only)

$$i''' = -16 \times \frac{3}{8} = -6\text{ A}$$

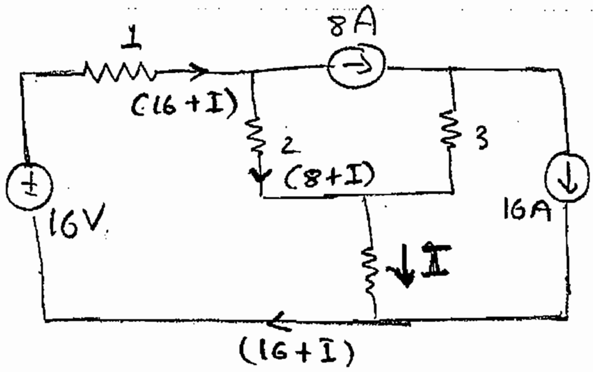
Using S.P.T

$$i = i' + i'' + i''' = 2 + 2 - 6 = -2\text{ A} \Rightarrow$$ 


So, $P_{Lost} = I_{net}^2 \times R$

$$= 4 \times 5 = \boxed{20\text{W}}$$

Check

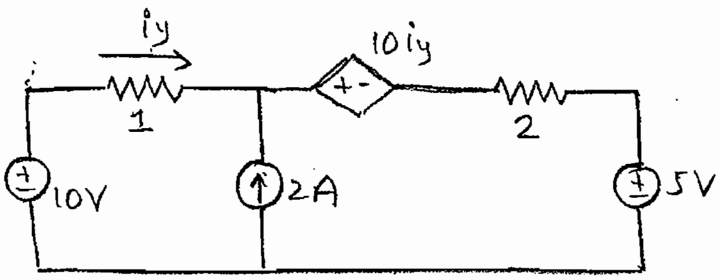



$$-16 + 1(16+I) + 2(8+I) + 5I = 0$$

$$\therefore 8I = -16$$

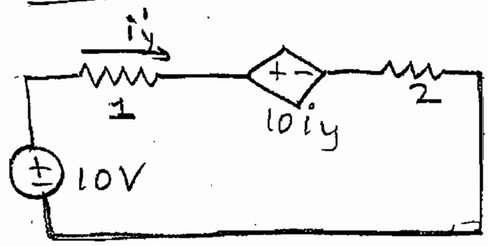
$$\therefore I = -2\text{ A}$$

$$P_{Lost} = 4 \times 5 = 20\text{W}$$

7) 


Find $i_y$ using S.P.T.

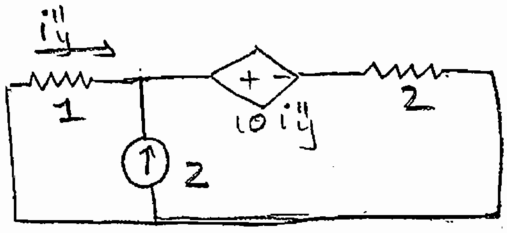
Step 1 (10V only)




$$10 - i_y' - 10 i_y' - 2 i_y' = 0$$

$$\therefore 13 i_y' = 10 \Rightarrow i_y' = \frac{10}{13}$$

Step 2 :- (2 A only)

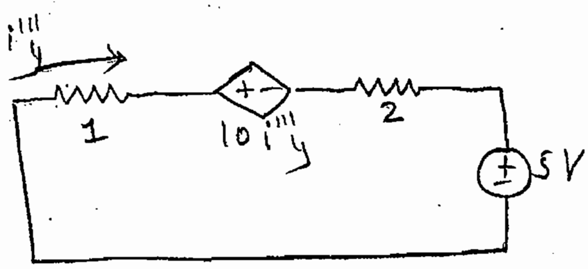



$$i_y'' + 10 i_y'' + 2(i_y'' + 2) = 0$$

$$\therefore 13 i_y'' = -4$$

$$\therefore i_y'' = \frac{-4}{13}\text{ A}$$

Step 3 :- (5 V only)



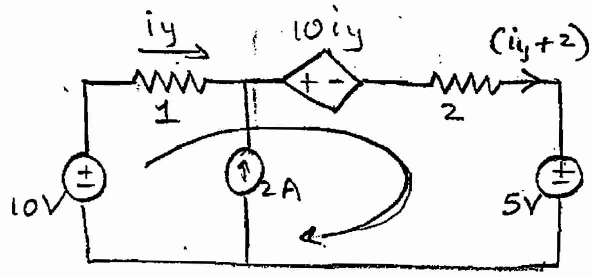

$$3 i_y''' + 10 i_y''' + 5 = 0$$

$$\therefore i_y''' = -\frac{5}{13}\text{ A}$$

## By SPT

$$i_y = \frac{10}{13} - \frac{4}{13} - \frac{5}{13} = \boxed{\frac{1}{13}\ A}$$

## Check

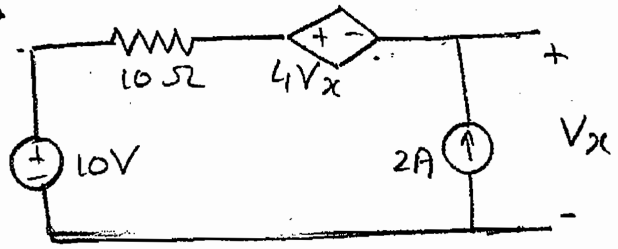



$$-10 + i_y + 10 i_y + 2(2+i_y) + 5 = 0$$

$$\therefore\ 13 i_y = 1$$

$$\therefore\ i_y = \frac{1}{13}\ A$$

8)




Find $V_x$ using S.P.T.

Step 1 (10 V only)

$$V_x' = 10 - 4V_x'$$

$$\therefore\ V_x' = 2\ V$$

Step 2 :- (2A only)

$$4V_x'' - 10(2) + V_x'' = 0$$

$$\therefore\ V_x'' = 4\ V$$

check

$$V_x + 4V_x - 10 - 20 = 0$$

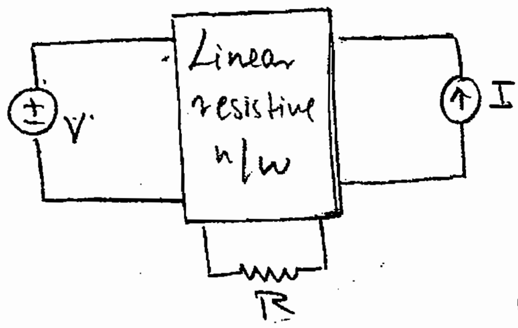
$$5V_x = 30$$

$$V_x = 6\ V$$

By SPT

$$V_x = 4 + 2 = \boxed{6\ V}$$

9)




The power lost in resistor are when vltg source alone act is 9 W & when current source alone act is 4W. What is total power lost in resistor R when both sources act simultaneously. (a) 1 W (b) 5W (c) 13W (d) 25W

V alone act :-

$$P_1 = I_1^2 R = 9 \Rightarrow |I_1| = \frac{3}{\sqrt{R}}$$

$I$ alone act :-

$P_2 = I_2^2 R = 4 \Rightarrow |I_2| = \frac{2}{\sqrt{R}}$

Now,

$I_{net} = \pm I_1 \pm I_2$

$P_T = [I_{net}]^2 R = [\pm I_1 \pm I_2]^2 R$

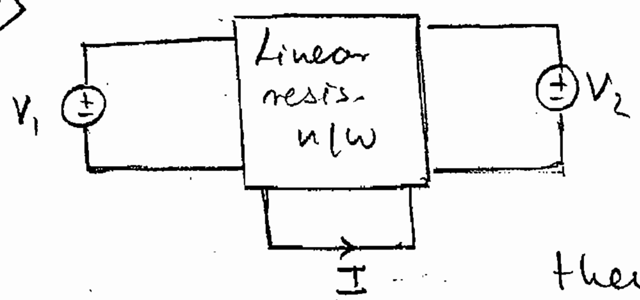
$= \left[\pm \frac{3}{\sqrt{R}} \pm \frac{2}{\sqrt{R}}\right]^2 R = [\pm 3 \pm 2]^2 \text{ W}$

$\therefore P_T = 1 \text{ W}$ ( $I_1$ & $I_2$ diff. dirn )

or

$25 \text{ W}$ ( $I_1$ & $I_2$ same dirn )

10)




If

| $V_1$ | $V_2$ | $I$ |
|---|---|---|
| 10 V | 0 V | 5 A |
| 0 V | −5 V | 1 A |

then if $V_1 = V_2 = 15$ V :

$I =$ ________

If $V_1$ alone act :-

$10 \longrightarrow 5A$

$\therefore 15 \longrightarrow \frac{15 \times 5}{10} = 7.5 \text{ A}$ (Homogenity)

If $V_2$ alone act

$-5 \longrightarrow 1 \text{ A}$

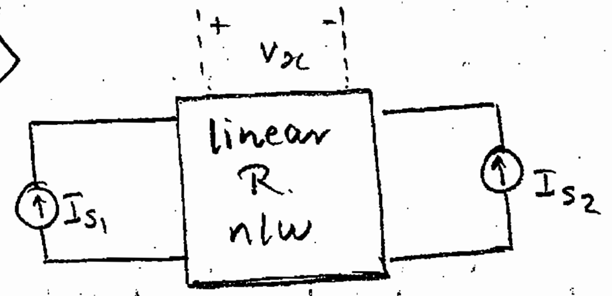
$15 \longrightarrow \frac{15}{-5} = -3 \text{ A}$ (Homogenity)

$\therefore$ By SPT

$I = 7.5 - 3$

$= 4.5 \text{ A}$

11)




If $I_{s_1} = 10A$ $I_{s_2} = 5A$.
then $V_x = 20$ V
If $I_{s_1} = 20A$, $I_{s_2} = -5A$
then $V_x = 10V$

Now if $I_{s_1} = I_{s_2} = 15$ A then $V_x$ = ______

$V_x = f(I_{s_1}; I_{s_2})$

$\therefore$ $\boxed{V_x = \alpha I_{s_1} + \beta I_{s_2}}$

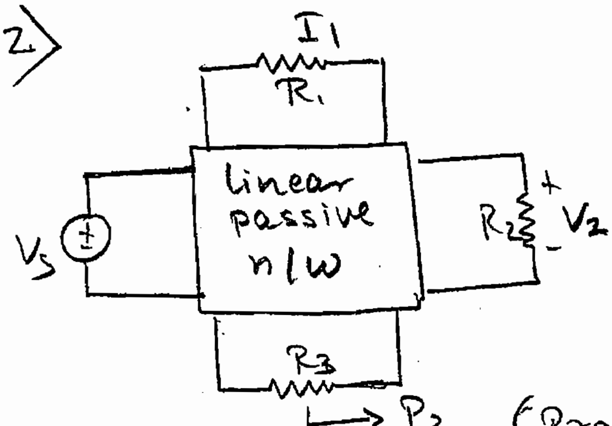
$$20 = \alpha(10) + \beta(5)$$
$$10 = \alpha(20) + \beta(-5)$$
$$30 = \alpha(30)$$

$\therefore$ $\boxed{\alpha = 1}$ $\Rightarrow$ $\boxed{\beta = 2}$

$$V_x = 1 I_{s_1} + 2 I_{s_2}$$
$$= 1 \times 15 + 2 \times 15$$

$\therefore$ $\boxed{V_x = 45 V}$

12)




If $V_s = 30$, then
$I_1 = 10A$, $V_2 = 20V$, $P_3 = 30W$
if $V_s = 50V$, then
$I_1 = ?$, $V_2 = ?$, $P_3 = ?$
(Proportionality, homogenity, ohm's Law).

$I_1$

$30 \rightarrow 10A$

$50 \rightarrow$ $I_{1\,new} = \dfrac{50 \times 10}{30} = 16.66$ A

$V_2$

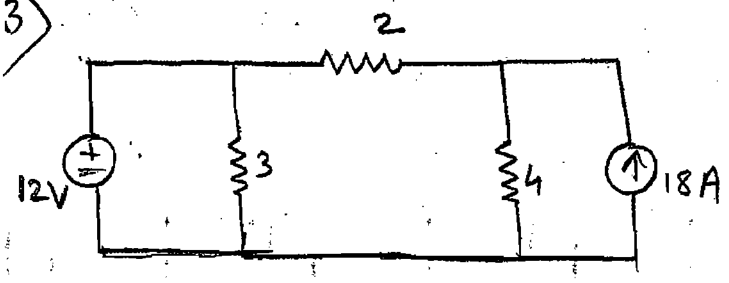
$30 \rightarrow 20$ V

$50 \rightarrow$ $V_{2\,new} = \dfrac{50 \times 20}{30} = 33.33$ V

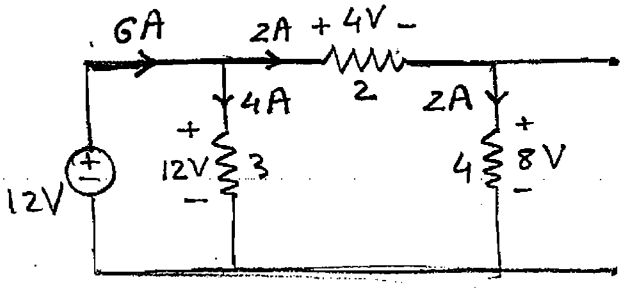
$P_3$ $P \propto V^2$

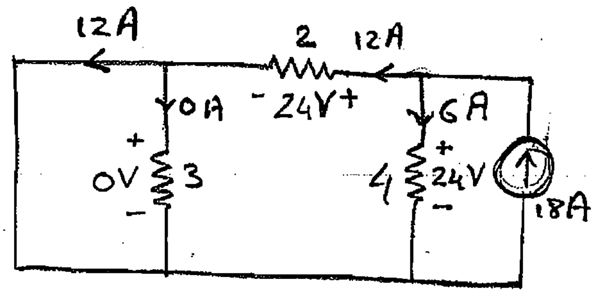
$(30)^2 \longrightarrow 30$

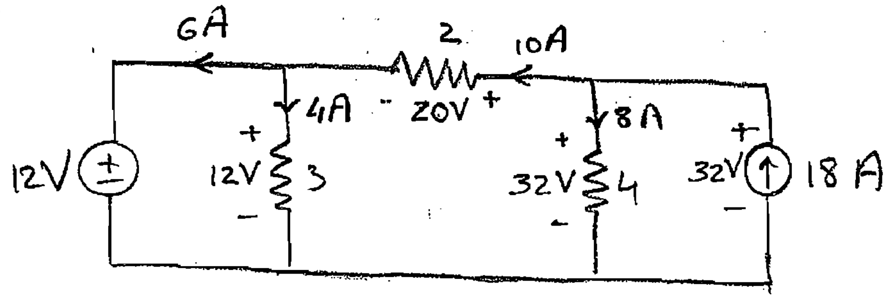
$(50)^2 \longrightarrow P_{3\,new} = \dfrac{(50)^2 (30)}{(30)^2} = \dfrac{250}{3}$ W

13) 


Determine vltg & current across all elements by using SPT.



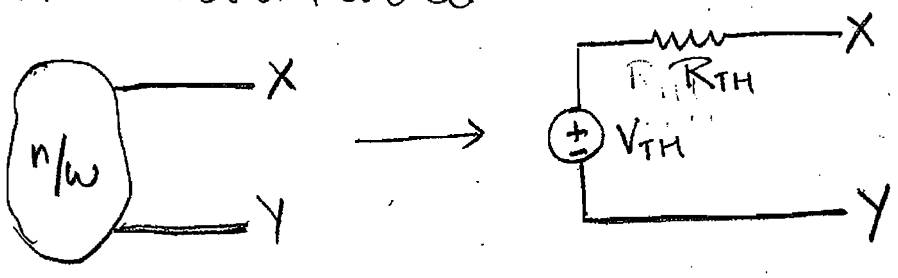




## Theorem 3:-

## Thevenins Theorem :-

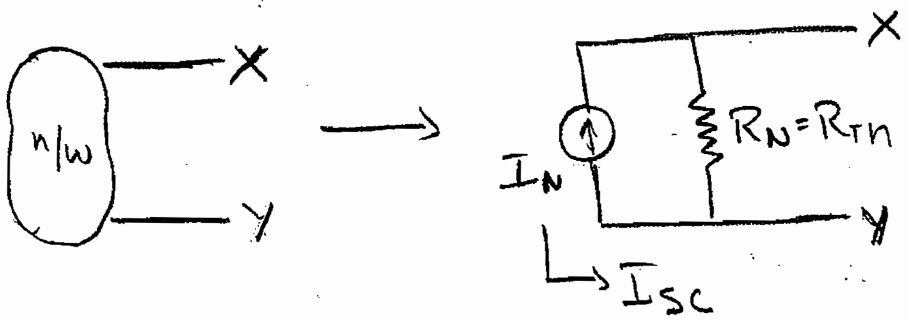
In any linear bilateral (active) n/w consisting of energy sources, resistors, etc with open o/p target terminal defined can be converted into a simple n/w consisting of vltg source in series with resistance



## Theorem 4 :-

# Nortons' Theorem :-

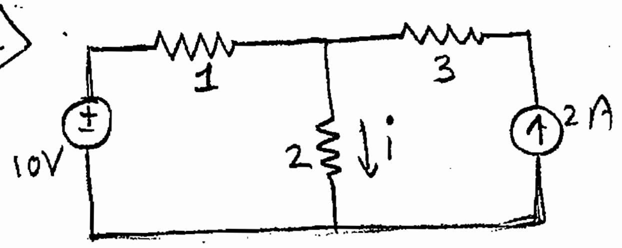
In any linear active bilateral n/w consisting of no. of energy sources, resist., etc with open o/p target terminal defined can be converted into a simple n/w consisting of current source in parallel with resistance.




Thevenins & Nortons equivalent are duals of each other.
i.e. They are source transformable.

## Category 1 :- Problems with only independent sources

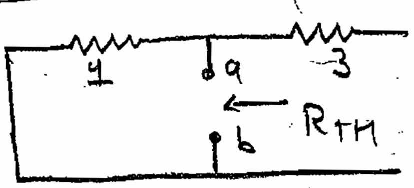
1)




Determine current i using ① Thevenin th.
② Norton th.

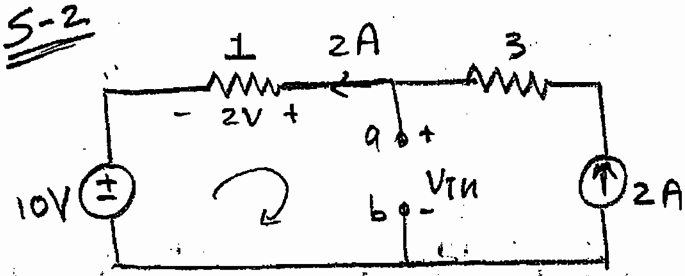
① Thevenin th.

S-1




$R_{TH} = 1\,\Omega$

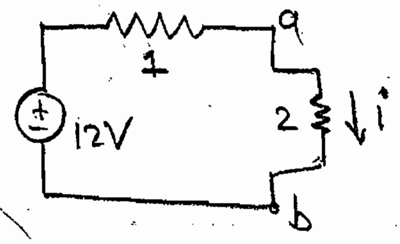
S-2




KVL

$-10-2+V_{TH}=0$

$V_{TH}=12\ V$

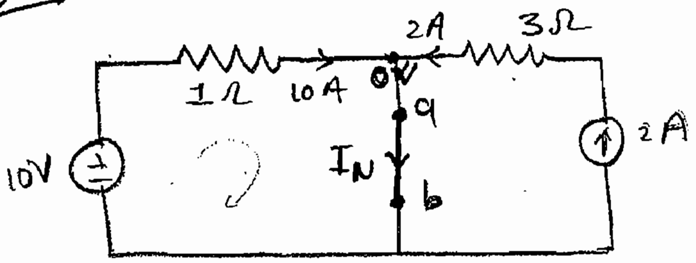



$i=\frac{12}{3}=4A$

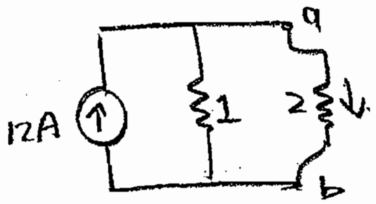
② Nortons' th.

S-1 $R_N=R_{TH}=1\ \Omega$

S-2

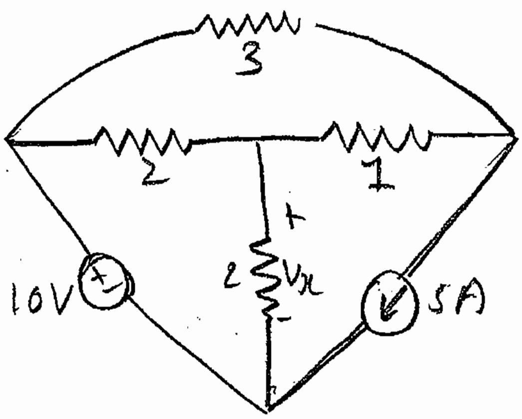



$I_N=10+2$
$=12\ A$




$i=12\left(\frac{1}{1+2}\right)=4A$

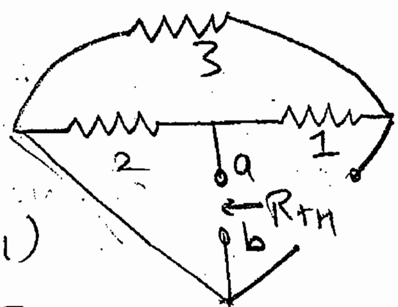
2>




Find $V_x$

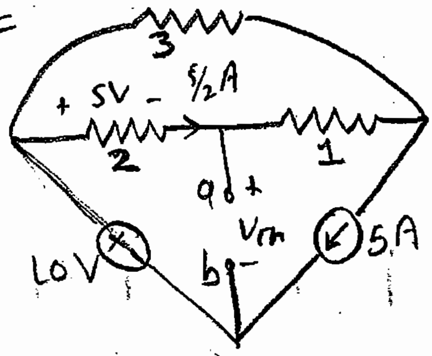
① Thevenins th.

S-1




$R_{TH}=(2\,||\,4)$
$=\frac{4}{3}\Omega$

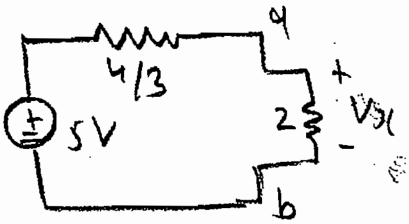
S-2




KVL

$-10 + 5 + V_{TH} = 0$

$\Rightarrow V_{TH} = 5V$

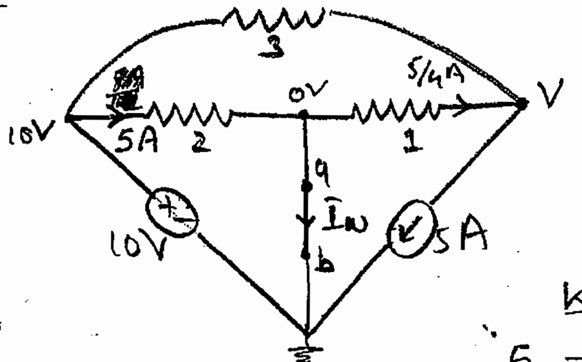



$V_x = 5\left(\frac{2}{2+4/3}\right) = 3\ V$

(2) Nortons the.

S-1 $R_N = R_{TH} = 4/3\ \Omega$

S-2




~~KCL~~ KCL

$5 + \frac{V}{1} + \frac{V-10}{3} = 0$

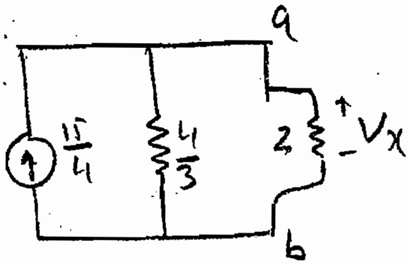
$4V = -5$

$\therefore V = -\frac{5}{4}\ V$

KCL

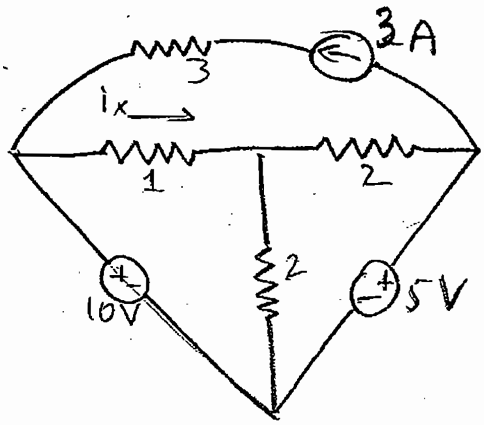
$5 = I_N + 5/4$

$I_N = \frac{15}{4}\ A$




$V_x = 2\left[\frac{15}{4}\left(\frac{4/3}{2+4/3}\right)\right] = 3\ V$

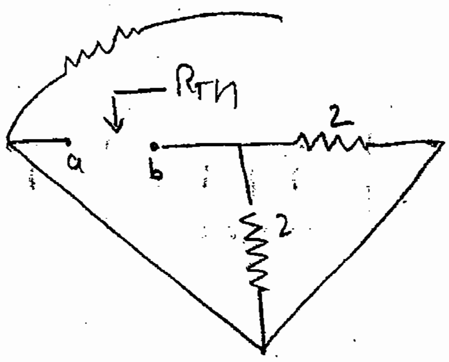
3)




Find ix using Thevenins & Nortons.

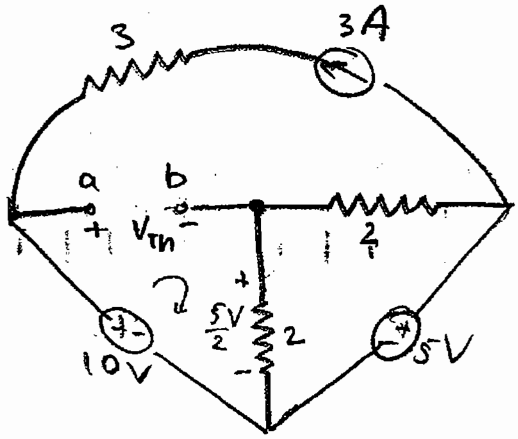
## ① Thevenins' th.

S-1




$R_{TH} = 2 \parallel 2$
$= 1\ \Omega$

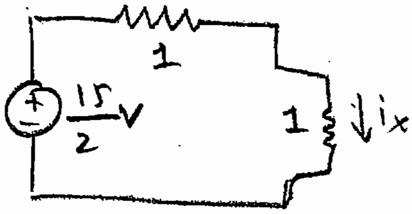
S-2




KVL

$10 - V_{TH} - \frac{5}{2}V = 0$

$\therefore\ V_{TH} = \frac{15}{2}\ V$

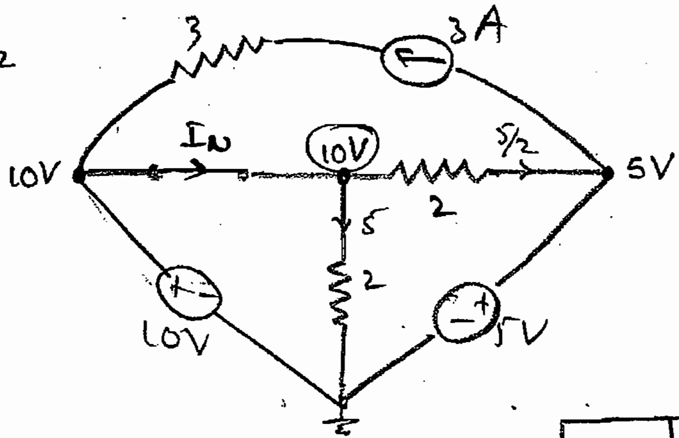



$\boxed{i_x = \frac{15}{4}\ A}$

## ② Nortons' the.

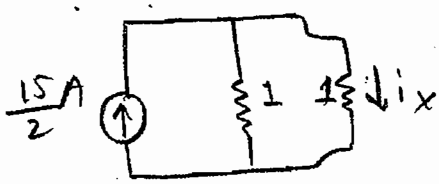
S-1 $R_N = R_{TH} = 1\ \Omega$

S-2



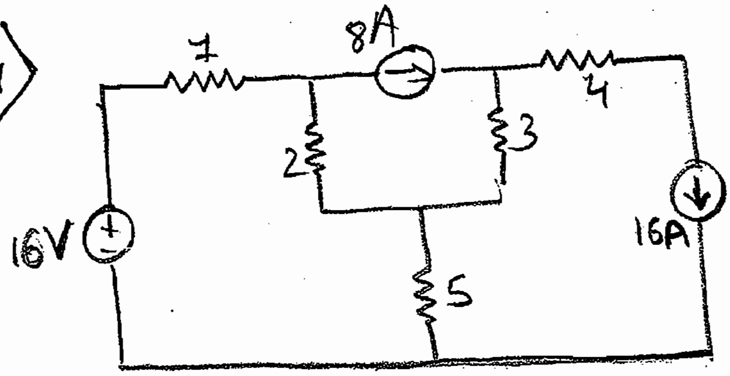

KCL

$I_N = 5 + \frac{5}{2}$
$= \frac{15}{2}\ A$




$i_x = \frac{15}{2}\left(\frac{1}{2}\right)$
$= \frac{15}{4}\ A$

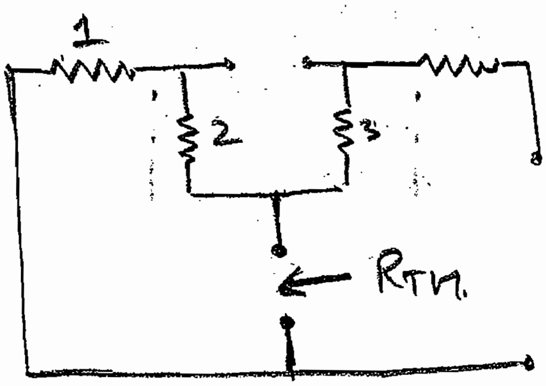
4)



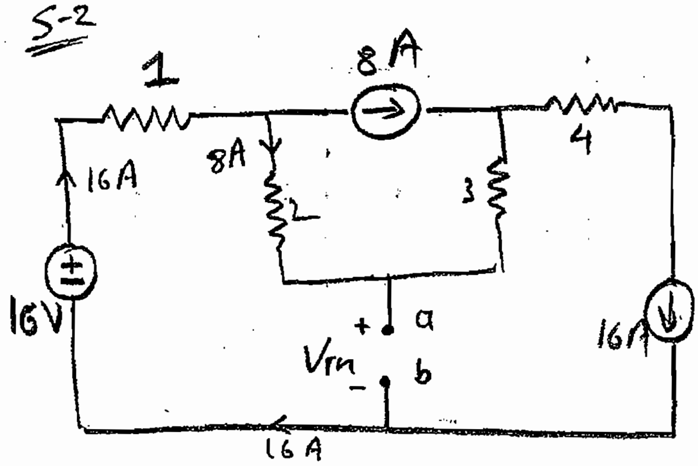

Find power lost in 5 Ω resis. using thevenin & Norton.

## ① Thevenins

S-1

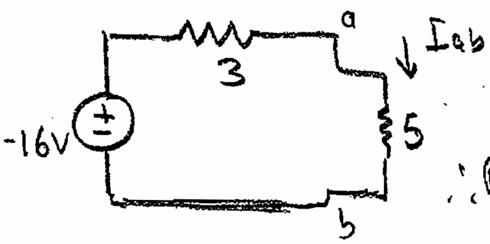



S-2




$$R_{TH} = 2 + 1$$
$$= 3\,\Omega$$

$$V_{TH} = 16 - 16(1) - 2(8)$$
$$= 16 - 32 = -16\,V$$



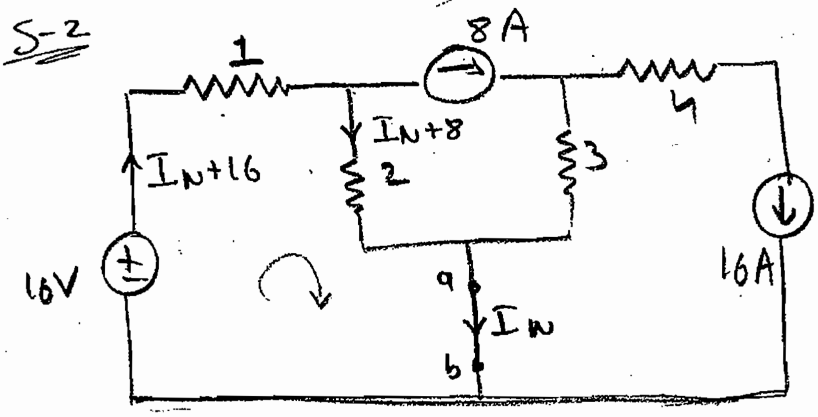

$$I_{ab} = \frac{-16}{8} = -2A$$

$$\therefore P_{Lost} = (-2)^2 \times 5 = 20\,W$$

## ② Nortons'

S-1 $R_N = R_{TH} = 3$

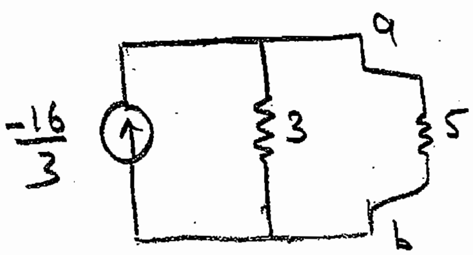
S-2




$$16 - (I_N+16) - 2(I_N+8) = 0$$

$$\therefore 3I_N = 16 - 32$$

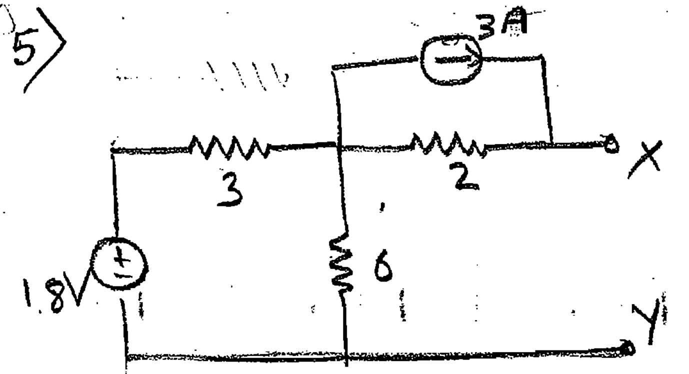
$$\therefore I_N = \frac{-16}{3}\,A$$




$$I_{ab} = \frac{-16}{3}\left(\frac{3}{8}\right) = -2\,A$$

$$\therefore P_{Lost} = (-2)^2 \times 5$$
$$= 20\,W$$

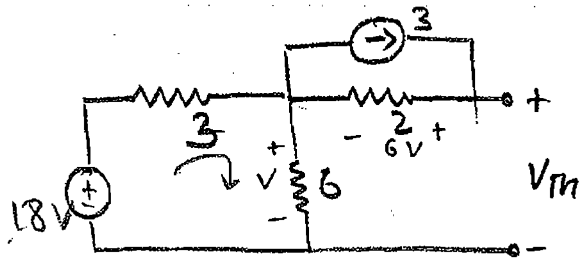
5) 




Determine Thevenin & Nortons equivalent b/w XY

① Thevenins'

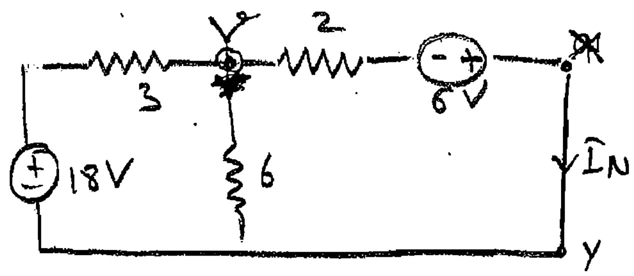
$R_{TH} = 2 + (6 \| 3) = 2 + 2 = 4\,\Omega$




$$V_{TH} = 6 + 18\left(\frac{6}{9}\right)$$
$$= 6 + 12$$
$$= 18\,V$$

② Nortons

$R_N = R_{TH} = 4\,\Omega$




KCL

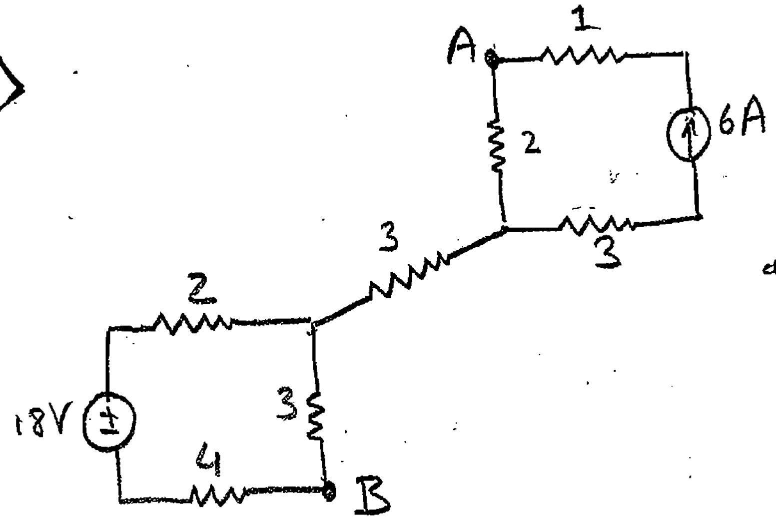
$$\frac{V-18}{3} + \frac{V}{6} + \frac{V+6}{2} = 0$$

$\therefore\ 2V - 36 + V + 3V + 18 = 0$

$\therefore\ V = 3$

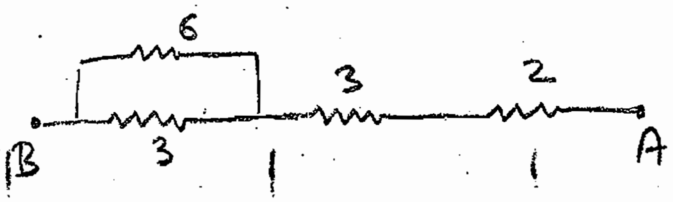
$$I_N = \frac{3+6}{2} = \frac{9}{2}\,A$$

6) 

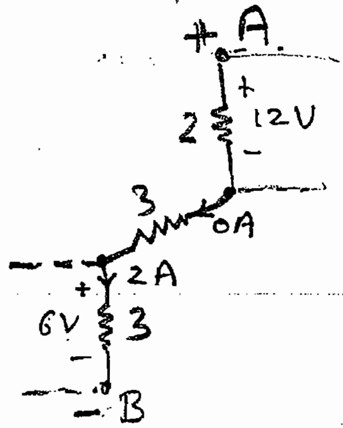



Determines Norton & Thevenin equivalent b/w A.B.

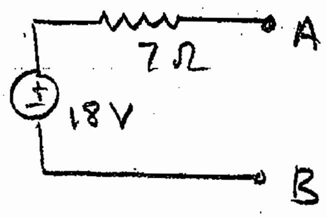
① Thevenins'

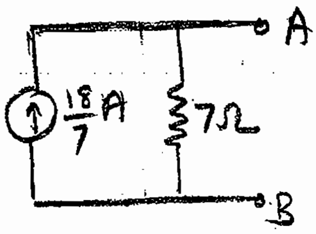



$$R_{TH} = 2 + 3 + (3 \| 6)$$
$$= 2 + 3 + 2$$
$$= 7\ \Omega$$



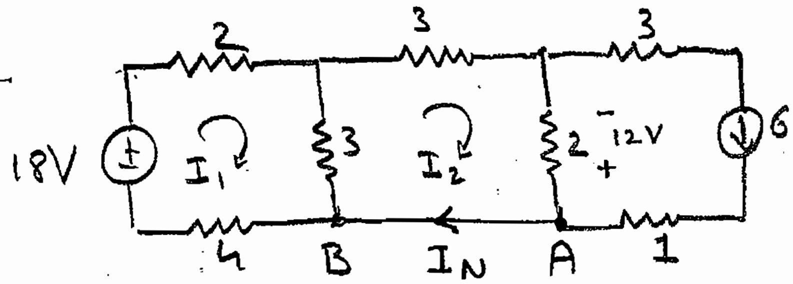

$$V_{TH} = 12 + 0 + 6$$
$$= 18\ V$$




② Nortons':

$R_N = R_{TH} = 7\ \Omega$




Now

$$18 - 9I_1 + 3I_2 = 0$$

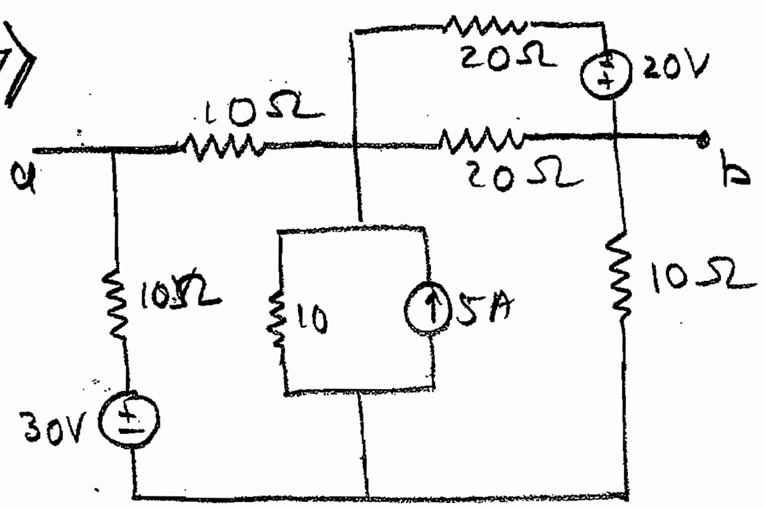
$$\therefore 3I_1 - I_2 = 6 \quad \text{—} ①$$

$$3I_1 - 8I_2 = -12 \quad \text{—} ②$$

From ① & ② : $7I_2 = 18 \Rightarrow I_2 = \frac{18}{7}$

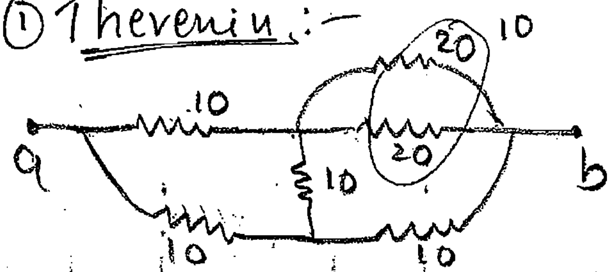
But $I_N = I_2 \Rightarrow I_N = \frac{18}{7}$ A

7)



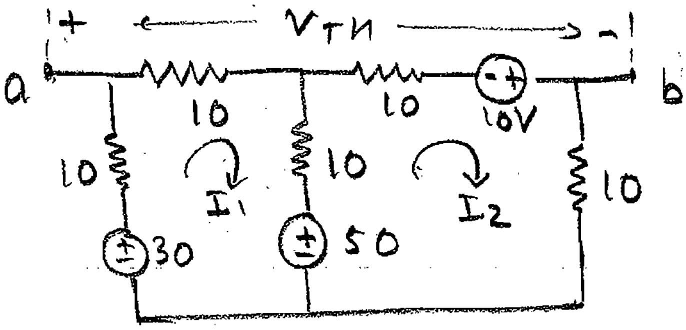

Determine Thevenine & Nortons equiv. across ab.

① Thevenin :-




$R_{TH} = 20 \| 20$

$= 10\,\Omega$




$30 - 30I_1 + 10I_2 = 50$

$\therefore -3I_1 + I_2 = 2$ — ①

$50 - 30I_2 + 10I_1 + 10 = 0$

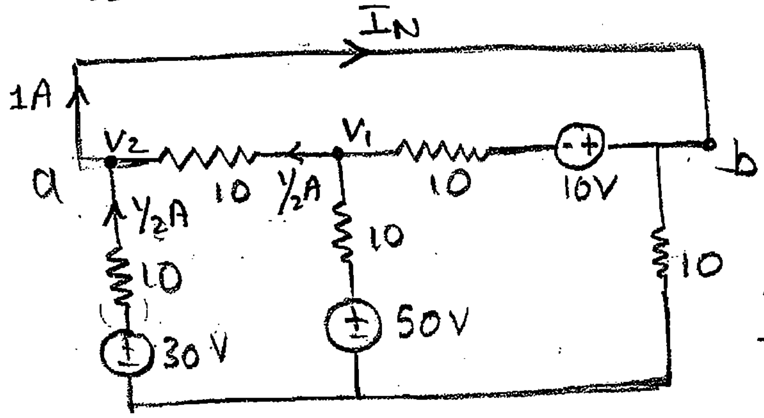
${}_{3}I_1 - \underset{9}{3}I_2 = -\underset{18}{6}$ — ②

From ① & ②; $-8I_2 = -16$ ⇒ $I_1 = 0$ A

$I_2 = \cancel{\frac{16}{8}}\ 2$ A

$\therefore V_{TH} = 0 + 10(2) - 10 = \boxed{10\text{ V}}$

② Nortons :-




Nodal

$$\frac{V_1-V_2}{10} + \frac{V_1-50}{10} + \frac{V_1-V_2+10}{10} = 0$$

$3V_1 - 2V_2 = 40$ — ①

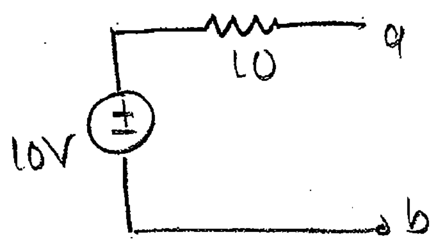
$$\frac{V_2-V_1}{10} + \frac{V_2-30}{10} + \frac{V_2}{10} + \frac{V_2-V_1-10}{10} = 0$$

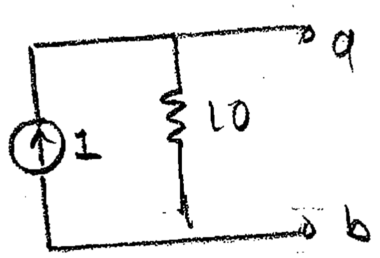
$\therefore -2V_1 + 4V_2 = 40$ — ②

From ① & ②;

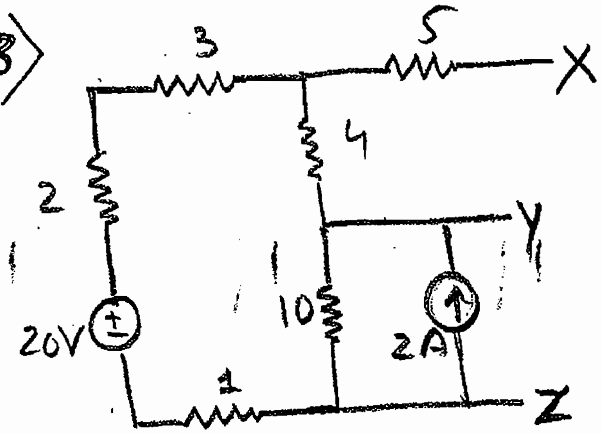
$4V_1 = 120 \Rightarrow \boxed{V_1 = 30V}$

$\boxed{V_2 = 25V}$



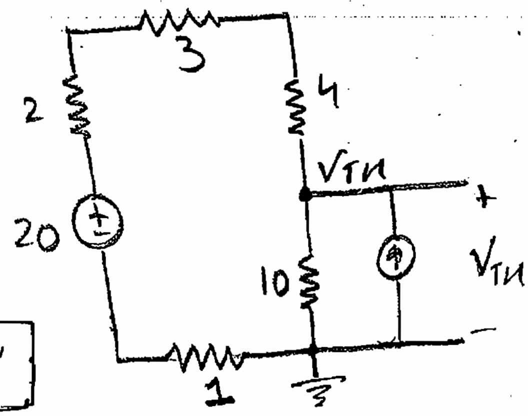



8) Determine Thevenin equivalent b/w Y & Z.




$R_{TH} = 10 \,//\, (3+2+1+4) = 10 \,//\, 10$

$= 5\ \Omega$

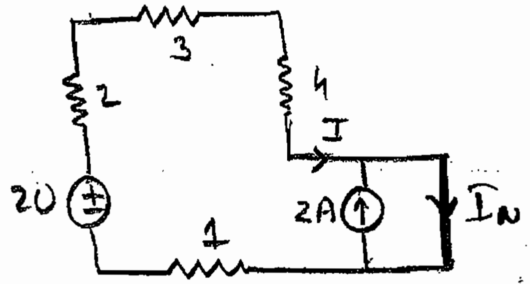



$V_{TH} \Rightarrow$ KCL

$$\frac{V_{TH}-20}{10} + \frac{V_{TH}}{10} - 2 = 0$$

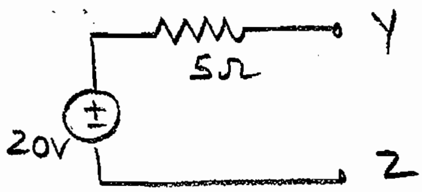
$\therefore 2V_{TH} = 40 \Rightarrow \boxed{V_{TH} = 20V}$

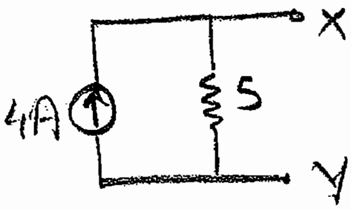
now,



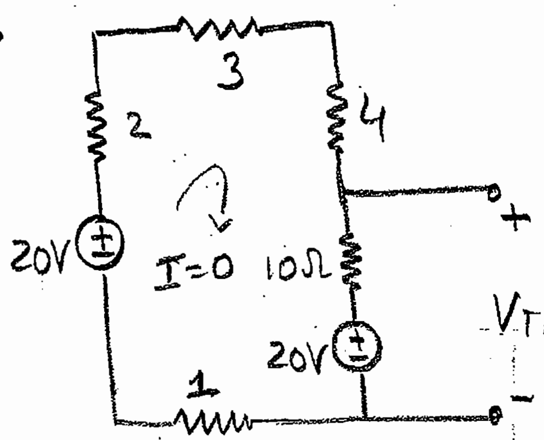

$R_N = R_{TH} = 5\ \Omega$

$I_N = 2 + \frac{20}{10} = 4\ A$




$\Rightarrow$



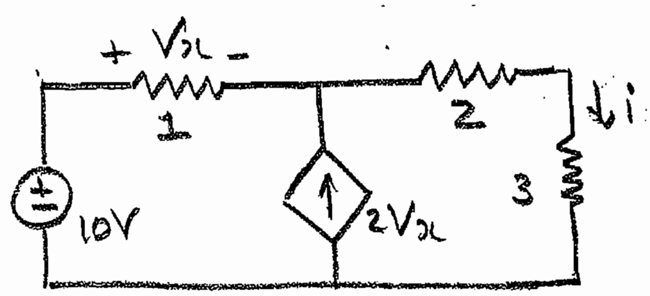

$V_{TH} = 20V$

# Category 2 :- Problems with both independent & dependent sources.

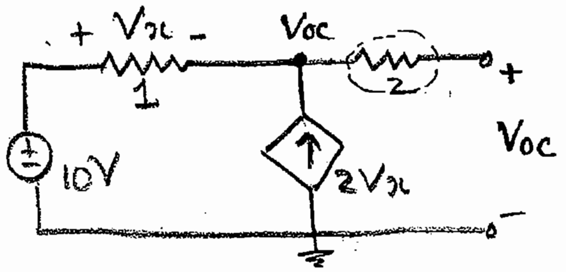
Dependent sources cannot be suppressed directly in terms of their resistances. So here, finding $R_{TH}$ or $R_N$ is not possible directly.

Hence we use Ohm's law where,

$$\boxed{R_{TH} = R_N = \frac{V_{oc}}{I_{sc}}}$$ at target terminal.

1) 


Find current $i$ using Thevenin & Nortons. th.

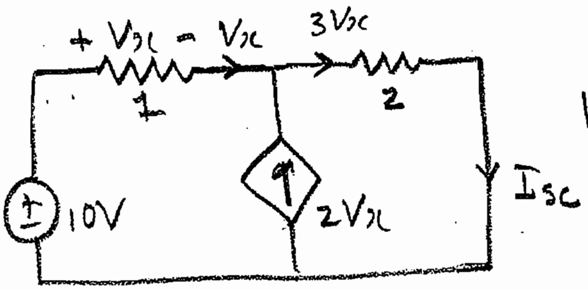



$$\frac{V_{oc} - 10}{1} - 2V_x = 0$$

$$V_x = 10 - V_{oc} \quad \text{—} \textcircled{1}$$

$$V_{oc} - 10 - 20 + 2V_{oc} = 0$$

$$\boxed{V_{oc} = 10\ V}$$




KVL

$$-10 + V_x + 6V_x = 0$$

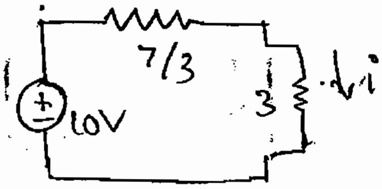
$$V_x = \frac{10}{7}$$

But $I_{sc} = 3V_x$

$$= 3 \times \frac{10}{7} \Rightarrow \boxed{I_{sc} = \frac{30}{7}\ A}$$

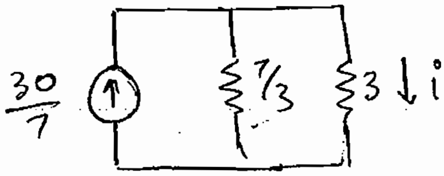
$$R_{TH} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{10 \times 7}{30} = \frac{7}{3}\,\Omega$$

T.E



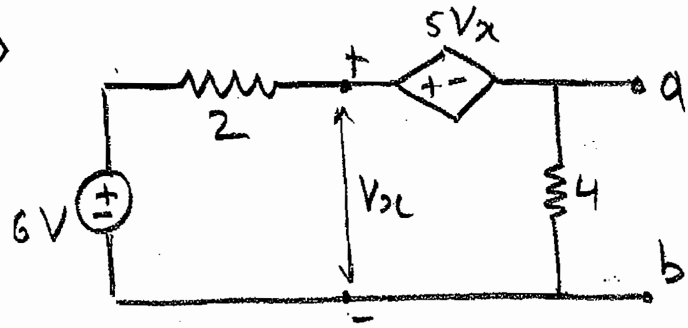

$$i = \frac{10}{3 + \frac{7}{3}} = \frac{15}{8}\ A$$

N.E

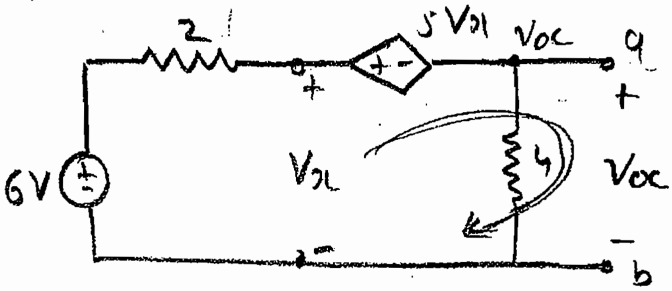



$$i = \frac{30}{7}\left[\frac{7/3}{3 + 7/3}\right] = \frac{15}{8}\ A$$

2)




Find Thevenin & Nortons equiv. b/w a & b.




Nodal

$$\frac{V_{oc} - 6 + 5V_x}{2} + \frac{V_{oc}}{4} = 0$$

$$2V_{oc} - 12 + 10V_x + V_{oc} = 0$$

$$\therefore\ 3V_{oc} + 10V_x = 12 \quad \text{—(1)}$$

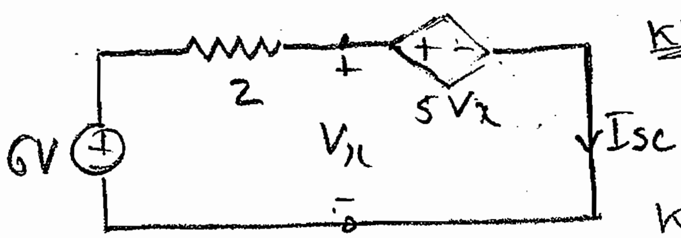
KVL

$$-V_x + 5V_x + V_{oc} = 0$$

$$\therefore\ V_{oc} = -4V_x \quad \text{—(2)}$$

From (1) & (2); $-12V_x + 10V_x = 12$

$$\therefore\ V_x = -6 \Rightarrow \boxed{V_{oc} = 24\,V}$$




KVL

$$6 - 2I_s - 5V_x = 0$$

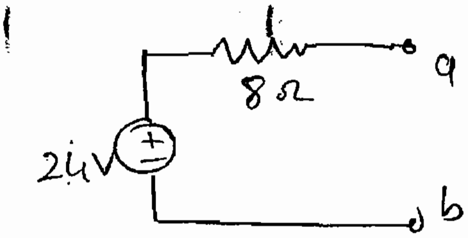
$$2I_s = 6 - 5V_x \quad \text{—(1)}$$

KVL

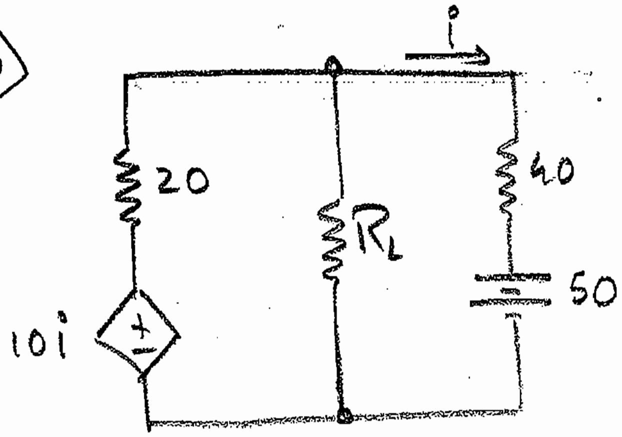
$$-V_x + 5V_x = 0 \Rightarrow \boxed{V_x = 0}$$

From ① & ②; $2I_{sc} = 6 \Rightarrow \boxed{I_{sc} = 3A}$

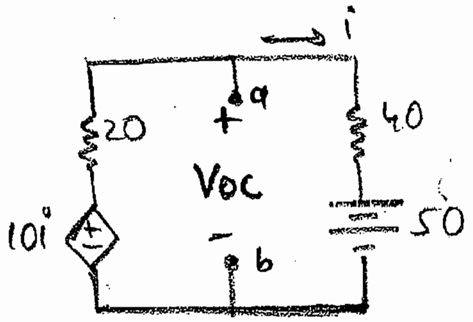
$$R_{TH} = R_N = \frac{V_{os}}{I_{sc}} = \frac{24}{3} = 8\,\Omega$$




3)




Determine Thevenin & Nortons equiv. across the load.

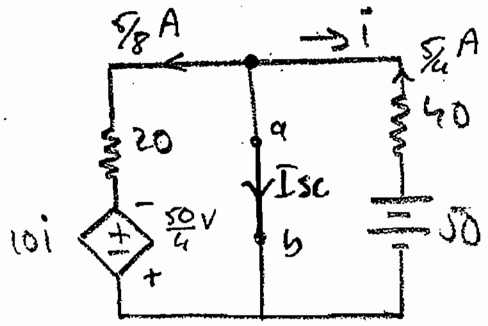



$10i = 60i + 50$

$\therefore i = -1A$

$\therefore V_{oc} = 50 + 40(-1)$

$= 10\,V$

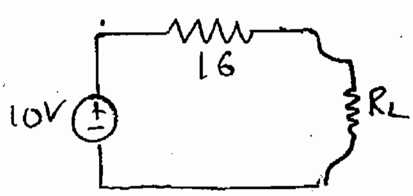



KCL

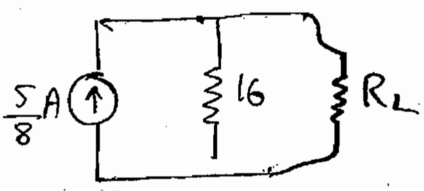
$$\frac{5}{4} = I_{sc} + \frac{5}{8}$$

$$\therefore I_{sc} = \frac{5}{8}\,A$$

$$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{10 \times 8}{5} = 16\,\Omega$$

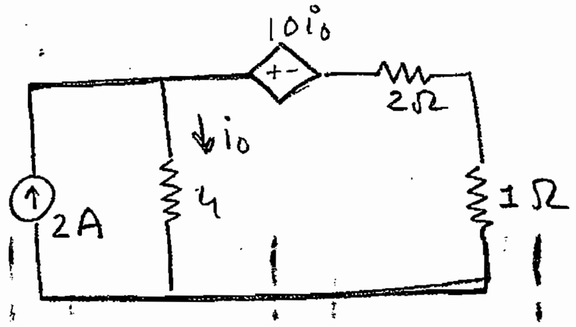



T.E

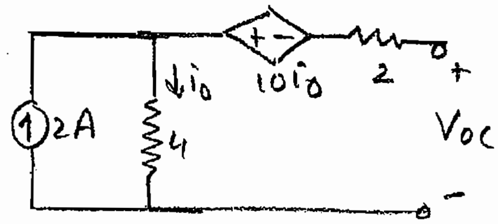



N.E

4)



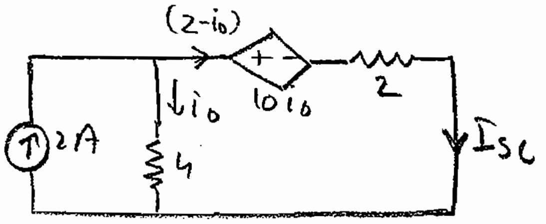

Find current through 1 Ω resis. using Nortons theorem & verify directly




$i_o = 2\ A$

$V_{oc} = 4(2) - 10(2)$

$= -12\ V$



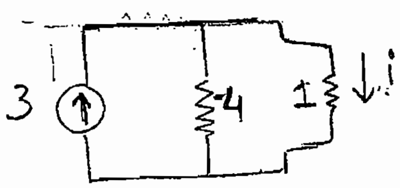

$-10i_o - 2(2-i_o) + 4i_o = 0$

$\therefore -4i_o = 4$

$\therefore i_o = -1\ A$

$I_{sc} = 2 - i_o = 2+1 = 3A$

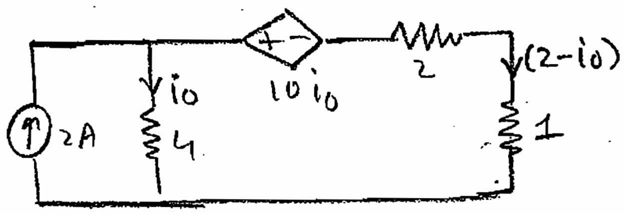
$\therefore R_N = \dfrac{+V_{oc}}{I_{sc}} = -4\ \Omega \Rightarrow \boxed{R_N = -4\Omega}$




$\cancel{i \neq 3 \times \frac{4}{5} \neq \frac{12}{5}\ 1A}$

$\boxed{i = 3 \times \left(\frac{-4}{-4+1}\right) = 4\ A}$

Direct approach :




$-4i_o + 10i_o + 3(2-i_o) = 0$

$\therefore 3i_o = -6$

$i_o = -2$

$I_{1\Omega} = 2 - (-2) = 4\ A$

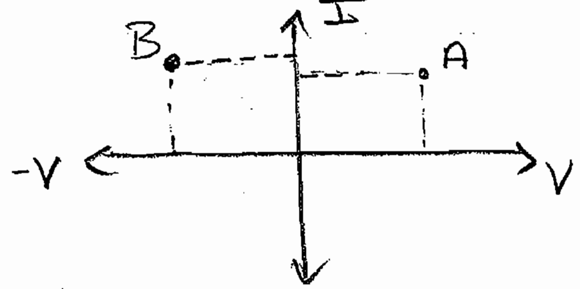
→ In above problem $R_{TH}$ or $R_N$ is -ve

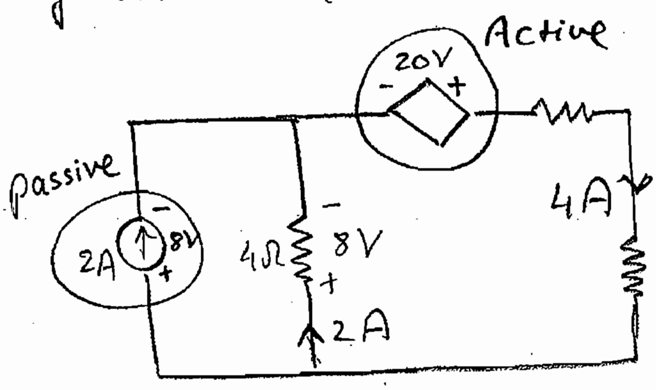
→ -ve resistance is a very powerful way of modelling active elements or active n/ws in circuit analysis.

eg: transistor as an ambifier is active,

thyristor is considered as very high current gain device,
optocouplers, etc.

The V-I charac. of this n/w is in quadrant 2.
Hence it is active.




## Category 3:- Problems with only dependent sources :-

Such n/w cannot function on their own as their is no independent active element to drive it.

In Thevenin's equivalent $V_{TH} = 0$

In Norton's equivalent $I_N = 0$

but they have only resistance.

This resis. can be ~~olive~~ indirectly determined by Ohm's Law by externally exciting them where

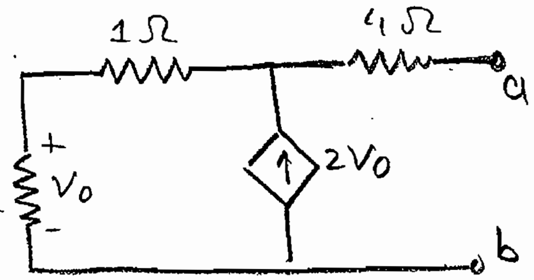
$$R_{TH} = R_N = \frac{1\,V}{I_T} = \frac{V_T}{1\,A}$$

But such models physically represent our n/w & electric devices.

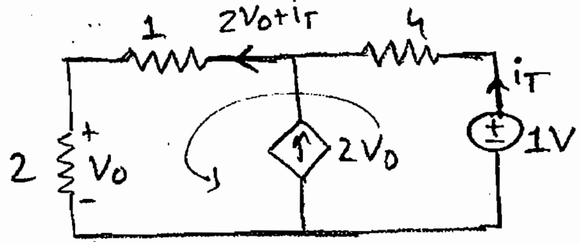
Eg:- ① H-parameter equivalent of BJT in common emitter ampr.

② ~~Piece-wise~~ PSPICE & MATLAB models of electronics devices, etc.

1)




Determine Thevenin & Nortons equiv. b/w a & b.




KVL

$-1 + 4i_T + 3(2V_0 + i_T) = 0$

$7i_T + 6V_0 = 1$ —— ①

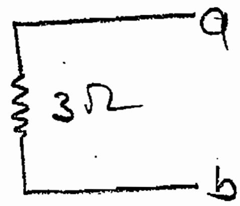
$V_0 = 2(V_0 + i_T)$

$3V_0 + 2i_T = 0$ —— ②

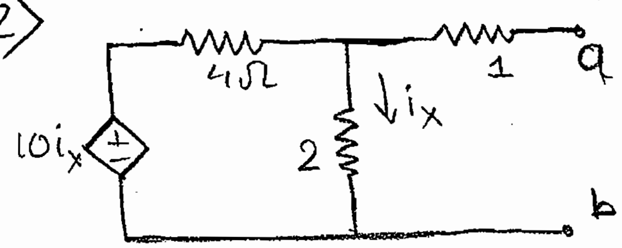
$\therefore \; 7i_T + 6\left(\frac{-2}{3}\right)i_T = 1$

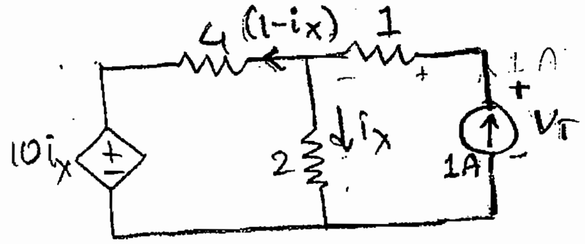
$3i_T = 1 \implies i_T = \frac{1}{3}$

$R_{TH} = R_N = \frac{1}{1/3} = 3\,\Omega$




2)



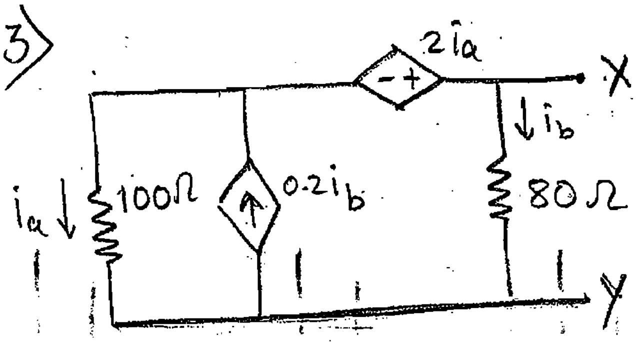

$10i_x + 4 - 4i_x - 2i_x = 0$

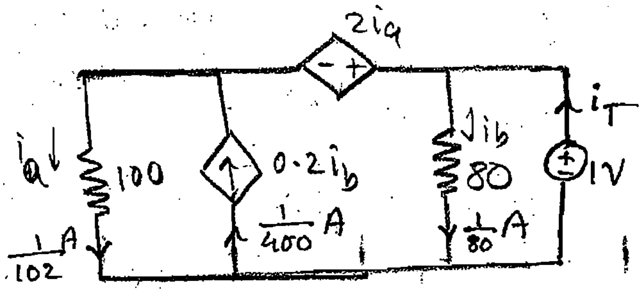
$\therefore \boxed{i_x = -1A}$

$V_T = 1 + 2(-1)$
$= -1\,V$

$R_N = \frac{-1}{1} = -1\,\Omega$

3)




KVL

$$-1 + 2i_a + 100i_a = 0$$

$$\therefore i_a = \frac{1}{102}$$

KCL

$$\frac{1}{102} + \frac{1}{80} = \frac{1}{400} + i_T$$

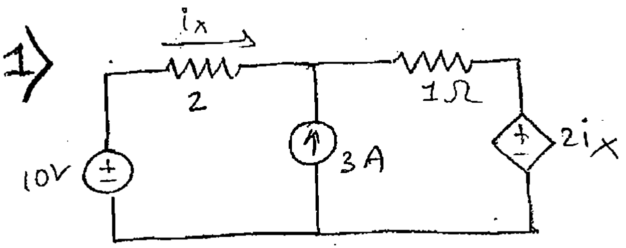
$$i_T = \frac{1}{80} + \frac{1}{102} - \frac{1}{400}$$

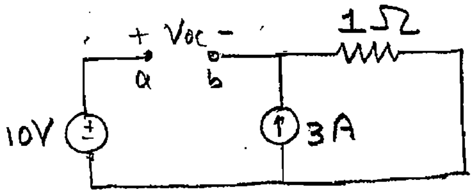
$$= 0.0198$$

$$R_{Th} = R_N = \frac{1}{i_T} \approx 50\,\Omega$$

# Special Models :-

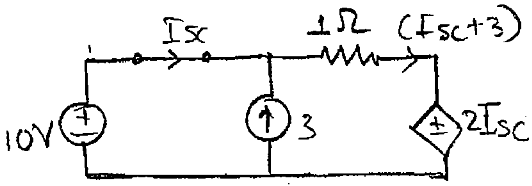
1) Find $i_x$ using Thevenin & Norton's theorem.




$$-10 + V_{oc} + 3 = 0$$

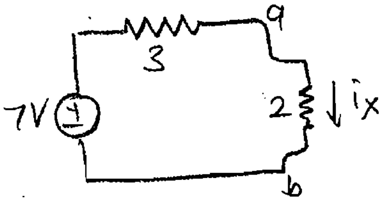
$$V_{oc} = 7\,V$$



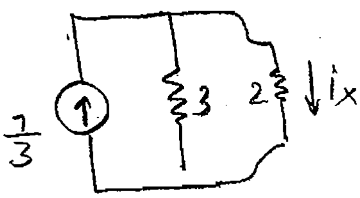

$$-10 + (I_{sc}+3) + 2I_{sc} = 0$$

$$3I_{sc} = 7 \Rightarrow I_{sc} = \frac{7}{3}A$$

$$R_{Th} = R_N = \frac{7}{7} \times 3 = 3\,\Omega$$

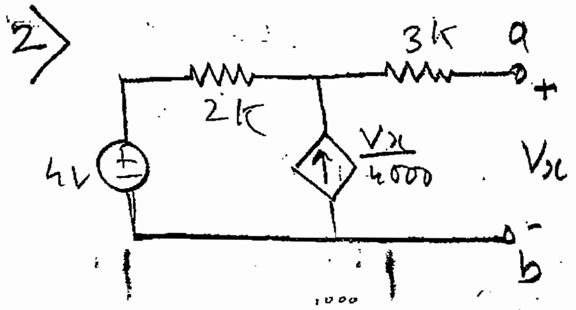



$$i_x = \frac{7}{5}A$$

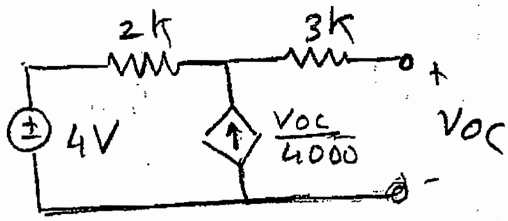



$$i_x = \frac{7}{3}\left(\frac{3}{5}\right) = \frac{7}{5}A$$

2)

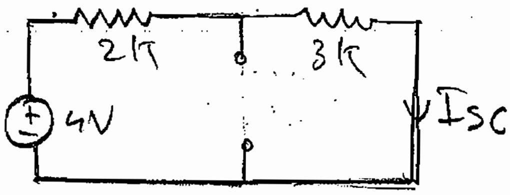



Determine Thevenin equivalent b/w a & b




$$V_{oc} = 2k\left(\frac{V_{oc}}{4000}\right) + 4.$$

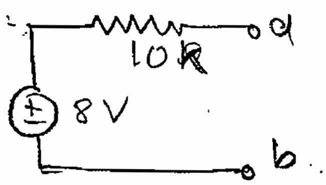
$$2V_{oc} = V_{oc} + 8 \Rightarrow V_{oc} = 8\ V$$




$$4 - 5I_{sc} = 0$$

$$\therefore I_{sc} = \frac{4}{5}\ mA$$

$$\therefore R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{8}{4} \times 5 = 10\,k\Omega$$

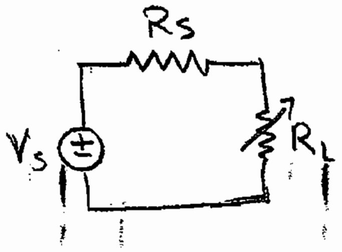



## Theorem 5 :-

## Maximum Power Transfer Theorem :-

In any linear active bilateral n/w consisting of no. of energy sources with their internal resistances, maximum power is transfered to the load when load resistance is equal to its equivalent resistance as seen by the load into the supply circuit.

It is indirectly application of Thevenins. theorem in designing the electrical loads to extract maximum power from source.




$$I_L = \frac{V_S}{R_S + R_L} \qquad P_L = I_L^2 \times R_L$$

$$P_L = \frac{V_S^2}{(R_S+R_L)^2} \cdot R_L$$

$$\frac{dP_L}{dR_L} = 0 \quad \Rightarrow \quad V_S^2 \left[ \frac{(R_S+R_L)^2 \cdot 1 - R_L \cdot 2 \cdot (R_S+R_L)}{(R_S+R_L)^4} \right] = 0$$

$$\therefore (R_S+R_L)^2 = R_L \cdot 2 \cdot (R_S+R_L)$$

$$\therefore \boxed{R_L = R_S}$$

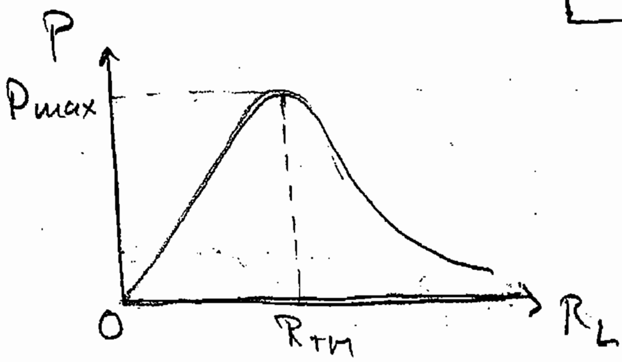
→ $P_{max} = \dfrac{V_S^2}{4R_S}$ W

In general :-


Reduce it to T.E.

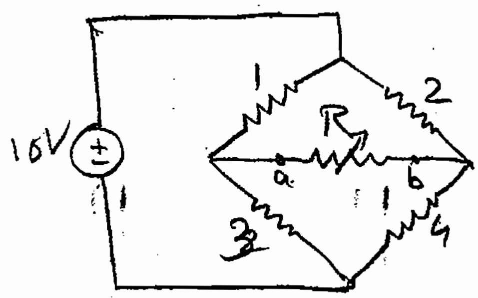
→ $P_{max}$ occurs in load when $R_L = R_{TH}$ &

$$\boxed{P_{max} = \frac{|V_{TH}|^2}{4R_{TH}}} \text{ W}$$

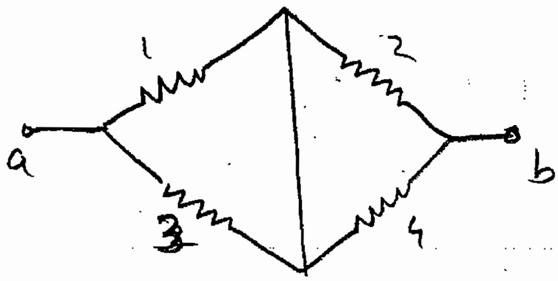



During $P_{max}$ transfer to the load
The o/p effeciency is 50%.

1)




What is the vltg for which maximum power is transferred to load $R_L$.

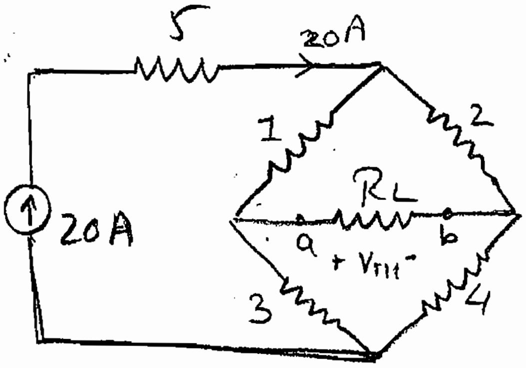



$$R_{TH} = (1 \| 3) + (2 \| 4)$$

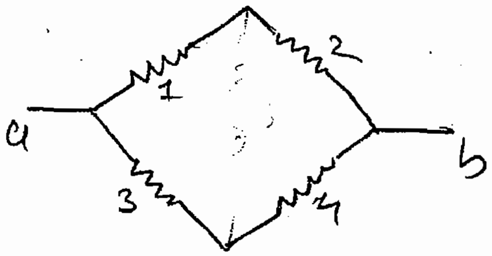
$$= \frac{3}{4} +$$

$$= \frac{25}{12}\ \Omega$$

2)




What is the max. power transfered to the load.

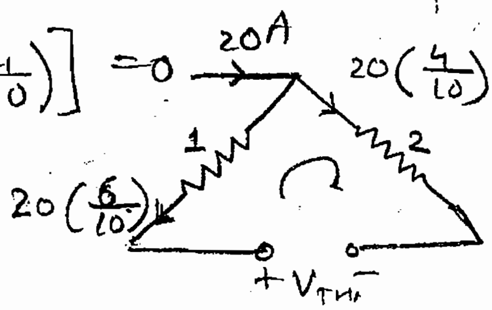



$$R_{TH} = 3 \| 7$$

$$= \frac{21}{10} = 2.1\ \Omega$$

KVL

$$-V_{TH} - 1\left[20\left(\frac{6}{10}\right)\right] + 2\left[20\left(\frac{4}{10}\right)\right] = 0$$

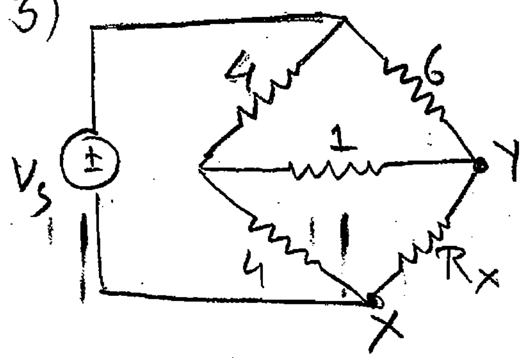



$$V_{TH} = -12 + 16$$

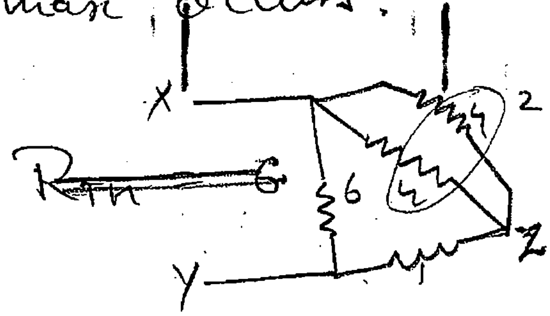
$$V_{TH} = 4\ V$$

$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{16 \times 10}{4 \times 21} = \frac{40}{21}\ W$$

3)




What is the value of $R_x$ for which $P_{max}$ occurs.

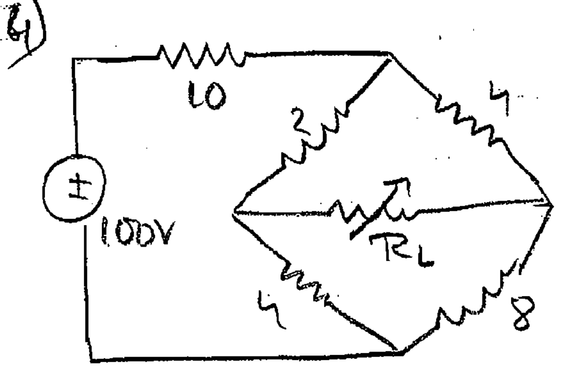



$$R_{TH} = (4 \parallel 4) + (6 \parallel R_x) = 2 + \frac{6R_x}{6+R_x}$$

$\therefore \boxed{R_x = 2\ \Omega}$ for $P_{max}$

$$R_{TH} = (2+1) \parallel 6 = 2\ \Omega$$

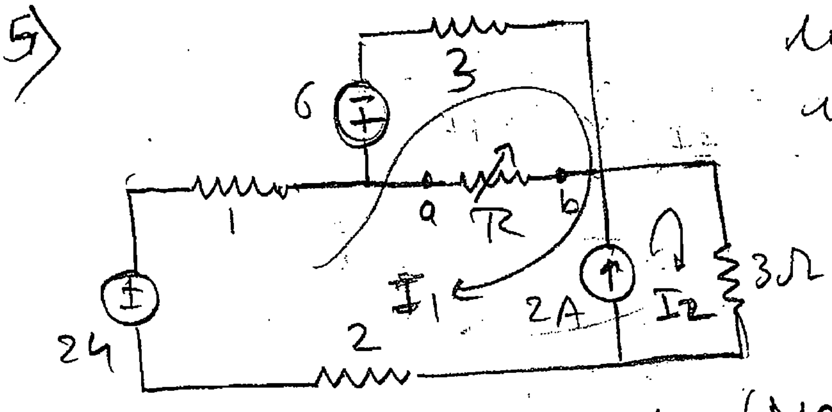
4)




Find the max. Power transfered across $R_L$.

