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Electronics & Communication Engineering

Fifth Edition

R. K. Kanodia
B.Tech.

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Preface

This book doesn't make promise but provides complete satisfaction to the readers. The market scenario is confusing and readers don't find the optimum quality books. This book provides complete set of problems appeared in competition exams as well as fresh set of problems.

The book is categorized into units which are then sub-divided into chapters and the concepts of the problems are addressed in the relevant chapters. The aim of the book is to avoid the unnecessary elaboration and highlights only those concepts and techniques which are absolutely necessary. Again time is a critical factor both from the point of view of preparation duration and time taken for solving each problem in the examination. So the problems solving methods in the book are those which take the least distance to the solution.

But however to make a comment that this book is absolute for GATE preparation will be an inappropriate one. The theory for the preparation of the examination should be followed from the standard books. But for a wide collection of problems, for a variety of problems and the efficient way of solving them, what one needs to go through is there in the book. Each unit (e.g. Networks) is subdivided into average seven number of chapters on an average each of which contains 40 problems which are selected so as to avoid unnecessary redundancy and highly needed completeness.

I shall appreciate and greatly acknowledge the comments and suggestion from the users of this book.

R. K. Kanodia

rajkumar.kanodia@nodia.co.in

Acknowledgments

My primary debt remains to my friends and all those wonderful people who provided real inspiration and encouragement in taking up this challenging project.

No amount of thanks can ever repay the great debt which I owe to my classmates Dinesh Bisht, Biju Deka, Nitin yadav, Bhupesh Kumar, Sushant Ranjan and Mayank Thakur who have supported me during the publishing of this book. A special word of thanks to Miss Shalvi Agarwal who have written the article for back page.

I am also indebted to my grandparents, mother and my sister for their emotional support they have provided while preparing this book.

R. K. Kanodia

SYLLABUS

ENGINEERING MATHEMATICS

Linear Algebra: Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.

Calculus: Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems.

Differential equations: First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

Complex variables: Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, Residue theorem, solution integrals.

Probability and Statistics: Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.

Numerical Methods: Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.

Transform Theory: Fourier transform, Laplace transform, Z-transform.

ELECTRONICS AND COMMUNICATION ENGINEERING

Networks: Network graphs: matrices associated with graphs; incidence, fundamental cut set and fundamental circuit matrices. Solution methods: nodal and mesh analysis. Network theorems: superposition, Thevenin and Norton's maximum power transfer, Wye-Delta transformation. Steady state sinusoidal analysis using phasors. Linear constant coefficient differential equations; time domain analysis of simple RLC circuits, Solution of network equations using Laplace transform: frequency domain analysis of RLC circuits. 2-port network parameters: driving point and transfer functions. State equations for networks.

Electronic Devices: Energy bands in silicon, intrinsic and extrinsic silicon. Carrier transport in silicon: diffusion current, drift current, mobility, and resistivity. Generation and recombination of carriers. p-n junction diode, Zener diode, tunnel diode, BJT, JFET, MOS capacitor, MOSFET, LED, p-I-n and avalanche photo diode, Basics of LASERs. Device technology: integrated circuits fabrication process, oxidation, diffusion, ion implantation, photolithography, n-tub, p-tub and twin-tub CMOS process.

Analog Circuits: Small Signal Equivalent circuits of diodes, BJTs, MOSFETs and analog CMOS. Simple diode circuits, clipping, clamping, rectifier. Biasing and bias stability of transistor and FET amplifiers. Amplifiers: single-and multi-stage, differential and operational, feedback, and power. Frequency response of amplifiers. Simple op-amp circuits. Filters. Sinusoidal oscillators; criterion for oscillation; single-transistor and op-amp configurations. Function generators and wave-shaping circuits, 555 Timers. Power supplies.

Digital circuits: Boolean algebra, minimization of Boolean functions; logic gates; digital IC families (DTL, TTL, ECL, MOS, CMOS). Combinatorial circuits: arithmetic circuits, code converters, multiplexers, decoders, PROMs and PLAs. Sequential circuits: latches and flip-flops, counters and shift-registers. Sample and hold circuits, ADCs, DACs. Semiconductor memories. Microprocessor(8085): architecture, programming, memory and I/O interfacing.

Signals and Systems: Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

Control Systems: Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

Communications: Random signals and noise: probability, random variables, probability density function, autocorrelation, power spectral density. Analog communication systems: amplitude and angle modulation and demodulation systems, spectral analysis of these operations, superheterodyne receivers; elements of hardware, realizations of analog communication systems; signal-to-noise ratio (SNR) calculations for amplitude modulation (AM) and frequency modulation (FM) for low noise conditions. Fundamentals of information theory and channel capacity theorem. Digital communication systems: pulse code modulation (PCM), differential pulse code modulation (DPCM), digital modulation schemes: amplitude, phase and frequency shift keying schemes (ASK, PSK, FSK), matched filter receivers, bandwidth consideration and probability of error calculations for these schemes. Basics of TDMA, FDMA and CDMA and GSM.

Electromagnetics: Elements of vector calculus: divergence and curl; Gauss and Stokes theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

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CHAPTER

1.1

BASIC CONCEPTS

1. A solid copper sphere, 10 cm in diameter is deprived of 10^{20} electrons by a charging scheme. The charge on the sphere is

(A) 160.2 C (B) –160.2 C

(C) 16.02 C (D) –16.02 C

2. A lightning bolt carrying 15,000 A lasts for 100 μs. If the lightning strikes an airplane flying at 2 km, the charge deposited on the plane is

(A) 13.33 μC (B) 75 C

(C) 1500 μC (D) 1.5 C

3. If 120 C of charge passes through an electric conductor in 60 sec, the current in the conductor is

(A) 0.5 A (B) 2 A

(C) 3.33 mA (D) 0.3 mA

4. The energy required to move 120 coulomb through 3 V is

(A) 25 mJ (B) 360 J

(C) 40 J (D) 2.78 mJ

5. $i = ?$

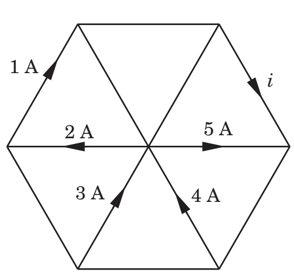


Fig. P.1.1.5

(A) 1 A (B) 2 A

(C) 3 A (D) 4 A

6. In the circuit of fig P1.1.6 a charge of 600 C is delivered to the 100 V source in a 1 minute. The value of v_1 must be

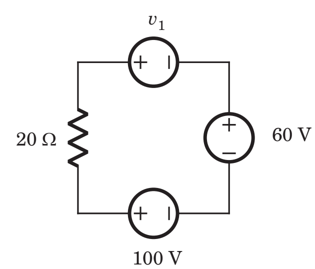


Fig. P.1.1.6

(A) 240 V (B) 120 V

(C) 60 V (D) 30 V

7. In the circuit of the fig P1.1.7, the value of the voltage source E is

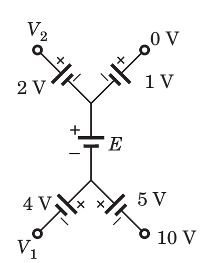


Fig. P.1.1.7

(A) –16 V (b) 4 V

(C) –6 V (D) 16 V

8. Consider the circuit graph shown in fig. P1.1.8. Each branch of circuit graph represent a circuit element. The value of voltage v_1 is

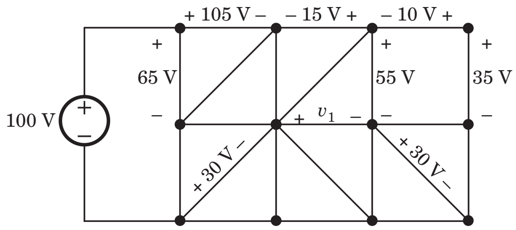


Fig. P.1.1.8

(A) –30 V
(B) 25 V
(C) –20 V
(D) 15 V

9. For the circuit shown in fig P.1.1.9 the value of voltage v_o is

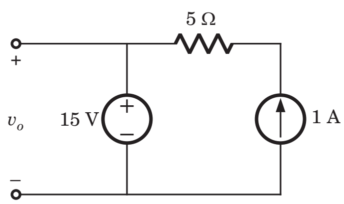


Fig. P.1.1.9

(A) 10 V
(B) 15 V
(C) 20 V
(D) None of the above

10. R_1 = ?

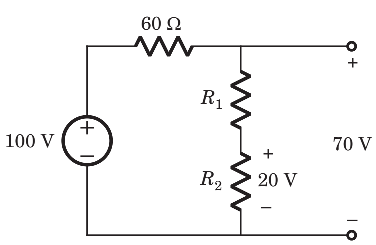


Fig. P.1.1.10

(A) 25 Ω
(B) 50 Ω
(C) 100 Ω
(D) 2000 Ω

11. Twelve 6 Ω resistor are used as edge to form a cube. The resistance between two diagonally opposite corner of the cube is

(A) $\frac{5}{6}$ Ω
(B) $\frac{6}{5}$ Ω
(C) 5 Ω
(D) 6 Ω

12. v_1 = ?

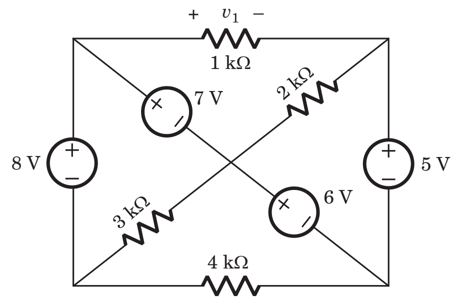


Fig. P1.1.12

(A) –11 V
(B) 5 V
(C) 8 V
(D) 18 V

13. The voltage v_o in fig. P1.1.11 is always equal to

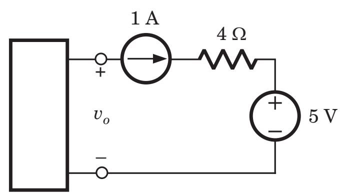


Fig. P1.1.11

(A) 1 V
(B) 5 V
(C) 9 V
(D) None of the above

14. R_{eq} = ?

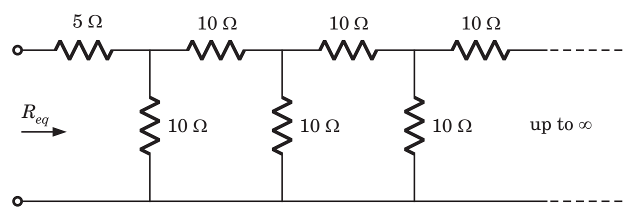


Fig. P1.1.14

(A) 11.86 Ω
(B) 10 Ω
(C) 25 Ω
(D) 11.18 Ω

15. v_s = ?

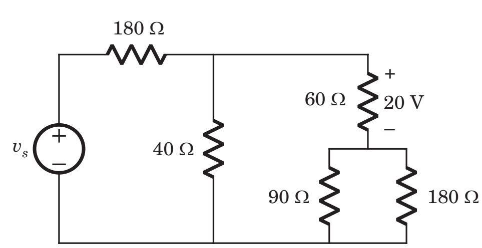


Fig. P.1.1.15

(A) 320 V
(B) 280 V
(C) 240 V
(D) 200 V

24. Let $i(t) = 3te^{-100t}$ A and $v(t) = 0.6(0.01 - t)e^{-100t}$ V for the network of fig. P.1.1.24. The power being absorbed by the network element at $t = 5$ ms is

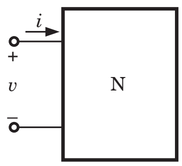


Fig. P.1.1.24

(A) 18.4 μW (B) 9.2 μW

(C) 16.6 μW (D) 8.3 μW

25. In the circuit of fig. P.1.1.25 bulb A uses 36 W when lit, bulb B uses 24 W when lit, and bulb C uses 14.4 W when lit. The additional A bulbs in parallel to this circuit, that would be required to blow the fuse is

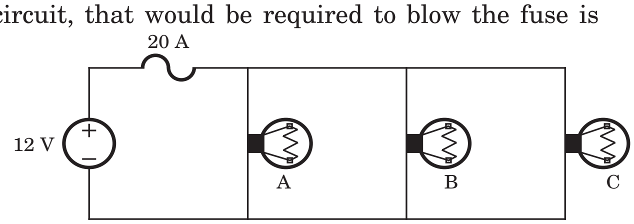


Fig. P.1.1.25

(A) 4 (B) 5

(C) 6 (D) 7

26. In the circuit of fig. P.1.1.26, the power absorbed by the load R_L is

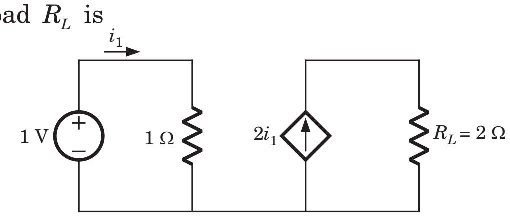


Fig. P.1.1.26

(A) 2 W (B) 4 W

(C) 6 W (D) 8 W

27. $v_o = ?$

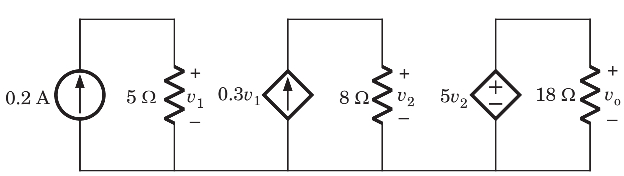


Fig. P.1.1.27

(A) 6 V (B) −6 V

(C) −12 V (D) 12 V

28. $v_{ab} = ?$

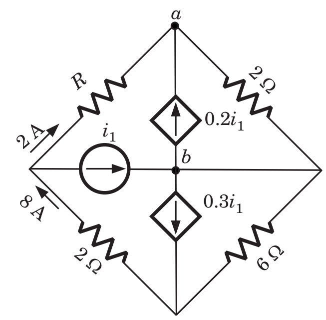


Fig. P.1.1.28

(A) 15.4 V (B) 2.6 V

(C) –2.6 V (D) 15.4 V

29. In the circuit of fig. P.1.1.29 power is delivered by

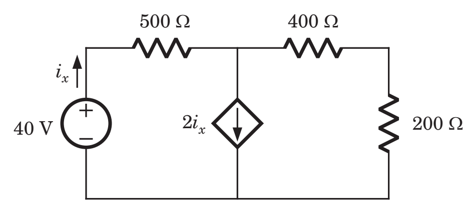


Fig. P.1.1.29

(A) dependent source of 192 W

(B) dependent source of 368 W

(C) independent source of 16 W

(D) independent source of 40 W

30. The dependent source in fig. P.1.1.30

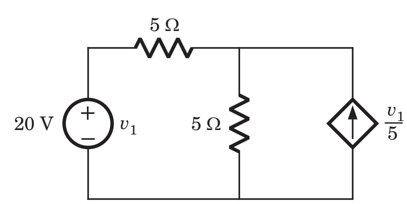


Fig. P.1.1.30

(A) delivers 80 W (B) delivers 40 W

(C) absorbs 40 W (D) absorbs 80 W

31. In the circuit of fig. P.1.1.31 dependent source

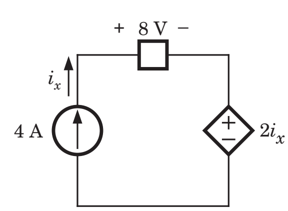


Fig. P.1.1.31

(A) supplies 16 W (B) absorbs 16 W

(C) supplies 32 W (D) absorbs 32 W

32. A capacitor is charged by a constant current of 2 mA and results in a voltage increase of 12 V in a 10 sec interval. The value of capacitance is

(A) 0.75 mF (B) 1.33 mF

(C) 0.6 mF (D) 1.67 mF

33. The energy required to charge a 10 μF capacitor to 100 V is

(A) 0.10 J (B) 0.05 J

(C) 5×10^{-9} J (D) 10×10^{-9} J

34. The current in a 100 μF capacitor is shown in fig. P.1.1.34. If capacitor is initially uncharged, then the waveform for the voltage across it is

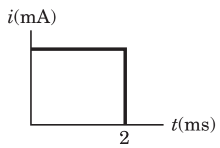
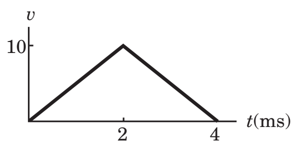
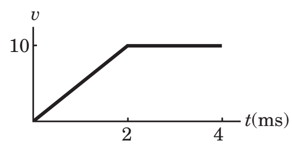


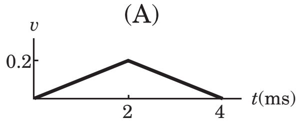
Fig. P. 1.1.34



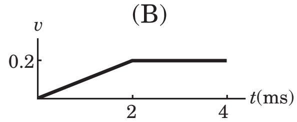
(A)



(B)



(C)



(D)

35. The voltage across a 100 μF capacitor is shown in fig. P.1.1.35. The waveform for the current in the capacitor is

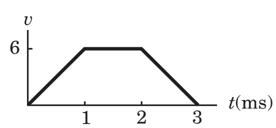
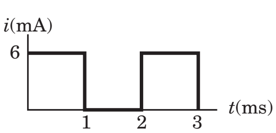
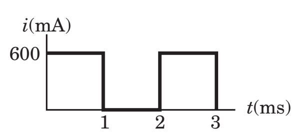


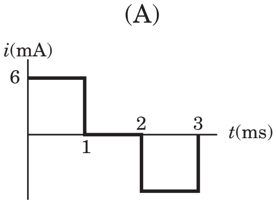
Fig. P.1.1.35



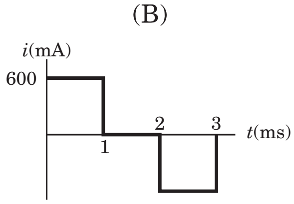
(A)



(B)



(C)



(D)

36. The waveform for the current in a 200 μF capacitor is shown in fig. P.1.1.36 The waveform for the capacitor voltage is

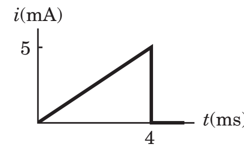
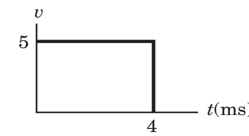
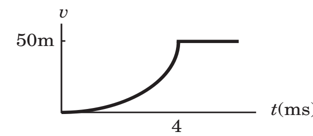


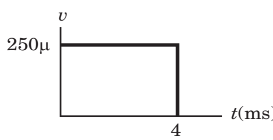
Fig. P. 1.1.36



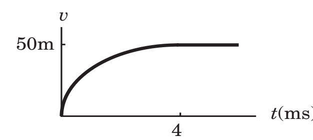
(A)



(B)



(C)



(D)

37. $C_{eq} = ?$

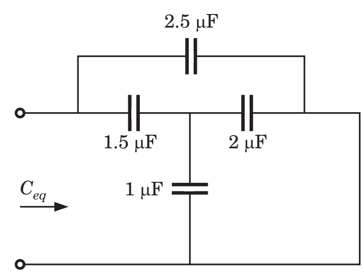


Fig. P.1.1.37

(A) 3.5 μF (B) 1.2 μF

(C) 2.4 μF (D) 2.6 μF

38. In the circuit shown in fig. P.1.1.38

$$i_{in}(t) = 300 \sin 20t \text{ mA, for } t \geq 0.$$

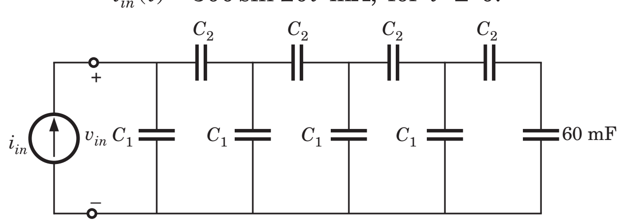


Fig. P. 1.1.38

Let $C_1 = 40$ μF and $C_2 = 30$ μF. All capacitors are initially uncharged. The $v_{in}(t)$ would be

(A) –0.25cos 20t V (B) 0.25cos 20t V

(C) –36cos 20t mV (D) 36cos 20t mV

SOLUTIONS

1. (C) $n = 10^{20}$, $Q = ne = e10^{20} = 16.02$ C

Charge on sphere will be positive.

2. (D) $\Delta Q = i \times \Delta t = 15000 \times 100\mu = 1.5$ C

3. (B) $i = \frac{dQ}{dt} = \frac{120}{60} = 2$ A

4. (B) $W = Qv = 360$ J

6. (A)

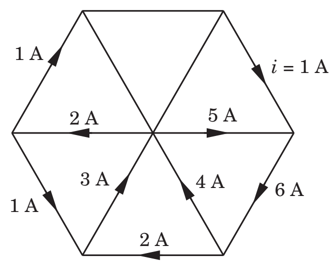


Fig. S 1.1.5

6. (A) In order for 600 C charge to be delivered to the 100 V source, the current must be anticlockwise.

$$i = \frac{dQ}{dt} = \frac{600}{60} = 10 \text{ A}$$

Applying KVL we get

$$v_1 + 60 - 100 = 10 \times 20 \text{ or } v_1 = 240 \text{ V}$$

7. (A) Going from 10 V to 0 V

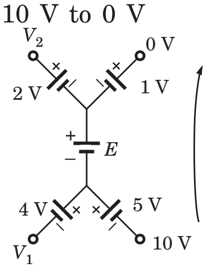


Fig. S 1.1.7

$10 + 5 + E + 1 = 0$ or $E = -16$ V

8. (D) $100 = 65 + v_2 \Rightarrow v_2 = 35$ V

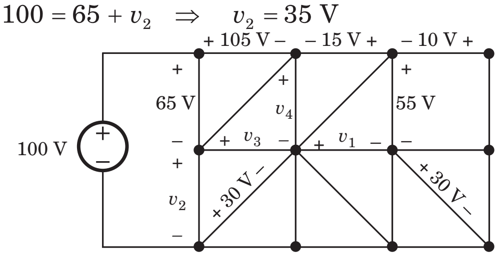


Fig. S 1.1.8

$v_3 - 30 = v_2 \Rightarrow v_3 = 65$ V

$105 + v_4 - v_3 - 65 = 0 \Rightarrow v_4 = 25$ V

$v_4 + 15 - 55 + v_1 = 0 \Rightarrow v_1 = 15$ V

9. (B) Voltage is constant because of 15 V source.

10. (C) Voltage across 60 Ω resistor = 30 V

Current $= \frac{30}{60} = 0.5$ A

Voltage across R_1 is $= 70 - 20 = 50$ V

$R_1 = \frac{50}{0.5} = 100\ \Omega$

11. (C) The current i will be distributed in the cube branches symmetrically

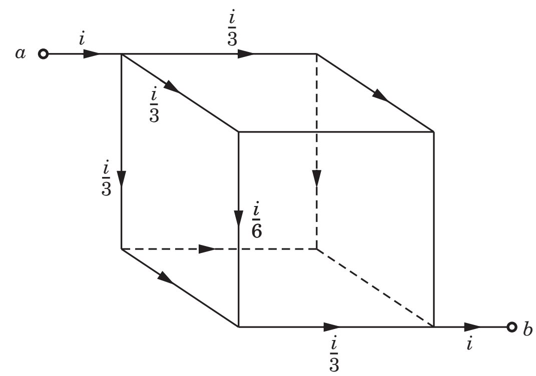


Fig. S. 1.1.11

$v_{ab} = \frac{6i}{3} + \frac{6i}{6} + \frac{6i}{3} = 5i,$

$R_{eq} = \frac{v_{ab}}{i} = 5\ \Omega$

12. (C) If we go from +side of 1 kΩ through 7 V, 6 V and 5 V, we get $v_1 = 7 + 6 - 5 = 8$ V

13. (D) It is not possible to determine the voltage across 1 A source.

14. (D) $R_{eq} = 5 + \frac{10(R_{eq} + 5)}{10 + 5 + R_{eq}}$

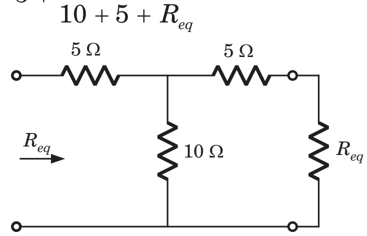


Fig. S 1.1.14

$\Rightarrow R_{eq}^2 + 15R_{eq} = 5R_{eq} + 75 + 10R_{eq} + 50$

$\Rightarrow R_{eq} = \sqrt{125} = 11.18\ \Omega$

$$\frac{v_o - 20}{5} + \frac{v_o}{5} = \frac{20}{5} \quad \Rightarrow \quad v_o = 20 \text{ V}$$

Power is $P = v_o \times \frac{v_1}{5} = 20 \times \frac{20}{5} = 80$ W

31. (D) Power $P = vi = 2i_x \times i_x = 2i_x^2$

$i_x = 4$ A, $P = 32$ W (absorb)

32. (D) $v_{t2} - v_{t1} = \frac{1}{C}\int_{t1}^{t2} idt \quad \Rightarrow \quad 12 = \frac{1}{C} 2\text{m}(t_2 - t_1)$

$\Rightarrow \quad 12C = 2\text{m} \times 10 \quad \Rightarrow \quad C = 1.67$ mF

33. (B) $E = \frac{1}{2} Cv^2 = 5 \times 10^{-6} \times 100^2 = 0.05$ J

34. (D) $v_c = \frac{1}{c}\int_0^{2\text{m}} idt = \frac{10 \times 10^{-3}}{100 \times 10^{-6}}(2 \times 10^{-3}) = 0.2$ V

This 0.2 V increases linearly from 0 to 0.2 V. Then current is zero. So capacitor hold this voltage.

35. (D) $i = C\frac{dv}{dt}$

For $0 < t < 1$, $C\frac{dv}{dt} = 100 \times 10^{-6} \times \frac{6 - 0}{10^{-3} - 0} = 600$ mA

For 1 ms $< t <$ 2 ms,

$C\frac{dv}{dt} = 100 \times 10^{-6} \times \frac{0 - 6}{(3 - 2)\text{m}} = -600$ mA

36. (B) For $0 \le t \le 4$,

$v_C = \frac{1}{C}\int idt = \frac{1}{200 \times 10^{-6}}\int \frac{5\text{m}}{4\text{m}} tdt = 3125t^2$

At $t = 4$ ms, $v_c = 0.05$ V

It will be parabolic path. at $t = 0$ t-axis will be tangent.

37. (A) 2 μF is in parallel with 1 μF and this combination is in series with 1.5 μF. $C_1 = \frac{1.5\,(2 + 1)}{1.5 + 2 + 1} = 1$μF, C_1 is in parallel with 2.5 μF

$C_{eq} = 1 + 2.5 = 3.5$ μF

38. (A) $C_a = \frac{30 \times 60}{30 + 60} = 20$ mF, $C_b = \frac{30(20 + 40)}{30 + 20 + 0} = 20$ mF

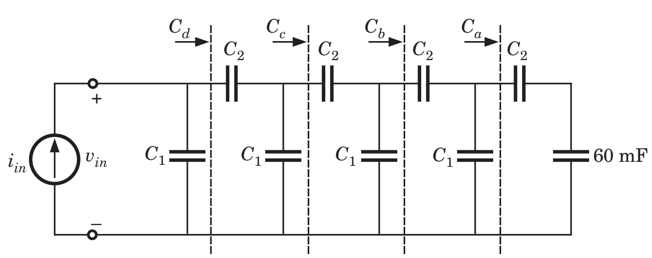


Fig. S 1.1.38

We can say $C_d = 20$ mF, $C_{eq} = 20 + 40 = 60$ mF

$v_C = \frac{1}{C}\int idt = \frac{1}{60\text{m}}\left(-\frac{300}{20}\cos 20t\right) \times 10^{-3} = -0.25\cos 20t$ V

39. (C) $i_{C1} = \frac{i_{in} C_1}{C_1 + C_2} = 0.8\sin 600t$ mA

At $t = 2$ ms, $i_{C1} = 0.75$ mA

40. (B) $v_{C1} = \frac{v_{in} C_2}{C_1 + C_2} = \frac{4v_{in}}{6 + 4} \quad \Rightarrow \quad \frac{v_{c1}}{v_{in}} = 0.4$

41. (D) $V = 2 + 3 + 5 = 10$, $Q = 1$ C, $C = \frac{Q}{V} = 0.1$ F

42. (A) $v_L = L\frac{di}{dt} \Rightarrow \quad 100\text{m} = L\frac{200\text{m}}{4\text{m}} \quad \Rightarrow \quad L = 2$ mH

43. (B) $v_L = L\frac{di}{dt} = 0.01 \times 2(377\cos 377t)$ V

$= 7.54\cos 377t$ V

44. (A) $i = \frac{1}{L}\int vdt = \frac{1}{0.01}\int 120\cos 3t\, dt = \frac{12000}{377}\sin 377t$

$P = vi = \frac{12000 \times 120}{377}(\sin 377t)(\cos 377t)$

$= 1910\sin 754t$ W

45. (D) $v_L = L\frac{di_L}{dt}$, $i_C = C\frac{dv_C}{dt}$

$v_C = 3v_L \Rightarrow \quad i_C = 3LC\frac{d^2 i_L}{dt} = -9.6\sin 4t$ A

46. (B) $v_L = L\frac{di_L}{dt}$

For $2 < t \le 4$, $\quad v_L = (0.05)\left(\frac{-100 - 0}{2}\right) = -2.5$ V

For $4 < t \le 8$, $\quad v_L = (0.05)\left(\frac{100 + 100}{4}\right) = 2.5$ V

For $8 < t \le 10$, $\quad v_L = (0.05)\left(\frac{0 - 100}{2}\right) = -2.5$ V

Thus (B) is correct option.

47. (C) Algebraic sum of the current entering or leaving a cutset is equal to 0.

$i_2 + i_4 + i_3 = 0 \quad \Rightarrow \quad \frac{6}{2} + \frac{16}{4} + i_3 = 0$

$i_3 = -7$ A, $\quad v_3 = -7 \times 3 = -21$ V

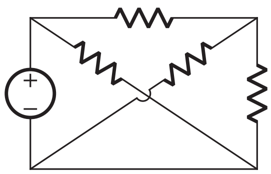
****

CHAPTER

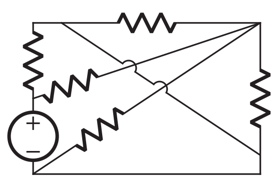
1.2

GRAPH THEORY

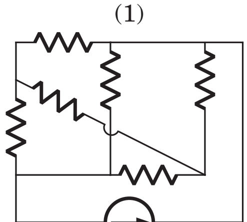
1. Consider the following circuits :



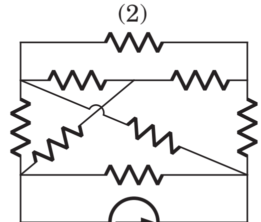
(1)



(2)



(3)



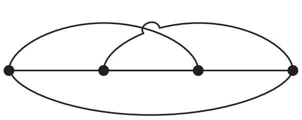
(4)

The planner circuits are

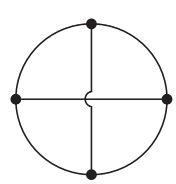
(A) 1 and 2 (B) 2 and 3

(C) 3 and 4 (D) 4 and 1

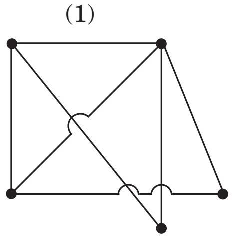
2. Consider the following graphs



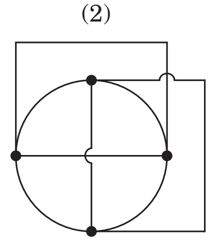
(1)



(2)



(3)



(4)

Non-planner graphs are

(A) 1 and 3 (B) 4 only

(C) 3 only (D) 3 and 4

3. A graph of an electrical network has 4 nodes and 7 branches. The number of links l, with respect to the chosen tree, would be

(A) 2 (B) 3

(C) 4 (D) 5

4. For the graph shown in fig. P.1.1.4 correct set is

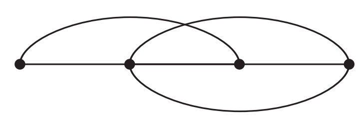


Fig. P.1.1.4

	Node	Branch	Twigs	Link
(A)	4	6	4	2
(B)	4	6	3	3
(C)	5	6	4	2
(D)	5	5	4	1

5. A tree of the graph shown in fig. P.1.2.5 is

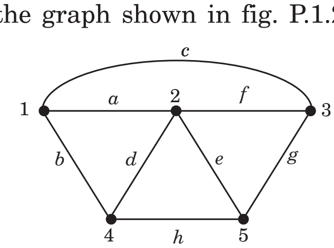


Fig. P.1.2.5

(A) $a\ d\ e\ h$ (B) $a\ c\ f\ h$

(C) $a\ f\ h\ g$ (D) $a\ e\ f\ g$

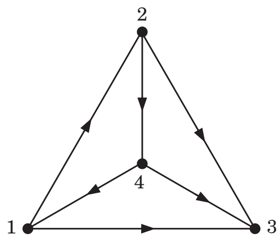
(A) $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

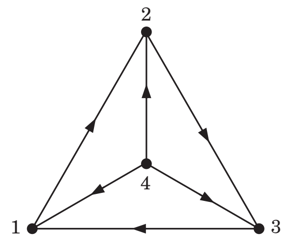
13. The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

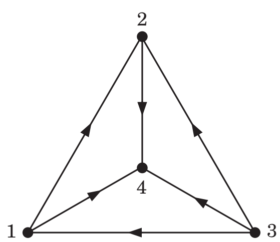
The graph is



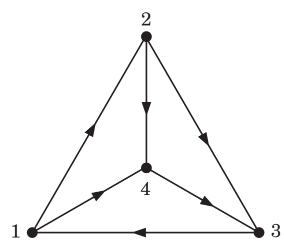
(A)



(B)



(C)

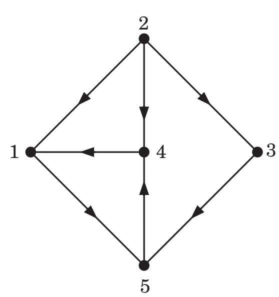


(D)

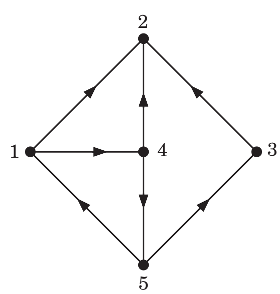
14. The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

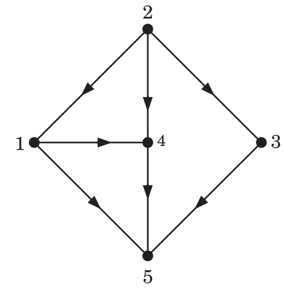
The graph is



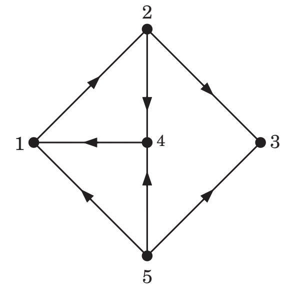
(A)



(B)



(C)

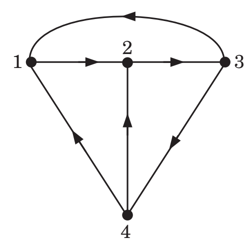


(D)

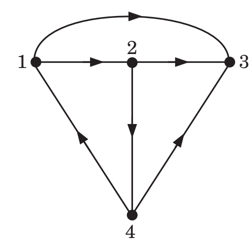
15. The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

The graph is



(A)



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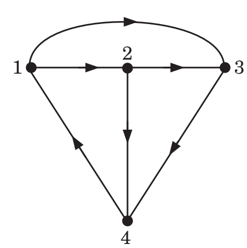
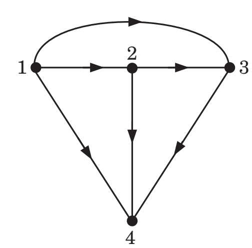


(C)



(D)

16. The graph of a network is shown in fig. P.1.1.16. The number of possible tree are

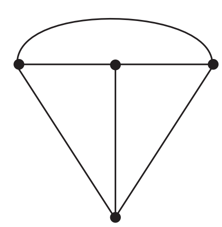


Fig. P.1.1.16

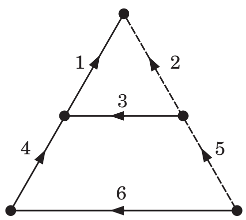
(A) 8 (B) 12

(C) 16 (D) 20

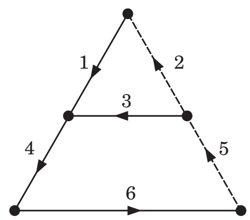
22. The fundamental cut-set matrix of a graph is

$$\mathbf{Q}_F = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

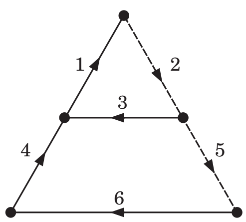
The oriented graph of the network is



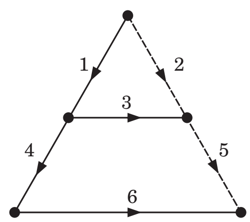
(A)



(B)



(C)



(D)

23. A graph is shown in fig. P.1.2.23 in which twigs are solid line and links are dotted line. For this chosen tree fundamental set matrix is given below.

$$\mathbf{B}_F = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

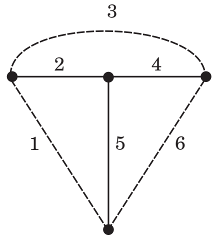
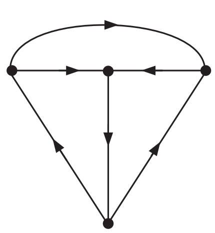
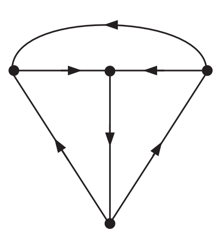


Fig. P. 1.2.23

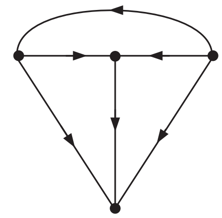
The oriented graph will be



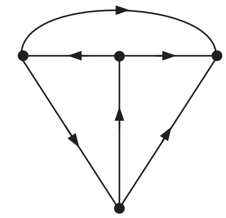
(A)



(B)



(C)



(D)

24. A graph is shown in fig. P.1.2.24 in which twigs are solid line and links are dotted line. For this tree fundamental loop matrix is given as below

$$\mathbf{B}_F = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

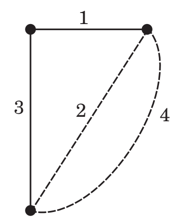
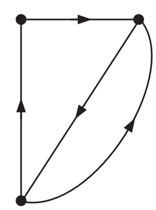
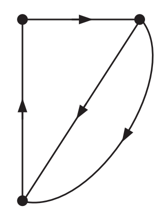


Fig. P.1.2.24

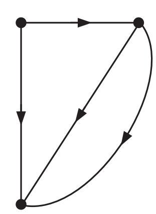
The oriented graph will be



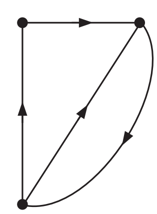
(A)



(B)



(C)



(D)

25. Consider the graph shown in fig. P.1.2.25 in which twigs are solid line and links are dotted line.

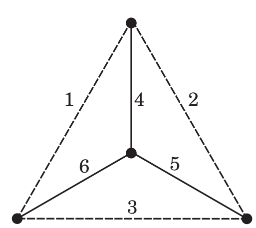
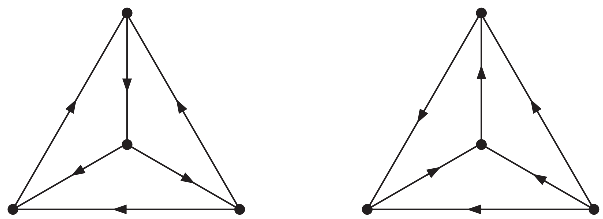


Fig. P. 1.2.25

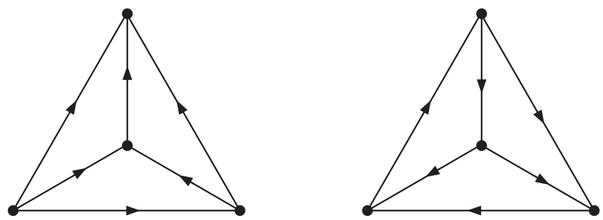
A fundamental loop matrix for this tree is given as below

$$\mathbf{B}_F = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

The oriented graph will be



(A) (B)



(C) (D)

26. In the graph shown in fig. P.1.2.26 solid lines are twigs and dotted line are link. The fundamental loop matrix is

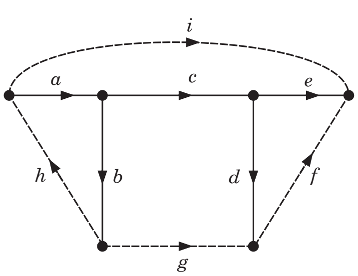


Fig. P.1.2.26

(A) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

27. Branch current and loop current relation are expressed in matrix form as

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where i_j represent branch current and I_k loop current. The number of independent node equation are

(A) 4 (B) 5

(C) 6 (D) 7

28. If the number of branch in a network is b, the number of nodes is n and the number of dependent loop is l, then the number of independent node equations will be

(A) $n + l - 1$ (B) $b - 1$

(C) $b - n + 1$ (D) $n - 1$

Statement for Q.29–30:

Branch current and loop current relation are expressed in matrix form as

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

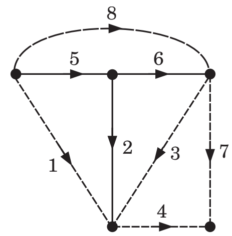
where i_j represent branch current and I_k loop current.

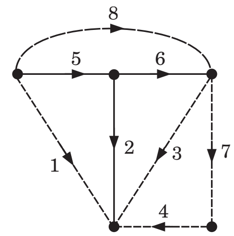
29. The rank of incidence matrix is

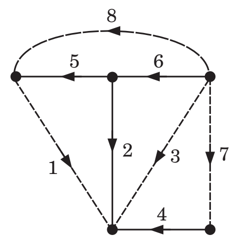
(A) 4 (B) 5

(C) 6 (D) 8

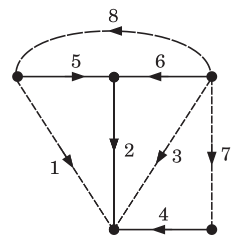
30. The directed graph will be







(C)



(D)

31. A network has 8 nodes and 5 independent loops. The number of branches in the network is

(A) 11 (B) 12

(C) 8 (D) 6

32. A branch has 6 node and 9 branch. The independent loops are

(A) 3 (B) 4

(C) 5 (D) 6

Statement for Q.33–34:

For a network branch voltage and node voltage relation are expressed in matrix form as follows:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

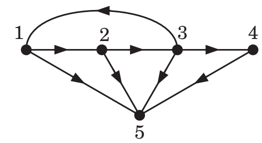
where v_i is the branch voltage and V_k is the node voltage with respect to datum node.

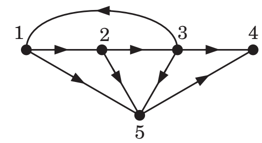
33. The independent mesh equation for this network are

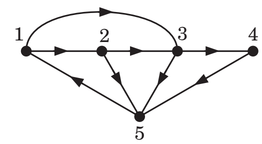
(A) 4 (B) 5

(C) 6 (D 7

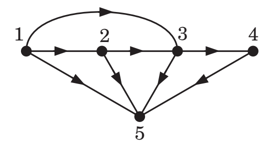
33. The oriented graph for this network is







(C)



(D)

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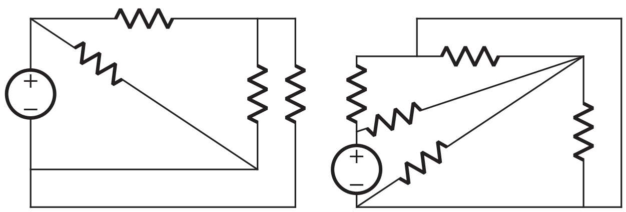
SOLUTIONS

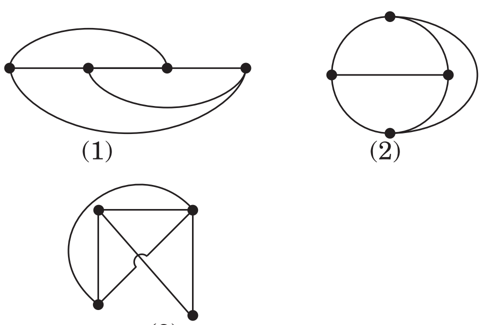
1. (A) The circuit 1 and 2 are redrawn as below. 3 and 4 can not be redrawn on a plane without crossing other branch.



(1) (2)

Fig. S1.2.1

2. (B) Other three circuits can be drawn on plane without crossing



(1) (2) (3)

Fig. S1.2.1

3. (C) $l = b - (n - 1) = 4$.

4. (B) There are 4 node and 6 branches.

$t = n - 1 = 3, \quad l = b - n + 1 = 3$

5. (C) From fig. it can be seen that $a\ f\ h\ g$ is a tree of given graph

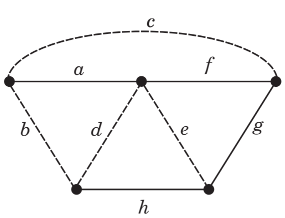


Fig. S 1.2.5

6. (B) From fig. it can be seen that $a\ d\ f$ is a tree.

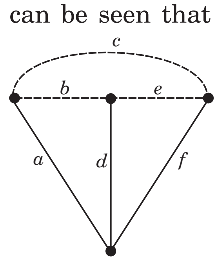


Fig. S. 1.2.6

7. (D) D is not a tree

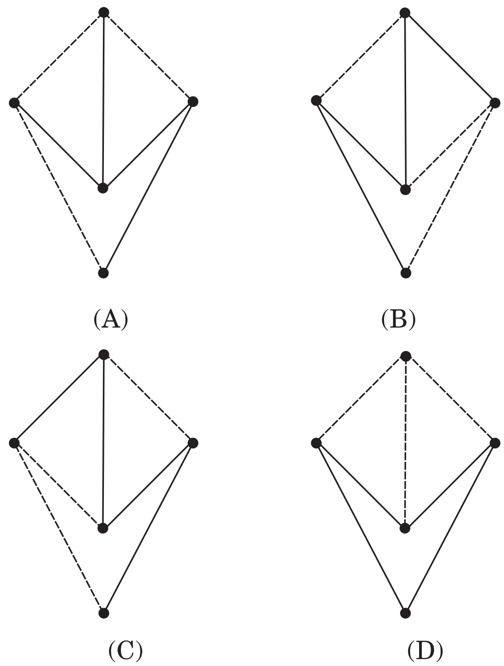


Fig. S .1.2.7

8. (D) it is obvious from the following figure that 1, 3, and 4 are tree

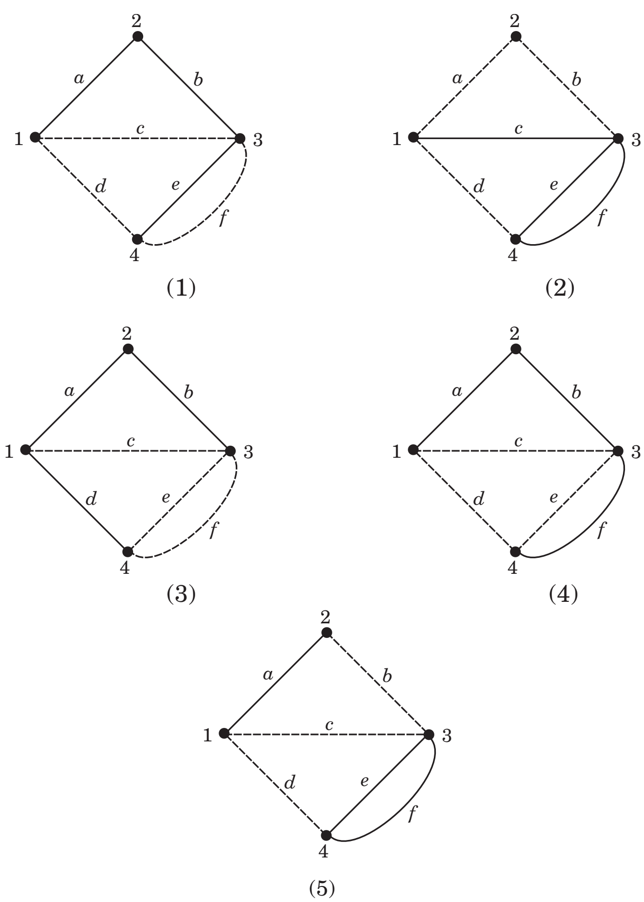


Fig. S. 1.2.8

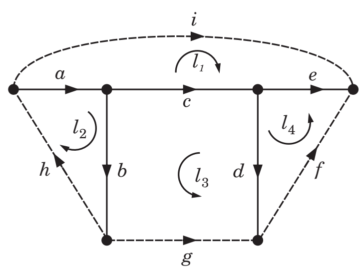


Fig. S 1.2.26

This in similar to matrix in (A). Only place of rows has been changed.

27. (A) Number of branch =8

Number of link =4

Number of twigs $=8-4=4$

Number of twigs =number of independent node equation.

28. (D) The number of independent node equation are $n-1$.

29. (A) Number of branch $b=8$

Number of link $l=4$

Number of twigs $t=b-l=4$

rank of matrix $=n-1=t=4$

30. (B) We know the branch current and loop current are related as

$[i_b]=[B^T][I_L]$

So fundamental loop matrix is

$$\mathbf{B}_f=\begin{bmatrix}0 & -1 & 0 & 1 & 0 & 1 & 1 & 0\\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0\\ 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1\end{bmatrix}$$

f-loop 1 include branch (2, 4, 6, 7) and direction of branch–2 is opposite to other (B only).

31. (B) Independent loops =link

$l=b-(n-1)$

$\Rightarrow \quad 5=b-7,\ b=12$

32. (B) Independent loop =link

$l=b-(n-1)=4$

33. (A) There are 8 branches and $4+1=5$ node

Number of link $=8-5+1=4$

So independent mesh equation =Number of link.

34. (D) We know that $[v_b]=A_r^T[V_n]$

So reduced incidence matrix is

$$\mathbf{A}_r=\begin{bmatrix}1 & 0 & 0 & 0 & 1 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1\\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0\end{bmatrix}$$

At node-1, three branch leaves so the only option is (D).

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CHAPTER

1.3

METHODS OF ANALYSIS

1. $v_1 = ?$

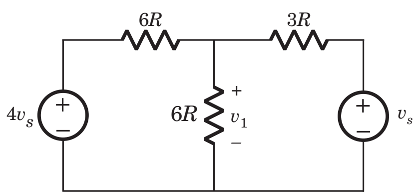

Fig. P1.3.1

(A) $0.4v_s$ (B) $1.5v_s$

(C) $0.67v_s$ (D) $2.5v_s$

2. $v_a = ?$

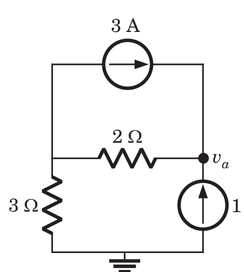

Fig. P1.3.2

(A) –11 V (B) 11 V

(C) 3 V (D) –3 V

3. $v_1 = ?$

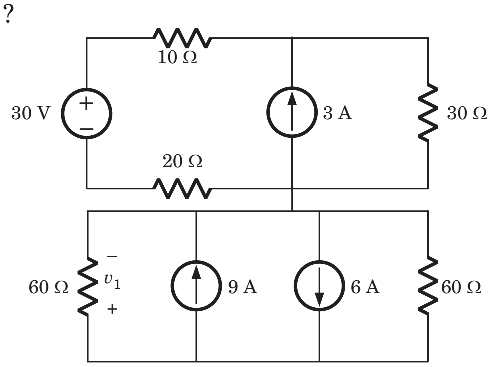

Fig. P1.3.3

(A) 120 V (B) –120 V

(C) 90 V (D) –90 V

4. $v_a = ?$

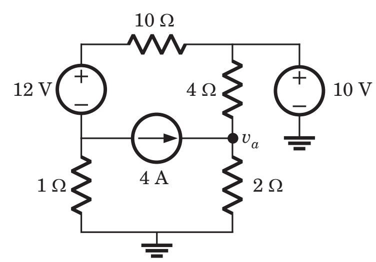

Fig. P1.3.4

(A) 4.33 V (B) 4.09 V

(C) 8.67 V (D) 8.18 V

5. $v_2 = ?$

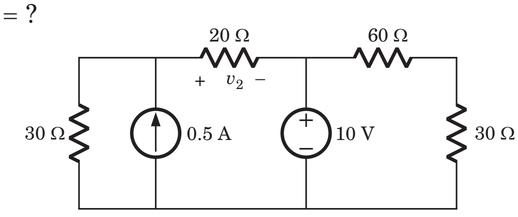

Fig. P1.3.5

(A) 0.5 V (B) 1.0 V

(C) 1.5 V (D) 2.0 V

6. $i_b = ?$

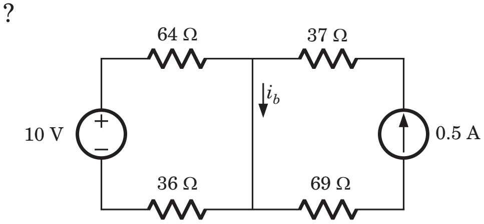

Fig. P1.3.6

(A) 0.6 A (B) 0.5 A

(C) 0.4 A (D) 0.3 A

7. $i_1 = ?$

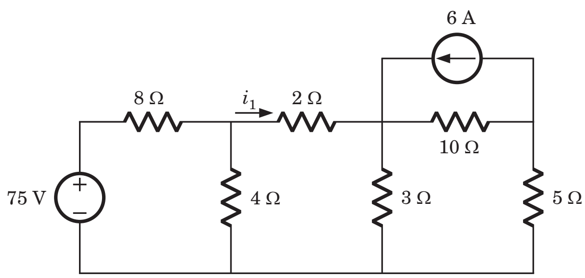

Fig. P1.3.7

(A) 3.3 A (B) 2.1 A

(C) 1.7 A (D) 1.1 A

8. $i_1 = ?$

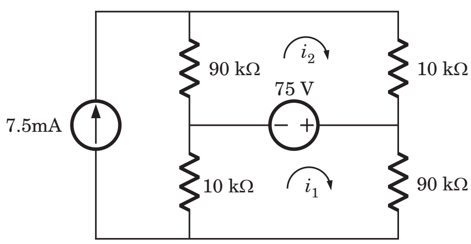

Fig. P1.3.8

(A) 1 mA (B) 1.5 mA

(C) 2 mA (D) 2.5 mA

9. $i_1 = ?$

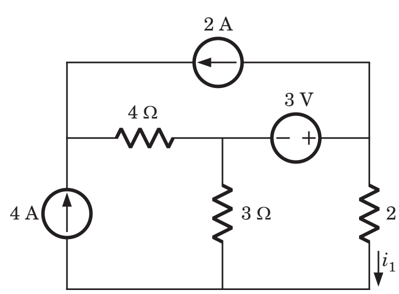

Fig. P1.3.9

(A) 4 A (B) 3 A

(C) 6 A (D) 5 A

10. $i_1 = ?$

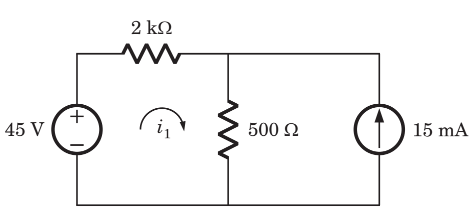

Fig. P1.3.10

(A) 20 mA (B) 15 mA

(C) 10 mA (D) 5 mA

11. $i_1 = ?$

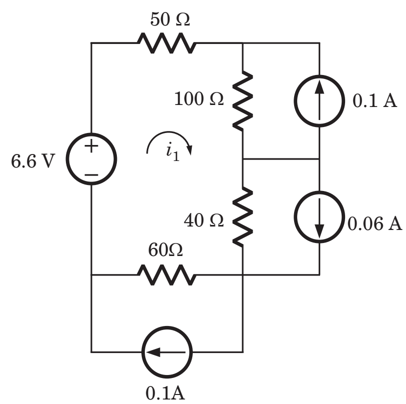

Fig. P1.3.11

(A) 0.01 A (B) –0.01 A

(C) 0.03 A (D) 0.02 A

12. The value of the current measured by the ammeter in Fig. P1.3.12 is

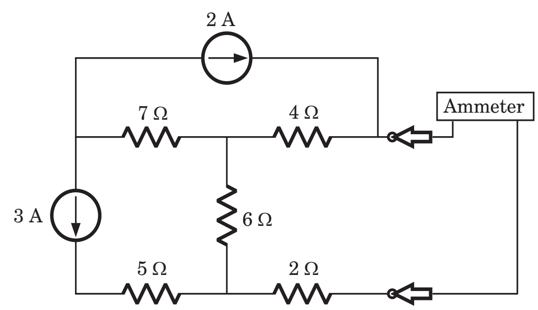

Fig. P1.3.12

(A) $\frac{2}{3}$ A (B) $\frac{5}{3}$ A

(C) $-\frac{5}{6}$ A (D) $\frac{2}{9}$ A

13. $i_1 = ?$

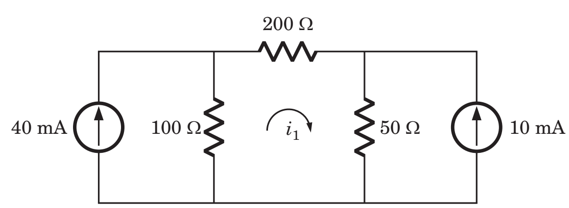

Fig. 1.3.13

(A) 10 mA (B) –10 mA

(C) 0.4 mA (D) –0.4 mA

14. The values of node voltage are $v_a = 12$ V, $v_b = 9.88$ V and $v_c = 5.29$ V. The power supplied by the voltage source is

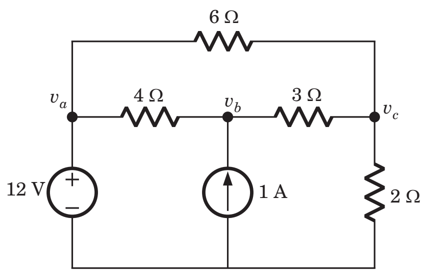

Fig. 1.3.14

(A) 19.8 W (B) 27.3 W

(C) 46.9 W (D) 54.6 W

15. i_1 , i_2 , i_3 = ?

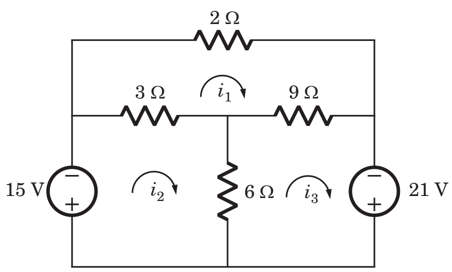

Fig. P1.3.15

(A) 3 A, 2 A, and 4 A (B) 3 A, 3 A, and 8 A

(C) 1 A, 3 A, and 4 A (D) 1 A, 2 A, and 8 A

16. v_o = ?

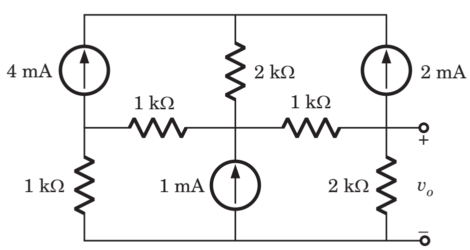

Fig. P1.3.16

(A) $\frac{6}{5}$ V (B) $\frac{8}{5}$ V

(C) $\frac{6}{7}$ V (D) $\frac{5}{7}$ V

17. The mesh current equation for the circuit in Fig. P1.3.17 are

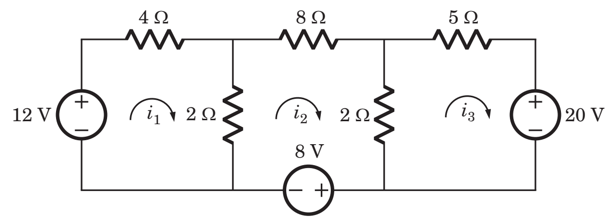

Fig. 1.3.17

(A) $\begin{bmatrix} 4 & -2 & 0 \\ -2 & 8 & -2 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ 20 \end{bmatrix}$

(B) $\begin{bmatrix} 6 & -2 & 0 \\ 2 & -12 & 2 \\ 0 & 2 & -7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 20 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & -2 & 0 \\ -2 & 12 & -2 \\ 0 & -2 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 20 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & -2 & 0 \\ 2 & -8 & 2 \\ 0 & 2 & -5 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 20 \end{bmatrix}$

18. For the circuit shown in Fig. P1.3.18 the mesh equation are

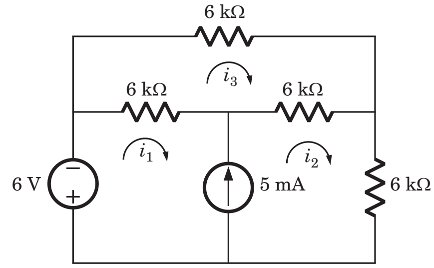

Fig. 1.3.18

(A) $\begin{bmatrix} 6k & -12k & -12k \\ -6k & 6k & -18k \\ -1k & -1k & 0k \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$

(B) $\begin{bmatrix} 6k & 12k & -12k \\ -6k & -6k & 18k \\ -1k & 1k & 0k \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$

(C) $\begin{bmatrix} -6k & -12k & 12k \\ 6k & -6k & 18k \\ 1k & 1k & 0k \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$

(D) $\begin{bmatrix} -6k & 12k & -12k \\ -6k & 6k & -18k \\ -1k & 1k & 0k \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$

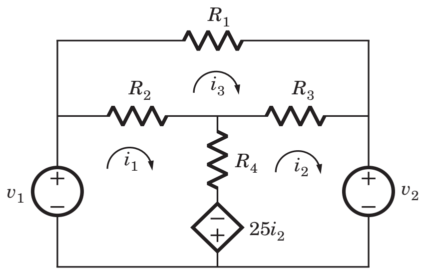

Fig. P1.3.25

The value of R_4 is

(A) 40 (B) 15

(C) 5 (D) 20

26. v_a = ?

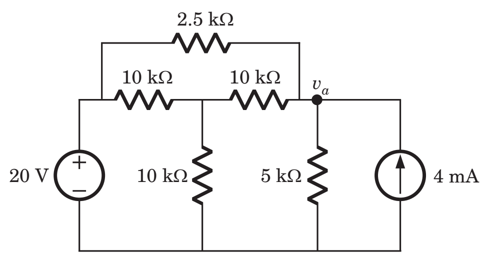

Fig. P1.3.26

(A) 26 V (B) 19 V

(C) 13 V (D) 18 V

27. v = ?

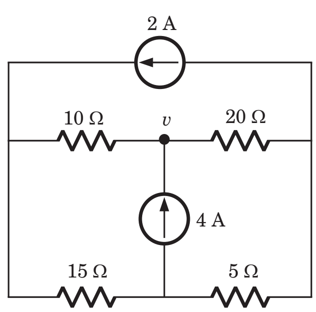

Fig. P.3.1.27

(A) 60 V (B) –60 V

(C) 30 V (D) –30 V

28. i_1 = ?

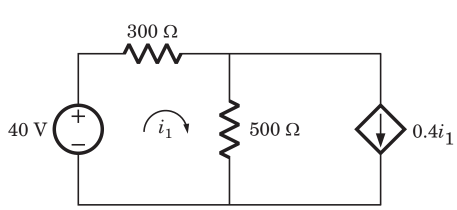

Fig. P1.3.28

(A) 66.67 mA (B) 46.24 mA

(C) 23.12 mA (D) 33.33 mA

29. v_a = ?

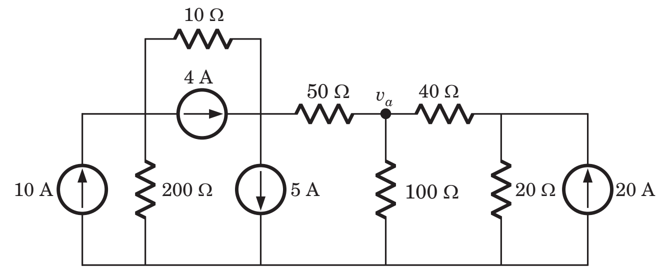

Fig. P1.3.29

(A) 342 V (B) 171 V

(C) 198 V (D) 396 V

30. i_a = ?

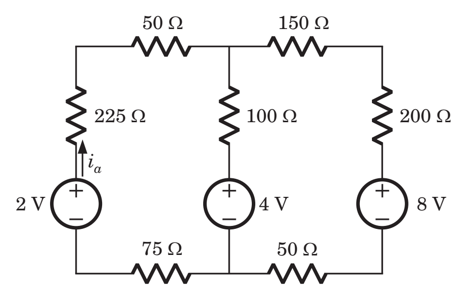

Fig. P1.3.30

(A) 14 mA (B) –6.5 mA

(C) 7 mA (D) –21 mA

31. v_2 = ?

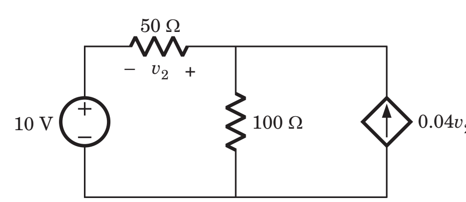

Fig. P1.3.31

(A) 5 V (B) 75 V

(C) 3 V (D) 10 V

32. i_1 = ?

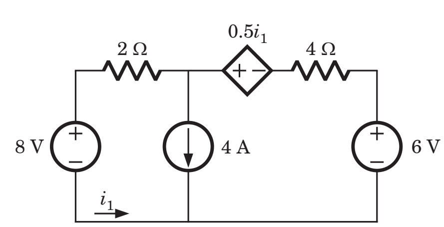

Fig. P1.3.32

(A) –1.636 A (B) –3.273 A

(C) –2.314 A (D) –4.628 A

33. $v_x = ?$

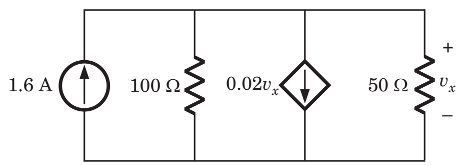

Fig. P1.3.33

(A) 32 V (B) –32 V

(C) 12 V (D) –12 V

34. $i_b = ?$

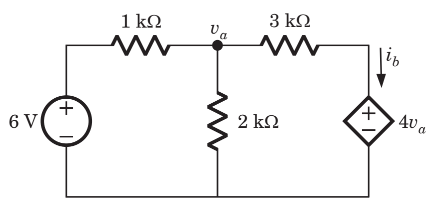

Fig. P1.3.34

(A) 4 mA (B) –4 mA

(C) 12 mA (D) –12 mA

35. $v_b = ?$

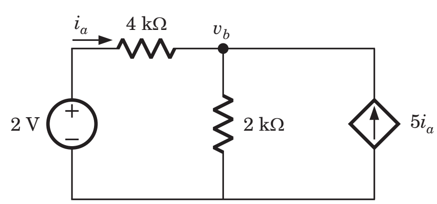

Fig. P1.3.35

(A) 1 V (B) 1.5 V

(C) 4 V (D) 6 V

36. $v_x = ?$

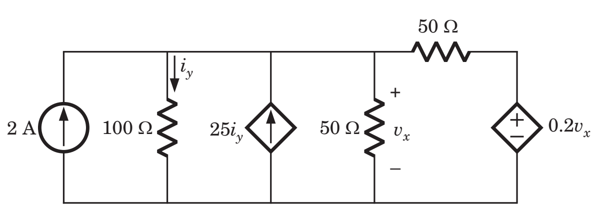

Fig. P1.3.36

(A) –3 V (B) 3 V

(C) 10 V (D) –10 V

37. $v_a = ?$

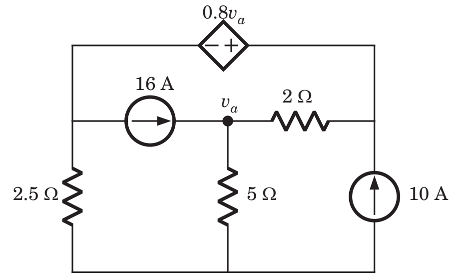

Fig. P1.3.37

(A) 25.91 V (B) –25.91 V

(C) 51.82 V (D) –51.82 V

38. For the circuit of Fig. P1.3.38 the value of v_s, that will result in $v_1 = 0$, is

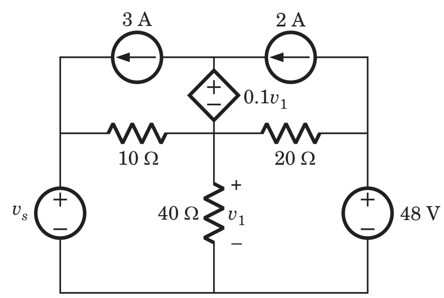

Fig. P1.3.38

(A) 28 V (B) –28 V

(C) 14 V (D) –14 V

39. i_1, $i_2 = ?$

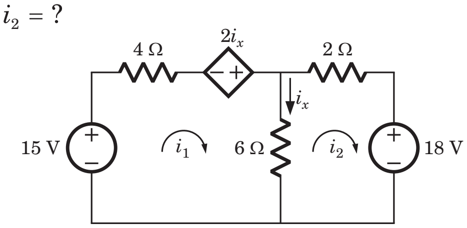

Fig. P1.3.39

(A) 2.6 A, 1.4 A (B) 2.6 A, –1.4 A

(C) 1.6 A, 1.35 A (D) 1.2 A, –1.35 A

40. $v_1 = ?$

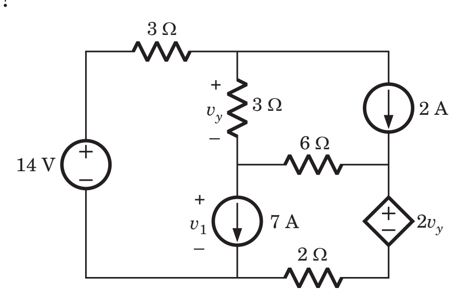

Fig. P1.3.40

(A) 10 V (B) –10 V

(C) 7 V (D) –7 V

41. $v_x = ?$

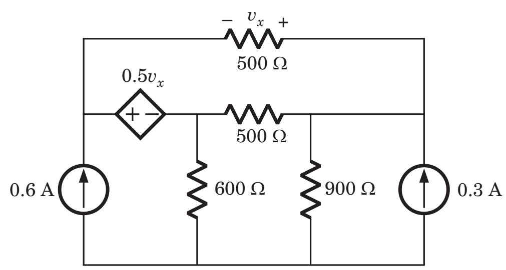

Fig. P1.3.41

(A) 9 V (B) –9 V

(C) 10 V (D) –10 V

42. The power being dissipated in the 2 Ω resistor in the circuit of Fig. P1.3.42 is

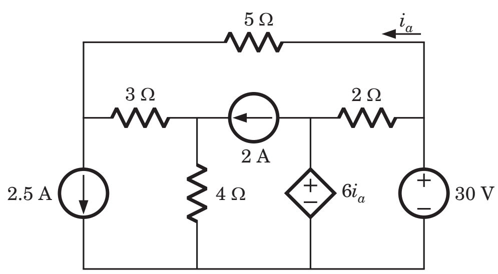

Fig. P1.3.42

(A) 76.4 W (B) 305.6 W

(C) 52.5 W (D) 210.0 W

43. $i_1 = ?$

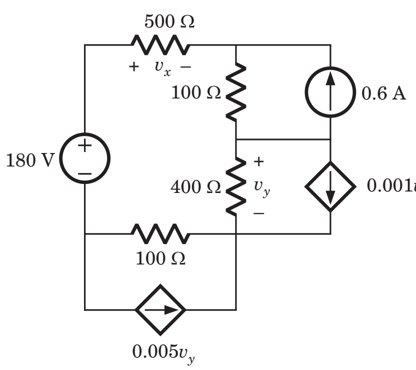

Fig. P1.3.43

(A) 0.12 A (B) 0.24 A

(C) 0.36 A (D) 0.48 A

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SOLUTIONS

1. (B) Applying the nodal analysis

$$v_1 = \frac{\frac{4v_s}{6R} + \frac{v_s}{3R}}{\frac{1}{6R} + \frac{1}{3R} + \frac{1}{6R}} = 1.5v_s$$

2. (C) $v_a = 2(3+1) + 3(1) = 11$ V

3. (D) $-\frac{v_1}{60} + \frac{-v_1}{60} + 6 = 9 \quad \Rightarrow \quad v_1 = -90$ V

4. (C) $\frac{v_a - 10}{4} + \frac{v_a}{2} = 4 \quad \Rightarrow \quad v_a = 8.67$ V

5. (D) $\frac{v_2}{20} + \frac{v_2 + 10}{30} = 0.5 \quad \Rightarrow \quad v_2 = 2$ V

6. (B) Using Thevenin equivalent and source transform

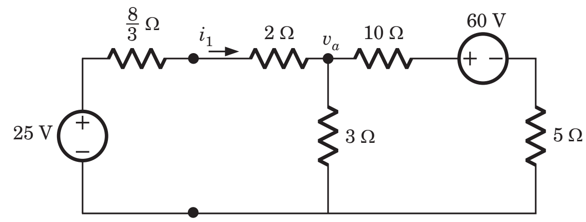

Fig. S.1.3.6

$$v_a = \frac{\frac{25}{\frac{8}{3}+2} + \frac{60}{15}}{\frac{3}{14} + \frac{1}{3} + \frac{1}{15}} = 15.23 \text{ V}$$

$$i_1 = \frac{25 - 15.23}{\frac{14}{3}} = 2.09 \text{ A}$$

7. (A) $i_b = \frac{10}{64 + 36} + 0.5 = 0.6$ A

8. (B) $75 = 90\text{k}i_1 + 10\text{k}(i_1 - 7.5\text{m})$

$150 = 100\text{k}i_1 \quad \Rightarrow \quad i_1 = 1.5$ mA

9. (B) $3 = 2i_1 + 3(i_1 - 4) \quad \Rightarrow \quad i_1 = 3$ A

10. (B) $45 = 2\text{k}i_1 + 500\,(i_1 + 15\text{m})$

$\Rightarrow \quad i_1 = 15$ mA

11. (D)

$6.6 = 50i_1 + 100(i_1 + 0.1) + 40(i_1 - 0.06) + 60(i_1 - 0.1)$

$i_1 = 0.02$ A

38. (D) If $v_1 = 0$, the dependent source is a short circuit

$$\frac{v_1}{40} + \frac{v_1 - v_s}{10} + \frac{v_1 - 48}{20} = 2 - 3 \quad \Rightarrow \quad v_1 = 0$$

$$-\frac{v_s}{10} - \frac{48}{20} = -1 \quad \Rightarrow \quad v_s = -14 \text{ V}$$

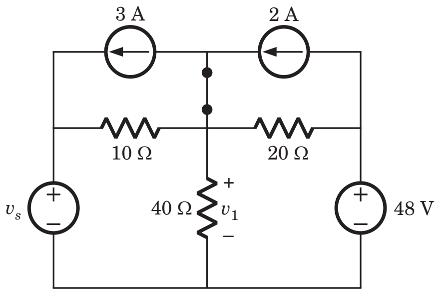

Fig. S1.3.38

39. (D) $i_x = i_1 - i_2$

$15 = 4i_1 - 2(i_1 - i_2) + 6(i_1 - i_2)$

$\Rightarrow \quad 8i_1 - 4i_2 = 15$ ...(i)

$-18 = 2i_2 + 6(i_2 - i_1)$

$\Rightarrow \quad 3i_1 - 4i_2 = 9$ ...(ii)

$i_1 = 1.2 \text{ A}, \; i_2 = -1.35 \text{ A}$

40. (B) $14 = 3i_1 + v_y + 6(i_1 - 2 - 7) + 2v_y + 2(i_1 - 7)$

$v_y = 3(i_1 - 2)$

$14 = 3i_1 + 9(i_1 - 2) + 6(i_1 - 9) + 2(i_1 - 7)$

$14 = 20i_1 - 18 - 54 - 14 \quad \Rightarrow \quad i_1 = 5 \text{ A}$

$v_1 = 6(5 - 2 - 7) + 2 \times 3(5 - 2) + 2(5 - 7) = -10 \text{ V}$

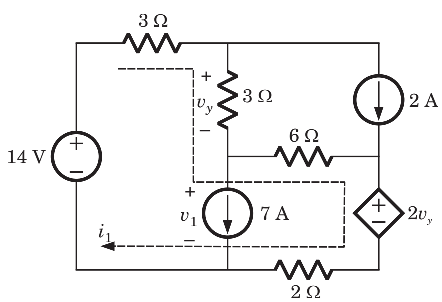

Fig. S1.3.40

41. (D) Let i_1 and i_2 be two loop current

$0.5v_x = 500i_1 + 500(i_1 - i_2),$

$v_x = -500i_1$

$\Rightarrow \quad 5i_1 - 2i_2 = 0$ ...(i)

$500(i_2 - i_1) + 900(i_2 + 0.3) + 600(i_2 - 0.6) = 0$

$-5i_1 + 20i_2 = 0.9$ ...(ii)

$i_1 = 20 \text{ mA}, \; v_x = -500 \times 20\text{m} = -10 \text{ V}$

42. (C) $30 = 5i_a + 3(i_a - 2.5) + 4(i_a - 2.5 + 2)$

$$i_a = \frac{30 + 7.5 + 2}{12} = 3.29 \text{ A} \quad \Rightarrow \quad 6i_a = 19.75 \text{ V}$$

voltage across 2 Ω resistor

$30 - 19.75 = 10.25 \text{ V},$

$$P = \frac{(10.25)^2}{2} = 52.53 \text{ W}$$

43. (A) $v_x = 500i_1$

$v_y = 400(i_1 - 0.001v_x) = 400\,(i_1 - 0.5i_1) = 200i_1$

$180 = 500i_1 + 100(i_1 - 0.6) + 200i_1 + 100(i_1 + 0.005v_y)$

$180 = 900i_1 - 60 + 100 \times 0.005 \times 200i_1$

$i_1 = 0.12 \text{ A}$

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CHAPTER

1.4

NETWORKS THEOREM

1. v_{TH} , R_{TH} = ?

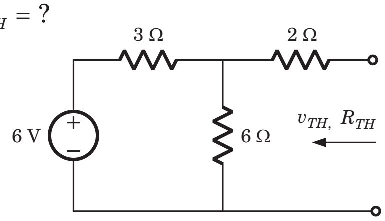


Fig. P.1.4.1

(A) 2 V, 4 Ω

(B) 4 V, 4 Ω

(C) 4 V, 5 Ω

(D) 2 V, 5 Ω

2. i_N , R_N = ?

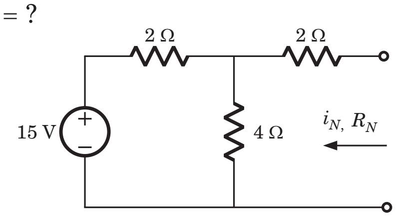


Fig. P.1.4.2

(A) 3 A, $\frac{10}{3}$ Ω

(B) 10 A, 4 Ω

(C) 1,5 A, 6 Ω

(D) 1.5 A, 4 Ω

3. v_{TH} , R_{TH} = ?

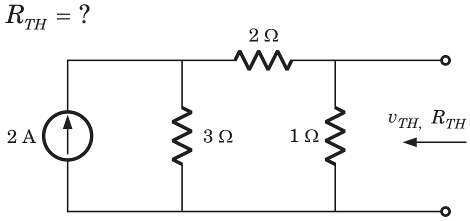


Fig. P.1.4.3

(A) –2 V, $\frac{6}{5}$ Ω

(B) 2 V, $\frac{5}{6}$ Ω

(C) 1 V, $\frac{5}{6}$ Ω

(D) –1 V, $\frac{6}{5}$ Ω

4. A simple equivalent circuit of the 2 terminal network shown in fig. P1.4.4 is

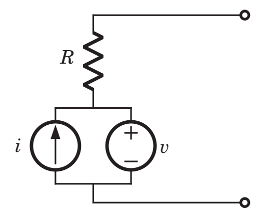
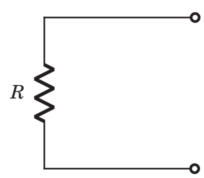
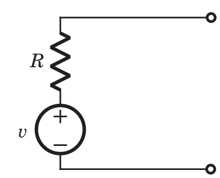


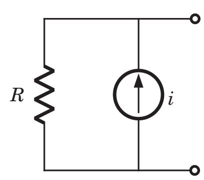
Fig. P.1.4.4



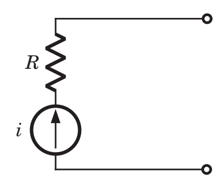
(A)



(B)



(C)



(D)

5. i_N , R_N = ?

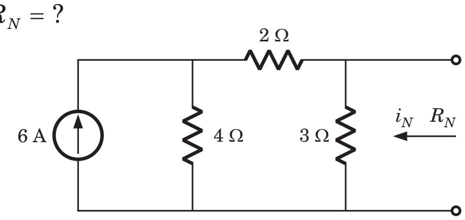


Fig. P.1.4.5

(A) 4 A, 3 Ω

(B) 2 A, 6 Ω

(C) 2 A, 9 Ω

(D) 4 A, 2 Ω

6. v_{TH} , R_{TH} = ?

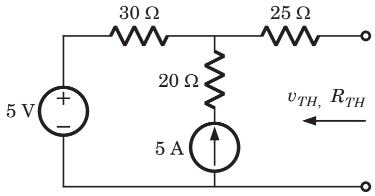


Fig. P.1.4.6

(A) –100 V, 75 Ω (B) 155 V, 55 Ω

(C) 155 V, 37 Ω (D) 145 V, 75 Ω

7. R_{TH} = ?

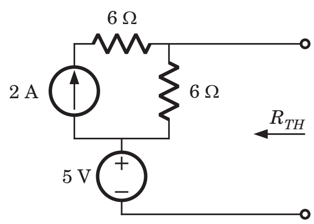


Fig. P.1.4.7

(A) 3 Ω (B) 12 Ω

(C) 6 Ω (D) ∞

8. The Thevenin impedance across the terminals *ab* of the network shown in fig. P.1.4.8 is

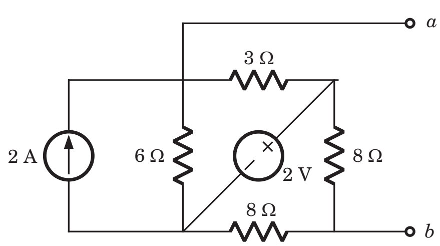


Fig. P.1.4.8

(A) 2 Ω (B) 6 Ω

(C) 6.16 Ω (D) $\frac{4}{3}$ Ω

9. For In the the circuit shown in fig. P.1.4.9 a network and its Thevenin and Norton equivalent are given

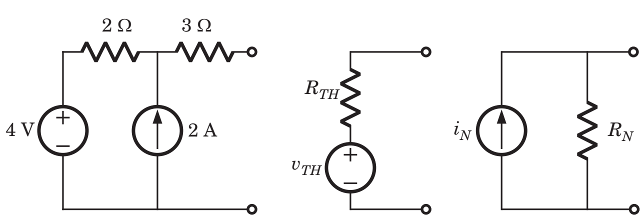


Fig. P.1.4.9

The value of the parameter are

	v_{TH}	R_{TH}	i_N	R_N
(A)	4 V	2 Ω	2 A	2 Ω
(B)	4 V	2 Ω	2 A	3 Ω
(C)	8 V	1.2 Ω	$\frac{30}{3}$ A	1.2 Ω
(D)	8 V	5 Ω	$\frac{8}{5}$ A	5 Ω

10. v_1 = ?

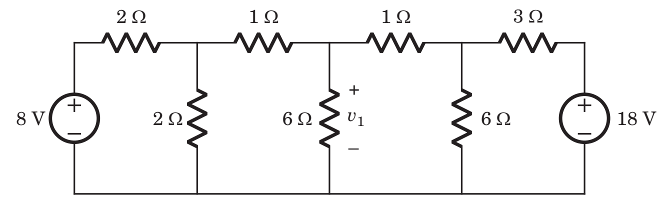


Fig. P.1.4.10

(A) 6 V (B) 7 V

(C) 8 V (D) 10 V

11. i_1 = ?

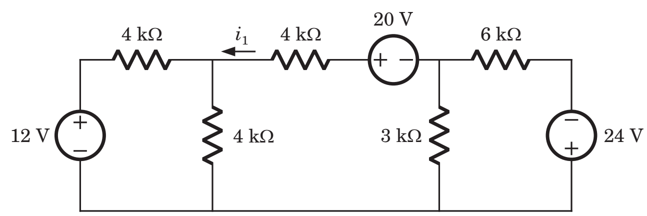


Fig. P.1.4.11

(A) 3 A (B) 0.75 mA

(C) 2 mA (D) 1.75 mA

Statement for Q.12–13:

A circuit is given in fig. P.1.4.12–13. Find the Thevenin equivalent as given in question..

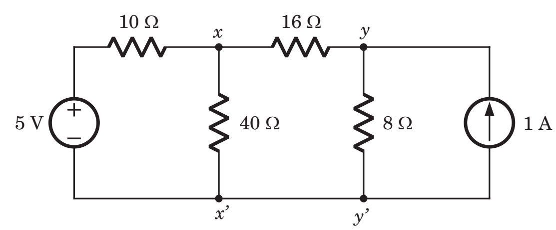


Fig. P.1.4.12–13

12. As viewed from terminal x and x' is

(A) 8 V, 6 Ω (B) 5 V, 6 Ω

(C) 5 V, 32 Ω (D) 8 V, 32 Ω

13. As viewed from terminal y and y' is

(A) 8 V, 32 Ω (B) 4 V, 32 Ω

(C) 5 V, 6 Ω (D) 7 V, 6 Ω

14. A practical DC current source provide 20 kW to a 50 Ω load and 20 kW to a 200 Ω load. The maximum power, that can drawn from it, is

(A) 22.5 kW (B) 45 kW

(C) 30.3 kW (D) 40 kW

Statement for Q.15–16:

In the circuit of fig. P.1.4.15–16 when $R = 0\,\Omega$, the current i_R equals 10 A.

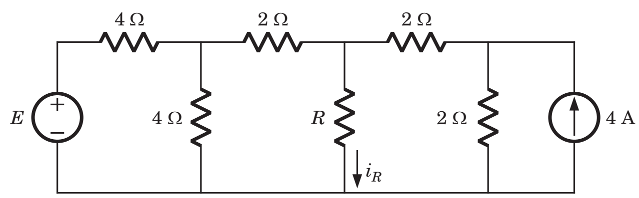


Fig. P.1.4.15–16.

15. The value of R, for which it absorbs maximum power, is

(A) 4 Ω (B) 3 Ω

(C) 2 Ω (D) None of the above

16. The maximum power will be

(A) 50 W (B) 100 W

(C) 200 W (D) value of E is required

17. Consider a 24 V battery of internal resistance $r = 4\,\Omega$ connected to a variable resistance R_L. The rate of heat dissipated in the resistor is maximum when the current drawn from the battery is i. The current drawn form the battery will be $i/2$ when R_L is equal to

(A) 2 Ω (B) 4 Ω

(C) 8 Ω (D) 12 Ω

18. i_N, $R_N = ?$

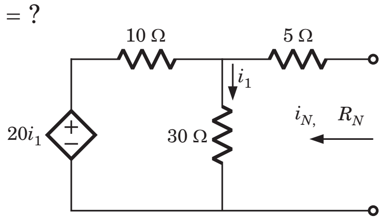


Fig. P.1.4.18

(A) 2 A, 20 Ω (B) 2 A, –20 Ω

(C) 0 A, 20 Ω (D) 0 A, –20 Ω

19. v_{TH}, $R_{TH} = ?$

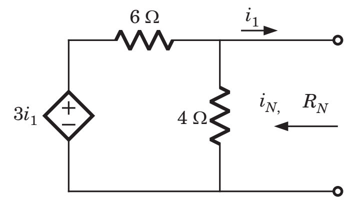


Fig. P1.4.19

(A) 0 Ω (B) 1.2 Ω

(C) 2.4 Ω (D) 3.6 Ω

20. v_{TH}, $R_{TH} = ?$

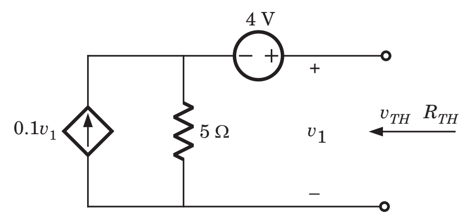


Fig. P.1.4.20

(A) 8 V, 5 Ω (B) 8 V, 10 Ω

(C) 4 V, 5 Ω (D) 4 V, 10 Ω

21. $R_{TH} = ?$

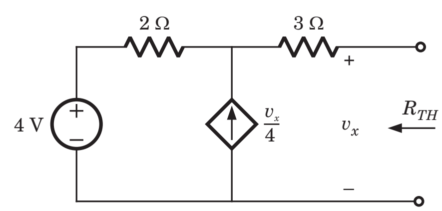


Fig. P.1.4.21

(A) 3 Ω (B) 1.2 Ω

(C) 5 Ω (D) 10 Ω

22. In the circuit shown in fig. P.1.4.22 the effective resistance faced by the voltage source is

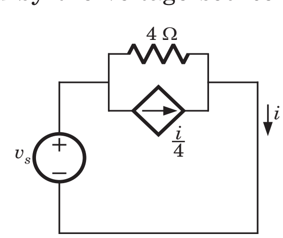


Fig. P.1.4.22

(A) 4 Ω (B) 3 Ω

(C) 2 Ω (D) 1 Ω

23. In the circuit of fig. P1.4.23 the value of R_{TH} at terminal ab is

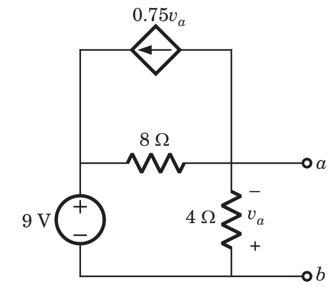


Fig. P.1.4.23

(A) –3 Ω (B) $\frac{9}{8}$ Ω

(C) $-\frac{8}{3}$ Ω (D) None of the above

24. R_{TH} = ?

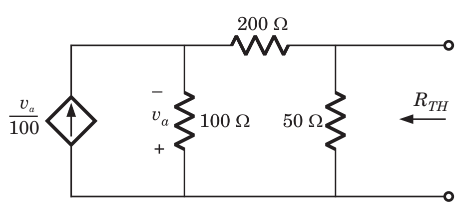


Fig. P.1.4.24

(A) ∞ (B) 0

(C) $\frac{3}{125}$ Ω (D) $\frac{125}{3}$ Ω

25. In the circuit of fig. P.1.4.25, the R_L will absorb maximum power if R_L is equal to

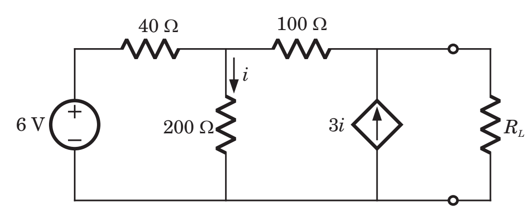


Fig. P.1.4.25

(A) $\frac{400}{3}$ Ω (B) $\frac{2}{9}$ kΩ

(C) $\frac{800}{3}$ Ω (D) $\frac{4}{9}$ kΩ

Statement for Q.26–27:

In the circuit shown in fig. P1.4.26–27 the maximum power transfer condition is met for the load R_L .

26. The value of R_L will be

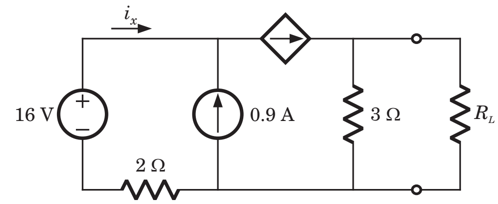


Fig. P.1.4.26–27

(A) 2 Ω (B) 3 Ω

(C) 1 Ω (D) None of the above

27. The maximum power is

(A) 0.75 W (B) 1.5 W

(C) 2.25 W (D) 1.125 W

28. R_{TH} = ?

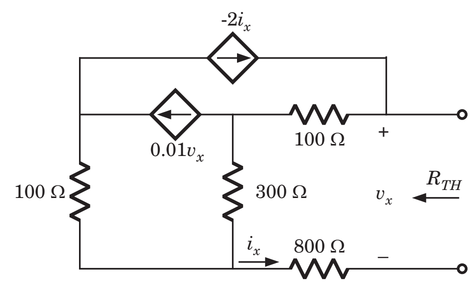


Fig. P.1.4.28

(A) 100 Ω (B) 136.4 Ω

(C) 200 Ω (D) 272.8 Ω

29. Consider the circuits shown in fig. P.1.4.29

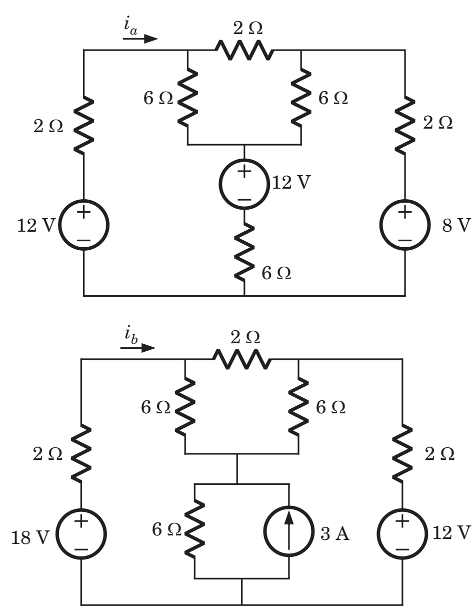


Fig. P.1.4.29a & b

The relation between i_a and i_b is

(A) $i_b = i_a + 6$ (B) $i_b = i_a + 2$

(C) $i_b = 1.5 i_a$ (D) $i_b = i_a$

30. $R_{eq} = ?$

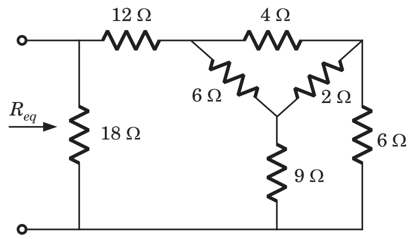


Fig. P.1.4.30

(A) 18 Ω (B) $\frac{72}{13}$ Ω

(C) $\frac{36}{13}$ Ω (D) 9 Ω

31. In the lattice network the value of R_L for the maximum power transfer to it is

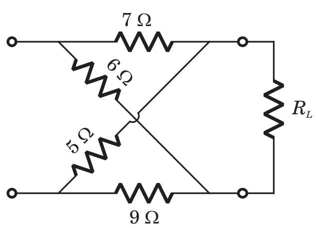


Fig. P.1.4.31

(A) 6.67 Ω (B) 9 Ω

(C) 6.52 Ω (D) 8 Ω

Statement for Q.32–33:

A circuit is shown in fig. P.1.4.32–33.

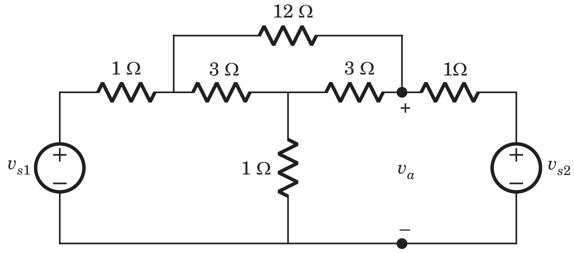


Fig. P.1.4.32–33

32. If $v_{s1} = v_{s2} = 6$ V then the value of v_a is

(A) 3 V (B) 4 V

(C) 6 V (D) 5 V

33. If $v_{s1} = 6$ V and $v_{s2} = -6$ V then the value of v_a is

(A) 4 V (B) –4 V

(C) 6 V (D) –6 V

34. A network N feeds a resistance R as shown in fig. P1.4.34. Let the power consumed by R be P. If an identical network is added as shown in figure, the power consumed by R will be

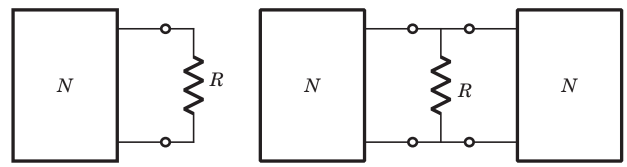


Fig. P.1.4.34

(A) equal to P (B) less than P

(C) between P and $4P$ (D) more than $4P$

35. A certain network consists of a large number of ideal linear resistors, one of which is R and two constant ideal source. The power consumed by R is P_1 when only the first source is active, and P_2 when only the second source is active. If both sources are active simultaneously, then the power consumed by R is

(A) $P_1 \pm P_2$ (B) $\sqrt{P_1} \pm \sqrt{P_2}$

(C) $(\sqrt{P_1} \pm \sqrt{P_2})^2$ (D) $(P_1 \pm P_2)^2$

36. A battery has a short-circuit current of 30 A and an open circuit voltage of 24 V. If the battery is connected to an electric bulb of resistance 2 Ω, the power dissipated by the bulb is

(A) 80 W (B) 1800 W

(C) 112.5 W (D) 228 W

37. The following results were obtained from measurements taken between the two terminal of a resistive network

Terminal voltage	12 V	0 V
Terminal current	0 A	1.5 A

The Thevenin resistance of the network is

(A) 16 Ω (B) 8 Ω

(C) 0 (D) ∞

38. A DC voltmeter with a sensitivity of 20 kΩ/V is used to find the Thevenin equivalent of a linear network. Reading on two scales are as follows

(a) 0 – 10 V scale : 4 V

(b) 0 –15 V scale : 5 V

The Thevenin voltage and the Thevenin resistance of the network is

(A) $\frac{16}{3}$ V, $\frac{200}{3}$ kΩ (B) $\frac{32}{3}$ V, $\frac{1}{15}$ MΩ

(C) 18 V, $\frac{2}{15}$ MΩ (D) 36 V, $\frac{200}{3}$ kΩ

39. Consider the network shown in fig. P.1.4.39.

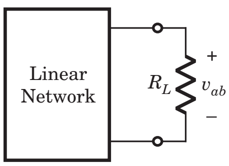


Fig. P.1.4.39

The power absorbed by load resistance R_L is shown in table :

R_L	10 kΩ	30 kΩ
P	3.6 MW	4.8 MW

The value of R_L , that would absorb maximum power, is

(A) 60 kΩ (B) 100 Ω

(C) 300 Ω (D) 30 kΩ

40. Measurement made on terminal ab of a circuit of fig.P.1.4.40 yield the current-voltage characteristics shown in fig. P.1.4.40. The Thevenin resistance is

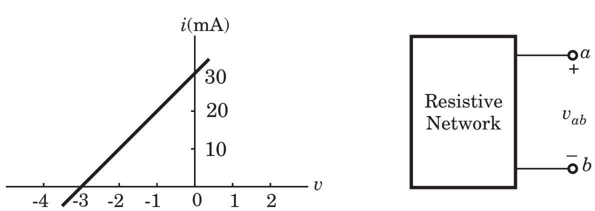


Fig. P.1.4.40

(A) 300 Ω (B) –300 Ω

(C) 100 Ω (D) –100 Ω

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SOLUTIONS

1. (B) $v_{TH} = \frac{(6)(6)}{3+6} = 4$ V,

$R_{TH} = (3 \| 6) + 2 = 4\,\Omega$

2. (A)

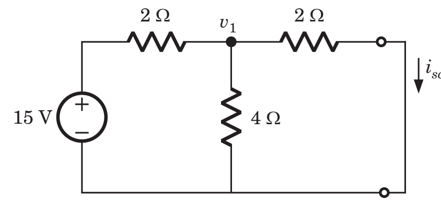

Fig. S.1.4.2

$R_N = 2 \| 4 + 2 = \frac{10}{3}\ \Omega,$

$$v_1 = \frac{\frac{15}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{4}} = 6 \text{ V}$$

$i_{sc} = i_N = \frac{v_1}{2} = 3$ A

3. (C) $v_{TH} = \frac{(2)(3)(1)}{3+3} = 1$ V,

$R_{TH} = 1 \| 5 = \frac{5}{6}\,\Omega$

4. (B) After killing all source equivalent resistance is R

Open circuit voltage $= v_1$

5. (D) The short circuit current across the terminal is

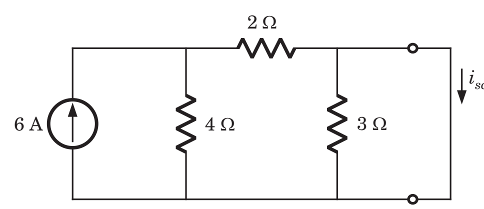

Fig. S1.4.5

$i_{sc} = \frac{6 \times 4}{4+2} = 4 \text{ A} = i_N,$

$R_N = 6 \| 3 = 2\,\Omega$

6. (B) For the calculation of R_{TH} if we kill the sources then 20 Ω resistance is inactive because 5 A source will be open circuit

$R_{TH} = 30 + 25 = 55\,\Omega,$

$v_{TH} = 5 + 5 \times 30 = 155$ V

7. (C) After killing the source, $R_{TH} = 6\,\Omega$

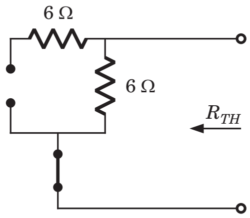

Fig. S.1.4.7

8. (B) After killing all source,

$R_{TH} = 3 \,||\, 6 + 8 \,||\, 8 = 6\,\Omega$

9. (D) $v_{oc} = 2 \times 2 + 4 = 8 \text{ V} = v_{TH}$

$R_{TH} = 2 + 3 = 5\,\Omega = R_N, \quad i_N = \dfrac{v_{TH}}{R_{TH}} = \dfrac{8}{5} \text{ A}$

10. (A) If we solve this circuit direct, we have to deal with three variable. But by simple manipulation variable can be reduced to one. By changing the LHS and RHS in Thevenin equivalent

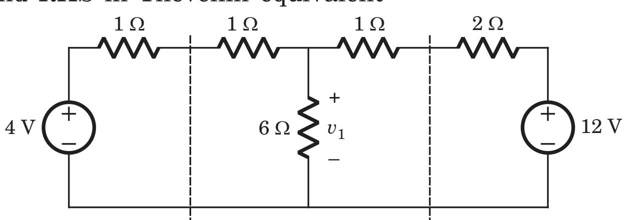

Fig. S1.4.10

$$v_1 = \frac{\dfrac{4}{1+1} + \dfrac{12}{1+2}}{\dfrac{1}{1+1} + \dfrac{1}{6} + \dfrac{1}{1+2}} = 6 \text{ V}$$

11. (B) If we solve this circuit direct, we have to deal with three variable. But by simple manipulation variable can be reduced to one. By changing the LHS and RHS in Thevenin equivalent

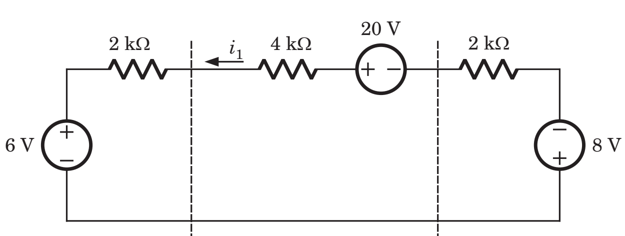

Fig. S1.4.11

$$i_1 = \frac{20 - 6 - 8}{2\text{k} + 4\text{k} + 2\text{k}} = 0.75 \text{ mA}$$

12. (B) We Thevenized the left side of xx' and source transformed right side of yy'

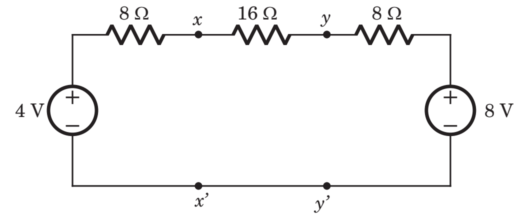

Fig. S1.4.12

$$v_{xx'} = v_{TH} = \frac{\dfrac{4}{8} + \dfrac{8}{24}}{\dfrac{1}{8} + \dfrac{1}{24}} = 5 \text{ V},$$

$R_{TH} = 8 \,||\, (16 + 8) = 6\,\Omega$

13. (D) Thevenin equivalent seen from terminal yy' is

$$v_{yy'} = v_{TH} = \frac{\dfrac{4}{24} + \dfrac{8}{8}}{\dfrac{1}{24} + \dfrac{1}{8}} = 7 \text{ V},$$

$R_{TH} = (8 + 16) \,||\, 8 = 6\,\Omega$

14. (A)

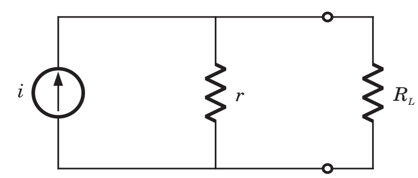

Fig. S1.4.14

$$\left(\frac{ir}{r+50}\right)^2 50 = 20\text{k}, \left(\frac{ir}{r+200}\right)^2 200 = 20\text{k}$$

$(r + 200)^2 = 4(r + 50)^2$

$\Rightarrow \quad r = 100\,\Omega$

$i = 30 \text{ A}, \quad P_{max} = \dfrac{(30)^2 \times 100}{4} = 22.5 \text{ kW}$

15. (C) Thevenized the circuit across R, $R_{TH} = 2\,\Omega$

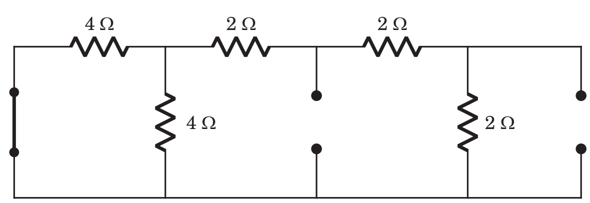

Fig. S1.4.15

16. (A) $i_{sc} = 10 \text{ A}, \; R_{TH} = 2\,\Omega,$

$$P_{\max} = \left(\frac{10}{2}\right)^2 \times 2 = 50 \text{ W}$$

Now in this circuit all straight-through connection have been cut as shown in fig. S1.4.32b

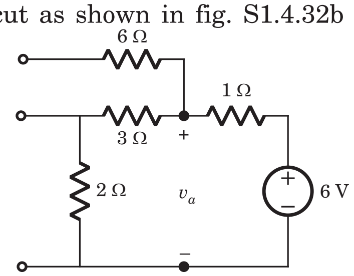

Fig. S.1.4.32b

$$v_a = \frac{6 \times (2+3)}{2+3+1} = 5 \text{ V}$$

33. (B) Since both source have opposite polarity, hence short circuit the all straight-through connection as shown in fig. S.1.4.33

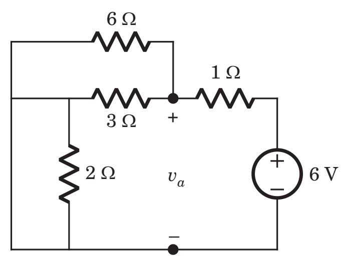

Fig. S1.4.33

$$v_a = -\frac{6 \times (6 \| 3)}{2+1} = -4 \text{ V}$$

34. (C) Let Thevenin equivalent of both network

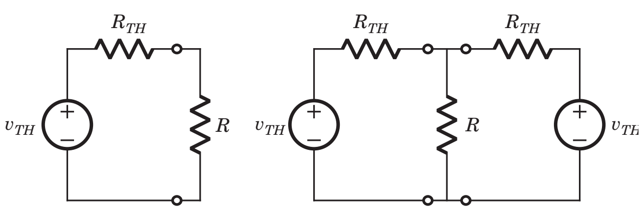

Fig. S1.4.34

$$P = \left(\frac{V_{TH}}{R_{TH}+R}\right)^2 R$$

$$P' = \left(\frac{V_{TH}}{R + \frac{R_{TH}}{2}}\right)^2 R = 4\left(\frac{V_{TH}}{2R + R_{TH}}\right)^2 R$$

Thus $P < P' < 4P$

35. (C) $i_1 = \sqrt{\frac{P_1}{R}}$ and $i_2 = \sqrt{\frac{P_2}{R}}$

using superposition $i = i_1 + i_2 = \sqrt{\frac{P_1}{R}} \pm \sqrt{\frac{P_2}{R}}$

$$i^2 R = (\sqrt{P_1} \pm \sqrt{P_2})^2$$

36. (C) $r = \frac{v_{oc}}{i_{sc}} = 1.2\ \Omega$

$$P = \frac{24^2}{(1.2+2)^2} \times 2 = 112.5 \text{ W}$$

37. (B) $R_{TH} = \frac{v_{oc}}{i_{sc}} = \frac{12}{1.5} = 8\ \Omega$

38. (A) Let $\frac{1}{\text{sensitivity}} = \frac{1}{20\text{k}} = 50\ \mu\text{A}$

For 0 –10 V scale $R_m = 10 \times 20\text{k} = 200\text{ k}\Omega$

For 0 –50 V scale $R_m = 50 \times 20\text{k} = 1\text{ M}\Omega$

For 4 V reading $i = \frac{4}{10} \times 50 = 20\ \mu\text{A}$

$v_{TH} = 20\mu R_{TH} + 20\mu \times 200\text{k} = 4 + 20\mu R_{TH}$ ...(i)

For 5 V reading $i = \frac{5}{50} \times 50\mu = 5\ \mu\text{A}$

$v_{TH} = 5\mu \times R_{TH} + 5\mu \times 1\text{M} = 5 + 5\mu R_{TH}$ ...(ii)

Solving (i) and (ii)

$v_{TH} = \frac{16}{3}$ V, $R_{TH} = \frac{200}{3}$ kΩ

39. (D) $v_{10\text{k}} = \sqrt{10\text{k} \times 3.6\text{m}} = 6$

$v_{30\text{k}} = \sqrt{30\text{k} \times 4.8\text{m}} = 12$ V

$6 = \frac{10}{10 + R_{TH}} v_{TH} \Rightarrow 10 v_{TH} = 6 R_{TH} + 60$

$12 = \frac{30\ v_{TH}}{30 + R_{TH}} \Rightarrow 5 v_{TH} = 2 R_{TH} + 60$

$R_{TH} = 30$ kΩ

40. (D) At $v = 0$, $i_{sc} = 30$ mA

At $i = 0$, $v_{oc} = -3$ V

$$R_{TH} = \frac{v_{oc}}{i_{sc}} = \frac{-3}{30\text{m}} = -100\ \Omega$$

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CHAPTER

1.6

THE RLC CIRCUITS

1. The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2}+2\frac{dv}{dt}+v=0,\quad v(0)=10,\quad \frac{dv(0)}{dt}=0.$$

The $v(t)$ is

(A) $10(1+t)e^{-t}$ V (B) $10(1-t)e^{-t}$ V

(C) $10e^{-t}$ V (D) $10te^{-t}$ V

2. The differential equation for the circuit shown in fig. P1.6.2. is

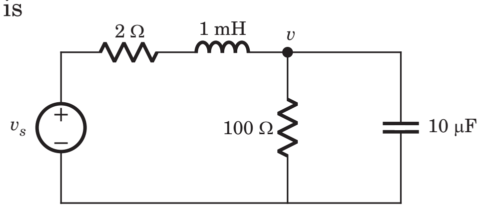

Fig. P1.6.2

(A) $v''(t)+3000v'(t)+1.02\times10^8v(t)=10^8v_s(t)$

(B) $v''(t)+1000v'(t)+1.02\times10^8v(t)=10^8v_s(t)$

(C) $\dfrac{v''(t)}{10^8}+\dfrac{2v'(t)}{10^5}+1.02v(t)=v_s(t)$

(D) $\dfrac{v''(t)}{10^8}+\dfrac{2v'(t)}{10^5}+1.98v(t)=v_s(t)$

3. The differential equation for the circuit shown in fig. P1.6.3 is

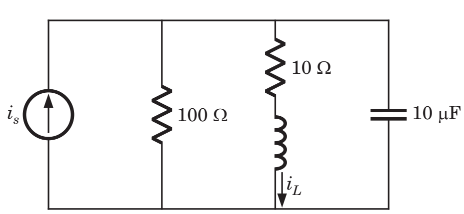

Fig. P.1.6.3

(A) $i_L''(t)+1100i_L''(t)+11\times10^8i_L(t)=10^8i_s(t)$

(B) $i_L''(t)+1100i_L''(t)+11\times10^8i_L(t)=10^8i_s(t)$

(C) $\dfrac{i_L''(t)}{10^8}+\dfrac{1.1i_L''(t)}{10^4}+1.1i_L(t)=i_s(t)$

(D) $\dfrac{i_L''(t)}{10^8}+\dfrac{11i_L'(t)}{10^4}+11i_L(t)=i_s(t)$

4. In the circuit of fig. P.1.6.4 $v_s=0$ for $t>0$. The initial condition are $v(0)=6$ V and $dv(0)/dt=-3000$ V/s. The $v(t)$ for $t>0$ is

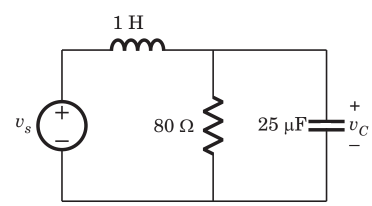

Fig. P1.6.4

(A) $-2e^{-100t}+8e^{-400t}$ V (B) $6e^{-100t}+8e^{-400t}$ V

(C) $6e^{-100t}-8e^{-400t}$ V (D) None of the above

5. The circuit shown in fig. P1.6.5 has been open for a long time before closing at $t=0$. The initial condition is $v(0)=2$ V. The $v(t)$ for $t>$ is

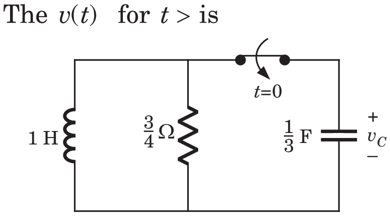

Fig. P.1.6.5

(A) $5e^{-t}-7e^{-3t}$ V (B) $7e^{-t}-5e^{-3t}$ V

(C) $-e^{-t}+3e^{-3t}$ V (D) $3e^{-t}-e^{-3t}$ V

Statement for Q.6–7:

Circuit is shown in fig. P.1.6. Initial conditions are $i_1(0) = i_2(0) = 11$ A

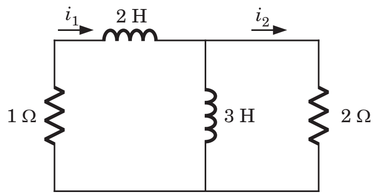

Fig. P1.6.6–7

6. $i_1(1\text{ s}) = ?$

(A) 0.78 A
(B) 1.46 A
(C) 2.56 A
(D) 3.62 A

7. $i_2(1\text{ s}) = ?$

(A) 0.78 A
(B) 1.46 A
(C) 2.56 A
(D) 3.62 A

8. $v_C(t) = ?$ for $t > 0$

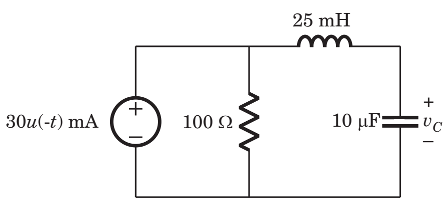

Fig. P1.6.8

(A) $4e^{-1000t} - e^{-2000t}$ V
(B) $(3 + 6000t)e^{-2000t}$ V
(C) $2e^{-1000t} + e^{-2000t}$ V
(D) $(3 - 6000t)e^{-2000t}$ V

9. The circuit shown in fig. P1.6.9 is in steady state with switch open. At $t = 0$ the switch is closed. The output voltage $v_C(t)$ for $t > 0$ is

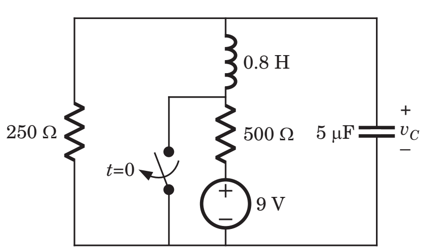

Fig. P1.6.9

(A) $-9e^{-400t} + 12e^{-300t}$
(B) $e^{-400t}[3\cos 300t + 4\sin 300t]$
(C) $e^{-300t}[3\cos 400t + 4\sin 300t]$
(D) $e^{-300t}[3\cos 400t + 2.25\sin 300t]$

10. The switch of the circuit shown in fig. P1.6.10 is opened at $t = 0$ after long time. The $v(t)$, for $t > 0$ is

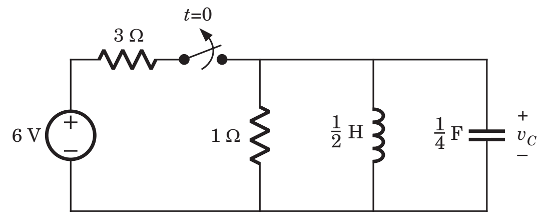

Fig. P1.6.10

(A) $4e^{-2t}\sin 2t$ V
(B) $-4e^{-2t}\sin 2t$ V
(C) $4e^{-2t}\cos 2t$ V
(D) $-4e^{-2t}\cos 2t$ V

11. In the circuit of fig. P1.6.23 the switch is opened at $t = 0$ after long time. The current $i_L(t)$ for $t > 0$ is

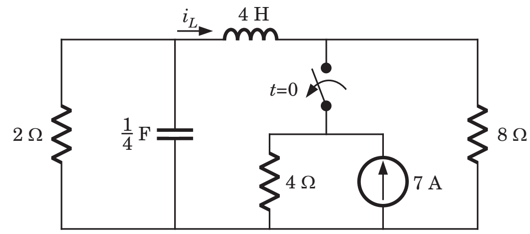

Fig. P1.6.11

(A) $e^{-2t}(2\cos t + 4\sin t)$ A
(B) $e^{-2t}(3\sin t - 4\cos t)$ A
(C) $e^{-2t}(-4\sin t + 2\cos t)$ A
(D) $e^{-2t}(2\sin t - 4\cos t)$ A

Statement for Q.12–14:

In the circuit shown in fig. P1.6.12–14 all initial condition are zero.

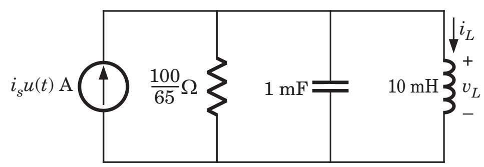

Fig. P1.5.12-14

12. If $i_s(t) = 1$ A, then the inductor current $i_L(t)$ is

(A) 1 A
(B) t A
(C) $t + 1$ A
(D) 0 A

13. If $i_s(t) = 0.5t$ A, then $i_L(t)$ is

(A) $0.5t + 3.25 \times 10^{-3}$ A
(B) $2t - 3250$ A
(C) $0.5t - 0.25 \times 10^{-3}$ A
(D) $2t + 3250$ A

14. If $i_s(t) = 2e^{-250t}$ A then $i_L(t)$ is

(A) $\frac{4000}{3} te^{-250t}$ A
(B) $\frac{4000}{3} e^{-250t}$ A
(C) $\frac{200}{7} e^{-250t}$ A
(D) $\frac{200}{7} te^{-250t}$ A

15. The forced response for the capacitor voltage $v_f(t)$ is

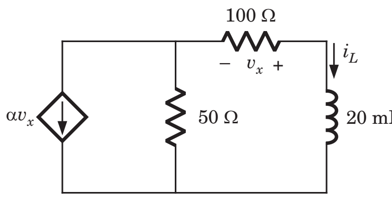

Fig. P1.6.15

(A) $0.2t + 1.17 \times 10^{-3}$ V (B) $0.2t - 1.17 \times 10^{-3}$ V

(C) $1.17 \times 10^{-3}t - 0.2$ V (D) $1.17 \times 10^{-3}\, t + 0.2$ V

16. For a *RLC* series circuit $R = 20\,\Omega$, $L = 0.6$ H, the value of C will be
[CD =critically damped, OD =over damped, UD =under damped].