Since the bridge is balanced so current can not flow through $R_L$. Here $V_{TH}$ across $R_L$ will be zero

5)




What is the max. power transfered to the R.

$$R_{TH} = 3 \parallel 6 = 2\ \Omega$$

$V_{TH}$ (Mesh)

$$24 - 6I_1 \mp 6 - 3I_2 = 0$$

$$2I_1 + I_2 = 6 \quad \text{——} \quad (1)$$

$$2I_1 + I_2 = 6 \quad \text{—— (1)}$$

$$-I_1 + I_2 = -2 \quad \text{—— (2)}$$

$$\therefore \; 3I_1 = 4 \Rightarrow I_1 = \frac{4}{3}\ \text{A}$$

**Now**

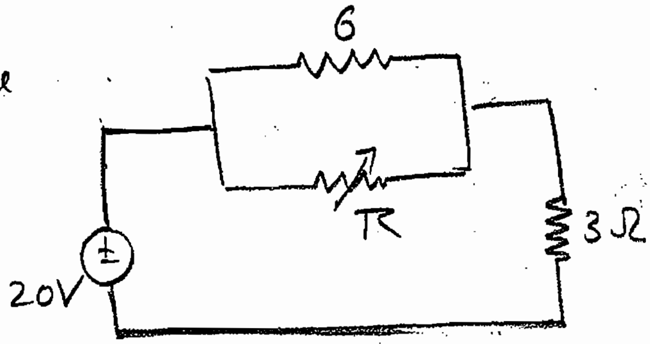
**KVL**

$$-V_{TH} + 6 + 3I_1 = 0$$

$$\therefore V_{TH} = 6 + 3\left(\frac{4}{3}\right) = 6 + 4 = 10\ \text{V}$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{10 \times 10}{4 \times 2} = \frac{25}{2}\ \text{W}$$

8) For what value of R, max power is transferred to 3 Ω resis.



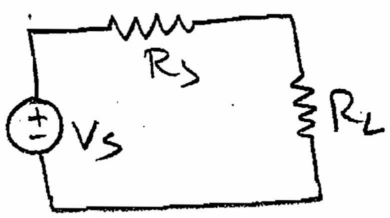

(a) 6 (b) 3 (c) 2 (d) infinity

**Acc. to concept**

$R_{TH}$ across $3\Omega = 3\Omega$

$6 \parallel R = 3$

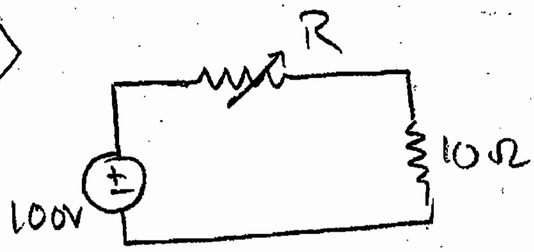
$\Rightarrow \boxed{R = 3\Omega}$




The value of $R_L$ for which $P_{max}$ occurs in $R_L = R_s\ \Omega$

The value of $R_s$ for which $P_{max}$ occurs in $R_L = 0\ \Omega$

9)



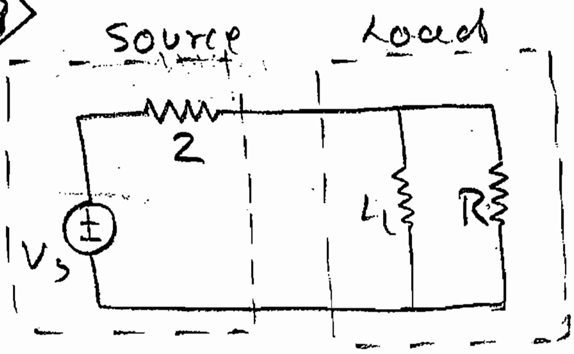

What is the max. power transfered to 10 Ω. If $1\Omega \le R \le 100\Omega$

Here $R = 1\ \Omega$

$$\therefore I_{10\Omega} = \frac{100}{10+1} = \frac{100}{11}\ A$$

$$P_{max} = \left(\frac{100}{11}\right)^2 \times 10 = 826.4\ W$$

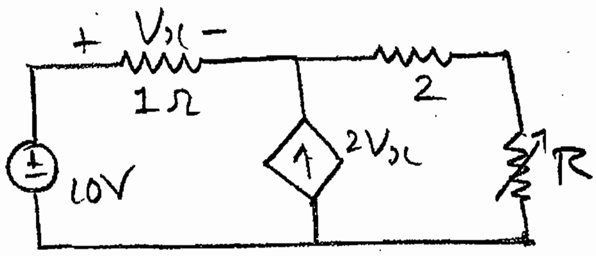
18)



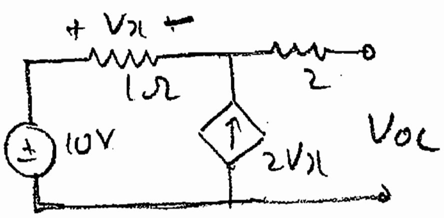

What is the value of R for which max. power is transfered from source to load.

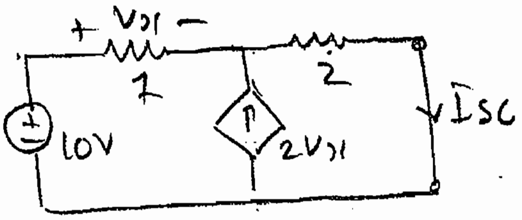
$$\boxed{R = 4\Omega}$$

9)




What is the max. power transfered to R



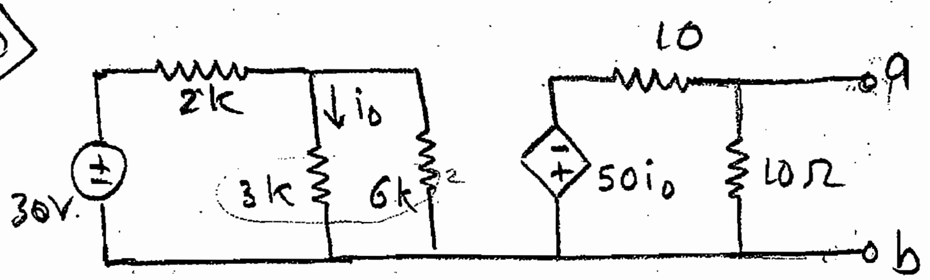

$V_{oc} = 10V$

$I_{sc} = \frac{30}{7}\ A$

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{7}{3}\ \Omega$$

$$P_{max} = \frac{10 \times 10}{4 \times \frac{7}{3}} = \frac{75}{7}\ W$$

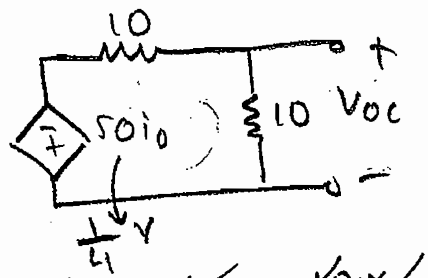
10)

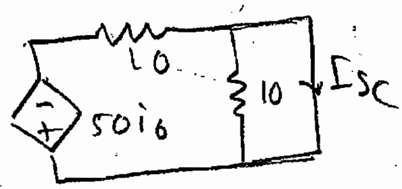



What resis. connected b/w a & b will draw maxi. power from n/w. & also find this max. power.

$$I = \frac{30}{4k} A \Rightarrow i_0 = \frac{30}{24k}\left(\frac{6k}{9k}\right) = 5\,mA$$

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$




KVL

$50 i_0 - 20 I = 0$

$\therefore 20 I = 50 \times 5$

$\therefore I = \frac{25}{2}\,mA$

$\therefore \cancel{V_{oc} = -10 \times \frac{25}{2}}$

$V_{oc} = -\frac{1}{4}\left[\frac{10}{20}\right]$

$= -\frac{1}{8}\,V.$

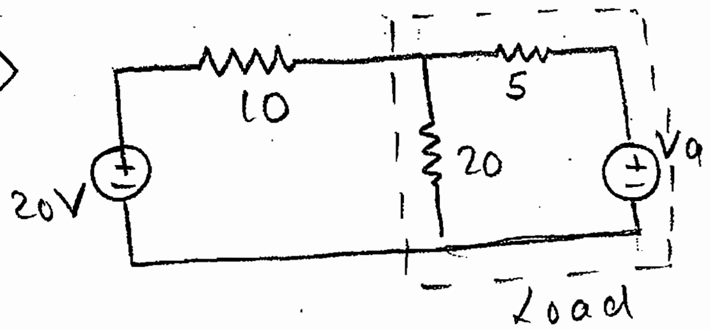
$I_{sc} = \frac{-1/4}{10}$

$= \frac{-1}{40}\,A$

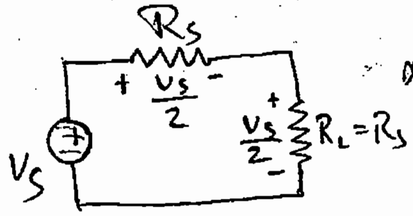
$$R_{TH} = \frac{-1/8}{-1/40} = 5\,\Omega$$

$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{(-1/8)^2}{4 \times 5} = 0.781\,W$$

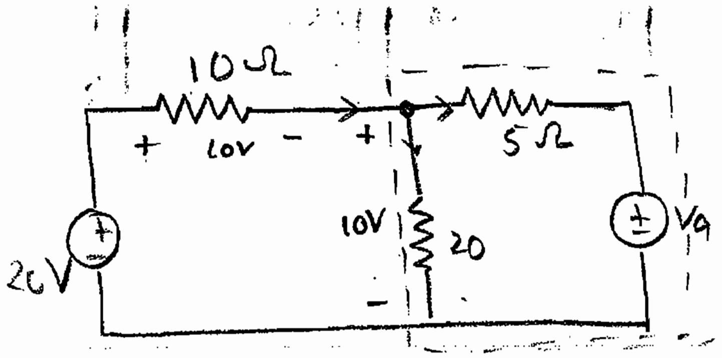
11)




Circuit show a typical battery charging circuit. What is value of Va for which Pmax is transfered to load.




During $P_{max}$ transfer to load vltg across load is 50% of supply ($V_{Th}$)



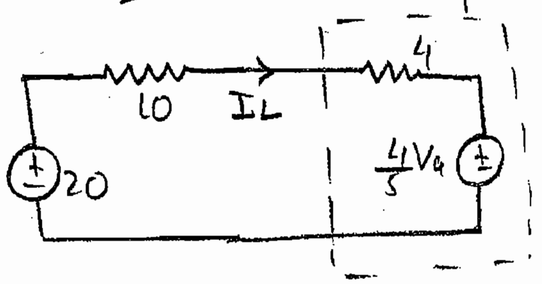

KCL

$$\frac{20-10}{10} = \frac{10}{20} + \frac{(10-V_a)}{5}$$

$$1 = \frac{1}{2} + \frac{10-V_a}{5}$$

$$\therefore V_a = 10 - \frac{5}{2} = 7.5\ V$$

Exact Method



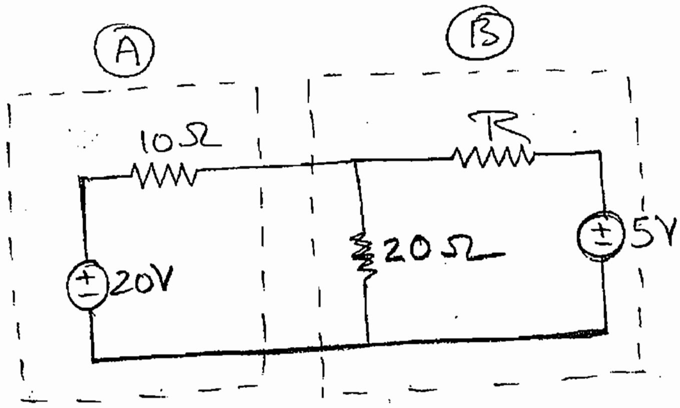

→ T.E. of load ckt as seen by source

$$I_L = \frac{(20 - \frac{4}{5}V_a)}{14} = \frac{100 - 4V_a}{14 \times 5}$$

$$= \frac{50 - 2V_a}{35}$$

$$\boxed{P_L = (I_L)^2\, 4 + \left(\frac{4}{5}V_a\right) \cdot I_L}$$

12)




For what value of R, max. power is transfered from ckt. A to ckt B

Shortcut: $\frac{20-10}{10} = \frac{10}{20} + \frac{10-5}{R} \Rightarrow R = 10\,\Omega$

⇒ Vltg is divided equally across source and load.

13)

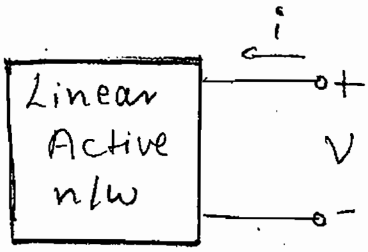



Fig (a)

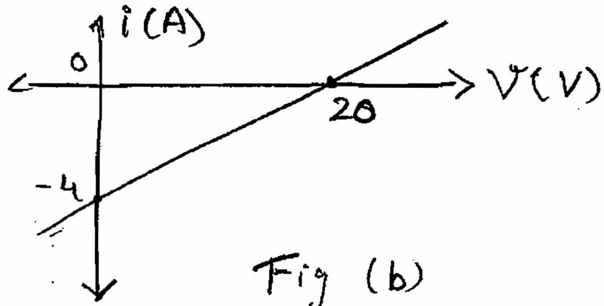
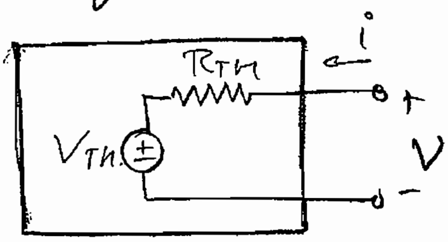



Fig (b)

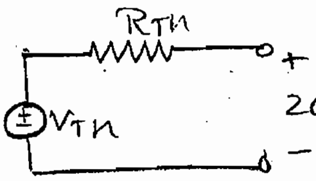
The static VI characteristics of n/w shown in fig (a) is plotted in fig (b). What is the max. power that can be drawn from the n/w.

→ 4th quadrant ⇒ active elements are present in n/w.




1st operating point

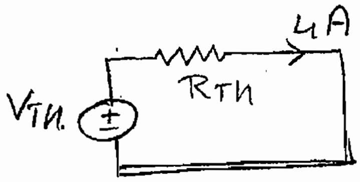
$I = 0$ , $V = 20V$




$\Rightarrow V_{TH} = 20V$

2nd operating point

$I = -4A$ , $V = 0$



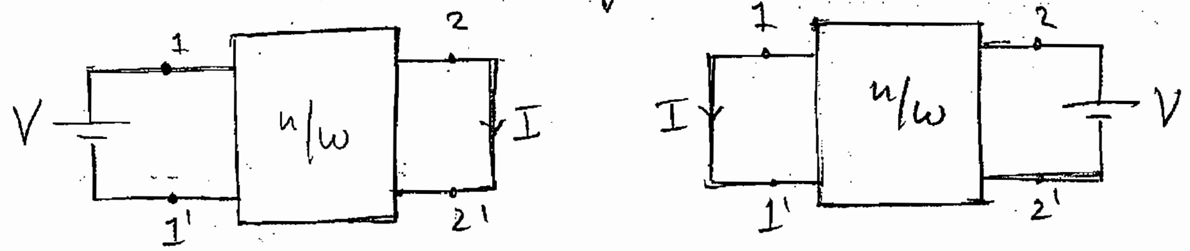

$V_{TH} = 4R_{TH} \Rightarrow R_{TH} = 5$

$$\therefore P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{20 \times 20}{4 \times 5} = 20W$$

Theorem 6 :-

# Reciprocity Theorem :-

"In any linear passive bilateral n/w excited with only a single source, the ratio of response to excitation remains const. even if the positions of source & load are interchanged.




$$\boxed{\frac{I}{V} = \text{const.}} \xrightarrow[\text{Homogenity principle}]{\text{By applying}} \boxed{\frac{I_1}{V_1} = \frac{I_2}{V_2}}$$

NOTE :-

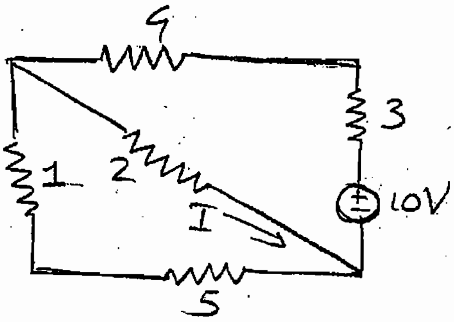
This theorem is valid for n/w excited with single source only.

This theorem can not be applied for n/ws with dependent source. Since dependent sources can make n/w active.

✓ While writing the Reciprocity n/w of given n/w, ideal independent voltage sources are connected in series to the target branch and ideal independent current sources are connected in parallel to target branch.

Eg:- Communication lines
Electrical power Tx n/w.

1) Verify Reciprocity theorem for a ckt. shown below in finding current I.




Step 1

Find I

$$I = \left[\frac{10}{7+6\|2}\right]\left(\frac{6}{2+6}\right)$$

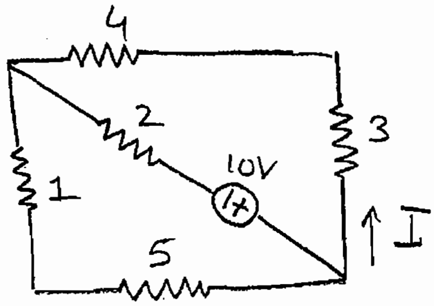
Total current $\left(\frac{V}{R_{eq}}\right)$

$$= \frac{10}{7+\frac{12}{8}} \times \frac{6}{8}$$

$$= \frac{15}{17}\ \text{A}$$

Step 2

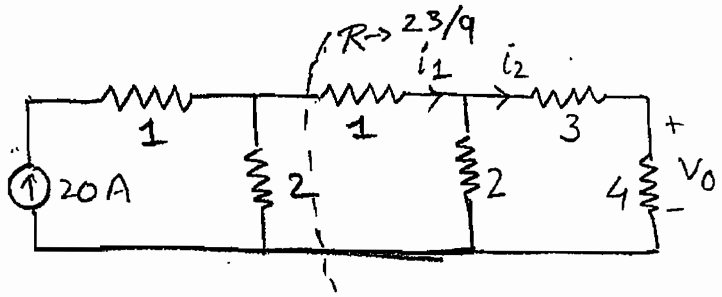
Reciprocal n/w




$$I = \left[\frac{10}{2+7\|6}\right]\left(\frac{6}{6+7}\right)$$

$$= \frac{10}{2+\frac{42}{13}}\left(\frac{6}{13}\right) = \frac{15}{17}\ \text{A}$$

2)




Verify R.T & find Vo

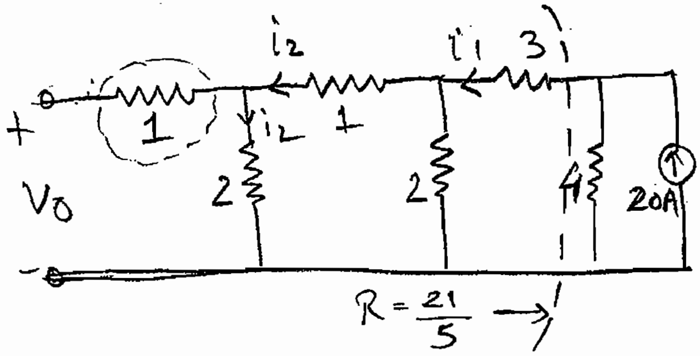
$$(7\|2)+1 = \frac{23}{9}\ ; \quad i_1 = 20\left[\frac{2}{2+23/9}\right]$$

$$i_2 = \left[20 \times \frac{2}{2+23/9}\right] \times \frac{2}{9}$$

$$V_o = 4\, i_2$$

$$= 4\left[20\left(\frac{2}{2+23/9}\right) \times \frac{2}{9}\right]$$

$$= \frac{320}{41}\ \text{V}$$




reciprocal n/w

$$3+[2\|3] = 3+\frac{6}{5} = \frac{21}{5}$$

$$i_1 = 20\left(\frac{4}{21/5+4}\right)\ ; \quad i_2 = 20\left[\frac{4}{4+21/5}\right]\left[\frac{2}{2+3}\right]$$

$$V_0 = i_2 R = 20\left[\frac{4}{4+21/8}\right]\left[\frac{2}{5}\right] \times 2$$

$$= \frac{20 \times 2 \times 4 \times 2}{\frac{41}{5} \times 5} = \frac{320}{41}\ V$$

3) Use the data given in fig A to find current I in fig. B.

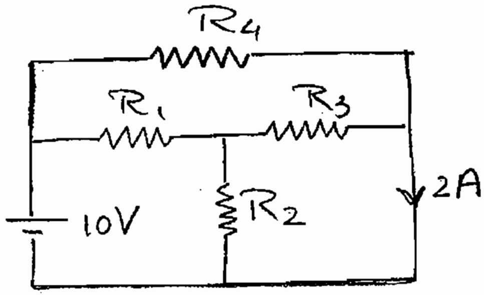



fig (A)

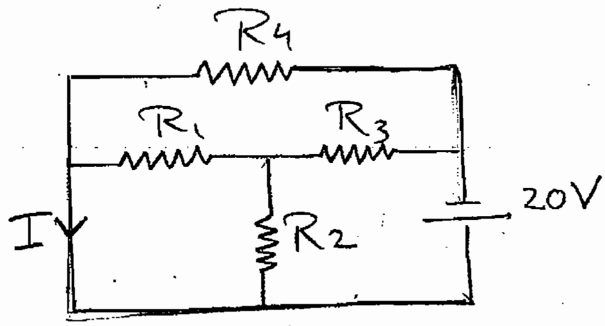
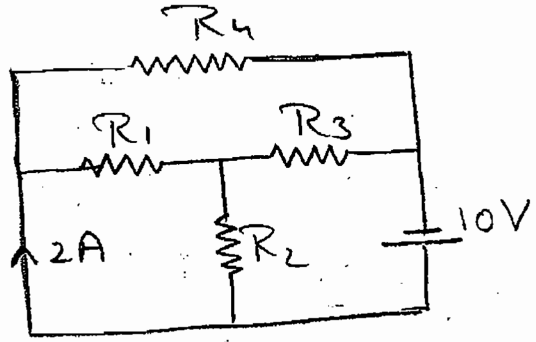



fig (B)

⇓ Reciprocal n/w




↓ Applying homogenity principle

$\therefore I = -4A$

4) Use the data given in fig (A) to find current i in fig. (B)

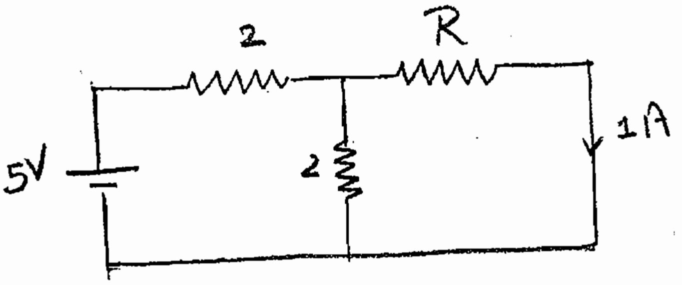



fig (A)

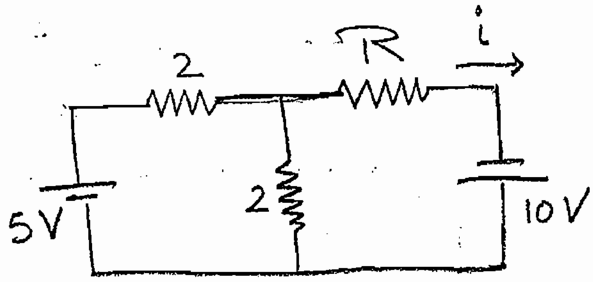
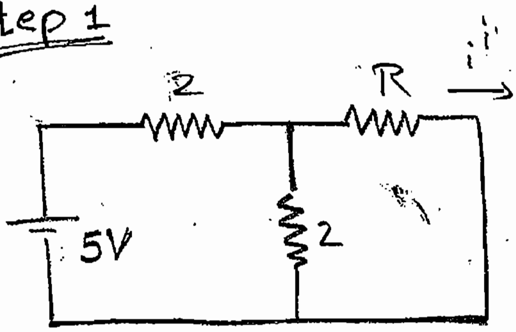



fig (B)

To solve Fig. B use superposition theorem.

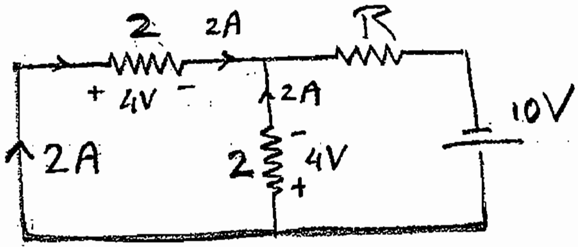
Step 1




(only 5 V)

$\Rightarrow i' = \cancel{1}A$
$= 1\,A$ (from fig A)

Step 2




(only 10 V)

2A → from reciprocity theorem and homogenity principle applied in fig A.

$\therefore i'' = 4\,A$

$\therefore i = i' + i'' = 1 + 4 = 5\,A$

5) Use the data in fig A to find current 'i' in fig B.

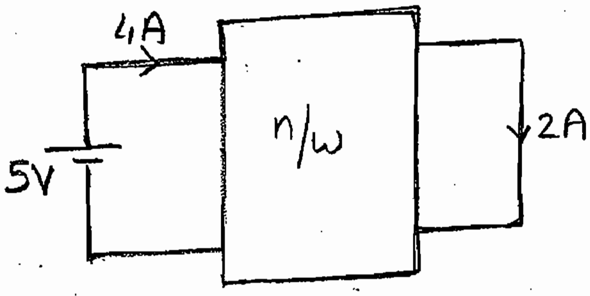



fig A.

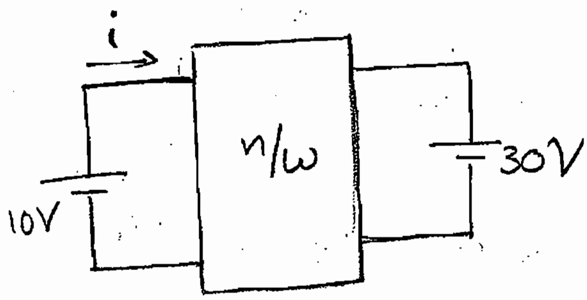
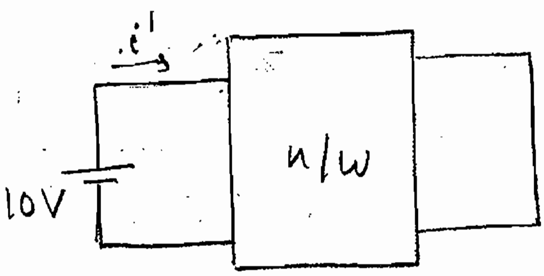



fig. B.

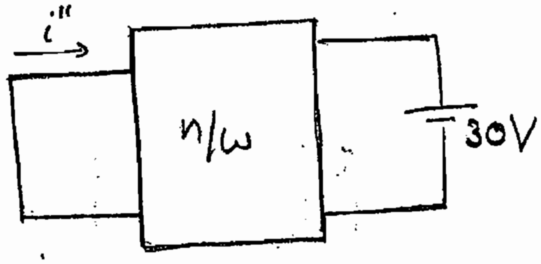
Apply superposition

Step 1




From fig. A
By homogenity principle
$i' = 8\,A$

Step 2:-




From reciprocal n/w of fig. A and applying Homogenity principle.

$i'' = -12A$

$i = i' + i'' = 8 - 12 = -4\,A$

## Theorem 7 :-

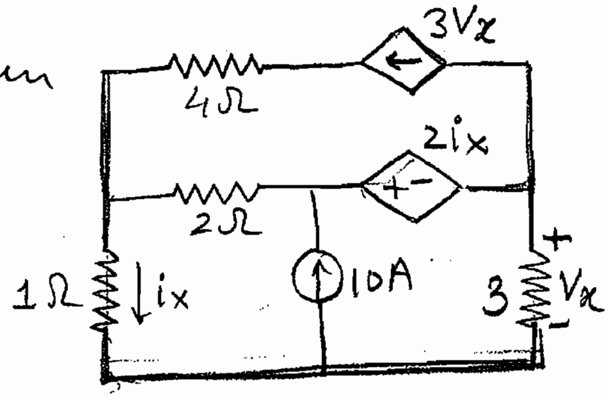
## Tellegen's Theorem :-

This theorem is verification of law of conservation of energy. However in any linear time invariant system, power at an instant is like an energy over a period. So in this theorem we need to verify. $\sum_{k=1}^{n} V_K \cdot I_K = 0$

where, $n \rightarrow$ no. of elements in n/w

i.e. $\sum VI \Big|_{source} = \sum VI \Big|_{sinks}$

1) Verify Tellegen's theorem for the circuit shown below.



## Mesh :

① $i_1 + 2(i_1 - i_3) + 2i_x + 3i_2 = 0$

② $-i_1 + i_2 = 0$

③ $i_3 = -3V_x$

④ $i_x = -i_1$

⑤ $V_x = 3i_2$

## Nodal :

① $\frac{V_1}{1} + \frac{V_1 - V_2}{2} - 3V_x = 0$

② $\frac{V_2 - V_1}{2} + \frac{V_3}{3} - 10 + 3V_x = 0$

③ $V_2 - V_3 = 2i_x$

④ $i_x = \frac{V_1}{1}$

⑤ $V_x = V_3$

$i_1 = -105/11$ A

$i_2 = 5/11$ A

$i_3 = -45/11$ A

$V_x = 15/11$ V

$i_x = 105/11$ A

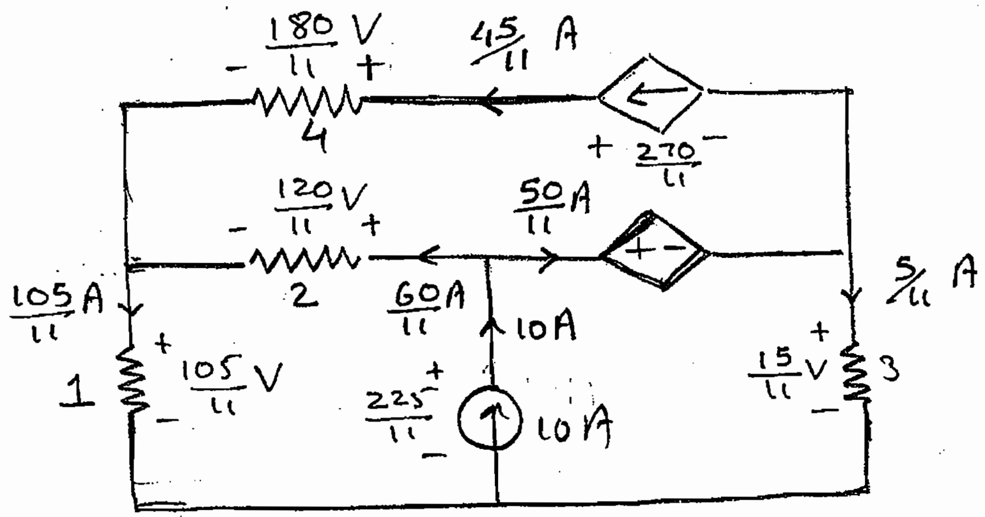
$V_1 = 105/11$ V

$V_2 = 225/11$ V

$V_3 = 15/11$ V

$i_x = 105/11$ A

$V_x = 15/11$ V




$$\sum P_{source} = \left(\frac{225}{11} \times 10\right) + \left(\frac{270}{11} \times \frac{45}{11}\right) = 304.95\ W$$

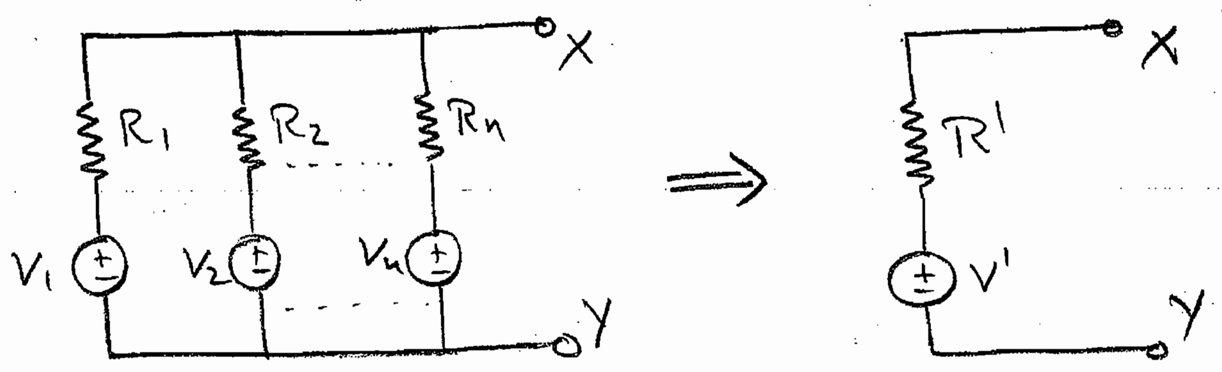
$$\sum P_{sink} = \left(\frac{105}{11} \times \frac{105}{11}\right) + \left(\frac{120}{11} \times \frac{60}{11}\right) + \left(\frac{15}{11} \times \frac{5}{11}\right) + \left(\frac{180}{11} \times \frac{45}{11}\right) + \left(\frac{210}{11} \times \frac{50}{11}\right)$$

$= 304.95$ W

Theorem 8 :-

# Millman's Theorem :-

## [Parallel Generator Theorem]

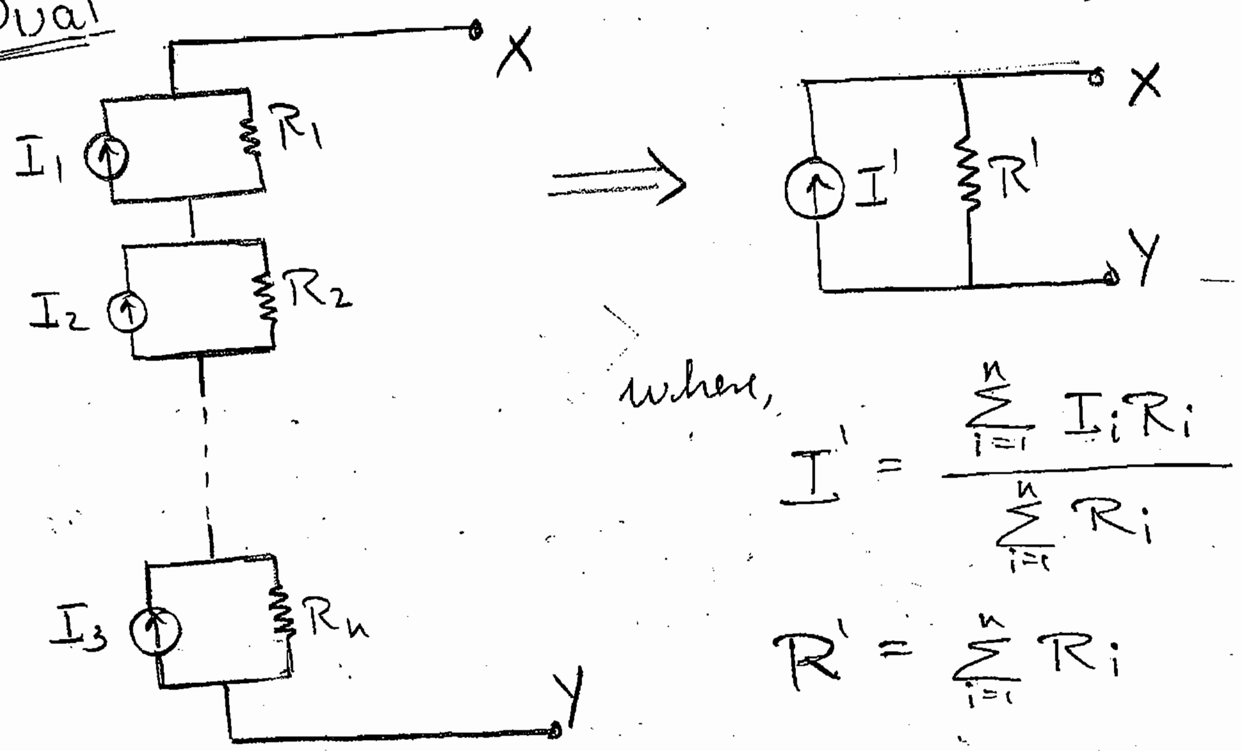



where $V' = \dfrac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}} = \dfrac{\sum_{i=1}^{n} V_i G_i}{\sum_{i=1}^{n} G_i}$

$R' = \dfrac{1}{\sum_{i=1}^{n} \frac{1}{R_i}} = \dfrac{1}{\sum_{i=1}^{n} G_i}$

Dual of Millman's theorem

### Dual




where, $I' = \dfrac{\sum_{i=1}^{n} I_i R_i}{\sum_{i=1}^{n} R_i}$

$R' = \sum_{i=1}^{n} R_i$

# ACE ENGINEERING ACADEMY

ACE
Engineering Academy

**SUBJECT NAME**

**ELECTRIC CIRCUITS** / ELECTRIC CIRCUITS & FIELDS **(E Engineering)**
**NETWORK THEORY** / NETWORK ANALYSIS & TRANSMISSION LINES **(E & T Engineering)**

**IMPORTANCE of SUBJECT**

**Unique Order of covering IES Syllabus**

Gate 3M
1) Fundamentals - Definitions, Notations, Symbols, Units, Formulas, Examples and Applications
2) DC Circuit Analysis - Resistor as Fundamental Component (MESH and NODAL Analysis)
3) DC Network Theorems and Applications

1M
4) Inductors and Capacitors

4M
5) AC Fundamentals - PHASOR, j-Operator, RMS and Average values of Time-Varying Waveforms
6) Concept of POWER in AC, AC Circuit Analysis (MESH and NODAL Analysis)
7) AC Network Theorems and Applications
8) Locus Diagrams, Duals and Duality in Electrical Networks
9) Resonance

10) Magnetic Circuits
11) Network Topology / Graph Theory
12) Transient Circuit Analysis (Time-Domain)
4 M 13) Solution of Network Equations using Laplace Transform
14) Network Functions and Filters Concepts
15) Two-Port Networks
16) Network Synthesis (for IES Only)*
17) State Equations for Networks (for IES Only)*
~~18)~~ Three Phase Circuits (for EE Only)**

**Reference Books**

IES/Gate Rank-1: Engineering Circuit Analysis, by William Hyat and Kemmerly, TMH Publications
Gate Rank-2: Fundamentals of Electric Circuits, by Alexander and Sadiku, TMH Publications
IES Rank-3: Network Analysis, by M E Van Valkenburg, PHI Publications
IES Rank-4: Networks and Systems, by D Roy Choudhury, New Age International Publications
Rank-5: Linear Circuit Analysis, by DeCarlo and Lin, OXFORD University Press

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"If I **HEAR**, I will **FORGET**"
"If I **SEE**, I will **REMEMBER**"
"If I **DO**, I will **UNDERSTAND**"

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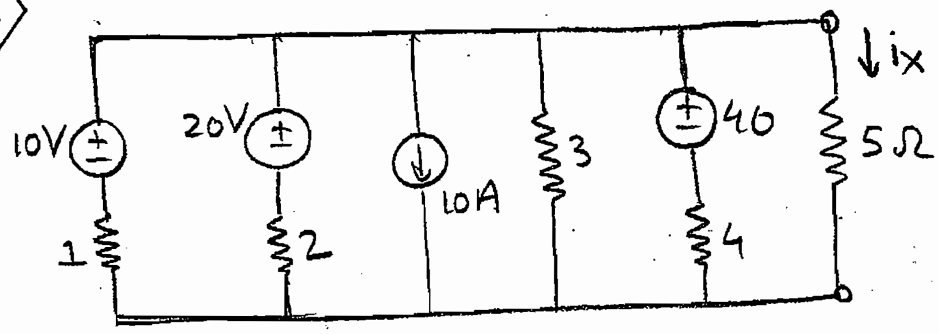
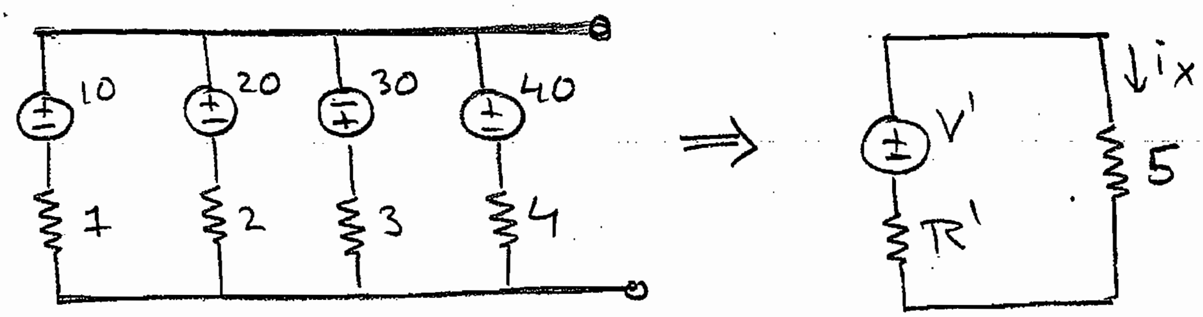
**IES = FUNDAMENTALS + CONFIDENCE**

1)




Find $i_x$




$$V' = \frac{\frac{10}{1} + \frac{20}{2} - \frac{30}{3} + \frac{40}{4}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

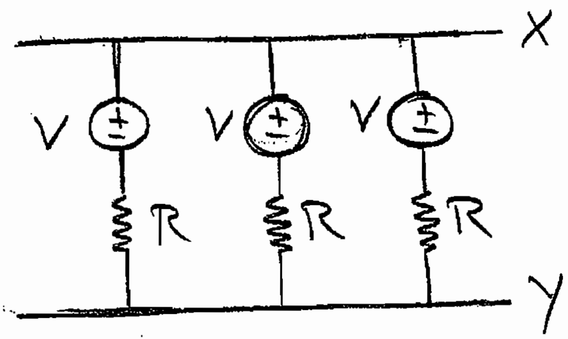
$$= 9.61 \text{ V}$$

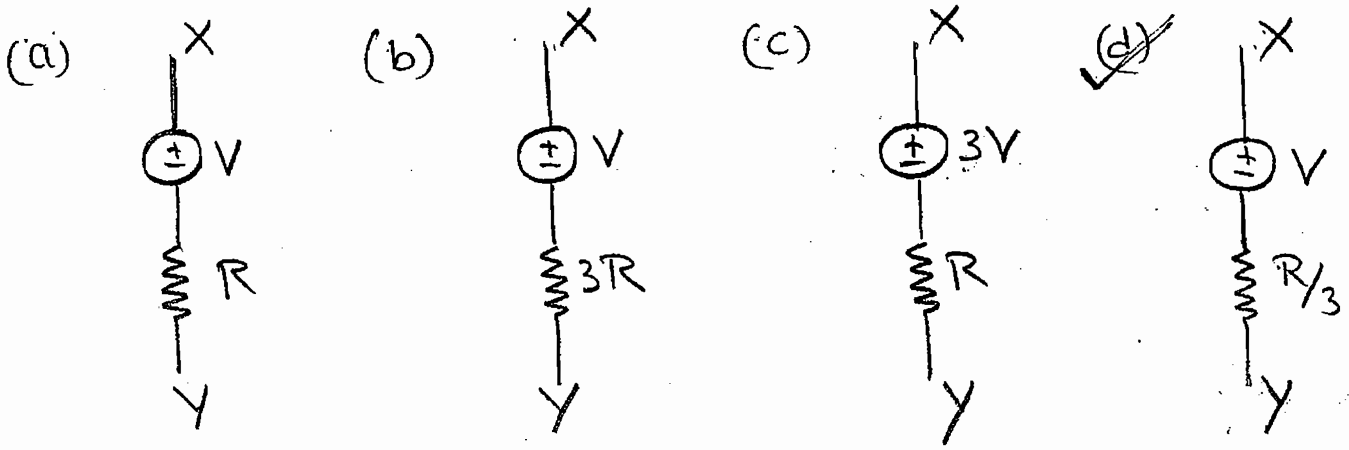
$$R' = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

$$= 0.48\,\Omega$$

$$\therefore\ i_x = \frac{9.61}{5.48} = 1.75 \text{ A}$$

2)



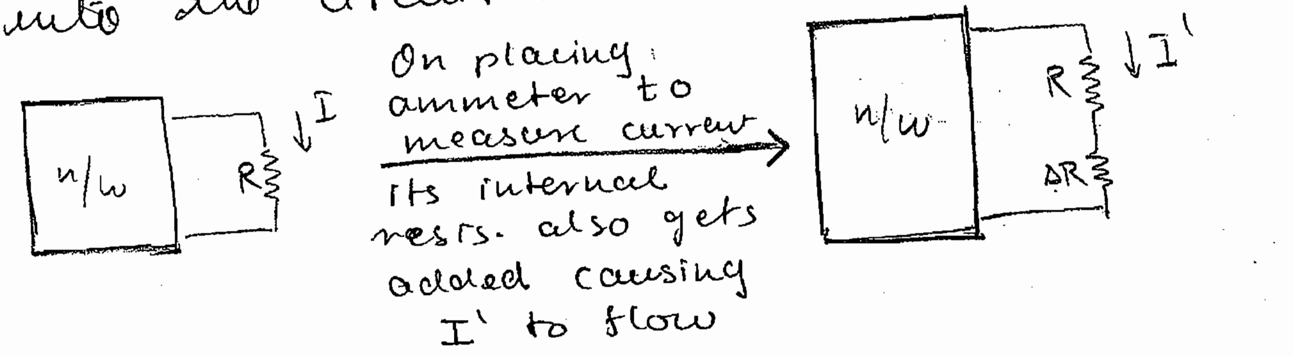



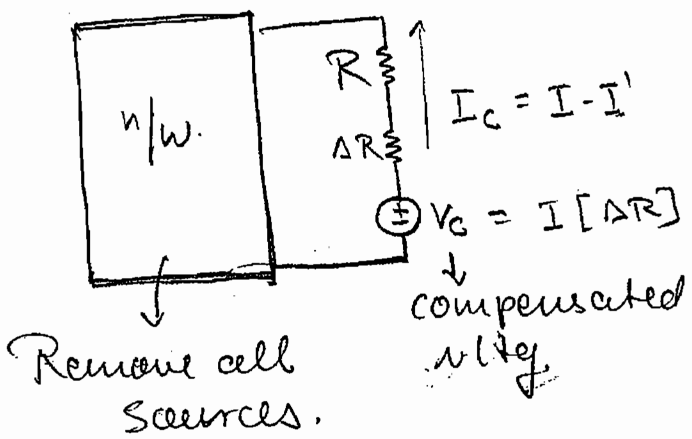
## Theorem 9:—

# Compensation theorem:—

→ This theorem allows us to calculate the correct value of electrical parameters such as voltages & current when they are subjected to parametric variations within the circuit.

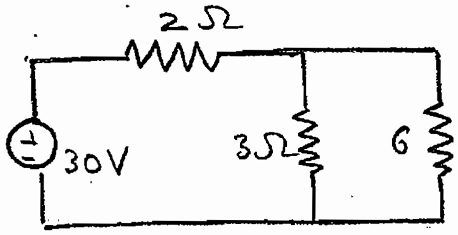
→ This theorem is exclusively used to determine steady state error in measuring instruments as practical meters with their internal resistances will alter the ideal values when they are connected into the circuit.




⟹ Compensated n/w

1) Find the change in current introduced by ammeter with an internal resis. of 0.15Ω while measuring the current in 6Ω resis. branch. Also determine steady state error introduced by the meter.



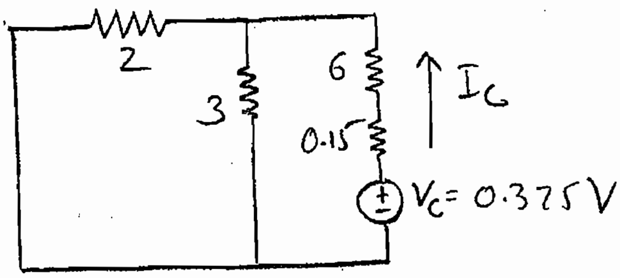

Step 1 :- Find 'I' → theoretically

$$I = \frac{30}{2+2} \times \frac{3}{9} = \frac{5}{2} A$$

Step 2 :- Calculate compensated vltg

$$V_c = I[\Delta R] = \frac{5}{2}(0.15) = 0.375 \text{ V}$$

Step 3 :- Compensated n/w :-




$$I_c = \frac{(0.375)}{6.15 + (2 \| 3)}$$

$$= \frac{0.375}{7.35}$$

$$= 0.051 \text{ A}$$

So by connecting an ammeter with internal resis. of 0.15 Ω the current in that 6Ω branch is reduced by 0.051 A.

Now,

$$\% \text{ Error} = \frac{I - I'}{I} \times 100\% = \frac{I_c}{I} \times 100\%$$

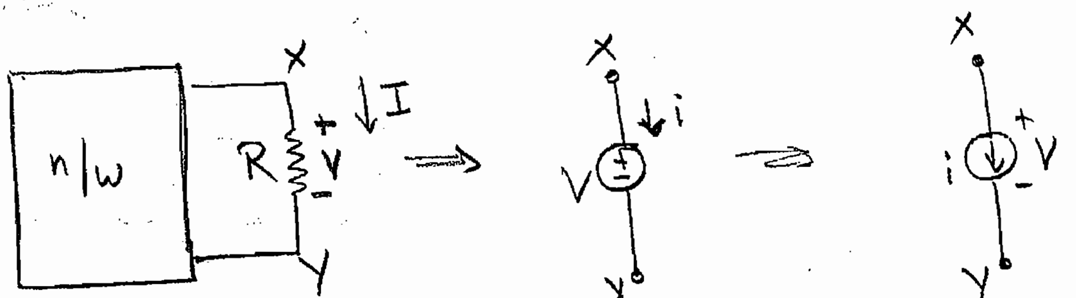
$$= \frac{0.051}{2.5} \times 100\% = \underline{2.05\%}$$

↳ acceptable

## Theorem 10 :-

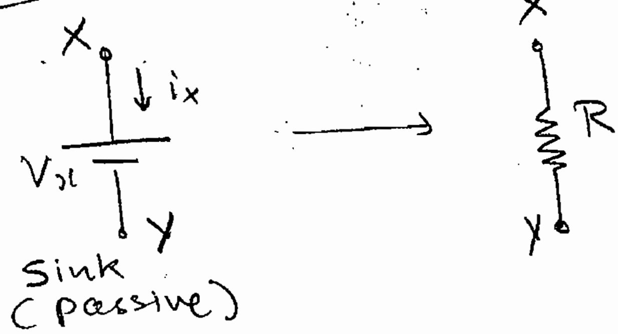
# Substitution theorem :-

In any linear active bilateral n/w consisting of no. of energy sources, passive elements, etc. any passive element can be substituted in terms of its equivalent vltg ~~and~~ or current for further analysis of n/w w/o disturbing the rest of the n/w provided the power absorbed by this passive element & its equivalently substituted source is same.



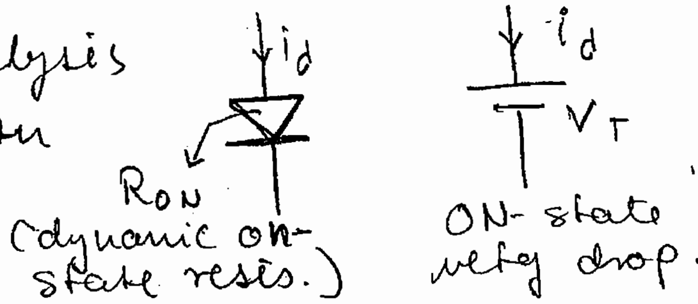

Vice-versa

$$P_{absorbed} = \frac{V^2}{R} = I^2R = V \cdot I$$

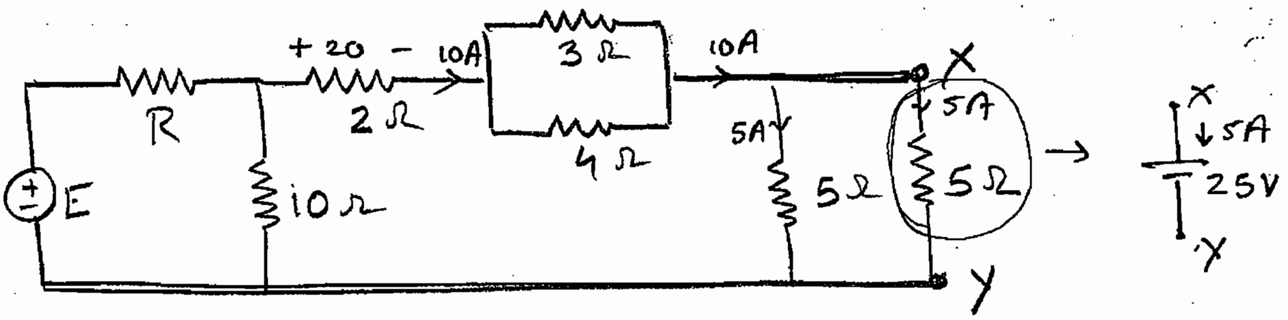



$$R = \frac{V_x}{i_x}\,\Omega$$

We can model a semiconductor device while conducting, as a passive element in terms of its on-state vltg drop for further analysis using substitution theorem.



1)

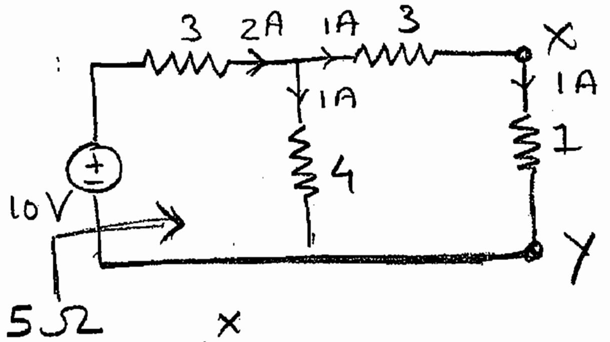


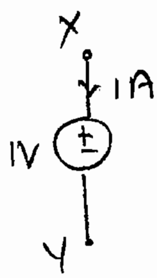

If vltg drop across 2 Ω resis is 20 V, the 5 Ω resis. branch b/w X & Y can be substituted by an equivalent vltg of _____ ?

$P_{absorbed} = (5)^2 \times 5$
$= 125\ W$

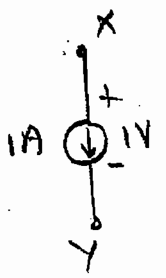
$P_{abs.} = 25 \times 5$
$= 125\ W$

2) Use substitution theorem to substitute 1 Ω branch in 5 diff. ways. (atleast)

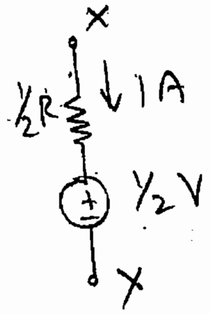



① 

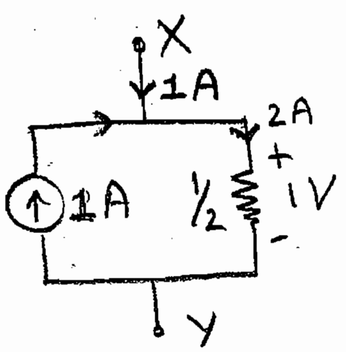

$P_{abs} = 1W$

② 

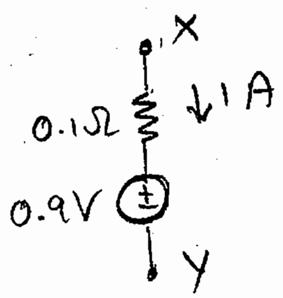

$P_{abs} = 1W$

③ 


$P_{abs} = (1)^2 \times \frac{1}{2} + \frac{1}{2} \times 1$
$= 1W$

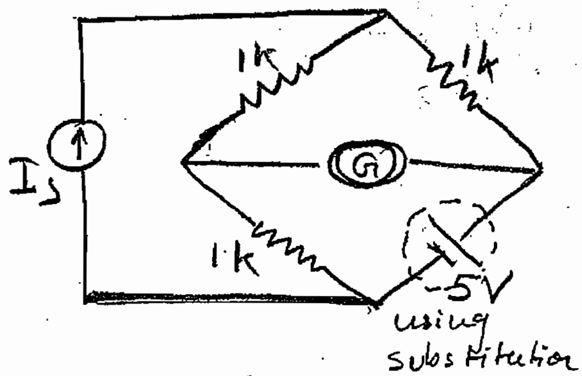
④ 


$P_{abs} = (2)^2 \times \frac{1}{2} = 2W$
$P_{del} = 1 \times 1 = 1W$
$\Big\}\ P_{abs} = 1W$

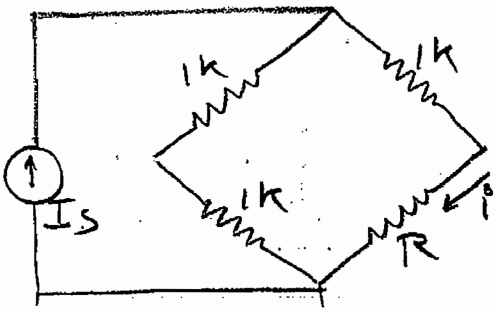
⑤ 


$P_{abs} = (1)^2 (0.1) + 0.9$
$= 1\ W$

3) For the balanced bridge determine the value of $I_s$.




$5 = iR$



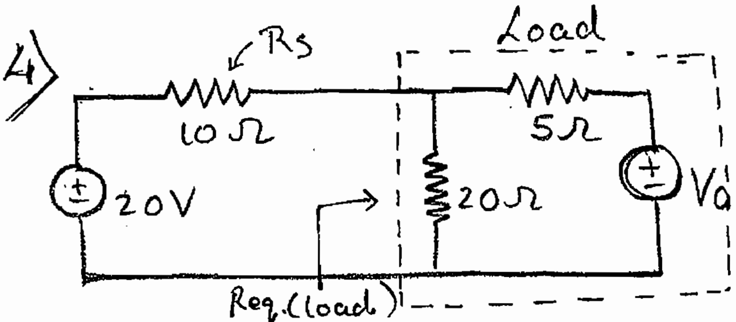

$\because$ Balanced bridge

$\therefore R = 1\,k\Omega$.

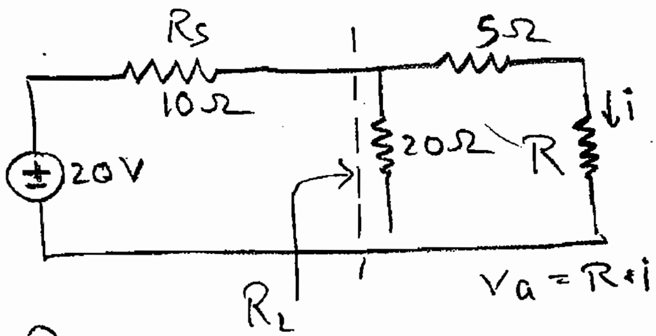
$i = \frac{5}{R} = \frac{5}{1k} = 5\,mA$

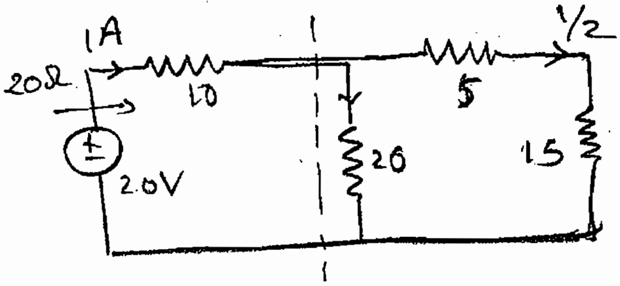
So, $I_s = 2 \times i = 2 \times 5\,mA = 10\,mA$.

4)




Use substitution theo. to find the value of vltg Va for which max. power is transfered to the load.




$P_{max}$

$R_s = R_L$

$\therefore\ 10 = \frac{20(5+R)}{25+R}$

$25 + R = 10 + 2R$

$\therefore\ R = 15\,\Omega$

$R_a = 15\,\Omega$

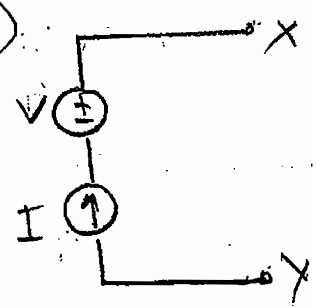
$i = 1/2\,A$

So, $V_a = i \cdot R$

$= \frac{1}{2} \times 15$

$= 7.5\,V$

5) What is applicable?




(a) T.E only ✓(b) N.E only

(c) Both (d) None.

## Thevenin theo.

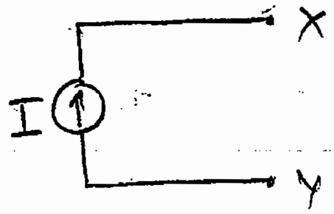
$R_{TH} = R_N = \infty$

$V_{TH}$ → cannot be determined

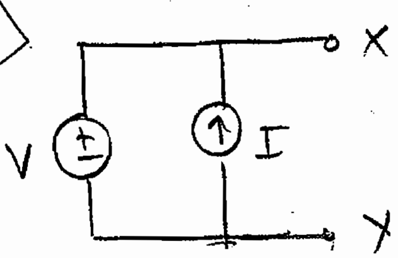
## Norton theo.

$I_N = I$

$R_N = \infty$



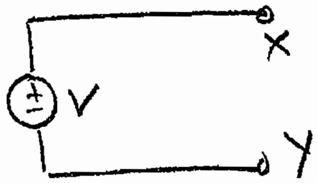

6)




$R_{TH} = R_N = 0\ \Omega$

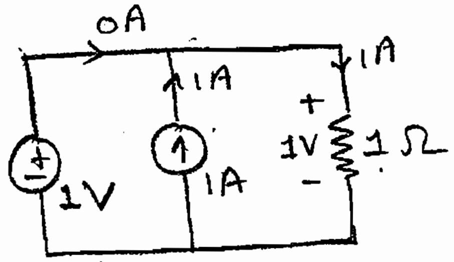
$V_{TH} = V$

$I_N$ → cannot be determined




Here Thevenin ~~theo.~~ ⇒ equival. is possible.

7)

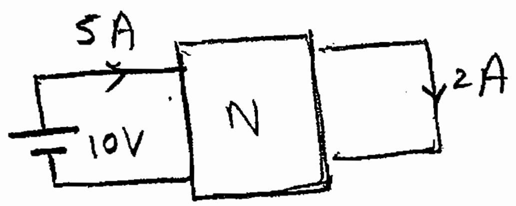



1 V delivers 0 W

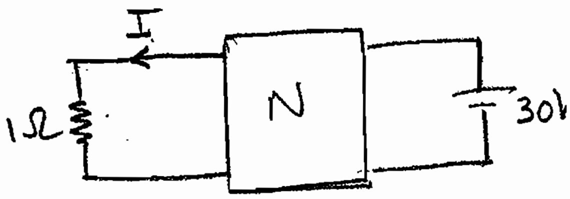
1 A delivers 1 W

1 Ω absorbs 1 W

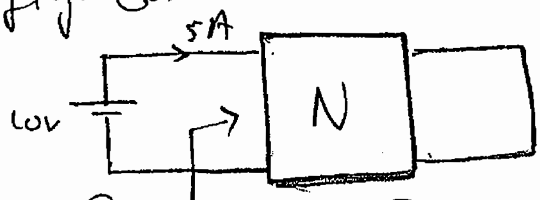
8) The n/w N consists of only the resistor. Use the data given in fig (a) to find current I in fig-(b).




(fig a)

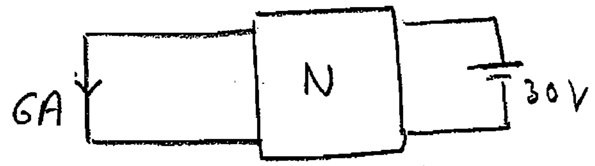



(fig b)

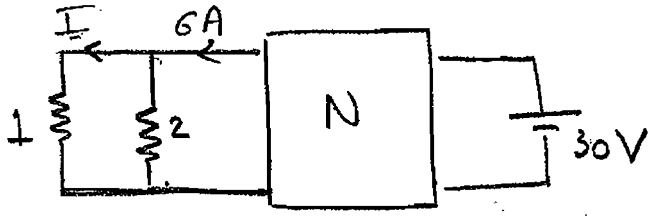


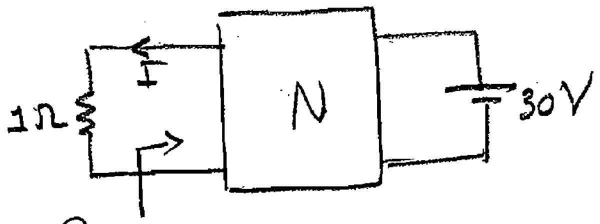

$R_{in} = 10/5 = 2\ \Omega$

Using reciprocity :-




When no resis. is present then ~~this~~ we get 6A current (using homogenity) where $R_{in}$ does not appear




$$I = 6\left(\frac{2}{3}\right) = 4A$$

$R_{in}$ appears.
(with addition of R)

Addition of 1 Ω resistance will allow to appear the input / port resistance.

# PROPERTIES OF INDUCTORS

Sine $V = L\dfrac{di}{dt}$

① For dc excitation, $\dfrac{di}{dt} = 0$,

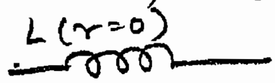
$V_L = 0 \longrightarrow$ inductor is S.C for ideal D.C

② An inductor never allows sudden change in current through it.

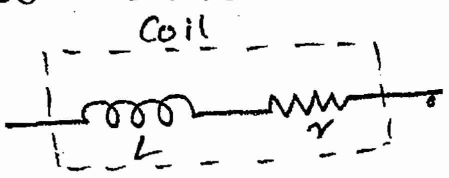
Eg:- This is the principle of operation of choke coil in flourescent lamp.

If inductor really allows sudden change in current through it, we get huge impulse voltages appear across it.

③ An ideal inductor is a coiled wire with zero internal resis., so power dissipated is zero.




④ Practical inductors will have small internal resistance & they are represented as coil shown below which allow some power loses.




⑤ Inductors are available in diff. shapes & sizes & they classified on the basis of the type of core material on which winding is done.

⑥ Inductors are used as filters, current limiting reactors, var ~~bar~~ compensators, etc in communication & power sys.

# PROPERTIES OF CAPACITORS

Since $i = C\dfrac{dV}{dt}$

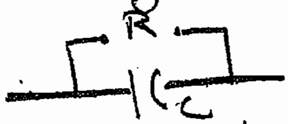
① For DC excitation, $\dfrac{dV}{dt} = 0$, $i_C = 0$

→ Capacitor acts as O.C for ideal D.C (in steady state)

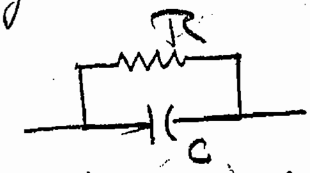
② Capacitor never allows sudden change in vltg across it.

③ Ideal capacitors are considered to have infinite dielectric ~~capaci~~ resistance b/w electrodes so dielectric losses are zero but conduction is through polarization.

$R \to \infty$



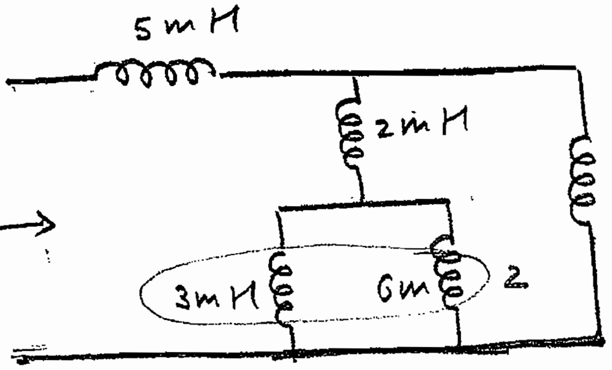

④ Practical capacitors are considered to have very large dielectric resis. b/w electrodes so they undergo losses.




⑤ Capacitors are available in diff shapes & sizes & they are classified on the basis of dielectric material b/w the electrodes.

⑥ They are used as filters, compensators, power factor correcting equipments, signal conditioning & wave shaping, etc in communication & power sys.

1)



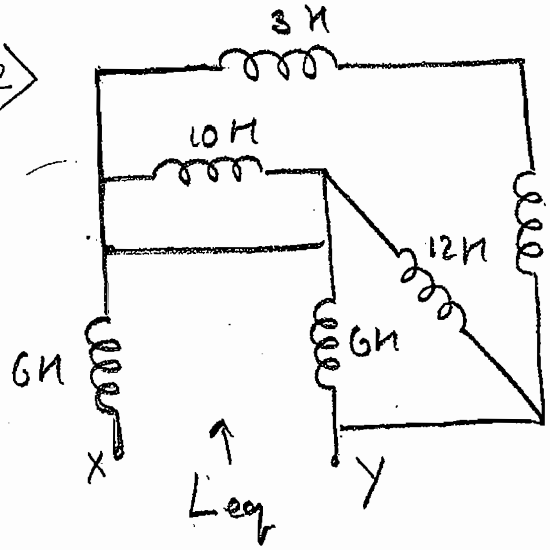

$L_{eq} = 5 + [[(3 \| 6) + 2] \| 4]$

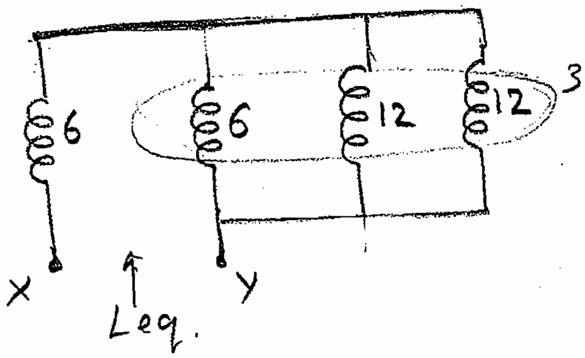
$\therefore L_{eq} = 5 + [(2+2) \| 4]$

$= 5 + 2$

$= 7$ mH.

2)



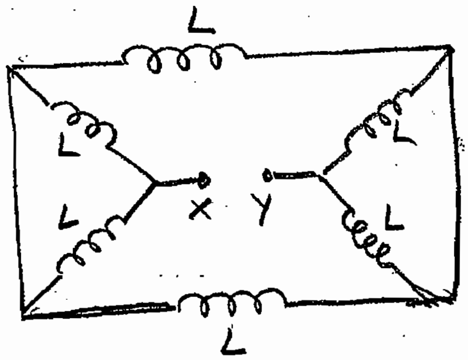




$\therefore L_{eq} = 6 + [6 \| 12 \| 12]$

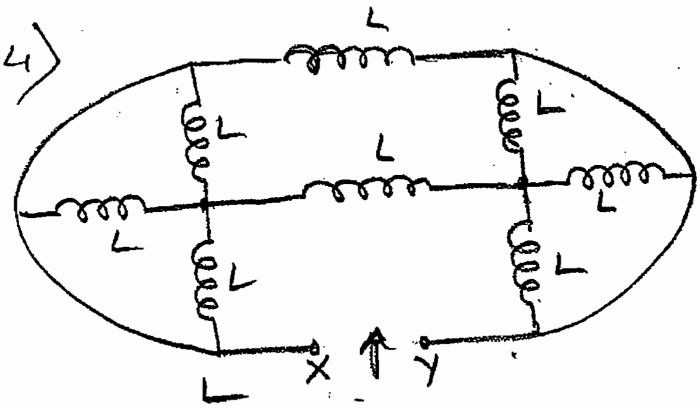
$= 6 + 3$

$= 9$ H

3)

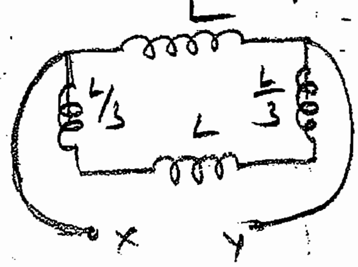



4)




3. $L_{eq} = \frac{L}{2} + \frac{L}{2} + \frac{L}{2}$

$L_{eq} = \frac{3L}{2}$

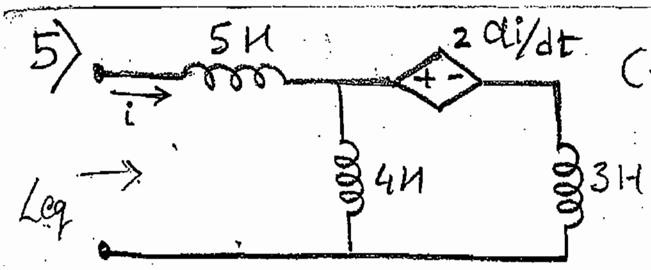


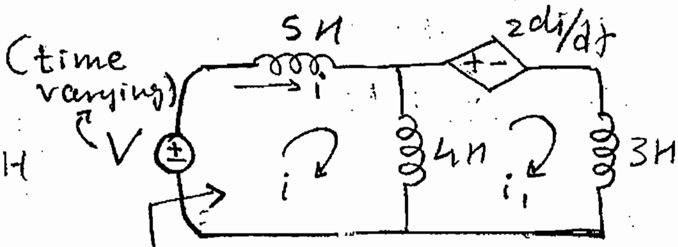

$\frac{5L}{3} \| L$

$= \frac{5L^2}{3 \times \frac{8L}{3}}$

$= \frac{5}{8} L$

5)




$$L_{eq} = \frac{V}{(di/dt)}$$

Mesh

$$-V + 5\frac{di}{dt} + 4\left[\frac{di}{dt} - \frac{di_1}{dt}\right] = 0$$

$$9X - 4Y = V \quad \text{——— (1)} \qquad ; \quad X = \frac{di}{dt}, \quad Y = \frac{di_1}{dt}$$

$$4\left[\frac{di_1}{dt} - \frac{di}{dt}\right] + 2\frac{di}{dt} + 3\frac{di_1}{dt} = 0$$

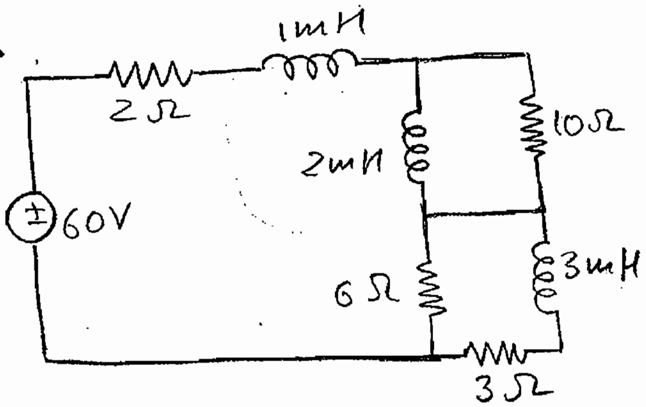
$$-2X + 7Y = 0 \quad \text{——— (2)}$$

$$X\left[9 - 4 \times \frac{2}{7}\right] = V$$

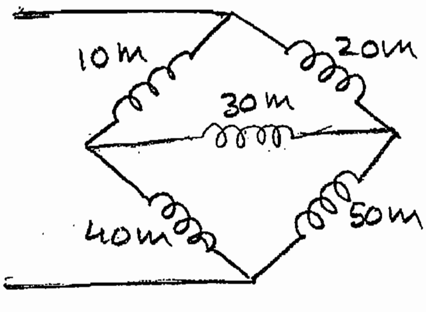
$$\therefore V = \left[\frac{63-8}{7}\right]X = \frac{55}{7}\frac{di}{dt}$$

$$\therefore L_{eq} = \frac{55}{7}\ H$$

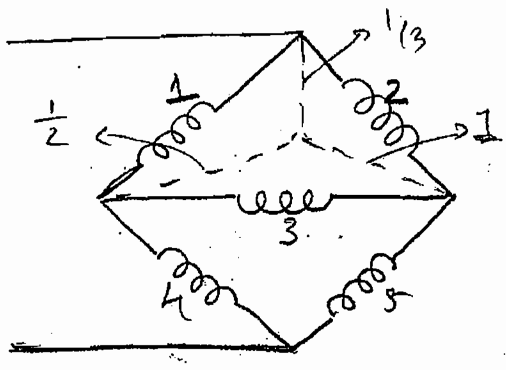
6)

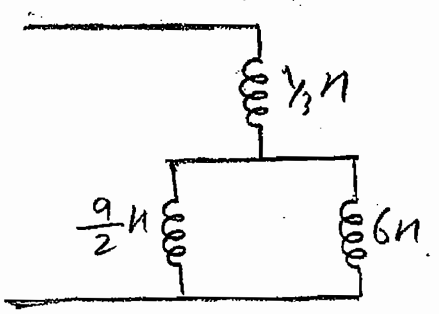



Find the steady state current through the inductors & also energy stored in it.




Scaling 'L' by 10mH $\Rightarrow$



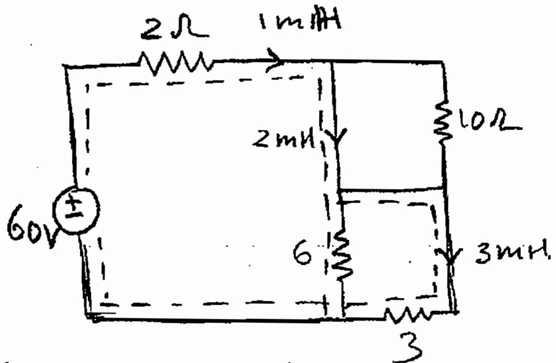


$$L_{eq} = \left[\frac{1}{3} + \left(\frac{9}{2} \parallel 6\right)\right] \times \underline{10\ mH}$$

$\downarrow$ scaled product

$$= 2.9 \times \underline{10\ mH}$$

$$= 29\ mH$$



$$i_{3m} = \frac{6}{9}(15) = 10A$$

$\rightarrow i_{1m} = 15A$

$$E_L = \frac{1}{2}(1m)(15)^2 = 112.5 mJ$$

$\rightarrow i_{2m} = 15A$

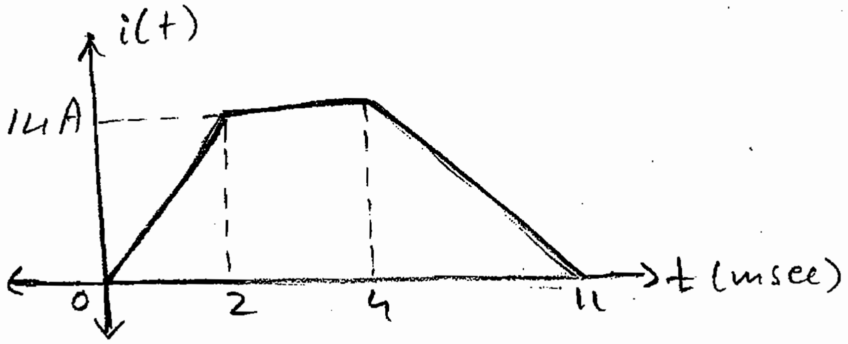
$$E_L = \frac{1}{2}(2m)(15)^2 = 225 mJ$$

$\rightarrow i_{3m} = 10A$

$$E_L = \frac{1}{2}(3m)(10)^2 = 150 mJ$$

These energies are in electromagnetic form (DC flux)

7) If the current flowing through 2H inductor is as shown, plot the vltg across it.



$0 < t < 2m$

$$i(t) = 7t \Rightarrow v = 2\frac{d}{dt}(7t) = +14\ V \text{ pulse}$$

$2m < t < 4m$

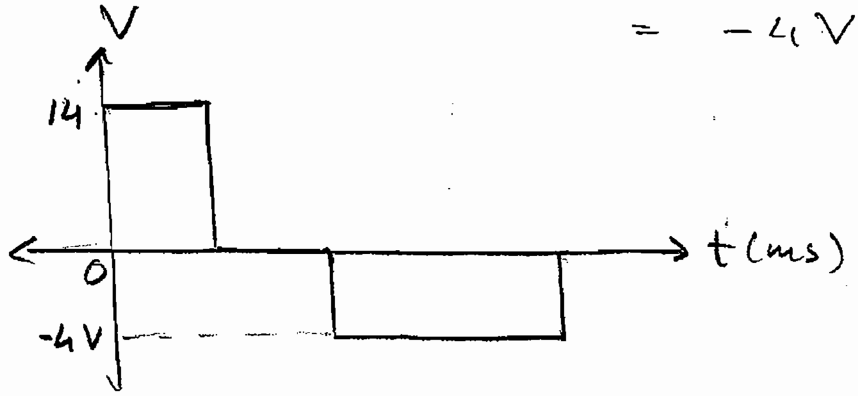
$$i(t) = 14 \Rightarrow v = 2\frac{d}{dt}(14) = 0V$$

(For DC L is shorted)

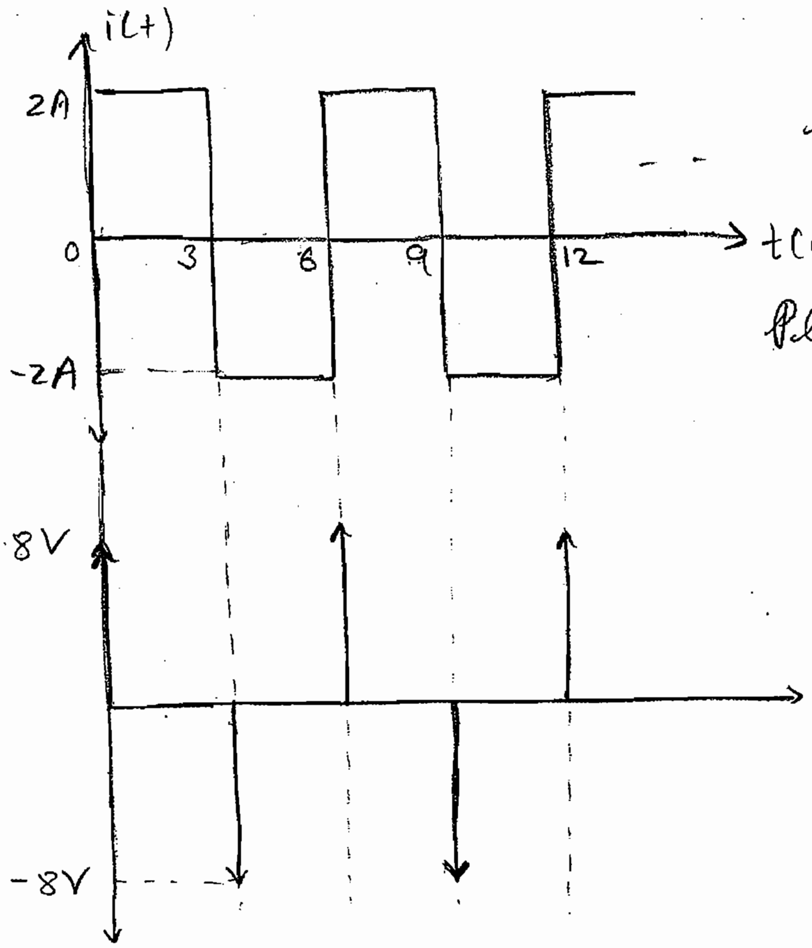
$4m < t < 11m$

$i(t) = -2t + c \Rightarrow v = 2\frac{d}{dt}(-2t+c)$

$= -4V$




8) If current flowing through 2H inductor is shown. Plot voltage across it.




$\infty$

$\boxed{V = L\frac{di}{dt}}$

impulse response

$dt \to 0$

Numerator restricts magnitude

$\infty \uparrow V = L\frac{[\Delta I]}{\Delta t} = L\left[\frac{i_{final} - i_{initial}}{\Delta t}\right]$

impulse

$\Delta t \to 0$

At $t = 0$ msec

$$\underset{\downarrow \atop \text{impulse}}{V} = 2\,\frac{[2-0]}{\underset{\downarrow \atop 0}{\Delta t}} = 4V \text{ impulse}$$

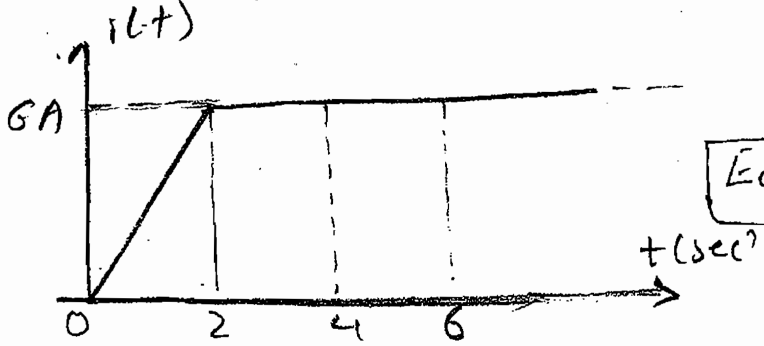
At $t = 3$ msec

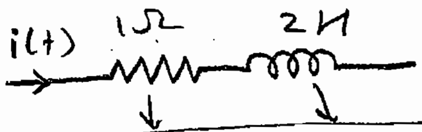
$$\underset{\downarrow \atop \text{impulse}}{V} = 2\left[\frac{-2\ -2}{\underset{\downarrow \atop 0}{\Delta t}}\right] = -8V$$

At $t = 6$ msec

$$V = 2\left[\frac{2-(-2)}{\Delta t}\right] = +8V$$

4) A practical coil has an inductance of 2H & resis. of 1 Ω. If this coil is excited with the current as shown below find the total energy absorbed by the coil upto $1^{st}$ 4 seconds.




$$\boxed{E_{abs} = E_{dissipated} + E_{stored}}$$

R

$$E_{diss} = \int I^2 R\, dt = \int_0^4 [i(t)]^2 \cancel{dt}\, R\, dt$$

$$= \int_0^2 (3t)^2 \times 1 \cdot dt + \int_2^4 6^2 dt$$

$$= 3\left[t^3\right]_0^2 + 36\,[4-2]$$

$$= 24 + 72$$

$$= 96 \text{ J}$$

$$E_{stored} = \int P_L\,dt = \int_0^4 L\,i\,\frac{di(t)}{dt}\,dt)$$

$$= \int_0^2 2(3t)\frac{d(3t)}{dt}\,dt + \int_2^4 2\cdot 6\,\frac{d}{dt}(0)\,dt.$$

$$= 9\left[t^2\right]_0^2 = \boxed{36\,J}$$

$$E_{absorbed} = 96 + 36 = 132\,J$$

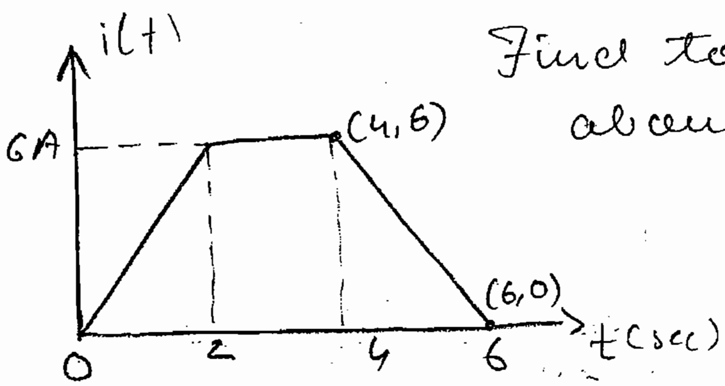
## NOTE

An inductor stores energy for some time variance occuring at any instance in that ckt. & retains this energy as long as excitation is given.

So energy stored here upto first 4 sec is the energy stored at the $4^{th}$ sec

$$E_L = \frac{1}{2}Li^2 = \frac{1}{2}(2)(6)^2 = \boxed{36\,J}$$

10) Find total energy about $6^{th}$ second.




$0 < t < 2$ $\quad y = mx = i(t) = 3t$

$2 < t < 4$ $\quad i(t) = 6$

$4 < t < 6$ $\quad (i(t) - 0) = \frac{(6-0)}{(4-6)}(t-6) \Rightarrow i(t) = -3t + 18$

In the above problem adding the limits from 4 to 6, we get

$$E_{dessi.} = 96 + \int_4^6 (-3t+18)^2 \, dt.$$

$$= 96 + 9\int_4^6 (t^2 - 12t + 36)\, dt.$$

$$= 96 + 24$$

$$= 120 \text{ J}$$

~~Eabsor.~~

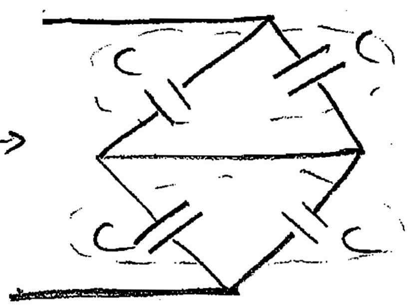
$$E_{stored} = 36 + \int_4^6 \frac{\cancel{2}}{\cancel{1}} (-3t+18) \frac{d}{dt}(-3t+18)\, dt$$

$$= 36 + 18\int_4^6 (-t+6)(-1)\, dt$$

$$= 36 - 36 = 0 \text{ J}$$

$\therefore$ $E_{absorbed} = \underline{\underline{120 \text{ J}}}$

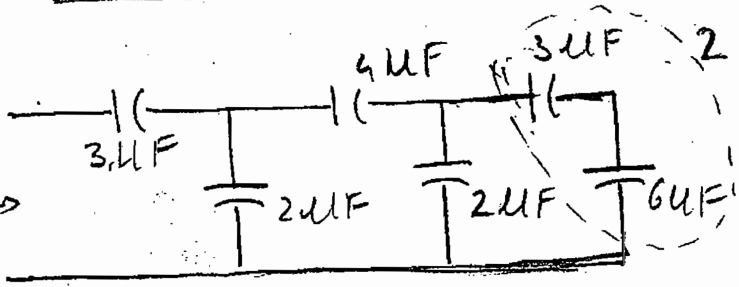
11)




$$C_{eq} = \frac{2C \times 2C}{4C}$$

$$= \frac{4C^2}{4C} = C$$

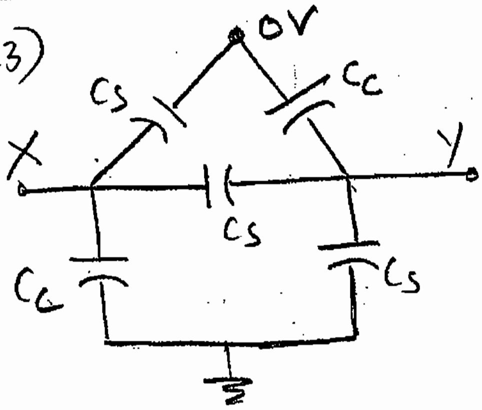
12)



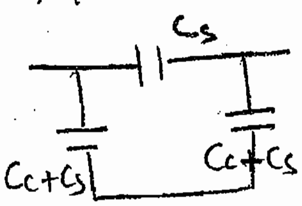

$$C_{eq} = \left[\frac{(2+2)\times 4}{8} + 2\right] \text{ series } (3\mu F)$$

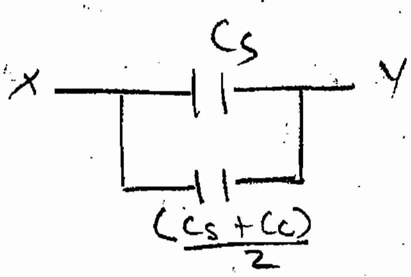
$$= \frac{4 \times 3}{7} = \frac{12}{7} \mu F$$

13)

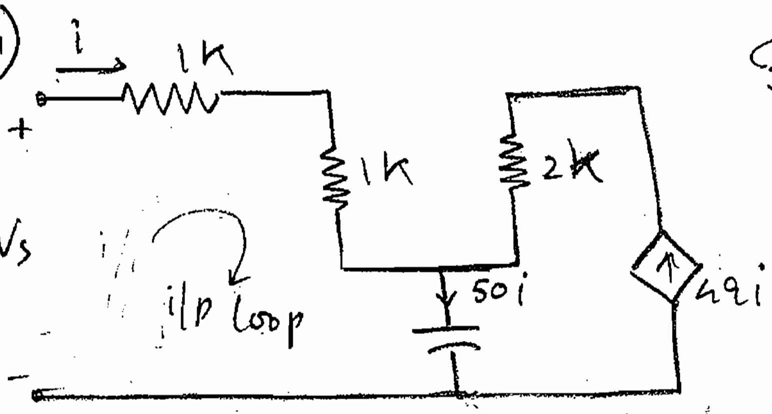



$C_{XY} =$




14)




Find the i/p loop Capacitance.

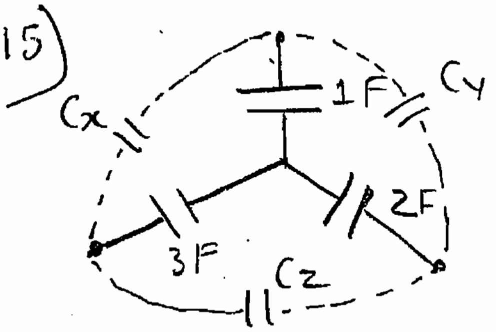
KVL

$V_s = (2000)\, i + \frac{1}{100\mu} \int 50\, i\, dt$

$\therefore V_s = \underset{R}{2000}\, i + \frac{1}{\underset{C}{2\mu}} \int i\, dt$

$\therefore C_{loop} = 2\,\mu F$ & also $R_{loop} = 2000\,\Omega$

15)




Convert star to delta

$\Rightarrow \frac{1}{C_x} = \frac{1}{1} + \frac{1}{3} + \frac{\frac{1}{1} \times \frac{1}{3}}{\frac{1}{2}}$

$= 1 + \frac{1}{3} + \frac{2}{3}$

$= 2$

$\therefore C_x = \frac{1}{2}$ F

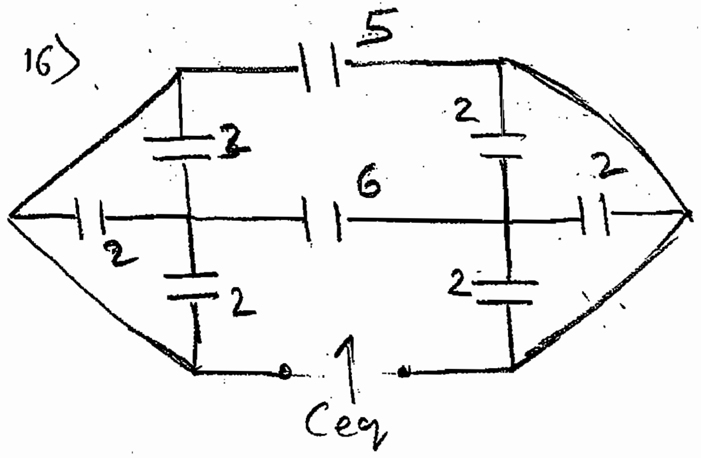
$\Rightarrow \frac{1}{C_y} = \frac{1}{1} + \frac{1}{2} + \frac{\frac{1}{1} \times \frac{1}{2}}{\frac{1}{3}} = 1 + \frac{1}{2} + \frac{3}{2}$

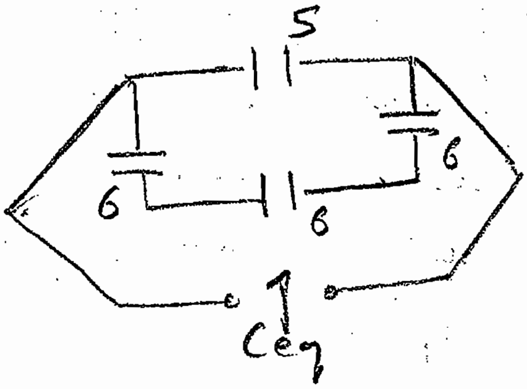
$\frac{1}{C_y} = \frac{6}{2} = 3 \Rightarrow C_y = \frac{1}{3}$ F

$\Rightarrow \frac{1}{C_z} = \frac{1}{3} + \frac{1}{2} + \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{1}} = \frac{1}{3} + \frac{1}{2} + \frac{1}{6}$

$\frac{1}{C_z} = 1 \Rightarrow C_z = 1$ F

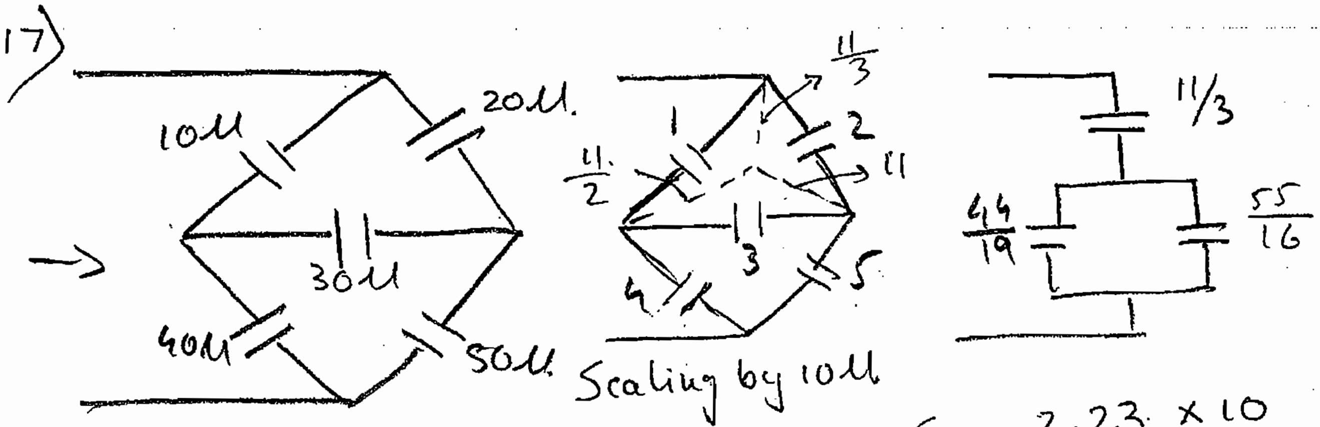
16)



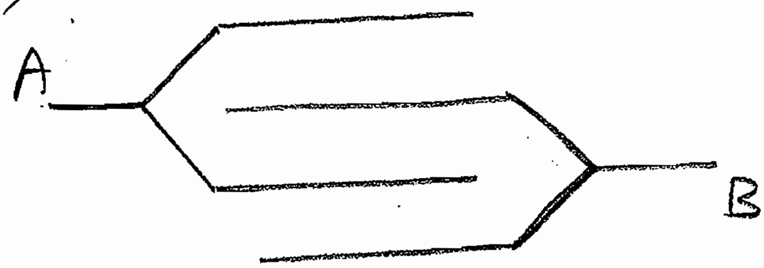

$\therefore C_{eq} = \frac{5 \times 2}{7} = \frac{10}{7}$ F

17)

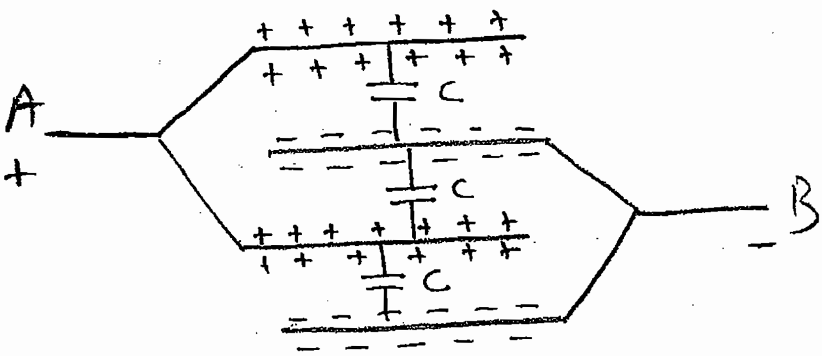


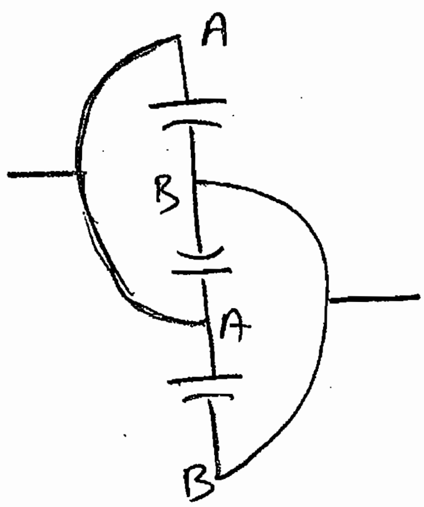

$C_{eq} = 2.23 \times 10$
$= 22.3\ \mu F$

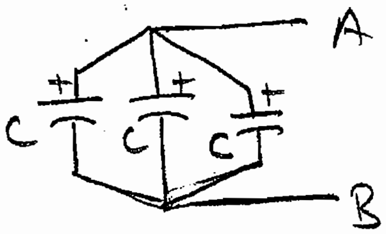
18)




If equivalent capacitance b/w the plates is 'C' then $C_{AB}$ = ______.

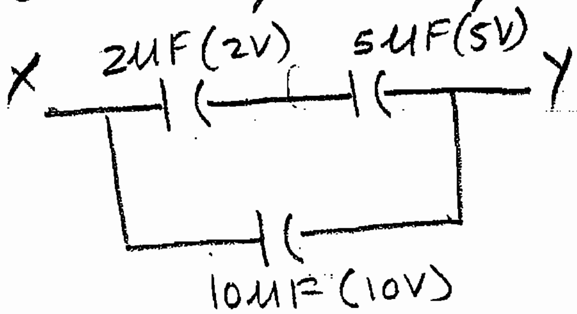



$\Rightarrow C_{AB} = 3C$

1) Three capacitors are arranged in fig. as shown below. The value of vltgs in bracket indicates their breakdown vltg limit. What the max vltg upto which ~~end~~ entire n/w can work w/o breakdown of any capacitor. & hence determine max. storage charge in the n/w.




For $10\,\mu F$

$$V = 10\,V$$

For $5\,\mu F$

$$V\left[\frac{2}{7}\right] = 5 \Rightarrow V = 17.5\,V$$

For $2\,\mu F$

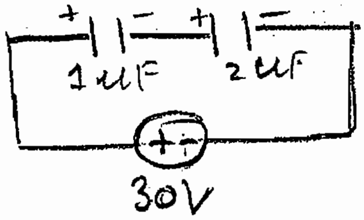
$$V\left[\frac{5}{7}\right] = 2 \Rightarrow V = 2.8\,V$$

$\therefore\ V_{max} = 2.8\,V$

Here $C_{eq} = \frac{10}{7} + 10 = \frac{80}{7}\,\mu F$

$$q_{max} = C_{eq} V_{max} = \frac{80}{7} \times \frac{28}{10} = 32\,\mu C$$

2) Two capacitors of $1\,\mu F$ & $2\,\mu F$ are connected in series across a 30V DC source. Find their steady vltgs & charge on each. Now if these 2 capacitors are disconnected from supply & connected with like polarities together, now determine steady state vltg & charge on ~~it~~ each



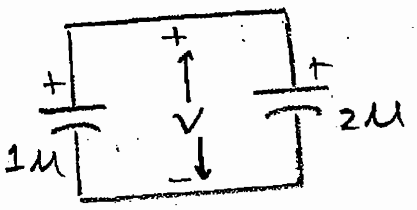

$V_{1\mu} = \frac{30(2)}{3} = 20\ V$

$V_{2\mu} = \frac{30(1)}{3} = 10\ V$

$q_{1\mu} = 1 \times 20 = 20\ \mu C$ $\quad q_{2\mu} = 2 \times 10 = 20\ \mu C$

In current electricity if current is equal in series connected elements then in static electricity the charges will be equal / same in series connected capacitors.

$\therefore q_1 = q_2 = 20\ \mu C.$




Here, $V_1 = V_2 = V$

$\therefore \frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{q_1}{1} = \frac{q_2}{2}$

$\therefore q_2 = 2q_1$

From law of conservation of charge

$q_1 + q_2 = 40$

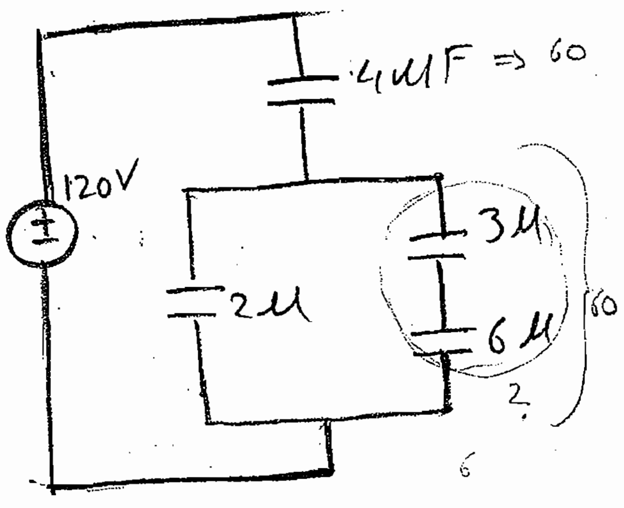
$\therefore q_1 + 2q_1 = 40 \Rightarrow q_1 = \frac{40}{3}\ \mu C$ ;

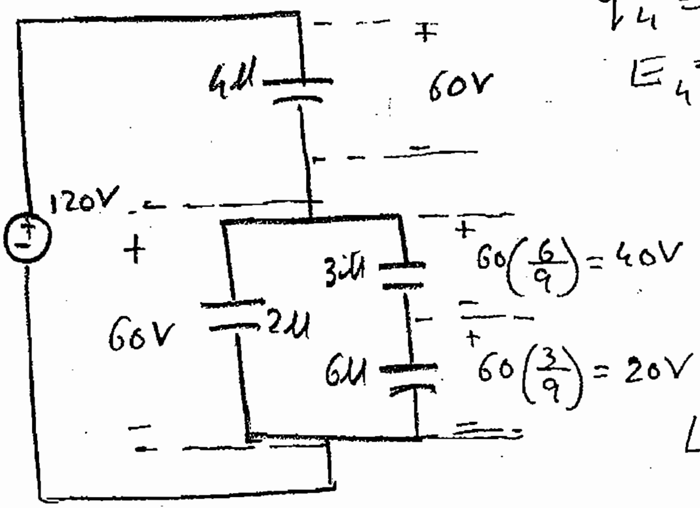
$\Rightarrow q_2 = \frac{80}{3}\ \mu C$

$\therefore V_1 = V_2 = \frac{q_1}{C_1} = \frac{q_2}{C_2}$

$= \frac{40}{3}$ volts.

3) Determine steady state vltgs across each capacitor & energy stored in each






$q_4 = CV = 4 \times 60 = 240\ \mu C$

$E_4 = \frac{1}{2}(4)(60)^2 = 7200\ \mu J$

$q_2 = 2 \times 60 = 120\ \mu C$

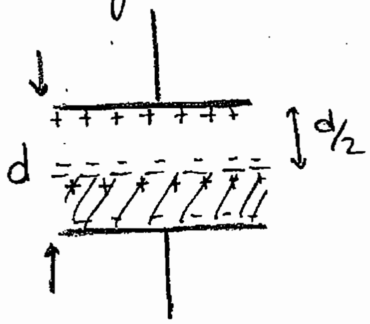
$E_2 = \frac{1}{2}(2)(60)^2 = 3600\ \mu J$

$q_3 = q_6 = 3 \times 40 = 120\ \mu C$

$E_3 = E_6 = \frac{1}{2}(3)(40)^2$

$= 2400\ \mu J$

4) A capacitor having a common cross section area b/w the plates of A m² & dist. b/w the plates as 'd' metres is now dipped in ethyl alchohol upto d/2 m. What is the ratio of capacitance before & after immersing it into the ethyl alchohol. [Consider $\varepsilon_r = 25$ for ethyl alcohol]




<u>Before</u>: $C = \frac{A\varepsilon_0\varepsilon_r}{d}$

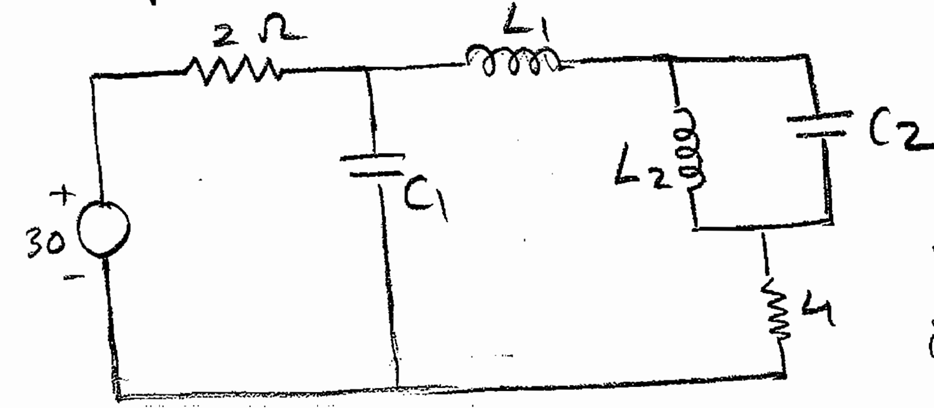
$C = \frac{A\varepsilon_0}{d}$ ($\because \varepsilon_r = 1$ (air))

<u>After</u>: $\boxed{\cancel{C' = \frac{\frac{A\varepsilon_0\varepsilon_r}{d/2} \times \frac{A\varepsilon_0\varepsilon_r}{d/2}}{2\frac{A\varepsilon_0\varepsilon_r}{d/2}} = \frac{A\varepsilon_0(25)}{4d}}}$

$$C' = \frac{\frac{A\varepsilon_0}{d/2} \times \frac{A\varepsilon_0\varepsilon_r}{d/2}}{\frac{A\varepsilon_0}{d/2}\left[1+\varepsilon_r\right]} = \underbrace{\frac{A\varepsilon_0}{d}}_{C} \times \frac{25\times 2}{\cancel{2}\times 26}$$

$\therefore \frac{C}{C'} = \cancel{\frac{25}{25}}\ \frac{26}{50}$

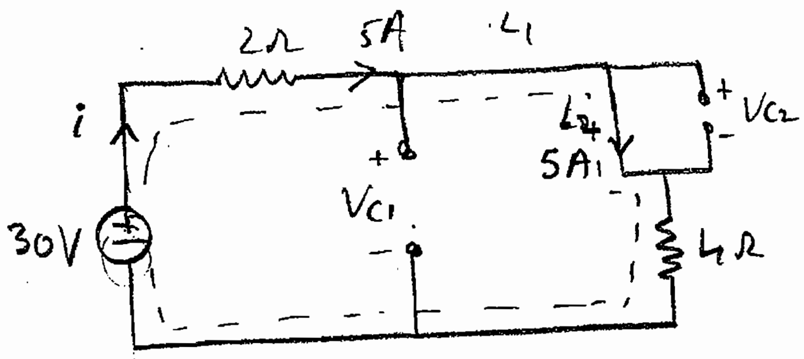
1) Determine steady state voltages across Capacitors & current through inductors




$R_T = 2 + 4 = 6$

$i = \frac{30}{6} = 5A$

$\therefore I_{L_1} = I_{L_2} = 5A$

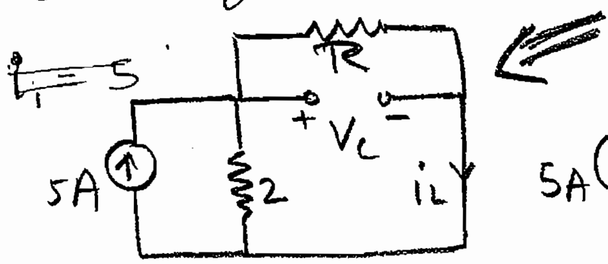


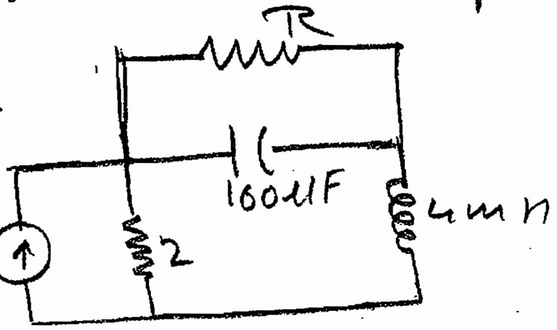

$-30 + 10 + V_{C_1} = 0$

$\Rightarrow V_{C_1} = 20V$

& $V_{C_2} = 0V$

2) What is value of R for which the energy stored in inductor & capacitor are equal.




$i_L = 5 \times \frac{2}{R+2} = \frac{10}{2+R}$

$V_c = i_L \times R = \frac{10R}{2+R}$

Now

$E_L = E_C$

$\frac{1}{2} L i_L^2 = \frac{1}{2} C V_c^2$

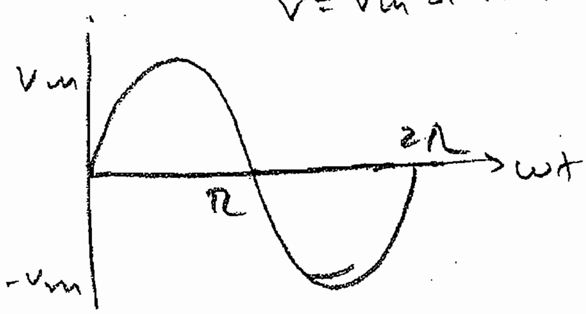
$\therefore \frac{1}{\cancel{2}} \times \frac{(4m)(1\cancel{00})}{(2+R)^2} = \frac{1}{\cancel{2}} \frac{(160\mu)(1\cancel{00}) R^2}{(2+R)^2}$

$R^2 = \frac{.1m}{40\mu} = 25 \Rightarrow R = 5\Omega$

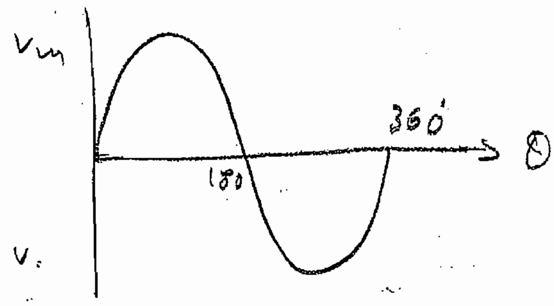
# STEADY STATE AC circuit Analysis :

Radian

$v = V_m \sin\omega t$




Degre

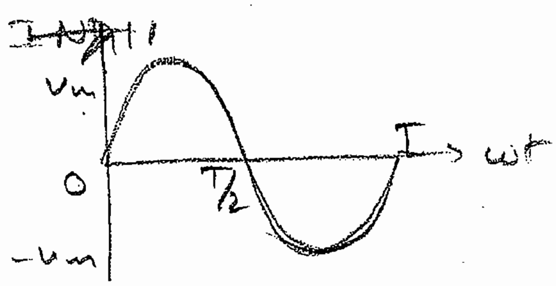



$v = V_m \sin\theta$

$V_m \rightarrow$ amplitude

$\omega \rightarrow$ angular freq. (rad/sec)

$\omega = 2\pi f = \frac{2\pi}{T}$




Time

$v = V_m \sin\left(\frac{2\pi}{T} \cdot t\right)$

India

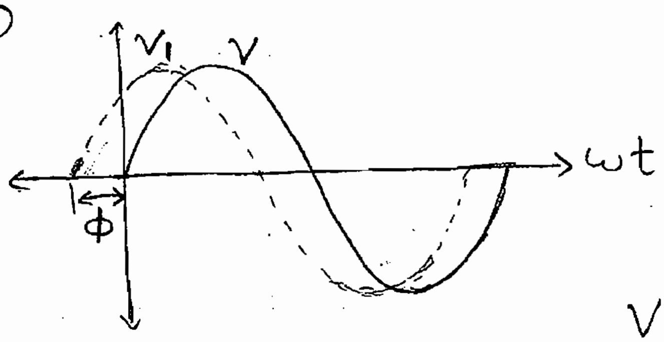
Power freq = 50 Hz $\Rightarrow$ $T = \frac{1}{50} = 20$ msec

$1T = 2\pi = 360° = 20\text{ms}$

## Standard sine wave :—

$V_x = V_m \sin(\omega t + \phi)$ ; $\phi$ = phase shift (deg)
↳ time shift

①



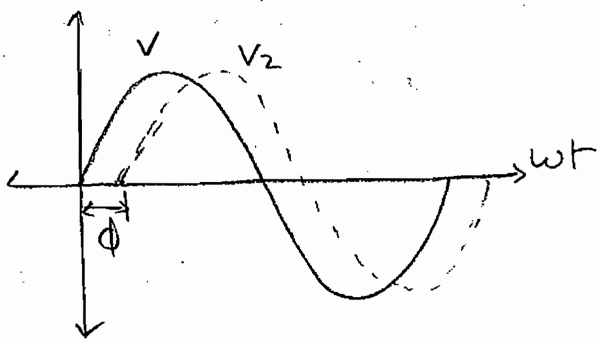

$V_1 = V_m \sin(\omega t + \phi)$

$V_1$ comes early than $V$ by $\phi°$

$V_1$ leads $V$ by $\phi°$

(+) → leading

②




$V_2 = V_m \sin(\omega t - \phi)$

$V_2$ lags $V$ by $\phi°$

(−) → lagging.

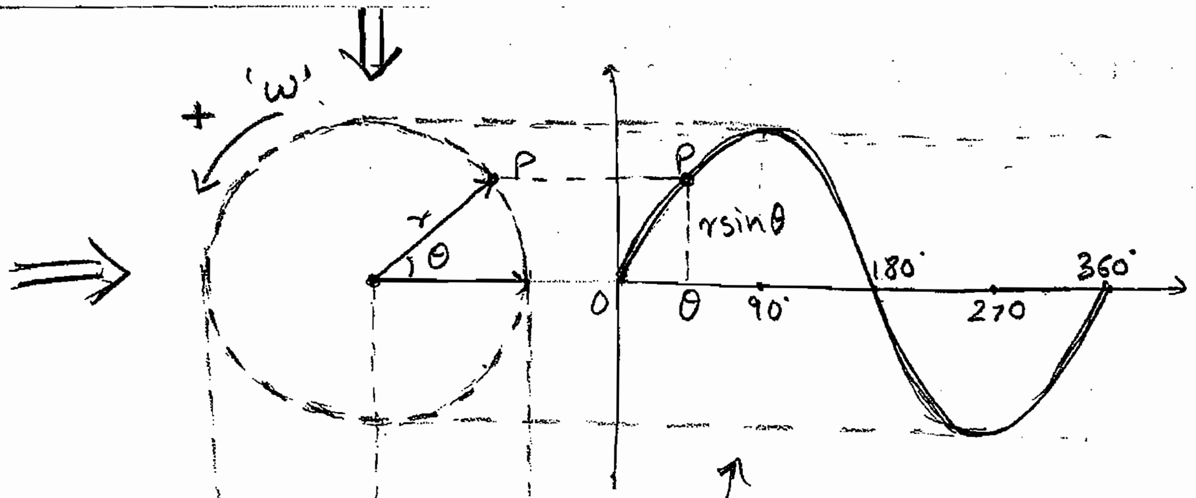
<u>INDIA</u> , $\phi = 60°$

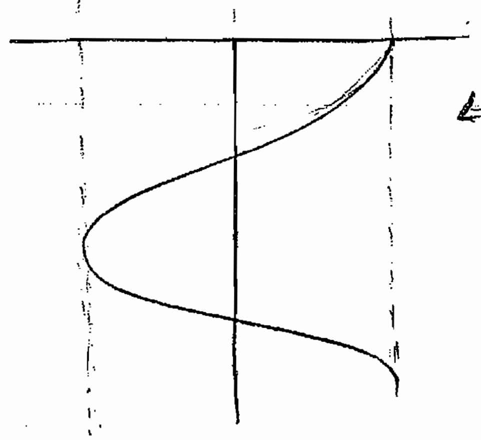
$360° \rightarrow 20$ msec

$60° \rightarrow t_{shift}$

$t_{shift} = \frac{60}{360°} \times 20$ msec $= 3.33$ msec.

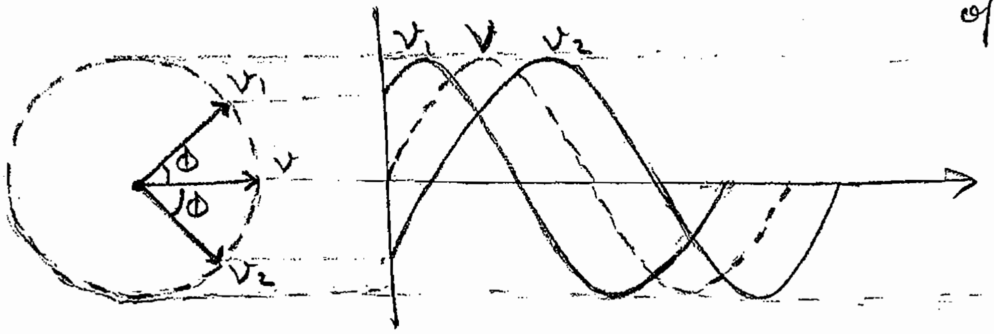
$\omega = 2\pi f = 2\pi(50) = 100\pi$ rad/sec



represents sinusoids.

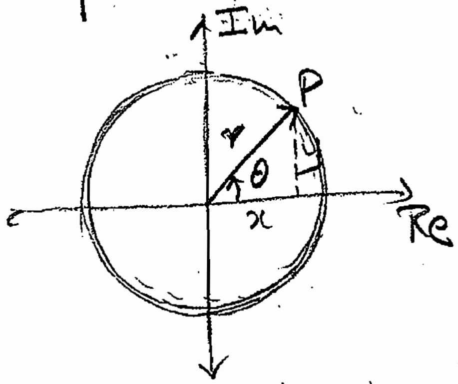
* Time shift can be easily understood from phase shift (i.e. using phasors).
It also indicate the amt of shift.




$V = |V_m| \angle 0°$

$V_1 = |V_m| \angle \phi$ $\qquad V_2 = |V_m| \angle -\phi$

S-plane




$\boxed{P} = x + iy \rightarrow$ Rectangular form

$\boxed{P} = r \angle \theta \rightarrow$ Polar form

$\boxed{P} = r e^{j\theta} \rightarrow$ Euler's form

$\boxed{R} \longrightarrow \boxed{P}$

$0 + j0 \longrightarrow 0\angle 0^\circ$

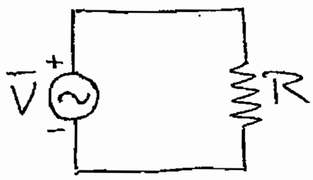
$1 + j0 \longrightarrow 1\angle 0^\circ$

$\boxed{0 + j1 \longrightarrow 1\angle 90^\circ} \rightarrow j \rightarrow$ powerful operator

$1 + j1 \longrightarrow \sqrt{2}\angle 45^\circ$ $\qquad +j \rightarrow$ leads

$0 - j1 \longrightarrow 1\angle -90^\circ$ $\qquad -j \rightarrow$ lags

## Phasor relationship b/w voltages & Current in passive elements :-

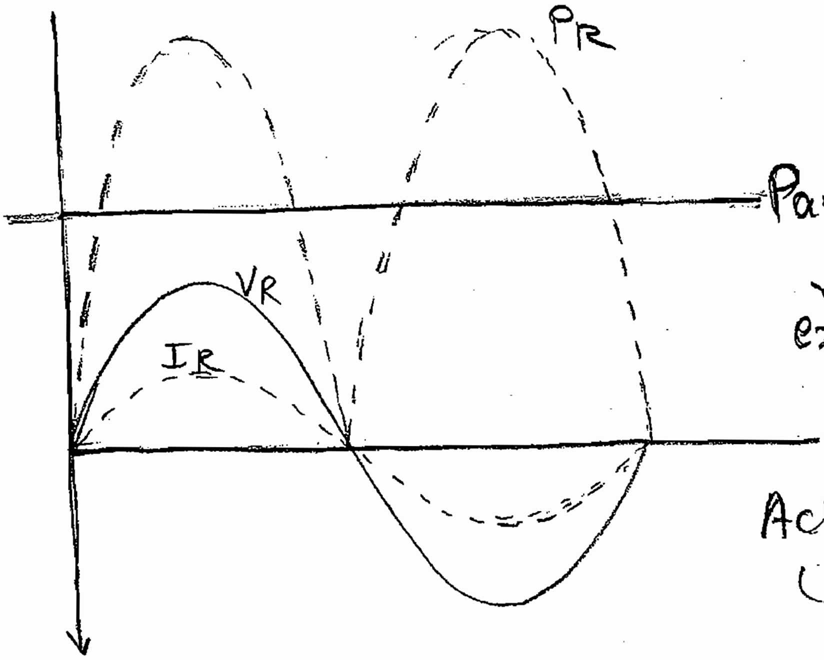



Let $\bar{V} = V_m \sin\omega t$ —— ①

$\boxed{V = IR}$

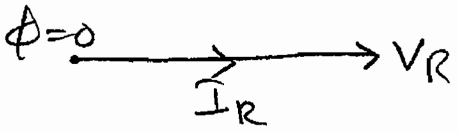
$\therefore \bar{I} = \dfrac{\bar{V}}{R} = \dfrac{V_m \sin\omega t}{R} = \dfrac{V_m}{R}\sin\omega t$

& $\bar{I} = I_m \sin\omega t$ —— ②




$P_{avg}$ ↓ exists ↓ Active | Real (in watts)

Phasor diagram




Power factor

$\cos\phi = \cos 0^\circ = 1$ (UPF)

$$P_{avg} = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$$

$$P_{avg} = V_m \sin\omega t * I_m \sin\omega t$$

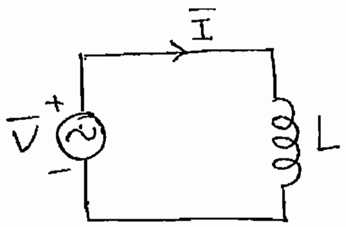
$$= V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$= \frac{V_m I_m}{2} - \underbrace{\frac{V_m I_m}{2}\cos 2\omega t}_{\to 0}$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V_{RMS} \cdot I_{RMS} \text{ watts.}$$

# Inductor :—




Let $\bar{I} = I_m \sin\omega t$ —— (1)

$$\boxed{V = L\frac{dI}{dt}}$$

$$V = L\frac{d}{dt}(I_m \sin\omega t) = \omega L I_m \cos\omega t$$

$$= \omega L I_m \sin(\omega t + 90^\circ) \quad \text{—— (2)}$$

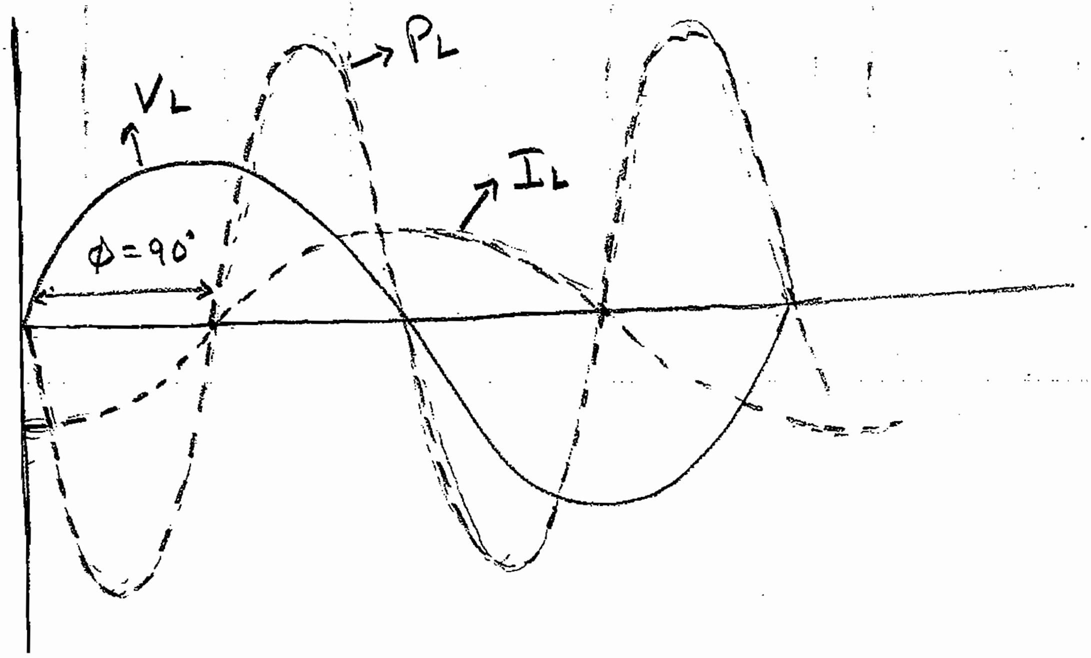
$$= \omega L I_m \sin(\omega t)\, [j]$$

$\bar{V} = j\omega L\bar{I}$ Vltg is "j operator" times the current.

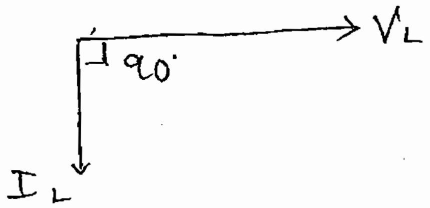
$\bar{V} = +jX_L\bar{I}$ i.e. V leads I by $90^\circ$

Where $\underset{\downarrow}{\underline{X_L}} = \omega L = 2\pi f L$

inductive reactance.



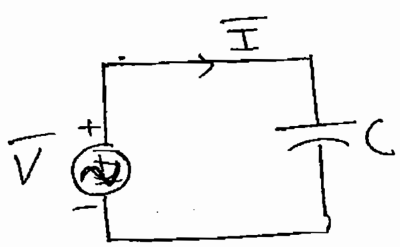

Phasor diag




Power factor

$PF = \cos\phi = \cos 90^\circ$
$= 0^\circ$

# Capacitor




Let $\bar{V} = V_m \sin\omega t$ —— ①

$$\boxed{I = C\frac{dV}{dt}}$$

$$I = C\frac{d}{dt}(V_m \sin\omega t) = C \cdot \omega V_m \cos\omega t$$

$$= \omega C V_m \sin(\omega t + 90^\circ) \quad \text{—— ②}$$

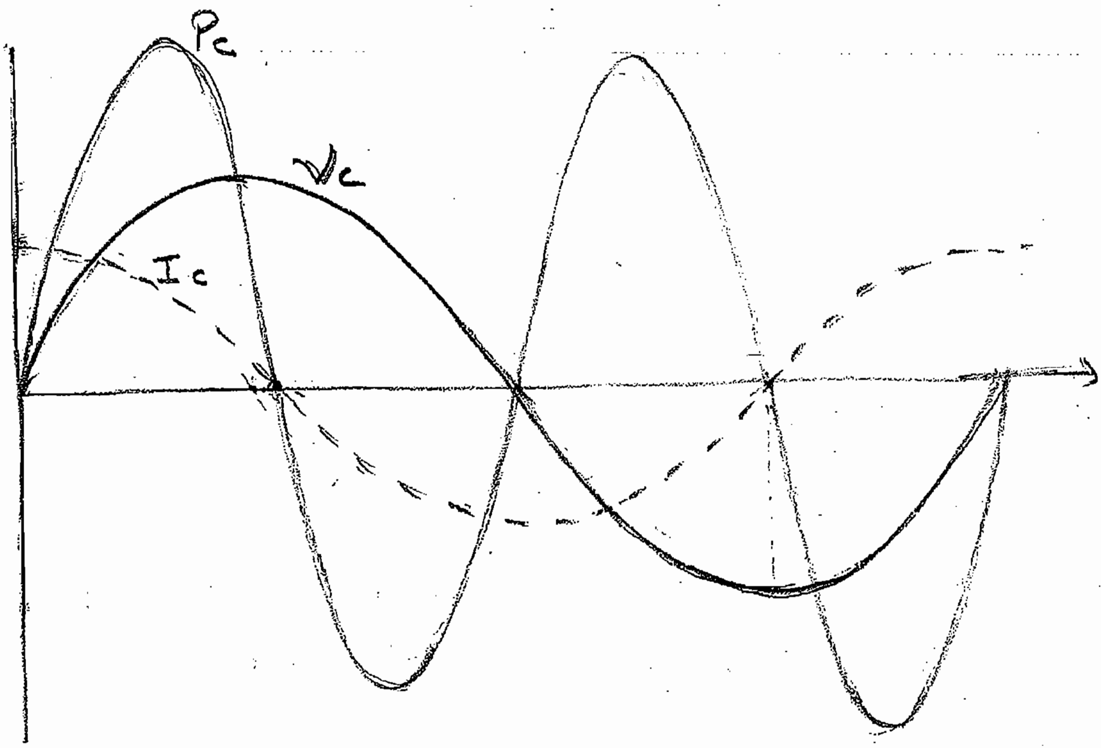
$$= \omega C V_m \sin\omega t\ [j]$$

$$\bar{I} = j\omega C V$$

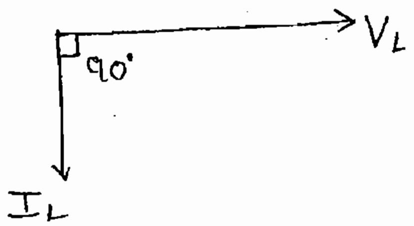
$$\bar{V} = \frac{\bar{I}}{j\omega C} = \frac{-j}{\omega C}\bar{I}$$

$$\bar{V} = -jX_c\bar{I}$$

where, $\underset{\substack{\downarrow \\ \text{capacitive reactance}}}{X_c} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$



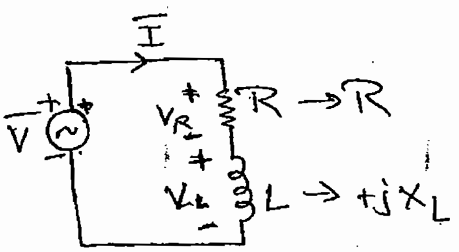

Phasor diag




Power factor

$$P.F. = \cos\phi = \cos 90°$$
$$= 0$$

# Series R-L

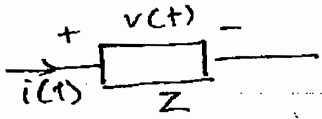



KVL: $-\bar{V} + \bar{V}_R + \bar{V}_L = 0$

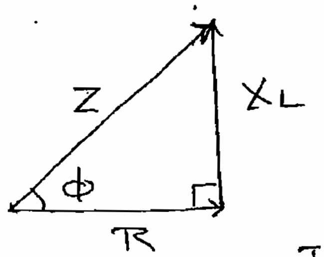
$\bar{V} = IR + jX_L I$

$\bar{V} = I[R + jX_L]$

$\boxed{V = IZ}$ ; $Z = R + jX_L$
↓
impedance (Ω)




## Impedance Δle




$|Z| = \sqrt{R^2 + X_L^2}$

$\phi = \tan^{-1}\left[\frac{X_L}{R}\right] = \tan^{-1}\left[\frac{\omega L}{R}\right]$
↓
Impedance of ckt

## Power factor

$\cos\phi = \frac{R}{|Z|}$ , $\sin\phi = \frac{X_L}{|Z|}$

$I^2 Z = I^2 R + j X_L I^2$

$VI = I^2 R + j I^2 X_L$

In AC ckt, all the convertable part of energy is represented by Resistance
eg: electricity → light
→ heat
→ sound

Generate — Do business — Control

$\boxed{S = P + j Q_L}$

$Q_L$ → Reactive power (VAR's)
(flux, charge)

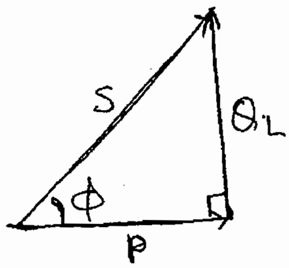
$P$ → Active / Real / True Power (watts)
(consumable part of energy)

$S$ → Total / Apparent Power (VA's)
(associated with source)

## Power Δle

$|S| = \sqrt{P^2 + Q_L^2}$

$\phi = \tan^{-1}\left[\frac{Q_L}{P}\right]$



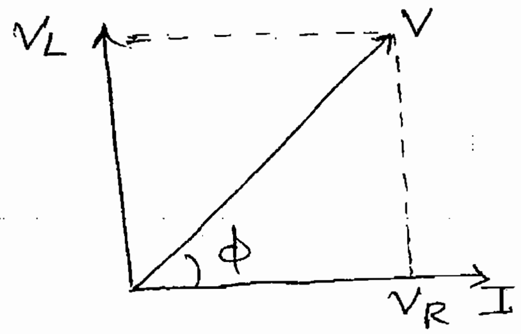
$$\cos\phi = \frac{P}{S} \Rightarrow P = \cos\phi \times S = VI\cos\phi \text{ watts}$$

$$\sin\phi = \frac{Q_L}{S} \Rightarrow Q_L = S\sin\phi = VI\sin\phi \text{ VAR's } \cancel{\text{watts}}.$$

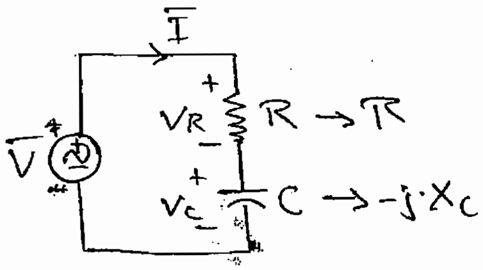
Here $V, I \rightarrow$ rms value

Phasor diagram :

$I$ lags $V$ by $\phi < 90°$




# Series RC




KVL

$$-\bar{V} + \bar{V}_R + \bar{V}_C = 0$$

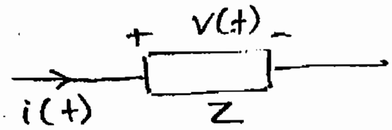
$$\bar{V} = IR - jX_C I$$

$$\bar{V} = \bar{I}[R - jX_C]$$

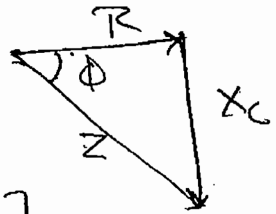
$$\boxed{V = IZ}$$

Here,

$Z = R - jX_C$
$\downarrow$
impedance $(\Omega)$




Impedance $\Delta$le :-




$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left[\frac{X_C}{R}\right] = \tan^{-1}\left[\frac{1}{\omega RC}\right]$$

$\downarrow$
impedance of ckt

Power factor :-

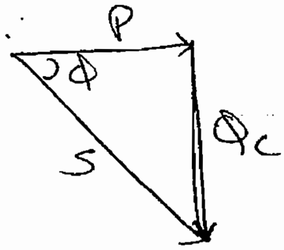
$$\cos\phi = \frac{R}{|Z|} \qquad \sin\phi = \frac{X_C}{|Z|}$$

$$IZ^2 = I^2R - jX_cI^2$$

$$VI = I^2R - jI^2X_c$$

$$\boxed{S = P - jQ_c}$$ → Reactive power (VAR's)

Power Δle



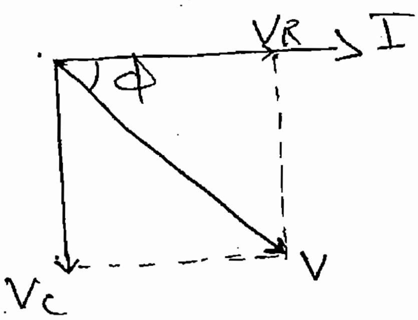

$$|S| = \sqrt{P^2 + Q_c^2}$$

$$\phi = \tan^{-1}\left[\frac{Q_c}{P}\right]$$

$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = VI\cos\phi \quad \text{(watts)}$$

$$\sin\phi = \frac{Q_c}{S} \Rightarrow Q_c = S\sin\phi = VI\sin\phi \quad \text{(VAR's)}$$

$V, I \rightarrow$ rms value



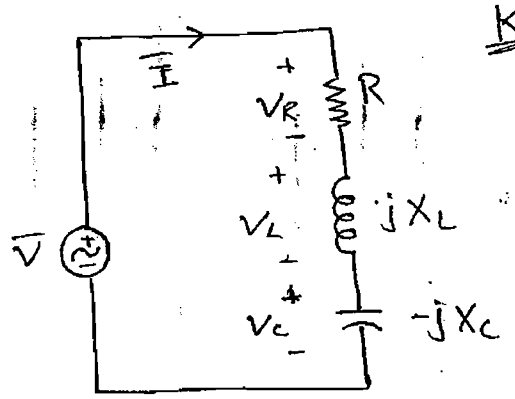

Phasor diag.

I leads V by $\phi < 90°$

(+) $Q_L \rightarrow$ absorbing VAR's (lagging VAR's)

(−) $Q_c \rightarrow$ generating VAR's (leading VAR's)

# R-L-C circuit




KVL $-\bar{V} + \bar{V}_R + \bar{V}_L + \bar{V}_C = 0$

$\bar{V} = IR + jX_L I - jX_C I$

$\bar{V} = \bar{I}\,[R + j(X_L - X_C)]$

$V = IZ$

Here $Z = R + j\underbrace{[X_L - X_C]}_{X_{net}}$

$X_{net} \rightarrow$ net reactances

$\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$

$\downarrow$

net impedance angle

$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

Power factor

$\cos\phi = \frac{R}{|Z|}$ $\sin\phi = \frac{|X_L - X_C|}{|Z|}$

**Case 1** If $X_L > X_C \implies$ (General nature of electrical sys.)

$Z = R + jX_{net}$

↳ series RL ckt

I lags V by $\phi < 90°$ (lagging PF)

**Case 2** If $X_L < X_C$

$Z = R - jX_{net}$

↳ series RC ckt

I leads V by $\phi < 90°$ (leading PF)

**Case 3** If $X_L = X_C$

$Z = R \implies$ purely resistive

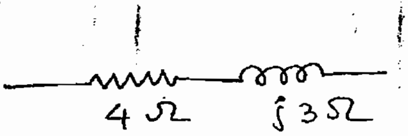
I in phase with V $\implies \phi = 0°$, P.F = 1 (UPF)

$$\boxed{Z = R \pm jX}$$

→ $Z = R + jX_L$

→ $Z = R - jX_C$

Eg:-




$Z = 4 + j3$

$$\boxed{Y = \frac{1}{Z}}$$ → admittance (℧) or S

$$Y = \frac{1}{4+j3} \times \frac{4-j3}{4-j3} = \frac{4-j3}{25}$$

$\therefore$ $Y = (0.16 - j0.12)$ ℧

$$\boxed{Y = G \pm jB}$$

→ $Y = G - jB_L$

when $B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L}$

↳ inductive susceptance (℧)

→ $Y = G + jB_C$

when $B_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C = 2\pi f C$

↳ capacitive susceptance (℧)

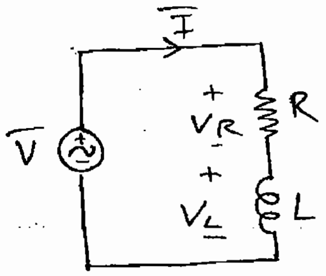
impedance ⟶ admittance

reactance ⟶ susceptance

# Phasor diagrams :—

## ⟨1⟩ Series circuits ⟹ I → 'ref'

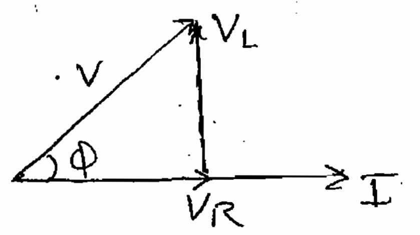
### (a) Series R-L ckts :




$V_R = IR\angle 0°$

$V_L = +j\,X_L I$

$= I \cdot X_L \angle +90°$

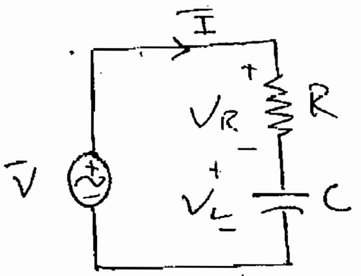



$|V| = \sqrt{V_R^2 + V_L^2}$

$\cos\phi = \dfrac{V_R}{V}$ (lagging)

$\phi = \tan^{-1}\left(\dfrac{V_L}{V_R}\right)$

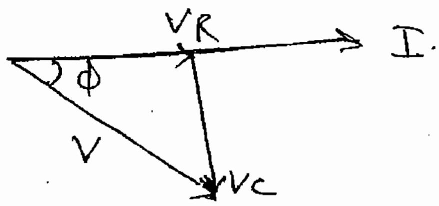
### (b) Series R-C ckt




$V_R = IR\angle 0°$

$V_c = -j\,X_c I$

$= X_c I \angle -90°$

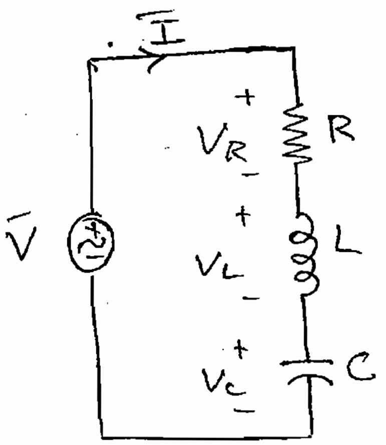



$|V| = \sqrt{V_R^2 + V_c^2}$

$\phi = \tan^{-1}\left(\dfrac{V_c}{V_R}\right)$

$\cos\phi = \dfrac{V_R}{V}$ (leading)

### (c) Series R-L-C




$V_R = IR\angle 0°$

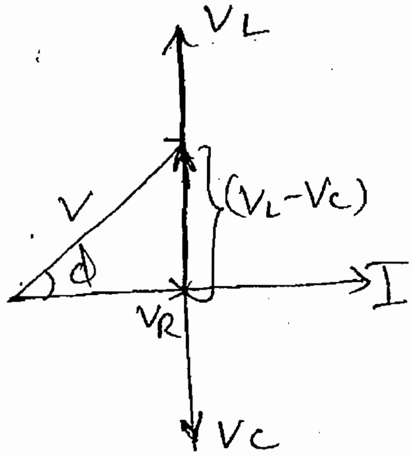
$V_L = IX_L\angle 90°$

$V_c = IX_c\angle -90°$

$|V| = \sqrt{V_R^2 + (V_L - V_c)^2}$

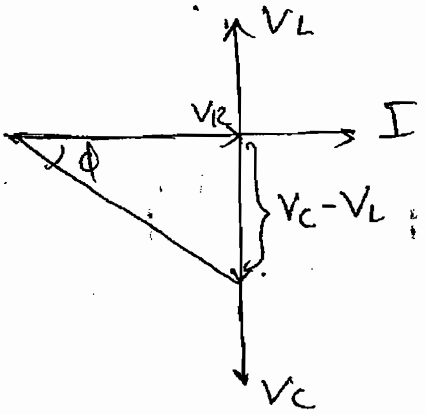
$\phi = \tan^{-1}\left(\dfrac{V_L - V_c}{V_R}\right)$

$\cos\phi = \dfrac{V_R}{V}$ (lagging)




(Case 1)

**Case 2:** $X_L < X_C \rightarrow V_L < V_C$

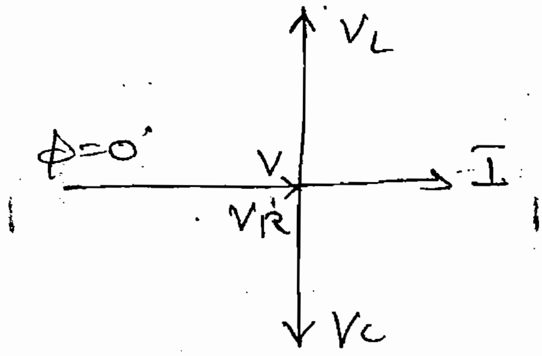



$$|V| = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$$

$$\cos\phi = \frac{V_R}{V} \text{ (leading)}$$

**Case 3:** $X_L = X_C \rightarrow V_L = V_C$



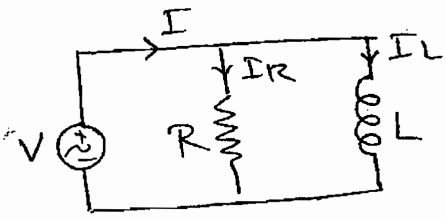

$$|V| = V_R$$

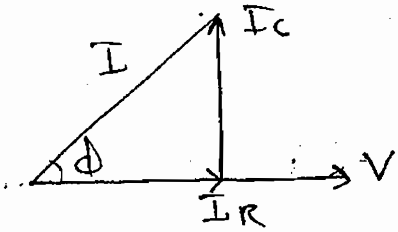
$$\phi = 0^\circ$$

$$\text{P.F.} = \cos\phi = \cos 0^\circ$$
$$= 1 \quad \text{(UPF)}$$

## ⟨2⟩ Parallel ckts $\Longrightarrow$ V → 'ref'

### (a) Parallel R-L




$$I_R = \frac{V}{R}\angle 0^\circ$$

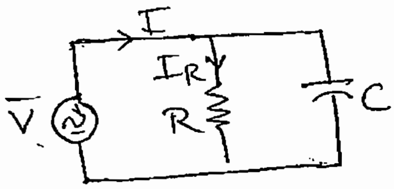
$$I_L = \frac{V}{jX_L} = \frac{V}{X_L}\angle -90^\circ$$

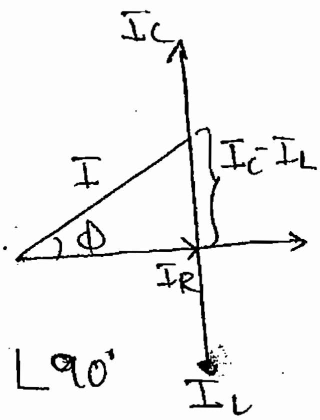
$$|I| = \sqrt{I_R^2 + I_C^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$\cos\phi = I_R / I \text{ (lagging)}$$

### (b) Parallel R-C




$$I_R = \frac{V}{R}\angle 0^\circ$$

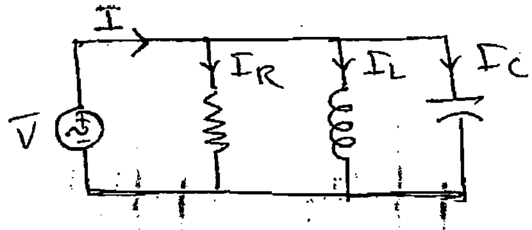
$$I_C = V/-jX_C = V/X_C \angle 90^\circ$$

$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$
(leading)

## (c) Parallel R-L-C

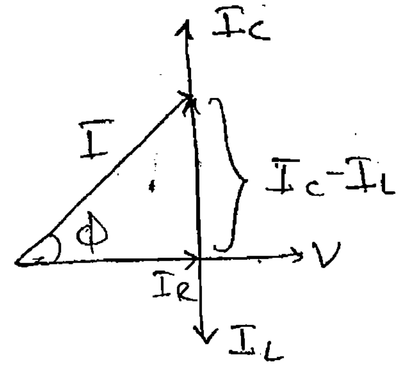



$$I_R = \frac{V}{R} \angle 0^\circ$$

$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

Case (1) : $X_L > X_C \Rightarrow I_L < I_C$

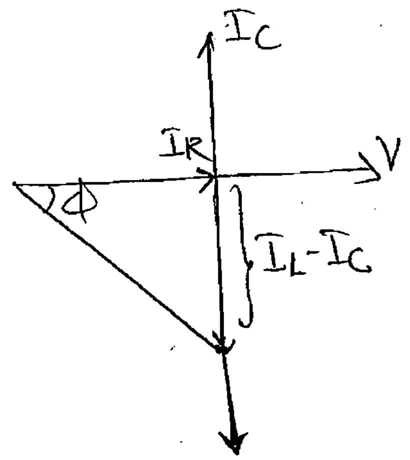



$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1}\left[\frac{I_C - I_L}{I_R}\right]$$

$$\cos\phi = \frac{I_R}{I} \quad \text{(leading)}$$

Case (2) : $X_L < X_C \Rightarrow I_L > I_C$




$$|I| = \sqrt{I_R^2 + (I_L - I_C)^2}$$

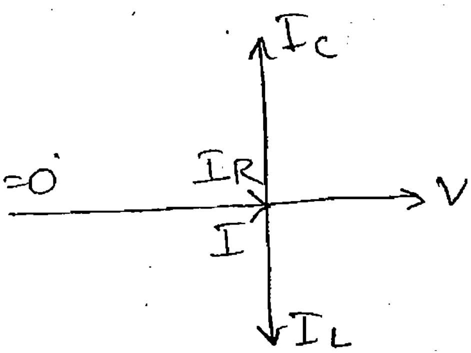
$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$

(lagging)

Case (3) : $X_L = X_C \Rightarrow I_L = I_C$

$\phi = 0^\circ$

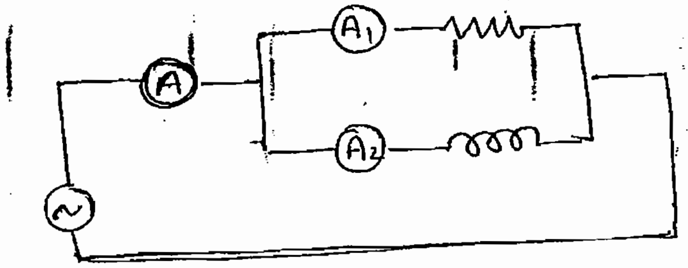


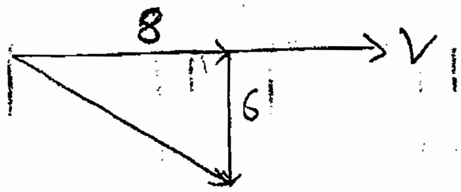

$$|I| = I_R$$

$$\phi = 0^\circ$$

$$\cos\phi = \cos 0^\circ = 1 \quad \text{(UPF)}$$

1) If $A_1 \rightarrow$ reads $\rightarrow 8A$
$A_2 \rightarrow$ reads $\rightarrow 6A$
$A \rightarrow$ reads $=$ ______ ckt P.F = ______

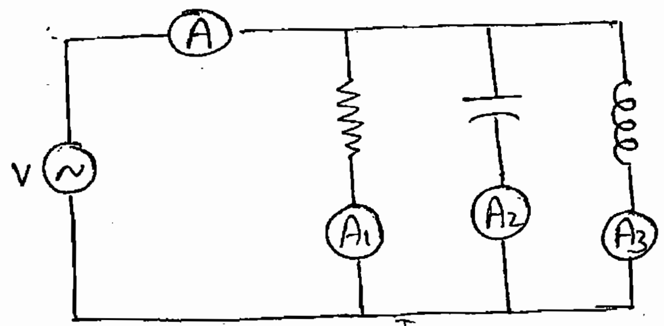


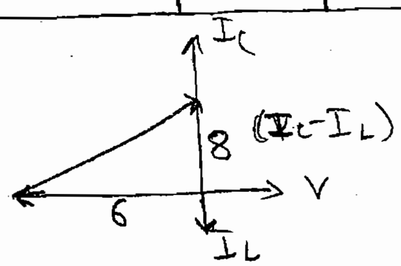




$$I = \sqrt{I_R^2 + I_L^2} = \sqrt{64+36} = \sqrt{100} = 10$$

$$P.F. = \frac{I_R}{I} = \frac{8}{10} = 0.8 \text{ (lagging)}$$

2) If $A_1 \rightarrow$ reads $\rightarrow 6A$
$A_2 \rightarrow$ reads $\rightarrow 18A$
$A_3 \rightarrow$ reads $\rightarrow 10A$
$A \rightarrow$ reads $=$ ______ ckt P.F. = ______

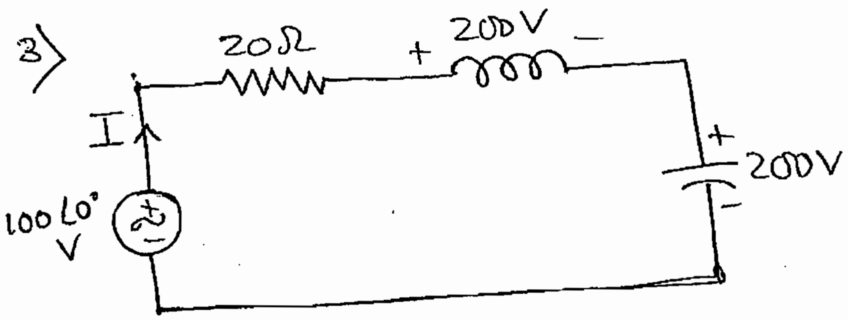



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100} = 10$$

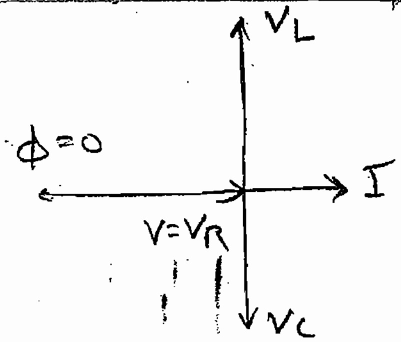
$$P.F. = \frac{I_R}{I} = \frac{6}{10}$$
$$= 0.6 \text{ (leading)}$$

3)




Find I = ______

Ckt PF = ______




$$I_R = \frac{V_R}{R} = \frac{100\angle 0^\circ}{20}$$

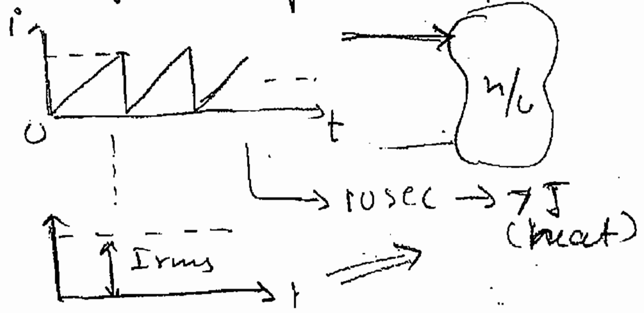
$$= 5\,A = I$$

$$\cos\phi = \cos 0^\circ = 1 \text{ (UPF)}$$

## RMS value / True / Effective value :—

It is that steady value of a time varying vltg or current waveform which could produce the same value of **heat** as given by the original waveform for definite period of time.

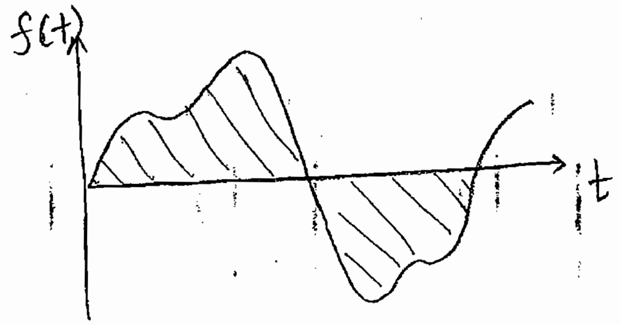
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2\,dt}$$




## Average value / Mean value

It is that steady ~~value~~ of equivalent value of time varying vltg or current waveform which could develope same amount of **charge** as given by the original waveform for a definite period of time in a ckt.

# Concept of Symmetry :-

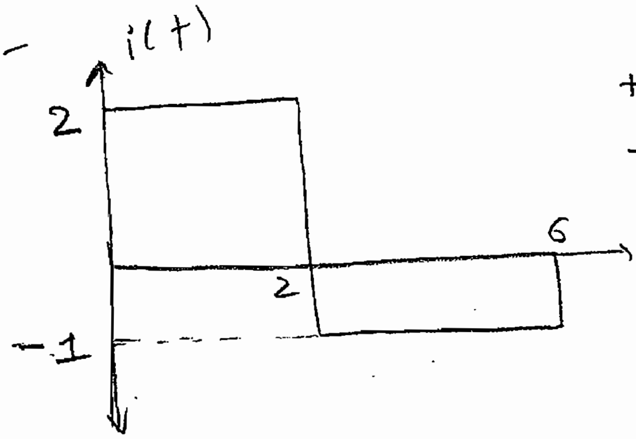



$|A_1| = |A_2| \Rightarrow$ Symmetrical

$|A_1| \neq |A_2|$

$\Rightarrow$ Asymmetrical

Eg:-




+ve area $= |2 \times 2| = 4$

-ve area $= |4 \times (-1)| = 4$

$\Rightarrow$ Symmetrical

The average value of any symmetrical waveform for one full cycle is always zero.

(a) For symmetrical waveform :-

$$V_{avg} = \begin{cases} 0 \longrightarrow \text{full cycle} \\ \dfrac{1}{(T/2)} \displaystyle\int_0^{T/2} v(t)\,dt \longrightarrow \text{half cycle} \end{cases}$$

(b) For asymmetrical waveform :-

$$V_{avg} = \frac{1}{T} \int_0^T v(t)\,dt \longrightarrow \text{Full cycle}$$

## Peak factor / Crest Factor :—

$$= \frac{V_{max}}{V_{RMS}}$$

## Form Factor / Shape Factor :—

$$= \frac{V_{RMS}}{V_{avg}}$$

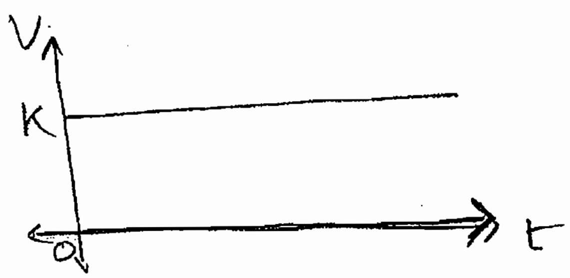
## Peak to Peak value :—

$$V_{p-p} = |V_{max} - V_{min}|$$

**Note:**—
Most of our electrical utilities application involves heat generation so we talk RMS values in general.
eg:- 1 Φ, Domestic supply in India
$= 230\ V \longrightarrow$ RMS value.

However applications like battery charging electro plating, electro refining process, etc involves charge, so we calculate "avg." values.