	CD	OD	UD
(A)	$C = 6$ mF	$C > 6$ mF	$C < 6$ mF
(B)	$C = 6$ mF	$C < 6$ mF	$C > 6$ mF
(C)	$C > 6$ mF	$C = 6$ mF	$C < 6$ mF
(D)	$C < 6$ mF	$C = 6$ mF	$C > 6$ mF

17. The circuit shown in fig. P1.6.17 is critically damped. The value of R is

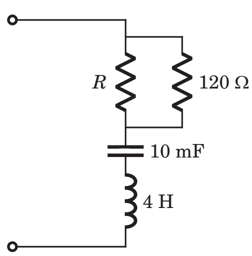

Fig. P1.6.17

(A) 40 Ω (B) 60 Ω

(C) 120 Ω (D) 180 Ω

18. The step response of an *RLC* series circuit is given by

$$\frac{d^2 i(t)}{dt} + \frac{2di(t)}{dt} + 5i(t) = 10,\ i(0^+) = 2,\ \frac{di(0^+)}{dt} = 4.$$

The $i(t)$ is

(A) $1 + e^{-t}\cos 4t$ A (B) $4 - 2e^{-t}\cos 4t$ A

(C) $2 + e^{-t}\sin 4t$ A (D) $10 + e^{-t}\sin 4t$ A

19. In the circuit shown in fig. P 1.5.19 $v(t)$ for $t > 0$ is

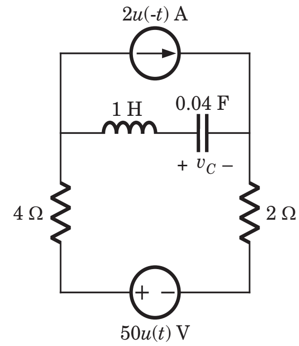

Fig. P1.6.19

(A) $50 - (46.5 \sin 3t + 62 \cos 3t)e^{-4t}$ V

(B) $50 + (46.5 \sin 3t + 62 \cos 3t)e^{-4t}$ V

(C) $50 + (62 \cos 4t + 46.5 \sin 4t)e^{-3t}$ V

(D) $50 - (62 \cos 4t + 46.5 \sin 4t)\, e^{-3t}$ V

20. In the circuit of fig. P1.6.20 the switch is closed at $t = 0$ after long time. The current $i(t)$ for $t > 0$ is

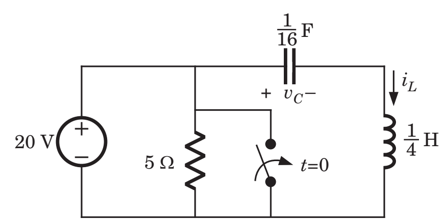

Fig. P1.6.20

(A) $-10 \sin 8t$ A (B) $10 \sin 8t$ A

(C) $-10 \cos 8t$ A (D) $10 \cos 8t$ A

21. In the circuit of fig. P1.6.21 switch is moved from 8 V to 12 V at $t = 0$. The voltage $v(t)$ for $t > 0$ is

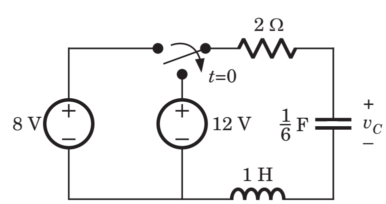

Fig. P1.6.21

(A) $12 - (4 \cos 2t + 2 \sin 2t)e^{-t}$ V

(B) $12 - (4 \cos 2t + 8 \sin 2t)e^{-t}$ V

(C) $12 + (4 \cos 2t + 8 \sin 2t)e^{-t}$ V

(D) $12 + (4 \cos 2t + 2 \sin 2t)e^{-t}$ V

22. In the circuit of fig. P1.5.22 the voltage $v(t)$ is

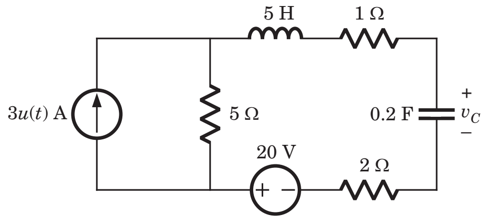

Fig. P1.6.22

(A) $40 - (20\cos 0.6t + 15\sin 0.6t)e^{-0.8t}$ V

(B) $35 + (15\cos 0.6t + 20\sin 0.6t)e^{-0.8t}$ V

(C) $35 - (15\cos 0.6t + 20\sin 0.6t)\, e^{-0.8t}$ V

(D) $35 - 15\cos 0.6t\, e^{-0.8t}$ V

23. In the circuit of fig. P1.6.23 the switch is opened at $t = 0$ after long time. The current $i(t)$ for $t > 0$ is

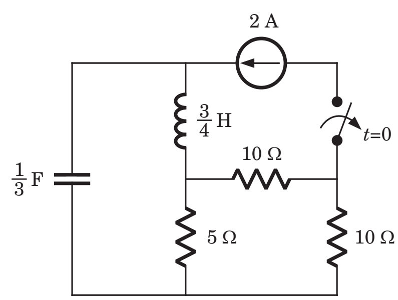

Fig. P1.6.23

(A) $e^{-2.306t} + e^{-0.869t}$ A

(B) $-e^{-2.306t} + 2e^{-0.869t}$ A

(C) $e^{-4.431t} + e^{-0.903t}$ A

(D) $2e^{-4.431t} - e^{-0.903t}$ A

24. In the circuit of fig. P1.6.24 switch is moved from position a to b at $t = 0$. The $i_L(t)$ for $t > 0$ is

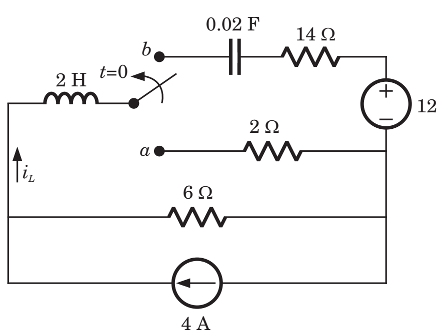

Fig. P1.6.24

(A) $(4 - 6t)e^{4t}$ A (B) $(3 - 6t)e^{-4t}$ A

(C) $(3 - 9t)e^{-5t}$ A (D) $(3 - 8t)e^{-5t}$ A

25. In the circuit shown in fig. P1.6.25 a steady state has been established before switch closed. The $i(t)$ for $t > 0$ is

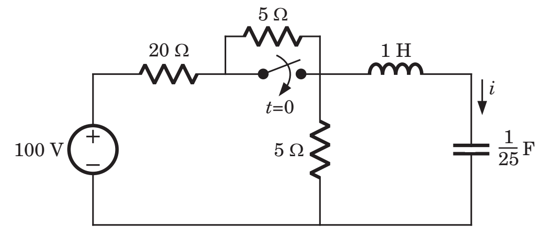

Fig. P1.6.25

(A) $0.73e^{-2t}\sin 4.58t$ A

(B) $0.89e^{-2t}\sin 6.38t$ A

(C) $0.73e^{-4t}\sin 4.58t$ A

(D) $0.89e^{-4t}\sin 6.38t$ A

26. The switch is closed after long time in the circuit of fig. P1.6.26. The $v(t)$ for $t > 0$ is

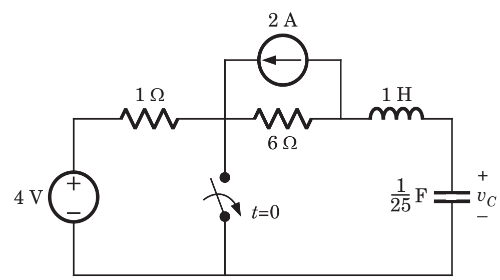

Fig. P1.6.26

(A) $-8 + 6e^{-3t}\sin 4t$ V

(B) $-12 + 4e^{-3t}\cos 4t$ V

(C) $-12 + (4\cos 4t + 3\sin 4t)e^{-3t}$ V

(D) $-12 + (4\cos 4t + 6\sin 4t)e^{-3t}$ V

27. $i(t) = ?$

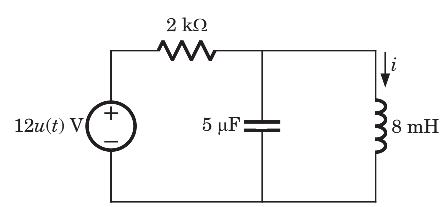

Fig. P1.6.27

(A) $6 - (6\cos 500t + 6\sin 5000t)e^{-50t}$ mA

(B) $8 - (8\cos 500t + 0.06\sin 5000t)e^{-50t}$ mA

(C) $6 - (6\cos 5000t + 0.06\sin 5000t)e^{-50t}$ mA

(D) $6e^{-50t}\sin 5000t$ mA

28. In the circuit of fig. P1.6.28 $i(0) = 1$ A and $v(0) = 0$. The current $i(t)$ for $t > 0$ is

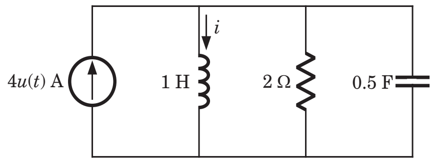

Fig. P1.6.28

(A) $4 + 6.38\,e^{-0.5t}$ A (B) $4 - 6.38\,e^{-0.5t}$ A

(C) $4 + (3\cos 1.32t + 1.13\sin 1.32t)e^{-0.5t}$ A

(D) $4 - (3\cos 1.32t + 1.13\sin 1.32t)e^{-0.5t}$ A

29. In the circuit of fig. P1.6.29 a steady state has been established before switch closed. The $v_o(t)$ for $t > 0$ is

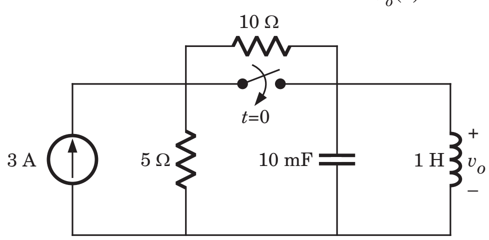

Fig. P1.6.29

(A) $100te^{-10t}$ V (B) $200te^{-10t}$ V

(C) $400te^{-50t}$ V (D) $800te^{-50t}$ V

30. In the circuit of fig. P1.6.30 a steady state has been established before switch closed. The $i(t)$ for $t > 0$ is

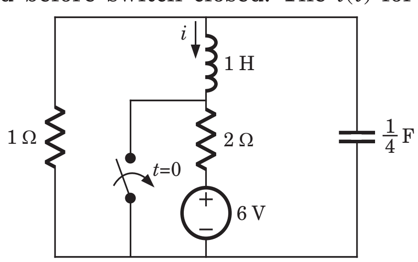

Fig. P1.6.30

(A) $2e^{-2t}\sin 2t$ A (B) $-e^{-2t}\sin 2t$ A

(C) $-2(1-t)e^{-2t}$ A (D) $2(1-t)e^{-2t}$ A

31. In the circuit of fig. P1.6.31 a steady state has been established. The $i(t)$ for $t > 0$ is

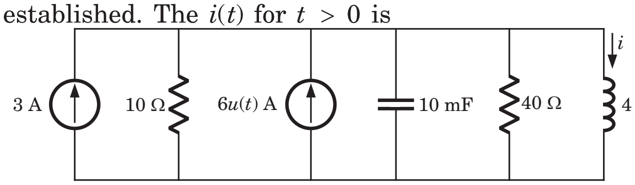

Fig. P1.6.31

(A) $9 + 2e^{-10t} - 8e^{-2.5t}$ A (B) $9 - 8e^{10t} + 2e^{-2.5t}$ A

(C) $9 + (2\cos 10t + \sin 10t)e^{-2.5t}$ A

(D) $9 + (\cos 10t + 2\sin 10t)e^{-2.5t}$ A

****

SOLUTIONS

1. (A) $s^2 + 2s + 1 = 0 \quad \Rightarrow \quad s = -1, -1,$

$v(t) = (A_1 + A_2 t)e^{-t}$

$v(0) = 10$ V, $\quad \dfrac{dv(0)}{dt} = 0 = -1A_1 + A_2$

$A_1 = A_2 = 10$

2. (A) $i_L = \dfrac{v}{100} + 10 \times 10^{-6}\dfrac{dv}{dt}$

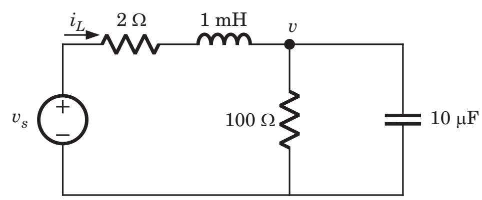

Fig. S1.6.2

$$v_s = 2i_L + 10^{-3}\frac{di_L}{dt} + v$$

$$= 2\left(\frac{v}{100} + 10^{-6} \times 10^{-t}\frac{dv}{dt}\right) + 10^{-3}\left(\frac{1}{100}\frac{dv}{dt} + 10 \times 10^{-6}\frac{d^2v}{dt^2}\right) + v$$

$$10^8 v_s(t) = v''(t) + 3000v'(t) + 1.02v(t)$$

3. (C) $i_s = \dfrac{v_C}{100} + i_L + 10\mu\dfrac{dv_C}{dt}$

$$v_C = 10i_L + 10^{-3}\frac{di_L}{dt}$$

$$i_s = 0.1i_L + 10^{-5}\frac{di_L}{dt} + i_L + 10^{-5}\frac{d}{dt}(10i_L + 10^{-3}\frac{di_L}{dt})$$

$$= 0.1i_L + 10^{-5}\frac{di_L}{dt} + i_L + 10^{-4}\frac{di_L}{dt} + 10^{-8}\frac{d^2i_L}{dt^2}$$

$$\Rightarrow \frac{i_L''(t)}{10^8} + \frac{1.1}{10^4}i_L'(t) + 1.1i_L(t) = i_s(t)$$

4. (A) $\dfrac{v}{80} + 25\mu\dfrac{dv}{dt} + \int (v - v_s)dt = 0$

$$\Rightarrow \frac{d^2v}{dt^2} + 500\frac{dv}{dt} + 40000 = 0$$

$s^2 + 500s + 40000 = 0$

$\Rightarrow \quad s = -100, -400,$

$v(t) = Ae^{-100t} + Be^{-400t}$

$A + B = 6, \ -100A - 400B = -3000 \quad \Rightarrow \quad B = 8, \ A = -2$

5. (C) The characteristic equation is $s^2 + \dfrac{s}{RC} + \dfrac{1}{LC} = 0$

After putting the values, $\quad s^2 + 4s + 3 = 0$

$v(t) = Ae^{-t} + Be^{-3t},$

$v(0^+) = 2 \text{ V} \quad \Rightarrow \quad A + B = 2$

$i_L(0^+) = 0 \Rightarrow \quad i_R(0) = \dfrac{2}{3/4} = \dfrac{8}{3},$

$-C\dfrac{dv(0^+)}{dt} = \dfrac{8}{3} \quad \Rightarrow \quad \dfrac{dv(0^+)}{dt} = -8,$

$-A - 3B = -8, \quad B = 3, \quad A = -1$

6. (D) $i_1 + 5\dfrac{di_1}{dt} - 3\dfrac{di_2}{dt} = 0,$

$2i_2 + \dfrac{3di_2}{dt} - 3\dfrac{di_1}{dt} = 0$

$(1 + 5s)i_1 - 3si_2 = 0, \; -3si_1 + (2 + 3s)i_2 = 0$

$(1 + 5s)i_1 - \dfrac{(3s)(3s)i_1}{2 + 3s} = 0$

$\Rightarrow \quad 6s^2 + 13s + 2 = 0$

$\Rightarrow \quad s = -\dfrac{1}{6}, -2$

$i_1 = A\,e^{-\frac{1}{6}t} + Be^{-2t}\,, \quad i(0) = A + B = 11$

In differential equation putting $t = 0$ and solving

$\dfrac{di_1(0^+)}{dt} = -\dfrac{33}{2}, \quad \dfrac{di_2(0^+)}{dt} = -\dfrac{143}{6}$

$-\dfrac{A}{6} - 2B = -\dfrac{33}{2}, \Rightarrow \quad A = 3, B = 8,$

$i_1 = 3e^{-\frac{t}{6}} + 8e^{-2t},$

$i_1(1\text{ s}) = 3e^{-\frac{1}{6}} + 8e^{-2} = 3.62 \text{ A}$

7. (A) $i_2 = Ce^{-\frac{t}{6}} + De^{-2t}$

$i_2(0) = 11 = C + D, \; \dfrac{di_2(0)}{dt} = \dfrac{-143}{6} = -\dfrac{C}{6} - 2D$

$C = -1$ and $D = 12$

$i_2 = -e^{-\frac{t}{6}} + 12e^{-2t} \text{ A}, \quad i_2\,(1\text{ s}) = e^{-\frac{1}{6}} + 12e^{-2} = 0.78 \text{ A}$

8. (B) $v_C(0^+) = 30\text{m} \times 100 = 3 \text{ V}$

$C\dfrac{dv_C(0^-)}{dt} = i_L(0^-) = 0 = i_L(0^+) = C\dfrac{dv_C(0^+)}{dt}$

$s^2 + \dfrac{100}{25 \times 10^{-3}}s + \dfrac{1}{25 \times 10^{-3} \times 10 \times 10^{-6}}$

$\Rightarrow \quad s = -2000, -2000$

$v_C(t) = (A_1 + A_2 t)e^{-2000t}$

$\dfrac{dv_C(t)}{dt} = A_2 e^{-2000t} + (A_1 + A_2 t)e^{-2000t}(-2000)$

$v_C(0^+) = A_1 = 3, \; \dfrac{dv_C(0)}{dt} = A_2 - 2000 \times 3 = 0$

$\Rightarrow \quad A_2 = 6000$

9. (B) $v_C(0^+) = 3 \text{ V}, \quad i_L(0^+) = -12 \text{ mA}$

$\dfrac{v_C}{250} + i_L + 5 \times 10^{-6}\dfrac{dv_C}{dt} = 0$

$\dfrac{3}{250} - 12\text{m} + 5 \times 10^{-6}\dfrac{dv_C(0^+)}{dt} = 0 \quad \Rightarrow \quad \dfrac{dv_C(0^+)}{dt} = 0$

$s^2 + \dfrac{s}{250 \times 5 \times 10^{-6}} + \dfrac{1}{0.8 \times 5 \times 10^{-6}} = 0$

$\Rightarrow \quad s^2 + 800s + 25 \times 10^4 = 0$

$\Rightarrow \quad s = -400 \pm j300$

$v_C(t) = e^{-400t}(A_1 \cos 300t + A_2 \sin 300t)$

$A_1 = 3, \; \dfrac{dv_C(0)}{dt} = -400A_1 + 300A_2\,, \quad A_2 = 4$

10. (B) $v(0^+) = 0, \quad i_L(0^+) = 2 \text{ A}, \quad \dfrac{1}{4}\dfrac{dv_C(0^+)}{xdt} = -2$

$s^2 + 4s + 8 = 0 \quad \Rightarrow \quad s = -2 \pm j2$

$v_C(t) = e^{-2t}(A_1 \cos 2t + A_2 \sin 2t)$

$A_1 = 0, \; \dfrac{dv_C\,(0^+)}{dt} = -8 = -2\,(0 + 0) + (0 + 2A_2), \; A_2 = -4$

11. (D) $i_L(0^+) = -4, \quad v_C(0^+) = 8 \text{ V}$

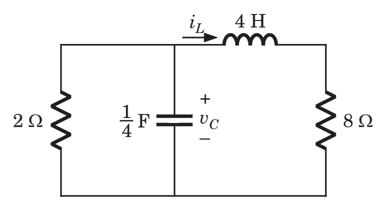

Fig. S1.6.11

$4\dfrac{di_L(0^+)}{dt} = 8 - (-4) \times 8 \quad \Rightarrow \quad \dfrac{di_L(0^+)}{dt} = 10$

$\dfrac{s}{4}v_C + \dfrac{1}{2}v_C + i_L = 0, \quad v_C = 4si_L + 8i_L$

$s^2 i_L + 4si_L + 5 = 0, \quad s = -2 \pm j$

$i_L(t) = e^{-2t}(A_1 \cos t + A_2 \sin t)$

$A_1 = -4, \; \dfrac{di_L(0^+)}{dt} = 10 = -2\,(A_1 + 0) + A_2, \; A_2 = 2$

12. (A) $i_s = \dfrac{v}{100/65} + 10^{-3}\dfrac{dv}{dt} + i_L, \; v = 10 \times 10^{-3}\dfrac{di_L}{dt}$

$i_s = \dfrac{65}{100}(10 \times 10^{-3})\dfrac{di_L}{dt} + 10^{-3}(10 \times 10^{-3})\dfrac{d^2 i_L}{dt} + i_L = 0$

$\dfrac{d^2 i_L}{dt} + 650\dfrac{di_L}{dt} + 10^5 i_L = 10^5 i_s$

Trying $i_L(t) = B$

$0 + 0 + 10^5\,B = 10^5, \qquad B = 1, \qquad i_L = 1 \text{ A}$

13. (A) Trying $i_L(t) = At + B,$

$0 + 650A + (At + B)10^5 = 10^5(0.5t), \; A = 0.5$

$$\frac{di(0^+)}{dt} = \frac{-16}{3} = -4.431\,A - 0.903B$$

$A = 1,\ B = 1$

24. (C) $v_C(0) = 0, \quad i_L(0) = \frac{4 \times 6}{6 + 2} = 3$

$$0.02\frac{dv_C(0)}{dt} = i_L(0) = 3 \quad \Rightarrow \quad \frac{dv_C(0)}{dt} = 150$$

$$\alpha = \frac{6 + 14}{2 \times 2} = 5, \quad \omega_o = \frac{1}{\sqrt{2 \times 0.02}} = 5$$

$\alpha = \omega_o$ critically damped

$v(t) = 12 + (A + Bt)e^{-5t}$

$0 = 12 + A, \quad 150 = -5A + B \quad \Rightarrow \quad A = -12, \quad B = 90$

$v(t) = 12 + (90t - 12)e^{-5t}$

$i_L(t) = 0.02(-5)\,e^{-5t}(90t - 12) + 0.02(90)e^{-5t} = (3 - 9t)e^{-5t}$

25. (A) $v(0^+) = \frac{100 \times 5}{5 + 5 + 20} = \frac{50}{3}, \quad i_L(0^+) = 0$

$i_f = 0$ A

$$\frac{di_L(0^+)}{dt} = 20 - \frac{50}{3} = \frac{10}{3}$$

$$\alpha = \frac{4}{2 \times 1} = 2, \ \omega_o = \frac{1}{\sqrt{1 \times \frac{1}{25}}} = 5$$

$s = -2 \pm \sqrt{4 - 25} = -2 \pm j4.58$

$i(t) = (A\cos 4.58t + B\sin 4.58t)e^{-2t}$

26.(A) $i_L(0^+) = 0, \ v_L(0^+) = 4 - 12 = -8$

$$\frac{1}{25}\frac{dv_L(0^+)}{dt} = i_L(0^+) = 0$$

$$\alpha = \frac{6}{2} = 3, \ W_o = \frac{1}{\sqrt{1 \times 1/25}} = 5$$

$\beta = -3 \pm \sqrt{9 - 25} = -3 \pm j4$

$v_1(t) = -12 + (A\cos 4t + B\sin 4t)e^{-3t}$

$v_L(0) = -8 = 12 + A, \quad \Rightarrow A = 4$

$$\frac{dv_L(0)}{dt} = 0 = -3A + 4B, \quad \Rightarrow B = 3$$

27. (C) $\alpha = \frac{1}{2RC} = \frac{1}{2 \times 2k \times 54} = 50$

$$W_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8m \times 5\mu}} = 5000$$

$\alpha < W_o$, underdamped response.

$s = -50 \pm \sqrt{50^2 - 5000^2} = -50 \pm j5000$

$i(t) = 6 + (A\cos 5000t + B\sin 5000t)e^{-50t}$ mA

$i(0) = 6 = 6 + A, \quad \Rightarrow A = -6$

$$\frac{di(0)}{dt} = -50A + 5000B = 0, \quad B = -0.06$$

28.(D) $i(0^+) = 1$ A, $v(0^+) = \frac{Ldi(0^+)}{dt}$

$$\alpha = \frac{1}{2 \times 2 \times 0.5} = 0.5, \quad W_o = \frac{1}{\sqrt{1 - 0.5}} = \sqrt{2}$$

$s = -0.5 \pm \sqrt{0.5^2 - 2} = 0.5 \pm j1.323$

$i(t) = 4 + (A\cos 1.32t + B\sin 1.32t)e^{-0.5}t$

$1 = 4 + A, \quad \Rightarrow A = -3$

$$\frac{di(0)}{dt} = 0 = 0.5A + 1.32B, \quad B = -1.13$$

29. (B) $V_o(0^+) = 0 \ ,i_L(0^+) = 1$ A

$$\frac{di_L(0^+)}{dt} = v_1(0) = 0$$

$$\alpha = \frac{1}{2 \times 5 \times 0.01} = 10, \quad W_o = \frac{1}{\sqrt{1 \times 0.01}} = 10$$

$\alpha = W_o$, so critically damped response

$s = -10, -10$

$i(t) = 3(A + Bt)e^{-10t}, \quad i(0) = 1 = 3 + A$

$$\frac{di(0^+)}{dt} = -10A + B$$

$i_L(t) = 3 - (2 + 20t)e^{-10t}, \quad v_o = \frac{Ldi_L(t)}{dt} = 200te^{-10t}$

30. (C) $i(0^+) = \frac{-6}{1 + 2} = -2$ A, $\quad v_c(0^+) = 2 \times 1 = 2 = \frac{di(0^+)}{dt}$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 1 \times 0.25} = 2, \quad W_o = \frac{1}{\sqrt{LC}} = 2$$

$\alpha = W_o$, critically damped response

$s = -2, -2$

$i(t) = (A + Bt)e^{-2t}, \quad A = -2$

$$\frac{di(t)}{dt} = (-2 + Bt)e^{2t}(-2) + (0 + B)e^{-2t}$$

At $t = 0, \ \Rightarrow B = -2$

31. (A) $i(0^+) = 3$ A, $v_C(0^+) = 0$ V $= \frac{4di(0^+)}{dt}$

$i_s = 9$ A, $R = 10 || 40 = 8\,\Omega$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 8 \times 0.01} = 6.25$$

$$W_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times 0.01}} = 5$$

$\alpha > W_o$, so overdamped response

$s = -6.25 \pm \sqrt{6.25^2 - 25} = -10, -2.5$

$i(t) = 9 + Ae^{-10t} + Be^{-2.5t}$

$3 = 9 + A + B, \quad 0 = -10A - 2.5B$

On solving, $A = 2, \ B = -8$

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CHAPTER

1.7

SINUSOIDAL STEADY STATE ANALYSIS

1. $i(t) = ?$

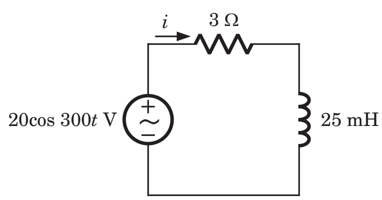

Fig. P1.7.1

(A) $20\cos(300t + 68.2°)$ A

(B) $20\cos(300t - 68.2°)$ A

(C) $2.48\cos(300t + 68.2°)$ A

(D) $2.48\cos(300t - 68.2°)$ A

2. $v_C(t) = ?$

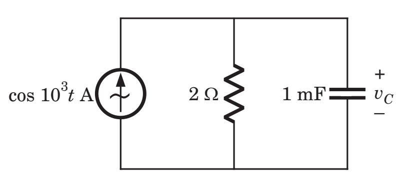

Fig. P1.7.2

(A) $0.89\cos(10^3t - 63.43°)$ V

(B) $0.89\cos(10^3t + 63.43°)$ V

(C) $0.45\cos(10^3t + 26.57°)$ V

(D) $0.45\cos(10^3t - 26.57°)$ V

3. $v_C(t) = ?$

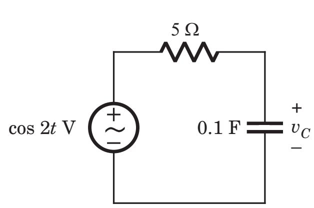

Fig. P1.7.3

(A) $\frac{1}{\sqrt{2}}\cos(2t - 45°)$ V

(B) $\frac{1}{\sqrt{2}}\cos(2t + 45°)$ V

(C) $\frac{1}{\sqrt{2}}\sin(2t - 45°)$ V

(D) $\frac{1}{\sqrt{2}}\sin(2t + 45°)$ V

4. $v_C(t) = ?$

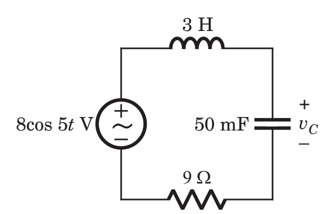

Fig. P1.7.4

(A) $2.25\cos(5t + 150°)$ V

(B) $2.25\cos(5t - 150°)$ V

(C) $2.25\cos(5t + 140.71°)$ V

(D) $2.25\cos(5t - 140.71°)$ V

5. $i(t) = ?$

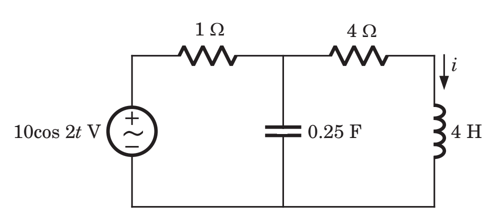

Fig. P1.7.5

(A) $2\sin(2t + 5.77°)$ A

(B) $\cos(2t - 84.23°)$ A

(C) $2\sin(2t - 5.77°)$ A

(D) $\cos(2t + 84.23°)$ A

13. In the bridge shown in fig. P1.7.13, $Z_1 = 300\ \Omega$, $Z_2 = (300 - j\,600)\ \Omega$, $Z_3 = (200 + j\,100)\Omega$. The Z_4 at balance is

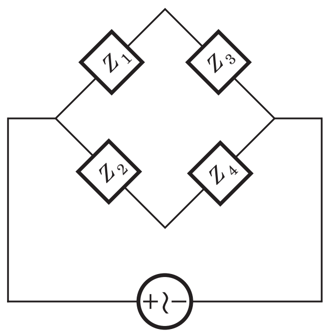

Fig. P1.7.13

(A) $400 + j300\ \Omega$ (B) $400 - j\ 300\ \Omega$

(C) $j100\ \Omega$ (D) $-\ j900\ \Omega$

14. In a two element series circuit, the applied voltage and the resulting current are $v(t) = 60 + 66\sin(10^3 t)$ V, $i(t) = 2.3\sin(10^3 t + 68.3°)$ A. The nature of the elements would be

(A) $R - C$ (B) $L - C$

(C) $R - L$ (D) $R - R$

15. $V_o = ?$

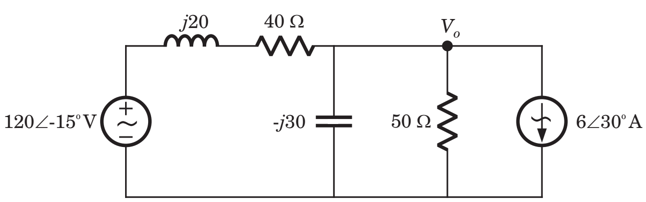

Fig. P1.7.15

(A) $223\angle -56°$ V (B) $223\angle 56°$ V

(C) $124\angle -154°$ V (D) $124\angle 154°$ V

16. $v_o(t) = ?$

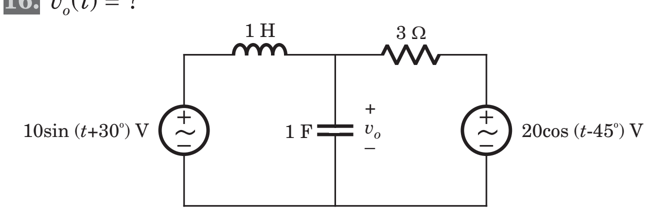

Fig. P1.7.16

(A) $31.5\cos(t + 112°)$ V

(B) $43.2\cos(t + 23°)$ V

(C) $31.5\cos(t - 112°)$ V

(D) $43.2\cos(t - 23°)$ V

Statement for Q.17-18:

The circuit is as shown in fig. P1.7.17-18

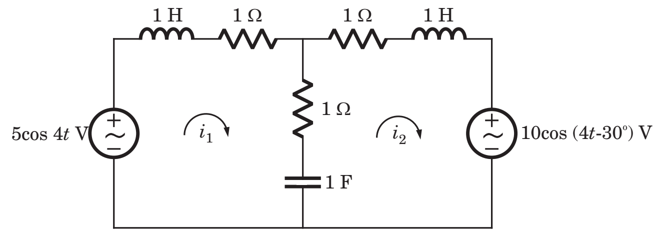

Fig. P1.7.17–18

17. $i_1(t) = ?$

(A) $2.36\cos(4t - 41.07°)$ A

(B) $2.36\cos(4t + 41.07°)$ A

(C) $1.37\cos(4t - 41.07°)$ A

(D) $2.36\cos(4t + 41.07°)$ A

18. $i_2(t) = ?$

(A) $2.04\sin(4t + 92.13°)$ A

(B) $-2.04\sin(4t + 2.13°)$ A

(C) $2.04\cos(4t + 2.13°)$ A

(D) $-2.04\cos(4t + 92.13°)$ A

19. $I_x = ?$

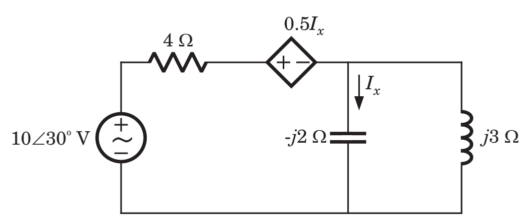

Fig. P1.7.19

(A) $3.94\angle 46.28°$ A (B) $4.62\angle 97.38°$ A

(C) $7.42\angle 92.49°$ A (D) $6.78\angle 49.27°$ A

20. $V_x = ?$

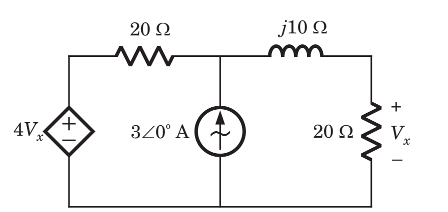

Fig. P1.7.20

(A) $29.11\angle 166°$ V (B) $29.11\angle -166°$ V

(C) $43.24\angle 124°$ V (D) $43.24\angle -124°$ V

Statement for Q.27–32:

Determine the complex power for hte given values in question.

27. $P = 269$ W, $Q = 150$ VAR (capacitive)

(A) 150 – j269 VA (B) 150 + j269 VA

(C) 269 – j150 VA (D) 269 + j150 VA

28. $Q = 2000$ VAR, $pf = 0.9$ (leading)

(A) 4129.8 + j2000 VA (B) 2000 + j4129.8 VA

(C) 2000 – j4129.8 VA (D) 4129.8 – j2000 VA

29. $S = 60$ VA, $Q = 45$ VAR (inductive)

(A) 39.69 + j45 VA (B) 39.69 – j45 VA

(C) 45 + j39.69 VA (D) 45 – j39.69 VA

30. $V_{rms} = 220$ V, $P = 1$ kW, $|Z| = 40\ \Omega$ (inductive)

(A) 1000 – j681.25 VA (B) 1000 + j681.25 VA

(C) 681.25 + j1000 VA (D) 681.25 – j1000 VA

31. $V_{rms} = 21\angle 20°$ V, $V_{rms} = 21\angle 20°$ V, $I_{rms} = 8.5\angle -50°$ A

(A) 154.6 + j89.3 VA (B) 154.6 – j89.3 VA

(C) 61 + j167.7 VA (D) 61 – j167.7 VA

32. $V_{rms} = 120\angle 30°$ V, $Z = 40 + j80\ \Omega$

(A) 72 + j144 VA (B) 72 – j144 VA

(C) 144 + j72 VA (D) 144 – j72 VA

33. V_o = ?

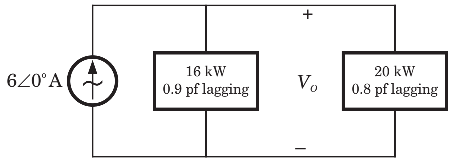

Fig. P1.7.33

(A) 7.1∠32.29° kV (B) 42.59∠32.29° kV

(C) 38.49∠24.39° kV (D) 38.49∠32.29° kV

34. A relay coil is connected to a 210 V, 50 Hz supply. If it has resistance of 30 Ω and an inductance of 0.5 H, the apparent power is

(A) 30 VA (B) 275.6 VA

(C) 157 VA (D) 187 VA

35. In the circuit shown in fig. P1.7.35 power factor is

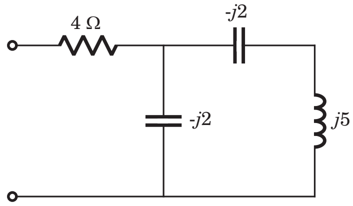

Fig. P1.7.35

(A) 56.31 (leading) (B) 56.31 (lagging)

(C) 0.555 (lagging) (D) 0.555 (leading)

36. The power factor seen by the voltage source is

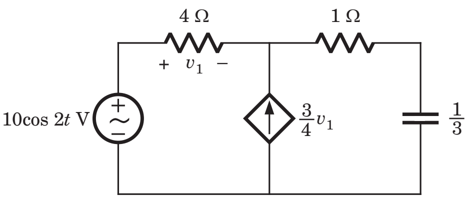

Fig. P1.7.36

(A) 0.8 (leading) (B) 0.8 (lagging)

(C) 36.9 (leading) (D) 39.6 (lagging)

37. The average power supplied by the dependent source is

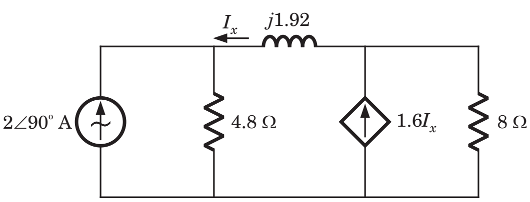

Fig. P1.7.37

(A) 96 W (B) –96 W

(C) 92 W (D) –192 W

38. In the circuit of fig. P1.7.38 the maximum power absorbed by Z_L is

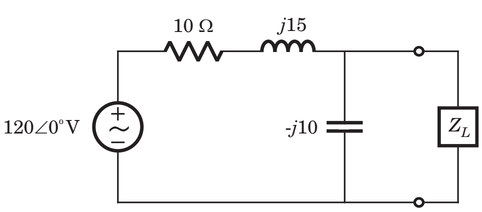

Fig. P1.7.38

(A) 180 W (B) 90 W

(C) 140 W (D) 700 W

39. The value of the load impedance, that would absorbs the maximum average power is

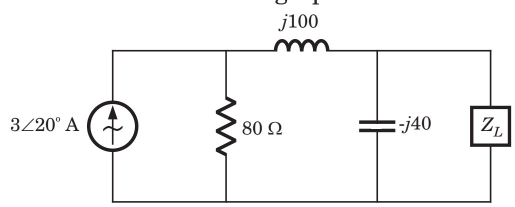

Fig. P1.7.39

(A) $12.8 - j49.6\,\Omega$ (B) $12.8 + j49.6\,\Omega$

(C) $33.9 - j86.3\,\Omega$ (D) $33.9 + j86.3\,\Omega$

Statement for Q.40–41:

In a balanced Y–connected three phase generator $V_{ab} = 400$ V_{rms}

40. If phase sequence is abc then phase voltage V_a, V_b, and V_c are respectively

(A) $231\angle 0°,\ 231\angle 120°,\ 231\angle 240°$

(B) $231\angle -30°,\ 231\angle -150°,\ 231\angle 90°$

(C) $231\angle 30°,\ 231\angle 150°,\ 231\angle -90°$

(D) $231\angle 60°,\ 231\angle 180°,\ 231\angle -60°$

41. If phase sequence is acb then phase voltage are

(A) $231\angle 0°,\ 231\angle 120°,\ 231\angle 240°$

(B) $231\angle -30°,\ 231\angle -150°,\ 231\angle 90°$

(C) $231\angle 30°,\ 231\angle 150°,\ 231\angle -90°$

(D) $231\angle 60°,\ 231\angle 180°,\ 231\angle -60°$

42. A balanced three-phase Y-connected load has one phase voltage $V_c = 277\angle 45°$ V. The phase sequence is abc. The line to line voltage V_{AB} is

(A) $480\angle 45°$ V (B) $480\angle -45°$ V

(C) $339\angle 45°$ V (D) $339\angle -45°$ V

43. A three-phase circuit has two parallel balanced Δ loads, one of the $6\,\Omega$ resistor and one of $12\,\Omega$ resistors. The magnitude of the total line current, when the line-to-line voltage is 480 V_{rms}, is

(A) 120 A_{rms} (B) 360 A_{rms}

(C) 208 A_{rms} (D) 470 A_{rms}

44. In a balanced three-phase system, the source has an abc phase sequence and is connected in delta. There are two parallel Y-connected load. The phase impedance of load 1 and load 2 is $4 + j4\,\Omega$ and $10 + j4\,\Omega$ respectively. The line impedance connecting the source to load is $0.3 + j0.2\,\Omega$. If the current in a phase of load 1 is $I = 10\angle 20°$ A_{rms}, the current in source in ab branch is

(A) $15\angle -122°$ A_{rms} (B) $8.67\angle -122°$ A_{rms}

(C) $15\angle 27.9°$ A_{rms} (D) $8.67\angle -57.9°$ A_{rms}

45. An abc phase sequence 3-phase balanced Y-connected source supplies power to a balanced Δ–connected load. The impedance per phase in the load is $10 + j8\,\Omega$. If the line current in a phase is $I_{aA} = 28.10\angle -28.66°$ A_{rms} and the line impedance is zero, the load voltage V_{AB} is

(A) $207.8\angle -140°$ V_{rms} (B) $148.3\angle 40°$ V_{rms}

(C) $148.3\angle -40°$ V_{rms} (D) $207.8\angle 40°$ V_{rms}

46. The magnitude of the complex power supplied by a 3-phase balanced Y-Y system is 3600 VA. The line voltage is 208 V_{rms}. If the line impedance is negligible and the power factor angle of the load is 25°, the load impedance is

(A) $5.07 + j10.88\,\Omega$ (B) $10.88 + j5.07\,\Omega$

(C) $43.2 + j14.6\,\Omega$ (D) $14.6 + j43.2\,\Omega$

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SOLUTIONS

1. (D) $Z = 3 + j(25\text{m})(300) = 3 + j7.5\ \Omega = 8.08\angle 68.2°$

$I = \dfrac{20\angle 0}{8.08\angle 68.2°} = 2.48\angle -68.2°\ \text{A}$

$i(t) = 2.48\cos(300t - 68.2°)\ \text{A}$

2. (A) $Y = \dfrac{1}{2} + j(1\text{m})(10^3) = 0.5 + j = 1.12\angle 63.43°$

$V_C = \dfrac{(1\angle 0)}{1.12\angle 63.43°} = 0.89\angle -63.43°\ \text{V}$

$v_C(t) = 0.89\cos(10^3 t - 63.43°)\ \text{V}$

3. (A) $Z = 5 + \dfrac{-j}{(0.1)(2)} = 5 - j5 = 5\sqrt{5}\angle -45°$

$V_C = \dfrac{(1\angle 0)(5\angle -90°)}{5\sqrt{2}\angle -45°} = \dfrac{1}{\sqrt{2}}\angle -45°\ \text{V}$

$v_C(t) = \dfrac{1}{\sqrt{2}}\cos(2t - 45°)\ \text{V}$

4. (D) $Z = 9 + j(3)(5) + \dfrac{-j}{(50\text{m})(5)} = 9 + j11$

$\Rightarrow \quad Z = 14.21\angle 50.71°\ \Omega$

$V_C = \dfrac{(8\angle 0)(4\angle -90°)}{14.21\angle 50.71°} = 2.25\angle 140.71°\ \text{V}$

$v_C(t) = 2.25\cos(5t - 140.71°)\ \text{V}$

5. (B) $V_a = \dfrac{\dfrac{10\angle 0}{1}}{\dfrac{1}{1} + \dfrac{1}{-j2} + \dfrac{1}{4 + j8}} = \dfrac{10\angle 0}{1.05 + j0.4}\ \text{V}$

$I = \dfrac{V_a}{4 + j8} = \dfrac{10\angle 0}{1 + j10} = 1\angle -84.23\ \text{A}$

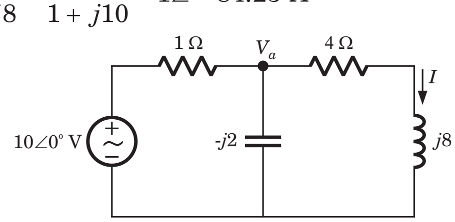

Fig. S1.7.5

$i(t) = \cos(2t - 84.23°)\ \text{A}$

6. (D) $\omega = 2\pi \times 10 \times 10^3 = 2\pi \times 10^4$

$Y = j(1\mu)(2\pi \times 10^4) + \dfrac{-j}{(160\mu)(2\pi \times 10^4)} + \dfrac{1}{36}$

$= 0.0278 - j0.0366\ \text{S}$

$Z = \dfrac{1}{Y} = 13.16 + j17.33\ \Omega$

7. (C) $Z = \left(\dfrac{-j}{\omega(22\mu)}\right) || (6 + j(27\text{m})\omega)$

$= \dfrac{\dfrac{-j10^6}{22\omega}(6 + j27 \times 10^{-3}\omega)}{6 + j(27\text{m}\omega - \dfrac{10^6}{22\omega})} = \dfrac{\dfrac{27 \times 10^3}{22} - \dfrac{j6 \times 10^6}{22\omega}}{6 + j\omega\left(27\text{m} - \dfrac{10^6}{22\omega^2}\right)}$

$-\dfrac{j36 \times 10^6}{\omega 22} - \dfrac{j27 \times 10^3}{22}\omega\left(27\text{m} - \dfrac{10^6}{22\omega^2}\right) = 0$

$\Rightarrow \quad \omega = 1278$

$f = \dfrac{\omega}{2\pi}\ \text{Hz} = \dfrac{1278}{2\pi} = 203\ \text{Hz}$

8. (C) $V_s = 7.68\angle 47°\ \text{V},\ V_2 = 7.51\angle 35°$

$V_1 = V_s - V_2 = 7.68\angle 47° - 7.51\angle 35° = 1.59\angle 125°$

9. (B) $v_{in} = \sqrt{3^2 + (14 - 10)^2} = 5$

10. (C) $I_1 = 744\angle -118°\ \text{mA},$

$I_2 = 540\angle 100°\ \text{mA}$

$I = I_1 + I_2 = 744\angle -118° + 540.5\angle 100°$

$= 460\angle -164°$

$i(t) = 460\cos(3t - 164°)\ \text{mA}$

11. (A) $\sqrt{2}\angle 45° = \dfrac{V_C}{-j4} + \dfrac{V_C - 20\angle 0}{j5 + 10}$

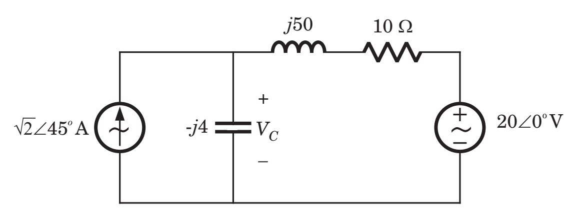

Fig. S1.7.11

$(1 + j)(-j4)(10 + j5) = V_C(10 + j5 - j4) + j8$

$\Rightarrow \quad 60 - j100 = V_C(10 + j)$

$\Rightarrow \quad V_C = 11.6\angle -64.7°$

12. (D) $X = X_L + X_C = 0$

So reactive power drawn from the source is zero.

13. (B) $Z_1 Z_4 = Z_3 Z_2$

$300 Z_4 = (300 - j600)(200 + j100)$

$\Rightarrow \quad Z_4 = 400 - j300$

14. (A) $R - C$ causes a positive phase shift in voltage

$Z = |Z|\angle\theta\ ,\ -90° < \theta < 0\ ,$

$I = \dfrac{V}{Z} = \dfrac{V}{|Z|}\angle -\theta$

15. (C) $V_o = \dfrac{\dfrac{120\angle 15^\circ}{40 + j20} - 6\angle 30^\circ}{\dfrac{1}{40 + j20} + \dfrac{1}{-j30} + \dfrac{1}{50}} = 124\angle -154^\circ$

16. (C) $10\sin(t + 30^\circ) = 10\cos(t - 60^\circ)$

$$V_o = \frac{\dfrac{10\angle -60^\circ}{j} + \dfrac{20\angle -45^\circ}{3}}{\dfrac{1}{j} + \dfrac{1}{-j} + \dfrac{1}{3}}$$

$= 30\angle -150^\circ + 20\angle -45^\circ$

$V_o = 31.5\angle -112^\circ$ V

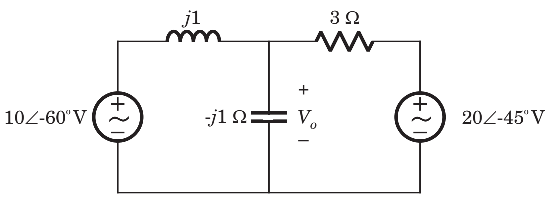

Fig. S.1.7.16

17. (C) $5\angle 0^\circ = I_1\left(j4 + 1 + 1 - \dfrac{j}{4}\right) - I_2\left(1 - \dfrac{j}{4}\right)$

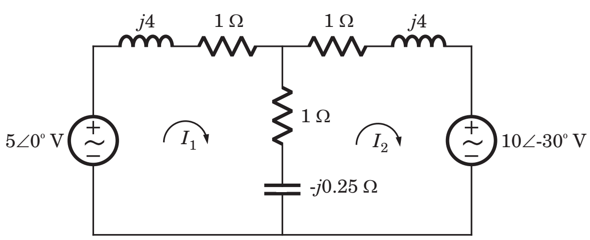

Fig. S.1.7.17

$\Rightarrow \quad (8 + j15)I_1 - (4 - j)I_2 = 20\angle 0$ ...(i)

$-10\angle -30^\circ = I_2\,(1 + j4 + 1 - \dfrac{j}{4}) - I_1\,(1 - \dfrac{j}{4})$

$\Rightarrow \quad (4 - j)I_1 - (8 + j15)I_2 = 40\angle -30^\circ$ ...(ii)

$I_1[(8 + j15)^2 - (4 - j)^2]$

$= (20\angle 0)(8 + j15) - (40\angle -30^\circ)(4 - j)$

$I_1(-176 + j248) = 41.43 + j414.64$

$\Rightarrow \quad I_1 = 1.03 - j0.9 = 1.37\angle -41.07$

18. (B) $I_2 = \dfrac{(8 + j15)(1.03 - j0.9) - 20\angle 0^\circ}{4 - j}$

$= -0.076 + j2.04 \quad \Rightarrow \quad I_2 = 2.04\angle 92.13^\circ$

19. (B) $10\angle 30^\circ = 4I_1 - 0.5I_x + (-j2)I_x$

$(-j2)I_x = (I_1 - I_x)j3,\ I_1 = \dfrac{I_x}{3}$

$10\angle 30^\circ = \left(\dfrac{4}{3} - 0.5 - j2\right)I_x \quad \Rightarrow \quad I_x = \dfrac{10\angle 30^\circ}{2.17\angle -67.38^\circ}$

20. (B) Let V_o be the voltage across current source

$\dfrac{V_o - 4V_x}{20} + \dfrac{V_o - V_x}{j10} = 3$

$V_o(20 + j10) - (20 + j40)V_x = j600$

$V_x = \dfrac{V_o(20)}{20 + j10} \quad \Rightarrow \quad V_o = \dfrac{V_x}{2}(2 + j)$

$V_x = \left(\dfrac{(2 + j)(20 + j10)}{2} - 20(1 + j2)\right) = j600$

$V_x = \dfrac{j600}{-5 - j20} = 29.22\angle -166^\circ$

21. (A) $I_1 = V_3\left(\dfrac{j}{2}\right) + \dfrac{V_3 - V_2}{j10} = j0.1V_2 + j0.4V_3$

$= (0.1\angle 90^\circ)(0.757\angle 66.7^\circ) + (0.4\angle 90^\circ)(0.606\angle -69.8^\circ)$

$\Rightarrow \quad I_1 = 0.196\angle 35.6^\circ$

22. (A) $\dfrac{V_o}{2} + \dfrac{V_o - 3V_o}{j4} = 4\angle -30^\circ$

$V_o(0.5 + j0.5) = 3.46 - j2 \quad \Rightarrow \quad V_o = 5.65\angle -75^\circ$

23. (D) $I_2 = 4\angle 90^\circ$, $I_3 = 2\angle 0^\circ$

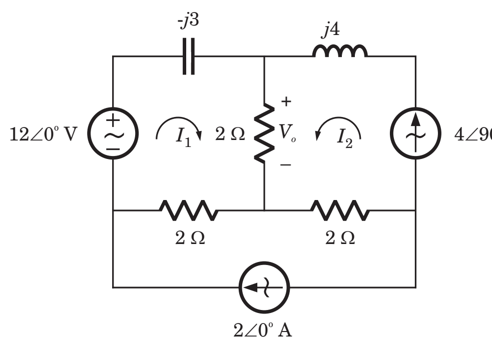

Fig. P1.7.23

$12\angle 0^\circ = I_1(-j3 + 2 + 2) + 8\angle 90^\circ - 4\angle 0^\circ$

$\Rightarrow \quad I_1 = 3.52 + j0.64$

$V_o = 2(3.52 + j0.64 + j4) = 11.65\angle 52.82^\circ$ V

24. (D) $I_2 = 3\angle 0^\circ$ A , $I_4 - I_3 = 6\angle 0^\circ$ A

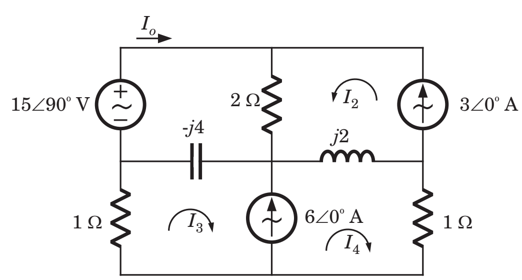

Fig. S.1.7.24

$I_3(1) + (I_3 - I_o)(-j4) + (I_4 + I_2)(j2) + I_4 = 0$

$I_3 + (I_3 - I_o)(-j4) + (I_3 + 6\angle 0° + 3\angle 0°)(j2) + I_3 + 60° = 0$

$I_3(2 - j2) + I_o(j4) = -18j - 6$

$$I_3 = \frac{-I_o(j2) - 3 - 9j}{(1 - j)}$$

$\Rightarrow \quad I_3 = I_o + 3 - j6$

$15\angle 90° = (I_o + 3\angle 0°)(2) + (I_o - I_3)(-j4)$

$\Rightarrow \quad j15 = 2I_o + 6 + (j4)(3 - j6)$

25. (A) $Z_{TH} = \dfrac{(j10)(8 - j5)}{8 + j10 - j5} = 9 + j4.4$

$V_{TH} = \dfrac{(32\angle 0°)(j10)}{8 + j10 - j5} = 33.9\angle 58°$ V

26 (D) $(600 - j300)I_1 + j300I_2 = 9$ ...(i)

$300I_2 = 3V_1$, $V_1 = (-j300)(I_1 - I_2)$

$\Rightarrow \quad I_2 = -j3(I_1 - I_2) \quad \Rightarrow \quad 3I_1 = (3 + j)I_2$ ...(ii)

Solving (i) and (ii) $I_2 = 12.36\angle -16°$ mA

$V_{oc} = 300I_2 = 3.71\angle -16°$

$-2V_1 - V_1 = 0 \quad \Rightarrow \quad V_1 = 0$

$\Rightarrow \quad I_{sc} = \dfrac{9\angle 0°}{600} = 15\angle 0°$ mA

$Z_{TH} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{3.11\angle -16°}{15\angle 0° \times 10^{-3}} = 247\angle -16°\,\Omega$

27. (C) $S = P - jQ = 269 - j150$ VA

28. (D) $pf = \cos\theta = 0.9 \quad \Rightarrow \quad \theta = 25.84°$

$Q = S\sin\theta \quad \Rightarrow \quad S = \dfrac{Q}{\sin\theta} = \dfrac{2000}{\sin 25.84°} = 4588.6$ VA

$P = S\cos\theta = 4129.8,$

$S = 4129.8 - j2000$

29. (A) $Q = S\sin\theta \quad \Rightarrow \quad \sin\theta = \dfrac{Q}{S} = \dfrac{45}{60}$ or

$\Rightarrow \quad \theta = 48.59°,$

$P = S\cos\theta = 39.69,$

$S = 39.69 + j45$ VA

30. (B) $S = \dfrac{|V_{rms}|^2}{|Z|} = \dfrac{(220^2)}{40} = 1210$

$\cos\theta = \dfrac{P}{S} = \dfrac{1000}{1210} = 0.8264$ or $\theta = 34.26°,$

$Q = S\sin\theta = 681.25,$

$S = 1000 + j681.25$ VA

31. (C) $S = V_{rms}I^*_{rms} = (21\angle 20°)(8.5\angle 50°)$

$= 61 + j167.7$ VA

32. (A) $S = \dfrac{|V|^2}{Z^*} = \dfrac{(120)^2}{40 - j80} = 72 + j144$ VA

33. (A) $S_1 = 16 + j\dfrac{16}{0.9}\sin(\cos^{-1}(0.9)) = 16 + j7.75$

$S_2 = 20 + j\dfrac{20}{0.8}\sin(\cos^{-1}(0.8)) = 20 + j15$

$S = S_1 + S_2 = 36 + j2.75 = 42.59\angle 32.29°$

$S = V_oI^* = 6V_o \quad \Rightarrow \quad V_o = 7.1\angle 32.29°$

34. (B) $Z = 30 + j(0.5)(2\pi)(50) = 30 + j157,$

$S = \dfrac{|V|^2}{Z^*} = \dfrac{(210)^2}{30 - j157}$

Apparent power $= |S| = \dfrac{(210)^2}{\sqrt{30^2 + 152^2}} = 275.6$ VA

35. (D) $Z = 4 + \dfrac{(-j2)(j5 - j2)}{-j2 + j5 - j2}$

$= 4 - j6 = 7.21\angle -56.31°,$

$pf = \cos 56.31° = 0.555$ leading

36. (A) $\dfrac{V_1}{4} + \dfrac{3}{4}V_1 = \dfrac{10 - V_1}{1 - j1.5} \quad \Rightarrow \quad V_1 = 4\angle 36.9°,$

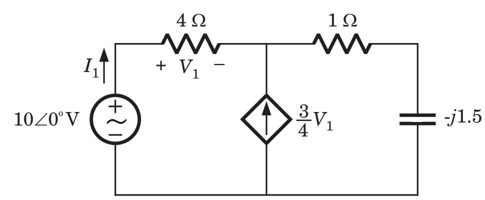

Fig. S.1.7.36

$I_1 = 1\angle 36.9°$

$S = \dfrac{(1\angle 36.9°)(10\angle 0°)}{2} = 5\angle -36.9°$

$pf = \cos 36.9° = 0.8$ leading

37. (A) $(2\angle -90°)4.8 = -I_x(4.8 + j1.92) + 0.6I_x(8)$

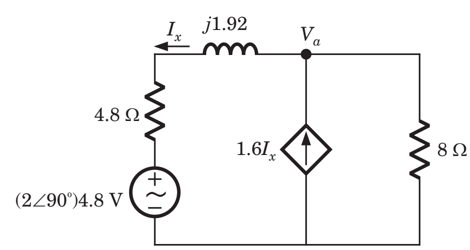

Fig. S.1.7.37

$I_x = 5\angle 0°, \qquad V_a = 0.6 \times 5 \times 8 = 24\angle 0°,$

$P_{ave} = \dfrac{1}{2} \times 24 \times 1.6 \times 5 = 96$

38. (A) $Z_{TH} = \frac{(-j10)(10+j15)}{10+j15-j10} = 8 - j14\,\Omega$

$V_{TH} = \frac{120(-j10)}{10+j5} = 107.3\angle -116.6°\ \text{V}$

$I_L = \frac{107.3\angle -116.6°}{16} = 6.7\angle -116.6°$

$P_{Lmax} = \frac{1}{2}(6.7)^2 \times 8 = 180\ \text{W}$

39. (B) $Z_{TH} = \frac{(-j40)(80+j100)}{80+j60} = 12.8 - j49.6\,\Omega$

40. (B) $V_a = \frac{400}{\sqrt{3}}\angle -30° = 231\angle -30°\ \text{V}$

$V_b = 231\angle -150°\ \text{V},\ V_c = 231\angle -270°\ \text{V}$

41. (C) For the *acb* sequence

$V_{ab} = V_a - V_b = V_p\angle 0° - V_p\angle 120°$

$400 = V_p\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = V_p\sqrt{3}\angle -30°$

$\Rightarrow \quad V_p = \frac{400}{\sqrt{3}}\angle 30°$

$V_a = V_p\angle 0° = 231\angle 30°\ \text{V},$

$V_b = V_p\angle 120° = 231\angle 150°\ \text{V}$

$V_c = V_p\angle 240° = 231\angle -90°\ \text{V}$

42. (B) $V_A = 277\angle(45° - 120°) = 277\angle -75°\ \text{V}$

$V_B = 277\angle(45° + 120°) = 277\angle 165°\ \text{V}$

$V_{AB} = V_A - V_B = 480\angle -45°\ \text{V}$

43. (C) $Z_A = 6\,||\,12 = 4,$

$I_P = \frac{480}{4} = 120\ \text{A}_{\text{rms}}$

$I_L = \sqrt{3}I_P = 208\ \text{A}_{\text{rms}}$

44 (B) $I = \frac{I_{aA}(10+j4)}{(10+j4)+(4+j4)} = 10\angle 20°$

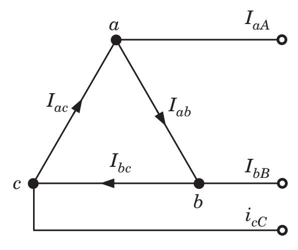

Fig. S.1.7.44

$I_{aA} = 15\angle -27.9°\ \text{A}_{\text{rms}}$

$I_{ab} = -\frac{|I_{aA}|}{\sqrt{3}}\angle(\theta + 30°) = 8.67\angle -122.1°\ \text{A}_{\text{rms}}$

45. (D) $I_{AB} = \frac{I_{aA}}{\sqrt{3}}\angle(\theta + 30°) = 16.22\angle 1.34°\ \text{A}_{\text{rms}}$

$V_{AB} = I_{AB}\cdot Z_{\Delta} = (16.22\angle 1.340°)(10 + j8)$

$= 207.8\angle 40°\ \text{V}_{\text{rms}}$

46. (B) $|S| = \sqrt{3}V_L I_L \quad \Rightarrow \quad I_L = \frac{3600}{208\sqrt{3}} = 10\ \text{A}_{\text{rms}}$

$Z_Y = \frac{208}{10\sqrt{3}}\angle 25° = 12\angle 25° = 10.88 + j5.07\,\Omega$

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CHAPTER

1.8

CIRCUIT ANALYSIS IN THE S-DOMAIN

1. $Z(s) = ?$

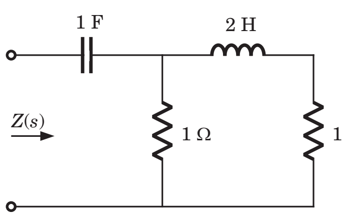


Fig. P1.8.1

(A) $\dfrac{s^2 + 1.5s + 1}{s(s+1)}$

(B) $\dfrac{s^2 + 3s + 1}{s(s+1)}$

(C) $\dfrac{2s^2 + 3s + 2}{s(s+1)}$

(D) $\dfrac{2s^2 + 3s + 1}{2s(s+1)}$

2. $Z(s) = ?$

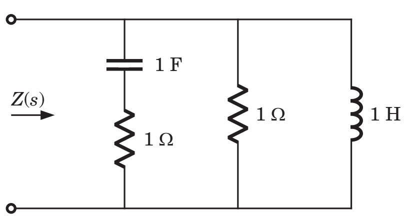


Fig. P1.8.2

(A) $\dfrac{s^2 + s + 1}{s(s+1)}$

(B) $\dfrac{2s^2 + s + 1}{s(s+1)}$

(C) $\dfrac{s(s+1)}{2s^2 + s + 1}$

(D) $\dfrac{s(s+1)}{s^2 + s + 1}$

3. $Z(s) = ?$

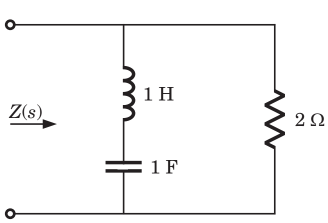


Fig. P1.8.3

(A) $\dfrac{s^2 + 1}{s^2 + 2s + 1}$

(B) $\dfrac{2(s^2 + 1)}{(s+1)^2}$

(C) $\dfrac{2s^2 + 1}{s^2 + 2s + 2}$

(D) $\dfrac{s^2 + 1}{3s + 2}$

4. $Z(s) = ?$

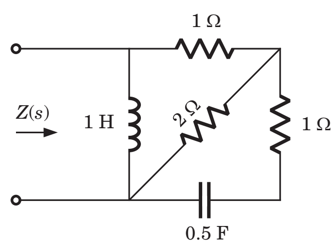


Fig. P1.8.4

(A) $\dfrac{3s^2 + 8s + 7}{s(5s + 6)}$

(B) $\dfrac{s(5s + 6)}{3s^2 + 8s + 7}$

(C) $\dfrac{3s^2 + 7s + 6}{s(5s + 6)}$

(D) $\dfrac{s(5s + 6)}{3s^2 + 7s + 6}$

5. The s-domain equivalent of the circuit of Fig.P1.8.5. is

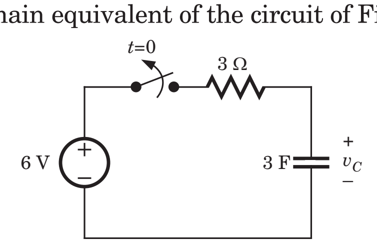
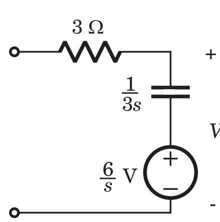
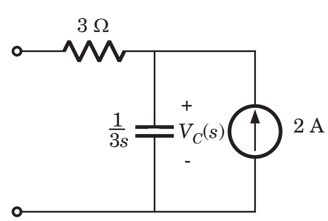


Fig. P1.8.5



(A)



(B)

(C) Both A and B

(D) None of these

6. The s–domain equivalent of the circuit shown in Fig. P1.8.6 is

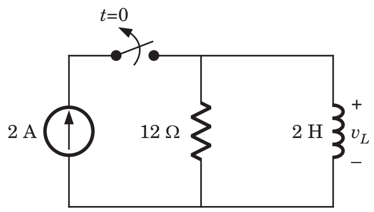
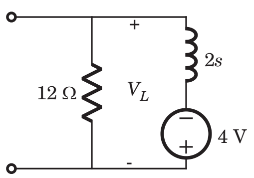
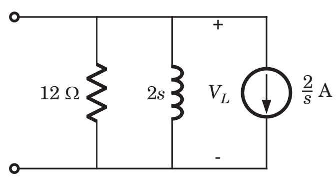


Fig. P1.8.6

(A)



(B)



(C) Both A and B

(D) None of these

Statement for Q.7-8:

The circuit is as shown in fig. P1.8.7–8. Solve the problem and choose correct option.

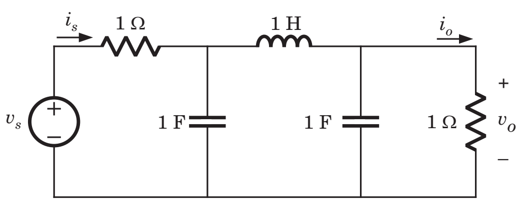


Fig. P1.8.7–8

7. $H_1(s) = \dfrac{V_o(s)}{V_s(s)} = ?$

(A) $s(s^3 + 2s^2 + 3s + 1)^{-1}$

(B) $(s^3 + 3s^2 + 2s + 1)^{-1}$

(C) $(s^3 + 2s^2 + 3s + 2)^{-1}$

(D) $s(s^3 + 3s^2 + 2s^2 + 2)^{-1}$

8. $H_2(s) = \dfrac{I_o(s)}{V_s(s)} = ?$

(A) $\dfrac{-s}{(s^3 + 3s^2 + 2s + 1)}$

(B) $-(s^3 + 3s^2 + 2s + 1)^{-1}$

(C) $\dfrac{-s}{(s^3 + 2s^2 + 3s + 1)}$

(D) $(s^3 + 2s^2 + 3s + 2)^{-1}$

9. For the network shown in fig. P1.8.9 voltage ratio transfer function G_{12} is

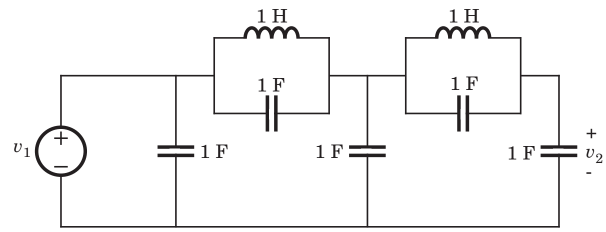


Fig. P1.8.9

(A) $\dfrac{(s^2 + 2)}{5s^4 + 5s^2 + 1}$

(B) $\dfrac{s^2 + 1}{5s^4 + 5s^2 + 1}$

(C) $\dfrac{(s^2 + 2)^2}{5s^4 + 5s^2 + 1}$

(D) $\dfrac{(s^2 + 1)^2}{5s^4 + 5s^2 + 1}$

10. For the network shown in fig. P1.8.10, the admittance transfer function is

$$Y_{12} = \frac{K(s+1)}{(s+2)(s+4)}$$

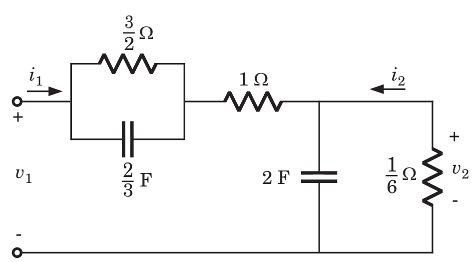


Fig. P1.8.10

The value of K is

(A) –3

(B) 3

(C) $\dfrac{1}{3}$

(D) $-\dfrac{1}{3}$

11. In the circuit of fig. P1.8.11 the switch is in position 1 for a long time and thrown to position 2 at $t = 0$. The equation for the loop currents $I_1(s)$ and $I_2(s)$ are

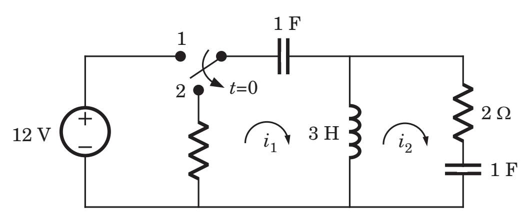


Fig. P1.8.11

(A) $\begin{bmatrix} 2 + 3s + \frac{1}{s} & -3s \\ -3s & 2 + \frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{12}{s} \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} 2+3s+\frac{1}{s} & -3s \\ -3s & 2+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\frac{12}{s} \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 2+3s+\frac{1}{s} & -3s \\ -3s & 2+3s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\frac{12}{s} \\ 0 \end{bmatrix}$

(D) $\begin{bmatrix} 2+3s+\frac{1}{s} & -3s \\ -3s & 2+3s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{12}{s} \\ 0 \end{bmatrix}$

12. In the circuit of fig. P1.8.12 at terminal *ab* Thevenin equivalent is

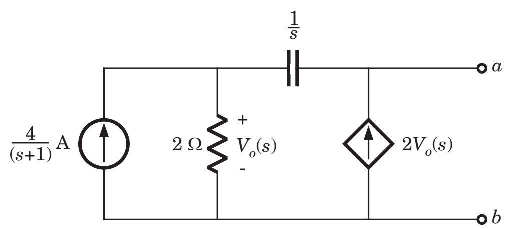


Fig. P1.8.12

(A) $V_{TH}(s) = \dfrac{-8(s+2)}{3s(s+1)}$, $Z_{TH}(s) = \dfrac{-(2s+1)}{3s}$

(B) $V_{TH}(s) = \dfrac{8(s+2)}{3s(s+1)}$, $Z_{TH}(s) = \dfrac{(2s+1)}{3s}$

(C) $V_{TH}(s) = \dfrac{4(s+3)}{3s(s+1)}$, $Z_{TH}(s) = \dfrac{(2s+1)}{6s}$

(D) $V_{TH}(s) = \dfrac{-4(s+3)}{3s(s+1)}$, $Z_{TH}(s) = \dfrac{-(2s+1)}{6s}$

13. In the circuit of fig. P1.8.13 just before the closing of switch at $t = 0$, the initial conditions are known to be $v_{C1}(0^-) = 1$ V, $v_{C2}(0^-) = 0$. The voltage $v_{C1}(t)$ is

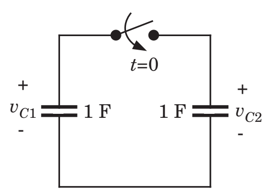


Fig. P1.8.13

(A) $u(t)$ V (B) $0.5u(t)$ V

(C) $0.5e^{-t}$ V (D) e^{-t} V

14. The initial condition at $t = 0^-$ of a switched capacitor circuit are shown in Fig. P1.8.14. Switch S_1 and S_2 are closed at $t = 0$. The voltage $v_a(t)$ for $t > 0$ is

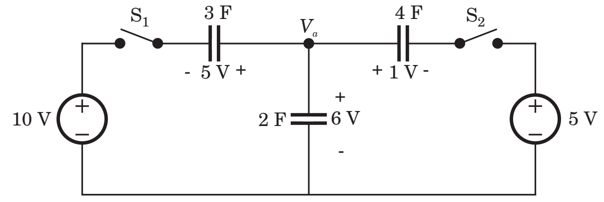


Fig. P1.8.14

(A) $\frac{9}{t}$ V (B) $9e^{-t}$ V

(C) 9 V (D) 0 V

15. A unit step current of 1 A is applied to a network whose driving point impedance is

$$Z(s) = \frac{V(s)}{I(s)} = \frac{(s+3)}{(s+2)^2}$$

The steady state and initial values of the voltage developed across the source would be respectively

(A) $\frac{3}{4}$ V, I V (B) $\frac{1}{4}$ V, $\frac{3}{4}$ V

(C) $\frac{3}{4}$ V, 0 V (D) 1 V, $\frac{3}{4}$ V

16. In the circuit of Fig. P1.8.16 $i(0) = 1$ A, $v_C(0) = 8$ V and $v_1 = 2e^{-2\times10^4 t}u(t)$. The $i(t)$ is

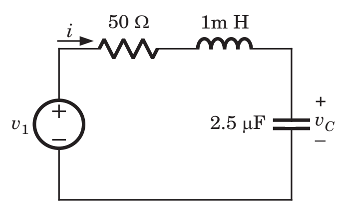


Fig. P1.8.16

(A) $\frac{1}{15}[10e^{-10^4 t} - 3e^{-2\times10^4 t} - 22e^{-4\times10^4 t}]u(t)$ A

(B) $\frac{1}{15}[-10e^{-10^4 t} + 3e^{-2\times10^4 t} + 22e^{-4\times10^4 t}]u(t)$ A

(C) $\frac{1}{3}[10e^{-10^4 t} + 3e^{-2\times10^4 t} + 22e^{-4\times10^4 t}]u(t)$ A

(D) $\frac{1}{3}[-10e^{-10^4 t} + 3e^{-2\times10^4 t} - 22e^{-4\times10^4 t}]u(t)$ A

17. In the circuit shown in Fig. P1.8.18 $v(0^-) = 8$ V and $i_{in}(t) = 4\delta(t)$. The $v_C(t)$ for $t \geq 0$ is

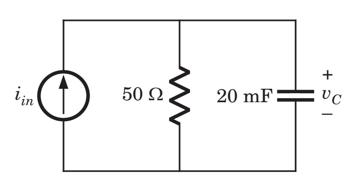


Fig. P1.8.17

(A) $164e^{-t}$ V (B) $208e^{-t}$ V

(C) $208(1 - e^{-3t})$ V (D) $164e^{-3t}$ V

18. The driving point impedance $Z(s)$ of a network has the pole zero location as shown in Fig. P1.8.18. If $Z(0) = 3$, the $Z(s)$ is

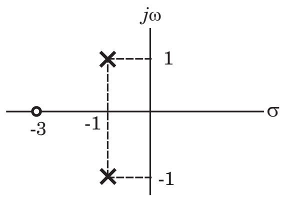


Fig. P1.8.18

(A) $\frac{4(s+3)}{s^2+s+1}$ (B) $\frac{2(s+3)}{s^2+2s+2}$

(C) $\frac{2(s+3)}{s^2+2s+2}$ (D) $\frac{4(s+3)}{s^2+s+2}$

Statement for Q.19-21:

The circuit is as shown in the fig. P1.8.19–21. All initial conditions are zero.

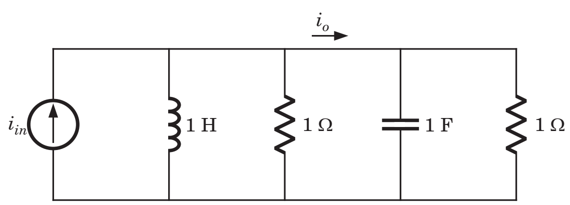


Fig. P1.8.19–21

19. $\frac{I_o(s)}{I_{in}(s)} = ?$

(A) $\frac{(s+1)}{2s}$ (B) $2s(s+1)^{-1}$

(C) $(s+1)s^{-1}$ (D) $s(s+1)^{-1}$

20. If $i_{in}(t) = 4\delta(t)$ then $i_o(t)$ will be

(A) $4\delta(t) - e^{-t}u(t)$ A

(B) $4\delta(t) - 4e^{-t}u(t)$ A

(C) $4e^{-t}u(t) - 4\delta(t)$ A

(D) $e^{-t}u(t) - \delta(t)$ A

21. If $i_{in}(t) = tu(t)$ then $i_o(t)$ will be

(A) $e^{-t}u(t)$ A (B) $(1 - e^{-t})u(t)$ A

(C) $u(t)$ A (D) $(2 - e^{-t})u(t)$ A

22. The voltage across 200 μF capacitor is given by

$$V_C(s) = \frac{2s+6}{s(s+3)}$$

The steady state voltage across capacitor is

(A) 6 V (B) 0 V

(C) ∞ (D) 2 V

23. The transformed voltage across the 60 μF capacitor is given by

$$V_C(s) = \frac{20s+6}{(10s+3)(s+4)}$$

The initial current through capacitor is

(A) 0.12 mA (B) – 0.12 mA

(C) 0.48 mA (D) – 0.48 mA

24. The current through an 4 H inductor is given by

$$I_L(s) = \frac{10}{s(s+2)}$$

The initial voltage across inductor is

(A) 40 V (B) 20 V

(C) 10 V (D) 5 V

25. The amplifier network shown in fig. P1.8.25 is stable if

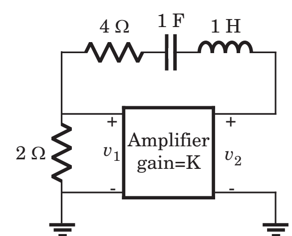


Fig.P1.8.25

(A) $K \le 3$ (B) $K \ge 3$

(C) $K \le \frac{1}{3}$ (D) $K \ge \frac{1}{3}$

26. The network shown in fig. P1.8.26 is stable if

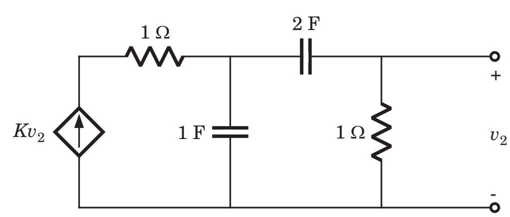


Fig.P1.8.26

(A) $K \ge \frac{5}{2}$ (B) $K \le \frac{5}{2}$

(C) $K \ge \frac{2}{5}$ (D) $K \le \frac{2}{5}$

27. A circuit has a transfer function with a pole $s = -4$ and a zero which may be adjusted in position as $s = -a$ The response of this system to a step input has a term of form Ke^{-4t}. The K will be (H= scale factor)

(A) $H\left(1-\frac{a}{4}\right)$ (B) $H\left(1+\frac{a}{4}\right)$

(C) $H\left(4-\frac{a}{4}\right)$ (D) $H\left(4+\frac{a}{4}\right)$

28. A circuit has input $v_{in}(t) = \cos 2t\, u(t)$ V and output $i_o(t) = 2\sin 2t\, u(t)$ A. The circuit had no internal stored energy at $t = 0$. The admittance transfer function is

(A) $\frac{2}{s}$ (B) $\frac{s}{2}$

(C) s (D) $\frac{1}{s}$

29. A two terminal network consists of a coil having an inductance L and resistance R shunted by a capacitor C. The poles of the driving point impedance function Z of this network are at $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ and zero at -1. If $Z(0) = 1$ the value of R, L, C are

(A) 3 Ω, 3 H, $\frac{1}{3}$ F (B) 2 Ω, 2 H, $\frac{1}{2}$ F

(C) 1 Ω, 2 H, $\frac{1}{2}$ F (D) 1 Ω, 1 H, 1 F

30. The current response of a network to a unit step input is

$$I_o = \frac{10(s+2)}{s^2(s+11s+30)}$$

The response is

(A) Under damped (B) Over damped

(C) Critically damped (D) None of the above

Statement for Q.31-33:

The circuit is shown in fig. P1.8.31-33.

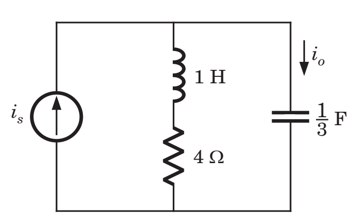


Fig. P1.8.31-33

31. The current ratio transfer function $\frac{I_o}{I_s}$ is

(A) $\frac{s(s+4)}{s^2+3s+4}$ (B) $\frac{s(s+4)}{(s+1)(s+3)}$

(C) $\frac{s^2+3s+4}{s(s+4)}$ (D) $\frac{(s+1)(s+3)}{s(s+4)}$

32. The response is

(A) Over damped (B) Under damped

(C) Critically damped (D) can't be determined

33. If input i_s is $2u(t)$ A, the output current i_o is

(A) $(2e^{-t} - 3te^{-3t})u(t)$ A (B) $(3te^{-t} - e^{-3t})u(t)$ A

(C) $(3e^{-t} - e^{-3t})u(t)$ A (D) $(e^{-3t} - 3e^{-t})u(t)$ A

34. In the network of Fig. P1.8.34, all initial condition are zero. The damping exhibited by the network is

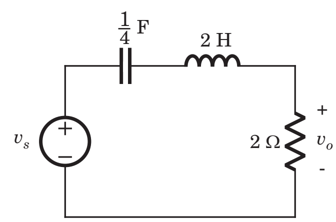


Fig. P1.8.34

(A) Over damped

(B) Under damped

(C) Critically damped

(D) value of voltage is requires

35. The voltage response of a network to a unit step input is

$$V_o(s) = \frac{10}{s(s^2+8s+16)}$$

The response is

(A) under damped (B) over damped

(C) critically damped (D) can't be determined

36. The response of an initially relaxed circuit to a signal v_s is $e^{-2t}u(t)$. If the signal is changed to $\left(v_s + \frac{2dv_s}{dt}\right)$, the response would be

(A) $5e^{-2t}u(t)$ (B) $-3e^{-2t}u(t)$

(C) $4e^{-2t}u(t)$ (D) $-4e^{-2t}u(t)$

37. Consider the following statements in the circuit shown in fig. P1.8.37

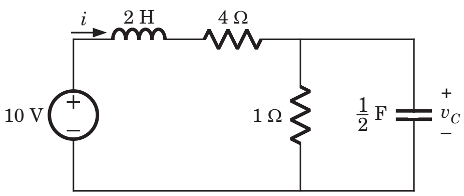


Fig. P1.8.37

1. It is a first order circuit with steady state value of $v_C = \frac{10}{3}$, $i = \frac{5}{3}$ A

2. It is a second order circuit with steady state of $v_C = 2$ V, $i = 2$ A

3. The network function $\frac{V(s)}{I(s)}$ has one pole.

4. The network function $\frac{V(s)}{I(s)}$ has two poles.

The true statements are

(A) 1 and 3 (B) 1 and 4

(C) 2 and 3 (D) 2 and 4

38. The network function $\frac{s^2 + 10s + 24}{s^2 + 8s + 15}$ represent a

(A) *RC* admittance (B) RL impedance

(C) LC impedance (D) None of the above

39. The network function $\frac{s(s+4)}{(s+1)(s+2)(s+3)}$ represents an

(A) *RC* impedance (B) *RL* impedance

(C) *LC* impedance (D) None of these

40. The network function $\frac{s(3s+8)}{(s+1)(s+3)}$ represents an

(A) *RL* admittance (B) *RC* impedance

(C) *RC* admittance (D) None of the above

41. The network function $\frac{(s+1)(s+4)}{s(s+2)(s+5)}$ is a

(A) *RL* impedance function

(B) *RC* impedance function

(C) *LC* impedance function

(D) Above all

42. The network function $\frac{s^2 + 7s + 6}{s+2}$ is a

(A) *RL* impedance function (B) *RL* admittance

(C) *LC* impedance function (D) *LC* admittance

43. A valid immittance function is

(A) $\frac{(s+4)(s+8)}{(s+2)(s-5)}$ (B) $\frac{s(s+1)}{(s+2)(s+5)}$

(C) $\frac{s(s+2)(s+3)}{(s+1)(s+4)}$ (D) $\frac{s(s+2)(s+6)}{(s+1)(s+4)}$

44. The network function $\frac{s^2 + 8s + 15}{s^2 + 6s + 8}$ is a

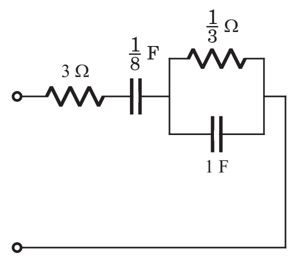
(A) *RL*admittance (B) *RC* admittance

(C) *LC* admittance (D) Above all

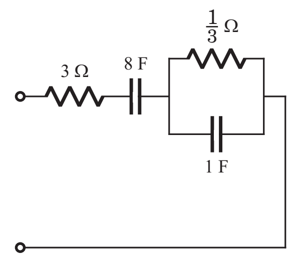
45. A impedance function is given as

$$Z(s) = \frac{3(s+2)(s+4)}{s(s+3)}$$

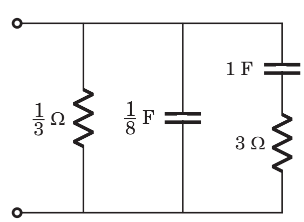
The network for this function is



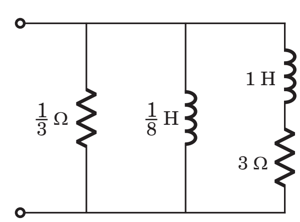
(A)



(B)



(C)



(D)

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18. (B) $Z(s)=\dfrac{K(s+3)}{(s-(-1+j))(s-(1-j))}=\dfrac{K(s+3)}{s^2+2s+2}$

$Z(0)=\dfrac{3K}{2}=3 \quad\Rightarrow\quad K=2$

19. (D) $\dfrac{I_o(s)}{I_{in}(s)}=\dfrac{\dfrac{s}{s+1}}{\dfrac{s}{s+1}+\dfrac{1}{s+1}}=\dfrac{s}{s+1}$

20. (B) $I_{in}(s)=4$

$I_o(s)=\dfrac{4s}{s+1}=4-\dfrac{4}{s+1} \quad\Rightarrow\quad i_o(t)=4\delta(t)-4e^{-t}u(t)$

21. (B) $I_{in}(s)=\dfrac{1}{s^2}$,

$I_o(s)=\dfrac{1}{s(s+1)}=\dfrac{1}{s}-\dfrac{1}{s+1}$

$i_o(t)=u(t)-e^{-t}u(t)=(1-e^{-t})u(t)$

22. (D) $v_C(\infty)=\lim\limits_{s\to 0} sV_C(s)=\lim\limits_{s\to 0}\dfrac{2s+6}{s+3}=2\text{ V}$

23. (D) $v_C(0^+)=\lim\limits_{s\to\infty} sV_C(s)=\dfrac{s(20s+6)}{(10s+3)(s+4)}=2\text{ V}$

$i_C=\dfrac{Cdv_C}{dt} \quad\Rightarrow\quad I_C(s)=C[sV_C(s)-v_C(0^+)]$

$=60\times10^{-6}\left(\dfrac{s(20s+6)}{(10s+3)(s+4)}-2\right)=\dfrac{-480\times10^{-6}(10s+3)}{10s^2+43s+12}$

$i_C(0^+)=\lim\limits_{s\to\infty} sI_C(s)=-480\times10^{-6}=-0.48\text{ mA}$

24. (A) $v_L=L\dfrac{di_L}{dt}\Rightarrow V_L(s)=L[sI_L(s)-i_L(0^+)]$

$i_L(0^+)=\lim\limits_{s\to\infty} sI_L(s)=\dfrac{10}{s+2}=0$

$V_L(s)=\dfrac{40s}{s(s+2)}=\dfrac{40}{s+2}$

$v_L(0^+)=\lim\limits_{s\to\infty} sV_L(s)=\dfrac{s40}{s+2}=40$

25. (A) $V_2(s)=KV_1(s)$

$\Rightarrow\quad \dfrac{V_1(s)}{2}+\dfrac{V_1(s)-KV_1(s)}{4+s+\dfrac{1}{s}}=0$

$4+s+\dfrac{1}{s}+2-2K=0$

$\Rightarrow\quad s^2+(6-2K)s+1=0$

$(6-2K)>0 \quad\Rightarrow\quad K<3$

26. (B) Let v_1 be the node voltage of middle node

$V_1(s)=\dfrac{KV_2(s)+2sV_2(s)}{1+2s+s}$

$\Rightarrow (3s+1)V_1(s)=(2s+K)V_2(s)$

$\Rightarrow\quad V_2(s)=\dfrac{2sV_1(s)}{2s+1}$

$\Rightarrow (2s+1)V_2(s)=2sV_1(s)$

$\Rightarrow\quad (3s+1)(2s+1)=2s(2s+K)$

$2s^2+(5-2K)s+1=0,$

$5-2K>0,\ K<\dfrac{5}{2}$

27. (A) $H(s)=\dfrac{H(s+a)}{s+4}$

$R(s)=\dfrac{H(s+a)}{s(s+4)}=\dfrac{Ha}{4s}+\dfrac{H\left(1-\dfrac{a}{4}\right)}{s+4}$

$r(t)=\dfrac{Ha}{4}u(t)+H\left(1-\dfrac{a}{4}\right)e^{-4t}$

28. (A) $V_{in}(s)=\dfrac{s}{s^2+1},\ I_o(s)=\dfrac{2}{s^2+1},\ \dfrac{I_o(s)}{V_{in}(s)}=\dfrac{2}{s}$

29. (D) $Z(s)=\dfrac{(sL+R)\dfrac{1}{sC}}{sL+R+\dfrac{1}{sC}}=\dfrac{\dfrac{1}{C}\left(s+\dfrac{R}{L}\right)}{s^2+\dfrac{R}{L}+\dfrac{1}{LC}}$

$Z(s)=\dfrac{K(s+1)}{\left(s+\dfrac{1}{2}+j\dfrac{\sqrt{3}}{2}\right)\left(s+\dfrac{1}{2}-j\dfrac{\sqrt{3}}{2}\right)}=\dfrac{K(s+1)}{(s^2+s+1)}$

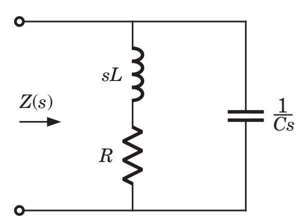

Fig. S1.8.29

Since $Z(0)=1$, thus $K=1$

$\dfrac{1}{C}=1,\ \dfrac{R}{L}=1,\ \dfrac{1}{LC}=1$

$\Rightarrow\quad C=1,\ L=1,\ R=1$

30. (B) The characteristic equation is

$s^2(s^2+11s+30)=0 \quad\Rightarrow\quad s^2(s+6)(s+5)=0$

$s=-6,\ -5$, Being real and unequal, it is overdamped.

31. (B) $\frac{I_o}{I_s} = \frac{s+4}{s+4+\frac{3}{s}} = \frac{s(s+4)}{(s+1)(s+3)}$

32. (A) The characteristic equation is $(s+1)(s+3) = 0$.

Being real and unequal root, it is overdamped response.

33. (C) $i_s = 2u(t) \quad \Rightarrow \quad I_s(s) = \frac{2}{s}$

$$I_o(s) = \frac{2(s+4)}{(s+1)(s+3)} = \frac{3}{s+1} - \frac{1}{s+3}$$

$i_o = (3e^{-t} - e^{-3t})u(t)$ A

34. (B) $\frac{V_o(s)}{V_s(s)} = \frac{2}{\frac{4}{s}+2s+2} = \frac{1}{s^2+s+2}$

The roots are imaginary so network is underdamped.

35. (C) The characteristic equation is

$s(s^2+8s+16) = 0,\ (s+4)^2 = 0,\ s = -4, -4$

Being real and repeated root, it is critically damped.

36. (B) $v_o = e^{-2t}u(t) \quad \Rightarrow \quad V_o(s) = H(s)V_s(s) = \frac{1}{s+2}$

$v_s' = v_s + \frac{2dv_s}{dt} \quad \Rightarrow \quad V_s'(s) = (1+2s)V_s(s)$

$V_o'(s) = H(s)V_s'(s) = (1+2s)V_s(s)H(s)$

$V_o'(s) = \frac{1+2s}{s+2} = 2 - \frac{3}{s+2} \quad \Rightarrow \quad v_o' = 2\delta(s) - 3e^{-2t}u(t)$

37. (C) It is a second order circuit. In steady state

$i = \frac{10}{4+1} = 2$ A , $v = 2\times 1 = 2$ V

$$I(s) = \frac{10}{2s+4+\frac{1}{1+\frac{1}{2}s}} = \frac{5(s+2)}{(s+2)^2+1}$$

$$V(s) = \frac{\frac{10}{1+\frac{1}{2}s}}{(2s+4)+\frac{1}{1+\frac{1}{2}s}} = \frac{10}{(s+2)^2+1}$$

$\frac{V(s)}{I(s)} = \frac{2}{s+2}$, It has one pole at $s = -2$

38. (D) $\frac{s^2+10s+24}{s^2+8s+15} = \frac{(s+4)(s+6)}{(s+3)(s+5)}$

The singularity near to origin is pole. So it may be RC impedance or RL admittance function.

39. (D) Poles and zero does not interlace on negative real axis so it is not a immittance function.

40. (C) The singularity nearest to origin is a zero. So it may be RL impedance or RC admittance function. Because of (D) option it is required to check that it is a valid RC admittance function. The poles and zeros interlace along the negative real axis. The residues of $\frac{Y_{RC}(s)}{s}$ are real and positive.

41. (B) The singularity nearest to origin is a pole. So it may be RC impedance or RL admittance function.

42. (A) $\frac{s^2+7s+6}{s+2} = \frac{(s+1)(s+6)}{(s+2)}$

The singularity nearest to origin is at zero. So it may be RC admittance or RL impedance function.

43. (D)

(A) pole lie on positive real axis

(B) poles and zero does not interlace on axis.

(C) poles and zero does not interlace on axis.

(D) is a valid immittance function.

44. (A) $\frac{s^2+8s+15}{s^2+6s+8} = \frac{(s+3)(s+5)}{(s+2)(s+4)}$

The singularity nearest to origin is a pole. So it may be a RL admittance or RC impedance function.

45. (A) The singularity nearest to origin is a pole. So this is RC impedance function.

$$Z(s) = 3 + \frac{8}{s} + \frac{1}{s+3} = 3 + \frac{8}{s} + \frac{1/3}{1+\frac{s}{3}}$$

****

CHAPTER

1.9

MAGNETICALLY COUPLED CIRCUITS

Statement for Q.1-2:

In the circuit of fig. P1.9.1-2 $i_1 = 4\sin 2t$ A, and $i_2 = 0$.

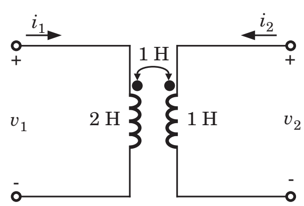

Fig. P1.9.1-2

1. v_1 = ?

(A) $-16\cos 2t$ V (B) $16\cos 2t$ V

(C) $4\cos 2t$ V (D) $-4\cos 2t$ V

2. v_2 = ?

(A) $2\cos 2t$ V (B) $-2\cos 2t$ V

(C) $8\cos 2t$ V (D) $-8\cos 2t$ V

Statement for Q.3-4:

Consider the circuit shown in Fig. P1.9.3-4

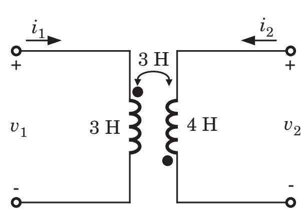

Fig. P1.9.5-6

3. If $i_1 = 0$ and $i_2 = 2\sin 4t$ A, the voltage v_1 is

(A) $24\cos 4t$ V (B) $-24\cos 4t$ V

(C) $1.5\cos 4t$ V (D) $-1.5\cos 4t$ V

4. If $i_1 = e^{-2t}$ V and $i_2 = 0$, the voltage v_2 is

(A) $-6e^{-2t}$ V (B) $6e^{-2t}$ V

(C) $1.5e^{-2t}$ V (D) $-1.5e^{-2t}$ V

Statement for Q.5-6:

Consider the circuit shown in fig. P19.5-6

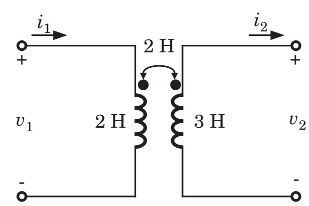

Fig. P1.9.5-6

5. If current $i_1 = 3\cos 4t$ A and $i_2 = 0$, then voltage v_1 and v_2 are

(A) $v_1 = -24\sin 4t$ V, $v_2 = -24\sin 4t$ V

(B) $v_1 = 24\sin 4t$ V, $v_2 = -36\sin 4t$ V

(C) $v_1 = 1.5\sin 4t$ V, $v_2 = \sin 4t$ V

(D) $v_1 = -1.5\sin 4t$ V, $v_2 = -\sin 4t$ V

6. If current $i_1 = 0$ and $i_2 = 4\sin 3t$ A, then voltage v_1 and v_2 are

(A) $v_1 = 24\cos 3t$ V, $v_2 = 36\cos 3t$ V

(B) $v_1 = 24\cos 3t$ V, $v_2 = -36\cos 3t$ V

(C) $v_1 = -24\cos 3t$ V, $v_2 = 36\cos 3t$ V

(D) $v_1 = -24\cos 3t$ V, $v_2 = -36\cos 3t$ V

Statement for Q.7-8:

In the circuit shown in fig. P1.9.7-8, $i_1 = 3\cos 3t$ A and $i_2 = 4\sin 3t$ A.

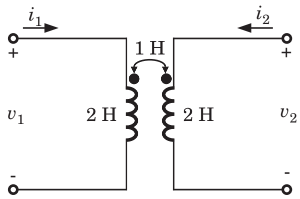

Fig. P1.9.7-8

7. v_1 = ?

(A) $6(-2\cos t + 3\sin t)$ V (B) $6(2\cos t + 3\sin t)$ V

(C) $-6(2\cos t + 3\sin t)$ V (D) $6(2\cos t - 3\sin t)$ V

8. v_2 = ?

(A) $3(8\cos 3t - 3\sin t)$ V (B) $6(2\cos t + 3\sin t)$ V

(C) $3(8\cos 3t + 3\sin 3t)$ V (D) $6(2\cos t - 3\sin t)$ V

Statement for Q.9-10:

In the circuit shown in fig. P1.9.9-10, $i_1 = 5\sin 3t$ A and $i_2 = 3\cos 3t$ A

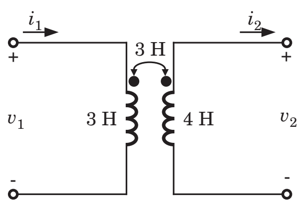

Fig. P1.9.9-10

9. v_1 =?

(A) $9(5\cos 3t + 3\sin 3t)$ V (B) $9(5\cos 3t - 3\sin 3t)$ V

(C) $9(4\cos 3t + 5\sin 3t)$ V (D) $9(5\cos 3t - 3\sin 3t)$ V

10. v_2 = ?

(A) $9(-4\sin 3t + 5\cos 3t)$ V (B) $9(4\sin 3t - 5\cos 3t)$ V

(C) $9(-4\sin 3t - 5\cos 3t)$ V (D) $9(4\sin 3t + 5\cos 3t)$ V

11. In the circuit shown in fig. P1.9.11 if current $i_1 = 5\cos(500t - 20°)$ mA and $i_2 = 20\cos(500t - 20°)$ mA, the total energy stored in system at $t = 0$ is

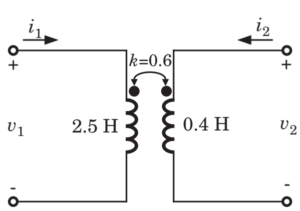

Fig. P1.9.11

(A) 151.14 μJ (B) 45.24 μJ

(C) 249.44 μJ (D) 143.46 μJ

12. L_{eq} = ?

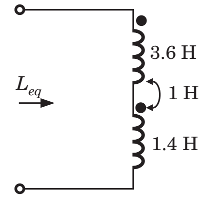

Fig. P1.9.12

(A) 4 H (B) 6 H

(C) 7 H (D) 0 H

13. L_{eq} = ?

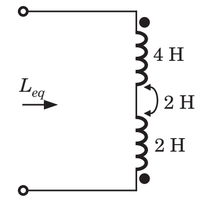

Fig. P1.9.13

(A) 2 H (B) 4 H

(C) 6 H (D) 8 H

14. L_{eq} = ?

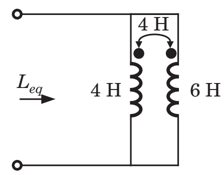

Fig. P1.9.14

(A) 8 H (B) 6 H

(C) 4 H (D) 2 H

15. L_{eq} = ?

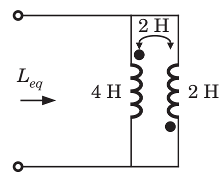

Fig. P1.9.15

(A) 0.4 H (B) 2 H

(C) 1.2 H (D) 6 H

16. The equivalent inductance of a pair of a coupled inductor in various configuration are

(a) 7 H after series adding connection

(b) 1.8 H after series opposing connection

(c) 0.5 H after parallel connection with dotted terminal connected together.

The value of L_1, L_2 and M are

(A) 3 H, 1.6 H, 1.2 H (B) 1.6 H, 3 H, 1.4 H

(C) 3.7 H, 0.7 H, 1.3 H (D) 2 H, 3 H, 3 H

17. L_{eq} = ?

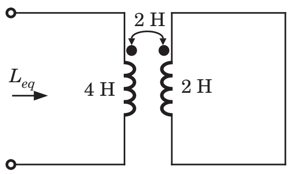

Fig. P1.9.17

(A) 0.2 H (B) 1 H

(C) 0.4 H (D) 2 H

18. L_{eq} = ?

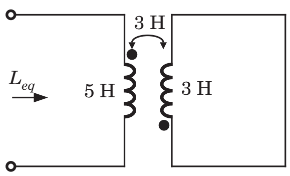

Fig. P1.9.18

(A) 1 H (B) 2 H

(C) 3 H (D) 4 H

19. In the network of fig. P1.9.19 following terminal are connected together

(i) none (ii) A to B

(iii) B to C (iv) A to C

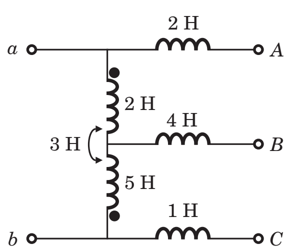

Fig. P1.9.19

The correct match for equivalent induction seen at terminal $a-b$ is

	(i)	(ii)	(iii)	(iv)
(A)	1 H	0.875 H	0.6 H	0.75 H
(B)	13 H	0.875 H	0.6 H	0.75 H
(C)	13 H	7.375 H	6.6 H	2.4375 H
(D)	1 H	7.375 H	6.6 H	2.4375 H

20. L_{eq} = ?

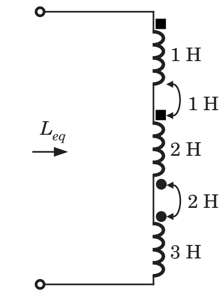

Fig. P1.9.20

(A) 1 H (B) 2 H

(C) 3 H (D) 4 H

21. L_{eq} = ?

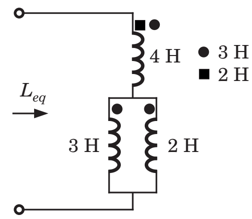

Fig. P1.9.21

(A) $\frac{41}{5}$ H (B) $\frac{49}{5}$ H

(C) $\frac{51}{5}$ H (D) $\frac{39}{5}$ H

Statement for Q.22-24:

Consider the circuit shown in fig. P1.9.22–24.