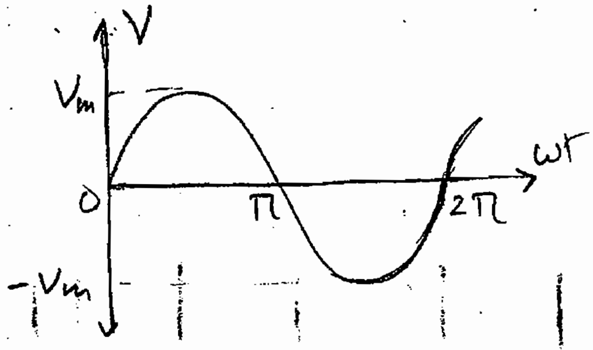
## Standard waveforms :—




$V_{RMS} = k$ F.F = 1

$V_{avg} = k$ $V_{PP} = k$

P.F. = 1

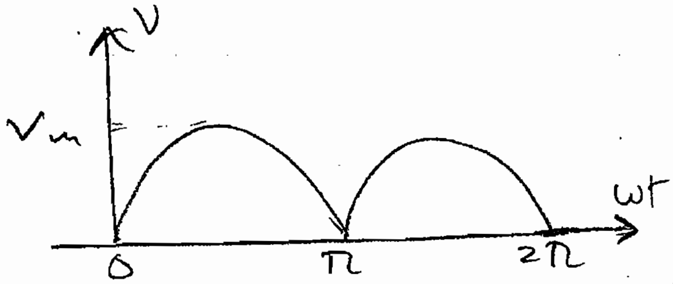



$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{avg} = 0 \longrightarrow \text{Full cycle}$$

$$= \frac{2V_m}{\pi} \longrightarrow \text{Half cycle}$$

$$P.F = \sqrt{2} \qquad F.F = \frac{\pi}{2\sqrt{2}} = 1.11 \qquad V_{pp} = 2V_m$$

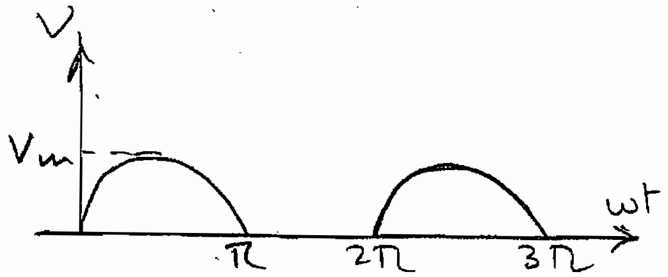



$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_{avg} = \frac{2V_m}{\pi}$$

$$PF = \sqrt{2} \qquad FF = 1.11$$

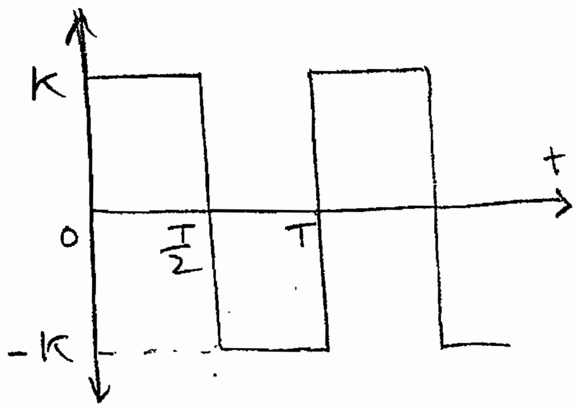
$$V_{pp} = V_m$$




$$V_{RMS} = \frac{V_m}{2} \qquad V_{avg} = \frac{V_m}{\pi}$$

$$P.F = 2 \qquad F.F = 1.57$$

$$V_{pp} = V_m$$



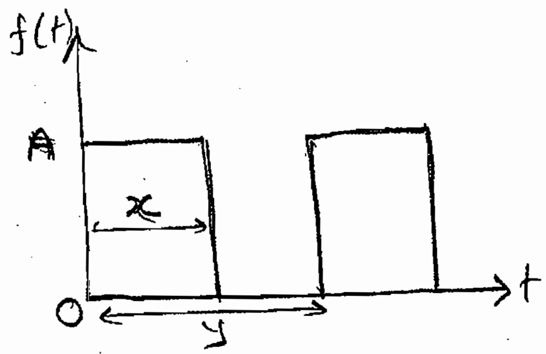

$$V_{RMS} = k$$

$$V_{avg} = 0 \longrightarrow \text{Full cycle}$$

$$= k \longrightarrow \text{Half cycle}$$

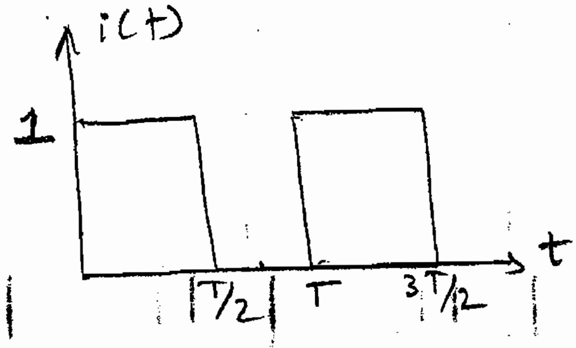
$$P.F = 1 \qquad FF = 1.57$$

$$V_{pp} = V_m$$




$$f_{RMS} = A\sqrt{\frac{x}{y}}$$

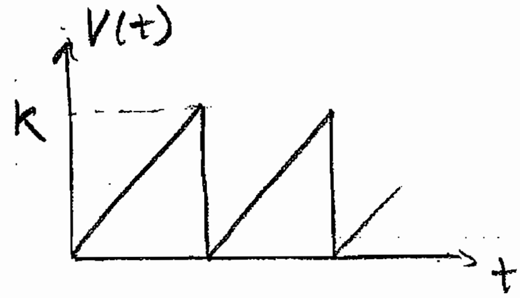
$$f_{avg} = A\left[\frac{x}{y}\right]$$




$I_{RMS} = \frac{I_0}{\sqrt{2}}$ $\quad$ $I_{avg} = \frac{I_0}{2}$

$P.F = \sqrt{2}$ $\quad$ $F.F = \sqrt{2}$

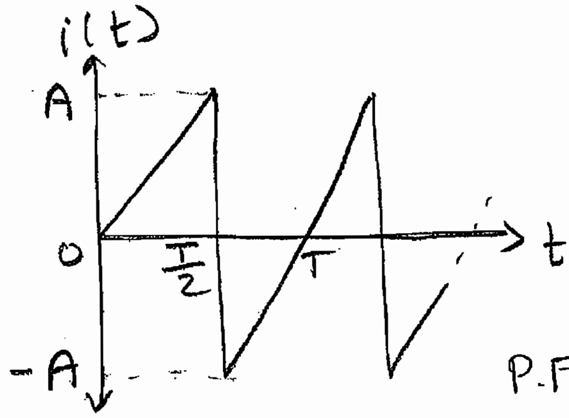
$I_{PP} = I_0$




$V_{RMS} = \frac{k}{\sqrt{2}}$ $\quad$ $V_{avg} = \frac{k}{2}$

$P.F = \sqrt{3}$ $\quad$ $F.F =$

$V_{PP} = k$

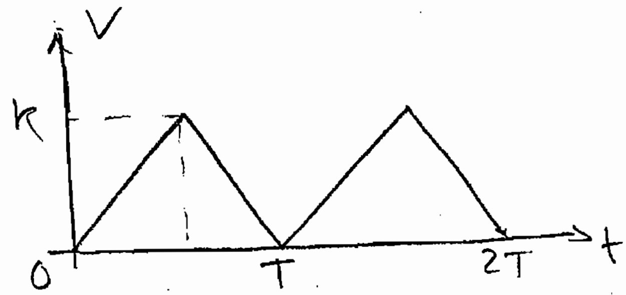



$I_{rms} = \frac{A}{\sqrt{3}}$

$I_{avg} = 0 \longrightarrow$ full cycle

$= \frac{A}{2} \rightarrow$ half cycle

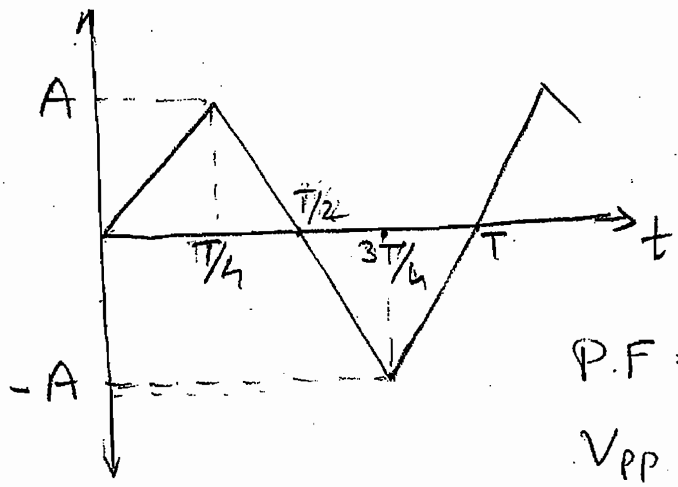
$P.F = \sqrt{3}$ $\quad$ $F.F = \frac{2}{\sqrt{3}}$ $\quad$ $I_{p.p} = 2A$




$V_{RMS} = \frac{k}{\sqrt{3}}$ $\quad$ $V_{avg} = \frac{k}{2}$

$P.F = \sqrt{3}$ $\quad$ $FF = \frac{2}{\sqrt{3}}$

$V_{PP} = k$




$V_{RMS} = \frac{A}{\sqrt{3}}$

$V_{avg} = 0 \rightarrow$ Full cycle

$= \frac{A}{2} \rightarrow$ Half cycle

$P.F = \sqrt{3}$ $\quad$ $FF = \frac{2}{\sqrt{3}}$

$V_{PP} = 2A$

**NOTE:-**

AC Analog Meters
↓
Moving Iron type
↓
RMS values

DC Analog Meters
↓
PMMC Type
↓
Average value

Rectifier Type

$$\begin{bmatrix}\text{final}\\ \text{value}\end{bmatrix} = \begin{bmatrix}\text{Avg.}\\ \text{value}\end{bmatrix} \times \underset{1.11}{\underbrace{\text{F.F.}}}$$ (of sinusoidal waveform)

→ Practical waveforms are not std.

So, we use Fourier series expansion to express these practical vltg & current waveforms into terms of sine (or) cosine.

eg: $v(t) = V_0 + V_1 \sin \omega t + V_2 \sin 2\omega t + \ldots\ldots$

$$V_{avg} = V_0$$

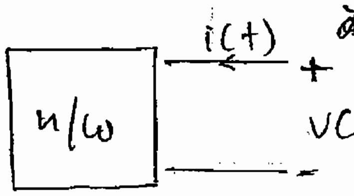
$$V_{rms} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \ldots\ldots}$$

$$i(t) = I_0 + I_1 \cos(\omega t - \phi_1) + I_3 \cos(3\omega t + \phi_3) + \ldots$$

$$\therefore I_{avg} = I_0$$

$$I_{RMS} = \sqrt{I_0^2 + \left(I_1/\sqrt{2}\right)^2 + \left(I_3/\sqrt{2}\right)^2 + \ldots}$$

i) Non sinusoidal excitation :-




let, $v(t) = V_0 + V_1 \sin\omega t + V_3 \sin 3\omega t + V_5 \sin 5\omega t + V_7 \sin 7\omega t$

$i(t) = I_0 + I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t + \phi_3) + I_7 \cos(7\omega t - \phi_7)$

$I_7 \sin(90 + 7\omega t - \phi_7)$

What is avg. power in n/w ?

NOTE :-

If freq. are same, don't use fourier series concept.

$$v(t) = V_1 \sin(\omega t + \phi) + V_2 \sin(\omega t - \phi)$$

$$= \underbrace{V_1 \angle \phi + V_2 \angle -\phi}_{\text{phasor addition}}$$

$$\Rightarrow V_{RMS} = \frac{V_{max}}{\sqrt{2}}$$

Solⁿ

$$P_{avg} = \frac{1}{T}\int_0^T v(t) \cdot i(t)\, dt$$

$$P_{avg} = V_0 I_0 + \frac{V_1}{\sqrt{2}} \cdot \frac{I_1}{\sqrt{2}} \cos\phi_1 + \frac{V_3}{\sqrt{2}} \frac{I_3}{\sqrt{2}} \cos\phi_3 + 0 + \frac{V_7}{\sqrt{2}} \cdot \frac{I_7}{\sqrt{2}} \cos(90 - \phi_7)$$

Avg. power is avg ~~a~~ value of as power waveform by itself the product of vltg & current waveform.

This avg power exist in resistive part of the n/w which is the consumable / convertible part.

So, simply avg power means **active power** in watts.

→ From power Δle avg power / active power for any combination of load can be expressed as.

$$P = V_{rms} \cdot I_{rms} \cos\phi$$

2) $v(t) = 10 + 5\sqrt{2}\cos\omega t + 3\sqrt{2}\sin 3\omega t$

$v(t) = 10 + 5\sqrt{2}\sin(90° + \omega t) + 3\sqrt{2}\sin 3\omega t$

~~$V_0 = 10$~~

$$V_{RMS} = \sqrt{100 + (5)^2 + (3)^2}$$

$$= \sqrt{134} = 11.57$$

$V_{avg} = 10$

3) $i(t) = 10 + 7\cos(\omega t - 10^\circ) + 5\cos[3\omega t + 30^\circ]$

$I_{avg} = 10$

$I_{RMS} = \sqrt{(10)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2}$

$= \sqrt{137} = \underline{11.7}$

4) $i(t) = 20 + 10\sin(\omega t - 30^\circ) + 7\cos(\omega t + 40^\circ)$

$= 20 + 10\sin(\omega t - 30^\circ) + 7\sin(\omega t + 130^\circ)$

$= 20 + [10\angle -30^\circ] + [7\angle +130^\circ]$

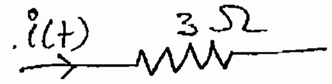
✓ $\cancel{= 20 + 8.66 - 5j + (-4.5 + 5.36j)}$

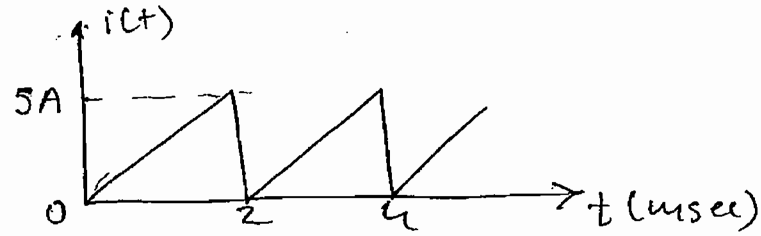
$= \cancel{24.16 + 0.36j}\ 20 + 4.16 + 0.36j$

$= 20 + [4.17\angle 4.94^\circ]$

$I_{rms} = \sqrt{(20)^2 + \left(\frac{4.17}{\sqrt{2}}\right)^2} = \underline{\underline{20.21}}$ $\qquad I_{avg} = 20$

5)



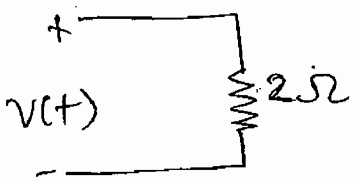


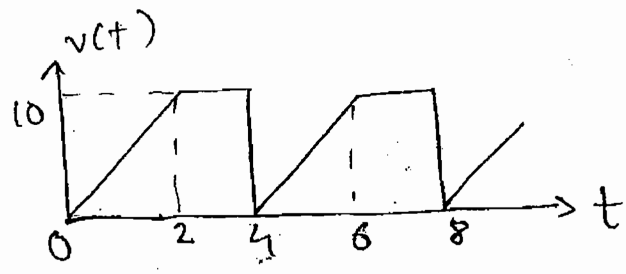

$P_{Lost} = I_{RMS}^2 \times R$

$= \left(\frac{5}{\sqrt{3}}\right)^2 \times 3$

$= 25W$

6)






$\underline{0 < t < 2}$

$v(t) = 5t$

$P_{Lost} = \left[\frac{V_{RMS}^2}{R}\right]$

$\underline{2 < t < 4}$

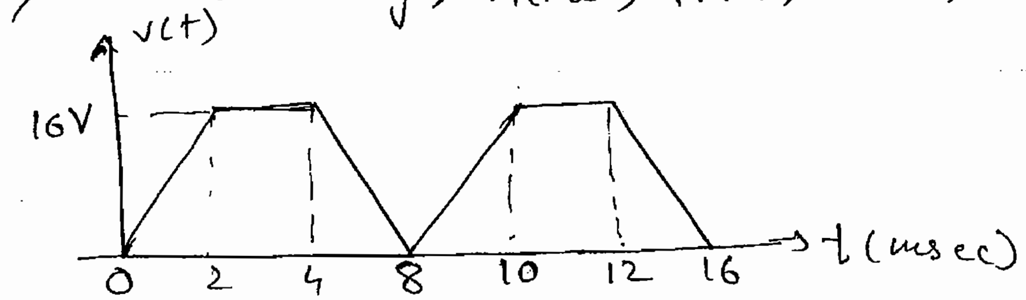
$v(t) = 10$

$$V_{RMS} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\left(\frac{8}{3}\right) + 100 \times 2\right]} = \sqrt{66.66}$$

$$P_{lost} = \frac{(V_{RMS})^2}{2} = \frac{66.66}{2} = 33.33\ W$$

7) Find $V_{avg}$, $V_{RMS}$, P.F., F.F., $V_{PP}$




→ asymmetrical

→ Calculate for full cycle

$0 < t < 2$ $\quad v(t) = 8t$

$2 < t < 4$ $\quad v(t) = 16$

$4 < t < 8$

$$(y-0) = \frac{(16-0)}{(4-8)}(x-8)$$

$$v(t) = -4t + 32$$

$$V_{avg} = \frac{1}{8}\left[\int_0^2 8t\,dt + \int_2^4 16\,dt + \int_4^8 (-4t+32)\,dt\right]$$

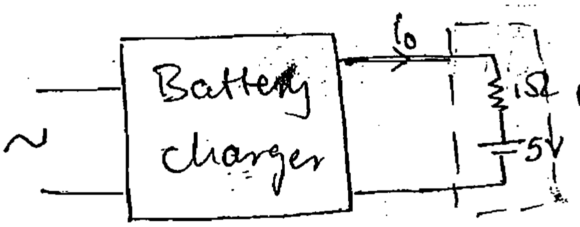
$$= \frac{1}{8}\left[4(4) + 16(2) + 32(4) - 2(48)\right] = 10\ V$$

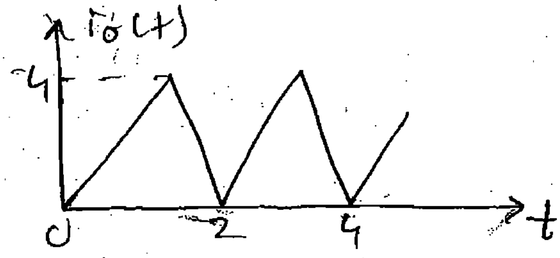
$$V_{rms} = \sqrt{\frac{1}{8}\left[\int_0^2 (8t)^2\,dt + \int_2^4 (16)^2\,dt + \int_4^8 (-4t+32)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{8}\left[\frac{64}{3}(8) + (16)^2(2) + (32)^2(4) + \frac{16}{2}(\cancel{48}\ 448) - 128(48)\right]}$$

$$= 8\sqrt{2}$$

$$PF = \frac{16}{8\sqrt{2}} = \sqrt{2} \qquad F.F = \frac{8\sqrt{2}}{10} = 1.13$$

8) 


Total power absorbed by cell = ?

$P_{absorbed} = (i_{o_{RMS}})^2 R + (i_{o_{avg}}) E$

$i_{o_{RMS}} = \frac{4}{\sqrt{3}}$ $\quad i_{o\,avg} = \frac{4}{2} = 2$

$\therefore P_{abs} = \left(\frac{4}{\sqrt{3}}\right)^2 \times 1 + (2) \times 5$

$= \frac{16}{3} + 10 = \frac{46}{3}$

$= 15.33\ W$

9) If a current of $i(t) = 5\cos(1000t + 100^\circ)$ A is flowing through an impedance of $(4+j3)\Omega$, the avg. power is ___ ?

$i(t) = 5\cos(1000t + 100^\circ)$

$z = 4 + j3 \Rightarrow R = 4$

$X_L = 3$

$P_{avg} = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt \longrightarrow$ exist only for resistive element

$\hookrightarrow$ active power.

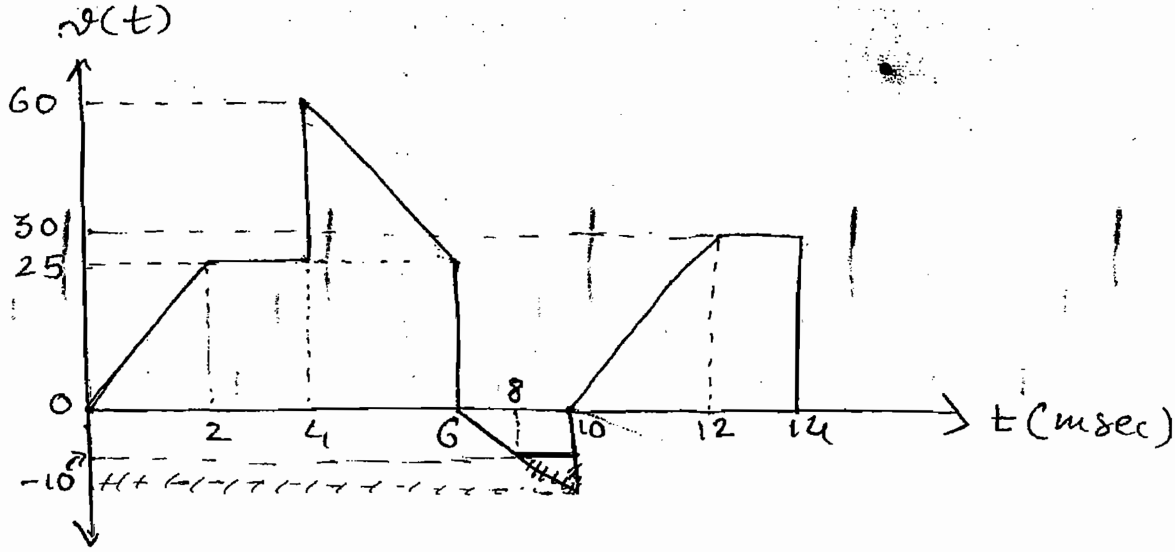
$\therefore P_{avg} = (i_{rms})^2 \times R$

$= \left(\frac{5}{\sqrt{2}}\right)^2 \times 4$

$= \frac{25}{2} \times \cancel{4}^{2}$

$= \underline{50\ W}$

10)




If the periodic vltg waveform shown is given to a ~~PMFC~~ PMMC type voltmeter then what is its readings.

PMMC $\xrightarrow[\text{provides}]{}$ Avg value

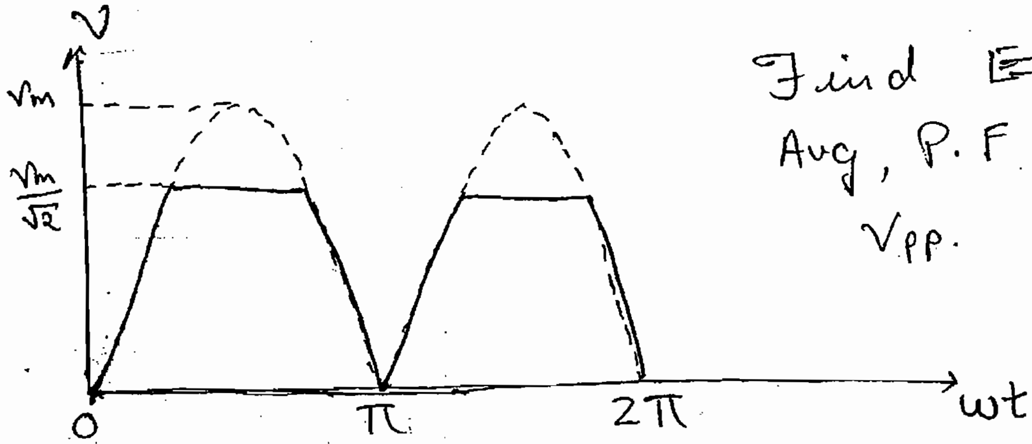
$\therefore V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$ which is area under the curve $v(t)$

$\therefore V_{avg} = \left\{\frac{1}{2}\times 25\times 2 + 2\times 25 + \frac{1}{2}\times 2\times 35 + 2\times 25 + \frac{1}{2}\times 2\times 25\right\} - \left\{\frac{1}{2}\times 2\times 10 + 2\times 10\right\}$

$\therefore$ ~~$V_{avg}$~~ Net area $= 250 - 30$
$= 220$

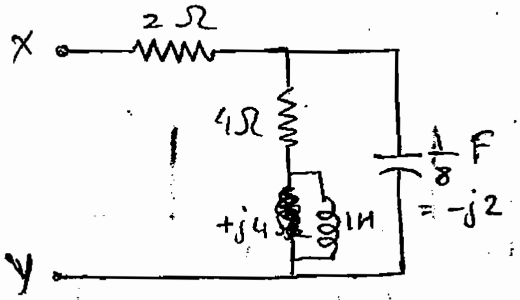
$\therefore V_{avg} = \frac{\text{Area}}{T} = \frac{220}{14} = 15.71$ V

11)




Find ~~EMF~~, RMS, Avg, P.F, FF, Vpp.

1) If $\omega = 4$ rad/sec. Find n/w P.F.



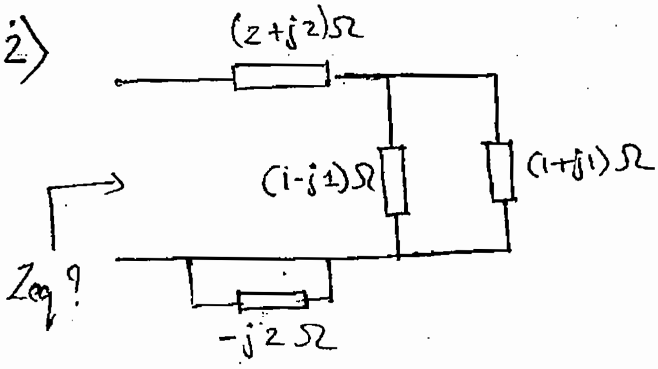

$$Z_{eq} = 2 + (4+j4)\,||\,(-2j)$$
$$= (2.8 - j2.4)\ \Omega$$

→ Overall R & n/w

$$\cos\phi = \frac{R}{|Z|} = \frac{2.8}{\sqrt{(2.8)^2+(2.4)^2}}$$

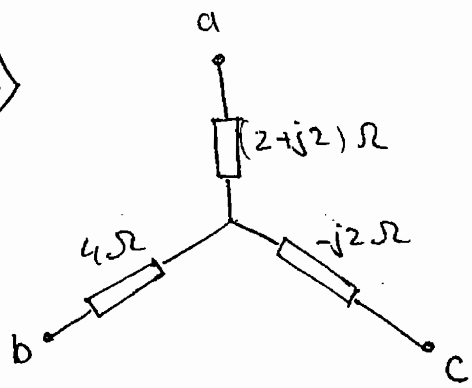
$\therefore$ PF = 0.76 (leading)

2)

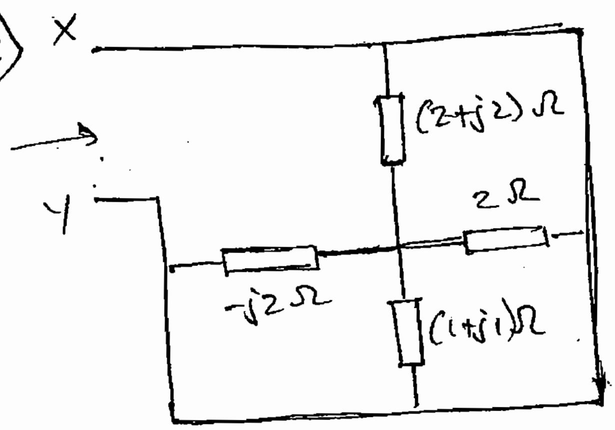



$$Z_{eq} = (2+j2) + [(1-j1)\,//\,(1+j1)]$$
$$= 2+j2+1$$
$$= 3+j2$$

3)

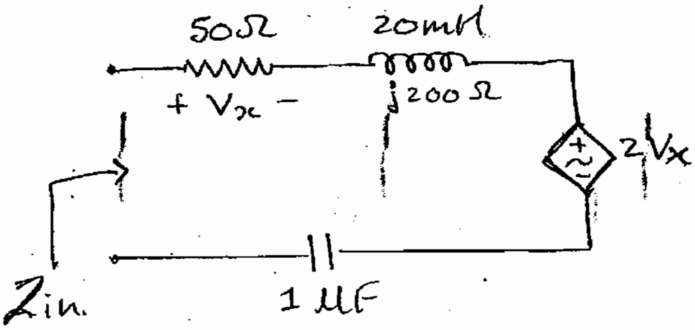


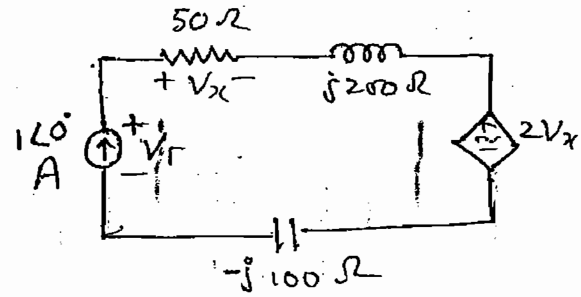

4)




$$Z_{eq} = 0$$

5) If $\omega = 10$ k rad/sec, $Z_{in}$ = ______



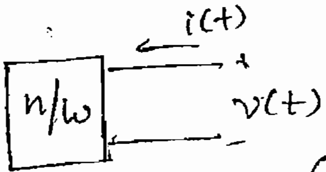




$\therefore V_x = 50(1) = 50\,V$

KVL $-V_T + [50 + j200 - j100)(1\angle 0°) + 100\angle 0° = 0$

$\therefore V_T = 150 + j100$

$Z_{in} = \dfrac{V_T}{1\angle 0} = (150 + j100)\ \Omega$

6) 


$v(t) = 20\cos(100t - 40°)$
$i(t) = 8\sin(100t + 10°)$
Determine n/w elements.

Here,
$v(t) = 20\cos(100t - 40)$
$= 20\sin(100t + 50°) \Rightarrow 20\angle 50°$

$i(t) = 8\sin(100t + 10°) \Rightarrow 8\angle 10°$

$Z = \dfrac{V}{I} = \dfrac{20\angle 50°}{8\angle 10°} = 2.5\angle 40°$
$= 1.91 + j1.6\ \Omega$

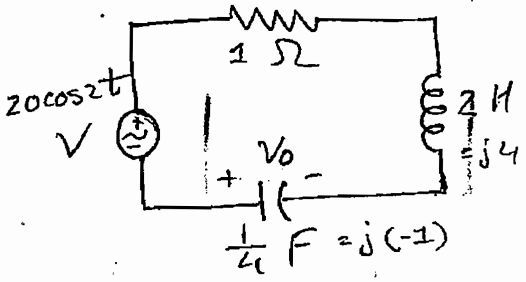
Simple form of n/w could be R-L series ckt

$\therefore R = 1.91\ \Omega$

$X_L = 1.6 \Rightarrow \omega L = 1.6 \Rightarrow L = 16\ mH$

It can also be a R-L-C n/w when $X_L > X_C$

7) Find $V_0$ = ______




Here $V = 20\cos 2t$

$\therefore \omega = 2$ rad/sec

$V_m = 20\angle 0°$

$$V_0 = \underbrace{\left[\frac{20\angle 0°}{\sqrt{2}}\right]}_{V_{rms}}\left[\frac{-j2}{1+j4-j2}\right]$$

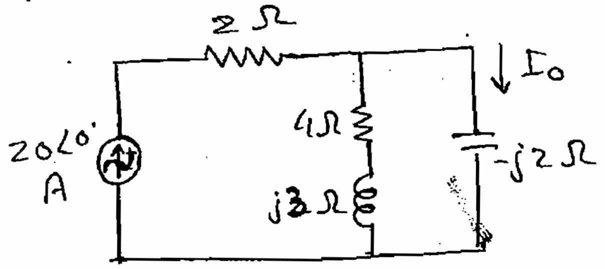
↓ RMS value

$$\therefore V_0 = \frac{(10\sqrt{2}\angle 0)(-j2)}{(1+j2)}$$

$\therefore V_0 = (11.3 + 5.6j)$ V

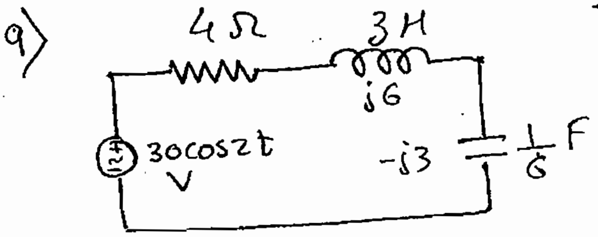
In phasor representation of polar quantity; $r\angle\theta$; '$r$' is clearly representing the max. value.

8) Find $I_0$ = ______




$$I_0 = \frac{(20\angle 0°)(4+j3)}{4+j1} =$$

9)




Find: ① Z ② Y ③ I

④ $V_R, V_L, V_C$

⑤ verify KVL

⑥ P, Q, S

⑦ Phasor diag.

⑧ P.F

① $Z = 4 + j6 - j3 = 4 + j3$

$|Z| = 5\ \Omega$

② $Y = \frac{1}{4+j3} = \frac{4-j3}{25} = (0.16 - j0.12)$ ℧

$\therefore |Y| = \frac{1}{|Z|} = 0.2$ S

(c) $I = \dfrac{\frac{30}{\sqrt{2}}\angle 0^\circ}{4+j3} = \dfrac{V}{Z} = \dfrac{30\angle 0^\circ}{5\sqrt{2}\angle 36.86^\circ}$

$\downarrow$ RMS

$= 4.24\angle -36.86^\circ$ A $\hookrightarrow$ impedance angle

$= I_R = I_L = I_C$

(d) $V_R = I_R R = (4.24\angle -36.86) \times 4$

$= 16.96\angle -36.86$ V

$V_L = I_L (jX_L) = (4.24\angle -36.86)(j6)$

$= 25.44\angle 53.14^\circ$ V

$V_C = I_C(-jX_C) = (4.24\angle -36.86)(-j3)$

$= \cancel{4.24\angle -36.86}$ $12.72\angle -126.86^\circ$ V

(e) $\bar{V}_S = \bar{V}_R + \bar{V}_L + \bar{V}_C$ [KVL $\rightarrow$ RMS value]

RHS $|\bar{V}_R| + |\bar{V}_L| + |\bar{V}_C|$

$= \cancel{16.96 + 25.44 + 12.72} = 21.16$

To be written in rect. form

LHS $V_S = \dfrac{30\cancel{\angle 0}}{\sqrt{2}} = 15\sqrt{2} = 21.2$

$\therefore$ LHS = RHS

$\therefore$ KVL verified.

(f) For calculating power consider RMS mag only

$P = I_R^2 R = \dfrac{V_R^2}{R} = V_R \cdot I_R$

$= \cancel{(16.96)^2}(4.42)^2 4 = \dfrac{(16.96)^2}{4} = (16.96)(4.42)$

$= 71.91$ watt.

$Q_{net} = +Q_L - Q_C$

$$Q_L = I_L^2 \cdot X_L = \frac{V_L^2}{X_L} = V_L \cdot I_L.$$

$$= (4.42)^2 (6) = \frac{(25.44)^2}{6} = (25.44)(4.42)$$

$$= 107.86 \text{ VAR (absorbing)}$$

$$Q_c = I_c^2 \cdot X_c = \frac{V_c^2}{X_c} = V_c \cdot I_c$$

$$= (4.42)^2 (3) = \frac{(12.72)^2}{3} = (12.72)(4.42)$$

$$= 53.93 \text{ VAR's (generating)}$$

$\therefore Q_{net} = 107.86 - 53.93$

$= +53.93$ VAR's (absorbing / lagging)

⑧ Total Power

$$S = V_s \cdot I_s = |I_s|^2 Z = \frac{|V_s|^2}{Z}$$

$$= (21.21)(4.24) = (4.24)^2 (5) = \frac{(21.21)^2}{5}$$

$$= 89.93 \text{ VA's}$$

⑨ Power factor $= \cos\phi = \dfrac{R}{Z} = \dfrac{P}{S}$

$$= \cos(36.36) = \frac{4}{5} = \frac{71.91}{89.93}$$

$$= 0.8 \text{ (lagging)}$$

Also check;

$$|Z| = \sqrt{R^2 + (X_L - X_c)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_c}{R}\right)$$

$$|S| = \sqrt{P^2 + (Q_L - Q_c)^2}$$

$$\phi = \tan^{-1}\left(\frac{Q_L - Q_c}{P}\right)$$

$$P = S\cos\phi = V_s I_s \cos\phi$$

$$Q_{net} = S\sin\phi = V_s I_s \cos\phi$$

Phasor diagram

$V_s = 21.21\angle 0^\circ$ ← Ref

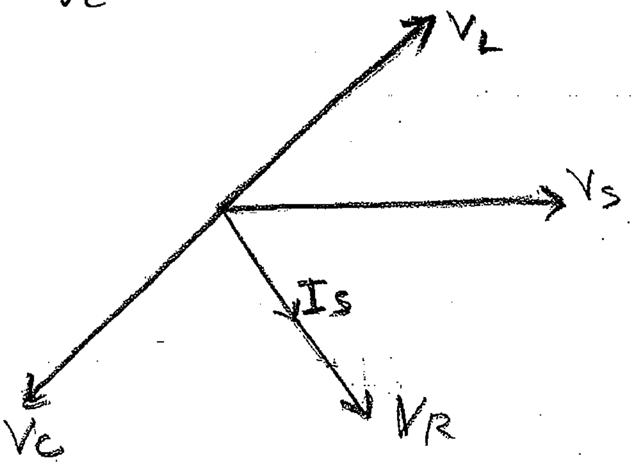
$I_s = I_R = I_L = I_C = 4.42\angle -36.86^\circ$ A

$V_R = 16.96\angle -36.86^\circ$ V

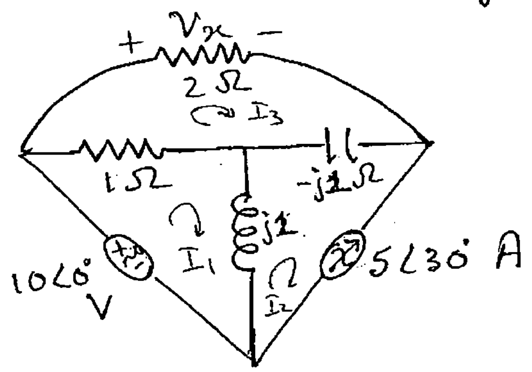
$V_L = 25.44\angle 53.14^\circ$ V

$V_C = 12.72\angle -126.86$ V

Normally we take I as ref. but here we take Vs as ref., becoz all calculations are done w.r.t Vs.




10) Find $V_x$ using mesh & nodal analysis.




Mesh

$$-[10\angle 0^\circ] + 1[I_1 - I_3] + j1[I_1 - I_2] = 0$$

$$(1+j)I_1 - jI_2 - I_3 = 10 \quad \text{——} (1)$$

$$I_2 = -[5\angle 30^\circ] \quad \text{——} (2)$$

$$2I_3 - j1[I_3 - I_2] + 1[I_3 - I_1] = 0$$

$$\therefore\ -I_1 + jI_2 + (3-j)I_3 = 0 \quad \text{——} (3)$$

$$(1+j)I_1 - I_3 = \underbrace{10 - 5\angle 120^\circ}_{12.5 - j4.33} \quad \text{——} (A)$$

$$-I_1 + (3-j)I_3 = \underbrace{5\angle 120^\circ}_{-2.5 + j4.33} \quad \text{——} (B)$$

$(A) \Rightarrow (1+j1) I_1 - I_3 = 12.5 - j4.33$

$(B)(1+j) \Rightarrow -(1+j1) I_1 + (4+2j) I_3 = -6.83 + j1.83$

$(3+j2) I_3 = 5.67 - j2.5$

$\therefore I_3 = 1.71 \angle -57.48^\circ$ A

then, $V_x = 2 I_3 = 3.42 \angle -57.48^\circ$ V

**Nodal**

$V_1 = 10$ —— (1)

$$\frac{V_2 - 10}{1} + \frac{V_2}{j} + \frac{V_2 - V_3}{-j1} = 0$$

$V_2 - 10 - jV_2 + j(V_2 - V_3) = 0$

$\therefore V_2 - jV_3 = 10$ —— (2)

$$-[5 \angle 30^\circ] + \frac{V_3 - V_2}{-j1} + \frac{V_3 - 10}{2} = 0$$

$$\frac{j(V_3 - V_2)}{1} + \frac{(V_3 - 10)}{2} = 5 \angle 30^\circ$$

$-j2 V_2 + (1+j2) V_3 = 10 + 10 \angle 30^\circ$

$= 18.66 + j5$ —— (3)

(2) $\times j2 \Rightarrow j2 V_2 + 2V_3 = j20$

(3) $\Rightarrow -j2 V_2 + (1+j2) V_3 = 18.66 + j5$

$$V_3 = \frac{18.66 + j5}{3 + j2}$$

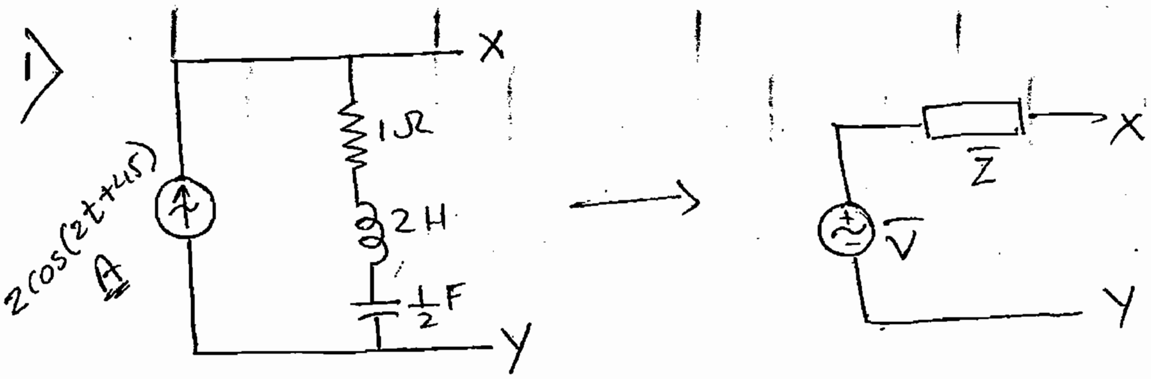
$\therefore V_3 = 8.65 \angle 19.57^\circ$

$V_x = \bar{V}_1 - \bar{V}_3 = 10 - [8.66 \angle 19.57^\circ]$

$= 3.43 \angle -57.44^\circ$ V

Theorem 1

# Source Transformation Technique :-

1) 



Here $I = 2\angle 45^\circ$ ; $\omega = 2$

$\therefore \bar{Z} = (1+j4) - j1) = (1+j3)\ \Omega$

$\bar{V} = \bar{I}\cdot Z = 2\angle 45^\circ \cdot (1+j3)$

$= 2\angle 45^\circ \times 3.16\angle 71.56^\circ$

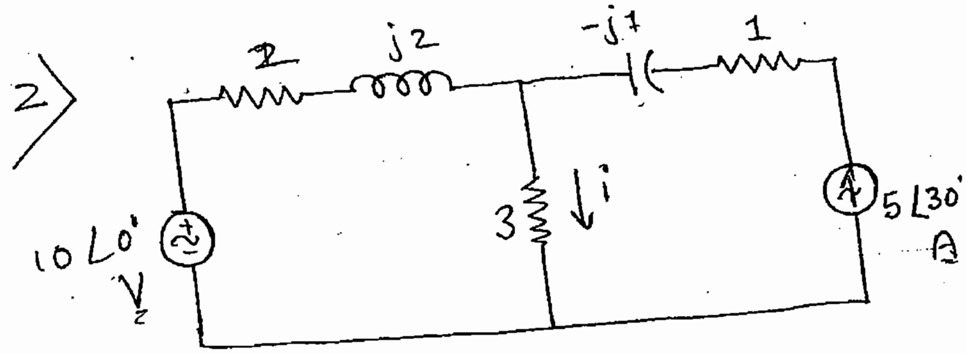
$= 6.32\angle 116.56$ V

$\therefore \bar{V} = 6.32\cos[2t + 116.56]$ V

Theorem 2 : Superposition theo.

Theorem 3 : Thevenin's theo.

Theorem 4 : Norton's theo.

2) 



Find i using Superposition Thevenin Norton

(1) Superposition :-

① 10 V only : $i' = \dfrac{10\angle 0^\circ}{5+j2} = \dfrac{10\angle 0^\circ}{5.38\angle 21.80^\circ}$

$\therefore i' = 1.857 \angle -21.8^\circ$

② 5A only:

$i'' = 5\angle 30^\circ \left[\dfrac{2+j2}{5+j2}\right] = 5\angle 30^\circ \times (0.482 + 0.2j)$

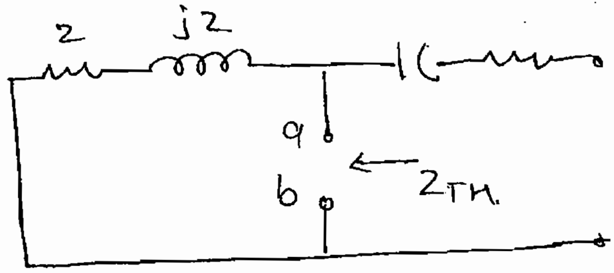
$= 2.61 \angle 53.19^\circ$

$\therefore i = i' + i''$

$= 3.58 \angle 23.2^\circ$ A

## ⟨2⟩ Thevenin

① ~~10V only~~



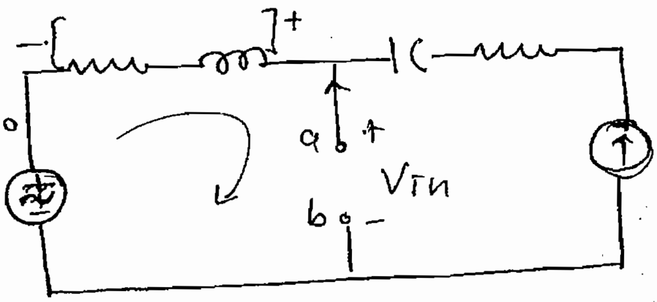

$Z_{TH} = (2+j2)\ \Omega$

②

KVL

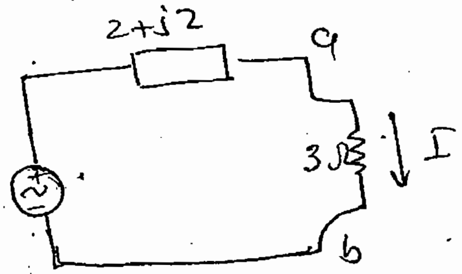
$-[10\angle 0^\circ] + V_{TH} - (2+j2)(5\angle 30^\circ) = 0$

$\therefore V_{TH} = 19.32\angle 45^\circ$




$I = \dfrac{19.32\angle 45^\circ}{5+j2}$

$= 3.58 \angle 23.2^\circ$ A

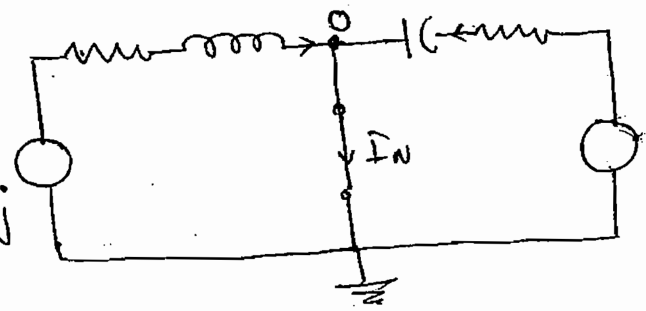



## ⟨3⟩ Nortons

$I_N = \dfrac{10\angle 0^\circ}{(2+j2)} + 5\angle 30^\circ$

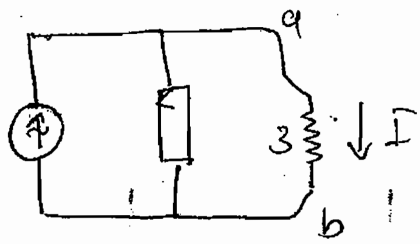
$= \dfrac{10\angle 0^\circ}{2.82\angle 45^\circ} + 4.33 + 2.5j$

$= 6.83 \angle 23.2^\circ$ A

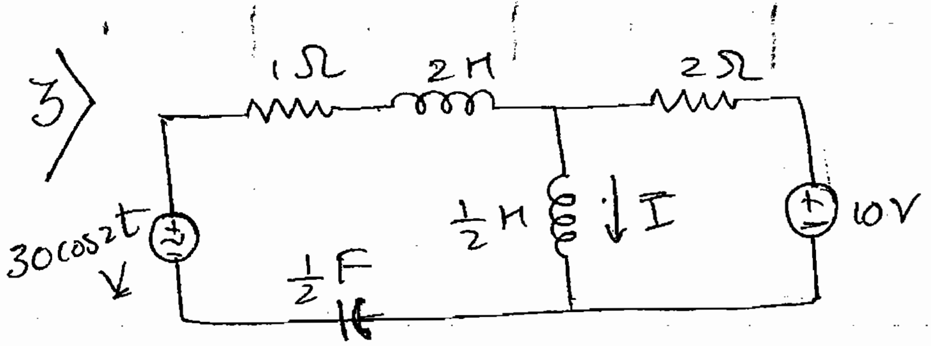


$$I = 6.83\angle 0 \left[\frac{2+j2}{5+j2}\right]$$

$$= 3.58\angle 23.2^\circ \ A$$



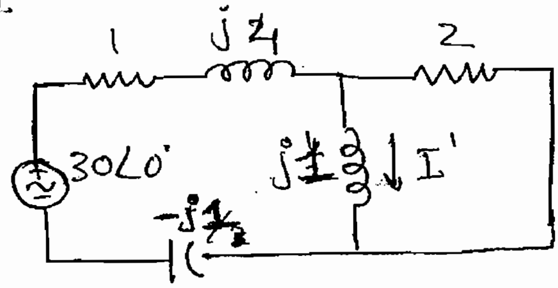

3)




Find I

This ckt is multi frequency excited. So in time domain only superposition theorem can provide the solution. But we can also do this problem in freq. domain by applying Laplace Transforms.

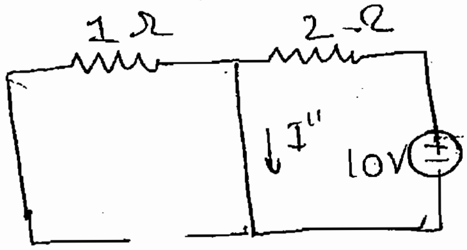
① 30 V AC source only




$$I' = \frac{30\angle 0^\circ}{(1+j3)+[2 \| j1]} \cdot \frac{2}{2+j1}$$

$$= 6.63\angle -96^\circ \ A$$

② 10V DC source only

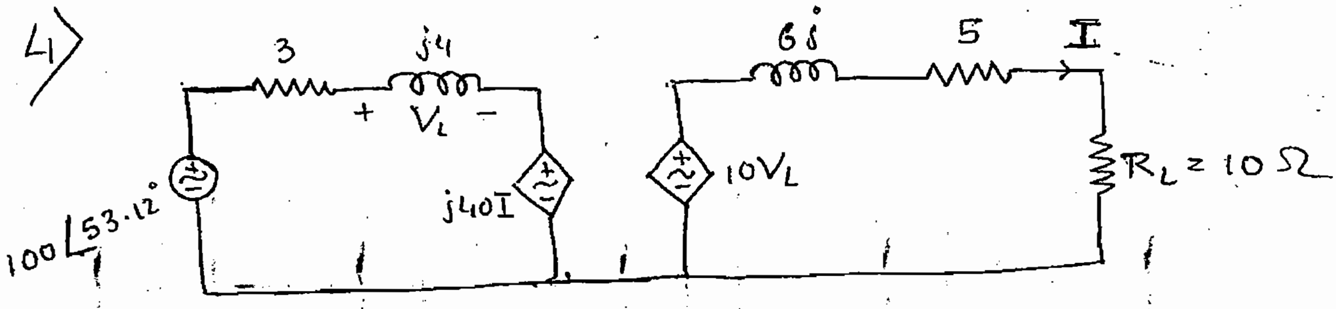



$$I'' = \frac{10}{2} = 5 \ A$$

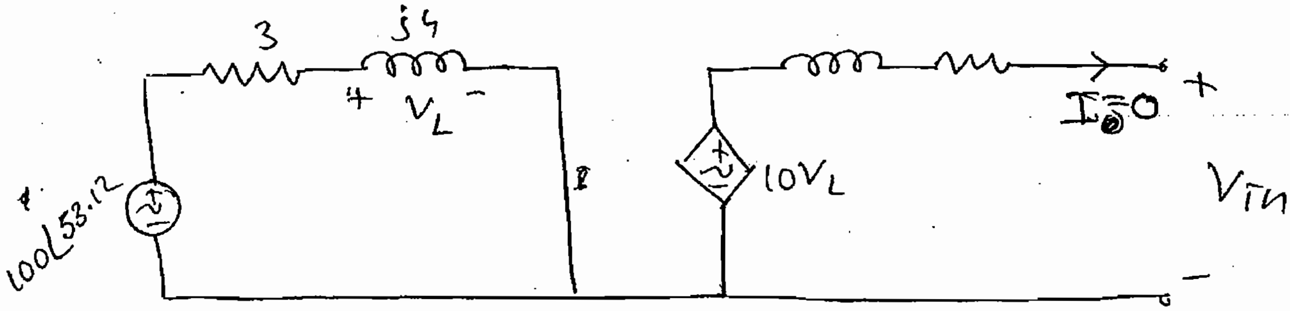
By SPT

$$I = 5 + 6.63\cos(2t - 96^\circ) \ A$$

4)



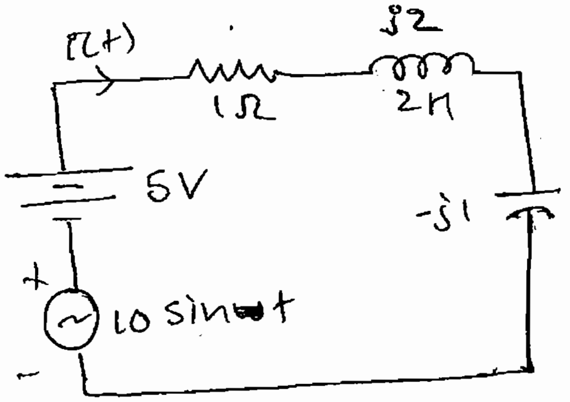

Thevinin equivalent vltg across the load.




$$V_L = 100\angle 53.12)\left[\frac{j4}{3+j4}\right] = 80\angle 90^\circ$$

$$\therefore V_{Th} = 10V_L = 800\angle 90^\circ$$

5)




$i(t) = ?$

For DC (5V)

$i'(t) = 0$

For 10V ac source:

$$i''(t) = \frac{10\angle 0^\circ}{1+j1} = 7.07\angle -45^\circ$$

∴ By SPT $\quad i(t) = 7.07\sin(t-45^\circ)$

6) $\sqrt{\dfrac{(10\angle 20^\circ)(30\angle 40^\circ)}{50\angle 70^\circ)}} = \sqrt{6\angle -10^\circ} = \sqrt{6e^{-j10}}$

$= \sqrt{6}\, e^{-j5} = \sqrt{6}\angle -5^\circ$

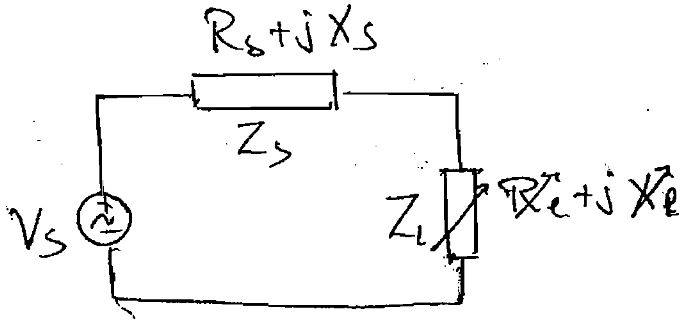
## Theorem 5

### Maximum Power Transfer theo.

Though there are 3 types of physical power existing in AC steady state n/ws, the power that is consumable / utilizable / convertible in any other form is Active power or Real power in watts.

So max. power transfer theo. is confined to active power only.

### General case :




In this case both the resistive part & reactive part are controllable.

Since our target is to maximise active power in the load. (i.e. $R_L$), here if we can compensate the net reactance so Total power = Active power

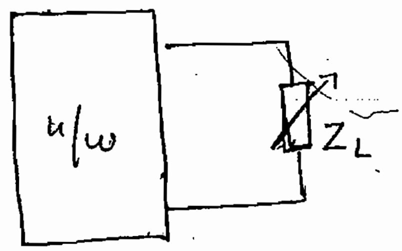
so $P_{max}$ occurs in the load if load

impedance is complex conjugate of equivalent source impedance seen by it. i.e. $Z_\ell = Z_s^* = R_s - jX_s$ &

$$P_{max} = \frac{|V_s|^2}{4R_s} \; W$$

rms value (heat concept for power)

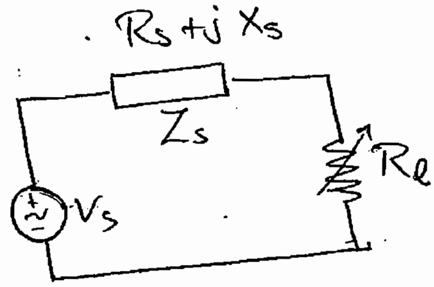
## In General :—




Pmax occurs in the load when $Z_\ell = Z_{TH}^*$

and. $P_{max} = \dfrac{|V_{TH}|^2}{4R_{TH}}$ ; $R_{TH}$ = Real part $[Z_{TH}^*]$

## Special case :— (a)




Since load is purely resistive but source has some reactance; ~~which~~ there we cannot avoid reactive power in the n/w. Since phase balancing of impedances is not possible so to extract max. power atleast balance the magnitude so Pmax occurs in the load. if:

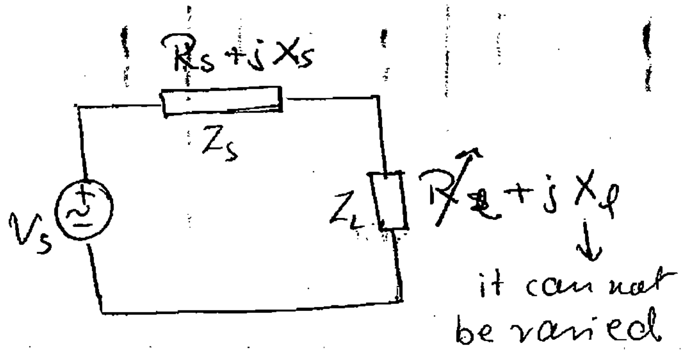
$$R_\ell = |Z_\ell| = \sqrt{R_s^2 + X_s^2}$$

Now to calculate Pmax, resubstitute the calculated value of $R_\ell$ to find rms current in it. Then calculate

$$P_{max} = I_{rms}^2 \times R_\ell$$

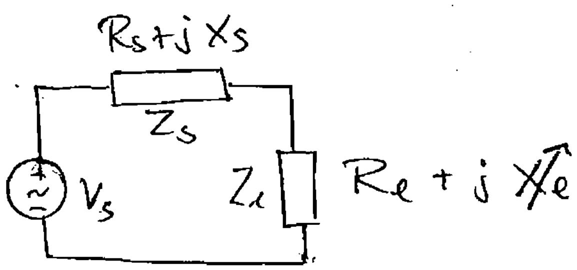
Use Thevenins concept to all other general circuit.

## Special Case :- (b)




$$R_\ell = |R_s + jX_s + jX_\ell|$$
$$= \sqrt{R_s^2 + (X_\ell + X_s)^2}$$

## Special Case :- (c)

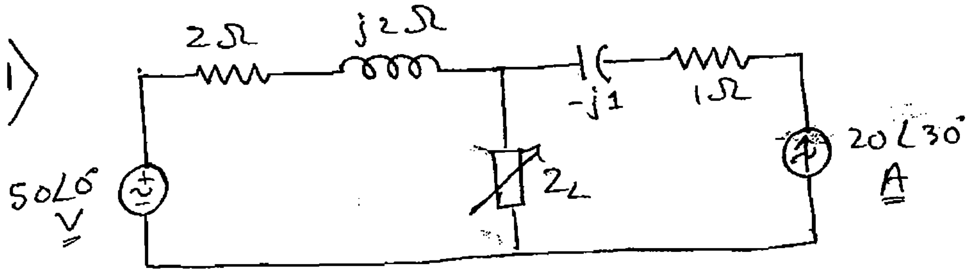



For $P_{max}$;

$$X_\ell = -X_s$$
$$X_\ell + X_s = 0$$

Here, we can nullify the net reactance & reactive power but still resistance can not be controlled so $P_{max}$ occurs in the load if $X_\ell = -X_s$.

1)




Value of $Z_\ell$ for which $P_{max}$ occurs in it & also find $P_{max}$.

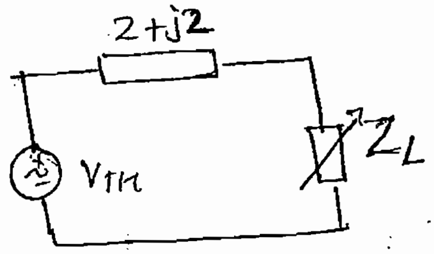
→ Determine Thevenin equi. across $Z_\ell$.

① $Z_{TH} = (2 + j2)\ \Omega$

② KVL $\quad -[50\angle 0°] - (2 + j2)(20\angle 30°) + V_{TH} = 0$

$\therefore V_{TH} = 84.64 \angle 40.2°$

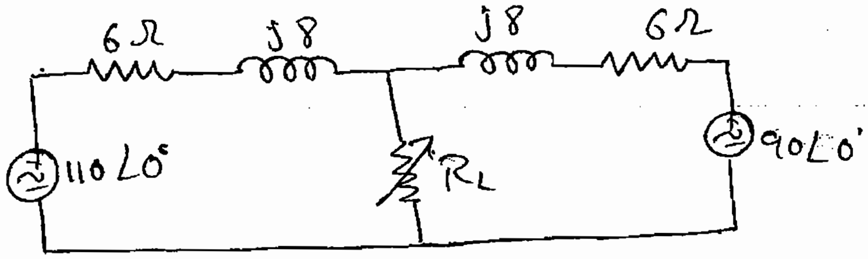
$Z_\ell = Z_{TH}^* = (2 - j2)\,\Omega$
$\uparrow$
$R_{TH}$




$$P_{max} = \frac{|V_{TH}|^2}{4 R_{TH}} = \frac{|84.64|^2}{4 \times 2}$$

$$= \underline{895}\ W$$

2)

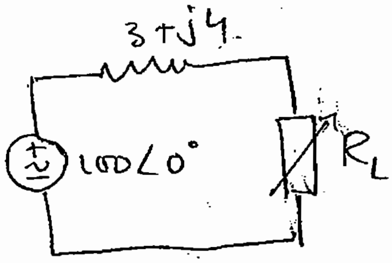



What is Pmax transfered to load.

① $Z_{TH} = (6 + j8) \,//\, (6 + j8) = (3 + j4)\,\Omega$

② ~~i * 100∠0°~~ .. Nodal Analysis :-

$$\frac{(V_{TH} - 110\angle 0^\circ)}{6 + j8} + \frac{V_{TH} - 90\angle 0^\circ}{6 + j8} = 0$$




$\therefore\ 2V_{TH} = 200\angle 0^\circ$

$V_{TH} = 100\angle 0^\circ$

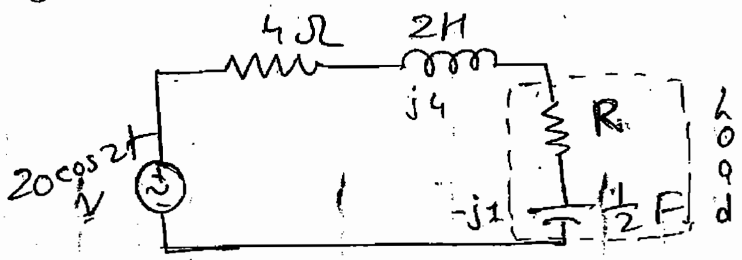
$R_\ell = |Z_{TH}| = \sqrt{3^2 + 4^2} = 5\,\Omega$

$$I_{5\Omega} = \frac{100\angle 0^\circ}{(3 + j4 + 5)} = 11.18\angle -26.56^\circ$$

$P_{max} = |11.18|^2 \times 5$

$= \underline{625\ W}$

3) What is $P_{max}$ transfered to load



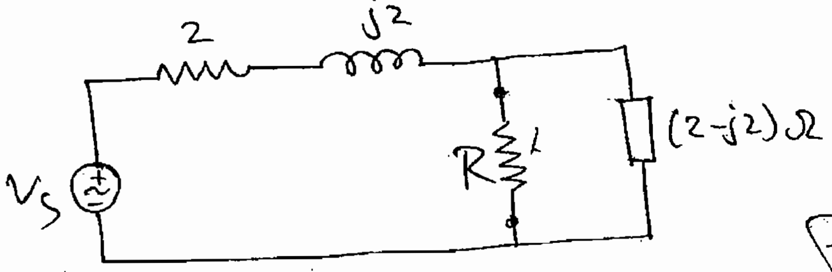

$$R = |4+j4-j1| = \sqrt{16+9} = 5\,\Omega$$

$$I_{Rms} = \frac{\frac{20}{\sqrt{2}}\angle 0^\circ}{(4+j4+5-j1)} = \frac{14.142\angle 0^\circ}{9.486\angle 18.43}$$

$$= 1.49\angle -18.43^\circ$$

$$P_{max} = |1.49|^2 \times 5 = \underline{11.10\ W}$$

4)



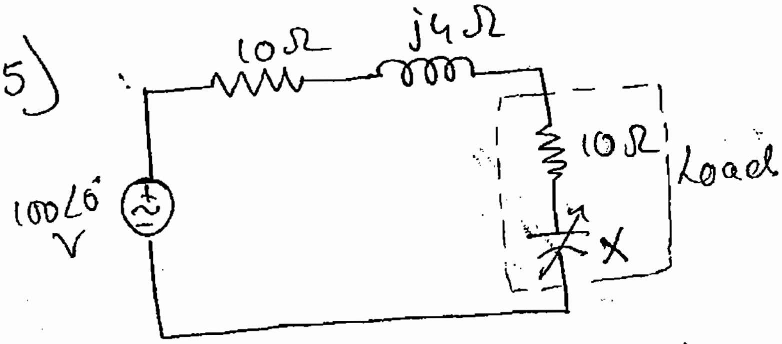

What is value of R for which $P_{max}$ occurs in it.

$$R = |Z_{TH}| =$$

$$Z_{TH} = (2+j2)\,||\,(2-j2) = \frac{8}{4} = 2$$

$$\therefore R = |2| = 2\,\Omega.$$

5)



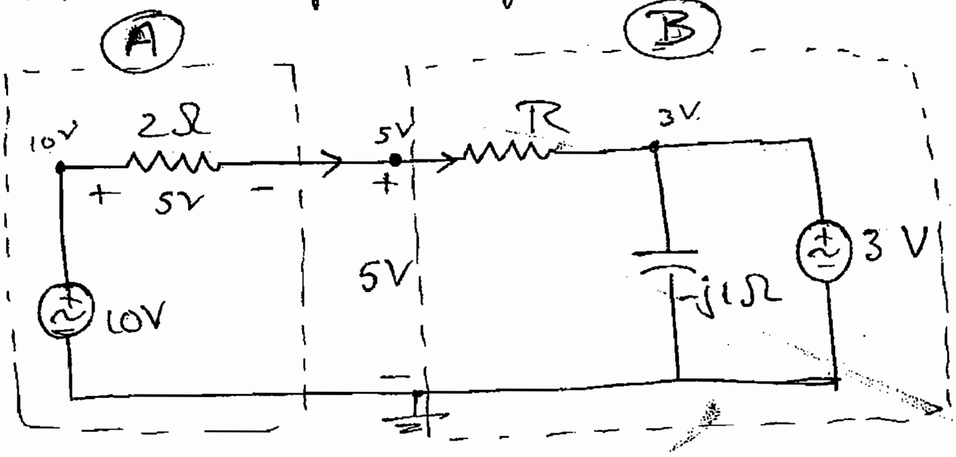

$P_{max}$ transfered to load?

$$Z_{TH} = 10+j4$$

$V_{TH}$ = ~~100∠0°~~ then $X = -j4$

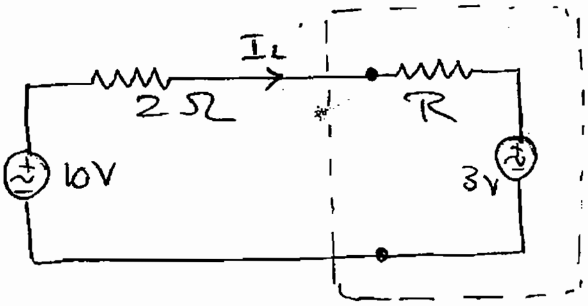
$$P_{max} = I^2 R$$
$$= \left[\frac{100\angle 0^\circ}{20}\right]^2 \times 10 = \underline{250\,W}$$

6) For what value of R max. power is transfered from ckt A to ckt B.




Short cut
$$\frac{10-5}{2} = \frac{5-3}{R} \Rightarrow R = \frac{2\times 2}{5} = 0.8\,\Omega$$

Exact proof




→ T.E. of ckt (B) as seen by ckt (A)

$$I_L = \frac{(10-3)}{2+R} = \frac{7}{2+R}$$

$$P_L = I_L^2 \cdot R + 3(I_L) = \frac{49}{(2+R)^2} \times R + 3\left(\frac{7}{2+R}\right)$$

$$\therefore P_L = \frac{7}{2+R}\left[\frac{7R}{2+R} + 3\right] = \frac{70R+42}{(2+R)^2}$$

For $P_{max}$; $\dfrac{dP_L}{dR} = 0$

$$\left[\frac{(2+R)^2(70)-(70R+42)\cdot(2)(2+R)}{(2+R)^4}\right]=0$$

$\therefore (2+R)^2(70) = 2(2+R)(70R+42)$

$\therefore 140+70R = 140R+84$

$\therefore 70R = 56$

$\therefore \boxed{R = 0.8\,\Omega}$

## Concept of Complex Power :— $(S^*)$

$S^* = V I^*$

It is a mathematical concept which removes the confusion in calculating the 3 physical powers: S, P, Q. in any n/w or element.

It will be very essential to calculate instantaneous power in AC ckts to estimate, design, control & analyse stability, fault calculations, etc in electrical power system since load is very dynamic in nature.

$I^*$ To verify Tellegens theorem, to calculate power at any instant, we need complex power.

Here, $I^* \rightarrow$ complex conjugate I

eg: $I = 10\angle 20^\circ$ A $\Rightarrow$ $I^* = 10\angle -20^\circ$ A

Units of $S^*$ : Volt-Ampere (VA's.

We can also write.

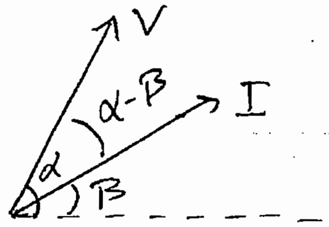
$S^* = V^* I$ but generally we write

$S^* = V I^*$ as $V \rightarrow$ is the cause &

$I \rightarrow$ is the effect.

Ex:1

In one element




$V = |V| \angle \alpha$

$I = |I| \angle \beta$

→ If we want to calculate inst. power

$V \cdot I = |V| |I| \angle \alpha + \beta$ ✗

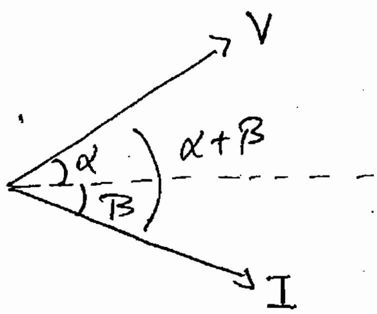
→ Correct way of calculating inst-power

$V \cdot I^* = |V| |I| \angle \alpha - \beta$ ✓

Ex:2

$V = |V| \angle \alpha$

$I = |I| \angle -\beta$



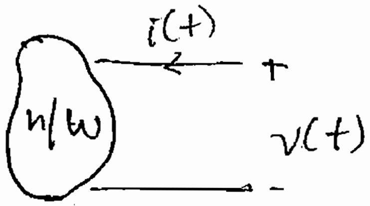

→ just VI product

$|V| |I| \angle \alpha - \beta$ ✗

→ Consider $S^* = V I^*$

$= |V| |I| \angle \alpha + \beta$ ✓

1)




Here, $v(t) = 20 + j12$

$i(t) = 5 + j4$

Calculate : P, Q, in the n/w

$v(t) \rightarrow 20 + j12 \rightarrow 23.32\angle 30.96^\circ$ V

$i(t) \rightarrow 5 + j4 \rightarrow 6.4\angle 38.65^\circ$ A

Complex Power $S^* = V I^*$

$$= 149.248 \angle -7.69^\circ$$

$$= \underset{\downarrow \, P}{147.9} - \underset{\downarrow \, Q_c}{19.97j} \ \underline{VA}$$

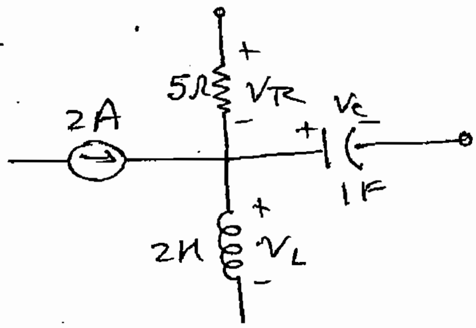
For our n/w :

$P = 147.9$ watts.

$Q_c = 19.97$ VAR's (generating.)

2) If $V_R = 5V$, $V_c = 4\sin 2t$ V, $V_L = ?$

$$2 + \frac{V_R}{5} = \frac{dV_c}{dt} + \frac{1}{2}\int V_L \, dt$$

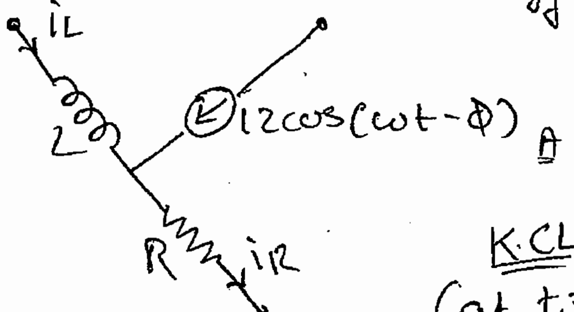



$$\frac{1}{2}\int V_L \, dt = 2 + 1 - 8\cos 2t$$

$$V_L = \frac{d}{dt}(6 - 16\cos 2t)$$

$$V_L = 32 \sin 2t \ \text{V}$$

3)




If. $i_R = (4e^{-3t} + 3e^{-4t})$ A

$i_L(0) = 1$ A.

$\Phi = \underline{\ ?\ }$

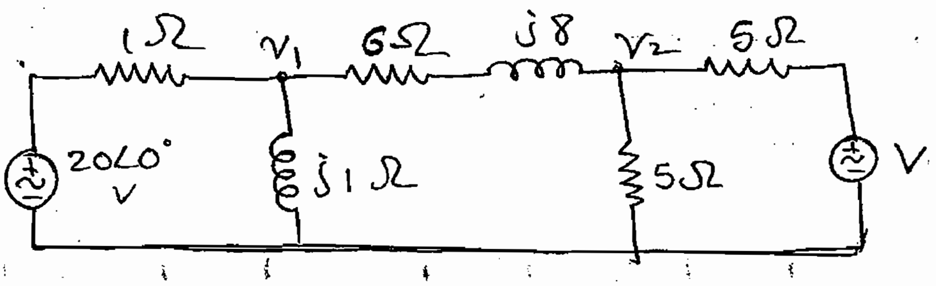
K.C.L (at t=0) : $i_L + 12\cos(\omega t - \Phi) = i_R$

$i_L(0) + 12\cos(-\Phi) = 4 + 3$

$1 + 12\cos\Phi = 7$

$$\therefore \cos\Phi = \frac{6}{12} = \frac{1}{2} \Rightarrow \boxed{\Phi = 60^\circ}$$

4)




If power dissipated in 6 Ω resis. is 0W then what is 'V'.

Here, $P_{6\Omega} = 0W \Rightarrow$ I across 6 Ω resis. is zero.

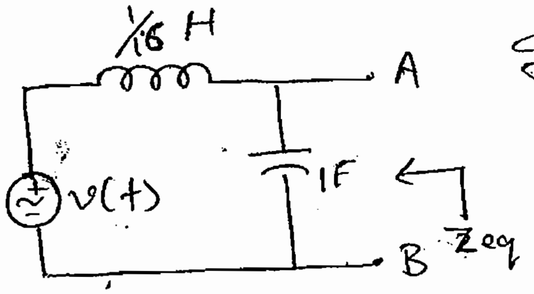
so, $V_1 = V_2$ (same in mag. & phase).

$$V_1 = 20\angle 0^\circ \left[\frac{j1}{1+j1}\right] = 10\sqrt{2}\angle 45^\circ$$

$$V_2 = 10\sqrt{2}\angle 45^\circ = V\left[\frac{5}{5+2}\right]$$

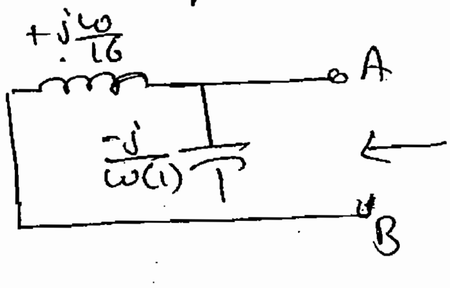
$$\therefore V = 2V_2 \Rightarrow V = 20\sqrt{2}\angle 45^\circ$$

5)




For what ckt frequency the n/w acts as ideal current source b/w A & B.

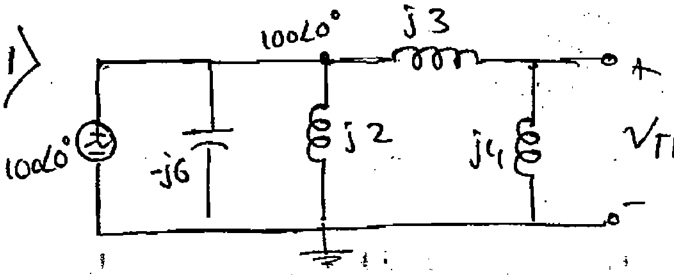
Ideal current source $\Rightarrow Z_{eq} = \infty$




$$Z_{eq} = \left(\frac{j\omega}{16}\right) \| \left(\frac{-j}{\omega}\right)$$

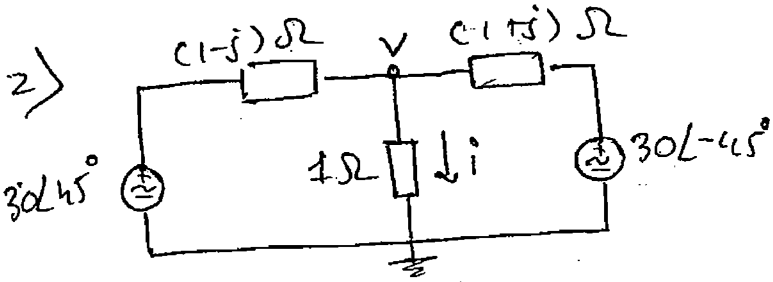
$$= \frac{1/16}{j\left[\frac{\omega}{16} - \frac{1}{\omega}\right]} \rightarrow \infty$$

$$\therefore \frac{\omega}{16} = \frac{1}{\omega} \Rightarrow \omega^2 = 16 \Rightarrow \boxed{\omega = 4}$$

1) 

$V_{TH} = ?$

$$V_{TH} = (100\angle 0^\circ)\,\frac{j4}{j5+j4} = \frac{400}{7}\angle 0^\circ \text{ V}$$

2) 

$i = ?$

NODAL $\quad \dfrac{V-[30\angle 45^\circ]}{1-j} + \dfrac{V-[30\angle -45^\circ]}{1+j} + \dfrac{V}{1} = 0$

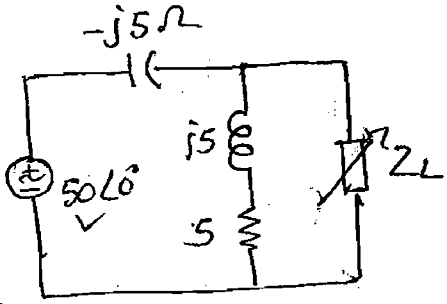
$$\therefore \frac{[V-(30\angle 45^\circ)](1+j)}{2} + \frac{[V-(30\angle -45^\circ)](1-j)}{2} + V = 0$$

$$\therefore 4V = (30\angle 45^\circ)(1+j) + (30\angle -45^\circ)(1-j)$$

$$= (30\sqrt{2}\angle 90^\circ) + (30\sqrt{2}\angle -90^\circ)$$

$$4V = 0 \Rightarrow V = 0 \Rightarrow \boxed{i = 0\text{ A}}$$

3) Whats the max. power transfered to the load impedance.



It is a general case prob.

$$\therefore Z_{TH} = (5+j5) \,//\, (-j5)$$

$$= \underline{5-j5}\ \Omega$$

For Pmax : $Z_L = Z_{TH}^{*} = \underline{5+j5}\ \Omega$

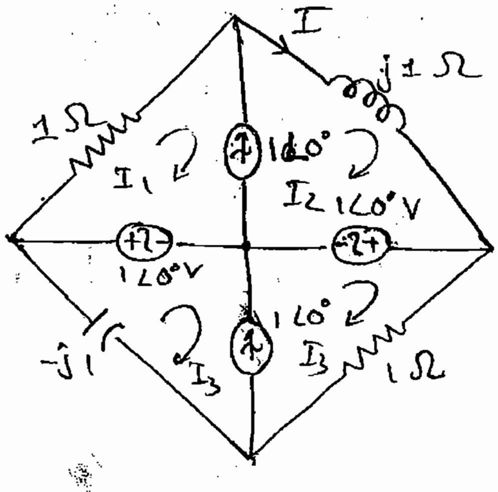
$$P_{max} = \frac{|V_{TH}|^2}{4R_{TH}}$$

$$V_{Th} = (50\angle 0^\circ)\cdot\left[\frac{5+j5}{5}\right] = 50\sqrt{2}\angle 45^\circ \text{ V}$$

$$\therefore P_{max} = \frac{(50\sqrt{2})^2}{4\times 5} = \underline{\underline{250\,W}}$$

2)

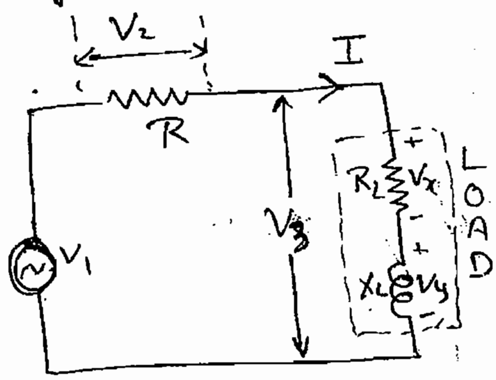
7) Find I.




<u>KVL</u>

$$I_1 + j I_2 = 0$$
$$-I_1 + I_2 = 1$$
$$(1+j) I_2 = 1$$
$$\therefore \; I_2 = \left(\frac{1}{1+j}\right) A$$

8) If. $V_1 = 220V$, $V_2 = 122V$, $V_3 = 136V$



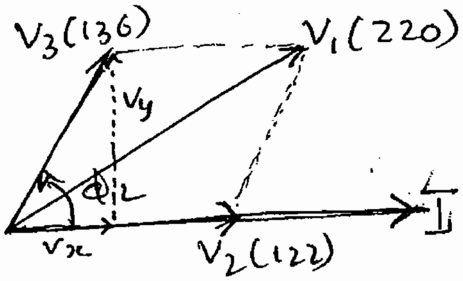

(a) Find Load Power factor

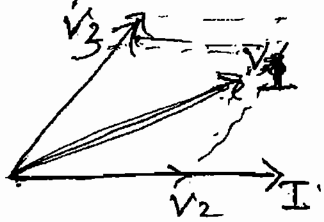
(b) If $R_L = 5\Omega$, what is avg power in the load. (i.e. active power in watts)

Load Power factor : $\cos\phi_L = \cos(V_3, I)$

N/w Power factor : $\cos\phi = \cos(V_1, I)$

→ This is based on phasor diagram:




<u>Formula</u> : $R_T = \sqrt{R_1^2 + R_2^2 + 2R_1R_2\cos(R_1, R_2)}$

$$\therefore \; 220 = \sqrt{(122)^2 + (136)^2 + 2(122)(136)\cos\phi_L}$$

$$\therefore \; \cos\phi_L = 0.45 \quad (\text{lagg})$$

Now, $\underline{R_L = 5\Omega}$

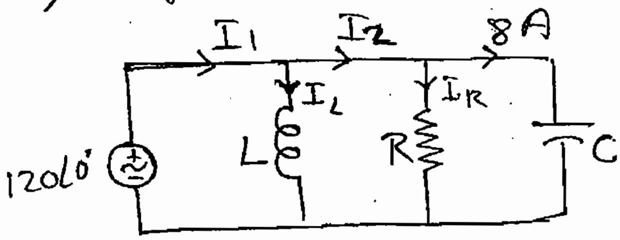
$$P = \frac{|V_x|^2}{R} = |I|^2 R = |I||V|$$

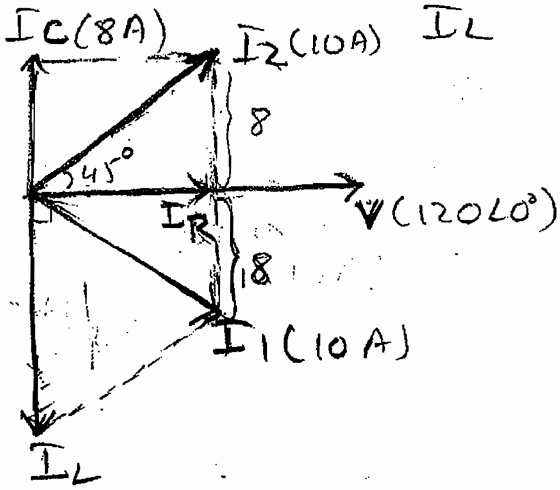
also; $\cos\phi_L = \frac{V_x}{V_y}$

$$\therefore V_x = V_3 \cos\phi_L = 136 \times 0.45$$
$$= 61.2$$

$$\therefore P = \frac{(61.2)^2}{5} = 750\,W.$$

9) Q. $|I_1| = |I_2| = 10\,A$ determine $I_R$ & $I_L$

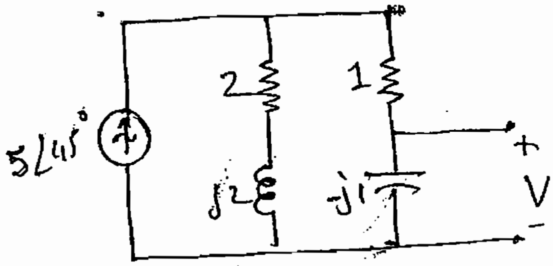



$$I_R = \sqrt{100-64} = \sqrt{36}$$
$$I_R = 6\,A$$

From phasor diag.
$$I_2 = 8+8 = 16\,A.$$

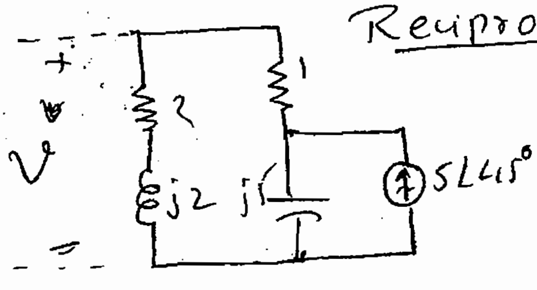
10) Verify Reciprocity theorem in find vltg V




Step 1

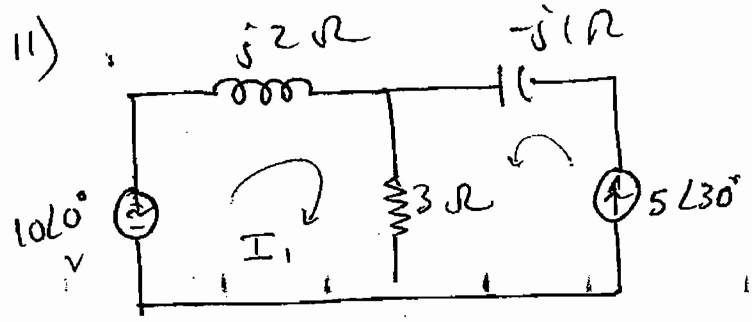
$$V = \left[5\angle 45^\circ \left(\frac{2+j2}{3+j1}\right)\right](-j1)$$

Step 2




Reciprocal n/w

$$V = (2+j2)\left[\frac{5\angle 45^\circ(-j1)}{(3+j1)}\right]$$

11) 

Verify Tellegens theo.

Here we verify instantaneous power theory!

So we find complex power $S^*$ in each element.

| Element | V | I | $I^*$ | $VI^*$ | Remarks |
|---|---|---|---|---|---|
| $V_s$ | $10\angle 0^\circ$ | $2.23\angle -145.4^\circ$ | " | $22.3\angle 145.4^\circ$ | Source → Apparent power |
| $I_s$ | $10\angle -3.63$ | $5\angle 30^\circ$ | $5\angle -30^\circ$ | $50\angle -33.63^\circ$ | Source → Apparant power |
| R | $8.34\angle 26.31$ | $2.78\angle 26.31^\circ$ | " | $23.18\angle 0^\circ$ | Sink → Active power |
| L | $4.46\angle -55.4^\circ$ | $2.23\angle -145.4^\circ$ | " | $9.94\angle 90^\circ$ | Sink → Reactive power $(+Q_L)$ |
| C | $5\angle -60^\circ$ | $5\angle 30^\circ$ | $5\angle -30^\circ$ | $25\angle -120^\circ$ | Sink → Reactive power $(-Q_c)$ |

KVL

$$-[10\angle 0^\circ] + j2I_1 + 3I_1 + 3(5\angle 30^\circ) = 0$$

$$\therefore (3+j2)I_1 + 12.99 + 7.5j = 0$$

$$I_1 = \cancel{10\angle 0^\circ + 5\angle 30^\circ} \; -\frac{12.99+7.5j}{3+j2} = 2.23\angle -145.4^\circ$$

$$I_R = I_1 + 5\angle 30^\circ = 2.78\angle 26.31^\circ$$

$$V_R = 3(I_R) = 8.34\angle 26.31$$

$$I_L = 2.23\angle -145.4^\circ$$

$$V_L = (j2)I_L = (2\angle 90^\circ)(2.23\angle -145.4^\circ)$$
$$= 4.46\angle -55.4^\circ$$

$$V_c = (-j)I_c = (1\angle -90^\circ)(5\angle 30^\circ)$$
$$= 5\angle -60^\circ$$

To verify Tellegens Theorem;

$$\sum VI^* \Big|_{source} = \sum VI^* \Big|_{sink}$$

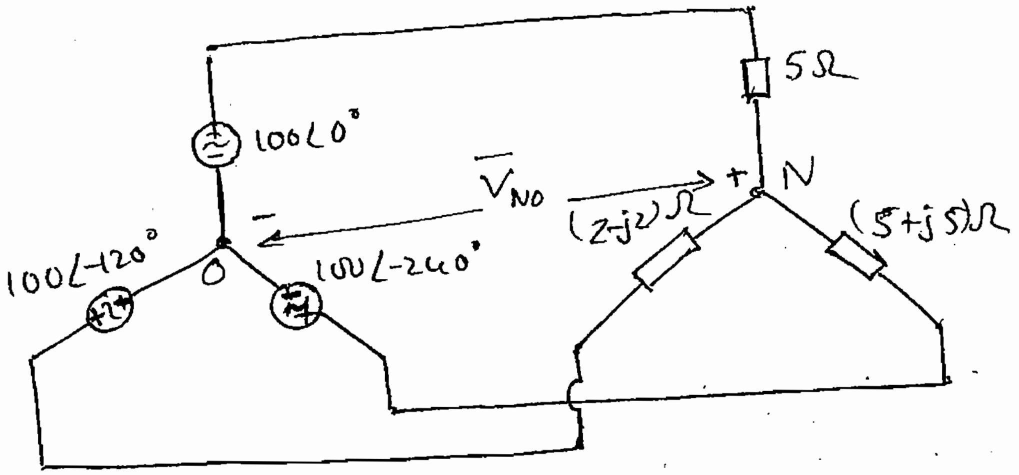
$$LHS = [22.3\angle 145.4^\circ] + [50\angle -33.60^\circ]$$

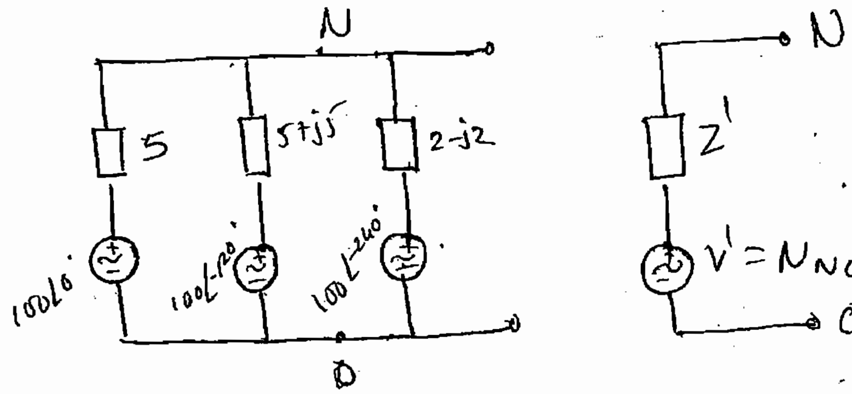
$$= 27.7\angle -32.8^\circ \text{ VA's}$$

$$RHS = [23.18\angle 0^\circ] + [9.94\angle 90^\circ] + [25\angle -90^\circ]$$

$$= 27.7\angle -32.8^\circ \text{ VA's}$$

12) Find the shift in the neutral vltg using Millman's theo. for the unbalanced 3-phase ckt shown.




$V' = V_{NO} \rightarrow$ shift in neutral vltg.

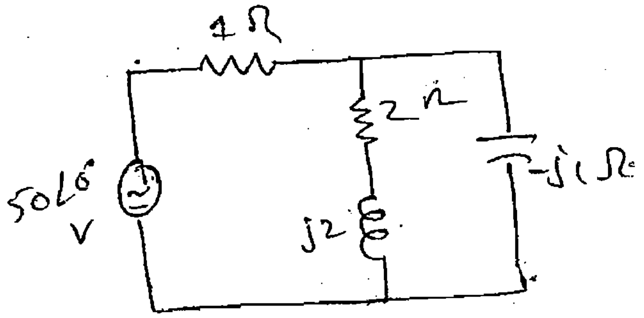
$$V' = \frac{\left(\frac{100\angle 0^\circ}{5}\right)\left(\frac{100\angle -120^\circ}{5+j5}\right)\left(\frac{100\angle -240^\circ}{2-j2}\right)}{\frac{1}{5} + \frac{1}{5+j5} + \frac{1}{2-j2}}$$

$$V = \frac{20\angle 0^\circ + 14.14\angle -165^\circ + 35.36\angle -195^\circ}{0.2 + 0.1 - 0.1i + 0.25 + 0.25i}$$

$$= \frac{-27.813\angle 168.82}{}$$

$$= 49.72\angle 153.7^\circ \text{ V}$$

12) Find change in current in capacitor when $1\,\Omega$ resistor is increased to $3\,\Omega$ (→ compensation theo.)




$$I_{1\Omega} = \frac{50\angle 0^\circ}{1 + [(2+j2)\,//\,(-j1)]}$$

$$= \frac{50\angle 0}{1 + 0.4 - 1.2i}$$

$$= 27.11\angle +40.6^\circ$$

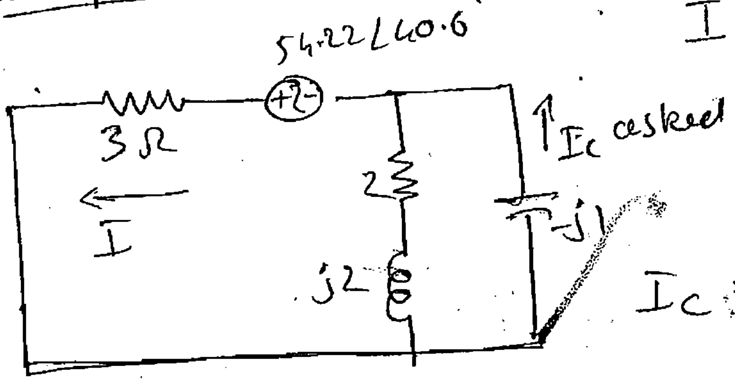
Now Find compensated vltg.

$$V_c = \bar{I}[\Delta Z]$$

$$= [27.11\angle 40.6][3-1]$$

$$= 54.22\angle 40.6^\circ$$

Compensated n/w




$$I = \frac{54.22\angle 40.6}{3 + [(2+j2)\,//\,-j1]}$$

$$= 15.0\angle 60$$

$$I_c = I\left[\frac{2+j2}{2+j2-j1}\right]$$

$$= 18.96\angle 78^\circ \text{ A}$$

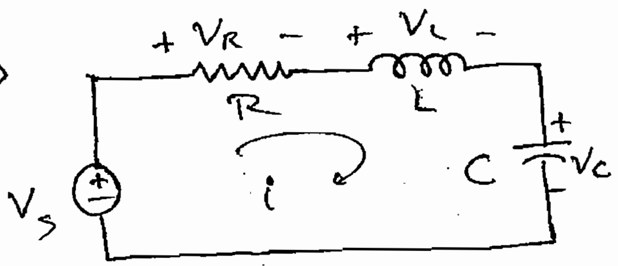
# DUALS & DUALITY

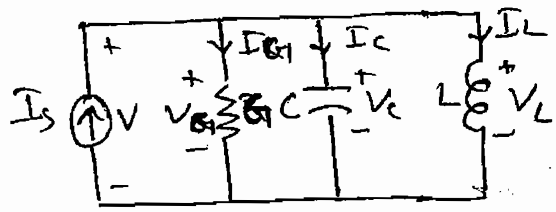
Two ckts are duals of each other if the mesh eq^n that characterize one of them has the same mathematical form as the nodal eq^n that characterize the ~~ord~~ other.

## Principle of Duality :—

Identical behaviour & pattern observed bet^n voltages & currents of 2 independent n/ws illustrate the priciple of Duality.

Eg)




**Mesh - KVL**

$$-V_s + V_R + V_L + V_C = 0$$

$$V_s = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

**Nodal - KCL**

$$-I_s + I_G + I_C + I_L = 0$$

$$\therefore I_s = V \cdot G + C\frac{dV}{dt} + \frac{1}{L}\int V\,dt$$

# Some Dual Elements

$V \longleftrightarrow I$

$v(t) \longleftrightarrow i(t)$

$V_m \sin\omega t \longleftrightarrow I_m \sin\omega t$

$R \longleftrightarrow G$

$L \longleftrightarrow C$

KVL $\longleftrightarrow$ KCL

Series $\longleftrightarrow$ Parallel

Mesh $\longleftrightarrow$ Node

$\phi$

$\pi \longleftrightarrow T$

Tree $\longleftrightarrow$ Co-Tree

*Twig $\longleftrightarrow$ Link / chord

$\dfrac{di}{dt} \longleftrightarrow \dfrac{dv}{dt}$

$\int v\,dt \longleftrightarrow \int i\,dt$

O.C $\longleftrightarrow$ S.C

Thevenin $\longleftrightarrow$ Nortons

* $\phi \longleftrightarrow q$

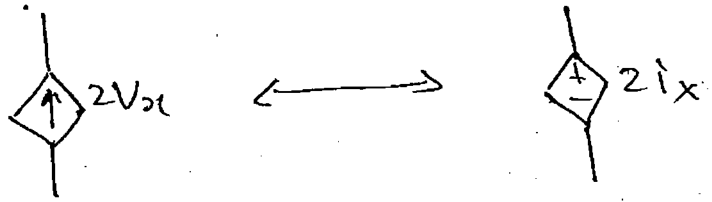
$Y \longleftrightarrow \Delta$

cutset $\longleftrightarrow$ Tie-set

$Z \longleftrightarrow Y$

$X \longleftrightarrow B$

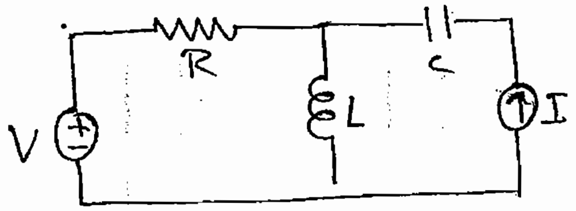
s/w in series (getting closed) $\longleftrightarrow$ s/w in parallel (getting opened)

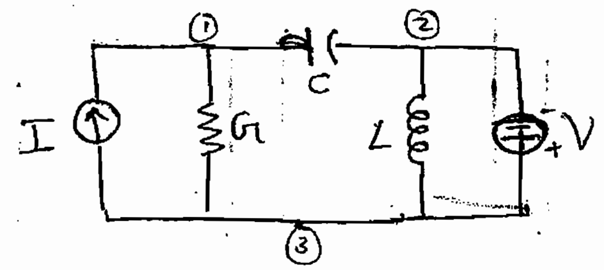



polarity (voltage) $\longleftrightarrow$ direction (current)

Eg Dual of $2\,\Omega$ resistance $\Rightarrow$ 2 Seimens conductance

1) Construct the dual of ckt shown below & verify by writing n/w equations.




Mesh

$$-V + i_1 R + L\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0$$

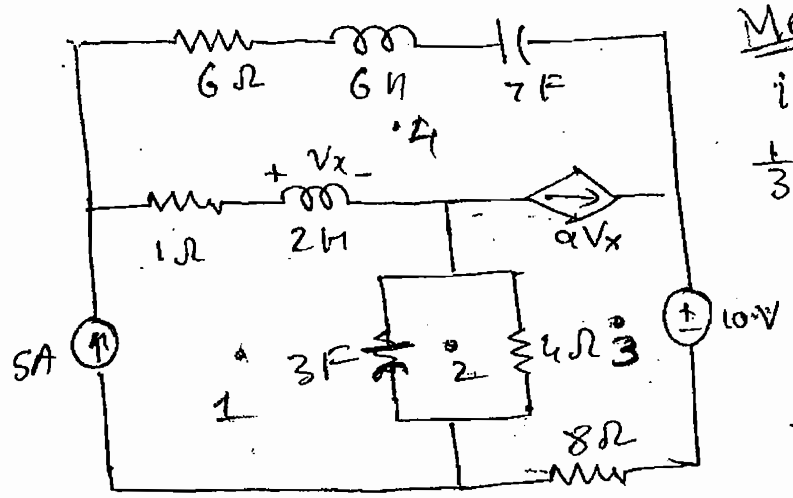
$$i_2 = -I$$

NODAL.

$$-I + GV_1 + C\left[\frac{dV_1}{dt} - \frac{dV_2}{dt}\right] = 0$$

$$V_2 = -V$$

2)




Mesh

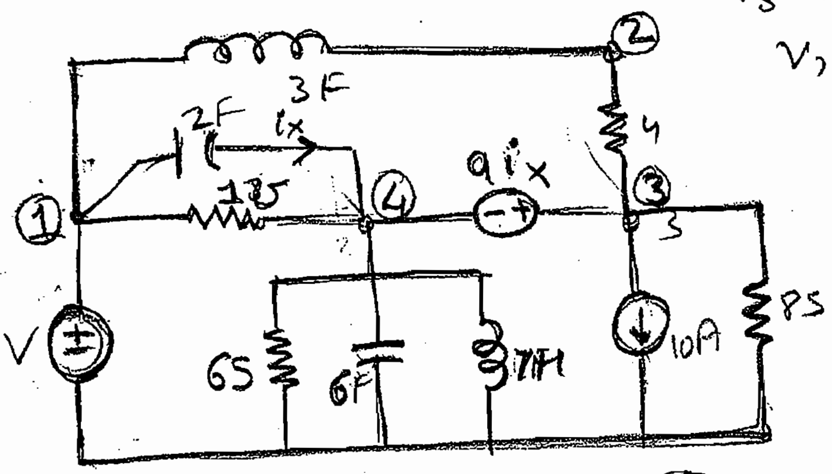
$$i_1 = 5 \quad \text{—} (1)$$

$$\frac{1}{3}\int (i_2 - i_1) + 4[i_2 - i_3] = 0 \quad \text{—} (2)$$

$$4[i_3 - i_2] + 2\left[\frac{di_4}{dt} - \frac{di_1}{dt}\right] + 1[i_4 - i_1] + 6 i_4 + 6\frac{di_4}{dt} + \frac{1}{7}\int i_4\, dt + 10 + 8i_3 = 0 \quad \text{—} (3)$$

$$i_3 - i_4 = aV_x \quad \text{—} (4)$$

$$V_x = 2\left[\frac{di_1}{dt} - \frac{di_4}{dt}\right] \quad \text{—} (5)$$




Nodal

$$V_1 = 5 \quad \text{—} (1)$$

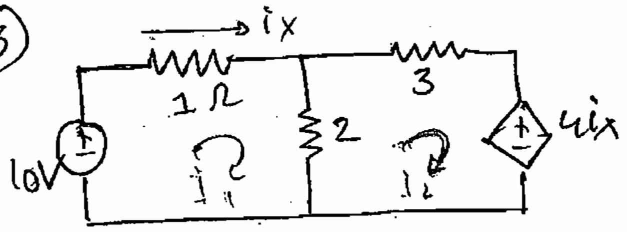
$$\frac{1}{3}\int (V_2 - V_1)\, dt + 4[V_2 - V_3] = 0 \quad \text{—} (2)$$

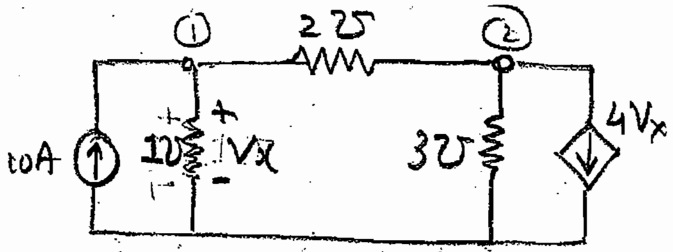
$$4[V_3 - V_2] + 2\left[\frac{dV_4}{dt} - \frac{dV_1}{dt}\right] + 1[V_4 - V_1] + 6 V_4 + 6\frac{dV_4}{dt} + \frac{1}{7}\int V_4\, dt + 10 + 8V_3 = 0 \quad \text{—} (3)$$

$$V_3 - V_4 = a\, i_x \quad \text{—} (4)$$

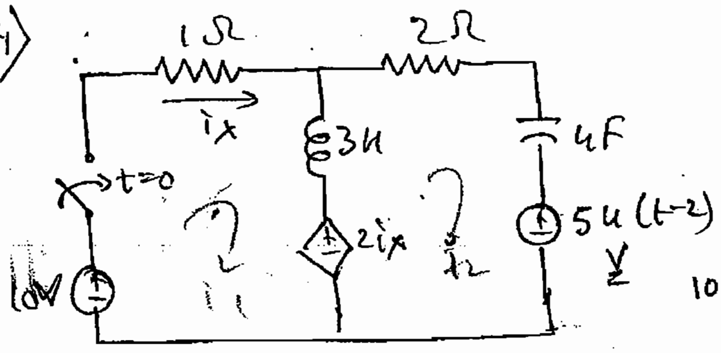
$$i_x = 2\left[\frac{dV_1}{dt} - \frac{dV_4}{dt}\right] \quad \text{—} (5)$$

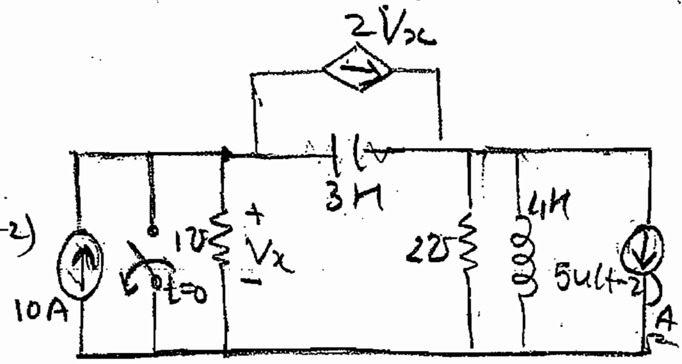
3)


ix
1 Ω
3
2
4ix
10V
i1
i2


1
2 ℧
2
10A
1℧
Vx
3℧
4Vx

4)


1 Ω
2 Ω
ix
3H
4F
t=0
2ix
5u(t-2)
V
10V
i1
i2

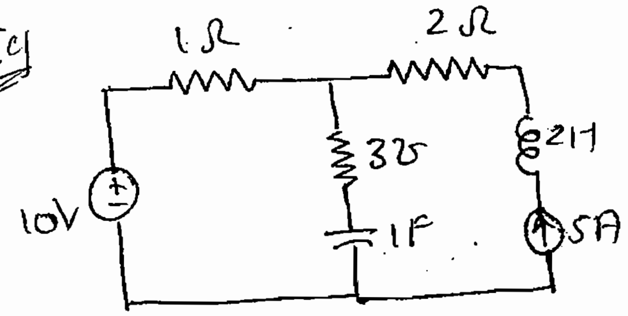

2Vx
3H
1℧
Vx
t=0
2℧
4H
5u(t-2)
A
10A

# NETWORK TOPOLOGY or GRAPH THEORY

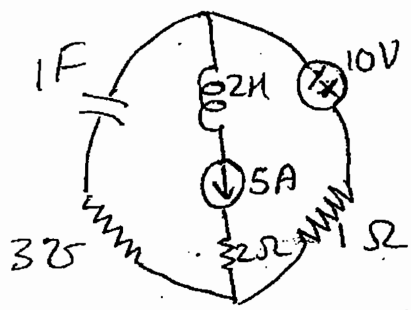
## Topology

Topology is a branch of geometry applicable to electrical ckts where even by bending, stretching, swaping, thrashing, tieing in knots, making ckt upside down, etc will not disturb the ckt property.

Eg




≅




⇒ Shape of a n/w ideally will not affect circuit analysis.

## Graph

A graph is a skeleton representation of a n/w where every element is suppressed by its nature & represented as a simple line - segment.

**NOTE**

Ideal vltg source → S.C

" current " → O.C

→ To simplify the order of matrix we generally consider principle node only.

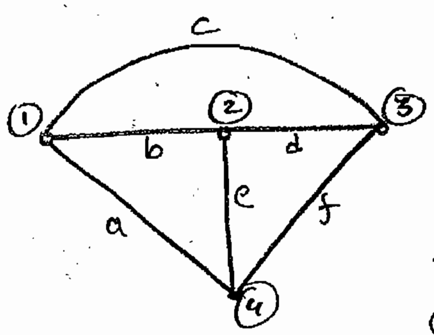
→ Nodes ⟶ vertices

Branches ⟶ edges.

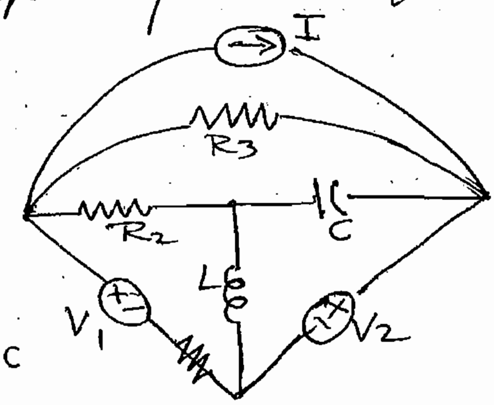
→ Nodes are numbers. → ①, ②, ③, ---.

Branches are named → a, b, c, ---.

**Ex** Construct the correct graph of the n/w shown below:




Ans ⟸




Vltg ⇒ S.C

Current source ⇒ O.C

Node (n)

Branch (b)

Mesh (m) → independent loops $\boxed{li = b-(n-1)}$

Loop (l)

$n = 4$

$b = 6$

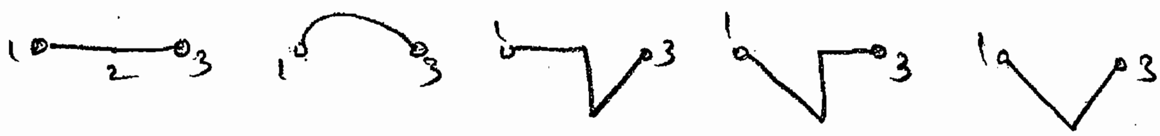
$m = 3 \rightarrow [6-(4-1)]$

$l = 3+4 = 7$

## Path

A path is a traversal from one node to another node without crossing the same node twice.

**Q** No. of possible path for the above graph from node ① to node ③ is 5

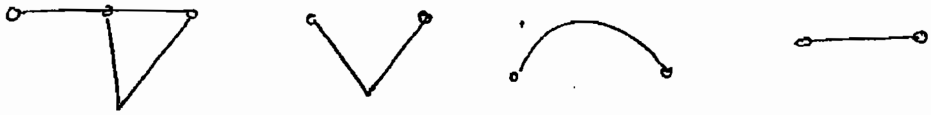


## Sub graph

A sub-graph consists of some the nodes & branches of main graph.

Ex Even a single edge can be subgraph of the main graph.

Ex



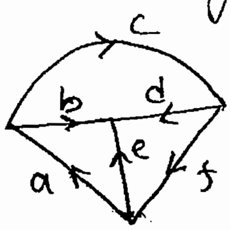
## Directed Graph

A graph is said to be directed if every edge is given a reference dir^n which is indicated by placing an arrow on every branch.

**Note:-**

This reference orientation need not necessarily indicate current dir^n.

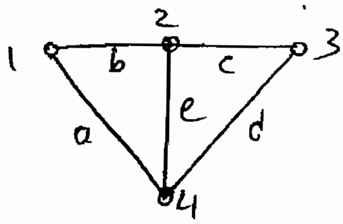
Ex




## Connected Graph.

A graph is said to be connected if there exist atleast one path from every node to every other node.

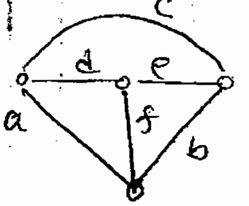
Ex



# Complete Graph / Completely Connected Graph.

A graph is said to be complete graph ∃ a direct path from every node to every other node.

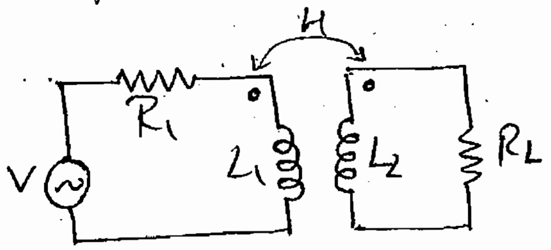
Ex

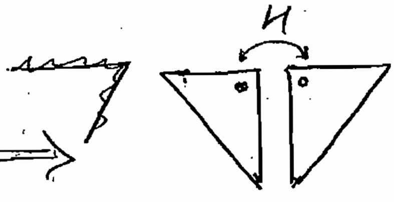



## Unconnected Graph :-

1) Wireless Communications n/ws
2) Magnetic circuits

Ex



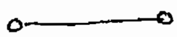
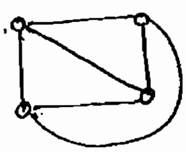
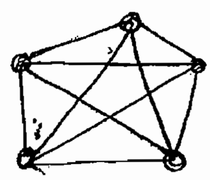




Graph

Q

1) The mini. no. of edges to make a graph complete with 'n' nodes is $^nC_2$

Ex

| nodes | Graph | Edges |
|---|---|---|
| 2 |  | $1 = \frac{2(2-1)}{2}$ |
| 3 |  | $3 = \frac{3(3-1)}{2}$ |
| 4 |  | $6 = \frac{4(4-1)}{2}$ |
| 5 |  | $10 = \frac{5(5-1)}{2}$ |

## Tree

A Tree is a sub graph of main graph which connects all the nodes without forming closed loops.

The rank of a tree with 'n' nodes is (n-1).

Any corresponding tree of a given graph with 'n' nodes will have (n-1) **edges**.

$$\text{No. of trees} = \begin{cases} n^{(n-2)} & ; \text{ for } n>2 \rightarrow \text{complete graph only} \\ \det\left[[A_r][A_r]^T\right] & ; \text{ for any graph} \end{cases}$$

where, $[A_r]$ = Reduced Incidence Matrix

The branch of a tree is specifically called as a Twig indicated by thick line segment

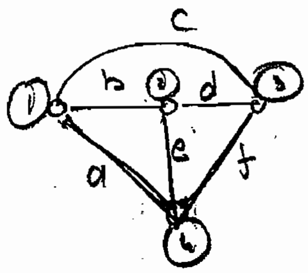
Any tree with 'n' nodes has (n-1) twigs.

## Co-Tree (Compliment of Tree)

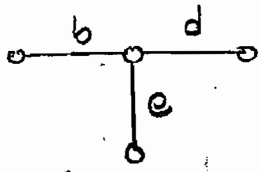
The set of branches other than Tree branches in a graph collectively form a co-tree

## Link / Chord :

The branch of Co-Tree is specifically called as a link which is indicated by dotted line. For any corresponding Co-Tree we have b-(n-1) links.




=




Tree

b, d, e → Twigs


Co-Tree

a, c, f → Links

GRAPH = TREE + CO-TREE

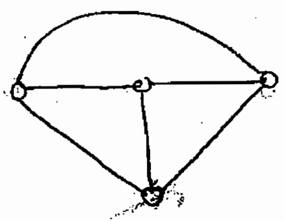
GRAPH ↓ edges ↓ $b$

TREE ↓ twigs ↓ $(n-1)$

CO-TREE ↓ Links ↓ $b-(n-1)$

$$b = (n-1) + b-(n-1)$$

1) The no. of possible trees for the n/w graph shown below is ______ & draw seperately.

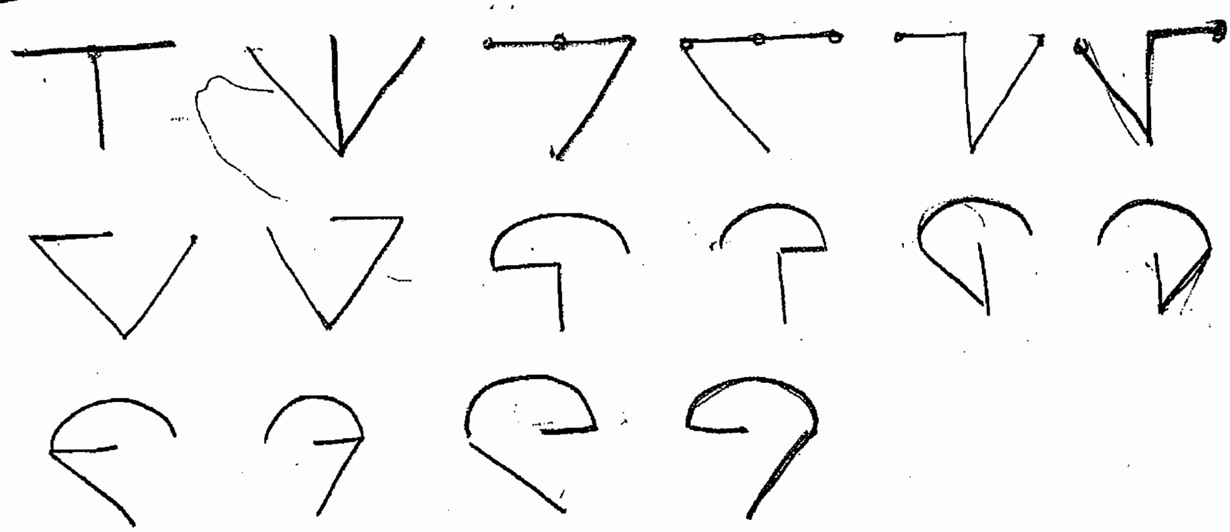


→ This is a complete graph

$\therefore$ No. of Trees $= n^{(n-2)}$

$= 4^2 = 16$

These are



# Incidence Matrix [A]

It is the matrix that gives relation b/w no. of nodes & no. of branches & the orientation of a particular branch w.r.t a node.

The order of this matrix is $(n \times b)$ or $(\underset{\text{vertices}}{v} \times \underset{\text{edges}}{e})$

The ~~rank~~ rank of Incidence Matrix with 'n' nodes is $(n-1)$

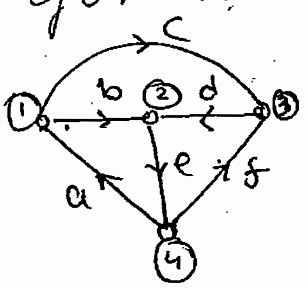
The elements of this matrix $[A] = [a_{ij}]_{n \times b}$

where $a_{ij} = +1$
if $j^{th}$ branch is incident with $i^{th}$ node & oriented away from it.

$a_{ij} = -1$ ; if incidented towards

$a_{ij} = 0$ ; if not incident

Q ⇒ Construct the complete Incidence Matrix for the oriented graph shown below:

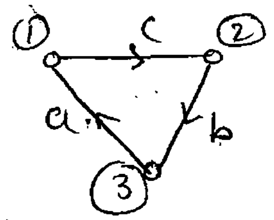



$$[A] = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \left[\begin{array}{cccccc} -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \\ +1 & 0 & 0 & 0 & -1 & 1 \end{array}\right]_{4 \times 6} \end{array}$$

The algebraic sum of the elements of every column vertically is zero

1) The determinant of Incidence Matrix of a closed ~~ato~~ loop graph is $\underline{0}$

Incidence matrix of a closed loop graph is of order $n \times n$

Eg 

$$[A] = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\det |A| = -1(-(-1)) + 1(1)$$
$$= -1 + 1 = \underline{\underline{0}}$$

⇒ In the Incidence Matrices of 2 independent n/w graphs are identical they they are said to obey the principle of Isomorphism.

Reduced Incidence Matrix [Ar]

If one of the node in a given graph is considered as reference & that particular raw is neglected while writing the incidence matrix, then it is a reduced incidence matrix, its order is ~~(n-1)/b~~ $(n-1) \times b$

In computer methods of electrical ckt analysis by considering Ar the memory space requirement & itteration time for solutions will be decreased.

Eg From above graph if node ④ is considered as ref. & that particular raw ④ is neglected, then the reduced incidence matrix is :

$$[A_r] = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{bmatrix}$$

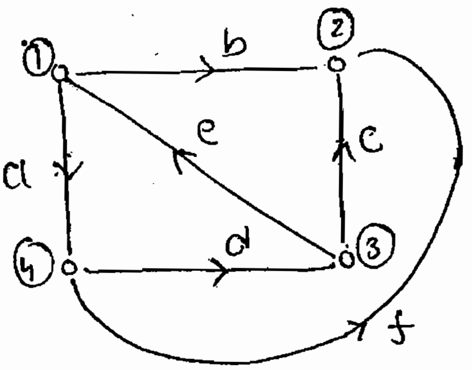
2) Construct the oriented graph of a n/w whose Incidence Matrix is given as : $\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix}$

→ The given is an $[A_r]$

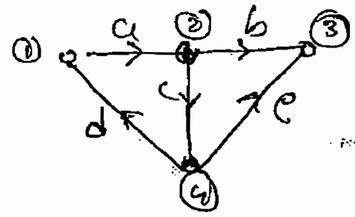
So make it (complete $[A]$)

$$\Rightarrow \begin{array}{c|c} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

∴ The n/w graph has nodes = 4 , edges = 6




3) The no. of possible trees from n/w graph shown below is ______ & draw them seperately.




→ Just connected graph

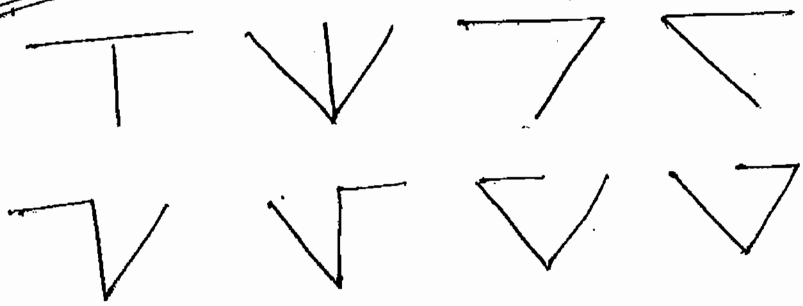
No. of trees $= \det |[A_r][A_r]^T|$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[A_r][A_r]^T = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

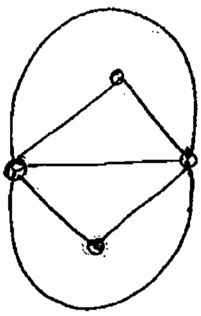
$$\det \left| [A_r][A_r]^T \right| = 2(6-1) - 1(2)$$
$$= 10 - 2 = \underline{\underline{8}} \text{ possible trees.}$$

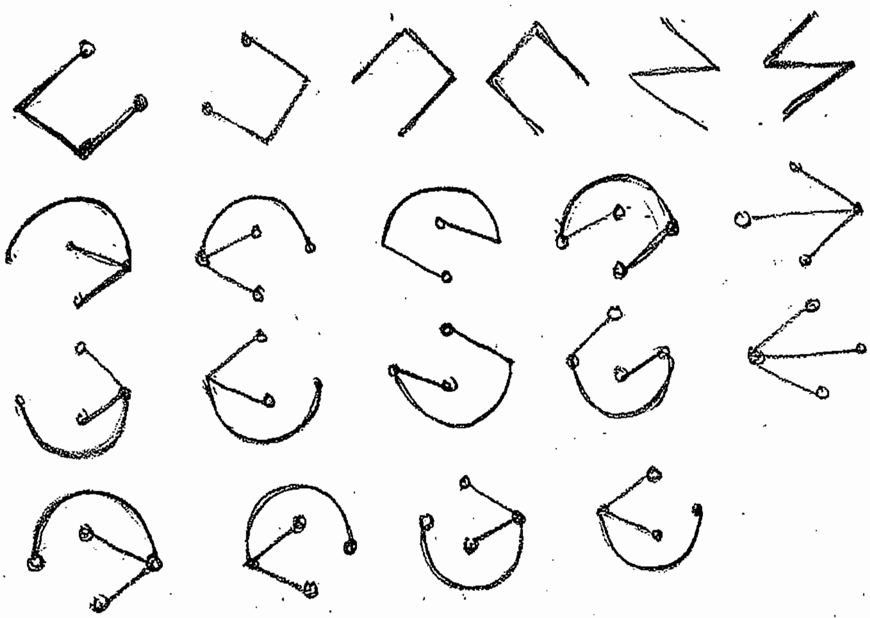
<u>These Trees are</u>



<u>HW</u>

1)





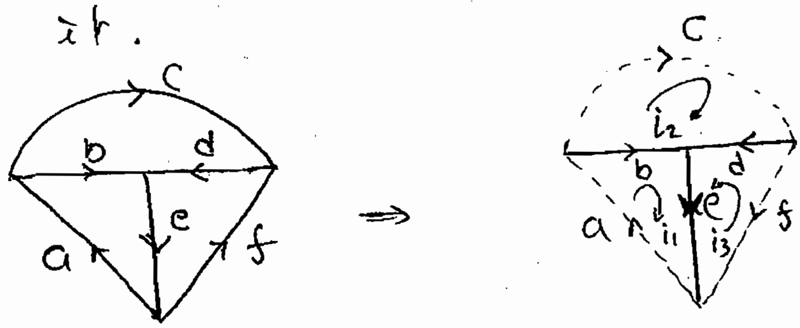
# Concept of Fundament Loops & TIE SET Currents :—

Fundamental loops are closed path of the graph which are formed by only one link & rest of them as twigs.

The no. of F-loops for any given graph = no. of links i.e. $b-(n-1)$

These fundamental loops currents are called Tie-set current & their orientation is governed by the link in it.

Eg :-

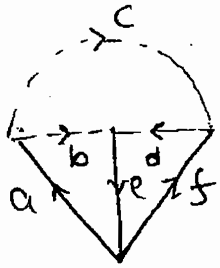



No. of f-loops = 6 − (4−1) = 3

$fl_1$ = a,b,e → $i_1$

$fl_2$ = b,c,d → $i_2$

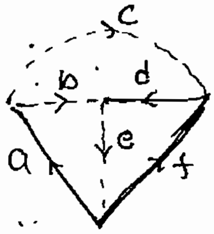
$fl_3$ = def → $i_3$

① 


$fl_1$ = abe → $i_1$

$fl_3$ = def → $i_2$

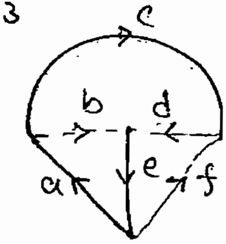
$fl_2$ = acf → $i_3$

② 


$fl_1$ = abdf → $i_1$

$fl_2$ = acf → $i_2$

$fl_3$ = def → $i_3$

③ 


$fl_1$ = abe → $i_1$

$fl_2$ = acde → $i_2$

$fl_3$ = acf → $i_3$

## Tie-Set Matrix [M]:-

It is a matrix that gives the relation b/w the branch currents & Tie-set currents where every branch current can be expressed in terms of Tie-set currents.

The order of this matrix is (links) × (branches)
i.e. $[b-(n-1)] \times b$

The no. of Tie-set Matrices possible for any graph = no. of Trees.

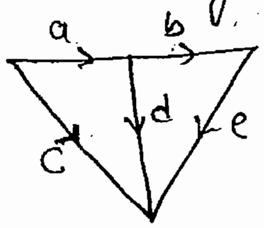
The elements of this matrix $[M] = [a_{ij}]_{\text{links} \times \text{branches}}$

where $a_{ij} = +1$ ; if $j^{th}$ branch current is incident with $i^{th}$ Tie set current & oriented in same dir$^n$

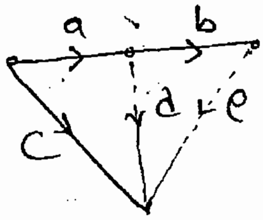
$a_{ij} = -1$ ; if incident & opp.

$a_{ij} = 0$ ; if not incident.

1) Construct the Tie-Set Matrix & the write the equilibrium eq$^n$ in std. KVL form for n/w graph shown below by considering a, b, c as tree branches.




$\Rightarrow$




No. of f-loops
$= 5-(4-1)$
$= 2$

$fl_1 \rightarrow a, c, d \rightarrow i_1$

$fl_2 \rightarrow a, b, e, e \rightarrow i_2$

$$[M] = \begin{array}{c} \\ i_1 \\ i_2 \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} +1 & 0 & -1 & +1 & 0 \\ +1 & +1 & -1 & 0 & +1 \end{array}\right]_{2\times 5} \end{array}$$

· Equilibrium Equations

let $j_a, j_b, j_c, j_d, j_e \longrightarrow$ Branch Currents

$V_a, V_b, V_c, V_d, V_e \longrightarrow$ Branch voltages

Set I KVL $[\longrightarrow]$

$$[M][V_b] = [0]$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}_{(2\times 5)} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}_{5\times 1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times 1}$$

Set II Relation b/w 'j' & 'i' $[\downarrow]$

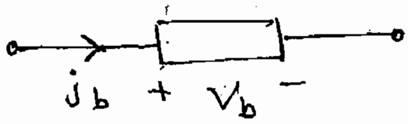
$$[M]^T [I_l] = [J_b]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{5\times 2} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_{2\times 1} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5\times 1}$$

$$\begin{bmatrix} i_1 + i_2 \\ i_2 \\ -i_1 - i_2 \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}$$

**NOTE :-**

In graph theory if graph is given, we consider every edge as a load set of reference (by default)

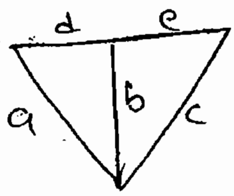



# Concept of cut-set :-

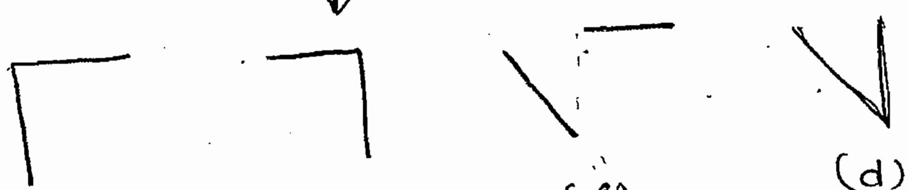
A cut set represents set of branches which when removed in a graph can be divided into 2 parts.

**Note** The no. of cut-sets simply represent the no. of possible ways that a graph can be divided in 2 parts.

1) Which of foll. set of branches represent a proper cut set for n/w graph shown below.




(1) a c d
(2) a c e
(3) b d c
(4) c d e



(a) (b) (c) (d)

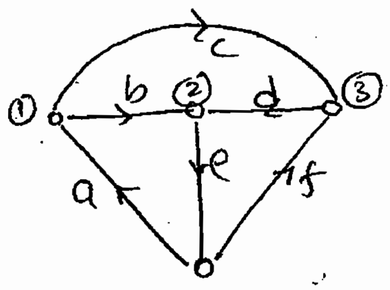
# Concept of ~~fundamental~~ ~~loops~~ cut-set & Cut-set voltages :—

Fundamental cut set are <u>cut through</u> of a graph which can divide in 2 parts in any dir$^n$ but in path of cutting it should cut only one twig & rest of them as links.

The no. of f-cut-set = no. of twigs (ie $n-1$)

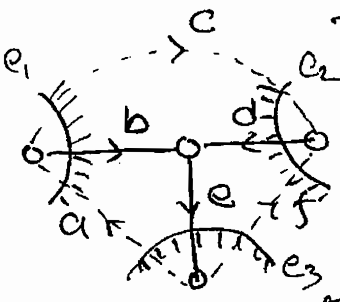
These f-cut sets form isopotential lines & their voltages are termed as cut set vltges.

The orientation of f-cut sets are governed by the twig in it.




GRAPH

TREE-1



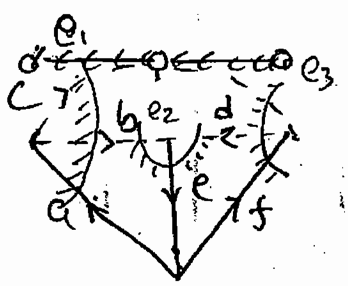

No. of ~~loops~~ f-cutsets $= 4-1 = 3$

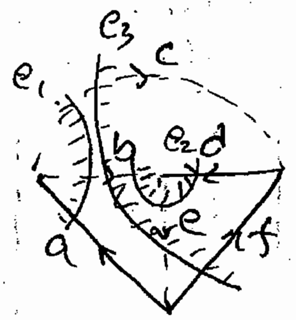
$fc_1 \rightarrow a, b, c \rightarrow \overrightarrow{e_1}$

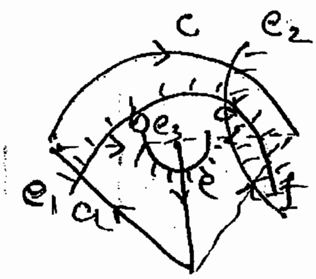
$fc_2 \rightarrow c, d, f \rightarrow \overleftarrow{e_2}$

$fc_3 \rightarrow a, e, f \rightarrow e_3 \downarrow$

TREE 2 | TREE-3 | TREE-4







| TREE 2 | TREE-3 | TREE-4 |
|---|---|---|
| $fC_1 \rightarrow a, b, c$ | $fC_1 \rightarrow a, b, c$ | $fC_1 \rightarrow a, b, d, f$ |
| $fC_2 \rightarrow b, d, e$ | $fC_2 \rightarrow b, d, e$ | $fC_2 \rightarrow c, d, f$ |
| $fC_3 \rightarrow c, d, f$ | $fC_3 \rightarrow b, c, e, f$ | $fC_3 \rightarrow b, d, e$ |

# Cut-set Matrix [C] :–

It is a matrix that gives relation b/w branch vltgs & cut-set vltgs where every branch vltg can be expressed in terms of cut-set vltgs.

The order of this matrix = (twigs) × (branches)
i.e. $(n-1) \times b$

The no. of possible cut set-matrix for any given graph = no. of trees.

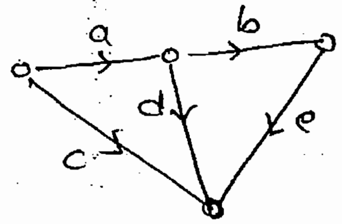
The elements of this matrix $[C] = [a_{ij}]_{(n-1)\times b}$

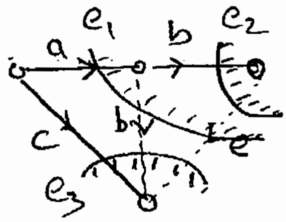
where $a_{ij} = 1$ ; if $j^{th}$ branch vltg is incident to $i^{th}$ cut set vltg & oriented in same dir$^n$.

$a_{ij} = -1$ ; if incident & opp.

$a_{ij} = 0$ ; if not incident.

1) Construct the cut-set matrix & write the equilibrium eq$^n$ in std. KCL form for the oriented graph shown below by considering branches a, b, c as tree branches.




No. of f-cut sets ~~fcut sets~~

$= 4 - 1 = 3$

$fC_1 = a, d, e \rightarrow \vec{e_1}$

$fC_2 = b, e \rightarrow \vec{e_2}$

$fC_3 = c, d, e \rightarrow \vec{e_3}$

$$[C] = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & +1 \end{bmatrix}_{3\times 5} \end{array}$$

Let, $j_a, j_b, j_c, j_d, j_e \rightarrow$ Branch currents

$V_a, V_b, V_c, V_d, V_e \rightarrow$ Branch voltages.

**5-1** KCL $[\longrightarrow]$

$$[C][J_b] = [0]$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}_{3\times 5} \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

$$\begin{bmatrix} j_a - j_d - j_e \\ j_b - j_e \\ j_c + j_d + j_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

**S-2** Relation b/w 'V' & 'e' [↓]

$$[C]^T [e_t] = [V_b]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix}_{5\times3} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}_{5\times1}$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ -e_1 + e_3 \\ -e_1 - e_2 + e_3 \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}$$

2) Mention the relation b/w Tie-set & cut-set matrices.

⇒ For the same given n/w; for that particular tree, for that particular orientation only we can compare Tie-set & Cut-set matrices.

Tie set Matrix → Links → KVL → Mesh
(f-loops)

$$[M] = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[ \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -1 \end{bmatrix}}_{M_{twigs}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{U_{twigs}} \right] \end{array}$$

Cut-set Matrix → Twigs → KCL → Nodal

$$[C] = \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \left[ \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{U_{twigs}} \underbrace{\begin{bmatrix} -1 & -1 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}}_{C_{links}} \right]$$

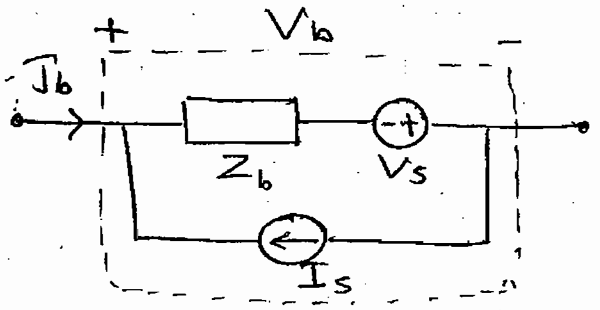
also,

$$[M_{twigs}] = -[C_{links}]^T$$

or $$[C_{links}] = -[M_{twigs}]^T$$

# Application of N/w Topology in Electrical ckt Analysis :-

Concept of std. branch :




## [I] Solution by KVL Eqⁿ [TIE-SET]

$$V_b = \underbrace{Z_b[j_b + I_s]}_{\text{Impedance form}} - V_s$$

But,

$$[M][V_b] = 0$$

$$\therefore \; M Z_b j_b + M Z_b I_s - M V_s = 0$$

But,

$$[M]^T [I_e] = [J_b]$$

$$[M][Z_b][M]^T [I_e] = [M]\cdot[V_s] - [M][Z_b][I_s]$$

Solve this ($[I_e]$)

Then branch currents can be calculated

$$[M]^T [I_e] = [J_b]$$

substitute this ($[I_e]$) ; $[J_b]$ → final ans.

## [II] Solution by KCL Eq$^n$ [Cut-set]

$$J_b = Y_b[V_b + V_s] - I_s$$

Admittance form.

But,

$$[C][J_b] = 0$$

$$C Y_b V_b + C Y_b V_s - C I_s = 0$$

But,

$$[C]^T [e_t] = [V_b]$$

$$[C][Y_b][C]^T \underset{\text{Solve this}}{[e_t]} = [C][I_s] - [C][Y_b][V_s]$$

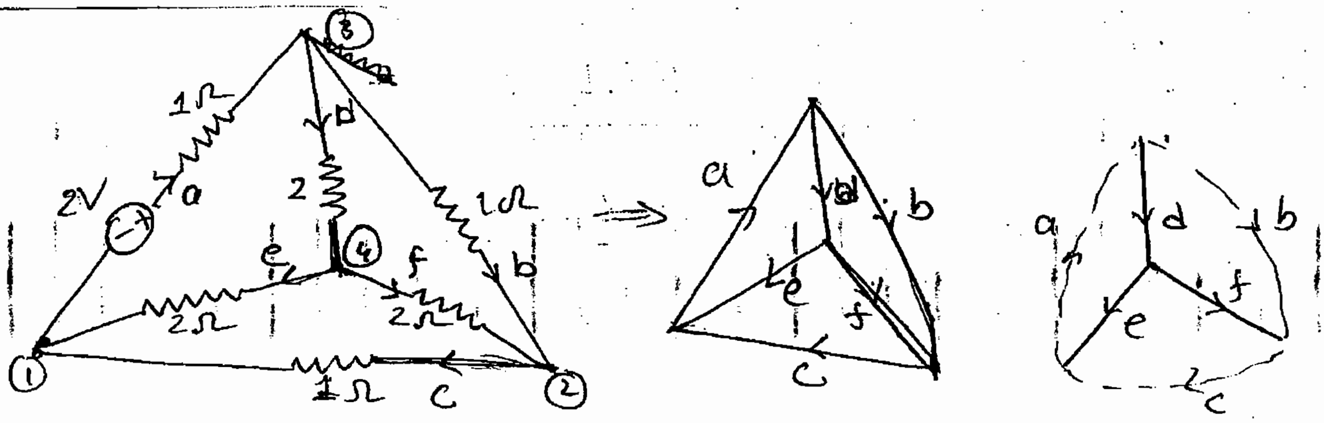
Then branch voltges can be determined by:

$$[C]^T \underset{\text{Substitute this}}{[e_t]} = \underset{\text{Final answer}}{[V_b]}$$

Q

1) Solve the n/w to find branch currents by writing the eq$^n$ in std KVL form

2) Solve the n/w to find all the branch vltg by writing eq$^n$ in std KCL form




## 1) TIE SET

$$[M][Z_b][M]^T[I_l] = [M][V_s] - [M][Z_b][I_s]$$

$$[M] = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \end{array}\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}_{3\times 6} \end{array}$$

$$[Z_b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6\times 6}$$

$$[I_l] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_{3\times 1} \qquad [V_s] = \begin{bmatrix} +2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1} \qquad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}$$

LHS

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix}_{3\times 6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}_{6\times 3} = \begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

RHS $[M][V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$

Final LHS

$$\begin{bmatrix} 5i_1 - 2i_2 - 2i_3 \\ -2i_1 + 5i_2 - 2i_3 \\ -2i_1 - 2i_2 + 5i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$ Final RHS

Solving $i_1 = \frac{6}{7}A$, $i_2 = \frac{4}{7}A$, $i_3 = \frac{4}{7}A$.

Then, Branch currents can be calculated by

$[M]^T[I_e] = [J_b]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 6/7 \\ 4/7 \\ 4/7 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \\ j_f \end{bmatrix}$$

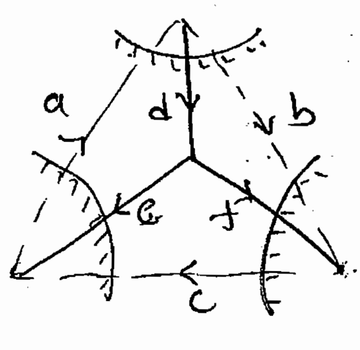
Final Ans

$j_a = \frac{6}{7}$, $j_b = \frac{4}{7}$, $j_c = \frac{4}{7}$, $j_d = \frac{2}{7}$

$j_e = \frac{2}{7}$, $j_f = 0$.

2) CUT-SET

$[C][Y_b][C]^T[e_t] = [C][I_s] - [C][Y_b][V_s]$



$$
[C] = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix}_{3\times 6}
\end{array}
$$

$$
[Y_b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}_{3\times 6}
$$

$$
[e_t] = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3\times 1} \qquad [V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1} \qquad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
$$

<u>LHS</u>

$$
\begin{bmatrix} -1 & 1 & 0 & 1/2 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1/2 \end{bmatrix}_{3\times 6}
\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{6\times 3}
= \begin{bmatrix} 5/2 & 1 & 1 \\ 1 & 5/2 & -1 \\ 1 & -1 & 5/2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}
$$

<u>RHS</u>

$$
-[C][Y_b][V_s]
$$

$$
-\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} +2 \\ +2 \\ 0 \end{bmatrix}
$$

<u>Final LHS</u> <u>Final RHS</u> <u>Solving</u>

$$
\begin{bmatrix} 2.5e_1 + e_2 + e_3 \\ e_1 + 2.5e_2 - e_3 \\ e_1 - e_2 + 2.5e_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}
$$

$e_1 = 4/7$

$e_2 = 4/7$

$e_3 = 0$

Then, Branch vltgs can be calculated by

$$[C]^T [e_t] = [V_b]$$

$$\begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4/7 \\ 4/7 \\ 0 \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix}$$

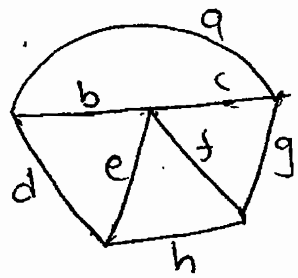
Final Ans

this indicates presence of source in branch

$V_a = -\frac{8}{7} V$ , $V_b = \frac{4}{7} V$ , $V_c = \frac{4}{7} V$ , $V_d = \frac{4}{7} V$

$V_e = \frac{4}{7} V$ , $V_f = 0 V$

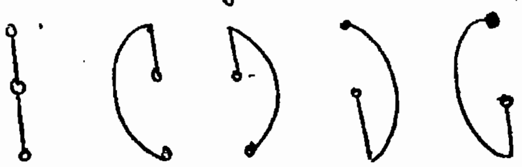
1) Which of the foll. set of branches is not a tree for the graph shown below.

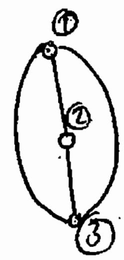



(a) aghe (b) bcgh

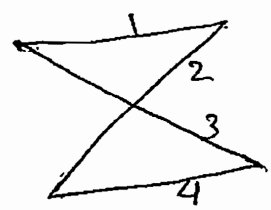
(c) defg ✓(d) abfg

2) No. of trees = 5



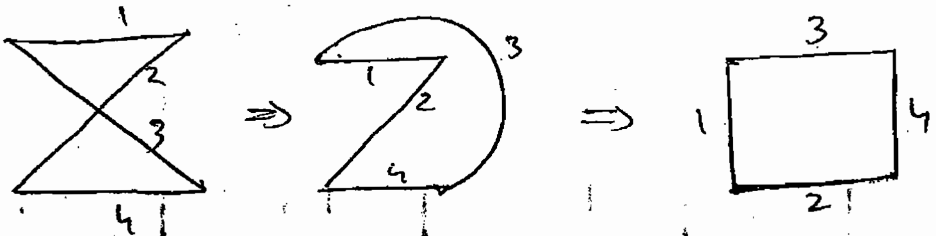



3) For the ~~oriented~~ n/w graph shown below if no of trees = p & no. of cut sets = q. then

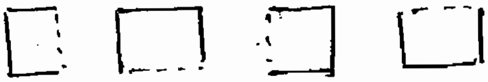



(a) P = 2 , Q = 2 (c) P = 4 Q = 6

(b) P = 4 Q = 4 (d) P = 4 Q = 10

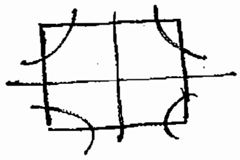


Here numbering indicates that there are 4 nodes only & the centre one is not a node.

No. of trees = 4 

No of cut sets = 6

i.e. no. of ways that the graph can be cut into 2 parts.



## Proper Tree :-

↳ Same defn of Tree + $\left[\begin{array}{l} C, V \rightarrow \text{Twigs} \\ L, I \rightarrow \text{Links} \end{array}\right]$

C & V to be linked with twigs.

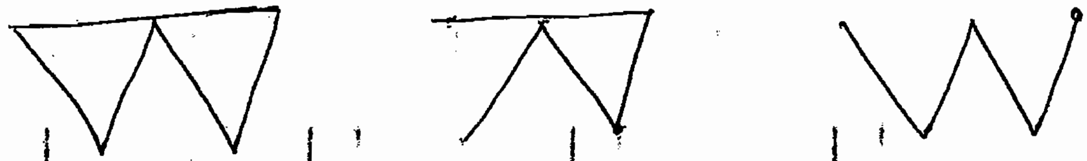
L & I to be linked with links.

## Spanning Graph :-

It is a sub-graph which connects all the nodes.

All trees are spanning graphs., but all spanning graphs are not trees.

Ex



Graph → spanning graph → but not Tree

→ Spanning graph → Also Tree

1) A n/w with 'n' nodes & 'b' branches. has ____ no. of node-pair voltages.

Ans $^{n}C_2 = \frac{n(n-1)}{2}$

# ELECTRICAL RESONANCE

- Resonance is the freq. response of a ckt / n/w when the ckt operates at its natural freq called resonance freq.
- Under resonance total supply vltg & supply current are in phase. So, $\phi = 0°$ & $PF = \cos\phi = 1$ (UPF)
- Under resonance the net impedance of the ckt becomes purely resistive & max power will be transfered to the ckt from source

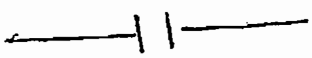
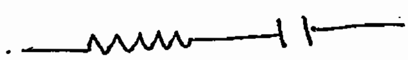
→ Resonance can occur in any electrical n/w provided we have 2 similar but opposite natured energy storage components i.e. L & C.

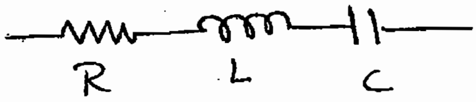
→ To undergo a good observable resonance for practical applications we need a good quality in these energy storage components which is measured as Quality factor or figure of merit given by :

$$\text{Q-factor} = 2\pi * \left[\frac{\text{Max. energy stored per cycle of supply}}{\text{Energy dissipated per cycle of supply}}\right]$$

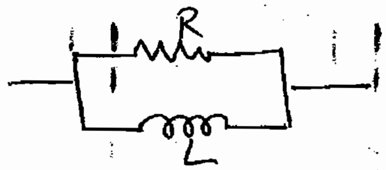
In practical applications $\boxed{Q \geq 10}$

→ Resonance phenomenon is useful in designing of passive filters, & antennas, receivers, SONARS, etc

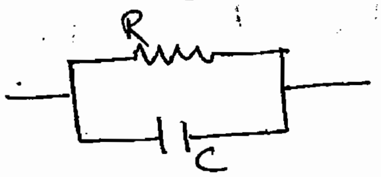
| Element | Q-Factor |
|---|---|
|  | 0 |
|  | $\infty$ |
|  | $\infty$ |
|  | $\frac{\omega L}{R}$ |
|  | $\frac{1}{\omega RC}$ |



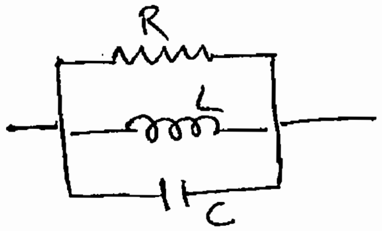

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$




$$\frac{R}{\omega L}$$

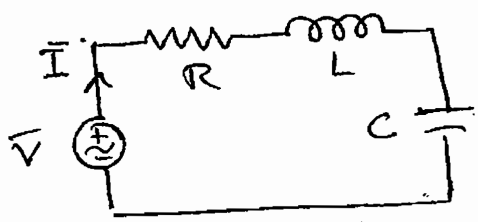



$$\omega RC$$




$$Q_0 = R\sqrt{\frac{C}{L}}$$

## ⟨1⟩ Series Resonance :—




At resonance ; $\omega = \omega_0$

$\bar{V}$ & $\bar{I}$ in phase.

$\therefore$ $\phi = 0°$ ; $Z = R$

But, $Z = R + j[X_L - X_C]$

but at resonance, net reactance $= 0$

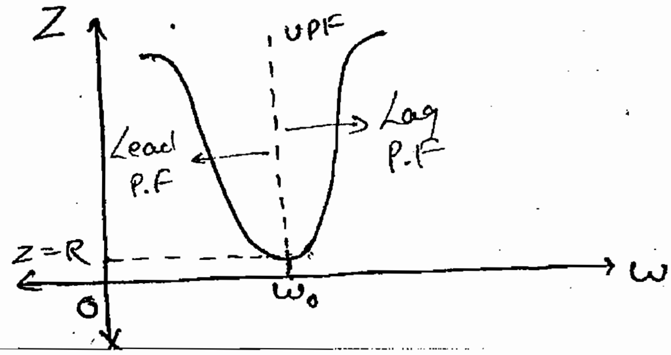
$\therefore$ $X_L - X_C = 0 \Rightarrow X_L = X_C$

$\therefore$ $\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad ; \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

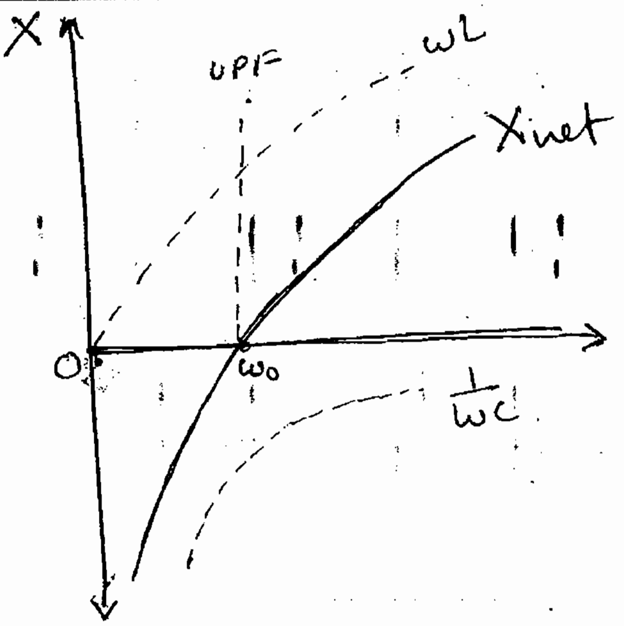
Graph ①

Z vs ω

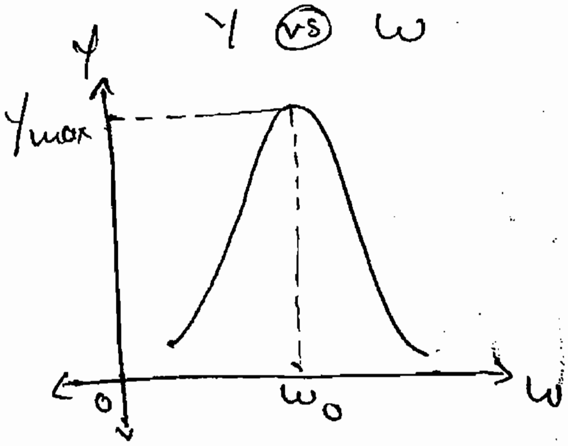


## Graph ② X vs ω

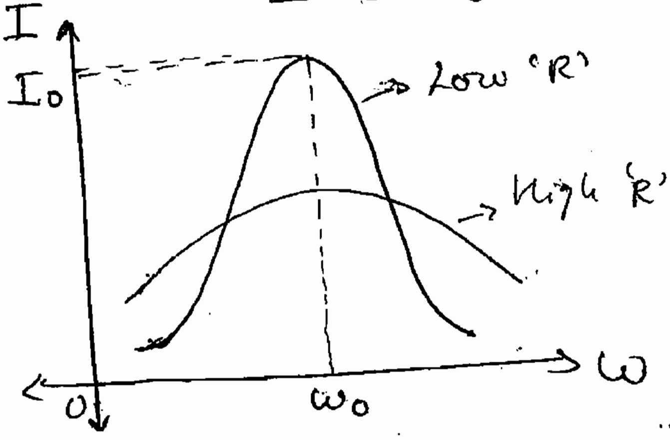
$$Z = R + j\underbrace{\left[\omega L - \frac{1}{\omega C}\right]}_{X_{net}}$$




## GRAPH ③ Y vs ω




## Graph ④ I vs ω




## Phasor diagram :-

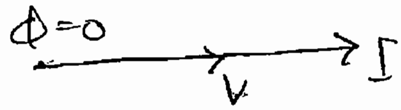
(a) $\omega = \omega_0$

$Z = R$

↳ purely resis.

'I' in phase with 'V'

$\phi = 0°$ [UPF)



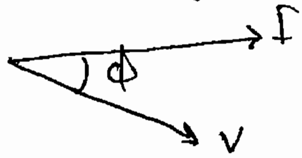

(b) $\omega < \omega_0$

$Z = R - jX_{net}$

↳ R-C ckt

'I' leads 'V' by $\phi < 90°$

(Leading PF)




$\phi$ is -ve with ~~w.r.t.~~ respect to I (ref.)

(c) $\omega > \omega_0$

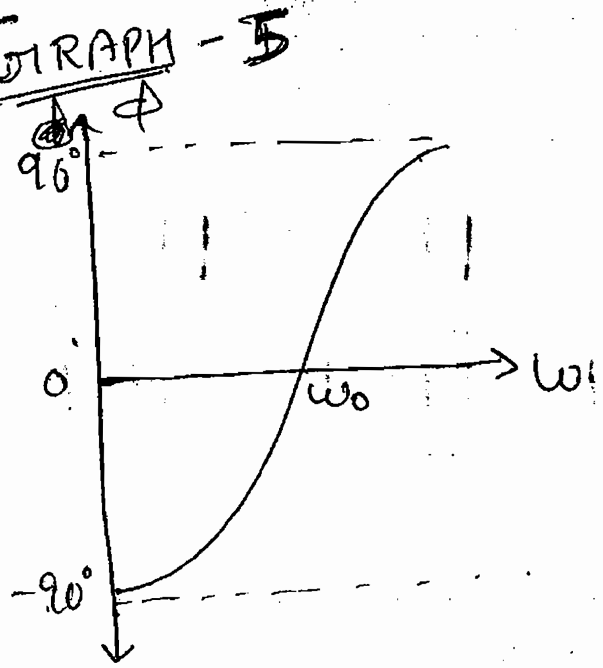
$Z = R + jX_{net}$

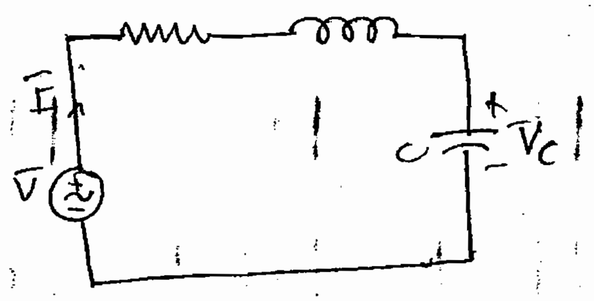
↳ R-L ckt

'I' lags 'V' by $\phi < 90°$

(lagging PF

GRAPH - 5

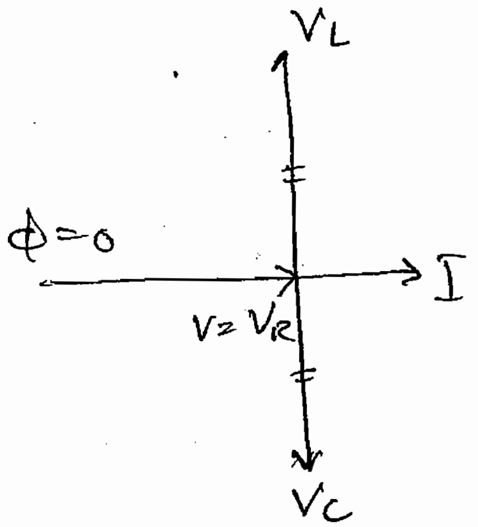



at Resonance $\omega = \omega_0$

$|X_L| = |X_C|$

$|V_L| = |V_C|$




Q - Factor at Resonance.

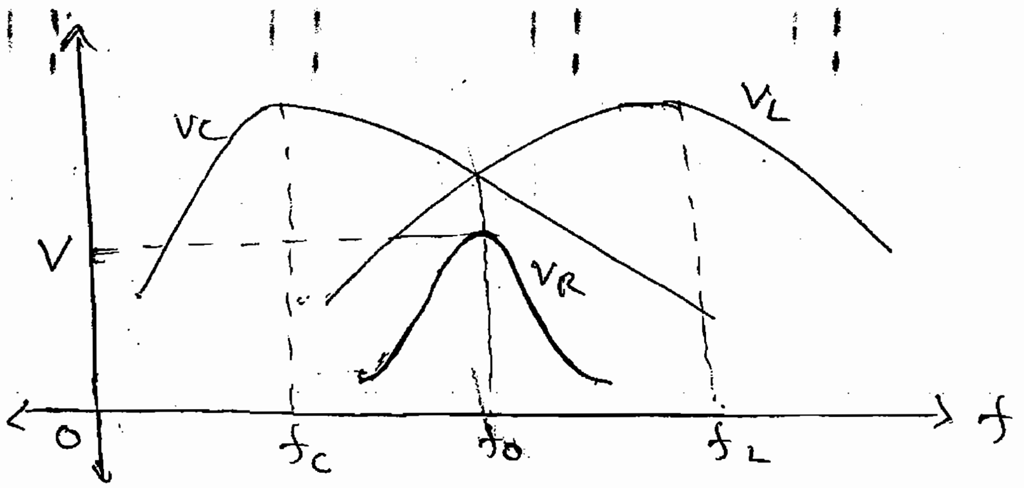
$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$

→ Under series resonace net impedance is min, so current is max. Hence it is called as acceptor ckt.

→ At series resonce freq. it is as if the total supply vltg appears across resistor: Hence series resonace is called vltg amplification ckt.

Variation of voltages across passive elements with change in freq.




The freq. at which max. vltg appears across capacitor

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{2L}}$$

$$f_C = f_0\sqrt{1 - \frac{R^2C}{2L}} \text{ Hz}$$

The freq. at which max. vltg appears across inductor:

$$f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}} = \frac{1}{2\pi\sqrt{LC\left[1 - \frac{R^2C}{2L}\right]}}$$

$$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} \text{ Hz}$$

From circuit

$$\bar{V} = \bar{I}\,\bar{Z} \Rightarrow \bar{I} = \frac{|V|}{|Z|}$$

But

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$|I| = \frac{|V|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

But at $\omega = \omega_0$

$$|I|_0 = \frac{|V|}{\sqrt{R^2 + 0^2}} \Rightarrow \text{Maximum}$$

So,

$$|I_0| = \frac{|V|}{R}$$

So, Power transfer also 'maximum'

$$P_0 = I_0^2 R = \frac{|V|^2}{R} \text{ W}$$

at $\omega = \omega_0$

$\boxed{R}$ $V_R = I_R \cdot R = I_0 R = \frac{|V|}{R} \cdot R = \boxed{|V| = V_R}$

$\boxed{L}$ $V_L = +jX_L I = j\omega_0 L I_0 = +j\omega_0 L \times \frac{|V|}{R}$

$$= +j\left[\frac{\omega_0 L}{R}\right]|V| \Rightarrow \boxed{V_L = +jQ_0|V|}$$

$\boxed{C}$ $V_C = -jX_C I = \frac{-j}{\omega_0 C} I_0 = \frac{-j}{\omega_0 C}\frac{|V|}{R} = -j\left[\frac{1}{\omega_0 RC}\right]|V|$

$$\Rightarrow \boxed{V_C = -jQ_0|V|}$$

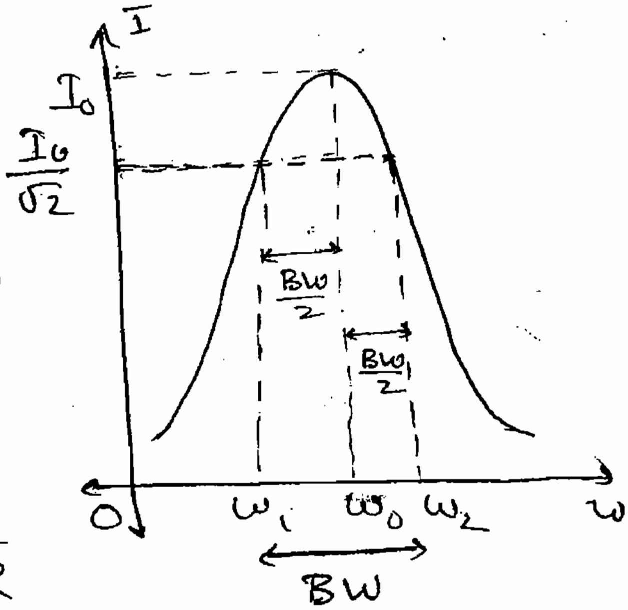
$Q_0 \rightarrow$ vltg magnification factor.

## Bandwidth :-

Bandwidth represents the range of frequencies for which the power level in the signal is at half of the ~~signal~~ max. power.

→ Half of max. power frequencies :

$$\frac{P_0}{2} = \frac{I_0^2 R}{2} = \left[\frac{I_0}{\sqrt{2}}\right]^2 R = (0.707\, I_0)^2 R$$




$\omega_1 \rightarrow$ lower cut-off / roll off corner ~~freq~~. freq.

$\omega_2 \rightarrow$ upper cut-off / roll off corner freq.

$$\frac{|V|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{2}R}$$

$$\therefore\ R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2 \Rightarrow \omega L - \frac{1}{\omega C} = \pm R$$

At $\omega = \omega_1$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{--- (1)}$$

At $\omega = \omega_2$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{--- (2)}$$

# Bandwidth :

$$\omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec}$$

$$f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$

B.W $\propto$ R

|BW| is independent to '$f_0$'

$\Rightarrow$ Resonance freq. is geometric mean of Bandwidth freq..

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$f_0 = \sqrt{f_1 f_2}$$

Also,

$$\omega_0 - \omega_1 = \frac{BW}{2} \Rightarrow \omega_1 = \omega_0 - \frac{R}{2L} \text{ rad/sec}$$

$$f_1 = f_0 - \frac{R}{4\pi L} \text{ Hz}$$

&

$$\omega_2 - \omega_0 = \frac{BW}{2} \Rightarrow \omega_2 = \omega_0 + \frac{R}{2L} \text{ rad/sec}$$

$$f_2 = f_0 + \frac{R}{4\pi L} \text{ Hz}$$

$\Rightarrow$ Resonance freq. is independent of resistor (R).

## Selectivity :- (S)

→ Selectivity is the ability of a ckt/n/w to distinguish or discriminate desired & undesired freq.

→ $S \propto \frac{1}{|BW|}$

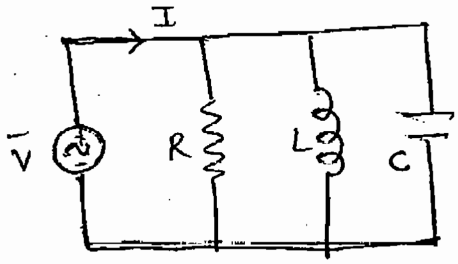
$$S = \frac{f_0}{|f_2 - f_1|} = \frac{\omega_0}{|\omega_2 - \omega_1|}$$

$$S = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi L}} = \frac{1}{R}\sqrt{\frac{L}{C}} = Q_0 \quad !!$$

→ The value of selectivity is Q-factor under Resonance.