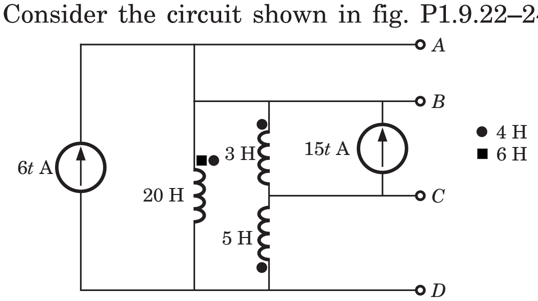

Fig. P1.9.22–24

22. The voltage V_{AG} of terminal AD is

(A) 60 V (B) –60 V

(C) 180 V (D) 240 V

23. The voltage v_{BG} of terminal BD is

(A) 45 V (B) 33 V

(C) 69 V (D) 105 V

24. The voltage v_{CG} of terminal CD is

(A) 30 V (B) 0 V

(C) –36 V (D) 36 V

33. In the circuit of fig. P1.9.33 the $\omega = 2$ rad/s. The resonance occurs when C is

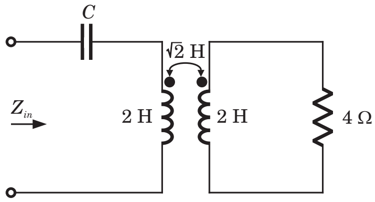

Fig. P1.9.33

(A) 1 F (B) $\frac{1}{2}$ F

(C) $\frac{1}{3}$ F (D) $\frac{1}{6}$ F

34. In the circuit of fig. P1.9.34, the voltage gain is zero at $\omega = 333.33$ rad/s. The value of C is

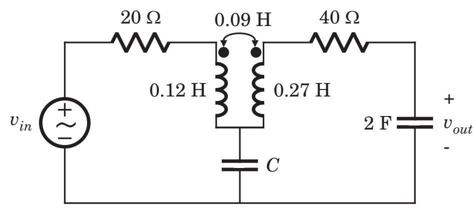

Fig. P1.9.34

(A) 100 μF (B) 75 μF

(C) 50 μF (D) 25 μF

35. In the circuit of fig. P1.9.35 at $\omega = 333.33$ rad/s, the voltage gain v_{out}/v_{in} is zero. The value of C is

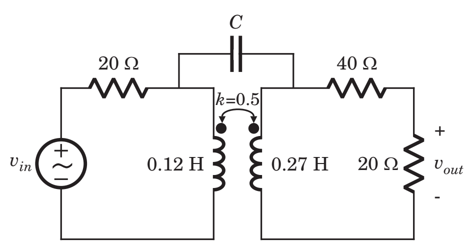

Fig. P1.9.35

(A) 3.33 mF (B) 33.33 mF

(C) 3.33 μF (D) 33.33 μF

36. The Thevenin equivalent at terminal ab for the network shown in fig. P1.9.36 is

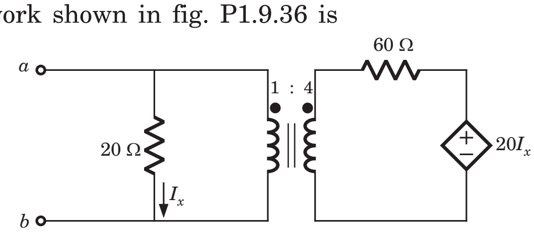

Fig. P1.9.36

(A) 6 V, 10 Ω (B) 6 V, 4 Ω

(C) 0 V, 4 Ω (D) 0 V, 10 Ω

37. In the circuit of fig. P1.9.37 the maximum power delivered to R_L is

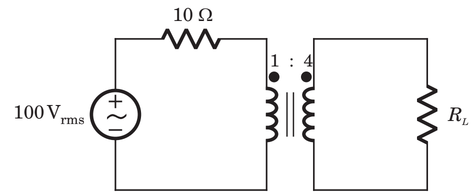

Fig. P1.9.37

(A) 250 W (B) 200 W

(C) 150 W (D) 100 W

38. The average power delivered to the 8 Ω load in the circuit of fig. P1.9.38 is

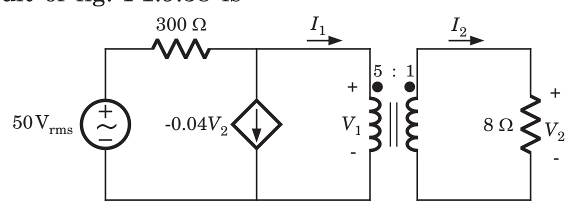

Fig. P1.9.38

(A) 8 W (B) 1.25 kW

(C) 625 kW (D) 2.50 kW

39. In the circuit of fig. P1.9.39 the ideal source supplies 1000 W, half of which is delivered to the 100 Ω load. The value of a and b are

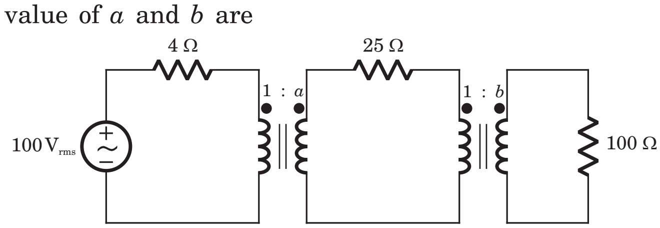

Fig. P1.9.39

(A) 6, 0.47 (B) 5, 0.89

(C) 0.89, 5 (D) 0.47, 6

40. $I_2 = ?$

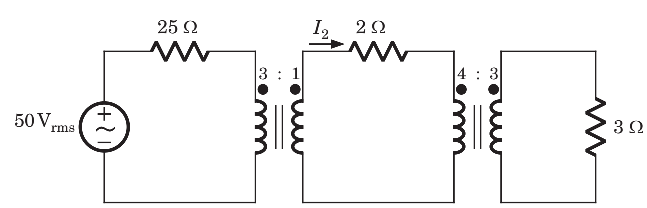

Fig. P1.9.40

(A) 1.65 A_{rms} (B) 0.18 A_{rms}

(C) 0.66 A_{rms} (D) 5.90 A_{rms}

41. $V_2 = ?$

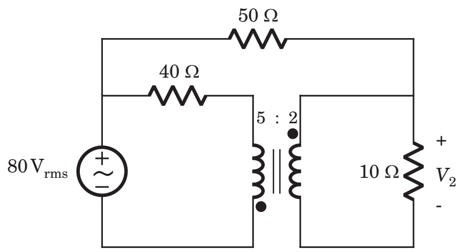

Fig. P1.9.41

(A) –12.31 V (B) 12.31 V

(C) –9.231 V (D) 9.231 V

42. The power being dissipated in 400 Ω resistor is

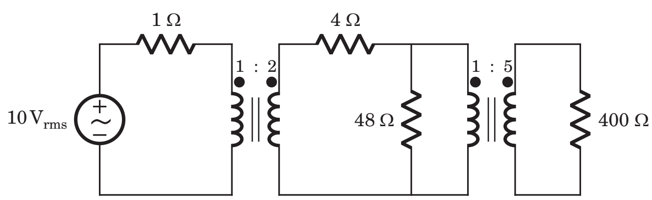

Fig. P1.9.42

(A) 3 W (B) 6 W

(C) 9 W (D) 12 W

43. $I_x = ?$

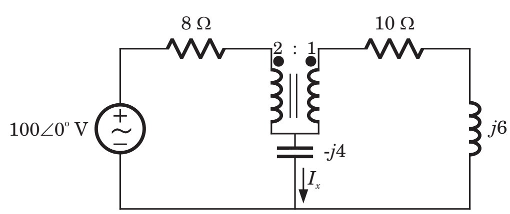

Fig. P1.9.43

(A) $1.921\angle 57.4°$ A (B) $2.931\angle 59.4°$ A

(C) $1.68\angle 43.6°$ A (D) $1.79\angle 43.6°$ A

44. $Z_{in} = ?$

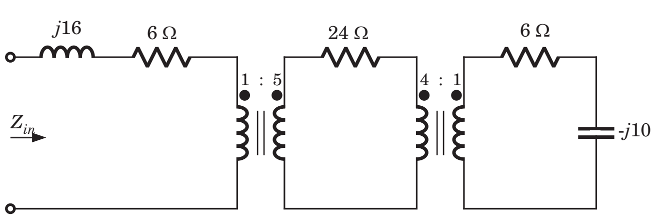

Fig. P1.9.44

(A) $46.3 + j6.8\ \Omega$ (B) $432.1 + j0.96\ \Omega$

(C) $10.8 + j9.6\ \Omega$ (D) $615.4 + j0.38\ \Omega$

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SOLUTIONS

1. (B) $v_1 = 2\frac{di_1}{dt} + 1\frac{di_2}{dt} = 2\frac{di_1}{dt} = 16\cos 2t$ V

2. (C) $v_2 = (1)\frac{di_2}{dt} + (1)\frac{di_1}{dt} = \frac{di_1}{dt} = 8\cos 2t$ V

3. (B) $v_1 = 3\frac{di_1}{dt} - 3\frac{di_2}{dt} = -3\frac{di_2}{dt} = -24\cos 4t$ V

4. (C) $v_2 = 4\frac{di_2}{dt} - 3\frac{di_1}{dt} = -3\frac{di_1}{dt} = 6e^{-2t}$ V

5. (A) $v_1 = 2\frac{di_1}{dt} - 2\frac{di_2}{dt} = 2\frac{di_1}{dt} = -24\sin 4t$ V

$v_2 = -3\frac{di_2}{dt} + 2\frac{di_1}{dt} = 2\frac{di_1}{dt} = -24\sin 4t$ V

6. (D) $v_1 = 2\frac{di_1}{dt} - 2\frac{di_2}{dt} = -2\frac{di_2}{dt} = -24\cos 3t$ V

$v_2 = 3\frac{di_2}{dt} + 2\frac{di_1}{dt} = -3\frac{di_2}{dt} = -36\cos 3t$ V

7. (A) $v_1 = 2\frac{di_1}{dt} + 1\frac{di_2}{dt}$

$= -18\sin t + 12\cos t = 6(2\cos t - 3\sin t)$ V

8. (A) $v_2 = 2\frac{di_2}{dt} + 1\frac{di_1}{dt}$

$= 24\cos 3t - 9\sin 3t = 3(8\cos 3t - 3\sin 3t)$ V

9. (A) $v_1 = 3\frac{di_1}{dt} - 3\frac{di_2}{dt}$

$= 45\cos 3t + 27\sin 3t = 9(5\cos 3t + 3\sin 3t)$ V

10. (D) $v_2 = -4\frac{di_2}{dt} + 3\frac{di_1}{dt}$

$= 36\sin 3t + 45\cos 3t = 9(4\sin 3t + 5\cos 3t)$ V

11. (A) $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$

At $t = 0$, $i_1 = 4\cos(-20°) = 4.7$ mA

$i_2 = 20\cos(-20°) = 18.8$ mA,

$M = 0.6\sqrt{2.5 \times 0.4} = 0.6$

$W = \frac{1}{2}(2.5)(4.7)^2 + \frac{1}{2}(0.4)(18.8)^2 + 0.6(4.7)(18.8)$

$= 151.3\ \mu$J

12. (C) $L_{eq} = L_1 + L_2 + 2M = 7$ H

13. (A) $L_{eq} = L_1 + L_2 - 2M = 4 + 2 - 2 \times 2 = 2$ H

14. (C) $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \dfrac{24 - 16}{6 + 4 - 8} = 4$ H

15. (A) $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \dfrac{8 - 4}{6 + 4} = 0.4$ H

16. (C) $L_1 + L_2 + 2M = 7,\ L_1 + L_2 - 2M = 1.8$

$\Rightarrow \quad L_1 + L_2 = 4.4,\ M = 1.3$

$\dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = 0.5,\ L_1 L_2 - 1.3^2 = 0.5 \times 1.8$

$L_1 L_2 = 2.59,\ (L_1 - L_2)^2 = 4.4^2 - 4 \times 2.59 = 9$

$L_1 - L_2 = 3,\ L_1 = 3.7,\ L_2 = 0.7$

17. (D) $L_{eq} = L_1 - \dfrac{M^2}{L_2} = 4 - \dfrac{4}{2} = 2$ H

18. (B) $L_{eq} = L_1 - \dfrac{M^2}{L_2} = 5 - \dfrac{9}{3} = 2$ H

19. (A)

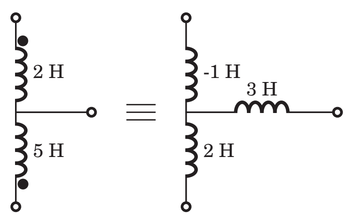


Fig. S.1.9.19

20. (D) $V_{L1} = 1sI + 1sI = 2sI$

$V_{L2} = 2sI + 1sI - 2sI = sI,$

$V_{L3} = 3sI - 2sI = sI$

$V_L = V_{L1} + V_{L2} + V_{L3} = 4sI \quad \Rightarrow \quad L_{eq} = 4$ H

21. (B) Let I_1 be the current through 4 H inductor and I_2 and I_3 be the current through 3 H, and 2 H inductor respectively

$I_1 = I_2 + I_3\ ,\ V_2 = V_3$

$3sI_2 + 3sI_1 = 2sI_3 + 2sI_1$

$\Rightarrow \quad 3I_2 + I_1 = 2I_3 \quad \Rightarrow \quad 4I_2 = I_3$

$\Rightarrow \quad I_2 = \dfrac{I_1}{5}\ ,\ I_3 = \dfrac{4}{5} I_1$

$V = 4sI_1 + 3sI_2 + 2sI_3 + 3sI_2 + 3s\,I_1$

$= 7sI_1 + \dfrac{6s}{5} I_1 + \dfrac{2 \times 4s}{5} I_3$

$V = \dfrac{49}{5} sI_1\ ,\ L_{eq} = \dfrac{49}{5}$ H

22. (C) $v_{AG} = 20 \dfrac{d(6t)}{dt} + 4 \dfrac{d(15t)}{dt} = 180$ V

23. (B) $v_{BG} = 3 \dfrac{d(15t)}{dt} + 4 \dfrac{d6(t)}{dt} - 6 \dfrac{d\,(6t)}{dt} = 33$ V

24. (C) $v_{CG} = -6 \dfrac{d(6t)}{dt} = -36$ V

25. (B) $Z = Z_{11} + \dfrac{\omega^2 M^2}{Z_{22}}$

$= 4 + j\,(50)\left(\dfrac{1}{10}\right) + \dfrac{(50)^2\left(\dfrac{1}{5}\right)^2}{5 + j\,(50)\dfrac{1}{2}}$

$= 4.77 + j\ 1.15\ \Omega$

26. (B) $V_s = j\,(0.8)10(1.2\angle 0) - j\,(0.2)(10)(2\angle 0)$

$+[3 + j(0.5)(10)](1.2\angle 0 + 2\angle 0)$

$= 9.6 + j21.6 = 26.64\angle 66.04^\circ$ V

27. (A) $[\,j(100\pi)\,(2) + 10\,]I_2 + j(100\pi)(0.4)\,(2\angle 0) = 0$

$\Rightarrow \quad I_2 = -0.4 - j0.0064,$

$V_o = 10I_2 = -4 - j0.064$

$= 4\angle -179.1^\circ$

$\Rightarrow \quad v_o = 4\cos\,(100\pi t - 179.1^\circ)$ V

28. (B) $30\angle 30^\circ = I(-j6 + j8 - j4 + j12 - j4 + 10)$

$\Rightarrow \quad I = \dfrac{30\angle 30^\circ}{(10 + j6)} = 2.57 - j0.043$

$V_o = I(\,j12 - j4 + 10)$

$= (2.57 - j0.043)(10 + 8\,j)$

$= 26.067 + j20.14 = 32.9\angle 37.7^\circ$ V

29. (A)

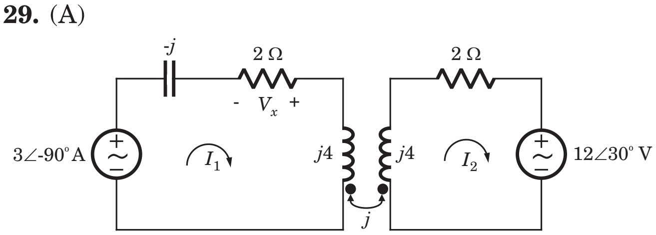


Fig. S1.9.29

$(-j + 2 + j4)I_1 - jI_2 = -j3$

$(\,j4 + 2)I_2 - jI_1 = -12\angle 30^\circ$ V

$I_1 = -1.45 - j0.56,$

$V_x = -2\,I_1 = 2.9 + j1.12$

$= 3.11\angle 21.12^\circ$ V

30. (D)

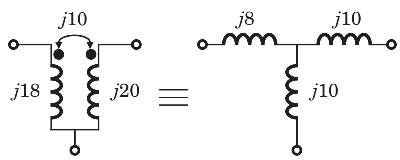


Fig. S.1.9.30

$$Z_{eq} = 10 + j8 + \frac{(j14)(j10 + 2 - j6)}{j14 + j10 + 2 - j6}$$

$$= 11.2 + j11.2\ \Omega$$

31. (C) $Z_{in} = (-j6) \,||\, (Z_o)$

$$Z_o = j20 + \frac{(12)^2}{(j30 + j5 - j2 + 4)} = 0.52 + j15.7$$

$$Z_{in} = \frac{(-j6)(0.52 + j15.7)}{(-j6 + 0.52 + j15.7)} = 0.20 - j9.7\ \Omega$$

32. (D) $M = k\sqrt{L_1 L_2}$, $\quad M^2 = 160 \times 10^{-12}$

$$Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$$

$$= j250 \times 10^3 \times 2 \times 10^{-6} + \frac{(250 \times 10^3)^2 \times 160 \times 10^{-12}}{2 + j10 + j \times 250 \times 10^3 \times 80 \times 10^{-6}}$$

$$Z_{in} = 0.02 + j0.17\ \Omega$$

33. (D)

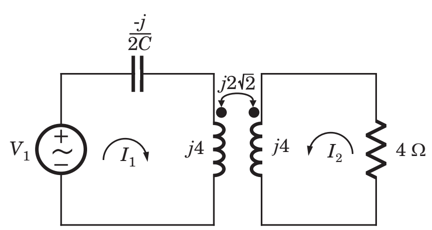


Fig. S.1.9.33

$$V_1 = -\frac{jI_1}{2C} + j4I_1 + j2\sqrt{2}\, I_2$$

$$0 = (4 + 4j)I_2 + j2\sqrt{2}I_1$$

$$\Rightarrow \quad I_2 = \frac{-j\sqrt{2}I_1}{2(1+j)}$$

$$\frac{V_1}{I_1} = \frac{-j}{2C} + j4 + \frac{2}{1+j} = \frac{-j + j8C + 2C - j2C}{2C}$$

$$Z_{in} = \frac{-j + j8C + 2C - j2C}{2C}$$

$$\text{Im}(Z_{in}) = 0 \quad \Rightarrow \quad -j + j8C - j2C = 0$$

$$\Rightarrow \quad C = \frac{1}{6}$$

34. (A) $j30 - \dfrac{3j}{1000C} = 0$, $C = 100\ \mu\text{F}$

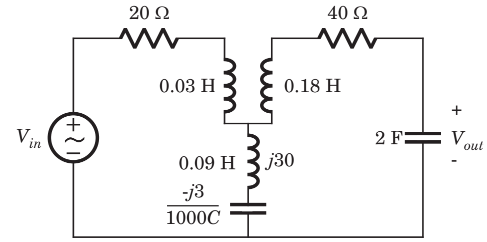


Fig. P1.9.34

35. (D) The π equivalent circuit of coupled coil is shown in fig. S1.9.35

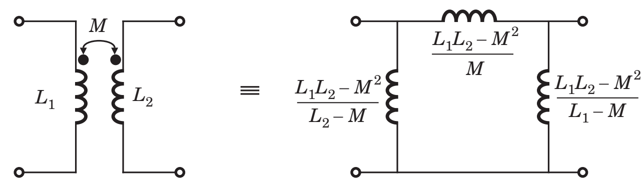


Fig. S1.9.35

$$\frac{L_1L_2 - M^2}{M} = \frac{\sqrt{L_1L_2}(1-k^2)}{k} = \frac{\sqrt{0.12 \times 0.27}(1.0.5^2)}{0.5} = 0.27$$

Output is zero if $\dfrac{-j}{0.27\omega} + jC\omega = 0$

$$C = \frac{1}{0.27\omega^2} = 33.33\ \mu\text{F}$$

36. (C) Applying 1 V test source at ab terminal,

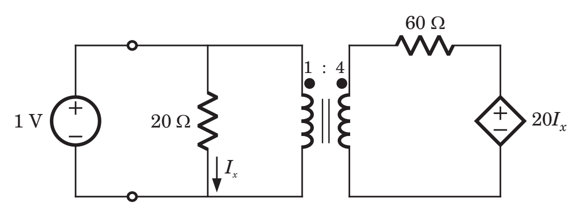


Fig. S1.9.36

$V_{ab} = 1$ V, $I_x = \dfrac{1}{20} = 0.05$ A, $V_2 = 4$ V ,

$4 = 60\, I_2 + 20 \times 0.05 \quad \Rightarrow \quad I_2 = 0.05$ A

$I_{in} = I_x + I_1 = I_x + 4I_2 = 0.25$ A

$R_{TH} = \dfrac{1}{I_{in}} = 4\ \Omega$, $\quad V_{TH} = 0$

37. (A) Impedance seen by $R_L = 10 \times 4^2 = 160\ \Omega$

For maximum power $R_L = 160\ \Omega$, $Z_o = 10\ \Omega$

$$P_{Lmax} = \left(\frac{100}{10 + 10}\right)^2 \times 10 = 250\ \text{W}$$

38. (B) $I_2 = \dfrac{V_2}{8}$, $I_1 = \dfrac{I_2}{5} = \dfrac{V_2}{40}$, $V_1 = 5V_2$

CHAPTER

1.10

TWO PORT NETWORK

Statement for Q.1-4:

The circuit is given in fig. P.1.10.1–4

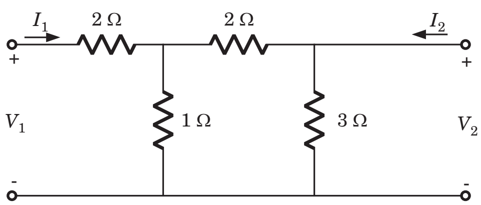


Fig. P.1.10.1–4

1. $[z] = ?$

(A) $\begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ -\frac{17}{6} & \frac{1}{2} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{17}{6} & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{17}{6} & -\frac{1}{2} \\ -\frac{1}{6} & \frac{3}{2} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{17}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{3}{2} \end{bmatrix}$

2. $[y] = ?$

(A) $\begin{bmatrix} \frac{3}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{17}{24} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{17}{24} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{17}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{17}{6} & -\frac{1}{2} \\ -\frac{1}{8} & \frac{3}{2} \end{bmatrix}$

3. $[h] = ?$

(A) $\begin{bmatrix} \frac{6}{17} & -\frac{3}{17} \\ \frac{3}{17} & \frac{24}{17} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{8}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{6}{17} & \frac{3}{17} \\ -\frac{3}{17} & \frac{24}{17} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{8}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

4. $[T] = ?$

(A) $\begin{bmatrix} \frac{17}{3} & 8 \\ 2 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} \frac{17}{3} & -8 \\ -2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{17}{3} & -8 \\ 2 & -3 \end{bmatrix}$

(D) $\begin{bmatrix} \frac{17}{3} & -8 \\ 2 & -3 \end{bmatrix}$

5. $[z] = ?$

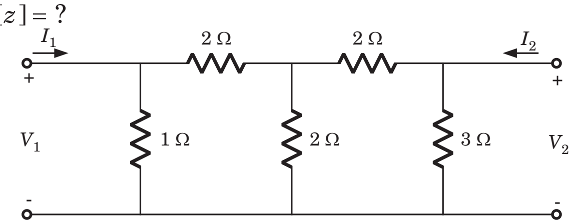


Fig. P.1.10.5

(A) $\begin{bmatrix} \frac{21}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{7}{12} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{7}{9} & \frac{1}{6} \\ \frac{1}{6} & \frac{7}{4} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{21}{16} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{7}{12} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{7}{9} & \frac{1}{3} \\ \frac{1}{3} & \frac{7}{4} \end{bmatrix}$

6. $[y] = ?$

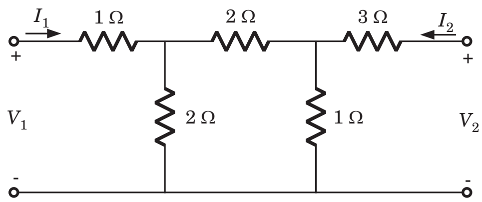


Fig. P.1.10.6

(A) $\begin{bmatrix} \frac{11}{41} & \frac{2}{41} \\ \frac{2}{41} & \frac{19}{41} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{11}{41} & -\frac{2}{41} \\ -\frac{2}{41} & \frac{19}{41} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{19}{41} & \frac{2}{41} \\ \frac{2}{41} & \frac{11}{41} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{19}{41} & -\frac{2}{41} \\ -\frac{2}{41} & \frac{11}{41} \end{bmatrix}$

Statement for Q.7-10:

A two port is described by $V_1 = I_1 + 2V_2$, $I_2 = -2I_1 + 0.4V_2$

7. $[z] = ?$

(A) $\begin{bmatrix} 11 & -5 \\ -5 & 2.5 \end{bmatrix}$ (B) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -2 \\ 5 & 0.4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$

8. $[y] = ?$

(A) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 \\ -2 & 4.4 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 4.4 \\ 4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 11 & -5 \\ -5 & 2.5 \end{bmatrix}$

9. $[h] = ?$

(A) $\begin{bmatrix} 3 & -6 \\ 4 & -4 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & -2 \\ -2 & 4.4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$ (D) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$

10. $[T] = ?$

(A) $\begin{bmatrix} 2.2 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}$ (B) $\begin{bmatrix} 2.2 & -0.5 \\ 0.2 & -0.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -2 \\ -2 & -0.4 \end{bmatrix}$

11. $[y] = ?$

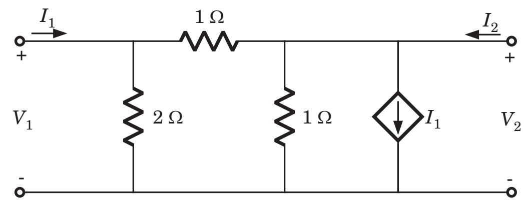


Fig. P.1.10.11

(A) $\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{3}{2} & -1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{3}{2} & -1 \\ \frac{1}{2} & 1 \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

12. $[z] = ?$

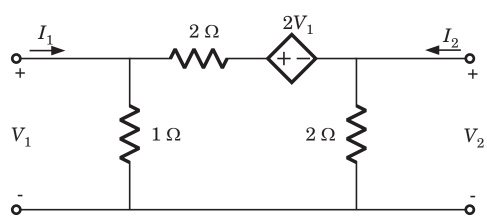


Fig. P.1.10.12

(A) $\begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{2}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{2}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

13. $[y] = ?$

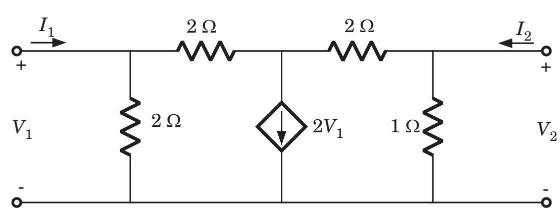


Fig. P.1.10.13

(A) $\begin{bmatrix} \frac{7}{4} & -\frac{1}{4} \\ \frac{1}{2} & \frac{5}{4} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{7}{4} & -\frac{1}{4} \\ \frac{3}{4} & \frac{5}{4} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{10}{19} & \frac{2}{19} \\ \frac{6}{19} & \frac{14}{19} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{6}{19} & \frac{14}{19} \\ \frac{10}{19} & \frac{2}{19} \end{bmatrix}$

14. $[z] = ?$

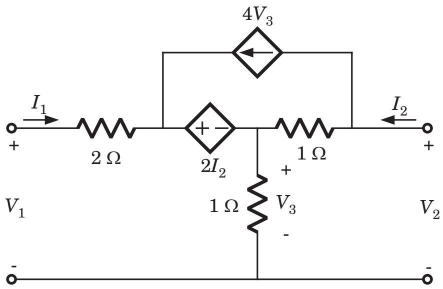


Fig. P.1.10.14

(A) $\begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 \\ 3 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 3 \\ -3 & -2 \end{bmatrix}$

15. $[z] = ?$

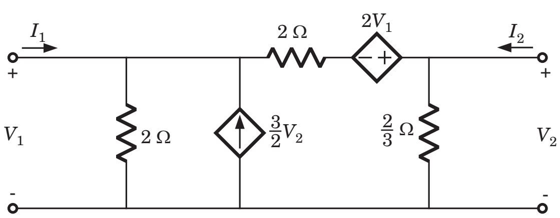


Fig. P.1.1.15

(A) $\begin{bmatrix} 2 & 2 \\ \frac{3}{2} & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -2 & \frac{3}{2} \\ 2 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & \frac{3}{2} \\ 2 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -2 \\ -\frac{3}{2} & 2 \end{bmatrix}$

16. $[y] = ?$

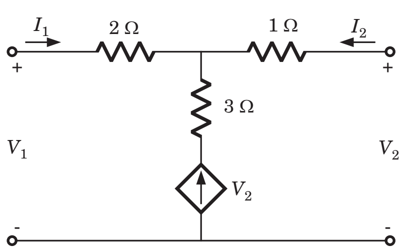


Fig. P.1.10.16

(A) $\begin{bmatrix} -1 & 1 \\ -1 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$

17. $[z] = ?$

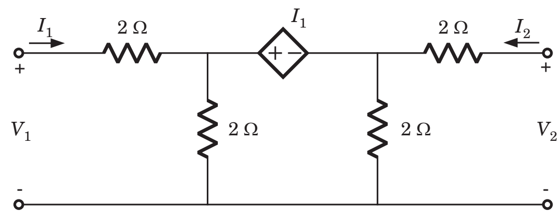


Fig. P.1.10.17

(A) $\begin{bmatrix} 3 & 2 \\ 6 & \frac{1}{7} \end{bmatrix}$ (B) $\begin{bmatrix} 6 & \frac{1}{7} \\ 3 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} \frac{7}{4} & 1 \\ \frac{1}{2} & 3 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & 3 \\ \frac{7}{4} & 1 \end{bmatrix}$

18. $[T] = ?$

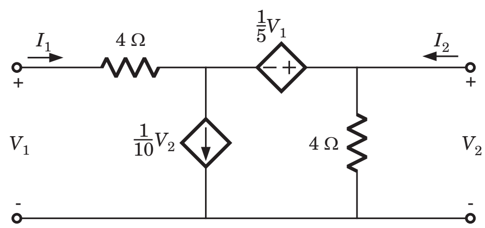


Fig. P.1.10.18

(A) $\begin{bmatrix} 0.35 & -1 \\ 2 & -3.33 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -3.33 \\ 0.35 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3.33 \\ 0.35 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0.35 & 1 \\ 2 & 3.33 \end{bmatrix}$

19. $[h] = ?$

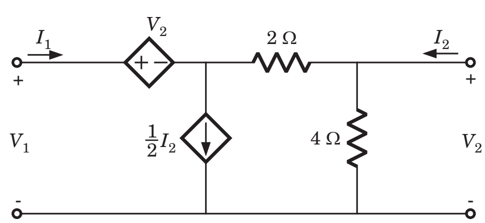


Fig. P.1.10.19

(A) $\begin{bmatrix} 4 & \frac{3}{2} \\ -2 & \frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} -2 & \frac{1}{2} \\ 4 & \frac{3}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 4 & -\frac{3}{2} \\ 2 & \frac{1}{2} \end{bmatrix}$ (D) $\begin{bmatrix} 2 & \frac{1}{2} \\ 4 & -\frac{3}{2} \end{bmatrix}$

(A) $\begin{bmatrix} Z_a + Z_{ab} & Z_{ab} \\ Z_{ab} & Z_b + Z_{ab} \end{bmatrix}$ (B) $\begin{bmatrix} Z_a - Z_{ab} & Z_{ab} \\ Z_{ab} & Z_b - Z_{ab} \end{bmatrix}$

(C) $\begin{bmatrix} Z_a + Z_{ab} & -Z_{ab} \\ -Z_{ab} & Z_b + Z_{ab} \end{bmatrix}$ (D) $\begin{bmatrix} Z_{ab} - Z_a & Z_{ab} \\ Z_{ab} & Z_{ab} - Z_b \end{bmatrix}$

27. $[y] = ?$

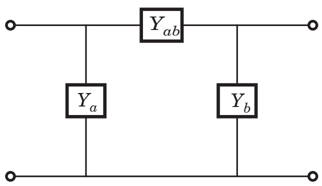


Fig. P.1.10.27

(A) $\begin{bmatrix} Y_a + Y_{ab} & -Y_{ab} \\ -Y_{ab} & Y_b + Y_{ab} \end{bmatrix}$ (B) $\begin{bmatrix} Y_a - Y_{ab} & Y_{ab} \\ Y_{ab} & Y_b - Y_{ab} \end{bmatrix}$

(C) $\begin{bmatrix} Y_{ab} - Y_a & Y_{ab} \\ Y_{ab} & Y_{ab} - Y_a \end{bmatrix}$ (D) $\begin{bmatrix} Y_a - Y_{ab} & -Y_{ab} \\ -Y_{ab} & Y_b - Y_{ab} \end{bmatrix}$

28. The y-parameters of a 2-port network are

$$[y] = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \text{S}$$

A resistor of 1 ohm is connected across as shown in fig. P.1.10.2 8. The new y –parameter would be

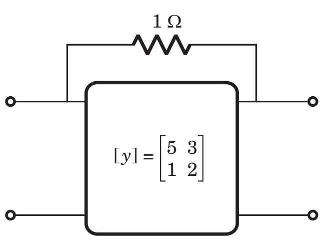


Fig. P.1.10.28

(A) $\begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix}$ S (B) $\begin{bmatrix} 6 & 2 \\ 0 & 3 \end{bmatrix}$ S

(C) $\begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$ S (D) $\begin{bmatrix} 4 & 4 \\ 2 & 1 \end{bmatrix}$ S

29. For the 2-port of fig. P.1.10.29, $[y_a] = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix}$ mS

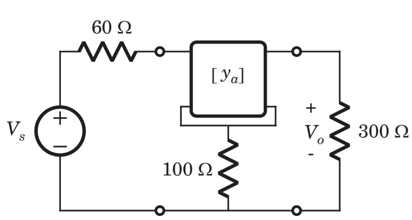


Fig. P.1.10.29

The value of $\frac{V_o}{V_s}$ is

(A) $\frac{3}{32}$ (B) $\frac{1}{16}$

(C) $\frac{2}{33}$ (D) $\frac{1}{17}$

30. The T-parameters of a 2-port network are

$$[T] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

If such two 2-port network are cascaded, the z –parameter for the cascaded network is

(A) $\begin{bmatrix} 2 & -2 \\ -\frac{1}{2} & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{5}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{5}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$

31. $[y] = ?$

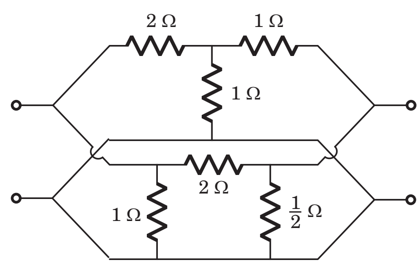


Fig. P.1.10.31

(A) $\begin{bmatrix} \frac{19}{10} & \frac{-9}{10} \\ -\frac{9}{10} & \frac{31}{10} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{19}{10} & \frac{-7}{10} \\ -\frac{7}{10} & \frac{31}{10} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{19}{10} & \frac{9}{10} \\ \frac{9}{10} & \frac{31}{10} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{19}{10} & \frac{7}{10} \\ \frac{7}{10} & \frac{31}{10} \end{bmatrix}$

32. $[y] = ?$

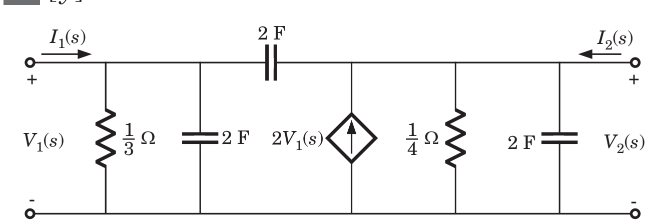


Fig. P.1.10.32

(A) $\begin{bmatrix} s+3 & 2s \\ 2s+2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} s+3 & -2s \\ -2s-2 & 4s+4 \end{bmatrix}$

(C) $\begin{bmatrix} s+3 & -2s \\ -2s-2 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 3s+3 & -2s \\ -2s-2 & 4s+4 \end{bmatrix}$

33. $h_{21} = ?$

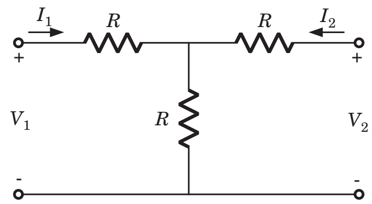


Fig. P.1.10.33

(A) $-\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) $\frac{3}{2}$

34. In the circuit shown in fig. P.1.10.34, when the voltage V_1 is 10 V, the current I is 1 A. If the applied voltage at port-2 is 100 V, the short circuit current flowing through at port 1 will be

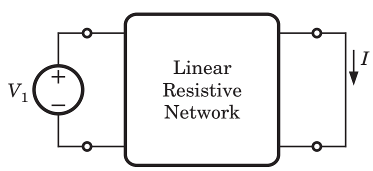


Fig. P.1.10.34

(A) 0.1 A (B) 1 A

(C) 10 A (D) 100 A

35. For a 2-port symmetrical bilateral network, if transmission parameters $A = 3$ and $B = 1\ \Omega$, the value of parameter C is

(A) 3 (B) 8 S

(C) 8Ω (D) 9

36. A 2-port resistive network satisfy the condition $A = D = \frac{3}{2}B = \frac{4}{3}C$. The z_{11} of the network is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$

(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

37. The circuit shown in fig. P.1.10.37 is reciprocal if a is

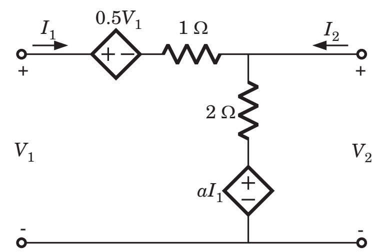


Fig. P.1.10.37

(A) 2 (B) –2

(C) 1 (D) –1

38. $Z_{in} = ?$

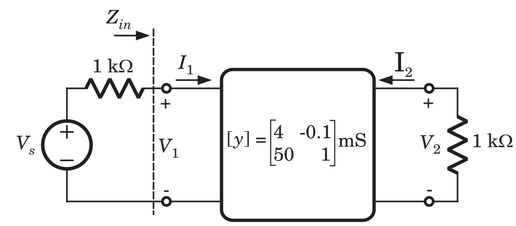


Fig. P.1.10.38

(A) 86.4 Ω (B) 64.3 Ω

(C) 153.8 Ω (D) 94.3 Ω

39. $V_1, V_2 = ?$

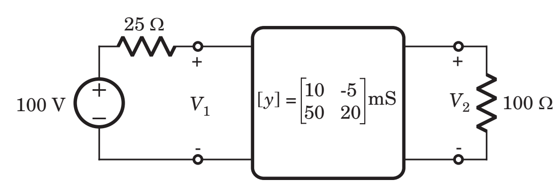


Fig. P.1.10.39

(A) –68.6 V, 114.3 V (B) 68.6 V, –114.3 V

(C) 114.3 V, –68.6 V (D) –114.3 V, 68.6 V

40. A 2-port network is driven by a source $V_s = 100$ V in series with 5 Ω, and terminated in a 25Ω resistor. The impedance parameters are

$$[z] = \begin{bmatrix} 20 & 2 \\ 40 & 10 \end{bmatrix}\Omega$$

The Thevenin equivalent circuit presented to the 25Ω resistor is

(A) 80 V, 2.8 Ω (B) 160 V, 6.8 Ω

(C) 100 V, 2.4 Ω (D) 120 V, 6.4 Ω

$$[z] = \begin{bmatrix} \frac{7}{4} & 1 \\ \frac{1}{2} & 3 \end{bmatrix}$$

18. (D) Let I_3 be the clockwise loop current in center loop

$I_1 = \frac{V_2}{10} + I_3$, $\quad V_2 = 4(I_2 + I_3) \quad \Rightarrow \quad I_3 = 0.25V_2 - I_2$

$\Rightarrow \quad I_1 = 0.35V_2 - I_2$ ...(i)

$V_1 = 4I_1 - 0.2V_1 + V_2$

$1.2V_1 = 4(0.35V_2 - I_2) + V_2 = 2.4V_2 - 4I_2$

$\Rightarrow \quad V_1 = 2V_2 - 3.33I_2$ ...(ii)

19. (A) $V_2 = 4\left(I_2 + I_1 - \frac{I_2}{2}\right) \Rightarrow \quad I_2 = -2I_1 + \frac{1}{2}V_2$ ...(ii)

$I_1 = \frac{I_2}{2} + \frac{(V_1 - V_2) - V_2}{2} = -I_1 + \frac{V_2}{4} + \frac{V_1}{2} - V_2$

$\Rightarrow \quad V_1 = 4I_1 + \frac{3}{2}V_2$ ...(i)

20. (B) $I_1 = -V_2 + \frac{V_1}{1} + \frac{V_1 - V_2}{2} = \frac{3}{2}V_1 - \frac{3}{2}V_2$ ...(i)

$I_2 = 2V_1 + \frac{V_2}{1} + \frac{V_2 - V_1}{2} = \frac{3}{2}V_1 + \frac{3}{2}V_2$ ...(ii)

21. (D) $I_1 = 2V_1 + jV_1 + j(V_1 - V_2)$

$\Rightarrow \quad I_1 = (2 + j2)V_1 - jV_2$ ...(i)

$I_2 = \frac{V_2}{1} + V_1 + j(V_2 - V_1) = (1 - j)V_1 + (1 + j)V_2$ ...(ii)

22. (B) $V_1 = \frac{I_1}{s} + sI_1 + sI_2 = \left(\frac{1}{s} + s\right)I_1 + sI_2$ ...(i)

$V_2 = 2I_2 + 2s\,I_2 + sI_1 \quad \Rightarrow \quad V_2 = sI_1 + (2 + 2s)I_2$ ...(ii)

23. (D) $Z_R = \frac{9}{n^2} = \frac{9}{9} = 1$

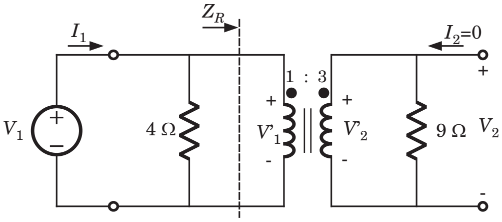


Fig. S1.10.23a

$V_1 = (4 \,||\, 1)I_1 = \frac{4}{5}I_1 \quad \Rightarrow \quad z_{11} = \frac{V_1}{I_1} = 0.8$

$V_2 = V'_2 = nV'_1 = 3\left(\frac{4}{5}I_1\right) \quad \Rightarrow \quad z_{21} = \frac{V_2}{I_1} = 2.4,$

$Z'_R = n^2 4 = 36$

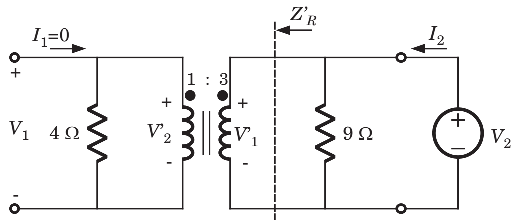


Fig. S1.10.23b

$V_2 = (36 \,||\, 9)I_2 = 7.2I_2 \quad \Rightarrow \quad z_{22} = \frac{V_2}{I_2} = 7.2,$

$z_{12} = z_{21} = 2.4$

24. (C) $V_1 = 3sI_1 + 3sI_1 - 3sI_1 + 3sI_1 + 2sI_2$

$\Rightarrow \quad V_1 = 6sI_1 + 2sI_2$ ...(i)

$V_2 = 3sI_2 + 2sI_1 \quad \Rightarrow \quad V_2 = 2sI_1 + 3sI_2$ ...(ii)

25. (C) $V_1 = \frac{V_2}{5} + 0(-I_2)$, $\quad I_1 = (0)V_2 + 5(-I_2)$

26. (A) $V_1 = (Z_a + Z_{ab})I_1 + Z_{ab}I_2$ ...(i)

$V_2 = (Z_a + Z_{ab})I_2 + Z_{ab}I_1 = Z_{ab}I_1 + (Z_a + Z_{ab})I_2$ ....(ii)

27. (A) $I_1 = (V_1 - V_2)Y_{ab} + V_1Y_a$

$\Rightarrow \quad I_1 = V_1(Y_a + Y_{ab}) - V_2Y_{ab}$ ...(i)

$I_2 = (V_2 - V_1)Y_{ab} + V_2Y_b = -V_1Y_{ab} + V_2(Y_b + Y_{ab})$ ...(ii)

28. (B) y-parameter of 1 Ω resistor network are $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

New y-parameter $= \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 0 & 3 \end{bmatrix}$.

29. (A) $[z_a] = \begin{bmatrix} 2\text{ mS} & 0 \\ 0 & 10\text{ mS} \end{bmatrix}^{-1} = \begin{bmatrix} 5000 & 0 \\ 0 & 100 \end{bmatrix}$

$[z] = \begin{bmatrix} 5000 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$

$V_1 = 600I_1 + 100I_2$, $V_2 = 100I_1 + 200I_2$

$V_s = 60I_1 + V_1 = 660I_1 + 100I_2$, $V_2 = V_o = -300I_2$

$V_o = 100I_1 - \frac{2}{3}V_o \quad \Rightarrow \quad I_1 = \frac{V_o}{60}$

$V_s = 11V_o - \frac{V_o}{3} \quad \Rightarrow \quad \frac{V_o}{V_5} = \frac{3}{32}$

30. (C) $[T_N] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$V_1 = 5V_2 - 3I_2$, $I_1 = 3V_2 - 2I_2$

$3V_1 - 5I_1 = I_2 \quad \Rightarrow \quad V_1 = \frac{5}{3}I_1 + \frac{1}{3}I_2$ ...(i)

$V_2 = \frac{1}{3}I_1 + \frac{2}{3}I_2$ ...(ii)

31. (B)

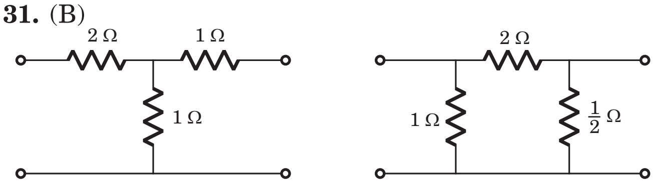


Fig. S.1.10.31a & b

$$[z_a] = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}, \; [y_a] = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}, \; [y_b] = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

$$[y] = [y_a] + [y_b] = \begin{bmatrix} \frac{19}{10} & -\frac{7}{10} \\ -\frac{7}{10} & \frac{31}{10} \end{bmatrix}$$

32. (D) $[y_a] = \begin{bmatrix} 3s & -2s \\ -2s & 4s \end{bmatrix}, [y_b] = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}$

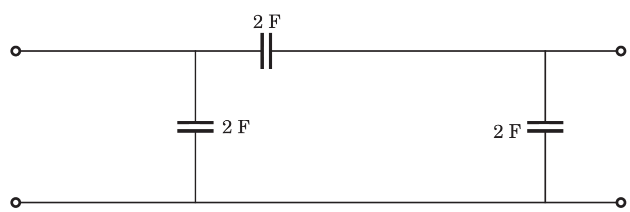


Fig. S.1.10.32a

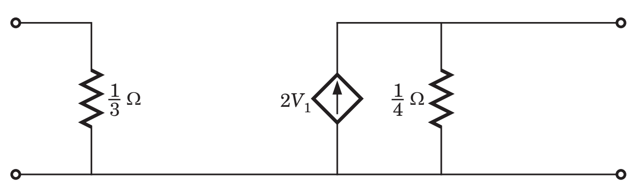


Fig. S.1.10.32b

$$[y] = [y_a] = [y_b] = \begin{bmatrix} 3s+3 & -2s \\ -2s-2 & 4s+4 \end{bmatrix}$$

33. (C) $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \; -I_2 = \frac{I_1 R}{R+R}, \; \frac{I_2}{I_1} = -\frac{1}{2}$

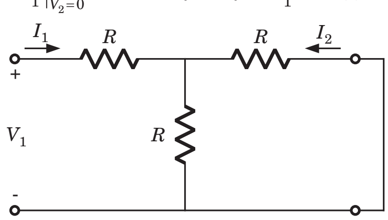


Fig. S.1.10.33

34. (C) $\left.\frac{I_2}{V_1}\right|_{V_2=0} = y_{21} = \frac{1}{10} = 0.1$

Interchanging the port $\frac{I_2'}{V_1'} = 0.1, \; I_2' = 100 \times 0.1 = 10$

35. (B) For symmetrical network $A = D = 3$

For bilateral $AD - BC = 1, \; 9 - C = 1, \quad C = 8$ S

36. (A) $z_{11} = \frac{A}{C} = \frac{4}{3}$

37. (A) $V_1 = 0.5V_1 + I_1 + 2(I_1 + I_2) + aI_1$

$\Rightarrow \quad V_1 = (6 + 2a)I_1 + 4I_2$ ...(i)

$V_2 = 2(I_1 + I_2) + aI_1 \quad \Rightarrow \quad V_2 = (2 + a)I_1 + 2I_2$ ...(ii)

For reciprocal network

$z_{12} = z_{21}, \; 4 = 2 + a \quad \Rightarrow \quad a = 2$

38. (C) $I_1 = 4 \times 10^3 V_1 - 0.1 \times 10^{-3} V_2$

$I_2 = 50 \times 10^{-3} V_1 + 10^{-3} V_2, \; V_2 = -10^3 I_2$

$-10^{-3} V_2 = 50 \times 10^{-3} V_1 + 10^{-3} V_2, \; V_2 = -25V_1$

$10^3 I_1 = 4V_1 + 2.5V_1, \; \frac{V_1}{I_1} = \frac{10^3}{6.5} = 153.8$

39. (B) $I_1 = 10 \times 10^{-3} V_1 - 5 \times 10^{-3} V_2,$

$100 = 25I_1 + V_1$

$100 - V_1 = 0.25V_1 - 0.125V_2 \quad \Rightarrow \quad 800 = 10V_1 - V_2$ ...(i)

$I_2 = 50 \times 10^{-3} V_1 + 20 \times 10^{-3} V_2, \; V_2 = -100I_2$

$V_2 = -5V_1 - 2V_2 \quad \Rightarrow \quad 3V_2 + 5V_1 = 0$ ...(ii)

From (i) and (ii) $V_1 = 68.6$ V, $\quad V_2 = -114.3$ V.

40. (B) $100 = 5I_1 + V_1, \quad V_1 = 20I_1 + 2I_2$

$\Rightarrow \quad 100 = 25I_1 + 2I_2, \; V_2 = 40I_1 + 10I_2$

$800 - 5V_2 = -34I_2 \quad \Rightarrow \quad V_2 = 160 + 6.8I_2$

$V_{TH} = 160$ V, $R_{TH} = 6.8\ \Omega$

41. (B) $V_1 = z_{11}I_1, \; V_2 = z_{21}I_1, \; \frac{V_2}{V_1} = \frac{z_{21}}{z_{11}}$

42. (B) $I_2 = y_{21}V_1 + y_{22}V_2, \; I_2 = -V_2Y_L$

$y_{21}V_1 + (y_{22} + Y_L)V_2 = 0, \; \frac{V_2}{V_1} = \frac{-y_{21}}{(y_{22} + y_L)}$

43. (A) $V_2 = z_{21}I_1 + z_{22}I_2, \quad V_2 = -Z_LI_2$

$V_2 = z_{21}I_1 + z_{22}\left(-\frac{V_2}{Z_L}\right)$

$V_2(Z_L + z_{22}) = z_{21}Z_LI_1, \quad \frac{V_2}{I_1} = \frac{z_{21}Z_L}{z_{22} + Z_L}$

****

CHAPTER

1.11

FREQUENCY RESPONSE

Statement for Q.1-3:

A parallel resonant circuit has a resistance of 2 kΩ and half power frequencies of 86 kHz and 90 kHz.

1. The value of capacitor is

(A) 6 μF (B) 20 nF
(C) 2 nF (D) 60 μF

2. The value of inductor is

(A) 4.3 mH (B) 43 mH
(C) 0.16 mH (D) 1.6 mH

3. The quality factor is

(A) 22 (B) 100
(C) 48 (D) 200

Statement for Q.4-5:

A parallel resonant circuit has a midband admittance of 25×10^{-3} S, quality factor of 80 and a resonant frequency of 200 krad/s.

4. The value of R is

(A) 40 Ω (B) 56.57 Ω
(C) 80 Ω (D) 28.28 Ω

5. The value of C is

(A) 2 μF (B) 28.1 μF
(C) 10 μF (D) 14.14 μF

6. A parallel *RLC* circuit has $R = 1$ kΩ and $C = 1$ μF. The quality factor at resonance is 200. The value of inductor is

(A) 35.4 μH (B) 25 μH
(C) 17.7 μH (D) 50 μH

7. A parallel circuit has $R = 1$ kΩ, $C = 50$ μF and $L = 10$ mH. The quality factor at resonance is

(A) 100 (B) 90.86
(C) 70.7 (D) None of the above

8. A series resonant circuit has an inductor $L = 10$ mH. The resonant frequency $\omega_o = 10^6$ rad/s and bandwidth is BW $= 10^3$ rad/s. The value of R and C will be

(A) 100 μF, 10 Ω (B) 100 pF, 10 Ω
(C) 100 pF, 10 MΩ (D) 100 μF, 10 MΩ

9. A series resonant circuit has $L = 1$ mH and $C = 10$ μF. The required R for the BW 15.9 Hz is

(A) 0.1 Ω (B) 0.2 Ω
(C) 15.9 mΩ (D) 500 Ω

10. For the *RLC* parallel resonant circuit when $R = 8$ kΩ, $L = 40$ mH and $C = 0.25$ μF, the quality factor Q is

(A) 40 (B) 20
(C) 30 (D) 10

11. The maximum voltage across capacitor would be

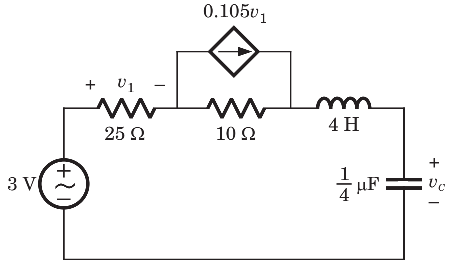


Fig. P1.11.11

(A) 3200 V (B) 3 V
(C) −3 V (D) 1600 V

12. For the circuit shown in fig. P1.1.11 resonant frequency f_o is

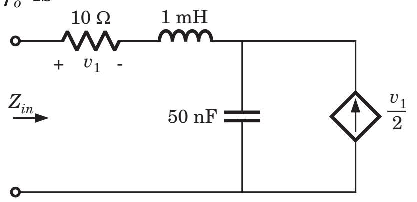


Fig. P1.11.12

(A) 346 kHz (B) 55 kHz

(C) 196 kHz (D) 286 kHz

13. For the circuit shown in fig. P1.11.13 the resonant frequency f_o is

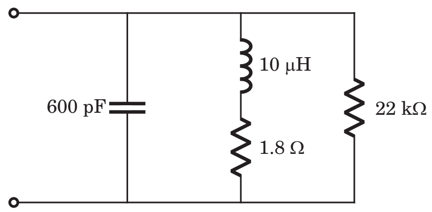


Fig. P1.11.13

(A) 12.9 kHz (B) 12.9 MHz

(C) 2.05 MHz (D) 2.05 kHz

14. The network function of circuit shown in fig.P1.11.14 is

$$H(\omega) = \frac{V_o}{V_1} = \frac{4}{1 + j0.01\omega}$$

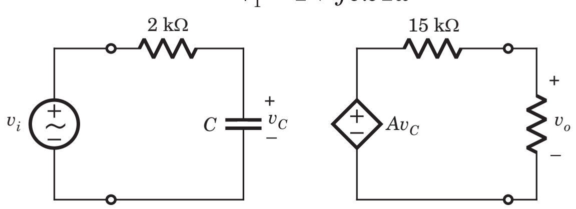


Fig. P1.11.14

The value of the C and A is

(A) 10 μF, 6 (B) 5 μF, 10

(C) 5 μF, 6 (D) 10 μF, 10

15. $H(\omega) = \frac{V_o}{V_i} = ?$

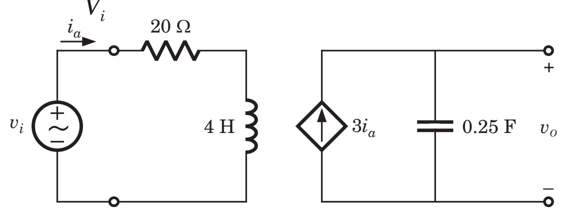


Fig. P1.11.15

(A) $\frac{0.6}{j\omega(1 + j0.2\omega)}$ (B) $\frac{0.6}{j\omega(5 + j\omega)}$

(C) $\frac{3}{j\omega(1 + j\omega)}$ (D) $\frac{3}{j\omega(20 + j4\omega)}$

16. $H(\omega) = \frac{V_o}{V_i} = ?$

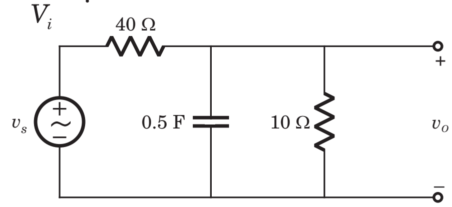


Fig. P1.11.16

(A) $(5 + j20\omega)^{-1}$ (B) $(5 + j4\omega)^{-1}$

(C) $(5 + j30\omega)^{-1}$ (D) $5(1 + j6\omega)^{-1}$

17. The value of input frequency is required to cause a gain equal to 1.5. The value is

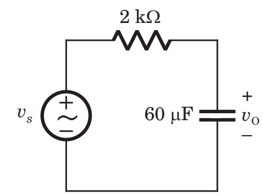


Fig. P1.11.17

(A) 20 rad/s (B) 20 Hz

(C) 10 rad/s (D) No such value exists.

18. In the circuit of fig. P1.11.18 phase shift equal to $-45°$ is required at frequency $\omega = 20$ rad/s . The value of R is

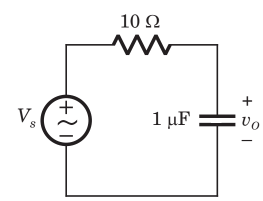


Fig. P1.11.18

(A) 200 kΩ (B) 150 kΩ

(C) 100 kΩ (D) 50 kΩ

19. For the circuit of fig. P1.11.19 the input frequency is adjusted until the gain is equal to 0.6. The value of the frequency is

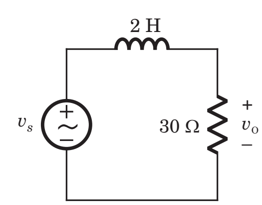


Fig. P1.11.19

(A) 20 rad/s (B) 20 Hz

(C) 40 rad/s (D) 40 Hz

31. Bode diagram of the network function V_o/V_s for the circuit of fig. P1.11.30 is

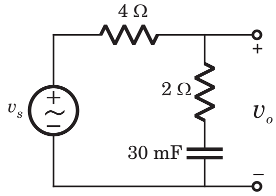
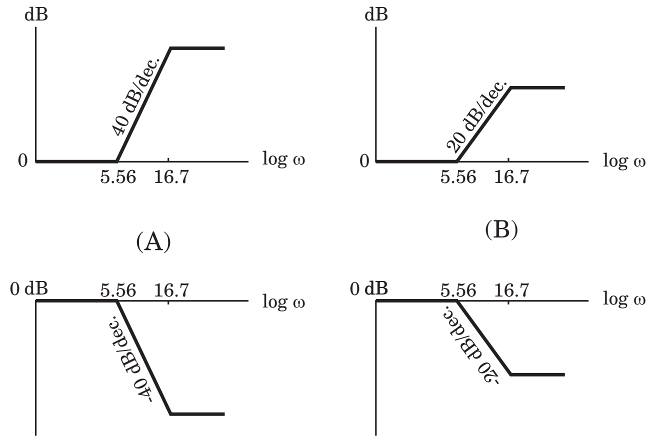


Fig.P1.11.30



(A) (B) (C) (D)

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SOLUTIONS

1. (B) $BW = \omega_2 - \omega_1 = 2\pi(90-86)k = 8\pi$ krad/s

$$BW = \frac{1}{RC} \Rightarrow C = \frac{1}{RBW} = \frac{1}{8\pi\times 10^3 \times 2\times 10^3}$$

$= 19.89$ nF

2. (C) $\omega_o = \dfrac{(\omega_1+\omega_2)}{2} = \dfrac{2\pi(90+86)k}{2} = 176\pi$ krad/s

$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_o^2} C$$

$$= \frac{1}{(176\pi\times 10^3)^2(20\times 10^{-9})} = 0.16 \text{ mH}$$

3. (A) $Q = \dfrac{\omega_o}{B} = \dfrac{176\pi k}{8\pi k} = 22$

4. (A) At mid-band frequency $Z = R$, $Y = \dfrac{1}{R}$

$$R = \frac{1}{25\times 10^{-3}} = 40\ \Omega$$

5. (C) $Q = \omega_o RC$

$$\Rightarrow C = \frac{Q}{\omega_o R} = \frac{80}{200\times 10^3 \times 40} = 10\ \mu\text{F}$$

6. (B) $Q_o = R\sqrt{\dfrac{C}{L}} \Rightarrow 200 = 10^3\sqrt{\dfrac{10^{-6}}{L}}$

$\Rightarrow L = 25\ \mu$H

7. (C) $Q_o = R\sqrt{\dfrac{C}{L}} = 10^3\sqrt{\dfrac{50\times 10^{-6}}{10\times 10^{-3}}} = 70.7$

8. (B) $\omega_o = \dfrac{1}{\sqrt{LC}}$

$$\Rightarrow C = \frac{1}{10\times 10^{-3}\times (10^6)^2} = 100 \text{ pF}$$

$BW = \dfrac{R}{L} \Rightarrow R = 10\times 10^{-3}\times 10^3 = 10$

9. (A) $BW = \dfrac{R}{L}$

$$\Rightarrow \frac{R}{1\times 10^{-3}} = 15.9\times 2\pi = 0.1\Omega$$

10. (B) $Q = R\sqrt{\dfrac{C}{L}}$

$$= 8\times 10^3\sqrt{\frac{0.25\times 10^{-6}}{40\times 10^{-3}}} = 20$$

11. (A) Thevenin equivalent seen by L–C combination

$$3 = v_1 + 10\left(\frac{v_1 - 0.105v_1}{125}\right) \Rightarrow v_1 = 100$$

$$I_{sc} = \frac{1100}{125} = 0.8 \text{ V}$$

Open Circuit : $v_1 = 0, \quad v_{oc} = 3$ V

$$R_{TH} = \frac{3}{0.8} = 3.75\,\Omega,\ \omega_o = \frac{1}{\sqrt{LC}} = 1000$$

$$Q_o = \frac{\omega_o L}{R} = \frac{1000\times 4}{3.75} = 1066.67$$

$$|v_C|_{max} = Q_o v_{TH} = 1066.67\times 3 = 3200 \text{ V}$$

12. (B) Applying 1 A at input port $V_1 = 10$ V voltage across 1 A source

$$V_{test} = 10 + j\omega 10^{-3} - \frac{j}{\omega 50\times 10^{-9}}(5+1)$$

$Z_{in} = V_{test}$

At resonance Im $\{Z_{in}\} = 0$

$$\Rightarrow \omega_o 10^{-3} = \frac{6}{\omega_o 50\times 10^{-9}} \Rightarrow \omega_o = 346 \text{ kHz}$$

$f_o = 55$ kHz

13. (C) $Y = j\omega 600\times 10^{-12} + \frac{1}{2\times 10^3} + \frac{1}{1.8 + j\omega 10^{-5}}$

$$= j\omega 6\times 10^{-10} + 45.45 + \frac{1.8 - j\omega 10^{-5}}{3.24 + \omega^2 10^{-50}}$$

At resonance Im $\{Y\} = 0$

$$\omega_o 6\times 10^{-10}(3.24 + \omega_o^2 10^{-10}) - \omega_o 10^{-5} = 0$$

$$3.24 + \omega_o^2 10^{-10} = 16.67\times 10^3 \omega_o = 12.9 \text{ Mrad/s}$$

$$f_o = \frac{\omega_o}{2\pi} = 2.05 \text{ MHz}$$

14. (C) $V_C = \dfrac{\frac{V_i}{jC\omega}}{2\times 10^3 + \frac{1}{jC\omega}} = \dfrac{V_i}{1 + j2\times 10^3 C\omega}$

$$V_o = AV_c\frac{(15\text{k})}{16\text{k} + 30\text{k}} = \frac{2AV_c}{3} = \frac{2AV_i}{3(1 + j2\times 10^3 C\omega)}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{\frac{2A}{3}}{1 + j2\pi\times 10^3 C\omega}$$

$$\frac{2A}{3} = 4 \Rightarrow A = 6, \quad 2\times 10^3 C = 0.01$$

$\Rightarrow C = 5\ \mu$F

15. (A) $I_a = \dfrac{V_i}{20 + j4\omega}, \quad V_o = \dfrac{3I_a}{0.25j\omega}$

$$\frac{V_o}{V_1} = \frac{3}{j\omega(5 + j\omega)} = \frac{0.6}{j\omega(1 + j0.2\omega)}$$

16. (A) $Z_1 = \dfrac{\frac{10}{j\omega(0.51)}}{\frac{1}{j\omega 0.5} + 10} = \dfrac{10}{1 + j3\omega}$

$$\frac{V_o}{V_i} = \frac{Z_1}{40 + Z_1} = \frac{\frac{10}{1 + j5\omega}}{\frac{10}{1 + j5\omega} + 40}$$

$$= \frac{10}{50 + j200\omega} = (5 + j20\omega)^{-1}$$

17. (D) $H(\omega) = \dfrac{V_o}{V_i} = \dfrac{1}{1 + j\omega RC}$

$$\text{gain} = \frac{1}{\sqrt{(1 + \omega^2 RC)}}$$

For any value of ω, R, C gain ≤ 1.

Thus (D) is correct option.

18. (D) $H(\omega) = \dfrac{V_o}{V_s} = \dfrac{1}{1 + j\omega CR}$

phase shift $= -\tan^{-1}\omega CR = -450°$

$\omega CR = 1,$

$20\times 1\times 10^{-6} R = 1 \Rightarrow R = 50$ kΩ.

19. (A) $H(\omega) = \dfrac{V_o}{V_s} = \dfrac{R}{\sqrt{1 + j\omega L}}$

$$\text{gain} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{30}{\sqrt{900^2 + 4\omega^2 + 0.6}}$$

$$\omega = \frac{\sqrt{50^2 - 30^2}}{2} = 20 \text{ rad/s}$$

20. (A) $H(\omega) = \dfrac{V_o}{V_s} = \dfrac{1}{1 + j\omega CR} = \dfrac{1}{1 + j}$

Phase shift $= -\tan^{-1}\omega CR = -45°$

$$\text{gain} = \frac{1}{|j + 1|} = \frac{1}{\sqrt{2}} = 0.707$$

21. (B) BW$= \omega_2 - \omega_1 = 2\pi(456 - 434) = 44\pi$

$\omega_o = 2\pi f_o = Q\text{BW} = 20\times 44\pi$

$f_o = 440$ Hz

22. (C) $f_o = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{360 \times 10^{-12} \times 240 \times 10^{-6}}} = 541 \text{ kHz}$$

$$f_o = \frac{1}{2\pi\sqrt{50 \times 10^{-12} \times 240 \times 10^{-6}}} = 1.45 \text{ MHz}$$

23. (B) $f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$$\frac{R}{L} = \frac{400}{240 \times 10^{-6}} = \frac{10^7}{6}$$

$$\frac{1}{LC} = \frac{1}{240 \times 10^{-6} \times 120 \times 10^{-12}} = \frac{10^{16}}{288}$$

$$\frac{R}{L} < \frac{1}{LC}, \ f_o = \frac{1}{2\pi\sqrt{LC}} = 938 \text{ kHz}$$

24. (B) $\omega_o = \frac{1}{RC}$, R and C should be as small as possible.

$$R = (3.3)\frac{(1.8)}{3.3 + 1.8} = 1.165 \text{ k}\Omega$$

$$C = (10)\frac{(30)}{(10 + 30)} = 7.5 \text{ pF}$$

$$\omega = \frac{1}{1.165 \times 7.5 \times 10^{-9}} = 114.5 \times 10^6 \text{ rad/s}$$

25. (D) $R' = K_m R = 800 \times 12 \times 10^3 = 9.6 \text{ M}\Omega$

$$L' = \frac{K_m}{K_f L} = \frac{800}{1000} 40 \times 10^{-6} = 32 \ \mu\text{F}$$

$$C' = \frac{C}{K_m} K_F = 30 \times \frac{10^{-9}}{80} \times 1000 = 0.375 \text{ pF}$$

26. (A) $L'C' = \frac{LC}{K_f^2} \Rightarrow K_f^2 = \frac{4 \times 20 \times 10^{-3} \times 10^{-6}}{1 \times 6}$

$\Rightarrow K_f = 2 \times 10^{-4}$

$$\frac{L'}{C'} = \frac{L}{C} K_m^2 \Rightarrow K_m^2 = \frac{(1)(20 \times 10^{-6})}{(2)(4 \times 10^{-3})}$$

$\Rightarrow K_m = 0.05$

27. (D) $\omega_c = 2\pi f_c = \frac{1}{RC}$

$$\Rightarrow R = \frac{1}{2\pi \times 20 \times 10^3 \times 0.5 \times 10^{-6}} = 15.9 \ \Omega$$

28. (A) R_{TH} across the capacitor is

$R_{TH} = (1\text{k} + 4\text{k}) || 5\text{k} = 2.5 \text{ k}\Omega$

$$f_c = \frac{1}{2\pi \times 2.5 \times 10^3 \times 40 \times 10^{-9}} = 1.06 \text{ kHz}$$

29. (B) $\omega_c = 2\pi f_c = \frac{1}{RC}$

$$\Rightarrow R = \frac{1}{2\pi \times 15 \times 10 \times 10^{-6}} = 1.06 \text{ k}\Omega$$

30. (B) $20 \log H = 20 \log \frac{1}{\omega^2} = -40 \log \omega$

31. (D) $\frac{V_o}{V_s} = \frac{2 + \frac{1}{j\omega 30 \times 10^{-3}}}{6 + \frac{1}{j\omega 30 \times 10^{-3}}} = \frac{1 + \frac{j\omega}{16.67}}{1 + \frac{j\omega}{3.56}}$

-20 dB/decade line starting from $\omega = 5.56$ rad/s

20 dB/decade line starting from $\omega = 16.67$ rad/s

Hence -20 dB/decade line for $5.56 < \omega < 16.67$

parallel to ω axis to $\omega > 16.67$

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CHAPTER

2.1

SEMICONDUCTOR PHYSICS

In the problems assume the parameter given in following table. Use the temperature $T = 300$ K unless otherwise stated.

Property	Si	GaAs	Ge
Bandgap Energy	1.12	1.42	0.66
Dielectric Constant	11.7	13.1	16.0
Effective density of states in conduction band N_c(cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band N_v(cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concertration n_i (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility Electron Hole	1350 480	8500 400	3900 1900

1. In germanium semiconductor material at $T = 400$ K the intrinsic concentration is

(A) 26.8×10^{14} cm^{-3} (B) 18.4×10^{14} cm^{-3}

(C) 8.5×10^{14} cm^{-3} (D) 3.6×10^{14} cm^{-3}

2. The intrinsic carrier concentration in silicon is to be no greater than $n_i = 1 \times 10^{12}$ cm^{-3}. The maximum temperature allowed for the silicon is ($E_g = 1.12$ eV)

(A) 300 K (B) 360 K

(C) 382 K (D) 364 K

3. Two semiconductor material have exactly the same properties except that material A has a bandgap of 1.0 eV and material B has a bandgap energy of 1.2 eV. The ratio of intrinsic concentration of material A to that of material B is

(A) 2016 (B) 47.5

(C) 58.23 (D) 1048

4. In silicon at $T = 300$ K the thermal-equilibrium concentration of electron is $n_0 = 5 \times 10^4$ cm^{-3}. The hole concentration is

(A) 4.5×10^{15} cm^{-3} (B) 4.5×10^{15} m^{-3}

(C) 0.3×10^{-6} cm^{-3} (D) 0.3×10^{-6} m^{-3}

5. In silicon at $T = 300$ K if the Fermi energy is 0.22 eV above the valence band energy, the value of p_0 is

(A) 2×10^{15} cm^{-3} (B) 10^{15} cm^{-3}

(C) 3×10^{15} cm^{-3} (D) 4×10^{15} cm^{-3}

6. The thermal-equilibrium concentration of hole p_0 in silicon at $T = 300$ K is 10^{15} cm^{-3}. The value of n_0 is

(A) 3.8×10^{8} cm^{-3} (B) 4.4×10^{4} cm^{-3}

(C) 2.6×10^{4} cm^{-3} (D) 4.3×10^{8} cm^{-3}

7. In germanium semiconductor at $T = 300$ K, the acceptor concentrations is $N_a = 10^{13}$ cm^{-3} and donor concentration is $N_d = 0$. The thermal equilibrium concentration p_0 is

(A) 2.97×10^{9} cm^{-3} (B) 2.68×10^{12} cm^{-3}

(C) 2.95×10^{13} cm^{-3} (D) 2.4 cm^{-3}

Statement for Q.8-9:

In germanium semiconductor at $T = 300$ K, the impurity concentration are

$$N_d = 5 \times 10^{15} \text{ cm}^{-3} \text{ and } N_a = 0$$

8. The thermal equilibrium electron concentration n_0 is

(A) 5×10^{15} cm^{-3} (B) 1.15×10^{11} cm^{-3}

(C) 1.15×10^{9} cm^{-3} (D) 5×10^{6} cm^{-3}

9. The thermal equilibrium hole concentration p_0 is

(A) 3.96×10^{13} (B) 1.95×10^{13} cm^{-3}

(C) 4.36×10^{12} cm^{-3} (D) 3.96×10^{13} cm^{-3}

10. A sample of silicon at $T = 300$ K is doped with boron at a concentration of 2.5×10^{13} cm^{-3} and with arsenic at a concentration of 1×10^{13} cm^{-3}. The material is

(A) p – type with $p_0 = 1.5 \times 10^{13}$ cm^{-3}

(B) p – type with $p_0 = 1.5 \times 10^{7}$ cm^{-3}

(C) n – type with $n_0 = 1.5 \times 10^{13}$ cm^{-3}

(D) n – type with $n_0 = 1.5 \times 10^{7}$ cm^{-3}

11. In a sample of gallium arsenide at $T = 200$ K, $n_0 = 5p_0$ and $N_a = 0$. The value of n_0 is

(A) 9.86×10^{9} cm^{-3} (B) 7 cm^{-3}

(C) 4.86×10^{3} cm^{-3} (D) 3 cm^{-3}

12. Germanium at $T = 300$ K is uniformly doped with an acceptor concentration of $N_a = 10^{15}$ cm^{-3} and a donor concentration of $N_d = 0$. The position of fermi energy with respect to intrinsic Fermi level is

(A) 0.02 eV (B) 0.04 eV

(C) 0.06 eV (D)0.08 eV

13. In germanium at $T = 300$ K, the donor concentration are $N_d = 10^{14}$ cm^{-3} and $N_a = 0$. The Fermi energy level with respect to intrinsic Fermi level is

(A) 0.04 eV (B) 0.08 eV

(C) 0.42 eV (D) 0.86 eV

14. A GaAs device is doped with a donor concentration of 3×10^{15} cm^{-3}. For the device to operate properly, the intrinsic carrier concentration must remain less than 5% of the total concentration. The maximum temperature on that the device may operate is

(A) 763 K (B) 942 K

(C) 486 K (D) 243 K

15. For a particular semiconductor at $T = 300$ K$E_g = 1.5$ eV, $m_p^* = 10m_n^*$ and $n_i = 1 \times 10^{15}$ cm^{-3}. The position of Fermi level with respect to the center of the bandgap is

(A) +0.045 eV (B) –0.046 eV

(C) +0.039 eV (D) –0.039 eV

16. A silicon sample contains acceptor atoms at a concentration of $N_a = 5 \times 10^{15}$ cm^{-3}. Donor atoms are added forming and n – type compensated semiconductor such that the Fermi level is 0.215 eV below the conduction band edge. The concentration of donors atoms added are

(A) 1.2×10^{16} cm^{-3} (B) 4.6×10^{16} cm^{-3}

(C) 3.9×10^{12} cm^{-3} (D) 2.4×10^{12} cm^{-3}

17. A silicon semiconductor sample at $T = 300$ K is doped with phosphorus atoms at a concentrations of 10^{15} cm^{-3}. The position of the Fermi level with respect to the intrinsic Fermi level is

(A) 0.3 eV (B) 0.2 eV

(C) 0.1 eV (D) 0.4 eV

18. A silicon crystal having a cross-sectional area of 0.001 cm^2 and a length of 20 μm is connected to its ends to a 20 V battery. At $T = 300$ K, we want a current of 100 mA in crystal. The concentration of donor atoms to be added is

(A) 2.4×10^{13} cm^{-3} (B) 4.6×10^{13} cm^{-3}

(C) 7.8×10^{14} cm^{-3} (D) 8.4×10^{14} cm^{-3}

19. The cross sectional area of silicon bar is 100 μm^2. The length of bar is 1 mm. The bar is doped with arsenic atoms. The resistance of bar is

(A) 2.58 mΩ (B) 11.36 kΩ

(C) 1.36 mΩ (D) 24.8 kΩ

20. A thin film resistor is to be made from a GaAs film doped n – type. The resistor is to have a value of 2 kΩ. The resistor length is to be 200 μm and area is to be 10^{-6} cm^2. The doping efficiency is known to be 90%. The mobility of electrons is 8000 cm^2/V – s. The doping needed is

(A) 8.7×10^{15} cm^{-3} (B) 8.7×10^{21} cm^{-3}

(C) 4.6×10^{15} cm^{-3} (D) 4.6×10^{21} cm^{-3}

21. A silicon sample doped n – type at 10^{18} cm^{-3} have a resistance of $10\ \Omega$. The sample has an area of 10^{-6} cm^2 and a length of $10\ \mu m$. The doping efficiency of the sample is ($\mu_n = 800\ cm^2/V-s$)

(A) 43.2% (B) 78.1%

(C) 96.3% (D) 54.3%

22. Six volts is applied across a 2 cm long semiconductor bar. The average drift velocity is 10^4 cm/s. The electron mobility is

(A) $4396\ cm^2/V-s$ (B) $3\times10^4\ cm^2/V-s$

(C) $6\times10^4\ cm^2/V-s$ (D) $3333\ cm^2/V-s$

23. For a particular semiconductor material following parameters are observed:

$\mu_n = 1000\ cm^2/V-s$,

$\mu_p = 600\ cm^2/V-s$,

$N_c = N_v = 10^{19}\ cm^{-3}$

These parameters are independent of temperature. The measured conductivity of the intrinsic material is $\sigma = 10^{-6}(\Omega-cm)^{-1}$ at $T = 300$ K. The conductivity at $T = 500$ K is

(A) $2\times10^{-4}\ (\Omega-cm)^{-1}$ (B) $4\times10^{-5}\ (\Omega-cm)^{-1}$

(C) $2\times10^{-5}\ (\Omega-cm)^{-1}$ (D) $6\times10^{-3}\ (\Omega-cm)^{-1}$

24. An n – type silicon sample has a resistivity of 5 $\Omega-cm$ at $T = 300$ K. The mobility is $\mu_n = 1350$ $cm^2/V-s$. The donor impurity concentration is

(A) $2.86\times10^{-14}\ cm^{-3}$ (B) $9.25\times10^{14}\ cm^{-3}$

(C) $11.46\times10^{15}\ cm^{-3}$ (D) $1.1\times10^{-15}\ cm^{-3}$

25. In a silicon sample the electron concentration drops linearly from 10^{18} cm^{-3} to 10^{16} cm^{-3} over a length of 2.0 μm. The current density due to the electron diffusion current is ($D_n = 35\ cm^2/s$).

(A) $9.3\times10^4\ A/cm^2$ (B) $2.8\times10^4\ A/cm^2$

(C) $9.3\times10^9 A/cm^2$ (D) $2.8\times10^9\ A/cm^2$

26. In a GaAs sample the electrons are moving under an electric field of 5 kV/cm and the carrier concentration is uniform at 10^{16} cm^{-3}. The electron velocity is the saturated velocity of 10^7 cm/s. The drift current density is

(A) $1.6\times10^4\ A/cm^2$ (B) $2.4\times10^4\ A/cm^2$

(C) $1.6\times10^8 A/cm^2$ (D) $2.4\times10^8\ A/cm^2$

27. For a sample of GaAs scattering time is $\tau_{sc} = 10^{-13}$ s and electron's effective mass is $m_e^* = 0.067m_o$. If an electric field of 1 kV/cm is applied, the drift velocity produced is

(A) 2.6×10^6 cm/s (B) 263 cm/s

(C) 14.8×10^6 cm/s (D) 482

28. A gallium arsenide semiconductor at $T = 300$ K is doped with impurity concentration $N_d = 10^{16}\ cm^{-3}$. The mobility μ_n is 7500 $cm^2/V-s$. For an applied field of 10 V/cm the drift current density is

(A) 120 A/cm^2 (B) 120 A/cm^2

(C) $12\times10^4\ A/cm^2$ (D) $12\times10^4 A/cm^2$

29. In a particular semiconductor the donor impurity concentration is $N_d = 10^{14}\ cm^{-3}$. Assume the following parameters,

$\mu_n = 1000\ cm^2/V-s$,

$$N_c = 2\times10^{19}\left(\frac{T}{300}\right)^{3/2}\ cm^{-3},$$

$$N_v = 1\times10^{19}\left(\frac{T}{300}\right)^{3/2}\ cm^{-3},$$

$E_g = 1.1$ eV.

An electric field of $E = 10$ V/cm is applied. The electric current density at 300 K is

(A) 2.3 A/cm^2 (B) 1.6 A/cm^2

(C) 9.6 A/cm^2 (D) 3.4 A/cm^2

Statement for Q.30-31:

A semiconductor has following parameter

$\mu_n = 7500\ cm^2/V-s$,

$\mu_p = 300\ cm^2/V-s$,

$n_i = 3.6\times10^{12}\ cm^{-3}$

30. When conductivity is minimum, the hole concentration is

(A) $7.2\times10^{11}\ cm^{-3}$ (B) $1.8\times10^{13}\ cm^{-3}$

(C) $1.44\times10^{11}\ cm^{-3}$ (D) $9\times10^{13}\ cm^{-3}$

31. The minimum conductivity is

(A) $0.6\times10^{-3}\ (\Omega-cm)^{-1}$ (B) $1.7\times10^{-3}\ (\Omega-cm)^{-1}$

(C) $2.4\times10^{-3}\ (\Omega-cm)^{-1}$ (D) $6.8\times10^{-3}\ (\Omega-cm)^{-1}$

32. A particular intrinsic semiconductor has a resistivity of 50 $(\Omega - \text{cm})$ at $T = 300$ K and 5 $(\Omega - \text{cm})$ at $T = 330$ K. If change in mobility with temperature is neglected, the bandgap energy of the semiconductor is

(A) 1.9 eV (B) 1.3 eV

(C) 2.6 eV (D) 0.64 eV

33. Three scattering mechanism exist in a semiconductor. If only the first mechanism were present, the mobility would be 500 $\text{cm}^2/\text{V}-\text{s}$. If only the second mechanism were present, the mobility would be 750 $\text{cm}^2/\text{V}-\text{s}$. If only third mechanism were present, the mobility would be 1500 $\text{cm}^2/\text{V}-\text{s}$. The net mobility is

(A) 2750 $\text{cm}^2/\text{V}-\text{s}$ (B) 1114 $\text{cm}^2/\text{V}-\text{s}$

(C) 818 $\text{cm}^2/\text{V}-\text{s}$ (D) 250 $\text{cm}^2/\text{V}-\text{s}$

34. In a sample of silicon at $T = 300$ K, the electron concentration varies linearly with distance, as shown in fig. P2.1.34. The diffusion current density is found to be $J_n = 0.19$ A/cm^2. If the electron diffusion coefficient is $D_n = 25$ cm^2/s, The electron concentration at is

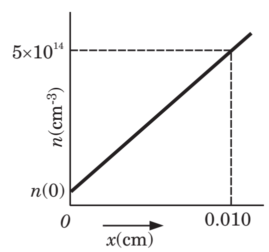

Fig. P2.1.34

(A) 4.86×10^8 cm^{-3} (B) 2.5×10^{13} cm^{-3}

(C) 9.8×10^{26} cm^{-3} (D) 5.4×10^{15} cm^{-3}

35. The hole concentration in $p-$type GaAs is given by

$$p = 10^{16}\left(1 - \frac{x}{L}\right) \text{cm}^{-3} \text{ for } 0 \le x \le L$$

where $L = 10$ μm. The hole diffusion coefficient is 10 cm^2/s. The hole diffusion current density at $x = 5$ μm is

(A) 20 A/cm^2 (B) 16 A/cm^2

(C) 24 A/cm^2 (D) 30 A/cm^2

36. For a particular semiconductor sample consider following parameters:

Hole concentration $p_0 = 10^{15} e^{\left(\frac{-x}{L_p}\right)}$ $\text{cm}^{-3}, x \ge 0$

Electron concentration $n_0 = 5 \times 10^{14} e^{\left(\frac{-x}{L_n}\right)}$ $\text{cm}^{-3}, x \le 0$

Hole diffusion coefficient $D_p = 10$ cm^2/s

Electron diffusion coefficients $D_n = 25$ cm^2/s

Hole diffusion length $L_p = 5 \times 10^{-4}$ cm,

Electron diffusion length $L_n = 10^{-3}$ cm

The total current density at $x = 0$ is

(A) 1.2 A/cm^2 (B) 5.2 A/cm^2

(C) 3.8 A/cm^2 (D) 2 A/cm^2

37. A germanium Hall device is doped with 5×10^{15} donor atoms per cm^3 at $T = 300$ K. The device has the geometry $d = 5 \times 10^{-3}$ cm, $W = 2 \times 10^{-2}$ cm and $L = 0.1$ cm. The current is $I_x = 250$ μA, the applied voltage is $V_x = 100$ mV, and the magnetic flux is $B_z = 5 \times 10^{-2}$ tesla. The Hall voltage is

(A) –0.31mV (B) 0.31 mV

(C) 3.26 mV (D) –3.26 mV

Statement for Q.38-39:

A silicon Hall device at $T = 300$ K has the geometry $d = 10^{-3}$ cm , $W = 10^{-2}$ cm, $L = 10^{-1}$ cm. The following parameters are measured: $I_x = 0.75$ mA, $V_x = 15$ V, $V_H = +5.8$ mV, tesla

38. The majority carrier concentration is

(A) 8×10^{15} cm^{-3}, $n-$type

(B) 8×10^{15} cm^{-3}, $p-$type

(C) 4×10^{15} cm^{-3}, $n-$type

(D) 4×10^{15} cm^{-3}, $p-$type

39. The majority carrier mobility is

(A) 430 $\text{cm}^2/\text{V}-\text{s}$ (B) 215 $\text{cm}^2/\text{V}-\text{s}$

(C) 390 $\text{cm}^2/\text{V}-\text{s}$ (D) 195 $\text{cm}^2/\text{V}-\text{s}$

40. In a semiconductor $n_0 = 10^{15}$ cm^{-3} and $n_i = 10^{10}$ cm^{-3}. The excess-carrier life time is 10^{-6} s. The excess hole concentration is $\delta p = 4 \times 10^{13}$ cm^{-3}. The electron-hole recombination rate is

(A) 4×10^{19} $\text{cm}^{-3}\text{s}^{-1}$ (B) 4×10^{14} $\text{cm}^{-3}\text{s}^{-1}$

(C) 4×10^{24} $\text{cm}^{-3}\text{s}^{-1}$ (D) 4×10^{11} $\text{cm}^{-3}\text{s}^{-1}$

41. A semiconductor in thermal equilibrium, has a hole concentration of $p_0 = 10^{16}$ cm^{-3} and an intrinsic concentration of $n_i = 10^{10}$ cm^{-3}. The minority carrier life time is 4×10^{-7}s. The thermal equilibrium recombination rate of electrons is

(A) 2.5×10^{22} cm^{-3}s^{-1} (B) 5×10^{10} cm^{-3}s^{-1}

(C) 2.5×10^{10} cm^{-3}s^{-1} (D) 5×10^{22} cm^{-3}s^{-1}

Statement for Q.42-43:

A n-type silicon sample contains a donor concentration of $N_d = 10^6$ cm^{-3}. The minority carrier hole lifetime is $\tau_{p0} = 10$ μs.

42. The thermal equilibrium generation rate of hole is

(A) 5×10^8 cm^{-3}s^{-1} (B) 10^4 cm^{-3}s^{-1}

(C) 2.25×10^9 cm^{-3}s^{-1} (D) 10^3 cm^{-3}s^{-1}

43. The thermal equilibrium generation rate for electron is

(A) 1.125×10^9 cm^{-3}s^{-1} (B) 2.25×10^9 cm^{-3}s^{-1}

(C) 8.9×10^{-10} cm^{-3}s^{-1} (D) 4×10^9 cm^{-3}s^{-1}

44. A n-type silicon sample contains a donor concentration of $N_d = 10^{16}$ cm^{-3}. The minority carrier hole lifetime is $\tau_{p0} = 20$ μs. The lifetime of the majority carrier is ($n_i = 1.5 \times 10^{10}$ cm^{-3})

(A) 8.9×10^6 s (B) 8.9×10^{-6} s

(C) 4.5×10^{-17} s (D) 1.13×10^{-7} s

45. In a silicon semiconductor material the doping concentration are $N_a = 10^{16}$ cm^{-3} and $N_a = 0$. The equilibrium recombination rate is $R_{p0} = 10^{11}$ cm^{-3}s^{-1}. A uniform generation rate produces an excess- carrier concentration of $\delta n = \delta p = 10^{14}$ cm^{-3}. The factor, by which the total recombination rate increase is

(A) 2.3×10^{13} (B) 4.4×10^{13}

(C) 2.3×10^9 (D) 4.4×10^9

****

SOLUTIONS

1. (C) $n_i^2 = N_c N_v e^{-\left(\frac{-E_g}{kT}\right)}$

$V_t = 0.0259\left(\frac{400}{300}\right) = 0.0345$

For Ge at 300 K,

$N_c = 1.04 \times 10^{19}$, $N_v = 6.0 \times 10^{18}$, $E_g = 0.66$ eV

$$n_i^2 = 1.04 \times 10^{19} \times 6.0 \times 10^{18} \times \left(\frac{400}{300}\right)^3 \times e^{-\left(\frac{0.66}{0.0345}\right)}$$

$\Rightarrow \quad n_i = 8.5 \times 10^{14}$ cm^{-3}

2. (C) $n_i^2 = N_c N_v e^{-\left(\frac{-E_g}{kT}\right)}$

$$(10^{12})^2 = 2.8 \times 10^{19} \times 1.04 \times 10^{19}\left(\frac{T}{300}\right)^3 e^{-\left(\frac{1.12e}{kT}\right)}$$

$T^3 e^{\frac{-13\times10^3}{T}} = 9.28 \times 10^{-8}$, By trial $T = 382$ K

3. (B) $$\frac{n_{iA}^2}{n_{iB}^2} = \frac{e^{-\frac{E_{gA}}{kT}}}{e^{-\frac{E_{gB}}{kT}}} = e^{-\left(\frac{E_{gA} - E_{gB}}{kT}\right)} = 2257.5 \quad \Rightarrow \quad \frac{n_{iA}}{n_{iB}} = 47.5$$

4. (A) $$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^4} = 4.5 \times 10^{15} \text{ cm}^{-3}$$

5. (A) $$p_0 = N_v e^{-\frac{(E_F - E_v)}{kT}} = 1.04 \times 10^{19} e^{\frac{-0.22}{0.0259}} = 2 \times 10^{15} \text{ cm}^{-3}$$

6. (B) $$p_0 = N_v e^{-\frac{(E_F - E_v)}{kT}} \quad \Rightarrow \quad E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right)$$

At 300 K, $N_v = 1.0 \times 10^{19}$ cm^{-3}

$$E_F - E_v = 0.0259 \ln\left(\frac{1.04 \times 10^{19}}{10^{15}}\right) = 0.239 \text{ eV}$$

$n_0 = N_c e^{\frac{(E_c - E_F)}{kT}}$

At 300 K, $N_c = 2.8 \times 10^{19}$ cm^{-3}

$E_c - E_F = 1.12 - 0.239 = 0.881$ eV

$n_0 = 4.4 \times 10^4$ cm^{-3}

7. (C) $$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(N_a - \frac{N_d}{2}\right)^2 + n_i^2}$$

For Ge $n_i = 2.4 \times 10^3$

$$p_0 = \frac{10^{13}}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + (2.4 \times 10^{13})^2} = 2.95 \times 10^{13} \text{ cm}^{-3}$$

$$=\frac{0.1}{(1.6\times10^{-19})(1100)(5\times10^{16})(100\times10^{-8})}=11.36\ \text{k}\Omega$$

20. (A) $R=\frac{L}{\sigma A}$, $\sigma\approx e\mu_n n_0$, $R=\frac{L}{e\mu_n n_0 A}$

$$\Rightarrow\ n_0=\frac{L}{e\mu_n AR}$$

$$n_0=0.9N_d$$

$$=\frac{20\times10^{-4}}{(0.9)(1.6\times10^{-19})(8000)(10^{-6})(2\times10^{3})}=8.7\times10^{15}\ \text{cm}^{-3}$$

21. (B) $\sigma\approx e\mu_n n_0$, $R=\frac{L}{\sigma A}$, $n_0=\frac{L}{e\mu_n AR}$

$$=\frac{10\times10^{-4}}{(1.6\times10^{-19})(800)(10^{-6})(10)}=7.81\times10^{17}\ \text{cm}^{-3}$$

$$\text{Efficiency}=\frac{n_0}{N_d}\times100=\frac{7.8\times10^{17}}{10^{18}}\times100=78.1\ \%$$

22. (D) $E=\frac{V}{L}=\frac{6}{2}=3$ V/cm, $v_d=\mu_n E$,

$$\mu_n=\frac{v_d}{E}=\frac{10^4}{3}=3333\ \text{cm}^2/\text{V}-\text{s}$$

23. (D) $\sigma_1=en_i(\mu_n+\mu_p)$

$$10^{-6}=(1.6\times10^{-19})(1000+600)n_i$$

At $T=300$ K, $n_i=3.91\times10^{9}\ \text{cm}^{-3}$

$$n_i^2=N_cN_v\,e^{-\left(\frac{E_g}{kT}\right)}\ \Rightarrow E_g=kT\ln\left(\frac{N_cN_v}{n_i^2}\right)$$

$$\Rightarrow E_g=2(0.0259)\ln\left(\frac{10^{19}}{3.91\times10^{9}}\right)=1.122\ \text{eV}$$

At $T=500$ K, $kT=0.0259\left(\frac{500}{300}\right)=0.0432$ eV,

$$n_i^2=(10^{19})^2\,e^{-\left(\frac{1.122}{0.0432}\right)}\ \text{cm}^{-3},$$

$$\Rightarrow\ n_i=2.29\times10^{13}\ \text{cm}^{-3}$$

$$=(1.6\times10^{-19})(2.29\times10^{13})(1000+600)$$

$$=5.86\times10^{-3}(\Omega-\text{cm})^{-1}$$

24. (B) $\rho=\frac{1}{\sigma}=\frac{1}{e\mu_n N_d}$

$$N_d=\frac{1}{\rho e\mu_n}=\frac{1}{5(1.6\times10^{-19})(1350)}=9.25\times10^{14}\ \text{cm}^{-3}$$

25. (B) $J_n=eD_n\frac{dn}{dx}$

$$=(1.6\times10^{-19})(35)\left(\frac{10^{18}-10^{16}}{2\times10^{-4}}\right)=2.8\times10^{4}\ \text{A/cm}^2$$

26. (A) $J=evn=(1.6\times10^{-19})(10^{7})(10^{16})=1.6\times10^{4}\ \text{A/cm}^2$

27. (A) $v_d=\frac{e\tau_{sc}E}{m_e^*}=\frac{(1.6\times10^{-19})(10^{-13})(10^{5})}{(0.067)(9.1\times10^{-31})}$

$$=26.2\times10^{3}\ \text{m/s}=2.6\times10^{6}\ \text{cm/s}$$

28. (A) $N_d>>n_i\ \Rightarrow\ n_0=N_d$

$$J=e\mu_n n_0 E=(1.6\times10^{-19})(7500)(10^{16})(10)=120\ \text{A/cm}^2$$

29. (D) $n_i^2=N_cN_v\,e^{-\left(\frac{-E_g}{kT}\right)}$

$$=(2\times10^{19})(1\times10^{19})\,e^{-\left(\frac{1.1}{0.0259}\right)}=7.18\times10^{19}$$

$$\Rightarrow\ n_i=8.47\times10^{9}\ \text{cm}^{-3}$$

$$N_d>>n_i\ \Rightarrow\ N_d=n_0$$

$$J=\sigma E=e\mu_n n_0 E$$

$$=(1.6\times10^{-19})(1000)(10^{14})(100)=1.6\ \text{A/cm}^2$$

30. (A) $\sigma=e\mu_n n_0+e\mu_p p_0$ and $n_0=\frac{n_i^2}{p_0}$

$$\Rightarrow\ \sigma=e\mu_n\frac{n_i^2}{p_0}+e\mu_p p_0\ ,$$

$$\frac{d\sigma}{dp_0}=0=\frac{(-1)e\mu_n n_i^2}{p_0^2}+e\mu_p$$

$$\Rightarrow\ p_0=n_i\left(\frac{\mu_n}{\mu_p}\right)^{\frac{1}{2}}=3.6\times10^{12}\left(\frac{7500}{300}\right)^{\frac{1}{2}}$$

$$=7.2\times10^{11}\ \text{cm}^{-3}$$

31. (B) $\sigma_{min}=\frac{2\sigma_i\sqrt{\mu_p\mu_n}}{\mu_p+\mu_n}=2\,en_i\sqrt{\mu_p\mu_n}$

$$=2\times1.6\times10^{-19}(3.6\times10^{12})\sqrt{(7500)(300)}$$

$$=1.7\times10^{-3}(\Omega-\text{cm})^{-1}$$

32. (B) $\sigma=\frac{1}{\rho}=e\mu n_i$,

$$\frac{\frac{1}{\rho_1}}{\frac{1}{\rho_2}}=\frac{n_{i1}}{n_{i2}}=\frac{e^{-\frac{E_g}{2kT_1}}}{e^{-\frac{E_g}{2kT_2}}}$$

$$0.1=e^{-\frac{E_g}{2k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$

$$\frac{E_g}{2k}\left(\frac{330-300}{330\times300}\right)=\ln 10$$

$$E_g=22(k300)\ln 10=1.31\ \ \text{eV}$$

33. (D) $\frac{1}{\mu}=\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3}$

CHAPTER

2.2

THE PN JUNCTION

In this chapter, N_d and N_a denotes the net donor and acceptor concentration in the individual n and p-region.

1. An abrupt silicon in thermal equilibrium at $T = 300$ K is doped such that $E_c - E_F = 0.21$ eV in the $n-$region and $E_F - E_v = 0.18$ eV in the $p-$region. The built-in potential barrier V_{bi} is

(A) 0.69 V (B) 0.83 V
(C) 0.61 V (D) 0.88 V

2. A silicon pn junction at $T = 300$ K has $N_d = 10^{14}$ cm^{-3}and $N_a = 10^{17}$ cm^{-3}. The built-in voltage is

(A) 0.63 V (B) 0.93 V
(C) 0.026 V (D) 0.038 V

3. In a uniformly doped GaAs junction at $T = 300$ K, at zero bias, only 20% of the total space charge region is to be in the p-region. The built in potential barrier is $V_{bi} = 1.20$ V. The majority carrier concentration in n-region is

(A) 1×10^{16} cm^{-3} (B) 4×10^{16} cm^{-3}
(C) 1×10^{22} cm^{-3} (D) 4×10^{22} cm^{-3}

Statement for Q.4–5:

An abrupt silicon pn junction at zero bias and $T = 300$ K has dopant concentration of $N_a = 10^{17}$ cm^{-3} and $N_d = 5 \times 10^{15}$ cm^{-3}.

4. The Fermi level on $n-$side is

(A) 0.1 eV (B) 0.2 eV
(C) 0.3 eV (D) 0.4 eV

5. The Fermi level on $p-$side is

(A) 0.2 eV (B) 0.1 eV
(C) 0.4 eV (D) 0.3 eV

Statement for Q.6–8:

A silicon pn junction at $T = 300$ K with zero applied bias has doping concentrations of $N_d = 5 \times 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}.

6. The width of depletion region extending into the n-region is

(A) 4×10^{-6} cm (B) 3×10^{-6} cm
(C) 4×10^{-5} cm (D)3×10^{-5} cm

7. The space charge width is

(A) 3.2×10^{-5} cm (B) 4.5×10^{-5} cm
(C) 4.5×10^{-4} cm (D) 3.2×10^{-4} cm

8. In depletion region maximum electric field $|E_{max}|$ is

(A) 1×10^4 V/cm (B) 2×10^4 V/cm
(C) 3×10^4 V/cm (D) 4×10^4 V/cm

9. An $n - n$ isotype doping profile is shown in fig. P2.2.9. The built-in potential barrier is ($n_i = 1.5 \times 10^{10}$ cm^{-3})

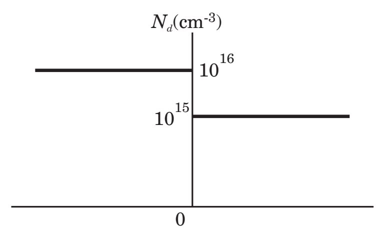

Fig. P2.2.9

(A) 0.66 V (B) 0.06 V
(C) 0.03 V (D) 0.33 V

Statement for Q.10–11:

A silicon abrupt junction has dopant concentration $N_a = 2 \times 10^{16}$ cm^{-3} and $N_d = 2 \times 10^{15}$ cm^{-3}. The applied reverse bias voltage is $V_R = 8$ V.

10. The maximum electric field $|E_{max}|$ in depletion region is

(A) 15×10^4 V/cm (B) 7×10^4 V/cm

(C) 3.5×10^4 V/cm (D) 5×10^4 V/cm

11. The space charge region is

(A) 2.5 μm (B) 25 μm

(C) 50 μm (D) 100 μm

12. A uniformly doped silicon *pn* junction has $N_a = 5 \times 10^{17}$ cm^{-3} and $N_d = 10^{17}$ cm^{-3}. The junction has a cross-sectional area of 10^{-4} cm^{-3} and has an applied reverse-bias voltage of $V_R = 5$ V. The total junction capacitance is

(A) 10 pF (B) 5 pF

(C) 7 pF (D) 3.5 pF

Statement for Q.13–14:

An ideal one-sided silicon n^+p junction has uniform doping on both sides of the abrupt junction. The doping relation is $N_d = 50N_a$. The built-in potential barrier is $V_{bi} = 0.75$ V. The applied reverse bias voltage is $V_R = 10$.

13. The space charge width is

(A) 1.8 μm (B) 1.8 mm

(C) 1.8 cm (D) 1.8 m

14. The junction capacitance is

(A) 3.8×10^{-9} F/cm^2 (B) 9.8×10^{-9} F/cm^2

(C) 2.4×10^{-9} F/cm^2 (D) 5.7×10^{-9} F/cm^2

15. Two p^+n silicon junction is reverse biased at $V_R = 5$ V. The impurity doping concentration in junction A are $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{-15}$ cm^{-3}, and those in junction B are $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The ratio of the space charge width is

(A) 4.36 (B) 9.8

(C) 19 (D) 3.13

16. The maximum electric field in reverse-biased silicon *pn* junction is $|E_{max}| = 3 \times 10^5$ V/cm. The doping concentration are $N_d = 4 \times 10^{16}$ cm^{-3} and $N_a = 4 \times 10^{17}$ cm^{-3}. The magnitude of the reverse bias voltage is

(A) 3.6 V (B) 9.8 V

(C) 7.2 V (D) 12.3 V

17. An abrupt silicon *pn* junction has an applied reverse bias voltage of $V_R = 10$ V. it has dopant concentration of $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. The *pn* junction area is 6×10^{-4} cm^2. An inductance of 2.2 mH is placed in parallel with the *pn* junction. The resonant frequency is

(A) 1.7 MHz (B) 2.6 MHz

(C) 3.6 MHz (D) 4.3 MHz

18. A uniformly doped silicon p^+n junction is to be designed such that at a reverse bais voltage of $V_R = 10$ V the maximum electric field is limited to $E_{max} = 10^6$ V/cm. The maximum doping concentration in the *n*-region is

(A) 3.2×10^{19} cm^{-3} (B) 3.2×10^{17} cm^{-3}

(C) 6.4×10^{17} cm^{-3} (D) 6.4×10^{19} cm^{-3}

19. A diode has reverse saturation current $I_s = 10^{-10}$ A and non ideality factor $\eta = 2$. If diode voltage is 0.9 V, then diode current is

(A) 11 mA (B) 35 mA

(C) 83 mA (D) 143 mA

20. A diode has reverse saturation current $I_s = 10^{-18}$ A and nonideality factor $\eta = 1.05$. If diode has current of 70 μA, then diode voltage is

(A) 0.63 V (B) 0.87 V

(C) 0.54 V (D) 0.93 V

21. An ideal *pn* junction diode is operating in the forward bais region. The change in diode voltage, that will cause a factor of 9 increase in current, is

(A) 83 mV (B) 59 mV

(C) 43 mV (D) 31 mV

22. An *pn* junction diode is operating in reverse bias region. The applied reverse voltage, at which the ideal reverse current reaches 90% of its reverse saturation current, is

(A) 59.6 mV (B) 2.7 mV

(C) 4.8 mV (D) 42.3 mV

23. For a silicon p^+n junction diode the doping concentrations are $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The minority carrier hole diffusion coefficient is $D_p = 12$ cm^2/s and the minority carrier hole life time is $\tau_{p0} = 10^{-7}$ s. The cross sectional area is $A = 10^{-4}$ cm^2. The reverse saturation current is

(A) 4×10^{-12} A (B) 4×10^{-15} A

(C) 4×10^{-11} A (D) 4×10^{-7} A

24. For an ideal silicon *pn* junction diode

$\tau_{no} = \tau_{po} = 10^{-7}$ s ,

$D_n = 25$ cm^2/s ,

$D_p = 10$ cm^2/s

The ratio of N_a/N_d , so that 95% of the current in the depletion region is carried by electrons, is

(A) 0.34 (B) 0.034

(C) 0.83 (D) 0.083

Statement for Q.25–26:

A ideal long silicon *pn* junction diode is shown in fig. P2.2.25–26. The n – region is doped with 10^{16} organic atoms per cm^3 and the p – region is doped with 5×10^{16} boron atoms per cm^3. The minority carrier lifetimes are $D_n = 23$ cm^2/s and $D_p = 8$ cm^2/s. The forward-bias voltage is $V_a = 0.61$ V.

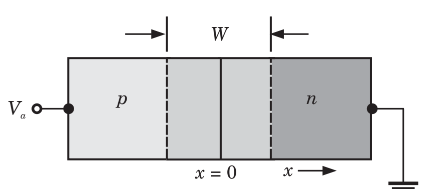

Fig. P.2.2.25-26

25. The excess hole concentration is

(A) $6.8 \times 10^{12} e^{-246x}$ cm^{-3}, $x \geq 0$

(B) $6.8 \times 10^{12} e^{-246x}$ cm^{-3}, $x \geq 0$

(C) $3.8 \times 10^{14} e^{-3534x}$ cm^{-3}, $x \geq 0$

(D) $3.8 \times 10^{14} e^{+3534x}$ cm^{-3}, $x \geq 0$

26. The hole diffusion current density at $x = 3$ μm is

(A) 0.6 A/cm^2 (B) 0.6×10^{-3} A/cm^2

(C) 0.4 A/cm^2 (D) 0.4×10^{-3} A/cm^2

27. The doping concentrations of a silicon *pn* junction are $N_d = 10^{16}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. The cross-sectional area is 10^{-3} cm^2. The minority carrier lifetimes are $\tau_{n0} = 1$ μs and $\tau_{p0} = 0.1$ μs. The minority carrier diffusion coefficients are $D_n = 35$ cm^2/s and $D_p = 10$ cm^2/s. The total number of excess electron in the p – region, if applied forward bias is $V_a = 0.5$ V, is

(A) 4×10^7 cm^{-3} (B) 6×10^{10} cm^{-3}

(C) 4×10^{10} cm^{-3} (D) 6×10^7 cm^{-3}

28. Two ideal *pn* junction have exactly the same electrical and physical parameters except for the band gap of the semiconductor materials. The first has a bandgap energy of 0.525 eV and a forward-bias current of 10 mA with $V_a = 0.255$ V. The second *pn* junction diode is to be designed such that the diode current $I = 10$ μA at a forward-bias voltage of $V_a = 0.32$ V. The bandgap energy of second diode would be

(A) 0.77 eV (B) 0.67 eV

(C) 0.57 eV (D) 0.47 eV

29. A *pn* junction biased at $V_a = 0.72$ V has DC bias current $I_{DQ} = 2$ mA. The minority carrier lifetime is 1 μs is both the n and p regions. The diffusion capacitance is in

(A) 49.3 nF (B) 38.7 nF

(C) 77.4 nF (D) 98.6 nF

30. A p^+n silicon diode is forward biased at a current of 1 mA. The hole life time in the n – region is 0.1 μs. Neglecting the depletion capacitance the diode impedance at 1 MHz is

(A) $38.7 + j12.1\,\Omega$ (B) $23.5 + j7.5\,\Omega$

(C) $38.7 - j12.1$ mΩ (D) $23.5 - j7.5$ Ω

31. The slope of the diffusion capacitance verses forward-bias current of a p^+n diode is 2.5×10^{-6} F/A. The hole lifetime is

(A) 1.3×10^{-7} s (B) 1.3×10^{-4} s

(C) 6.5×10^{-8} s (D) 6.5×10^{-4} s

32. A silicon *pn* junction with doping profile of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3} has a cross sectional area of 10^{-2} cm^2. The length of the p – region is 2 mm and length of the n – region is 1 mm. The approximately series resistance of the diode is

(A) 62 Ω (B) 43 Ω

(C) 72 Ω (D) 81 Ω

33. A gallium arsenide *pn* junction is operating in reverse-bias voltage $V_R = 5$ V. The doping profile are $N_a = N_d = 10^{16}$ cm^{-3}. The minority carrier life- time are $\tau_{p0} = \tau_{n0} = \tau_0 = 10^{-8}$ *s*. The reverse-biased generation current density is $(\varepsilon_r = 13.1,\ n_i = 1.8 \times 10^6)$

(A) 1.9×10^{-8} A/cm^2 (B) 1.9×10^{-9} A/cm^2

(C) 1.4×10^{-8} A/cm^2 (D) 1.4×10^{-9} A/cm^2

34. For silicon the critical electric field at breakdown is approximately $E_{crit} = 4 \times 10^5$ V/cm. For the breakdown voltage of 25 V, the maximum $n-$type doping concentration in an abrupt p^+n-junction is

(A) 2×10^{16} cm^{-3} (B) 4×10^{16} cm^{-3}

(C) 2×10^{18} cm^{-3} (D) 4×10^{18} cm^{-3}

35. A uniformly doped silicon *pn* junction has dopant profile of $N_a = N_d = 5 \times 10^{16}$ cm^{-3}. If the peak electric field in the junction at breakdown is $E = 4 \times 10^5$ V/cm, the breakdown voltage of this junction is

(A) 35 V (B) 30 V

(C) 25 V (D) 20 V

36. An abrupt silicon p^+n junction has an $n-$region doping concentration of $N_d = 5 \times 10^{15}$ cm^{-3}. The minimum $n-$region width, such that avalanche breakdown occurs before the depletion region reaches an ohmic contact, is ($V_B \approx 100$ V)

(A) 5.1 μm (B) 3.6 μm

(C) 7.3 μm (D) 6.4 μm

37. A silicon *pn* junction diode has doping profile $N_a = N_d = 5 \times 10^{19}$ cm^{-3}. The space charge width at a forward bias voltage of $V_a = 0.4$ V is

(A) 102 A° (B) 44 A°

(C) 153 A° (D) 62 A°

38. A GaAs pn^+ junction LED has following parameters

$D_n = 25$ cm^2/s, $D_p = 12$ cm^2/s

$N_d = 5 \times 10^{17}$ cm^{-3}, $N_a = 10^{16}$ cm^{-3}

$\tau_{n0} = 10$ ns , $\tau_{p0} = 10$ ns

The injection efficiency of the LED is

(A) 0.83 (B) 0.99

(C) 0.64 (D) 0.46

39. A GaAs laser has a threshold density of 500 A/cm^2. The laser has dimensions of $10\ \mu m \times 200\ \mu m$. The active region is $d_{Las} = 100$ A°. The electron-hole recombination time at threshold is 1.5 ns. The current density of $5J_{th}$ is injected into the laser. The optical power emitted, if emitted photons have an energy of 1.43 eV, is

(A) 143 mW (B) 71.5 mW

(C) 62.3 mW (D) 124.6 mW

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$= 3.01 \times 10^{-5}$ cm,

$$C_T = \frac{\varepsilon A}{W} = \frac{11.7 \times 8.85 \times 10^{-14} \times 10^{-4}}{3.01 \times 10^{-5}} = 3.5 \times 10^{-12} \text{ F}$$

13. (A) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$0.751 = 0.0259 \ln\left(\frac{50 N_a^2}{2.25 \times 10^{20}}\right)$$

$N_a = 4.2 \times 10^{15}$ cm^{-3}, $N_d = 2.1 \times 10^{17}$ cm^{-3}

$$W = \left\{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{1}{N_a} + \frac{1}{N_a}\right]\right\}^{\frac{1}{2}},$$

$$N_d >> N_a \quad \Rightarrow \quad W \approx \left[\frac{2\,\varepsilon (V_{bi} + V_R)}{e}\left(\frac{1}{N_a}\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{2 \times (11.7 \times 8.85 \times 10^{-4})(10.752)}{1.6 \times 10^{-19} \times 4.2 \times 10^{15}}\right]^{\frac{1}{2}} = 1.8 \times 10^{-4} \text{ cm}$$

$= 1.8$ μm

14. (D) $C' = \left[\frac{e\varepsilon N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right]^{\frac{1}{2}}$

For $N_d >> N_a$, $C' = \left[\frac{e\varepsilon N_a}{2(V_{bi} + V_R)}\right]^{\frac{1}{2}}$

$$= \left[\frac{1.6 \times 10^{-19} \times 11.7 \times 8.85 \times 10^{-4} \times 4.2 \times 10^{15}}{2\,(10 + 0.754)}\right]^{\frac{1}{2}}$$

$= 5.7 \times 10^{-9}$ F/cm^2

15. (D) $W = \left\{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{1}{N_a} + \frac{1}{N_a}\right]\right\}^{\frac{1}{2}}$

$$\frac{W_A}{W_B} = \left[\frac{(V_{bia} + V_R)}{(V_{bib} +_R)}\frac{(N_{aA} + N_{dA})}{(N_{aB} + N_{dB})}\frac{N_{aB} N_{dB}}{N_{aA} N_{dB}}\right]^{\frac{1}{2}}$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$V_{biA} = 0.0259 \ln\left(\frac{10^{18} \times 10^{15}}{2.25 \times 10^{20}}\right) = 0.754 \text{ V}$$

$$V_{biB} = 0.0259 \ln\left(\frac{10^{18} \times 10^{16}}{2.25 \times 10^{20}}\right) = 0.814 \text{ V}$$

$$\frac{W_A}{W_B} = \left[\left(\frac{5.754}{5.814}\right)\left(\frac{10^{18} + 10^{15}}{10^{18} + 10^{16}}\right)\left(\frac{10^{16}}{10^{15}}\right)\right]^{\frac{1}{2}} = 3.13$$

16. (C) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= 0.0259 \ln\left(\frac{4 \times 10^{16} \times 4 \times 10^{17}}{2.25 \times 10^{20}}\right) = 0.826 \text{ V}$$

$$|E_{max}| = \left[\frac{2e(V_{bi} + V_R)}{\varepsilon}\frac{N_a N_d}{(N_a + N_d)}\right]^{\frac{1}{2}}$$

$$V_{bi} + V_R = \frac{\varepsilon E_{\max}^2}{2e}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)$$

$$= \frac{(11.7 \times 8.85 \times 10^{-14})(3 \times 10^5)^2}{2 \times 1.6 \times 10^{-19}}\left(\frac{1}{4 \times 10^{16}} + \frac{1}{4 \times 10^{17}}\right) \text{V}$$

$= 8.008$ V

$V_R = 8.008 - 0.826 = 7.18$ V

17. (B) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= 0.0259 \ln\left(\frac{10^{18} \times 10^{15}}{2.25 \times 10^{20}}\right) = 0.754 \text{ V}$$

$$C' = \left[\frac{e\varepsilon N_a N_d}{2\,(V_i + V_R)(N_a + N_d)}\right]^{\frac{1}{2}}$$

For $N_a >> N_d$, $C' = \left[\frac{e\varepsilon N_d}{2\,(V_{bi} + V_R)}\right]^{\frac{1}{2}}$

$$= \left[\frac{1.6 \times 10^{-19} \times 11.7 \times 8.85 \times 10^{-4} \times 10^{15}}{2\,(10 + 0.754)}\right]^{\frac{1}{2}}$$

$= 2.77 \times 10^{-9}$ F/cm^2

$C = AC' = 6 \times 10^{-4} \times 2.77 \times 10^{-9} = 1.66 \times 10^{-12}$ F

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2.2 \times 10^{-3} \times 1.66 \times 10^{-12}}} = 2.6 \text{ MHz.}$$

18. (B) $|E_{\max}| = \frac{eN_a x_n}{\varepsilon}$

For a $p^+ n$ junction, $x_n \approx \left[\frac{2\varepsilon(V_{bi} + V_R)}{eN_d}\right]$

So that $|E_{max}| = \left[\frac{2\,eN_d}{\varepsilon_s}\,(V_{bi} + V_R)\right]^{\frac{1}{2}}$

Assuming $V_{bi} << V_R$,

$$N_d = \frac{\varepsilon E_{\max}^2}{2eV_R} = \frac{(11.7 \times 8.85 \times 10^{-14})(10^6)^2}{2(1.6 \times 10^{-14})(10)}$$

$= 3.24 \times 10^{17}$ cm^{-3}

19. (B) $I_D = I_s\left(e^{\frac{V_D}{\eta V_t}} - 1\right) = 10^{-10}\left(e^{\frac{0.9}{2 \times (0.0259)}} - 1\right) = 35$ mA

20. (B) $I_D = I_s\left(e^{\frac{V_D}{\eta V_t}} - 1\right) \quad \Rightarrow \quad V_D = \eta V_t \ln\left(1 + \frac{I_D}{I_s}\right)$

$$=(1.05)(0.0259)\ln\left(1+\frac{70\times10^{-6}}{10^{-18}}\right)=0.87\ \text{V}$$

21. (B) $I_d \approx I_s e^{\left(-\frac{V}{V_t}\right)}$, $\quad \frac{I_{d1}}{I_{d2}}=\frac{e^{-\frac{V_1}{V_t}}}{e^{-\frac{V_2}{V_t}}}=e^{-\frac{(V_1-V_2)}{V_t}}$

$$V_1-V_2=V_t\ln\left(\frac{I_{d2}}{I_{d1}}\right)=0.0259\ln 10=59.6\ \text{mV}$$

22. (A) $I=I_s\left(e^{\frac{V}{V_t}}-1\right) \Rightarrow V=V_t\ln\left(\frac{I}{I_s}+1\right)$

$\frac{I}{I_s}=-0.90$ (–ive due to reverse current)

$V=0.0259\ln(1-0.9)=-59.6\ \text{mV}$

23. (B) $I_s=Aen_i^2\frac{1}{N_d}\sqrt{\frac{D}{\tau_{po}}}$

$$=\frac{(10^{-4})(1.6\times10^{-19})(1.5\times10^{10})^2}{10^{16}}\sqrt{\frac{12}{10^{-7}}}=3.94\times10^{-15}\ \text{A}$$

24. (D) $\frac{J_n}{J_n+J_p}=0.95$,

$J_n=en_i^2\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}}$, $\quad J_p=en_i^2\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}$

$$\frac{\sqrt{D_n}}{\sqrt{D_n}+\frac{N_a}{N_d}\sqrt{D_p}}=0.95,\qquad \frac{5}{5+\frac{N_a}{N_d}\sqrt{10}}=0.95$$

$$\Rightarrow \frac{N_a}{N_d}=0.083$$

25. (C) $\delta p_n=p_n-p_{n0}=p_{n0}\left[e^{\left(\frac{eV_a}{kt}\right)}-1\right]\left[e^{\left(-\frac{x}{L_p}\right)}\right]$

$$p_{n0}=\frac{n_i^2}{N_d}=\frac{(1.5\times10^{10})^2}{10^{16}}=2.25\times10^4\ \text{cm}^{-3}$$

$$L_p=\sqrt{D_p\tau_{p0}}=\sqrt{(8)(1\times10^{-8})}=2.83\times10^{-4}\ \text{cm}$$

$$\delta p_n=2.25\times10^4\left[e^{\left(\frac{0.61}{0.0259}\right)}-1\right]\left[e^{\left(-\frac{x}{2.83\times10^{-4}}\right)}\right]$$

$$=3.8\times10^{14}e^{-3534x}\ \text{cm}^{-3}$$

26. (A) $J_p=-eD_p\frac{\partial(\delta p_n)}{\partial x}=eD_p(3.8\times10^{14})(3534)e^{-3534x}$

$x=3\ \mu\text{m}=3\times10^{-4}\ \text{cm}$

$J_p=(1.6\times10^{-19})(18)(3.8\times10^{14})(3534)e^{-(3534)(3\times10^{-4})}$

$=0.6\ \text{A/cm}^2$

27. (A) $N_p=AL_n n_{p0}\left[e^{\left(\frac{eV_a}{kT}\right)}-1\right]$

$$n_{p0}=\frac{n_i^2}{N_a}=\frac{(1.5\times10^{10})^2}{8\times10^{15}}=2.8\times10^4\ \text{cm}^{-3}$$

$$L_n=\sqrt{D_n\tau_{no}}=\sqrt{35\times10^{-6}}=5.9\times10^{-3}\ \text{Cm}$$

$$N_p=10^{-3}\times5.9\times10^{-3}\times2.8\times10^4\left[e^{\left(\frac{0.5}{0.0259}\right)}-1\right]$$

$$=4\times10^7\ \text{cm}^{-3}$$

28. (A) $I\propto n_i^2 e^{\left(\frac{V_a}{V_t}\right)}\propto e^{\left(\frac{-E_g}{V_t}\right)}e^{\left(\frac{V_a}{V_t}\right)}$

$$\Rightarrow I\propto e^{\left(\frac{V_a-E_g}{V_t}\right)}$$

$$\frac{I_1}{I_2}=\frac{e^{\left(\frac{V_a-E_{g1}}{V_t}\right)}}{e^{\left(\frac{V_{a2}-E_{g2}}{V_t}\right)}}=e^{\frac{1}{V_t}(V_{a1}-V_{a2}-E_{g1}+E_{g2})}$$

$$\frac{10\times10^{-3}}{10\times10^{-6}}=e^{\frac{(0.255-0.32-0.525+E_{g2})}{(0.0259)}}$$

$$10^3=e^{\left(\frac{E_{g2}-0.59}{0.0259}\right)}$$

$$E_{g2}=0.59+0.0259\ln 10^3=0.769\ \text{EV}$$

29. (B) $C_d=\left(\frac{I_{p0}\tau_{p0}+I_{n0}\tau_{n0}}{2V_t}\right)$

$\tau_{n0}=\tau_{p0}=10^{-6}\ \text{s}$, $\qquad I_{p0}+I_{n0}=I_{dQ}=2\ \text{mA}$

$$C_d=\frac{2\times10^{-3}\times10^{-6}}{2(0.0259)}=3.86\times10^{-8}\ \text{F}$$

30. (D) $g_d=\frac{I_{dQ}}{V_t}=\frac{10^{-3}}{0.0259}=3.86\times10^{-2}\ \text{S}$

$$C_d=\frac{I_{dQ}\tau_{p0}}{2V_t}=\frac{10^{-3}\times10^{-7}}{2\times(0.0259)}=1.93\times10^{-9}\ \text{F}$$

$$Z=\frac{1}{Y}=\frac{1}{g_d+j\omega C_d}=23.5-j7.5\ \Omega$$

31. (A) For a p^+n diode $I_{p0}>>I_{n0}$

$$C_d=\left(\frac{1}{2V_t}\right)(I_{po}\tau_{po}),\qquad \frac{\tau_{p0}}{2V_t}=2.5\times10^{-6}$$

$$\Rightarrow \tau_{p0}=2\times0.0259\times2.5\times10^{-6}=1.3\times10^{-7}\ s$$

32. (C) $R_P=\frac{\rho_p L}{A}=\frac{L}{A(e\mu_p N_a)}$

$$=\frac{0.2}{(10^{-2})(1.6\times10^{-19})(480)(10^{16})}=26\ \Omega$$

$R_n = \frac{\rho_n L}{A} = \frac{L}{Ae(\mu_n N_d)}$

$= \frac{0.10}{(10^{-3})(1.6\times10^{-19})(1350)(10^{15})} = 46.3\,\Omega$

$R = R_p + R_n = 72.3\,\Omega$

33. (B) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$= 2(0.0259)\ln\left(\frac{10^{16}}{1.8\times10^{6}}\right) = 1.16 \text{ V}$

$W = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{1}{N_a}+\frac{1}{N_a}\right]\right\}^{\frac{1}{2}}$

$= \left[\frac{2\times(13.1\times8.85\times10^{-14})(6.16)}{1.6\times10^{-19}}\left(\frac{2}{10^{16}}\right)\right]^{\frac{1}{2}} = 1.34\times10^{-4} \text{ cm}$

$J_{gen} = \frac{en_i W}{2\tau_o}$

$= \frac{1.6\times10^{-19}\times1.8\times10^{6}\times1.34\times10^{-4}}{2\times10^{-8}} = 1.93\times10^{-9} \text{ A/cm}^2$

34. (A) $V_B = \frac{\varepsilon E_{crit}^2}{2eN_B}$

$25 = \frac{(11.7\times8.85\times10^{-4})(4\times10^{5})^2}{2\times1.6\times10^{-19}\times N_B}$

$N_B = N_d = 2\times10^{16} \text{ cm}^{-3}$

35. (D) $E_{max} = \frac{eN_d x_n}{\varepsilon} \Rightarrow x_n = \frac{\varepsilon E_{max}}{eN_d}$

$= \frac{(11.7\times8.85\times10^{-14})(4\times10^{5})}{(1.6\times10^{-19})(5\times10^{16})} = 5.18\times10^{-5} \text{ cm}$

$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = 2(0.0259)\ln\left(\frac{5\times10^{6}}{1.5\times10^{10}}\right) = 0.778 \text{ V}$

$x_n = \left\{\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{\frac{1}{2}}$

$(5.18\times10^{-5})^2 = \left[\frac{2(11.7\times8.85\times10^{-4})(V_{bi}+V_R)}{(1.6\times10^{-19})(2\times5\times10^{6})}\right]$

$\Rightarrow \quad V_{bi}+V_R = 20.7, \quad V_R = 19.9 \text{ V}, \quad V_R = V_B$

36. (A) For a p^+n diode, Neglecting V_i compared to V_B,

$x_n \approx \left[\frac{2\varepsilon V_B}{eN_d}\right]^{\frac{1}{2}} = \left[\frac{2(11.7\times8.85\times10^{-14})(100)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{\frac{1}{2}} = 5.1\ \mu\text{m}$

37. (D) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$= 2(0.0259)\ln\left(\frac{5\times10^{19}}{1.5\times10^{10}}\right) = 1.14 \text{ V}$

$W = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{1}{N_a}+\frac{1}{N_a}\right]\right\}^{\frac{1}{2}}$

$= \left[\frac{2\times(11.7\times8.85\times10^{-14})(1.14-0.4)}{1.6\times10^{-14}}\left(\frac{2}{5\times10^{19}}\right)\right]^{\frac{1}{2}}$

$= 6.19\times10^{-7} \text{ cm} = 62 \text{ A}^\circ$

38. (B) $L_n = \sqrt{D_n\tau_{n0}}$, $L_p = \sqrt{D_p\tau_{p0}}$

$\eta_{inj} = \frac{\frac{D_n n_{p0}}{L_n}}{\frac{D_n n_{p0}}{L_n}+\frac{D_p p_{n0}}{L_p}} = \frac{n_{p0}\sqrt{\frac{D_n}{\tau_{n0}}}}{n_{p0}\sqrt{\frac{D_n}{\tau_{n0}}}+p_{n0}\sqrt{\frac{D_p}{\tau_{p0}}}}$

$n_{p0} = \frac{n_i^2}{N_a} = \frac{(1.8\times10^6)^2}{10^{16}} = 3.24\times10^{-4} \text{ cm}^{-3}$

$p_{n0} = \frac{n_i^2}{N_d} = \frac{(1.8\times10^6)^2}{5\times10^{17}} = 6.48\times10^{6} \text{ cm}^{-3}$

$\sqrt{\frac{D_n}{\tau_{n0}}} = \sqrt{\frac{25}{10\times10^{-9}}} = 5\times10^4,$

$\sqrt{\frac{D_p}{\tau_{p0}}} = \sqrt{\frac{12}{10\times10^{-9}}} = 3.5\times10^4$

$\eta_{inj} = \frac{(5\times10^4)(3.24\times10^{-4})}{(5\times10^4)(3.24\times10^{-4})+(3.5\times10^4)(6.48\times10^{-6})}$

$= 0.986$

39. (B) The areal density at threshold is

$n_{2D} = \frac{J_{th}\tau_r}{e} = \frac{(500)(1.5\times10^{-9})}{1.6\times10^{-19}} = 4.69\times10^{12} \text{ cm}^{-3}$

The carrier density is

$n_{th} = \frac{n_{2D}}{d_{Las}} = \frac{4.69\times10^{12}}{10^{-6}} = 4.69\times10^{18} \text{ cm}^{-3}$

Once the threshold is reached, the carrier density does not change. When $J > J_{th}$ the electron hole recombination is

$\tau_r(J) = \frac{J_{th}}{J}\tau_r(J_{th}) = \frac{1.5\times10^{-9}}{5} = 3\times10^{-10} \text{ s}$

The optical power produced is $p = \frac{JA}{e}h\omega$

$= \frac{(5\times500)(2\times10^{-5})(1.43\times1.6\times10^{-19})}{1.6\times10^{-19}} = 71.5 \text{ MW}$

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CHAPTER 2.3

THE BIPOLAR JUNCTION TRANSISTOR

Statement for Q.1-2:

The parameters in the base region of an *npn* bipolar transistor are as follows $D_n = 20$ cm^2/s, $n_{B0} = 10^4$ cm^{-3}, $x_B = 1\ \mu$m, $A_{BE} = 10^{-4}$ cm^2.

1. If $V_{BE} = 0.5$ V, then collector current I_C is

(A) 7.75 μA (B) 1.6 μA

(C) 0.16 μA (D) 77.5 μA

2. If $V_{BE} = 0.7$ V, then collector current I_C is

(A) 418 μA (B) 210 μA

(C) 17.5 μA (D) 98 μA

3. In bipolar transistor biased in the forward-active region the base current is $I_B = 50\ \mu$A and the collector currents is $I_C = 2.7$ mA. The α is

(A) 0.949 (B) 54

(C) 0.982 (D) 0.018

4. A uniformly doped silicon *npn* bipolar transistor is to be biased in the forward active mode with the B-C junction reverse biased by 3 V. The transistor doping are $N_E = 10^{17}$ cm^{-3}, $N_B = 10^{16}$ cm^{-3} and $N_C = 10^{15}$ cm^{-3}. The BE voltage, at which the minority carrier electron concentration at $x = 0$ is 10% of the majority carrier hole concentration, is

(A) 0.94 V (B) 0.64 V

(C) 0.48 V (D) 0.24 V

5. A uniformly doped *npn* bipolar transistor is biased in the forward-active region. The transistor doping concentration are $N_E = 5 \times 10^{17}$ cm^{-3}, $N_B = 10^{16}$ cm^{-3} and $N_C = 10^{15}$ cm^{-3}. The minority carrier concentration p_{E0}, n_{B0} and p_{C0} are

(A) 4.5×10^2, 2.25×10^4, 2.25×10^5 cm^{-3}

(B) 2.25×10^4, 2.25×10^5, 4.5×10^2 cm^{-3}

(C) 2.25×10^4, 2.25×10^5, 4.5×10^4 cm^{-3}

(D) 4.5×10^4, 2.25×10^4, 2.25×10^5 cm^{-3}

6. A uniformly doped silicon *pnp* transistor is biased in the forward-active mode. The doping profile is $N_E = 10^{18}$ cm^{-3}, $N_B = 5 \times 10^{16}$ cm^{-3} and $N_C = 10^{15}$ cm^{-3}. For $V_{EB} = 0.6$ V, the p_B at $x = 0$ is (See fig. P2.3.7-8)

(A) 5.2×10^{19} cm^{-3} (B) 5.2×10^{13} cm^{-3}

(C) 5.2×10^{16} cm^{-3} (D) 5.2×10^{11} cm^{-3}

Statement for Q.7-8:

An *npn* bipolar transistor having uniform doping of $N_E = 10^{18}$ cm^{-3} $N_B = 10^{16}$ cm^{-3} and $N_C = 6 \times 10^{15}$ cm^{-3} is operating in the inverse-active mode with $V_{BE} = -2$ V and $V_{BC} = 0.6$ V. The geometry of transistor is shown in fig P2.3.7-8.

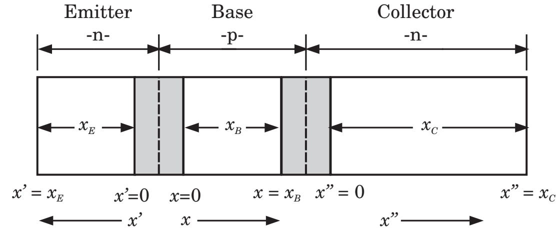

Fig. P2.3.7-8

7. The minority carrier concentration at $x = x_B$ is

(A) 4.5×10^{14} cm^{-3} (B) 2.6×10^{12} cm^{-3}

(C) 2.6×10^{14} cm^{-3} (D) 3.9×10^{14} cm^{-3}

8. The minority carrier concentration at $x'' = 0$ is

(A) 3.9×10^{14} cm^{-3} (B) 2.7×10^{12} cm^{-3}

(C) 2.7×10^{14} cm^{-3} (D) 4.5×10^{14} cm^{-3}

9. An *pnp* bipolar transistor has uniform doping of $N_E = 6 \times 10^{17}$ cm^{-3}, $N_B = 2 \times 10^{16}$ cm^{-3} and $N_C = 5 \times 10^{14}$ cm^{-3}. The transistor is operating is inverse-active mode. The maximum V_{CB} voltage, so that the low injection condition applies, is

(A) 0.86 V (B) 0.48 V

(C) 0.32 V (D) 0.60 V

Statement for Q.10-12:

The following currents are measured in a uniformly doped *npn* bipolar transistor:

$I_{nE} = 1.20$ mA, $I_{pE} = 0.10$ mA, $I_{nC} = 1.18$ mA

$I_R = 0.20$ mA, $I_G = 1\ \mu$A, $I_{pC0} = 1\ \mu$A

10. The α is

(A) 0.667 (B) 0.733

(C) 0.787 (D) 0.8

11. The β is

(A) 3.69 (B) 0.44

(C) 2.27 (D) 8.39

12. The γ is

(A) 0.816 (B) 0.923

(C) 1.083 (D) 0.440

13. A silicon *npn* bipolar transistor has doping concentration of $N_E = 2 \times 10^{18}$ cm^{-3}, $N_B = 10^{17}$ cm^{-3} and $N_C = 1.5 \times 10^{16}$ cm^{-3}. The area is 10^{-3} cm^2 and neutral base width is 1 μm. The transistor is biased in the active region at $V_{BE} = 0.5$ V. The collector current is

($D_B = 20$ cm^2/s)

(A) 9 μA (B) 17 μA

(C) 22 μA (D) 11 μA

14. A uniformly doped *npn* bipolar transistor has following parameters:

$N_E = 10^{18}$ cm^{-3}, $N_B = 5 \times 10^{16}$ cm^{-3},

$N_C = 2 \times 10^{19}$ cm^{-3},

$D_E = 8$ cm^2/s , $D_B = 15$ cm^2/s , $D_C = 14$ cm^2/s

$x_E = 0.8\ \mu$m, $x_B = 0.7\ \mu$m

The emitter injection efficiency γ is

(A) 0.999 (B) 0.977

(C) 0.982 (D) 0.934

15. A uniformly doped silicon epitaxial *npn* bipolar transistor is fabricated with a base doping of $N_B = 3 \times 10^{16}$ cm^{-3} and a heavily doped collector region with $N_C = 5 \times 10^{17}$ cm^{-3}. The neutral base width is $x_B = 0.7\ \mu$m when $V_{BE} = V_{BC} = 0$. The V_{BC} at punch-through is

(A) 26.3 V (B) 18.3 V

(C) 12.2 V (D) 6.3 V

16. A silicon *npn* transistor has a doping concentration of $N_B = 10^{17}$ cm^{-3} and $N_C = 7 \times 10^{15}$ cm^{-3}. The metallurgical base width is 0.5 μm. Let $V_{BE} = 0.6$ V. Neglecting the B–E junction depletion width the V_{CE} at punch-through is

(A) 146 V (B) 70 V

(C) 295 V (D) 204 V

17. A uniformly doped silicon *pnp* transistor is to designed with $N_E = 10^{19}$ cm^{-3} and $N_C = 10^{16}$ cm^{-3}. The metallurgical base width is to be 0.75 μm. The minimum base doping, so that the minimum punch-through voltage is $V_{pt} = 25$ V, is

(A) 4.46×10^{15} cm^{-3} (B) 4.46×10^{16} cm^{-3}

(C) 1.95×10^{15} cm^{-3} (D) 1.95×10^{16} cm^{-3}

18. For a silicon *npn* transistor assume the following parameters:

$I_E = 0.5$ mA, $\beta = 48$

$x_B = 0.7\ \mu$A, $x_{dc} = 2\ \mu$m

$C_s = C_\mu = 0.08$ pF, $C_{je} = 0.8$ pF

$D_n = 25$ cm^2/s, $r_c = 30\ \Omega$

The carrier cross the space charge region at a speed of 10^7 cm/s. The total delay time τ_{ec} is

(A) 164.2 ps (B) 234.4 ps

(C) 144.2 ps (D) 298.4 ps

19. In a bipolar transistor, the base transit time is 25% of the total delay time. The base width is 0.5 μm and base diffusion coefficient is $D_B = 20\ \text{cm}^2/\text{s}$. The cut-off frequency is

(A) 637 MHz (B) 436 MHz

(C) 12.8 GHz (D) 46.3 GHz

20. The base transit time of a bipolar transistor is 100 ps and carriers cross the 1.2 μm B–C space charge at a speed of 10^7 cm/s . The emitter-base junction charging time is 25 ps and the collector capacitance and resistance are 0.10 pF and 10 Ω, respectively. The cutoff frequency is

(A) 43.8 GHz (B) 32.6 GHz

(C) 3.26 GHz (D) 1.15 GHz

Statement for Q.21-22:

Consider the circuit shown in fig. P2.3.21-22. If voltage $V_s = 0.63$V, the currents are $I_C = 275\ \mu\text{A}$ and $I_B = 5\ \mu\text{A}$.