## ⟨2⟩ Parallel Resonance :—

### ① General ckt




At resonance
V & I (in phase)
$\phi = 0^\circ$ ; PF = 1 (UPF)

$$Y_T = Y_R + Y_L + Y_C$$

$$Y_T = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

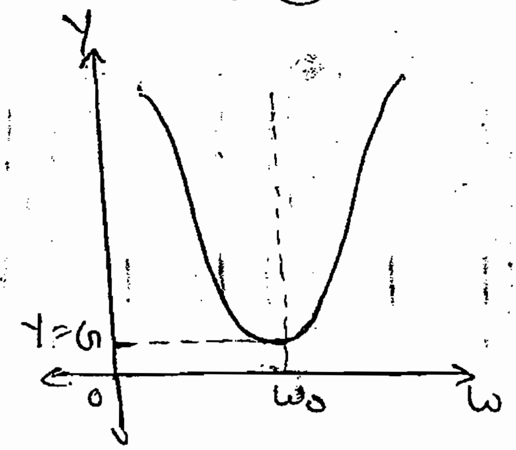
$$= \frac{1}{R} + \frac{j}{X_C} - \frac{j}{X_L} = \frac{1}{R} + j\left[\frac{1}{X_C} - \frac{1}{X_L}\right]$$

At resonance $(\omega = \omega_0)$
Net susceptance $= 0$

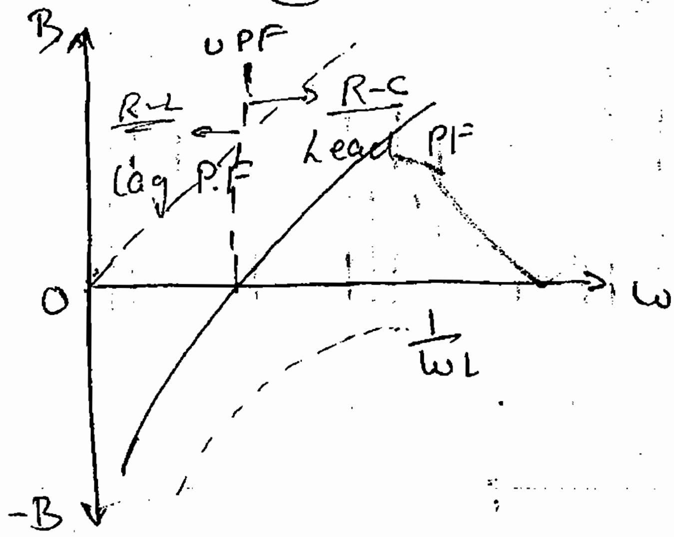
$$\frac{1}{X_C} - \frac{1}{X_L} = 0 \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

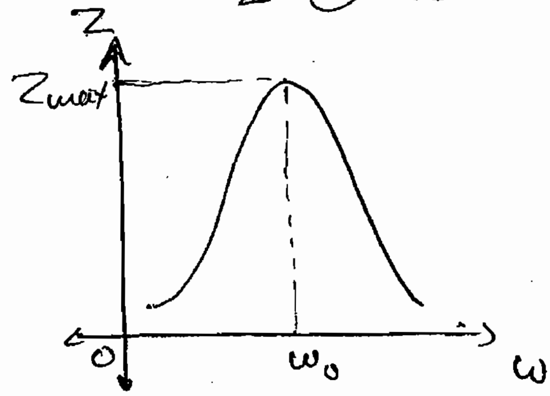
Graph 1 — Y vs ω



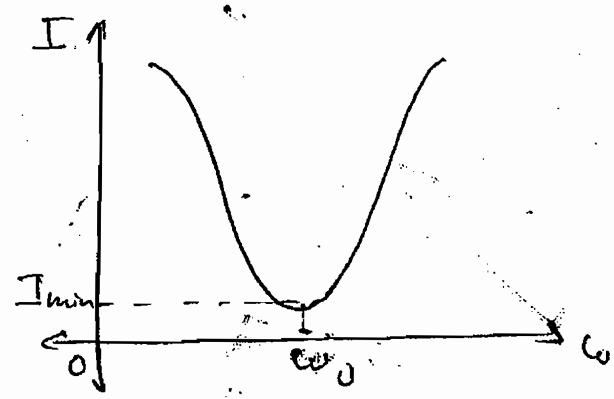

Graph 2 — B vs ω




Graph 3 — Z vs ω




Graph 4 — I vs ω




## Phasor diagrams:

(a) $\omega = \omega_0$

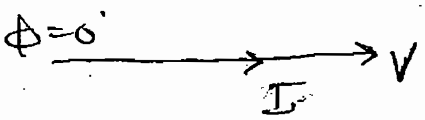
$Y = G$

↳ pure resis.

'I' in phase with 'V'.

$\phi = 0°$

PF = 1 (UPF



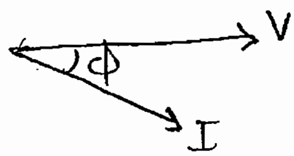

(b) $\omega < \omega_0$

$Y = G - jB_{net}$

↳ RL parallel ckt

'I' lags 'V' by $\phi < 90°$

(lagging PF)




$\phi$ is −ve w.r.t V (ref)

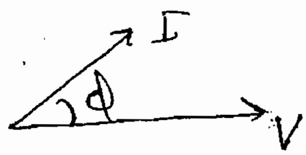
(c) $\omega > \omega_0$

$Y = G + jB_{net}$

↳ RC ||$^{el}$ ckt

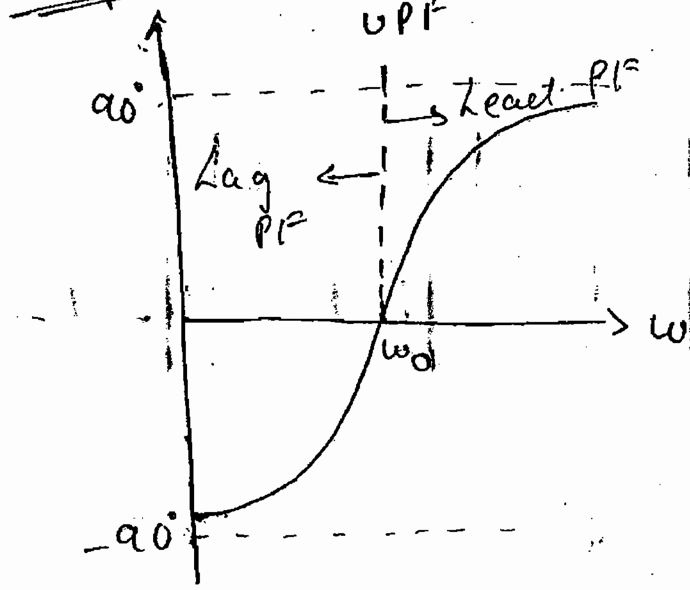
'I' leads 'V' by $\phi < 90°$

(leading PF)

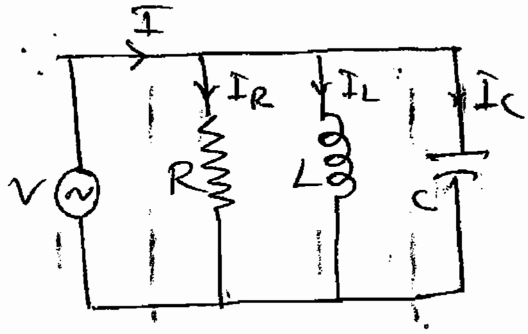



$\phi$ is ~~+ve~~ w.r.t V(ref.)

## Graph ⑤



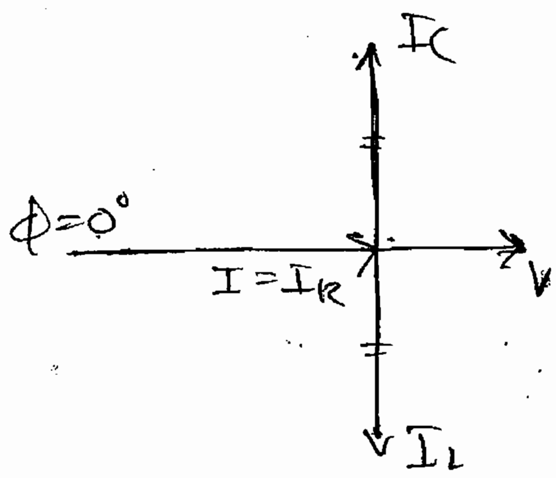

Complete phasor diag




At resonance

$|B_L| = |B_C|$

$|I_L| = |I_C|$




$I = I_R$

$\phi = 0°$

$PF = 1$ (UPF)

Q-factor at Resonance

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q_0 = \frac{R}{\frac{1}{\sqrt{LC}} \times L} = \frac{1}{\sqrt{LC}} \times RC$$

$$Q_0 = R\sqrt{\frac{C}{L}}$$

At parallel resonance condition, net impedance is max. so current is minimum. Hence it is called rejector ckt.

At ll^el resonance freq. it is as if the total current flows only through resis. Hence it is called as current amplification circuit.

So, from circuit

$$|V| = |I||Z| \Rightarrow |V| = \frac{|I|}{|Y|}$$

But, $$|Y| = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\therefore \quad |V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

At $\omega = \omega_0$

$$|V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + 0^2}}$$

So,

$$|V| = |I| R$$

At $\omega = \omega_0$

$\boxed{R}$ $$I_R = \frac{V_R}{R} = \frac{V}{R} = \frac{|I|R}{R} \Rightarrow I_R = |I|$$

$\boxed{L}$ $$I_L = \frac{V_L}{+jX_L} = \frac{V}{+j\omega_0 L} = -j\left[\frac{R}{\omega L}\right] \cdot I$$

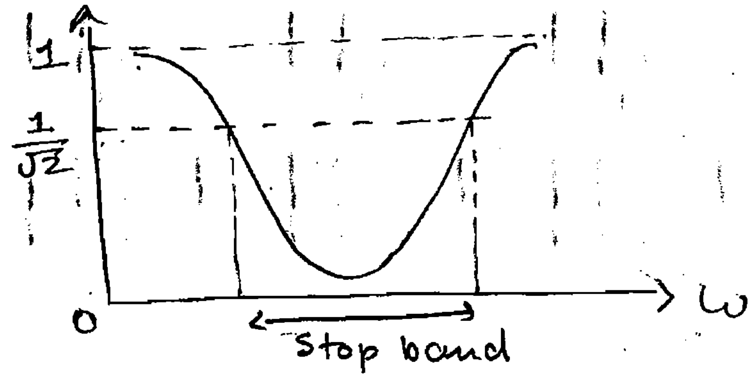
$$\Rightarrow I_L = -j\,Q_0 |I|$$

$\boxed{C}$ $$I_C = \frac{V_C}{-jX_C} = \frac{V}{-j/\omega_0 C} = +j\,[\omega_0 RC] \cdot I$$

$$\Rightarrow I_C = +j\,Q_0 |I|$$

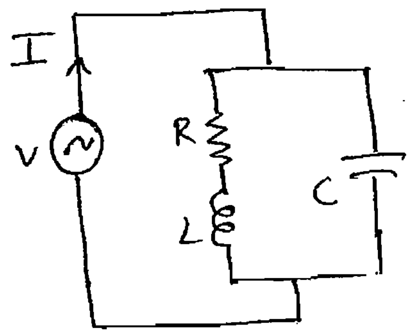
$Q_0 \rightarrow$ current amplification factor.

Parallel resonance phenomenon is used in design of Band Stop Filter.




## Practical Parallel Resonance :-

(Tank Circuit)




$$Y_T = Y_1 + Y_2$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R+jX_L} = \frac{R-jX_L}{R^2+X_L^2}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y_T = \frac{R}{(R^2+X_L^2)} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]$$

At Resonance $\omega = \omega_0$

⇒ Net susceptance = 0

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2} \Rightarrow R^2 + X_L^2 = \omega_0 L \times \frac{1}{\omega_0 C}$$

$$\therefore \quad R^2 + X_L^2 = \frac{L}{C}$$

So, $R^2 + \omega_0^2 L^2 = \frac{L}{C}$

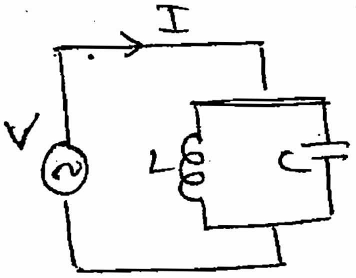
$$\omega_0^2 L^2 = \frac{L}{C} - R^2 \Rightarrow \omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \quad \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad Hz$$

Ideal Tank ckt :—

(R=0)




$$Y_T = Y_L + Y_C \qquad Y_C = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y_L = \frac{1}{jX_L} = \frac{-j}{X_L}$$

$$Y_T = +j\left[\frac{1}{X_C} - \frac{1}{X_L}\right]$$

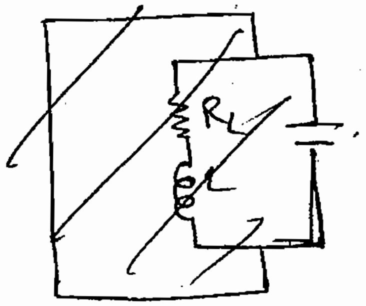
At resonance $\omega = \omega_0$

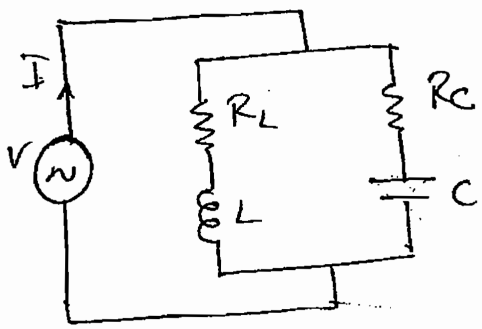
$\Rightarrow$ Net susceptance $= 0$

$$\frac{1}{X_C} = \frac{1}{X_L} \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

1) Determine $f_0$ = ______.




$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$$

# Concept of Dynamic Impedance
## (Dynamic Resistance)

→ It is the resis. offered by the ckt to the i/p at resonant freq.

① Series R-L-C ckt

$$Z_{dyn} = R$$

② Generally parallel R-L-C ckt.

$$Z_{dyn} = R$$

③ Tank ckt :- $Z_{dyn} = \frac{L}{RC}$

practically $Z_{dyn} >> R$

④ Ideal tank ckt :

$$Z_{dyn} = \infty \rightarrow 0°C$$

1) Two practical coils with internal resistance $R_1$, $R_2$ have Q-factor $Q_1$, $Q_2$ resp. If these coils are connected in series then total Q-factor is ——

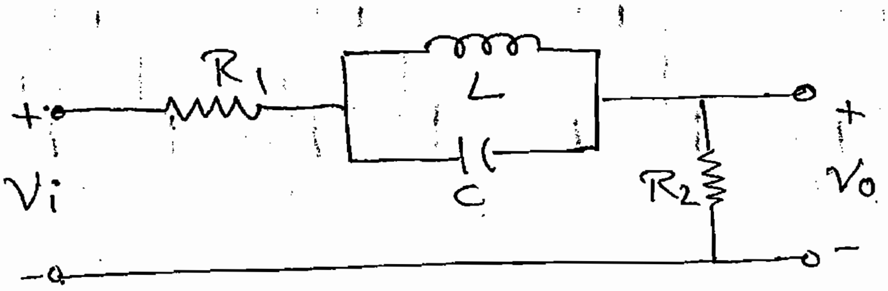
$$Q_1 = \frac{\omega L_1}{R_1}$$

$$Q_2 = \frac{\omega L_2}{R_2}$$

$$Q_T = \frac{\omega (L_1 + L_2)}{R_1 + R_2} = \frac{\frac{\omega L_1}{R_1} \times R_1 + \frac{\omega L_2}{R_2} \times R_2}{R_1 + R_2}$$

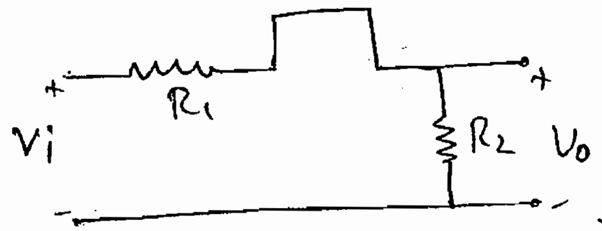
$$\therefore Q_T = \frac{Q_1 R_1 + Q_2 R_2}{R_1 + R_2}$$

2) The n/w below acts as ______




(a) Low pass filter
(b) High pass "
(c) Band pass "
(d) Band stop " ✓

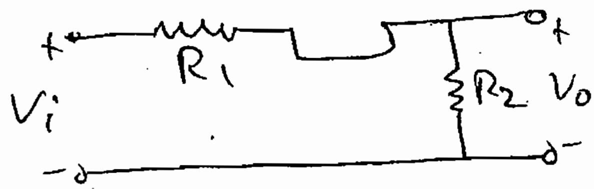
At $\omega = 0$




$$V_o = V_i \left[\frac{R_2}{R_1 + R_2}\right]$$

$$H(\omega) = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

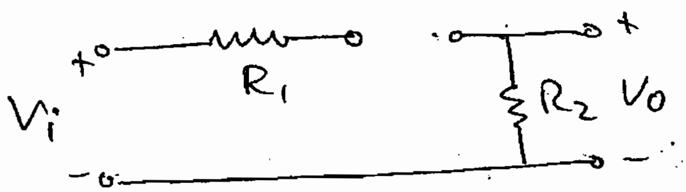
At $\omega = \infty$




$$V_o = V_i \left[\frac{R_2}{R_1 + R_2}\right]$$

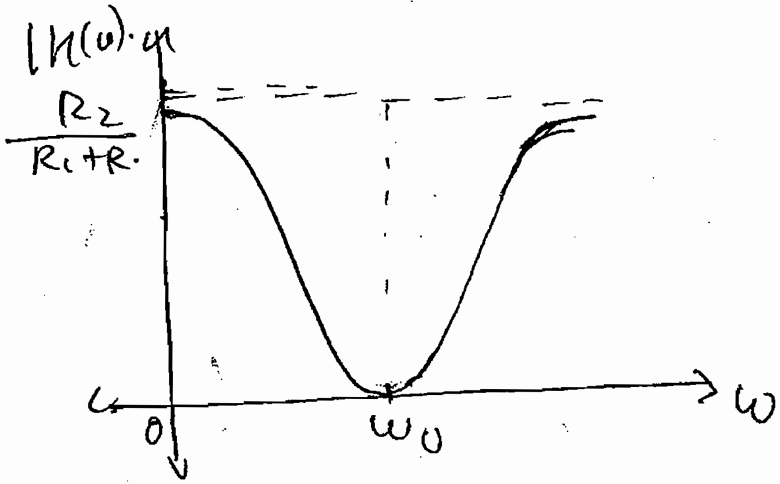
$$H(\omega) = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

At $\omega = \omega_0$




$V_o = 0$

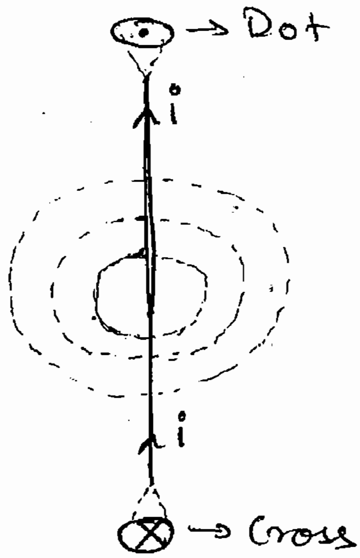
$H(\omega) = 0$



# MAGNETIC CIRCUITS

Charges at rest produce only Electro-static field but charges in motion produce both Electric & Magnetic field

## Amperes Right Hand Thumb Rule :-



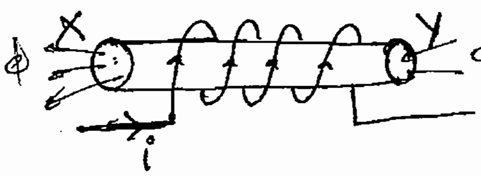

**Permanent Magnet**

↳ Rare Earth material

→ **Alnico's**

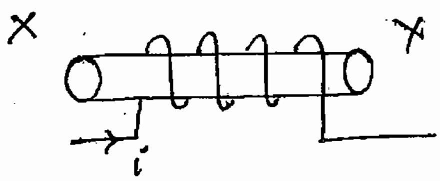
↳ High Retentivity

## Electro-magnet



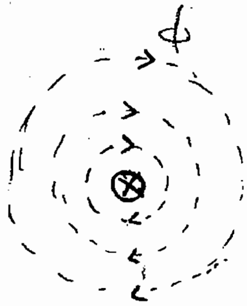

X = N pole

Y = S pole

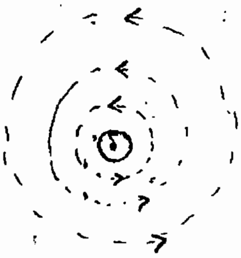



X = S pole

Y = N pole




ϕ = clock-wise oriented



ϕ = anti clock wise oriented.

X i   X = S pole

Y i   Y = N pole

# Analogy b/w Electric & Magnetic ckts

| Electric Circuit | Magnetic Circuit- |
|---|---|
| ① Voltage / EMF is the 'cause' . units : volts | ① MMF → Magneto motive force → cause $\boxed{MMF = N \cdot i}$ units : Ampere-turns (AT) |
| ② 'Current' is the 'effect' I units : Ampere. (A) | ② 'Flux' is the 'effect' ϕ , units : webbers (wb) |
| ③ Ohms Law : $R = \frac{V}{I}$ ↳ Resistance ; unit : Ω | ③ Ohms Law : $S = \frac{MMF}{\phi} = \frac{Ni}{\phi}$ ↳ Reluctance ; units : $\frac{AT}{wb}$ |
| ④ $\boxed{R = \frac{\rho l}{a}}$ → Electrical Material | ④ $\boxed{S = \frac{l}{\mu a}}$ → Magnetic material |
| ⑤ Electric Field Intensity : $\boxed{E = \frac{V}{d}}$ unit : volt/m | ⑤ Magnetic Field intensity $\boxed{H = \frac{MMF}{l} = \frac{N \cdot i}{l}}$ units : AT/m |

## ⑥ Electric Current Density

$$\boxed{J = \frac{I}{a}}$$ unit : $\frac{A}{m^2}$

⑦ $\boxed{D = \varepsilon E}$

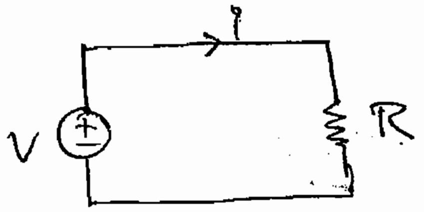
↳ Electric flux density

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

⑧ $R_S = R_1 + R_2$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

⑨




↳ Electric Conductor

(Cu, Al, etc.)

⑩ KVL

KCL

VDR (series)

CDR ($\parallel^{el}$)

## ⑥ Magnetic Flux Density

$$\boxed{B = \frac{\phi}{a}}$$ units: $\frac{wb}{m^2}$ (or) Tesla (T)

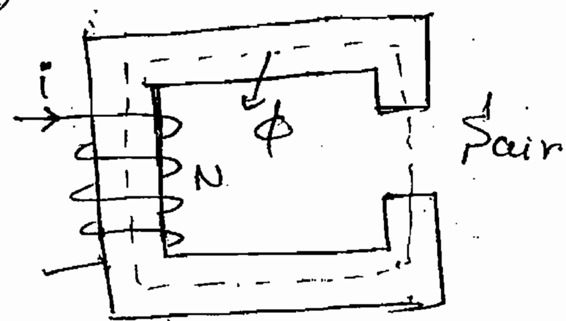
⑦ $\boxed{B = \mu H}$

$$\boxed{\mu = \mu_0 \mu_r}$$

⑧ $S_s = S_1 + S_2$

$$\frac{1}{S_P} = \frac{1}{S_1} + \frac{1}{S_2}$$

⑨




↳ Magnetic Conductor

(Fe, SiFe, etc)

⑩ K MMF L

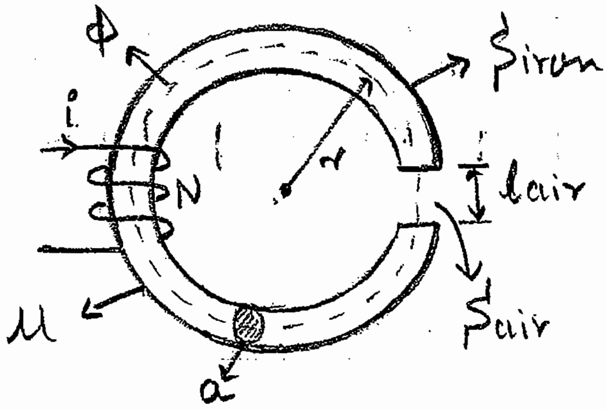
K Flux L

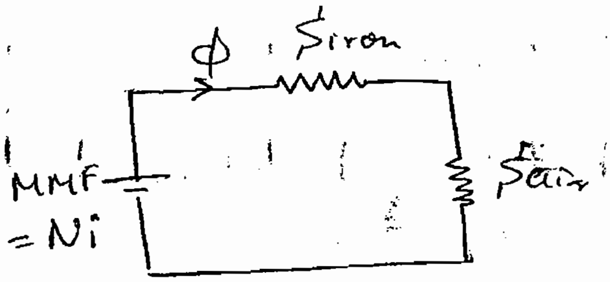
MMF · D · R (Series)

Flux · D · R ($\parallel^{el}$)

# Composite Magnetic Circuits :—

## Series Magnetic Circuit




$$S_{iron} = \frac{l_{iron}}{\mu_0 \mu_r a}$$

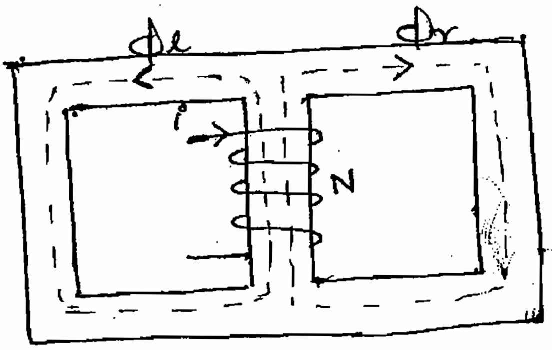
$$S_{air} = \frac{l_{air}}{\mu_0 (1) a}$$

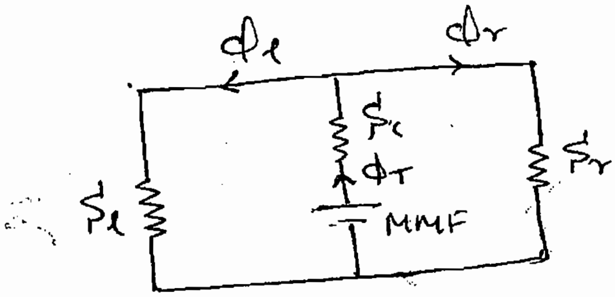
$$MMF = Ni = \phi [S_T]$$

$$Ni = \phi [S_{iron} + S_{air}]$$

$$l = 2\pi r$$

$$l_{iron} + l_{air} = 2\pi r$$




$$[MMF - \phi_T S_c] = \phi_l S_l = \phi_r S_r$$

# Faraday's Law of Electromagnetic Induction :—

1. "There should be change in the flux linkage with the coil in order to induce emf in it."

2. "The magnitude of this induced emf is proportional the rate of change of flux."

Mathematically Faradays law is given by:

$$e = -\frac{d\psi}{dt}$$

But, $\psi = N\phi \Rightarrow$ Flux linkages (wb-Turns)

$$\therefore \boxed{e = -N\frac{d\phi}{dt}}$$

$\frac{d\phi}{dt}$ → Rate of change of flux (wb/sec)

$N$ → No. of turns.

$-$ → -ve sign due to Lenzs' law

$e$ → induced emf in coil (volts)

$$e = -N\left[\frac{\phi_{final} - \phi_{initial}}{\Delta t}\right]$$

## Dynamically Induced emf :-

Ex Motors, generator.

## Statically Induced emf :-

Ex Transformers.

⇒ Flux is a function of current

$$MMF = Ni = \phi S$$

$$\therefore \frac{\phi}{i} = \frac{N}{S}$$

$N$ → const

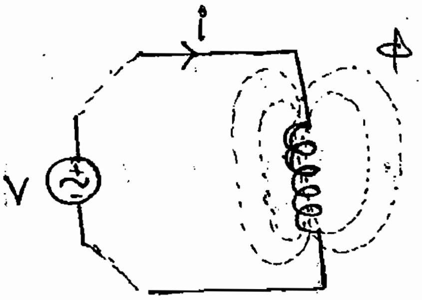
$S$ → const.

$$\therefore \boxed{\frac{\phi}{i} = k} \Rightarrow \phi = ki$$

$\phi \propto i$

$\phi$ → Flux, a fun of current

# Concept of self induced emf & Self inductance :-




$$\phi = \frac{\phi}{i} \times i$$

$$\frac{d\phi}{dt} = \frac{\phi}{i} \frac{di}{dt}$$

$$e = -N \frac{d\phi}{dt}$$

$$\therefore \quad e = -\frac{N\phi}{i} \cdot \frac{di}{dt}$$

$$e = -L\frac{di}{dt} \text{ volts} \qquad \left( v = -e = +L\frac{di}{dt} \right)$$

$$L = \frac{N\phi}{i} = \frac{\psi}{i} \qquad \text{(self inductance H)}$$

$$e = -L\left[\frac{i_{final} - i_{initial}}{\Delta t}\right] \text{ volts.}$$

$$\boxed{\psi = Li} \longrightarrow \text{Ohms Law}$$

Also

$$MMF = \phi S$$

$$Ni = \phi S$$

$$\therefore \quad N\left[\frac{N\phi}{L}\right] = \phi S$$

$$L = \frac{N^2}{S}$$

$$L \propto N^2$$

Also

$$S = \frac{l}{\mu a}$$

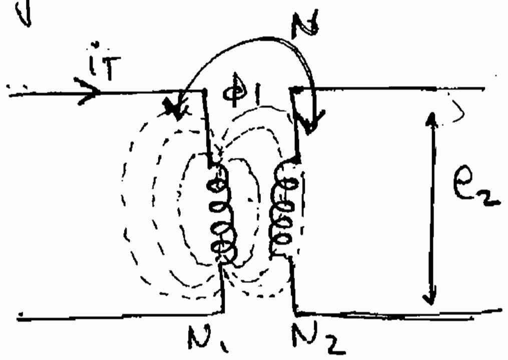
$$L = \frac{N^2}{l/\mu a}$$

$$L = \frac{\mu N^2 a}{l} \quad \underline{H}$$

# Concept of Mutual Inductance & Mutually Induced Emf :—

Self inductance is w.r.t the same coil & its own turns, current & flux.

However mutual inductance is b/w sep. of coils (mini. two).



$$\boxed{\phi_{12} = k\phi_1}$$

$$0 \le k \le 1$$

$$\phi_{12} = \phi_{12} \times \frac{i_1}{i_1}$$

$$\phi_{12} = \frac{\phi_{12}}{i_1} \cdot i_1$$

$$\frac{d\phi_{12}}{dt} = \left[\frac{\phi_{12}}{i_1}\right] \frac{di_1}{dt} \quad \text{—— (1)}$$

$$\phi_1 = \phi_{11} + \phi_{12}$$

Total flux, Leakage flux, Common flux (Mutual flux)

$$e_2 = -N \frac{d\phi_{12}}{dt} \quad \text{—— (2)}$$

$$e_2 = -\left[\frac{N\phi_{12}}{i_1}\right] \frac{di_1}{dt} \Rightarrow \boxed{e_2 = -[M_{12}] \frac{di_1}{dt}}$$

↳ Mutually induced emf (volts)

$$M_{12} = \frac{N_2\phi_{12}}{i_1} = \frac{k\phi_1 N_2}{i_1}$$

Mutual inductance in H.

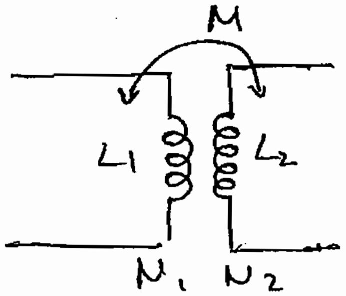
Also vice-versa

$$\phi_{21} = k\phi_2$$

$$e_1 = -\left[\frac{N_1\phi_{21}}{i_2}\right] \frac{di_2}{dt}$$

$$e = -[M_{21}]\frac{di_2}{dt} \qquad M_{21} = \frac{N_1 \phi_{21}}{i_2} = \frac{k\phi_2 N_1}{i_2}$$

* If dist. b/w coils & permeability of medium b/w coils is const., then : $M_{12} = M_{21} = M$




$$L_1 = \frac{N_1\phi_1}{i_1}$$

$$L_2 = \frac{N_2\phi_2}{i_2}$$

$$M = \frac{k\phi_1 N_2}{i_1} = \frac{k\phi_2 N_1}{i_2}$$ } Mutual inductance b/w coils.

## Relation b/w self & Mutual inductances :—

$$M * M = \left[\frac{k\phi_1 N_2}{i_1}\right] * \left[\frac{k\phi_2 N_1}{i_2}\right]$$

$$M^2 = k^2\left[\frac{N_1\phi_1}{i_1}\right]\left[\frac{N_2\phi_2}{i_2}\right]$$

$$M^2 = k^2 [L_1][L_2]$$

$$\boxed{M = k\sqrt{L_1 L_2}}$$

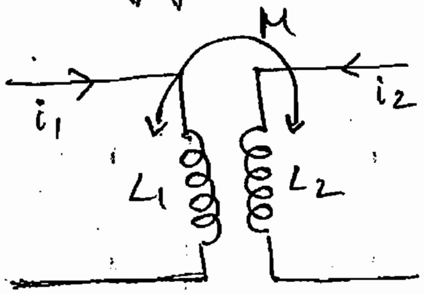
But, $0 \leq k \leq 1 \Rightarrow \boxed{M \leq \sqrt{L_1 L_2}}$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

↳ co-eff of coupling

For ideal transf. $k = 1$

Energy stored in system of 2-coils :—




$$E_T = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

$+ \rightarrow$ Mutually adding flux
$- \rightarrow$ Mutually opposing flux

Mutual inductance is always ~~a~~ a ~~+ve~~ quantity.

But, mutually induced emf may be +ve & -ve

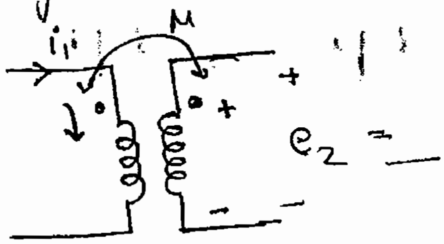
Determining the correct polarity of mutual induced emf is not possible directly & hence we use Dot convention.

① If current enters the dotted terminal of the 1st coil, then the polarity of mutual vltg is +ve at the dotted terminal of 2nd coil.

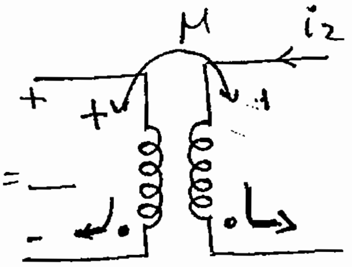
② If current leaves dotted terminal of 1st coil, then the polarity of mutual vltg is -ve at the dotted terminal of the 2nd coil.

## Ex

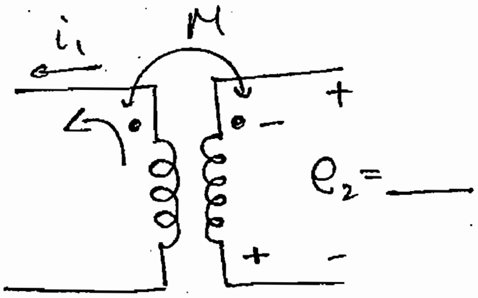
1) Determine the correct magnitude & polarity of the mutual voltage w.r.t the given reference voltage for the sys. of coils shown below.

① 

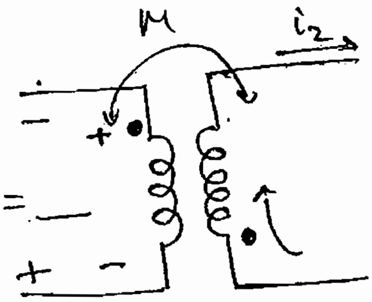

$$e_2 = +M\frac{di_1}{dt}$$

② 


$$e_1 = +M\frac{di_2}{dt}$$

③ 

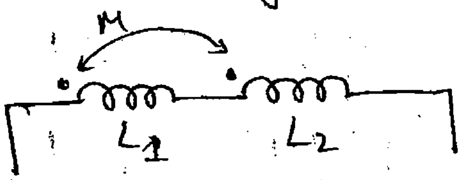

$$e_2 = -M\frac{di_1}{dt}$$

④ 


$$e_1 = -M\frac{di_2}{dt}$$

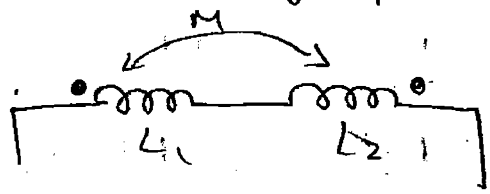
# Coils in Series

Mutually adding




$$L_{eq} = L_1 + L_2 + 2M$$

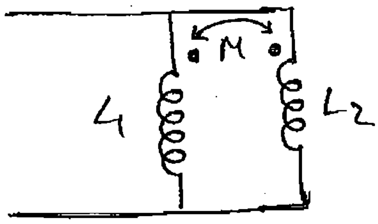
Mutually opposing




$$L_{eq} = L_1 + L_2 - 2M$$

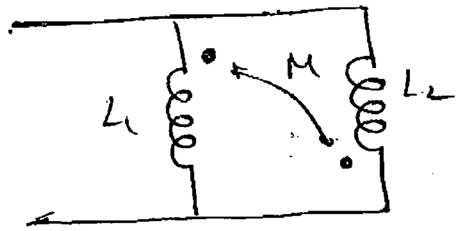
# Coils in Parallel

Mutually adding



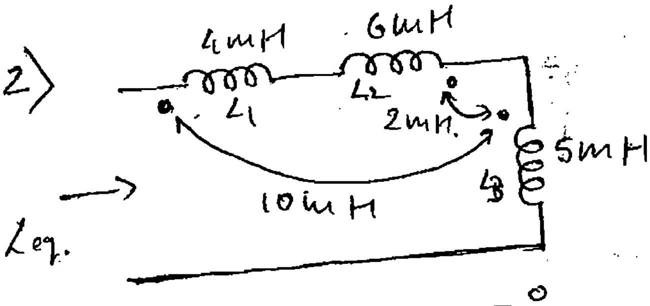

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Mutually opposing




$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

2)



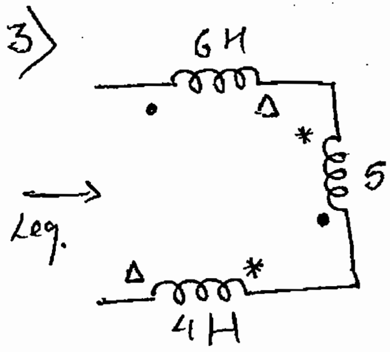

$L_1 + M_{13} - \overset{0}{M_{12}} + L_2 - \overset{0}{M_{21}}$
$- M_{23} + L_3 + M_{31} - M_{32}$ (or)

$= (4+6+5) + 2(10) - 2(2)$
$= 15 + 20 - 4$
$= 31$ mH

$V_1 = 4 + 10 = 14$
$V_2 = 6 - 2 = 4$
$V_3 = 5 + 10 - 2 = 13$

$\Big\}$ 31 mH

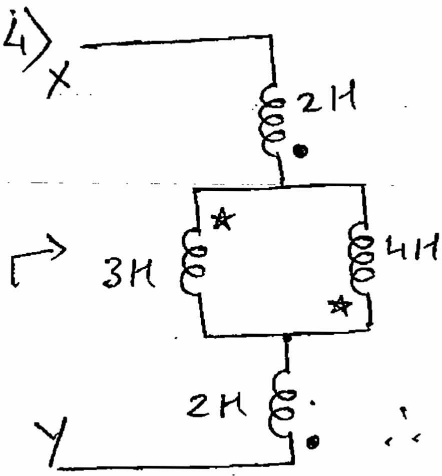
3)



• 3H
Δ 2H
* 1H

$$L_{eq} = (6-3+2) + (5-3+1) + (4+1+2)$$
$$= 5+3+7$$
$$L_{eq} = 15\ \text{H}$$

4)



• 1H
★ 2H

S.A

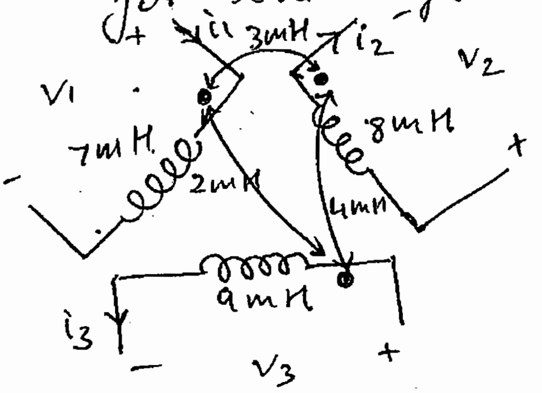
$$2+2+2 = 6\ \text{H}$$

D.O

$$\frac{12-4}{3+4+4} = \frac{8}{11}$$

$$\therefore L_{eq} = 6 + \frac{8}{11} = \frac{74}{11}\ \text{H}$$

5) Write the complete inductance Matrix for the systems of coils below



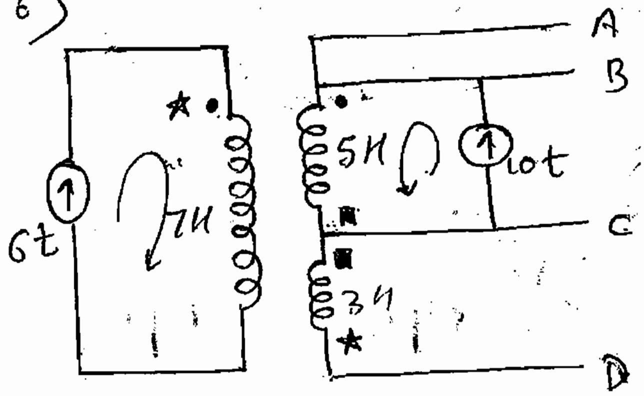
$$v_1 = 7\frac{di_1}{dt} - 3\frac{di_2}{dt} + 2\frac{di_3}{dt}$$

$$v_2 = -3\frac{di_1}{dt} + 8\frac{di_2}{dt} - 4\frac{di_3}{dt}$$

$$v_3 = 2\frac{di_1}{dt} - 4\frac{di_2}{dt} + 9\frac{di_3}{dt}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 2 \\ -3 & 8 & -4 \\ 2 & -4 & 9 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \\ di_3/dt \end{bmatrix}$$

6)



$\cdot$ 4H

$\bigstar$ 2H

$\blacksquare$ 1H

$V_{BD} = ?$

$V_{BD} = V_{BC} + V_{CD}$

$V_{BC} = V_5 + V_{M(\cdot)}$

$= 5\frac{d}{dt}(10t) + 4\frac{d}{dt}(6t)$

$= 50 + 24 = 74$ volts

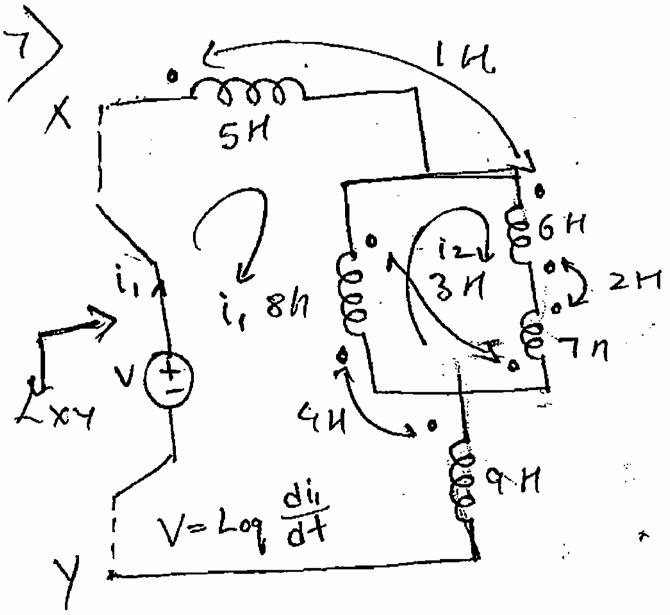
$V_{CD} = V_{M(\bigstar)} + V_{M(\blacksquare)}$ (No ~~individual~~ self inductance as current does not flow)

$= -2\frac{d}{dt}(6t) - 1\frac{d}{dt}(10t)$

$= -12 - 10 = -22$ volts

$\therefore V_{BD} = 74 - 22 = \underline{52 \text{ volts}}$.

7)



Rules [Mutual Vltg]

① Which Mesh?

② Which two dots?

③ ⊕ or ⊖

④ Resultant current

M-I

$-V + 5\frac{di_1}{dt} + 8\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] + 9\frac{di_1}{dt} + 1\frac{di_2}{dt} - 3\frac{di_2}{dt}$

$-4\frac{di_1}{dt} - 4\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0$

$\therefore \; 14X - 6Y = V$ ——— ①

M-2

$$8\left[\frac{di_2}{dt} - \frac{di_1}{dt}\right] + 6\frac{di_2}{dt} + 7\frac{di_2}{dt} + 4\frac{di_1}{dt} + 3\frac{di_2}{dt} + 1\frac{di_1}{dt}$$
$$-2\frac{di_2}{dt} + 3\left[\frac{di_2}{dt} - \frac{di_1}{dt}\right] - 2\frac{di_2}{dt} = 0$$

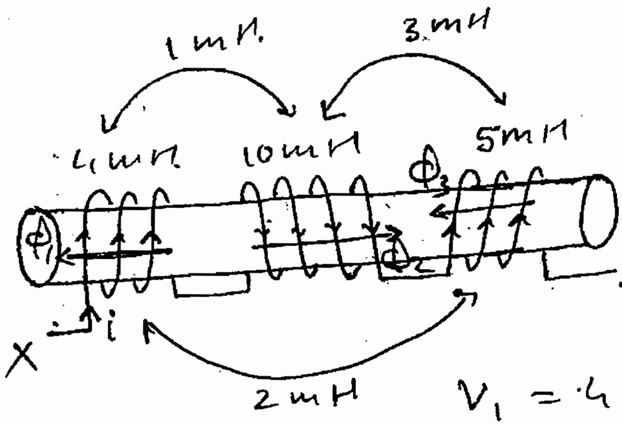
$\therefore -6X + 23Y = 0$ ——— ②

From ① & ②

$$X\left[14 - \frac{6\times 6}{23}\right] = V$$

$$\therefore V = \left[\frac{286}{23}\right]\frac{di_1}{dt} \Rightarrow L_{eq} = \frac{286}{23}\ H.$$

8)



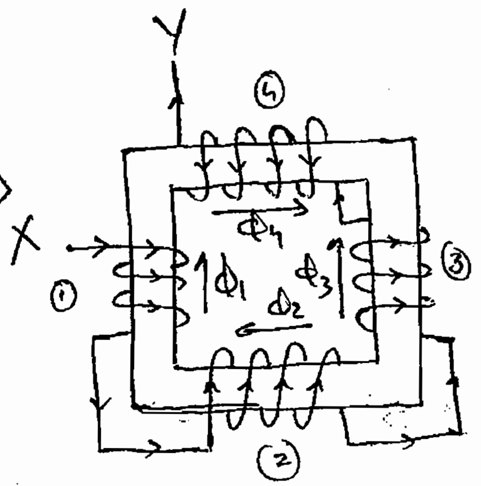

$L_{XY} = ?$

Using Right Hand Thumb rule we decide the dirⁿ of flux

$V_1 = 4 - 1 + 2 = 5$
$V_2 = 10 - 1 - 3 = 6$
$V_3 = 5 + 2 - 3 = 4$

$\Big\}$ 15 mH

9)




Find $L_{XY}$ if:

$L_{11} = L_{22} = L_{33} = L_{44} = 5\ mH$

$M_{12} = M_{23} = M_{34} = 2\ mH$

$M_{13} = M_{14} = M_{24} = 1\ mH$

$V_1 = 5 + 2 - 1 + 1 = 7$

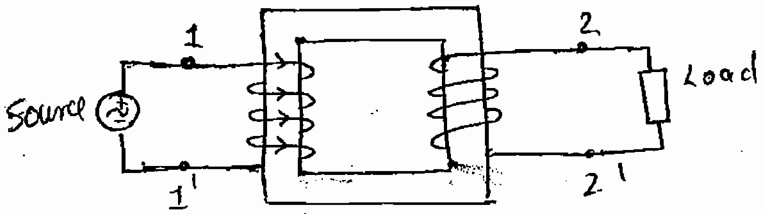
$V_2 = 5 + 2 - 2 + 1 = 6$

$V_3 = 5 - 1 - 2 - 2 = 0$

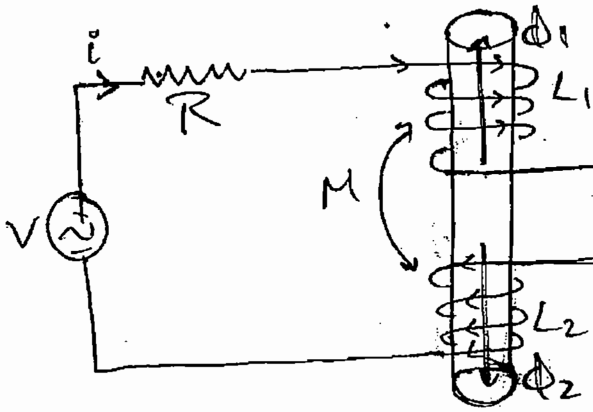
$V_4 = 5 + 1 + 1 - 2 = 5$

$\therefore L_{xy} = 7 + 6 + 0 + 5 = 18 \text{ mH}$

10) Place correct dot convention b/w 2 coils




11) Write KVL governing the circuit below & find resonance frequency '$f_0$' if $R = 10\Omega$, $L_1 = L_2 = 10 \text{ mH}$, $M = 2 \text{ mH}$, $C = 0.1 \mu F$




KVL (Exact form)

$$-V + iR + (L_1 + L_2 - 2M)\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

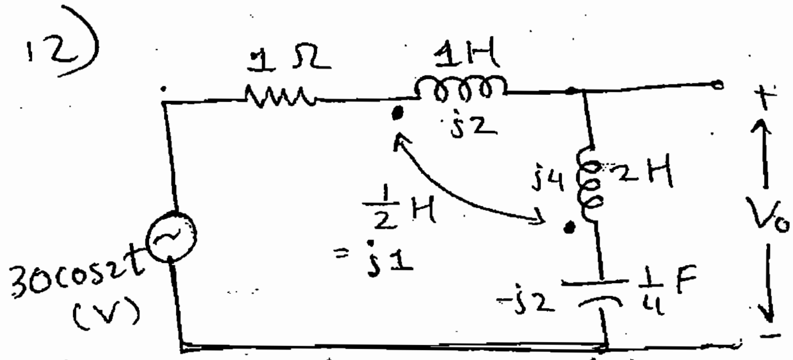
KVL (S.S AC form)

$$-V + IR + j\omega(L_1 + L_2 - 2M)I - \frac{j}{\omega C} I = 0$$

$$f_0 = \frac{1}{2\pi\sqrt{(L_1 + L_2 - 2M)C}}$$

$=$

12)




<u>KVL</u> $-[30\angle 0^\circ] + (1+j2+j4-j2)\,I - (j1\times 2)\,I = 0$

$(1+j2)I = 30\angle 0^\circ$

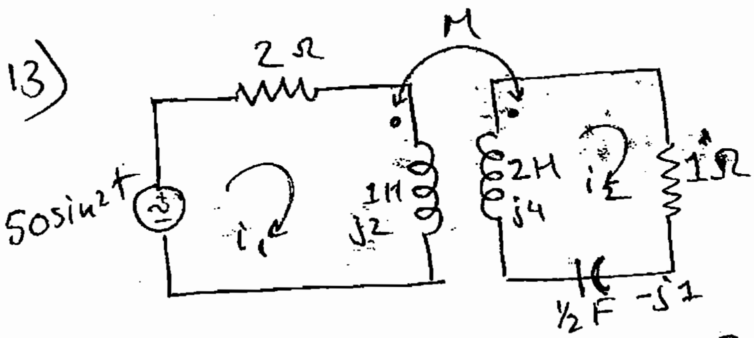
$\therefore\ I = \dfrac{30\angle 0^\circ}{1+j2}$

$V_0 = I[+j4-j2] - j1[I] = jI$

$= \dfrac{30\angle 90^\circ}{1+j2} = 13.41\angle 26.56^\circ$

$\therefore V_0 = 13.41\cdot\cos(2t+26.56)$

13)




Find power lost in 1Ω resis.

$-[50\angle 0^\circ] + (2+j2)\,I_1 - j1[I_2] = 0$ —— ①

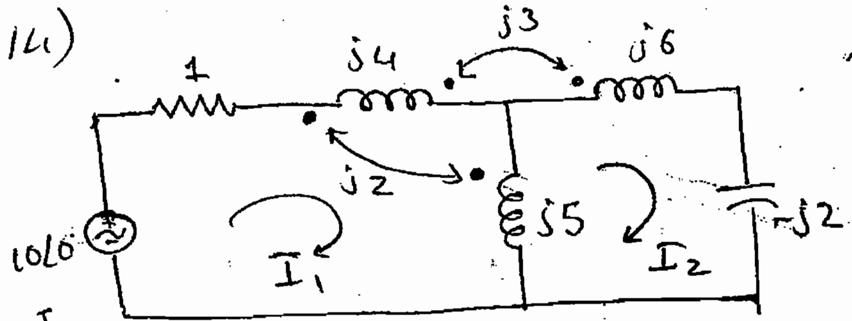
$(1+j4-j1)\,I_2 - j1[I_1] = 0$ —— ②

Solving for $I_2$ = ______

$I_{2\,(RMS)} = \dfrac{\quad}{\sqrt{2}}$

$P_{1\Omega} = |I_{2\,(rms)}|^2 * R$

14)




Write Mesh eq^n governing the circuit below.

M-I

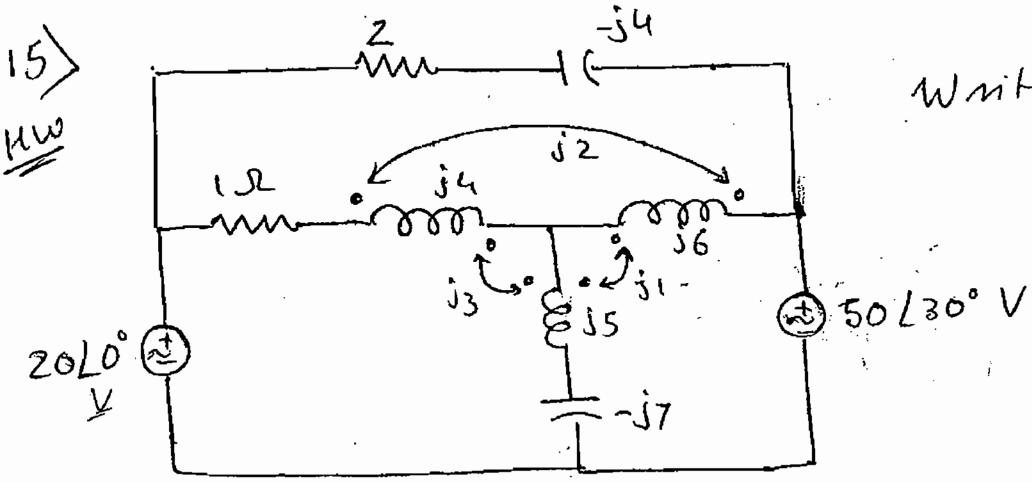
$$-[10\angle 0^\circ] + (1+j9)I_1 - j5 I_2 + j2[I_1 - I_2] - j3[I_2] + j2[I_1] = 0$$

M-II

$$j5[I_2 - I_1] + j6 I_2 - j2 I_2 - j2[I_1] - j3[I_2] = 0$$

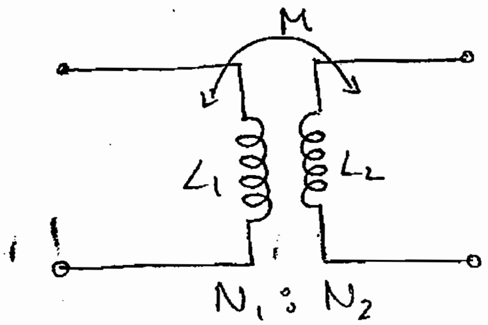
15)

HW




Write mesh eq^ns.

# IDEAL TRANSFORMER (Circuit Concept)




it can store / transfer ideally electrical energy

$L_1 \to \infty$, $M \to \infty$

$L_2 \to \infty$

$K = 1 \longrightarrow$ co. eff. of coupling.

→ No losses

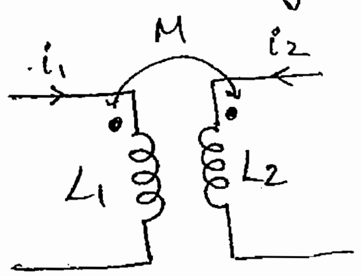
→ 100 % $\eta$

$$\boxed{L_1 : L_2 : M = N_1^2 : N_2^2 : N_1 N_2}$$

Turns Ratio: $\dfrac{N_1}{N_2} = \sqrt{\dfrac{L_1}{L_2}} = \dfrac{L_1}{M} = \dfrac{M}{L_2}$

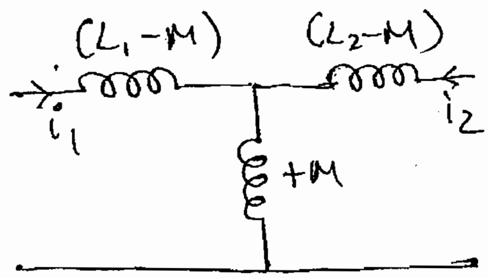
## T-equivalent representation of Ideal transformer (Circuit concept)

(A) Mutually Adding




KVL: $V_1 = L_1 \dfrac{di_1}{dt} + M \cdot \dfrac{di_2}{dt}$

$V_2 = L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt}$

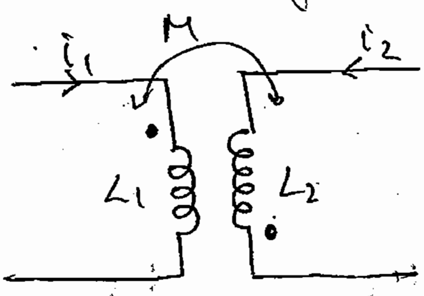


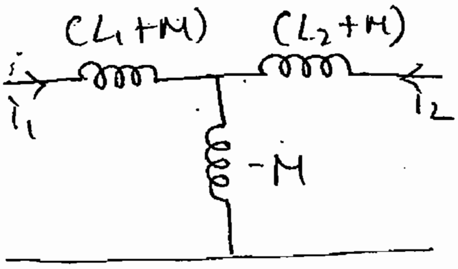

KVL

$V_1 = (L_1 - M)\dfrac{di_1}{dt} + M\left[\dfrac{di_1}{dt} + \dfrac{di_2}{dt}\right]$

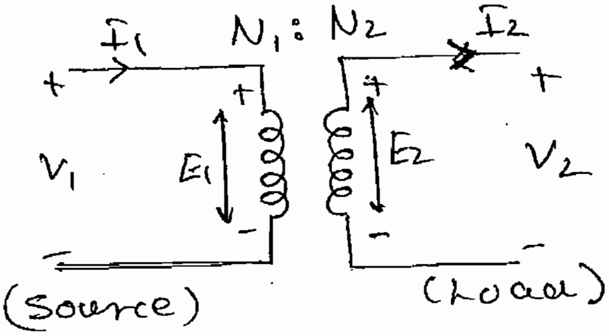
$V_2 =$

(b) Mutually opposing.




# IDEAL TRANSFORMER Machine Concept :-




But,

$$a = \frac{1}{K} = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

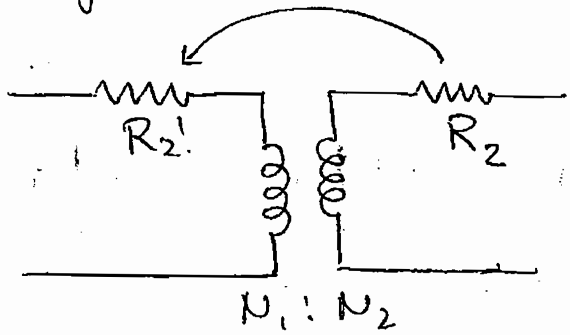
↳ Turns ratio

Here.

$$\boxed{K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_2}{I_2}}$$

↳ Voltage transformation ratio

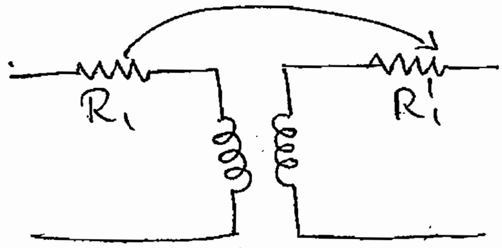
## Concept of Reflecting Primary (or) Secondary circuit parameters to simplify the T/f as a single ckt:




$$\boxed{R_2' = \frac{R_2}{K^2}} \quad \left(\because k = \frac{N_2}{N_1}\right)$$

↳ secondary resis. reflected to primary

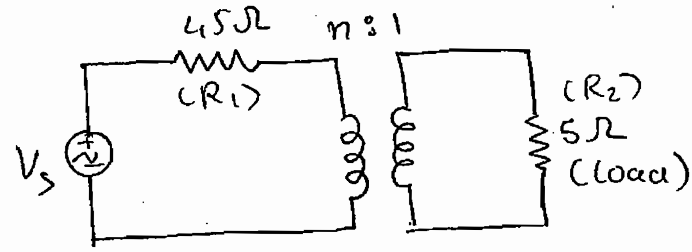
Also; $X_2' = \dfrac{X_2}{k^2}$ , $Z_2' = \dfrac{Z_2}{K^2}$




$$\boxed{R_1' = k^2 R_1}$$

↳ primary resis. reflected to secondary

1) What is the value of 'n' for which max. power is transfered to the load.

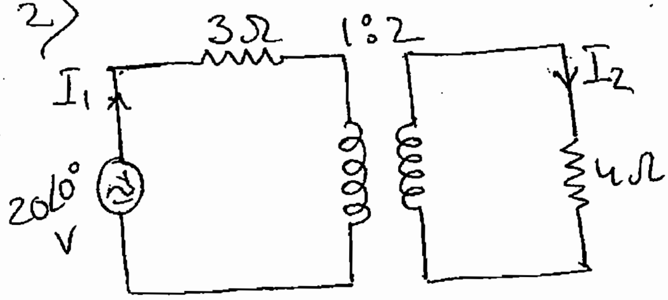



$$R_2' = \frac{R_2}{k^2} = \frac{R_2}{(1/n)^2}$$
$$= 5n^2$$

For Pmax

$R_L = R_S \Rightarrow 5n^2 = 45 \Rightarrow n^2 = 9 \Rightarrow n = 3$

2)




$I_1 = ?$ $\quad I_2 = ?$

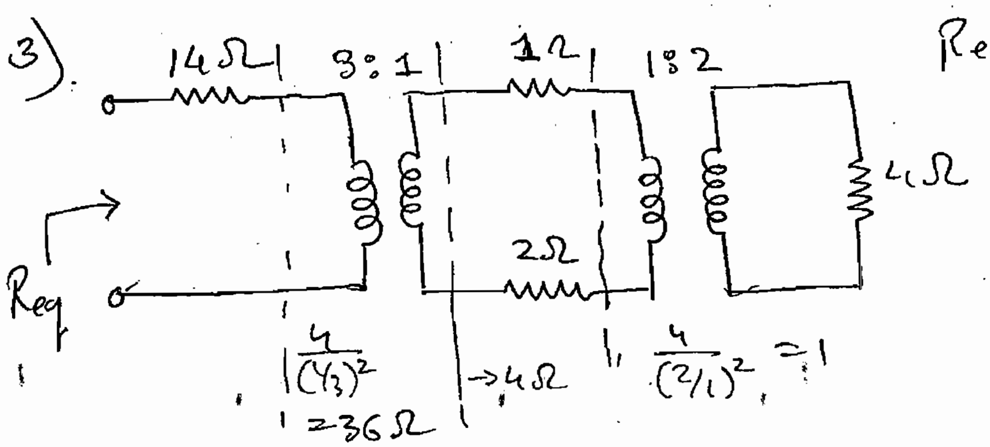
$$R_1 + R_2' = 3 + \frac{4}{(2/1)^2} = 4\,\Omega$$

$$I_1 = \frac{V_S}{R_T} = \frac{20\angle 0^\circ}{4} = \underline{5\angle 0^\circ\ A}$$

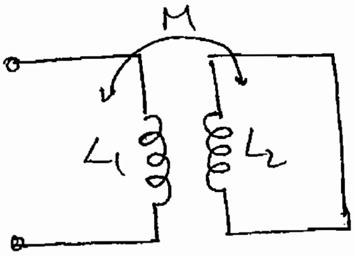
$$k = \frac{I_1}{I_2} = \frac{2}{1}$$

$$I_2 = \frac{I_1}{2} = \frac{5\angle 0^\circ}{2} = \underline{2.5\angle 0^\circ\ A}$$

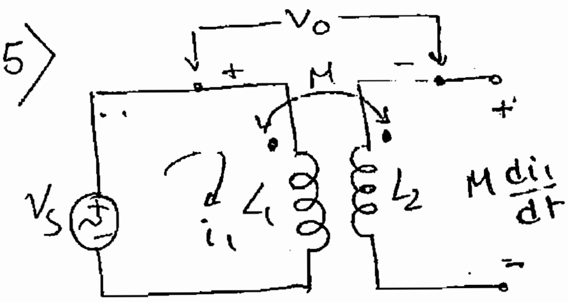
3) $R_{eq} = \underline{\quad ? \quad}$




$R_{eq} = 14 + 36 = 50\ \Omega$

4) 


$L_{eq} = ?$ can be solved by both methods: Mutually adding
Mutually opposing

5) 


Find $\underline{V_o}$

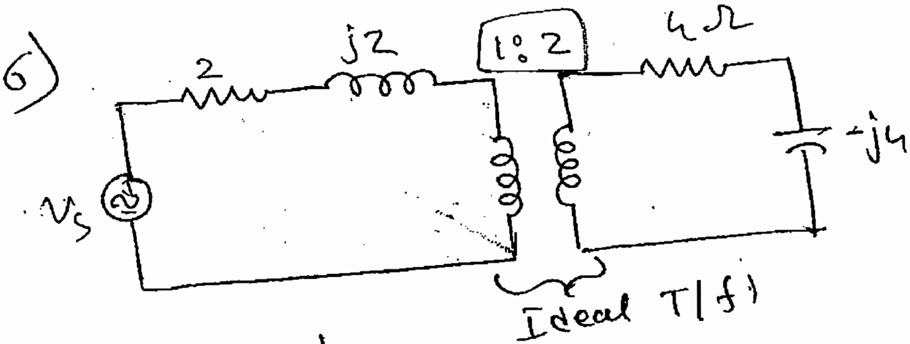
(a) $V_s\left[1-\frac{M}{L_1}\right]$ (b) $V_s\left[1-\frac{M}{L_2}\right]$

(c) $V_s\left[1-\frac{L_1}{M}\right]$ (d) $V_s\left[1-\frac{L_2}{M}\right]$

$V_s = L_1 \frac{di_1}{dt}$

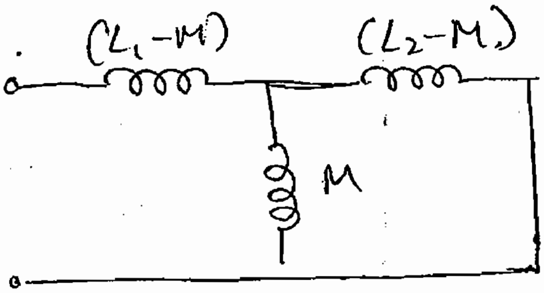
$V_o = V_s - M\frac{di_1}{dt} = V_s - M\left(\frac{V_s}{L_1}\right)$

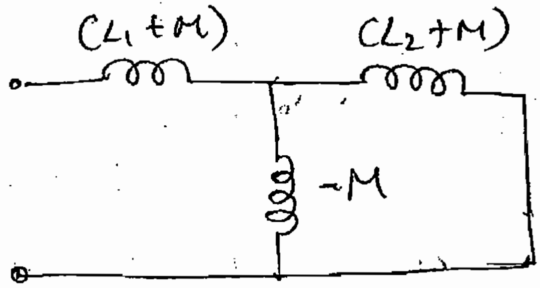
$\therefore V_o = V_s\left[1-\frac{M}{L_1}\right]$

6) 


$Z_T' = Z_1 + Z_2'$

$= (2+j2) + \frac{(4-j4)}{(2/1)^2} = (3+j)\ \Omega$

4) 


$$L_{eq} = L_1 - M + \left[\frac{M(L_2-M)}{M+L_2-M}\right]$$

$$= L_1 - M + \frac{ML_2}{L_2} - \frac{M^2}{L_2}$$

$$= L_1 - \frac{M^2}{L_2}$$

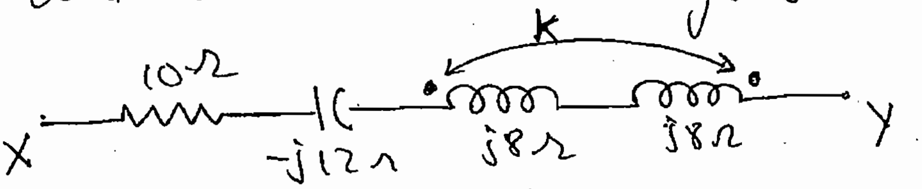
$$L_{eq} = L_1 + M + \left[\frac{-M(L_2+M)}{-M+L_2+M}\right]$$

$$= L_1 + M - \frac{ML_2}{L_2} - \frac{M^2}{L_2}$$

$$= L_1 - \frac{M^2}{L_2}$$

⇒ Dot convention will not affect circuit performance.
Its just used for analysis purpose.

5) What is the value of k for which the branch undergoes resonance.




~~For~~ resonance : $X_L = X_C$

$\therefore$ ~~$j12 = (j8 + j8 - 2M)\omega$~~

$$X_{L_{eq}} = \omega L_{eq}$$

$$= \omega[L_1 + L_2 - 2M]$$

$$= \omega L_1 + \omega L_2 - 2\omega k\sqrt{L_1 L_2}$$

$$= \omega L_1 + \omega L_2 - 2k\sqrt{(\omega L_1)(\omega L_2)}$$

$$= 8 + 8 - 2k\sqrt{8 \times 8}$$

$$= 16 - 16k$$

so, $X_L = X_C$

$\therefore$ $16 - 16k = 12$ $\Rightarrow$ $16k = 4$

$\boxed{k = 0.25}$

# ELECTRICAL TRANSIENTS

→ Transients are considered as sudden changes in the state of a ckt or n/w which are indicated by the switch operation.

→ Transients occur in any electrical n/w or sys. as none of the system's are adaptable for quick sudden changes in time.

→ Transients are also considered as an argument b/w the i/p command to o/p response as a n/w changes from previous steady state to next steady state w.r.t the state variable. (I, L or V, C)

→ Though transients occur for very short duration in time, their impact is huge in determining the entire Steady state resp.

→ Though customers look into steady state performance of an electrical device or n/w, the designers are more interested in transient state performance as they give the critical design specification values.

# State Variables :—

These are the critical parameters that must be observed to determine the transient state solution in any n/w.

(a) In a capacitor :—

$$\boxed{i = C\frac{dV}{dt}}$$

(KCL → Nodal) ↳ Voltage across capacitor is correct S.V.

(b) In an inductor :—

$$\boxed{V = L\frac{di}{dt}}$$

(KVL → Mesh) ↳ Current through inductor is correct S.V.

→ The response of any ckt / n/w when a source is present is called as forced response.

This response is independent to the nature of passive elements & it can be different for diff. types of i/ps.

Eg:- DC ckt Analysis, AC ckt analysis, etc.

The response of any ckt or n/w without any source is called as Natural resp. This response is independent to i/p but purely depents upon the nature of passive elements. & it is always unique determined by the characteristics eqn governing the n/w.

This source free response is possible, provided the n/w has initial stored energy (inductor stores energy in electromagnetic form : $\psi = Li \rightarrow I_0$

capacitor stores energy in electrostatic form : $q = CV \rightarrow V_0$ )

So,

$$\begin{bmatrix}\text{Complete resp.}\\ \text{of a sys.}\end{bmatrix} = \underbrace{\begin{bmatrix}\text{Forced}\\ \text{resp.}\end{bmatrix}}_{\text{Zero state response}} + \underbrace{\begin{bmatrix}\text{Natural}\\ \text{resp.}\end{bmatrix}}_{\text{Zero i/p response}}$$

Any n/w or sys. reaches steady state after overcoming transient state. So in this chapter, as we are determining state response, all the solutions of voltage, current, power, energy, etc are w.r.t time.

## Initial Conditions :—

These are critical values of vltg across capacitors & current through inductors from the previous steady state of a n/w which are specifically indicated as :

$t = 0^-$ → instant just before s/w operation (steady state before s/w operation)

$t = 0^+$ → instant just after s/w operation. (TRANSIENT state operation)

$t \to \infty \Rightarrow$ Steady state after s/w operation

## Order of ckt or n/w :—

The no. of energy storage components available in distributed form in any ckt represents its order

Eg:- R-L, R-C $\to$ 1st order n/w

R-L-C, L-L-R, C-C-R, L-C $\to$ 2nd order n/w

NOTE :-

① A capacitor will never allow sudden change in vltg across it.

$$V_c(0^-) = V_c(0) = V_c(0^+)$$

② ~~An current will~~ inductor will never allow sudden change in current through it.

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

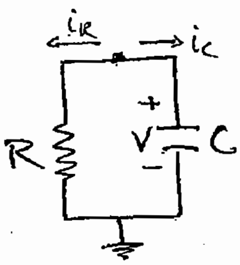
# Behaviour of passive elements in Transient state in comparision to Steady state :—

If analyzing of n/w as $t \to 0^+$, is considered as transient solution then, analysing the same n/w as $s \to \infty$ (Steady state freq. ~~resp.~~ sol$^n$) is also considered as Transient solution Hence L.T. are powerfull tools to analyze the n/w during TRANSIENT STATE.

| Element | D.C SS $(s=0)$ | A.C SS $(s=j\omega)$ | Transient State $(t\to 0^+)$ $(s\to\infty)$ | $s=j\omega$ ↳ complex freq. |
|---|---|---|---|---|
| R | R | R | R | $Z_R = R$ |
| L | S.C | 'i' lags 'V' by $\phi = 90°$ | O.C | $Z_L = +j\omega L$ $= sL$ |
| C | O.C | 'i' leads 'V' by $\phi = 90°$ | S.C | $Z_C = \frac{1}{j\omega C}$ $= \frac{1}{sC}$ |

# [I] Source Free $1^{st}$ Order :—

## (a) R-C circuit :—




Let $V(0) = V_0$

KCL $\quad i_L + i_c = 0$

$\therefore \; C\frac{dV}{dt} + \frac{V}{R} = 0$

$\therefore \; C\frac{dV}{dt} = -\frac{V}{R} \Rightarrow \int \frac{dV}{V} = \int -\frac{dt}{RC}$

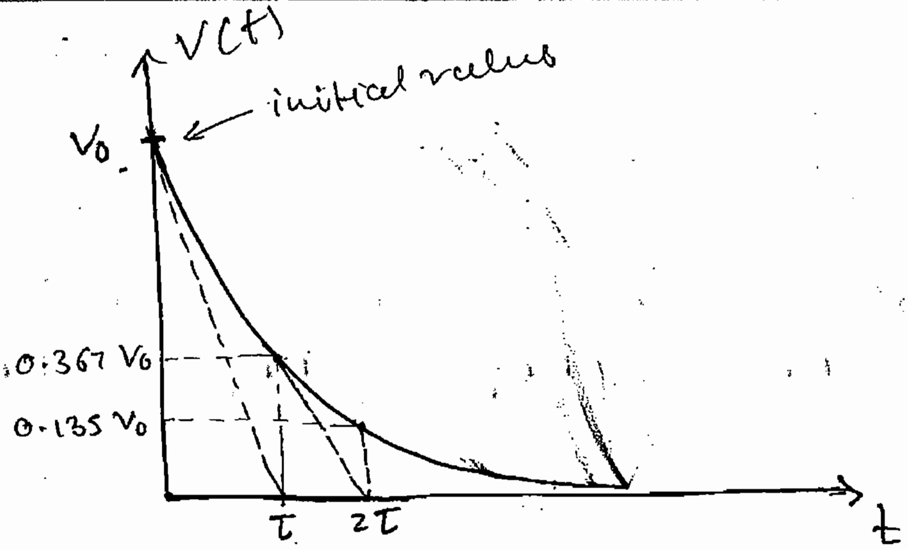
$\therefore \; \ln[V] = -\frac{t}{RC} + \ln[A]$

$\therefore \; \ln\left[\frac{V}{A}\right] = -\frac{t}{RC} \Rightarrow V = A\, e^{\frac{-t}{RC}}$

But at $t=0,\; V=V_0$ then $A = V_0$

$\therefore \; \cancel{V(t)} = V_0\, e^{\frac{-t}{RC}} \qquad V(t) = V_0\, e^{\frac{-t}{\tau}}$

$\tau = RC \rightarrow$ Time const. of R-C n/w




$v(t=0) \longrightarrow V_0$

$v(t=\tau) = e^{-1} V_0 = 0.367\, V_0$

$v(t=2\tau) = e^{-2} V_0 = 0.135\, V_0$

$v(t=3\tau) = e^{-3} V_0 = 0.049\, V_0$

$v(t=4\tau) = e^{-4} V_0 = 0.018\, V_0$

$0 < t \leq 4\tau$
Transient state

$v(t=5\tau) = e^{-5} V_0 = 0.006\, V_0 \rightarrow \quad t \geq 5\tau$
(steady state)

Expression of current through capacitor.

$$i_c = C\frac{dV}{dt} = C\frac{d}{dt}\left[V_0 e^{-t/\tau}\right] = CV_0 e^{-t/\tau} \times \left(\frac{-1}{\tau}\right)$$

$$i_c = -\frac{V_0}{R} e^{-t/\tau}$$

Expression of power descipated through R

$$P_R(t) = \frac{[V(t)]^2}{R} = \frac{\left[V_0 e^{-t/\tau}\right]^2}{R}$$

$$= \frac{V_0^2}{R} e^{-t/(\tau/2)}$$

Expression for energy decay in the capacitor:

$$E_c(t) = \frac{1}{2} C [V(t)]^2$$

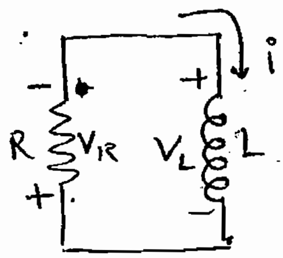
$$= \frac{1}{2} C V_0^2 e^{-t/(\tau/2)}$$

Expression for charge

$$q(t) = C\,V(t)$$

$$q = C V_0 e^{-t/\tau}$$

(b) R-L circuit :—




Let $i(0) = I_0$

KVL

$$V_R + V_L = 0$$

$$iR + L\frac{di}{dt} = 0$$

$$\therefore L\frac{di}{dt} = -iR$$

$$\int \frac{di}{i} = \int -\frac{R}{L}\, dt$$

$$\ln[i] = -\frac{R}{L} t + \ln[A]$$

$$\therefore \cancel{\text{Lin}}\ \ln\left[\frac{i}{A}\right] = -\frac{R}{L} t$$

$$\therefore i = A e^{-\frac{R}{L}t}$$

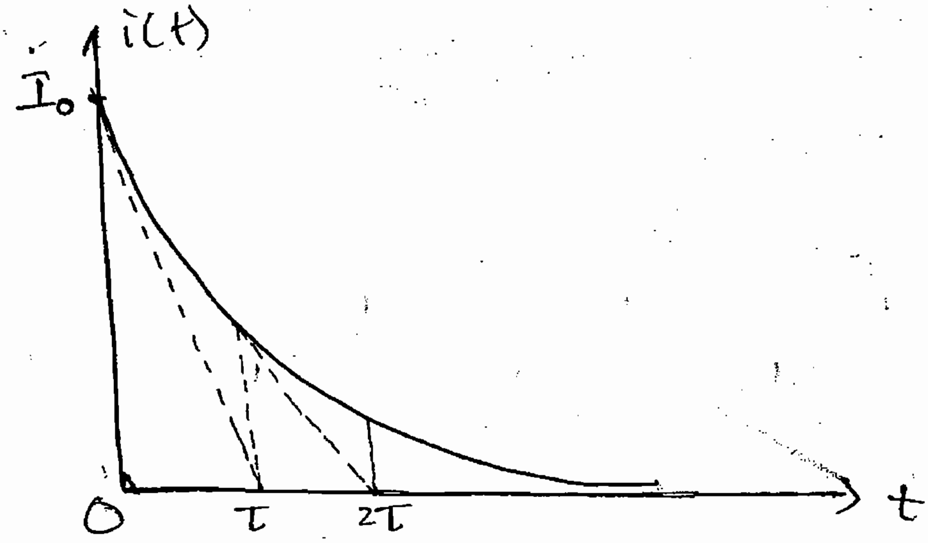
at $t=0$, $i = I_0$ then $A = I_0$

$$i(t) = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/(L/R)}$$

$$i(t) = I_0 e^{-t/\tau}$$

$\tau = \frac{L}{R}$

time const. of R-L n/w




Expression of vltg across inductor.

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}(I_0 e^{-t/\tau})$$

$$= -I_0 R e^{-t/\tau}$$

$$V_R + V_L = 0$$

$$V_R(t) = -V_L(t) = I_0 R e^{-t/\tau}$$

Expression for power dissipated in the restistor.

$$P_R = [i(t)]^2 R = (I_0 e^{-t/\tau})^2 R$$

$$P_R(t) = I_0^2 R e^{-t/(\tau/2)} \quad \underline{\underline{W}}$$

Expression for energy decay in inductor

$$E_L(t) = \frac{1}{2} L [i(t)]^2 = \frac{1}{2} L (I_0 e^{-t/\tau})^2$$

$$= \frac{1}{2} L I_0^2 e^{-t/(\tau/2)} \quad \underline{\underline{J}}$$

Expression for Flux linkages

$$\psi(t) = L\, i(t)$$
$$= L I_0 e^{-t/\tau} \quad \text{wb-T}$$

Here the power dissipated by the resis. & energy decay in inductor or capacitor is 2 times faster than current or vltg respectively.

Time Constant :- It is the time taken by the resp to reach 36.7 % of its initial value or It is also defined as time taken by the resp. to reach 63.4 % to its final value.

The unit of Time const. is : seconds

Then, w.r.t to units only :-

$$\sec = \sec$$
$$\tau_i = \tau_v$$
$$\therefore \boxed{\frac{L}{R} = RC}$$

→ Unit of $\frac{L}{RC} = \Omega$

Unit of $R^2C = H$

Units of $\frac{L}{R^2} = F$

Units of $\frac{RC}{L} = \mho$

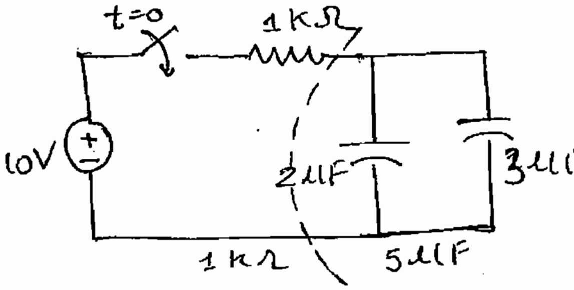
Units of $\frac{L}{C} = (\Omega)^2$

Units of $\sqrt{\frac{L}{C}} = \Omega$

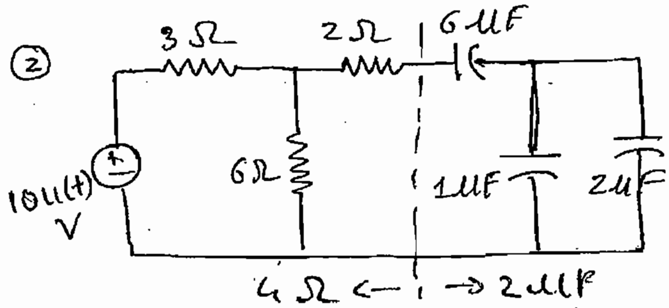
Units of $\frac{R^2C}{L} = 1$

Time const. determines the time with which the state variables responds.

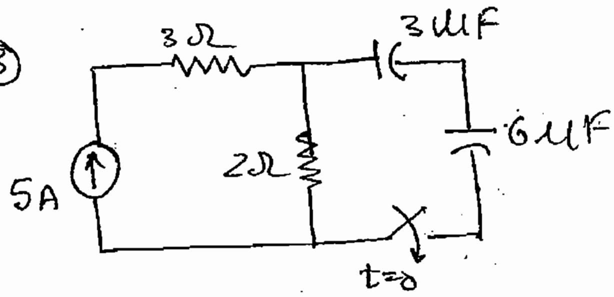
1) Determine the time const. of ckt shown below

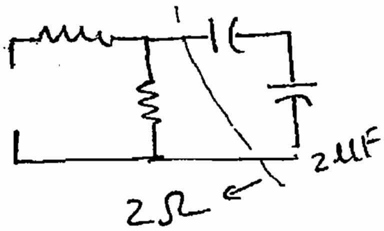
① 


$\tau = R_{eq}.C_{eq}.$
$= 1k \times 5\mu = 5\ msec$

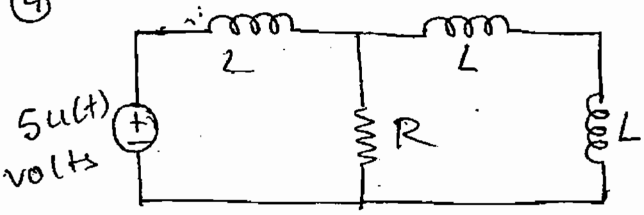
② 

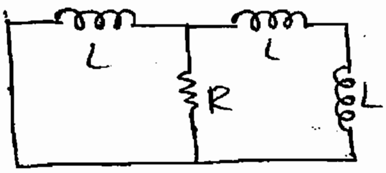

$\tau = 4 \times 2$
$= 8\ \mu sec$

③ 

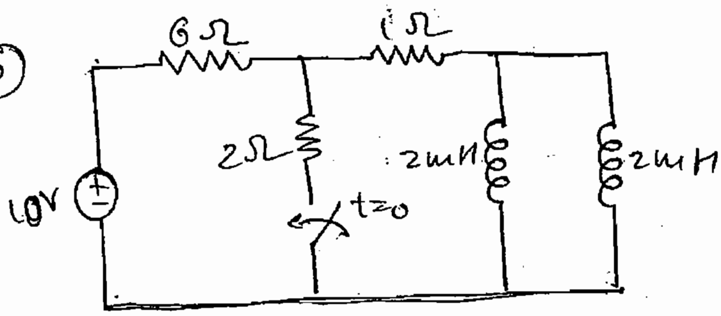

$\tau = 2 \times 2 = 4\mu sec$

④ 

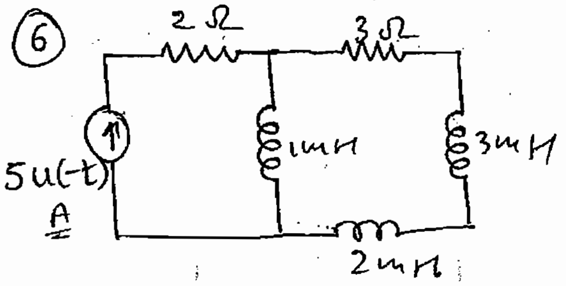

$L_{eq} = L \parallel 2L = \frac{2}{3}L$

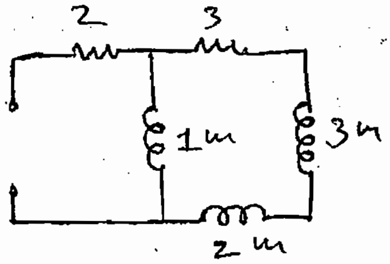
$\tau = \frac{L}{R} = \frac{2}{3}\frac{L}{R}$

⑤ 


$\tau = \frac{1m}{7/2} = \frac{2}{7}\ msec$

(6)



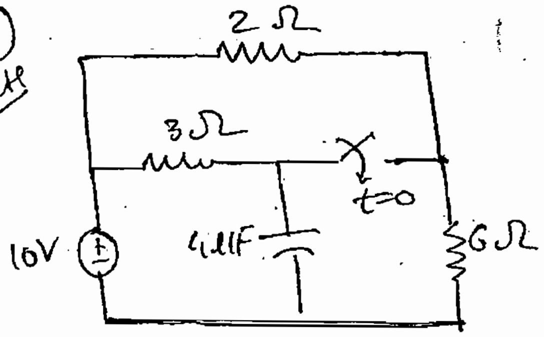



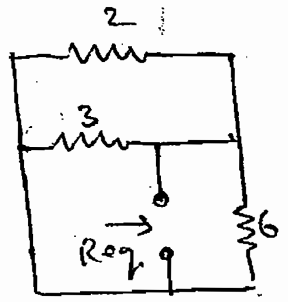

$L_{eq} = 6\,mH$

$R = 3\,\Omega$

$\tau = 2\,msec.$

(7) Gate

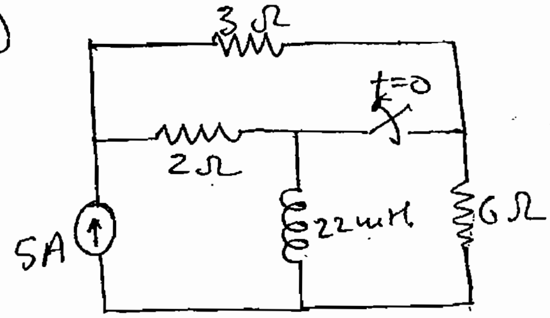


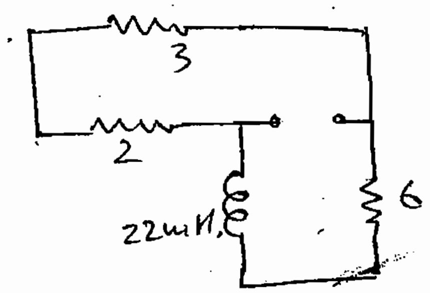




$R_{eq} = 6 \| 3 \| 2$
$= 1\,\Omega$

$\tau = R_{eq} \cdot C$
$= 1 \cdot 4 = 4\,\mu sec$

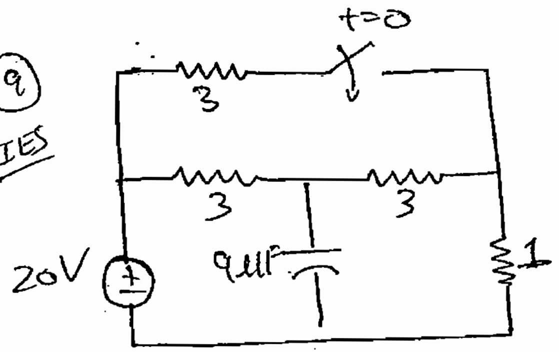
(8)

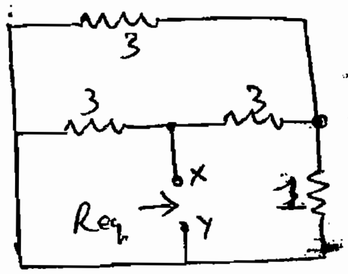


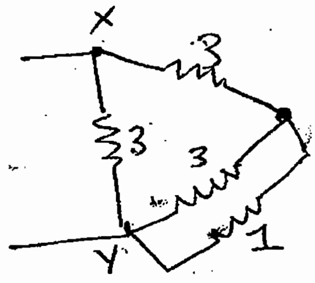




$$\tau = \frac{22\,mH}{11} = 2\,msec.$$

(9) IES



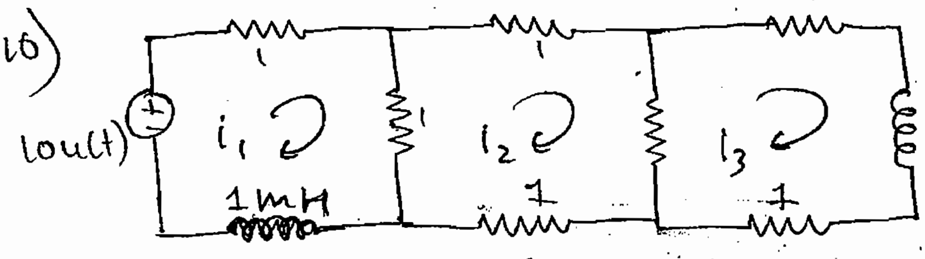




$$R_{eq} = 3 \,\|\, [3 + 3/4] = 3 \,\|\, \frac{15}{4} = \frac{45}{4} \times \frac{4}{27} = \frac{5}{3}\,\Omega$$

$$\tau = \frac{5}{3} \times 9 = 15\,\mu sec$$

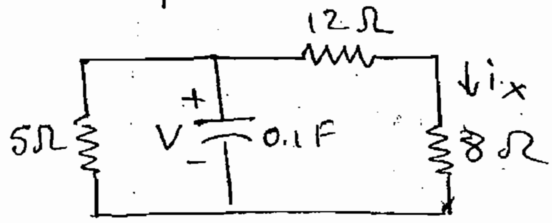
10)



Here the inductors & ~~capa~~ resistors cannot be lumped together. This is a second order L-L-R circuit.

This ckt has multiple time const. in multiple segments & the solution to these state variables can be determined by solving simultaneous differential eq^n or in a simple way by using Laplace Transforms (L.T.)

(11) If $V(0) = 15V$ find complete expression for '$i_x$'.




This is a source-free 1st order R-C n/w

S.V → 'V'

$$V(t) = V_0 e^{-t/\tau}$$

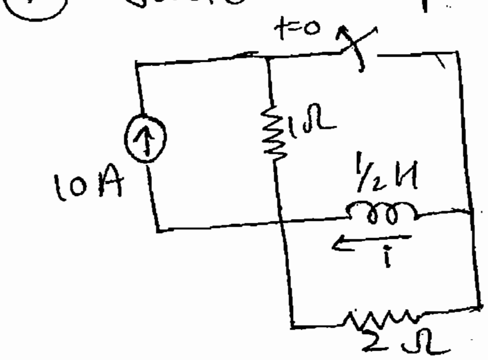
$$V_0 = 15V \quad \text{(given)}$$

$$\tau = R_{eq}.C = (5 \| 20) \cdot 0.1 = 0.4 \text{ sec}$$

$$\therefore V(t) = 15 \cdot e^{-t/0.4} = 15 e^{-2.5t} \; \underline{V}$$

But $$i_x(t) = \frac{V(t)}{12+8} = 0.75 e^{-2.5t} \; \underline{A}$$

(12) Find complete expression for I.

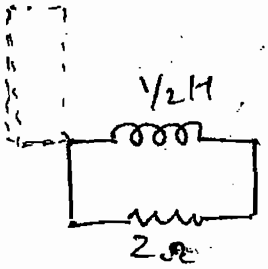


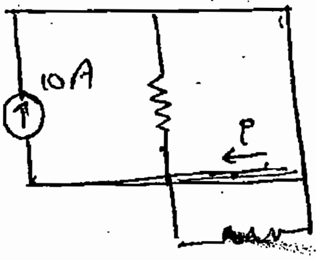

This is a source free, 1st order R-L ckt.

S.V → 'i'

$i(t) = I_0 e^{-t/\tau}$

~~$\tau = R_{eq} \cdot C$~~ $R_{eq} = 2\Omega$




$\tau = \frac{L}{R} = \frac{1/2}{2}$

$\therefore \tau = \frac{1}{4}$ sec

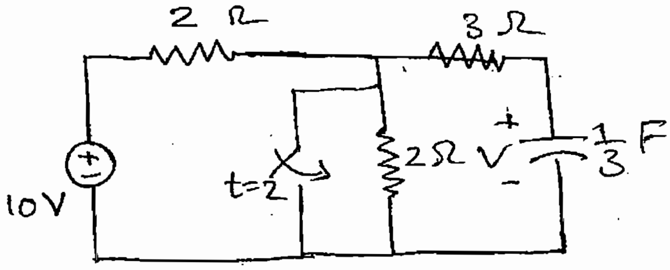
$i(0^-) = 10A = I_0$

($\because$ inductor does not allow sudden change in current)

$i(t) = 10 e^{-t/1/4}$

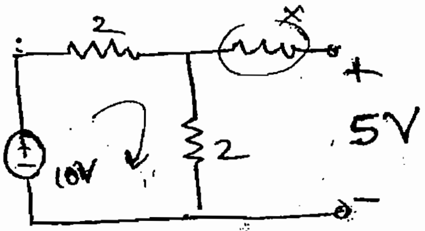
$= 10 e^{-4t}$ A.

2) Determine the complete expression for V & determine the energy stored in the capacitor upto 3rd second.




This is a source free 1st order R-C n/w (delayed)

State variable → V



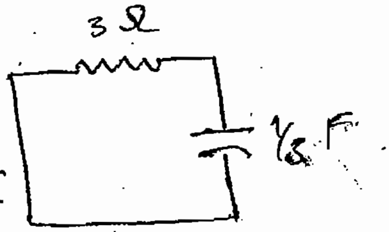

$0 < t \leq 2$ → Steady state only.

$t > 2$ → Transient solution.

$V = V_0 e^{-t/\tau}$

Here $V_0$ → means → $V(2^-)$. (from previous state)

$\therefore V_0 = 5$ V



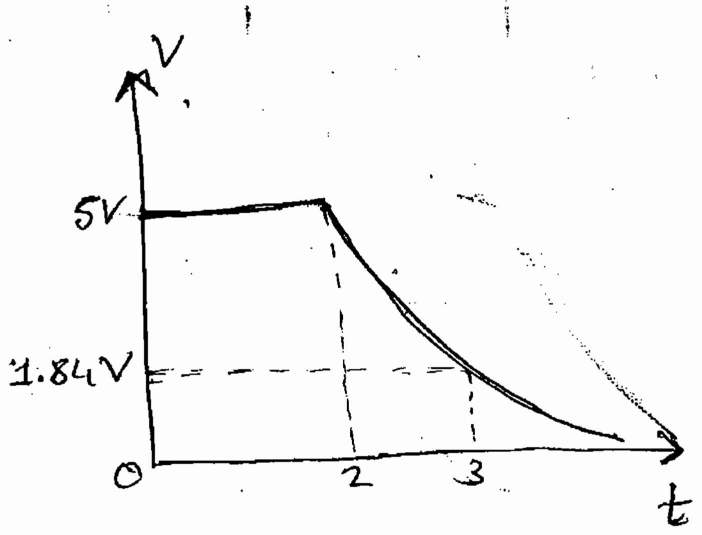

$\tau$; $t > 2$ : $\tau = R \cdot C = \frac{1}{3} \times 3 = 1$ sec

$v(t) = 5 e^{\frac{-(t-2)}{1}}$

The complete expression

$$v(t) = \begin{cases} 5V & ; \ 0 < t \leq 2 \\ 5e^{-(t-2)} & ; \ t > 2. \end{cases}$$

$E_c(t=3) = \frac{1}{2} C (V)^2_{\to t=3}$



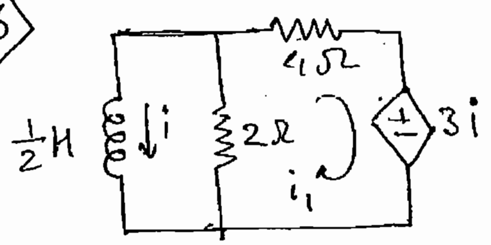

Now, $V(t=3) = 5e^{-(3-2)}$

$\therefore \quad = \underline{1.84 \ V}$

$E_C = \frac{1}{2} \times \frac{1}{3} \times (1.84)^2$

$= \underline{0.56} \ J$

3)




If $i(0) = 10A$, find its complete expression.

This is a $1^{st}$ order, source free R-L ckt. $\underline{S.V \to i}$

Classical Method

$\frac{1}{L} \frac{di}{dt} + 2[i + i_1] = 0$ —— ①

$\therefore 4 i_1 + 3i + 2[i + i_1] = 0$

$\therefore \ 5i = -6 i_1$ —— ②

$\frac{1}{L} \frac{di}{dt} + 2i + 2\left(\frac{-5}{6}\right) i = 0$

$\int \frac{1}{L} \frac{dI}{dt} = \int -\frac{i}{3} \Rightarrow \ln[i] = -\frac{2}{3} t + \ln[A]$

$\therefore \ i = A e^{-\frac{2}{3}t}$

At $t=0$, $i = 10$

So $A = 10$

$\therefore \ \boxed{i(t) = 10 e^{-\frac{2}{3}t}} \ \underline{A}$

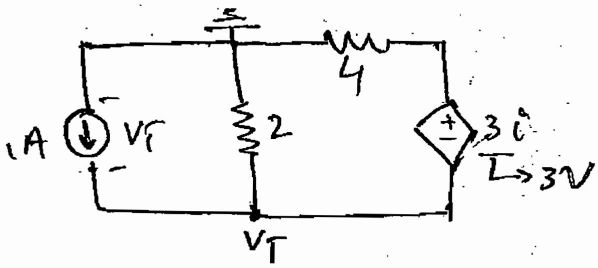
Shortcut

$i(t) = I_0 e^{-t/\tau}$

$I_0 = 10$ A (given)

$\tau = \frac{L}{R_{eq}}$

$= \frac{1/2}{1/3} = \frac{3}{2}$ sec



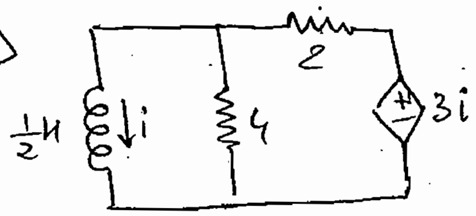
$-1 + \frac{V_T}{2} + \frac{V_T+3}{4} = 0$

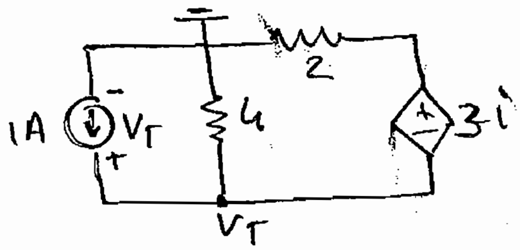
$\therefore 3V_T = 1 \Rightarrow V_T = \frac{1}{3}$

$\Rightarrow R = \frac{1}{3}\,\Omega$

$\therefore \boxed{i(t) = 10e^{-\frac{2}{3}t}}$ A

4)





$i(t) = I_0 e^{-t/\tau}$

$I_0 = 10$ A (given)

$\tau = \frac{L}{R_{eq}}$

$= -\frac{3}{2} \times \frac{1}{2} = -\frac{3}{4}$ sec

$-1 + \frac{V_T}{4} + \frac{V_T+3}{2} = 0$

$\therefore -4 + V_T + 2V_T + 6 = 0$

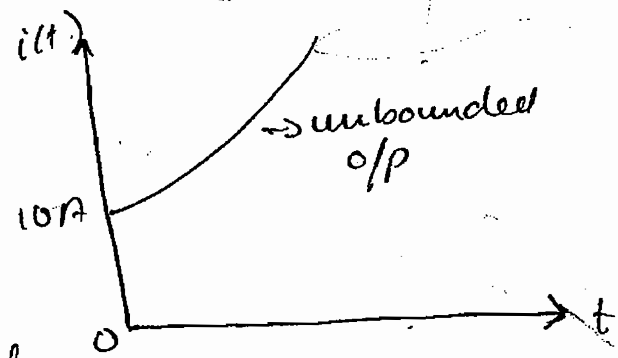
$3V_T = -2$

$V_T = -2/3$

$\therefore R = -\frac{2}{3}\,\Omega$

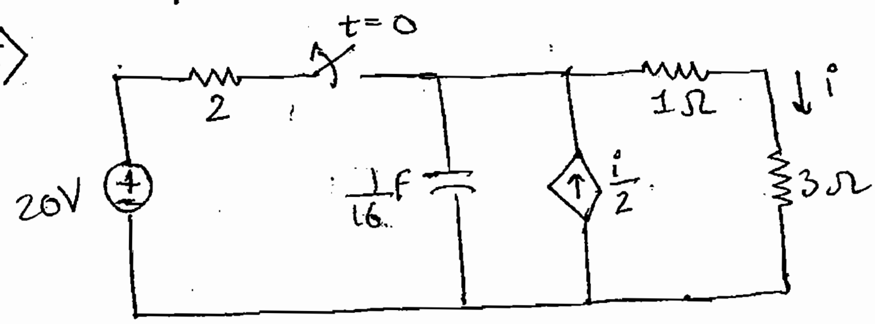
$\therefore i(t) = I_0 e^{-t/-3/4}$

$\boxed{i(t) = I_0 e^{\frac{4}{3}t}}$ A



→ Such circuits are conceptual & not practical.

5)

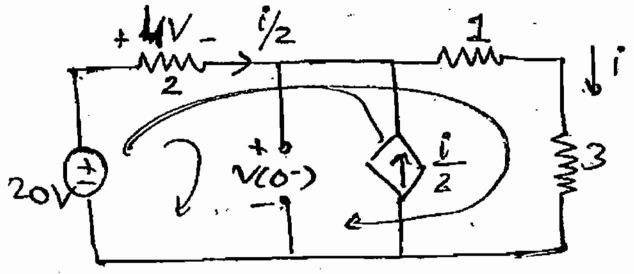


Find out complete expression for $i$

This is a 1st order, ~~...~~ source free R-C ckt.

S.V → 'v'

**t = 0⁻**



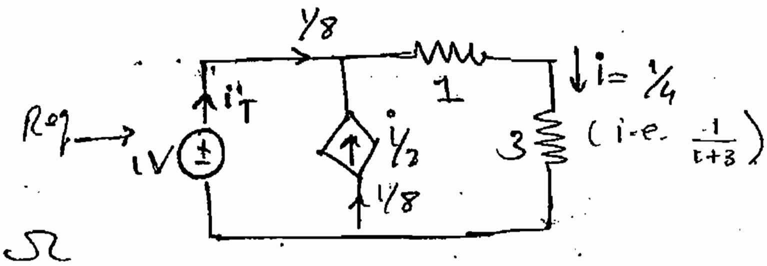

KVL $-20 + i + 4i = 0$

$i = 4$

KVL $-20 + 4 + v(0^-) = 0$

$\therefore v(0^-) = 16\ V = V_0$

**I, t > 0**




So, $i_T = \frac{1}{8}$

$R_{eq} = \frac{1}{i_T} = \frac{1}{1/8} = 8\,\Omega$

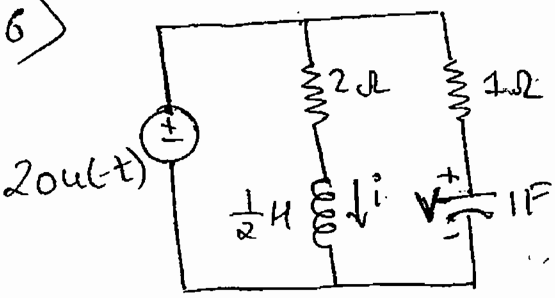
$\tau = R_{eq} \cdot C = 8 \times \frac{1}{16} = \frac{1}{2}$ sec.

$v(t) = 16\, e^{-2t}$ V.

Then

$i(t) = \frac{v(t)}{1+3} = 4\, e^{-2t}$ A

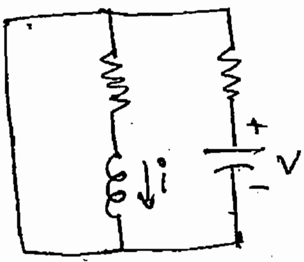
6)



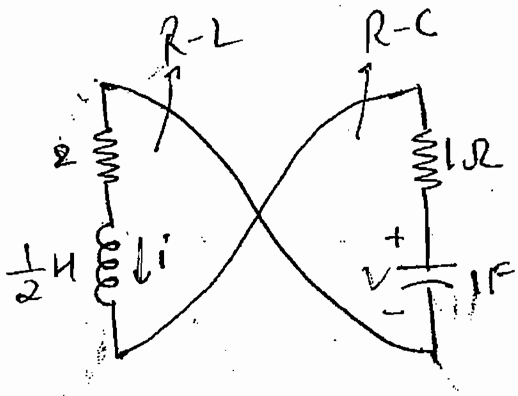

Find $\frac{I_v}{I_i}$,

Find complete expression for v & i

t > 0




$\Rightarrow$

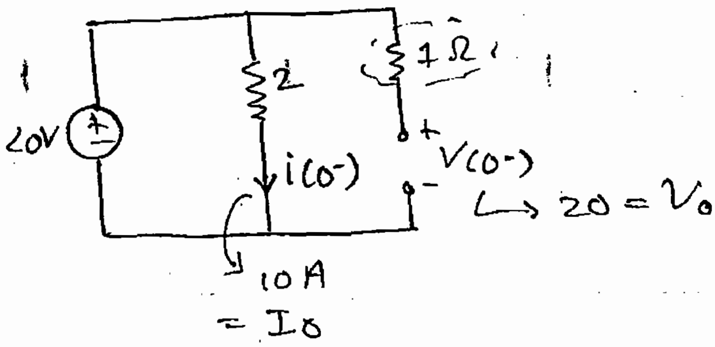


$$\tau_i = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4} \text{ sec} \qquad \tau_v = RC = 1 \text{ sec}$$

(a) $$\frac{\tau_v}{\tau_i} = \frac{1}{1/4} = 4.$$

(b) $v(t) = V_0 \cdot e^{-t/\tau}$

$i(t) = I_0 e^{-t/\tau}$



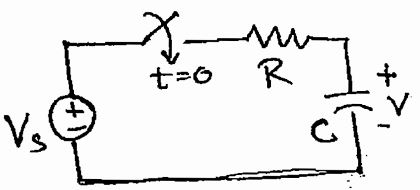

$\therefore v(t) = 20 e^{-t} \text{ V}$

$i(t) = 10 e^{-4t} \text{ A}$

Here current is decaying faster than vltg

# [II] Step Response of 1st order ckt :-

## (a) Series R-C ckt :—




$v(t) = V_{ss}(t) + V_{tr}(t)$

$v(t)$ → Total response

$V_{ss}(t)$ ↳ steady state resp.

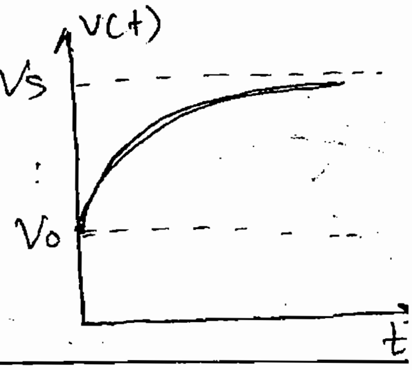
$V_{tr}(t)$ ↳ Transient resp.

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}}$$

where, $v(0)$ → vltg across capacitor before switch operation & steady state

$v(\infty)$ → vltg across capacitor after switch operation & steady state

$\tau = RC$




Case ① with initial condition given :

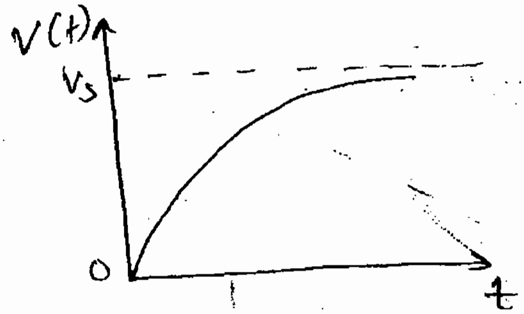
Let $v(0) = V_0$

$v(t) = V_s + [V_0 + V_s]\, e^{-t/\tau}$

Case ② without initial condition:

let $V(0) = 0$

$$V(t) = V_s\left[1 - e^{-t/\tau}\right]$$

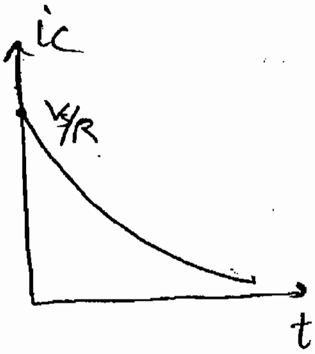



Expression for '$i_c$' :-

$$i_c(t) = C\frac{d}{dt}\left[V_s(1 - e^{-t/\tau})\right]$$

$$= C V_s\left[0 - e^{-t/\tau} \times (-1/\tau)\right]$$

$$i_c(t) = \frac{V_s}{R} e^{-t/\tau}$$



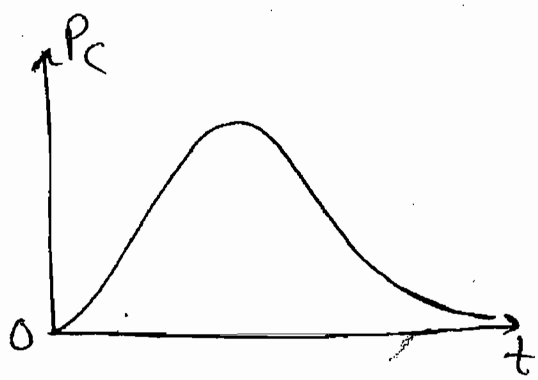

Expression for power :-

$$P_c(t) = V_c(t) \cdot i_c(t)$$

$$= \frac{V_s^2}{R}\left[e^{-t/\tau} - e^{-2t/\tau}\right]$$

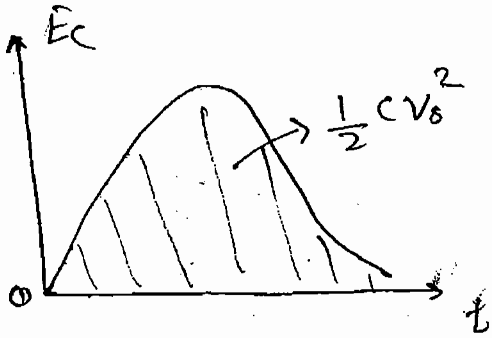
at $t = 0 \Rightarrow P_c = 0$

at $t \to \infty \Rightarrow P_c = 0$




Expression for Energy

$$E_c = \int_0^{\infty} P_c \, dt = \frac{1}{2} C [V_s]^2$$




$\rightarrow$ $i_R(t) = \frac{V_s}{R} e^{-t/\tau}$

$\rightarrow$ KVL

$$-V_s + V_R + V_c = 0$$

$$V_R = V_s - \left[V_s(1 - e^{-t/\tau})\right]$$

$$V_R(t) = V_s e^{-t/\tau}$$

$\rightarrow P_R(t) = V_R(t) \cdot i_R(t)$

$$= \frac{V_s^2}{R} \cdot e^{-2t/\tau} \quad \underline{\underline{W}}$$

$\rightarrow E_R(t) = \int_0^\infty P_R \, dt = \frac{1}{2} C V_s^2 \quad \underline{\underline{J}}$

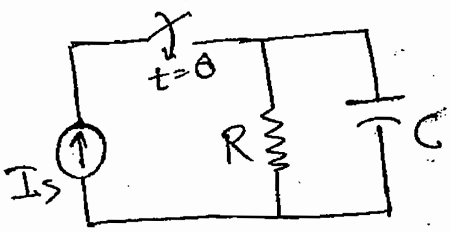
$$\% \text{ Efficiency} = \frac{o/p}{i/p} \times 100\%$$

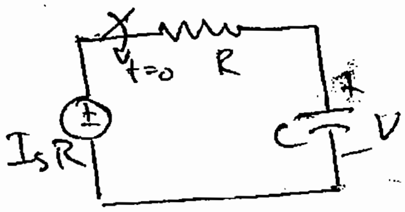
$$= \frac{E_C}{E_R + E_C} \times 100\%$$

$$= \frac{\frac{1}{2} C V_s^2}{\frac{1}{2} C V_s^2 + \frac{1}{2} C V_s^2} \times 100\% = 50\%$$

$\therefore \boxed{\eta = 50\%}$

<u>Note</u> (Parallel R-C ckt)



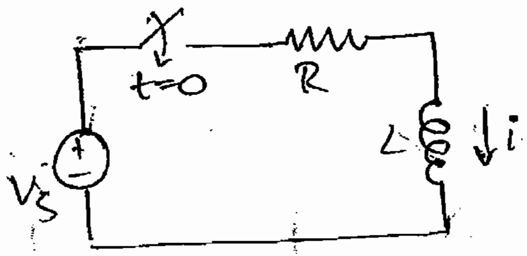


$$V(t) = I_S R + [V_0 - I_S R] e^{-t/\tau} \quad \& \quad \tau = RC.$$

$\Rightarrow$ Here we cannot connect a vltg source across capacitor as it will cause sudden change of vltg across capacitor ~~will~~ which will result in large current which may damage the ckt.

$\therefore$ The ckt behaviour cannot be determined at $t = 0^+$

## (b) R-L ckt :-




$$i(t) = i_{ss}(t) + i_{tr}(t)$$

$$\boxed{i(t) = I(\infty) + [I(0) - I(\infty)]\, e^{-t/\tau}}$$

$I(0) \rightarrow$ current through inductor before switch operation & steady state.

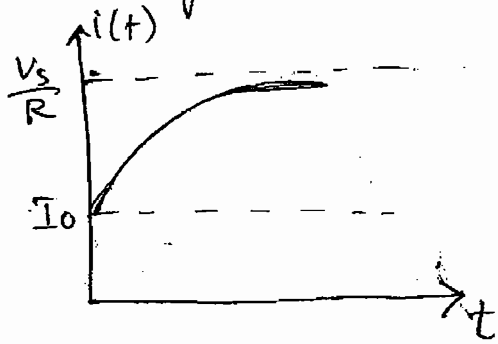
$I(\infty) \rightarrow$ current through inductor after switch operation & steady state.

$$\boxed{\tau = \frac{L}{R}}$$

Case ① with initial condition given:-

Let $I(0) = I_0$

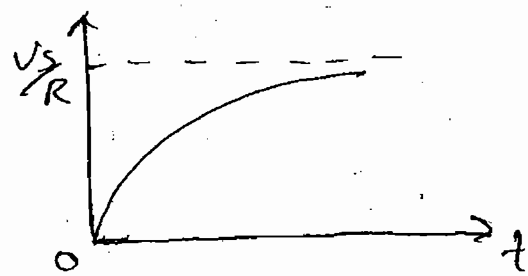
$$i_L(t) = \frac{V_S}{R} + \left[I_0 - \frac{V_S}{R}\right] e^{-t/\tau}$$




Case ② without initial conditions:-

$I(0) = 0$

$$i_L(t) = \frac{V_S}{R}\left[1 - e^{-t/\tau}\right]$$

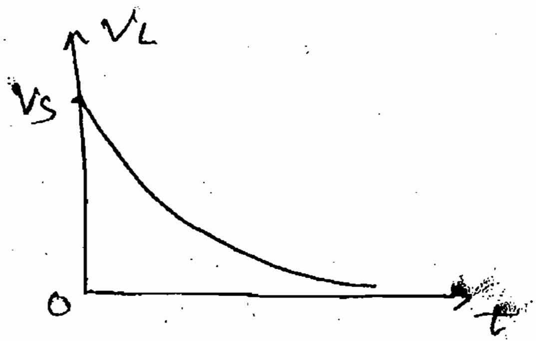



Expression for vltg across inductor:-

$$V_L = L \frac{d}{dt}\left[\frac{V_S}{R}(1 - e^{-t/\tau})\right]$$

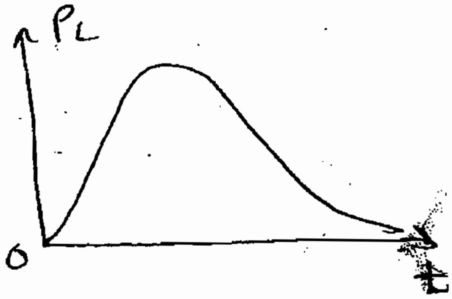
$$= \frac{L V_S}{R}\left[0 - e^{-t/\tau} \times -\frac{1}{\tau}\right]$$

$$\therefore V_L(t) = V_S e^{-t/\tau}$$



$$\Rightarrow P_L(t) = v_L(t) \cdot i_L(t)$$

$$= \frac{V_s^2}{R}\left[e^{-t/\tau} - e^{-2t/\tau}\right]$$




$$\Rightarrow E_L(t) = \int_0^{\infty} P_L(t)\,dt$$

$$= \frac{1}{2} L[i(\infty)]^2$$

$$= \frac{1}{2} L\left[\frac{V_s}{R}\right]^2 \text{ J}$$

$$\Rightarrow i_R(t) = \frac{V_s}{R}\left[1 - e^{-t/\tau}\right]$$

KVL

$$-V_s + V_R + V_L = 0$$

$$\Rightarrow V_R(t) = V_s - V_L(t) = V_s - V_s e^{-t/\tau}$$

$$= V_s\left[1 - e^{-t/\tau}\right]$$

$$\Rightarrow P_R(t) = V_R \cdot i_R = \frac{V_s^2}{R}\left[1 - e^{-t/\tau}\right]^2 \text{ W}$$

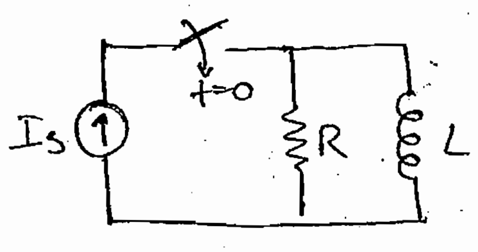
$$\Rightarrow E_R(t) = \int_0^{\infty} P_R(t)\,dt = \infty \text{ J}$$

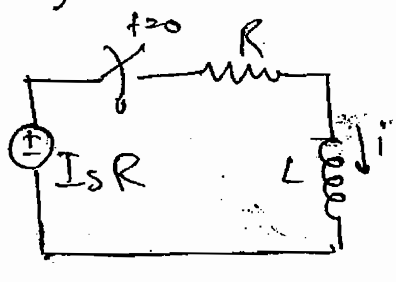
$$\Rightarrow \%\,\eta = \frac{E_L}{E_L + E_R} \times 100\%$$

$$= \frac{\frac{1}{2} L\left(V_s/R\right)^2}{\frac{1}{2} L\left(\frac{V_s}{R}\right)^2 + \infty} \times 100\%$$

$$\boxed{\%\,\eta = 0\%}$$

Note (Paralle R-L ckt)

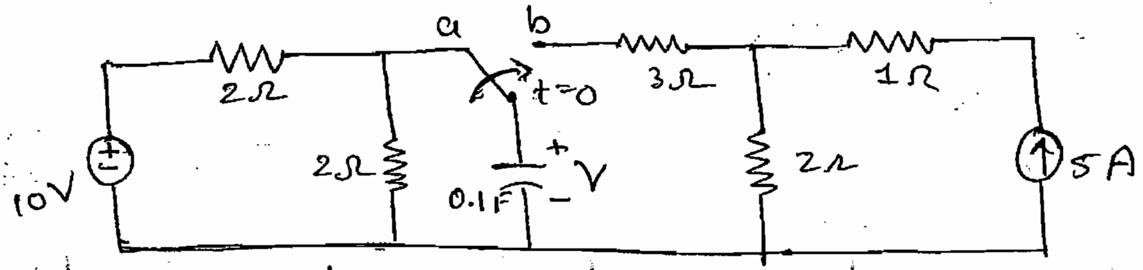



$$i(t) = I_s + \left[I_0 - I_s\right] e^{-t/\tau}$$

$$\tau = L/R$$

1) Find the complete expression for V

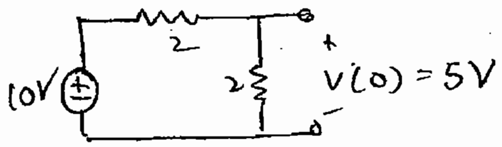



This is step response of $1^{st}$ order R-C n/w

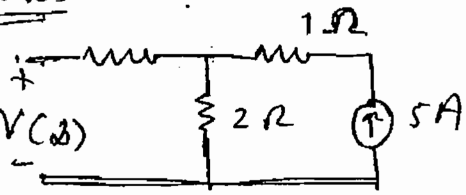
S.V → 'V'

$$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$$

→ $V(0) = ?$

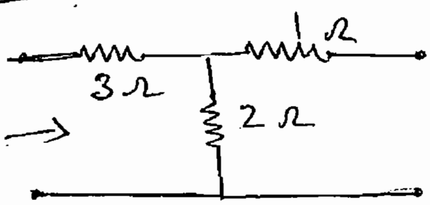



⇒ $V(\infty) = ?$




↳ $V(\infty) = 5 \times 2 = 10V$

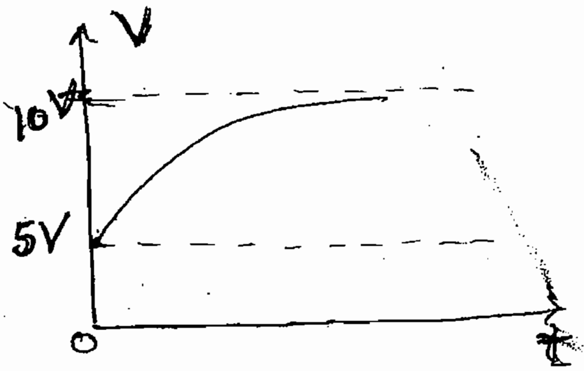
$\tau$, $t > 0$



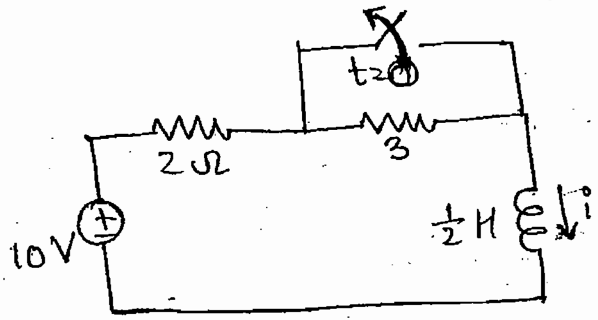

Req
↳ 5Ω

$$\tau = R_{eq}\, C$$
$$= 5 \times 0.1$$
$$= \frac{1}{2} \text{ sec}$$

$$V(t) = 10 + [5 - 10]\, e^{-t/1/2}$$
$$= 10 - 5e^{-2t} \text{ A}$$




2) Find complete expression for I.




This is step response of $1^{st}$ order R-L n/w.

S.V → i

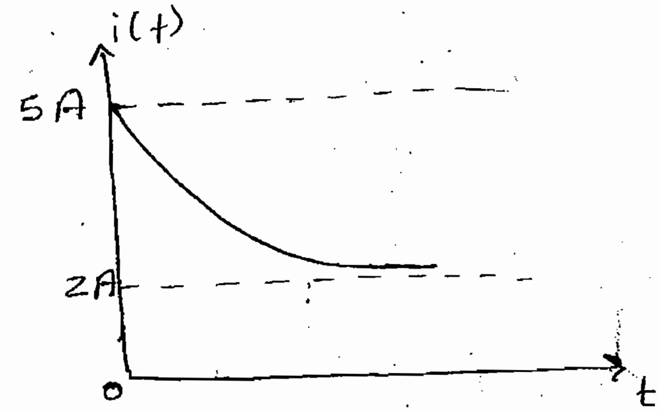
$$i(t) = I(\infty) + [I(0) - I(\infty)]\, e^{-t/\tau}$$

$\rightarrow I(0) = \frac{10}{2} = 5A$

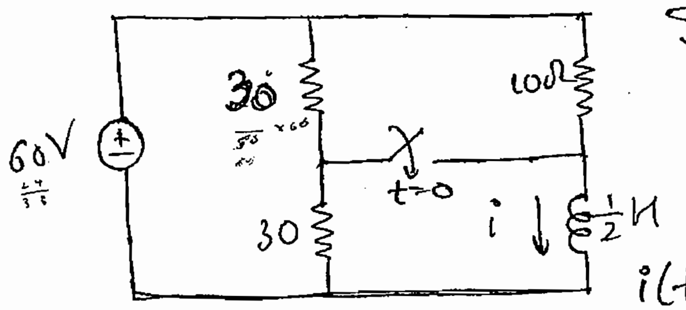
$\rightarrow I(\infty) = \frac{10}{5} = 2A$

$\rightarrow \tau = \frac{L}{R} = \frac{1/2}{5} = \frac{1}{10}$ sec

$\boxed{i(t) = 2 + 3e^{-10t}}$ A




3) Find the complete expression for i




This is a 1st order, step-response R-L ckt

S.V → I

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$

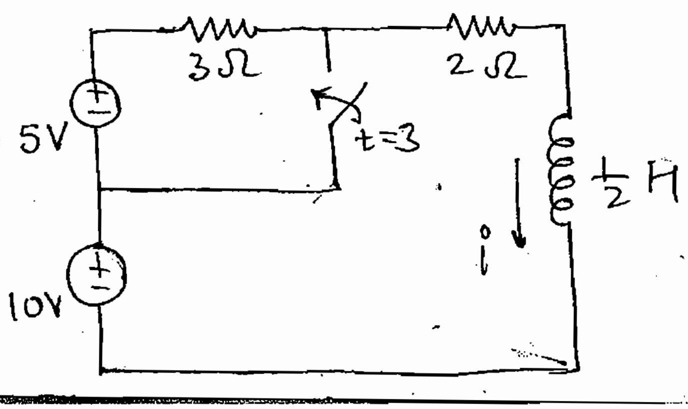
$i(0) = \frac{(50)}{50+10} \times \frac{60 \times (50+10)}{50 \times 10} = 6$ A.

$i(\infty) = ?$ $\quad i_{(\infty)} = \frac{60 \times 40}{30 \times 10} = 8$ A.

$\tau = \frac{1/2}{30 \| 30 \| 10} = \frac{1}{12}$ sec

$\therefore \boxed{i(t) = 8 - 2e^{-12t}}$ A

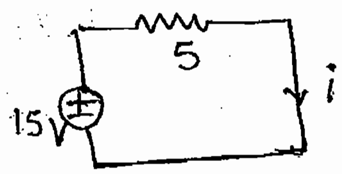
4) Find the complete expression for i & the energy stored in the inductor upto 3.25 sec.




This is 1st order, step response, R-L circuit. (delayed)

S.V → i

$0 < t \leq 3 \longrightarrow$ Steady State




$i = 3\,A$

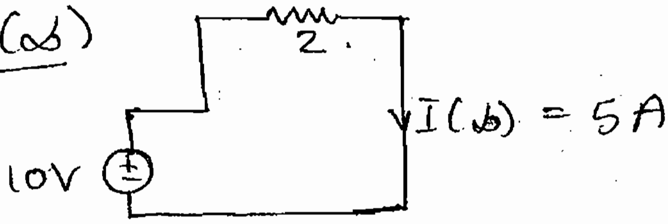
$t > 3 \longrightarrow$ Transient Solution

$i(t) = I(\infty) + [I(0) - I(\infty)]\, e^{-t/\tau}$

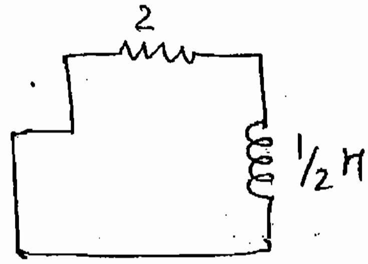
$\rightarrow$ Here $I(0) \longrightarrow i(3^-) \longrightarrow$ from previous state

$i(3^-) = 3A$

$\Rightarrow$ $I(\infty)$




$\Rightarrow$ $\tau$, ~~t/3~~ $t > 3$

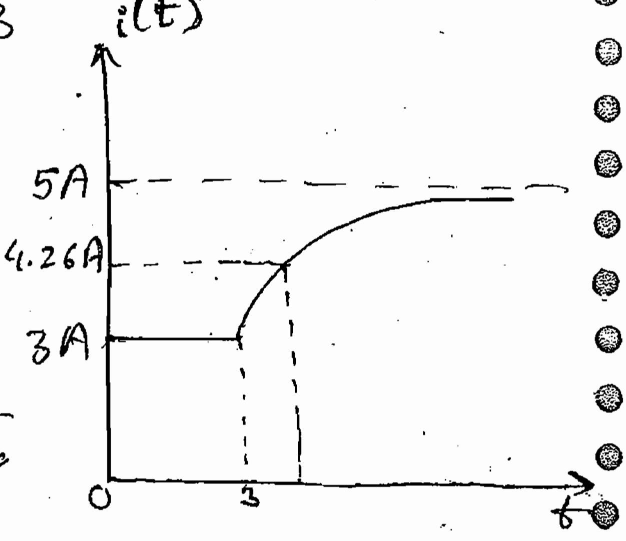



$\tau = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4}$ sec

$i(t) = 5 + [3 - 5]\, e^{-(t-3)/(1/4)}$

Complete expression

$$i(t) = \begin{cases} 3\,A, & ; 0 < t \leq 3 \\ 5 - 2e^{-4(t-3)} & ; t > 3 \end{cases}$$



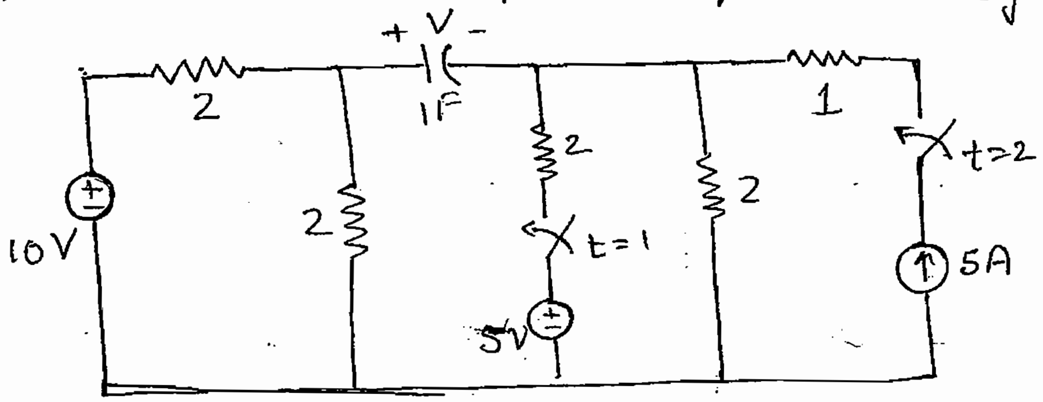

$E_L(t=3.25) = \frac{1}{2} L i^2$

$i(t=3.25) = 5 - 2e^{-4(3.25-3)}$

$= 4.26\ A$

$E_L = \frac{1}{2} \times \frac{1}{2} \times (4.26)^2 = 4.545\ J$

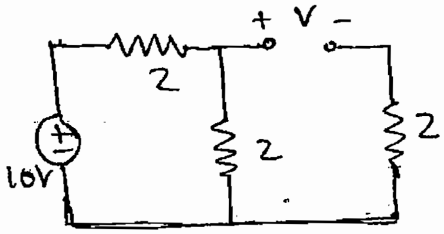
5) Find the complete expression of 'V'




This is Step response, $1^{st}$ order R-C ckt.

S.V $\longrightarrow$ 'V'

<u>$0 < t \leq 1$</u> (Steady state)




$\therefore$ V = 5 V

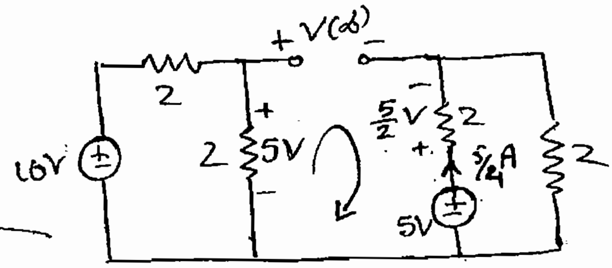
<u>$1 < t \leq 2$</u> ($1^{st}$ part of T.R.)

$$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$$

Here $V(0) \rightarrow V(1^-) \rightarrow$ from previous state.

$\therefore V(1^-) = 5V$

$\Rightarrow$ <u>$V(\infty)$</u>

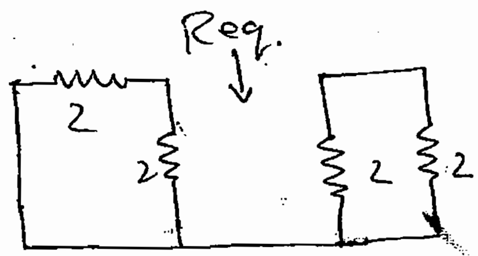



<u>KVL</u>

$$-5 + V(\infty) + 5 - \frac{5}{2} = 0$$

$\therefore V(\infty) = \frac{5}{2}$ V

$\Rightarrow$ <u>$\tau$</u>




$\tau = RC$

$= 2 \times 1 = 2$ sec

$$\underline{V(t) = 2.5\left[1 + e^{-\frac{(t-1)}{2}}\right]}$$

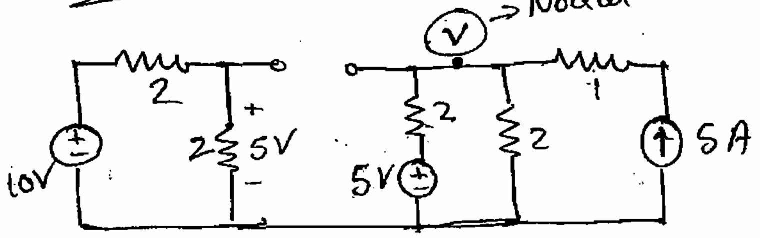
<u>$t > 2$</u> ($2^{nd}$ part of T.R)

$$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$$

Here $V(0) = V(2^-) \rightarrow$ from previous state

$$v(2^-) = 2.5\left[1+e^{-1/2}\right] = \underline{4}\,V$$

$\rightarrow$ $\underline{V(\infty)}$




$$\frac{(V-5)}{2} + \frac{V}{2} - 5 = 0$$

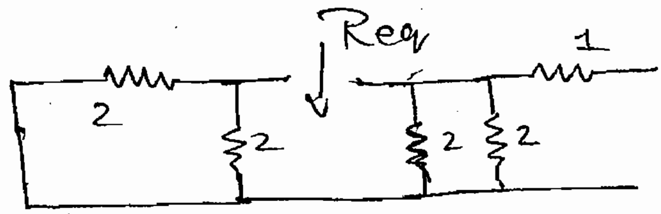
$$2V = 15$$

$$V = \frac{15}{2}$$

$\underline{KVL}$ $\quad -5 + V(\infty) + \frac{15}{2} = 0$

$$V(\infty) = \frac{-5}{2} = -2.5\ V$$

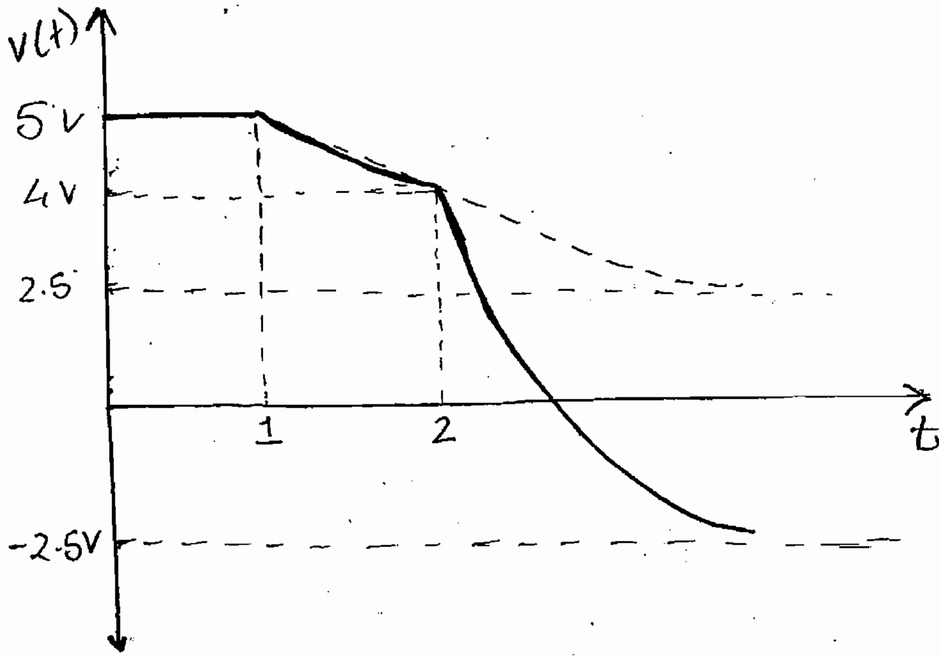
$\rightarrow$ $\underline{\tau}$



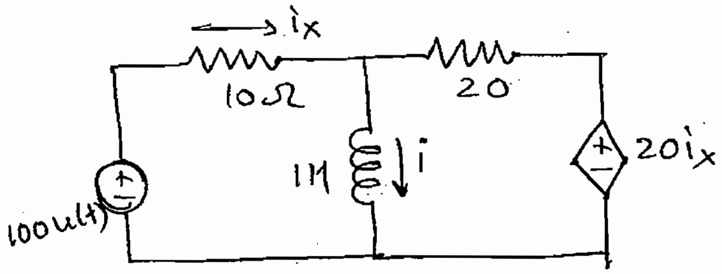

$$\tau = R \cdot C = 2 \cdot 1 = 2\ sec$$

$$\underline{v(t) = -2.5 + 6.5\, e^{-\frac{(t-2)}{2}}\ V}$$

$$v(t) = \begin{cases} 5\ V & ; \quad 0 \le t \le 1 \\ 2.5\left[1 + e^{-\frac{(t-1)}{2}}\right] \underline{V} & ; \quad 1 < t \le 2 \\ -2.5 + 6.5\, e^{-\frac{(t-2)}{2}}\ \underline{V} & ; \quad 2 < t \end{cases}$$

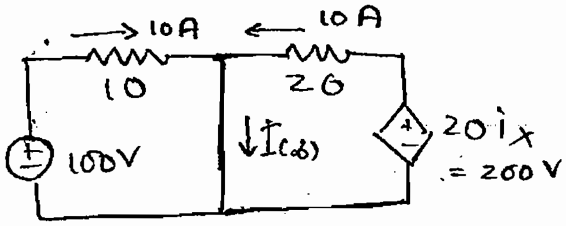


6) Find the complete expression for 'i'



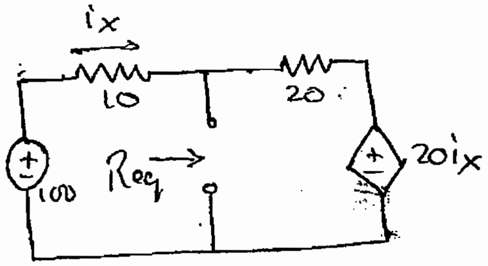

$\rightarrow I(0) = 0A$ (initially relaxed)

$\rightarrow \underline{I(\infty)}$

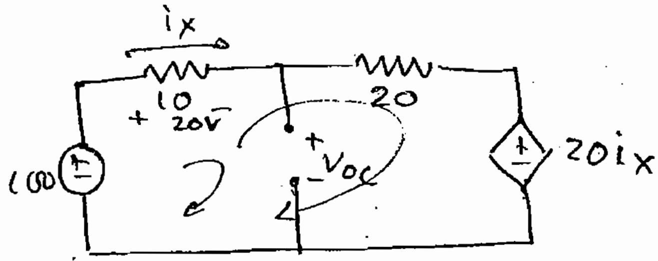



KCL $I(\infty) = 20A$

$\Rightarrow \tau = \frac{L}{R_{eq}}$




S-I




S-II $I_{sc} = I(\infty)$

$\therefore I_{sc} = 20A$

KVL $-100 + 30i_x + 20i_x = 0$

$i_x = 2$

KVL $-100 + 20 + V_{oc} = 0$

$V_{oc} = 80\ V$

$$R_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{80}{20} = 4\Omega$$

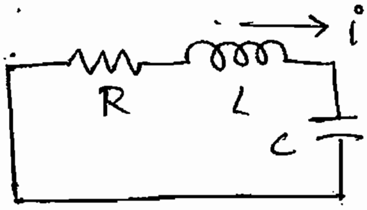
$$\tau = \frac{L}{R} = \frac{1}{4}\ \text{sec.}$$

$$i(t) = 20[1 - e^{-4t}]\ A$$

# III] Source free 2nd order circuits :—

(Canonical form)

## (a) Series R-L-C




Dominant S.V. $\rightarrow i$

KVL

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}\cdot i = 0$$

$\rightarrow$ Use Laplace T/f. (homogeneous)

$$Ls^2 I(s) + RsI(s) + \frac{I(s)}{C} = 0$$

$$I(s)\left[Ls^2 + Rs + \frac{1}{C}\right] = 0$$

$$I(s)\left[s^2 + \frac{R}{2}s + \frac{1}{LC}\right] = 0 \qquad (\text{As } I(s) \neq 0)$$

$$\therefore \quad s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

The 2 roots are : $s_1, s_2 = \dfrac{\frac{-R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$

$$\therefore \quad s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Let $\alpha = \dfrac{R}{2L}$ } Damping factor

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ } Undamped natural freq

$\therefore \quad s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

Case ①: If $\alpha > \omega_0 \longrightarrow$ overdamped

$$\boxed{\frac{R}{2L} > \frac{1}{\sqrt{LC}}}$$

→ Two roots are: -ve, real & unequal

then,
$$\boxed{i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

→ $A_1, A_2$ are arbitrary const. that can be determined from initial condition.

Case ②: If $\alpha = \omega_0 \longrightarrow$ critically damped

$$\boxed{\frac{R}{2L} = \frac{1}{\sqrt{LC}}}$$

→ The 2 roots are: -ve, real & equal

Then,
$$\boxed{i(t) = e^{-\alpha t}[A_1 + A_2 t]}$$

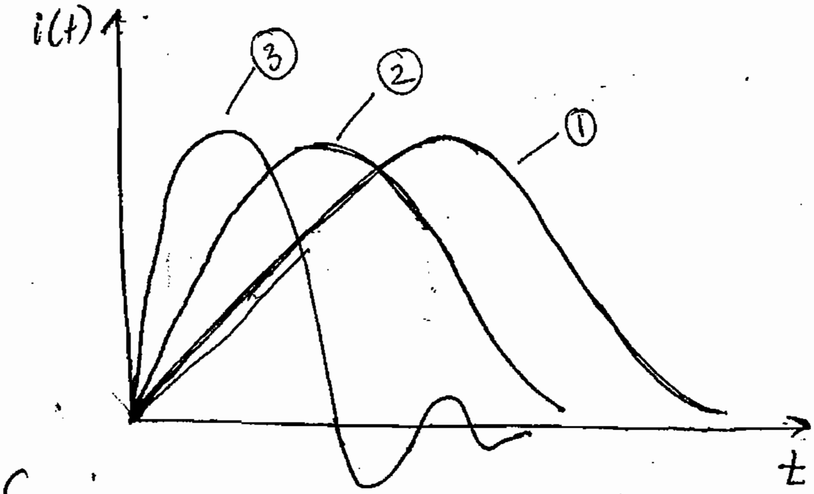
Case ③: If $\alpha < \omega_0 \longrightarrow$ underdamped

$$\boxed{\frac{R}{2L} < \frac{1}{\sqrt{LC}}}$$

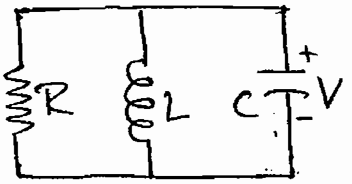
→ The 2 roots are: complex conjugate with -ve, real

Then,
$$\boxed{i(t) = e^{-\alpha t}\left[A_1 \cos\omega_d t + A_2 \sin\omega_d t\right]}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$
$\hookrightarrow$ damped freq.




## (b) Parallel R-L-C :




Dominant S.V $\rightarrow$ V

KCL

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = 0$$

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{V}{L} = 0$$

$\rightarrow$ Use Laplace T/f (homogeneous)

$$Cs^2V(s) + \frac{1}{R}sV(s) + \frac{V(s)}{L} = 0$$

$$V(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = 0$$

$$\therefore\ s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

The 2 roots are : $s_1, s_2 = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$

Let, $\alpha = \frac{1}{2RC}$ } damping factor

$\omega_0 = \frac{1}{\sqrt{LC}}$ } undamped natural freq.

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Case ① If $\alpha > \omega_o \longrightarrow$ overdamped

$$\frac{1}{2RC} > \frac{1}{\sqrt{LC}}$$

→ The two roots are -ve, real & unequal.

Then,
$$\boxed{v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

→ $A_1, A_2$ are arbitary const- that can be determined from initial conditions.

Case ② If $\alpha = \omega_o \longrightarrow$ critically damped

$$\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

→ The two roots are -ve, real & equal

Then,
$$\boxed{v(t) = e^{-\alpha t}[A_1 + A_2 t]}$$

Case ③ If $\alpha < \omega_o \longrightarrow$ Underdamped

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}}$$

⇒ The 2 roots are : complex conjugate with -ve, real

then,
$$\boxed{v(t) = e^{-\alpha t} \cdot [A_1 \cos \omega_d t + A_2 \sin \omega_d t]}$$

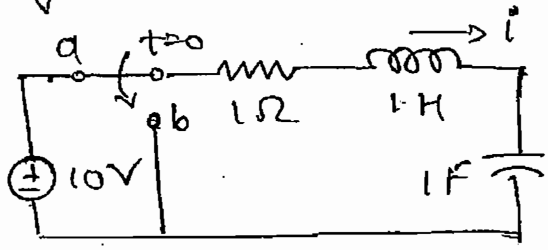
$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

1) The nature of response of $i(t)$, $t>0$ is:

(a) UD ✓ (c) OD

(b) CD

(d) Sinusoidal

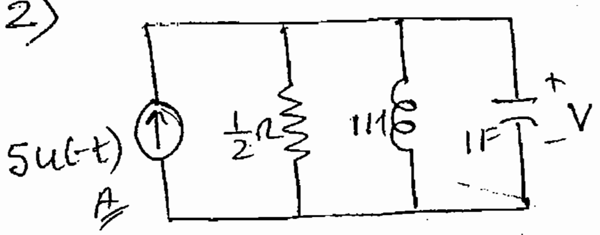



→ This is 2nd order, source free, series R-L-C circuit.

$$\alpha = \frac{R}{2L} = \frac{1}{2\times 1} = \frac{1}{2}$$

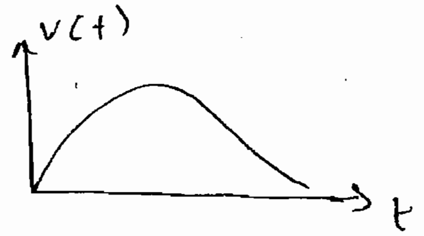
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 1}} = 1$$

$\Rightarrow \alpha < \omega_0$

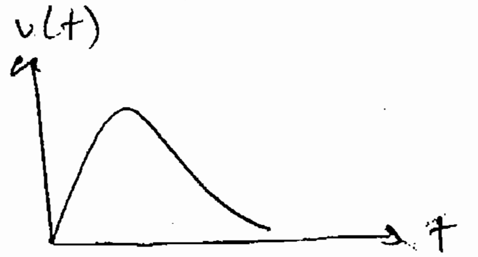
2) 


The nature of resp. of $V(t)$, $t>0$ is ___.

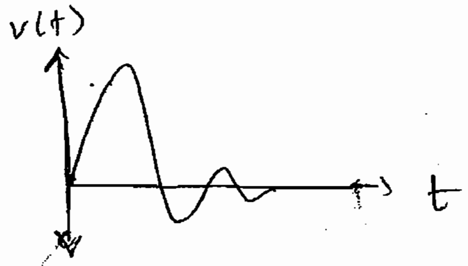
(a)



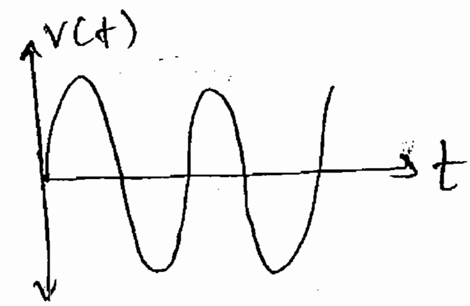

(b) ✓




(c)




(d)




→ This is 2nd order, source free, parallel R-L-C circuit.

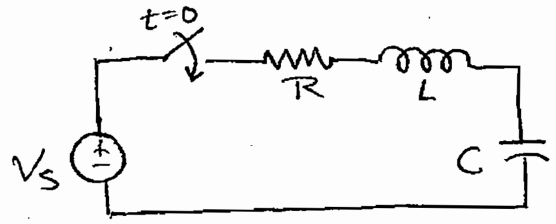
$$\alpha = \frac{1}{2RC} = \frac{1}{2\times\frac{1}{2}\times 1} = 1$$

$$= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 1}} = 1$$

$\Rightarrow \alpha = \omega_0$

# IV] Step Response of $2^{nd}$ Order sys.

## (a) Series R-L-C

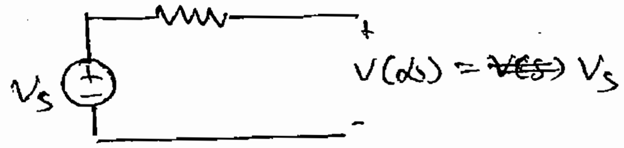



Dominant variable → 'V'
(i.e. vltg across capacitor in steady state condition).

$$v(t) = V_{ss}(t) + V_{tr}(t)$$

<u>But</u>

$V_{ss}(t)$, is as $t \to \infty$

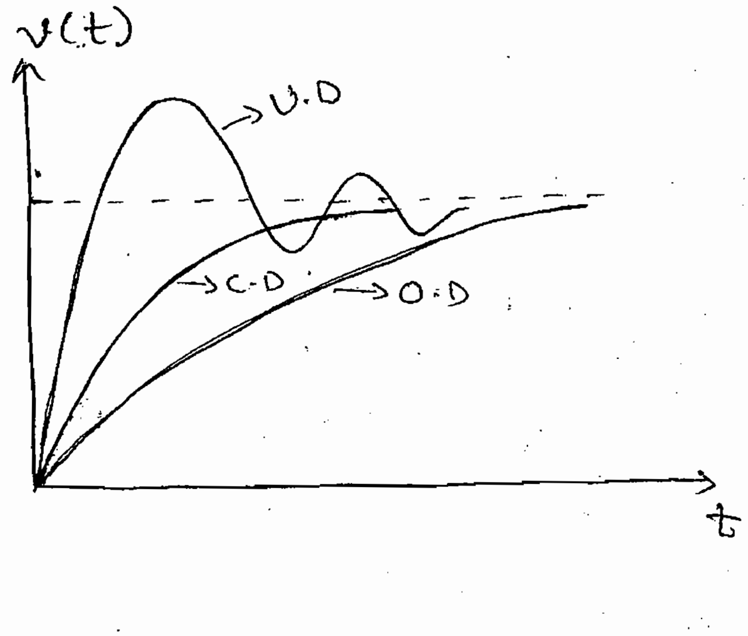



<u>But</u> $V_{tr}(t)$ depends upon the values of R-L-C

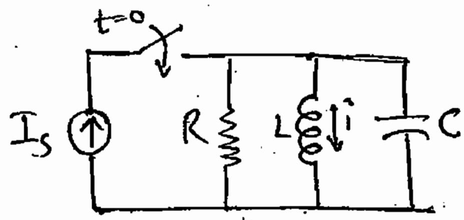
So here, $\alpha = \dfrac{R}{2L}$ $\qquad \omega_0 = \dfrac{1}{\sqrt{LC}}$

So the complete solution is:

$$v(t) = \begin{cases} V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \rightarrow \text{overdamped } (\alpha > \omega_0) \\ (\text{or}) \\ V_s + e^{-\alpha t}[A_1 + A_2 t] \rightarrow \text{critically damped } (\alpha = \omega_0) \\ (\text{or}) \\ V_s + e^{-\alpha t}[A_1 \cos\omega_d t + A_2 \sin\omega_d t] \rightarrow \text{underdamped } (\alpha < \omega_0 \end{cases}$$



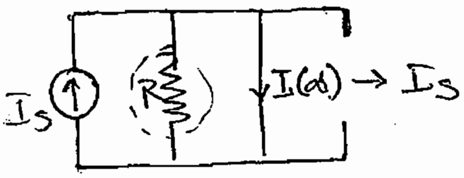
(b) Parallel R-L-C




Dominant S.V. → 'i'

$i(t) = I_{ss}(t) + I_{tr}(t)$

But $I_{ss}(t)$ is as $t \to \infty$

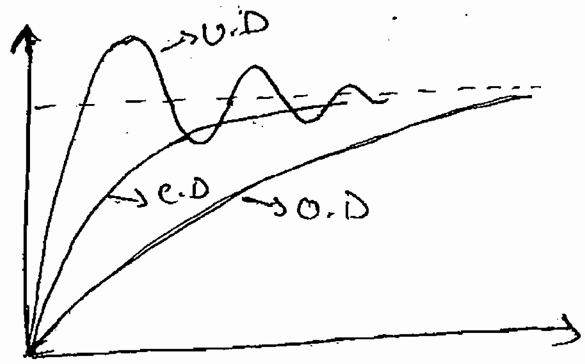



But $I_{tr}(t)$, depends upon the values of R-L-C.

So here, $\alpha = \frac{1}{2RC}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

So, complete solution is,

$$i(t) = \begin{cases} I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \to \text{over damped } (\alpha > \omega_0) \\ I_s + e^{-\alpha t}[A_1 + A_2 t] \to \text{critically damped } (\alpha = \omega_0) \\ I_s + e^{-\alpha t}[A_1 \cos\omega_d t + A_2 \sin\omega_d t] \to \text{under damped } (\alpha < \omega_0) \end{cases}$$




Note :-

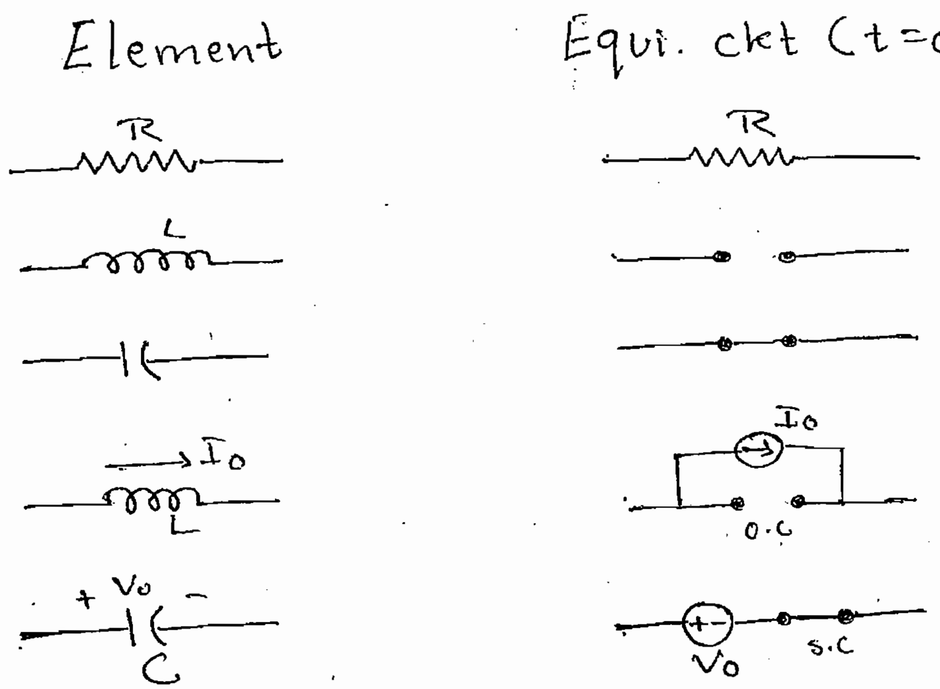
In determining the solution to 2nd order ckts, we need to find the correct values of const. & A1 & A2 through initial conditions.

In 2nd order ckt, there are 2 const., so we require mini. 2 eq^ns.

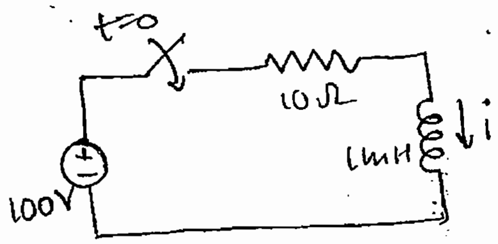
## V] Initial Condition Problems :-

(Transient State Problems, i.e problems at $t = 0^+$ )

Equivalent ~~rep~~ ckt representation of passive elements during transient state (at $t=0^+$) :-




1) Find $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2 i(0^+)}{dt^2}$




At $t=0^-$, s/w was open

$\therefore i(0^-) = 0\,A = i(0^+)$

($\because$ inductor cannot allow sudden change in current.)