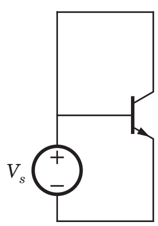

Fig.P2.3.21-22

21. The forward common-emitter gain β_F is

(A) 56 (B) 55

(C) 0.9821 (D) 0.9818

22. The forward current gain α_F is

(A) 0.9821 (B) 0.9818

(C) 55 (D) 56

23. Consider the circuit shown in fig P2.3.23. If $V_s = 0.63$ V, $I_1 = 275\ \mu\text{A}$ and $I_2 = 125\ \mu\text{A}$, then the value of I_3 is

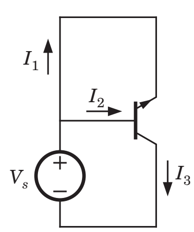

Fig. P2.3.23

(A) – 400 μA (B) 400 μA

(C) – 600 μA (D) 600 μA

Statement for Q.24-26:

For the transistor in circuit of fig. P2.3.24-26. The parameters are $\beta_R = 1, \beta_F = 100$, and $I_s = 1$ fA .

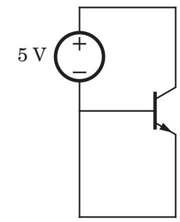

Fig. P2.3.24-26

24. The current I_C is

(A) 1 fA (B) 2 fA

(C) 1.384 fA (D) 0 A

25. The current I_E is

(A) 1 fA (B) -1 fA

(C) 2 fA (D) -2 fA

26. The current I_B is

(A) 2 fA (B) -2 fA

(C) 1 fA (D) -1 fA

27. For the transistor in fig. P2.3.27, $I_S = 10^{-15}$ A, $\beta_F = 100, \beta_R = 1$. The current I_{CBO} is

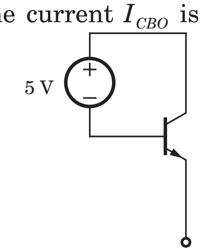

Fig.P2.3.27

(A) 1.01×10^{-14} A (B) 2×10^{-14} A

(C) 1.01×10^{-15} A (D) 2×10^{-15} A

Statement for Q.28-31:

Determine the region of operation for the transistor shown in circuit in question.

28.

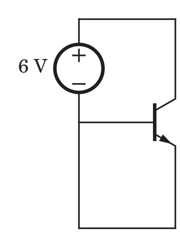

Fig.P2.3.28

(A) Forward-Active (B) Reverse-Active

(C) Saturation (D) Cutoff

29.

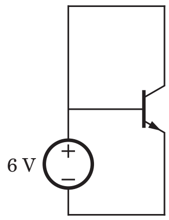

Fig. P2.3.29

(A) Forward-Active (B) Reverse-Active

(C) Saturation (D) Cutoff

30.

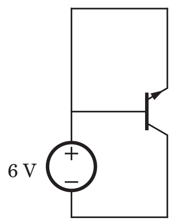

Fig.P2.3.30

(A) Forward-Active (B) Reverse-Active

(C) Saturation (D) Cutoff

31.

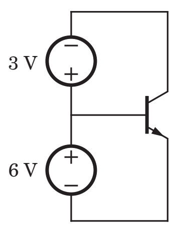

Fig.P2.3.31

(A) Forward-Active (B) Reverse-Active

(C) Saturation (D) Cutoff

Statement for Q.32-33:

For the circuit shown in fig. P2.3.32-33, let the value of $\beta_R = 0.5$ and $\beta_F = 50$. The saturation current is 10^{-16} A.

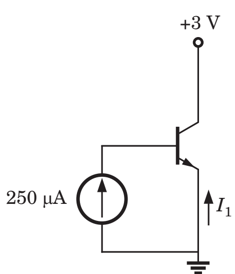

Fig. P2.3.32-33

32. The base-emitter voltage is

(A) 0.53 V (B) 0.7 V

(C) 0.84 V (D) 0.98 V

33. The current I_1 is

(A) –12.75 mA (B) 12.75 mA

(C) 12.5 mA (D) – 12.5 mA

Statement for Q.34–35:

The leakage current of a transistor are $I_{CBO} = 5\ \mu$ A and $I_{CEO} = 0.4$ mA, and $I_B = 30\ \mu$A.

34. The value of β is

(A) 79 (B) 81

(C) 80 (D) None of the above

35. The value of I_C is

(A) 2.4 mA (B) 2.77 mA

(C) 2.34 mA (D) 1.97 mA

Statement for Q.36–37:

For a BJT, $I_C = 5$ mA, $I_B = 50\ \mu$A and $I_{CBO} = 0.5\ \mu$A.

36. The value of β is

(A) 103 (B) 91

(C) 83 (D) 51

37. The value of I_E is

(A) 5.25 mA (B) 5.4 mA

(C) 5.65 mA (D) 5.1 mA

****

SOLUTIONS

1. (A) $I_C = I_s e^{\left(\frac{V_{BE}}{V_b}\right)}$

$$I_s = \frac{eD_n A_{BE} n_{B0}}{x_B}$$

$$= \frac{(1.6\times10^{-19})(20)(10^{-4})(10^4)}{10^{-4}} = 3.2\times10^{-14}\ \text{A}$$

$$I_C = 3.2\times10^{-14} e^{\left(\frac{0.5}{0.0259}\right)} = 7.75\ \mu\text{A}$$

2. (C) $I_C = 3.2\times10^{-14} e^{\left(\frac{0.7}{0.0259}\right)} = 17.5\ \text{mA}$

3. (C) $\beta_F = \frac{I_C}{I_B}$, $\alpha_F = \frac{\beta_F}{1+\beta_F}$

$$\alpha_F = \frac{I_C}{I_C + I_B} = \frac{2.7\text{m}}{2.7\text{m} + 50\mu} = 0.982$$

4. (B) $n_{p0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$

At $x = 0$, $n_p(0) = n_{p0} e^{\left(\frac{V_{BE}}{V_t}\right)}$

$$\Rightarrow \quad V_{BE} = V_T \ln\left(\frac{n_p(0)}{n_{p0}}\right)$$

$$n_p(0) = \frac{10}{100}\times N_B = \frac{10^{16}}{10} = 10^{15}$$

$$V_{BE} = 0.0259 \ln\left(\frac{10^{15}}{2.25\times10^4}\right) = 0.635\ \text{V}$$

5. (A) $p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{5\times10^{17}} = 450\ \text{cm}^{-3}$

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{5\times10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$$

$$p_{C0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{5\times10^{15}} = 2.25\times10^5\ \text{cm}^{-3}$$

6. (B) $p_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{5\times10^{16}} = 4.5\times10^3\ \text{cm}^{-3}$

$$p_B(0) = p_{B0} e^{\left(\frac{V_{EB}}{V_t}\right)} = 4.5\times10^3 e^{\left(\frac{0.6}{0.0259}\right)} = 5.2\times10^{13}\ \text{cm}^{-3}$$

7. (C) $n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$

$$n_B(x = x_B) = n_{B0} e^{\left(\frac{V_{BC}}{V_t}\right)}$$

$$= 2.25\times10^4 e^{\frac{0.6}{0.0259}} = 2.6\times10^{14}\ \text{cm}^{-3}$$

8. (D) $p_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{6\times10^{15}} = 3.75\times10^4\ \text{cm}^{-3}$

$$p_C(x'' = 0) = p_{C0} e^{\left(\frac{V_{BC}}{V_t}\right)}$$

$$= 3.75\times10^4 e^{\left(\frac{0.6}{0.0259}\right)} = 4.31\times10^{14}\ \text{cm}^{-3}$$

9. (B) Low injection limit is reached when

$p_C(0) = 0.10 N_C = 5\times10^{13}\ \text{cm}^{-3}$,

$$p_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{5\times10^{14}} = 4.5\times10^5$$

$$p_C(0) = p_{C0} e^{\left(\frac{V_{CB}}{V_t}\right)} \quad \Rightarrow \quad V_{CB} = V_t \ln\left(\frac{p_C(0)}{p_{C0}}\right)$$

$$= 0.0259 \ln\left(\frac{5\times10^{13}}{4.5\times10^5}\right) = 0.48\ \text{V}$$

10. (C) $\alpha = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}} = \frac{I_{nC}}{I_{nE} + I_R + I_{pE}}$

$$= \frac{1.18}{1.2 + 0.2 + 0.1} = 0.787$$

11. (A) $\beta = \frac{\alpha}{1-\alpha} == \frac{0.787}{1-0.787} = 3.69$

12. (B) $\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}} = \frac{I_{nE}}{I_{nE} + I_{pE}} = \frac{1.2}{1.2 + 0.1} = 0.923$

13. (B) $n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3\ \text{cm}^{-3}$

$$n_B(0) = n_{B0} e^{\left(\frac{V_{BE}}{V_t}\right)} = 2.25\times10^3 e^{\left(\frac{0.5}{0.0259}\right)} = 5.45\times10^{11}\ \text{cm}^{-3}$$

$$I_C = \frac{eD_B A n_B(0)}{x_B}$$

$$= \frac{(1.6\times10^{-19})(20)(10^{-3})(5.45\times10^{11})}{10^{-4}} = 17.4\ \mu\text{A}$$

14. (B) $\gamma = \frac{1}{1 + \frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}}$

$$= \frac{1}{1 + \frac{5\times10^{16}}{10^{18}}\ \frac{8}{15}\ \frac{0.7}{0.8}} = 0.977$$

15. (B) $V_{bi} = V_t \ln\left(\frac{N_B N_C}{n_i^2}\right)$

$$= 0.0259 \ln\left(\frac{3\times10^{16}\times5\times10^{17}}{(1.5\times10^{10})^2}\right) = 0.824\ \text{V}$$

At punch-through

27. (C) $I_E = I_S\left(e^{\left(\frac{V_{BE}}{V_t}\right)} - e^{\left(\frac{V_{BC}}{V_t}\right)}\right) + \frac{I_S}{\beta_F}\left(e^{\left(\frac{V_{BE}}{V_t}\right)} - 1\right) = 0$

$$\Rightarrow \quad e^{\left(\frac{V_{BE}}{V_t}\right)} = \frac{1}{1+\beta_F} + \frac{\beta_R}{1+B_F} e^{\left(\frac{V_{BC}}{V_t}\right)}$$

$$I_C = I_S\left[e^{\left(\frac{V_{BE}}{V_t}\right)} - e^{\left(\frac{V_{BC}}{V_t}\right)}\right] - \frac{I_S}{\beta_R}\left[e^{\left(\frac{V_{BC}}{V_t}\right)} - 1\right]$$

$$I_{CBO} = \frac{I_S}{1+\beta_F}\left[1 - e^{\left(\frac{V_{BC}}{V_t}\right)}\right] - \frac{I_S}{\beta_R}\left[e^{\left(\frac{V_{BC}}{V_t}\right)} - 1\right]$$

$V_{BC} = -5\text{ V},\ V_t = 0.0259\text{ V}$

$$I_{CBO} = \frac{I_s}{101}(1-0) - \frac{I_S}{1}(0-1) = 1.01 I_S = 1.01\times 10^{-15}\text{ A}$$

28. (D)

B-E junction V_{BE}	B-C Junction V_{BC}	
	Reverse Bias	Forward bias
Forward bias	Forward-Active	Saturation
Reverse Bias	Cut-off	Reverse-Active

$V_{BE} = 0$, $V_{BC} < 0$, Thus both junction are in reverse bias. Hence cutoff region.

29.(A) $V_{BE} > 0$, $V_{BC} = 0$, Base-Emitter junction forward bais, Base-collector junction reverse bias, Hence forward-active region.

30. (B) $V_{BE} = 0$, $V_{BC} > 0$, Base-Emitter junction reverse bais , Base-collector junction forward bias, Hence reverse-active region.

31. (C) $V_{BE} = 6$ V, $V_{BC} = 3$ V, Both junction are forward biase, Hence saturation region.

32. (C) The current source will forward bias the base-emitter junction and the collector base junction will then be reverse biased. Therefore the transistor is in the forward active region

$$I_C = I_S e^{\left(\frac{V_{BE}}{V_t}\right)}$$

$$I_C = \beta_F I_B = 50\times 250\times 10^{-6} = 12.5\times 10^{-3}\text{ A}$$

$$V_{BE} = V_t \ln\left(\frac{I_C}{I_S}\right) = 0.0259\ln\left(\frac{12.5\times 10^{-3}}{10^{-16}}\right) = 0.84\text{ V}$$

33. (A) $I_E = (\beta_F + 1) I_B = 12.75$ mA

$I_1 = -I_E = -12.75$ mA.

34. (A) $I_{CEO} = (\beta + 1) I_{CBO}$

$$\beta + 1 = \frac{0.4\text{m}}{5\mu} = 80 \quad \Rightarrow \quad \beta = 79$$

35. (B) $I_C = \beta I_B + I_{CEO} = 79(30\mu) + 0.4\text{m} = 2.77$ mA

36. (A) $I_C = \beta I_B + I_{CEO} = \beta I_B + (\beta+1) I_{CBO}$

$$\beta = \frac{I_C - I_{CBO}}{I_B + I_{CBO}} = \frac{5.2\text{m} - 0.5\mu}{50\mu + 0.5\mu} \approx 103.96$$

37. (A) $\alpha = \frac{\beta}{\beta+1} = 0.9904$

$$I_E = \frac{I_C - I_{CBO}}{\alpha} = \frac{5.2\text{m} - 0.5\mu}{0.9904} = 5.25\text{ MA}$$

****

11. In n-well CMOS fabrication substrate is

(A) lightly doped n –type

(B) lightly doped p –type

(C) heavily doped n –type

(D) heavily doped p –type

12. The chemical reaction involved in epitaxial growth in IC chips takes place at a temperature of about

(A) 500° C (B) 800° C

(C) 1200° C (D) 2000° C

13. A single monolithic IC chip occupies area of about

(A) 20 mm^2 (B) 200 mm^2

(C) 2000 mm^2 (D) 20,000 mm^2s

14. Silicon dioxide layer is used in IC chips for

(A) providing mechanical strength to the chip

(B) diffusing elements

(C) providing contacts

(D) providing mask against diffusion

15. The p-type substrate in a monolithic circuit should be connected to

(A) any dc ground point

(B) the most negative voltage available in the circuit

(C) the most positive voltage available in the circuit

(D) no where, i.e. be floating

16. The collector-substrate junction in the epitaxial collector structure is, approximately

(A) a step-graded junction

(B) a linearly graded junction

(C) an exponential junction

(D) None of the above

17. The sheet resistance of a semiconductor is

(A) an important characteristic of a diffused region especially when used to form diffused resistors

(B) an undesirable parasitic element

(C) a characteristic whose value determines the required area for a given value of integrated capacitance

(D) a parameter whose value is important in a thin-film resistance

18. Monolithic integrated circuit system offer greater reliability than discrete-component systems because

(A) there are fewer interconnections

(B) high-temperature metalizing is used

(C) electric voltage are low

(D) electric elements are closely matched

19. Silicon dioxide is used in integrated circuits

(A) because of its high heat conduction

(B) because it facilitates the penetration of diffusants

(C) to control the location of diffusion and to protect and insulate the silicon surface.

(D) to control the concentration of diffusants.

20. Increasing the yield of an IC

(A) reduces individual circuit cost

(B) increases the cost of each good circuit

(C) results in a lower number of good chips per wafer

(D) means that more transistor can be fabricated on the same size wafer.

21. The main purpose of the metalization process is

(A) to act as a heat sink

(B) to interconnect the various circuit elements

(C) to protect the chip from oxidation

(D) to supply a bonding surface for mounting the chip

22. In a monolithic-type IC

(A) each transistor is diffused into a separate isolation region

(B) all components are fabricated into a single crystal of silicon

(C) resistors and capacitors of any value may be made

(D) all isolation problems are eliminated

23. Isolation in ICs is required

(A) to make it simpler to test circuits

(B) to protect the transistor from possible ``thermal run away”

(C) to protect the components mechanical damage

(D) to minimize electrical interaction between circuit components

24. Almost all resistor are made in a monolithic IC

(A) during the base diffusion

(B) during the collector diffusion

(C) during the emitter diffusion

(D) while growing the epitaxial layer

25. The equation governing the diffusion of neutral atom is

(A) $\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial x^2}$ (B) $\frac{\partial N}{\partial x} = D\frac{\partial^2 N}{\partial t^2}$

(C) $\frac{\partial^2 N}{\partial t^2} = D\frac{\partial N}{\partial x}$ (D) $\frac{\partial^2 N}{\partial x^2} = D\frac{\partial N}{\partial t}$

26. The true statement is

(A) thick film components are vacuum deposited

(B) thin film component are made by screen-and- fire process

(C) thin film resistor have greater precision and are more stable

(D) thin film resistor are cheaper than the thin film resistor

27. The False statement is

(A) Capacitor of thin film capacitor made with proper dielectric is not voltage dependent

(B) Thin film resistors and capacitor need to be biased for isolation purpose

(C) Thin film resistors and capacitor have smaller stray capacitances and leakage currents.

(D) None of the above

28. Consider the following two statements

S_1 : The dielectric isolation method is superior to junction isolation method.

S_2 : The beam lead isolation method is inferior to junction isolation method.

The true statements is (are)

(A) S_1 , S_2 (B) only

(C) only (D) Neither nor S_2

29. If P is passivation, Q is n-well implant, R is metallization and S is source/drain diffusion, then the order in which they are carried out in a standard n-well CMOS fabrication process is

(A) S - R - Q - P (B) R - P - S - Q

(C) Q - S - R - P (D) P - Q - R - S

30. For the circuit shown in fig. P2.5.30, the minimum number and the maximum number of isolation regions are respectively

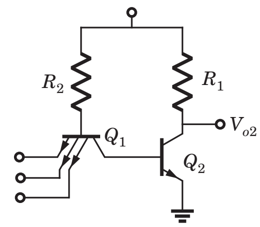

Fig. P2.5.32

(A) 2, 6 (B) 3, 6

(C) 2, 4 (D) 3, 4

31. For the circuit shown in fig. P2.5.31, the minimum number of isolation regions are

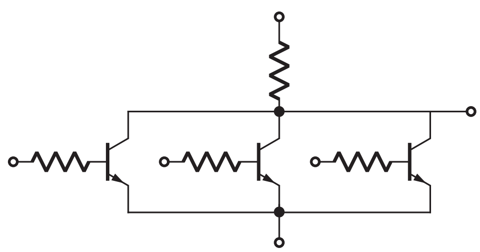

Fig. P2.5.31

(A) 2 (B) 3

(C) 4 (D) 7

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SOLUTIONS

1. (D)	2. (D)	3. (B)	4. (B)	5. (C)	6. (B)
7. (C)	8. (B)	9. (C)	10. (D)	11. (B)	12. (C)
13. (C)	14. (D)	15. (B)	16. (A)	17. (A)	18. (A)
19. (C)	20. (A)	21. (B)	22. (B)	23. (D)	24. (A)
25. (A)	26. (C)	27. (B)	28. (B)	29. (C)	

30. (D) The minimum number of isolation region is 3 one containing Q_1 , one containing and one containing both and . The maximum number of isolation region is 4, or one per component.

31. (A) The minimum number of isolation region is two. One for transistor and one for resistor.

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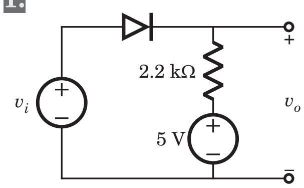
CHAPTER 3.1

DIODE CIRCUITS

Statement for Q.1–4:

In the question a circuit and a waveform for the input voltage is given. The diode in circuit has cutin voltage $V_\gamma = 0$. Choose the option for the waveform of output voltage v_o .

1.



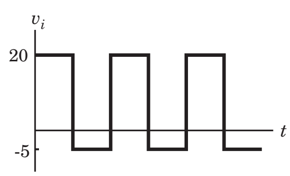
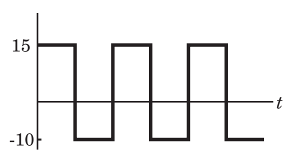


Fig.3.1.1



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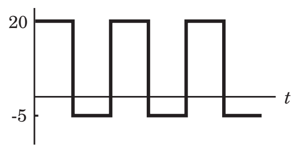

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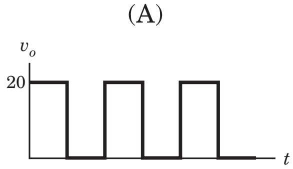
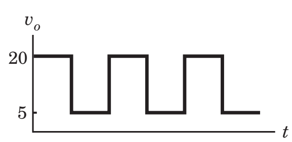


(A)



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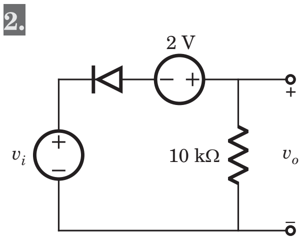
2.



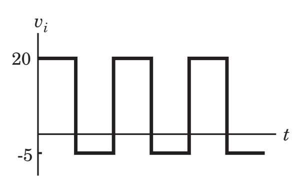
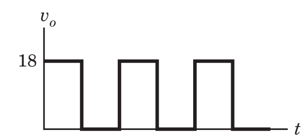
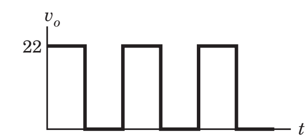


Fig.3.1.2



(A)



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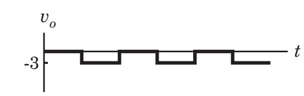

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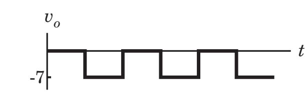

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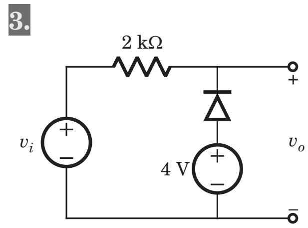


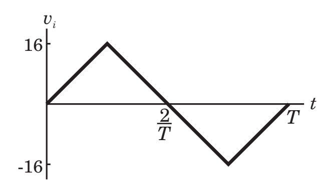
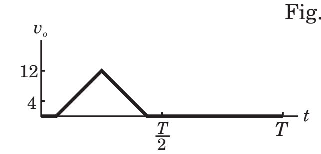
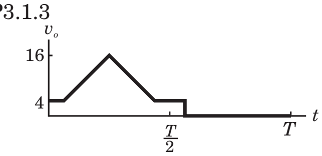


Fig. P3.1.3



(A)



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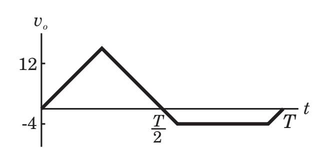

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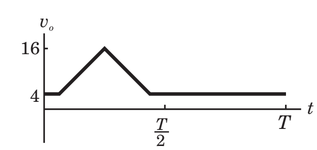

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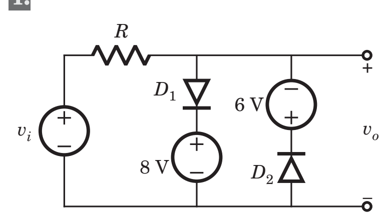


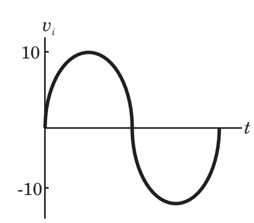
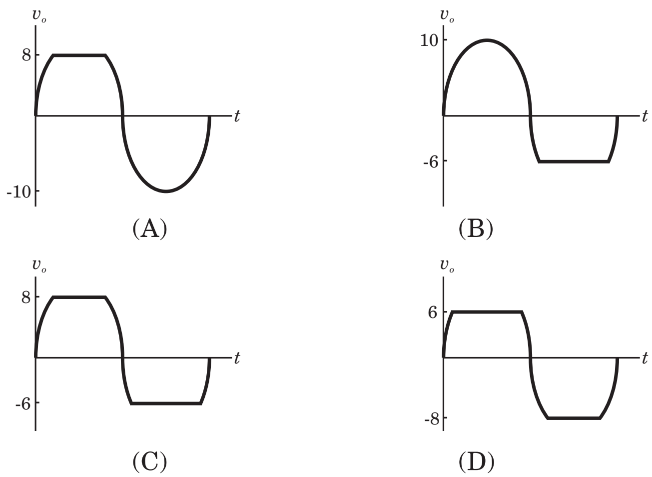


Fig.3.1.4



5. For the circuit in fig. P.3.1.5, let cutin voltage $V_\gamma = 0.7$ V. The plot of v_o verses v_i for $-10 \le v_i \le 10$ V is

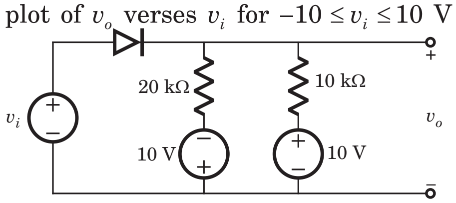
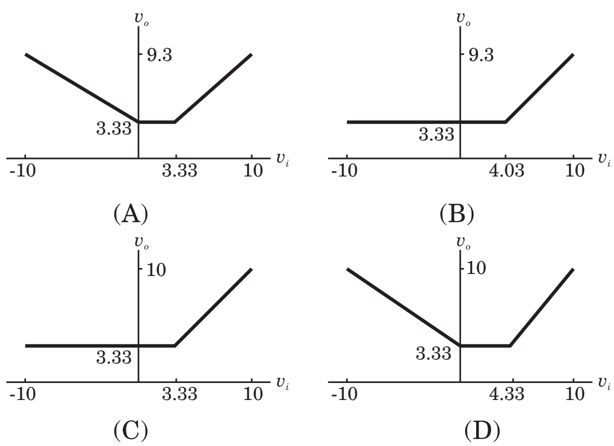


Fig. P3.1.5



6. For the circuit in fig. P3.1.6 the cutin voltage of diode is $V_\gamma = 0.7$ V. The plot of v_o versus v_i is

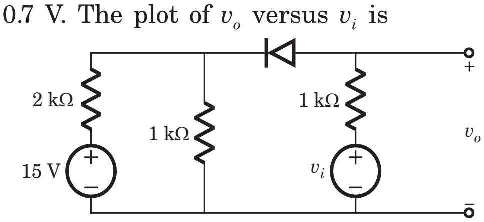
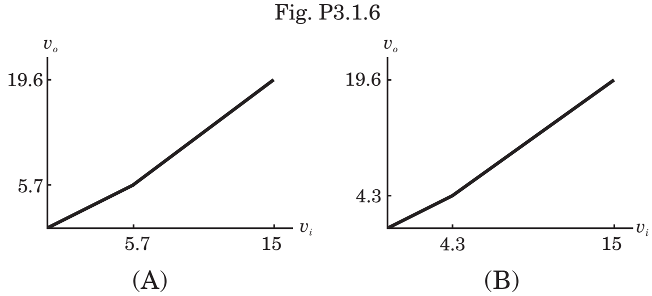
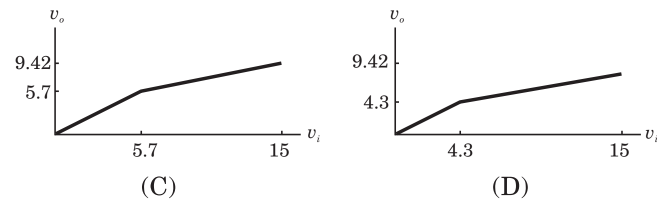


Fig. P3.1.6





7. For the circuit shown in fig. P3.1.7, each diode has $V_\gamma = 0.7$ V. The v_o for $-10 \le v_s \le +10$ V is

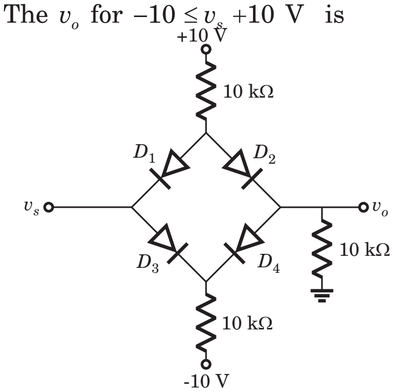
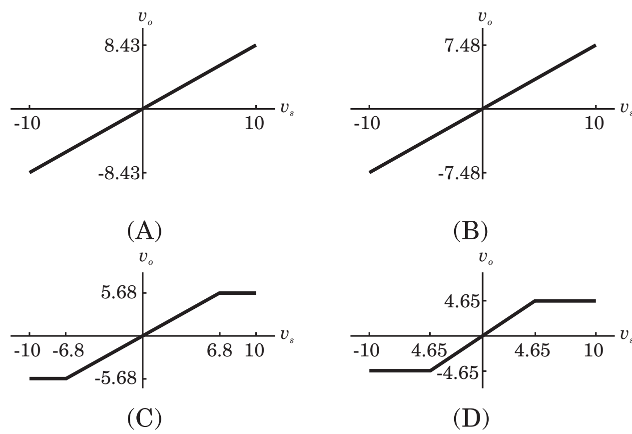


Fig. P3.1.7.



8. A symmetrical 5 kHz square wave whose output varies between +10 V and –10 V is impressed upon the clipping circuit shown in fig. P3.1.8. If diode has $r_f = 0$ and $r_r = 2$ MΩ and $V_\gamma = 0$, the output waveform is

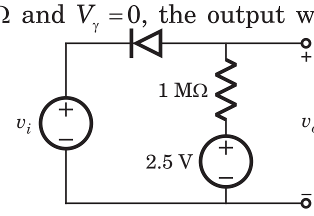
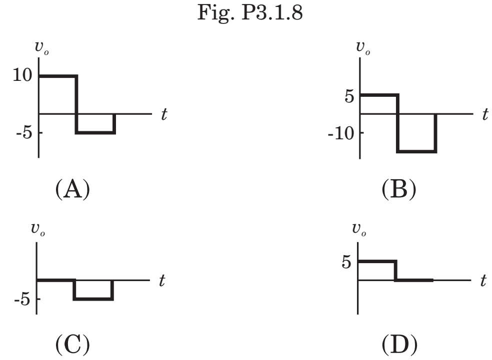


Fig. P3.1.8



9. In the circuit of fig. P3.1.9, the three signals of fig are impressed on the input terminals. If diode are ideal then the voltage v_o is

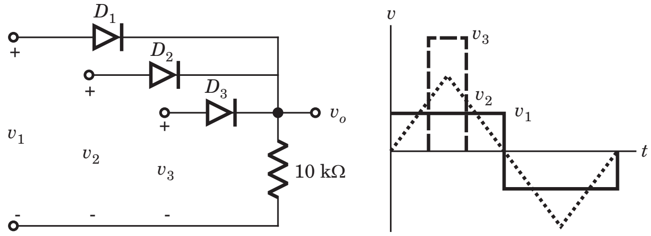
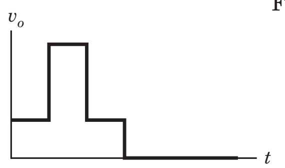
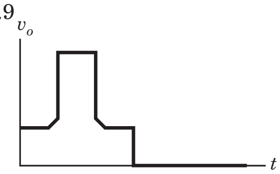


Fig. P.3.1.9



(A)



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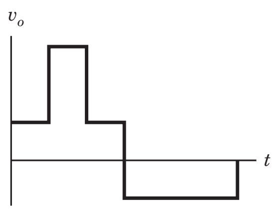

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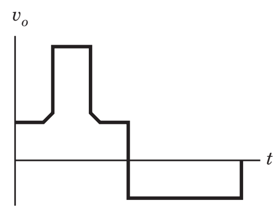


(D)

10. For the circuit shown in fig. P3.1.10 the input voltage v_i is as shown in fig. Assume the RC time constant large and cutin voltage of diode $V_\gamma = 0$. The output voltage v_o is

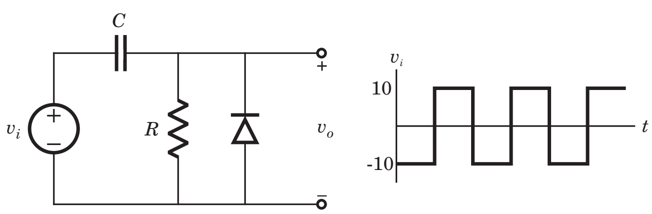
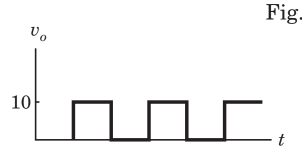
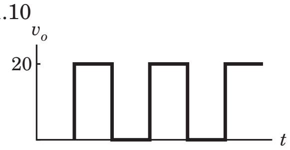


Fig. P.3.1.10



(A)



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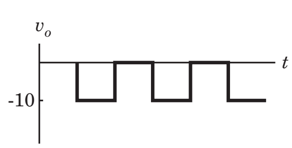

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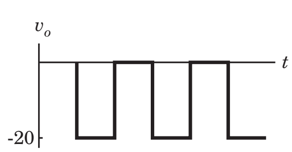


(D)

11. For the circuit shown in fig. P.3.1.11, the input voltage v_i is as shown in fig. Assume the RC time constant large and cutin voltage $V_\gamma = 0$. The output voltage v_o is

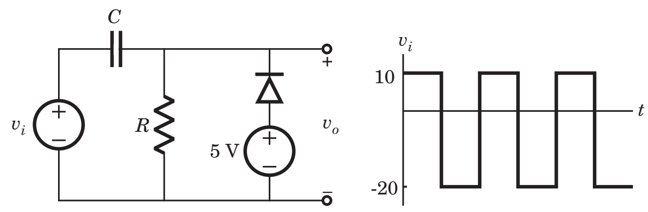
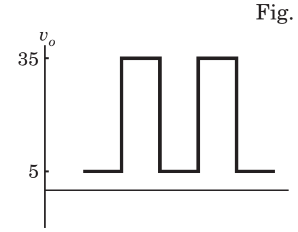
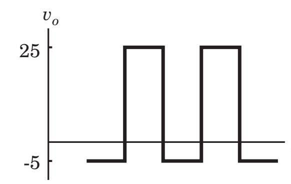


Fig. P.3.1.11



(A)



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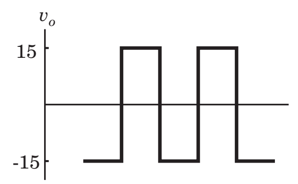

(C)

(D) None of the above

12. In the circuit of fig. P3.1.12, D_1 and D_2 are ideal diodes. The current i_1 and i_2 are

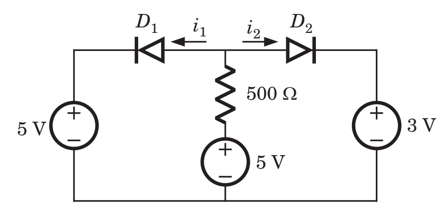

Fig. P3.1.12

(A) 0, 4 mA (B) 4 mA, 0

(C) 0, 8 mA (D) 8 mA, 0

13. In the circuit of Fig. P3.1.13 diodes has cutin voltage of 0.6 V. The diode in ON state are

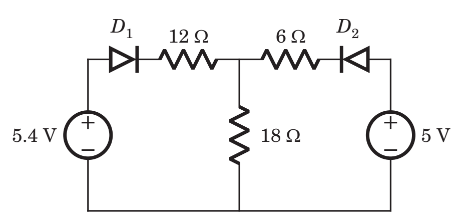

Fig. P3.1.13

(A) only D_1 (B) only D_2

(C) both D_1 and D_2 (D) None of the above

22. The diodes in the circuit in fig. P3.1.22 has parameters $V_\gamma = 0.6$ V and $r_f = 0$. The current i_{D2} is

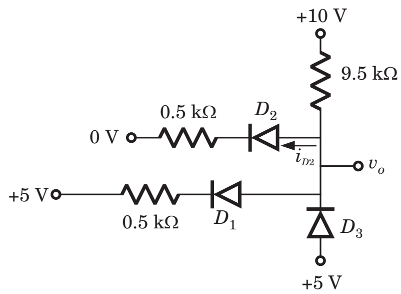

Fig. P3.1.22

(A) 8.4 mA (B) 10 mA

(C) 7.6 mA (D) 0 mA

Statement for Q.23–25:

The diodes in the circuit in fig. P3.1.23-25 have linear parameter of $V_\gamma = 0.6$ V and $r_f = 0$.

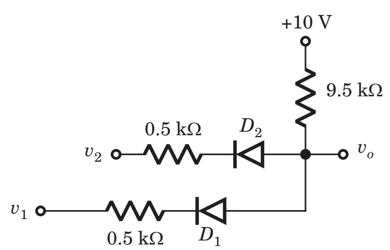

Fig. P3.1.23–25

23. If $v_1 = 10$ V and $v_2 = 0$ V, then v_o is

(A) 8.93 V (B) 7.82 V

(C) 1.07 V (D) 2.18 V

24. If $v_1 = 10$ V and $v_2 = 5$ V, then v_o is

(A) 9.13 V (B) 0.842 V

(C) 5.82 V (D) 1.07 V

25. If $v_1 = v_2 = 0$, then output voltage v_o is

(A) 0.964 V (B) 1.07 V

(C) 10 V (D) 0.842 V

Statement for Q.26–28:

The diodes in the circuit of fig. P3.1.26–28 have linear parameters of $V_\gamma = 0.6$ V and $r_f = 0$.

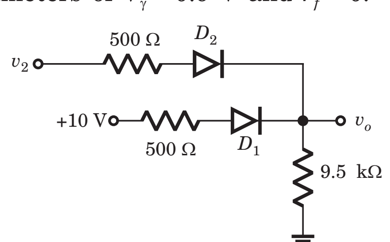

Fig. P3.1.26–28

26. If $v_2 = 0$, then output voltage v_o is

(A) 6.43 V (B) 9.43 V

(C) 7.69 V (D) 8.93 V

27. If $v_2 = 5$ V, then v_o is

(A) 8.93 V (B) 12.63 V

(C) 18.24 V (D) 10.56 V

28. If $v_2 = 10$ V, then v_o is

(A) 10 V (B) 9.16 V

(C) 8.43 V (D) 12.13 V

Statement for Q.29–30:

The diode in the circuit of fig. P3.1.29–30 has the non linear terminal characteristic as shown in fig. Let the voltage be $v_s = \cos \omega t$ V.

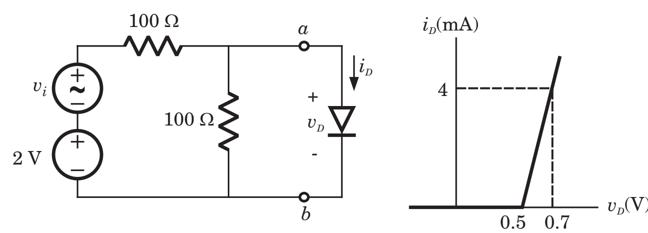

Fig. P3.1.29–30

29. The current i_D is

(A) $2.5(1 + \cos \omega t)$ mA (B) $5(0.5 + \cos \omega t)$ mA

(C) $5(1 + \cos \omega t)$ mA (D) $5(1 + 0.5 \cos \omega t)$ mA

30. The voltage v_D is

(A) $0.25(3 + \cos \omega t)$ V (B) $0.25(1 + 3\cos \omega t)$ V

(C) $0.5(3 + 1\cos \omega t)$ V (D) $0.5(2 + 3\cos \omega t)$ V

31. The circuit inside the box in fig. P3.1.31. contains only resistor and diodes. The terminal voltage v_o is connected to some point in the circuit inside the box. The largest and smallest possible value of v_o most nearly to is respectively

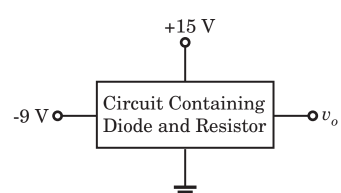

Fig. 3.1.31

(A) 15 V, 6 V (B) 24 V, 0 V

(C) 24 V, 6 V (D) 15 V, –9 V

32. In the voltage regulator circuit in fig. P3.1.32 the maximum load current i_L that can be drawn is

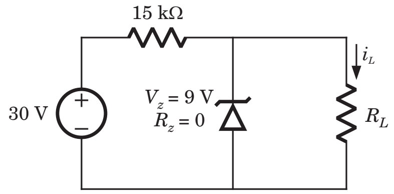

Fig. 3.1.32

(A) 1.4 mA (B) 2.3 mA

(C) 1.8 mA (D) 2.5 mA

33. In the voltage regulator shown in fig. P3.1.33 the power dissipation in the Zener diode is

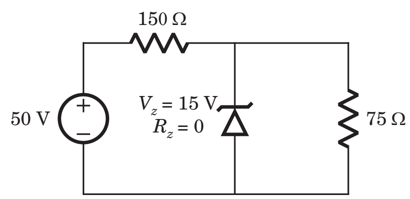

Fig. P3.1.33

(A) 1 W (B) 1.5 W

(C) 2 W (D) 0.5 W

34. The Q-point for the Zener diode in fig. P3.1.34 is

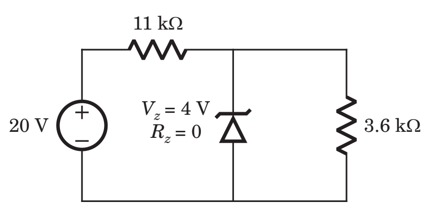

Fig. P3.1.34

(A) (0.34 mA, 4 V) (B) (0.34 mA, 4.93 V)

(C) (0.94 mA, 4 V) (D) (0.94 mA, 4.93 V)

35. In the voltage regulator circuit in fig. P3.1.35 the power rating of Zener diode is 400 mW. The value of R_L that will establish maximum power in Zener diode is

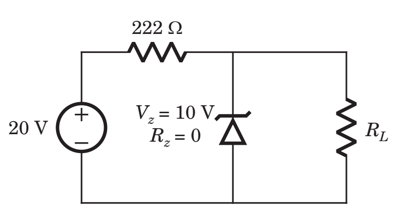

Fig. P3.1.35

(A) 5 kΩ (B) 2 kΩ

(C) 10 kΩ (D) 8 kΩ

Statement for Q.36–38:

In the voltage regulator circuit in fig. P3.1.36–38 the Zener diode current is to be limited to the range $5 \le i_z \le 100$ mA.

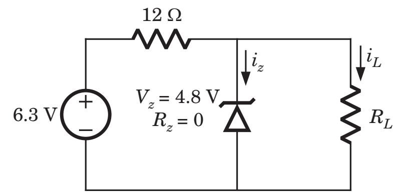

Fig. P3.1.36–38

36. The range of possible load current is

(A) $5 \le i_L \le 130$ mA (B) $25 \le i_L \le 120$ mA

(C) $10 \le i_L \le 110$ mA (D) None of the above

37. The range of possible load resistance is

(A) $60 \le R_L \le 372\,\Omega$ (B) $60 \le R_L \le 200\,\Omega$

(C) $40 \le R_L \le 192\,\Omega$ (D) $40 \le R_L \le 360\,\Omega$

38. The power rating required for the load resistor is

(A) 576 mW (B) 360 μW

(C) 480 mW (D) 75 μW

39. The secondary transformer voltage of the rectifier circuit shown in fig. P3.1.39 is $v_s = 60 \sin 2\pi 60t$ V. Each diode has a cut in voltage of $V_\gamma = 0.6$ V. The ripple voltage is to be no more than $V_{rip} = 2$ V. The value of filter capacitor will be

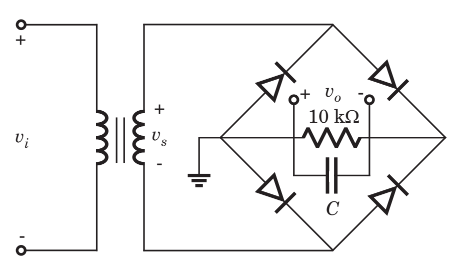

Fig. P3.1.39

(A) 48.8 μF (B) 24.4 μF

(C) 32.2 μF (D) 16.1 μF

40. The input to full-wave rectifier in fig. P3.1.40 is $v_i = 120 \sin 2\pi 60t$ V. The diode cutin voltage is 0.7 V. If the output voltage cannot drop below 100 V, the required value of the capacitor is

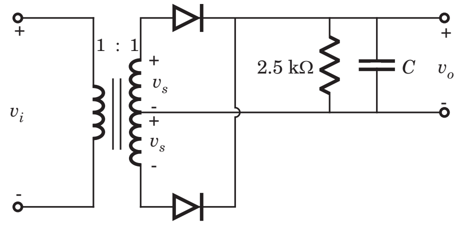

Fig. P3.1.40

(A) 61.2 μF (B) 41.2 μF

(C) 20.6 μF (D) 30.6 μF

41. For the circuit shown in fig. P3.1.41 diode cutin voltage is $V_{in} = 0$. The ripple voltage is to be no more than $v_{rip} = 4$ V. The minimum load resistance, that can be connected to the output is

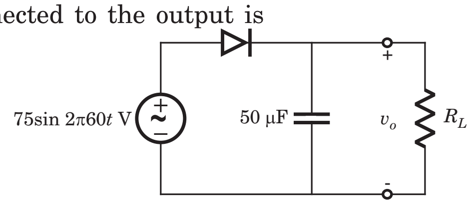

Fig. P3.1.41

(A) 6.25 kΩ (B) 12.50 kΩ

(C) 30 kΩ (D) None of the above

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SOLUTIONS

1. (D) Diode is off for $v_i < 5$ V. Hence $v_o = 5$ V.

For $v_i > 5$ V, $v_o = v_i$, Therefore (D) is correct option.

2. (C) Diode will be off if $v_i + 2 > 0$.Thus $v_o = 0$

For $v_i + 2 < 0$ V, $v_i < -2$, $v_o = v_i + 2 = -3$ V

Thus (C) is correct option.

3. (D) For $v_i < 4$ V the diode is ON and output $v_o = 4$ V.

For $v_i > 4$ V diode is off and output $v_o = v_i$.

Thus (D) is correct option.

4. (C) During positive cycle when $v_s < 8$ V, both diode are OFF $v_o = v_s$. For $v_s > 8$ V , $v_o = 8$ V, D_1 is ON. During negative cycle when $|v_s| < 6$ V, both diode are OFF, $v_o = v_s$. For $|v_s| > 6$ V, D_2 is on $v_o = -6$ V. Therefore (C) is correct.

5. (B) For D off , $v_o = \dfrac{\frac{10}{20} - \frac{10}{20}}{\frac{1}{20} + \frac{1}{10}} = 3.33$ V.

For $v_i \le 3.33 + 0.7 = 4.03$ V, $v_o = 3.33$ V

For $v_i > 4.03$ V , $v_o = v_i - 0.7$

For $v_i = 10$ V, $v_o = 9.3$ V

6. (C) Let v_1 be the voltage at n-terminal of diode,

$$v_1 = \frac{15 \times 1}{2 + 1} = 5 \text{ V}$$

For $v_i \le 5.7$ V, $v_o = v_i$

$$\frac{v_1 - 15}{2k} + \frac{v_1}{1k} + \frac{v_o - v_i}{1k} = 0 \quad \Rightarrow \quad 3v_1 + 2v_o - 2v_i = 15$$

$v_o = v_1 + 0.7$

$5v_o - 2v_1 = 15 + 2.1 = 17.9 \quad \Rightarrow \quad v_o = 0.4\, v_i + 3.42$

7. (D) For $v_s > 0$, when D_1 is OFF, Current through D_2 is

$$i = \frac{10 - 0.7}{10 + 10} = 0.465 \text{ mA}, \; v_o = 10\text{k}i = 4.65 \text{ V}$$

$v_o = v_s$ for $0 < v_s < 4.65$ V.

For negative values of v_s , the output is negative of positive part. Thus (D) is correct option.

8. (B) The diode conducts (zero resistance) when $v_i < 2.5$ V and $v_o = v_i$. Diode is open (2 MΩ resistance) when $v_i > 2.5$ V and $v_o = 2.5 + \dfrac{v_i - 2.5}{3} = 5$ V.

$= 50 \times 5\,(1 + \cos\,\omega t) \times 10^{-3} + 0.5$

$= 0.75 + 0.25 \cos\,\omega t = 0.25(3 + \cos\,\omega t)$ V

31. (D) The output voltage cannot exceed the positive power supply voltage and cannot be lower than the negative power supply voltage.

32. (A) At regulated power supply $i_s = \dfrac{30 - 9}{15k} = 1.4$ mA i_L will remain less than 1.4 mA.

33. (D) $v_{TH} = \dfrac{75(50)}{75 + 150} = \dfrac{50}{3}$ V

$\dfrac{50}{3} > V_Z$, $R_{TH} = 150\,||\,75 = 50\ \Omega$

$i_Z = \dfrac{1}{50}\left(\dfrac{50}{3} - 15\right) = 33$ mA, $P = 15 i_Z = 0.5$ W

34. (A) $v_{TH} = \dfrac{3.6(20)}{11 + 3.6} = 4.93$ V $> V_Z$,

$R_{TH} = 11\ ||\ 3.6 = 2.71$ kΩ, $i_Z = \dfrac{4.93 - 4}{2.71k} = 0.34$ mA

35. (B) $i_{Z(max)} = \dfrac{400m}{10} = 40$ mA

$i_L + i_Z = \dfrac{20 - 10}{222} = 45$ mA

$i_{L(min)} = 45 - 40 = 5$ mA, $R_L = \dfrac{10}{5m} = 2$ kΩ

36. (B) Current through 12 Ω resistor is

$i = \dfrac{6.3 - 4.8}{12} = 125$ mA

$i_L = i - i_Z = 125 - i_Z \quad \Rightarrow \quad 25 \le i_L \le 120$ mA

37. (C) $25 \le i_L \le 120$ mA, $i_L R_L = 4.8$ V

$25 \le \dfrac{4.8}{R_L} \le 120$ mA $\quad \Rightarrow \quad 40 \le R_L \le 192\ \Omega$

38. (A) $P_L = i_L V_Z = (120m)(4.8) = 576$ mV

39. (B) $v_s = 60 \sin 2\pi 60 t$ V

$v_{max} = 60 - 1.4 = 58.6$ V

$C = \dfrac{v_{max}}{2 f R V_{rip}} = \dfrac{58.6}{2(60)10 \times 10^3 \times 2} = 24.4$ μF

40. (C) Full wave rectifier

$v_s = v_i = 120 \sin 2\pi 60 t$ V

$v_{max} = 120 - 0.7 = 119.3$ V

$V_{rip} = 119.3 - 100 = 19.3$ V

$C = \dfrac{v_{max}}{2 f k V_{rip}} = \dfrac{119.3}{2(60)2.5 \times 10^3 \times 14.4} = 20.6$ μF

41. (A) $V_{rip} = \dfrac{v_{max}}{f R_L C}$

$R_L = \dfrac{v_{max}}{f C V_{rip}} = \dfrac{75}{60 \times 50 \times 10^{-5} \times 4} = 6.25$ kΩ

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CHAPTER

3.2

BASIC BJT CIRCUITS

Use $V_{BE(ON)} = 0.7$ V, $V_{CE(Sat)} = 0.2$ V for *npn* transistor if not given in problem.

Statement for Q.1-4:

The common-emitter current gain of the transistor is β =75. The voltage V_{BE} in ON state is 0.7 V.

1. I_E , R_C =?

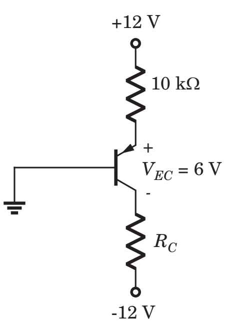

Fig. P3.3.1

(A) 1.46 mA, 6.74 kW

(B) 0.987 mA, 3.04 kW

(C) 1.13 mA,, 5.98 kW

(D) None of the above

2. V_{EC} =?

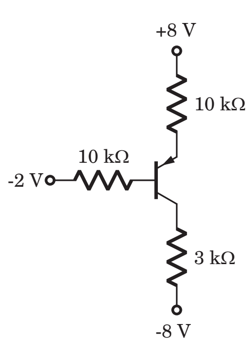

Fig. P3.3.2

(A) 8.4 V

(B) 6.2 V

(C) 4.1 V

(D) None of the above

3. I_C , R_C =?

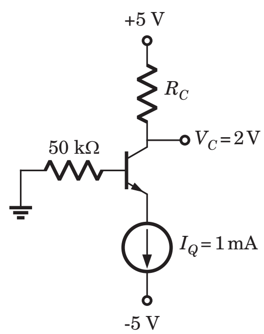

Fig. P3.3.3

(A) 0.987 mA, 3.04 kΩ

(B) 1.013 mA, 2.96 kΩ

(D) 0.946 mA, 4.18 kΩ

(D) 1.057 mA, 3.96 kΩ

4. V_C =?

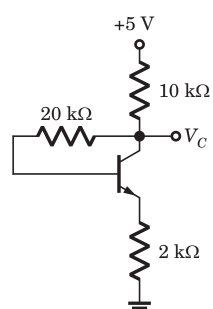

Fig. P3.3.4

(A) 1.49 V

(B) 2.9 V

(C) 1.78 V

(D) 2.3 V

Statement for Q.5-6:

In the circuit of fig.P3.3.5-6 $V_B = -1$ V

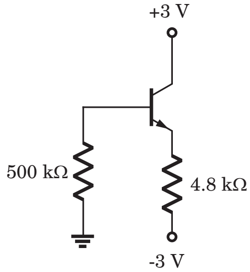

Fig. P3.3.5-6

5. $\beta = ?$

(A) 103.4 (B) 135.5
(C) 134.5 (D) 102.4

6. $V_{CE} = ?$

(A) 6.4 V (B) 4.7 V
(C) 1.3 V (D) 4.2 V

7. In the circuit shown in fig. P3.3.7 voltage $V_E = 4$ V. The value of α and β are respectively

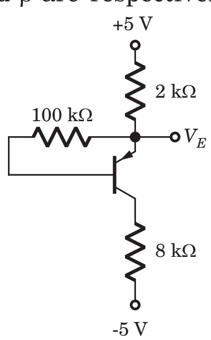

Fig. P3.3.7

(A) 0.943, 17.54 (B) 0.914, 17.54
(C) 0.914, 11.63 (D) 0.914, 11.63

Statement for Q.8-10:

For the transistor in circuit shown in fig. P3.3.8-10, $\beta = 200$. Determine the value of I_E and I_C for given value of V_B in question.

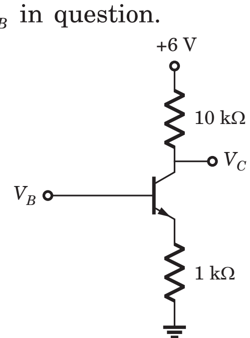

Fig. P3.3.8-10

8. $V_B = 0$ V

(A) 6.43 mA, 2.4 V (B) 2.18 mA, 3.4 V
(C) 0 A, 6 V (D) None of the above

9. $V_B = 1$ V

(A) 4 V (B) 3 V
(C) 1 V (D) 1.9 V

10. $V_B = 2$ V

(A) –7 V (B) 1.5 V
(C) 2.6 V (D) None of the above

Statement for Q.11-12:

The transistor in circuit shown in fig. P3.3.11-12 has $\beta = 200$. Determine the value of voltage V_o for given value of V_{BB}.

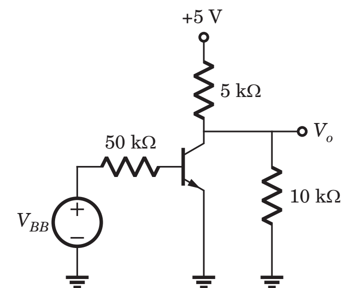

Fig. P3.3.11-12

11. $V_{BB} = 0$

(A) 2.46 V (B) 1.83 V
(C) 3.33 V (D) 4.04 V

12. $V_{BB} = 1$ V

(A) 4.11 V (B) 1.83 V
(C) 2.46 V (D) 3.44 V

13. $V_{BB} = 2$ V

(A) 3.18 V (B) 1.46 V
(C) 0.2 V (D) None of the above

Statement for Q.14-16:

The transistor shown in the circuit of fig. P3.3.14-16 has $\beta = 150$. Determine V_o for given value of I_Q in question.

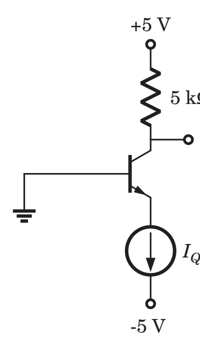

Fig. P3.3.14-16

14. $I_Q = 0.1$ mA

(A) 1.4 V (B) 4.5 V
(C) 3.2 V (D) None of the above

15. $I_Q = 0.5$ mA

(A) 3.16 V (B) 2.52 V

(C) 2.14 V (D) 3.94V

16. $I_Q = 2$ mA

(A) 4.9 V (B) –4.9 V

(C) 0.5 V (D) –0.5 V

17. For the circuit in fig. P3.3.17 $V_B = V_C$ and $\beta = 50$. The value of V_B is

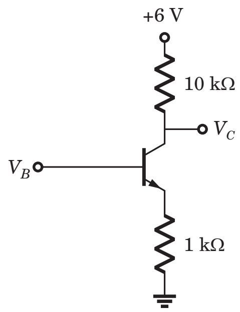

Fig. P3.317

(A) 0.9 V (B) 1.19 V

(C) 2.14 V (D) 1.84 V

18. For the circuit shown in fig. P3.3.18, $V_{CB} = 0.5$ V and $\beta = 100$. The value of I_Q is

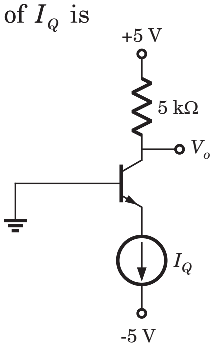

Fig. P3.3.18

(A) 1.68 mA (B) 0.909 mA

(C) 0.134 mA (D) None of the above

19. For the circuit shown in fig. P3.3.19 the emitter voltage is $V_E = 2$ V. The value of α is

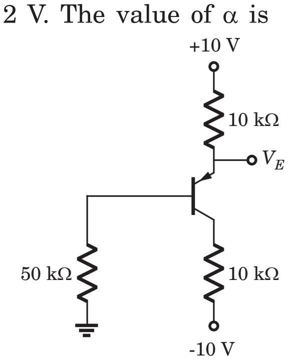

Fig. P3.3.19

(A) 0.991 (B) 0.939

(C) 0.968 (D) 0.914

20. For the transistor in fig. P3.3.20 , $\beta = 50$. The value of voltage V_{EC} is

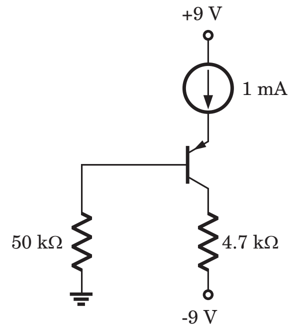

Fig. P3.3.20

(A) 3.13 V (B) 4.24 V

(C) 5.18 V (D) 6.07 V

21. In the circuit shown in fig. P3.3.21 if $\beta = 50$, the power dissipated in the transistor is

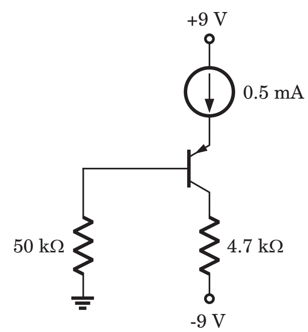

Fig. P3.3.21

(A) 3.87 mW (B) 10.46 mW

(C) 7.49 mW (D) 18.74 mW

22. For the circuit shown in fig. P3.3.22 the Q-point is $V_{CEQ} = 12$ V and $I_{CQ} = 2$ A when $\beta = 60$. The value of resistor R_C and R_B are

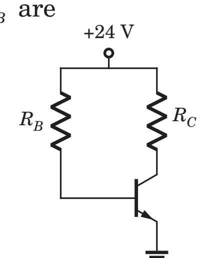

Fig. P3.3.22

(A) 10 kΩ, 241 kΩ (B) 10 kΩ, 699 kΩ

(C) 6 kΩ, 699 kΩ (D) 6 kΩ, 241 kΩ

29. For the transistor in the circuit of fig. P3.3.29, $\beta = 100$. The voltage V_B is

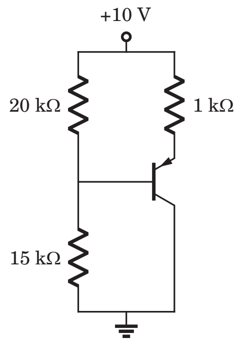

Fig. P3.3.29

(A) 3.6 V (B) 4.29 V

(C) 3.9 V (D) 4.69 V

30. The current gain of the transistor shown in the circuit of fig. P3.3.30 is $\beta = 125$. The Q-point values (I_{CQ}, V_{CEQ}) are

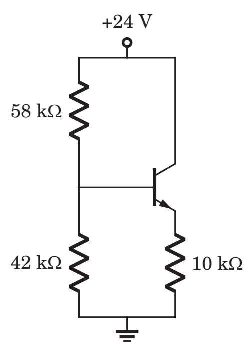

Fig. P3.3.30

(A) (0.418 mA, 20.4 V) (B) (0.915 mA, 14.8 V)

(C) (0 .915 mA, 16.23 V) (D) (0.418 mA, 18.43 V)

31. For the circuit shown in fig. P3.3.31, let $\beta = 75$. The Q-point (I_{CQ}, V_{CEQ}) is

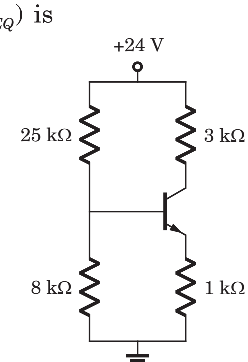

Fig. P3.3.31

(A) (4.68 mA, 16.46 V) (B) (3.12 mA, 1.86 V)

(C) (3.12 mA, 8.46 V) (D) (4.68 mA , 5.22 V)

32. The current gain of the transistor shown in the circuit of fig.P3.3.32 is $\beta = 100$. The values of Q-point (I_{CQ}, V_{CEQ}) is

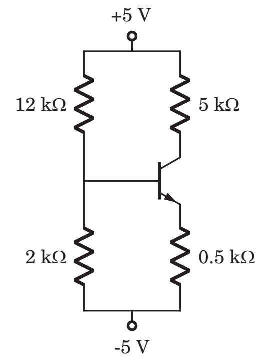

Fig. P3.3.32

(A) (1.8 mA, 2.1 V) (B) (1.4 mA, 2.3 V)

(C) (1.4 mA , 1.8 V) (D) (1.8 mA, 1.4 V)

33. For the circuit in fig. P3.3.33, let $\beta = 60$. The value of V_{ECQ} is

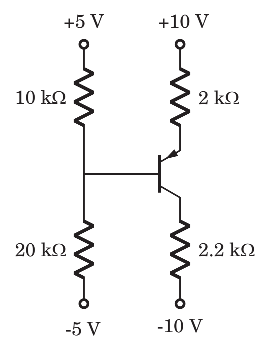

Fig.P3.3.33

(A) 2.68 V (B) 4.94 V

(C) 3.73 V (D) 5.69 V

34. In the circuit of fig. P3.3.34 Zener voltage is $V_Z = 5$ V and $\beta = 100$. The value of I_{CQ} and V_{CEQ} are

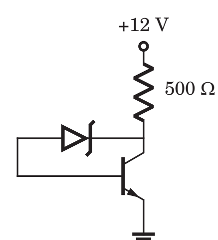

Fig. P3.3.34

(A) 12.47 mA, 4.3 V (B) 12.47 mA, 5.7 V

(C) 10.43 A, 5.7 V (D) 10.43 A , 4.3 V

35. The two transistor in fig. P3.2.35 are identical. If $\beta = 25$, the current I_{C2} is

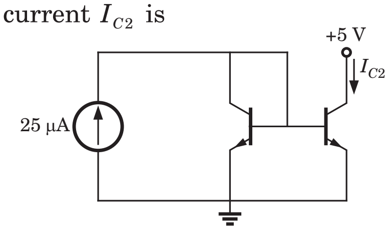

Fig. P3.2.35

(A) 28 μA (B) 23.2 μA

(C) 26 μA (D) 24 μA

36. In the shunt regulator of fig. P3.2.26, the $V_Z = 8.2$ V and $V_{BE} = 0.7$ V. The regulated output voltage V_o is

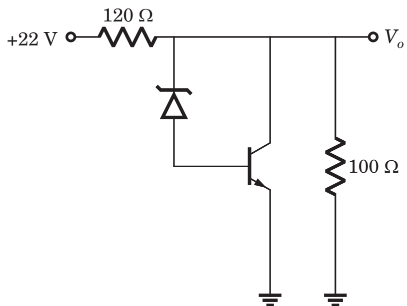

Fig. P3.2.36

(A) 11.8 V (B) 7.5 V

(C) 12.5 V (D) 8.9 V

37. In the series voltage regulator circuit of fig. P3.2.37 $V_{BE} = 0.7$ V, $\beta = 50$, $V_Z = 8.3$ V. The output voltage V_o is

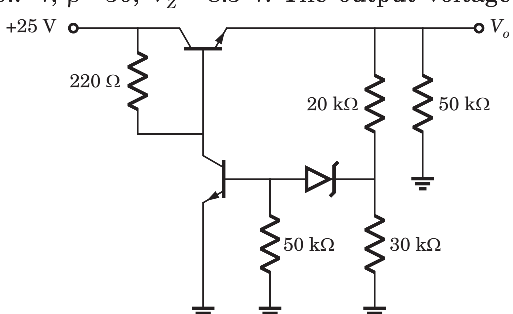

Fig. P3.2.37

(A) 25 V (B) 25.7 V

(C) 15 V (D) 15.7 V

38. In the regulator circuit of fig. P3.2.38 $V_Z = 12$ V, $\beta = 50$, $V_{BE} = 0.7$ V. The Zener current is

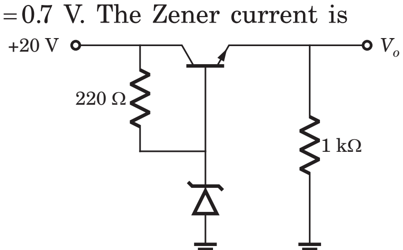

Fig. P3.2.38

(A) 36.63 mA (B) 36.17 mA

(C) 49.32 mA (D) 49.78 mA

39. In the bipolar current source of fig. P3.2.39 the diode voltage and transistor BE voltage are equal. If base current is neglected then collector current is

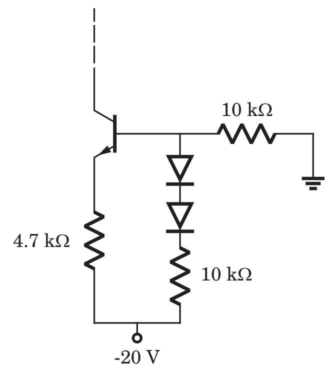

Fig. P3.2.39

(A) 6.43 mA (B) 2.13 mA

(C) 1.48 mA (D) 9.19 mA

40. In the current mirror circuit of fig. P3.2.40. the transistor parameters are $V_{BE} = 0.7$ V, $\beta = 50$ and the Early voltage is infinite. Assume transistor are matched. The output current I_o is

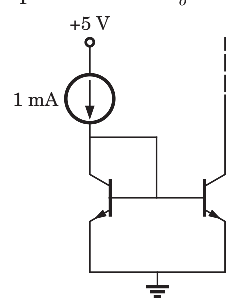

Fig. P3.2.40

(A) 1.04 mA (B) 1.68 mA

(C) 962 μA (D) 432 μA

41. All transistor in the N output mirror in fig. P3.2.41 are matched with a finite gain β and early voltage $V_A = \infty$. The expression for each load current is

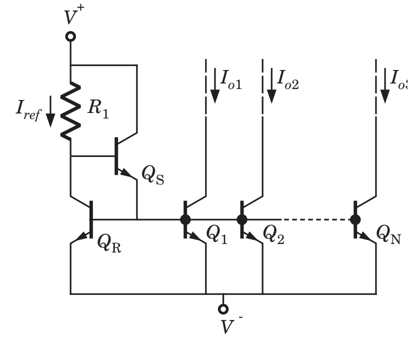

Fig. P32.41

(A) $\dfrac{I_{ref}}{\left(1+\dfrac{(1+N)}{\beta(\beta+1)}\right)}$ (B) $\dfrac{I_{ref}}{\left(1+\dfrac{N}{(\beta+1)}\right)}$

(C) $\dfrac{\beta I_{ref}}{\left(1+\dfrac{(1+N)}{(\beta+1)}\right)}$ (D) $\dfrac{\beta I_{ref}}{\left(1+\dfrac{N}{\beta+1}\right)}$

42. Consider the basic three transistor current source in fig. P3.2.42. Assume all transistor are matched with finite gain and early voltage $V_A=\infty$. The expression for I_o is

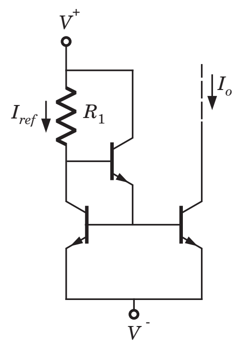

Fig. P3.2.42

(A) $\dfrac{I_{ref}}{\left(1+\dfrac{2}{(1+\beta)}\right)}$ (B) $\dfrac{I_{ref}}{\left(1+\dfrac{1}{(2+\beta)}\right)}$

(C) $\dfrac{I_{ref}}{\left(1+\dfrac{2}{\beta(1+\beta)}\right)}$ (D) $\dfrac{I_{ref}}{\left(1+\dfrac{1}{\beta(2+\beta)}\right)}$

43. Consider the wilder current source of fig. P3.2.43. Both of transistor are identical and $\beta>>1$ and $V_{BE1}=0.7$ V. The value of resistance R_1 and R_E to produce $I_{ref}=1$ mA and $I_o=12\ \mu$A is ($V_t=0.026$)

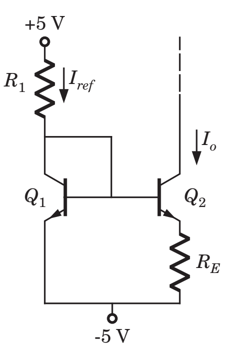

Fig. P3.2.43

(A) 9.3 kΩ, 18.23 kΩ (B) 9.3 kΩ , 9.58 kΩ

(C) 15.4 kΩ , 16.2 kΩ (D) 15.4 kΩ , 32.4 kΩ

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SOLUTIONS

1. (C) $I_E=\dfrac{12-0.7}{10\text{k}} \Rightarrow I_E=1.13\text{ mA}$

$I_C=\left(\dfrac{75}{75+1}\right)(1.13)=1.12\text{ mA}$

$V_{CE}=12-1.13\times10-1.12R_C-(-12)=6\text{ V}$

$R_C=5.98\text{ k}\Omega$

2. (C) $8=10\times(75+1)I_B+0.7+10I_B-2$

$I_B=\dfrac{9.3}{10+760}=12.08\ \mu\text{A},$

$I_C=\beta I_B=0.906\text{ mA},\ I_E=(\beta+1)I_B=0.918\text{ mA}$

$8=10(0.918)+V_{EC}+3(0.906)-8$

$\Rightarrow V_{EC}=4.1\text{ V}$

3. (A) $I_C=\left(\dfrac{75}{75+1}\right)I_E=\dfrac{75}{76}(1\text{m})=0.987\text{ mA}$

$R_C=\dfrac{5.2}{0.987\text{m}}=3.04\text{ k}\Omega$

4. (A) $5=(1+\beta)10\text{k}I_B+20\text{k}I_B+0.7+\beta 2\text{k}I_B$

$5=(760\text{k}+20\text{k}+150\text{k})I_B+0.7$

$\Rightarrow I_B=4.62\ \mu\text{A},$

$I_C=\beta I_B=0.347\text{ mA}$

$V_C=5-(\beta+1)I_BR_C=5-760\times4.62\times10^{-3}=1.49\text{ V}$

5. (C) $V_B=-I_BR_B$

$\Rightarrow I_B=\dfrac{-V_B}{R_B}=\dfrac{1}{500\text{k}}=2.0\ \mu\text{A}$

$V_E=-1-0.7=-1.7\text{ V}$

$I_E=\dfrac{V_E-(-3)}{4.8\text{k}}=\dfrac{-1.7+3}{4.8\text{k}}=0.271\text{ mA}$

$\dfrac{I_E}{I_B}=(\beta+1)=\dfrac{0.271\text{m}}{2\mu}$

$\Rightarrow \beta=134.5$

6. (B) $V_{CE}=3-V_E=3-(-1.7)=4.7\text{ V}$

7. (C) $I_E=\dfrac{5.4}{2\text{k}}=0.5\text{ mA}$

$4=0.7+I_BR_B+I_CR_C-5,\ I_C\approx I_E,$

$8.3=100I_B+0.5\times8$

$\Rightarrow I_B=43\ \mu\text{A},$

$\dfrac{I_E}{I_B}=1+\beta=\dfrac{0.5\text{m}}{43\mu}=11.63$

$\beta = 10.63,\ \alpha = \dfrac{\beta}{1+\beta} \rightarrow \alpha = 0.914$

8. (C) $V_B = 0$ Transistor is in cut-off region.

$I_E = 0,\ V_C = 6\ \text{V}$

9. (B) $V_B = 1\ \text{V}\ ,\ I_E = \dfrac{1-0.7}{1\text{k}} = 0.3\ \text{mA}$

$I_C \approx I_E = 0.3\ \text{mA}$

$V_C = 6 - I_C R_C = 6 - (0.3)(10) = 3\ \text{V}$

10. (B) $V_B = 2\ \text{V},\ I_E = \dfrac{2-0.7}{1} = 1.3\ \text{mA},$

$I_C \approx I_E = 1.3\ \text{mA}$

$V_C = 6 - (1.3)(10) = -7\ \text{V}$

Transistor is in saturation. The saturation voltage

$V_{CE} = 0.2\ \text{V}$

$V_E = (1.3)(1) = 1.3\ \text{V}\ ,\ V_C = V_{CE} + V_E = 1.5\ \text{V}$

11. (C) $V_{BB} =\ 0$, Transistor is in cutoff region

$V_o = \dfrac{R_L}{R_C + R_L} V_{CC} = \dfrac{10(5)}{10} + 5 = 3.33\ \text{V}$

12. (B) $I_B = \dfrac{1-0.7}{50\text{k}} = 6\ \mu\text{A}$

$I_C = \beta I_B = 75 \times 6\mu = 0.45\ \text{mA}$

$\dfrac{5 - V_o}{5\text{k}} = I_C + \dfrac{V_o}{10k}$

$(1 - 0.45) = \dfrac{V_o}{5} + \dfrac{V_o}{10},\quad \Rightarrow \quad V_o = 1.83\ \text{V}$

13. (C) $I_B = \dfrac{2-0.7}{50\text{k}} = 26\ \mu\text{A}$

$I_C = \beta I_B = 75 \times 26\ \mu\text{A} = 1.95\ \text{mA}$

$V_C = 5 - I_C R_C = 5 - 5 \times 1.95 = -4.75\ \text{V}$

Transistor is in saturation, $V_{CE} = 0.2\ \text{V} = V_C = V_o$

14. (B) $I_E = 0.1\ \text{mA}$

$I_C = \dfrac{\beta}{(\beta+1)} I_E = \dfrac{150}{151}(0.1) = 0.099\ \text{mA}$

$V_o = 5 - R_C I_C = 5 - 5(0.099) = 4.50\ \text{V}$

15. (B) $I_E = I_Q = 0.5\ \text{mA}$

$I_C = \left(\dfrac{150}{150+1}\right)(0.5\text{m}) = 0.497\ \text{mA}$

$V_o = 5 - R_C I_C = 2.517\ \text{V}$

16. (D) Transistor is in saturation

$V_o = V_{CE(sat)} - V_{BE} = 0.2 - 0.7 = -0.5\ \text{V}$

17. (B) $I_E = \dfrac{V_B - 0.7}{1\text{k}}$

$I_C = \left(\dfrac{\beta}{\beta+1}\right) I_E = \left(\dfrac{50}{51}\right)(V_B - 0.7)\ \text{mA}$

$I_C = \dfrac{6 - V_C}{10}\ \text{mA},\ V_C = V_B$

$\dfrac{50}{51}(V_B - 0.7) = \dfrac{6 - V_B}{10}$

$10.8\ V_B = 12.86\ ,\ V_B = 1.19\ \text{V}$

18. (B) $V_{CB} = 0.5\ \text{V}\ ,\ V_C = 0.5\ \text{V}$

$I_C = \dfrac{5 - 0.5}{5\text{k}} = 0.9\ \text{mA},\ I_Q = \left(\dfrac{101}{100}\right)0.9 = 0.909\ \text{mA}$

19. (C) $I_E = \dfrac{10 - V_E}{10\text{k}} = 0.8\ \text{mA}$

$V_B = V_E - 0.7 = 1.3\ \text{V}$

$I_B = \dfrac{V_B}{R_B} = \dfrac{1.3}{50\text{k}} = 26\ \mu\text{A}$

$\beta + 1 = \dfrac{I_E}{I_B} = \dfrac{0.8\text{m}}{26\mu} = 30.77 \quad \Rightarrow \quad \beta = 29.77$

$\alpha = \dfrac{\beta}{\beta+1} = \dfrac{29.77}{30.77} = 0.968$

20. (D) $I_C = \left(\dfrac{\beta}{\beta+1} I_E\right) = \dfrac{50}{51}\ \text{mA} = 0.98\ \text{mA}$

$V_C = I_C R_C - 9 = (0.98)(4.7) - 9 = -4.394\ \text{V}$

$I_B = \dfrac{I_E}{(\beta+1)} = \dfrac{1}{51}\ \text{mA} = 19.6\ \mu\text{A}$

$V_E = I_B R_B + V_{EB} = 50(0.0196) + 0.7 = 1.68\ \text{V}$

$V_{EC} = 1.68 - (-4.394) = 6.074\ \text{V}$

21. (A) $I_C = \left(\dfrac{\beta}{\beta+1}\right) I_E = \dfrac{50}{51}(0.5) = 0.49\ \text{mA}$

$I_B = \dfrac{0.5}{51} = 9.8\ \mu\text{A}$

$V_E = I_B R_B + V_{EB} = (0.0098)(50) + 0.7 = 1.19\ \text{V}$

$V_C = I_C R_C - 9 = (0.49)(4.7) - 9 = -6.7\ \text{V}$

$V_{EC} = 1.19 - (-6.7) = 7.89\ \text{V}$

$P_Q = I_C V_{EC} + I_B V_{EB}$

$= (0.49)(7.89) + (0.0098)(0.7)\ \text{mW} = 3.87\ \text{mW}$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{15}{20+15}\right)(10) = 4.29 \text{ V}$$

$$10 = I_E(1\text{k}) + V_{EB} + \frac{I_E}{\beta+1}(8.57\text{k}) + 4.29$$

$$10 = I_E + 0.7 + \frac{I_E}{101}(8.57\text{k}) + 4.29$$

$$\Rightarrow \quad I_E = 4.62 \text{ MA}$$

$$I_B = \frac{I_E}{\beta+1} = 0.046 \text{ mA}$$

$$V_B = (8.57)(0.046) + 4.29 = 4.69 \text{ V}$$

30. (B) $R_1 = 58\ \text{k}\Omega,\ R_2 = 42\ \text{k}\Omega$

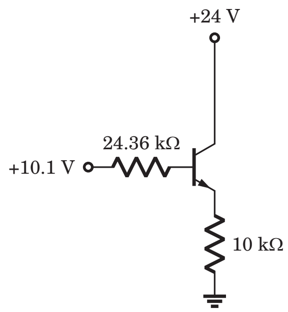

Fig. S3.3.30

$$R_{TH} = 58 \,||\, 42 = 24.36\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{42}{42+58}\right)(24) = 10.1 \text{ V}$$

$$10.1 = I_{BQ}(24.36\text{k}) + V_{BE} + (\beta+1)I_{BQ}(10\text{k})$$

$$10.1 - 0.7 = I_{BQ}(24.36\text{k} + 1260\text{k})$$

$$I_{BQ} = 7.32\ \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = 0.915 \text{ mA}$$

$$I_{EQ} = (\beta+1)I_{BQ} = 0.922 \text{ mA}$$

$$V_{CEQ} = 24 - (0.922)(10) = 14.8 \text{ V}$$

31. (D) $R_1 = 25\ \text{k}\Omega,\ R_2 = 8\ \text{k}\Omega$

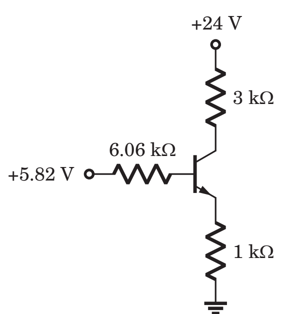

Fig. S3.3.31

$$R_{TH} = 25 \,||\, 8 = 6.06\ \text{k}\Omega,\ V_{TH} = \left(\frac{8}{25+8}\right)(24) = 5.82 \text{ V}$$

$$5.82 = (6.06\text{k})(I_{BQ}) + V_{BE} + (\beta+1)I_B(1\text{k})$$

$$5.82 - 0.7 = (6.06\text{k} + 76\text{k})I_{BQ}$$

$$\Rightarrow \quad I_{BQ} = 62.4\ \mu\text{A}$$

$$I_{EQ} = (\beta+1)\, I_{BQ} = 4.74 \text{ mA}$$

$$I_{CQ} = \beta I_{BQ} = 4.68 \text{ mA}$$

$$V_{CEQ} = 24 - I_{CQ}R_C - I_{EQ}R_E$$

$$= 24 - (4.68)(3) - (4.74)(1) = 5.22 \text{ V}$$

32. (B) $R_1 = 12\ \text{k}\Omega,\ R_2 = 2\ \text{k}\Omega$

$$R_{TH} = R_1 \,||\, R_2 = 12 \,||\, 2 = 1.71\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{2}{12+2}\right)(10) - 5 = -3.57 \text{ V}$$

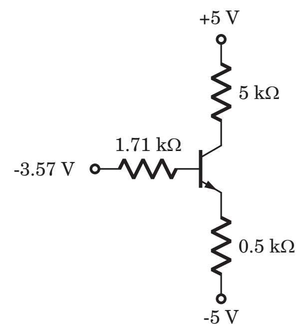

Fig. S3.3.32

$$-3.57 = I_{BQ}(1.71\text{k}) + V_{BE} + (\beta+1)I_{BQ}(0.5\text{k}) - 5$$

$$5 - 3.57 - 0.7 = (1.71 + 50.5)I_{BQ}$$

$$\Rightarrow \quad I_{BQ} = 14\ \mu\text{A}$$

$$I_{EQ} = (100+1)I_{BQ} = 1.412 \text{ mA}$$

$$I_{CQ} = 100 I_{BQ} = 1.4 \text{ mA}$$

$$V_{CEQ} = 5 - R_C I_{CQ} - R_E I_{EQ} + 5$$

$$= 5 - (5)(1.4) - (0.5)(1.412) + 5 = 2.3 \text{ V}$$

33. (B) $R_{TH} = 20 \,||\, 10 = 6.67\ \text{k}\Omega$

$$V_{TH} = \left(\frac{20}{10+20}\right)10 - 5 = 1.67 \text{ V}$$

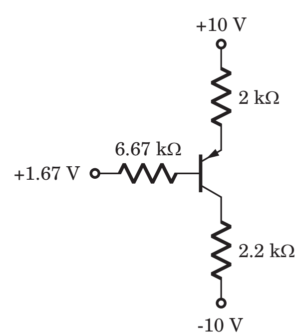

Fig. S3.3.33

$$10 = (1+\beta)I_{BQ}(2) + V_{EB} + I_{BQ}(6.67) + 1.67$$

$$10 - 1.67 - 0.7 = I_{BQ}(6.67 + 122)$$

41. (A) $I_{ref} = I_{CR} + I_{BS} = I_{CR} + \dfrac{I_{ES}}{1+\beta}$

$I_{ES} = I_{BR} + I_{B1} + I_{B2} + \dots\dots I_{BN}$

$I_{BR} = I_{Bi}\,,\quad I_{CR} = I_{Ci} = I_{oi}$

$I_{ES} = (1+N)I_{BR} = \dfrac{(1+N)I_{CR}}{\beta}$

Then $I_{ref} = I_{CR} + \dfrac{I_{ES}}{\beta+1} = I_{CR} + \dfrac{(1+N)\,I_{CR}}{\beta\,(\beta+1)}$

$= I_{Oi}\left(1 + \dfrac{(1+N)}{\beta(\beta+1)}\right)$

$$I_{oi} = \frac{I_{ref}}{\left(1 + \dfrac{(1+N)}{\beta(\beta+1)}\right)}$$

42. (C) $I_{ref} = I_{C1} + I_{B3}\,,\ I_{B1} = I_{B2}\,,\ I_{E3} = 2I_{B2}$

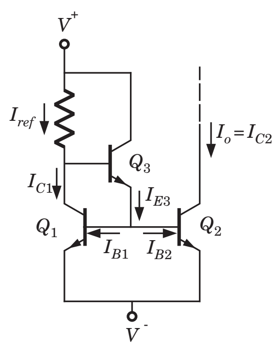

Fig. S3.2.42

$I_{E3} = (1+\beta)I_{B3}$

$I_{ref} = I_{C1} + \dfrac{I_{E3}}{(1+\beta)} = I_{C1} + \dfrac{2I_{B2}}{(1+\beta)}$

$I_{C1} = I_{C2} = \beta I_{B2}$

$I_{ref} = I_{C2} + \dfrac{2I_{C2}}{\beta(1+\beta)} = I_{C2}\left(1 + \dfrac{2}{\beta(1+\beta)}\right)$

$$I_{C2} = I_o = \frac{I_{ref}}{\left(1 + \dfrac{2}{\beta(1+\beta)}\right)}$$

43. (B) If $\beta >> 1$ and transistor are identical

$I_{ref} \approx I_{C1} = I_S e^{\frac{V_{BE1}}{V_t}}\,,\ I_o = I_{C2} = I_S e^{\frac{V_{BE2}}{V_t}}$

$V_{BE1} = V_t \ln\left(\dfrac{I_{ref}}{I_S}\right),\ V_{BE2} = V_t \ln\left(\dfrac{I_o}{I_S}\right)$

$V_{BE1} - V_{BE2} = V_t \ln\left(\dfrac{I_{ref}}{I_o}\right)$

From the circuit,

$V_{BE1} - V_{BE2} = I_{E2}R_E \approx I_o R_E$

$I_o R_E = V_t \ln\left(\dfrac{I_{ref}}{I_o}\right)$

$R_E = \dfrac{0.026}{12\times 10^{-6}} \ln\left(\dfrac{1\times 10^{-3}}{12\times 10^{-6}}\right) = 9.58\ \text{k}\Omega$

$R_1 = \dfrac{V^+ - V_{BE1} - V^-}{I_{ref}} = \dfrac{5 - 7 - (-5)}{1\text{m}} = 9.3\ \text{k}\Omega$

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CHAPTER 3.3

BASIC FET CIRCUITS

Statement for Q.1–3:

In the circuit shown in fig. P3.3.1–3 the transistor parameters are as follows:

Threshold voltage $V_{TN} = 2$ V

Conduction parameter $K_n = 0.5$ mA / V^2

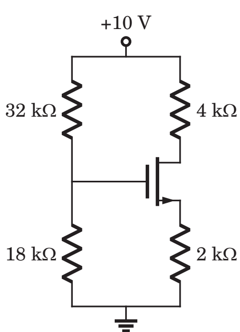

Fig. P3.3.1–3

1. $V_{GS} = ?$

(A) 2.05 V (B) 6.43 V

(C) 4.86 V (D) 3.91 V

2. $I_D = ?$

(A) 1.863 mA (B) 1.485 mA

(C) 0.775 mA (D) None of the above

3. $V_{DS} = ?$

(A) 4.59 V (B) 3.43 V

(C) 5.35 V (D) 6.48 V

Statement for Q.4–6:

In the circuit shown in fig. P3.3.4–6 the transistor parameter are as follows:

$$V_{TN} = 2 \text{ V}, \quad k_n' = 60 \ \mu\text{A / V}^2, \quad \frac{W}{L} = 60$$

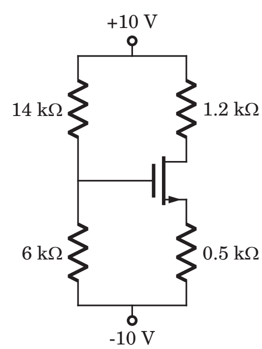

Fig. P3.3.4–6

4. $V_{GS} = ?$

(A) –3.62 V (B) 3.62 V

(C) –0.74 V (D) 0.74 V

5. $I_D = ?$

(A) 13.5 mA (B) 10 mA

(C) 19.24 mA (D) 4.76 mA

6. $V_{DS} = ?$

(A) 2.95 V (B) 11.9 V

(C) 3 V (D) 12.7 V

16. The parameter of the transistor in fig. P3.3.16 are $V_{TN} = 1.2$ V, $K_n = 0.5$ mA / V^2 and $\lambda = 0$. The voltage V_{DS} is

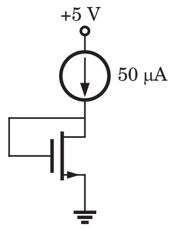

Fig. P3.3.16

(A) 1.69 V (B) 1.52 V

(C) 1.84 V (D) 0

17. The parameter of the transistor in fig. P3.3.17 are $V_{TN} = 0.6$ V and $K_n = 0.2$ mA / V^2. The voltage V_S is

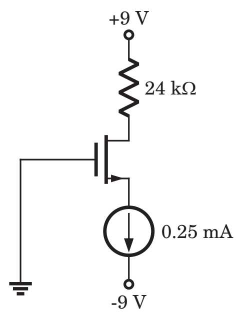

Fig. P3.3.17

(A) 1.72 V (B) –1.72 V

(C) 7.28 V (D) –7.28 V

18. In the circuit of fig. P3.3.18 the transistor parameters are $V_{TN} = 1.7$ V and $K_n = 0.4$ mA / V^2.

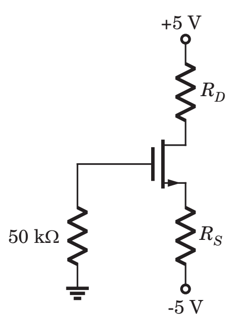

Fig. P3.3.18

If $I_D = 0.8$ mA and $V_D = 1$ V, then value of resistor R_S and R_D are respectively

(A) 2.36 kΩ, 5 kΩ (B) 5 kΩ, 2.36 kΩ

(C) 6.43 kΩ, 8.4 kΩ (D) 8.4 kΩ, 6.43 kΩ

19. In the circuit of fig. P.3.3.19 the PMOS transistor has parameter $V_{TP} = -1.5$ V, $k'_p = 25$ μA / V^2, $L = 4$ μm and $\lambda = 0$. If $I_D = 0.1$ mA and $V_{SD} = 2.5$ V, then value of W will be

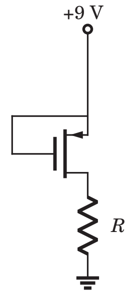

Fig. P3.3.19

(A) 15 μm (B) 1.6 μm

(C) 32 μm (D) 3.2 μm

20. The PMOS transistor in fig. P3.3.20 has parameters

$$V_{TP} = -1.2 \text{ V}, \quad \frac{W}{L} = 20, \text{ and } k'_p = 30 \ \mu\text{A / V}^2.$$

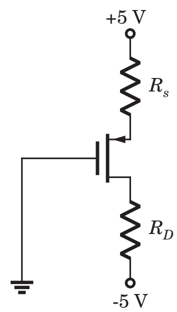

Fig. P3.3.20

If $I_D = 0.5$ mA and $V_D = -3$ V, then value of R_S and R_D are

(A) 4 kΩ, 5.8 kΩ (B) 4 kΩ, 5 kΩ

(C) 5.8 kΩ, 4 kΩ (D) 5 kΩ, 4 kΩ

21. The parameters for the transistor in circuit of fig. P3.3.21 are $V_{TN} = 2$ V and $K_n = 0.2$ mA / V^2. The power dissipated in the transistor is

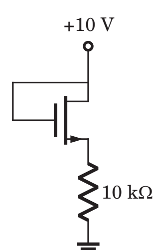

Fig. P3.3.21

(A) 5.84 mW (B) 2.34 mW

(C) 0.26 mW (D) 58.4 mW

Statement for Q.22–23:

Consider the circuit shown in fig. P3.2.22–33.

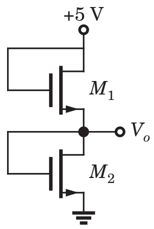

Fig. P3.3.22-23

The both transistor have parameter as follows

$V_{TN} = 0.8$ V, $k_n' = 30$ μA / V²

22. If the width-to-length ratios of M_1 and M_2 are

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 40$$

The output V_o is

(A) –2.5 V (B) 2.5 V

(C) 5 V (D) 0 V

23. If the ratio is $\left(\frac{W}{L}\right)_1 = 40$ and $\left(\frac{W}{L}\right)_2 = 15$, then V_o is

(A) 2.91 V (B) 2.09 V

(C) 3.41 V (D) 1.59 V

24. In the circuit of fig. P3.324. the transistor parameters are $V_{TN} = 1$ V and $k_n' = 36$ μA / V². If $I_D = 0.5$ mA, $V_1 = 5$ V and $V_2 = 2$ V then the width to-length ratio required in each transistor is

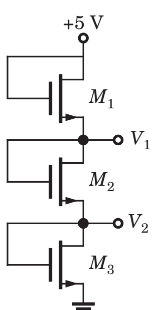

Fig. P3.3.24

	$\left(\frac{W}{L}\right)_1$	$\left(\frac{W}{L}\right)_2$	$\left(\frac{W}{L}\right)_3$
(A)	1.75	6.94	27.8
(B)	4.93	10.56	50.43
(C)	35.5	22.4	8.53
(D)	56.4	38.21	12.56

25. The transistors in the circuit of fig. P3.3.25 have parameter $V_{TN} = 0.8$ V, $k_n' = 40$ μA / V² and $\lambda = 0$. The width-to-length ratio of M_2 is $\left(\frac{W}{L}\right)_2 = 1$. If $V_o = 0.10$ V when $V_i = 5$ V, then $\left(\frac{W}{L}\right)_1$ for M_1 is

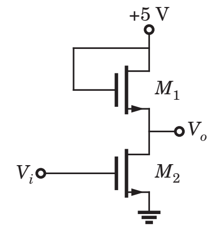

Fig. P3.3.25

(A) 47.5 (B) 28.4

(C) 40.5 (D) 20.3

Statement for Q.26–27:

All transistors in the circuit in fig. P3.3.26–27 have parameter $V_{TN} = 1$ V and $\lambda = 0$.

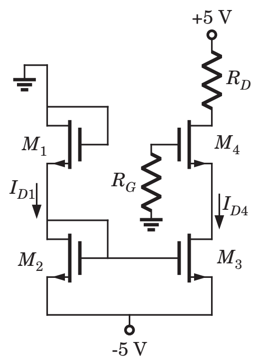

Fig. P3.3.26–27

The conduction parameter are as follows:

$K_{n1} = 400$ μA / V²

$K_{n2} = 200$ μA / V²

$K_{n3} = 100$ μA / V²

$K_{n4} = 80$ μA / V²

26. $I_{D1} = ?$

(A) 0.23 mA (B) 0.62 mA

(C) 0.46 mA (D) 0.31 mA

27. $I_{D4} = ?$

(A) 0.62 mA (B) 0.31 mA

(C) 0.46 mA (D) 0.92 mA

28. For the circuit in fig. P3.3.28 the transistor parameter are $V_{TN} = 0.8$ V and $k'_n = 30\ \mu A / V^2$. If output voltage is $V_o = 0.1$ V, when input voltage is $V_i = 4.2$ V, the required transistor width-to length ratio is

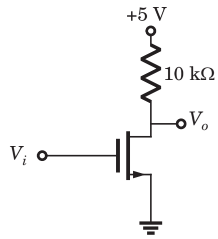

Fig. P3.3.28

(A) 1.568 (B) 0.986

(C) 0.731 (D) 1.843

29. For the transistor in fig. P3.3.29 parameters are $V_{TN} = 1$ V and $K_n = 12.5\ \mu A / V^2$. The Q-point (I_D, V_{DS}) is

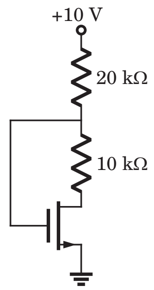

Fig. P3.3.29.

(A) (1 mA, 8 V) (B) (0.2 mA, 4 V)

(C) (1.17 mA, 8 V) (D) (0.23 mA, 3.1V)

30. For an n-channel JFET, the parameters are $I_{DSS} = 6$ mA and $V_P = -3$ V. If $V_{DS} > V_{DS(sat)}$ and $V_{GS} = -2$ V, then I_D is

(A) 16.67 mA (B) 0.67 mA

(C) 5.55 mA (D) 1.67 mA

31. For the circuit in fig. P3.3.32 the transistor parameters are $V_p = -3.5$ V, $I_{DSS} = 18$ mA, and $\lambda = 0$. The value of V_{DS} is

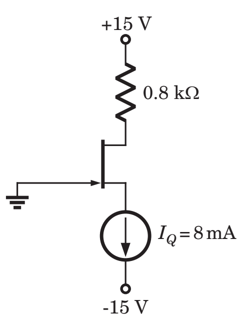

Fig. P3.3.32

(A) 7.43 V (B) 8.6 V

(C) –1.17 V (D) 1.17 V

32. A p-channel JFET biased in the saturation region with $V_{SD} = 5$ V has a drain current of $I_D = 2.8$ mA, and $I_D = 0.3$ mA at $V_{GS} = 3$ V. The value of I_{DSS} is

(A) 10 mA (B) 5 mA

(C) 7 mA (D) 2 mA

Statement for Q.33–34:

For the p-channel transistor in the circuit of fig. P3.3.33–34 the parameters are $I_{DSS} = 6$ mA, $V_P = 4$ V and $\lambda = 0$.

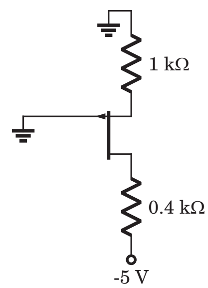

Fig. P3.3.33–34

33. The value of I_{DQ} is

(A) 8.86 mA (B) 6.39 mA

(C) 4.32 mA (D) 1.81 mA

34. The value of V_{SD} is

(A) –4.28 V (B) 2.47 V

(C) 4.28 V (D) 2.19 V

35. The transistor in the circuit of fig. P3.3.35 has parameters $I_{DSS} = 8$ mA and $V_P = -4$ V. The value of V_{DSQ} is

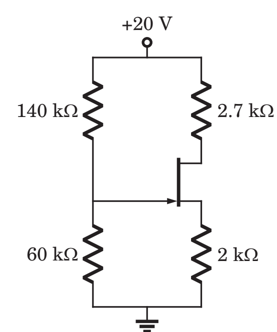

Fig. P3.3.35

(A) 2.7 V (B) 2.85 V

(C) –1.30 V (D) 1.30 V

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SOLUTIONS

1. (A) $R_1 = 32\ k\Omega, \quad R_2 = 18\ k\Omega, \quad V_{DD} = 10\ V$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{18}{18+32}\right)10 = 3.6\ V$$

Assume that transistor in saturation region

$$I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2$$

$R_S = 2\ k\Omega, \quad K_n = 0.5\ mA/V^2$

$3.6 - V_{GS} = (2)(0.5)(V_{GS} - 0.8)^2 \quad \Rightarrow \quad V_{GS} = 2.05\ V$

2. (C) $I_D = \dfrac{V_G - V_{GS}}{R_S} = \dfrac{3.6 - 2.05}{2k} = 0.775\ mA$

3. (C) $V_{DS} = V_{DD} - I_D(R_D + R_S)$

$= 10 - 0.775(4 + 2) = 5.35\ V$

$V_{DS(sat)} = V_{GS} - V_{TN} = (2.05 - 0.8) = 1.25\ V$

$V_{DS} > V_{DS(sat)}$ as assumed.

4. (B) $R_1 = 14\ k\Omega,\ R_2 = 6\ k\Omega,\ R_S = 0.5\ k\Omega,\ R_D = 1.2\ k\Omega$

$$V_G = \left(\frac{R_L}{R_1 + R_2}\right)(20) - 10 = \left(\frac{6}{14+6}\right)(20) - 10 = -4\ V$$

Assume transistor in saturation

$$I_D = \frac{V_S - (-10)}{R_S} = \frac{V_G - V_{GS} + 10}{R_S} = K_n(V_{GS} - V_{TN})^2$$

$$K_n = \frac{k_n'}{2}\frac{W}{L} = \frac{(60)(60 \times 10^{-6})}{2} = 1.8\ mA/V^2$$

$\Rightarrow \quad -4 - V_{GS} + 10 = (0.5)(1.8)(V_{GS} - 2)^2$

$\Rightarrow \quad V_{GS} = 3.62,\ -0.74\ V,\ V_{GS}$ will be positive.

5. (D) $I_D = \dfrac{V_G - V_{GS} + 10}{R_S} = \dfrac{-4 - 3.62 + 10}{0.5k} = 4.76\ mA$

6. (B) $10 = I_D(R_S + R_D) + V_{DS} - 10$

$V_{DS} = 20 - 4.76(1.2 + 0.5) = 11.9\ V$

$V_{DS(sat)} = V_{GS} - V_{TN} = 3.62 - 2 = 1.62\ V$

$V_{DS} = 11.9\ V > V_{DS(sat)}$, Assumption is correct.

7. (B) $R_1 = 8\ k\Omega, \quad R_L = 22\ k\Omega, \quad R_S = 0.5\ k\Omega, \quad R_D = 2\ k\Omega$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(20) - 10 = \left(\frac{22}{8+22}\right)(20) - 10 = 4.67\ V$$

Assume transistor in saturation

$$I_D = \frac{10 - V_S}{R_S} = K_P(V_{SG} + V_{TP})^2$$

$V_S = V_G + V_{SG}$

$10 - (4.67 + V_{SG}) = (0.5)(1)(V_{GS})$

$\Rightarrow \quad V_{SG} = 3.77\ V,\ -1.77\ V,\ V_{SG}$ is positive voltage.

8. (A) $I_D = \dfrac{10 - V_S}{R_S} = \dfrac{10 - (V_G + V_{GS})}{R_S}$

$= \dfrac{10 - (4.67 + 3.77)}{0.5} = 3.12\ mA$

9. (C) $10 = I_D(R_S + R_D) + V_{SD} - 10$

$V_{SD} = 20 - I_D(R_S + R_D) = 20 - 2.12(2 + 0.5) = 12.2\ V$

10. (C) Assume transistor in saturation.

$I_D = 0.4\ mA, \quad 0.4 = K_P(V_{GS} + V_{TP})^2$

$0.4 = (0.2)(V_{SG} - 0.8)^2 \quad \Rightarrow \quad V_{SG} = \sqrt{2} + 0.8 = 2.21\ V$

$V_G = 0, \quad V_{SG} = V_S - V_G = V_S$

11. (A) $V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3\ V$

$V_{SD} = V_S - V_D = 2.21 - (-3) = 5.21\ V$

12. (C) $R_1 = 14.5\ k\Omega, \quad R_2 = 5.5\ k\Omega,$

$R_S = 0.6\ k\Omega,\ R_D = 0.8\ k\Omega,$

$$V_G = \left(\frac{R_L}{R_1 + R_2}\right)(10) - 5 = \left(\frac{5.5}{14.5 + 5.5}\right)(10) - 5 = -2.25\ V$$

Assume transistor in saturation.

$$I_D = \frac{V_S - (-5)}{R_S} = K_n(V_{GS} - V_{TN})^2$$

$V_S = V_G - V_{GS}$

$-2.25 - V_{GS} + 5 = (0.6)(0.5)(V_{GS} - (-1))^2$

$\Rightarrow \quad V_{GS} = 1.24,\ -6.58\ V$

V_{GS} is positive. Thus (D) is correct option.

13. (D) $I_D = \dfrac{V_S + 5}{R_S} = \dfrac{V_G - V_{GS} + 5}{R_S} = \dfrac{-2.25 - 1.24 + 5}{0.6k}$

$= 2.52\ mA$, Therefore (D) is correct option.

14. (B) $5 = I_D(R_S + R_D) + V_{DS} - 5$

$V_{DS} = 10 - I_D(R_S + R_D) = 10 - 2.52(0.8 + 0.6) = 6.47\ V$

$V_{DS(sat)} = V_{GS} - V_{TH} = 1.24 - (-1) = 2.24$

$V_{DS} > V_{DS(sat)}$, Assumption is correct.

15. (B) $I_S = 50\ \mu A = I_D,\ I_D = K_n(V_{GS} - V_{TN})^2$

$\Rightarrow \quad 50 \times 10^{-6} = 0.5 \times 10^{-3}(V_{GS} - 1.2)^2 \quad \Rightarrow \quad V_{GS} = 1.516\ V,$

$V_G = 0,\ V_S = V_G - V_{GS} = -1.516\ V$

$V_{DS} = V_D - V_S = 5 - (-1.516) = 6.516\ V$

16. (B) $I_D = 50\mu = K_n(V_{GS} - V_{TN})^2$

$\Rightarrow \quad 50 \times 10^{-6} = 0.5 \times 10^{-3}(V_{GS} - 1.2)^2$

$V_{GS} = 1.52$ V, $V_{GS} = V_{DS}$

17. (B) $I_D = K_n(V_{GS} - V_{TN})^2$

$\Rightarrow \quad 0.25 = 0.2(V_{GS} - 0.6)^2 \quad \Rightarrow \quad V_{GS} = 1.72$ V,

$V_{GS} = V_G - V_S, \quad V_G = 0, \quad V_S = -1.72$ V

18. (A) $I_D = \dfrac{5 - V_D}{R_D} = 0.8$ mA, $R_D = \dfrac{6-1}{0.8\text{m}} = 5\ \text{k}\Omega$

$I_D = K_n\ (V_{GS} - V_{TN})^2$

$\Rightarrow \quad 0.8 = (0.4)(V_{GS} - 1.7)^2 \quad \Rightarrow \quad V_{GS} = 3.11$ V

$V_{GS} = V_G - V_S\ , \quad V_G = 0, \quad V_S = -3.11$ V

$I_D = 0.8\ \text{mA} = \dfrac{-3.11 - (-5)}{R_S} \quad \Rightarrow \quad R_S = 2.36\ \text{k}\Omega$

19. (C) $V_{SD} = V_{SG}, \quad I_D = \dfrac{k'_p}{2}\dfrac{W}{L}(V_{GS} + V_{TP})^2$

$10^{-4} = \left(\dfrac{25}{2}\right)\left(\dfrac{W}{4}\right)(2.5 - 1.5)^2 \quad \Rightarrow \quad W = 32\ \mu\text{m}$

20. (D) $K_p = \left(\dfrac{30 \times 10^{-6}}{2}\right)(20) = 0.3\ \text{mA / V}^2$

$I_D = K_p(V_{SG} + V_{TP})^2 \quad \Rightarrow \quad 0.5 = 0.3(V_{SG} - 1.2)^2$

$\Rightarrow \quad V_{SG} = 2.49$ V, $V_G = 0$

$V_S = V_{SG} = 2.49$ V

$I_D = \dfrac{5 - V_S}{R_S} \quad \Rightarrow \quad R_S = \dfrac{5 - 2.49}{0.5\text{m}} = 5.02\ \text{k}\Omega$

$I_D = \dfrac{V_D - (-5)}{R_D} \quad \Rightarrow R_D = \dfrac{-3+5}{0.5\text{m}} = 4\ \text{k}\Omega$

21. (B) Assume transistor in saturation

$I_D = \dfrac{10 - V_{GS}}{10\text{k}} = K_n(V_{GS} - V_{TN})^2$

$10 - V_{GS} = (10)(0.2)(V_{GS} - 2)^2$

$\Rightarrow \quad V_{GS} = 3.77$ V, -0.27 V, V_{GS} will be 3.77 V

$V_{GS} = V_{DS} = 3.77$ V

$I_D = \dfrac{10 - 3.77}{10\text{k}} = 0.623$ mA

Power $= I_D V_{DS} = 2.35$ mW

$V_{DS} > V_{GS} - V_{TN}$ assumption is correct.

22. (B) For both transistor $V_{DS} = V_{GS}$,

$V_{DS} > V_{GS} - V_{TN}$ Therefore both transistor are in saturation.

$I_{D1} = I_{D2} \quad \Rightarrow \quad K_{n1}(V_{GS1} - V_{TN1})^2 = K_{n2}(V_{GS2} - V_{TN2})^2$

$K_{n1} = K_{n2}\ , \quad V_{TN1} = V_{TN2}$

$V_{GS1} = V_{GS2} = \dfrac{5}{2}$ V

$V_o = V_{GS2} = 2.5$ V

23. (A) $\left(\dfrac{W}{L}\right)_1 > \left(\dfrac{W}{L}\right)_2$ thus $V_{GS1} < V_{GS2}$

$40(V_{GS1} - 0.8)^2 = 15(V_{GS2} - 0.8)^2$

$V_{GS2} = 5 - V_{GS1}$

$1.63(V_{GS1} - 0.8) = (5 - V_{GS1} - 0.8)$

$V_{GS1} = 2.09, \ V_{GS2} = 2.91$ V, $V_o = V_{GS2} = 2.91$ V

24. (A) Each transistor is biased in saturation because

$V_{DS} = V_{GS}$ and $V_{DS} > V_{GS} - V_{TN}$

For M_3 , $V_2 = 2\ \text{V} = V_{GS3}$

$I_D = 0.5 = \left(\dfrac{36 \times 10^{-3}}{2}\right)\left(\dfrac{W}{L}\right)_3 (2-1)^2 \quad \Rightarrow \quad \left(\dfrac{W}{L}\right)_3 = 27.8$

For M_2 , $V_{GS2} = V_1 - V_2 = 5 - 2 = 3$ V

$I_D = 0.5 = \left(\dfrac{36 \times 10^{-3}}{2}\right)\left(\dfrac{W}{L}\right)_2 (3-1)^2 \quad \Rightarrow \quad \left(\dfrac{W}{L}\right)_2 = 6.94$

For M_1 , $V_{GS1} = 10 - V_1 = 10 - 5 = 5$ V

$I_D = 0.5 = \left(\dfrac{36}{2} \times 10^{-5}\right)\left(\dfrac{W}{L}\right)_1 (5-1)^2 \quad \Rightarrow \quad \left(\dfrac{W}{L}\right)_1 = 1.74$

25. (D) M_2 is in saturation because

$V_{GS2} = V_{DS2} > V_{GS2} - V_{TN}$

M_1 is in non saturation because

$V_{GS1} = V_i = 5$ V, $V_{DS1} = V_D = 0$ V

$V_{DS1} < V_{GS1} - V_{TN}\ , \quad I_{D1} = I_{D2}$

$\left(\dfrac{W}{L}\right)_1 [2(V_{GS1} - V_{TN1})V_{DS1} - V_{DS2}^2] = \left(\dfrac{W}{L}\right)_2 (V_{GS2} - V_{TN2})^2$

$\Rightarrow \left(\dfrac{W}{L}\right)_1 [2(5-0.8)(0.1) - (0.1)^2] = (1)(5 - 0.1 - 0.8)^2$

$\left(\dfrac{W}{L}\right)_1 (0.83) = 16.81 \quad \Rightarrow \quad \left(\dfrac{W}{L}\right)_1 = 20.3$

26. (B) $I_{D1} = K_{n1}\ (V_{GS1} - V_{TN})^2 = K_{n2}\ (V_{GS2} - V_{TN})^2$

$V_{GS1} = 5 - V_{GS2} \quad \Rightarrow \quad (5 - V_{GS2} - 1)^2 = 200\ (V_{GS2} - 1)^2$

$\Rightarrow \quad V_{GS2} = 2.76$ V, $\quad V_{GS1} = 2.24$ V

$I_{D1} = 400 \times 10^{-6}\ (2.24 - 1)^2 = 0.62$ mA

27. (B) $V_{GS2} = V_{GS3} = 2.76$ V

$I_{D4} = K_{n4}\ (V_{GS4} - V_{TN})^2 = K_{n3}\ (V_{GS3} - V_{TN})^2$

$= 100 \times 10^{-6}\ (2.76 - 1)^2 = 0.31$ mA

28. (C) $V_{GS} = 4.2$ V, $\quad V_{DS} = 0.1$ V

$V_{DS} < V_{GS} - V_{TN}$, Thus transistor is in non saturation.

$I_D = \dfrac{5 - 0.1}{10\text{k}} = 0.49$ mA

$I_D = \dfrac{k'_n}{2}\dfrac{W}{L}\{2\ (V_{GS} - V_{TN})\ V_{DS} - V_{DS}^2\}$

CHAPTER

3.4

AMPLIFIERS

1. If the transistor parameter are $\beta = 180$ and Early voltage $V_A = 140$ V and it is biased at $I_{CQ} = 2$ mA, the values of hybrid-π parameter g_m, r_π and r_o are respectively

(A) 14 A/V, 2.33 kΩ, 90 kΩ

(B) 14 A/V, 90 kΩ , 2.33 kΩ

(C) 77 mA/V, 2.33 kΩ , 70 kΩ

(D) 77.2 A/V, 70 kΩ, 2.33 kΩ

Statement for Q.2–3.

Consider the circuit of fig. P3.4.2–3. The transistor parameters are $\beta = 120$ and $V_A = \infty$.

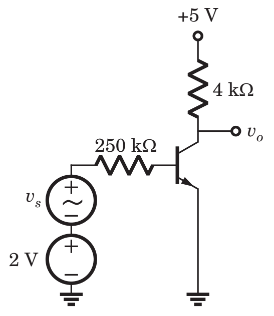

Fig. P3.4.2–3

2. The hybrid-π parameter values of g_m , r_π and r_o are

(A) 24 mA/V, ∞, 5 kΩ

(B) 24 mA/V, 5 kΩ , ∞

(C) 48 mA/V, 10 kΩ , 18.4 kΩ

(D) 48 mA/V, 18.4 kΩ, 10 kΩ

3. The small signal votlage gain $A_v = v_o/v_s$ is

(A) –4.38 (B) 4.38

(C) –1.88 (D) 1.88

4. The nominal quiescent collector current of a transistor is 1.2 mA. If the range of β for this transistor is $80 \le \beta \le 120$ and if the quiescent collector current changes by ±10 percent, the range in value for r_π is

(A) 1.73 kΩ $< r_\pi <$ 2.59 kΩ

(B) 1.93 kΩ $< r_\pi <$ 2.59 kΩ

(C) 1.73 kΩ $< r_\pi <$ 2.59 kΩ

(D) 1.56 kΩ $< r_\pi <$ 2.88 kΩ

Statement for Q.5–6:

Consider the circuit in fig. P3.4.5.6. The transistor parameter are $\beta = 100$ and $V_A = \infty$.

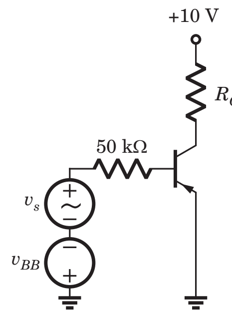

Fig. P3.4.5–6

5. If Q-point is in the center of the load line and $I_{CQ} = 0.5$ mA, the values of V_{BB} and R_C are

(A) 10 kΩ , 0.95 V (B) 10 kΩ , 1.45 V

(C) 48 kΩ , 0.95 V (D) 48 kΩ , 1.45 V

Statement for Q.14–15:

Consider the common Base amplifier shown in fig. P3.4.14–15. The parameters are $g_m = 2$ mS and $r_o = 250$ kΩ. Find the Thevenin equivalent faced by load resistance R_L .

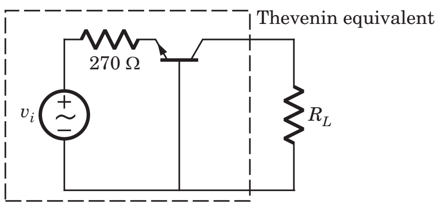

Fig. P3.4.14–15

14. The Thevenin voltage v_{TH} is

(A) $263v_i$ (B) $132v_i$

(C) $346v_i$ (D) $498v_i$

15. The Thevenin equivalent resistance R_{TH} is

(A) 384 kΩ (B) 697 kΩ

(C) 408 kΩ (D) 915 kΩ

Statement for Q.16–17:

The common-base amplifier is drawn as a two-port in fig. P3.4.16–17. The parameters are $\beta = 100$, $g_m = 3$ mS, and $r_o = 800$ kΩ.

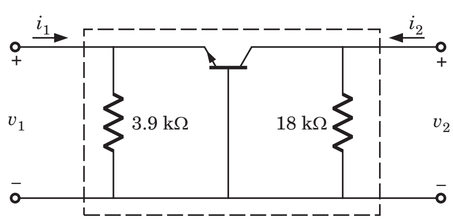

Fig. P3.4.16–17

16. The h-parameter h_{21} is

(A) 2.46 (B) 0.9

(C) 0.5 (D) 0.67

17. The h-parameter h_{12} is

(A) 3.8×10^{-4} (B) 4.83×10^{-3}

(C) 3.8×10^{4} (D) 4.83×10^{3}

18. For an n-channel MOSFET biased in the saturation region, the parameters are $K_n = 0.5$ mA/V^2 , $V_{TN} = 0.8$ V and $\lambda = 0.01$ V^{-1}, and $I_{DQ} = 0.75$ mA. The value of g_m and r_o are

(A) 0.68 mS, 603 kΩ (B) 1.22 mS, 603 kΩ

(C) 1.22 mS, 133 kΩ (D) 0.68 mS, 133 kΩ

19. For an n-channel MOSFET biased in the saturation region, the parameters are $V_{TN} = 1$ V, $\frac{1}{2}\mu_n C_{ox} = 18$ mA/V^2 and $\lambda = 0.015$ V^{-1} and $I_{DQ} = 2$ mA. If transconductance is $g_m = 3.4$ mA/V, the width-to-length ratio is

(A) 80.6 (B) 43.2

(C) 190 (D) 110

20. In the circuit of fig. P3.4.20, the parameters are $g_m = 1$ mA / V, $r_o = 50$ kΩ. The gain $A_v = v_o/v_s$ is

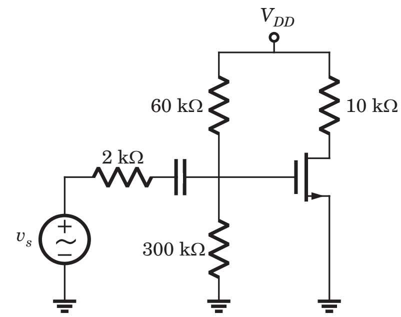

Fig. P3.4.20

(A) –8.01 (B) 8.01

(C) 14.16 (D) –14.16

Statement for Q.21–23:

For the circuit shown in fig. P3.4.21–23 transistor parameters are $V_{TN} = 2$ V, $K_n = 0.5$ mA / V^2 and $\lambda = 0$. The transistor is in saturation.

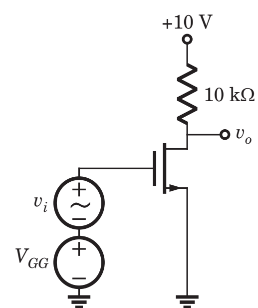

Fig. P3.4.21–23

21. If I_{DQ} is to be 0.4 mA, the value of V_{GSQ} is

(A) 5.14 V (B) 4.36 V

(C) 2.89 V (D) 1.83 V

22. The values of g_m and r_o are

(A) 0.89 mS, ∞ (B) 0.89 mS, 0

(C) 1.48 mS, 0 (D) 1.48 mS, ∞

23. The small signal voltage gain A_v is

(A) 14.3 (B) –14.3

(C) –8.9 (D) 8.9

(A) 4.44 (B) –4.44

(C) 2.22 (D) –2.22

Statement for Q.33–34:

Consider the source-follower circuit in fig. P3.4.33-34. The values of parameter are $g_m = 2$ mS and $r_o = 100$ kΩ.

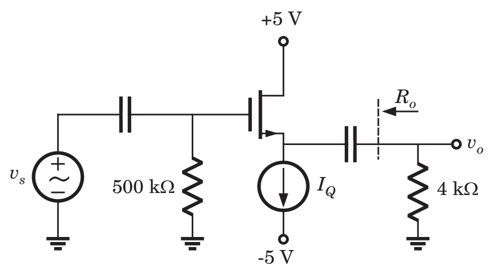

Fig. P3.4.33-34

33. The voltage gain A_v is

(A) 0.89 (B) –0.89

(C) 2.79 (D) –2.79

34. The output resistance R_o is

(A) 100 kΩ (B) 0.498 kΩ

(C) 1.33 kΩ (D) None of the above

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SOLUTIONS

1. (C) $g_m = \frac{I_{CQ}}{V_t} = \frac{2m}{0.0259} = 77.2$ mA/V

$$r_\pi = \frac{\beta V_t}{I_{CQ}} = \frac{\beta}{g_m} = \frac{180}{77.2m} = 2.33 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{140}{2m} = 70 \text{ k}\Omega$$

2. (B) $I_{BQ} = \frac{2 - 0.7}{250k} = 5.2\ \mu A$

$$I_{CQ} = \beta I_B = (120)(5.2\mu) = 0.642 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_t} = \frac{0.624}{0.0259} = 24 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_t}{I_{CQ}} = \frac{\beta}{g_m} = \frac{120}{24m} = 5 \text{ k}\Omega,\ r_o = \infty$$

3. (C) $A_v = -g_m R_C \left(\frac{r_\pi}{r_\pi + R_B}\right) = \frac{\beta R_C}{r_\pi + R_B}$

$$= -(24m)(4k)\left(\frac{5k}{5k + 250k}\right) = -1.88$$

4. (D) $r_\pi = \frac{\beta V_T}{I_{CQ}}$,

$$r_{\pi(max)} = \frac{(120)(0.0259)}{1.08m} = 2.88 \text{ k}\Omega,$$

$$r_{\pi(min)} = \frac{(80)(0.0259)}{1.32m} = 1.56 \text{ k}\Omega$$

5. (A) $V_{ECQ} = \frac{1}{2} V_{CC} = 5$ V

$$V_{ECQ} = 10 - I_{CQ} R_C = 5$$

$$\Rightarrow \quad 10 - (0.5m) R_C = 5$$

$$R_C = 10 \text{ k}\Omega,$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 5\ \mu A$$

$$V_{EB(ON)} + I_{BQ} R_B = V_{BB}$$

$$\Rightarrow \quad 0.7 + (5\mu)(50k) = 0.95 \text{ V}$$

6. (D) $g_m = \frac{I_{CQ}}{V_t} = \frac{0.5}{0.0259} = 19.3$ mA/V

$$r_\pi = \frac{\beta V_t}{I_{CQ}} = \frac{(100)(0.0259)}{0.5m} = 5.18 \text{ k}\Omega,\ r_o = \infty$$

7. (B) $I_{CQ} = \left(\frac{100}{1001}\right)(0.35) = 0.347$ mA

The small-signal equivalent circuit is as shown in fig. S3.4.7

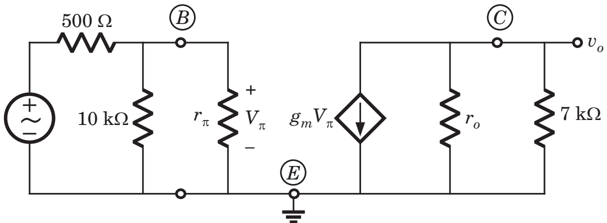

Fig. S3.4.7

$$V_\pi = \frac{r_\pi \,||\, 10\text{k}}{500 + r_\pi \,||\, 10\text{k}}(v_s),\ v_o = -g_m V_\pi (r_o \,||\, 7\text{k})$$

$$\frac{v_o}{v_s} = -g_m\left(\frac{r_\pi \,||\, 10\text{k}}{500 + r_\pi \,||\, 10k}\right)(r_o \,||\, 7\text{k})$$

$$g_m = \frac{I_{CQ}}{V_t} = \frac{0.347\text{m}}{0.0259} = 13.13\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_t}{I_{CQ}} = \frac{100}{13.13\text{m}} = 7.6\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.347\text{m}} = 288\ \text{k}\Omega$$

$$r_o \,||\, 7\text{k} = \frac{288 \times 7}{288 + 7} = 6.83\ \text{k}\Omega,\ r_\pi \,||\, 10\text{k} = \frac{7.6 \times 10}{7.6 + 10} = 4.32\ \text{k}\Omega$$

$$A_v = -13.13\,\text{m}\left(\frac{4.32\text{k}}{500 + 4.32\text{k}}\right)(6.83\text{k}) = -80$$

8. (C) DC Analysis: $I_{CQ} = I_{EQ}$

$$V_{CEQ} = 5 = 10 - I_{CQ}(R_C + R_E)$$

$$\Rightarrow 5 = 10 - I_{CQ}(1.2\text{k} + 0.2\text{k}) \quad \Rightarrow \quad I_{CQ} = 3.57\ \text{mA}$$

$$I_{BQ} = \frac{3.57}{150} = 23.8\ \mu\text{A}$$

AC Analysis:

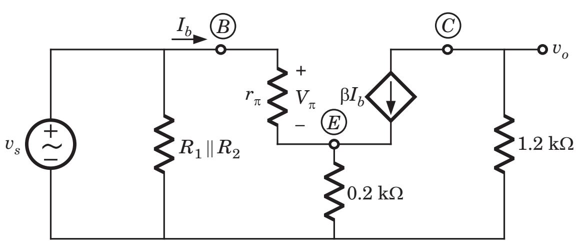

Fig. S3.4.8

$$r_\pi = \frac{\beta V_t}{I_{CQ}} = (150)\frac{(0.0259)}{3.57\text{m}} = 1.09\ \text{k}\Omega\ ,\ r_o = \infty$$

$$A_v = \frac{v_o}{v_s} = \frac{-(\beta I_b)R_C}{v_s},\ v_s = I_b r_\pi + (\beta + 1)R_E I_b$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = \frac{-(150)(1.2)\text{k}}{1.09\text{k} + (151)(0.2\text{k})} = -5.75$$

9. (A) DC Analysis: $V_{TH} = \frac{50}{10 + 50}(12) = 10\ \text{V}$

$$R_{TH} = 11 \,||\, 50 = 8.33\ \text{k}\Omega$$

$$I_{BQ} = \frac{12 - 0.7 - 10}{8.33\text{k} + (101)1\text{k}} = 11.9\ \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = 1.19\ \text{mA},\ I_{EQ} = 1.2\ \text{mA}$$

$$V_{ECQ} = 12 - (1.20)1 - (1.19)2 = 8.42\ \text{V}$$

AC Analysis:

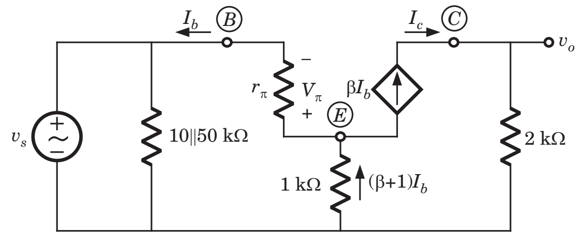

Fig. S3.4.9

$$r_\pi = \frac{\beta V_t}{I_{CQ}} = \frac{(100)(0.0259)}{1.19\text{m}} = 2.18\ \text{k}\Omega$$

$$v_o = \beta I_b(2\text{k})\ ,\ v_s = -(\beta + 1)I_b(1\text{k}) + I_b(r_\pi)$$

$$A_v = \frac{v_o}{v_s} = \frac{-\beta(2\text{k})}{r_\pi + (\beta + 1)1\text{k}} = \frac{-(100)(2\text{k})}{2.18\text{k} + (100)(1\text{k})} = -1.96$$

10. (B) $V_{ECQ} = 8.42$ V,

For $1 \le v_{EC} \le 11$ V, $\Delta v_{EC} = 11 - 8.42 = 2.58$ V

$\Rightarrow$ Output voltage swing $= 5.16$ V peak to peak.

11. (B) Since the B–C junction is not reverse biased, the transistor continues to operate in the forward-active -mode

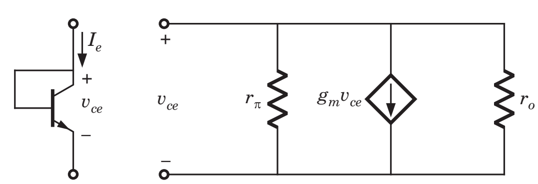

Fig. S 3.4.11

$$r = \frac{v_{ce}}{g_m V_{ce}} = \frac{1}{g_m}\ ,\ \text{So}\ r_\pi \,||\left(\frac{1}{g_m}\right)||\, r_o$$

$$r_\pi = \frac{(100)(0.0259)}{2\text{m}} = 2.33\ \text{k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_t} = \frac{2\text{m}}{0.0259} = 77.2\ \text{mA/V}$$

$$\frac{1}{g_m} = 12.95\ ,\ r_o = \frac{V_A}{I_{CQ}} = \frac{150}{2\text{m}} = 75\ \text{k}\Omega$$

$$r_e = (2.33\text{k}) \,||\, (12.95) \,||\, (75\text{k}) = 12.87\ \Omega$$

12. (C) $r_o = \dfrac{V_A}{I_{CQ}} \Rightarrow I_{CQ} = \dfrac{V_A}{r_o} = \dfrac{75}{200\text{k}} = 0.375 \text{ mA}$

13. (B) $r_\pi = \dfrac{\beta V_t}{I_{CQ}} = \dfrac{75(0.0259)}{1\text{m}} = 1.94 \text{ k}\Omega$

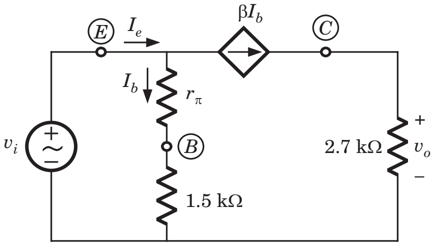

Fig. S 3.4.13

$$v_i = I_b(r_\pi + 1.5\text{k}),\ I_{in} = I_e = (\beta+1)I_b$$

$$R_{in} = \frac{V_i}{I_e} = \frac{(r_\pi + 1.5\text{k})}{(\beta+1)} = \frac{1.94 + 1.5\text{k}}{76} = 45\ \Omega$$

14. (D) The equivalent circuit is shown below

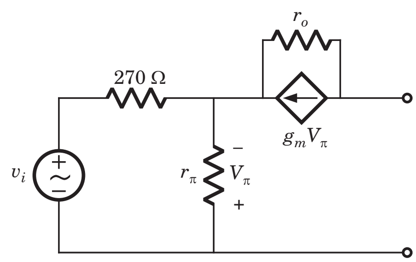

Fig. S 3.4.14

Removing the R_L , $-V_\pi = \dfrac{v_i r_\pi}{270 + r_\pi}$

$$v_{TH} = -r_o g_m V_\pi - V_\pi = \frac{v_i r_\pi(1 + g_m r_o)}{270 + r_\pi}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{2\text{m}} = 50 \text{ k}\Omega$$

$$v_{TH} = \frac{v_i 50\text{k}(1 + (2\text{m})(250\text{k}))}{270 + 50\text{k}} = 498\ v_i$$

15. (A) The equivalent small-signal circuit is shown in fig. S3.4.15

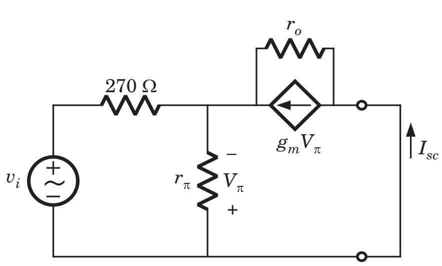

Fig. S 3.4.15

$$I_{sc} = g_m V_\pi + \frac{V_\pi}{r_o} = 2\text{m}V_\pi + \frac{V_\pi}{250\text{k}} = 2.004 \text{ m}V_\pi$$

$$\frac{V_\pi}{r_\pi} + \frac{V_\pi}{r_o} + g_m V_\pi + \frac{v_i + V_{\pi i}}{270} = 0$$

$$\frac{V_\pi}{50\text{k}} + \frac{V_\pi}{250\text{k}} + 2\text{m}V_\pi + \frac{v_i + V_{pi}}{270} = 0 \Rightarrow V_\pi = -0.647 v_i\ ,$$

$$I_{sc} = 1.297 \text{ m}v_i$$

$$R_{TH} = \frac{v_{TH}}{I_{SC}} = \frac{498 v_i}{1.297 \text{m} v_i} = 384 \text{ k}\Omega$$

16. (B) The equivalent small-signal circuit is shown in fig. S3.4.16

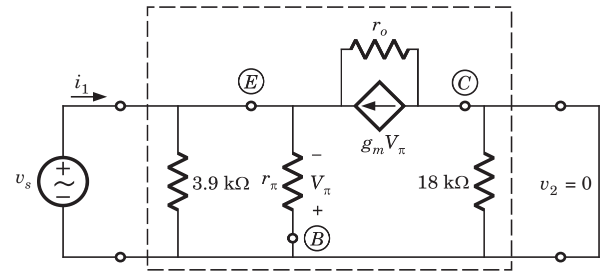

Fig. S 3.4.16

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{3\text{m}} = 33.3 \text{ k}\Omega$$

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0}\ ,\quad i_2 = \frac{V_\pi}{r_0} + g_m V_\pi$$

$$i_1 = -\frac{V_\pi}{3.9\text{k}} - \frac{V_\pi}{r_\pi} - \frac{V_\pi}{r_o} - g_m V_\pi\ ,\quad \frac{V_\pi}{r_o} \text{ can be neglected}$$

$$h_{21} = \frac{i_2}{i_1} = \frac{-g_m}{\frac{1}{3.9\text{k}} + \frac{1}{r_\pi} + g_m} = \frac{-g_m r_\pi 3.9\text{k}}{r_\pi + 3.9\text{k} + g_m r_\pi 3.9\text{k}} = 0.91$$

17. (A) $v_1 = -V_\pi\ ,\quad \dfrac{v_1}{3.9\text{k}} + \dfrac{v_1}{r_\pi} + \dfrac{v_1 - v_2}{r_o} = g_m V_\pi$

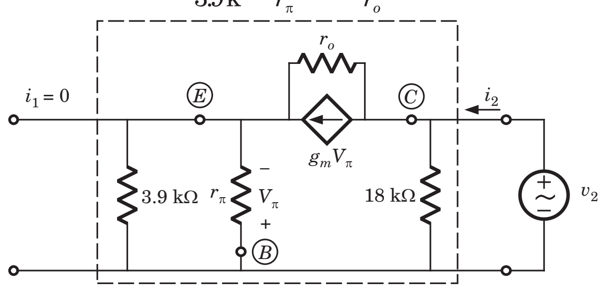

Fig. S 3.4.17

$$v_1\left(\frac{1}{3.9\text{k}} + \frac{1}{r_\pi} + \frac{1}{r_o}\right) - \frac{v_2}{r_o} = -g_m v_1$$

$$\frac{v_1}{v_2} = \frac{\frac{1}{r_o}}{\frac{1}{3.9\text{k}} + \frac{1}{r_\pi} + \frac{1}{r_o} + g_m} = \frac{\frac{1}{800\text{k}}}{\frac{1}{3.9\text{k}} + \frac{1}{33.3\text{k}} + \frac{1}{800\text{k}} + 3\text{m}}$$

$$= 3.8 \times 10^{-4}$$

$R_S = 4\ \text{k}\Omega,\ v_{gs} = 0.84 v_i$,

$v_o = -g_m v_{gs}(r_o \,||\, R_D)$

$= -(1.41\text{m})(0.84 v_i)(100\text{k} \,||\, 5\text{k})$

$\Rightarrow \quad \dfrac{v_o}{v_i} = A_v = -5.6$

28. (A) $R_o = R_D \,||\, r_o \,||\, 100\text{k} = 4.76\ \text{k}\Omega$

29. (A) As shown in fig. S3.4.27, $R_i = R_1 \,||\, R_2 = 20.6\ \text{k}\Omega$

30. (C) From the DC analysis:

$V_{GSQ} = 1.5\ \text{V},\ I_{DQ} = 0.5\ \text{mA}$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1\text{m})(1.5 - 0.8) = 1.4\ \text{mA/V}$

$r_o = [\lambda I_{DQ}]^{-1} = \infty$

The resulting small-signal equivalent circuit is shown in fig. S5.4.30

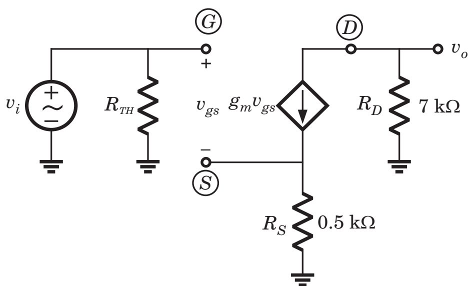

Fig. S 3.4.30

$v_o = -g_m v_{gs} R_D,\ v_i = v_{gs} + g_m v_{gs} R_S$

$$\Rightarrow \quad \frac{v_o}{v_i} = \frac{-g_m R_D}{1 + g_m\ R_S} = -(1.4\text{m})\frac{(7\text{k})}{1 + (1.4\text{m})(0.5\text{k})} = -5.76$$

31. (B) Since the DC gate current is zero, $V_S = -V_{GSQ}$

$I_{DQ} = I_Q = K_n(V_{GSQ} - V_{TN})^2$

$\Rightarrow \quad 0.5 = 1(V_{GSQ} - 0.8)^2$

$V_{GSQ} = 1.51\ \text{V} = -V_S$

$V_{DSQ} = 5 - (0.5\text{m})(7\text{k}) - (-1.51) = 3.01\ \text{V}$

The transistor is therefore biased in the saturation region. The small-signal equivalent circuit is shown in fig.S3.4.31.

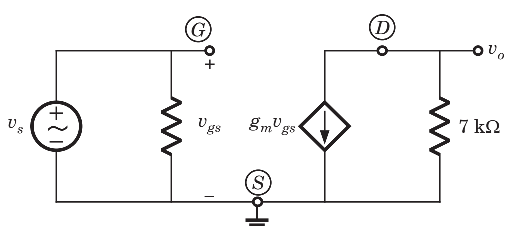

Fig. S3.4.31

$v_o = -g_m v_{gs}(7\text{k})$

$v_{gs} = v_i$, $\dfrac{v_o}{v_i} = A_v = -g_m(7\text{k})$

$g_m = 2K_n(V_{GS} - V_{TN})$

$= 2\,(1\text{m})(1.51 - 0.8) = 1.42\ \text{mS}$

$A_v = -(1.42\text{m})\,(7\text{k}) = -9.9$

32. (A) The small-signal equivalent circuit is shown in fig. S.3.4.34

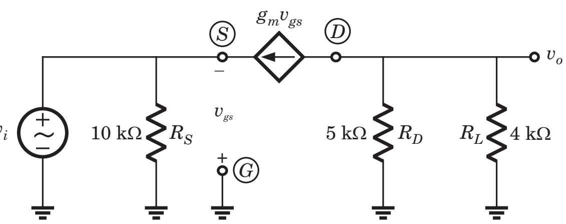

Fig. S3.4.32

$v_o = -g_m v_{gs}(R_D \,||\, R_L),\ v_i = -v_{gs}$

$A_v = \dfrac{v_o}{v_i} = g_m(R_D \,||\, R_L) = (2\text{m})(5\text{k} \,||\, 4\text{k}) = 4.44$

33. (A) The small-signal equivalent circuit is shown in fig. S3.4.33

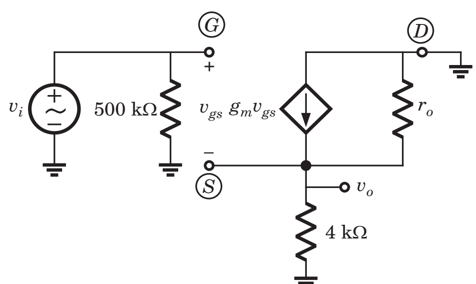

Fig. S3.4.33

$v_o = g_m v_{gs}(R_L \,||\, r_o)$

$v_i = v_{gs} + v_o = v_{gs} + g_m v_{gs}(R_L \,||\, r_o)$

$v_{gs} = \dfrac{1}{1 + g_m(R_L \,||\, r_o)}$

$\dfrac{v_o}{v_i} = A_v = \dfrac{g_m(R_L \,||\, r_o)}{1 + g_m(R_L \,||\, r_o)}$

$R_L \,||\, r_o = 4\text{k} \,||\, 100\text{k} = \dfrac{100\text{k}}{26} = 3.86\ \text{k}\Omega$

$A_v = \dfrac{(2\text{m})(3.85\text{k})}{1 + (2\text{m})(3.85\text{k})} = 0.89$

34. (B) $R_o = \dfrac{1}{g_m} \,||\, r_o$

$= \left(\dfrac{1}{2}\right) || (100) \approx 0.498\ \text{k}\Omega$

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CHAPTER

3.5

OPERATIONAL AMPLIFIERS

1. $A_v = \frac{v_o}{v_i} = ?$

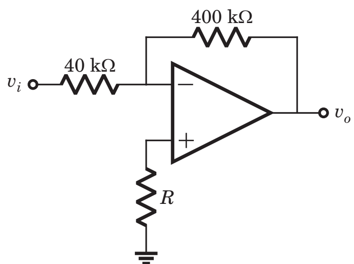


Fig. P3.5.1

(A) –10 (B) 10

(C) –11 (D) 11

2. $A_v = \frac{v_o}{v_i} = ?$

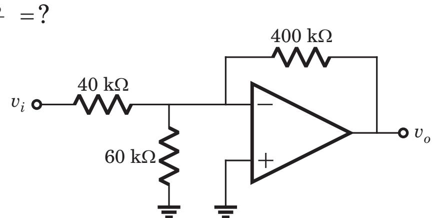


Fig. P3.5.2

(A) –10 (B) 10

(C) 13.46 (D) –13.46

3. The input to the circuit in fig. P3.5.3 is $v_i = 2 \sin \omega t$ mV. The current i_o is

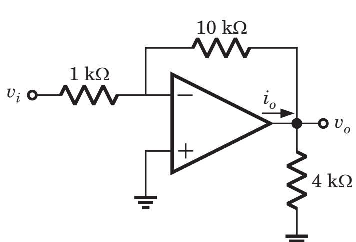


Fig. P3.5.3

(A) $-2 \sin \omega t$ μA (B) $-7 \sin \omega t$ μA

(C) $-5 \sin \omega t$ μA (D) 0

4. In circuit shown in fig. P3.5.4, the input voltage v_i is 0.2 V. The output voltage v_o is

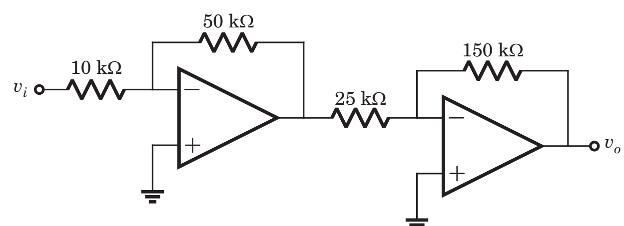


Fig. P3.5.4

(A) 6 V (B) –6 V

(C) 8 V (D) –8 V

5. For the circuit shown in fig. P3.5.5 gain is $A_v = v_o/v_i = -10$. The value of R is

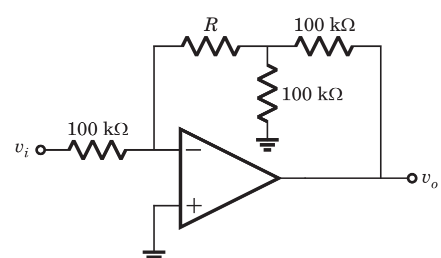


Fig. P3.5.5

(A) 600 kΩ (B) 450 kΩ

(C) 4.5 MΩ (D) 6 MΩ

6. For the op-amp circuit shown in fig. P3.5.6 the voltage gain $A_v = v_o/v_i$ is

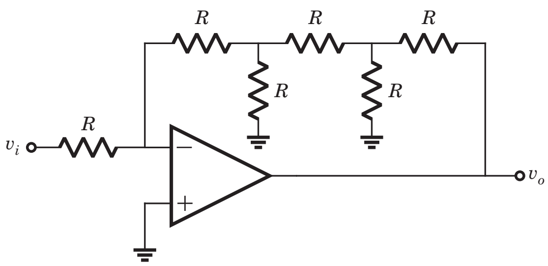


Fig. P3.5.6

(A) –8 (B) 8

(C) –10 (D) 10

7. For the op-amp shown in fig. P3.5.7 open loop differential gain is $A_{od} = 10^3$. The output voltage v_o for $v_i = 2$ V is

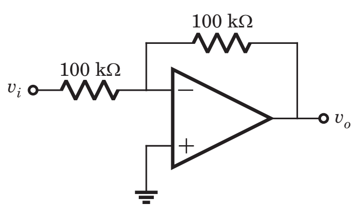


Fig. P3.5.7

(A) –1.996 (B) –1.998

(C) –2.004 (D) –2.006

8. The op-amp of fig. P3.5.8 has a very poor open-loop voltage gain of 45 but is otherwise ideal. The closed-loop gain of amplifier is

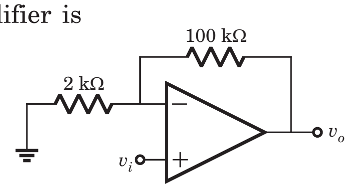


Fig. P3.5.8

(A) 20 (B) 4.5

(C) 4 (D) 5

9. For the circuit shown in fig. P3.5.9 the input voltage v_i is 1.5 V. The current i_o is

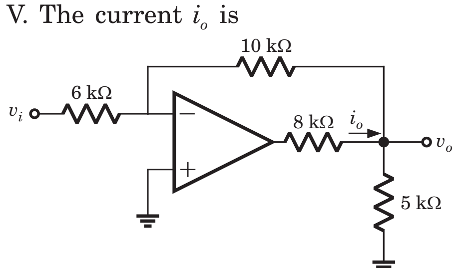


Fig. P3.5.9

(A) –1.5 mA (B) 1.5 mA

(C) –0.75 mA (D) 0.75 mA

10. In the circuit of fig. P3.5.10 the output voltage v_o is

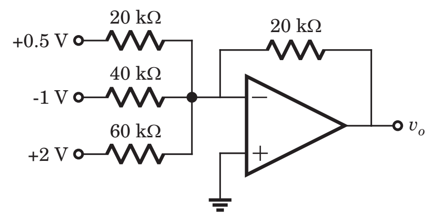


Fig. P3.5.10

(A) 2.67 V (B) –2.67 V

(C) –6.67 V (D) 6.67 V

11. In the circuit of fig. P3.5.11 the voltage v_{i1} is $(1 + 2\sin \omega t)$ mV and $v_{i2} = -10$ mV. The output voltage v_o is

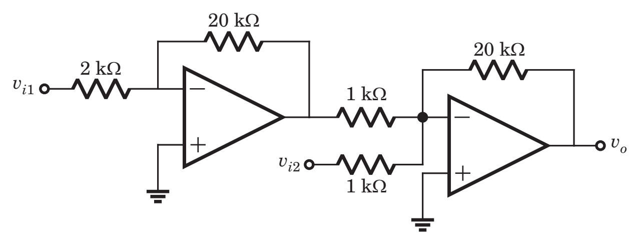


Fig. P3.5.11

(A) $-0.4(1 + \sin \omega t)$ mV (B) $0.4(1 + \sin \omega t)$ mV

(C) $0.4(1 + 2\sin \omega t)$ mV (D) $-0.4(1 + 2\sin \omega t)$ mV

12. For the circuit in fig. P3.5.12 the output voltage is $v_o = 2.5$ V in response to input voltage $v_i = 5$ V. The finite open-loop differential gain of the op-amp is

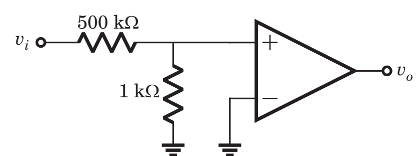


Fig. P3.5.12

(A) 5×10^4 (B) 250.5

(C) 2×10^4 (D) 501

13. $v_o = ?$

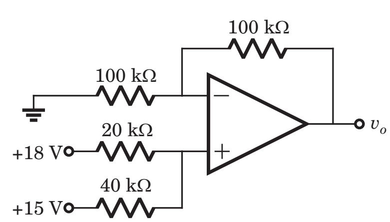


Fig. P3.5.13

(A) 34 V (B) –17 V

(C) 32 V (D) –32 V

28. For the circuit shown in fig. P3.5.28 the input resistance is

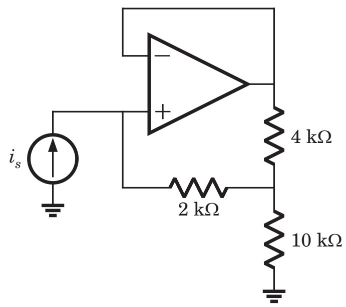


Fig. P3.5.28

(A) 38 kΩ (B) 17 kΩ

(C) 25 kΩ (D) 47 kΩ

29. In the circuit of fig. P3.5.29 the op-amp slew rate is $SR = 0.5$ V/μs. If the amplitude of input signal is 0.02 V, then the maximum frequency that may be used is

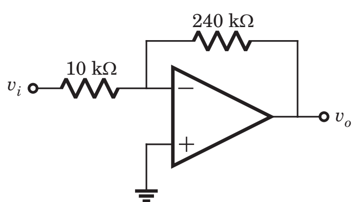


Fig. P3.5.29

(A) 0.55×10^6 rad/s (B) 0.55 rad/s

(C) 1.1×10^6 rad/s (D) 1.1 rad/s

30. In the circuit of fig. P3.5.30 the input offset voltage and input offset current are $V_{io} = 4$ mV and $I_{io} = 150$ nA. The total output offset voltage is

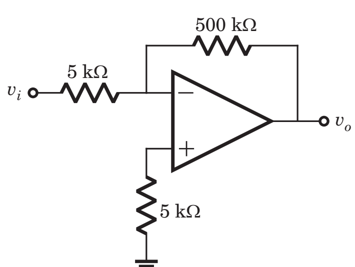


Fig. P3.5.30

(A) 479 mV (B) 234 mV

(C) 168 mV (D) 116 mV

31. $i_o = ?$

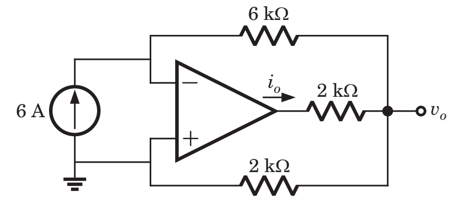


Fig. S3.5.31

(A) –18 A (B) 18 A

(C) –36 A (D) 36 A

Statement for Q.32–33:

Consider the circuit shown below

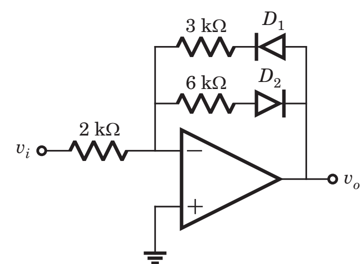


Fig. P3.5.32–33

32. If $v_i = 2$ V, then output v_o is

(A) 4 V (B) –4 V

(C) 3 V (D) –3 V

33. If $v_i = -2$ V, then output v_o is

(A) –6 V (B) 6 V

(C) –3 V (D) 3 V

34. $v_o(t) = ?$

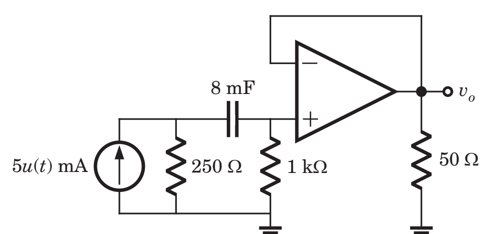


Fig. P3.5.34

(A) $e^{-\frac{t}{10}} u(t)$ V (B) $-e^{-\frac{t}{10}} u(t)$ V

(C) $e^{-\frac{t}{1.6}} u(t)$ V (D) $-e^{-\frac{t}{1.6}} u(t)$ V

35. The circuit shown in fig. P3.5.35 is at steady state before the switch opens at $t = 0$. The $v_C(t)$ for $t > 0$ is

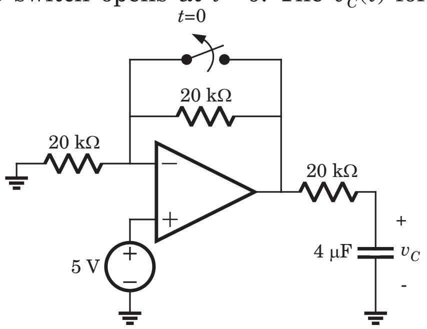


Fig. P3.5.35

(A) $10 - 5e^{-12.5t}$ V (B) $5 + 5e^{-12.5t}$ V

(C) $5 + 5e^{-\frac{t}{12.5}}$ V (D) $10 - 5e^{-\frac{t}{12.5}}$ V

36. The LED in the circuit of fig. P3.5.36 will be on if v_i is

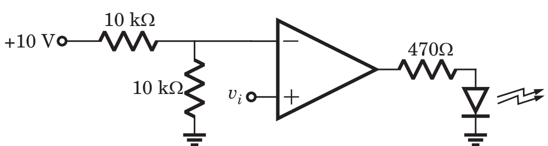


Fig. P3.5.36

(A) >10 V (B) <10 V

(C) >5 V (D) <5 V

37. In the circuit of fig. P3.5.37 the CMRR of the op-amp is 60 dB. The magnitude of the v_o is

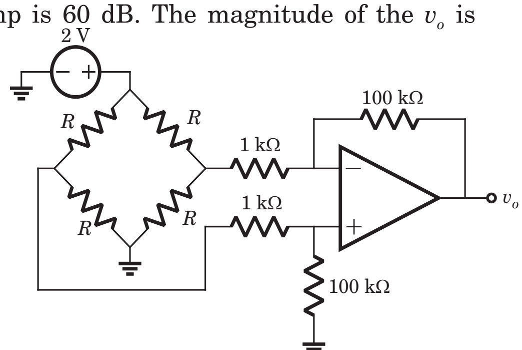


Fig. P3.5.37

(A) 1 mV (B) 100 mV

(C) 200 mV (D) 2 mV

38. The analog multiplier X of fig. P.3.5.38 has the characteristics $v_p = v_1 v_2$. The output of this circuit is

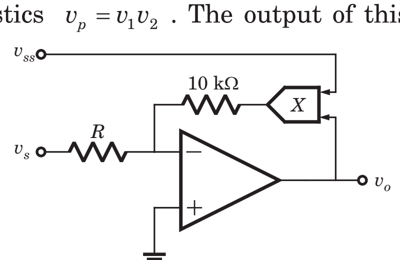


Fig. P3.5.38

(A) $v_s v_{ss}$ (B) $-v_s v_{ss}$

(C) $-\frac{v_s}{v_{ss}}$ (D) $\frac{v_s}{v_{ss}}$

39. If the input to the ideal comparator shown in fig. P3.5.39 is a sinusoidal signal of 8 V (peak to peak) without any DC component, then the output of the comparator has a duty cycle of

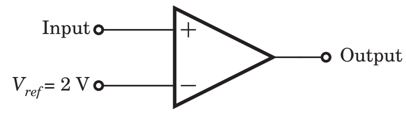


Fig. P3.5.39

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{12}$

40. In the op-amp circuit given in fig. P3.5.40 the load current i_L is

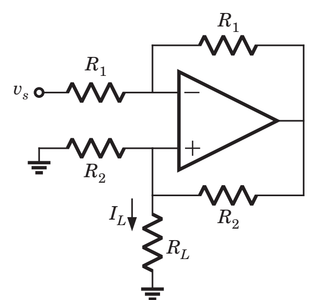


Fig. P3.5.40

(A) $-\frac{v_s}{R_2}$ (B) $\frac{v_s}{R_2}$

(C) $-\frac{v_s}{R_L}$ (D) $\frac{v_s}{R_L}$

41. In the circuit of fig. P3.5.41 output voltage is $|v_o| = 1$ V for a certain set of ω, R, an C. The $|v_o|$ will be 2 V if

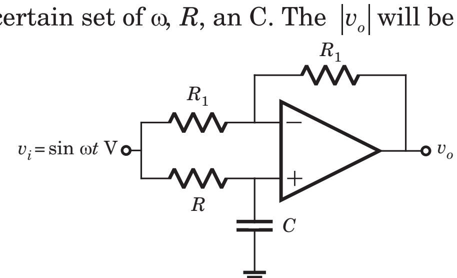


Fig. P3.5.41

(A) ω is doubled (B) ω is halved

(C) R is doubled (D) None of the above

42. In the circuit of fig. P3.5.42. the 3 dB cutoff frequency is

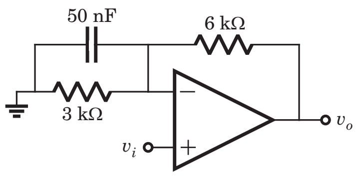


Fig. P3.5.42

(A) 10 kHz (B) 1.59 kHz

(C) 354 Hz (D) 689 Hz

43. The phase shift oscillator of fig. P3.5.43 operate at $f = 80$ kHz. The value of resistance R_F is

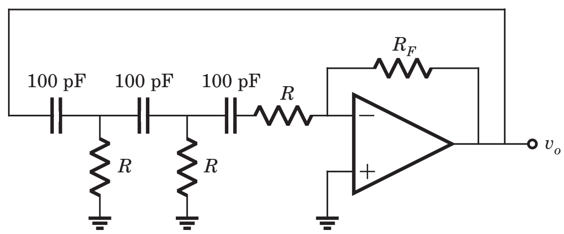


Fig. P3.5.43

(A) 148 kΩ (B) 236 kΩ

(C) 438 kΩ (D) 814 kΩ

44. The value of C required for sinusoidal oscillation of frequency 1 kHz in the circuit of fig. P3.5.44 is

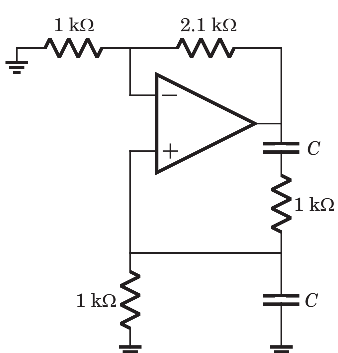


Fig. P3.5.44

(A) $\frac{1}{2\pi}$ μF (B) 2π μF

(C) $\frac{1}{2\pi\sqrt{6}}$ μF (D) $2\pi\sqrt{6}$ μF

45. In the circuit shown in fig. P3.5.45 the op-amp is ideal. If $\beta_F = 60$, then the total current supplied by the 15 V source is

(A) 123.1 mA (B) 98.3 mA

(C) 49.4 mA (D) 168 mA

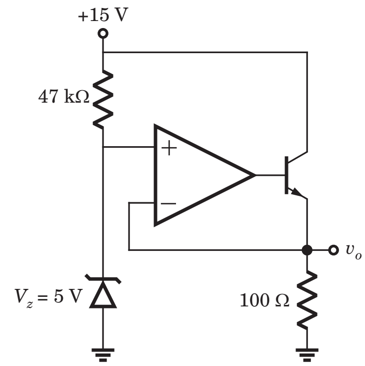


Fig. P3.5.45

46. In the circuit in fig. P3.5.46 both transistor Q_1 and Q_2 are identical. The output voltage at $T = 300$ K is

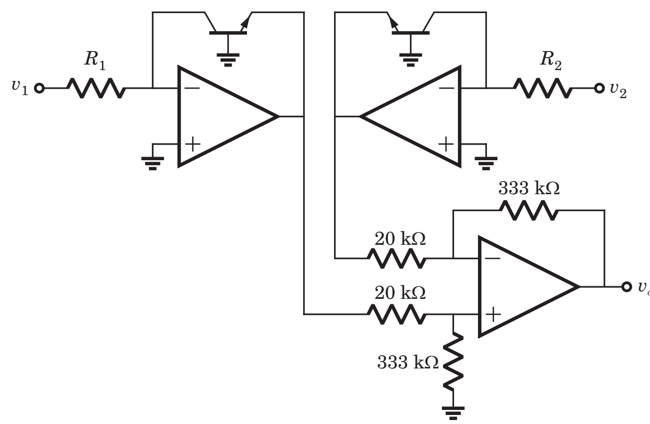


Fig. P3.5.46

(A) $2\log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$ (B) $\log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$

(C) $2.303\log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$ (D) $4.605\log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$

47. In the op-amp series regulator circuit of fig. P8.3.47 $V_z = 6.2$ V, $V_{BE} = 0.7$ V and $\beta = 60$. The output voltage v_o is

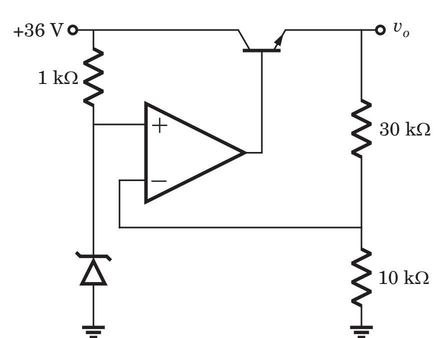


Fig. P3.5.47

(A) 35.8 V (B) 24.8 V

(C) 29.8 V (D) None of the above

****

26. (C) $v_{2+} = v_{2-} = 0$ V, current through 6 V source

$$i = \frac{6}{3k} = 2 \text{ mA}, \ v_o = -2m(3k + 2k) = -10 \text{ V}$$

27. (D) $v_+ = \frac{v_o(1)}{1+3} = \frac{v_o}{4}, \ v_- = \frac{v_i(2)}{2+1} + \frac{v_o(1)}{2+1}$

$$v_+ = v_- , \ \frac{v_o}{4} = \frac{v_o}{3} + \frac{2v_i}{3} , \ \frac{v_o}{v_i} = -8$$

28. (B) Since op-amp is ideal

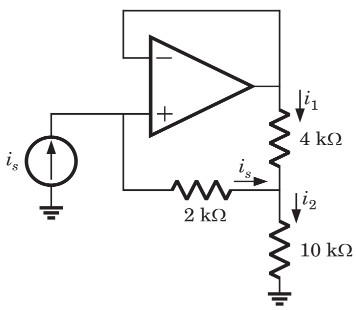


Fig. S3.5.28

$$v_- = v_+ , \ 2ki_s = 4ki_1 \quad \Rightarrow \quad i_s = 2i_1$$
$$v_s = 2ki_s + 10ki_2$$
$$i_2 = i_s + i_1 , \ v_s = 2ki_s + 10k(i_s + i_1), \ i_1 = \frac{i_s}{2}$$
$$v_s = 2ki_s + 10k\left(i_s + \frac{i_s}{2}\right) \quad \Rightarrow \quad \frac{v_s}{i_s} = 17k = R_{in}$$

29. (C) Closed loop gain $A = \left|\frac{R_F}{R_1}\right| = \frac{240k}{10k} = 24$

The maximum output voltage $v_{om} = 24 \times 0.02 = 0.48$ V

$$\omega \le \frac{SR}{v_{om}} = \frac{0.5/\mu}{0.48} = 1.1 \times 10^6 \text{ rad/s}$$

30. (A) The offset due to V_{io} is $v_o = \left(1 + \frac{R_1}{R_1}\right) V_{io}$

$$= \left(1 + \frac{500}{5}\right) 4m = 404 \text{ mV}$$

Due to I_{io}, $v_o = R_F I_{io} = (500k)(150n) = 75$ mV

Total offset voltage $v_o = 404 + 75 = 479$ mV

31. (A) $6 = \frac{-v_o}{6k} , \quad i_o = -6 + \frac{v_o}{3k}$

$$i_o = -6 + \frac{-6(6k)}{3k} = -18 \text{ A.}$$

32. (B) If $v_i > 0$, then $v_o < 0$, D_1 blocks and D_2 conducts

$$A_v = -\frac{6k}{3k} = -2 \quad \Rightarrow \quad v_o = (-2)(2) = -4 \text{ V}$$

33. (D) If $v_i < 0$, then $v_o > 0$, D_2 blocks and D_1 conduct

$$A_v = -\frac{3k}{2k} = -1.5, \ v_o = (-2)(-1.5) = 3 \text{ V}$$

34. (A) Voltage follower $v_o = v_- = v_+$

$$v_+(0^+) = 5m(250 \ || \ 1000) = 1 \text{ V}, \ v_+(\infty) = 0$$
$$\tau = 8m(1000 + 250) = 10 \text{ s}$$

35. (A) $v_c(0^-) = 5 \text{ V} = v_c(0^+) = 5$ V

For $t > 0$ the equivalent circuit is shown in fig. S3.5.35

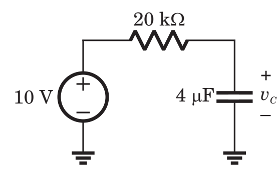


Fig. S3.5.35

$$\tau = 20k \times 4\mu = 0.08 \text{ s}$$
$$v_c = 10 + (5 - 10)e^{-\frac{t}{0.08}} = 10 - 5e^{-12.5t} \text{ V} \quad \text{for } t > 0$$

36. (C) $v_- = \frac{(10)(10k)}{10k + 10k} = 5$ V

When $v_+ > 5$ V, output will be positive and LED will be on. Hence (C) is correct.

37. (B) $v_+ = (2)\frac{R}{2R} = 1 \text{ V}, \ v_- = (2)\frac{R}{2R} = 1 \text{ V}, \ v_d = 0$

$$V_{CM} = \frac{v_+ + v_-}{2} = 1, \quad v_o = \frac{R_F}{1} \frac{V_{CM}}{CMRR}$$
$$CMRR = 60 \text{ dB} = 10^3 , \ v_o = \frac{100}{1} \frac{1}{10^3} = 100 \text{ mV}$$

38. (C) $v_+ = 0 = v_-$,

Let output of analog multiplier be v_p.

$$\frac{v_s}{R} = -\frac{v_p}{R} \quad \Rightarrow \quad v_s = -v_p , \ v_p = v_{ss}v_o$$
$$v_s = -v_{ss}v_o , \ v_o = -\frac{v_s}{v_{ss}}$$

39. (B) When $v_i > 2$ V, output is positive. When $v_i < 2$ V, output is negative.

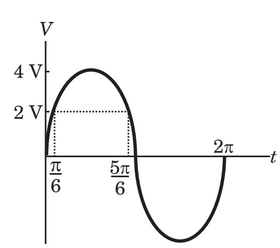


Fig. S3.5.39

$$\text{Duty cycle} = \frac{T_{ON}}{T} = \frac{\frac{5\pi}{6} - \frac{\pi}{6}}{2\pi} = \frac{1}{3}$$

40.(A) $\frac{v_s - v_-}{R_1} = \frac{v_- - v_o}{R_1} \Rightarrow 2v_1 = v_s + v_o$

$$\frac{v_+}{R_2} + \frac{v_+}{R_L} + \frac{v_+ - v_o}{R_2} = 0 \Rightarrow v_o = \left(2 + \frac{R_2}{R_L}\right)v_+$$

$$2v_- = v_s + \left(2 + \frac{R_2}{R_L}\right)v_+ \ , \ v_- = v_+$$

$$\Rightarrow 0 = v_s + \frac{R_2}{R_L}v_+$$

$$v_+ = -\frac{R_L}{R_2}v_s \ , \quad i_L = \frac{v_+}{R_L} \ , \quad i_L = -\frac{v_s}{R_2}$$

41. (D) This is a all pass circuit

$$\frac{v_o}{v_i} = H(j\omega) = \frac{1 - j\omega RC}{1 + j\omega RC} \ , \quad |H(j\omega)| = \frac{\sqrt{1 + (\omega R^2 C)^2}}{\sqrt{1 + (\omega RC)^2}} = 1$$

Thus when ω and R is changed, the transfer function is unchanged.

42. (B) Let $R_1 = 3\text{ k}\Omega$, $R_2 = 6\text{ k}\Omega$, $C = 50\text{ nF}$

$$\frac{v_i}{R_1 \,||\left(\frac{1}{sC}\right)} + \frac{v_i - v_o}{R_2} = 0 \Rightarrow \frac{v_i}{\left(\frac{R_1}{1 + sR_1C}\right)} + \frac{v_i}{R_2} = \frac{v_o}{R_2}$$

$$v_i\left[\frac{R_2}{R_1}(1 + sR_1C) + 1\right] = v_o$$

$$\frac{v_i}{R_1}\left[R_2 + R_1 + sR_1R_2C\right] = v_o$$

$$\frac{v_o}{v_i} = \frac{R_2 + R_1}{R_1}\left[1 + \frac{sR_1R_2C}{R_1 + R_2}\right]$$

$$\Rightarrow \frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right)\left(1 + s(R_1 \,||\, R_2)C\right)$$

$$f_{3dB} = \frac{1}{2\pi(R_1 \,||\, R_2)C}$$

$$= \frac{1}{2\pi(3\text{k} \,||\, 6\text{k})50\text{n}} = \frac{1}{2\pi(2\text{k})50\text{n}} = 1.59\text{ kHz}$$

43. (B) The oscillation frequency is

$$f = \frac{1}{2\pi\sqrt{6}RC} \Rightarrow 80\text{k} = \frac{1}{2\pi\sqrt{6}R(100\pi)}$$

$$\Rightarrow R = \frac{1}{(80\text{k})(2\pi\sqrt{6})(100\pi)} = 8.12\text{ k}\Omega$$

$$\frac{R_F}{R} = 29 \Rightarrow R_F = (8.12\text{k})(29) = 236\text{ k}\Omega$$

44. (A) This is Wien-bridge oscillator. The ratio $\frac{R_2}{R_1} = \frac{2.1\text{k}}{1\text{k}} = 2.1$ is greater than 2. So there will be oscillation

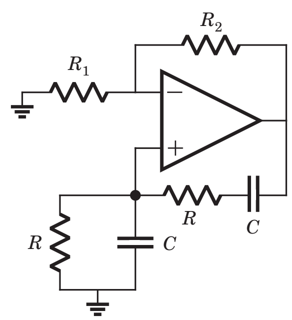


Fig. S3.5.44

$$\text{Frequency} = \frac{1}{2\pi RC} \Rightarrow 1 \times 10^3 = \frac{1}{2\pi(1\text{k})C}$$

$$C = \frac{1}{2\pi}\,\mu\text{F}$$

45. (C) $v_+ = 5\text{ V} = v_- = v_E$,

The input current to the op-amp is zero.

$$i_{+15V} = i_Z + i_C = i_Z + \alpha_F i_E$$

$$= \frac{15 - 5}{47\text{k}} + \frac{60}{61}\left(\frac{5}{100}\right) = 49.4\text{ mA}$$

46. (B) $v_o = \frac{333}{20}(v_{o1} - v_{o2})$

$$v_{o1} = -v_{BE1} - V_t \ln\left(\frac{i_{c1}}{i_s}\right), \ v_{o2} = -v_{BE2} - V_t \ln\left(\frac{i_{c2}}{i_s}\right)$$

$$v_{o1} - v_{o2} = -V_t \ln\left(\frac{i_{c1}}{i_{c2}}\right) = V_t \ln\left(\frac{i_{c2}}{i_{c1}}\right)$$

$$i_{c1} = \frac{v_1}{R_1} \ , \quad i_{c2} = \frac{v_2}{R_2}$$

$$v_{o1} - v_{o2} = V_t \ln\left(\frac{v_2}{R_2}\frac{R_1}{v_1}\right), \quad V_t = 0.0259\text{ V}$$

$$v_o = \frac{333}{20}(v_{o1} - v_{o2}) = \frac{333}{20}(0.0259)\ln\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$$

$$= 0.4329\ln\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right) = 0.4329(2.3026)\log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$$

$$= \log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$$

47. (B) $v_+ = v_-, \ v_Z = \frac{10v_o}{10 + 30} = \frac{v_o}{4}$

$$v_o = 4v_z = 6.2 \times 4 = 24.8\text{ V}$$

****

CHAPTER

4.1

NUMBER SYSTEMS & BOOLEAN ALGEBRA

1. The 100110_2 is numerically equivalent to

1. 26_{16} 2. 36_{10} 3. 46_8 4. 212_4

The correct answer are

(A) 1, 2, and 3 (B) 2, 3, and 4

(C) 1, 2, and 4 (D) 1, 3, and 4

2. If $(211)_x = (152)_8$, then the value of base x is

(A) 6 (B) 5

(C) 7 (D) 9

3. 11001, 1001 and 111001 correspond to the 2's complement representation of the following set of numbers

(A) 25, 9 and 57 respectively

(B) –6, –6 and –6 respectively

(C) –7, –7 and –7 respectively

(D) –25, –9 and –57 respectively

4. A signed integer has been stored in a byte using 2's complement format. We wish to store the same integer in 16-bit word. We should copy the original byte to the less significant byte of the word and fill the more significant byte with

(A) 0

(B) 1

(C) equal to the MSB of the original byte

(D) complement of the MSB of the original byte.

5. A computer has the following negative numbers stored in binary form as shown. The wrongly stored number is

(A) –37 as 1101 1011 (B) –89 as 1010 0111

(C) –48 as 1110 1000 (D) –32 as 1110 0000

6. Consider the signed binary number $A = 01010110$ and $B = 1110\,1100$ where B is the 1's complement and MSB is the sign bit. In list-I operation is given, and in list-II resultant binary number is given.

List–I	List-II
P. $A + B$	1. 0 1 0 0 0 0 1 1
	2. 0 1 1 0 1 0 0 1
Q. $B - A$	3. 0 1 0 0 0 0 1 0
	4. 1 0 0 1 0 1 0 1
R. $A - B$	5. 1 0 1 1 1 1 0 0
	6. 1 0 0 1 0 1 1 0
S. $-A - B$	7. 1 0 1 1 1 1 0 1
	8. 0 1 1 0 1 0 1 0

The correct match is

	P	Q	R	S
(A)	3	4	2	5
(B)	3	6	8	7
(C)	1	4	8	7
(D)	1	6	2	5

28. The simplified form of a logic function $Y = A(B + C\overline{(AB + AC)})$ is

(A) $\overline{A}\,\overline{B}$ (B) AB

(C) $\overline{A}B$ (D) $A\overline{B}$

29. The reduced form of the Boolean expression of $Y = \overline{(A\overline{B})}\cdot\overline{(\overline{A}B)}$ is

(A) $A + B$ (B) $\overline{A} + \overline{B}$

(C) $A\overline{B} + \overline{A}B$ (D) $\overline{A}\,\overline{B} + AB$

30. If $X\overline{Y} + \overline{X}Y = Z$ then $X\overline{Z} + \overline{X}Z$ is equal to

(A) $\overline{Y}$ (B) Y

(C) 0 (D) 1

31. If $XY = 0$ then $X \oplus Y$ is equal to

(A) $X + Y$ (B) $\overline{X} + \overline{Y}$

(C) XY (D) $\overline{X}\,\overline{Y}$

32. From a four-input OR gate the number of input condition, that will produce HIGH output are

(A) 1 (B) 3

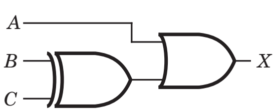
(C) 15 (D) 0

33. A logic circuit control the passage of a signal according to the following requirements :

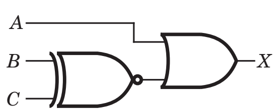
1. Output X will equal A when control input B and C are the same.

2. X will remain HIGH when B and C are different.

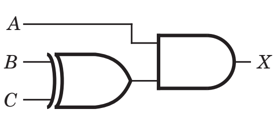
The logic circuit would be



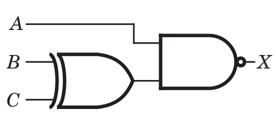
(A)



(B)

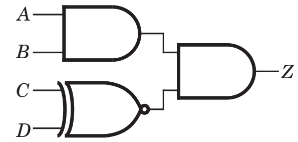


(C)

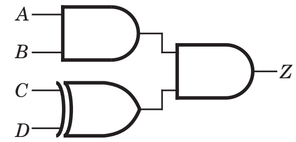


(D)

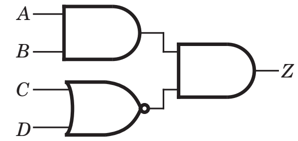
34. The output of logic circuit is HIGH whenever A and B are both HIGH as long as C and D are either both LOW or both HIGH. The logic circuit is



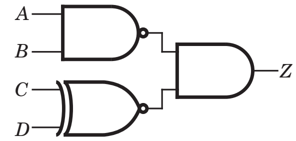
(A)



(B)



(C)



(D)

35. In fig. P.4.1.35 the input condition, needed to produce $X = 1$, is

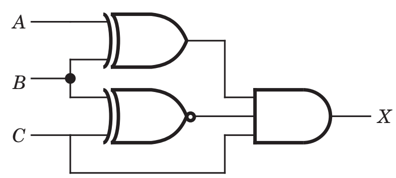


Fig. P4.1.34

(A) $A = 1,\ B = 1,\ C = 0$ (B) $A = 1,\ B = 1,\ C = 1$

(C) $A = 0,\ B = 1,\ C = 1$ (D) $A = 1,\ B = 0,\ C = 0$

36. Consider the statements below:

1. If the output waveform from an OR gate is the same as the waveform at one of its inputs, the other input is being held permanently LOW.

2. If the output waveform from an OR gate is always HIGH, one of its input is being held permanently HIGH.

The statement, which is always true, is

(A) Both 1 and 2 (B) Only 1

(C) Only 2 (D) None of the above

37. To implement $y = ABCD$ using only two-input NAND gates, minimum number of requirement of gate is

(A) 3 (B) 4

(C) 5 (D) 6

38. If the X and Y logic inputs are available and their complements $\overline{X}$ and $\overline{Y}$ are not available, the minimum number of two-input NAND required to implement $X \oplus Y$ is

(A) 4 (B) 5

(C) 6 (D) 7

Statement for Q.39–40:

A Boolean function $Z = A\overline{B}C$ is to be implement using NAND and NOR gate. Each gate has unit cost. Only A, B and C are available.

39. If both gate are available then minimum cost is

(A) 2 units (B) 3 units

(C) 4 units (D) 6 units

40. If NAND gate are available then minimum cost is

(A) 2 units (B) 3 units

(C) 4 units (D) 6 units

41. In fig. P4.1.41 the LED emits light when

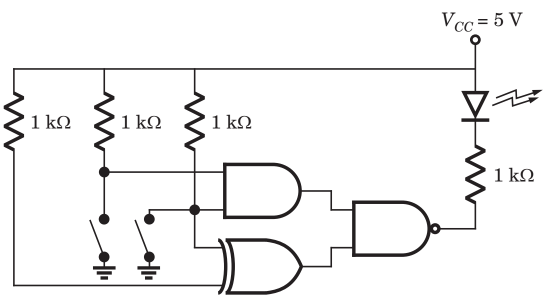


Fig. P4.1.41

(A) both switch are closed

(B) both switch are open

(C) only one switch is closed

(D) LED does not emit light irrespective of the switch positions

42. If the input to the digital circuit shown in fig. P.4.1.42 consisting of a cascade of 20 XOR gates is X, then the output Y is equal to

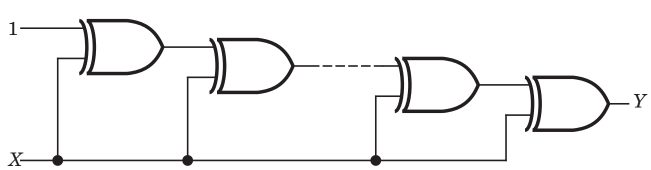


Fig. P4.1.42

(A) X (B) $\overline{X}$

(C) 0 (D) 1

43. A Boolean function of two variables x and y is defined as follows :

$$f(0,0) = f(0,1) = f(1,1) = 1;\ f(1,0) = 0$$

Assuming complements of x and y are not available, a minimum cost solution for realizing f using 2-input NOR gates and 2-input OR gates (each having unit cost) would have a total cost of

(A) 1 units (B) 2 units

(C) 3 units (D) 4 units

44. The gates G_1 and G_2 in Fig. P.4.2.44 have propagation delays of 10 ns and 20 ns respectively.

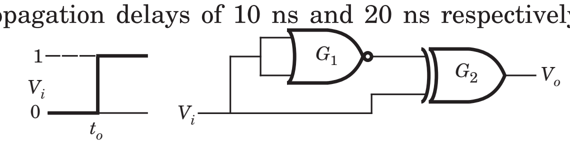
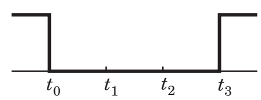
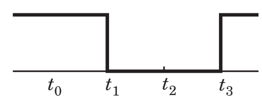


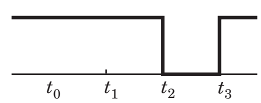
Fig. P4.1.44

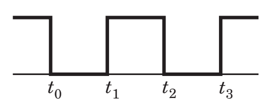
If the input V_i makes an abrupt change from logic 0 to 1 at $t = t_0$ then the output waveform V_o is

$[t_1 = t_0 + 10 \text{ ns},\ t_2 = t_1 + 10 \text{ ns},\ t_3 = t_2 + 10 \text{ ns}]$

(A)


(B)


(C)


(D)


45. In the network of fig. P4.1.45 f can be written as

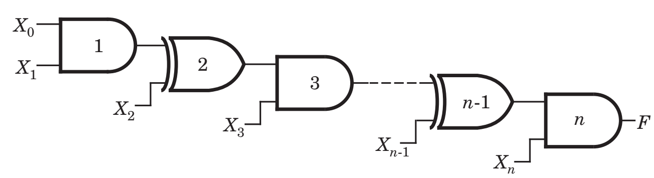


Fig. P4.1.45

(A) $X_0X_1X_3X_5 + X_2X_4X_5 \dots X_{n-1} + \dots X_{n-1}X_n$

(B) $X_0X_1X_3X_5 + X_2X_3X_4 \dots X_n + \dots X_{n-1}X_n$

(C) $X_0X_1X_3X_5 \dots X_n + X_2X_3X_5 \dots X_n + \dots + X_{n-1}X_n$

(D) $X_0X_1X_3X_5 \dots X_{n-1} + X_2X_3X_5 \dots X_n + \dots + X_{n-1}X_{n-2} + X_n$

****

SOLUTIONS

1. (D) $100110_2 = 2^5 + 2^2 + 2^1 = 38_{10}$

$26_{16} = 2 \times 16 + 6 = 38_{10}$

$46_8 = 4 \times 8 + 6 = 38_{10}$

$212_4 = 2 \times 4^2 + 4^1 = 38_{10}$

So 36_{10} is not equivalent.

2. (C) $2x^2 + x + 1 = 64 + 5 \times 8 + 2 \quad \Rightarrow \quad x = 7$

3. (C) All are 2's complement of 7

$$\begin{array}{lcrl} 11001 & \Rightarrow & 00110 & \\ & & +\quad 1 & \\ \hline & & 00111 & = 7_{10} \\ 1001 & \Rightarrow & 0110 & \\ & & +\quad 1 & \\ \hline & & 0111 & = 7_{10} \\ 111001 & \Rightarrow & 000110 & \\ & & +\quad 1 & \\ \hline & & 000111 & = 7_{10} \end{array}$$

4. (C) See a example

42 in a byte	0 0 1 0 1 0 1 0
42in a word	0 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0
–42 in a byte	1 1 0 1 0 1 1 0
–42 in a word	1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 0

Therefore (C) is correct.

5. (C)

$$\begin{array}{rr} 48_{10} = & 00110000_2 \\ -48_{10} = & 1100\,1111 \\ & +\qquad 1 \\ \hline & 11010000 \end{array}$$

6. (D) Here $\overline{A}$, $\overline{B}$ are 1's complement

$$\begin{array}{llr} A + B, & A & 01010110 \\ & B & +\ 1110\,1100 \\ \hline & & 10100\,0010, \\ & & +\qquad 1 \\ \hline & & 0100\,0011 \end{array}$$

$$\begin{array}{llr} B - A = B + \overline{A}, & B & 1110\,1100 \\ & \overline{A} & +\ 1010\,1001 \\ \hline & & 110010101 \\ & & +\qquad 1 \\ \hline & & 10010110 \end{array}$$

$$\begin{array}{llr} A - B = A + \overline{B}, & A & 010\,10110 \\ & \overline{B} & +\ 00010011 \\ \hline & & 0110\,1001 \end{array}$$

$$\begin{array}{llr} -A - B = \overline{A} + \overline{B}, & \overline{A} & 1010\,1001 \\ & \overline{B} & +\ 00010011 \\ \hline & & 10111100 \end{array}$$

7. (B) Here $\overline{A}$, $\overline{B}$ are 2's complement

$$\begin{array}{llr} A + B, & A & 0100\,0110 \\ & B & +\ 1101\,0011 \\ \hline & & 1\ 0001\,1001 \end{array}$$

Discard the carry 1

$$\begin{array}{llr} A - B = A + \overline{B}, & A & 0100\,0110 \\ & \overline{B} & +\ 0010\,1101 \\ \hline & & 0111\,0011 \end{array}$$

$$\begin{array}{llr} B - A, & B & 1101\,0011 \\ & \overline{A} & +\ 1011\,1010 \\ \hline & & 1\ 1000\,1101 \end{array}$$

Discard the carry 1

$$\begin{array}{llr} -A - B = \overline{A} + \overline{B}, & \overline{A} & 1011\,1010 \\ & \overline{B} & +\ 0010\,1101 \\ \hline & & 1110\,0111 \end{array}$$

8. (B) $11_{10} = 1011_2$

0.3	$2F_{i-1}$	B_i	F_i
	0.6	0	0.6
	1.2	1	0.2
	0.4	0	0.4
	0.8	0	0.8
	1.6	1	0.6

Repeat from the second line $0.3_{10} = 0.0\overline{1001}_2$

9. (C)

	b_4	b_3	b_2	p_3	b_1	p_2	p_1
Received	1	1	0	1	1	0	0

$C_1^* = b_4 \oplus b_2 \oplus b_1 \oplus p_1 = 0$

$C_2^* = b_4 \oplus b_3 \oplus b_1 \oplus p_2 = 1$

$C_3^* = b_4 \oplus b_3 \oplus b_2 \oplus p_3 = 1$

$C_3^* C_2^* C_1^* = 110$ which indicate position 6 in error Transmitted code 1001100.

10. (D) $X = MNQ + M\overline{N}Q + \overline{M}NQ$

$= MQ + \overline{M}NQ = Q(M + \overline{M}N) = Q(M + N)$

11. (A) The logic circuit can be modified as shown in fig. S. 4.1.11

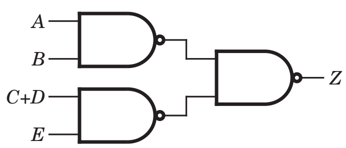


Fig. S4.1.11

Now $Z = AB + (C + D)E$

12. (D) You can see that input to last XNOR gate is same. So output will be HIGH.

13. (D) $Z = \overline{A} + (\overline{AB} + \overline{BC}) + \overline{C}$

$= \overline{A} + (\overline{A} + \overline{B} + \overline{B} + \overline{C}) + \overline{C} = \overline{A} + \overline{B} + \overline{C}$

$\overline{ABC} = \overline{A} + \overline{B} + \overline{C}$

$\overline{AB} + \overline{BC} + \overline{AC} = \overline{A} + \overline{B} + \overline{B} + \overline{C} + \overline{A} + \overline{C} = \overline{A} + \overline{B} + \overline{C}$

14. (C) $(X + \overline{Y})(\overline{X} + Y) = XY + \overline{X}\,\overline{Y}$

$(X + Y)(X + \overline{Y})(\overline{X} + Y) = (X + Y)(XY + \overline{X}\,\overline{Y})$

$= XY + XY = XY$

15. (B) Using duality

$(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$

Thus (B) is correct option.

16. (B) $Z = \overline{(\overline{AB})(\overline{CD})(\overline{EF})} = AB + CD + EF$

17. (A) $X = (A\,\overline{B} + \overline{A}B)\overline{(A + \overline{B})} = (A\overline{B} + \overline{A}\,B)(\overline{A}B) = \overline{A}B$

18. (B) $Y = \overline{(A \oplus B) \cdot C} = \overline{(A\overline{B} + \overline{A}C) \cdot C}$

$= \overline{(A\overline{B} + \overline{A}B)} + \overline{C} = \overline{A}\,\overline{B} + AB + \overline{C}$

19. (C) $Z = A(A + \overline{A})BC = ABC$

20. (A) $Z = AB(\overline{B} + C) = ABC$

21. (A) $Z = \overline{(\overline{A} + \overline{B})} \cdot BC = (AB) \cdot BC = ABC$

22. (A) $A(A + \overline{B})(A + \overline{B} + C)$

$= (AA + A\overline{B})(A + \overline{B} + C) = A(A + \overline{B} + C) = A$

Therefore No gate is required to implement this function.

23. (A)

A	B	C	$(A + BC)$	$(A + B)(A + C)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1

Fig. S 4.1.23

24. (B) $\overline{X} = \overline{A}\overline{B}C + A\overline{B}C + AB\overline{C} = \overline{B}C + AB\overline{C}$

25. (B) $\overline{(A + B)}\,\overline{(B + \overline{C})} = (\overline{A}\overline{B})(\overline{B}C) = \overline{A}\overline{B}C$

$\overline{\overline{(A + B)}\,\overline{(B + \overline{C})}} = (A + B) + (B + \overline{C}) = A + B + \overline{C}$

$\overline{(\overline{A} + B)\overline{(B + C)}} = \overline{(\overline{A} + B)} + (B + C)$

$= A\overline{B} + B + C = A + B + C$

From truth table $Z = A + B + \overline{C}$

Thus (B) is correct.

26. (D) $AC + B\overline{C} = AC(B + \overline{B}) + (A + \overline{A})B\overline{C}$

$= ABC + A\overline{B}C + AB\overline{C} + \overline{A}B\overline{C}$

27. (D) $F = A + \overline{A}B + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}(D + \overline{D}E)$

$= A + \overline{A}B + \overline{A}\,\overline{B}(C + \overline{C}(D + E))$

$= A + \overline{A}(B + \overline{B}(C + D + E)) = A + B + C + D + E$

28. (B) $A(B + C\,\overline{(AB + AC)}) = AB + AC\,(\overline{AB} \cdot \overline{AC})$

$= AB + AC[(\overline{A} + \overline{B})(\overline{A} + \overline{C})]$

$= AB + AC\,(\overline{A} + \overline{A}\overline{C} + \overline{A}\overline{B} + \overline{B}\,\overline{C}) = AB$

29. (C) $\overline{\overline{(A\overline{B})} \cdot \overline{(\overline{A}B)}} = \overline{\overline{A\overline{B}}} + \overline{\overline{\overline{A}B}} = A\overline{B} + \overline{A}B$

30. (B) $X\overline{Z} + \overline{X}Z = X\overline{(X\overline{Y} + \overline{X}Y)} + \overline{X}(X\overline{Y} + \overline{X}Y)$

$= X(XY + \overline{X}\,\overline{Y}) + \overline{X}Y = XY + \overline{X}Y = Y$

31. (A) $X \oplus Y = X\overline{Y} + \overline{X}Y = \overline{(XY + \overline{X}\overline{Y})} = \overline{(\overline{X}\overline{Y})} = X + Y$

(A) $(\overline{w} + y)(x + \overline{y} + z)(w + x + z)$

(B) $(w + \overline{x})(w + \overline{z})(\overline{x} + \overline{y})(\overline{y} + \overline{z})$

(C) $(x + z)(\overline{w} + y)$

(D) $(\overline{x} + \overline{z})(w + \overline{y})$

7. A function with don't care condition is as follows

$$f(a, b, c, d) = \Sigma m(0, 2, 3, 5, 7, 8, 9, 10, 11) + \Sigma dc(4, 15)$$

The minimized expression for this function is

(A) $a\overline{b} + \overline{b}\,\overline{d} + cd + \overline{a}b\overline{c}$ (B) $a\overline{b} + \overline{b}\,\overline{d} + cd + \overline{a}b\overline{d}$

(C) $a\overline{b} + \overline{b}\,\overline{d} + \overline{b}c + \overline{a}bd$ (D) Above all

8. A function with don't cares is as follows :

$$g(X, Y, Z) = \Sigma m(5, 6) + \Sigma dc(1, 2, 4)$$

For above function consider following expression

1. $XY\overline{Z} + X\overline{Y}Z$
2. $X\overline{Y} + X\overline{Z}$
3. $X\overline{Z} + \overline{X}Z + \overline{Y}Z$
4. $Y\overline{Z} + \overline{Y}Z$

The solution for g are

(A) 1, 2, and 3 (B) 1, 2, and 4

(C) 1, and 4 (D) 1, and 3

9. A logical function of four variable is given as

$$f(A, B, C, D) = (\overline{A} + B\,C)(B + CD)$$

The function as a sum of product is

(A) $\overline{A} + BC + A\overline{C}D + BCD$

(B) $\overline{A} + BC + \overline{A}CD + BCD$

(C) $\overline{A}B + BC + \overline{A}CD + BCD$

(D) $AB + A\overline{B} + A\overline{C}D + BCD$

10. A combinational circuit has input A, B, and C and its K-map is as shown in fig. P4.2.10. The output of the circuit is given by

A \ CD	00	01	11	10
00		1		1
01	1		1	

Fig. P4.2.1

(A) $(\overline{A}B + A\overline{B})\overline{C}$ (B) $(AB + \overline{A}\,\overline{B})\overline{C}$

(C) $\overline{A}\,\overline{B}\,\overline{C}$ (D) $A \oplus B \oplus C$

11. The Boolean Expression $Y = (A + B)(\overline{A} + C)$ is equal to

(A) $AC + \overline{A}B$ (B) $AC + \overline{A}B + BC$

(C) $\overline{A}B + BC + A\overline{B}C$ (D) Above all

12. In the logic circuit of fig. P4.2.12 the redundant gate is

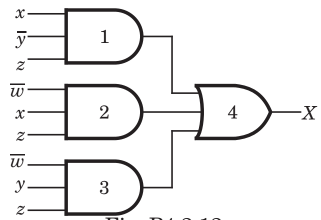


Fig. P4.2.12

(A) 1 (B) 2

(C) 3 (D) 4

13. If function W, X, Y, and Z are as follow

$$W = R + \overline{P}Q + \overline{R}S$$

$$X = PQ\overline{R}\,\overline{S} + \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S} + P\overline{Q}\,\overline{R}\,\overline{S}$$

$$Y = RS + \overline{PR + P\overline{Q} + \overline{P}\,\overline{Q}}$$

$$Z = R + S + \overline{PQ + \overline{P}\,\overline{Q}\,\overline{R} + P\overline{Q}\,\overline{S}}$$

Then

(A) $W = Z,\ X = \overline{Z}$ (B) $W = Z,\ X = Y$

(C) $W = Y$ (D) $W = Y = \overline{Z}$

14. In a certain application four inputs A, B, C, D are fed to logic circuit, producing an output which operates a relay. The relay turns on when $f(A, B, C, D) = 1$ for the following states of the inputs ($ABCD$) : 0000, 0010, 0101, 0110, 1101 and 1110. States 1000 and 1001 do not occur, and for the remaining states the relay is off. The minimized Boolean expression f is

(A) $\overline{A}C\overline{D} + BC\overline{D} + B\overline{C}D$ (B) $\overline{B}\,\overline{C}D + BC\overline{D} + \overline{A}C\overline{D}$

(C) $ABD + \overline{B}C\overline{D} + \overline{B}\,\overline{C}D$ (D) $\overline{A}\,\overline{B}\,\overline{D} + B\overline{C}D + BC\overline{D}$

15. There are four Boolean variables x_1, x_2, x_3 and x_4. The following function are defined on sets of them

$$f(x_3, x_2, x_1) = \Sigma m(3, 4, 5)$$

$$g(x_4, x_3, x_2) = \Sigma m(1, 6, 7)$$

$$h(x_4, x_3, x_2, x_1) = fg$$

Then $h(x_4, x_3, x_2, x_1)$ is

(A) $\Sigma m(3,\ 12,\ 13)$ (B) $\Sigma m(3,\ 6)$

(C) $\Sigma m(3,\ 12)$ (D) 0

Statement for Q.16–17:

A switching function of four variable, $f(w, x\ y, z)$ is to equal the product of two other function f_1 and f_2, of the same variable $f = f_1 f_2$. The function f and f_1 are as follows :

$$f = \Sigma m(4, 7, 15)$$

$$f_1 = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9, 10, 11, 15)$$

16. The number of full specified function, that will satisfy the given condition, is

(A) 32 (B) 16

(C) 4 (D) 1

17. The simplest function for f_2 is

(A) x (B) $\bar{x}$

(C) y (D) $\bar{y}$

18. A four-variable switching function has minterms m_6 and m_9. If the literals in these minterms are complemented, the corresponding minterm numbers are

(A) m_3 and m_0 (B) m_9 and m_6

(C) m_2 and m_0 (D) m_6 and m_9

19. The minimum function that can detect a "divisible by 3" 8421 BCD code digit (representation $D_8 D_4 D_2 D_1$) is given by

(A) $D_8 D_1 + D_4 D_2 + D_8 D_2 D_1$

(B) $D_8 D_1 + D_4 D_2 \overline{D_1} + \overline{D_4} D_2 D_1 + \overline{D_8}\,\overline{D_4}\,\overline{D_2}\,\overline{D_1}$

(C) $D_4 D_1 + D_4 D_2 + D_8 \overline{D_1}\,\overline{D_2} D_1$

(D) $D_4 D_2 \overline{D_1} + \overline{D_4} D_2 D_1 + D_8 D_4 D_2 D_1$

20. $f(x_2, x_1, x_0) = ?$

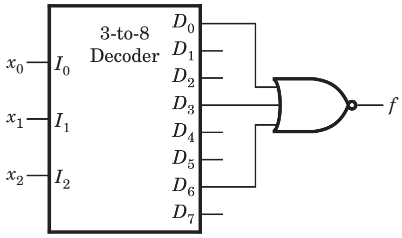


Fig. P4.2.21

(A) $\Pi(1, 2, 4, 5, 7)$ (B) $\Sigma(1, 2, 4, 5, 7)$

(C) $\Sigma(0, 3, 6)$ (D) None of Above

21. For a binary half subtractor having two input A and B, the correct set of logical expressions for the outputs $D = (A - B)$ and X (borrow) are

(A) $D = AB + \bar{A}\bar{B},\ X = \bar{A}B$

(B) $D = \bar{A}B + A\bar{B},\ X = A\bar{B}$

(C) $D = \bar{A}B + A\bar{B},\ X = \bar{A}B$

(D) $D = AB + \bar{A}\bar{B},\ X = A\bar{B}$

22. $f_1 f_2 = ?$

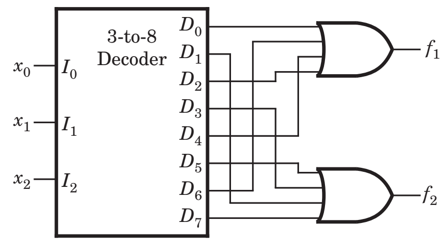


Fig. P4.2.22

(A) $x_0 x_1 x_2$ (B) $x_0 \oplus x_1 \oplus x_2$

(C) 1 (D) 0

23. The logic circuit shown in fig. P4.2.23 implements

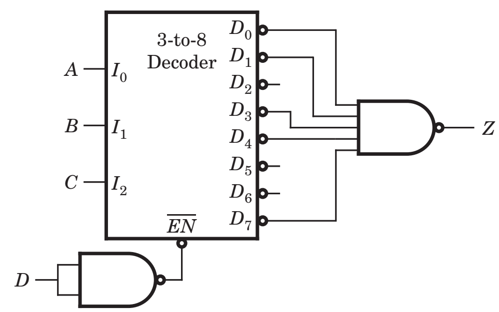


Fig. P4.2.23

(A) $D(A \text{ u } C + \bar{A}\bar{C})$ (B) $D(B \oplus C + \bar{A}\bar{C})$

(C) $D(B \oplus C + \bar{A}\bar{B})$ (D) $D(B \text{ u } C + \bar{A}\bar{B})$

Statement for Q.24–25:

The building block shown in fig. P4.2.24–25 is a active high output decoder.

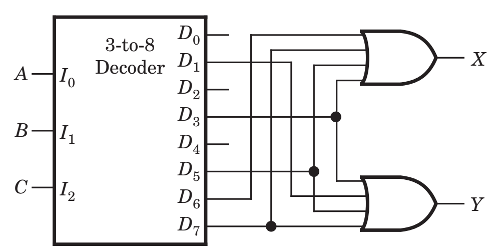


Fig. P4.2.24-25

24. The output X is

(A) $AB + BC + CA$ (B) $A + B + C$

(C) ABC (D) None of the above

25. The output Y is

(A) $A + B$ (B) $B + C$

(C) $C + A$ (D) None of the above

26. A logic circuit consist of two 2×4 decoder as shown in fig. P4.2.26.

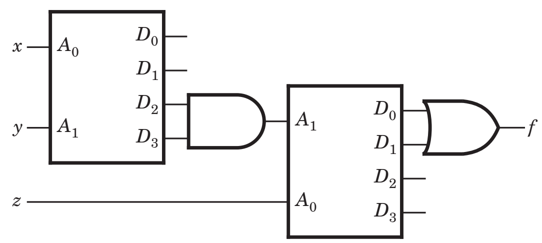


Fig. P4.2.26

The output of decoder are as follow

$$D_0 = 1 \text{ when } A_0 = 0, \quad A_1 = 0$$

$$D_1 = 1 \text{ when } A_0 = 1, \quad A_1 = 0$$

$$D_2 = 1 \text{ when } A_0 = 0, \quad A_1 = 1$$

$$D_3 = 1 \text{ when } A_0 = 1, \quad A_1 = 1$$

The value of $f(x, y, z)$ is

(A) 0 (B) z

(C) $\bar{z}$ (D) 1

Statement for Q.27-29:

A MUX network is shown in fig. P4.2.27-29.

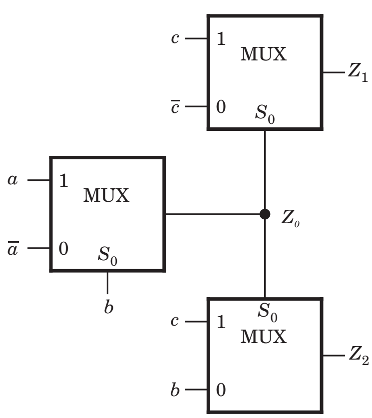


Fig. P4.2.27-29

27. $Z_1 = ?$

(A) $a + b + c$ (B) $ab + ac + bc$

(C) $a \text{ u } b \text{ u } c$ (D) $a \oplus b \oplus c$

28. $Z_2 = ?$

(A) $ab + bc + ca$ (B) $a + b + c$

(C) abc (D) $a \text{ u } b \text{ u } c$

29. This circuit act as a

(A) Full adder (B) Half adder

(C) Full subtractor (D) Half subtractor

30. The network shown in fig. P4.2.30 implements

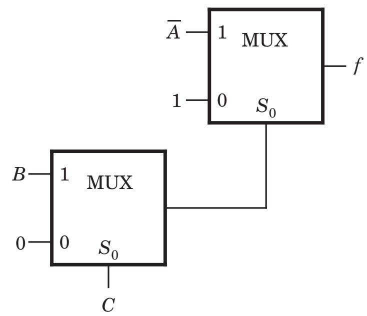


Fig. P4.2.30

(A) NOR gate (B) NAND gate

(C) XOR gate (D) XNOR gate

31. The MUX shown in fig. P4.2.31 is 4×1 multiplexer. The output Z is

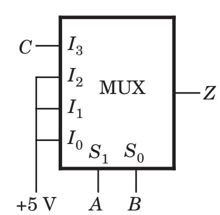


Fig. P4.2.31

(A) ABC B) $A \oplus B \oplus C$

(C) $A \text{ u } B \text{ u } C$ (D) $A + B + C$

32. The output of the 4×1 multiplexer shown in fig. P4.2.32 is

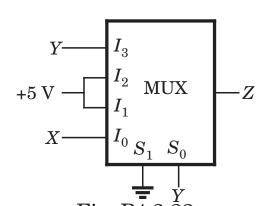


Fig. P4.2.32

(A) $X + Y$ (B) $\overline{X} + \overline{Y}$

(C) $\overline{XY} + X$ (D) $X\overline{Y}$

33. The MUX shown in fig. P4.2.33 is a 4×1 multiplexer. The output Z is

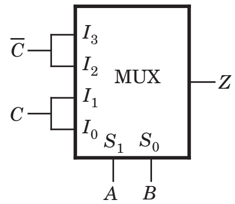


Fig. P4.2.33

(A) $A \oplus C$ (B) $A \,\mathrm{u}\, C$

(C) $B \oplus C$ (D) $B \,\mathrm{u}\, C$

34. $f = ?$

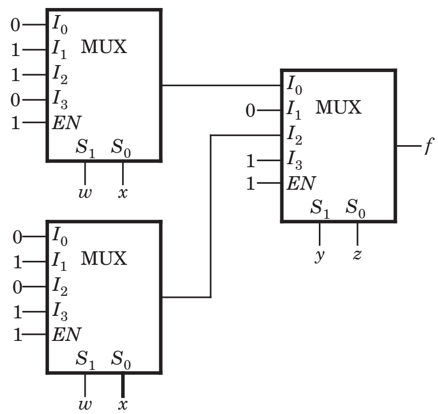


Fig. P4.2.34

(A) $\overline{w}x\overline{y}\overline{z} + w\overline{x}\,\overline{y}\overline{z} + xy + yz$

(B) $w\overline{x}yz + \overline{w}xyz + \overline{x}\,\overline{y} + \overline{yz}$

(C) $w\overline{x}\,\overline{y}\overline{z} + \overline{w}\,\overline{x}\,\overline{y}z + y\overline{z} + zx$

(D) $\overline{w}xyz + wxy\overline{z} + gz + \overline{zx}$

35. For the logic circuit shown in fig. P4.2.35 the output Y is

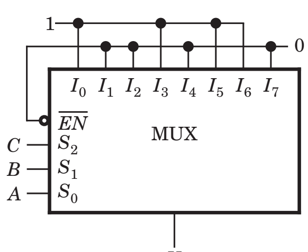


Fig. P4.2.35

(A) $A \oplus B$ (B) $\overline{A \oplus B}$

(C) $A \oplus B \oplus C$ (D) $\overline{A \oplus B \oplus C}$

36. The 4–to–1 multiplexer shown in fig. P4.2.36 implements the Boolean expression

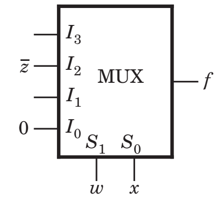


Fig. P4.2.36

$$f(w, x, y, z) = \Sigma m(4, 5, 7, 8, 10, 12, 15)$$

The input to I_1 and I_3 will be

(A) $y\overline{z}$, $\overline{y} + \overline{z}$ (B) $\overline{y} + z$, $y \,\mathrm{u}\, z$

(C) $\overline{y} + z$, $y \oplus z$ (D) $x + \overline{y}$, $y \oplus z$

37. The 8-to-1 multiplexer shown in fig. P4.2.37 realize the following Boolean expression

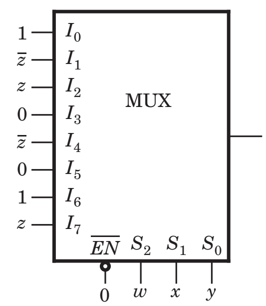


Fig. P4.2.37

(A) $w\overline{x}z + \overline{w}\,\overline{x}\,\overline{z} + wyz + x\overline{y}\,\overline{z}$

(B) $wxz + \overline{w}yz + wy\overline{z} + \overline{w}\,\overline{x}\,\overline{y}$

(C) $\overline{w}\,\overline{x}\,\overline{z} + w\overline{y}\,\overline{z} + \overline{w}\,\overline{y}z + wxz$

(D) MUX is not enable

Statement for Q.38-40:

A PLA realization is shown in fig. P4.2.38–40

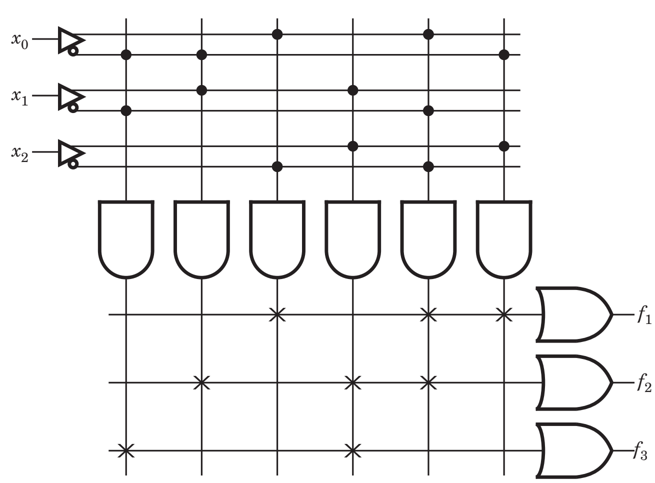


Fig. P.4.2.38-40

38. $f_1(x_2, x_1, x_0) = ?$

(A) $x_2\bar{x}_0 + x_1x_0$ (B) $x_2x_0 + x_1\bar{x}_2$

(C) $x_2 \oplus x_0$ (D) x_2 U x_0

39. $f_2(x_2, x_1, x_0) = ?$

(A) $\Sigma m(1, 2, 5, 6)$ (B) $\Sigma m(1, 2, 6, 7)$

(C) $\Sigma m(2, 3, 4)$ (D) None of the above

40. $f_3(x_2, x_1, x_0) = ?$

(A) $\Pi M(0, 4, 6, 7)$ (B) $\Pi M(2, 4, 5, 7)$

(C) $\Pi M(1, 2, 3, 5)$ (D) $\Pi M(2, 3, 4, 7)$

41. If the input $X_3X_2X_1X_0$ to the ROM in fig. P4.2.41 are 8–4–2–1 BCD numbers, then output $Y_3Y_2Y_1Y_0$ are

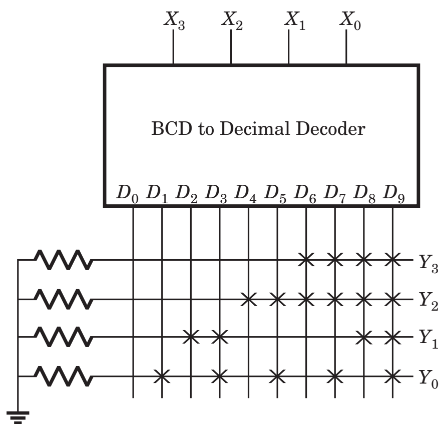


Fig. P4.2.41

(A) 2–4–2–1 BCD number (B) gray code number

(C) excess 3 code converter (D) none of the above

42. It is desired to generate the following three Boolean function

$$f_1 = a\bar{b}c + \bar{a}b\bar{c} + bc$$
$$f_2 = a\bar{b}c + ab + \bar{a}b\bar{c},$$
$$f_3 = \bar{a}\bar{b}\bar{c} + abc + \bar{a}c$$

by using an OR gate array as shown in fig. P4.2.42 where P_1 and P_5 are the product terms in one or more of the variable $a, \bar{a}, b, \bar{b}, c$ and $\bar{c}$.

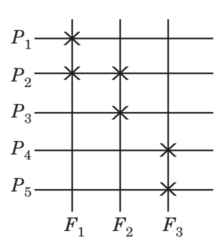


Fig. P4.2.42

The terms P_1, P_2, P_3, P_4 and P_5 are

(A) $\bar{a}b, ac, b\bar{c}, bc, \bar{a}\bar{b}$ (B) $a\bar{b}, b\bar{c}, ac, \bar{a}\bar{b}, bc$

(C) $ac, \bar{a}b, b\bar{c}, \bar{a}\bar{b}, bc$ (D) Above all

43. The circuit shown in fig. P.4.2.43 has 4 boxes each described by input P, Q, R and output Y, Z with $Y = P \oplus Q \oplus R$ and $Z = RQ + \bar{P}R + Q\bar{P}$.

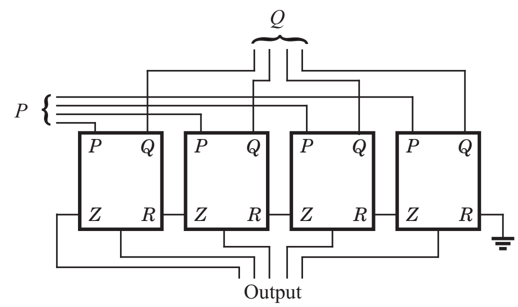


Fig. P4.2.43

The circuit act as a 4 bit

(A) adder giving $P + Q$

(B) subtractor giving $P - Q$

(C) subtractor giving $Q - P$

(D) adder giving $P + Q + R$

44. The circuit shown in fig. P4.2.44 converts

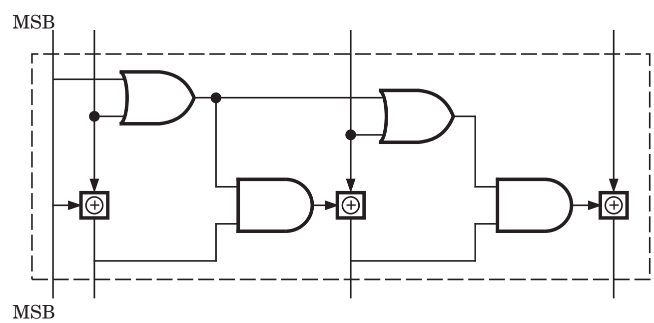


Fig. P4.2.44

(A) BCD to binary code

(B) Binary to excess

(C) Excess–3 to Gray Code

(D) Gray to Binary code

****

PQ \ RS	00	01	11	10
00			1	
01	1	1	1	1
11	1	1	1	
10			1	

Fig. S 4.2.13c

$Z = R + S + \overline{PQ + \overline{P}\,\overline{Q}\,\overline{R} + P\overline{Q}S} = R + S + \overline{PQ}\,\overline{\overline{P}\,\overline{Q}\,\overline{R}}\,\overline{P\overline{Q}\,S}$

$= R + S + (\overline{P} + \overline{Q})(P + Q + R)(\overline{P} + Q + S)$

$= R + S + (\overline{P}Q + \overline{P}R + \overline{Q}P + \overline{Q}R)(\overline{P} + Q + S)$

$= R + S + \overline{P}Q + \overline{P}QS + \overline{P}R + \overline{P}RQ + \overline{P}RS + \overline{Q}PS + \overline{Q}\overline{P}R + \overline{Q}RS$

PQ \ RS	00	01	11	10
00		1	1	1
01	1	1	1	1
11		1	1	1
10		1	1	1

Fig. S 4.2.13d

$= R + S + \overline{P}Q$

We can see that $W = Z$, $X = \overline{Z}$

14. (D) $\overline{A}\overline{B}\overline{D} + B\overline{C}D + BC\overline{D}$

AB \ CD	00	01	11	10
00	1			1
01		1		1
11		1		1
10	×	×		

Fig. S 4.2.14

15. (A) $f = \overline{x}_3x_2x_1 + x_3\overline{x}_2\overline{x}_1 + x_3\overline{x}_2x_1 = \overline{x}_3x_2x_1 + x_3\overline{x}_2$

$g = \overline{x}_4\overline{x}_3x_2 + x_4x_3\overline{x}_2 + x_4x_3x_2 = \overline{x}_4\overline{x}_3x_2 + x_4x_3$

$fg = \overline{x}_4\overline{x}_3x_2x_1 + x_4x_3\overline{x}_2$

$= \overline{x}_4\overline{x}_3x_2x_1 + x_4x_3\overline{x}_2\overline{x}_1 + x_4x_3\overline{x}_2x_1$

$h = \Sigma m(3, 12, 13)$

16. (A) $f = \Sigma m(4, 7, 15)$,

$f_1 = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9, 10, 11, 15)$

$f_2 = \Sigma m(4, 7, 15) + \Sigma dc(5, 6, 12, 13, 14)$

There are 5 don't care condition. So $2^5 = 32$ different functions f_2 .

17. (A) $f_2 = \Sigma m(4, 7, 15) + \Sigma dc(5, 6, 12, 13, 14)$, $f_2 = x$

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	×	1	×
11	×	×	1	×
10	0	0	0	0

Fig. S 4.2.17

18. (B) $m_6 = \overline{A}BC\overline{D}$, $m_9 = A\overline{B}\overline{C}D$

After complementing literal

$m_6' = A\overline{B}\overline{C}D = m_9$, $m_9' = \overline{A}BC\overline{D} = m_6$

19. (B) 0, 3, 6 and 9 are divisible by 3

D_8D_4 \ D_2D_1	00	01	11	10
00	1		1	
01				1
11	×	×	×	×
10		1	×	×

Fig. S 4.2.19

$f = D_8D_1 + D_4D_2\overline{D_1} + \overline{D_4}D_2D_1 + \overline{D_8}\,\overline{D_4}\,D_2\,D_1$

20. (B) $f = \overline{\Sigma m(0, 3, 6)} = \Sigma m(1, 2, 4, 5, 7)$

21. (C) $D = A\overline{B} + \overline{A}B$, $X = \overline{A}B$

A	B	D	X
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Fig. S 4.2.20

22. (D) $f_1 = \Sigma m(0, 2, 4, 6)$,

$f_2 = \Sigma m(1, 3, 5, 7)$, $f_1 f_2 = 0$

23. (D) $Z = D(\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}\overline{C} + ABC)$

$= D(\overline{A}\overline{B}(\overline{C} + C) + BC(\overline{A} + A) + A\overline{B}\overline{C})$

$= D(\overline{A}\overline{B} + BC + A\overline{B}\overline{C}) = D(\overline{B}(\overline{A} + A\overline{C}) + BC)$

$= D(\overline{B}\overline{A} + \overline{B}\overline{C} + BC) = D(B \,\mathrm{u}\, C + \overline{A}\overline{B})$

24. (A) $X = \Sigma m(3, 5, 6, 7)$, $X = AB + BC + CA$

25. (D) $Y = \Sigma m(1, 3, 5, 7)$, $Y = C$

A \ BC	00	01	11	10
00			1	
01		1	1	1

Fig. S4.2.24

26. (D) $D_0 = \overline{A}_1\overline{A}_0$, $D_1 = \overline{A}_1 A_0$, $D_2 = A_1\overline{A}_0$,

A \ BC	00	01	11	10
00		1	1	
01		1	1	

Fig. S4.2.25

$D_3 = A_1 A_0$

For first decoder $A_0 = x$, $A_1 = y$, $D_2 = y\overline{x}$, $D_3 = xy$

For second decoder $A_1 = D_2 D_3 = y\overline{x}xy = 0$, $A_0 = z$

$f = D_0 + D_1 = \overline{A}_1\overline{A}_0 + \overline{A}_1 A_0 = \overline{A}_1 = 1$

27. (D) The output of first MUX is

$Z_o = ab + \overline{a}\overline{b} = \overline{(a \oplus b)}$

This is input to select S_0 of both second-level MUX

$Z_1 = CS_0 + \overline{C}\overline{S}_0 = C \oplus \overline{S}_0 = a \oplus b \oplus c$

28. (A) $Z_2 = bS_0 + c\overline{S}_0$

$= b(ab + \overline{a}\overline{b}) + c(\overline{a}b + a\overline{b}) = ab + a\overline{b}c + \overline{a}bc$

$= a(b + \overline{b}c) + \overline{a}bc = ab + ac + \overline{a}bc$

$= ab + ac + bc$

29. (A) The equation of Z_1 is the equation of sum of A and B with carry and equation of 2 is the resultant carry. Thus, it is a full adder.

30. (B) $f_1 = \overline{C}D + CB = CB$, $S = F_1$

$f = \overline{f}_1 + f_1\overline{A} = \overline{CB} + CB\overline{A} = \overline{CB} + \overline{A}$

$= \overline{C} + \overline{B} + \overline{A} = \overline{ABC}$

31. (D) $Z = \overline{A}\overline{B}C + \overline{A}B + A\overline{B} + AB$

$= \overline{A}(\overline{B}C + B) + A(\overline{B} + B) = \overline{A}(B + C) + A = A + B + C$

32. (A) $Z = \overline{X}Y + X\overline{Y} + XY$, $Z = X + Y$

33. (A) $Z = \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C}$

$= \overline{A}C + A\overline{C} = A \oplus C$

34. (A) The output from the upper first level multiplexer is f_a and from the lower first level multiplexer is f_b

$f_a = \overline{w}x + w\overline{x}$, $f_b = \overline{w}x + wx = x$

$f = f_a\overline{y}\overline{z} + f_b y\overline{z} + yz = (\overline{w}x + w\overline{x})\overline{y}\overline{z} + xy\overline{z} + yz$

$= \overline{w}x\overline{y}\overline{z} + w\overline{x}\overline{y}\overline{z} + xy + yz$

35. (D) Output is 1 when even parity

C \ BA	00	01	11	10
00	1		1	
01		1		1

Fig. S4.2.35

Therefore $Y = \overline{A \oplus B \oplus C}$

36. (B) $I_1 = \overline{y} + z$, $I_3 = y \quad z$

S_1S_0 ↕↕ $w\,x$ \ yz	00	01	11	10	
00	0	0	0	0	$I_0 = 0$
01	1	1	1	0	$I_1 = \overline{y} + z$
11	1	0	1	0	$I_3 = yz + \overline{y}\overline{z} = y \,\mathrm{u}\, z$
10	1	0	0	1	$I_2 = \overline{z}$

Fig. S 4.2.36

37. (C) Let $z=0$, Then

$f=\overline{w}\,\overline{x}\,\overline{y}+\overline{w}\,\overline{x}y+w\overline{x}\,\overline{y}+wx\overline{y}=\overline{w}\ \overline{x}+w\ \overline{y}$

If we put $z=0$ in given option then

(A) $=\overline{w}\,\overline{x}+x\overline{y}$ (B) $=wy+\overline{w}\ \overline{x}\ \overline{y}$ (C) $=\overline{w}\,\overline{x}+w\overline{y}$

Since MUX is enable so option (C) is correct.

38. (C)$f=x_0\overline{x}_2+x_0\overline{x}_1\overline{x}_2+\overline{x}_0x_2=x_0\overline{x}_2(1+\overline{x}_1)+\overline{x}_0x_2$

$=x_0\overline{x}_2+\overline{x}_0x_2$

39. (B) $f_2=\overline{x}_0x_1+x_1x_2+x_0\overline{x}_1\overline{x}_2$

$=\overline{x}_0x_1x_2+\overline{x}_0x_1\overline{x}_2+x_1x_2x_0+x_1x_2\overline{x}_0+x_0\overline{x}_1\overline{x}_2$

$=x_2x_1\overline{x}_0+\overline{x}_2x_1\overline{x}_0+\overline{x}_2\overline{x}_1x_0+x_2x_1x_0$

$f_2(x_2,\ x_1,\ x_0)=\Sigma m(1,\ 2,\ 6,\ 7)$

40. (C) $f_3=\overline{x}_0\overline{x}_1+x_1x_2$

$=x_2\overline{x}_1\overline{x}_0+\overline{x}_2\overline{x}_1\overline{x}_0+x_2x_1x_0+x_2x_1\overline{x}_0$

$f_3(x_2,\ x_1,\ x_0)=\Sigma m(0,\ 4,\ 6,\ 7)$

$f_3(x_2,\ x_1,\ x_0)=\Pi M(1,\ 2,\ 3,\ 5)$

41. (A)

Let $X_3X_2X_1X_0$ be 1001 then $Y_3Y_2Y_1Y_0$ will be 1111.

Let $X_3X_2X_1X_0$ be 1000 then $Y_3Y_2Y_1Y_0$ will be 1110

Let $X_3X_2X_1X_0$ be 0110 then $Y_3Y_2Y_1Y_0$ will be 1100

a \ bc	00	01	11	10
00			1	1
01		1	1	

Fig. S4.2.42a

a \ bc	00	01	11	10
00				1
01		1	1	1

Fig. S4.2.42b

a \ bc	00	01	11	10
00	1	1	1	
01			1	

Fig. S4.2.42b

So this converts 2–4–2–1 BCD numbers.

42. (A) $f_1=a\overline{b}c+\overline{a}b\overline{c}+bc=ac+\overline{a}b$

$f_2=a\overline{b}c+ab+\overline{a}b\overline{c}=ac+b\ \overline{c}$

$f_3=\overline{a}\overline{b}\overline{c}+abc+\overline{a}c=\overline{a}\overline{b}+bc$

Thus $P_1=\overline{a}b$, $P_2=ac$, $P_3=b\overline{c}$, $P_4=bc$, $P_5=\overline{a}\overline{b}$

43. (B) Let P = 1001 and Q = 1010 then

$Y_n=P_n\oplus Q_n\oplus R_n$, $Z_n=R_nQ_n+\overline{P}_nR_n+Q_n\overline{P}_n$

output is 1111 which is 2's complement of –1. So it gives $P-Q$. Let another example $P=1101$ and $Q=0110$ then output is 00111. It gives P–Q.

So (B) is correct.

	P_n	Q_n	R_n	Z_n	Y_n
$n=1$	1	0	0	0	1
$n=2$	0	1	0	1	1
$n=3$	0	0	1	1	1
$n=4$	1	1	1	1	1
					1

Fig. S4.2.43a

	P_n	Q_n	R_n	Z_n	Y_n
$n=1$	1	0	0	0	1
$n=2$	0	1	0	1	1
$n=3$	1	1	1	1	1
$n=4$	1	0	1	0	0
					0

Fig. S4.2.43b

44. (D) Let input be 1010

output will be 1101

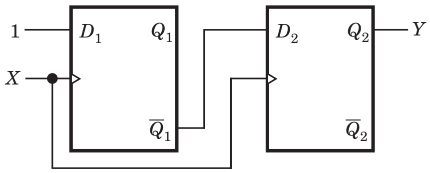
Let input be 0110

output will be 0100

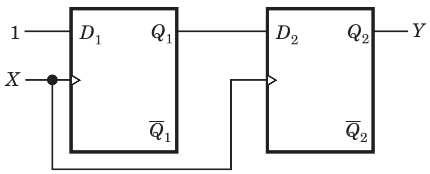
This convert gray to Binary code.

****

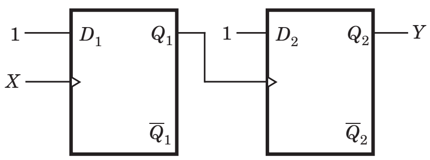
(B)



(C)



(D)



11. The circuit shown in fig. P4.3.11 is

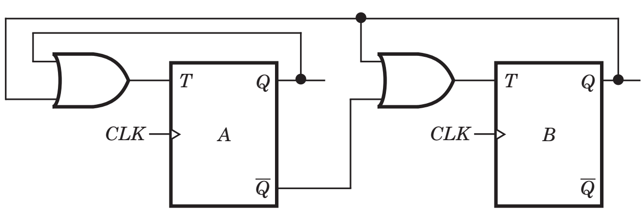


Fig.P4.3.11

(A) a MOD–2 counter

(B) a MOD–3 counter

(C) generate sequence 00, 10, 01, 00.....

(D) generate sequence 00, 10, 00, 10, 00 ......

12. The counter shown in fig. P4.3.12 is a

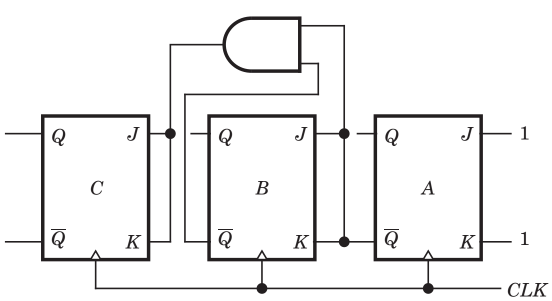


Fig.P4.3.12

(A) MOD–8 up counter

(B) MOD–8 down counter

(C) MOD–6 up counter

(D) MOD–6 down counter

13. The counter shown in fig. P4.3.13 counts from

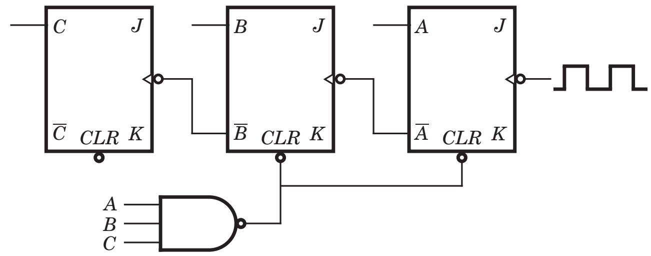


Fig.P4.3.13

(A) 0 0 0 to 1 1 1 (B) 1 1 1 to 0 0 0

(C) 1 0 0 to 0 0 0 (D) 0 0 0 to 1 0 0

14. The mod-number of the asynchronous counter shown in fig. P4.2.13 is

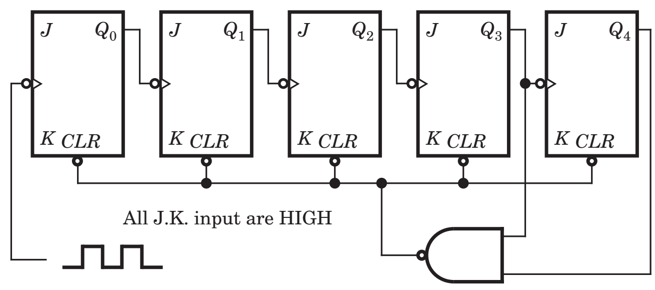


Fig.P4.3.14

(A) 24 (B) 48

(C) 25 (D) 36

15. The frequency of the pulse at z in the network shown in fig. P4.3.15. is

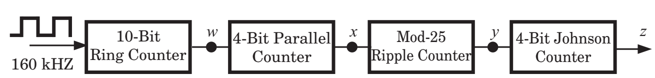


Fig.P4.3.15

(A) 10 Hz (B) 160 Hz

(C) 40 Hz (D) 5 Hz

16. The three-stage Johnson counter as shown in fig. P4.2.16 is clocked at a constant frequency of f_c from the starting state of $Q_2Q_1Q_0 = 101$. The frequency of output $Q_2Q_1Q_0$ will be

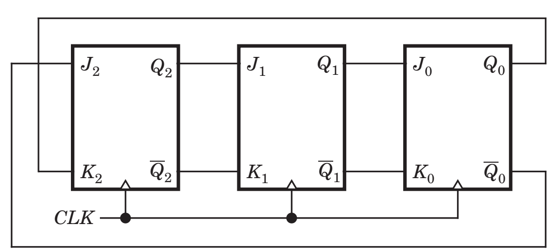


Fig.P4.3.16

(A) $\frac{f_c}{8}$ (B) $\frac{f_c}{6}$

(C) $\frac{f_c}{3}$ (D) $\frac{f_c}{2}$

17. The counter shown in the fig. P4.3.17 has initially $Q_2Q_1Q_0 = 000$. The status of $Q_2Q_1Q_0$ after the first pulse is

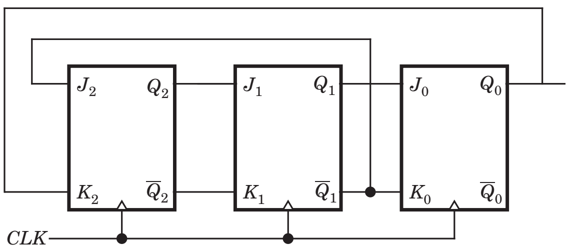


Fig.P4.3.17

(A) 0 0 1 (B) 0 1 0

(C) 1 0 0 (D) 1 0 1

18. A 4 bit ripple counter and a 4 bit synchronous counter are made by flips flops having a propagation delay of 10 ns each. If the worst case delay in the ripple counter and the synchronous counter be R and S respectively, then

(A) $R = 10$ ns, $S = 40$ ns (B) $R = 40$ ns, $S = 10$ ns

(C) $R = 10$ ns, $S = 30$ ns (D) $R = 30$ ns, $S = 10$ ns

19. A 4 bit modulo–6 ripple counter uses JK flip-flop. If the propagation delay of each FF is 50 ns, the maximum clock frequency that can be used is equal to

(A) 5 MHz (B) 10 MHz

(C) 4 MHz (D) 20 Mhz

20. The initial contents of the 4-bit serial-in-parallel-out right-shift, register shown in fig. P4.3.20 is 0 1 1 0. After three clock pulses are applied, the contents of the shift register will be

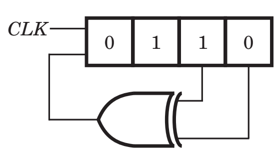


Fig.P4.3.20

(A) 0 0 0 0 (B) 0 1 0 1

(C) 1 1 1 1 (D) 1 0 1 0

21. In the circuit shown in fig. P4.3.21 is PIPO 4-bit register, which loads at the rising edge of the clock. The input lines are connected to a 4 bit bus. Its output acts as the input to a 16 × 4 ROM whose output is floating when the enable input E is 0. A partial table of the contents of the ROM is as follows

Address	0	2	4	6	8	10	12
Data	0011	1111	0100	1010	1011	1000	0010

The clock to the register is shown below, and the data on the bus at time t_1 is 0110.

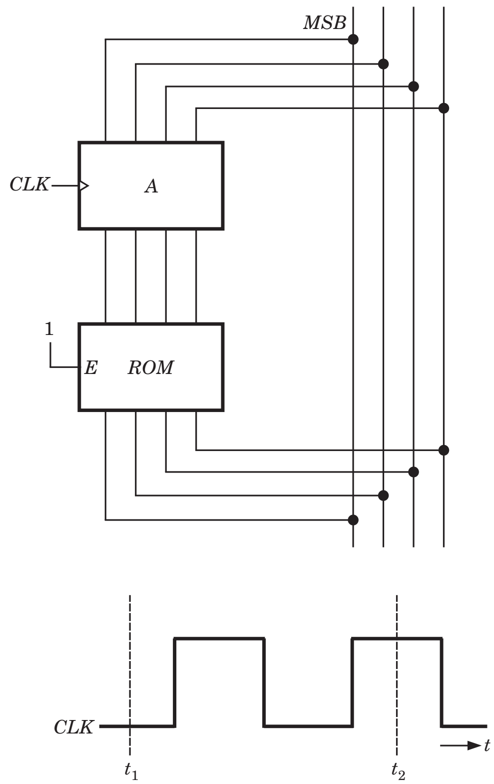


Fig. P4.3.21

The data on the bus at time t_2 is

(A) 1 1 1 1 (B) 1 0 1 1

(C) 1 0 0 0 (D) 0 0 1 0

22. A 4-bit right shift register is initialized to value 1000 for (Q_3, Q_2, Q_1, Q_0). The D input is derived from Q_0, Q_2 and Q_3 through two XOR gates as shown in fig. P4.2.22. The pattern 1000 will appear at

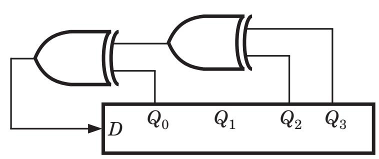


Fig.P4.3.22

(A) 3rd pulse (B) 7th pulse

(C) 6th pulse (D) 4th pulse

Statement for Q.23–24:

The 8-bit left shift register and D-flip-flop shown in fig. P4.3.22–23 is synchronized with same clock. The

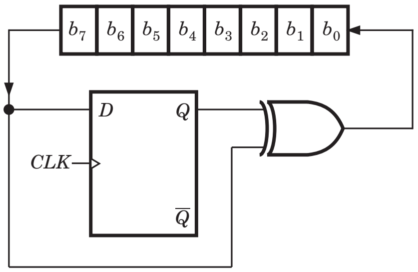


Fig.P4.3.23-24

D flip-flop is initially cleared.

23. The circuit act as

(A) Binary to 2's complement converter

(B) Binary to Gray code converter

(C) Binary to 1's complement converter

(D) Binary to Excess–3 code converter

24. If initially register contains byte B7, then after 4 clock pulse contents of register will be

(A) 73 (B) 72

(C) 7E (D) 74

Statement for Q.25–26:

A Mealy system produces a 1 output if the input has been 0 for at least two consecutive clocks followed immediately by two or more consecutive 1's.

25. The minimum state for this system is

(A) 4 (B) 5

(C) 8 (D) 9

26. The flip-flop required to implement this system are

(A) 2 (B) 3

(C) 4 (D) 5

27. The output of a Mealy system is 1 if there has been a pattern of 11000, otherwise 0. The minimum state for this system is

(A) 4 (B) 5

(C) 6 (D) 7

28. To count from 0 to 1024 the number of required flip-flop is

(A) 10 (B) 11

(C) 12 (D) 13

29. Four memory chips of 16×4 size have their address buses connected together. This system will be of size

(A) 64×4 (B) 32×8

(C) 16×16 (D) 256×1

30. The address bus width of a memory of size 1024×8 bits is

(A) 10 bits (B) 13 bits

(C) 8 bits (D) 18 bits

31. For the circuit of Fig. P4.3.31 consider the statement:

Assertion (A) : The circuit is sequential

Reason (R) : There is a loop in circuit

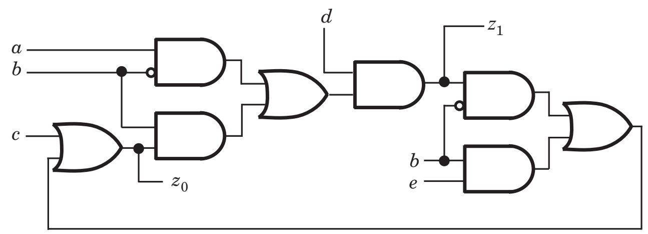


Fig.P4.3.131

Choose correct option

(A) Both A and R true and R is the correct explanation of A

(B) Both A and R true but R is not a correct explanation on of A

(C) A is true but R is false

(D) A is false

****

SOLUTIONS

1. (C) Given FF is a negative edge triggered T flip-flop. So at the negative edge of clock V_i FF will invert the output if there is 1 at input.

2. (A) At first rising edge of clock, D is HIGH. So Q will be high till 2nd rising edge of clock. At 2nd rising edge, D is low so Q will be LOW till 3rd rising edge of clock. At 3rd rising edge, D is HIGH, so Q will be HIGH till 4th rising edge. At 4th rising edge D is HIGH so Q will be HIGH till 5th rising. edge. At 5th rising edge, D is LOW, so Q will be LOW till 6th rising edge.

3. (C)

x	Q	S	R	Q^+
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	0	1	0

Fig. S4.3.3

4. (D) $Q^+ = x \oplus Q$

$Q_1^+ = x_1 \oplus Q_0 = x_1\overline{0} + \overline{x}_1 0 = x_1$

$Q_2^+ = x_2 \oplus x_1$, $Q_3^+ = x_3 \oplus x_2 \oplus x_1$

$Q_4^+ = x_4 \oplus x_3 \oplus x_2 \oplus x_1$

So this generate the even parity and check odd parity.

5. (C)

A	B	S	R	Q	Q^+
0	0	1	0	0	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	0	1	1	0
1	0	0	0	0	0
1	0	0	0	1	1
1	1	1	1	0	×
1	1	1	1	1	×

Fig. S4.3.5

$Q^+ = \overline{A}\overline{B} + AQ \quad = \overline{A}\overline{B} + \overline{B}Q$

6. (D) $Q^+ = L\overline{M} + LM\overline{Q}$

$= L(\overline{M} + M\overline{Q})$

$= L\,\overline{M} + L\,\overline{Q}$

L	M	Q^+
0	0	0
0	1	0
1	0	1
1	1	$\overline{Q}_1$

Fig. S4.3.6

7. (D)

	J	K	Q	$\overline{Q}$	Q_{n+1}	$\overline{Q}_{n+1}$
Initially		1	0	1		
Clock 1st	1	1	0	1	1	0
2nd	0	1	1	0	0	1
3rd	1	1	0	1	1	0
4th	0	1	1	0	0	1
5th	1	1	0	1	1	0

Fig. S4.3.7

Therefore sequence is 010101.

8. (A)

A	B	X	Y
1	1	0	1
1	0	0	1

X and Y are fixed at 0 and 1.

9. (D) $Z = \overline{X}Q + Y\overline{Q}$

X	Y	Z
0	0	Q
0	1	0
1	0	1
1	1	$\overline{Q}_1$

Fig. S4.3.9

Comparing from the truth table of J–K FF

$Y = J,\ X = K$

10. (C)

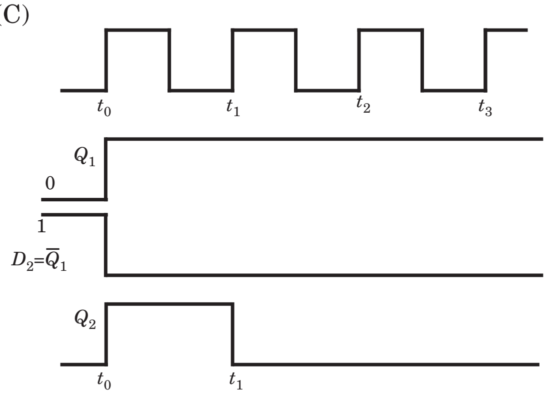


Fig.S4.3.10

11. (B)

Present State	FF Input	Next State
Q_A Q_B	T_A T_B	Q_A^+ Q_B^+
0 0	0 1	0 1
0 1	1 1	1 0
1 0	1 0	0 0
1 1	1 1	0 0

Fig. S4.3.11

From table it is clear that it is a MOD–3 counter.

12. (B) It is a down counter because 0 state of previous FFs change the state of next FF. You may trace the following sequence, let initial state be 0 0 0

FF C	FF B	FF A	
$J\,K\,\overline{C}$	$J\,K\,\overline{B}$	$J\,K\,\overline{A}$	$C^+B^+A^+$
1 1 1	1 1 1	1 1 1	1 1 1
0 0 0	0 0 0	1 1 0	1 1 0
0 0 0	1 1 0	1 1 1	1 0 1
0 0 0	0 0 1	1 1 0	1 0 0
1 1 1	1 1 1	1 1 1	0 1 1
0 0 1	0 0 0	1 1 0	0 1 0
0 0 1	1 1 0	1 1 1	0 0 1
0 0 0	0 0 1	1 1 0	0 0 0

Fig. S4.3.12

13. (C) It is a down counter because the inverted FF output drive the clock inputs. The NAND gate will clear FFs A and B when the count tries to recycle to 111. This will produce as result of 100. Thus the counting sequence will be 100, 011, 010, 001, 000, 100 etc.

14. (A) It is a 5 bit ripple counter. At 11000 the output of NAND gate is LOW. This will clear all FF. So it is a Mod–24 counter. Note that when 11000 occur, the CLR input is activated and all FF are immediately cleared. So it is a MOD 24 counter not MOD 25.