KVL $\quad -100 + 10i + 1m\frac{di}{dt} = 0 \longrightarrow$ exact form

At $t = 0^+$

$$-100 + 10[\underbrace{i(0^+)}_{0A}] + 1m\frac{di(0^+)}{dt} = 0$$

$$\therefore \frac{di(0^+)}{dt} = 100\,k\ A/sec$$

Differentiating,

$$10\frac{di}{dt} + 1m\frac{d^2i}{dt^2} = 0$$

At $t=0^+$

$$10\underbrace{\frac{di(0^+)}{dt}}_{100k} + 1m\frac{di^2}{dt^2}(0^+) = 0$$

$$\frac{d^2i(0^+)}{dt^2} = -1000\ mA/sec^2$$

→ Suppose in this problem find complete solution.

$$i(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

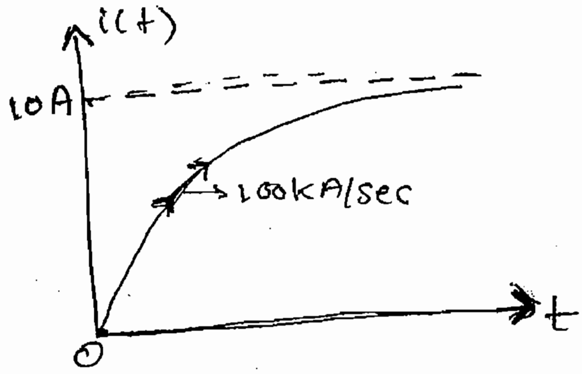
Here,

$$I(0) = 0A$$

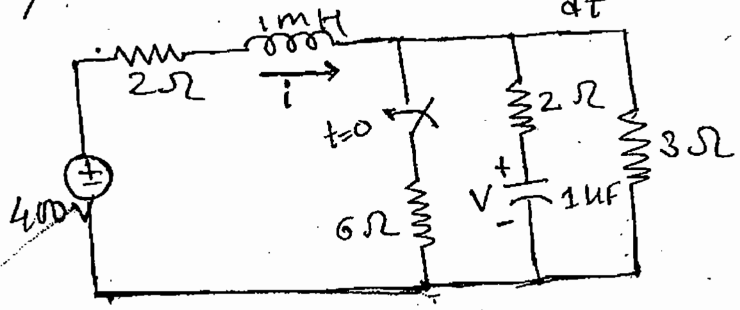
$$I(\infty) = \frac{100}{10} = 10A$$

$$\tau = \frac{L}{R} = \frac{1m}{10} = 10^{-4}$$

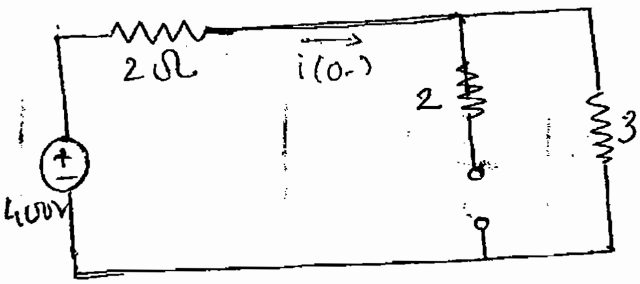
$$i(t) = 10[1 - e^{-1000t}]\ A$$




2) Find $i(0^+)$, $v(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$; $i(\infty)$, $v(\infty)$



① Past $(t=0^-)$

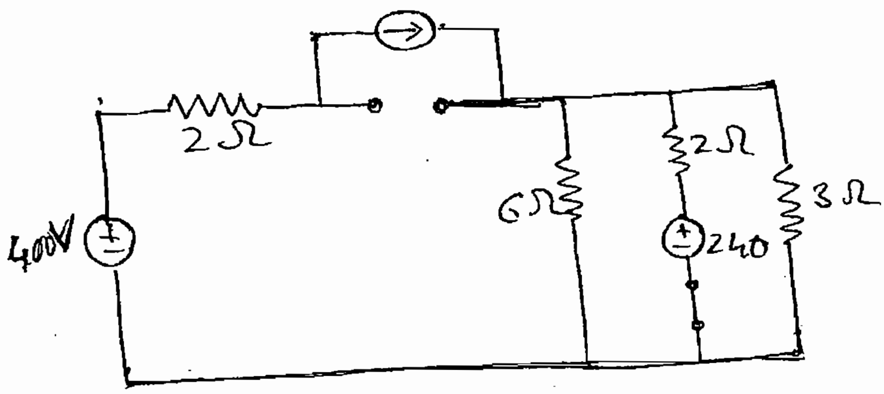



$$i(0^-) = \frac{400}{5} = 80\,A$$

$$\therefore\ i(0^+) = 80\,A$$

$$V(0^-) = 240\,V \longrightarrow V(0^+)$$

② Present State $(t=0^+)$




$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

$$\frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

Nodal

$$-80 + \frac{V}{6} + \frac{(V-240)}{2} + \frac{V}{3} = 0$$

$$-480 + V + 3V - 720 + 2V = 0$$

$$\therefore\ \boxed{V = 200\,V}$$

KVL $\quad -400 + 160 + V_L(0^+) + 200 = 0$

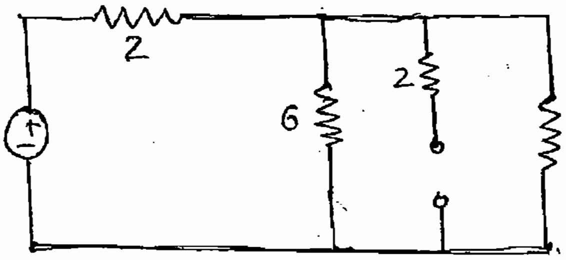
$$V_L(0^+) = 40$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{40}{1m} = 40\,kA/sec$$

$$i_c(0^+) = \frac{(V-240)}{2} = \frac{(200-240)}{2} = -20$$

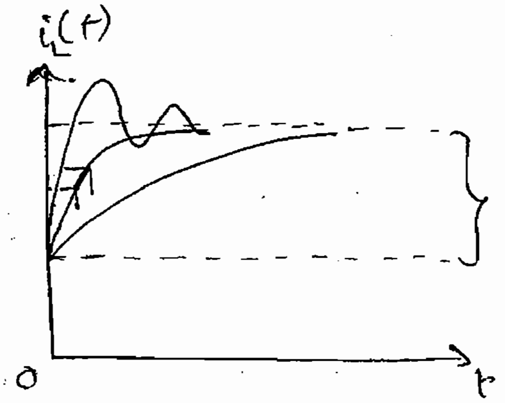
$$\frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-20}{1\mu} = -20\,MV/sec$$

③ Future ($t \to \infty$)

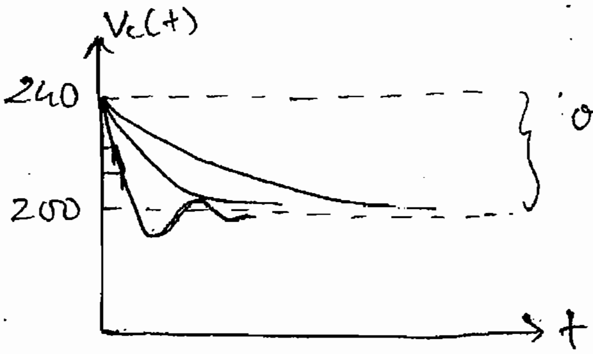



$I(\infty) = \frac{400}{4} = 100\ A$

$V(\infty) = 200\ V$

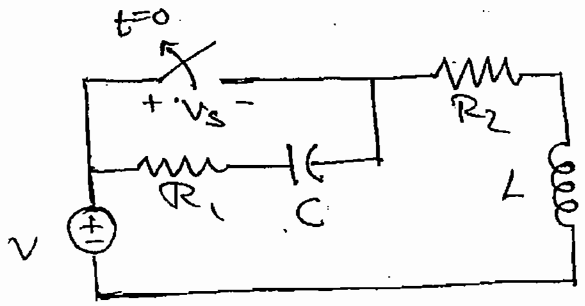



one of this is correct solution




one of this is correct solution

3) Find $V_s(0^+)$




(a) $V\left[\frac{R_1}{R_1+R_2}\right]$

(b) $V\left[\frac{R_2}{R_1+R_2}\right]$

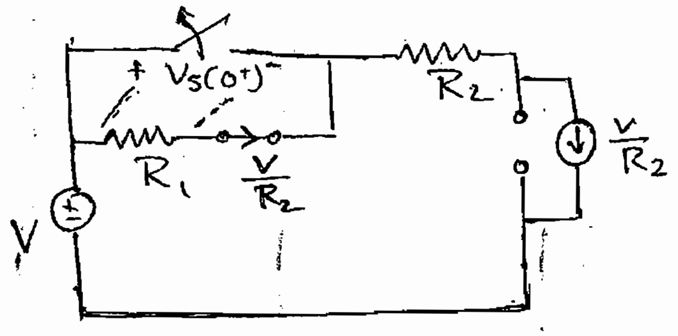
(c) $V\left[\frac{R_1}{R_2}\right]$ (d) $V\left[\frac{R_2}{R_1}\right]$

① At ($t=0^-$)

$i(0^-) = \frac{V}{R_2}\ A$ $V(0^-) = 0\ V$ $V_s(0^-) = 0\ V$

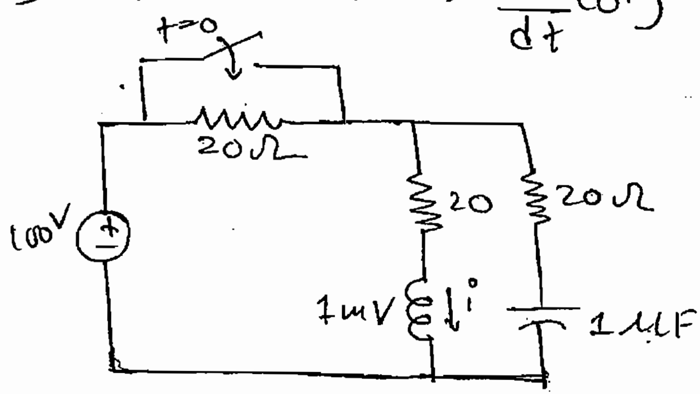
$= i(0^+)$ $= V(0^+)$

② At $t = 0^+$

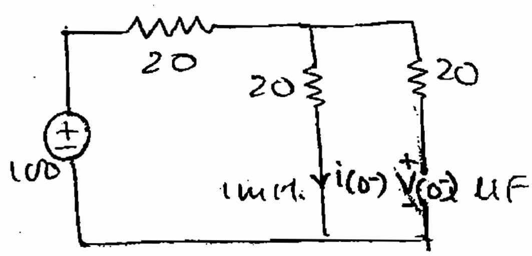



$$V_s(0^+) = \frac{V}{R_2} \times R_1$$

2) Find ~~$V_i(0^+)$~~ $\frac{d\,i}{dt}(0^+)$




① at $t=0^-$

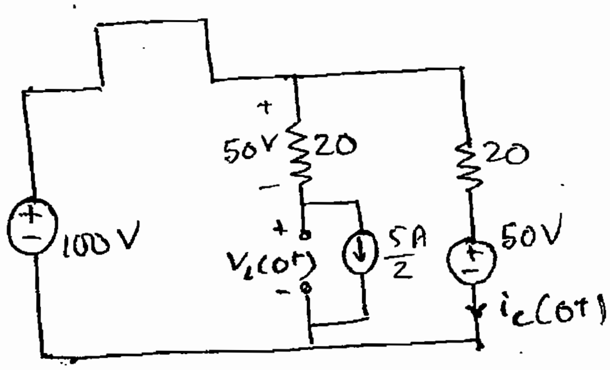



$$i(0^-) = \frac{100}{40} = \frac{5}{2}\,A$$

$$V(0^-) = \frac{20}{40} \times 100$$

$$= 50\,V$$

② at $t=0^+$




KVL $\quad -100 + 50 + V_L(0^+) = 0$

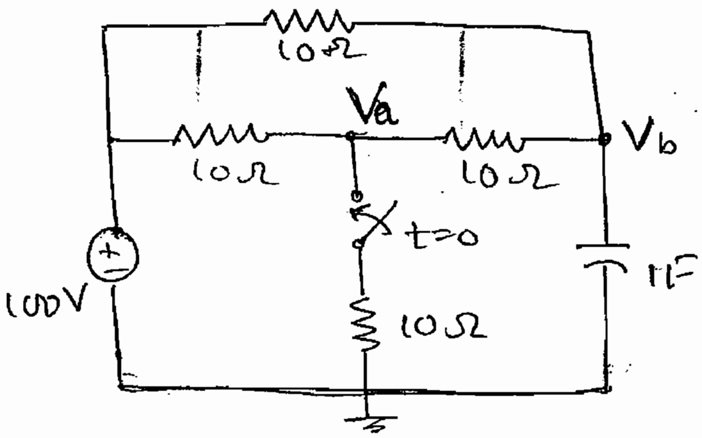
$$V_L(0^+) = 50$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

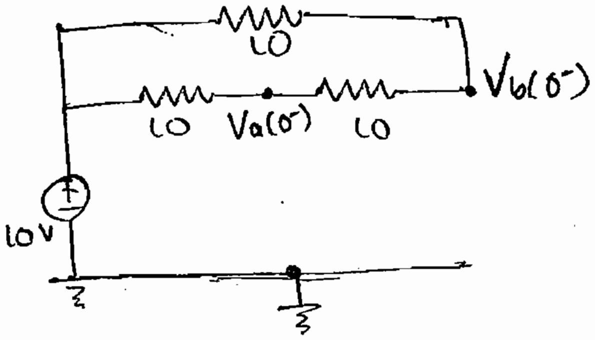
$$= \frac{50}{1m}$$

$$= 50\ kA/sec$$

3) $V_a(0^-)$, $V_b(0^-)$
$V_a(0^+)$, $V_b(0^+)$



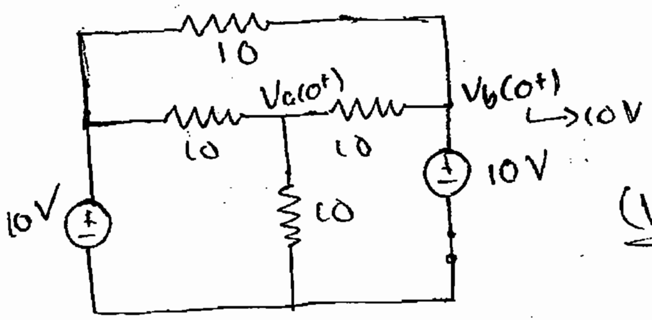

① At $(t = 0^-)$




$V_a(0^-) = 10V$

$V_b(0^-) = 10V$

② At $(t = 0^+)$



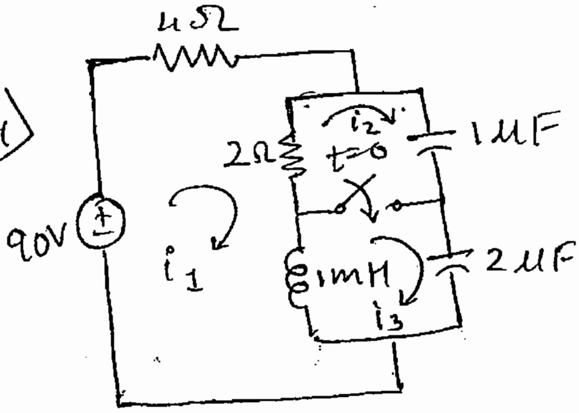

$V_b(0^+) = 10V$

Nodal

$$\frac{(V_a - 10)}{10} + \frac{V_a}{10} + \frac{(V_a - 10)}{10} = 0$$

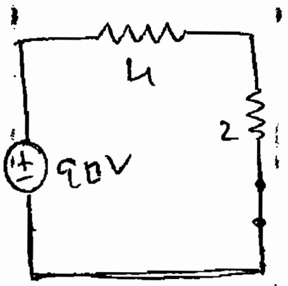
$$V_a(0^+) = \frac{20}{3}\ V$$

4)




Find $i_1(0^+)$, $i_2(0^+)$, $i_3(0^+)$

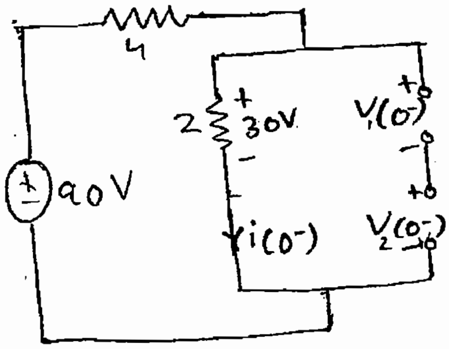
① At $t=0^-$




$$i_1(0^-) = \frac{90}{6} = 15\ A$$

$$i_2(0^-) = 0\ A \qquad i_3(0^-) = 0\ A$$

② ~~At $t=0^+$~~

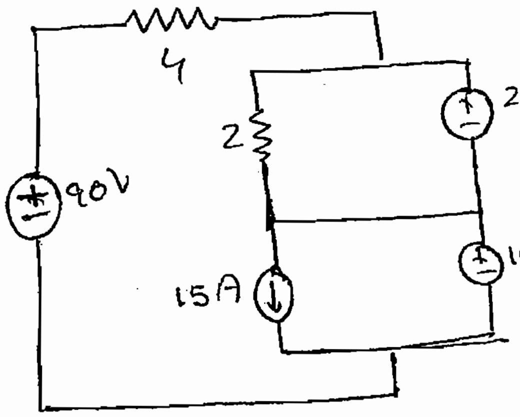



$$i_1(0^+) = 15\ A$$

$$V_1(0^-) = 30\left[\frac{2}{3}\right] = 20\ V$$

$$V_2(0^-) = 30\left[\frac{1}{3}\right] = 10\ V$$

② At $t=0^+$




Mesh

$$-90 + 4i_1 + 2[i_1 - i_2] + 10 = 0 \quad \text{— ①}$$

$$2[i_2 - i_1] + 20 = 0 \quad \text{— ②}$$

$$i_1 - i_3 = 15 \quad \text{— ③}$$

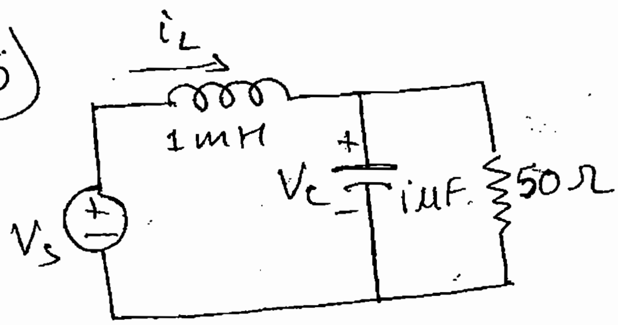
Solve

$$i_1(0^+) = 15\ A$$

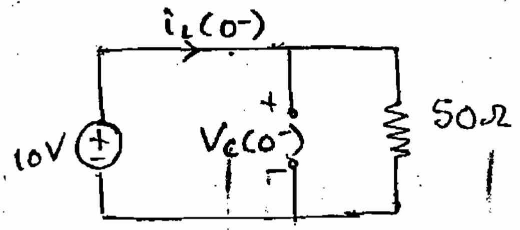
$$i_2(0^+) = 5\ A$$

$$i_3(0^+) = 0\ A$$

5)



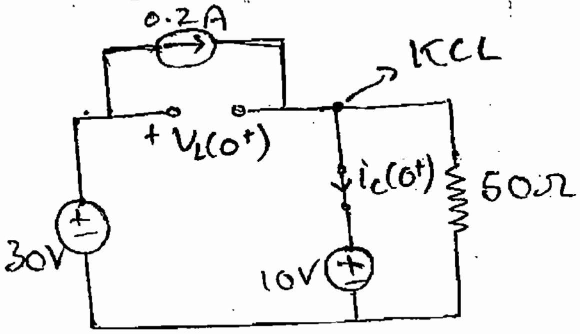
① At $t=0^-$




$$i(0^-) = \frac{10}{50} = 0.2\,A$$

$$V(0^-) = 10\,V$$

② At $t=0^+$




KVL

$$-30 + V_L(0^+) + 10 = 0$$

$$V_L(0^+) = 20$$

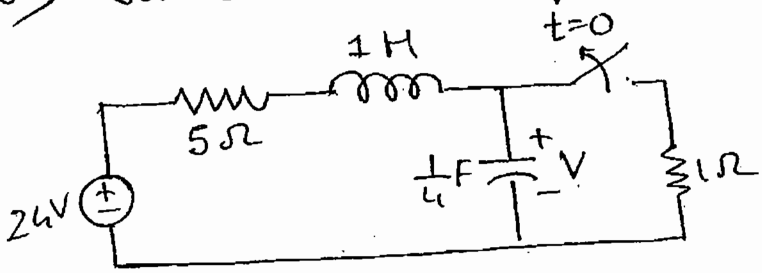
KCL

$$i_c(0^+) = 0\,A$$

$$\rightarrow \frac{dV_c(0^+)}{dt} = \frac{0}{C} = 0\ V/sec$$

$$\rightarrow \frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{20}{1m} = 20\,kA/sec$$

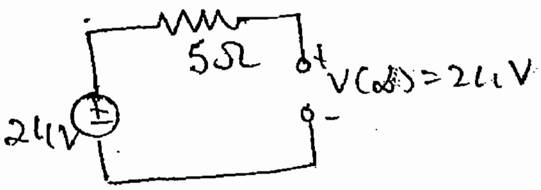
6) Find the complete expression for V




This is a step resp., 2nd order, series RLC. (canonical form)

$$V(t) = V_{ss}(t) + V_{tr}(t)$$

$\rightarrow V_{ss}(t)$ as $t \rightarrow \infty$




But

$V_{tr}(t)$ depends upon values of R, L, C

$$\alpha = \frac{R}{2L} = \frac{5}{2\times 1} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times \frac{1}{4}}} = 2$$

$\alpha > \omega_0 \Rightarrow$ over damped 'R'

→ So complete solution is of the form

$$V(t) = 24 + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

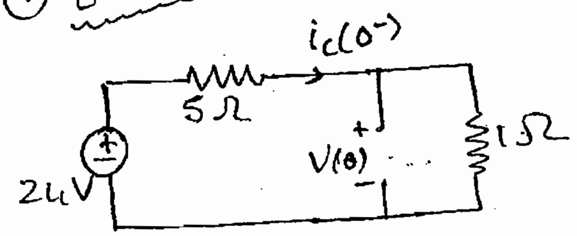
$$= -2.5 \pm \sqrt{(2.5)^2 - (2)^2}$$

$$= -4, -1$$

So,

$$V(t) = 24 + A_1 e^{-4t} + A_2 e^{-t} \quad \text{———} \quad (A)$$

① $t = 0^-$



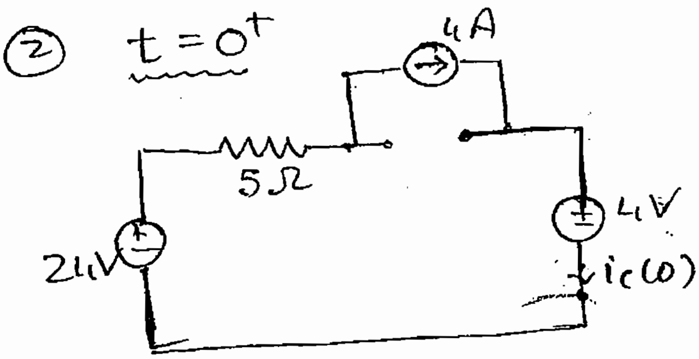

$$i(0^-) = \frac{24}{6} = 4A$$

$$V(0^-) = 4\times 1 = 4V$$

Substituting in (A) ; $4 = 24 + A_1 + A_2$

$$\therefore A_1 + A_2 = -20 \quad \text{———} \quad ①$$

② $t = 0^+$




$$\frac{dV(0)}{dt} = \frac{i_c(0)}{C} = \frac{4}{1/4}$$

$$= 16 \text{ V/sec}$$

$$\frac{dV}{dt} = -4A_1 e^{-4t} - A_1 e^{-t}$$

at $(t=0)$

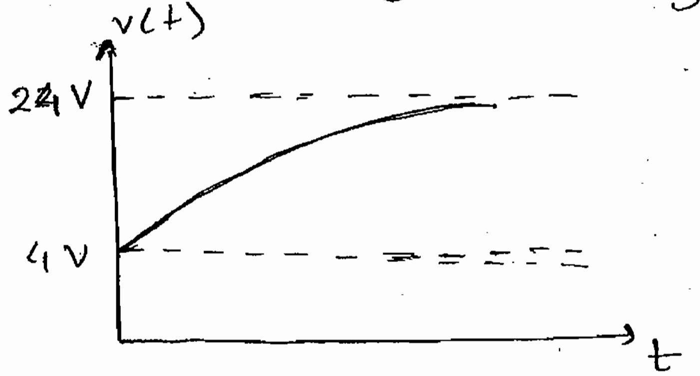
$$16 = -4A_1 - A_2 \quad \text{———} \quad ②$$

Solving ① & ②

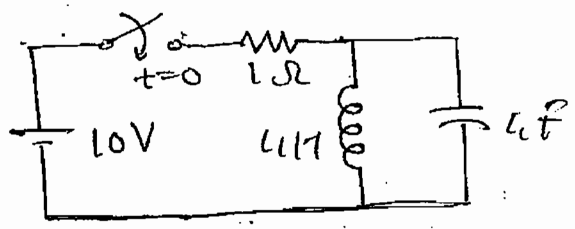
$A_1 = \frac{4}{3}$  $A_2 = \frac{-64}{3}$

Then complete solⁿ /expression for v(t) is,

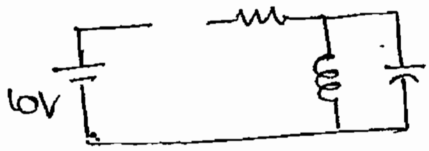
$$v(t) = 24 + \frac{4}{3} e^{-4t} - \frac{64}{3} e^{-t}$$




7) Find current through battery at $t=0^+$ and $t \to \infty$.




① $t=0^-$

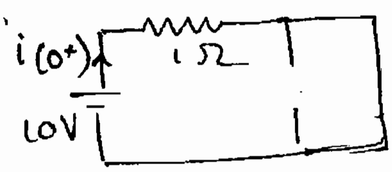



$i_L(0^-) = 0V$

$V_c(0^-) = 0V$

⇒ It is initially relaxed (i.e. zero state)

② $t=0^+$

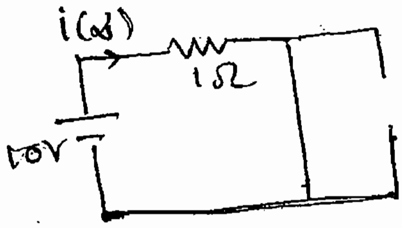



$\therefore\ i(0^+) = \frac{10}{1}$

$= 10\ A$

Now

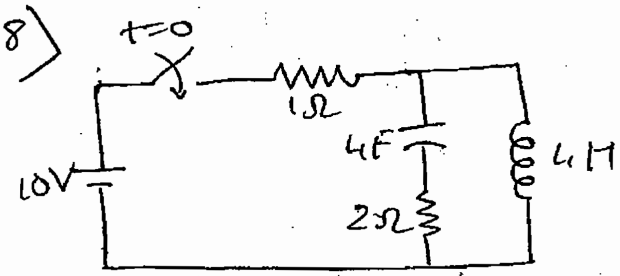
$t \to \infty$




$i(\infty) = \frac{10}{1}$

$= 10\ A$

8) Find current through the battery at $t=0^+$ & $t \to \infty$

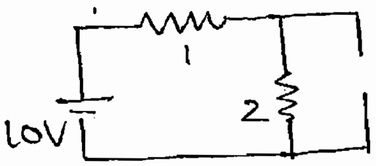



$t=0^-$ (initially relaxed)

$V(0^-) = 0\,V$

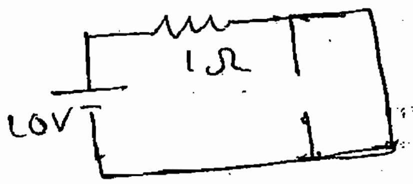
$i(0^-) = 0\,A$

① $t=0^+$



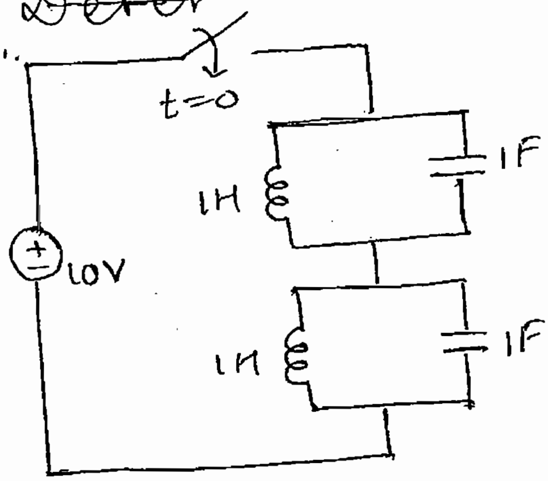

$i(0^+) = \frac{10}{3}\,A$

② $t \to \infty$




$i(\infty) = 10\,A$

9) ~~Deter~~ Find current through the battery at $t=0^+$ & $t \to \infty$

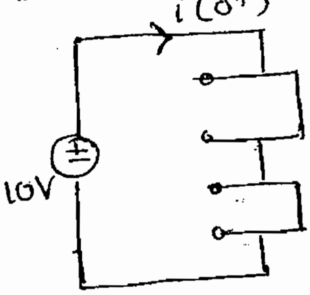



At $t=0^-$

$i(0-) = 0$

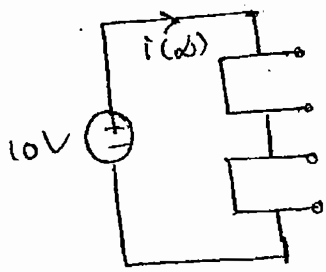
$v(0-) = 0$

At $t=0^+$



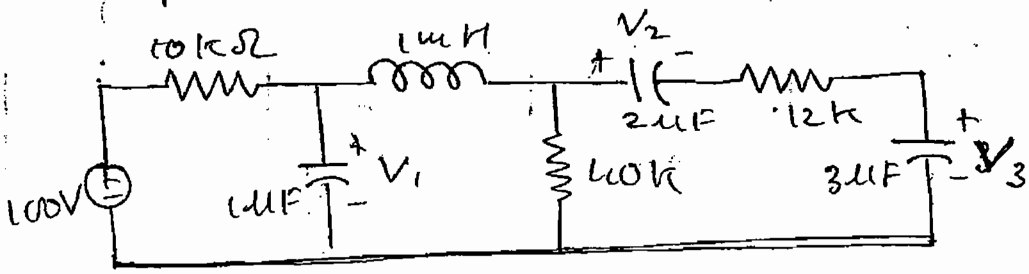

$i(0^+) \to$ very high practically

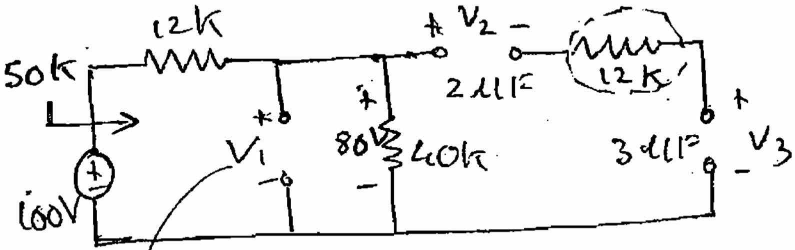
At $t=\infty$




$i(\infty) \to$ very high practically

10) Determine steady state voltages across capacitor.



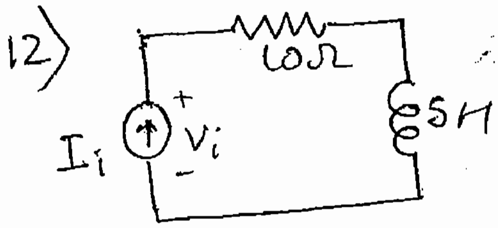




$V_2 = 80\left(\frac{3}{5}\right) = 48\,V$

$V_3 = 80\left(\frac{2}{5}\right) = 32\,V$

$V_1 = 100\left[\frac{40k}{80k}\right] = 8\,V$

12)




If $I_1 = 0.4t^2$ then, $V_i(t)$ ~~at~~ $t = 1$ sec

KVL

$$-V_i + 10\,I_i + 5\frac{dI_i}{dt} = 0$$

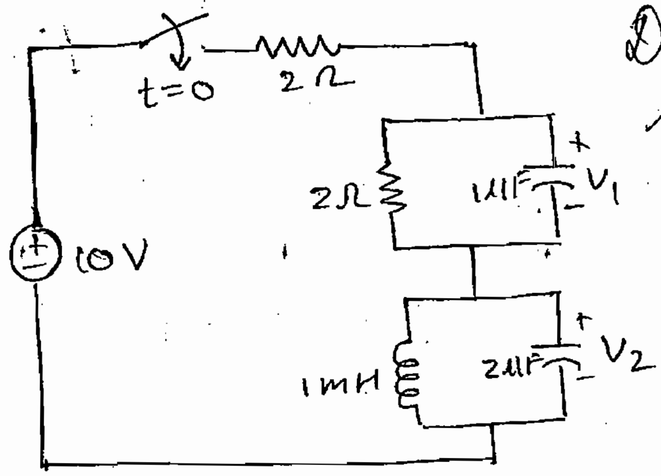
$$V_i = 10(0.4t^2) + 5\frac{d}{dt}(0.4t^2)$$

$$V_i(t) = 4t^2 + 4t$$

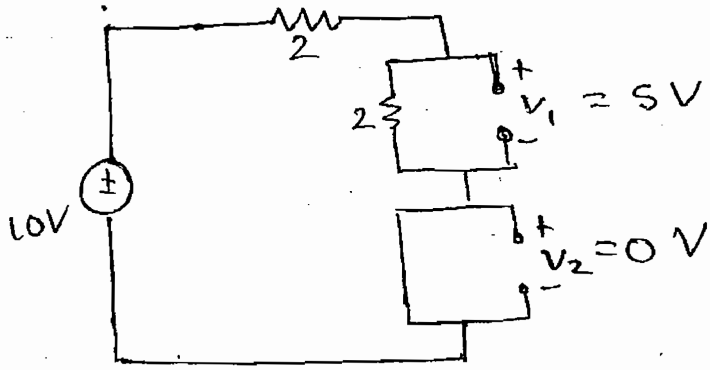
$\hookrightarrow$ at $t = 1$

$\therefore \boxed{V_i = 8 \text{ Volt}}$

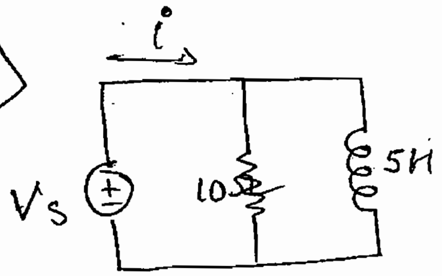
11)




Determine steady state voltages across capacitors.




13)




If $V_s = 40t$, $t \geq 0$
If $i_L(0) = 5A$ then
$i(t)$ at $t = 2$ sec. = ?

$$i = i_R + i_L$$

$$i = \frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} V\,dt$$

$$i = \frac{40t}{10} + \frac{1}{5}\Big[\underbrace{\int_{-\infty}^{0} V\,dt}_{\text{initial current}} + \int_{0}^{t} V\,dt\Big]$$

$$= 4t + i(0) + \frac{1}{5}\int_{0}^{t} 40t\,dt$$

$$= 4t + 5 + \frac{8t^2}{2}$$

$$i(t) = 4t^2 + 4t + 5$$

at $t=2$ $\quad i = 4(2)^2 + 4(2) + 5 = 29A$

# VI] AC Transients

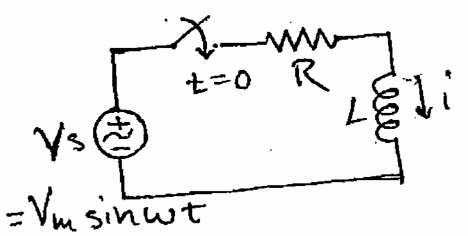
AC Transient are less effective than DC Transients because:

1. Once the equipment in AC is designed for peak rated values, operating at any point other than peak value, the equipment or n/w is safe.

2. Even the surges that occur can hardly travel half a cycle & get naturally suppressed.

3. Since there are natural zero vltg or current instances, we can avoid transients completely if we can operate the switch exactly at those instants of time when current or vltg is zero.

## (a) R-L circuit




$$i(t) = i_{tr}(t) + i_{ss}(t)$$

KVL

$$-V_s + iR + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} + Ri = V_m \sin\omega t$$

→ The nature of solution for i(t) is:

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t - \phi) + Ae^{-t/\tau}$$

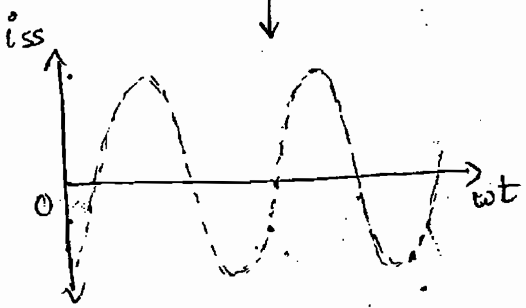
Here, $|Z| = \sqrt{R^2 + (\omega L)^2}$ $\quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

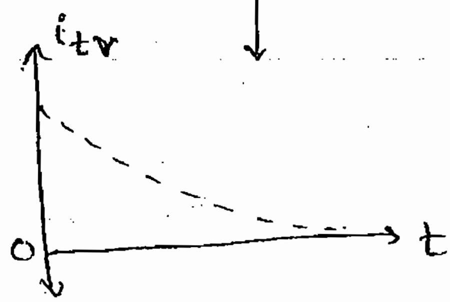
$\tau = \frac{L}{R}$

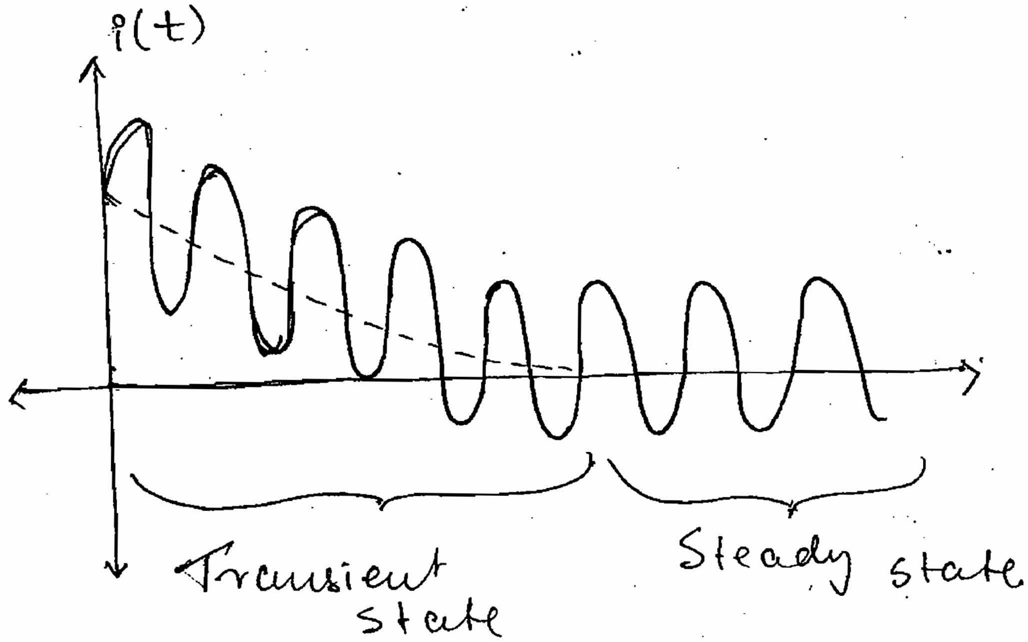
But at $t=0$, $i=0$

$0 = \frac{V_m}{|Z|}\sin(-\phi) + A \implies A = \frac{V_m}{|Z|}\sin\phi$

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t - \phi) + \frac{V_m}{|Z|}\sin\phi \, e^{-t/\tau}$$




**NOTE:-**

① Let $V_s = V_m \sin(\omega t + \theta)$

Then, the solution for $i(t)$ is like,

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t + \theta - \phi) + A e^{-t/\tau}$$

But at $t=0$, $i=0$

$$0 = \frac{V_m}{|Z|}\sin(\theta-\phi) + A$$

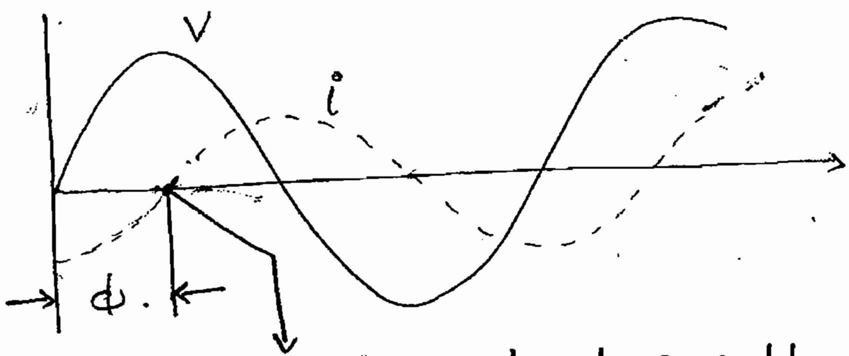
$$\therefore A = \frac{-V_m}{|Z|}\sin(\theta-\phi)$$

So, complete sol$^n$ is of the form

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t+\theta-\phi) - \frac{V_m}{|Z|}\sin(\theta-\phi)\, e^{-t/\tau}$$

② If i/p excitation is in cosine Terms, then express the o/p sol$^n$ for $i(t)$ also in cosine terms.

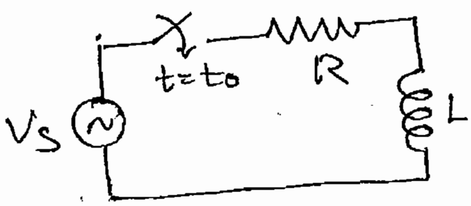
③ For general R-L Load




This is the instant exactly when $i=0$

So we can operate the s/w exactly at this instant of time, we can avoid.

TRANSIENTS

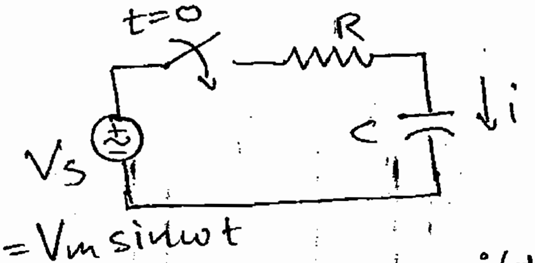



At $\omega t_0 = \phi$

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

↳ This is switch time, when we can avoid Transients.

## (b) Series R-C ckt:-




$$i(t) = i_{ss}(t) + i_{tr}(t)$$

The natural solⁿ of $i(t)$ is

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t + \phi) + A e^{-t/\tau}$$

where, $|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ $\qquad \phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$

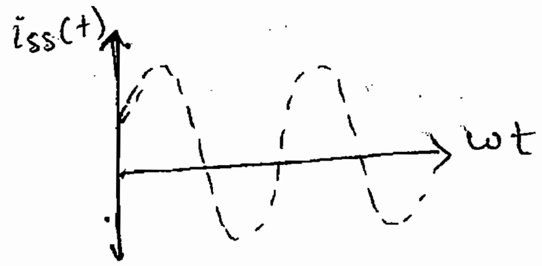
$$\tau = RC$$

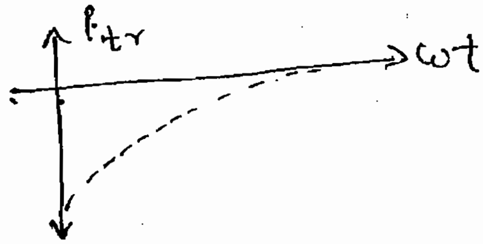
At $t = 0$, $i = 0$

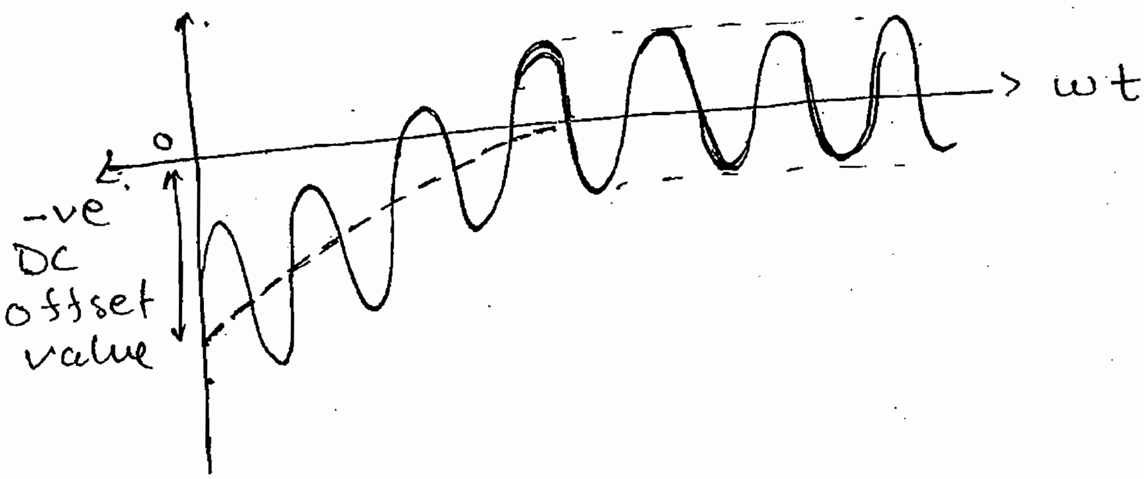
$$0 = \frac{V_m}{|Z|}\sin\phi + A \implies A = \frac{-V_m}{|Z|}\sin\phi$$

→ So complete solⁿ is:

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t + \phi) - \frac{V_m}{|Z|}\sin\phi \, e^{-t/\tau}$$



NOTE :-

① Here if $V_s = V_m \sin(\omega t + \theta)$

Then, $i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) + A e^{-t/\tau}$

at $t=0$, $i=0$;

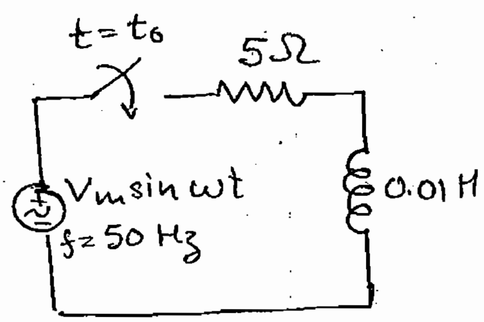
$0 = \frac{V_m}{|Z|} \sin(\theta + \phi) + A \implies A = -\frac{V_m}{|Z|} \sin(\theta + \phi)$

→ The complete solⁿ is:

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) - \frac{V_m}{|Z|} \sin(\theta + \phi)\, e^{-t/\tau}$$

② If i/p excitation is in cosine terms then express the o/p solⁿ for $i(t)$ also in cosine terms

1) At what switching instant time '$t_0$', the current in the circuit has transient free resp.




$\omega t_0 = \phi$

$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$2\pi(50) t_0 = \underbrace{\tan^{-1}\left[\frac{2\pi(50)(0.01)}{5}\right]}_{\text{radians}}$

$100\pi t_0 = 0.56$

$\therefore t_0 = 1.78$ msec

So, here if we can operate the switch exactly at 1.78 ms from the instant where vltg became zero, if can completely avoid transients.

2) In the above problem if $V_s = V_m \sin(\omega t - 10^\circ)$ then determine the value of to which results into transient free resp.

So, $\omega t_0 \mp 10^\circ = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$2\pi(50)t_0 \mp 10 \times \frac{\pi}{180} = \tan^{-1}\left[\frac{2\pi(50)(0.01)}{5}\right]$$

$$\therefore \quad 100\pi t_0 = 0.73$$

$$\therefore \quad t_0 = 2.33 \text{ msec.}$$

# VII] Laplace Transform & its Application to ckt Analysis :—

When to use L.T. methods in ckt analysis?

1) If determination of I is difficult
2) If order of ckt $\geq 2$
3) Non-canonical form of ckts
4) Non-standard excitation
   (i.e. impulse, pulse, ramp, parabolic, step, exponential, etc)

$$L[f(t)] \longleftrightarrow F(s)$$

$\boxed{s = j\omega}$

↳ Complex freq

$v(t) \longleftrightarrow V(s)$

$i(t) \longleftrightarrow I(s)$

$V \longleftrightarrow \frac{V}{s}$

$I \longleftrightarrow \frac{I}{s}$

$V_m \sin\omega t \longleftrightarrow \frac{V_m \omega}{s^2+\omega^2}$

$V_m \cos\omega t \longleftrightarrow \frac{I_s s}{s^2+\omega^2}$

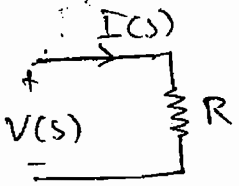
$R \longleftrightarrow R$

$L \longleftrightarrow sL$

$C \longleftrightarrow \frac{1}{Cs}$

## Equivalent ckt representation of passive elements in Laplace domain:-

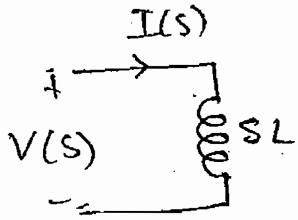
(a). Resistor




$V(s) = I(S)\cdot R$

$I(S) = \frac{V(S)}{R}$

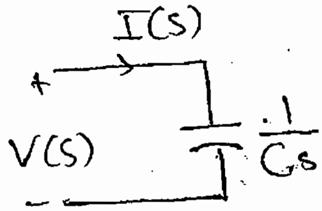
(b) Inductor




$V(s) = I(S)\cdot sL$

$I(s) = \frac{V(S)}{sL}$

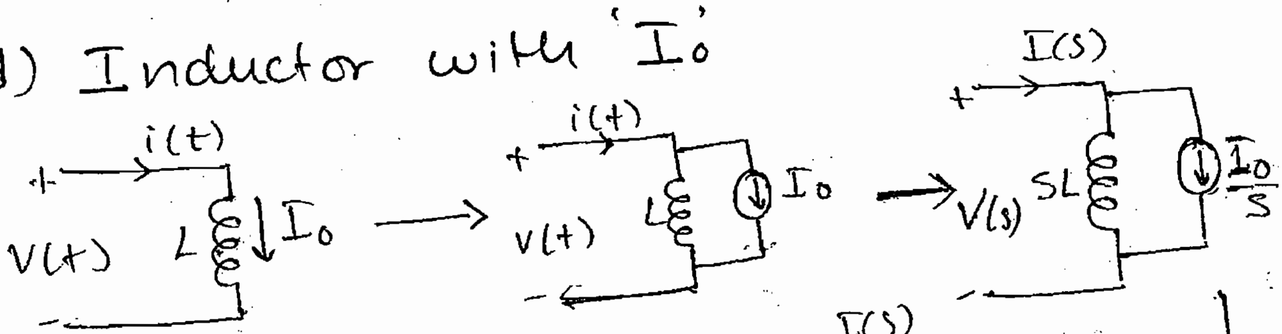
(c) Capacitor

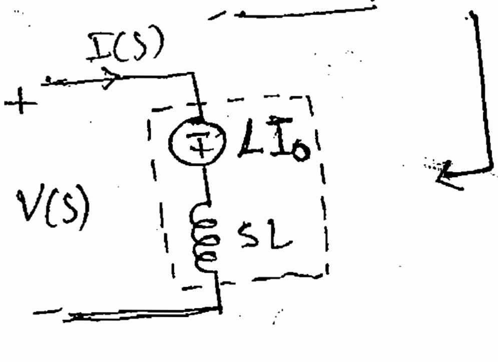



$V(S) = \frac{I(S)}{CS}$

$I(S) = V(S)\cdot CS$

(d) Inductor with '$I_0$'

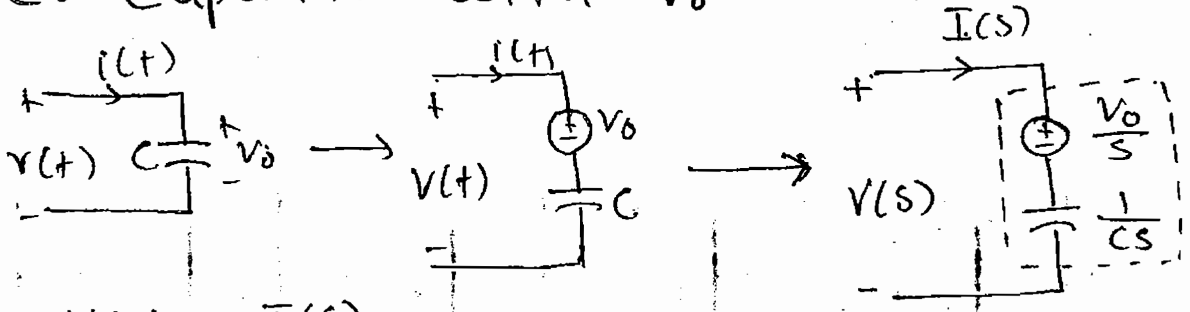



$V(S) = sLI(S) - LI_0$

$I(s) = \frac{V(S)}{SL} + \frac{I_0}{S}$

(c) Capacitor with '$V_0$'




$$V(s) = \frac{I(s)}{Cs} + \frac{V_0}{s}$$

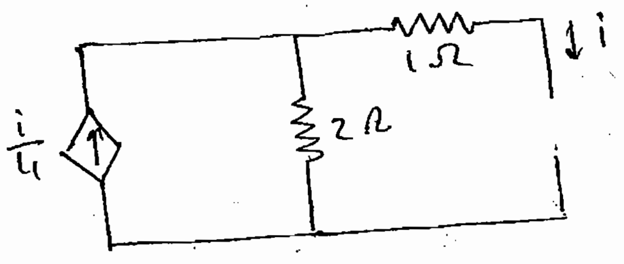
$$I(s) = Cs\,V(s) - C V_0$$

1) If $I_0 = 10\,A$, find the complete ~~exe~~ expression of I
(a) in Time domain
(b) in Laplace domain

(a) $i(t) = I_0 e^{-t/\tau}$

$I_0 = 10\,A$ given.

$\tau = \frac{L}{R_{eq}}$




KVL

$\frac{3}{2} + 1 - V_T = 0$

$V_T = \frac{5}{2}$

$R_{eq} = \frac{V_T}{1} = \frac{5}{2}\,\Omega$

$\tau = \frac{1/2}{5/2} = \frac{1}{5}$ sec

So, $i(t) = 10\,e^{-5t}$

(b) KVL

$\frac{3}{2} I(s) + I(s) - 5 + I(s) \cdot \frac{5}{2} = 0$

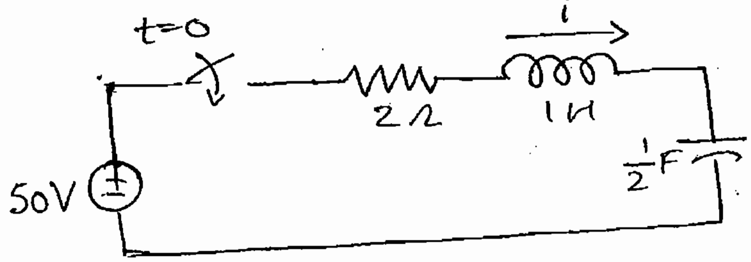
$$I(s)\left[\frac{s}{2}+\frac{s}{2}\right]=5$$

$$I(s)=\frac{10}{s+5}$$

$$i(t)=\mathcal{L}^{-1}\left[\frac{10}{s+5}\right]=10e^{-5t}$$

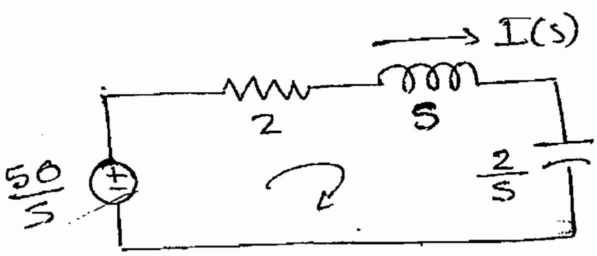
2) Find $i(t),\ t>0$

→ This ~~sg~~ ckt is initially relaxed.




KVL

$$\frac{-50}{s}+I(s)\left[2+s+\frac{2}{s}\right]=0$$




$$I(s)\left[\frac{s^2+2s+2}{s}\right]=\frac{50}{s}$$

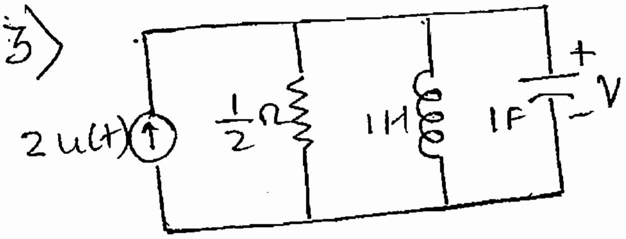
$$I(s)=\frac{50}{s^2+2s+2}$$

$$I(s)=\frac{50}{(s+1)^2+(1)^2}$$

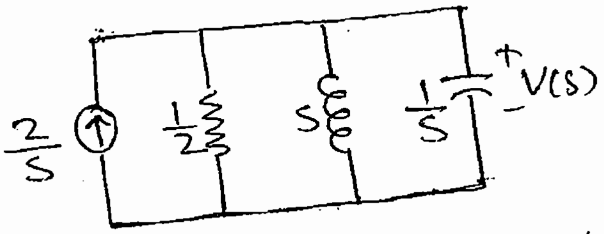
$$i(t)=\mathcal{L}^{-1}[I(s)]=50e^{-t}\sin t \text{ A}$$

→ This ckt is initially relaxed.

3) Find $v(t),\ t>0$




→ This is initially relaxed circuit




KCL

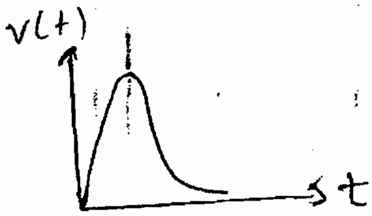
$$-\frac{2}{s}+\frac{V(s)}{1/2}+\frac{V(s)}{s}+\frac{V(s)}{1/s}=0$$

$$V(s)\left[2+\frac{1}{s}+s\right]=\frac{2}{s}$$

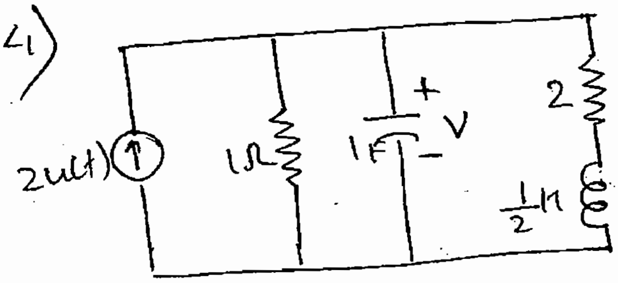
$$V(s) = \left[\frac{s^2+2s+1}{s}\right] = \frac{2}{s}$$

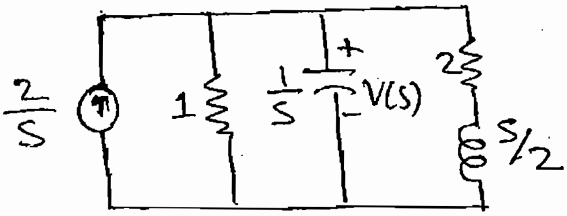
$$V(s) = \frac{2}{(s+1)^2} \Rightarrow v(t) = \underline{2te^{-t}}$$

↳ critically damped




4)




KCL

$$-\frac{2}{s} + \frac{V(s)}{1} + \frac{V(s)}{1/s} + \frac{V(s)}{\left(\frac{s+4}{2}\right)} = 0$$

$$\therefore V(s)\left[1 + s + \frac{s}{s+4}\right] = \frac{2}{s}$$

$$V(s) = \frac{2}{s}\,\frac{(s+4)}{s^2+5s+6}$$

$$V(s) = \frac{2(s+4)}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

where,

$$A = \frac{2(0+4)}{(0+2)(0+3)} = \frac{8}{6} = \frac{4}{3}$$

$$B = \frac{2(-2+4)}{(-2)(-2+3)} = \frac{4}{-2} = -2$$

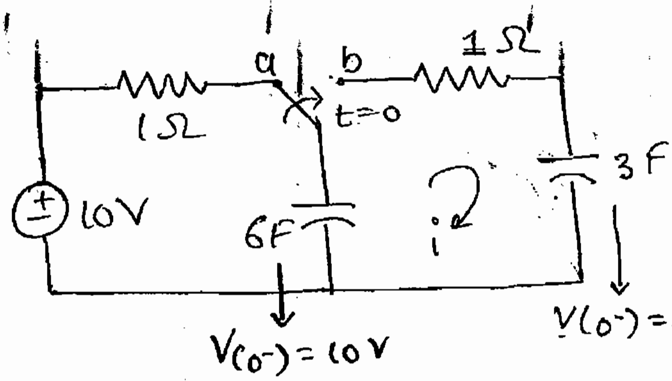
$$C = \frac{2(-3+4)}{(-3)(-3+2)} = \frac{+2}{3} = \frac{2}{3}$$

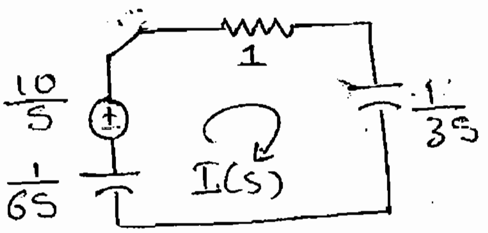
$$\therefore V(s) = \frac{4/3}{s} - \frac{2}{s+2} + \frac{2/3}{s+3}$$

$\therefore\ v(t) = \frac{4}{3}u(t) - 2e^{-2t} + \frac{2}{3}e^{-3t}$

$\Rightarrow$ over damped step resp.

5) Find $i(t)$, $t \geq 0$




KVL

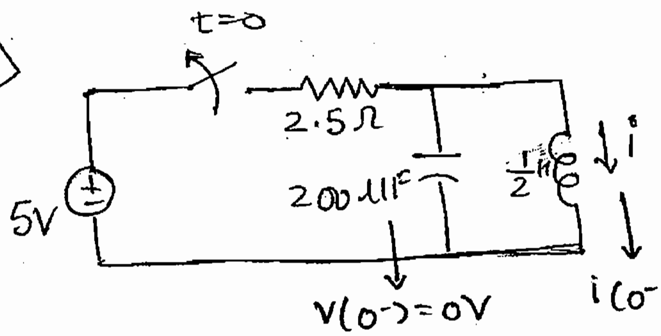
$$-\frac{10}{s} + I(s)\left[1 + \frac{1}{6s} + \frac{1}{3s}\right] = 0$$

$$I(s)\left[\frac{6s+3}{6s}\right] = \frac{10}{s}$$

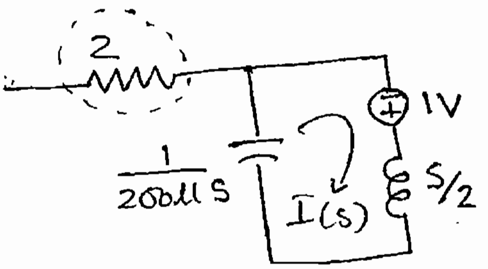
$$\therefore\ I(s) = \frac{60}{6s+3} = \frac{20}{2s+1} = \frac{10}{s+1/2}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \underline{10e^{-t/2}}\ A$$

6) Find $i(t)$, $t \geq 0$




$$i(0^-) = \frac{5}{2.5} = 2A$$




KVL

$$I(s)\left[\frac{s}{2} + \frac{1}{200\mu s}\right] = 1$$

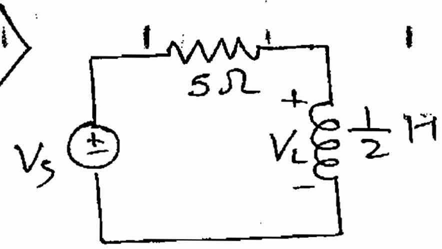
$$I(s)\left[s + \frac{10^6}{100s}\right] = 2$$

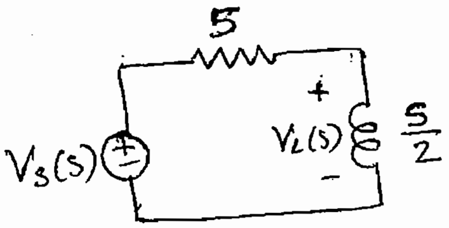
$$I(s)\left[s + \frac{10^4}{s}\right] = 2 \Rightarrow I(s)\left[\frac{s^2+10^4}{s}\right] = 2$$

$$I(s) = \frac{2 \cdot s}{s^2 + (100)^2}$$

$i(t) = \underbrace{2 \cos 100t}$ → sinusoidal o/p.

7) Find $V_L$ if (a) $V_s(t) = \delta(t)$
(b) $V_s(t) = e^{-t} u(t)$




$$V_L(s) = V_s(s)\left[\frac{s/2}{s/2 + 5}\right]$$

$$V_L(s) = V_s(s)\left[\frac{s}{s+10}\right]$$

(a) $V_s(t) = \delta(t)$
$V_s(s) = 1$

So, $V_L(s) = 1 \times \frac{s}{s+10} = \frac{s+10-10}{s+10} = 1 - \frac{10}{s+10}$

$\therefore v_L(t) = \delta(t) - 10e^{-10t}$ $\underline{\underline{V}}$

(b) $V_s(t) = e^{-t} u(t)$
$V_s(s) = \frac{1}{s+1}$

So, $V_L(s) = \frac{1}{(s+1)} \frac{s}{(s+10)} = \frac{A}{s+1} + \frac{B}{s+10}$

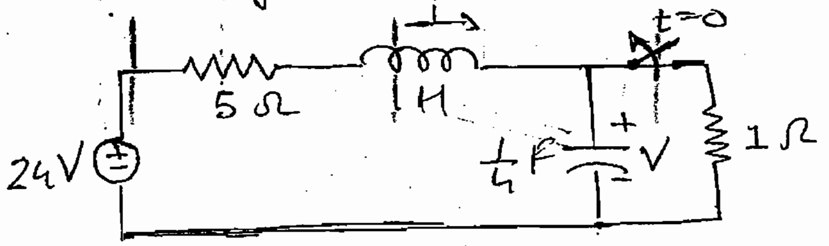
where, $A = \frac{-1}{-1+10} = \frac{-1}{9}$

$B = \frac{-10}{-10+1} = \frac{10}{9}$

So,
$$V_L(s) = \frac{10}{9(s+10)} - \frac{1}{9(s+1)}$$

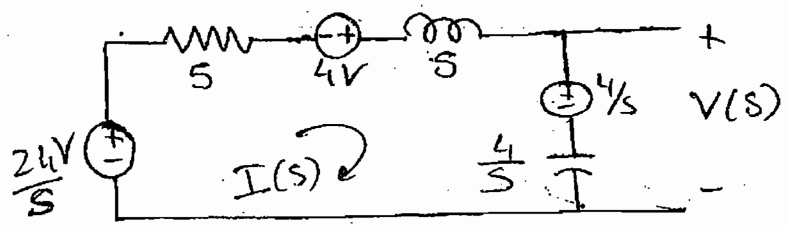
$\therefore V_L(t) = \frac{10}{9} e^{-10t} - \frac{1}{9} e^{-t}$

8) Find the complete expression for 'V' using L.T method.




$i(0^-) = 4A$

$v(0^-) = 4V$




$$\frac{-24}{S} - 4 + \frac{4}{S} + I(S)\left[5 + S + \frac{4}{S}\right] = 0$$

$$I(s)\left[\frac{S^2+5S+4}{S}\right] = \frac{4S+20}{S}$$

$$I(S) = \frac{4S+20}{S^2+5S+4}$$

$$V(S) = \frac{4}{S} + I(S) \times \frac{4}{S}$$

$$= \frac{4}{S} + \frac{4}{S}\frac{(4S+20)}{(S^2+5S+4)}$$

$$= \frac{4(S^2+5S+4) + 16S + 80}{S(S+1)(S+4)}$$

$$= \frac{4S^2+36S+96}{S(S+1)(S+4)}$$

$$V(S) = \frac{A}{S} + \frac{B}{S+4} + \frac{C}{S+1}$$

where, $A = \frac{96}{(1)(4)} = 24$

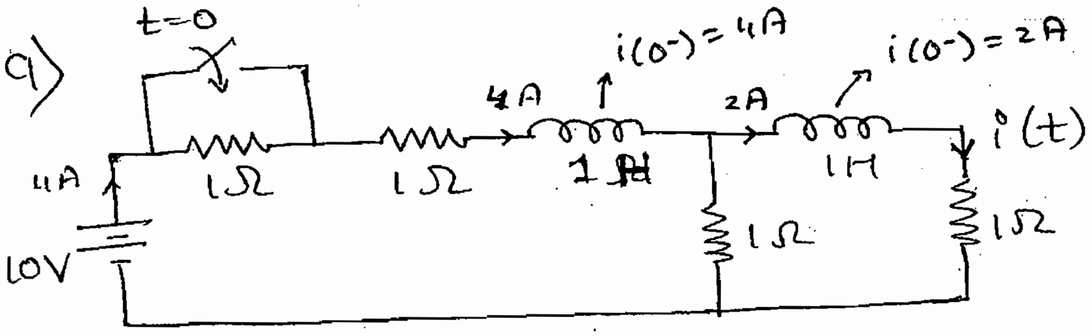
$$B = \frac{64 - 144 + 96}{12} = \frac{4}{3}$$

$$C = \frac{4 - 36 + 96}{-3} = -\frac{64}{3}$$

$$V(S) = \frac{24}{S} + \frac{4}{3(S+4)} + \frac{-64}{3(S+1)}$$

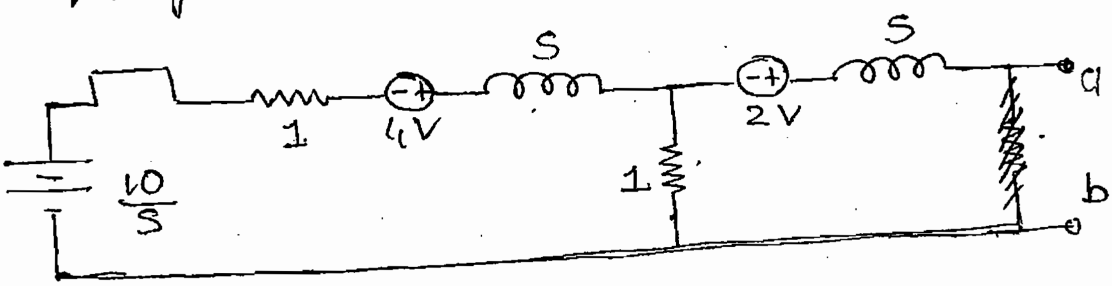
$$v(t) = 24 + \frac{4}{3} e^{-4t} - \frac{64}{3} e^{-t}$$

9)




Apply Thevenins theo. to obtain the expression of current i(t) in 1 Ω resis.

→ Thevenins' theo. can be applied to simplify the circuit in steady state. But, this is a transient problem. So, we can convert this problem in steady state freq. domain & then apply Thevenins theo.



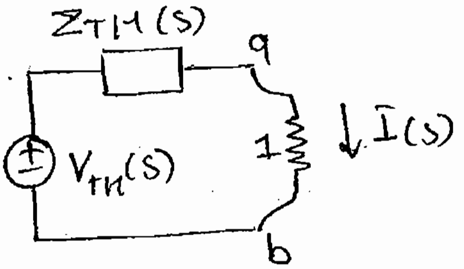

$$Z_{TH} = S + [1 \,||\, (S+1)]$$

$$= S + \frac{S+1}{S+2} = \frac{S^2+3S+1}{S+2}$$

$$V_{TH} = 2 + \left(\frac{10}{s} + 4\right)\left[\frac{1}{s+2}\right]$$

$$= 2 + \left(\frac{10+4s}{s}\right)\left(\frac{1}{s+2}\right)$$

$$= 2 + \frac{4s+10}{s(s+2)} = \frac{2s^2+8s+10}{s(s+2)}$$




$$I(s) = \frac{V_{TH}(s)}{Z_{TH}(s)+1}$$

$$\therefore I(s) = \frac{(2s^2+8s+10)\cancel{(s+2)}}{\cancel{(s+2)}\, s\left[(s^2+3s+1)+s+2\right]}$$

$$= \frac{2s^3+8s^2+10s+4s^2+16s+20}{}$$

$$= \frac{2s^2+8s+10}{s(s^2+4s+3)}$$

$$= \frac{2s^2+8s+10}{s(s+3)(s+1)}$$

$$I(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

where,

$$A = \frac{10}{3\times 1} = \frac{10}{3}$$

$$B = \frac{18-24+10}{(-3)(-2)} = \frac{4}{6} = \frac{2}{3}$$

$$C = \frac{2-8+10}{(-1)(2)} = \frac{4}{-2} = -2$$

$$I(s) = \frac{10}{3}\cdot\frac{1}{s} + \frac{2}{3}\,\frac{1}{s+3} - 2\,\frac{1}{s+1}$$

$\therefore\ i(t) = \frac{10}{3} + \frac{2}{3} e^{-3t} - 2 e^{-t}$

# NETWORK FUNCTIONS & FILTER CONCEPTS

If the magnitude of i/p supply supply is kept const. but freq. is varied, then the o/p is defined as the complete freq. resp. of the ckt or n/w.

eg:- Resonance

Freq. resp. of any n/w gives its complete steady state performance which is useful in design & analysis. & synthesis of filters, antennas, SONARS, radars, etc in communication engg.

To obtain the complete freq. resp. of the n/w, we need to build the n/w's transfer function.

$$X(\omega) \longrightarrow \boxed{n/w} \longrightarrow Y(\omega) \qquad \begin{matrix}N/w \\ T.F.\end{matrix} = H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$s = j\omega$
↳ complex

In n/ws, there are only 4 types of T.F that can defined.

(a) Voltage Gain T.F.

$$G(s) = \frac{V_o(s)}{V_i(s)}$$

(b) Current Gain T.F.

$$\alpha(s) = \frac{I_o(s)}{I_i(s)}$$

(c) Transfer Impedance

$$Z(s) = \frac{V_o(s)}{I_i(s)}$$

(d) Transfer Admittance

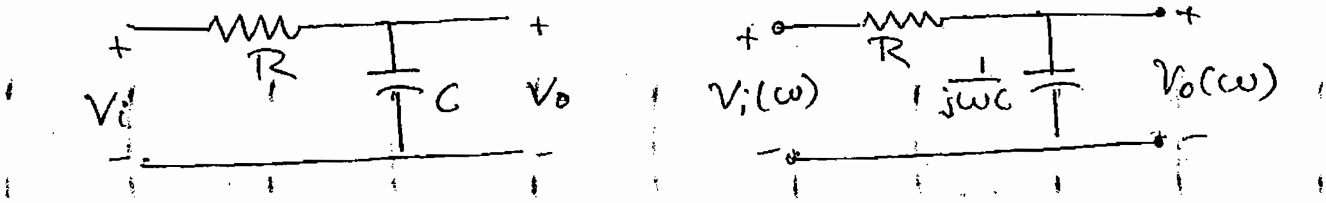
$$Y(s) = \frac{I_o(s)}{V_i(s)}$$

→ In general, impedance & admittance together are called as Immitance fun. then, in analysing filter ckts, we generally consider V.G.T.F : $H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$

→ The n/w func. $H(\omega)$ is complex quantity so, it has magnitude ~~modulous~~ $|H(\omega)|$ & phase of $|\phi(\omega)|$

To obtain complete freq. resp. of n/w T.F., we need to plot both mag. & phase as $\omega$ is varied from 0 to $\infty$.

1) Obtain the complete freq. resp. of the voltage gain T.F. ckt shown below.



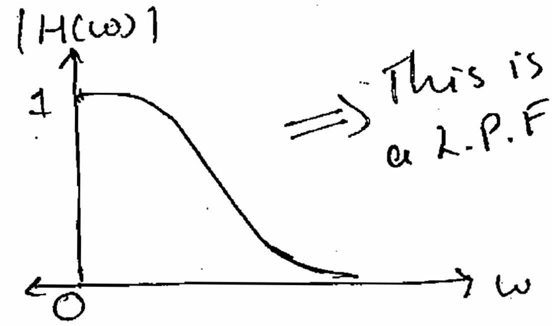

$$V_o(\omega) = V_i(\omega) \times \frac{1/j\omega C}{R + \frac{1}{j\omega C}}$$

$$\therefore\ H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{1 + j\omega RC}$$

$\rightarrow$ Magnitude $\rightarrow$ $|H(\omega)| = \dfrac{1}{\sqrt{1+\omega^2 R^2 C^2}}$

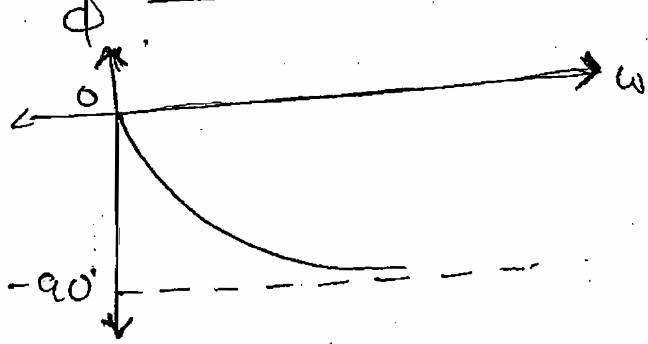
$\rightarrow$ Phase $\rightarrow$ $\phi(\omega) = -\tan^{-1}(\omega RC)$

$\rightarrow$ Magnitude plot

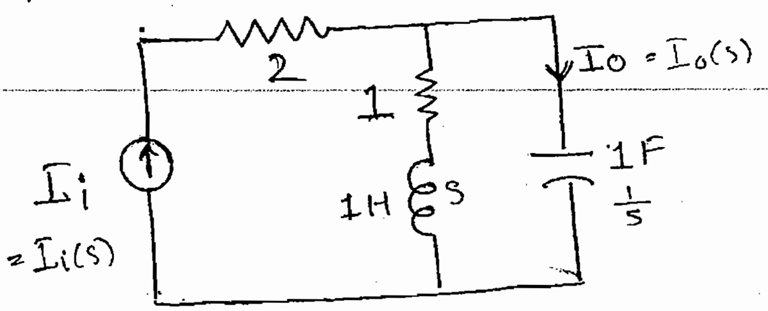



$\Rightarrow$ This is a L.P.F

Phase plot

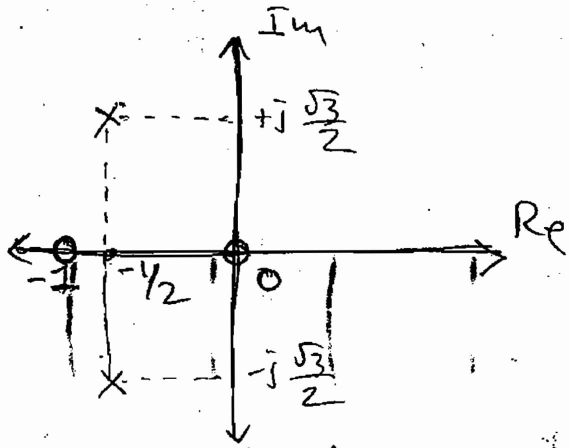



2) Obtain the current gain T.F of the ckt shown and plot the location of poles & zero in s-plane

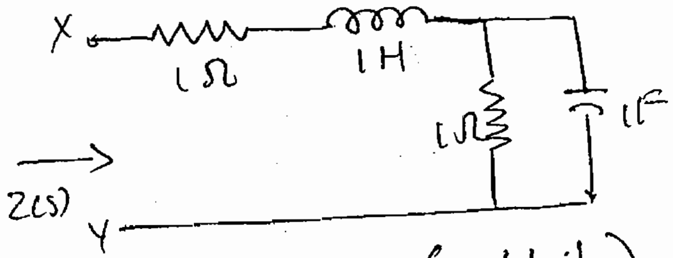


$$I_o(s) = I_i(s)\left[\frac{s+1}{s+1+\frac{1}{s}}\right]$$

$$\alpha(s) = \frac{I_o(s)}{I_i(s)} = \frac{s(s+1)}{s^2+s+1}$$

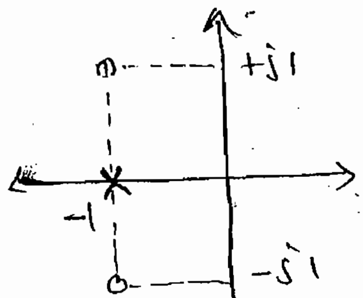



3) Obtain the driving pt. impedance of the n/w shown below.

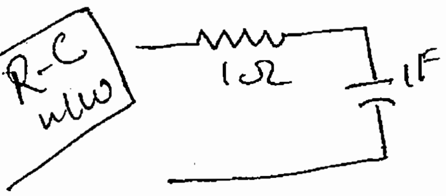


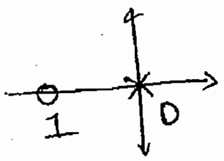

~~Z(s) = (1+s) || (1 || ~~

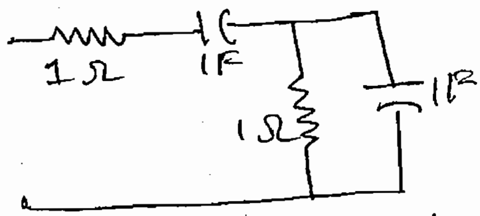
$$Z(s) = (1+s) + \left(1 \,||\, \frac{1}{s}\right) = \frac{s^2+2s+2}{s+1}$$




4) Obtain the driving pt. impedance of n/w shown below & plot the location of poles & zero in s-plane



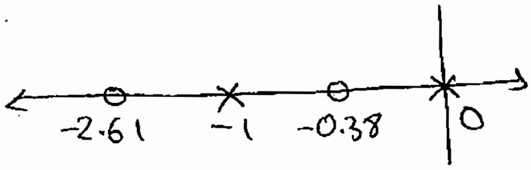



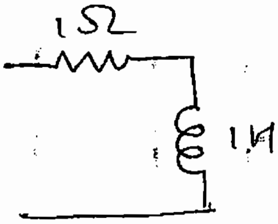

$$Z(s) = 1 + \frac{1}{s} = \frac{s+1}{s}$$

$$Z(s) = 1 + \frac{1}{s} + \frac{1/s}{\frac{s+1}{s}} = \frac{s^2+s+s+1+s}{s(s+1)}$$

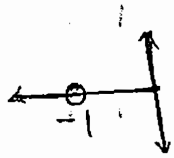
$$Z(s) = \frac{s^2+3s+1}{s(s+1)}$$

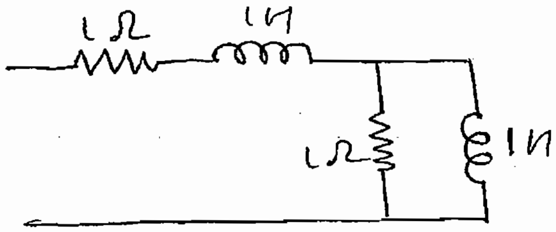


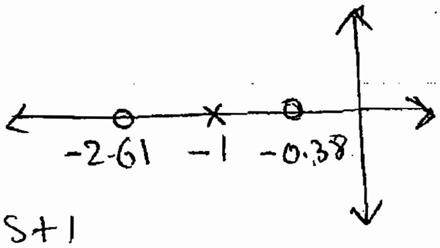

$\boxed{\text{R-L n/w}}$




$$Z(s) = \frac{s+1}{1}$$

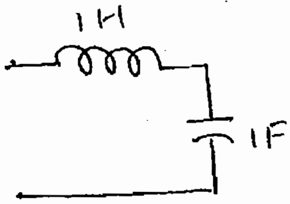


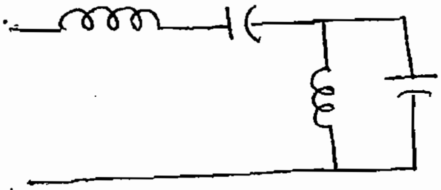




$$Z(s) = s+1+\frac{s}{s+1} = \frac{s^2+3s+1}{s+1}$$

$\boxed{\text{L-C n/w}}$




$$Z(s) = s+\frac{1}{s}$$
$$= \frac{s^2+1}{s}$$



$$Z(s) = s+\frac{1}{s}+\frac{s\times 1/s}{s+\frac{1}{s}}$$
$$= s+\frac{1}{s}+\frac{s}{s^2+1}$$

→ For any R-L-C n/w & its driving pt. impedance the poles and zeroes on left hand side of s-plane & scattered throughout the plane.

→ For any R-C n/w & its driving pt. impedance poles & zeros are located on the -ve real axis only. & are alternately places. And the nearest critical freq. to origin is a pole.

→ For any R-L n/w & its driving pt. impedance all the poles & zeros are located on the -ve real axis only & are alternately placed & the nearest critical freq to origin is a zero.

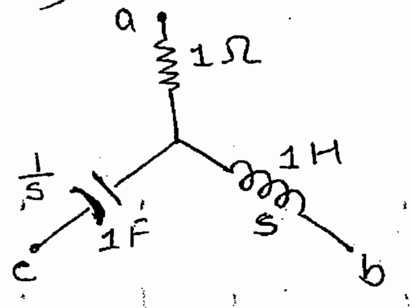
→ For any pure L-C n/w & its driving pt. impedance all poles & zeros are located on the imaginary axis only. & are alternately placed.

→ The pole-zero pattern of R-L driving pt. impedance is similar to R-C driving pt. admittance.

→ Ily, the pole-zero pattern of R-C driving pt. impedance func is similar to R-L driving pt. admittance func.

→ In general, these driving pt. impedance & admittance func. are together called as Imittance func.

1) Convert into $\Delta$ delta.

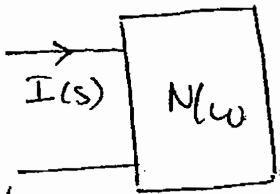



$$Z_{ab} = 1 + s + \frac{s}{1/s} = s^2 + s + 1 \ \Omega$$

$$Z_{ac} = 1 + \frac{1}{s} + \frac{1/s}{s} = \frac{s^2+s+1}{s^2} \ \Omega$$

$$Z_{bc} = s + \frac{1}{s} + \frac{1}{1} = \frac{s^2+s+1}{s} \ \Omega$$

2) If $I(s) = \dfrac{s+4}{(s+2)(s+3)}$




Find the initial value of current.

$I(s)$ From initial value theo. :

$$i(0) = \lim_{s\to\infty} \frac{s(s+4)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{1+4/s}{(1+2/s)(1+3/s)}$$

$$= \frac{1+0}{(1)(1)} = 1 \ A$$

3) Find L.T. of

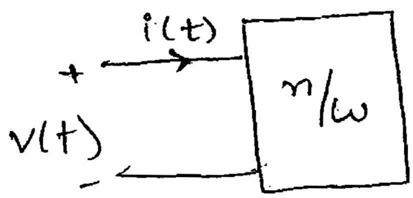

$$i(t) = 2t\,[u(t) - u(t-2)]$$

$$= 2tu(t) - 2(t-2+2)u(t-2)$$

$$\therefore I(s) = \frac{2}{s^2} - \frac{2\cdot e^{-2s}}{s^2} - \frac{4e^{-2s}}{s}$$

4) If $i(t) = e^{-3t}$ A for $v(t) = u(t)$ V, $t \geq 0$.

Determine the n/w elements




$$\frac{v(t)}{i(t)} = \frac{u(t)}{e^{-3t}}$$

$$\frac{V(s)}{I(s)} = \frac{1/s}{1/(s+3)} = \frac{s+3}{s} = 1 + \frac{3}{s}$$

$$Z(s) = 1 + \frac{1}{\frac{1}{3}s} \Rightarrow \left.\begin{array}{l} R = 1\,\Omega \\ C = \frac{1}{3}\,F \end{array}\right\}$$ Series RC n/w

**Note :-**

$$Z(S) = k_0 + k_1 S + \frac{k_2}{s}$$

Here,

$k_0 \rightarrow$ resistance $\rightarrow$ value $\rightarrow k_0 \,\Omega$

$k_1 \rightarrow$ inductance $\rightarrow$ value $\rightarrow k_1 \, H$

$k_2 \rightarrow$ capacitance $\rightarrow$ value $\rightarrow \frac{1}{k_2} \, F$

5) Find $V_o$ = ____

$$V_i = 2\sin t \longrightarrow \boxed{\frac{S}{S+1}} \longrightarrow V_o$$

$$V_o = V_i\left(\frac{S}{S+1}\right)$$

But $s = j\omega$ & here $\omega = 1 \Rightarrow S = j$

$$\therefore V_o = V_i\left(\frac{j}{j+1}\right) = 2\sin t\left[\frac{1\angle 90^\circ}{\sqrt{2}\angle 45^\circ}\right]$$

$$\therefore V_o = \sqrt{2}\sin(t + 45^\circ) \ \underline{V}$$

# Passive Filters :-

Filters are ckts which operate for a particular range of frequencies & attenuate other frequencies

Passive filter are ckts which are designed based on passive elements : R, L, C.

Active filter are ckts at electronic / signal level based on OP Amp. & digital filters also perform signal processing & they are based on DSP.

Passive filters are still used at power level to control harmonics & stablize the power fed to the load. eg: laptop charges,