15. (D) 10-bit ring counter is a MOD–10, so it divides the 160 kHz input by 10. therefore, $w = 16$ kHz. The four-bit parallel counter is a MOD–16. Thus, the frequency at $x = 1$ kHz. The MOD–25 ripple counter produces a frequency at $y = 40$ Hz. (1 kHz/25 = 40 Hz). The four-bit Johnson Counter is a MOD-8. This, the frequency at $z = 5$ Hz.

16. (D)

$\overline{Q}_0\,Q_0$	$Q_2\,\overline{Q}_2$	$Q_1\,\overline{Q}_1$	
$J_2\,K_2$	$J_1\,K_1$	$J_0\,K_0$	$Q_2^+\,Q_1^+\,Q_0^+$
			1 0 1
0 1	1 0	0 1	0 1 0
1 0	0 1	1 0	1 0 1
0 1	1 0	0 1	0 1 0
1 0	0 1	1 0	1 0 1

Fig. S4.3.16

We see that 1 0 1 repeat after every two cycles, hence frequency will be $f_c / 2$.

17. (C) At first cycle

$J_2K_2 = 10 \quad \Rightarrow \quad Q_2 = 1,$

$J_1K_1 = 00 \quad \Rightarrow \quad Q_1 = 1,$

$J_0K_0 = 00 \quad \Rightarrow \quad Q_0 = 0$

18. (B) In ripple counter delay $4T_d$ = 40 ns.

The synchronous counter are clocked simultaneously, then its worst delay will be equal to 10 ns.

19. (A) 4 bit uses 4 FF

Total delay $Nt_d = 4 \times 50 \text{ ns} = 200 \times 10^{-9}$

$$f = \frac{1}{200 \times 10^{-9}} = 5 \text{ Mhz}$$

20. (D) At pulse 1 input, $1 \oplus 0 = 1$

So contents are 1 0 1 1,

At pules 2 input $1 \oplus 1 = 0$‘

So contents are 0 1 0 1,

At pules 3 input $0 \oplus 1 = 1$, contents are 1 0 1 0

CHAPTER

4.4

DIGITAL LOGIC FAMILIES

Statement for Q.1–2:

Consider the DL circuit of fig. P4.4.1–2.

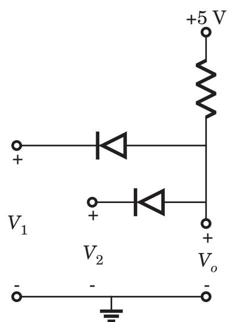


Fig. P4.4.1-2

1. For positive logic the circuit is a

(A) AND (B) OR

(C) NAND (D) NOR

2. For negative logic the circuit is a

(A) AND (B) OR

(C) NAND (D) NOR

3. The diode logic circuit of fig. P4.4.3 is a

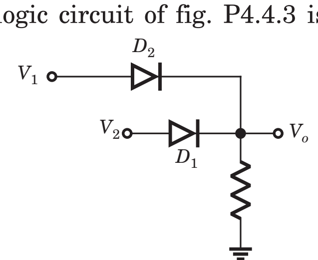


Fig. P4.4.3

(A) AND (B) OR

(C) NAND (D) NOR

4. In the circuit shown in fig. P.4.4.4. the output Z is

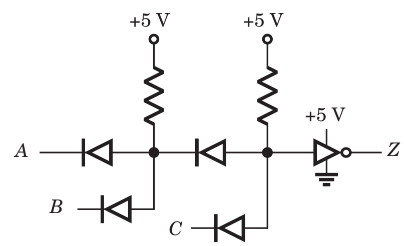


Fig. P4.4.4

(A) $AB+\overline{C}$ (B) $\overline{AB}C$

(C) $\overline{ABC}$ (D) ABC

Statement for Q.5–7:

Consider the AND circuit shown in fig. P4.4.5–7. The binary input levels are $V(0)=0$ V and $V(1)=25$ V. Assume ideal diodes. If $V_1=V(0)$ and $V_2=V(1)$, then V_o is to be at 5 V. However, if $V_1=V_2=V(1)$, then V_o is to rise above 5 V.

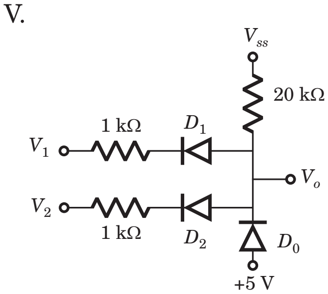


Fig. P4.4.5-7

5. If $V_{ss}=20$ V and $V_1=V_2=V(1)$, the diode current I_{D1}, I_{D2}, and I_{D0} are

(A) 1 mA, 1 mA, 4 mA (B) 1 mA, 1 mA, 5 mA

(C) 5 mA, 5mA, 1 mA (D) 0, 0, 0

6. If $V_{ss} = 40$ V and both input are at HIGH level then, diode current I_{D1}, I_{D2} and I_{D0} are respectively

(A) 0.4 mA, 0.4 mA, 0 (B) 0, 0, 1 mA

(C) 0.4 mA, 0.4 mA, 1 mA (D) 0, 0, 0

7. The maximum value of V_{ss} which may be used is

(A) 30 V (B) 25 V

(C) 125 V (D) 20 V

8. The ideal inverter in fig. P4.4.8 has a reference voltage of 2.5 V. The forward voltage of the diode is 0.75 V. The maximum number of diode logic circuit, that may be cascaded ahead of the inverter without producing logic error, is

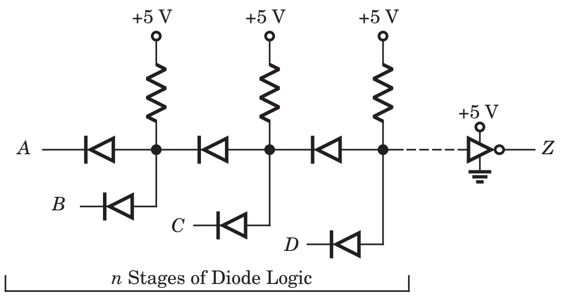


Fig. P4.4.8

(A) 3 (B) 4

(C) 5 (D) 9

9. Consider the TTL circuit in fig. P4.4.9. The value of V_H and V_L are respectively

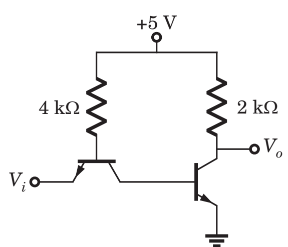


Fig. P4.4.9

(A) 5 V, 0 V (B) 4.8 V, 0 V

(C) 4.8 V, 0.2 V (D) 5 V, 0.2 V

Statement Q.10–11:

Consider the resistor transistor logic gate of fig. P4.4.10-11.

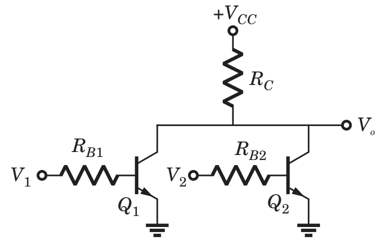


Fig. P4.4.10-11

10. For positive logic the gate is

(A) AND (B) OR

(C) NAND (D) NOR

11. For negative logic the gate is

(A) AND (B) OR

(C) NAND (D) NOR

Statement for Q.12–13:

Consider the RTL circuit of fig. P4.4.12–13.

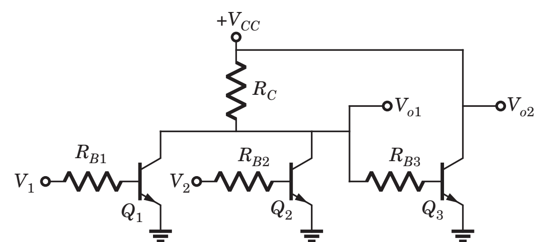


Fig. P4.4.12-13

12. If V_{o1} is taken as the output, then circuit is a

(A) AND (B) OR

(C) NAND (D) NOR

13. If V_{o2} is taken as output, then circuit is a

(A) AND (B) OR

(C) NAND (D) NOR

Statement for Q.14–15:

Consider the TTL circuit of fig. P4.4.14. If either or both V_1 and V_2 are logic LOW, Q_1 is driven to saturation.

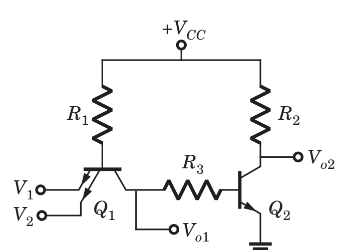


Fig. P4.4.14-15

22. The circuit shown in fig. P4.4.22 is

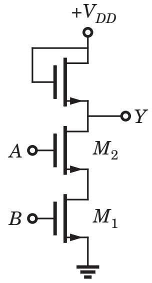


Fig. P4.4.22

(A) NAND (B) NOR

(C) AND (D) OR

23. The circuit shown in fig. P4.4.23 acts as a

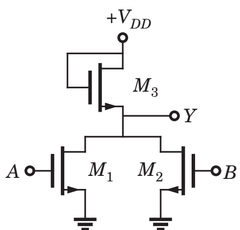


Fig. P4.4.23

(A) NAND (B) NOR

(C) AND (D) OR

24. The circuit shown in fig. P4.4.24 implements the function

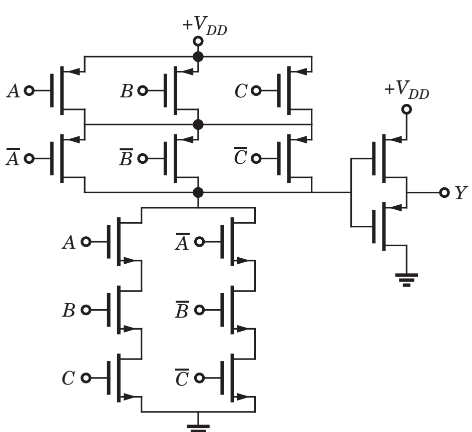


Fig. P4.4.24

(A) $ABC + \overline{ABC}$ (B $ABC + \overline{(A + B + C)}$

(C) $\overline{ABC} + \overline{(A + B + C)}$ (D) None of the above

25. The circuit shown in fig. P4.4.25. implements the function

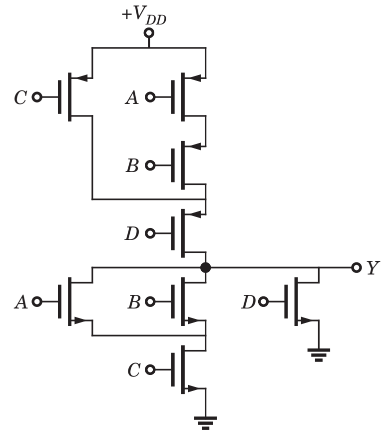


Fig. P4.4.25

(A) $(A + B)C + D$ (B) $\overline{(AB + C)D}$

(C) $\overline{(A + B)C + D}$ (D) $(AB + C)D$

26. Consider the CMOS circuit shown in fig. P4.4.26. The output Y is

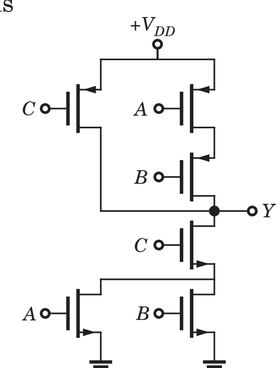


Fig. P4.4.26

(A) $\overline{(A + C)B}$ (B) $\overline{(A + B)C}$

(C) $AB + C$ (D) $AB + \overline{C}$

27. The CMOS circuit shown in fig. P4.4.27 implement

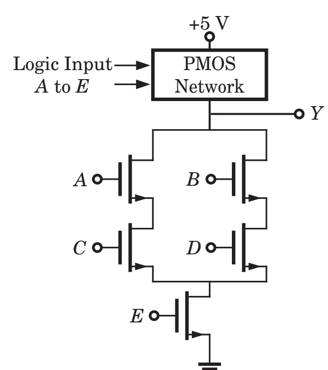


Fig. P4.4.27

(A) $\overline{AB + CD + E}$ (B) $\overline{(A + B)(C + D)E}$

(C) $AB + CD + E$ (D) $(A + B)(C + D)E$

V_1		V_2		V_o	
Actual	Logic	Actual	Logic	Actual	Logic
V_H	1	V_H	1	$V_{CE(sat)}$	0
V_L	0	V_L	0	V_{CE}	1
V_H	1	V_L	0	$V_{CE(sat)}$	0
V_L	0	V_H	1	$V_{CE(sat)}$	0

13. (B) The Q_3 stage is simply an inverter (a NOT gate). Thus output V_{o2} is the logic complement of V_{o1}. Therefore this is a OR gate.

14. (A) When Q_1 is saturated, V_{o1} is logic LOW otherwise V_{o1} is logic HIGH. The following truth table shows AND logic

V_1	V_2	V_{o1}
1	1	1
0	1	0
1	0	0
0	0	0

15. (C) The Q_2 stage is simply an inverter. Thus output V_{o2} is the logic complement of V_{o1}.

16. (C) If $V_1 = V_2 \leq V_L$, $V_{o1} \approx V_{CC}$. If $V_1(V_2) > V_H$, while $V_2(V_1) \leq V_L$, $Q_1(Q_2)$ is ON and $Q_2(Q_1)$ is OFF and $V_{o1} \approx V_{CC}$. If $V_1 = V_2 > V_H$, both Q_1 and Q_2 are ON and $V_{o1} \approx 2V_{CE(sat)}$. The truth table shows NAND logic

V_1		V_2		V_o	
Actual	Logic	Actual	Logic	Actual	Logic
V_L	0	V_L	0	V_{CC}	1
V_H	1	V_L	0	V_{CC}	1
V_L	0	V_H	1	V_{CC}	1
V_H	1	V_H	1	$2V_{CE(sat)}$	0

17. (A) The Q_3 stage is simple an inverter. Hence AND logic.

18. (C) For each successive gate, that has a transistor in saturation, the current required is

$$I_{B(sat)} = \frac{I_{C(sat)}}{\beta} = \frac{V_{CC} - V_{CE(sat)}}{\beta R_C} = \frac{5 - 0.2}{50(640)} = 0.15 \text{ mA}$$

For n attached gate $I_o = nI_{B(sat)}$.

To assure no logic error $V_o = V_{CC} - I_o R_C > V_H = 3.5$ V

$$n \leq \frac{V_{CC} - 3.5}{R_C I_{B(sat)}} = \frac{5 - 3.5}{640(0.15\text{m})} = 15.6 \quad \Rightarrow \quad n \leq 15$$

19. (A) Let $V_1 = V_2 = 0$ V, then M_3 will be ON, M_1 and M_2 OFF and M_4 ON, hence $V_o = -V_{DD}$. Let $V_1 = 0$ V and $V_2 = -V_{DD}$ then M_3 will be ON, M_1 OFF M_4 OFF, M_2 ON, hence $V_o = -V_{DD}$. Let $V_1 = -V_{DD}$ and $V_2 = 0$ V, then M_3 OFF, M_4 ON, M_2 OFF hence $V_o = -V_{DD}$. Finally if $V_1 = V_2 = -V_{DD}$, M_3 and M_4 will be OFF and M_1, M_2 will be ON, hence $V_o = 0$ V. Thus the given CMOS gate satisfies the function of a negative NAND gate.

20. (C) If $V_A = -V_{DD}$ then M_1 is ON and $V_Y = 0$ V. If $V_B = V_C = -V_{DD}$ and $V_A = 0$ V then M_3 and M_2 are ON but M_1 is OFF hence $V_Y = 0$ V. If $V_A = 0$ V and either or both V_B, V_C are 0 V then M_1 is OFF and either or both M_2 and M_3 will be OFF, which implies no current flowing through M_4 hence $V_Y = -V_{DD}$. Thus given circuit satisfies the logic equation $\overline{A + BC}$.

21. (A) Let $V_1 = V_2 = 0$ V $= V(0)$ then M_4 and M_3 will be ON and M_2, M_1 OFF hence $V_o = V_{DD} = V(1)$. Let $V_1 = 0$ V, $V_2 = V_{DD}$ then M_4 and M_2 will be ON but M_3 and M_1 will be OFF hence $V_o = 0 = V(0)$. Let $V_1 = V_{DD}$, $V_2 = 0$ V, then M_4 and M_3 will be OFF and M_1 ON hence $V_o = 0$ V $= V(0)$. Finally if $V_1 = V_2 = V_{DD}$, M_1 and M_2 will be ON but M_4 will be OFF hence $V_o = 0$ V $= V(0)$. Thus the given CMOS satisfy the function of a positive NOR gate.

22. (A) If either one or both the inputs are $V(0) = 0$ V the corresponding FET will be OFF, the voltage across the load FET will be 0 V, hence the output is V_{DD}. If boths inputs are $V(1) = V_{DD}$, both M_1 and M_2 are ON and the output is $V(0) = 0$ V. It satisfy NAND gate.

23. (B) If both the inputs are at $V(0) = 0$ V, the transistor M_1 and M_2 are OFF, hence the output is $V(1) = V_{DD}$. If either one or both of the inputs are at $V(1) = V_{DD}$, the corresponding FET will be ON and the output will be $V(0) = 0$ V. Hence it is a NOR gate.

24. (B) If all inputs A, B and C are HIGH, then input to invertor is LOW and output Y is HIGH. If all inputs are LOW, then input to inverter is also LOW and output Y is HIGH. In all other case the input to inverter is HIGH and output Y is LOW.

Hence $Y = ABC + \overline{A}\overline{B}\overline{C} = ABC + \overline{(A + B + C)}$

25. (C) The operation of circuit is given below

A B C D	P_A P_B P_C P_D	N_A N_B N_C N_D	Y
× × × 1	× × × OFF	× × × ON	LOW
× × 0 0	× × ON ON	× × OFF OFF	HIGH
0 0 1 0	ON ON OFF ON	OFF OFF ON OFF	HIGH
0 1 1 0	ON OFF OFF ON	OFF ON ON OFF	LOW
1 0 1 0	OFF ON OFF ON	ON OFF ON OFF	LOW
1 1 1 0	OFF OFF OFF ON	ON ON ON OFF	LOW

$Y = \overline{(A + B)C + D}$

26. (B) The operation of this circuit is given below :

A B C	P_A P_B P_C	N_A N_B N_C	Y
× × 0	× × ON	× × OFF	HIGH
0 0 1	ON ON OFF	OFF OFF ON	HIGH
× 1 1	× OFF OFF	× ON ON	LOW
1 × 1	OFF × OFF	ON × ON	LOW

$Y = \overline{(A + B)C}$

27. (B) If input E is LOW, output will not be LOW. It must be HIGH. Option (B) satisfy this condition.

28. (A) In this circuit parallel combination are OR gate and series combination are AND gate.

Hence $Y = \overline{(A + B)(C + D)(E + F)}$

29. (A) When an output is HIGH, it may be as low as $V_{OH(min)} = 2.4$ V. The minimum voltage that an input will respond to as a HIGH is $V_{IH(min)} = 2.0$ V. A negative noise spike that can drive the actual voltage below 2.0 V if its amplitude is greater than

$V_{NH} = V_{OH(min)} - V_{IH(min)} = 2.4 - 2.0 = 0.4$ V

30. (A) When an output is LOW, it may be as high as $V_{OL(max)} = 0.4$ V. The maximum voltage that an input will respond to as a LOW is $V_{IL(max)} = 0.8$ V. A positive noise spike can drive the actual voltage above the 0.8 V level if its amplitude is greater than

$V_{NL} = V_{IL(max)} - V_{OL(max)} = 0.8 - 0.4 = 0.4$ V

31. (B) A positive noise spike can drive the voltage above 1.0 V level if the amplitude is greater than

$V_{NL} = V_{IL(max)} - V_{OL(max)} = 1 - 0.1 = 0.9$ V,

A negative noise spike can drive the voltage below 3.5 V if the amplitude is greater than

$V_{NH} = V_{OH(min)} - V_{IH(min)} = 4.9 - 3.5 = 1.4$ V

32. (B) $V_{IH(min)} = V_{OH(min)} - V_{NH} = -0.8 - 0.5 = -1.3$ V

$V_{IL(max)} = V_{OL(max)} + V_{NL} = 0.5 + (-2) = -1.5$ V

33. (C) $V_{NH} = V_{OH(min)} - V_{IH(min)}$, $V_{NL} = V_{IL(max)} - V_{OL(max)}$

$V_{NH} = 2.7$ (for LS) -2.0 (for ALS) $= 0.7$ V

$V_{NL} = 0.8$ (for ALS) -0.5 (for LS) $= 0.3$ V

34. (B) $V_{NH} = 2.5$ (for ALS) $-$ 2.0 (for LS) $= 0.5$ V

$V_{NL} = 0.8$ (for LS) $-$ 0.4 (for ALS) $= 0.4$ V

35. (D) $V_{NH(min)} = 0.5$ V , $V_{NL(min)} = 0.3$ V

36. (B) fanout (LOW) $= \dfrac{I_{OL(max)}}{I_{IL(max)}} = \dfrac{8m}{0.1m} = 80$

fanout (HIGH) $= \dfrac{I_{OH(max)}}{I_{IH(max)}} = \dfrac{400\mu}{20\mu} = 20$

The fanout is chosen the smaller of the two.

37. (B) In HIGH state the loading on the output of gate 1 is equivalent to six 74LS input load.

Hence load $= 6 \times I_{IH} = 6 \times 20\mu = 120\ \mu$A

38. (C) The NAND gate represent only a single input load in the LOW state. Hence only five loads in the LOW state.

load $= 5I_{IL} = 5 \times 0.4 = 2$ mA

****

CHAPTER

4.6

MICROPROCESSOR

1. After an arithmetic operation, the flag register of 8085 μP has the following contents

D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0
1	0	×	1	×	0	×	1

The contents of accumulator after operation may be

(A) 75 (B) 6C

(C) DB (D) B6

2. In an 8085 microprocessor, the instruction CMP B has been executed while the contents of accumulator is less than that of register B. As a result carry flag and zero flag will be respectively

(A) set, reset (B) reset, set

(C) reset, reset (D) set, set

3. Consider the following 8085 instruction

```
MVI     A, A9H
MVI     B, 57H
ADD     B
ORA     A
```

The flag status (S, Z, CY) after the instruction ORA A is executed, is

(A) (0, 1, 1) (B) (0, 1, 0)

(C) (1, 0, 0) (D) (1, 0, 1)

4. Consider the following set of 8085 instructions

```
          MVI     A, 8EH
          ADI     73H
          JC      DSPLY
          OUT     PORT1
          HLT
DSPLY :   XRA     A
          OUT     PORT1
          HLT
```

The output at PORT1 is

(A) 00 (B) FEH

(C) 01H (D) 11H

5. Consider the following 8085 assembly program

```
          MVI     A, DATA1
          MOV     B, A
          SUI     51H
          JC      DLT
          MOV     A, B
          SUI     82H
          JC      DSPLY
DLT :     XRA     A
          OUT PORT1
          HLT
DSPLY :   MOV     A, B
          OUT PORT2
          HLT
```

This program will display

(A) the bytes from 51H to 82H at PORT2

(B) 00H AT PORT1

(C) all byte at PORT1

(D) the bytes from 52H to 81H at PORT 2

6. It is desired to mask is the high order bits $(D_7 - D_4)$ of the data bytes in register C. Consider the following set of instruction

```
(a)       MOV     A, C
          ANI     F0H
          MOV     C, A
          HLT
```

(b)
```
MOV  A, C
MVI  B, F0H
ANA  B
MOV  C, A
HLT
```

(c)
```
MOV  A, C
MVI  B, 0FH
ANA  B
MOV  C, A
HLT
```

(d)
```
MOV  A, C
ANI  0FH
MOV  C, A
HLT
```

The instruction set, which execute the desired operation are

(A) a and b (B) c and d

(C) only a (D) only d

7. Consider the following 8085 instruction

```
XRA  A
MVI  B, 4AH
SUI  4FH
ANA  B
HLT
```

The contents of register A and B are respectively

(A) 05, 4A (B) 4F, 00

(C) B1, 4A (D) None of the above

8. Consider the following 8085 assembly program :

```
MVI  B, 89H
MOV  A, B
MOV  C, A
MVI  D, 37H
OUT  PORT1
HLT
```

The output at PORT1 is

(A) 89 (B) 37

(C) 00 (D) None of the above

9. Consider the sequence of 8085 instruction given below

```
LXI  H, 9258H
MOV  A, M
CMA
MOV  M, A
```

By this sequence of instruction the contents of memory location

(A) 9258H are moved to the accumulator

(B) 9258H are compared with the contents of the accumulator

(C) 8529H are complemented and stored at location 529H

(D) 5829H are complemented and stored at location 85892H

10. Consider the sequence of 8085 instruction

```
MVI  A, 5EH
ADI  A2H
MOV  C, A
HLT
```

The initial contents of resistor and flag are as follows

A	C	S	Z	CY
××	××	0	0	0

After execution of the instructions the contents of register and flags are

	A	C	S	Z	CY
(A)	10H	10H	0	0	1
(B)	10H	10H	1	0	0
(C)	00H	00H	1	1	0
(D)	00H	00H	0	1	1

11. It is desired to multiply the number 0AH by 0BH and store the result in the accumulator. The numbers are available in register B and C respectively. A part of the 8085 program for this purpose is given below :

```
       MVI  A, 00H
LOOP : ---------------
       ---------------
       ---------------
       ---------------
       ---------------
       HLT
       END
```

The sequence of instruction to complete the program would be

(A)
```
JNZ  LOOP
ADD  B
DCR  C
```

(B)
```
ADD  B
JNZ  LOOP
DCR  C
```

(C)
```
DCR  C
JNZ  LOOP
ADD  B
```

(D)
```
ADD  B
DCR  C
JNZ  LOOP
```

12. Consider the following assembly language program:

```
        MVI   B, 87H
        MOV   A, B
START : JMP   NEXT
        MVI   B, 00H
        XRA   B
        OUT   PORT1
        HLT
NEXT :  XRA   B
        JP    START1
        OUT   PORT2
        HLT
```

The execution of the above program in an 8085 will result in

(A) an output of 87H at PORT1

(B) an output of 87H at PORT2

(C) infinite looping of the program execution with accumulator data remaining at 00H

(D) infinite looping of the program execution with accumulator data alternating between 00H and 87H.

13. Consider the following 8085 program

```
        MVI   A, DATA1
        ORA   A,
        JM    DSPLY
        OUT   PORT1
        CMA
DSPLY : ADI   01H
        OUT   PORT1
        HLT
```

If DATA1 = A7H, the output at PORT1 is

(A) 47H (B) 58H

(C) 00 (D) None of the above

Statement for Q.14–15:

Consider the following program of 8085 assembly language:

```
        LXI   H 4A02H
        LDA   4A00H
        MOV   B, A
        LDA   4A01H
        CMP   B
        JZ    FNSH
        JC    GRT
        MOV   M, A
        JMP   FNSH
        MOV   M, B
FNSH :  HLT
```

14. If the contents of memory location 4A00H, 4A01H and 4A02H, are respectively A7H, 98H and 47H, then after the execution of program contents of memory location 4A02H will be respectively

(A) A7H (B) 98H

(C) 47H (D) None of the above

15. The memory requirement for this program is

(A) 20 Byte (B) 21 Byte

(C) 23 Byte (D) 18 Byte

16. The instruction, that does not clear the accumulator of 8085, is

(A) XRA A (B) ANI 00H

(C) MVI A, 00H (D) None of the above

17. The contents of some memory location of an 8085 μP based system are shown

Address Hex.	Contents (Hex.)
3000	02
3001	30
3002	00
3003	30

Fig. P4.6.17

The program is as follows

```
LHLD   3000H
MOV    E, M
INX    H
MOV    D, M
LDAX   D
MOV    L, A
INX    D
LDAX   D
MOV    H, A
```

The contents if HL pair after the execution of the program will be

(A) 0030 H (B) 3000 H

(C) 3002 H (D) 0230H

18. Consider the following loop

```
        XRA   A
        LXI   B, 0007H
LOOP :  DCX   B
        JNZ   LOOP
```

This loop will be executed

(A) 1 times (B) 8 times

(C) 7 times (D) infinite times

19. Consider the following loop

```
        LXI   H, 000AH
LOOP :  DCX   B
        MOV   A, B
        ORA   C
        JNZ   LOOP
```

This loop will be executed

(A) 1 time (B) 10 times

(C) 11 times (D) infinite times

20. The contents of accumulator after the execution of following instruction will be

```
MVI   A, A7H
ORA   A
RLC
```

(A) CFH (B) 4FH

(C) 4EH (D) CEH

21. The contents of accumulator after the execution of following instructions will be

```
MVI   A, B7H
ORA   A
RAL
```

(A) 6EH (B) 6FH

(C) EEH (D) EFH

22. The contents of the accumulator after the execution of the following program will be

```
MVI   A, C5H
ORA   A
RAL
```

(A) 45H (B) C5H

(C) C4H (D) None of the above

23. Consider the following set of instruction

```
MVI   A, BYTE1
RLC
MOV   B, A
RLC
RLC
ADD   B
```

If BYTE1 = 07H, then content of A, after the execution of program will be

(A) 46H (B) 70H

(C) 38H (D) 68H

24. Consider the following program

```
MVI   A, BYTE1
RRC
RRC
```

If BYTE1 = 32H, the contents of A after the execution of program will be

(A) 08H (B) 8CH

(C) 12H (D) None of the above

25. Consider the following program

```
        MVI   A, DATA
        MVI   B, 64H
        MVI   C, C8H
        CMP   B
        JC    RJCT
        CMP   C
        JNC   RJCT
        OUT   PORT1
        HLT
RJCT :  SUB A
        OUT   PORT1
        HLT
```

If the following sequence of byte is loaded in accumulator,

DATA (H)	58	64	73	B4	C8	FA

then sequence of output will be

(A) 00, 00, 73, B4, 00, FA

(B) 58, 64, 00, 00, C8, FA

(C) 58, 00, 00, 00, C8, FA

(D) 00, 64, 73, B4, 00, FA

26. Consider the following instruction to be executed by a 8085 μp. The input port has an address of 01H and has a data 05H to input:

```
IN    01H
ANI   80H
```

After execution of the two instruction the contents of flag register are

(A)	1	0	×	1	×	1	×	0
(B)	0	1	×	0	×	1	×	0
(C)	0	1	×	1	×	1	×	0
(D)	0	1	×	1	×	0	×	0

```
          ORA     A          ;Set flag
          JM      DSPLY      ;If negative jump to
                             ;DSPLY
          OUT     PORT1      ;A → PORT1
DSPLY :   CMA                ;Complement A
          ADI     01H        ;A+1 → A
          OUT     PORT1      ;A → PORT1
          HLT
```

This program displays the absolute value of DATA1. If DATA1 is negative, it determine the 2's complements and display at PORT1.

14. (A)
```
          LXI     H, 4A02H   ;Store destination address
                             ;in HL pair
          LDA     4A00H      ;Load A with contents of
                             ;memory location A00H
          MOV     B, A       ;A → B
          LDA     4A01H      ;Load A with contents of
                             ;memory location 4A01H
          CMA     B          ;Compare A and B
          JZ      FNSH       ;Jump to FNSH if two
                             ;number are equal
          JC      GRT        ;If CY = 1, (A <B) jump
                             ;to GRT
          MOV     M, A       ;Otherwise A → (4A02H)
          JMP     FNSH
GRT :     MOV     M, B
FNSH :    HLT
```

This program find the larger of the two number stored in location 4A00H and 4A01H and store it in memory location 4A002.

A7H > 98H   Thus A7H will be stored at 4A02H.

15. (C) Operand R, M or implied : 1–Byte instruction

Operand 8–bit : 2–Byte instruction

Operand 16–bit : 3–Byte instruction

3–Byte instruction are: LXI, LDA, JZ, JC, JMP

P–Byte instruction are : MOV, CMP, HLT

Hence memory $= 3 \times 6 + 1 \times 5 = 23$ Byte.

16. (D) All instruction clear the accumulator

```
          XRA     A          ;A ⊕ A
          ANI     00H        ;A AND 00
          MVI     A          ;00 → A
```

17. (C)
```
          LHLD    3000H      ;(3000A) → HL = 3002H
          MOV     E, M       ;(3002H) → E = 00
          INX     H          ;HL +1 → HL = 3003H
          MOV     D, M       ;M → D=(3003H) = 30H
          LDAX    D          ;(DE) → A=(3000H) = 02H
          MOV     L, A       ;A → L = 02H
          INX     D          ;DE +1 → DE = 3001H
          LDAX    D          ;(DE) → A = (3001) = 30H
          MOV     H, A       ;A → H = 30H
```

Hence HL pair contain 3002H.

18. (A) The instruction XRA will set the Z flag. LXI and DCX does not alter the flag. Hence this loop will be executed 1 times.

19. (B)
```
          LXI     B, 000AH   ;00 → C, 0AH → B
LOOP :    DCX     B          ; CB – 1 → B,
                             ;flag not affected
          MOV     A, B       ;B → A
          ORA     C          ;A OR C → A, set flag
          JNZ     LOOP
```

Hence this loop will be executed 0AH or ten times.

20. (B)
```
          MVI     A, B7H     ;B7H → A
          ORA     A          ;Set Flags, CY = 1
          RLC                ;Rotate accumulator left
```

The contents of bit D_7 are placed in bit D_0.

	Accumulator
Before RLC	10100111
After RLC	01001111

21. (A) RAL instruction rotate the accumulator left through carry.

$D_7 \to CY$, $CY \to D_0$, ORA reset the carry.

	Accumulator	CY
Before RAL	10110111	0
After RAL	01101110	1

22. (A) RRC instruction rotate the accumulator right and D_0 is placed in D_7.

```
          MVI     A, C5H     ;C5H → A
          ORA     A          ;Reset Carry flag
          RAL                ;Rotate A left through
                             ;carry, A = 8AH
          RRC                ;Rotate A right, A = 45H
```

23. (A) This program multiply BYTE1 by 10. Hence content of A will be 46H.

$07H = 07_{10}$, $7 \times 10 = 70$, $70_{10} = 46H$

24. (B) Contents of Accumulator A = 0011 0010

After First RRC = 0001 1001

After second RRC = 1000 1100

25. (D) This program will display the number between 64H to C8H including 64H. C8H will not be displayed. Thus (D) is correct option.

26. (C) 05H AND 80H =00

After the ANI instruction S, Z and P are modified to reflect the result of operation. CY is reset and AC is set . Thus,

S = 0, Z = 1, AC = 1, P =1, CY = 0

27. (B) ACI 56H ;A + 56H + CY $\rightarrow$ A

37H + 56H + 1 =8EH

28. (C) Instruction load the register pairs HL with 01FFH. SHLD instruction store the contents of L in the memory location 2050H and content of H in the memory location 2051H. Contents of HL are not altered.

29. (B) At a time 8085 can drive only a digit. In a second each digit is refreshed 500 times. Thus time given to each digit $= \dfrac{1}{(5 \times 500)} = 0.4$ ms.

30. (C) The stack pointer register SP point to the upper memory location of stack. When data is pushed on stack, it stores above this memory location.

31. (B) Line 5 push the content of HL register pair on stack. The contents of L will go to 03FFH and contents of H will go to 03FEH. Hence memory location 03FEH contain 22H.

32. (C) Contents of register pair B lie on the top of stack when POP H is executed, HL pair will be loaded with the contents of register pair B.

33. (C) The instruction PUSH B store the contents of BC at stack. The POP PSW instruction copy the contents of BC in to PSW. The contents of register C will be copied into flag register.

$D_0 = 1 =$ carry flag, $D_6 = 0 =$ zero flag.

Hence zero flag will be reset and carry will be set.

34. (A)

```
      MVI A  DATA1    ;DATA1 → A
      ORA    A        ; Set flag
      JP     DSPLY    ;If A is positive, then
                      ;jump to DSPLY
      XRA    A        ; Clear A
DSPLY OUT    PORT1    ; A → PORT2
      HLT
```

If DATA1 is positive, it will be displayed at port1 otherwise 00.

****

30. $y(t) = u(t) * h(t)$, where $h(t) = \begin{cases} e^{2t}, & t < 0 \\ e^{-3t}, & t > 0 \end{cases}$

(A) $\frac{1}{2}e^{-2t}u(-t-1) + \frac{5}{6} - \frac{1}{3}e^{-3t}u(-t)$

(B) $\frac{1}{2}e^{2t}u(-t-1) + \frac{5}{6} - \frac{1}{3}e^{-3t}u(-t)$

(C) $\frac{1}{2}e^{2t} + \frac{1}{6}[5 - 3e^{2t} - 2e^{-3t}]u(t)$

(D) $\frac{1}{2}e^{2t} + \frac{1}{6}[5 - 3e^{2t} - 2e^{-3t}]u(-t)$

Statement for Q.31-34:

The impulse response of LTI system is given. Determine the step response.

31. $h(t) = e^{-|t|}$

(A) $2 + e^{t} - e^{-t}$ (B) $e^{t}u(-t+1) + 2 - e^{-t}$

(C) $e^{t}u(-t+1) + [2 - e^{-t}]u(t)$ (D) $e^{t} + [2 - e^{-t} - e^{t}]u(t)$

32. $h(t) = \delta^{(2)}(t)$

(A) 1 (B) $u(t)$

(C) $\delta^{(3)}(t)$ (D) $\delta(t)$

33. $h(t) = u(t) - u(t-4)$

(A) $tu(t) + (1-t)u(t-4)$ (B) $tu(t) + (1-t)u(t-4)$

(C) $1 + t$ (D) $(1+t)u(t)$

34. $h(t) = y(t)$

(A) $u(t)$ (B) t

(C) 1 (D) $tu(t)$

Statement for Q.35-38:

The system described by the differential equations has been specified with initial condition. Determine the output of the system and choose correct option.

35. $\frac{dy(t)}{dx} + 10y(t) = 2x(t),\ y(0^-) = 1,\ x(t) = u(t)$

(A) $\frac{1}{5}(1 + 4e^{-10t})u(t)$ (B) $\frac{1}{5}(1 + 4e^{-10t})$

(C) $-\frac{1}{5}(1 + 4e^{-10t})u(t)$ (D) $-\frac{1}{5}(1 + 4e^{-10t})$

36. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt},$

$$y(0^-) = 0,\ \left.\frac{dy(t)}{dt}\right|_{0^-} = 1,\ x(t) = \sin t\, u(t)$$

(A) $\frac{5}{34}\sin t + \frac{3}{34}\cos t + \frac{1}{6}e^{-t} - \frac{13}{61}e^{-4t},\ t \ge 0$

(B) $\frac{5}{34}\sin t + \frac{3}{34}\cos t - \frac{13}{51}e^{-4t} + \frac{1}{6}e^{-t},\ t \ge 0$

(C) $\frac{3}{34}\sin t + \frac{5}{34}\cos t - \frac{13}{51}e^{-4t} + \frac{1}{6}e^{-t},\ t \ge 0$

(D) $\frac{3}{34}\sin t + \frac{5}{34}\cos t + \frac{1}{6}e^{-4t} - \frac{13}{51}e^{-4t},\ t \ge 0$

37. $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t),$

$$y(0^-) = -1,\ \left.\frac{dy(t)}{dt}\right|_{0^-} = 1,\ x(t) = e^{-t}u(t)$$

(A) $\frac{2}{3}e^{-t} - \frac{5}{2}e^{-2t} + \frac{5}{6}e^{-4t},\ t \ge 0$

(B) $\frac{2}{3} + \frac{5}{2}e^{-2t} + \frac{5}{6}e^{-4t},\ t \ge 0$

(C) $4 + 5(3e^{-2t} + e^{-4t}),\ t \ge 0$

(D) $4 - 5(3e^{-2t} + e^{-4t}),\ t \ge 0$

38. $\frac{d^2y(t)}{dt^2} + y(t) = \frac{3dx(t)}{dt},$

$$y(0^-) = -1,\ \left.\frac{dy(t)}{dt}\right|_{0^-} = 1,\ x(t) = 2te^{-t}u(t)$$

(A) $\sin t + 4\cos t - 3te^{-3t} + t,\ t \ge 0$

(B) $4\sin t - \cos t - 3te^{-t},\ t \ge 0$

(C) $\sin t - 4\cos t + 3te^{-3t} + t,\ t \ge 0$

(D) $4\sin t + \cos t - 3te^{-t},\ t \ge 0$

39. The raised cosine pulse $x(t)$ is defined as

$$x(t) = \begin{cases} \frac{1}{2}(\cos \omega t + 1), & -\frac{\pi}{\omega} \le t \le \frac{\pi}{\omega} \\ 0, & \text{otherwise} \end{cases}$$

The total energy of $x(t)$ is

(A) $\frac{3\pi}{4\omega}$ (B) $\frac{3\pi}{8\omega}$

(C) $\frac{3\pi}{\omega}$ (D) $\frac{3\pi}{2\omega}$

40. The sinusoidal signal $x(t) = 4\cos(200t + \pi/6)$ is passed through a square law device defined by the input output relation $y(t) = x^2(t)$. The DC component in the signal is

(A) 3.46 (B) 4

(C) 2.83 (D) 8

41. The impulse response of a system is $h(t) = \delta(t-0.5)$. If two such systems are cascaded, the impulse response of the overall system will be

(A) $0.5\delta(t-0.25)$ (B) $\delta(t-0.25)$

(C) $\delta(t-1)$ (D) $0.5\delta(t-1)$

42. Fig. P5.1.40 show the input $x(t)$ to a LTI system and impulse response $h(t)$ of the system.

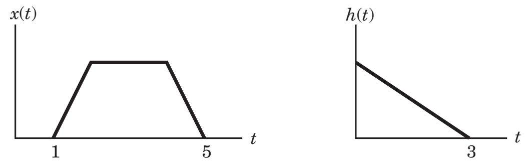


Fig P5.1.42

The output of the system is zero every where except for the

(A) $0 < t < 5$ (B) $0 < t < 8$

(C) $1 < t < 5$ (D) $1 < t < 8$

43. Consider the impulse response of two LTI system

$S_1 : h_1(t) = e^{-(1-2j)t}u(t)$

$S_2 : h_2(t) = e^{-t}\cos 2t\, u(t)$

The stable system is

(A) S_1 (B) S_2

(C) Both S_1 and S_2 (D) None

44. The non-invertible system is

(A) $y(t) = x(t-4)$ (B) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(C) $y(t) = \dfrac{dx(t)}{dt}$ (D) None of the above

45. A continuous-time linear system with input $x(t)$ and output $y(t)$ yields the following input-output pairs:

$$x(t) = e^{j2t} \Leftrightarrow y(t) = e^{j5t}$$

$$x(t) = e^{-j2t} \Leftrightarrow y(t) = e^{-j5t}$$

If $x_1(t) = \cos(2t-1)$, the corresponding $y_1(t)$ is

(A) $\cos(5t-1)$ (B) $e^{-j}\cos(5t-1)$

(C) $\cos 5(t-1)$ (D) $e^{j}\cos(5t-1)$

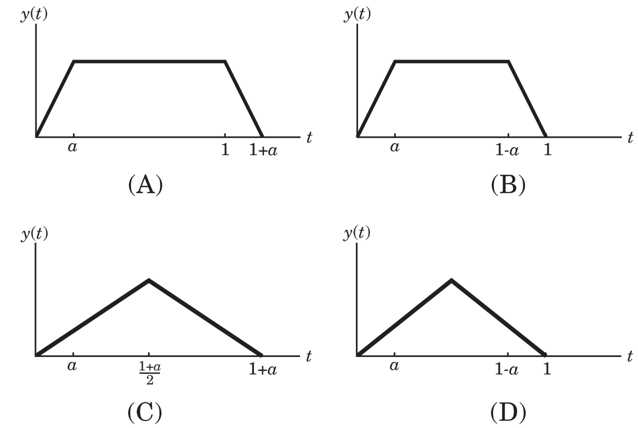
Statement for Q.46–47:

Suppose that

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and}$$

$$h(t) = x\left(\frac{t}{a}\right), \text{ where } 0 < a \le 1.$$

46. The $y(t) = x(t) * h(t)$ is



47. If $dy(t)/dt$ contains only three discontinuities, the value of a is

(A) 1 (B) 2

(C) 3 (D) 0

48. Consider the signal $x(t) = \delta(t+2) - \delta(t-2)$. The value of E_∞ for the signal $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ is

(A) 4 (B) 2

(C) 1 (D) ∞

49. The response of a system S to a complex input $x(t) = e^{j5t}$ is specified as $y(t) = te^{j5t}$. The system

(A) is definitely LTI

(B) is definitely not LTI

(C) may be LTI

(D) information is insufficient

50. The response of a system S to a complex input $x(t) = e^{j8t}$ is specified as $y(t) = \cos 8t$. The system

(A) is definitely LTI

(B) is definitely not LTI

(C) may be LTI

(D) information is insufficient.

51. The auto-correlation of the signal $x(t) = e^{-t}u(t)$ is

(A) $\frac{1}{2}e^{t}u(-t) + \frac{1}{2}e^{-t}u(t)$ (B) $\frac{e^{t}}{2} + \frac{1}{2}(e^{-t} - e^{t})u(t)$

(C) $\frac{1}{2}e^{-t}u(-t) + \frac{1}{2}e^{-t}u(t)$ (D) $\frac{1}{2}e^{t}u(-t) - \frac{1}{2}e^{-t}u(t)$

****

SOLUTIONS

1. (A) $\frac{2\pi}{T} = 60\pi \quad \Rightarrow \quad T = \frac{\pi}{30}$

2. (C) $T_1 = \frac{2\pi}{5}$ s, $T_2 = \frac{2\pi}{7}$ s, LCM $\left(\frac{2\pi}{5}, \frac{2\pi}{7}\right) = 2\pi$

3. (D) Not periodic because of t.

4. (D) Not periodic because least common multiple is infinite.

5. (C) $y(t)$ is not periodic although $\sin t$ and $6\cos 2\pi t$ are independently periodic. The fundamental frequency can't be determined.

6. (C) This is energy signal because

$$E_\infty = \int_{-\infty}^{\infty} |x(t)| dt < \infty \;= \int_{-\infty}^{\infty} e^{-4t} u(t) dt \;= \int_{0}^{\infty} e^{-4t} dt = \frac{1}{4}$$

7. (A) $|x(t)| = 1,\ E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt \;= \infty$

So this is a power signal not a energy.

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \;= 1$$

8. (D) $v(t)$ is sum of 3 unit step signal starting from, 1, 2, and 3, all signal ends at 4.

9. (A) The function 1 does not describe the given pulse. It can be shown as follows :

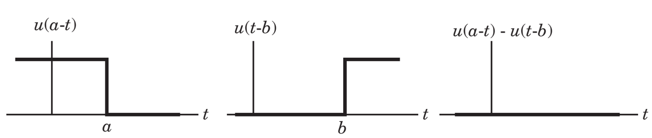


Fig S5.1.3.9

10. (B)

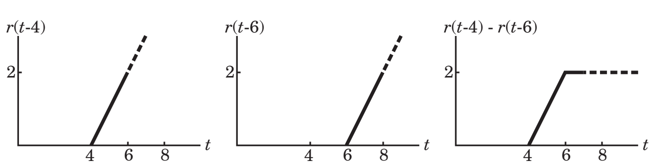


Fig S5.1.10

11. (C)

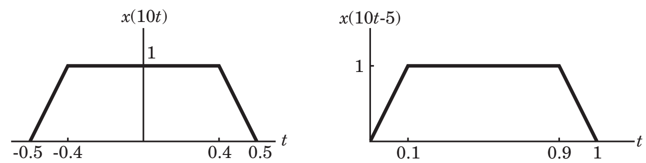


Fig S5.1.11

12. (D) Multiplication by 5 will bring contraction on time scale. It may be checked by $x(5 \times 0.8) = x(4)$.

13. (A) Division by 5 will bring expansion on time scale. It may be checked by $y(t) = x\left(\frac{20}{5}\right) = x(4)$.

14. (C) $y(t) = \begin{cases} 1, & \text{for } -5 < t < -4 \\ -1, & \text{for } \quad 4 < t < 5 \\ 0, & \text{otherwise} \end{cases}$

$$E = \int_{-5}^{-4} (1)^2 dt + \int_{4}^{5} (-1)^2 dt \;= 2$$

15. (D) $E = 2\int_0^5 x^2(t) dt \;= 2\int_0^4 (1)^1 dt + 2\int_4^5 (5-t)^2 dt$

$$= 8 + \frac{2}{3} = \frac{26}{3}$$

16. (B) Let $x_1(t) = v(t)$ then $y_1(t) = u\{v(t)\}$

Let $x_2(t) = kv(t)$ then $y_2(t) = u\{kv(t)\} \neq k y_1(t)$

(Not homogeneous not linear)

$y_1(t) = u\{v(t)\}$,

$y_2(t) = u\{v(t - t_o)\} = y_1(t - t_o)$ (Time invariant)

The response at any time depends only on the excitation at time $t = t_o$ and not on any future value.

(Causal)

17. (C) $y_1(t) = v(t-5) - v(3-t)$

$y_2(t) = kv(t-5) - kv(3-t) = ky_1(t)$ (Homogeneous)

Let $x_1(t) = v(t)$ then $y_1(t) = v(t-5) - v(3-t)$

Let $x_2(t) = 2w(t)$ then $y_2(t) = w(t-5) - w(3-t)$

Let $x_3(t) = x(t) + w(t)$

Then $y_3(t) = v(t-5) + w(t-5) - v(3-t) - w(3-t)$

$= y_1(t) + y_2(t)$ (Additive)

Since it is both homogeneous and additive, it is also linear.

$y_1(t) = v(t-5) - v(3-t)$

$y_2(t) = v(t - t_o - 5) - v(3 - t + t_o) = y_1(t - t_o)$

(Time invariant)

At time, $t = 0$, $y(0) = x(-5) - x(3)$. Therefore the response at time, $t = 0$ depends on the excitation at a later time $t = 3$. (Not causal)

If $x(t)$ is bounded then $x(t-5)$ and $x(3-t)$ are bounded and so is $y(t)$. (Stable)

18. (D) $y_1(t) = v\left(\frac{t}{2}\right)$, $y_2(t) = kv\left(\frac{t}{2}\right) = ky_1(t)$

(Homogeneous)

$x_3 = v(t) + w(t)$ then

$y_3(t) = v\left(\frac{t}{2}\right) + w\left(\frac{t}{2}\right) = y_1(y) + y_2(t)$ (Additive)

Since it is both homogeneous and additive, it is also linear

$y_1(t) = v\left(\frac{t}{2}\right)$, $y_2\left(\frac{t}{2} - t_o\right) \neq y(t - t_o) = v\left(\frac{t - t_o}{2}\right)$

(Time variant)

At time $t = -2$, $y(-2) = x(-1)$, therefore, the response at time $t = -2$, depends on the excitation at a later time, $t = -1$. (Not causal)

It $x(t)$ is bounded then $y(t)$ is bounded. (Stable)

19. (C) $y_1(t) = \cos 2\pi t\ v(t)$

$y_2(t) k \cos 2\pi t\ v(t) = ky_1(t)$ (Homogeneous)

$x_3(t) = v(t) + w(t)$

$y_3(t) = \cos 2\pi t\,[v(t) + w(t)] = y_1(t) + y_2(t)$ (Additive)

Since it is both homogeneous and additive. It is also linear.

$y_1(t) = \cos 2\pi t\ v(t)$

$y_2(t) = \cos 2\pi t\,(t - t_o) \neq y(t - t_o)$

$= \cos\,[2\pi(t - t_o)]v(t - t_o)$ (Time Variant)

The response at any time $t = t_o$ depends only on the excitation at that time and not on the excitation at any later time. (Causal)

If $x(t)$ is bounded then $y(t)$ is bounded. (Stable)

20. (C) $y_1(t) = |v(t)|$, $y_2(t) = |kv(t)| = |k| y_1(t)$

If k is negative $|k| y_1(t) \neq ky_1(t)$

(Not Homogeneous Not linear).

$y_1(t) = |v(t)|$, $y_2(t) = |y(t - t_o)| = y_1(t - t_o)$

(Time Invariant)

The response at any time $t = t_o$ depends only on the excitation at that time and not on the excitation at any later time. (Causal)

If $x(t)$ is bounded then $y(t)$ is bounded. (Stable)

21. (C) All option are linear. So it is not required to check linearity.

Let $x_1(t) = v(t)$ then $t\frac{d}{dt}y_1(t) - 8y_1(t) = v(t)$

Let $x_2(t) = v(t - t_o)$ then $t\frac{d}{dt}y_2(t) - 8y_2(t) = v(t - t_o)$

The first equation can be written as

$$(t - t_o)\frac{d}{dt}y(t - t_o) - 8y(t - t_o) = v(t - t_o)$$

This equation is not satisfied if $y_2(t) = y_1(t - t_o)$ therefore $y_2(t) \neq y_1(t - t_o)$ (Time Variant)

The system can be written as

$$y(t) = \int_{-\infty}^{t}\frac{x(\lambda)}{\lambda}d\lambda + 8\int_{-\infty}^{t}\frac{y(\lambda)}{\lambda}d\lambda$$

So the response at any time, $t = t_o$ depends on the excitation at $t \le t_o$, and not on any future values.

(Causal)

The Homogeneous solution to the differential equation is of the form $y(t) = kt^8$. If there is no excitation but the zero excitation, response is not zero. The response will increases without bound as time increases.

(Unstable)

22. (C) $y_1(t) = \int_{-\infty}^{t+3} v(\lambda)d\lambda$

$y_2(t) = \int_{-\infty}^{t+3} kv(\lambda)d\lambda = k\int_{-\infty}^{t+3} v(\lambda)d\lambda = ky_1(t)$ (Homogeneous)

$x_3(t) = v(t) + w(t)$

$y_3(t) = \int_{-\infty}^{t+3}[v(\lambda) + w(\lambda)]d\lambda = \int_{-\infty}^{t+3} v(\lambda)d\lambda + \int_{-\infty}^{t+3} w(\lambda)d\lambda$

$= y_1(t) + y_2(t)$ (Additive)

Since it is Homogeneous and additive, it is also linear.

$y_1(t) = \int_{-\infty}^{t+3} v(\lambda)d\lambda$

$y_2(t) = \int_{-\infty}^{t+3} v(\lambda - t_o)d\lambda = \int_{-\infty}^{t - t_o + 3} v(\lambda)d\lambda = y_1(t - t_o)$

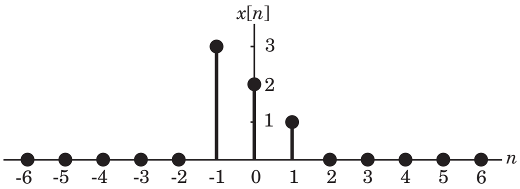
(Time invariant)

The response at any time, $t = t_o$, depends partially on the excitation at time to $t_o < t < (t_o + 3)$ which are in future. (Not causal)

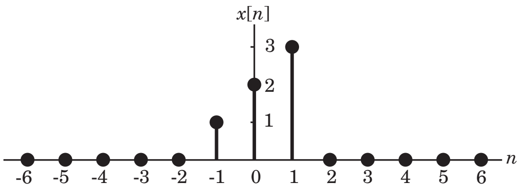
If $x(t)$ is a constant k, then $y(t) = \int_{-\infty}^{t+3} kd\lambda = k\int_{-\infty}^{t+3} d\lambda$ and as $t \to \infty$, $y(t)$ increases without bound. (unstable)

11. $x[n+2]y[n-2]$

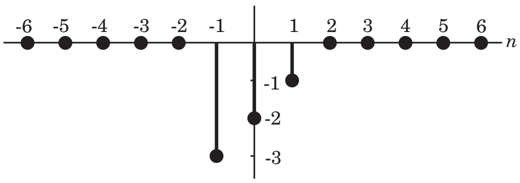
(A)



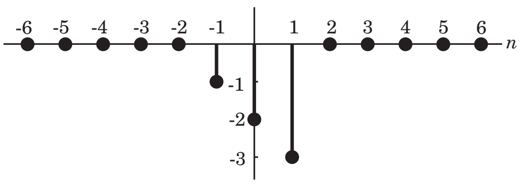
(B)



(C)



(D)



Statement for Q.12–15:

A discrete-time signal is given. Determine the period of signal and choose correct option.

12. $x[n]=\cos\frac{\pi n}{9}+\sin\left(\frac{\pi n}{7}+\frac{1}{2}\right)$

(A) periodic with period $N=126$

(B) periodic with period $N=32$

(C) periodic with period $N=252$

(D) Not periodic

13. $x[n]=\cos\left(\frac{n}{8}\right)\cos\left(\frac{\pi n}{8}\right)$

(A) Periodic with period 16π

(B) periodic with period $16(\pi+1)$

(C) periodic with period 8

(D) Not periodic

14. $x[n]=\cos\left(\frac{\pi n}{2}\right)-\sin\left(\frac{\pi n}{8}\right)+3\cos\left(\frac{\pi n}{4}+\frac{\pi}{3}\right)$

(A) periodic with period 16

(B) periodic with period 4

(C) periodic with period 2

(D) Not periodic

15. $x[n]=2e^{j\left(\frac{n}{6}-\pi\right)}$

(A) periodic with 12π (B) periodic with 12

(C) periodic with 11π (D) Not periodic

16. The sinusoidal signal has fundamental period $N=10$ samples. The smallest angular frequency, for which $x[n]$ is periodic, is

(A) $\frac{1}{10}$ rad/cycle (B) 10 rad/cycle

(C) 5 rad/cycle (D) $\frac{\pi}{5}$ rad/cycle

17. Let $x[n], -5\le n\le 3$ and $h[n], 2\le n\le 6$ be two finite duration signals. The range of their convolution is

(A) $-7\le n\le 9$ (B) $-3\le n\le 9$

(C) $2\le n\le 3$ (D) $-5\le n\le 6$

Statement for Q.18–26:

$x[n]$ and $h[n]$ are given in the question. Compute the convolution $y[n]=x[n]*h[n]$ and choose correct option.

18. $x[n]=\{1, 2, 4\}$, $h[n]=\{1, 1, 1, 1, 1\}$

(A) $\{1, 3, 7, 7, 7, 6, 4\}$

(B) $\{1, 3, 3, 7, 7, 6, 4\}$

(C) $\{1, 2, 4\}$

(D) $\{1, 3, 7\}$

19. $x[n]=\{1, 2, 3, 4, 5\}$, $h[n]=\{1\}$

(A) $\{1, 3, 6, 10, 15\}$ (B) $\{1, 2, 3, 4, 5\}$

(C) $\{1, 4, 9, 16, 20\}$ (D) $\{1, 4, 6, 8, 10\}$

20. $x[n]=\{1, 2, -1\}$, $h[n]=x[n]$

(A) $\{1, 4, 1\}$ (B) $\{1, 4, 2, -4, 1\}$

(C) $\{1, 2, -1\}$ (D) $\{2, 4, -2\}$

21. $x[n] = \{\underset{\uparrow}{1}, -2, 3\}$, $h[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

(A) $\{\underset{\uparrow}{1}, -2, 4, 1, 1, 1\}$

(B) $\{\underset{\uparrow}{0}, 0, 3\}$

(C) $\{\underset{\uparrow}{0}, 0, 3, 1, 1, 1, 1\}$

(D) $\{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

22. $x[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$, $h[n] = \{1, \underset{\uparrow}{-2}, 3\}$

(A) $\{0, \underset{\uparrow}{0}, 1, -1, 2, 2, 1, 3\}$

(B) $\{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

(C) $\{1, \underset{\uparrow}{-2}, 3, 1, 1, 2, 1, 1\}$

(D) $\{\underset{\uparrow}{1}, -2, 3, 1, 1, 1, 1\}$

23. $x[n] = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h[n] = \{1, -2, \underset{\uparrow}{-3}, 4\}$

(A) $\{1, -1, -2, \underset{\uparrow}{4}, 1, 1\}$

(B) $\{\underset{\uparrow}{1}, -1, -2, 4, 1, 1\}$

(C) $\{1, -1, \underset{\uparrow}{-5}, 2, 3, -5, 1, 4\}$

(D) $\{1, -1, -5, 2, 3, \underset{\uparrow}{-5}, 1, 4\}$

24. $x[n] = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$, $h[n] = x[n]$

(A) $\{\underset{\uparrow}{1}, 4, 4, 4, 10, 4, 4, 4, 1\}$

(B) $\{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4, 1\}$

(C) $\{1, 4, \underset{\uparrow}{4}, 10, 4, 4, 4, 1\}$

(D) $\{1, 4, 4, 10, 4, 4, 4, \underset{\uparrow}{1}\}$

25. $x[n] = \{1, 4, \underset{\uparrow}{-3}, 6, 4\}$, $h[n] = \{2, \underset{\uparrow}{-4}, 3\}$

(A) $\{2, 4, -19, \underset{\uparrow}{36}, -25, 2, 12\}$

(B) $\{4, -19, \underset{\uparrow}{36}, -25\}$

(C) $\{1, 4, \underset{\uparrow}{-3}, 6, 4\}$

(D) $\{\underset{\uparrow}{1}, 4, -3, 6, 4\}$

26. $x[n] = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0 & \text{elsewhere} \end{cases}$

$h(n) = \delta[n] - \delta[n-1] + \delta[n-4]$

(A) $\delta[n] - 2\delta[n-1] + 4\delta[n-4] + \delta[n-5]$

(B) $\delta[n+2] + \delta[n+1] - \delta[n] + 2\delta[n-3] + \delta[n-4] + \delta[n-5]$

(C) $\delta[n+2] - \delta[n+1] + \delta[n] + 2\delta[n-3] - \delta[n-4] + 2\delta[n-5]$

(D) $\delta[n] + 2\delta[n-1] + 4\delta[n-5] + \delta[n-5]$

Statement for Q.27–30:

In question $y[n]$ is the convolution of two signal. Choose correct option for $y[n]$.

27. $y[n] = (-1)^n * 2^n u[2n+2]$

(A) $\frac{4}{6}$

(B) $\frac{4}{6} u[-n+2]$

(C) $\frac{8}{3}(-1)^n u[-n+2]$

(D) $\frac{8}{3}(-1)^n$

28. $y[n] = \frac{1}{4^n} u[n] * u[n+2]$

(A) $\left(\frac{1}{3} - \frac{1}{4^n}\right) u[n]$

(B) $\left(\frac{1}{3} - \frac{12}{4^n}\right) u[n+2]$

(C) $\left(\frac{4}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^n\right) u[n+2]$

(D) $\left(\frac{16}{3} - \frac{1}{4^n}\right) u[n+2]$

29. $y[n] = 3^n u[-n+3] * u[n-2]$

(A) $\begin{cases} \frac{3^n}{2}, & n \le 5 \\ \frac{83}{2}, & n \ge 6 \end{cases}$

(B) $\begin{cases} 3^n, & n \le 5 \\ \frac{83}{2}, & n \ge 6 \end{cases}$

(C) $\begin{cases} \frac{3^n}{2}, & n \le 5 \\ \frac{81}{2}, & n \ge 6 \end{cases}$

(D) $\begin{cases} \frac{3^n}{6}, & n \le 5 \\ \frac{81}{2}, & n \ge 6 \end{cases}$

30. $y[n] = u[n+3] * u[n-3]$

(A) $(n+1)u[n]$ (B) $nu[n]$

(C) $(n-1)u[n]$ (D) $u[n]$

31. The convolution of $x[n] = \cos(\frac{\pi}{2}n)\, u[n]$ and $h[n] = u[n-1]$ is $f[n]u[n-1]$. The function $f[n]$ is

(A) $\begin{cases} 1, & n = 4m+1, \quad 4m+2 \\ 0, & n = 4m, \quad 4m+3 \end{cases}$

(B) $\begin{cases} 0, & n = 4m+1, \quad 4m+2 \\ 1, & n = 4m, \quad 4m+3 \end{cases}$

(C) $\begin{cases} 1, & n = 4m+1, \quad 4m+3 \\ 0, & n = 4m, \quad 4m+2 \end{cases}$

(D) $\begin{cases} 0, & n = 4m+1, \quad 4m+3 \\ 1, & n = 4m, \quad 4m+2 \end{cases}$

Statement for Q.32–38:

Let P be linearity, Q be time invariance, R be causality and S be stability. In question discrete time input $x[n]$ and output $y[n]$ relationship has been given. In the option properties of system has been given. Choose the option which match the properties for system.

32. $y[n] = rect(x[n])$

(A) P, Q, R (B) Q, R, S

(C) R, S, P (D) S , P, Q

33. $y[n] = nx[n]$

(A) P, Q, R, S (B) Q, R, S

(C) P, R (D) Q, S

34. $y[n] = \sum_{m=-\infty}^{n+1} u[m]$

(A) P, Q, R, S (B) R, S

(C) P, Q (D) Q, R

35. $y[n] = \sqrt{x[n]}$

(A) Q, R, S (B) R, S, P

(C) S, P, Q (D) P, Q, R

36. $x[n]$ as shown in fig. P5.2.36

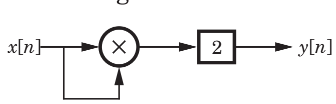


Fig. P5.2.36

(A) P, Q, R, S (B) Q, R, S

(C) P, Q (D) R, S

37. $x[n]$ as shown in fig. P5.2.37

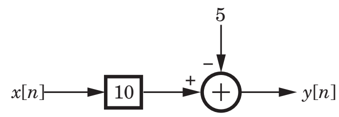


Fig. P5.2.37

(A) P, Q, R, S (B) Q, R, S

(C) P, R, S (D) P, Q, S

38. $x[n]$ as shown in fig. P5.2.38

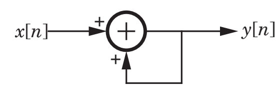


Fig. P5.2.38

(A) P, Q, R, S (B) P, Q, R

(C) P, Q (D) Q, R, S

Statement for Q.39–41:

Two discrete time systems S_1 and S_2 are connected in cascade to form a new system as shown in fig. P5.2.39–41.

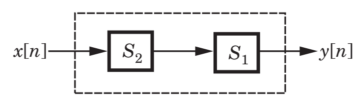


Fig. P5.2.39–41.

39. Consider the following statements

(a) If S_1 and S_2 are linear, the S is linear

(b) If S_1 and S_2 are nonlinear, then S is nonlinear

(c) If S_1 and S_2 are causal, then S is causal

(d) If S_1 and S_2 are time invariant, then S is time invariant

True statements are :

(A) a, b, c (B) b, c, d

(C) a, c, d (D) All

40. Consider the following statements

(a) If S_1 and S_2 are linear and time invariant, then interchanging their order does not change the system.

(b) If S_1 and S_2 are linear and time varying, then interchanging their order does not change the system.

True statement are

(A) Both a and b (B) Only a

(C) Only b (D) None

41. Consider the statement

(a) If S_1 and S_2 are noncausal, the S is non causal

(b) If S_1 and/or S_2 are unstable, the S is unstable.

True statement are :

(A) Both a and b (B) Only a

(C) Only b (D) None

42. The following input output pairs have been observed during the operation of a time invariant system :

$$x_1[n] = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{S} y_1[n] = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2[n] = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{S} y_2[n] = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3[n] = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{S} y_3[n] = \{1, \underset{\uparrow}{2}, 1\}$$

The conclusion regarding the linearity of the system is

(A) System is linear

(B) System is not linear

(C) One more observation is required.

(D) Conclusion cannot be drawn from observation.

43. The following input output pair have been observed during the operation of a linear system:

$$x_1[n] = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{S} y_1[n] = \{\underset{\uparrow}{1}, 2, -1, 0, 1\}$$

$$x_2[n] = \{1, \underset{\uparrow}{-1}, -1\} \xleftrightarrow{S} y_2[n] = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3[n] = \{\underset{\uparrow}{0}, 1, 1\} \xleftrightarrow{S} y_3[n] = \{\underset{\uparrow}{1}, 2, 1\}$$

The conclusion regarding the time invariance of the system is

(A) System is time-invariant

(B) System is time variant

(C) One more observation is required

(D) Conclusion cannot be drawn from observation

44. The stable system is

(A) $y[n] = x[n] + 1.1y[n-1]$

(B) $y[n] = x[n] - \frac{1}{2}(y[n-1] + y[n-2])$

(C) $y[n] = x[n] - (1.5y[n-1] + 0.4y[n-2])$

(D) Above all

45. The system shown in fig. P5.2.45 is

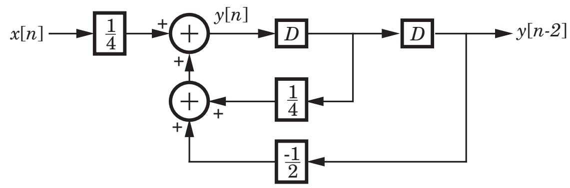


Fig. P5.2.45

(A) Stable and causal

(B) Stable but not causal

(C) Causal but unstable

(D) unstable and not causal

46. The impulse response of a LTI system is given as

$$h[n] = \left(-\frac{1}{2}\right)^n u[n].$$

The step response is

(A) $\frac{1}{3}\left(2 - \left(-\frac{1}{2}\right)^{n+1}\right)u[n]$ (B) $\frac{1}{3}\left(2 - \left(-\frac{1}{2}\right)^{n}\right)u[n]$

(C) $\frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^{n+1}\right)u[n]$ (D) $\frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^{n}\right)u[n]$

47. The difference equation representation for a system is

$$y[n] - \frac{1}{2}y[n-1] = 2x[n], \quad y[-1] = 3$$

The natural response of system is

(A) $\frac{3}{2}\left(-\frac{1}{2}\right)^n u[n]$ (B) $\frac{2}{3}\left(-\frac{1}{2}\right)^n u[n]$

(C) $\frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$ (D) $\frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$

48. The difference equation representation for a system is

$$y[n] - 2y[n-1] + y[n-2] = x[n] - x[n-1]$$

If $y[n] = 0$ for $n < 0$ and $x[n] = \delta[n]$, then $y[2]$ will be

(A) 2 (B) –2

(C) –1 (D) 0

49. Consider a discrete-time system S whose response to a complex exponential input $e^{j\pi n/2}$ is specified as

$$S : e^{j\pi n/2} \Rightarrow e^{j\pi 3n/2}$$

24. (B) $y[n] = \{1, \quad 4, \quad 4, \quad 10, \quad 4, \quad 4, \quad 4, \quad 1\}$
$\uparrow$

	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

Fig. S5.2.24

25. (A) $y[n] = \{2, \quad 4, \quad -19, \quad 36, \quad -25, \quad 2, \quad 12\}$
$\uparrow$

	1	4	-3	6	4
2	2	8	-6	12	8
-4	-4	-16	12	-24	-16
3	3	12	-9	18	12

Fig. S5.2.25

26. (B) $x[n] = \{1, \quad 2, \quad 1, \quad 1\}$, $h[n] = \{1, \quad -1, \quad 0, \quad 0, \quad 1\}$
$\uparrow$ $\qquad$ $\uparrow$

	1	2	1	1
1	1	2	1	-1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1

Fig. S5.2.26

$y[n] = \{1, \quad 1, \quad -1, \quad 0, \quad 0, \quad 2, \quad 1, \quad 1\}$
$\uparrow$

$$y[n] = \delta[n+2] + \delta[n+1] - \delta[n] + 2\delta[n-3] + \delta[n-4] + \delta[n-5]$$

27. (D) $y[n] = \sum_{k=n-2}^{\infty} (-1)^k 2^{n-k} = 2^n \sum_{k=n-2}^{\infty} \left(-\frac{1}{2}\right)^k$

$$= \frac{2^n\left(-\frac{1}{2}\right)^{n-2}}{1+\frac{1}{2}} = \frac{8}{3}(-1)^n$$

28. (C) For $n+2 < 0$ or $n < -2$, $y[n] = 0$

for $n + 2 \geq 0$ or $n \geq -2$, $y[n] = \sum_{k=0}^{n+2} \frac{1}{4}k = \frac{4}{3} - \frac{1}{12}\frac{1}{4^n}$,

$$\Rightarrow \quad y[n] = \left(\frac{4}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^n\right) u[n+2]$$

29. (D) For $n - 2 \leq 3$ or $n \leq 5$, $y[n] = \sum_{k=\infty}^{n-2} 3^k = \frac{3^n}{6}$

for $n - 2 \geq 4$ or $n \geq 6$, $y[n] = \sum_{k=-\infty}^{3} 3^k = \frac{81}{2}$,

$$\Rightarrow \quad y[n] = \begin{cases} \frac{3^n}{6}, & n \leq 5 \\ \frac{81}{2}, & n \geq 6 \end{cases}$$

30. (A) For $n - 3 < -3$ or $n < 0$, $y[n] = 0$

for $n - 3 \geq -3$ or $n > 0$, $y[n] = \sum_{k=-3}^{n-3} 1 = n + 1$,

$y[n] = (n+1)u[n]$

31. (A) For $n - 1 < 0$ or $n < 1$, $y[n] = 0$

For $n - 1 \geq 0$ or $n \geq 1$, $y[n] = \sum_{k=0}^{n-1} \cos\left(\frac{\pi}{2}k\right)$

$$\Rightarrow \quad y[n] = \begin{cases} 1, & n = 4m+1, \quad 4m+2 \\ 0, & n = 4m, \quad 4m+3 \end{cases}$$

32. (B) $y_1[n] = rect\,(v[n])$, $\quad y_2[n] = rect\,(kv[n])$

$y_2[n] \neq k\, y_1[n]$ (Not Homogeneous not linear)

$y_1[n] = rect\,(v[n])$, $\quad y_2[n] = rect\,(v[n-n_o])$

$y_1[n-n_o] = rect\,(v[n-n_o]) = y_2[n]$ (Time Invariant)

At any discrete time $n = n_o$, the response depends only on the excitation at that discrete time. (Causal)

No matter what values the excitation may have the response can only have the values zero or one.

(Stable)

33. (C) $y_1[n] = nv[n]$, $\quad y_2[n] = nkv[n]$

$ky_1[n] = y_2[n]$ (Homogeneous)

Let $x_1[n] = v[n]$ then $y_1[n] = nv[n]$

Let $x_2[n] = w[n]$ then $y_2[n] = nw[n]$

Let $x_3[n] = v[n] + w[n]$ then

$y_3[n] = n(v[n] + w[n]) = nv[n] + nw[n]$

$= y_1[n] + y_2[n]$ (Additive)

Since the system is homogeneous and additive, it is also linear.

$y_1[n-n_o] = (n-n_o)v[n-n_o] \neq y_n[n] = nv[n-n_o]$

(Time variant)

At any discrete time, $n = n_o$ the response depends only on the excitation at that same time. (Causal)

If the excitation is a constant, the response is unbounded as n approaches infinity. (Unstable)

34. (C) $y_1[n] = \sum_{m=-\infty}^{n+1} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n+1} kv[m]$

$y_2[n] = ky_1[n]$ (Homogeneous)

$y_1[n] = \sum_{n=-\infty}^{n+1} v[m]$, $y_2[n] = \sum_{n=-\infty}^{n+1} w[m]$

$y_3[n] = \sum_{m=-\infty}^{n+1} (v[n] + w[m])$

$= \sum_{m=-\infty}^{n+1} v[m] + \sum_{m=-\infty}^{n+1} w[n] = y_1[n] + y_2[n]$ (Additive)

Since the system is homogeneous and additive it is also linear

$y_1[n] = \sum_{m=-\infty}^{n+1} v[n]$, $y_2[n] = \sum_{m=-\infty}^{n+1} v[m - n_o]$

$y_1[n - n_o] = \sum_{m=-\infty}^{n-n_o+1} v[m] = \sum_{q=-\infty}^{n+1} v[q - n_o] = y_2[n]$

(Time Invariant)

At any discrete time, $n = n_o$, the response depends on the excitation at the next discrete time in future.

(Anti causal)

If the excitation is a constant, the response increases without bound. (Unstable)

35. (A) $y_1[n] = \sqrt{v[n]}$, $y_2 = \sqrt{kv[n]} = \sqrt{k}\sqrt{v[n]}$

$ky_1[n] = k\sqrt{v[n]} \neq y_2[n]$ (Not Homogeneous Not linear)

$y_1[n] = \sqrt{v[n]}$, $y_2[n] = \sqrt{v[n - n_o]}$

$y_1[n - n_o] = \sqrt{v[n - n_o]} = y_2[n]$ (Time Invariant)

At any discrete time $n = n_o$, the response depends only on the excitation at that time (Causal)

If the excitation is bounded, the response is bounded.

(Stable).

36. (B) $y[n] = 2x^2[n]$

Let $x_1[n] = v[n]$ then $y_1[n] = 2v^2[n]$

Let $x_2[n] = kv[n]$ then $y_2[n] = 2k^2v^2[n]$

$ky[n] \neq y_2[n]$ (Not homogeneous Not linear)

Let $x_1[n] = v[n]$ then $y_1[n] = 2v^2[n]$

Let $x_2[n] = v[n - n_o]$ then $y_2[n] = 2v^2[n - n_o]$

$y_1[n - n_o] = 2v[n - n_o] = y_2[n]$ (Time invariant)

At any discrete time, $n = n_o$, the response depends only on the excitation at that time. (Causal)

If the excitation is bounded, the response is bounded.

(Stable).

37. (B) $y_1[n] = 10v[n] - 5$, $y_2[n] = 10kv[n] - 5$

$y_2[n] \neq ky_1[n]$ (Not Homogeneous so not linear)

$y_1[n] = 10v[n] - 5$, $y_2[n] = 10v[n - n_o] - 5$

$y_1[n - n_o] = 10v[n - n_o] - 5, = y_2[n]$ (Time Invariant)

At any discrete time, $n = n_o$ the response depends only on the excitation at that discrete time and not on any future excitation. (Causal)

If the excitation is bounded, the response is bounded.

(Stable).

38. (B) $y[n] = x[n] + y[n-1]$, $y[n-1] = x[n-1] + y[n-2]$

$y[n] = x[n] + x[n-1] + y[n-2]$, Then by induction

$y[n] = x[n-1] + x[n-2] + \ldots x[n-k] + \ldots = \sum_{k=0}^{\infty} x[n-k]$

Let $m = n - k$ then $y[n] = \sum_{m=n}^{-\infty} x[m] = \sum_{m=-\infty}^{n} x[m]$

$y_1[n] = \sum_{m=-\infty}^{n} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n} kv[m] = ky_1[n]$

(Homogeneous)

$y_3[n] = \sum_{m=-\infty}^{n} (v[m] + w[m]) = \sum_{m=-\infty}^{\infty} v[m] + \sum_{m=-\infty}^{n} w[m]$

$= y_1[n] + y_2[n]$ (Additive)

System is Linear.

$y_1[n] = \sum_{m=-\infty}^{\infty} v[m]$, $y_2 = \sum_{m=-\infty}^{n} v[n - n_o]$

$y_1\,[n]$ can be written as

$y_1[n - n_o] = \sum_{m=-\infty}^{n-n_o} v[m] = \sum_{q=-\infty}^{n} v[q - n_o] = y_2[n]$

(Time Invariant)

At any discrete time $n = n_o$ the response depends only on the excitation at that discrete time and previous discrete time. (Causal)

If the excitation is constant, the response increase without bound. (Unstable)

39. (C) Only statement (b) is false. For example

$S_1 : y[n] = x[n] + b$, and $S_2 : y[n] = x[n] - b$, where $b \neq 0$

$S\{x[n]\} = S_2\{S_1\{x[n]\}\} = S_2\{x[n] + b\} = x[n]$

Hence S is linear.

40. (B) For example

$S_1 : y[n] = nx[n]$ and $S_2 : y[n] = nx[n+1]$

If $x[n] = \delta[n]$ then $S_2\{S_1\{\delta[n]\}\} = S_2[0] = 0$,

CHAPTER

5.3

THE LAPLACE TRANSFORM

Statement for Q.1-12:

Determine the Laplace transform of given signal.

1. $x(t) = u(t-2)$

(A) $\frac{-e^{-2s}}{s}$

(B) $\frac{e^{-2s}}{s}$

(C) $\frac{e^{-2s}}{1+s}$

(D) 0

2. $x(t) = u(t+2)$

(A) $\frac{1}{s}$

(B) $-\frac{1}{s}$

(C) $\frac{e^{-2s}}{s}$

(D) $\frac{-e^{-2s}}{s}$

3. $x(t) = e^{-2t}u(t+1)$

(A) $\frac{1}{s+2}$

(B) $\frac{e^{-s}}{s+2}$

(C) $\frac{e^{-(s+2)}}{s+2}$

(D) $\frac{-e^{-s}}{s+2}$

4. $x(t) = e^{2t}u(-t+2)$

(A) $\frac{e^{2(s-2)}-1}{s-2}$

(B) $\frac{e^{-2s}}{s+2}$

(C) $\frac{1-e^{-2(s-2)}}{s-2}$

(D) $\frac{e^{-2s}}{s-2}$

5. $x(t) = \sin 5t$

(A) $\frac{5}{s^2+5}$

(B) $\frac{s}{s^2+5}$

(C) $\frac{5}{s^2+25}$

(D) $\frac{s}{s^2+25}$

6. $x(t) = u(t) - u(t-2)$

(A) $\frac{e^{-2s}-1}{s}$

(B) $\frac{1-e^{-2s}}{s}$

(C) $\frac{2}{s}$

(D) $\frac{-2}{s}$

7. $x(t) = \frac{d}{dt}\{te^{-t}u(t)\}$

(A) $\frac{1}{s(s+1)^2}$

(B) $\frac{s}{(s+1)^2}$

(C) $\frac{e^{-s}}{s+1}$

(D) $\frac{e^{-s}}{(s+1)^2}$

8. $x(t) = tu(t) * \cos 2\pi t\, u(t)$

(A) $\frac{1}{s(s^2+4\pi^2)}$

(B) $\frac{2\pi}{s^2(s^2+4\pi^2)}$

(C) $\frac{1}{s^2(s^2+4\pi^2)}$

(D) $\frac{s^3}{s^2+4\pi^2}$

9. $x(t) = t^3u(t)$

(A) $\frac{3}{s^4}$

(B) $\frac{-3}{s^4}$

(C) $\frac{6}{s^4}$

(D) $-\frac{6}{s^4}$

10. $x(t) = u(t-1) * e^{-2t}u(t-1)$

(A) $\frac{e^{-2(s+1)}}{2s+1}$

(B) $\frac{e^{-2(s+1)}}{s+1}$

(C) $\frac{e^{-(s+2)}}{s+2}$

(D) $\frac{e^{-2(s+1)}}{s+2}$

11. $x(t) = \int_0^t e^{-3\tau} \cos 2\tau \, d\tau$

(A) $\frac{-(s+3)}{s((s+3)^2+4)}$ (B) $\frac{(s+3)}{s((s+3)^2+4)}$

(C) $\frac{s(s+3)}{(s+3)^2+4}$ (D) $\frac{-s(s+3)}{(s+3)^2+4}$

12. $x(t) = t\frac{d}{dt}\{e^{-t}\cos t\, u(t)\}$

(A) $\frac{-(s^2+4s+2)}{(s^2+2s+2)^2}$ (B) $\frac{(s^2+4s+2)}{(s^2+2s+2)^2}$

(C) $\frac{(s^2+2s+2)}{(s^2+4s+2)^2}$ (D) $\frac{-(s^2+2s+2)}{(s^2+4s+2)^2}$

Statement for Q.13–24:

Determine the time signal $x(t)$ corresponding to given $X(s)$ and choose correct option.

13. $X(s) = \frac{s+3}{s^2+3s+2}$

(A) $(2e^{-2t}+e^{-t})u(t)$ (B) $(2e^{-t}-e^{-2t})u(t)$

(C) $(2e^{-2t}-e^{-t})u(t)$ (D) $(2e^{-t}+e^{-2t})u(t)$

14. $X(s) = \frac{2s^2+10s+11}{s^2+5s+6}$

(A) $2\delta(t)+(e^{-3t}-e^{-2t})u(t)$

(B) $2\delta(t)+(e^{-2t}-e^{-3t})u(t)$

(C) $2\delta(t)+(e^{-2t}+e^{-3t})u(t)$

(D) $2\delta(t)-(e^{-2t}+e^{-3t})u(t)$

15. $X(s) = \frac{2s-1}{s^2+2s+1}$

(A) $(3e^{-t}-2te^{-t})u(t)$

(B) $(3e^{-t}+2te^{-t})u(t)$

(C) $(2e^{-t}-3te^{-t})u(t)$

(D) $(2e^{-t}+3te^{-t})u(t)$

16. $X(s) = \frac{5s+4}{s^3+3s^2+2s}$

(A) $(2+e^{-t}+3e^{-2t})u(t)$

(B) $(2+e^{-t}-3e^{-2t})u(t)$

(C) $(3+e^{-t}-3e^{-2t})u(t)$

(D) $(3+e^{-t}+3e^{-2t})u(t)$

17. $X(s) = \frac{s^2-3}{(s+2)(s^2+2s+1)}$

(A) $(e^{-2t}-2te^{-t})u(t)$ (B) $(e^{-2t}+2te^{-t})u(t)$

(C) $(e^{-t}-2te^{-2t})u(t)$ (D) $(e^{-t}+2te^{-2t})u(t)$

18. $X(s) = \frac{3s+2}{s^2+2s+10}$

(A) $\left(3e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t\right)u(t)$

(B) $\left(3e^{-t}\sin 3t - \frac{1}{3}e^{-t}\cos 3t\right)u(t)$

(C) $(3e^{-t}\cos 3t - e^{-t}\sin 3t)u(t)$

(D) $(3e^{-t}\sin 3t + 3e^{-t}\cos 3t)u(t)$

19. $X(s) = \frac{4s^2+8s+10}{(s+2)(s^2+2s+5)}$

(A) $(2e^{-2t}+2e^{-t}\sin 2t-2e^{-t}\cos 2t)u(t)$

(B) $(2e^{-2t}+2e^{-t}\cos 2t-2e^{-t}\sin 2t)u(t)$

(C) $(2e^{-2t}+2e^{-t}\cos 2t-e^{-t}\sin 2t)u(t)$

(D) $(2e^{-2t}+2e^{-t}\sin 2t-e^{-t}\cos 2t)u(t)$

20. $X(s) = \frac{3s^2+10s+10}{(s+2)(s^2+6s+10)}$

(A) $(e^{-2t}+2e^{-3t}\cos t+2e^{-3t}\sin t)u(t)$

(B) $(e^{-2t}+2e^{-3t}\cos t-6e^{-3t}\sin t)u(t)$

(C) $(e^{-2t}+2e^{-3t}\cos t-2e^{-3t}\sin t)u(t)$

(D) $(9e^{-2t}-6e^{-3t}\cos t+3e^{-3t}\sin t)u(t)$

21. $X(s) = \frac{2s^2+11s+16+e^{-2s}}{(s^2+5s+6)}$

(A) $2\delta(t)+(3e^{-2t}-2e^{-3t})u(t-2)$

(B) $2\delta(t)+(2e^{-2t}-e^{-3t}+e^{-2(t-2)}+e^{-3(t-2)})u(t)$

(C) $2\delta(t)+(2e^{-2t}-e^{-3t})u(t)+(e^{-2t}-e^{-3t})u(t-2)$

(D) $2\delta(t)+(2e^{-2t}-e^{-3t})u(t)+(e^{-2(t-2)}-e^{-3(t-2)})u(t-2)$

22. $X(s) = s\frac{d^2}{ds^2}\left(\frac{1}{s^2+9}\right)+\frac{1}{s+3}$

(A) $\left(e^{-3t}+\frac{2t}{3}\sin 3t+\frac{t^2}{9}\cos 3t\right)u(t)$

(B) $(e^{-3t}+2t\sin 3t+t^2\cos 3t)u(t)$

(C) $\left(e^{-3t}+\frac{2t}{3}\sin 3t+t^2\cos 3t\right)u(t)$

(D) $(e^{-3t}+t^2\sin 3t+2t\cos 3t)u(t)$

23. $X(s)=\dfrac{1}{(2s+1)^2+4}$

(A) $e^{-0.5t}\sin t\,u(t)$ (B) $\frac{1}{2}e^{-t}\sin t\,u(t)$

(C) $\frac{1}{4}e^{-0.5t}\sin t\,u(t)$ (D) $e^{-t}\sin t\,u(t)$

24. $X(s)=e^{-2s}\dfrac{d}{ds}\left(\dfrac{1}{(s+1)^2}\right)$

(A) $-te^{-t}u(1-t)$ (B) $-te^{-t}u(t-1)$

(C) $-(t-2)^2e^{-(t-2)}u(t-2)$ (D) $te^{-t}u(t-1)$

Statement for Q.25–29:

Given the transform pair below. Determine the time signal $y(t)$ and choose correct option.

$$\cos 2t\,u(t)\overset{L}{\longleftrightarrow}X(s).$$

25. $Y(s)=(s+1)X(s)$

(A) $[\cos 2t-2\sin 2t]u(t)$ (B) $\left(\cos 2t+\dfrac{\sin 2t}{2}\right)u(t)$

(C) $[\cos 2t+2\sin 2t]u(t)$ (D) $\left(\cos 2t-\dfrac{\sin 2t}{2}\right)u(t)$

26. $Y(s)=X(3s)$

(A) $\cos\left(\frac{2}{3}t\right)u(t)$ (B) $\frac{1}{3}\cos\left(\frac{2}{3}t\right)u(t)$

(C) $\cos 6t\,u(t)$ (D) $\frac{1}{3}\cos 6t\,u(t)$

27. $Y(s)=X(s+2)$

(A) $\cos 2(t-2)\,u(t)$ (B) $e^{2t}\cos 2t\,u(t)$

(C) $\cos 2(t+2)\,u(t)$ (D) $e^{-2t}\cos 2t\,u(t)$

28. $Y(s)=\dfrac{X(s)}{s^2}$

(A) $4\cos 2t\,u(t)$ (B) $\dfrac{1-\cos 2t}{4}u(t)$

(C) $t^2\cos 2t\,u(t)$ (D) $\dfrac{\cos 2t}{t^2}u(t)$

29. $Y(s)=\dfrac{d}{ds}[e^{-3s}X(s)]$

(A) $t\cos 2(t-3)\,u(t-3)$ (B) $t\cos 2(t-3)\,u(t)$

(C) $-t\cos 2(t-3)\,u(t-3)$ (D) $-t\cos 2(t-3)\,u(t)$

Statement for Q.30–33:

Given the transform pair

$$x(t)u(t)\overset{L}{\longleftrightarrow}\frac{2s}{s^2+2}.$$

Determine the Laplace transform $Y(s)$ of the given time signal in question and choose correct option.

30. $y(t)=x(t-2)$

(A) $\dfrac{2se^{-2s}}{s^2+2}$ (B) $\dfrac{2se^{2s}}{s^2+2}$

(C) $\dfrac{2(s-2)}{(s-2)^2+1}$ (D) $\dfrac{2(s+2)}{(s+2)^2+1}$

31. $y(t)=x(t)*\dfrac{dx(t)}{dt}$

(A) $\dfrac{4s^3}{(s^2+2)^2}$ (B) $\dfrac{4}{(s^2+2)^2}$

(C) $\dfrac{-4s^3}{(s^2+2)^2}$ (D) $\dfrac{4}{(s^2+2)^2}$

32. $y(t)=e^{-t}x(t)$

(A) $\dfrac{2(s+1)}{(s+1)^2+2}$ (B) $\dfrac{2(s+1)}{s^2+2s+2}$

(C) $\dfrac{2(s+1)}{s^2+2s+4}$ (D) $\dfrac{2(s+1)}{s^2+2s}$

33. $y(t)=2tx(t)$

(A) $\dfrac{8-4s^2}{(s^2+2)^2}$ (B) $\dfrac{4s^2-8}{(s^2+2)^2}$

(C) $\dfrac{4s^2}{s^2+1}$ (D) $\dfrac{s^2}{s^2+1}$

Statement for Q.34–43:

Determine the bilateral laplace transform and choose correct option.

34. $x(t)=e^{-t}u(t+2)$

(A) $\dfrac{e^{2(s+1)}}{s+1}$, $\mathrm{Re}(s)>-1$

(B) $\dfrac{1}{1+s}$, $\mathrm{Re}(s)<-1$

(C) $\dfrac{e^{2(s+1)}}{s+1}$, $\mathrm{Re}(s)<-1$

(D) $\dfrac{1}{1+s}$, $\mathrm{Re}(s)>-1$

35. $x(t) = u(-t+3)$

(A) $\frac{1-e^{-3s}}{s}$, $\quad$ Re $(s) > 0$

(B) $\frac{-e^{-3s}}{s}$, $\quad$ Re $(s) < 0$

(C) $\frac{1-e^{-3s}}{s}$, $\quad$ Re $(s) < 0$

(D) $\frac{-e^{-3s}}{s}$, $\quad$ Re $(s) > 0$

36. $y(t) = \delta(t+1)$

(A) e^{s}, Re$(s) > 0$ $\quad$ (B) e^{s}, Re $(s) < 0$

(C) e^{s}, all s $\quad$ (D) None of above

37. $x(t) = \sin t\, u(t)$

(A) $\frac{1}{(1+s^2)}$, $\quad$ Re $(s) < 0$

(B) $\frac{1}{(1+s^2)}$, $\quad$ Re $(s) > 0$

(C) $\frac{-1}{(1+s^2)}$, $\quad$ Re $(s) < 0$

(D) $\frac{-1}{(1+s^2)}$, $\quad$ Re $(s) > 0$

38. $x(t) = e^{-\frac{t}{2}}u(t) + e^{-t}u(t) + e^{t}u(-t)$

(A) $\frac{6s^2+2s-2}{(2s+1)(s^2-1)}$, $\quad$ Re $(s) < -0.5$

(B) $\frac{6s^2+2s-2}{(2s+1)(s^2-1)}$, $\quad$ $-1 >$ Re $(s) > 1$

(C) $\frac{1}{s+0.5} + \frac{1}{s+1} + \frac{1}{s-1}$, $\quad$ $-1 <$ Re $(s) < 1$

(D) $\frac{1}{s+0.5} + \frac{1}{s+1} - \frac{1}{s-1}$, $\quad$ $-0.5 <$ Re $(s) < 1$

39. $x(t) = e^{t}\cos 2t\, u(-t) + e^{-t}u(t) + e^{\frac{t}{2}}u(t)$

(A) $\frac{(1-s)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad$ $0.5 <$ Re $(s) < 1$

(B) $\frac{(1-s)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad$ $-1 <$ Re $(s) < 1$

(C) $\frac{(s-1)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad$ $0.5 <$ Re $(s) < 1$

(D) $\frac{(s-1)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad$ $-1 <$ Re $(s) < 1$

40. $x(t) = e^{(3t+6)}u(t+3)$

(A) $\frac{e^{3s}}{s-3}$, $\quad$ Re $(s) > 3$

(B) $\frac{e^{3s}}{s-3}$, $\quad$ Re $(s) < 3$

(C) $\frac{e^{3(s-1)}}{s-3}$, $\quad$ Re $(s) > 3$

(D) $\frac{e^{3(s-1)}}{s-3}$, $\quad$ Re $(s) < 3$

41. $x(t) = \cos 3t\, u(-t) * e^{-t}u(t)$

(A) $\frac{-s}{(s+1)(s^2+9)}$, $\quad$ Re $(s) > 0$

(B) $\frac{-s}{(s+1)(s^2+9)}$, $\quad$ $-1 <$ Re $(s) < 0$

(C) $\frac{s}{(s+1)(s^2+9)}$, $\quad$ $-1 <$ Re $(s) < 0$

(D) $\frac{s}{(s+1)(s^2+9)}$, $\quad$ Re $(s) > 0$

42. $x(t) = e^{t}\sin(2t+4)\, u(t+2)$

(A) $\frac{e^{2(s-1)}}{(s-1)^2+4}$, $\quad$ Re $(s) > 1$

(B) $\frac{e^{2(s-1)}}{(s-1)^2+4}$, $\quad$ Re $(s) < 1$

(C) $\frac{e^{(s-2)}}{(s-1)^2+4}$, $\quad$ Re $(s) > 1$

(D) $\frac{e^{(s-2)}}{(s-1)^2+4}$, $\quad$ Re $(s) < 1$

43. $x(t) = e^{t}\frac{d}{dt}[e^{-2t}u(-t)]$

(A) $\frac{1-s}{s+1}$, $\quad$ Re $(s) < -1$

(B) $\frac{1-s}{s+1}$, $\quad$ Re $(s) > -1$

(C) $\frac{s-1}{s+1}$, $\quad$ Re $(s) < -1$

(D) $\frac{s-1}{s+1}$, $\quad$ Re $(s) > -1$

Statement for Q.44–49:

Determine the corresponding time signal for given bilateral Laplace transform.

44. $X(s) = \frac{e^{5s}}{s+2}$ with ROC: Re $(s) < -2$

(A) $e^{-2(t+5)}u(t+5)$

(A) $\frac{1}{2}e^{-t}\sin t\,u(t)$ (B) $2e^{-t}\cos t\,u(t)$

(C) $2e^{-t}\cos t\,u(t)+\frac{1}{2}e^{-t}\sin t\,u(t)$

(D) $\frac{1}{2}e^{-t}\cos t\,u(t-1)+2e^{-t}\sin t\,u(t-1)$

58. $\frac{d^3y(t)}{dt^3}+4\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}=x(t)$

All initial condition are zero, $x(t)=10e^{-2t}$

(A) $\left[\frac{5}{3}+5e^{-t}-5e^{-2t}+\frac{5}{3}e^{-3t}\right]u(t)$

(B) $\left[\frac{5}{3}-5e^{-t}+5e^{-2t}+\frac{5}{3}e^{-3t}\right]u(t)$

(C) $\frac{5}{3}u(t)-5u(t-1)+5u(t-2)+\frac{5}{3}u(t-3)$

(D) $\frac{5}{3}u(t)+5u(t-1)-5u(t-2)+\frac{5}{3}u(t-3)$

59. The transform function $H(s)$ of a causal system is

$$H(s)=\frac{2s^2+2s-2}{s^2-1}$$

The impulse response is

(A) $2\delta(t)-(e^{-t}+e^{t})u(-t)$

(B) $2\delta(t)-(e^{-t}+e^{t})u(t)$

(C) $2\delta(t)+e^{-t}u(t)-e^{t}u(-t)$

(D) $2\delta(t)+(e^{-t}+e^{t})u(t)$

60. The transfer function $H(s)$ of a stable system is

$$H(s)=\frac{2s-1}{s^2+2s+1}$$

The impulse response is

(A) $2u(-t+1)-3tu(-t+1)$

(B) $(3te^{-t}-2e^{-t})u(t)$

(C) $2u(t+1)-3tu(t+1)$

(D) $(2e^{-t}-3te^{-t})u(t)$

61. The transfer function $H(s)$ of a stable system is

$$H(s)=\frac{s^2+5s-9}{(s+1)(s^2-2s+10)}$$

The impulse response is

(A) $-e^{-t}u(t)+(e^{t}\sin 3t+2e^{t}\cos 3t)u(t)$

(B) $-e^{-t}u(t)-(e^{t}\sin 3t+2e^{t}\cos 3t)u(-t)$

(C) $-e^{-t}u(t)-(e^{t}\sin 3t+2e^{t}\cos 3t)u(t)$

(D) $-e^{-t}u(t)+(e^{t}\sin 3t+2e^{t}\cos 3t)u(-t)$

62. A stable system has input $x(t)$ and output $y(t)=e^{-2t}\cos t\,u(t)$. The impulse response of the system is

(A) $\delta(t)-(e^{-2t}\cos t+e^{-2t}\sin t)u(t)$

(B) $\delta(t)-(e^{-2t}\cos t+e^{-2t}\sin t)u(t-2)$

(C) $\delta(t)-(e^{2t}\cos t+e^{2t}\sin t)u(t)$

(D) $\delta(t)-(e^{2t}\cos t+e^{2t}\sin t)u(t+2)$

63. The relation ship between the input $x(t)$ and output $y(t)$ of a causal system is described by the differential equation

$$\frac{dy(t)}{dt}+10y(t)=10x(t)$$

The impulse response of the system is

(A) $-10e^{-10t}u(-t+10)$ (B) $10e^{-10t}u(t)$

(C) $10e^{-10t}u(-t+10)$ (D) $-10e^{-10t}u(t)$

64. The relationship between the input $x(t)$ and output $y(t)$ of a causal system is defined as

$$\frac{d^2y(t)}{dt^2}-\frac{dy(t)}{dt}-2y(t)=-4x(t)+5\frac{dx(t)}{dt}.$$

The impulse response of system is

(A) $3e^{-t}u(t)+2e^{2t}u(-t)$

(B) $(3e^{-t}+2e^{2t})u(t)$

(C) $3e^{-t}u(t)-2e^{2t}u(-t)$

(D) $(3e^{-t}-2e^{2t})u(-t)$

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SOLUTIONS

1. (B) $X(s)=\int_0^\infty x(t)e^{-st}dt=\int_2^\infty e^{-st}dt=\frac{e^{-2s}}{s}$

2. (A) $X(s)=\int_0^\infty x(t)e^{-3t}dt=\int_0^\infty u(t+2)^{-3t}dt=\int_0^\infty e^{-3t}dt=\frac{1}{s}$

3. (A) $X(s)=\int_0^\infty e^{-2t}e^{-st}dt=\frac{1}{s+2}$

4. (C) $X(s)=\int_0^\infty x(t)e^{-st}dt=\int_0^\infty e^{2t}u(-t+2)e^{-st}dt$

$=\int_0^2 e^{t(2-s)}dt=\frac{e^{2(2-s)}-1}{2-s}=\frac{1-e^{-2(2-s)}}{s-2}$

5. (C) $X(s)=\int_0^\infty \frac{(e^{j5t}-e^{-j5t})}{2j}e^{-st}dt=\frac{5}{s^2+25}$

6. (B) $X(s)=\int_0^2 e^{-st}dt=\frac{1-e^{-2s}}{s}$

7. (B) $p(t)=te^{-t}u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{(s+1)^2}$

$x(t)=\frac{d}{dt}p(t)\overset{L}{\longleftrightarrow}X(s)=\frac{s}{(s+1)^2}$

8. (A) $p(t)=tu(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{s^2}$

$q(t)=\cos 2\pi t\,u(t)\overset{L}{\longleftrightarrow}Q(s)=\frac{s}{s^2+4\pi^2}$

$x(t)=p(t)*q(t)\overset{L}{\longleftrightarrow}X(s)=P(s)Q(s)$

$\Rightarrow\quad X(s)=\frac{1}{s(s^2+4\pi^2)}$

9. (C) $p(t)=tu(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{s^2}$

$q(t)=-tp(t)\overset{L}{\longleftrightarrow}Q(s)=\frac{d}{ds}P(s)=\frac{-2}{s^3}$

$x(t)=-tq(t)\overset{L}{\longleftrightarrow}X(s)=\frac{d}{ds}Q(s)=\frac{6}{s^4}$

$t^n u(t)\overset{L}{\longleftrightarrow}\frac{n!}{s^{n+1}}$

10. (D) $p(t)=u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{s}$

$q(t)=u(t-1)\overset{L}{\longleftrightarrow}Q(s)=\frac{e^{-s}}{s^2}$

$r(t)=e^{-2t}u(t)\overset{L}{\longleftrightarrow}R(s)=\frac{1}{s+2}$

$v(t)=e^{-2t}u(t-1)\overset{L}{\longleftrightarrow}V(s)=\frac{e^{-(s+2)}}{s^2}$

$x(t)=q(t)*v(t)\overset{L}{\longleftrightarrow}X(s)=Q(s)V(s)$

$\Rightarrow\quad X(s)=\frac{e^{-2(s+1)}}{s+2}$

11. (B) $p(t)=e^{-3t}\cos 2t\,u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{s+3}{(s+3)^2+4}$

$\int_{-\infty}^t p(\tau)d\tau\overset{L}{\longleftrightarrow}\frac{1}{s}\int_{-\infty}^0 p(\tau)d\tau+\frac{P(s)}{s}$

$\Rightarrow\quad X(s)=\frac{(s+3)}{s[(s+3)^2+4]}$

12. (A) $p(t)=e^{-t}\cos t\,u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{s+1}{(s+1)^2+1}$

$q(t)=\frac{d}{dt}p(t)\overset{L}{\longleftrightarrow}Q(s)=\frac{s(s+1)}{(s+1)^2+1}$

$x(t)=tq(t)\overset{L}{\longleftrightarrow}X(s)=-\frac{d}{ds}Q(s)$

$\Rightarrow\quad X(s)=\frac{-(s^2+4s+2)}{(s^2+2s+2)^2}$

13. (B) $X(s)=\frac{s+3}{(s^2+3s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$

$A=\left.\frac{s+3}{s+2}\right|_{s=-1}=2,\ B=\left.\frac{s+3}{s+1}\right|_{s=-2}=-1$

$x(t)=[2e^{-t}-e^{-2t}]u(t)$

14. (A) $X(s)=2-\frac{1}{(s+2)(s+3)}=2-\frac{1}{(s+2)}+\frac{1}{(s+3)}$

$x(t)=2\delta(t)+(e^{-3t}-e^{-3t})u(t)$

15. (C) $X(s)=\frac{2s-1}{s^2+2s+1}=\frac{A}{(s+1)}+\frac{B}{(s+1)^2}$

$B=(2s-2)|_{s=-1}=-3,\ A=2$

$x(t)=x(t)=[2e^{-t}-3te^{-t}]u(t)$

16. (B) $X(s)=\frac{5s+4}{s^3+3s^2+2s}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$

$A=sX(s)|_{s=0}=2,\ B=(s+1)X(s)|_{s=-1}=1,$

$C=(s+2)X(s)|_{s=-2}=-3$

$x(t)=[2+e^{-t}-3e^{-2t}]u(t)$

17. (C) $X(s)=\frac{s^2-3}{(s+2)(s^2+2s+1)}$

$=\frac{A}{(s+2)}+\frac{B}{(s+1)}+\frac{C}{(s+1)^2}$

$A = (s+2)X(s)\big|_{s=-2} = 1,\ C = (s+1)^2 X(s)\big|_{s=-1} = -2$

$A + B = 1 \Rightarrow B = 0$

$x(t) = [e^{-2t} - te^{-t}]u(t)$

18. (A) $X(s) = \dfrac{3s+2}{s^2+2s+10} = \dfrac{3(s+1)}{(s+1)^2+3^2} - \dfrac{1}{(s+1)^2+3^2}$

$x(t) = \left[3e^{-t}\cos 3t - \dfrac{1}{3}e^{-t}\sin 3t\right]u(t)$

19. (C) $X(s) = \dfrac{4s^2+8s+10}{(s+2)(s^2+2s+5)}$

$= \dfrac{A}{(s+2)} + \dfrac{B(s+1)}{(s+1)^2+2^2} + \dfrac{C}{(s+1)^2+2^2}$

$A = (s+2)X(s)\big|_{s=-2} = 2$

$A + B = 4 \Rightarrow B = 2$

$5A + 2B + 2C = 10 \Rightarrow C = -2$

$x(t) = [2e^{-2t} + 2e^{-t}\cos 2t - e^{-t}\sin 2t]u(t)$

20. (B) $X(s) = \dfrac{3s^2+10s+10}{(s+2)(s^2+6s+10)}$

$= \dfrac{A}{(s+2)} + \dfrac{B(s+3)}{(s+3)^2+1} + \dfrac{C}{(s+3)^2+1}$

$A = (s+2)X(s)\big|_{s=-2} = 1,\ A + B = 3 \Rightarrow B = 2$

$10A + 6B + 2C = 10 \Rightarrow C = -6$

$x(t) = [e^{-2t} + 2e^{-3t}\cos t - 6e^{-3t}\sin t]u(t)$

21. (D) $X(s) = \dfrac{2s^2+11s+16+e^{-2s}}{(s^2+5s+6)}$

$= 2 + \dfrac{A}{(s+2)} + \dfrac{B}{(s+3)} + \dfrac{e^{-2s}}{(s+2)} - \dfrac{e^{-2s}}{(s+3)}$

$A = \dfrac{(s+2)(2s^2+11s+16)}{(s^2+5s+6)}\bigg|_{s=-2} = 2$

$B = \dfrac{(s+3)(2s^2+11s+16)}{(s^2+5s+6)}\bigg|_{s=-3} = -1$

$x(t) = 2\delta(t) + [2e^{-2t} - e^{-3t}]u(t) + [e^{-2(t-2)} - e^{-3(t-2)}]u(t-2)$

22. (C) $P(s) = \dfrac{1}{s^2+9} \overset{L}{\longleftrightarrow} p(t) = \dfrac{1}{3}\sin 3t\, u(t)$

$Q(s) = \dfrac{d^2}{ds^2}P(s) \overset{L}{\longleftrightarrow} q(t) = (-1)^2 t^2 p(t) = \dfrac{t^2}{3}\sin 3t\, u(t)$

$R(s) = sQ(s) \overset{L}{\longleftrightarrow} r(t) = \dfrac{d}{dt}q(t) - q(0^-)$

$= \dfrac{2t}{3}\sin 3t\, u(t) + t^2\cos 3t\, u(t)$

$V(s) = \dfrac{1}{s+3} \overset{L}{\longleftrightarrow} v(t) = e^{-3t}u(t)$

$x(t) = v(t) + r(t) = \left[\dfrac{2t}{3}\sin 3t\, u(t) + t^2\cos 3t\, u(t) + e^{-3t}\right]u(t)$

23. (C) $P\left(\dfrac{s}{a}\right) \overset{L}{\longleftrightarrow} ap(at)$

$\dfrac{1}{(s+1)^2+4} \overset{L}{\longleftrightarrow} \dfrac{1}{2}e^{-t}\sin 2t\, u(t)$

$x(t) \overset{L}{\longleftrightarrow} \dfrac{1}{4}e^{-0.5t}\sin t\, u(t)$

24. (C) $P(s) = \dfrac{1}{(s+1)^2} \overset{L}{\longleftrightarrow} p(t) = te^{-t}u(t)$

$Q(s) = \dfrac{d}{ds}P(s) \overset{L}{\longleftrightarrow} q(t) = -tp(t) = -t^2e^{-t}u(t)$

$X(s) = e^{-2s}Q(s) \overset{L}{\longleftrightarrow} x(t) = q(t-2)$

$\Rightarrow x(t) = -(t-2)\,e^{(t-2)}u(t-2)$

25. (A) $sX(s) + X(s) \overset{L}{\longleftrightarrow} \dfrac{dx(t)}{dt} + x(t)$

$\Rightarrow y(t) = (-2\sin 2t + \cos 2t)u(t)$

26. (B) $X\left(\dfrac{s}{a}\right) \overset{L}{\longleftrightarrow} ax(at)$

$X(3s) \overset{L}{\longleftrightarrow} \dfrac{1}{3}\cos\left(\dfrac{2}{3}t\right)u(t)$

27. (D) $X(s+2) \overset{L}{\longleftrightarrow} e^{-2t}x(t)$

28. (B) $P(s) = \dfrac{X(s)}{s} \overset{L}{\longleftrightarrow} \displaystyle\int_{-\infty}^{t} x(\tau)d\tau$

$\overset{L}{\longleftrightarrow} \displaystyle\int_{-\infty}^{t}\cos 2\tau\, u(\tau)d\tau = \dfrac{\sin 2t}{2}$

$\dfrac{P(s)}{s} \overset{L}{\longleftrightarrow} \displaystyle\int_{0}^{t}\dfrac{\sin 2\tau}{2}d\tau = \dfrac{1-\cos 2t}{4}u(t),$

29. (C) $P(s) = e^{-3s}X(s) \overset{L}{\longleftrightarrow} p(t) = x(t-3)$

$= \cos 2(t-3)\, u(t-3)$

$Q(s) = \dfrac{d}{ds}P(s) \overset{L}{\longleftrightarrow} q(t) = -p(t)$

$= -t\cos 2(t-3)\, u(t-3).$

30. (A) $x(t-2) \overset{L}{\longleftrightarrow} e^{-2s}X(s),\ Y(s) = \dfrac{2se^{-2s}}{s^2+2}$

31. (A) $p(t) = \dfrac{d}{dt}x(t) \overset{L}{\longleftrightarrow} P(s) = sX(s)$

$y(t) = x(t) * p(t) \overset{L}{\longleftrightarrow} Y(s) = P(s)X(s) = s(X(s))^2$

32. (A) $e^{-t}x(t) \overset{L}{\longleftrightarrow} X(s+1) = \dfrac{2(s+1)}{(s+1)^2+2}$

33. (B) $2tx(t) \overset{L}{\longleftrightarrow} -2\dfrac{d}{ds}X(s) = \dfrac{4s^2-8}{(s^2+2)^2}$

34. (A) $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

$= \int_{-2}^{\infty} e^{-t}e^{-st}dt = \int_{-2}^{\infty} e^{-t(s+1)}dt = \dfrac{e^{2(s+1)}}{s+1}$, $\text{Re}(s) > -1$

35. (B) $X(s) = \int_{-\infty}^{\infty} u(-t+3)e^{-st}dt = \int_{-\infty}^{3} e^{-st}dt = \dfrac{-e^{-3s}}{s}$ $\text{Re}(s) < 0$

36. (C) $Y(s) = \int_{-\infty}^{\infty} \delta(t+1)e^{-st}dt = e^{s}$, All s

37. (B) $X(s) = \int_{0}^{\infty} \dfrac{(e^{jt}-e^{-jt})}{2j}e^{-st}dt$

$= \dfrac{1}{2j}\int_0^{\infty} e^{t(j-s)}dt - \dfrac{1}{2j}\int_0^{\infty} e^{-t(j+s)}dt = \dfrac{1}{1+s^2}$, $\text{Re}(s) > 0$

38. (D) $X(s) = \int_0^{\infty} e^{-\frac{t}{2}}e^{-st} + \int_0^{\infty} e^{-t}e^{-st}dt + \int_{-\infty}^{0} e^{t}e^{-st}dt$

$= \dfrac{1}{s+0.5} + \dfrac{1}{s+1} - \dfrac{1}{s-1}$

$\text{Re}(s) > -0.5$, $\text{Re}(s) > -1$, $\text{Re}(s) < 1$

$\Rightarrow -0.5 < \text{Re}(s) < 1$

39. (A) $X(s) = \int_{-\infty}^{0} e^{t}\dfrac{(e^{jt}-e^{-jt})}{2j}e^{-st}dt + \int_0^{\infty} e^{-t}e^{-st}dt + \int_0^{\infty} e^{-\frac{t}{2}}e^{-st}dt$

$\text{Re}(s) < 1$, $\text{Re}(s) > -1$, $\text{Re}(s) > 0.5$

Therefore $0.5 < \text{Re}(s) < 1$

$X(s) = -\dfrac{s-1}{(s-1)^2+4} + \dfrac{1}{(s+1)} + \dfrac{1}{s-0.5}$

40. (C) $x(t) = e^{-3}e^{-3(t+3)}u(t+3)$

$p(t) = e^{3t}u(t) \overset{L}{\longleftrightarrow} P(s) = \dfrac{1}{s-3}$

$q(t) = p(t+3) \overset{L}{\longleftrightarrow} Q(s) = e^{3s}P(s) = \dfrac{e^{3s}}{s-3}$

$X(s) = \dfrac{e^{3(s-1)}}{s-3}$, $\text{Re}(s) > 3$

41. (B) $p(t) * q(t) \overset{L}{\longleftrightarrow} P(s)Q(s)$

$X(s) = \dfrac{-s}{s^2+9}\left(\dfrac{1}{s+1}\right)$

$\text{Re}(s) > -1$, $\text{Re}(s) < 0$

$\Rightarrow -1 < \text{Re}(s) < 0$

42. (A) $x(t) = e^{-2}e^{t+2}\sin(2t+4)\,u(t+2)$

$p(t+2) \overset{L}{\longleftrightarrow} e^{2s}P(s)$,

$X(s) = \dfrac{e^{2s}e^{-2}}{(s-1)^2+4}$, $\text{Re}(s) > 1$

43. (A) $p(t) = e^{-2t}u(-t) \overset{L}{\longleftrightarrow} P(s) = \dfrac{-1}{s+2}$, $\text{Re}(s) < -2$

$q(t) = \dfrac{d}{dt}p(t) \overset{L}{\longleftrightarrow} Q(s) = sP(s)$

$x(t) = e^{t}q(t) \overset{L}{\longleftrightarrow} X(s) = Q(s-1) = \dfrac{1-s}{1+s}$

$\text{Re}(s) < -1$ thus left-sided .

44. (C) Left-sided

$P(s) = \dfrac{1}{s+2} \overset{L}{\longleftrightarrow} p(t) = -e^{-2t}u(-t)$

$X(s) = e^{5s}P(s) \overset{L}{\longleftrightarrow} x(t) = p(t+5)$

$\Rightarrow x(t) = -e^{-2(t+5)}u(-(t+5))$

45. (A) Right-sided

$P(s) = \dfrac{1}{(s-3)} \overset{L}{\longleftrightarrow} p(t) = e^{3t}u(t)$

$X(s) = \dfrac{d^2}{ds^2}P(s) \overset{L}{\longleftrightarrow} x(t) = t^2e^{3t}u(t)$

46. (D) Left-sided

$x(t) = -u(-t) + u(-t+1) + \delta(t+2)$

47. (C) Right-sided, $P(s) = \dfrac{1}{s} \overset{L}{\longleftrightarrow} p(t) = u(t)$

$Q(s) = e^{-3s}P(s) \overset{L}{\longleftrightarrow} q(t) = p(t-3) = u(t-3)$

$R(s) = \dfrac{d}{ds}Q(s) \overset{L}{\longleftrightarrow} r(t) = -tq(t) = -tu(t-3)$

$V(s) = \dfrac{1}{s}R(s) \overset{L}{\longleftrightarrow} v(t) = \int_{-\infty}^{t} r(\tau)d\tau$

$\Rightarrow v(t) = -\int_3^t tdt = -\dfrac{1}{2}(t^2-9)$

$X(s) = \dfrac{1}{s}v(s) \overset{L}{\longleftrightarrow} x(t) = -\dfrac{1}{2}\int_{-\infty}^{t}(t^2-9)$

$\Rightarrow x(t) = \left[\dfrac{-1}{6}(t^3-27) + \dfrac{9}{2}(t-3)\right]u(t-3)$

48. (B) $X(s) = \dfrac{-s-4}{s^2+3s+2} = \dfrac{-3}{(s+1)} + \dfrac{2}{s+2}$

Left-sided, $x(t) = 3e^{-t}u(-t) - 2e^{-2t}u(-t)$

49. (A) $X(s) = \dfrac{5}{(s+1)} - \dfrac{1}{(s+1)^2}$

Left-sided, $x(t) = -5u(-t) + te^{-t}u(-t)$

50. (D) $x(0^+) = \lim_{x\to\infty} sX(s) = \dfrac{s}{s^2+5s-2} = 0$

51. (A) $x(0^+) = \lim_{s\to\infty} sX(s) = \dfrac{s^2+2s}{s^2+2s-3} = 1$

52. (D) $x(0^+) = \lim_{s\to\infty} sX(s) = \dfrac{e^{-2s}(6s^3+s^2)}{s^2+2s-2} = 0$

53. (A) $x(\infty) = \lim_{s\to 0} sX(s) = \dfrac{2s^3+3s}{s^2+5s+1} = 0$

54. (C) $x(\infty) = \lim_{s\to 0} sX(s) = \dfrac{s+2}{s^2+3s+1} = 2$

55. (B) $x(\infty) = \lim_{s\to 0} sX(s) = \dfrac{e^{-3s}(2s^2+1)}{s^2+5s+4} = \dfrac{1}{4}$

56. (C) $sY(s) - y(0^-) + 10Y(s) = 10(s)$

$y(0^-) = 1, \quad X(s) = \dfrac{1}{s}$

$Y(s) = \dfrac{10}{s(s+1)} + \dfrac{1}{(s+1)} = \dfrac{1}{s}$

$\Rightarrow \quad y(t) = u(t)$

57. (C) $s^2Y(s) - 2s + 2sY(s) - 2 + 5Y(s) = 1$

$(s^2+2s+5)Y(s) = 3 + 2\,s$

$Y(s) = \dfrac{2s+3}{s^2+2s+5} = \dfrac{2(s+1)}{(s+1)^2+2^2} + \dfrac{1}{(s+1)^2+2^2}$

$\Rightarrow \quad y(t) = 2e^{-t}\cos t\, u(t) + \dfrac{1}{2}e^{-t}\sin t\, u(t)$

58. (B) $s^3Y(s) + 4s^2Y(s) + 3sY(s) = \dfrac{10}{(s+2)}$

$Y(s) = \dfrac{10}{s(s+1)(s+2)(s+3)} = \dfrac{A}{s} + \dfrac{B}{(s+1)} + \dfrac{C}{(s+2)} + \dfrac{D}{s+3}$

$A = sY(s)\big|_{s=0} = \dfrac{5}{3},\ B = (s+1)Y(s)\big|_{s=-1} = -5,$

$C = (s+2)Y(s)\big|_{s=-2} = 5,\ D = (s+3)Y(s)\big|_{s=0} = \dfrac{5}{3}$

$\Rightarrow \quad y(t) = \left[\dfrac{5}{3} - 5e^{-t} + 5e^{-2t} + \dfrac{5}{3}e^{-3t}\right]u(t)$

59. (D) For a causal system $h(t) = 0$ for $t < 0$

$H(s) = 2 + \dfrac{1}{s+1} + \dfrac{1}{s-1}$

$\Rightarrow \quad h(t) = 2\delta(t) + (e^{-t} + e^{t})u(t)$

60. (D) $H(s) = \dfrac{2}{s+1} - \dfrac{3}{(s+1)^2}$, System is stable

$h(t) = (2e^{-t} - 3te^{-t})u(t).$

61. (A) $H(s) = \dfrac{-1}{(s+1)} + \dfrac{2(s-1)}{(s-1)^2+3^2} + \dfrac{3}{(s-1)^2+3^2}$

System is stable

$h(t) = -e^{-t}u(t) + (2e^{t}\cos 3t + e^{t}\sin 3t)u(-t)$

62. (A) $X(s) = \dfrac{1}{s+1}, \quad Y(s) = \dfrac{(s+2)}{(s+2)^2+1}$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{(s+1)(s+2)}{(s+2)^2+1}$

$= 1 - \dfrac{(s+2)}{(s+2)^2+1} - \dfrac{1}{(s+2)^2+1}$

$h(t) = \delta(t) - (e^{-2t}\cos t + e^{-2t}\sin t)u(t)$

63. (B) $sY(s) + 10Y(s) = 10X(s)$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{10}{s+10}$

$\Rightarrow \quad h(t) = 10e^{-10t}u(t)$

64. (B) $Y(s)(s^2-s-2) = X(s)(5s-4)$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{5s-4}{s^2-s-2} = \dfrac{3}{s+1} + \dfrac{2}{s-2}$

$h(t) = 3e^{-t}u(t) + 2e^{2t}u(t).$

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CHAPTER

5.4

THE Z-TRANSFORM

Statement forQ.1-12:

Determine the z-transform and choose correct option.

1. $x[n] = \delta[n-k], k > 0$

(A) $z^{k}, \quad z > 0$

(B) $z^{-k}, \quad z > 0$

(C) $z^{k}, \quad z \neq 0$

(D) $z^{-k}, \quad z \neq 0$

2. $x[n] = \delta[n+k], k > 0$

(A) $z^{-k}, \quad z \neq 0$

(B) $z^{k}, \quad z \neq 0$

(C) $z^{-k}, \quad \text{all } z$

(D) $z^{k}, \quad \text{all } z$

3. $x[n] = u[n]$

(A) $\frac{1}{1-z^{-1}}, \ |z| > 1$

(B) $\frac{1}{1-z^{-1}}, \ |z| < 1$

(C) $\frac{z}{1-z^{-1}}, \ |z| < 1$

(D) $\frac{z}{1-z^{-1}}, \ |z| > 1$

4. $x[n] = \left(\frac{1}{4}\right)^{n}(u[n] - u[n-5])$

(A) $\frac{z^{5} - 0.25^{5}}{z^{4}(z-0.25)}, \ z > 0.25$

(B) $\frac{z^{5} - 0.25^{5}}{z^{4}(z-0.25)}, \ z > 0$

(C) $\frac{z^{5} - 0.25^{5}}{z^{3}(z-0.25)}, \ z < 0.25$

(D) $\frac{z^{5} - 0.25^{5}}{z^{4}(z-0.25)}, \ \text{all } z$

5. $x[n] = \left(\frac{1}{4}\right)^{4} u[-n]$

(A) $\frac{4z}{4z-1}, \ |z| > \frac{1}{4}$

(B) $\frac{4z}{4z-1}, \ |z| < \frac{1}{4}$

(C) $\frac{1}{1-4z}, \ |z| > \frac{1}{4}$

(D) $\frac{1}{1-4z}, \ |z| < \frac{1}{4}$

6. $x[n] = 3^{n} u[-n-1]$

(A) $\frac{z}{3-z}, \ |z| > 3$

(B) $\frac{z}{3-z}, \ |z| < 3$

(C) $\frac{3}{3-z}, \ |z| > 3$

(D) $\frac{3}{3-z}, \ |z| < 3$

7. $x[n] = \left(\frac{2}{3}\right)^{|n|}$

(A) $\frac{-5z}{(2z-3)(3z-2)}, \ -\frac{3}{2} < z < -\frac{2}{3}$

(B) $\frac{-5z}{(2z-3)(3z-2)}, \ \frac{2}{3} < |z| < \frac{3}{2}$

(C) $\frac{5z}{(2z-3)(3z-2)}, \ \frac{2}{3} < |z| < \frac{2}{3}$

(D) $\frac{5z}{(2z-3)(3z-2)}, \ -\frac{3}{2} < z < -\frac{2}{3}$

8. $x[n] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{4}\right)^{n} u[-n-1]$

(A) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}, \ \frac{1}{4} < |z| < \frac{1}{2}$

(B) $\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}}, \ \frac{1}{4} < |z| < \frac{1}{2}$

(C) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}, \ |z| > \frac{1}{2}$

(D) None of the above

33. $X(z)=\dfrac{1}{1-4z^{-2}}, \quad |z|<\dfrac{1}{4}$

(A) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[-n-2(k+1)]$

(B) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[-n+2(k+1)]$

(C) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)]$

(D) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n-2(k+1)]$

34. $X(z)=\ln(1+z^{-1}), \quad |z|>0$

(A) $\dfrac{(-1)^{k-1}}{k}\delta[n-1]$ (B) $\dfrac{(-1)^{k-1}}{k}\delta[n+1]$

(C) $\dfrac{(-1)^{k}}{k}\delta[n-1]$ (D) $\dfrac{(-1)^{k}}{k}\delta[n+1]$

35. If z-transform is given by

$$X(z)=\cos(z^{-3}), \ |z|>0,$$

The value of $x[12]$ is

(A) $-\dfrac{1}{24}$ (B) $\dfrac{1}{24}$

(C) $-\dfrac{1}{6}$ (D) $\dfrac{1}{6}$

36. $X(z)$ of a system is specified by a pole zero pattern in fig. P.5.4.36.

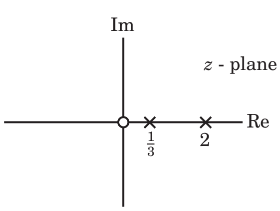

Fig. P.5.4.36

Consider three different solution of $x[n]$

$$x_1[n]=\left[2^n-\left(\frac{1}{3}\right)^n\right]u[n]$$

$$x_2[n]=-2^n u[n-1]-\frac{1}{3^n}u[n]$$

$$x_3[n]=-2^n u[n-1]+\frac{1}{3^n}u[-n-1]$$

Correct solution is

(A) $x_1[n]$ (B) $x_2[n]$

(C) $x_3[n]$ (D) All three

37. Consider three different signal

$$x_1[n]=\left[2^n-\left(\frac{1}{2}\right)^n\right]u[n]$$

$$x_2[n]=-2^n u[-n-1]+\frac{1}{2^n}u[-n-1]$$

$$x_3[n]=-2^n u[-n-1]-\frac{1}{2^n}u[n]$$

Fig. P.5.4.37 shows the three different region. Choose the correct option for the ROC of signal

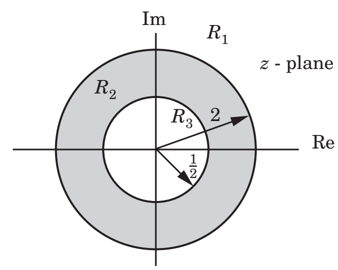

Fig. P5.4.37

	R_1	R_2	R_3
(A)	$x_1[n]$	$x_2[n]$	$x_3[n]$
(B)	$x_2[n]$	$x_3[n]$	$x_1[n]$
(C)	$x_1[n]$	$x_3[n]$	$x_2[n]$
(D)	$x_3[n]$	$x_2[n]$	$x_1[n]$

38. Given

$$X(z)=\frac{1+\frac{7}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}$$

For three different ROC consider there different solution of signal $x[n]$:

(a) $|z|>\dfrac{1}{2}$, $x[n]=\left[\dfrac{1}{2^{n-1}}-\left(\dfrac{-1}{3}\right)^n\right]u[n]$

(b) $|z|<\dfrac{1}{3}$, $x[n]=\left[\dfrac{-1}{2^{n-1}}+\left(\dfrac{-1}{3}\right)^n\right]u[-n+1]$

(c) $\dfrac{1}{3}<|z|<\dfrac{1}{2}$, $x[n]=-\dfrac{1}{2^{n-1}}u[-n-1]-\left(\dfrac{-1}{3}\right)^n u[n]$

Correct solutions are

(A) (a) and (b) (B) (a) and (c)

(C) (b) and (c) (D) (a), (b), (c)

39. $X(z)$ has poles at $z = 1/2$ and $z = -1$. If $x[1] = 1$ $x[-1] = 1$, and the ROC includes the point $z = 3/4$. The time signal $x[n]$ is

(A) $\frac{1}{2^{n-1}} u[n] - (-1)^n u[-n-1]$

(B) $\frac{1}{2^n} u[n] - (-1)^n u[-n-1]$

(C) $\frac{1}{2^{n-1}} u[n] + u[-n+1]$

(D) $\frac{1}{2^n} u[n] + u[-n+1]$

40. $x[n]$ is right-sided, $X(z)$ has a signal pole, and $x[0] = 2$, $x[2] = 1/2$. $x[n]$ is

(A) $\frac{u[-n]}{2^{n-1}}$ (B) $\frac{u[n]}{2^{n-1}}$

(C) $\frac{u[-n]}{2^{n+1}}$ (D) $\frac{u[-n]}{2^{n+1}}$

41. The z-transform function of a stable system is given as

$$H(z) = \frac{2 - \frac{3}{2} z^{-1}}{(1 - 2z^{-1})(1 + \frac{1}{2} z^{-1})}$$

The impuse response $h[n]$ is

(A) $2^n u[-n+1] - \left(\frac{1}{2}\right)^n u[n]$

(B) $2^n u[-n-1] + \left(\frac{-1}{2}\right)^n u[n]$

(C) $-2^n u[-n-1] + \left(\frac{-1}{2}\right)^n u[n]$

(D) $2^n u[n] - \left(\frac{1}{2}\right)^n u[n]$

42. Let $x[n] = \delta[n-2] + \delta[n+2]$. The unilateral z-transform is

(A) z^{-2} (B) z^2

(C) $-2z^{-2}$ (D) $2z^2$

43. The unilateral z-transform of signal $x[n] = u[n+4]$ is

(A) $1 + z + z^2 + 3z + z^4$ (B) $\frac{1}{1-z}$

(C) $1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$ (D) $\frac{1}{1-z^{-1}}$

44. The transfer function of a causal system is given as

$$H(z) = \frac{5z^2}{z^2 - z - 6}$$

The impulse response is

(A) $(3^n + (-1)^n 2^{n+1}) u[n]$

(B) $(3^{n+1} + 2(-2)^n) u[n]$

(C) $(3^{n-1} + (-1)^n 2^{n+1}) u[n]$

(D) $(3^{n-1} - (-2)^{n+1}) u[n]$

45. A causal system has input

$x[n] = \delta[n] + \frac{1}{4}\delta[n-1] - \frac{1}{8}\delta[n-2]$ and output

$y[n] = \delta[n] - \frac{3}{4}\delta[n-1]$.

The impulse response of this system is

(A) $\frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right] u[n]$

(B) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n + 2\left(\frac{-1}{4}\right)^n\right] u[n]$

(C) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n - 2\left(\frac{-1}{4}\right)^n\right] u[n]$

(D) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n + 2\left(\frac{1}{4}\right)^n\right] u[n]$

46. A causal system has input $x[n] = (-3)^n u[n]$ and output

$$y[n] = \left[4(2)^n - \left(\frac{1}{2}\right)^n\right] u[n].$$

The impulse response of this system is

(A) $\left[7\left(\frac{1}{2}\right)^n - 10\left(\frac{1}{2}\right)^n\right] u[n]$ (B) $\left[7(2^n) - 10\left(\frac{1}{2}\right)^n\right] u[n]$

(C) $\left[10\left(\frac{1}{2}\right)^2 - 7(2)^n\right] u[n]$ (D) $\left[10(2^n) - 7\left(\frac{1}{2}\right)^n\right] u[n]$

47. A system has impulse response

$$h[n] = \frac{1}{2^n} u[n]$$

The output $y[n]$ to the input $x[n]$ is given by $y[n] = 2\delta[n-4]$. The input $x[n]$ is

(A) $2\delta[-n-4] - \delta[-n-5]$ (B) $2\delta[n+4] - \delta[n+5]$

(C) $2\delta[-n+4] - \delta[-n+5]$ (D) $2\delta[n-4] - \delta[n-5]$

48. A system is described by the difference equation

$$y[n]-\frac{1}{2}y[n-1]=2x[n-1]$$

The impulse response of the system is

(A) $\frac{-1}{2^{n-2}}u[n-1]$ (B) $\frac{1}{2^{n-2}}u[n+1]$

(C) $\frac{1}{2^{n-2}}u[n-2]$ (D) $\frac{-1}{2^{n-2}}u[n-2]$

49. A system is described by the difference equation

$$y[n]=x[n]-x[n-2]+x[n-4]-x[n-6]$$

The impulse response of system is

(A) $\delta[n]-2\delta[n+2]+4\delta[n+4]-6\delta[n+6]$

(B) $\delta[n]+2\delta[n-2]-4\delta[n-4]+6\delta[n-6]$

(C) $\delta[n]-\delta[n-2]+\delta[n-4]-\delta[n-6]$

(D) $\delta[n]-\delta[n+2]+\delta[n+4]-\delta[n+6]$

50. The impulse response of a system is given by

$$h[n]=\frac{3}{4^n}u[n-1].$$

The difference equation representation for this system is

(A) $4y[n]-y[n-1]=3x[n-1]$

(B) $4y[n]-y[n+1]=3x[n+1]$

(C) $4y[n]+y[n-1]=-3x[n-1]$

(D) $4y[n]+y[n+1]=3x[n+1]$

51. The impulse response of a system is given by

$$h[n]=\delta[n]-\delta[n-5]$$

The difference equation representation for this system is

(A) $y[n]=x[n]-x[n-5]$ (B) $y[n]=x[n]-x[n+5]$

(C) $y[n]=x[n]+5x[n-5]$ (D) $y[n]=x[n]-5x[n+5]$

52. The transfer function of a system is given by

$$H(z)=\frac{z(3z-2)}{z^2-z-\frac{1}{4}}$$

The system is

(A) Causal and Stable

(B) Causal, Stable and minimum phase

(C) Minimum phase

(D) None of the above

53. The z-transform of a signal $x[n]$ is given by

$$X(z)=\frac{3}{1-\frac{10}{3}z^{-1}+z^{-2}}$$

If $X(z)$ converges on the unit circle, $x[n]$ is

(A) $-\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n-1]$

(B) $\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n-1]$

(C) $\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n]$

(D) $-\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n]$

54. The transfer function of a system is given as

$$H(z)=\frac{4z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)^2},\ |z|>\frac{1}{4}$$

The $h[n]$ is

(A) Stable (B) Causal

(C) Stable and Causal (D) None of the above

55. The transfer function of a system is given as

$$H(z)=\frac{2\left(z+\frac{1}{2}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}.$$

Consider the two statements

Statement(i) : System is causal and stable.

Statement(ii) : Inverse system is causal and stable.

The correct option is

(A) (i) is true

(B) (ii) is true

(C) Both (i) and (ii) are true

(D) Both are false

56. The impulse response of a system is given by

$$h[n]=10\left(\frac{-1}{2}\right)^n u[n]-9\left(\frac{-1}{4}\right)^n u[n]$$

For this system two statement are

Statement (i): System is causal and stable

Statement (ii): Inverse system is causal and stable.

The correct option is

(A) (i) is true (B) (ii) is true

(C) Both are true (D) Both are false

57. The system

$$y[n] = cy[n-1] - 0.12y[n-2] + x[n-1] + x[n-2]$$

is stable if

(A) $c < 1.12$ (B) $c > 1.12$

(C) $|c| < 1.12$ (D) $|c| > 1.12$

58. Consider the following three systems

$$y_1[n] = 0.2y[n-1] + x[n] - 0.3x[n-1] + 0.02x[n-2]$$

$$y_2[n] = x[n] - 0.1x[n-1]$$

$$y_3[n] = 0.5y[n-1] + 0.4x[n] - 0.3x[n-1]$$

The equivalent system are

(A) $y_1[n]$ and $y_2[n]$ (B) $y_2[n]$ and $y_3[n]$

(C) $y_3[n]$ and $y_1[n]$ (D) all

59. The z-transform of a causal system is given as

$$X(z) = \frac{2 - 1.5z^{-1}}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

The $x[0]$ is

(A) -1.5 (B) 2

(C) 1.5 (D) 0

60. The z-transform of a anti causal system is

$$X(z) = \frac{12 - 21z}{3 - 7z + 12z^2}$$

The value of $x[0]$ is

(A) $-\frac{7}{4}$ (B) 0

(C) 4 (D) Does not exist

61. Given the z-transforms

$$X(z) = \frac{z(8z-7)}{4z^2 - 7z + 3}$$

The limit of $x[\infty]$ is

(A) 1 (B) 2

(C) ∞ (D) 0

62. The impulse response of the system shown in fig. P5.4.62 is

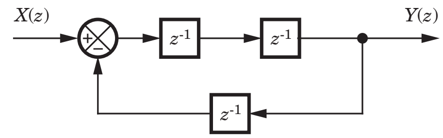

Fig. P5.4.62

(A) $2^{\left(\frac{n}{2}-2\right)}(1 + (-1)^n)u[n] + \frac{1}{2}\delta[n]$

(B) $\frac{2^n}{2}(1 + (-1)^n)u[n] + \frac{1}{2}\delta[n]$

(C) $2^{\left(\frac{n}{2}-2\right)}(1 + (-1)^n)u[n] - \frac{1}{2}\delta[n]$

(D) $\frac{2^n}{2}[1 + (-1)^n]u[n] - \frac{1}{2}\delta[n]$

63. The system diagram for the transfer function

$$H(z) = \frac{z}{z^2 + z + 1}$$

is shown in fig. P5.4.63. This system diagram is a

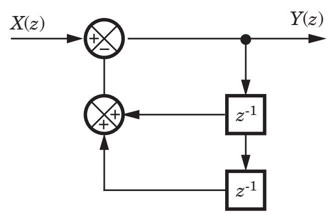

Fig. P5.4.63

(A) Correct solution

(B) Not correct solution

(C) Correct and unique solution

(D) Correct but not unique solution

****

24. (A) $X(z)=\dfrac{z^2-3z}{z^2+\frac{3}{2}z^{-1}}=\dfrac{1-3z^{-1}}{1+\frac{3}{2}z^{-1}-z^{-2}}$

$=\dfrac{2}{1+2z^{-1}}-\dfrac{1}{1-\frac{1}{2}z^{-1}}$, ROC $:\frac{1}{2}<|z|<2$

$\Rightarrow \quad x[n]=-2(2)^n u[-n-1]-\dfrac{1}{2^n}u[n]$

25. (A) $x[n]$ is right sided

$X(z)=\dfrac{z-\frac{1}{4}z^{-1}}{1-16z^{-1}}=\dfrac{\frac{49}{32}}{1+4z^{-1}}+\dfrac{\frac{47}{32}}{1-4z^{-1}}$

$\Rightarrow \quad x[n]=\left[\dfrac{49}{32}(-4)^n+\dfrac{47}{32}4^n\right]u[n]$

26. (C) $x[n]$ is right sided

$X(z)=\left(2+\dfrac{1}{1+z^{-1}}+\dfrac{-1}{1-z^{-1}}\right)z^2$

$\Rightarrow \quad x[n]=2\delta[n+2]+((-1)^n-1)u[n+2]$

27. (A) $\delta[n]+2\delta[n-6]+4\delta[n-8]$

28. (B) $x[n]$ is right sided, $x[n]=\sum_{k=5}^{10}\dfrac{1}{k}\delta[n-k]$

29. (D) $x[n]$ is right sided signal

$X(z)=1+3z^{-1}+3z^{-2}+z^{-3}$

$\Rightarrow \quad x[n]=\delta[n]+3\delta[n-1]+3\delta[n-2]+\delta[n-3]$

30. (A)

$x[n]=\delta[n+6]+\delta[n+2]+3\delta[n]+2\delta[n-3]+\delta[n-4]$

31. (B) $X(z)=1+z+\dfrac{z^2}{2!}+\dfrac{z^3}{3!}+.........$

$+1+\dfrac{1}{z}+\dfrac{1}{2!z^2}+\dfrac{1}{3!z^3}.........$

$x[n]=\delta[n]+\delta[n+1]+\dfrac{\delta[n+2]}{2!}+\dfrac{\delta[n+3]}{3!}......$

$+\delta[n]+\delta[n-1]+\dfrac{\delta[n-2]}{2!}+\dfrac{\delta[n-3]}{3!}.........$

$x[n]=\delta[n]+\dfrac{1}{n!}$

32. (A) $X(z)=1+\dfrac{z^{-2}}{4}+\left(\dfrac{z^{-2}}{4}\right)^2=\sum_{k=0}^{\infty}\left(\dfrac{1}{4}z^{-2}\right)^k$

$\Rightarrow \quad x[n]=\sum_{k=0}^{\infty}\left(\dfrac{1}{4}\right)^k\delta[n-2k]$

$=\begin{cases}\left(\dfrac{1}{4}\right)^{\frac{n}{2}}, & n \text{ even and } n\geq 0\\ 0, & n \text{ odd}\end{cases}$

$=\begin{cases}2^{-n}, & n \text{ even and } n\geq 0\\ 0, & n \text{ odd}\end{cases}$

33. (C) $X(z)=-4z^2\sum_{k=0}^{\infty}(2z)^{2k}=-\sum_{k=0}^{\infty}2^{2(k+1)}z^{2(k+1)}$

$\Rightarrow \quad x[n]=-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)]$

34. (A) $\ln(1+\alpha)=\sum_{k=1}^{\infty}\dfrac{(-1)^{k-1}}{k}(\alpha)^k$

$X(z)=\sum_{k=1}^{\infty}\dfrac{(-1)^{k-1}}{k}(z^{-1})^k$

$\Rightarrow \quad x[n]=\sum_{k=1}^{\infty}\dfrac{(-1)^{k-1}}{k}\delta[n-1]$

35. (B) $\cos\alpha=\sum_{k=0}^{\infty}\dfrac{(-1)^k}{(2k)!}\alpha^{2k}$

$X(z)=\sum_{k=0}^{\infty}\dfrac{(-1)^k}{(2k)!}(z^{-3k})^{2k}$

$\Rightarrow \quad x[n]=\sum_{k=0}^{\infty}\dfrac{(-1)^k}{(2k)!}\delta[n-6k]$

$n=12 \quad\Rightarrow\quad 12-6k=0,\ k=2,$

$x[12]=\dfrac{(-1)^2}{4!}=\dfrac{1}{24}$

36. (D) All gives the same z transform with different ROC. So all are the solution.

37. (C) $x_1[n]$ is right-sided signal

$z_1>2,\ z_1>\dfrac{1}{2}$ gives $z_1>2$

$x_2[n]$ is left-sided signal

$z_2<2,\ z_2<\dfrac{1}{2}$ gives $z_2<\dfrac{1}{2}$

$x_3[n]$ is double sided signal

$z_3>\dfrac{1}{2}$ and $z_3<2$ gives $\dfrac{1}{2}<z_3<2$

38. (B) $X(z)=\dfrac{2}{1-\frac{1}{2}z^{-1}}+\dfrac{-1}{1+\frac{1}{z}z^{-1}},$

$|z|>\dfrac{1}{2}$ (Right-sided) $\Rightarrow \quad x[n]=\dfrac{2}{2^n}u[n]-\left(\dfrac{-1}{3}\right)^n u[n]$

$|z|<\dfrac{1}{3}$ (Left-sided) $\Rightarrow \quad x[n]=\left[\dfrac{-2}{2^n}+\left(\dfrac{-1}{3}\right)^n\right]u[-n-1]$

$\frac{1}{3} < |z| < \frac{1}{2}$ (Two-sided) $x[n] = -\frac{2}{2^n} u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$

So (b) is wrong.

39. (A) Since the ROC includes the $z = \frac{3}{4}$, ROC is

$\frac{1}{2} < |z| < 1,$

$$X(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + z^{-1}}$$

$$\Rightarrow \quad x[n] = \frac{A}{2^n} u[n] \cdot B\,(-1)^n u[-n-1]$$

$$1 = \frac{A}{2} \quad \Rightarrow \quad A = 2\ ,$$

$$x[-1] = 1 = (-1)B(-1) \quad \Rightarrow \quad B = 1$$

$$\Rightarrow \quad x[n] = \frac{1}{2^{n-1}} u[n] - (-1)^n u[-n-1]$$

40. (B) $x[n] = Cp^n u[n]$, $x[0] = 2 = C$

$$x[2] = \frac{1}{2} = 2p^2 \quad \Rightarrow \quad p = \frac{1}{2}\ ,$$

$$x[n] = 2\left(\frac{1}{2}\right)^n u(n)$$

41. (B) $H(z) = \frac{1}{1 - 2z^{-1}} + \frac{1}{1 + \frac{1}{2}z^{-1}}$

$h[n]$ is stable, so ROC includes $|z| = 1$

ROC : $\frac{1}{2} < |z| < 2$,

$$h[n] = (2)^n u[-n-1] + \left(\frac{-1}{2}\right)^n u\,[n]$$

42. (A) $X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \delta[n-2]z^{-n} = z^{-2}$

43. (D) $X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}}$

44. (B) $H(z) = \frac{3}{1 - 3z^{-1}} + \frac{2}{1 + 2z^{-1}}$

$h[n]$ is causal so ROC is $|z| > 3$,

$$\Rightarrow \quad h[n] = [3^{n+1} + 2\,(-2)^n]u[n]$$

45. (A) $X(z) = 1 + \frac{z^{-1}}{4} - \frac{z^{-2}}{8}$, $\quad Y(z) = 1 - \frac{3z^{-1}}{4}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{-2}{3}}{1 - \frac{1}{4}z^{-1}} + \frac{\frac{5}{3}}{1 + \frac{1}{2}z^{-1}}\ ,$$

$$\Rightarrow \quad h[n] = \frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right]u[n]$$

46. (D) $X(z) = \frac{1}{1 + 3z^{-1}}$

$$Y(z) = \frac{4}{1 - 2z^{-1}} - \frac{1}{1 - \frac{1}{2}z^{-1}} = \frac{3}{(1 - 2z^{-1})\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{10}{1 - 2z^{-1}} + \frac{-7}{1 - \frac{1}{2}z^{-1}}$$

$$\Rightarrow \quad h[n] = \left[10(2)^n - 7\left(\frac{1}{2}\right)^n\right]u(n)$$

47. (D) $H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$, $Y(z) = 2z^{-4}$

$$X(z) = \frac{Y(z)}{H(z)} = 2z^{-4} - z^{-5}$$

$$\Rightarrow \quad x[n] = 2[\delta - 4] - \delta[n-5]$$

48. (A) $Y(z)\left[1 - \frac{z^{-1}}{2}\right] = 2z^{-1}X\,(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-1}}{1 - \frac{z^{-1}}{2}}$$

$$\Rightarrow \quad h[n] = 2\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

49. (C) $H(z) = \frac{Y(z)}{X(z)} = (1 - z^{-2} + z^{-4} - z^{-6})$

$$\Rightarrow \quad h[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6]$$

50. (A) $h[n] = \frac{3}{4}\left(\frac{1}{4}\right)^{n-1} u[n-1]$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{4}z^{-1}}{1 - \frac{1}{4}z^{-1}}$$

$$Y(z) - \frac{1}{4}z^{-1}Y(z) = \frac{3}{4}z^{-1}X(z)$$

$$\Rightarrow \quad y[n] - \frac{1}{4}y[n-1] = \frac{3}{4}x[n-1]$$

51. (A) $H(z) = \dfrac{Y(z)}{X(z)} = 1 - z^{-5}$

$\Rightarrow \quad y[n] = x[n] - x[n-5]$

52. (D) Zero at : $z = 0, \dfrac{2}{3}$, poles at $z = \dfrac{1 \pm \sqrt{2}}{2}$

(i) Not all poles are inside $|z| = 1$, the system is not causal and stable.

(ii) Not are poles and zero are inside $|z| = 1$, the system is not minimum phase.

53. (A) $X(z) = \dfrac{-\frac{3}{8}}{1 - \frac{1}{3}z^{-1}} + \dfrac{\frac{27}{8}}{1 - 3z^{-1}}$

Since $X(z)$ converges on $|z| = 1$. So ROC must include this circle.

ROC : $\dfrac{1}{3} < |z| < 3$,

$x[n] = -\dfrac{1}{3^{n-1}8} u[n] - \dfrac{3^{n+3}}{8} u[-n-1]$

54. (C) $h[n] = 16n\left(\dfrac{1}{4}\right)^n u[n]$. So system is both stable and causal. ROC includes $z = 1$.

55. (C) Pole of system at : $z = -\dfrac{1}{2}, \dfrac{1}{3}$

Pole of inverse system at : $z = -\dfrac{1}{2}$

For this system and inverse system all poles are inside $|z| = 1$. So both system are both causal and stable.

56. (A) $H(z) = \dfrac{10}{1 + \frac{1}{2}z^{-1}} - \dfrac{9}{1 + \frac{1}{4}z^{-1}}$

$= \dfrac{1 - 2z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$

Pole of this system are inside $|z| = 1$. So the system is stable and causal.

For the inverse system not all pole are inside $|z| = 1$. So inverse system is not stable and causal.

57. (C) $|a_2| = 0.12 < 1$, $a_1 = |-c| < 1 + 0.12$, $|c| < 1.12$

58. (A) $Y_1(z) = 1 - 0.1z^{-1}$, $Y_2(z) = 1 - 0.1z^{-1}$

$Y_3(z) = \dfrac{0.4 - 0.3z^{-1}}{1 - 0.5z^{-1}}$

So y_1 and y_2 are equivalent.

59. (B) Causal signal $x[0] = \lim_{z \to \infty} X(z) = 2$

60. (C) Anti causal signal, $x[0] = \lim_{z \to \infty} X(z) = 4$

61. (A) The function has poles at $z = 1, \dfrac{3}{4}$. Thus final value theorem applies.

$$\lim_{n \to \infty} x(n) = \lim_{z \to 1} (z-1)X(z) = (z-1)\frac{z(2z - \frac{7}{4})}{(z-1)\left(z - \frac{3}{4}\right)} = 1$$

62. (C) $[2Y(z) + X(z)]z^{-2} = Y(z)$

$H(z) = \dfrac{z^{-2}}{1 - 2z^{-2}}$

$\Rightarrow \quad h[n] = -\dfrac{1}{2} + \dfrac{\frac{1}{4}}{1 - \sqrt{2}z^{-1}} + \dfrac{\frac{1}{4}}{1 + \sqrt{2}z^{-1}}$

$= -\dfrac{1}{2}\delta[n] + \dfrac{1}{4}\{(\sqrt{2})^n + (-\sqrt{2})^n\}u[n]$

63. (D) $Y(z) = X(z)z^{-1} - \{Y(z)z^{-1} + Y(z)z^{-2}\}$

$\dfrac{Y(z)}{X(z)} = \dfrac{z^{-1}}{1 + z^{-1} + z^{-2}} = \dfrac{z}{z^2 + z + 1}$

So this is a solution but not unique. Many other correct diagrams can be drawn.

****

CHAPTER 5.6

THE DISCRETE-TIME FOURIER TRANSFORM

Statement for Q.1–9:

Determine the discrete-time Fourier Transform for the given signal and choose correct option.

1. $x[n] = \begin{cases} 1, & |n| \le 2 \\ 0, & \text{otherwise} \end{cases}$

(A) $\frac{\sin 5\Omega}{\sin \Omega}$

(B) $\frac{\sin 4\Omega}{\sin \Omega}$

(C) $\frac{\sin 2.5\Omega}{\sin \Omega}$

(D) None of the above

2. $x[n] = \left(\frac{3}{4}\right)^n u[n-4]$

(A) $\frac{\left(\frac{3}{4}e^{-j\Omega}\right)^4}{1-\frac{3}{4}e^{-j\Omega}}$

(B) $\frac{\left(\frac{3}{4}e^{j\Omega}\right)^4}{1-\frac{3}{4}e^{j\Omega}}$

(C) $\frac{\left(\frac{3}{4}e^{-j\Omega}\right)^4}{1+\frac{3}{4}e^{j\Omega}}$

(D) None of the above

3. $x[n] = u[n-2] - u[n-6]$

(A) $e^{3j\Omega} + e^{3j\Omega} + e^{4j\Omega} + e^{5j\Omega}$

(B) $\frac{e^{-2j\Omega}(1-e^{3j\Omega})}{1-e^{j\Omega}}$

(C) $e^{-2j\Omega} + e^{-3j\Omega} + e^{-4j\Omega} + e^{-5j\Omega}$

(D) $\frac{e^{-2j\Omega}(1-e^{-3j\Omega})}{1-e^{-j\Omega}}$

4. $x[n] = a^{|n|}, \quad |a| < 1$

(A) $\frac{1-a^2}{1-2a\sin\Omega + a^2}$

(B) $\frac{1-a^2}{1-2a\cos\Omega + a^2}$

(C) $\frac{1-a^2}{1-2ja\sin\Omega + a^2}$

(D) None of the above

5. $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$

(A) $\frac{e^{j\Omega}}{2-e^{-j\Omega}}$

(B) $\frac{2e^{j\Omega}}{2-e^{-j\Omega}}$

(C) $\frac{e^{j\Omega}}{2-e^{j\Omega}}$

(D) $\frac{2e^{j\Omega}}{2-e^{j\Omega}}$

6. $x[n] = 2\delta[4-2n]$

(A) $2e^{-j2\Omega}$

(B) $2e^{j2\Omega}$

(C) 1

(D) None of the above

7. $x[n] = u[n]$

(A) $\pi\delta(\Omega) - \frac{1}{1+e^{-j\Omega}}$

(B) $\frac{1}{1-e^{-j\Omega}}$

(C) $\pi\delta(\Omega) + \frac{1}{1-e^{-j\Omega}}$

(D) $\frac{1}{1+e^{-j\Omega}}$

8. $x[n] = \{-2, \ -1, \ \underset{\uparrow}{0}, \ 1, \ 2\}$

(A) $2j(2\sin 2\Omega + \sin\Omega)$

(B) $2(2\cos 2\Omega - \cos\Omega)$

(C) $-2j(2\sin 2\Omega + \sin\Omega)$

(D) $-2(2\cos 2\Omega - \cos\Omega)$

9. $x[n] = \sin\left(\frac{\pi}{2}n\right)$

(A) $\pi(\delta[\Omega - \pi/2] - \delta[\Omega + \pi/2])$

(B) $\frac{j}{2}(\delta[\Omega + \pi/2] - \delta[\Omega - \pi/2])$

(C) $2\pi(\delta[\Omega - \pi/2] - \delta[\Omega + \pi/2])$

(D) $j\pi(\delta[\Omega + \pi/2] - \delta[\Omega - \pi/2])$

Statement for Q.10–21:

Determine the signal having the Fourier transform given in question.

10. $X(e^{j\Omega}) = \dfrac{1}{(1-ae^{-j\Omega})^2}$, $|a|<1$

(A) $(n-1)a^n u[n]$
(B) $(n+1)a^n u[n]$
(C) $na^n u[n]$
(D) None of the above

11. $X(e^{j\Omega}) = 8\cos^2 \omega$

(A) $(\delta[n+2] + 2\delta[n] + \delta[n-2])$
(B) $2(\delta[n+2] + 2\delta[n] + \delta[n-2])$
(C) $-4(\delta[n+2] + \delta[n] + \delta[n-2])$
(D) $\frac{1}{2}(\delta[n+2] + \delta[n] + \delta[n-2])$

12. $X(e^{j\Omega}) = \begin{cases} 2j, & 0<\Omega\le\pi \\ -2j, & -\pi<\Omega\le 0 \end{cases}$

(A) $-\dfrac{4}{\pi n}\sin^2\left(\dfrac{\pi n}{2}\right)$
(B) $\dfrac{4}{\pi n}\sin^2\left(\dfrac{\pi n}{2}\right)$
(C) $\dfrac{8}{\pi n}\sin^2\left(\dfrac{\pi n}{2}\right)$
(D) $-\dfrac{8}{\pi n}\sin^2\left(\dfrac{\pi n}{2}\right)$

13. $X(e^{j\Omega}) = \begin{cases} 1, & \dfrac{\pi}{4}\le|\Omega|<\dfrac{3\pi}{4} \\ 0, & 0\le|\Omega|<\dfrac{\pi}{4},\ \dfrac{3\pi}{4}\le|\Omega|\le\pi \end{cases}$

(A) $\dfrac{2}{n}\left(\sin\left(\dfrac{3\pi n}{4}\right) - \sin\left(\dfrac{\pi n}{4}\right)\right)$
(B) $\dfrac{1}{\pi n}\left(\sin\left(\dfrac{3\pi n}{4}\right) - \sin\left(\dfrac{\pi n}{4}\right)\right)$
(C) $\dfrac{2}{n}\left(\cos\left(\dfrac{3\pi n}{4}\right) + \cos\left(\dfrac{\pi n}{4}\right)\right)$
(D) $\dfrac{1}{\pi n}\left(\cos\left(\dfrac{3\pi n}{4}\right) + \cos\left(\dfrac{\pi n}{4}\right)\right)$

14. $X(e^{j\Omega}) = e^{-\frac{j\Omega}{2}}$ for $-\pi \le \Omega \le \pi$

(A) $\pi\delta[n-1/2]$
(B) $2\pi\delta[n-1/2]$
(C) $\dfrac{(-1)^{n+1}}{\pi(n-\frac{1}{2})}$
(D) None of the above

15. $X(e^{j\Omega}) = \cos 2\Omega + j\sin 2\Omega$

(A) $2\pi\delta[n+2]$
(B) $\delta[n+2]$
(C) 0
(D) None of the above

16. $X(e^{j\Omega}) = j4\sin 4\Omega - 1$

(A) $4\pi\delta[n+4] - 4\pi\delta[n-4] - 2\pi\delta[n]$
(B) $2\delta[n+4] - 2\delta[n-4] - \delta[n]$
(C) $\delta[n+4] - \delta[n-4] - \delta[n]$
(D) None of the above

17. $X(e^{j\Omega}) = \dfrac{2}{-e^{-j2\Omega} + e^{-j\Omega} + 6}$

(A) $\dfrac{5}{2^{-n}}\left(1+\left(\dfrac{-2}{3}\right)^{n+1}\right)u[n]$
(B) $2^{-n}\left(1-\left(\dfrac{-2}{3}\right)^{n+1}\right)u[n]$
(C) $\dfrac{2^{-n}}{5}\left((-1)^n + \left(\dfrac{2}{3}\right)^{n+1}\right)u[n]$
(D) None of the above

18. $X(e^{j\Omega}) = \dfrac{2+\frac{1}{4}e^{-j\Omega}}{-\frac{1}{8}e^{-j2\Omega} + \frac{1}{4}e^{-j\Omega} + 1}$

(A) $2^{-n+1}[1+(-2)^{-n}]u[n]$
(B) $2^{-n}[1+(-2)^{-n}]u[n]$
(C) $2^{-n+1}[(-1)^n + 2^{-n}]u[n]$
(D) $2^{-n}[(-1)^n + 2^{-n}]u[n]$

19. $X(e^{j\Omega}) = \dfrac{2e^{-j\Omega}}{1-\frac{1}{4}e^{-j2\Omega}}$

(A) $2^{n-1}[1+(-1)^n]u[n]$
(B) $2^{1-n}[1+(-1)^n]u[n]$
(C) $2^{1-n}[1-(-1)^n]u[n]$
(D) $2^{n-1}[1-(-1)^n]u[n]$

20. $X(e^{j\Omega}) = \dfrac{1-\frac{1}{3}e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-2j\Omega}}$

(A) $\left(\dfrac{2}{9}\left(\dfrac{1}{2}\right)^n + \dfrac{7}{9}\left(-\dfrac{1}{4}\right)^n\right)u[n]$
(B) $\left(\dfrac{2}{9}\left(-\dfrac{1}{2}\right)^n + \dfrac{7}{9}\left(\dfrac{1}{4}\right)^n\right)u[n]$
(C) $\left(\dfrac{2}{9}\left(-\dfrac{1}{2}\right)^n - \dfrac{7}{9}\left(\dfrac{1}{4}\right)^n\right)u[n]$
(D) $\left(\dfrac{2}{9}\left(\dfrac{1}{2}\right)^n - \dfrac{7}{9}\left(-\dfrac{1}{4}\right)^n\right)u[n]$

$$x[n]=\left(-\frac{1}{2}\right)^n u[n]+\left(\frac{1}{4}\right)^n u[n]=2^{-n}[(-1)^n+2^{-n}]u[n]$$

19. (C) $X(e^{j\Omega})=\dfrac{2e^{-j\Omega}}{1-\frac{1}{4}e^{-j2\Omega}}=\dfrac{2}{1-\frac{1}{2}e^{-j\Omega}}-\dfrac{2}{1+\frac{1}{2}e^{-j\Omega}}$

$$x[n]=2\left(\frac{1}{2}\right)^n u[n]-2\left(-\frac{1}{2}\right)^n u[n]$$

$$=\frac{1}{2^{n-1}}[1-(-1)^n]u[n]=2^{1-n}[1-(-1)^n]u[n]$$

20. (A) $X(e^{j\Omega})=\dfrac{1-\frac{1}{3}e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}-\frac{1}{8}e^{-2j\Omega}}$

$$=\frac{\frac{2}{9}}{1-\frac{1}{2}e^{-j\Omega}}+\frac{\frac{7}{9}}{1+\frac{1}{4}e^{-j\Omega}}$$

$$x[n]=\frac{2}{9}\left(\frac{1}{2}\right)^n u[n]+\frac{7}{9}\left(-\frac{1}{4}\right)^n u[n]$$

21. (C) $X(e^{j\Omega})=\dfrac{(b-a)e^{j\Omega}}{e^{j2\Omega}-(a+b)e^{j\Omega}+ab}$

$$=\frac{(b-a)e^{-j\Omega}}{1-(a+b)e^{-j\Omega}+abe^{-j2\Omega}}=\frac{1}{1-be^{-j\Omega}}+\frac{-1}{1-ae^{-j\Omega}}$$

$x[n]=b^n u[n]+a^n u[-n-1]$.

22. (D) The signal must be read and odd. Only signal (h) is real and odd.

23. (A) The signal must be real and even. Only signal (c) and (e) are real and even.

24. (A) $Y(e^{j\Omega})=e^{j\alpha\Omega}X(e^{j\Omega})$, $\quad y[n]=x[n+\alpha]$

If $Y(e^{j\Omega})$ is real, then $y[n]$ is real and even (if $x[n]$ is real.). Therefore $x[n+\alpha]$ is even and $x[n]$ has to be symmetric about α.This is true for signal (a), (c), (e), (f) and (g).

25. (D) $\int_{-\pi}^{\pi} X(e^{j\Omega})\,d\Omega=2\pi x[0]$,

$x[0]=0$ is for signal (c), (f), (g) and (h).

26. (D) $X(e^{j\Omega})$ is always periodic with period 2π. Therefore all signals satisfy the condition.

27. (D) $X(e^{j0})=\sum_{n=-\infty}^{\infty}x[n]$, This condition is satisfied only if the samples of the signal add up to zero. This is true for signal (b) and (h).

28. (A) $X(e^{j0})=\sum_{n=-\infty}^{\infty}x[n]=6$

29. (A) $y[n]=x[n+2]$ is an even signal. Therefore $Y(e^{j\Omega})$ is real and even.

$Y(e^{j\Omega})=e^{j2\Omega}X(e^{j\Omega}) \quad\Rightarrow\quad X(e^{j\Omega})=e^{-j2\Omega}Y(e^{j\Omega})$,

Since $Y(e^{j\Omega})$ is real. This imply $\arg\{Y(e^{j\Omega})\}=0$

Thus $\arg\{X(e^{j\Omega})\}=-2\Omega$

30. (C) $\int_{-\pi}^{\pi}X(e^{j\Omega})\,d\Omega=2\pi x[0]=4\pi$

31. (A) $X(e^{j\pi})=\sum_{n=-\infty}^{\infty}(-1)^n x[n]=2$

32. (C) $\text{Ev}\{x[n]\}\overset{DTFT}{\longleftrightarrow}\text{Re}\{X(e^{j\Omega})\}$

$$\text{Ev}\{x[n]=\frac{(x[n]+x[-n])}{2}$$

$$=\left\{-\frac{1}{2},\ 0,\ \frac{1}{2},\ 1,\ 0,\ 0,\ 1,\ \underset{\uparrow}{2},\ 1,\ 0,\ 0,\ 1,\ \frac{1}{2},\ 0,\ -\frac{1}{2}\right\}$$

33. (D) $\int_{-\pi}^{\pi}\left|X(e^{j\Omega})\right|^2=2\pi\sum_{n=-\infty}^{\infty}\left|x[n]\right|^2=28\pi$

34. (C) $nx[n]\overset{DTFT}{\longleftrightarrow}j\dfrac{d\,X(e^{j\Omega})}{d\Omega}$

$$\int_{-\pi}^{\pi}\left|\frac{dX(e^{j\Omega})}{d\Omega}\right|=2\pi\sum_{n=-\infty}^{\infty}|n|^2\left|x[n]\right|^2=316\pi$$

35. (A) $Y(e^{j\Omega})=e^{-j4\Omega}X(e^{j\Omega})$

$$y[n]=x[n-4]=(n-4)\left(\frac{3}{4}\right)^{|n-4|}$$

36. (C) Since $x[n]$ is real and odd, $X(e^{j\Omega})$ is purely imaginary. Thus $y[n]=0$.

37. (D) $X_2(e^{j\Omega})=X(e^{j2\Omega})$

$$X(e^{j2\Omega})\overset{DTFT}{\longleftrightarrow}x_2[n]=\begin{cases}x[n], & n \text{ even}\\ 0, & \text{otherwise}\end{cases}$$

$$y[n]=\begin{cases}-jn^2\left(\frac{3}{4}\right)^{|n|}, & n\text{ even}\\ 0, & \text{otherwise}\end{cases}$$

38. (B) $Y(e^{j\Omega}) = X(e^{j\Omega}) * X(e^{j(\Omega-\pi/2)})$

$y[n] = 2\pi x[n]x_1[n],\ x_1[n] = e^{j\pi n/2}x[n]$,

$$\Rightarrow \quad y[n] = 2\pi n^2 e^{j\pi n/2}\left(\frac{3}{4}\right)^{2|n|}$$

39. (C) $Y(e^{j\Omega}) = \dfrac{d}{d\Omega}X(e^{j\Omega})$

$$\Rightarrow \quad y[n] = -jnx[n] = -jn^2\left(\frac{3}{4}\right)^{|n|}$$

40. (B) $Y(e^{j\Omega}) = X(e^{j\Omega}) + X(e^{-j\Omega})$

$$\Rightarrow \quad y[n] = x[n] + x[-n] = 0$$

41. (C) For a real signal $x[n]$

$\text{od}\{x[n]\} \xleftrightarrow{DTFT} j\text{Im}\{X(e^{j\Omega})\}$

$j\text{Im}\{X(e^{j\Omega})\} = j\sin\Omega - j\sin 2\Omega,$

$$= \frac{1}{2}\left(e^{j\Omega} - e^{-j\Omega} - e^{2j\Omega} + e^{-2j\Omega}\right)$$

Therefore $\text{od}\{x[n]\} = \text{F}^{-1}\{j\text{Im}\{X(e^{j\Omega})\}\}$

$$= \frac{1}{2}\left(\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right)$$

$$\text{Od}\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

Since $x[n] = 0$ for $n > 0$,

$x[n] = 2\text{od}\{x[n]\} = \delta[n+1] - \delta[n+2]$ For $n < 0$

Using Parseval's relation

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|X(e^{j\Omega})\right|^2 d\Omega = \sum_{n=-\infty}^{\infty}|x[n]|^2$$

$$3 = \sum_{n=-\infty}^{-1}|x[n]|^2 = (x[0])^2 + 2$$

$x[0] = \pm 1$, But $x[0] = 0$, Hence $x[0] = 1$

$x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$

42. (C) $\left(\dfrac{1}{4}\right)^n u[n] \xleftrightarrow{DTFT} \dfrac{1}{1-\frac{1}{4}e^{-j\Omega}}\left(\dfrac{1}{4}\right)^n$

$$n\left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{DTFT} j\frac{d}{d\Omega}\left(\frac{1}{1-\frac{1}{4}e^{-j\Omega}}\right) = \frac{\frac{1}{4}e^{-j\Omega}}{\left(1-\frac{1}{4}e^{-j\Omega}\right)^2}$$

$$\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \sum_{n=-\infty}^{\infty} x[n] = X(e^{j0}) = \frac{4}{9}$$

43. (A) For all pass system $\left|H(e^{j\Omega})\right| = 1$ for all Ω

$$H(e^{j\Omega}) = \frac{b + e^{-j\Omega}}{1 - ae^{-j\Omega}}, \quad \left|b + e^{-j\Omega}\right| = \left|1 - a\,e^{-j\Omega}\right|$$

$$1 + b^2 + 2b\cos\Omega = 1 + a^2 - 2a\cos\Omega$$

This is possible only if $b = -a$.

44. (A) For $x[n] = \delta[n]$, $X(e^{j\Omega}) = 1$, $\dfrac{dX(e^{j\Omega})}{d\Omega} = 0$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{j\Omega})\,e^{j\Omega n}\,d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\Omega}\,e^{j\Omega n}\,d\Omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\Omega(n-1)}d\Omega = \frac{\sin\pi(n-1)}{\pi(n-1)}$$

45. (B) $H(e^{j\Omega}) = H_1(e^{j\Omega}) + H_2(e^{j\Omega})$

$$\frac{-12 + 5e^{-j\Omega}}{12 - 7e^{-j\Omega} + e^{-j2\Omega}} = \frac{1}{1-\frac{1}{3}e^{-j\Omega}} + \frac{-2}{1-\frac{1}{4}e^{-j\Omega}}$$

$$H_2(e^{j\Omega}) = \frac{1}{1-\frac{1}{3}e^{-j\Omega}}, \qquad h_2[n] = \left(\frac{1}{3}\right)^n u[n]$$

46. (D) $H(e^{j\Omega}) = \dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})}$,

$$\left(\frac{2}{3}\right)^n u[n] \xleftrightarrow{DTFT} \frac{1}{1-\frac{2}{3}e^{-j\Omega}}$$

$$n\left(\frac{2}{3}\right)^n u[n] \xleftrightarrow{DTFT} j\frac{d}{d\Omega}\left(\frac{1}{1-\frac{2}{3}e^{-j\Omega}}\right) = \frac{\frac{2}{3}e^{-j\Omega}}{1-\frac{2}{3}e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{\frac{2}{3}e^{-j\Omega}}{1-\frac{2}{3}e^{-j\Omega}} = \frac{2e^{-j\Omega}}{3-2e^{-j\Omega}}$$

47. (B) $H(e^{j\Omega}) = \dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \dfrac{\frac{2}{3}e^{-j\Omega}}{1-\frac{2}{3}e^{-j\Omega}}$

$$\Rightarrow \quad \left(1-\frac{2}{3}e^{-j\Omega}\right)Y(e^{j\Omega}) = \frac{2}{3}e^{-j\Omega}X(e^{j\Omega})$$

$$\Rightarrow \quad y[n] - \frac{2}{3}y[n-1] = \frac{2}{3}x[n-1]$$

$$\Rightarrow \quad 3y[n] - 2y[n-1] = 2x[n-1].$$

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CHAPTER

5.7

THE CONTINUOUS-TIME FOURIER SERIES

Statement for Q.1-5:

Determine the Fourier series coefficient for given periodic signal $x(t)$.

1. $x(t)$ as shown in fig. P5.7.1

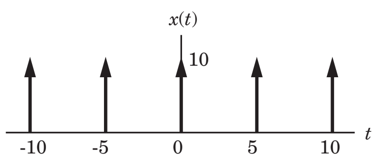

Fig. P5.7.1

(A) 1

(B) $\cos\left(\frac{\pi}{2}k\right)$

(C) $\sin\left(\frac{\pi}{2}k\right)$

(D) 2

2. $x(t)$ as shown in fig. P5.7.2

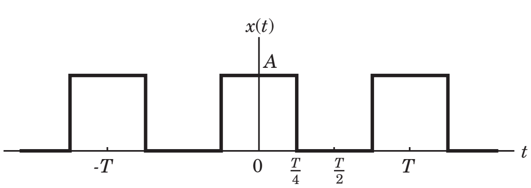

Fig. P5.7.2

(A) $\frac{A}{j\pi k}\sin\left(\frac{\pi}{2}k\right)$

(B) $\frac{A}{j\pi k}\cos\left(\frac{\pi}{2}k\right)$

(C) $\frac{A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$

(D) $\frac{A}{\pi k}\cos\left(\frac{\pi}{2}k\right)$

3. $x(t)$ as shown in fig. P5.7.3

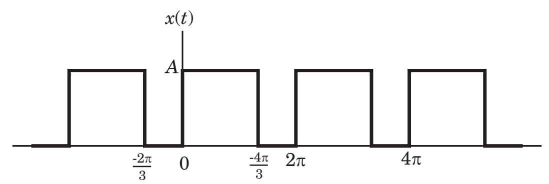

Fig. P5.7.3

(A) $\frac{A}{2\pi k}\left(e^{-j\left(\frac{4\pi k}{3}\right)}-1\right)$

(B) $j\frac{A}{2\pi k}\left(e^{-j\left(\frac{4\pi k}{3}\right)}-1\right)$

(C) $-j\frac{A}{2\pi k}\left(e^{-j\left(\frac{4\pi k}{3}\right)}-1\right)$

(D) $\frac{-A}{2\pi k}\left(e^{-j\left(\frac{4\pi k}{3}\right)}-1\right)$

4. $x(t)$ as shown in fig. P5.7.4

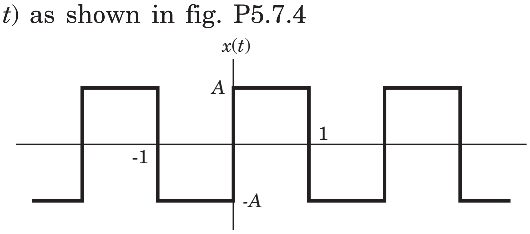

Fig. P5.7.4

(A) $\frac{A}{k\pi}(1-(-1)^k)$

(B) $\frac{A}{k\pi}(1+(-1)^k)$

(C) $\frac{A}{jk\pi}(1-(-1)^k)$

(D) $\frac{A}{jk\pi}(1+(-1)^k)$

5. $x(t)=\sin^2 t$

(A) $-\frac{1}{4}\delta[k-1]+\frac{1}{2}\delta[k]-\frac{1}{4}\delta[k+1]$

(B) $-\frac{1}{4}\delta[k-2]+\frac{1}{2}\delta[k]-\frac{1}{4}\delta[k+2]$

(C) $-\frac{1}{2}\delta[k-1]+\delta[k]-\frac{1}{2}\delta[k+1]$

(D) $-\frac{1}{2}\delta[k-2]+\delta[k]-\frac{1}{2}\delta[k+2]$

Statement for Q.6-11:

In the question, the FS coefficient of time-domain signal have been given. Determine the corresponding time domain signal and choose correct option.

7. $X[k] = j\delta[k-1] - j\delta[k+1] + \delta[k+3] + \delta[k-3]$, $\omega_o = 2\pi$

(A) $2(\cos 3\pi t - \sin \pi t)$ (B) $-2(\cos 3\pi t - \sin \pi t)$

(C) $2(\cos 6\pi t - \sin 2\pi t)$ (D) $-2(\cos 6\pi t - \sin 2\pi t)$

8. $X[k] = \left(\dfrac{-1}{3}\right)^{|k|}$, $\omega_o = 1$

(A) $\dfrac{4}{5 + 3\sin t}$ (B) $\dfrac{5}{4 + 3\sin t}$

(C) $\dfrac{5}{4 + 3\cos t}$ (D) $\dfrac{4}{5 + 3\cos t}$

9. $X[k]$ as shown in fig. P5.7.9 , $\omega_o = \pi$

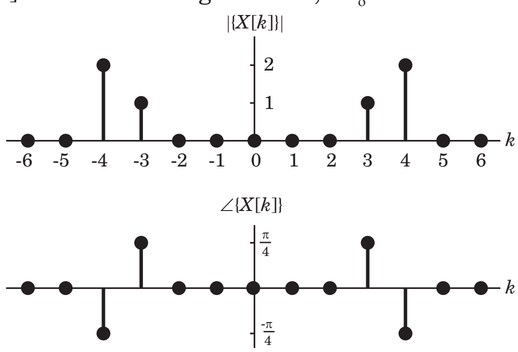

Fig. P5.7.9

(A) $6\cos\left(2\pi t + \dfrac{\pi}{4}\right) - 3\cos\left(3\pi t - \dfrac{\pi}{4}\right)$

(B) $4\cos\left(4\pi t - \dfrac{\pi}{4}\right) - 2\cos\left(3\pi t + \dfrac{\pi}{4}\right)$

(C) $2\cos\left(2\pi t + \dfrac{\pi}{4}\right) - 2\cos\left(3\pi t - \dfrac{\pi}{4}\right)$

(D) $4\cos\left(4\pi t + \dfrac{\pi}{4}\right) + 2\cos\left(3\pi t - \dfrac{\pi}{4}\right)$

10. $X[k]$ As shown in fig. P5.7.10 , $\omega_o = 2\pi$

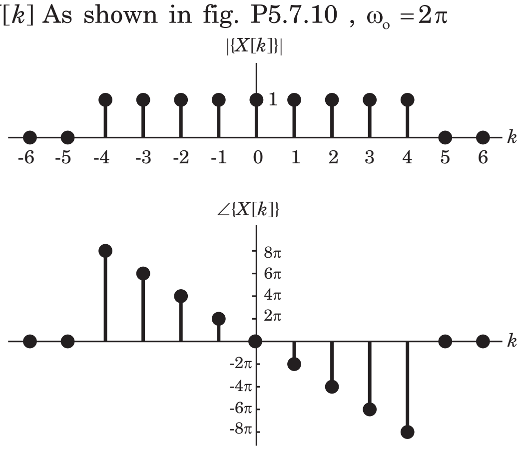

Fig. P5.7.10

(A) $\dfrac{\sin 9\pi t}{\sin \pi t}$ (B) $\dfrac{\sin 9\pi t}{\pi \sin \pi t}$

(C) $\dfrac{\sin 18\pi t}{2\sin \pi t}$ (D) None of the above

11. $X[k]$ As depicted in fig. P5.7.11, $\omega_o = \pi$

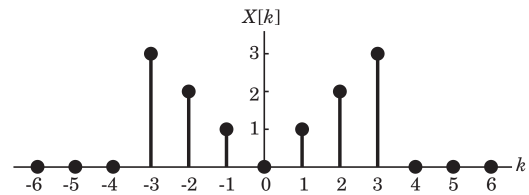

Fig. P5.7.11

(A) $3\cos 3\pi t + 2\cos 2\pi t + \cos \pi t$

(B) $3\sin 3\pi t + 2\sin 2\pi t + \sin \pi t$

(C) $6\sin 3\pi t + 4\sin 2\pi t + 2\sin \pi t$

(D) $6\cos 3\pi t + 4\cos 2\pi t + 2\cos \pi t$

Statement for Q.12-16:

Consider a continuous time periodic signal $x(t)$ with fundamental period T and Fourier series coefficients $X[k]$. Determine the Fourier series coefficient of the signal $y(t)$ given in question and choose correct option.

12. $y(t) = x(t - t_o) + x(t - t_o)$

(A) $2\cos\left(\dfrac{2\pi}{T}kt_o\right)X[k]$ (B) $2\sin\left(\dfrac{2\pi}{T}kt_o\right)X[k]$

(C) $e^{-t_o}X[k] + e^{t_o}X[-k]$ (D) $e^{-t_o}X[-k] + e^{t_o}X[k]$

13. $y(t) = \text{Ev}\{x(t)\}$

(A) $\dfrac{X[k] + X[-k]}{2}$ (B) $\dfrac{X[k] - X[-k]}{2}$

(C) $\dfrac{X[k] + X^*[-k]}{2}$ (D) $\dfrac{X[k] + X^*[-k]}{2}$

14. $y(t) = \text{Re}\{x(t)\}$

(A) $\dfrac{X[k] + X[-k]}{2}$ (B) $\dfrac{X[k] - X[-k]}{2}$

(C) $\dfrac{X[k] + X^*[-k]}{2}$ (D) $\dfrac{X[k] + X^*[-k]}{2}$

15. $y(t) = \frac{d^2x(t)}{dt^2}$

(A) $\left(\frac{2\pi k}{T}\right)^2 X[k]$ (B) $-\left(\frac{2\pi k}{T}\right)^2 X[k]$

(C) $j\left(\frac{2\pi k}{T}\right)^2 X[k]$ (D) $-j\left(\frac{2\pi k}{T}\right)^2 X[k]$

16. $y(t) = x(4t-1)$

(A) $\frac{8\pi}{T} X[k]$ (B) $\frac{4\pi}{T} X[k]$

(C) $e^{-jk\left(\frac{8\pi}{T}\right)} X[k]$ (D) $e^{jk\left(\frac{8\pi}{T}\right)} X[k]$

17. Consider a continuous-time signal

$$x(t) = 4\cos 100\pi t \sin 1000\pi t$$

with fundamental period $T = \frac{1}{50}$. The nonzero FS coefficient for this function are

(A) $X[-4], X[4], X[-7], X[7]$

(B) $X[-1], X[1], X[-10], X[10]$

(C) $X[-3], X[3], X[-4], X[4]$

(D) $X[-9], X[9], X[-11], X[11]$

18. A real valued continuous-time signal $x(t)$ has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are

$$X[1] = X[-1] = 4,\ X[3] = X^*[-3] = 4j$$

The signal $x(t)$ would be

(A) $4\cos\left(\frac{\pi}{4}t\right) + 4j\sin\left(\frac{3\pi}{4}t\right)$

(B) $4\cos\left(\frac{\pi}{4}t\right) - 4j\cos\left(\frac{3\pi}{4}t\right)$

(C) $8\cos\left(\frac{\pi}{4}t\right) + 8\cos\left(\frac{3\pi}{4}t + \frac{\pi}{2}\right)$

(D) None of the above

19. The continuous-time periodic signal is given as

$$x(t) = 4 + \cos\left(\frac{2\pi}{3}t\right) + 6\sin\left(\frac{5\pi}{3}t\right)$$

The nonzero Fourier coefficients are

(A) $X[0], X[-1], X[1], X[-5], X[5]$

(B) $X[0], X[-2], X[2], X[-5], X[5]$

(C) $X[0], X[-4], X[4], X[-10], X[10]$

(D) None of the above

Statement for Q.20-21:

Let $x_1(t)$ and $x_2(t)$ be continuous time periodic signal with fundamental frequency ω_1 and ω_2, Fourier series coefficients $X_1[k]$ and $X_2[k]$ respectively. Given that $x_2(t) = x_1(t-1) + x_1(1-t)$

20. The relation between ω_1 and ω_2 is

(A) $\omega_2 = \frac{\omega_1}{2}$ (B) $\omega_2 = \omega_1^2$

(C) $\omega_2 = \omega_1$ (D) $\omega_2 = \sqrt{\omega_1}$

21. The Fourier coefficient $X_2[k]$ will be

(A) $(X_1[k] - jX_1[-k])e^{-j\omega_1 k}$

(B) $(X_1[-k] - jX_1[k])e^{-j\omega_1 k}$

(C) $(X_1[k] + jX_1[-k])e^{-j\omega_1 k}$

(D) None of the above

Statement for Q.22-23:

Consider three continuous-time periodic signals whose Fourier series representation are as follows.

$$x_1(t) = \sum_{k=0}^{100}\left(\frac{1}{3}\right)^k e^{-jk\frac{2\pi}{50}t}$$

$$x_2(t) = \sum_{k=-100}^{100}\cos k\pi\, e^{-jk\frac{2\pi}{50}t}$$

$$x_3(t) = \sum_{k=-100}^{100} j\sin\left(\frac{k\pi}{2}\right) e^{-jk\frac{2\pi}{50}t}$$

22. The even signals are

(A) $x_2(t)$ only (B) $x_2(t)$ and $x_3(t)$

(C) $x_1(t)$ and $x_3(t)$ (D) $x_1(t)$ only

23. The real valued signals are

(A) $x_1(t)$ and $x_2(t)$ (B) $x_2(t)$ and $x_3(t)$

(C) $x_3(t)$ and $x_1(t)$ (D) $x_1(t)$ and $x_3(t)$

24. Suppose the periodic signal $x(t)$ has fundamental period T and Fourier coefficients $X[k]$. Let $Y[k]$ be the Fourier coefficient of $y(t)$ where $y(t) = dx(t)/dt$. The Fourier coefficient $X[k]$ will be

(A) $\frac{TY[k]}{j2\pi k}, k \neq 0$ (B) $\frac{TY[k]}{j2\pi k}$

(C) $\frac{TY[k]}{jk}, k \neq 0$ (D) $\frac{TY[k]}{jk}$

25. Suppose we have given the following information about a signal $x(t)$:

1. $x(t)$ is real and odd.
2. $x(t)$ is periodic with $T = 2$
3. Fourier coefficients $X[k] = 0$ for $|k| > 1$
4. $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

The signal, that satisfy these condition, is

(A) $\sqrt{2}\sin \pi t$ and unique

(B) $\sqrt{2}\sin \pi t$ but not unique

(C) $2\sin \pi t$ and unique

(D) $2\sin \pi t$ but not unique

26. Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = \frac{\sin 4\omega}{\omega}$$

The input to this system is a periodic signal

$$x(t) = \begin{cases} 2, & 0 \le t \le 4 \\ -2, & 4 \le t \le 8 \end{cases}$$

with period $T = 8$. The output $y(t)$ will be

(A) $1 + \sin^2\left(\frac{\pi t}{4}\right)$

(B) $1 + \cos^2\left(\frac{\pi t}{4}\right)$

(C) $1 + \sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right)$

(D) 0

27. Consider a continuous-time ideal low pass filter having the frequency response

$$H(j\omega) = \begin{cases} 1, & |\omega| \le 80 \\ 0, & |\omega| > 80 \end{cases}$$

When the input to this filter is a signal $x(t)$ with fundamental frequency $\omega_0 = 12$ and Fourier series coefficients $X[k]$, it is found that $x(t) \xleftrightarrow{S} y(t) = x(t)$. The largest value of $|k|$, for which $X[k]$ is nonzero, is

(A) 6

(B) 80

(C) 7

(D) 12

28. A continuous-time periodic signal has a fundamental period $T = 8$. The nonzero Fourier series coefficients are as,

$$X[1] = X^*[-1] = j\,,\; X[5] = X[-5] = 2,$$

The signal will be

(A) $4\cos\left(\frac{\pi}{4}t\right) - 2\sin\left(\frac{\pi}{4}t\right)$

(B) $2\cos\left(\frac{\pi}{4}t\right) + 4\sin\left(\frac{\pi}{4}t\right)$

(C) $2\cos\left(\frac{\pi}{4}t\right) + 2\sin\left(\frac{\pi}{4}t\right)$

(D) None of the above

Statement for Q.29-31:

Consider the following three continuous-time signals with a fundamental period of $T = 1$

$$x(t) = \cos 2\pi t\;,\; y(t) = \sin 2\pi t\;,\; z(t) = x(t)y(t)$$

29. The Fourier series coefficient $X[k]$ of $x(t)$ are

(A) $\frac{1}{2}(\delta[k+1] + \delta[k-1])$

(B) $\frac{1}{2}(\delta[k+1] - \delta[k-1])$

(C) $\frac{1}{2}(\delta[k-1] - \delta[k+1])$

(D) None of the above

30. The Fourier series coefficient of $y(t)$, $Y[k]$ will be

(A) $\frac{j}{2}(\delta[k+1] + \delta[k+1])$

(B) $\frac{j}{2}(\delta[k+1] - \delta[k-1])$

(C) $\frac{j}{2}(\delta[k-1] - \delta[k+1])$

(D) $\frac{1}{2j}(\delta[k+1] + \delta[k+1])$

31. The Fourier series coefficient of $z(t)$, $Z[k]$ will be

(A) $\frac{1}{4j}(\delta[k-2] - \delta[k+2])$

(B) $\frac{1}{2j}(\delta[k-2] - \delta[k+2])$

(C) $\frac{1}{2j}\,\delta[k+2] - \delta[k-2])$

(D) None of the above

32. Consider a periodic signal $x(t)$ whose Fourier series coefficients are

$$X[k] = \begin{cases} 2, & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise} \end{cases}$$

Consider the statements

1. $x(t)$ is real.
2. $x(t)$ is even
3. $\frac{dx(t)}{dt}$ is even

The true statements are

(A) 1 and 2

(B) only 2

(C) only 1

(D) 1 and 3

Statement for Q.33-36:

A waveform for one peroid is depicted in figure in question. Determine the trigonometric Fourier series and choose correct option.

33.

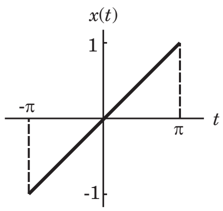

Fig. P5.7.33

(A) $\frac{2}{\pi}(\cos t + \frac{1}{2}\cos 2t + \frac{1}{3}\cos 3t + \frac{1}{4}\cos 4t + \ldots.)$

(B) $\frac{2}{\pi}(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \ldots.)$

(C) $\frac{2}{\pi}(\sin t + \cos t - \frac{1}{2}\sin 2t - \frac{1}{2}\cos 2t + \frac{1}{3}\sin 3t + \ldots.)$

(D) $\frac{2}{\pi}(\sin t + \cos t + \frac{1}{3}\sin 3t + \frac{1}{3}\cos 3t + \frac{1}{5}\sin 5t + \ldots.)$

34.

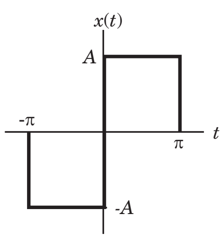

Fig. P5.7.34

(A) $\frac{A}{2} + \frac{4A}{\pi}\left(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \ldots.\right)$

(B) $\frac{A}{2} + \frac{4A}{\pi}\left(\cos t + \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t + \ldots.\right)$

(C) $\frac{4A}{\pi}\left(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \ldots.\right)$

(D) $\frac{4A}{\pi}\left(\cos t + \frac{1}{2}\cos 2t + \frac{1}{3}\cos 3t + \ldots.\right)$

35.

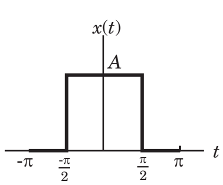

Fig. P5.7.35

(A) $\frac{A}{2} + \frac{2A}{\pi}(\sin t - \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t \ldots.)$

(B) $\frac{A}{2} + \frac{2A}{\pi}(\cos t - \frac{1}{2}\cos 2t + \frac{1}{3}\cos 3t \ldots.)$

(C) $\frac{A}{2} + \frac{2A}{\pi}(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t \ldots.)$

(D) $\frac{A}{2} + \frac{2A}{\pi}(\sin t + \cos t + \frac{1}{3}\sin 3t + \frac{1}{3}\cos 3t \ldots.)$

36.

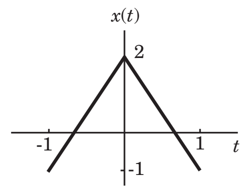

Fig. P5.7.36

(A) $\frac{1}{2} + \frac{12}{\pi^2}(\cos \pi t + \frac{1}{9}\cos 3\pi t + \frac{1}{25}\cos 5\pi t + \ldots.)$

(B) $3 + \frac{12}{\pi^2}(\cos \pi t + \frac{1}{9}\cos 3\pi t + \frac{1}{25}\cos 5\pi t + \ldots.)$

(C) $\frac{1}{2} + \frac{12}{\pi^2}(\sin \pi t - \frac{1}{9}\sin 3\pi t + \frac{1}{25}\sin 5\pi t - \ldots.)$

(D) $3 + \frac{12}{\pi^2}(\sin \pi t - \frac{1}{9}\sin 3\pi t + \frac{1}{25}\sin 5\pi t - \ldots.)$

*****

SOLUTIONS

1. (D) $X[k]=\frac{1}{T}\int_{-T/2}^{T/2}A\delta(t)e^{-jk\omega_o t}dt=\frac{A}{T}$,

$A=10$, $T=5$, $X[k]=2$

2. (C) $X[k]=\frac{1}{T}\int_{-T/2}^{T/2}x(t)e^{-jk\omega_o t}dt=\frac{1}{T}\int_{-T/4}^{T/4}Ae^{-jk\omega_o t}dt$

$=\frac{A}{T}\left[\frac{e^{-jk\omega_o t}}{-jk\omega_o}\right]_{\frac{-T}{4}}^{\frac{T}{4}}=\frac{A}{\pi k}\sin\left(\frac{\pi k}{2}\right)$

3. (B) $T=2\pi$, $\omega_o=\frac{2\pi}{2\pi}=1$, $x(t)=\begin{cases}A, & 0<t<\frac{4\pi}{3}\\ 0, & \frac{4\pi}{3}<t<2\pi\end{cases}$

$X[k]=\frac{1}{2\pi}\int_0^{2\pi}x(t)e^{-jkt}dt=\frac{1}{2\pi}\int_0^{4\pi/3}Ae^{-jkt}dt=\frac{jA}{2\pi k}\left[e^{-j\left(\frac{4\pi k}{3}\right)}-1\right]$

4. (C) $T=2$, $\omega_o=\frac{2\pi}{2}=\pi$, $x(t)=\begin{cases}-A, & -1<t<0\\ A, & 0<t<1\end{cases}$

$X[k]=\frac{1}{2}\int_{-1}^{1}x(t)e^{-jkt}dt=\frac{1}{2}\left(\int_{-1}^{0}-Ae^{-jk\pi t}dt+\int_0^1 Ae^{-jk\pi t}dt\right)$

$=\frac{A}{2}\left(\frac{1-e^{jk\pi}}{jk\pi}+\frac{e^{-jk\pi}-1}{-jk\pi}\right)=\frac{A}{jk\pi}(1-(-1)^k)$

5. (A) $\sin^2 t=\left(\frac{e^{jt}-e^{-jt}}{2j}\right)^2=\frac{-1}{4}(e^{2jt}-2+e^{-2jt})$

The fundamental period of $\sin^2(t)$ is π and $\omega_o=\frac{2\pi}{\pi}=2$,

$X[k]=\frac{-1}{4}\delta[k-1]+\frac{1}{2}\delta[k]-\frac{1}{4}\delta[k+1]$

7. (C) $x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{j2\pi kt}=je^{j2\pi t}-je^{-j2\pi t}+e^{j6\pi t}+e^{-j6\pi t}$

$=-2\sin 2\pi t+2\cos 6\pi t$

8. (D) $x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{jkt}=\sum_{k=-\infty}^{-1}\left(\frac{-1}{3}\right)^{-k}e^{jkt}+\sum_{k=0}^{\infty}\left(\frac{-1}{3}\right)^{k}e^{jkt}$

$=\frac{\frac{-1}{3}e^{-jt}}{1+\frac{1}{3}e^{-jt}}+\frac{1}{1+\frac{1}{3}e^{jt}}=\frac{4}{5+3\cos t}$

9. (D) $x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{j\pi kt}$

$=2e^{-j\frac{\pi}{4}}e^{j(-4)\pi t}+e^{j\frac{\pi}{4}}e^{j(-3)\pi t}+e^{-\frac{j\pi}{4}}e^{j(3)\pi t}+2e^{j\frac{\pi}{4}}e^{j(4)\pi t}$

$=2(e^{-j(4\pi t+\pi/4)}+e^{j(4\pi t+\pi/4)})+(e^{-j(3\pi t-\pi/4)}+e^{j(3\pi t-\pi/4)})$

$=4\cos(4\pi t+\pi/4)+2\cos(3\pi t-\pi/4)$

10. (A) $X[k]=e^{-j2\pi k}, -4\le k\le 4$

$x(t)=\sum_{k=-4}^{4}e^{-j2\pi k}e^{j\pi kt}=\sum_{k=-4}^{4}e^{-j2\pi k(t-1)}=\frac{\sin 9\pi t}{\sin \pi t}$

11. (D) $X[k]=|k|,\ -3\le k\le 3$

$x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{j\pi kt}$

$=3e^{j(-3)\pi t}+2e^{j(-2)\pi t}+e^{j(-1)\pi t}+e^{j(1)\pi t}+2e^{j(2)\pi t}+3e^{j(3)\pi t}$

$=6\cos 3\pi t+4\cos 2\pi t+2\cos \pi t$

12. (A) $x(t-t_o)$ is also periodic with T. The Fourier series coefficients $X_1[k]$ of $x(t-t_o)$ are

$X_1[k]=\frac{1}{T}\int_T x(t-t_o)e^{-jk\omega_o t}dt=\frac{e^{-jk\omega_o t_o}}{T}\int_T x(\tau)\,e^{-jk\omega_o\tau}d\tau$

$=e^{-jk\omega_o t_o}X[k]$

Similarly, the FS coefficients of $x(t+t_o)$ are

$X_2[k]=e^{jk\omega_o t_o}X[k]$

The FS coefficients of $x(t-t_o)+x(t+t_o)$ are

$Y[k]=X_1[k]+X_2[k]=e^{-jk\omega_o t_o}X[k]+e^{jk\omega_o t_o}X[k]$

$=2\cos(\omega_o k t_o)X[k]$

13. (A) $\mathrm{Ev}\{x(t)\}=\frac{x(t)+x(-t)}{2}$,

The FS coefficients of $x(t)$ are

$X_1[k]=\frac{1}{T}\int_T x(-t)e^{-jk\omega_o t}dt=\frac{1}{T}\int_T x(\tau)\,e^{jk\omega_o\tau}d\tau=X[-k]$

Therefore, the FS coefficients of $\mathrm{Ev}\{x(t)\}$ are

$Y[k]=\frac{X[k]+X[-k]}{2}$

14. (C) $\mathrm{Re}\{x(t)\}=\frac{x(t)+x^*(t)}{2}$,

The FS coefficient of $x^*(t)$ is

$X_1[k]=\frac{1}{T}\int_T x^*(t)\,e^{-jk\omega_o t}dt=X_1^*[-k]$

$X_1^*[k]=\frac{1}{T}\int_T x(t)e^{jk\omega_o t}dt=X[-k]$

$X_1[k]=X^*[-k]$

$Y[k]=\frac{X[k]+X^*[-k]}{2}$

13. $X[k] = \cos\left(\frac{10\pi}{19}k\right) + 2j\sin\left(\frac{4\pi}{19}k\right)$

(A) $\frac{19}{2}(\delta[n+5]+\delta[n-5]) + 19(\delta[n+2]-\delta[n-2]), |n| \le 9$

(B) $\frac{1}{2}(\delta[n+5]+\delta[n-5]) + (\delta[n+2]-\delta[n-2]), |n| \le 9$

(C) $\frac{9}{2}(\delta[n+5]+\delta[n-5]) + 9(\delta[n+2]-\delta[n-2]), |n| \le 9$

(D) $\frac{1}{2}(\delta[n+5]+\delta[n-5]) + (\delta[n+2]-\delta[n-2]), |n| \le 9$

14. $X[k] = \cos\left(\frac{\pi k}{21}\right)$

(A) $\frac{21}{2}(\delta[n+4]+\delta[n-4]), |n| \le 10$

(B) $\frac{1}{2}(\delta[n+4]+\delta[n-4]), |n| \le 10$

(C) $\frac{21}{2}(\delta[n+4]-\delta[n-4]), |n| \le 10$

(D) $\frac{1}{2}(\delta[n+4]-\delta[n-4]), |n| \le 10$

Statement for Q.15-20:

Consider a periodic signal $x[n]$ with period N and FS coefficients $X[k]$. Determine the FS coefficients $Y[k]$ of the signal $y[n]$ given in question.

15. $y[n] = x[n - n_o]$

(A) $e^{j\left(\frac{2\pi}{N}\right)n_o k}X[k]$ (B) $e^{-j\left(\frac{2\pi}{N}\right)n_o k}X[k]$

(C) $kX[k]$ (D) $-kX[k]$

16. $y[n] = x[n] - x[n-2]$

(A) $\sin\left(\frac{4\pi}{N}k\right)X[k]$ (B) $\cos\left(\frac{4\pi}{N}k\right)X[k]$

(C) $\left(1 - e^{-j\left(\frac{4\pi}{N}\right)k}\right)X[k]$ (D) $\left(1 - e^{j\left(\frac{4\pi}{N}\right)k}\right)X[k]$

17. $y[n] = x[n] + x[n + N/2]$, (assume that N is even)

(A) $2X[2k-1]$, for $0 \le k \le \left(\frac{N}{2} - 1\right)$

(B) $2X[2k-1]$, for $0 \le k \le \frac{N}{2}$

(C) $2X[2k]$, for $0 \le k \le \left(\frac{N}{2} - 1\right)$

(D) $2X[2k]$, for $0 \le k \le \frac{N}{2}$

18. $y[n] = x[n] - x[n - N/2]$, (assume that N is even)

(A) $(1-(-1)^{k+1})X[2k]$ (B) $(1-(-1)^{k})X[k]$

(C) $(1-(-1)^{k+1})X[k]$ (D) $(1-(-1)^{k\backslash})X[2k]$

19. $y[n] = x^*[-n]$

(A) $-X^*[k]$ (B) $-X^*[-k]$

(C) $X^*[k]$ (D) $X^*[-k]$

20. $y[n] = (-1)^n x[n]$, (assume that N is even)

(A) $X\left[k - \frac{N}{2}\right]$ (B) $X\left[k + \frac{N}{2}\right]$

(C) $X\left[k - \frac{N}{2} + 1\right]$ (D) $X\left[k + \frac{N}{2} - 1\right]$

Statement for Q.21-23:

Consider a discrete-time periodic signal

$$x[n] = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & 8 \le n \le 9 \end{cases}$$

with period $N = 10$. Also $y[n] = x[n] - x[n-1]$

21. The fundamental period of $y[n]$ is

(A) 9 (B) 10

(C) 11 (D) None of the above

22. The FS coefficients of $y[n]$ are

(A) $\frac{1}{10}\left(1 - e^{j\left(\frac{8\pi}{5}\right)k}\right)$ (B) $\frac{1}{10}\left(1 - e^{-j\left(\frac{8\pi}{5}\right)k}\right)$

(C) $\frac{1}{10}\left(1 - e^{j\left(\frac{4\pi}{5}\right)k}\right)$ (D) $\frac{1}{10}\left(1 - e^{-j\left(\frac{4\pi}{5}\right)k}\right)$

23. The FS coefficients of $x[n]$ are

(A) $-\frac{j}{2}e^{-j\left(\frac{\pi k}{10}\right)}\operatorname{cosec}\left(\frac{\pi k}{10}\right)Y[k], k \ne 0$

(B) $\frac{j}{2}e^{-j\left(\frac{\pi k}{10}\right)}\operatorname{cosec}\left(\frac{\pi k}{10}\right)Y[k], k \ne 0$

(C) $-\frac{1}{2}e^{-j\left(\frac{\pi k}{10}\right)}\sec\left(\frac{\pi k}{10}\right)Y[k]$

(D) $\frac{1}{2}e^{-j\left(\frac{\pi k}{10}\right)}\sec\left(\frac{\pi k}{10}\right)Y[k]$

Statement for Q.24-27:

Consider a discrete-time signal with Fourier representation.

$$x[n] \xleftrightarrow{DTFS;\frac{\pi}{10}} X[k]$$

In question the FS coefficient $Y[k]$ is given. Determine the corresponding signal $y[n]$ and choose correct option.

24. $Y[k] = X[k-5] + X[k+5]$

(A) $2\sin\left(\frac{\pi}{5}n\right)x[n]$ (B) $2\cos\left(\frac{\pi}{5}n\right)x[n]$

(C) $2\sin\left(\frac{\pi}{2}n\right)x[n]$ (D) $2\cos\left(\frac{\pi}{2}n\right)x[n]$

25. $Y[k] = \cos\left(\frac{\pi k}{5}\right)X[k]$

(A) $\frac{1}{2}(x[n+5]+x[n+5])$ (B) $\frac{1}{2}(x[n+2]+x[n-2])$

(C) $\frac{1}{2}(x[n+10]+x[n+10])$ (D) None of the above

26. $Y[k] = X[k] * X[k]$

(A) $\frac{(x[n])^2}{2\pi}$ (B) $j2\pi(x[n])^2$

(C) $(x[n])^2$ (D) $2\pi(x[n])^2$

27. $Y[k] = \mathrm{Re}\{X[k]\}$

(A) $\frac{x[n]+x[-n]}{2}$ (B) $\frac{x[n]-x[-n]}{2}$

(C) $\frac{x[n]-x[-n]}{2\pi}$ (D) $\frac{x[n]+x[-n]}{2\pi}$

28. Consider a sequence $x[n]$ with following facts :

1. $x[n]$ is periodic with $N = 6$
2. $\sum_{n=0}^{5} x[n] = 2$
3. $\sum_{n-2}^{7} (-1)^n x[n] = 1$
4. $x[n]$ has the minimum power per period among the set of signals satisfying the preceding three condition.

The sequence would be..

(A) $\left\{\dots\underset{\uparrow}{\frac{1}{2}}, \frac{1}{6}, \frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \dots\right\}$ (B) $\left\{\dots\underset{\uparrow}{0}, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$

(C) $\left\{\dots\underset{\uparrow}{\frac{1}{3}}, \frac{1}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{3}, \dots\right\}$ (D) $\{\dots\underset{\uparrow}{0}, 1, 2, 3, 4, \dots\}$

29. A real and odd periodic signal $x[n]$ has fundamental period $N = 7$ and FS coefficients $X[k]$. Given that $X[15] = j$, $X[16] = 2j$, $X[17] = 3j$. The values of $X[0]$, $X[-1]$, $X[-2]$, and $X[-3]$ will be

(A) $0, j, 2j, 3j$ (B) 1, 1, 2, 3

(C) 1, –1, –2, –3 (D) $0, -j, -2j, -3j$

30. Consider a signal $x[n]$ with following facts

1. $x[n]$ is a real and even signal
2. The period of $x[n]$ is $N = 10$
3. $X[11] = 5$
4. $\frac{1}{10}\sum_{n=0}^{9}|X[k]|^2 = 50$

The signal $x[n]$ is

(A) $5\cos\left(\frac{\pi}{10}n\right)$ (B) $5\sin\left(\frac{\pi}{10}n\right)$

(C) $10\cos\left(\frac{\pi}{5}n\right)$ (D) $10\sin\left(\frac{\pi}{5}n\right)$

31. Each of two sequence $x[n]$ and $y[n]$ has a period $N = 4$. The FS coefficient are

$$X[0] = X[3] = \frac{1}{2}X[1] = \frac{1}{2}X[2] = 1 \text{ and}$$

$$Y[0], Y[1], Y[2], Y[3] = 1$$

The FS coefficient $Z[k]$ for the signal $z[n] = x[n]y[n]$ will be

(A) 6 (B) $6|k|$

(C) $6^{|k|}$ (D) $e^{j\frac{\pi}{2}k}$

32. Consider a discrete-time periodic signal

$$x[n] = \frac{\sin\left(\frac{11\pi}{20}n\right)}{\sin\left(\frac{\pi}{20}n\right)}$$

with a fundamental period $N = 20$. The Fourier series coefficients of this function are

(A) $\frac{1}{20}(u[k+5]-u[k-6])$, $|k| \le 10$

(B) $\frac{1}{20}(u[k+5]-u[k-5])$, $|k| \le 10$

(C) $(u[k+5]-u[k+6])$, $|k| \le 10$

(D) $(u[k+5]-u[k-6])$, $|k| \le 10$

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11. (D) $N = 7,\ \Omega_o = \dfrac{2\pi}{7}$,

$$x[n] = \sum_{n=-3}^{3} X[k] e^{j\left(\frac{2\pi}{7}\right)kn} = 2e^{j(-1)\left(\frac{2\pi}{7}\right)n} - 1 + 2e^{j(1)\left(\frac{2\pi}{7}\right)n}$$

$$= 4\cos\left(\frac{2\pi}{7}n\right) - 1$$

12. (C) $N = 12,\ \Omega_o = \dfrac{\pi}{6}$, $X[k] = e^{-j\left(\frac{\pi}{6}\right)k}$

$$x[n] = \sum_{k=-6}^{6} e^{-j\left(\frac{\pi}{6}\right)k} e^{j\left(\frac{\pi}{6}\right)kn} = \sum_{k=-6}^{6} e^{j\left(\frac{\pi}{6}\right)k(n-1)}$$

$$= \frac{e^{j(-4)\frac{\pi}{6}(n-1)}\left(1 - e^{j\frac{9\pi}{6}(n-1)}\right)}{\left(1 - e^{j\frac{\pi}{6}(n-1)}\right)} = \frac{\sin\left(\frac{3\pi}{4}(n-1)\right)}{\sin\left(\frac{\pi}{12}(n-1)\right)}$$

13. (A) $N = 19,\ \Omega_o = \dfrac{2\pi}{19}$

$$X[k] = \cos\left(\frac{10\pi}{19}k\right) + 2j\sin\left(\frac{10\pi}{19}k\right)$$

$$= \frac{1}{2}\left(e^{-j(-5)\frac{2\pi}{19}k} + e^{-j(5)\frac{2\pi}{19}k}\right) + \left(e^{-j(-2)\frac{2\pi}{19}k} + e^{-j(2)\frac{2\pi}{19}k}\right)$$

By inspection

$$x[n] = \frac{19}{2}(\delta[n+5] + \delta[n-5]) + 19(\delta[n+2] - \delta[n-2]),$$

Where $|n| \le 9$

14. (A) $N = 21,\ \Omega_o = \dfrac{2\pi}{21}$

$$X[k] = \cos\left(\frac{8\pi}{21}k\right) = \frac{1}{2}\left(e^{-j(-4)\frac{2\pi}{21}k} + e^{-j(4)\frac{2\pi}{21}k}\right)$$

Since $X[k] = \dfrac{1}{N}\sum_{n=N} x[n] e^{-jk\Omega_o n}$, By inspection

$$x[n] = \begin{cases} \dfrac{21}{2}, & n = \pm 4 \\ 0, & \text{otherwise } n \in \{-10,\ -9, \ldots\ldots 9.\ 10\} \end{cases}$$

15. (B) $Y[k] = \dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1} x[n - n_o] e^{-j\left(\frac{2\pi}{N}\right)kn}$

$$= \frac{1}{N} e^{-j\left(\frac{2\pi}{N}\right)kn_o} \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi}{N}\right)kn} = e^{-j\left(\frac{2\pi}{N}\right)kn_o} X[k]$$

16. (C) $Y[k] = X[k] - e^{-j\left(\frac{2\pi}{N}\right)2k} X[k] = \left(1 - e^{-j\left(\frac{4\pi}{N}\right)k}\right) X[k]$

17. (C) Note that $y[n] = x[n] + x[n + N/2]$ has a period of $N/2$ and N has been assumed to be even,

$$Y[k] = \frac{2}{N}\sum_{n=0}^{N/2-1} (x[n] + x[n + N/2])\, e^{-j\left(\frac{4\pi}{N}\right)kn}$$

$= 2X[2k]$ for $0 \le k \le (N/2 - 1)$

18. (B) $y[n] = x[n] - x[n - N/2]$

$$Y[k] = \left(1 - e^{-j\left(\frac{2\pi}{N}\right)\frac{N}{2}k}\right) X[k] = (1 - e^{-j\pi k}) X[k]$$

$$= \begin{cases} 0, & k \text{ even} \\ 2X[k], & k \text{ odd} \end{cases}$$

19. (C) $y[n] = x^*[-n]$

$$Y[k] = \frac{1}{N}\sum_{n=0}^{N-1} x^*[-n] e^{-j\left(\frac{2\pi}{N}\right)kn} = X^*[k]$$

20. (A) With N even

$$y[n] = (-1)^n x[n] = e^{j\pi n} x[n] = e^{j\left(\frac{2\pi}{N}\right)\frac{N}{2}} x[n]$$

$$Y[k] = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}\right)\frac{N}{2}} x[n] e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi}{N}\right)n\left(k - \frac{N}{2}\right)} = X[k - N/2]$$

21. (B) $y[n]$ is shown is fig. S5.8.21. It has fundamental

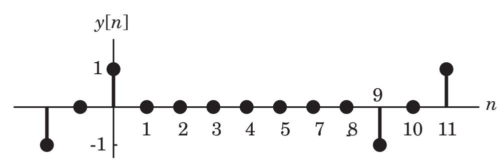

Fig. S5.8.21

period of 10.

22. (B) $Y[k] = \dfrac{1}{10}\displaystyle\sum_{n=0}^{9} y[n] e^{-j\left(\frac{2\pi}{10}\right)kn}$

$$= \frac{1}{10}\left(1 - e^{-j\left(\frac{2\pi}{10}\right)k8}\right) = \frac{1}{10}\left(1 - e^{-j\left(\frac{8\pi}{5}\right)k}\right)$$

23. (A) $y[n] = x[n] - x[n-1]$

$$Y[k] = X[k] - e^{-j\left(\frac{2\pi}{10}\right)k} X[k] \quad \Rightarrow \quad X[k] = \frac{Y[k]}{1 - e^{-j\left(\frac{\pi}{5}\right)k}}$$

$$\Rightarrow \quad X[k] = \frac{e^{j\left(\frac{\pi}{10}\right)k} Y[k]}{e^{j\left(\frac{\pi}{10}\right)k} - e^{-j\left(\frac{\pi}{10}\right)k}} = \frac{e^{j\left(\frac{\pi}{10}\right)k} Y[k]}{2j\sin\left(\frac{\pi k}{10}\right)}$$

$$= \frac{-j}{2} e^{-j\left(\frac{\pi}{10}k\right)} \operatorname{cosec}\left(\frac{\pi}{10}k\right) Y[k]$$

24. (D) $\Omega_o = \frac{\pi}{10}$, $Y[k] = X[k-5] + X[k+5]$

$$\Rightarrow \quad y[n] = \left(e^{j(5)\frac{\pi}{10}n} + e^{j(-5)\frac{\pi}{10}n}\right) x[n] = 2\cos\left(\frac{\pi}{2}n\right) x[n]$$

25. (B) $Y[k] = \cos\left(\frac{\pi}{5}k\right) X[k] = \left(\frac{e^{j\frac{\pi}{5}k} + e^{-j\frac{\pi}{5}k}}{2}\right) X[k]$

$$= \frac{1}{2}\left(e^{j(2)\frac{\pi}{10}k} + e^{j(-2)\frac{\pi}{10}k}\right) X[k]$$

$$\Rightarrow \quad y[n] = \frac{1}{2}(x[n-2] + x[n+2])$$

26. (C) $Y[k] = X[k] * X[k] \Rightarrow y[n] = x[n]x[n] = (x[n])^2$

27. (A) $Y[k] = \text{Re}\{X[k]\} \Rightarrow y[n] = \text{Ev}\{x[n]\} = \frac{x[n] + x[-n]}{2}$

28. (A) $N = 6$, $\Omega_o = \frac{2\pi}{6}$,

From fact 2, $\sum_{n=0}^{5} x[n] = 2$

$$\Rightarrow \quad \frac{1}{6}\sum_{n=0}^{5} e^{j\left(\frac{2\pi}{6}\right)(0)k} x[n] = \frac{1}{3} \quad \Rightarrow \quad X[0] = \frac{1}{3} ,$$

From fact 3, $\sum_{n=2}^{7} (-1)^n x[n] = 1$

$$\Rightarrow \quad \frac{1}{6}\sum_{n=0}^{5} e^{j\left(\frac{2\pi}{6}\right)(3)k} x[n] = \frac{1}{6}, \quad X[3] = \frac{1}{6}$$

By Parseval's relation, the average power in $x[n]$ is

$P = \sum_{k=0}^{5} |X[k]|^2$,

The value of P is minimized by choosing

$X[1] = X[2] = X[4] = X[5] = 0$

Therefore

$$x[n] = X[0] + X[3]e^{\left(\frac{2\pi}{6}\right)3n} = \frac{1}{3} + (-1)^n \frac{1}{6} = \frac{1}{3} + (-1)^n \frac{1}{6}$$

$$x[n] = \left\{\dots \frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \dots\right\}$$

$\uparrow$

29. (D) Since the FS coefficient repeat every N. Thus

$X[1] = X[15]$, $X[2] = X[16]$, $X[3] = X[17]$

The signal real and odd, the FS coefficient $X[k]$ will be purely imaginary and odd. Therefore $X[0] = 0$

$X[-1] = -X[1]$, $X[-2] = -X[2]$, $X[-3] = -X[3]$

Therefore (D) is correct option.

30. (C) Since $N = 10$, $X[11] = X[1] = 5$

Since $x[n]$ is real and even $X[k]$ is also real and even.

Therefore $X[1] = X[-1] = 5$.

Using Parseval's relation $\sum_{N} |X[k]|^2 = 50 = \sum_{k=-1}^{8} |X[k]|^2$

$$|X[-1]|^2 + |X[1]|^2 + |X[0]|^2 + \sum_{k=2}^{8} |X[k]|^2 = 50$$

$$|X[0]|^2 + \sum_{k=2}^{8} |X[k]|^2 = 0$$

Therefore $X[k] = 0$ for $k = 0, 2, 3, \dots .. 8$.

$$x[n] = \sum_{N} X[k] e^{j\left(\frac{2\pi}{N}\right)kn} = \sum_{k=-1}^{8} X[k] e^{j\left(\frac{2\pi}{10}\right)kn}$$

$$= 5\left(e^{-j\left(\frac{2\pi}{10}\right)n} + e^{j\left(\frac{2\pi}{10}\right)n}\right) = 10\cos\left(\frac{\pi}{5}n\right)$$

31. (A) $z[n] = x[n]y[n] \xleftrightarrow{DTFS} \sum_{k=<N>} X[l]Y[k-l]$

$$\Rightarrow \quad Z[k] = \sum_{l=0}^{3} X[l]Y[k-l]$$

$$\Rightarrow \quad Z[k] = X[0]Y[k] + X[1]Y[k-1] + X[2]Y[k-2] + X[3]Y[k-3]$$

$= Y[k] + 2Y[k-1] + 2Y[k-2] + Y[k-3]$

Since $Y[k]$ is 1 for all values of k.

Thus $Z[k] = 6$, for all k.

32. (A) $N = 20$ We know that

$$\begin{cases} 1, & |n| \le 5 \\ 0, & 5 < |n| \le 10 \end{cases} \xleftrightarrow{DTFS;\frac{\pi}{10}} \frac{\sin\left(\frac{11\pi}{20}k\right)}{\sin\left(\frac{\pi}{20}k\right)}$$

Using duality

$$\frac{\sin\left(\frac{11\pi}{20}n\right)}{\sin\left(\frac{\pi}{20}n\right)} \xleftrightarrow{DTFS;\frac{\pi}{10}} \frac{1}{20}\begin{cases} 1, & |k| \le 5 \\ 0, & 5 < |k| \le 10 \end{cases}$$

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CHAPTER

6.1

TRANSFER FUNCTION

1. The equivalent transfer function of three parallel blocks

$$G_1(s) = \frac{1}{s+1}\ ,\ G_2(s) = \frac{1}{s+4} \text{ and } G_3(s) = \frac{s+3}{s+5} \text{ is}$$

(A) $\dfrac{(s^3 + 10s^2 + 34s + 37)}{(s+1)(s+4)(s+5)}$

(B) $\dfrac{(s+3)}{(s+1)(s+4)(s+5)}$

(C) $\dfrac{-(s^3 + 10s^2 + 34s + 37)}{(s+1)(s+4)(s+5)}$

(D) $\dfrac{-(s+3)}{(s+1)(s+4)(s+5)}$

2. The block having transfer function

$$G_1(s) = \frac{1}{s+2}\ ,\ G_2(s) = \frac{1}{s+5}\ ,\ G_3(s) = \frac{s+1}{s+3}$$

are cascaded. The equivalent transfer function is

(A) $\dfrac{(s^3 + 10s^2 + 37s^2 + 31)}{(s+2)(s+3)(s+5)}$

(B) $\dfrac{s+1}{(s+2)\,(s+3)\,(s+5)}$

(C) $\dfrac{-(s^3 + 10s^2 + 37s^2 + 31)}{(s+2)(s+3)(s+5)}$

(D) $\dfrac{-(s+1)}{(s+2)(s+3)(s+5)}$

3. For a negative feedback system shown in fig. P.6.1.3

$$G(s) = \frac{s+1}{s(s+2)} \text{ and } H(s) = \frac{s+3}{s+4}$$

The equivalent transfer function is

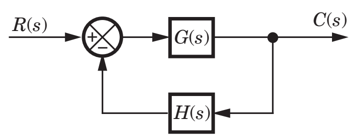


Fig. P.6.1.3

(A) $\dfrac{s(s+2)(s+3)}{s^3 + 7s^2 + 12s + 3}$

(B) $\dfrac{s(s+2)(s+3)}{s^3 + 5s^2 + 4s - 3}$

(C) $\dfrac{(s+1)(s+4)}{s^3 + 7s^2 + 12s + 3}$

(D) $\dfrac{(s+1)(s+4)}{s^3 + 5s^2 + 4s - 3}$

4. A feedback control system is shown in fig. P.6.1.4. The transfer function for this system is

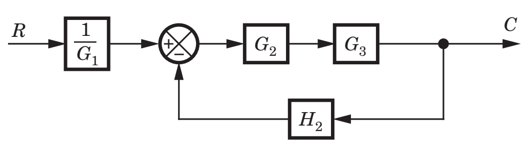


Fig. P.6.1.4

(A) $\dfrac{G_1G_2}{1 + H_1G_1G_2G_3}$

(B) $\dfrac{G_2G_3}{G_1(1 + H_1G_2G_3)}$

(C) $\dfrac{G_2G_3}{1 + H_1G_1G_2G_3}$

(D) $\dfrac{G_2G_3}{G_1(1 + H_1G_2G_3)}$

5. Consider the system shown in fig. P.6.1.5.

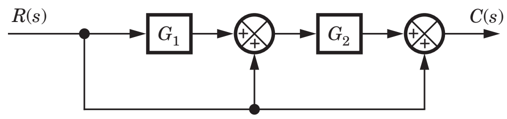
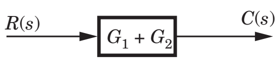


Fig. P.6.1.5

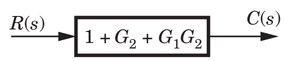
The input output relationship of this system is

(A)

(B)



(C)



(D)

6. A feedback control system shown in fig. P.6.1.6 is subjected to noise $N(s)$.

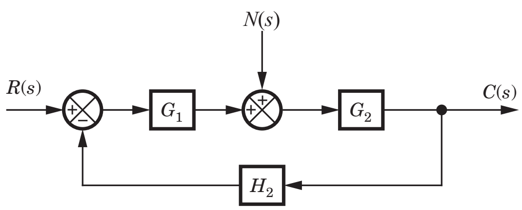


Fig. P.6.1.6

The noise transfer function $\dfrac{C_N(s)}{N(s)}$ is

(A) $\dfrac{G_2}{1+G_1G_2H}$

(B) $\dfrac{G_2}{1+G_1H}$

(C) $\dfrac{G_2}{1+G_2H}$

(D) None of the above

7. A system is shown in fig. P6.1.7. The transfer function for this system is

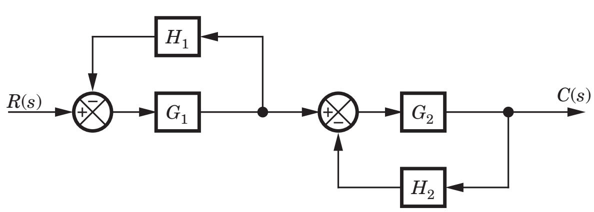


Fig. P.6.1.7

(A) $\dfrac{G_1G_2}{1+G_1G_1H_2+G_2H_1}$

(B) $\dfrac{G_1G_2}{1+G_1G_2+H_1H_2}$

(C) $\dfrac{G_1G_2}{1-G_1H_1-G_2H_2+G_1G_2H_1H_2}$

(D) $\dfrac{G_1G_2}{1+G_1H_1+G_2H_2+G_1G_2H_1H_2}$

8. The closed loop gain of the system shown in fig. P6.1.8 is

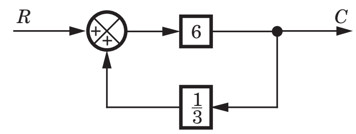


Fig.P6.1.8

(A) –2

(B) 6

(C) –6

(D) 2

9. The block diagrams shown in fig. P.6.1.9 are equivalent if G is equal to

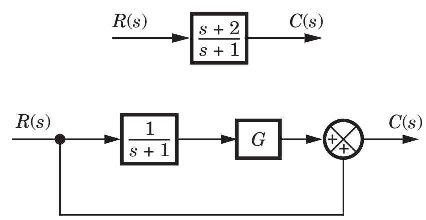


Fig. P.6.1.9

(A) $s+1$

(B) 2

(C) $s+2$

(D) 1

10. Consider the systems shown in fig. P.6.1.10. If the forward path gain is reduced by 10% in each system, then the variation in C_1 and C_2 will be respectively

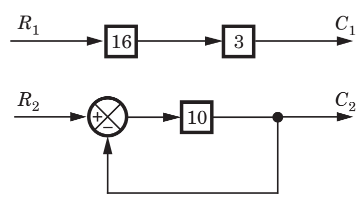


Fig. P.6.1.10

(A) 10% and 1%

(B) 2% and 10%

(C) 10% and 0%

(D) 5% and 1%

11. The transfer function $\frac{C}{R}$ of the system shown in the fig. P.6.1.11 is

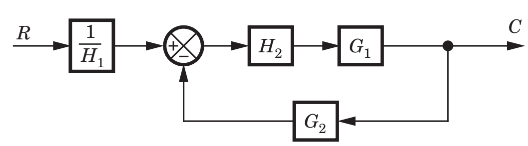


Fig. P.6.1.11

(A) $\frac{G_1 H_2}{H_1(1+G_1 G_2 H_2)}$ (B) $\frac{G_1 G_2 H_2}{H_1(1+G_1 G_2 H_2)}$

(C) $\frac{G_2 G_1}{1+H_1 H_2 G_1 G_2}$ (D) $\frac{G_1 G_2}{H_1(1+G_1 G_2 H_2)}$

12. In the signal flow graph shown in fig. P6.1.12 the sum of loop gain of non-touching loops is

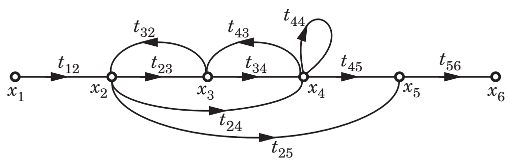


Fig. P.6.1.12

(A) $t_{32}t_{23} + t_{44}$ (B) $t_{23}t_{32} + t_{34}t_{43}$

(C) $t_{24}t_{43}t_{32} + t_{44}$ (D) $t_{23}t_{32} + t_{34}t_{43} + t_{44}$

13. For the SFG shown in fig. P.6.1.14 the graph determinant Δ is

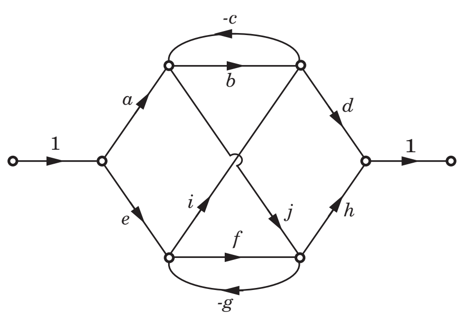


Fig. P.6.1.13

(A) $1 - bc - fg - bcfg + cigj$

(B) $1 - bc - fg - cigj + bcfg$

(C) $1 + bc + fg + cig\ j - bcfg$

(D) $1 + bc + fg + bcfg - cigj$

14. The sum of the gains of the feedback paths in the signal flow graph shown in fig. P.6.1.13 is

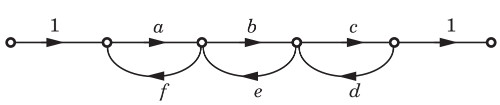


Fig. P.6.1.14

(A) $af + be + cd + abef + bcde$

(B) $af + be + cd$

(C) $af + be + cd + abef + abcdef$

(D) $af + be + cd + cbef + bcde + abcdef$

15. A closed-loop system is shown in fig. P.6.1.15. The noise transfer function $C_n(s)/N(s)$ is approximately

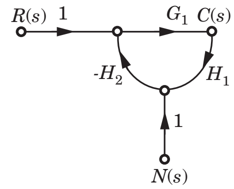


Fig. P.6.1.15

(A) $\frac{1}{G_1(s)H_1(s)}$ For $|G_1(s)H_1(s)H_2(s)| << 1$

(B) $\frac{1}{-H_1(s)}$ For $|G_1(s)H_1(s)H_2(s)| >> 1$

(C) $\frac{1}{H_1(s)H_2(s)}$ For $|G_1(s)H_1(s)H_2(s)| >> 1$

(D) $\frac{1}{G_1(s)H_1(s)H_2(s)}$ For $|G_1(s)H_1(s)H_2(s)| << 1$

16. The overall transfer function $\frac{C}{R}$ of the system shown in fig. P.6.1.16 will be

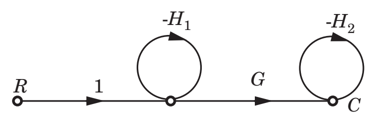


Fig. P.6.1.16

(A) G (B) $\frac{G}{1+H_2}$

(C) $\frac{G}{(1+H_1)(1+H_2)}$ (D) $\frac{G}{1+H_1+H_2}$

17. Consider the signal flow graphs shown in fig. P6.1.17. The transfer 2 is of the graph

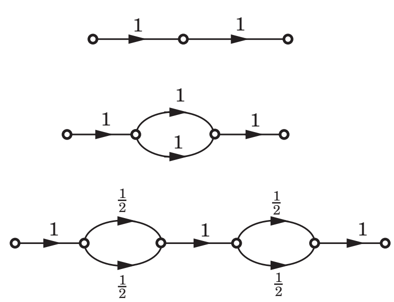
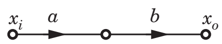
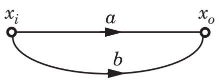
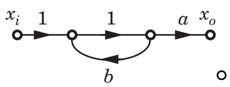
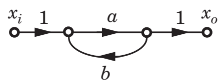


Fig. P.6.1.17

(A) a (B) b

(C) b and c (D) a, b and c

18. Consider the List I and List II

List I (Signal Flow Graph)	List II (Transfer Function)
P. 	1. $a + b$
Q. 	2. ab
S. 	3. $\frac{a}{(1-ab)}$
R. 	4. $\frac{a}{1-b}$

The correct match is

	P	Q	R	S
(A)	2	1	3	4
(B)	2	1	4	3
(C)	1	2	4	3
(D)	1	2	3	4

19. For the signal flow graph shown in fig. P6.1.19 an equivalent graph is

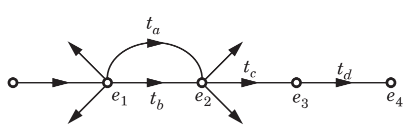
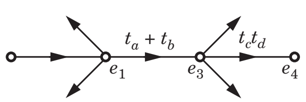
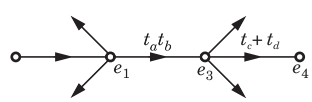


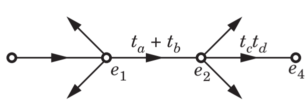
Fig. P.6.1.19



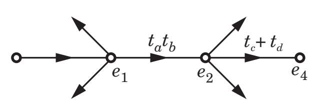
(A)



(B)



(C)



(D)

20. Consider the block diagram shown in figure P.6.1.20

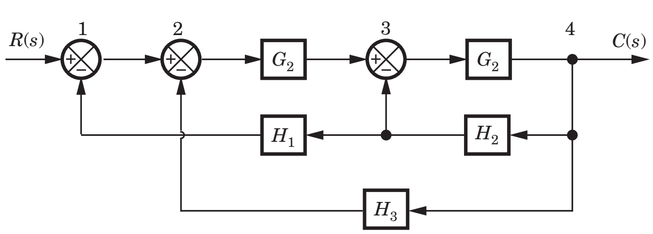
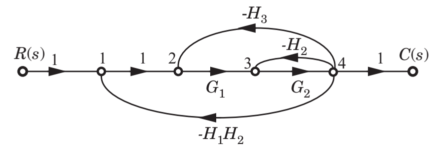
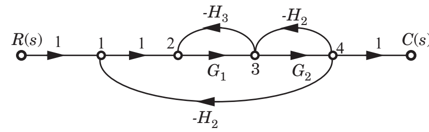
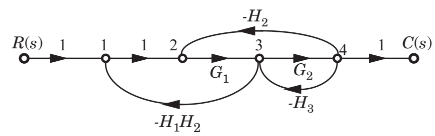


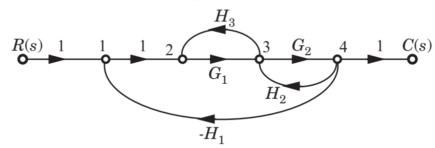
Fig. P.6.1.20

For this system the signal flow graph is

(A) 

(B) 

(C) 

(D) 

21. The block diagram of a system is shown in fig. P.6.1.21. The closed loop transfer function of this system is

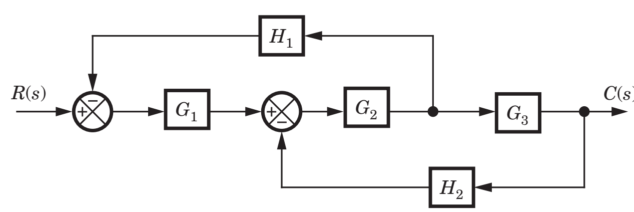


Fig. P.6.1.21

(A) $\frac{G_1G_2G_3}{1 + G_1G_2G_3H_1}$

(B) $\frac{G_1G_2G_3}{1 + G_1G_2G_3H_1H_2}$

(C) $\frac{G_1G_2G_3}{1 + G_1G_2H_1 + G_2G_3H_2}$

(D) $\frac{G_1G_2G_3}{1 + G_1G_2H_1 + G_1G_3H_2 + G_2G_3H_1}$

22. For the system shown in fig. P6.1.22 transfer function $C(s)/R(s)$ is

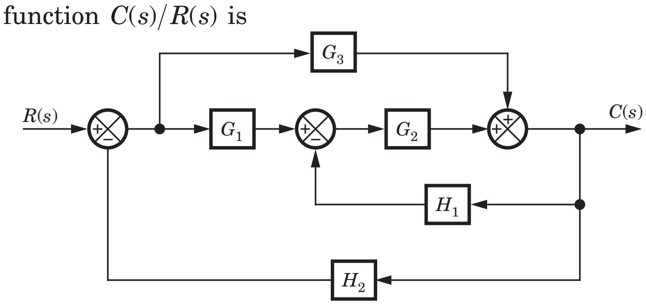


Fig. P.6.1.22

(A) $\dfrac{G_3}{1-H_1G_2-H_2G_3-G_1G_2H_2}$

(B) $\dfrac{G_3+G_1G_2}{1+H_1G_2+H_2G_3+G_1G_2H_2}$

(C) $\dfrac{G_3}{1+H_1G_2+H_2G_3+G_1G_2H_2}$

(D) $\dfrac{G_3}{1-H_1G_2-H_2G_3-G_1G_2H_2}$

23. In the signal flow graph shown in fig. P6.1.23 the transfer function is

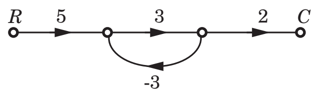


Fig. P.6.1.23

'(A) 3.75 (B) –3

(C) 3 (D) –3.75

24. In the signal flow graph shown in fig. P6.1.24 the gain C/R is

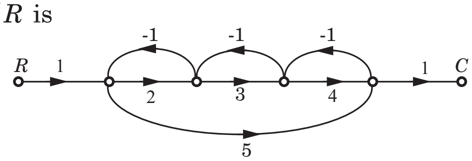


Fig. P.6.1.24

(A) $\dfrac{44}{23}$ (B) $\dfrac{29}{19}$

(C) $\dfrac{44}{19}$ (D) $\dfrac{29}{11}$

25. The gain $C(s)/R(s)$ of the signal flow graph shown in fig. P.6.1.25 is

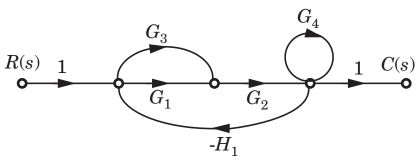


Fig. P.6.1.25

(A) $\dfrac{G_1G_2+G_2G_3}{1+G_1G_2H_1+G_2G_3H_1+G_4}$

(B) $\dfrac{G_1G_2+G_2G_3}{1+G_1G_3H_1+G_2G_3H_1-G_4}$

(C) $\dfrac{G_1G_3+G_2G_3}{1+G_1G_3H_1+G_2G_3H_1-G_4}$

(D) $\dfrac{G_1G_3+G_2G_3}{1+G_1G_3H_1+G_2G_3H_1+G_4}$

26. The transfer function of the system shown in fig. P.6.1.26 is

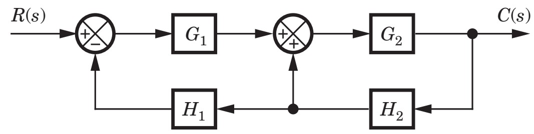


Fig. P.6.1.26

(A) $\dfrac{G_1G_2}{1-G_1G_2H_1-G_1G_2H_2}$ (B) $\dfrac{G_1G_2}{1-G_2H_2-G_1G_2H_1}$

(C) $\dfrac{G_1G_2}{1-G_2H_2+G_1G_2H_1H_2}$ (D) $\dfrac{G_1G_2}{1-G_1G_2H_1H_2}$

27. For the block diagram shown in fig. P.6.1.27 transfer function $C(s)/R(s)$ is

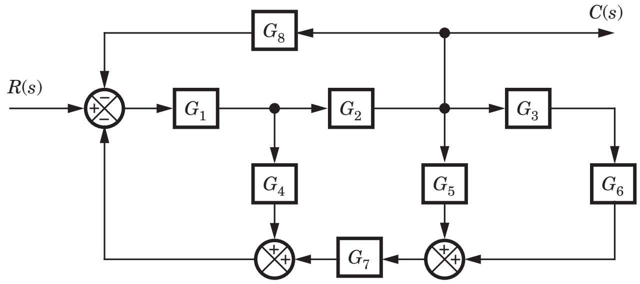


Fig. P.6.1.27

(A) $\dfrac{G_1G_2}{1+G_1G_2+G_1G_7G_3+G_1G_2G_8G_6+G_1G_2G_3G_7G_5}$

(B) $\dfrac{G_1G_2}{1+G_1G_4+G_1G_2G_8+G_1G_2G_5G_7+G_1G_2G_3G_6G_7}$

(C) $\dfrac{G_1+G_2}{1+G_1G_4+G_1G_2G_8+G_1G_2G_5G_7+G_1G_2G_3G_6G_7}$

(D) $\dfrac{G_1+G_2}{1+G_1G_2+G_3G_6G_7+G_1G_3G_4G_5+G_1G_2G_3G_6G_7G_8}$

28. For the block diagram shown in fig. P.6.1.28 the numerator of transfer function is

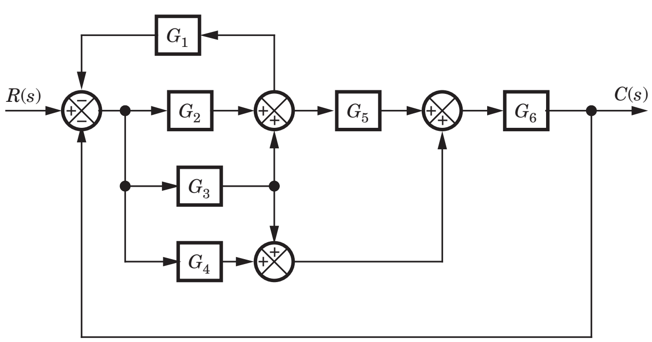


Fig. P.6.1.28

(A) $G_6[G_4+G_3+G_5(G_3+G_2)]$

(B) $G_6[G_2+G_3+G_5(G_3+G_4)]$

(C) $G_6[G_1 + G_2 + G_3(G_4 + G_5)]$

(D) None of the above

29. For the block diagram shown in fig. P.6.1.29 the transfer function $C(s)/R(s)$ is

(A) $\dfrac{50(s-2)}{s^3 + s^2 + 150s - 100}$ (B) $\dfrac{50(s-2)}{s^3 + s^2 + 150s}$

(C) $\dfrac{50s}{s^3 + s^2 + 150s - 100}$ (D) $\dfrac{50}{s^2 + s + 150}$

30. For the SFG shown in fig. P.6.1.30 the transfer function $\frac{C}{R}$ is

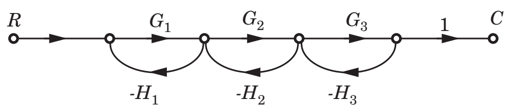


Fig. P.6.1.30

(A) $\dfrac{G_1 + G_2 + G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3}$

(B) $\dfrac{G_1 + G_2 + G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1G_3H_1H_3}$

(C) $\dfrac{G_1G_2G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3}$

(D) $\dfrac{G_1G_2G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1G_3H_1H_3}$

31. Consider the SFG shown in fig. P6.1.31. The Δ for this graph is

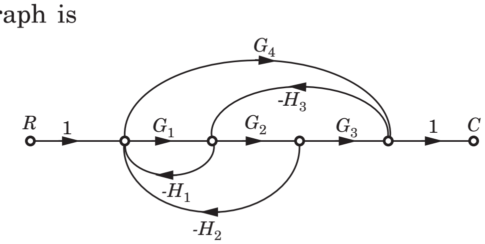


Fig. P.6.1.31

(A) $1 + G_1H_1 + G_2G_3H_3 + G_1G_3H_2$

(B) $1 + G_1H_1 - G_2G_3H_3 - G_1G_3H_3 + G_2G_4H_2H_3$

(C) $1 + G_1H_1 + G_2G_3H_3 + G_1G_3H_3 - G_2G_4H_2H_3$

(D) $1 + G_1H_1 + G_2G_3H_3 + G_1G_3H_3 + G_2G_4H_2H_3$

32. The transfer function of the system shown in fig. P.6.1.32 is

(A) $\dfrac{G_2G_3 + G_1G_3}{1 - G_3H_1 + G_2G_3}$ (B) $\dfrac{G_2G_3 + G_1G_3}{1 + G_3H_1 - G_2G_3}$

(C) $\dfrac{G_2G_3 + G_1G_3}{1 + G_3H_1 + G_2G_3}$ (D) $\dfrac{G_2G_3 + G_1G_3}{1 - G_3H_1 - G_2G_3}$

33. The closed loop transfer function of the system

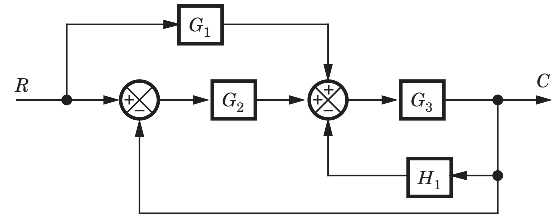


Fig. P.6.1.32

shown in fig. P6.1.33 is

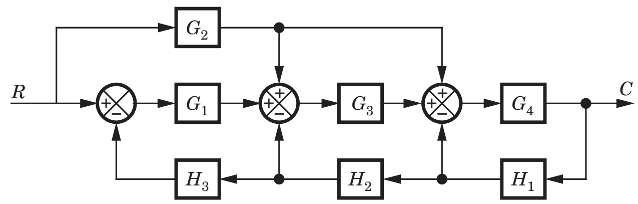


Fig. P.6.1.33

(A) $\dfrac{G_1G_2G_3 + G_2G_3G_4 + G_1G_4}{1 + G_1G_3G_4H_1H_2H_3 + G_2H_4H_1H_2 + G_4H_1}$

(B) $\dfrac{G_2G_4 + G_1G_2G_3}{1 + G_1G_3H_1H_2H_3 + G_4H_1 + G_3G_4H_1H_2}$

(C) $\dfrac{G_1G_3G_4 + G_2G_4}{1 + G_3G_4H_1H_2 + G_4H_1 + G_1G_3H_3H_2}$

(D) $\dfrac{G_1G_3G_4 + G_2G_3G_4 + G_2G_4}{1 + G_1G_3G_4H_1H_2H_3 + G_3G_4H_1H_2 + G_4H_1}$

Statement for Q.34-37:

A block diagram of feedback control system is shown in fig. P6.1.34-37

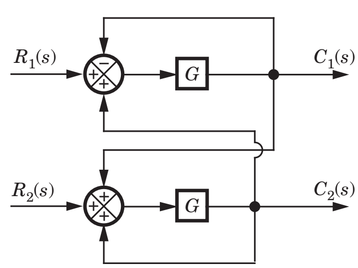


Fig. P.6.1.34-37

34. The transfer function $\left.\dfrac{C_1}{R_1}\right|_{R_2=0}$ is

(A) $\dfrac{G}{1-2G^2}$ (B) $\dfrac{G(1-G)}{1-2G^2}$

(C) $\dfrac{G(1-2G)}{1-G^2}$ (D) $\dfrac{G}{1-G^2}$

35. The transfer function $\left.\frac{C_1}{R_2}\right|_{R_1=0}$ is

(A) $\frac{G}{1-2G^2}$ (B) $\frac{G}{1-G^2}$

(C) $\frac{G^2}{1-2G^2}$ (D) $\frac{G^2}{1-G^2}$

36. The transfer function $\left.\frac{C_2}{R_1}\right|_{R_2=0}$ is

(A) $\frac{G(1+G)}{1-2G^2}$ (B) $\frac{G^2}{1-2G^2}$

(C) $\frac{G^2}{1-G^2}$ (D) $\frac{G}{1-G^2}$

37. The transfer function $\left.\frac{C_2}{R_2}\right|_{R_1=0}$ is

(A) $\frac{G(1+G)}{1-2G^2}$ (B) $\frac{G}{1-2G^2}$

(C) $\frac{G}{1+G}$ (D) $\frac{G}{1-G^2}$

Statement for Q.38–39:

A signal flow graph is shown in fig. P.6.1.38–39.

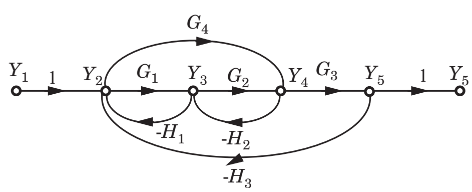


Fig. P.6.1.38-39

38. The transfer function $\frac{Y_2}{Y_1}$ is

(A) $\frac{1}{\Delta}$ (B) $\frac{1+G_2H_2}{\Delta}$

(C) $\frac{G_1G_2G_3}{\Delta}$ (D) None of the above

39. The transfer function $\frac{Y_5}{Y_2}$ is

(A) $\frac{G_1G_2G_3+G_4G_3}{\Delta}$ (B) $G_1G_2G_3+G_4G_3$

(C) $\frac{G_1G_2G_3+G_4G_3}{G_1G_2G_3}$ (D) $\frac{G_1G_2G_3+G_4G_3}{1+G_2H_2}$

Statement for Q.40–41:

A block diagram is shown in fig. P6.1.40–41.

40. The transfer function for this system is

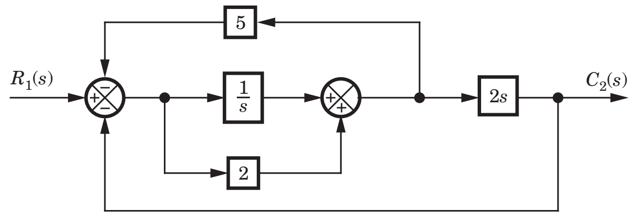


Fig. P.6.1.40-41

(A) $\frac{2s(2s+1)}{2s^2+3s+5}$ (B) $\frac{2s(2s+1)}{2s^2+13s+5}$

(C) $\frac{2s(2s+1)}{4s^2+13s+5}$ (D) $\frac{2s(2s+1)}{4s^2+3s+5}$

41. The pole of this system are

(A) $-0.75 \pm j1.39$ (B) $-0.41, -6.09$

(C) $-0.5, \quad -1.67$ (D) $-0.25 \pm j0.88$

****

SOLUTIONS

1. (A) $G_e(s) = G_1(s) + G_2(s) + G_3(s)$

$$= \frac{1}{(s+1)} + \frac{1}{(s+4)} + \frac{s+3}{(s+5)}$$

$$= \frac{s^2+9s+20+s^2+6s+5+s^3+5s^2+4s+3s^2+15s+12}{(s+1)(s+4)(s+5)}$$

$$= \frac{s^3+10s^2+34s+37}{(s+1)(s+4)(s+5)}$$

2. (B) $G_e(s) = G_1(s)G_2(s)G_3(s) = \dfrac{(s+1)}{(s+2)(s+5)(s+3)}$

3. (C) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+H(s)G(s)}$

$$= \frac{\frac{s+1}{s(s+2)}}{1+\frac{(s+3)}{(s+4)}\frac{(s+1)}{s(s+2)}} = \frac{(s+1)(s+4)}{s^3+7s^2+12s+3}$$

4. (B) Multiply G_2 and G_3 and apply feedback formula and then again multiply with $\dfrac{1}{G_1}$.

$$T(s) = \frac{G_2G_3}{G_1(1+G_2G_3H_1)}$$

5. (D) $T(s) = G_2(1+G_1)+1 = 1+G_1+G_1G_2$

6. (A) Open-loop gain $= G_2$

Feed back gain $= HG_1$ $\quad T_N(s) = \dfrac{G_2}{1+G_1G_2H}$

7. (D) Apply the feedback formula to both loop and then multiply

$$T(s) = \left(\frac{G_1}{1+G_1H_1}\right)\left(\frac{G_2}{1+G_2H_2}\right)$$

$$= \frac{G_1G_2}{1+G_1H_1+G_2H_2+G_1G_2H_1H_2}$$

8. (C) For positive feedback $\dfrac{C}{R} = \dfrac{6}{1-\frac{6\times 1}{3}} = -6$

9. (D) For system (b) closed loop transfer function

$\dfrac{G}{s+1} + 1 = \dfrac{G+s+1}{s+1}$, $\quad \dfrac{G+s+1}{s+1} = \dfrac{s+2}{s+1}$, $\quad$ Hence $G = 1$

10. (A) In open loop system change will be 10% in C_1 also but in closed loop system change will be less

$C_2 = \dfrac{10}{10+1} = \dfrac{10}{11}$, $\quad C_2' = \dfrac{9}{9+1} = \dfrac{9}{10}$, C_2 is reduced by 1%.

11. (A) Apply the feedback formula and then multiply by $\dfrac{1}{H_1}$,

$$\frac{C}{R} = \frac{(H_2G_1)\left(\frac{1}{H_1}\right)}{1+H_2G_1G_2} = \frac{H_2G_1}{H_1(1+G_1G_2H_2)}$$

12. (A) There cannot be common subscript because subscript refers to node number. If subscript is common, that means that node is in both loop.

13. (D) $L_1 = -bc,\ L_2 = -fg,\ L_3 = jgic,\ L_1L_3 = bcfg$

$\Delta = 1-(-bc-fg+cigj)+bcfg = 1+bc+fg-cig\,j+bcfg$

14. (A) In this graph there are three feedback loop. *abef* is not a feedback path because between path x_2 is a summing node.

15. (B) By putting $R(s) = 0$

$P_1 = -H_2G_1$, $L_1 = -G_1H_2H_1$, $\Delta_1 = 1$, $\quad T_n(s) = \dfrac{-H_2G_1}{1+G_1H_2H_1}$

if $|G_1H_2H_1| >> 1$, $T_n(s) = \dfrac{-H_2G_1}{G_1H_2H_1} = \dfrac{-1}{H_1}$

16. (C) $P_1 = G$, $L_1 = -H_1$, $L_2 = -H_2$, $L_1L_2 = H_1H_2$, $\Delta_1 = 1$

$$T(s) = \frac{G}{1+H_1+H_2+H_1H_2} = \frac{G}{(1+H_1)(1+H_2)}$$

17. (B) $G_a = 1$, $G_b = 1+1 = 2$, $G_c = \dfrac{1}{4}+\dfrac{1}{4}+\dfrac{1}{4}+\dfrac{1}{4} = 1$

There are no loop in any graph. So option (B) is correct.

18. (B)

P. $P_1 = ab$, $\Delta = 1$, $\quad L = 0$, $T = ab$

Q. $P_1 = a$, $P_2 = b$, $\Delta = 1$, $\quad L = \Delta_k = 0$, $T = a+b$

R. $P_1 = a$, $L_1 = b$, $\Delta = 1-b$, $\Delta_1 = 1$, $T = \dfrac{a}{a-b}$

S. $P_1 = a$, $L_1 = ab$, $\Delta = 1-ab$, $\Delta_1 = 1$, $T = \dfrac{a}{1-ab}$

19. (A) Between e_1 and e_2, there are two parallel path. Combining them gives $t_a + t_b$. Between e_2 and e_4 there is a path given by total gain t_ct_d. So remove node e_3 and place gain t_ct_d of the branch e_2e_4. Hence option (A) is correct.

20. (A) Option (A) is correct. Best method is to check the signal flow graph. In block diagram there is feedback from 4 to 1 of gain $-H_1H_2$. The signal flow graph of option (A) has feedback from 4 to 1 of gain $-H_1H_2$.

21. (C) Consider the block diagram as SFG. There are two feedback loop $-G_1G_2H_1$ and $-G_2G_3H_2$ and one forward path $G_1G_2G_3$. So (D) is correct option.

22. (B) Consider the block diagram as a SFG. Two forward path G_1G_2 and G_3 and three loops
$-G_1G_2H_2$, $-G_2H_1$, $-G_3H_2$.
There are no nontouching loop. So (B) is correct.

23. (C) $P_1 = 5 \times 3 \times 2 = 30$, $\Delta = 1 - (3 \times -3) = 10$

$\Delta_1 = 1$, $\frac{C}{R} = \frac{30}{10} = 3$

24. (A) $P_1 = 2 \times 3 \times 4 = 24$, $P_2 = 1 \times 5 \times 1 = 5$

$L_1 = -2$, $L_2 = -3$, $L_3 = -4$, $L_4 = -5$,

$L_1L_3 = 8$, $\Delta = 1 - (-2 - 3 - 4 - 5) + 8 = 23$,

$\Delta_1 = 1$, $\Delta_2 = 1 - (-3) = 4$,

$$\frac{C}{R} = \frac{24 + 5 \times 4}{24} = \frac{44}{23}$$

25. (B) $P_1 = G_1G_2$, $P_2 = G_3G_2$

$L_1 = -G_3G_2H_1$, $L_2 = -G_1G_2H_1$, $L_3 = G_4$, $\Delta_1 = \Delta_2 = 1$

There are no nontouching loop.

$$T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{1 - (L_1 + L_2 + L_3)} = \frac{G_1G_2 + G_2G_3}{1 + G_1G_2H_1 + G_2G_3H_1 - G_4}$$

26. (C) $P_1 = G_1G_2$, $L_1 = -G_1G_2H_1H_2$, $L_2 = G_2H_2$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1 + G_1G_2H_1H_2 - G_2H_2}$$

27. (B) There is one forward path G_1G_2.

Four loops $-G_1G_4$, $-G_1G_2G_8$, $-G_1G_2G_5G_7$ and $-G_1G_2G_3G_6G_7$.

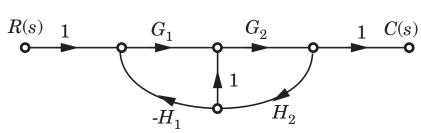


Fig. S6.1.27

There is no nontouching loop. So (B) is correct.

28. (A) SFG:

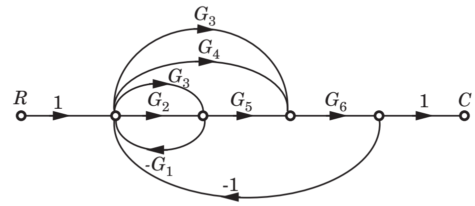


Fig. S6.1.28

$P_1 = G_2G_5G_6$, $P_2 = G_3G_5G_6$, $P_3 = G_3G_6$, $P_4 = G_4G_6$

If any path is deleted, there would not be any loop.

Hence $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

$$\frac{C}{R} = \frac{G_4G_6 + G_3G_6 + G_3G_5G_6 + G_2G_5G_6}{\Delta}$$

29. (A)

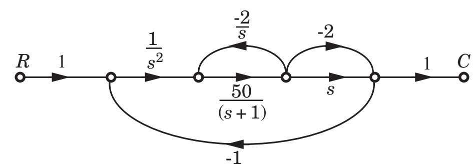


Fig. S6.1.29

$$P_1 = \frac{1}{s^2} \cdot \frac{50}{(s+1)} \cdot s = \frac{50}{s(s+1)}$$

$$P_2 = \frac{1}{s^2} \cdot \frac{50}{s+1} \cdot (-2) = \frac{-100}{s^2(s+1)}$$

$$L_1 = \frac{50}{s+1} \cdot \frac{-2}{s} = \frac{-100}{s(s+1)}$$

$$L_2 = \frac{1}{s^2} \cdot \frac{50}{s+1} \cdot s \cdot (-1) = \frac{-50}{s(s+1)}$$

$$L_3 = \frac{1}{s^2} \cdot \frac{50}{s+1} \cdot (-2) \cdot (-1) = \frac{100}{s^2(s+1)}$$

$$\Delta = 1 + \frac{100}{s(s+1)} + \frac{50}{s(s+1)} - \frac{100}{s^2(s+1)}$$

$\Delta_1 = \Delta_2 = 1$

$$\frac{C}{R} = \frac{P_1 + P_2}{\Delta} = \frac{50(s-2)}{s^3 + s^2 + 150s - 100}$$

30. (D) $P_1 = G_1\ G_2\ G_3$

$L_1 = -G_1H_1$, $L_2 = -G_2H_2$, $L_3 = -G_3H_3$

$L_1L_3 = G_1G_3H_1H_3$

$\Delta = 1 - (-G_1H_1 - G_2H_2 - G_3H_3) + G_1G_3H_1H_3$

$\Delta = 1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1G_3H_1H_3$

$\Delta_1 = 1$

$$\frac{C}{R} = \frac{G_1G_2G_3}{1 + G_1H_! + G_2H_2 + G_3H_3 + G_1G_3H_1H_3}$$

CHAPTER

6.2

STABILITY

1. Consider the system shown in fig. P6.2.1. The range of K for the stable system is

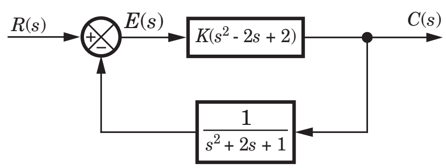


Fig. P6.2.1

(A) $-1 < K < -\frac{1}{2}$ (B) $-\frac{1}{2} < K < 1$

(C) $-1 < K < 1$ (D) Unstable

2. The forward transfer function of a *ufb* system is

$$G(s) = \frac{K(s^2 + 1)}{(s+1)(s+2)}$$

The system is stable for

(A) $K < -1$ (B) $K > -1$

(C) $K < -2$ (D) $K > -2$

3. The open-loop transfer function with *ufb* are given below for different systems. The unstable system is

(A) $\frac{2}{s+2}$ (B) $\frac{2}{s^2(s+2)}$

(C) $\frac{2}{s(s+2)}$ (D) $\frac{2(s+1)}{s(s+2)}$

4. Consider a *ufb* system with forward-path transfer function

$$G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$$

The range of K to ensure stability is

(A) $K > \frac{6}{8}$ (B) $K < -1$ or $K > \frac{3}{4}$

(C) $K < -1$ (D) $-1 < K < \frac{3}{4}$

5. Consider a *ufb* system with forward-path transfer function

$$G(s) = \frac{K(s+3)}{s^4(s+2)}$$

The system is stable for the range of K

(A) $K > 0$ (B) $K < 0$

(C) $K > 1$ (D) Always unstable

6. The open-loop transfer function of a *ufb* control system is

$$G(s) = \frac{K(s+2)}{(s+1)(s-7)}$$

For $K > 6$, the stability characteristic of the open-loop and closed-loop configurations of the system are respectively

(A) stable and unstable

(B) stable and stable

(C) unstable and stable

(D) unstable and unstable

7. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{K(s^2 - 4)}{s^2 + 3}$$

For the system to be stable the range of K is

(A) $K > -1$ (B) $K < \frac{3}{4}$

(C) $-1 < K < \frac{3}{4}$ (D) marginal stable

8. A *ufb* system have the forward-path transfer function

$$G(s) = \frac{K(s+6)}{s(s+1)(s+3)}$$

The system is stable for

(A) $K < 6$ (B) $-6 < K < 0$

(C) $0 < K < 6$ (D) $K > 6$

9. The feedback control system shown in the fig. P6.2.8.

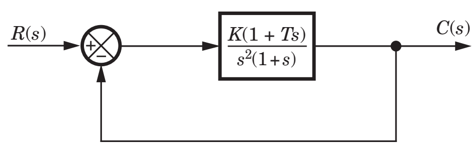


Fig. P6.2.9

is stable for all positive value of K, if

(A) $T = 0$ (B) $T < 0$

(C) $T > 1$ (D) $0 < T < 1$

10. Consider a *ufb* system with forward-path transfer function

$$G(s) = \frac{K}{(s+15)(s+27)(s+38)}$$

The system will oscillate for the value of K equal to

(A) 23690 (B) 2369

(C) 144690 (D) 14469

11. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{K(s-2)(s+4)(s+5)}{(s^2+3)}$$

For system to be stable, the range of K is

(A) $K > \frac{1}{54}$ (B) $K < \frac{3}{40}$

(C) $\frac{1}{54} < K < \frac{3}{40}$ (D) Unstable

12. The closed loop system shown in fig. P6.2.12 become marginally stable if the constant K is chosen to be

(A) 30 (B) –30

(C) 10 (D) –10

13. The open-loop transfer function of a *ufb* system is

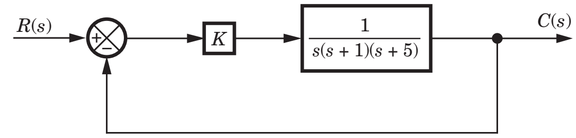


Fig. P6.2.12

$$G(s) = \frac{K(s+10)(s+20)}{s^2(s+2)}$$

The closed loop system will be stable if the value of K is

(A) 2 (B) 3

(C) 4 (D) 5

Statement for Q.14–15:

A feedback system is shown in fig. P6.14-15.

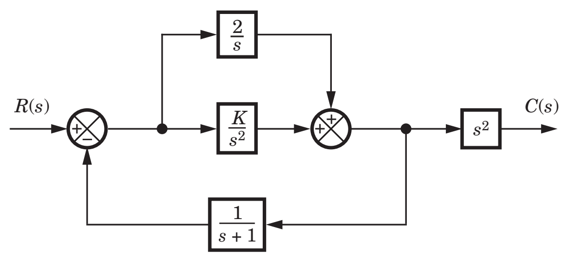


Fig. P6.2.14–15

14. The closed loop transfer function for this system is

(A) $\dfrac{s^5 + s^4 + 2s^3 + (K+2)s^2 + (K+2)s + K}{s^3 + s^2 + 2s + K}$

(B) $\dfrac{2s^4 + (K+2)s^3 + Ks^2}{s^3 + s^2 + 2s + K}$

(C) $\dfrac{s^3 + s^2 + 2s + K}{s^5 + s^4 + 2s^3 + (K+2)s^2 + (K+2)s + K}$

(D) $\dfrac{s^3 + s^2 + 2s + K}{2s^4 + (K+2)s^3 + Ks^2}$

15. The poles location for this system is shown in fig. P6.2.15. The value of K is

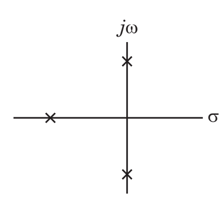


Fig. P6.2.15

(A) 4 (B) –4

(C) 2 (D) –2

(A) stable (B) unstable

(C) marginally stable

(D) More information is required.

27. The forward path transfer of *ufb* system is

$$G(s) = \frac{1}{4s^2(s^2+1)}$$

The system is

(A) stable (B) unstable

(C) marginally stable

(D) More information is required

28. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{G(s)}{2s^4 + 5s^3 + s^2 + 2s}$$

The system is

(A) stable (B) unstable

(C) marginally stable

(D) more information is required.

29. The open loop transfer function of a system is as

$$G(s)H(s) = \frac{K(s+0.1)}{s(s-0.2)(s^2+s+0.6)}$$

The range of K for stable system will be

(A) $K > 0.355$ (B) $0.149 < K < 0.355$

(C) $0.236 < K < 0.44$ (D) $K > 0.44$

30. The open-loop transfer function of a *ufb* control system is given by

$$G(s) = \frac{K}{s(sT_1+1)(sT_2+1)}$$

For the system to be stable the range of K is

(A) $0 < K < \left(\frac{1}{T_1} + \frac{1}{T_2}\right)$ (B) $K > \left(\frac{1}{T_1} + \frac{1}{T_2}\right)$

(C) $0 < K < T_1T_2$ (D) $K > T_1T_2$

31. The closed loop transfer function of a system is

$$T(s) = \frac{s^3 + 4s^2 + 8s + 16}{s^5 + 3s^4 + 5s^2 + s + 3}$$

The number of poles in right half-plane and in left half-plane are

(A) 3, 2 (B) 2, 3

(C) 1, 4 (D) 4, 1

32. The closed loop transfer function of a system is

$$T(s) = \frac{(s+8)(s+6)}{s^5 - s^4 + 4s^3 - 4s^2 + 3s - 2}$$

The number of poles in RHP and in LHP are

(A) 4, 1 (B) 1, 4

(C) 3, 2 (D) 2, 3

33. The closed loop transfer function of a system is

$$T(s) = \frac{s^3 + 3s^2 + 7s + 24}{s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4}$$

The number of poles in LHP, in RHP, and on $j\omega$ axis are

(A) 2, 1, 2 (B) 0, 1, 4

(C) 1, 0, 4 (D) 1, 2, 2

34. For the system shown in fig. P6.2.34. the number of poles on RHP, LHP, and imaginary axis are

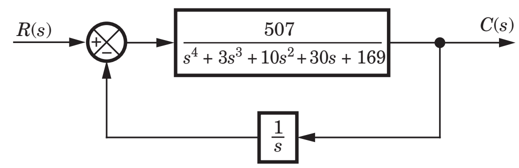


Fig. P6.2.34

(A) 2, 3, 0 (B) 3, 2, 0

(C) 2, 1, 2 (D) 1, 2, 2

35. A Routh table is shown in fig. P6.2.36. The location of pole on RHP, LHP and imaginary axis are

s^7	1	2
s^5	1	2
s^5	3	4
s^4	1	–1

Fig. P6.2.35

(A) 1, 2, 4 (B) 1, 6, 0

(C) 1, 0, 6 (D) None of the above

36. For the open loop system of fig. P6.2.35 location of poles on RHP, LHP, and an $j\omega$-axis are

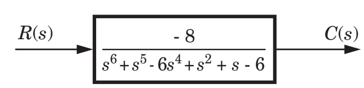


Fig. P6.2.35

(A) 3, 3, 0 (B) 1, 3, 2

(C) 1, 1, 4 (D) 3, 1, 2

****

11. (C) $T(s) = \dfrac{G(s)}{1+G(s)}$

$$= \frac{K(s-2)(s+4)(s+5)}{Ks^3 + (7K+1)s^2 + 2Ks + (3-40K)}$$

Routh table is as shown in fig. S.6.211

s^3	K	$2K$
s^2	$7K+1$	$3-40K$
s^1	$\frac{54K^2-K}{7K+1}$	
s^0	$3-40K$	

Fig. S.6.2.11

$K > 0$,

$$\left.\begin{aligned} 7K+1>0 &\quad\Rightarrow\quad K > -\frac{1}{7} \\ \frac{54K^2-K}{7K+1} > 0 &\quad\Rightarrow\quad K > \frac{1}{54} \\ 3-40K>0 &\quad\Rightarrow\quad K < \frac{3}{40} \end{aligned}\right\} \Rightarrow \frac{1}{54} < K < \frac{3}{40}$$

12. (A) $T(s) = \dfrac{1}{s^3 + 6s^2 + 5s + K}$

Routh table is as shown in fig. S.6.212

s^3	1	5
s^2	6	K
s^1	$30-K$	
s^0	K	

Fig. S.6.2.12

13. (D) $T(s) = \dfrac{K(s+10)(s+20)}{s^3 + (K+2)s^2 + 30Ks + 200K}$

Routh table is as shown in fig. S.6.2.13

s^3	1	$30K$
s^2	$K+2$	$200K$
s^1	$30K^2 - 140K$	
s^0	$200K$	

Fig. S.6.2.13

$200K > 0 \quad \rightarrow \quad K > 0, \quad 30K^2 - 140K > 0$

$\Rightarrow \quad K > \dfrac{14}{3}$, 5 satisfy this condition.

14. (B) First combine the parallel loop $\dfrac{K}{s^2}$ and $\dfrac{2}{s}$ giving $\dfrac{K}{s^2} + \dfrac{2}{s}$. Then apply feedback formula with $\left(\dfrac{K}{s^2} + \dfrac{2}{s}\right)$ and $\dfrac{1}{(s+1)}$, and then multiply with s^2.

$$T(s) = \frac{s^2\left(\dfrac{K}{s^2} + \dfrac{2}{s}\right)}{1 + \dfrac{1}{s+1}\left(\dfrac{K}{s^2} + \dfrac{2}{s}\right)} = \frac{2s^4 + (K+2)s^3 + Ks^2}{s^3 + s^2 + 2s + K}$$

15. (C) Denominator $= s^3 + s^2 + 2s + K$

Routh table is as shown in fig. S.6.2.15

s^3	1	5
s^2	1	K
s^1	$2-K$	
s^0	K	

Fig. S.6.2.15

Row of zeros when $K = 2$,

$s^2 + 2 = 0, \quad \Rightarrow \quad s = -1,\ j\sqrt{2},\ -j\sqrt{2}$

16. (D) Applying the feedback formula on the inner loop and multiplying by K yield

$$G_e(s) = \frac{K}{s(s^2 + 5s + 7)},$$

$$T(s) = \frac{K}{s^3 + 5s^2 + 7s + K}$$

17. (B) Routh table is as shown in fig. S.6.2.17

s^3	1	7
s^2	5	K
s^1	$\frac{35-K}{5}$	
s^0	K	

Fig. S.6.2.17

$K > 0, \quad \dfrac{35-K}{5} > 0 \quad \Rightarrow \quad K < 35$

18. (C) At $K = 35$ system will oscillate.

Auxiliary equation $5s^2 + 35 = 0, \quad \Rightarrow \quad s = \pm j\sqrt{7}$

19. (B) For inner loop

$$G_i(s) = \frac{K}{(s-a)(s+3a)(s+4a)} = \frac{K}{P(s)}, \qquad T_i(s) = \frac{K}{P(s)+K}$$

For outer loop, $G_o(s) = T_i(s) = \dfrac{K}{P(s)+K}$,

$$T_o(s) = \frac{K}{P(s) + 2K},$$

Therefore if inner loop is stable for $X < K < Y$, then outer loop will be stable for $X < 2K < Y$

$$\Rightarrow \frac{X}{2} < K < \frac{Y}{2}.$$

20. (D) $T(s) = \dfrac{K(s+2)}{s^4 + 3s^3 - 3s^2 + (K+3)s + (2K-4)}$

Routh table is as shown in fig. S.6.2.20

s^4	1	–3	$2K-4$
s^3	3	$K+3$	
s^2	$-\frac{(K+12)}{3}$	$2K-4$	
s^1	$\frac{K(K+33)}{K+12}$		
s^0	$2K-4$		

Fig. S.6.2.20

$$\frac{-(K+12)}{3} > 0 \Rightarrow K < -12, \quad 2K - 4 > 0$$

$\Rightarrow K > 2$ and $K > -33$, These condition can not be met simultaneously. System is unstable for any value of K

21. (D) Routh table is as shown in fig. S.6.2.21

s^4	1	1	1
s^3	K	1	
s^2	$\frac{K-1}{K}$	1	
s^1	$\frac{K-1-K^2}{K-1}$		
s^0	1		

Fig. S.6.2.21

$$K > 0, \quad K - 1 > 0 \Rightarrow K > 1, \quad \frac{K-1-K^2}{K-1} > 0,$$

But for $K > 1$ third term is always -ive. Thus the three condition cannot be fulfilled simultaneously.

22. (D) Routh table is as shown in fig. S.6.2.22

s^4	1	$4+K$	25
s^3	2	9	
s^2	$\frac{2K-1}{2}$	25	
s^1	$\frac{18K-109}{2K-1}$		
s^0	25		

Fig. S.6.2.22

$$\left.\begin{aligned} \frac{2K-1}{2} > 0 &\quad\Rightarrow\quad K > \frac{1}{2} \\ \frac{18K-109}{2K-1} > 0 &\quad\Rightarrow\quad K > \frac{109}{18} \end{aligned}\right\} \Rightarrow K > \frac{109}{18}$$

23. (B) Characteristic equation

$s^4 + 9s^3 + 20s^2 + Ks + K = 0$

Routh table is as shown in fig. S.6.2.23

s^4	1	20	K
s^3	9	K	
s^2	$\frac{180-K}{9}$	K	
s^1	$\frac{K(K-99)}{K-180}$		
s^0	K		

Fig. S.6.2.23

For stability $0 < K < 99$

24. (C) $T(s) = \dfrac{K(s+2)}{s^4 + 3s^2 - 3s^2 + (K+3)s + (2K-4)}$

Routh table is as shown in fig. S.6.2.24

s^4	1	–3	$2K-4$
s^3	3	$K+3$	
s^2	$\frac{-K+12}{3}$	$2K-4$	
s^1	$\frac{K(K+33)}{K+12}$		
s^0	$2K-4$		

Fig. S.6.2.24

For $K < -33$, 1 sign change

For $-33 < K < -12$, 1 sign change

For $-12 < K < 0$, 1 sign change

For $0 < K < 2$, 3 sign change

For $K > 2$, 2 sign change

Therefore $K > 2$ yield two RHP pole.

25. (B) Routh table is as shown in fig. S.6.2.25

s^4	1	8	15
s^3	4	20	
s^2	3	15	
s^1	6		ROZ
s^0	15		

Fig. S.6.2.25

$P(s) = 3s^2 + 15, \frac{d\,P(s)}{ds} = 6s$, No sign change from s^2 to s^0

on $j\omega$-axis 2 roots, RHP 0, LHP 2.

26. (B) Closed-loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{240}{s^4 + 10\,s^3 + 35\,s^2 + 50s + 264}$$

Routh table is as shown in fig. S.6.2.26

s^4	1	35	264
s^3	10	50	
s^2	30	264	
s^1	–386		ROZ
s^0	264		

Fig. S.6.2.26

Two sign change. RHP-2 poles. System is not stable.

27. (C) Closed loop transfer function

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{4s^4 + 4s^2 + 1}$$

Routh table is as shown in fig. S.6.2.27

s^4	4	4	1
s^3	16	8	ROZ
s^2	2	1	
s^1	46		ROZ
s^0	1		

Fig. S.6.2.27

$P(s) = 4s^4 + 4s + 1, \quad \frac{dp(s)}{ds} = 16s^3 + 3s$

There is no sign change. So all pole are on $j\omega$–axis. So system is marginally stable.

28. (B) Closed loop transfer function

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + 1}$$

Routh table is as shown in fig. S.6.2.28

s^4	2	1	1
s^3	5	2	
s^2	$\frac{1}{5}$	1	
s^1	–23		
s^0	1		

Fig. S.6.2.28

2 RHP poles so unstable.

29. (B) The characteristic equation is $1 + G(s)H(s) = 0$

$\Rightarrow \quad s(s-0.2)(s^2 + s + 0.6) + K(s + 0.1) = 0$

$s^4 + 0.8\,s^3 + 0.4s^2 + (K - 0.12)s + 0.1K = 0$

Routh table is as shown in fig. S.6.2.29

s^4	2	0.4	1
s^3	0.8	$K - 0.12$	
s^2	$0.55 - 1.25K$	$0.1K$	
s^1	$\frac{-1.25K^2 + 0.63K - 0.066}{0.55 - 1.25K}$		
s^0	$0.1K$		

Fig. S.6.2.29

$K > 0, \quad 0.55 - 1.25K > 0 \quad \Rightarrow \quad K < 0.44$

$-1.25K^2 + 0.63K - 0.066 > 0$

$(K - 0.149)(K - 0.355) < 0, \ 0.149 < K < 0.355$

30. (A) Characteristic equation

$s(sT_1 + 1)(sT_2 + 1) + K = 0$

$T_1T_2s^3 + (T_1 + T_2)s^2 + s + K = 0$

Routh table is as shown in fig. S.6.2.30

s^3	T_1T_2	1
s^2	$T_1 + T_2$	K
s^1	$\frac{(T_1 + T_2) - T_1T_2K}{T_1 + T_2}$	
s^0	K	

Fig. S.6.2.30

$K > 0, \quad (T_1 + T_2) - T_1T_2K > 0 \quad \Rightarrow \quad 0 < K < \left(\frac{1}{T_1} + \frac{1}{T_2}\right)$

31. (B) Routh table is as shown in fig. S.6.2.31

s^5	1	5	1
s^4	3	4	3
s^3	3.67	0	
s^2	4	3	
s^1	–2.75		
s^0	3		

In RHP –2 poles. In LHP –3 poles.

32. (C) Routh table is as shown in fig. S.6.2.32

ε + −
+ +
− −
+ −
+ −
+ +
− −

s^5	1	4	3
s^4	–1	–4	–2
s^3	ε	1	
s^2	$\frac{1-4\varepsilon}{\varepsilon}$	–2	
s^1	$\frac{2\varepsilon^2+1-4\varepsilon}{1-4\varepsilon}$		
s^0	–2		

Fig. S.6.2.32

3 RHP, 2 LHP poles.

33. (B) Routh table is as shown in fig. S.6.2.33

s^5	1	3	2
s^4	–2	–6	–4
s^3	–2	–3	ROZ
s^2	–3	–4	
s^1	$-\frac{1}{3}$		
s^0	–4		

Fig. S.6.2.33

$P(s) = -2s^4 - 6s^2$, $\frac{dP(s)}{ds} = -8s^3 - 12s$, –2 , –3

No sign change exist from the s^4 row down to the s^0 row. Thus, the even polynomial does not have RHP poles. Therefore because of symmetry all four poles must be on $j\omega$-axis.

$j\omega$-axis 4 pole
RHP 1 pole (1 sign change)
LHP 0 pole

34. (D) Closed loop transfer function

$$T(s) = \frac{G(s)}{1 + G(s) + H(s)}$$

$$= \frac{507s}{s^5 + 3s^4 + 10s^3 + 30s^2 + 169s + 507}$$

Routh table is as shown in fig. S.6.2.34

s^5	1	10	69
s^4	3	30	57
s^3	12	60	ROZ
s^2	15	507	
s^1	–345.6		
s^0	507		

Fig. S.6.2.33

$P(s) = 3s^4 + 30s^2 + 507$, $\frac{dP(s)}{ds} = 12s^3 + 60$

From s^4 row down to s^0 there is one sign change. So LHP–$1 + 1 = 2$ pole. RHP–1 pole, $j\omega$-axis –2 pole.

35. (A) Notice that in s^5 row there would be zero. In this row coefficient of $\frac{dP(s)}{ds}$, where $P(s) = s^6 + 2s^4 - s^2 - 2$ have been entered. From s^6 to row down to the s^0 row, there is one sign change. So there is one pole on RHP. Corresponding to this pole there is a pole on LHP. Corresponding to this pole there is a pole on LHP. Rest 4 out of 6 poles are on imaginary axis. Rest 1 pole is on LHP.

36. (A) Routh table is as shown in fig. S.6.2.36

ε + −
+ +
+ +
− −
− −
+ −
− +
− −

s^6	1	–6	–6
s^5	1	0	
s^4	–6	0	
s^3	–24	0	ROZ
s^2	ε		
s^1	$-\frac{144}{\varepsilon}$		
s^0	–6		

Fig. S.6.2.36

$P(s) = -6s^4 - 6$, $\frac{dP(s)}{ds} = -24s^3$,

There is two sign change from the s^4 row down to the s^0 row. So two roots are on RHS. Because of symmetry rest two roots must be in LHP. From s^6 to s^4 there is 1 sign change so 1 on RHP and 1 on LHP.

Total LHP 3 root, RHP 3 root.

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8. A system is shown in fig. P6.3.8. The rise time and settling time for this system is

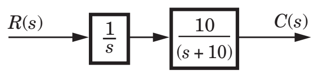


Fig. P6.3.8

(A) 0.22s, 0.4s (B) 0.4s, 0.22s

(C) 0.12s, 0.4s (D) 0.4s, 0.12s

9. For a second order system settling time is $T_s = 7$ s and peak time is $T_p = 3$ s. The location of poles are

(A) $-0.97 \pm j0.69$ (B) $-0.69 \pm j0.97$

(C) $-1.047 \pm j0.571$ (D) $-0.571 \pm j1.047$

10. For a second order system overshoot $= 10\%$ and peak time $T_p = 5$ s. The location of poles are

(A) $-0.46 \pm j0.63$ (B) $-0.63 \pm j0.46$

(C) $-0.74 \pm j0.92$ (D) $-0.92 \pm j0.74$

11. For a second-order system overshoot $= 12\ \%$ and settling time $= 0.6$ s. The location of poles are

(A) $-9.88 \pm j6.67$ (B) $-6.67 \pm j9.88$

(C) $-4.38 \pm j6.46$ (D) $-6.46 \pm j4.38$

Statement for Q.12–13:

A system has a damping ratio of 1.25, a natural frequency of 200 rad/s and DC gain of 1.

12. The response of the system to a unit step input is

(A) $1 + \frac{5}{3}e^{-50t} - \frac{2}{3}e^{-150t}$ (B) $1 - \frac{4}{3}e^{-100t} + \frac{1}{3}e^{-400t}$

(C) $1 + \frac{1}{3}e^{-100t} - \frac{4}{3}e^{-400t}$ (D) $1 + \frac{2}{3}e^{-50t} - \frac{5}{3}e^{-150t}$

13. The system is

(A) overdamped (B) under damped

(C) critically damped (D) None of the above

14. Consider the following system

a. $T(s) = \dfrac{5}{(s+3)(s+6)}$

b. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

c. $T(s) = \dfrac{20}{s^2 + 6s + 144}$

d. $T(s) = \dfrac{s+2}{s^2+9}$

e. $T(s) = \dfrac{(s+5)}{(s+10)^2}$

Consider the following response

1. Overdamped 2. Under damped

3. Undamped 4. Critically damped.

The correct match is

	1	2	3	4
(A)	a	c	d	e
(B)	b	a	d	e
(C)	c	a	e	d
(D)	c	b	e	d

15. The forward-path transfer of a *ufb* control system is

$$G(s) = \frac{1000}{(1+0.1s)(1+10s)}$$

The step, ramp, and parabolic error constants are

(A) 0, 1000, 0 (B) 1000, 0, 0

(C) 0, 0, 0 (D) 0, 0, 1000

16. The open-loop transfer function of a *ufb* control system is

$$G(s) = \frac{K(1+2s)(1+4s)}{s^2(s^2+2s+8)}$$

The position, velocity and acceleration error constants are respectively

(A) 0, 0, $4K$ (B) ∞, $\frac{K}{8}$, 0

(C) 0, $4K$, ∞ (D) ∞, ∞, $\frac{K}{8}$

17. The open-loop transfer function of a unit feedback system is

$$G(s) = \frac{50}{(1+0.1s)(1+2s)}$$

The position, velocity and acceleration error constants are respectively

(A) 0, 0, 250 (B) 50, 0, 0

(C) 0, 250, ∞ (D) ∞, 50, 0

Statement for Q.18–19:

The forward-path transfer function of a unity feedback system is

$$G(s) = \frac{K}{s^n (s+a)}$$

The system has 10% overshoot and velocity error constant $K_v = 100$.

18. The value of K is

(A) 237×10^3 (B) 144

(C) 14.4×10^3 (D) 237

19. The value of a is

(A) 23.7×10^3 (B) 237

(C) 14.4×10^3 (D) 144

20. For the system shown in fig. P6.3.20 the steady state error component due to unit step disturbance is 0.000012 and steady state error component due to unit ramp input is 0.003. The values of K_1 and K_2 are respectively

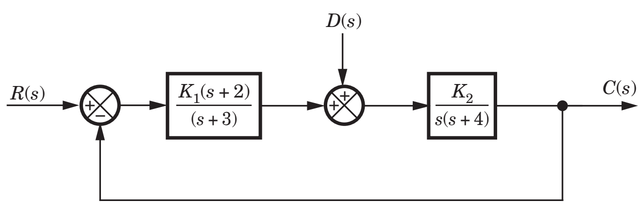


Fig. P6.3.20

(A) 16.4, 1684 (B) 1250, 2.4

(C) 125×10^3, 0.016 (D) 463, 3981

21. The transfer function for a single loop nonunity feedback control system is

$$G(s) = \frac{1}{s^2 + s + 2}, \quad H(s) = \frac{1}{(s+1)}$$

The steady state error due to unit step input is

(A) $\frac{6}{7}$ (B) $\frac{6}{5}$

(C) $\frac{2}{3}$ (D) 0

22. For the system of fig. P6.3.22 the total steady state error due to a unit step input and a unit step disturbance is

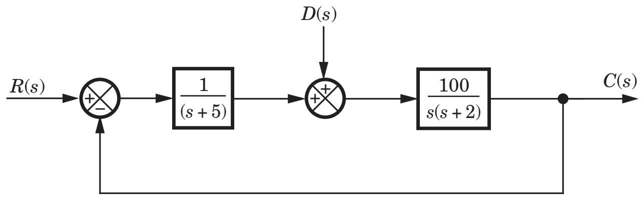


Fig. P6.3.22

(A) $-\frac{49}{11}$ (B) $\frac{49}{11}$

(C) $-\frac{63}{11}$ (D) $\frac{63}{11}$

23. The forward path transfer function of a *ufb* system is

$$G(s) = \frac{K}{s(s+4)(s+8)(s+10)}$$

If a unit ramp is applied, the minimum possible steady-state error is

(A) 0.16 (B) 6.25

(C) 0.14 (D) 7.25

24. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{1000(s^2 + 4s + 20)(s^2 + 20s + 15)}{s^3 (s+2)(s+10)}$$

The system has $r(t) = t^3$ applied to its input. The steady state error is

(A) 4×10^{-4} (B) 0

(C) ∞ (D) 2×10^{-5}

25. The transfer function of a *ufb* system is

$$G(s) = \frac{10^5 (s+3)(s+10)(s+20)}{s(s+25)(s+a)(s+30)}$$

The value of a to yield velocity error constant $K_v = 10^4$ is

(A) 4 (B) 0

(C) 8 (D) 16

26. A system has position error constant $K_p = 3$. The steady state error for input of $8tu(t)$ is

(A) 2.67 (B) 2

(C) ∞ (D) 0

27. The forward path transfer function of a unity feedback system is

$$G(s) = \frac{1000}{(s+20)(s^2+4s+10)}$$

For input of $60u(t)$ steady state error is

(A) 0 (B) 300

(C) ∞ (D) 10

28. For *ufb* system shown in fig. P6.3.28 the transfer function is

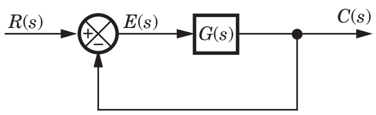


Fig. P6.3.28

$$G(s) = \frac{20(s+3)(s+4)(s+8)}{s^2(s+2)(s+15)}$$

If input is $30t^2$, then steady state error is

(A) 0.9375 (B) 0

(C) ∞ (D) 64

29. The forward-path transfer function of a *ufb* control system is

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

The steady state errors for the test input $37tu(t)$ is

(A) 0 (B) 0.061

(C) ∞ (D) 609

30. In the system shown in fig. P6.3.30, $r(t) = 1 + 2t$, $t > 0$. The steady state error $e(t)$ is equal to

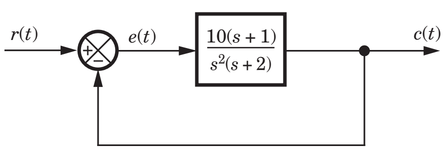


Fig. P6.3.30

(A) $\frac{1}{5}$ (B) 5

(C) 0 (D) ∞

31. A *ufb* control system has a forward path transfer function

$$G(s) = \frac{10(1+4s)}{s^2(1+s)}$$

If the system is subjected to an input $r(t) = 1 + t + \frac{1}{2}t^2$, $t > 0$ the steady state error of the system will be

(A) 0 (B) 0.1

(C) 10 (D) ∞

32. The system shown in fig. P6.3.32 has steady-state error 0.1 to unit step input. The value of K is

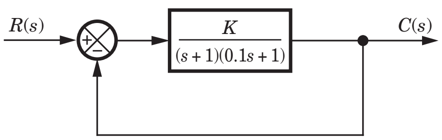


Fig. P6.3.32

(A) 0.1 (B) 0.9

(C) 1.0 (D) 9.0

Statement for Q.33–34:

Block diagram of a position control system is shown in fig.P6.3.33–34.

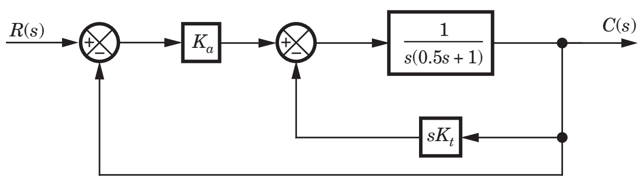


Fig. P6.3.33–34

33. If $K_t = 0$ and $K_a = 5$, then the steady state error to unit ramp input is

(A) 5 (B) 0.2

(C) ∞ (D) 0

34. If the damping ratio of the system is increased to 0.7 without affecting the steady state error, then the value of K_a and K_t are

(A) 86, 12.8 (B) 49, 9.3

(C) 24.5, 3.9 (D) 43, 6.4

35. A system has the following transfer function

$$G(s) = \frac{100(s+15)(s+50)}{s^4(s+12)(s^2+3s+10)}$$

The type and order of the system are respectively

(A) 7 and 5 (B) 4 and 5

(C) 4 and 7 (D) 7 and 4

SOLUTIONS

1. (D) Characteristic equation is $s^2 + 9s + 18$.

$\omega_n^2 = 18, \quad 2\xi\omega_n = 9$

Therefore $\xi = 1.06, \quad \omega_n = 4.24$ rad/s

2. (A) $T(s) = \dfrac{1}{6}\dfrac{0.6}{(s+0.8)^2 + (0.6)^2}$

$\omega_n\sqrt{1-\xi^2} = 0.6, \quad \xi\omega_n = 0.8$

Hence $\omega_n = 1, \quad \xi = 0.8$

3. (A) Characteristic equation is

$\Delta s = \{s-(-3+j4)\}\{s-(-3-j4)\} = (s+3)^2 + 4^2.$

$= s^2 + 6s + 25, \quad \omega_n^2 = 25 \quad \Rightarrow \quad \omega_n = 5$ rad/s

$2\xi\omega_n = 6, \quad \xi = \dfrac{6}{2\times 5} = 0.6$

4. (A) $T(s) = \dfrac{16}{(4s^2 + 8s + 16)} = \dfrac{4}{(s^2 + 2s + 4)}$

$\omega_n^2 = 4 \quad \Rightarrow \quad \omega_n = 2, \quad 2\xi\omega_n = 2, \quad \xi = 0.5$

5. (D) $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = \dfrac{5}{100} = 0.05,$

$\dfrac{\xi\pi}{\sqrt{1-\xi^2}} = 3 \quad \Rightarrow \quad \xi = 0.69,$

$T(s) = \dfrac{1}{1+G(s)} = \dfrac{K}{s^2 + 2s + K}$

$2\xi\omega_n = 2, \quad \omega_n = \dfrac{1}{0.69} = 1.45$

Peak time,

$T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{-\xi^2}} = \dfrac{\pi}{1.45\sqrt{(1-0.69^2)}} = 3$ sec

But the peak time T_p given is 1 sec. Hence these two specification cannot be met.

6. (C) $T(s) = \dfrac{K_1}{s^2 + (K_2 + s) + K_1}$,

$\omega_n^2 = K_1, \quad 2\xi\omega_n = 1 + K_2$

$\omega_d = 0.10, \quad \xi = 0.6, \quad \omega_d = \omega_n\sqrt{1-0.6^2} = 10$

$\omega_n = 12.5 \quad \Rightarrow \quad K_1 = 156.25,$

$2\omega_n 3 = K_2 + 1$

$2\times 12.5 \times 0.6 = K_2 + 1 \quad \Rightarrow \quad K_2 = 14$

7. (A) $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$, At $\xi = 0, \quad M_p = 1 = 100\%$

8. (A) $C(s) = \dfrac{10}{s(s+10)} = \dfrac{1}{s} - \dfrac{1}{s+10}$

$\Rightarrow \quad c(t) = 1 - e^{-10t}$

$a = 10$, Rise time $T_r = \dfrac{2.2}{a} = \dfrac{2.2}{10} = 0.22$s

Settling time $T_s = \dfrac{4}{a} = 0.4$s

9. (D) $\xi\omega_n = \dfrac{4}{T_s} = 0.571, \; \omega_n\sqrt{1-\xi^2} = \dfrac{\pi}{T_p} = 1.047$

Poles $= -0.571 \pm j1.047$

10. (A) $0.1 = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \quad \Rightarrow \quad \xi = 0.59$

$\omega_n = \dfrac{\pi}{T_p}\sqrt{1-\xi^2} = 0.779,$

Poles $= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -0.46 \pm j0.63$

11. (B) $0.12 = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \quad \Rightarrow \quad \xi = 0.56, \; \omega_n = \dfrac{4}{\xi T_s} = 11.92$

Therefore Poles $= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -6.67 \pm j9.88$

Note :

$T_s = \dfrac{3.2}{\xi\omega_n}$, For $0 < \xi < 0.69$

$T_s = \dfrac{4.5}{\xi\omega_n}$, For $\xi > 0.69$

12. (B) $T(s) = \dfrac{\omega_n^2}{s + 2\xi\omega_n s + \omega_n^2} = \dfrac{40000}{s^2 + 500s + 40000}$

$= \dfrac{40000}{(s+100)(s+400)}$

$R(s) = \dfrac{40000}{s(s+100)(s+400)} = \dfrac{1}{s} - \dfrac{4}{3(s+100)} + \dfrac{1}{3(s+400)}$

$r(t) = 1 - \dfrac{4}{3}e^{-100t} + \dfrac{1}{3}e^{-400t}$

13. (A) System has two different poles on negative real axis. So response is over damped.

14. (A) 1. Overdamped response (a, b)

Poles : Two real and different on negative real axis.

2. Underdamped response (c)

Poles : Two complex in left half plane

3.Undamped response (d)

Poles : Two imaginary.

4.Critically damped (e)

Poles : Two real and same on negative real axis.

15. (B) $K_p = \lim_{s\to 0} G(s) = 1000$

$K_v = \lim_{s\to 0} sG(s) = 0$

$K_a = \lim_{s\to 0} s^2 G(s) = 0$

16. (D) $H(s) = 1, \quad K_p = \lim_{s\to 0} G(s)\cdot H(s) = \infty$

$K_v = \lim_{s\to} sG(s)\cdot H(s) = \infty$

$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = \dfrac{K}{8}$

17. (B) $H(s) = 1, \quad K_p = \lim_{s\to 0} G(s)\cdot H(s) = 50$

$K_v = \lim_{s\to 0} sG(s)\cdot H(s) = 0$

$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = 0.$

18. (C) System type $= 1$, so $n = 1$

$K_v = \lim_{s\to 0} sG(s) = \dfrac{K}{a} = 100$

For 10% overshoot,

$0.1 = = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \quad \xi = 0.6$

$T(s) = \dfrac{G(s)}{1+G(s)} = \dfrac{K}{s^2 + as + K}$

$2\xi\omega_n = a, \quad \omega_n^2 = K \quad \Rightarrow \quad 2\times 0.6\sqrt{K} = a$

$\dfrac{K}{2}\times 0.6\sqrt{K} = 100 \quad \Rightarrow \quad K = 14400$

19. (D) $\dfrac{K}{a} = 100, \; K = 14400,$

$\dfrac{14400}{a} = 100 \quad \Rightarrow \quad a = 144$

20. (C) If $R(s) = 0$

$$T_D(s) = \frac{\dfrac{K_2}{s(s+4)}}{1 + \dfrac{K_1K_2(s+2)}{s(s+4)(s+3)}}$$

$$= \frac{K_2(s+3)}{s(s+3)(s+4) + K_1K_2(s+2)}$$

Error in output due to disturbance

$E(s) = T_D(s)D(s),$

If $D(s) = \dfrac{1}{s}$,

$$e_{ssD} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot T_D(s) = \lim_{s\to 0} T_D(s) = \frac{3}{2K_1}$$

$\dfrac{3}{2K_1} = 0.000012 \quad \Rightarrow \quad K_1 = 125\times 10^3$

Error due to ramp input

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)},$$

$R(s) = \dfrac{1}{s^2}, \; G(s) = \dfrac{K_1K_2(s+2)}{s(s+3)(s+4)}$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s + \dfrac{K_1K_2(s+2)}{(s+3)(s+4)}} = \frac{6}{K_1K_2}$$

$\dfrac{6}{125\times 10^3 K_2} = 0.003 \quad \Rightarrow \quad K_2 = 0.016$

21. (C) $E(s) = R(s) - C(s)H(s)$

$$= R(s) - \frac{R(s)G(s)H(s)}{1+G(s)H(s)} = \frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{\dfrac{s}{s}}{1 + \dfrac{1}{(s^2+s+2)}\dfrac{1}{(s+1)}} = \frac{2}{3}$$

22. (A) $e_{ss} = \lim_{s\to 0} \dfrac{sR(s) - sD(s)G_2(s)}{1 + G_1(s)G_2(s)}$

where $\quad G_1(s) = \dfrac{1}{s+5}$ and $G_2(s) = \dfrac{100}{s+2}$

$R(s) = D(s) = \dfrac{1}{s}$

$$e_{ss} = \frac{1 - \dfrac{100}{2}}{1 + \dfrac{1}{5}\times\dfrac{100}{2}} = \frac{-49}{11}$$

23. (A) Using Routh-Hurwitz Criterion, system is stable for $0 < K < 2000$

maximum $\quad K_v = \lim_{s\to 0} sG(s) = \dfrac{2000}{4\times 8\times 10} = 6.25$

minimum possible error $\quad \dfrac{1}{K_v} = \dfrac{1}{6.25} = 0.16$

24. (A) $R(s) = \dfrac{6}{s^4}, \quad E(s) = \dfrac{R(s)}{1+G(s)}$

$e_{ss} = \lim_{s\to 0} sE(s)$

$$= \lim_{s\to 0} \frac{\dfrac{6s}{s^4}}{1 + \dfrac{1000\,(s^2+4s+20)\,(s^2+20s+15)}{s^3(s+2)\,(s+10)}}$$

$$= \lim_{s\to 0} \frac{6}{s^3 + \dfrac{1000\,(s^2+4s+20)\,(s^2+20s+15)}{(s+2)\,(s+10)}}$$

$$= \frac{6}{0 + \frac{1000\times 20\times 15}{2\times 10}} = 4\times 10^{-4}$$

25. (A) $K_v = \lim_{s\to 0} sG(s)$

$$10^4 = \frac{10^4 \times 3 \times 10 \times 20}{25 \times a \times 30} \quad \Rightarrow \quad a = 4$$

26. (C) System is zero type $K_v = 0$, $e_{ss} = \frac{1}{K_v} = \infty$

27. (D) $K_p = \lim_{s\to 0} G(s) = 5$

For input $60u(t)$, $e_{ss} = \frac{60}{1+K_p} = 10$

28. (A) $K_a = \lim_{s\to 0} s^2 G(s) = 64$

$$e_{ss} = \frac{30 \times 2}{64} = 0.9375$$

29. (B) $K_v = \lim_{s\to 0} sG(s) = 609.02$

$$e_{ss} = \frac{37}{K_v} = 0.0607$$

30. (C) The system is type 2. Thus to step and ramp input error will be zero.

$$E(s) = R(s) - C(s) = R(s) - \frac{G(s)R(s)}{1+G(s)} = \frac{R(s)}{1+G(s)}$$

$$R(s) = \frac{1}{s} + \frac{2}{s^2} = \frac{s+2}{s^2}$$

$$E(s) = \frac{s+2}{s^2 + \frac{10(s+1)}{(s+2)}}$$

$$e_{ss}(t) = \lim_{s\to 0} sE(s) = 0$$

31. (C) System is type 2. Therefore error due to $1+t$ would be zero and due to $\frac{t^2}{2}$ would be $\frac{1}{K_a}$.

$$K_a = \lim_{s\to 0} s^2 G(s) = 10, \qquad e_{ss}(t) = \frac{1}{10} = 0.1$$

Note that you may calculate error from the formula

$$e_{ss}(t) = \lim_{s\to 0} sE(s) = \frac{sR(s)}{1+G(s)}$$

32. (D) $K_p = \lim_{s\to 0} G(s) = K$

$$e_{ss}(t) = \frac{1}{1+K_p} = \frac{1}{1+K} = 0.1 \quad \Rightarrow \quad K = 9.$$

33. (B) $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$

When $K_t = 0$ and $K_a = 5$

$$G(s) = \frac{5}{s(0.5\,s+1)}, \; H(s) = 1, \; R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{5}{s(0.5s+1)}} = \frac{1}{5} = 0.2$$

34. (C) The equivalent open-loop transfer function

$$G_e = \frac{\frac{K_a}{s(0.5\,s+1)}}{1 + \frac{sK_t}{s(0.5\,s+1)}} = \frac{K_a}{s(0.5\,s+1+K_t)}$$

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K_a}{0.5s^2 + s(1+K_t) + K_a}$$

$$= \frac{2K_a}{s^2 + 2s(1+K_t) + 2K_a}$$

$$\omega_n^2 = 2\,K_a \Rightarrow \omega_n = \sqrt{2K_a}$$

$$2\xi\,\omega_n = 2(1+K_t)$$

$$\xi = 1 + \frac{K_t}{\sqrt{2K_a}} = 0.7 \qquad \text{... (i)}$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G_e(s)}, \; R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s\left(1 + \frac{K_a}{s(0.5\,s+1+K_t)}\right)} = \frac{1+K_t}{K_a}$$

$$e_{ss} = \frac{1+K_t}{K_a} = 0.2 \qquad \text{...(ii)}$$

Solving (i) and (ii)

$K_a = 24.5$, $K_t = 3.9$

35. (C) The s has power of 4 and denominator has order of 7. So Type 4 and Order 7.

36. (D) For $8u(t)$, $e_{ss} = \frac{8}{1+K_p} = 2.$

For $8tu(t)$, $e_{ss} = \infty$, since the system is type 0.

37. (A) For equivalent unit feedback system the forward transfer function is

$$G_e = \frac{G(s)}{1+G(s)H(s)-G(s)} = \frac{\frac{10(s+10)}{s(s+2)}}{1 + \frac{10(s+10)(s+3)}{s(s+2)}}$$

$$= \frac{10(s+10)}{11s^2 + 132\,s + 300}$$

The system is of Type 0. Hence step input will produce a constant error constant.

CHAPTER 6.5

FREQUENCY-DOMAIN ANALYSIS

Statement for Q.1–2:

An under damped second order system having a transfer function of the form

$$T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

has a frequency response plot shown in fig. P6.5.1–2.

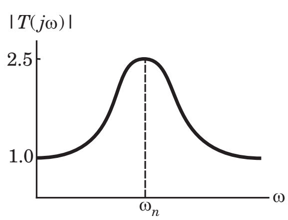


Fig. P6.5.1-2

1. The system gain K is

(A) 1 (B) 2

(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

2. The damping factor ξ is approximately

(A) 0.6 (B) 0.2

(C) 1.8 (D) 2.4

3. For the transfer function

$$G(s)H(s) = \frac{1}{s(s+1)(s+0.5)}$$

the phase cross-over frequency is

(A) 0.5 rad/sec (B) 0.707 rad/sec

(C) 1.732 rad/sec (D) 2 rad/sec

4. The gain-phase plots of open-loop transfer function of four different system are shown in fig. P6.5.4. The correct sequence of the increasing order of stability of these four system will be

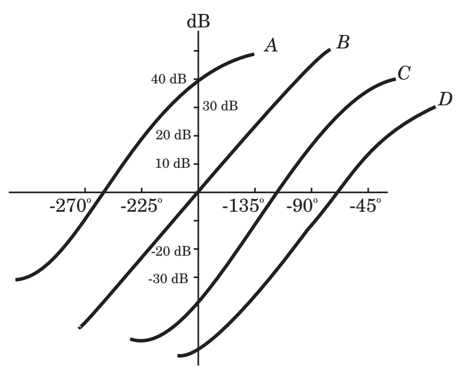


Fig. P6.5.4

(A) D, C, B, A (B) A, B, C, D

(C) B, C, A, D (D) A, D, B, C

5. The open-loop frequency response of a unity feedback system is shown in following table

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
2	8.5	–119°
3	6.4	–128°
4	4.8	–142°
5	2.56	–156°
6	1.4	–164°
8	1.00	–172°
10	0.63	–180°

Fig. P6.5.5

The gain margin and phase margin of the system are

(A) 2 dB, 8° (B) 2 dB, –172°

(C) 4 dB, 8° (D) 4 dB, –172°

Statement for Q.6–7:

Consider the gain-phase plot shown in fig. P6.5.6–7.

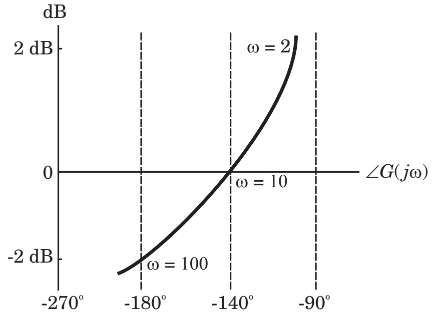


Fig. P6.5.6-7

6. The gain margin and phase margin are

(A) –2 dB, 40° (B) 2 dB, 40°

(C) 2 dB, 140° (D) –2 dB, 140°

7. The gain crossover and phase crossover frequency are respectively

(A) 10 rad/sec, 100 rad/sec

(B) 100 rad/sec, 10 rad/sec

(C) 10 rad/sec, 2 rad/sec

(D) 100 rad/sec, 2 rad/sec

8. The phase margin of a system with the open loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+s)(3+s)} \quad \text{is}$$

(A) 68.3° (B) 90°

(C) 0 (D) ∞

9. Consider a *ufb* system having an open-loop transfer function

$$G(s) = \frac{K}{s(0.2s+1)(0.05s+1)}$$

For $K = 1$, the gain margin is 28 dB. When gain margin is 20 dB, K will be equal to

(A) 2 (B) 4

(C) 5 (D) 2.5

10. The gain margin of the *ufb* system

$$G(s) = \frac{2}{(s+1)(s+2)} \quad \text{is}$$

(A) 1.76 dB (B) 3.5 dB

(C) –3.5 dB (D) –1.76 dB

11. The open-loop transfer function of a system is

$$G(s)H(s) = \frac{K}{s(1+2s)(1+3s)}$$

The phase crossover frequency is

(A) 6 rad/sec (B) 2.46 rad/sec

(C) 0.41 rad/sec (D) 3.23 rad/sec

12. The open-loop transfer function of a *ufb* system is

$$G(s) = \frac{1+s}{s(1+0.5s)}$$

The corner frequencies are

(A) 0 and 2 (B) 0 and 1

(C) 0 and –1 (D) 1 and 2

13. In the Bode-plot of a unity feedback control system, the value of magnitude of $G(j\omega)$ at the phase crossover frequency is $\frac{1}{2}$. The gain margin is

(A) 2 (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) 3

14. In the Bode-plot of a *ufb* control system, the value of phase of $G(j\omega)$ at the gain crossover frequency is –120°. The phase margin of the system is

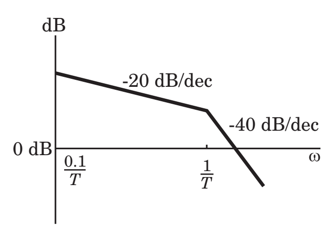
(A) –120° (B) 60°

(C) –60° (D) 120°

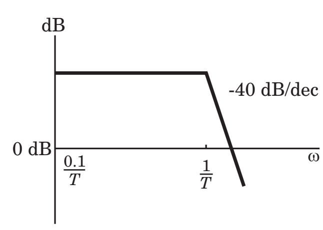
15. The transfer function of a system is given by

$$G(s) = \frac{K}{s(sT+1)} \;;\; K < \frac{1}{T}$$

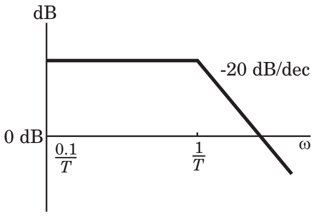
The Bode plot of this function is



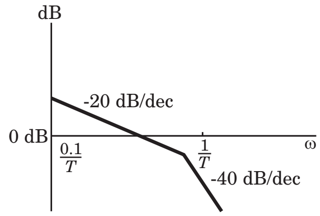
(A)



(B)



(C)



(D)

16. The asymptotic approximation of the log-magnitude versus frequency plot of a certain system is shown in fig. P6.5.16. Its transfer function is

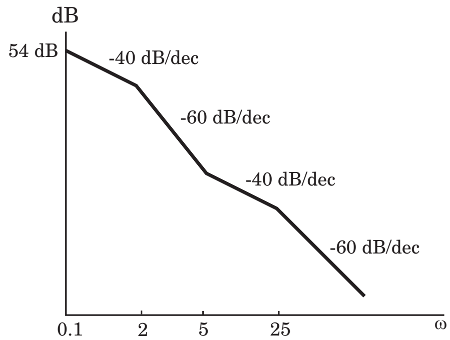


Fig. P6.5.16

(A) $\frac{50(s+5)}{s^2(s+2)(s+25)}$ (B) $\frac{20(s+5)}{s^2(s+2)(s+25)}$

(C) $\frac{10s^2(s+5)}{(s+2)(s+25)}$ (D) $\frac{20(s+5)}{s(s+2)(s+25)}$

17. For the Bode plot shown in fig. P6.5.17 the transfer function is

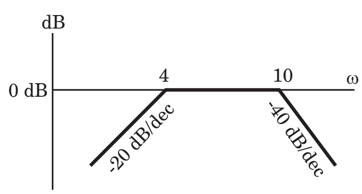


Fig. P6.5.17

(A) $\frac{100s}{(s+4)(s+10)^2}$ (B) $\frac{100(s+4)}{s(s+10)^2}$

(C) $\frac{100}{(s+4)(s+10)}$ (D) $\frac{100}{s^2(s+4)(s+10)}$

18. Bode plot of a stable system is shown in the fig. P6.3.18. The open-loop transfer function of the *ufb* system is

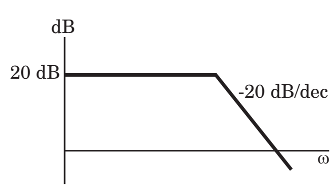


Fig. P6.5.18

(A) $\frac{100}{s+10}$ (B) $\frac{10}{s+10}$

(C) $\frac{1}{s+10}$ (D) None of the above

19. Consider the asymptotic Bode plot of a minimum phase linear system given in fig. P6.5.19. The transfer function is

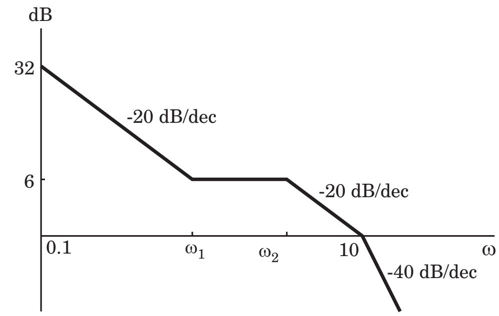


Fig. P6.5.19

(A) $\frac{8s(s+2)}{(s+5)(s+10)}$ (B) $\frac{4(s+5)}{(s+2)(s+10)}$

(C) $\frac{4(s+2)}{s(s+5)(s+10)}$ (D) $\frac{8s(s+5)}{(s+2)(s+10)}$

20. The Bode plot shown in fig. P6.5.20 represent

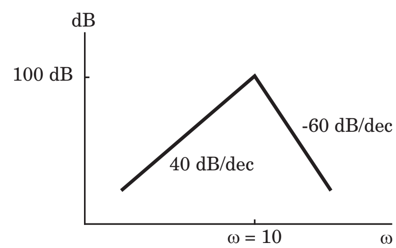


Fig. P6.5.20

(A) $\frac{100s^2}{(1+0.1s)^3}$ (B) $\frac{1000s^2}{(1+0.1s)^3}$

(C) $\frac{100s^2}{(1+0.1s)^5}$ (D) $\frac{1000s^2}{(1+0.1s)^5}$

Statement for Q.21–22:

The Bode plot of the transfer function $K/(1+sT)$ is given in the fig. P6.5.21–22.

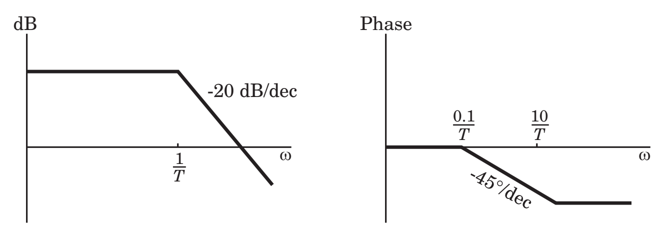


Fig. P6.5.21-22

Statement for Q.29–30:

Consider the Bode plot of a *ufb* system shown in fig. P6.5.29–30.

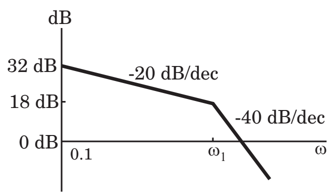


Fig. P6.5.29-30

29. The steady state error corresponding to a ramp input is

(A) 0.25 (B) 0.2

(C) 0 (D) ∞

30. The damping ratio is

(A) 0.063 (B) 0.179

(C) 0.483 (D) 0.639

31. The Nyquist plot of a open-loop transfer function $G(j\omega)H(j\omega)$ of a system encloses the $(-1, j0)$ point. The gain margin of the system is

(A) less than zero (B) greater than zero

(C) zero (D) infinity

32. Consider a *ufb* system

$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)(1+sT_3)}$$

The angle of asymptote, which the Nyquist plot approaches as $\omega \to 0$, is

(A) $-90°$ (B) $90°$

(C) $180°$ (D) $-45°$

33. If the gain margin of a certain feedback system is given as 20 dB, the Nyquist plot will cross the negative real axis at the point

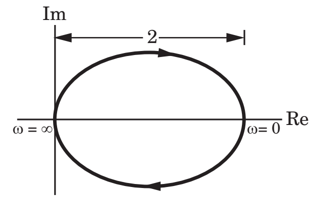
(A) $s = -0.05$ (B) $s = -0.2$

(C) $s = -0.1$ (D) $s = -0.01$

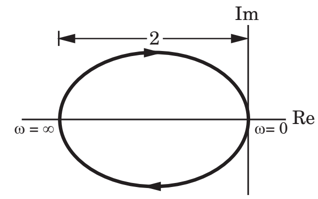
34. The transfer function of an open-loop system is

$$G(s)H(s) = \frac{s+2}{(s+1)(s-1)}$$

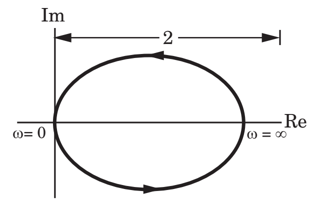
The Nyquist plot will be of the form



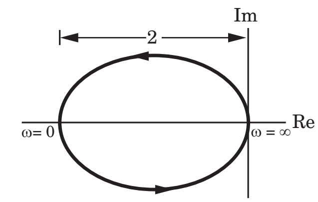
(A)



(B)



(C)

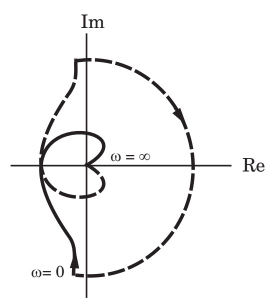


(D)

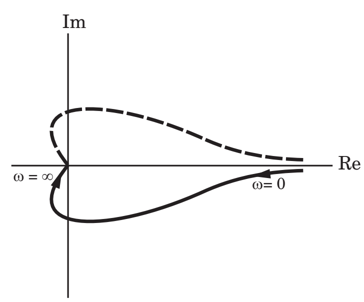
35. Consider a *ufb* system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s^2+2s+2)}$$

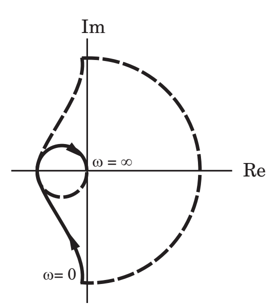
The Nyquist plot for this system is



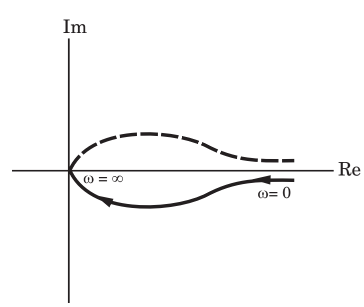
(A)



(B)



(C)

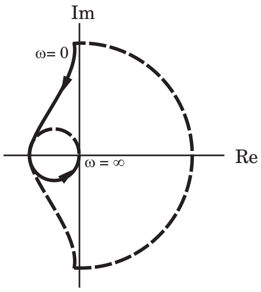


(D)

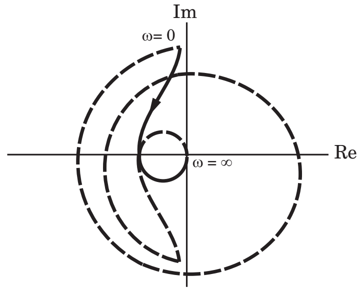
36. The open loop transfer function of a system is

$$G(s)H(s) = \frac{K(1+s)^2}{s^3}$$

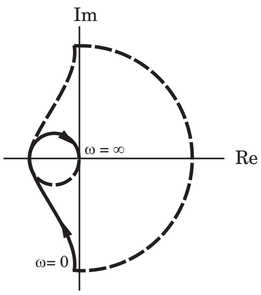
The Nyquist plot for this system is



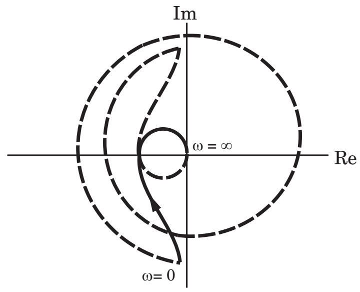
(A)



(B)



(C)

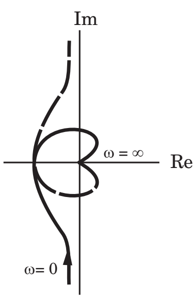


(D)

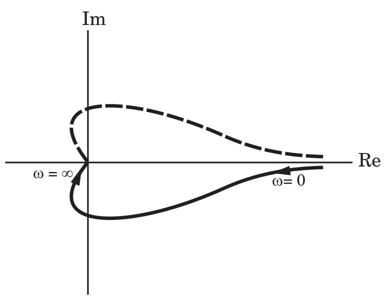
37. For the certain unity feedback system

$$G(s) = \frac{K}{s(s+1)(2s+1)(3s+1)}$$

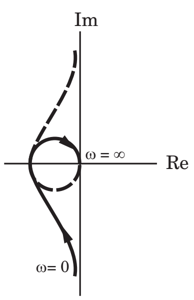
The Nyquist plot is



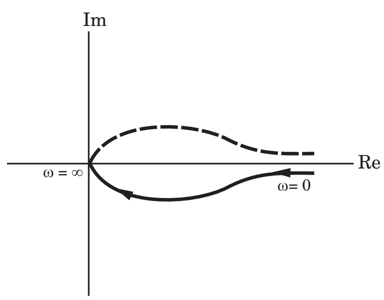
(A)



(B)



(C)



(D)

38. The Nyquist plot of a system is shown in fig. P6.5.38. The open-loop transfer function is

$$G(s)H(s) = \frac{4s+1}{s^2(s+1)(2s+1)}$$

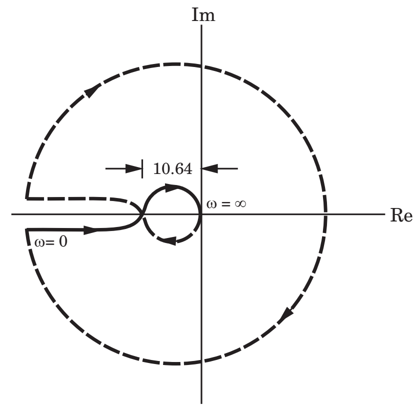


Fig. P6.5.38

The no. of poles of closed loop system in RHP are

(A) 0 (B) 1

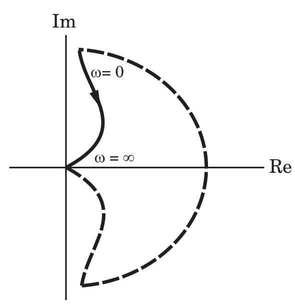
(C) 2 D) 4

Statement for Q.39–40:

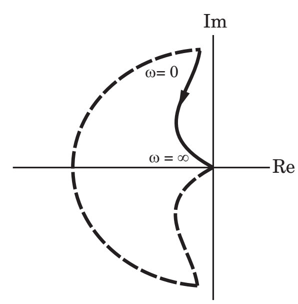
The open-loop transfer function of a feedback control system is

$$G(s)H(s) = \frac{-1}{2s(1-20s)}$$

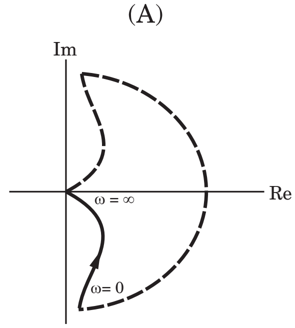
39. The Nyquist plot for this system is



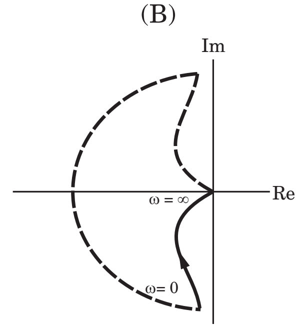
(A)



(B)



(C)



(D)

40. Regarding the system consider the statements

1. Open-loop system is stable
2. Closed-loop system is unstable
3. One closed-loop poles is lying on the RHP

The correct statements are

(A) 1 and 2 (B) 1 and 3

(C) only 2 (D) All

41. The Nyquist plot shown in the fig. P6.5.41 is for

(A) type–0 system (B) type–1 system

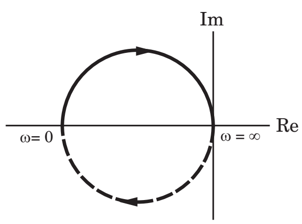
(C) type–2 system (D) type–3 system

Statement for Q.42–43:

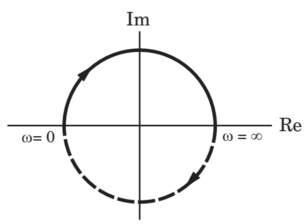
The open-loop transfer function of a feedback system is

$$G(s)H(s) = \frac{K(1+s)}{(1-s)}$$

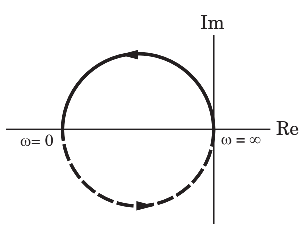
42. The Nyquist plot of this system is



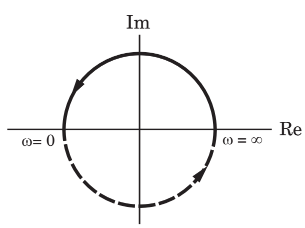
(A)



(B)



(C)



(D)

43. The system is stable for K

(A) $K > 1$ (B) $K < 1$

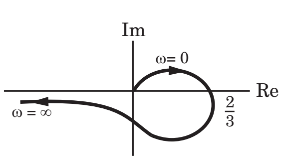
(C) any value of K (D) unstable

Statement for Q.44–46:

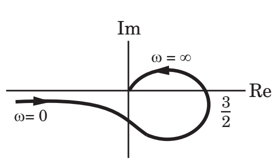
A unity feedback system has open-loop transfer function

$$G(s) = \frac{1}{s(2s+1)(s+1)}$$

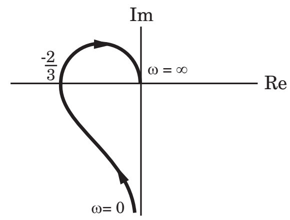
44. The Nyquist plot for the system is



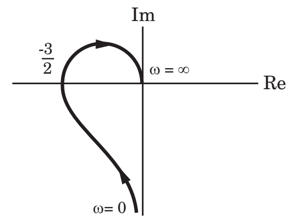
(A)



(B)



(C)



(D)

45. The phase crossover and gain crossover frequencies are

(A) 1.414 rad/sec, 0.57 rad/sec

(B) 1.414 rad/sec, 1.38 rad/sec

(C) 0.707 rad/sec, 0.57 rad/sec

(D) 0.707 rad/sec, 1.38 rad/sec

46. The gain margin and phase margin are

(A) –3.52 dB, –168.5° (B) –3.52 dB, 11.6°

(C) 3.52 dB, –168.5° (D) 3.52 dB, 11.6°

****

SOLUTIONS

1. (A) $T(s) = \dfrac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$T(j\omega) = \dfrac{K\omega_n^2}{-\omega^2 + 2j\xi\omega_n\omega + \omega_n^2}$

$|T(j\omega)|^2 = \dfrac{K^2\omega_n^4}{(\omega_n^2 - \omega^2)^2 + 4\xi^2\omega_n^2\omega^2}$

From the fig. P6.5.1–2, $|T(j0)| = 1$

$|T(j0)|^2 = \dfrac{K^2\omega_n^4}{\omega_n^4} = K^2 = 1 \quad \Rightarrow \quad K = 1$

2. (B) The peak value of $T(j\omega)$ occurs when the denominator of function $|T(j\omega)|^2$ is minimum i.e. when

$\omega_n^2 - \omega^2 = 0 \quad \Rightarrow \quad \omega = \omega_n$

$|T(j\omega_n)|^2 = \dfrac{K^2\omega_n^4}{4\xi^2\omega_n^4} = \dfrac{K^2}{4\xi^2} \quad \Rightarrow \quad |T(j\omega_n)| = \dfrac{K}{2\xi} = 2.5$

$\xi = \dfrac{K}{5} = 0.2$

3. (B) $G(j\omega)H(j\omega) = \dfrac{1}{j\omega(j\omega + 1)(0.5 + j\omega)}$

$\phi = -90° - \tan^{-1} 2\omega - \tan^{-1}\omega$

At phase cross over point $\phi = -180°$

$-\tan^{-1} 2\omega - \tan^{-1}\omega - 90° = -180°$

$\tan^{-1} 2\omega + \tan^{-1}\omega = 90°$

$\dfrac{2\omega + \omega}{1 - (2\omega)(\omega)} = \tan 90° = \infty$

$1 - (2\omega)\omega = 0 \Rightarrow \quad \omega = \dfrac{1}{\sqrt{2}} = 0.707$ rad/sec

4. (B) For a stable system gain at 180° phase must be negative in dB. More magnitude more stability.

5. (C) At 180° gain is 0.63. Hence gain margin is

$= 20 \log \dfrac{1}{0.63} = 4$ dB

At unity gain phase is –172°,

Phase margin $= 180° - 172° = 8°$

6. (A) At $\angle G(j\omega) = 180°$ gain is –2 dB. Hence gain margin is 2 dB. At 0 dB gain phase is –140°. Hence phase margin is $180° - 140° = 40°$.

7. (A) At $\omega = 100$ rad/sec phase is 180°. Phase cross-over frequency $\omega_\pi = 100$ rad/sec.

At $\omega = 10$ rad/sec gain is 0 dB. Gain cross over frequency $\omega = 10$ rad/sec.

8. (D) $|GH(j\omega)| \neq 1$, for any value of ω. Thus phase margin is ∞.

9. (D) For 28 dB gain Nyquist plot intersect the real axis at a,

$20 \log \dfrac{1}{a} = 28 \quad \Rightarrow \quad a = 0.04$

For 20 dB gain Nyquist plot should intersect at b,

$20 \log \dfrac{1}{b} = 20 \quad \Rightarrow \quad b = 0.1.$

This is achieved if the system gain is increased by factor $\dfrac{0.1}{0.04} = 2.5$. Thus $K = 2.5$.

10. (B) Here $K = 2, \quad T_1 = 1, \quad T_2 = \dfrac{1}{2}$

Gain Margin $= \left[\dfrac{KT_1T_2}{T_1 + T_2}\right]^{-1} = \left[\dfrac{(2)(0.5)}{1 + 0.5}\right]^{-1} = 1.5 = 3.5$ dB

11. (C) For phase crossover frequency

$\angle GH(j\omega) = -180°$

$GH(j\omega) = \dfrac{K}{j\omega(1 + 2j\omega)(1 + 3j\omega)}$

$-90° - \tan^{-1} 2\omega_\pi - \tan^{-1} 3\omega_\pi = -180°$

$\tan^{-1} 2\omega_\pi + \tan^{-1} 3\omega_\pi = 90°$

$\dfrac{2\omega_\pi + 3\omega_\pi}{1 - (2\omega_\pi)(3\omega_\pi)} = \tan 90°$

$1 - 6\omega_\pi^2 = 0 \quad \Rightarrow \quad \omega_\pi = 0.41$ rad/s

12. (D) $G(s) = \dfrac{s + 1}{s(1 + 0.5s)} = \dfrac{s + 1}{s\left(\dfrac{s}{2} + 1\right)}$

The Bode plot of this function has break at $\omega = 1$ and $\omega = 2$. These are the corner frequencies.

13. (A) G.M. $= \dfrac{1}{|GH(j\omega_\pi)|} = \dfrac{1}{1/2} = 2$

14. (B) P.M. $= 180° + \angle GH(j\omega_1) = 180° - 120° = 60°$

15. (D) Due to pole at origin initial plot has a slope of –20 dB/decade. At $s = j\omega = \dfrac{1}{T}$. Slope increases to –40 dB/decade. At $\omega = \dfrac{1}{T}$,

$|G(j\omega)| \approx KT < 1$,Gain in dB < 0.

27. (C) Initially slope is –20 dB/decade. Hence there is a pole at origin and system type is 1. For type–1 system position error coefficient is ∞.

$20\log K = 6 \Rightarrow K = 2,$

28. (B) The system is type –0,

$20\log K_p = 40,\ K_p = 100,\ e_{step}(\infty) = \frac{1}{1+K_p} = \frac{1}{101}.$

29. (A) The Bode plot is as shown in fig. S6.5.29

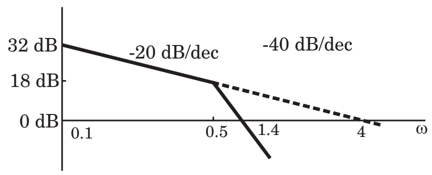


Fig.S6.5.29

$K_v = 4,\ e_{ramp}(\infty) = \frac{1}{K_v} = \frac{1}{4} = 0.25$

30. (B) From fig. S6.5.29 $\xi = \frac{\omega_2}{2\omega_3} = \frac{0.5}{2(1.4)} = 0.179$

31. (A) If Nyquist plot encloses the point $(-1, j0)$, the system is unstable and gain margin is negative.

32.(A) $GH(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_2)}$

$\lim_{\omega\to 0} GH(j\omega) = \lim_{\omega\to 0}\frac{K}{j\omega} = \lim_{\omega\to 0}\frac{K}{\omega}\angle -90^\circ$

Hence, the asymptote of the Nyquist plot tends to an angle of -90° as $\omega \to 0$.

33. (C) $20\log\frac{1}{|GH(j\omega)|} = 20$

$\frac{1}{|GH(j\omega)|} = 10 \Rightarrow |GH(j\omega)| = 0.1$

Since system is stable, it will cross at $s = -0.1$.

34. (B) $GH(s) = \frac{s+2}{(s^2-1)}$

$GH(j\omega) = \frac{j\omega+2}{(-1-\omega^2)}$

At $\omega = 0$, $GH(j\omega) = 2\angle -180^\circ$

At $\omega = \infty$, $GH(j\omega) = 0\angle -270^\circ$

Hence (B) is correct option.

35. (C) $GH(j\omega) = \frac{K}{j\omega(-\omega^2+2j\omega+2)}$

$\angle GH(j\omega) = -\tan^{-1}\frac{2\omega}{2-\omega^2} - 90^\circ$

$|GH(j\omega)| = \frac{K}{\omega\sqrt{(2-\omega^2)^2+4\omega^2}}$

At $\omega = 0$, $GH(j\omega) = \infty\angle -90^\circ$,

At $\omega = \infty$ $GH(j\omega) = 0\angle -270^\circ$,

At $\omega = 1$, $GH(j\omega) = \frac{K}{\sqrt{5}}\angle -153.43^\circ$,

At $\omega = 2$, $GH(j\omega) = \frac{K}{2\sqrt{18}}\angle -206.6^\circ$,

Due to s there will be a infinite semicircle. Hence (C) is correct option.

36. (B) $GH(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$

$|GH(j\omega)| = \frac{K(1+\omega^2)}{\omega^3}$

$\angle GH(j\omega) = -270^\circ + 2\tan^{-1}\omega$

For $\omega = 0$, $GH(j\omega) = \infty\angle -270^\circ$

For $\omega = 1$, $\angle GH(j\omega) = -180^\circ$

For $\omega = \infty$, $GH(j\omega) = 0\angle -90^\circ$

As ω increases from 0 to ∞, phase goes -270° to -90°. Due to s^3 term there will be 3 infinite semicircle.

37. (A) $|GH(j\omega)| = \frac{K}{\sqrt{1+\omega^2}\sqrt{1+4\omega^2}\sqrt{1+9\omega^2}}$,

$\angle GH(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega - \tan^{-1}3\omega$,

For $\omega = 0$, $GH(j\omega) = \infty\angle -90^\circ$,

For $\omega = \infty$, $GH(j\omega) = 0\angle -360^\circ$,

Hence (A) is correct option.

38. (C) The open-loop poles in RHP are $P = 0$. Nyquist path enclosed 2 times the point $(-1+j0)$. Taking clockwise encirclements as negative $N = -2$.

$N = P - Z,\ -2 = 0 - Z,\ Z = 2$ which implies that two poles of closed-loop system are on RHP.

39. (B) $G(s)H(s) = \frac{-1}{2s(1-20s)}$,

$|GH(j\omega)| = \frac{1}{2\omega\sqrt{1+400\omega^2}}$

$\angle GH(j\omega) = 180^\circ - 90^\circ - \tan^{-1}\frac{-20\omega}{1}$,

At $\omega = 0$ $GH(j\omega) = \infty\angle 90^\circ$

At $\omega = \infty$ $GH(j\omega) = 0\angle 180^\circ$

At $\omega = 0.1$ $GH(j\omega) = 2.24\angle 153.43^\circ$

At $\omega = 0.01$ $GH(j\omega) = 49\angle 91.15^\circ$

40. (C) One open-loop pole is lying on the RHP. Thus open-loop system is unstable and $P = 1$. There is one clockwise encirclement. Hence $N = -1$.

$Z = P - N = 1 - (-1) = 2,$

Hence there are 2 closed-loop poles on the RHP and system is unstable.

41. (B) There is one infinite semicircle. Which represent single pole at origin. So system type is1.

42. (D) $|GH(j\omega)| = \dfrac{K\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}} = K$

$\angle GH(j\omega) = \tan^{-1}\omega - \tan^{-1} - \dfrac{-\omega}{1}$

At $\omega = 0$ $GH(j\omega) = K\angle 0°$,

At $\omega = 1$ $GH(j\omega) = K\angle 90°$,

At $\omega = 2$ $GH(j\omega) = K\angle 127°$,

At $\omega = \infty$ $GH(j\omega) = K\angle 180°$,

43. (A) RHP poles of open-loop system $P = 1$, $Z = P - N$.

For closed loop system to be stable,

$Z = 0, \quad 0 = 1 - N \;\Rightarrow\; N = 1$

There must be one anticlockwise rotation of point $(-1 + j0)$. It is possible when $K > 1$.

44. (C) $G(s) = \dfrac{1}{s(2s+1)(s+1)}, \quad H(s) = 1$

$$GH(s) = \frac{1}{s(2s+1)(s+1)}$$

$$GH(j\omega) = \frac{1}{j\omega(2j\omega+1)(j\omega+1)}$$

$$\lim_{\omega\to 0} GH(j\omega) = \lim_{\omega\to 0}\frac{1}{j\omega} = \infty\angle -90°$$

$$\lim_{\omega\to\infty} GH(j\omega) = \lim_{\omega\to\infty}\omega\frac{1}{2(j\omega)^3} = 0\angle -270°$$

The intersection with the real axis can be calculated as $\mathrm{Im}\{GH(j\omega)\} = 0$, The condition gives $\omega(2\omega^2 - 1) = 0$

i.e. $\omega = 0, \ \dfrac{1}{\sqrt{2}}, \quad GH\left(j\dfrac{1}{\sqrt{2}}\right) = \dfrac{-2}{3}$

With the above information the plot in option (C) is correct.

45. (C) The Nyquist plot crosses the negative real axis at $\omega = \dfrac{1}{\sqrt{2}}$ rad/sec. Hence phase crossover frequency is

$\omega_\pi = \dfrac{1}{\sqrt{2}} = 0.707$ rad/sec.

The frequency at which magnitude unity is

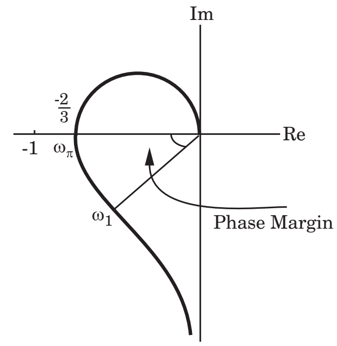


Fig.S6.5.44

$\omega_1^2(1+\omega_1^2)(1+4\omega_1^2) = 1$

$\omega^2 = 0.326, \quad \omega_1 = 0.57$ rad/sec

46. (D) G.M. $= 20\log\dfrac{1}{|GH(j\omega_\pi)|}, \qquad |GH(j\omega_\pi)| = \dfrac{2}{3}$

Gain Margin $= 20\log\dfrac{3}{2} = 3.52$ dB.

$\angle GH(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}2\omega$,

At unit gain $\omega_1 = 0.57$ rad/sec,

Phase at this frequency is

$\angle GH(j\omega_1) = -90° - \tan^{-1}0.57 - \tan^{-1}2(0.57) = -168.42°$

Phase margin $= -168.42° + 180° = 11.6°$

Note that system is stable. So gain margin and phase margin are positive value. Hence only possible option is (D).

****

CHAPTER

6.6

DESIGN OF CONTROL SYSTEMS

1. The term 'reset control' refers to

(A) Integral control (B) Derivative control

(C) Proportional control (D) None of the above

2. If stability error for step input and speed of response be the criteria for design, the suitable controller will be

(A) P controller (B) PI controller

(C) PD controller (D) PID controller

3. The transfer function $\frac{1+0.5s}{1+s}$ represent a

(A) Lag network

(B) Lead network

(C) Lag–lead network

(D) Proportional controller

4. A lag compensation network

(a) increases the gain of the original network without affecting stability.

(b) reduces the steady state error.

(c) reduces the speed of response

(d) permits the increase of gain of phase margin is acceptable.

In the above statements, which are correct

(A) a and b (B) b and c

(C) b, c, and d (D) all

5. Derivative control

(A) has the same effect as output rate control

(B) reduces damping

(C) is predictive in nature

(D) increases the order of the system

6. Consider the List I and List II

List I	List II
P. Derivative control	1. Improved overshoot response
Q. Integral control	2. Less steady state errors
R. Rate feed back control	3. Less stable
S. Proportional control	4. More damping

The correct match is

	P	Q	R	S
(A)	1	2	3	4
(B)	4	3	1	2
(C)	2	3	1	4
(D)	1	2	4	3

7. Consider the List–I (Transfer function) and List–II (Controller)

List I	List II
P.	1. P–controller
Q.	2. PI–controller
R. $\frac{K_1 s + K_2}{K_3 s}$	3. PD–controller
S. $\frac{K_1}{K_2 s}$	4. PID–controller

The correct match is

	P	Q	R	S
(A)	3	4	2	1
(B)	4	3	1	2
(C)	3	2	1	4
(D)	4	1	2	3

8. The transfer function of a compensating network is of form $(1+\alpha Ts)/(1+Ts)$. If this is a phase–Lag network, the value of α should be

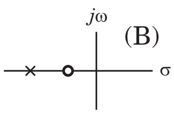
(A) greater than 1

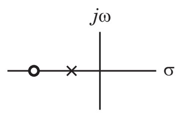
(B) between 0 and 1

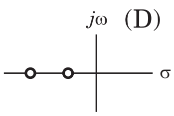
(C) exactly equal to 1

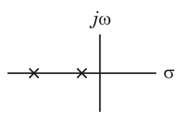
(D) exactly equal to 0

9. The poll–zero configuration of a phase–lead compensator is given by

(A)


(B)


(C)


(D)


10. While designing controller, the advantage of pole–zero cancellation is

(A) The system order is increased

(B) The system order is reduced

(C) The cost of controller becomes low

(D) System's error reduced to optimum levels

11. A proportional controller leads to

(A) infinite error for step input for type 1 system

(B) finite error for step input for type 1 system

(C) zero steady state error for step input for type 1 system

(D) zero steady state error for step input for type 0 system

12. The transfer function of a phase compensator is given by $(1+aTs)/(1+Ts)$ where $a>1$ and $T>0$. The maximum phase shift provided by a such compensator is

(A) $\tan^{-1}\left(\frac{a+1}{a-1}\right)$ (B) $\sin^{-1}\left(\frac{a-1}{a+1}\right)$

(C) $\tan^{-1}\left(\frac{a-1}{a+1}\right)$ (D) $\cos^{-1}\left(\frac{a-1}{a+1}\right)$

13. For an electrically heated temperature controlled liquid heater, the best controller is

(A) Single–position controller

(B) Two–position controller

(C) Floating controller

(D) Proportional–position controller

14. In case of phase–lag compensation used is system, gain crossover frequency, band width and undamped frequency are respectively

(A) decreased, decreased, decreased

(B) increased, increased, increased

(C) increased, increased, decreased

(D) increased, decreased, decreased

15. A process with open–loop model

$$G(s)=\frac{Ke^{-sT_D}}{\tau s+1}$$

is controlled by a PID controller. For this purpose

(A) the derivative mode improves transient performance

(B) the derivative mode improves steady state performance

(C) the integral mode improves transient performance

(D) the integral mode improves steady state performance.

The correct statements are

(A) (a) and (c) (B) (b) and (c)

(C) (a) and (d) (D) (b) and (d)

16. A lead compensating network

(a) improves response time

(b) stabilizes the system with low phase margin

(c) enables moderate increase in gain without affecting stability.

(d) increases resonant frequency

In the above statements, correct are

(A) (a) and (b) (B) (a) and (c)

(C) (a), (c) and (d) (D) All

17. A Lag network for compensation normally consists of

(A) R, L and C elements

(B) R and L elements

(C) R and C elements

(D) R only

18. The pole–zero plot given in fig.P6.6.18 is that of a

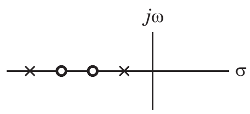


Fig. P6.6.18

(A) PID controller

(B) PD controller

(C) Integrator

(D) Lag–lead compensating network

19. The correct sequence of steps needed to improve system stability is

(A) reduce gain, use negative feedback, insert derivative action

(B) reduce gain, insert derivative action, use negative feedback

(C) insert derivative action, use negative feedback, reduce gain

(D) use negative feedback, reduce gain, insert derivative action.

20. In a derivative error compensation

(A) damping decreases and setting time decreases

(B) damping increases and setting time increases

(C) damping decreases and setting time increases

(D) damping increases and setting time decreases

21. An ON–OFF controller is a

(A) P controller

(B) PID controller

(C) integral controller

(D) non linear controller

****

SOLUTIONS

1. (A)	2. (D)	3. (A)	4. (D)	5. (B)
6. (D)	7. (A)	8. (B)	9. (A)	10. (B)
11. (C)	12. (B)	13. (C)	14. (D)	15. (C)
16. (D)	17. (C)	18. (D)	19. (D)	20. (D)
21. (D)				

CHAPTER

6.7

THE STATE-VARIABLE ANALYSIS

1. Consider the SFG shown in fig. P6.7.1

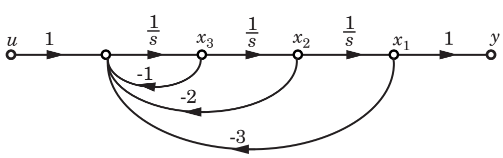


Fig. P6.7.1

For this system dynamic equation is

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$

(D) None of the above

Statement for Q.2–4:

Represent the given system in state-space equation $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}$. Choose the correction option for matrix $\mathbf{A}$.

2.

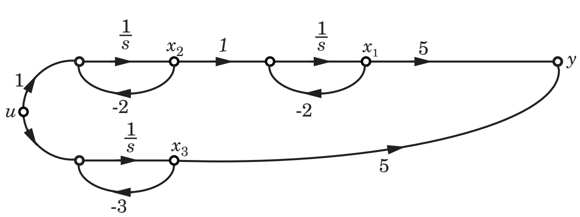


Fig. P6.7.2

(A) $\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 & -2 \\ 0 & -2 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & -1 & 2 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

3.

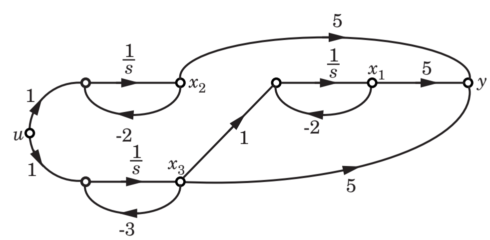


Fig. P6.7.3

(A) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & -2 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

4.

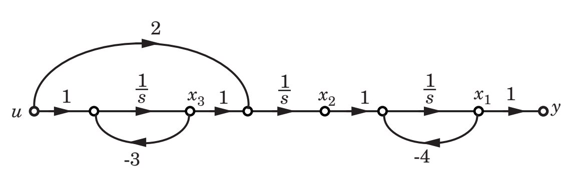


Fig. P6.7.4

(A) $\begin{bmatrix} 0 & 1 & -4 \\ 1 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -1 & 4 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

5. The state equation of a LTI system is represented by

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{u}$$

The Eigen values are

(A) $-1, +1$ (B) $-0.5 \pm j1.323$

(C) $-1, -1$ (D) None of the above

6. The state equation of a LTI system is

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}$$

The state-transition matrix $\Phi(t)$ is

(A) $\begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$ (B) $\begin{bmatrix} -e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$

(C) $\begin{bmatrix} -e^{-3t} & 0 \\ 0 & -e^{-3t} \end{bmatrix}$ (D) $\begin{bmatrix} e^{-3t} & 0 \\ 0 & -e^{-3t} \end{bmatrix}$

7. The state equation of a LTI system is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}$$

The state transition matrix is

(A) $\begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$ (B) $\begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$

(C) $\begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix}$ (D) $\begin{bmatrix} \sin 2t & -\cos 2t \\ \cos 2t & \sin 2t \end{bmatrix}$

Statement for Q.8–9:

The state-space representation of a system is given by $\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t)$, where

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

If $\mathbf{x}(0)$ is the initial state vector, and the component of the input vector $\mathbf{u}(t)$ are all unit step function, then the state transition equation is given by $\dot{\mathbf{x}}(t) = \Phi(t)\mathbf{x}(0) + \theta(t)$, where $\Phi(t)$ is a state transition matrix and $\theta(t)$ is a vector matrix.

8. The $\Phi(t)$ is

(A) $\begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$ (B) $\begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$

(C) $\begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix}$ (D) $\begin{bmatrix} \sin 2t & -\cos 2t \\ \cos 2t & \sin 2t \end{bmatrix}$

9. The $\theta(t)$ is

(A) $\begin{bmatrix} 0.5(1-\sin 2t) \\ 0.5\cos 2t \end{bmatrix}$ (B) $\begin{bmatrix} \sin 2t \\ \cos 2t \end{bmatrix}$

(C) $\begin{bmatrix} 0.5(1-\cos 2t) \\ 0.5\sin 2t \end{bmatrix}$ (D) $\begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix}$

10. From the following matrices, the state-transition matrices can be

(A) $\begin{bmatrix} -e^{-t} & 0 \\ 0 & 1-e^{-t} \end{bmatrix}$ (B) $\begin{bmatrix} 1-e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 1-e^{-t} & e^{-t} \end{bmatrix}$ (D) $\begin{bmatrix} 1-e^{-t} & e^{-t} \\ 0 & e^{-t} \end{bmatrix}$

Statement for Q.11–13:

A system is described by the dynamic equations $\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t)$, $y(t) = \mathbf{C}\cdot\mathbf{x}(t)$ where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{C} = [1\ \ 0\ \ 0]$$

11. The Eigen values of **A** are

(A) $0.325, -1.662 \pm j0.562$

(B) $2.325, 0.338 \pm j0.562$

(C) $-2.325, -0.338 \pm j0.562$

(D) $-0.325, 1.662 \pm j0.562$

12. The transfer-function relation between $X(s)$ and $U(s)$ is

(A) $\dfrac{1}{s^3+3s^2+2s-1}\begin{bmatrix} 1 \\ -s \\ s^2 \end{bmatrix}$ (B) $\dfrac{1}{s^3+3s^2+2s+1}\begin{bmatrix} 1 \\ s \\ s^2 \end{bmatrix}$

(C) $\dfrac{1}{s^3+3s^2+2s+1}\begin{bmatrix} 1 \\ -s \\ s^2 \end{bmatrix}$ (D) None of the above

13. The output transfer function $Y(s)/U(s)$ is

(A) $s(s^3+3s^2+2s+1)^{-1}$ (B) $s(s^3+3s^2+2s-1)^{-1}$

(C) $(s^3+3s^2+2s+1)^{-1}$ (D) None of the above

14. A system is described by the dynamic equation $\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t),\ y(t) = \mathbf{C}\cdot\mathbf{x}(t)$ where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \mathbf{C} = [1 \quad 1]$$

The output transfer function $Y(s)/U(s)$ is

(A) $\frac{(s+1)}{(s+2)^2}$ (B) $\frac{s+1}{s+2}$

(C) $\frac{(s+2)}{(s+1)}$ (D) None of the above

15. The state-space representation of a system is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\mathbf{u}(t),\ y(t) = [1 \quad 1]\mathbf{x}(t)$$

The transfer function of this system is

(A) $(s^2 + 3s + 2)^{-1}$ (B) $(s+2)^{-1}$

(C) $s(s^2 + 3s + 2)^{-1}$ (D) $(s+1)^{-1}$

16. The state-space representation for a system is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}\mathbf{u},\ y = [1 \quad 0 \quad 0]\mathbf{x}$$

The transfer function $Y(s)/U(s)$ is

(A) $\frac{10(2s^2 + 3s + 1)}{s^3 + 3s^2 + 2s + 1}$ (B) $\frac{10(2s^2 + 3s + 1)}{s^3 + 2s^2 + 3s + 1}$

(C) $\frac{10(2s^2 + 3s + 2)}{s^3 + 3s^2 + 2s + 1}$ (D) $\frac{10(2s^2 + 3s + 2)}{s^3 + 2s^2 + 3s + 1}$

Statement for Q.17–18:

Determine the state-space representation for the transfer function given in question. Choose the state variable as follows

$$x_1 = c = y,\ x_2 = \frac{dc}{dt} = \dot{c},\ x_3 = \frac{d^2c}{dt^2} = \ddot{c},\ x_4 = \frac{d^3c}{dt^2} = \dddot{c}$$

17. $\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 24 & 26 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 9 & 26 & 24 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

(D) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & -26 & -24 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

18. $\frac{C(s)}{R(s)} = \frac{100}{s^4 + 20s^3 + 10s^2 + 7s + 100}$

(A) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 100 & 7 & 10 & 20 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}\mathbf{r},$

$y = [1\ 0\ 0\ 0]\mathbf{x}$

(B) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -100 & -7 & -10 & -20 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}\mathbf{r}$

$y = [1\ 0\ 0\ 0]\mathbf{x}$

(C) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 20 & 10 & 7 & 100 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{r}$

$y = [100\ 0\ 0\ 0]\mathbf{x}$

(D) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -20 & -10 & -7 & -100 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{r}$

$y = [100\ 0\ 0\ 0]\mathbf{x}$

19. A state-space representation of a system is given by $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}\mathbf{x},\ y = [1 \quad -1]\mathbf{x}$, and $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

The time response of this system will be

(A) $\sin\sqrt{2}t$ (B) $\frac{3}{\sqrt{2}}\sin\sqrt{2}t$

(C) $-\frac{1}{\sqrt{2}}\sin\sqrt{2}t$ (D) $\sqrt{3}\sin\sqrt{2}t$

20. For the transfer function

$$\frac{Y(s)}{U(s)} = \frac{s+3}{(s+1)(s+2)}$$

The state model is given by $\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\mathbf{u}$, $y = \mathbf{C}\cdot\mathbf{x}$. The $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are

29. $\dot{\mathbf{x}} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\mathbf{r},\ y = [-1\ \ 2\ \ 1]\mathbf{x}$

	$e_{step}(\infty)$	$e_{ramp}(\infty)$
(A)	1.0976	0
(B)	1.0976	∞
(C)	0	1.0976
(D)	∞	1.0976

30. $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{r},\ y = [1\ \ 0\ \ 0]\mathbf{x}$

	$e_{step}(\infty)$	$e_{ramp}(\infty)$
(A)	0	0.714
(B)	∞	0.714
(C)	0	4.86
(D)	∞	4.86

Statement for Q.31–33:

Consider the system shown in fig. P6.7.31-33

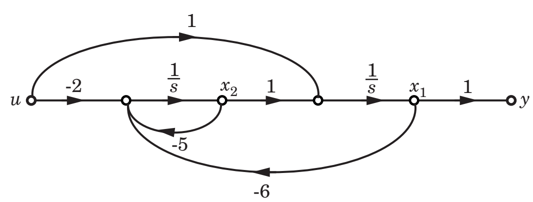


Fig. P6.7.31-33

31. The controllability matrix is

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

32. The observability matrix is

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

33. The system is

(A) Controllable and observable

(B) Controllable only

(C) Observable only

(D) None of the above

Statement for Q.34–36:

Consider the system shown in fig. P6.7.34-36.

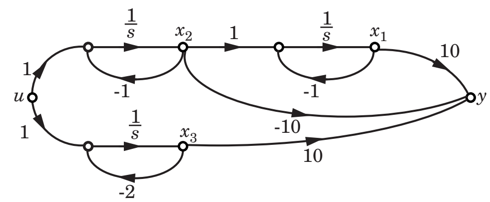


Fig. P6.7.34-36

34. The controllability matrix for this system is

(A) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -20 \\ 10 & -20 & 40 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & 20 \\ 10 & -20 & -40 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 6 & -1 \\ 1 & -4 & -4 \end{bmatrix}$

35. The observability matrix is

(A) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -20 \\ 10 & -10 & 40 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & 20 \\ 10 & -10 & -40 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

36. The system is

(A) Controllable and observable

(B) Controllable only

(C) Observable only

(D) None of the above

Statement for Q.37–38:

A state flow graph is shown in fig. P6.7.37-38.

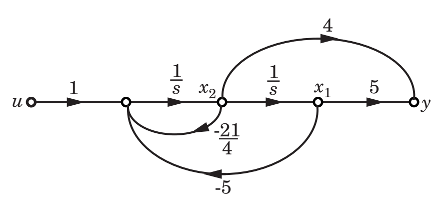


Fig. P6.7.37-38

37. The state and output equation for this system is

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 5 & \frac{21}{4} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u},\ y = [5\ \ 4]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}, \ y = [5 \ \ 4] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 5 & \frac{21}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u}, \ y = [4 \ \ 5] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(D) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u}, \ y = [4 \ \ 5] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

38. The system is

(A) Observable and controllable

(B) Controllable only

(C) Observable only

(D) None of the above

39. Consider the network shown in fig. P6.7.39. The state-space representation for this network is

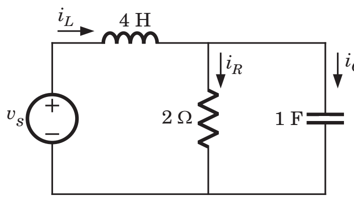


Fig. P6.7.39

(A) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.25 & 1 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 1 \\ 0.25 \end{bmatrix} v_s, \ i_R = [0.5 \ \ 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

(B) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ -0.25 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 1 \end{bmatrix} v_s, \ i_R = [0.5 \ \ 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

(C) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 1 & 0.25 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} v_s, \ i_R = [0.5 \ \ 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

(D) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -1 & 0.25 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} v_s, \ i_R = [0.5 \ \ 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

40. For the network shown in fig. P6.7.40. The output is

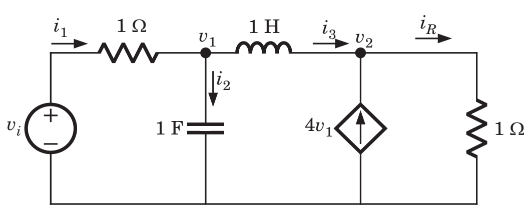


Fig. P6.7.40

$i_R(t)$. The state space representation is

(A) $\begin{bmatrix} \dot{v}_1 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i, \ i_R = [4 \ \ 1] \begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$

(B) $\begin{bmatrix} \dot{v}_1 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i, \ i_R = [4 \ \ 1] \begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$

(C) $\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} v_i, \ i_R = [1 \ \ 4] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

(D) $\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -1 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} v_i, \ i_R = [1 \ \ 4] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

Statement for Q.41–43:

Consider the network shown in fig. P6.7.41-43. This system may be represented in state space representation $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}$

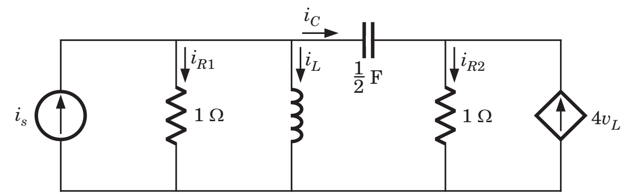


Fig. P6.7.41-43

41. The state variable may be

(A) i_{R1}, i_{R2}

(B) i_L, i_C

(C) v_C, i_L

(D) None of the above

42. If state variable are chosen as in previous question, then the matrix **A** is

(A) $\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 3 \\ 1 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

43. The matrix **B** is

(A) $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(C) $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Statement for Q.44–47:

Consider the network shown in fig. P6.7.44-47

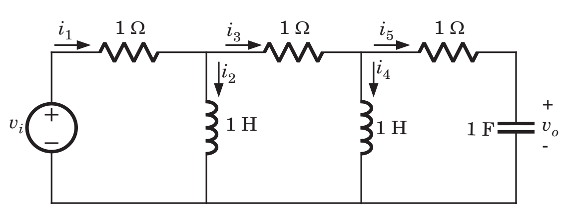


Fig. P6.7.44-47

44. The state variable may be

(A) i_2 , i_4 (B) i_2 , i_4 , v_o

(C) i_1 , i_3 (D) i_1 , i_3 , i_5

45. In state space representation matrix **A** is

(A) $\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

46. The matrix **B** is

(A) $\begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

47. If output is v_o , then matrix **C** is

(A) [–1 0 0] (B) [1 0 0]

(C) [0 0 –1] (D) [0 0 1]

****

SOLUTIONS

1. (B) From the SFG

$$\dot{x}_3 = -3x_1 - 2x_2 - x_3 + u$$

$$x_2 = \frac{x_3}{s} \quad \Rightarrow \quad \dot{x}_2 = x_3$$

$$x_1 = \frac{x_2}{s} \quad \Rightarrow \quad \dot{x}_1 = x_2$$

2. (A) $\dot{x}_1 = -2x_1 + x_3$, $\dot{x}_2 = -2x_2 + u$, $\dot{x}_3 = -3x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$$

3. (C) $\dot{x}_1 = -2x_1 + x_3$, $\dot{x}_2 = -2x_2 + u$, $\dot{x}_3 = -3x_3 + u$

$$y = 5x_1 + 5x_2 + 5x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \mathbf{u}$$

4. (C) $\dot{x}_1 = -4x_1 + x_2$, $\dot{x}_2 = x_3 + 2u$, $\dot{x}_3 = -3x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \mathbf{u}$$

5. (B) $\Delta s = |s\mathbf{I} - \mathbf{A}| = s^2 + s + 2 = 0 \Rightarrow s = -0.5 \pm j1.323$

6. (A) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s+3 & 0 \\ 0 & s+3 \end{bmatrix}$, $|s\mathbf{I} - \mathbf{A}| = \frac{1}{(s+3)^2}$

$$(s\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} \frac{1}{s+3} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\Phi(t) = \mathrm{L}^{-1}\{(s\mathbf{I} - \mathbf{A})\} == \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$$

7. (A) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 \\ 2 & s \end{bmatrix}$, $|s\mathbf{I} - \mathbf{A}| = s^2 + 4$

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^2+4} \begin{bmatrix} s & 2 \\ -2 & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2+4} & \frac{2}{s^2+4} \\ \frac{-2}{s^2+4} & \frac{s}{s^2+4} \end{bmatrix}$$

$$\Phi(t) = \mathrm{L}^{-1}\{(s\mathbf{I} - \mathbf{A})\} = \begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$$

8. (A) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 \\ 2 & s \end{bmatrix}$, $\Delta s = |s\mathbf{I} - \mathbf{A}| = s^2 + 4$

$$(s\mathbf{I}-\mathbf{A})^{-1}=\frac{1}{s^2+4}\begin{bmatrix} s & 2\\ -2 & s\end{bmatrix}=\begin{bmatrix}\frac{s}{s^2+4} & \frac{2}{s^2+4}\\ \frac{-2}{s^2+4} & \frac{s}{s^2+4}\end{bmatrix}$$

$$\Phi(t)=\mathrm{L}^{-1}\{(s\mathbf{I}-\mathbf{A})\}=\begin{bmatrix}\cos 2t & \sin 2t\\ -\sin 2t & \cos 2t\end{bmatrix}$$

9. (C) $\theta(t)=\mathrm{L}^{-1}\{(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}R(s)\}$

$$=\mathrm{L}^{-1}\left\{\frac{1}{s^2+4}\begin{bmatrix} s & 2\\ -2 & s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\frac{1}{s}\right\}=\mathrm{L}^{-1}\left\{\frac{1}{s(s^2+4)}\begin{bmatrix}2\\s\end{bmatrix}\right\}$$

$$=\mathrm{L}^{-1}\left\{\begin{bmatrix}\frac{2}{s(s^2+4)}\\ \frac{1}{(s^2+4)}\end{bmatrix}\right\}=\begin{bmatrix}0.5(1-\cos 2t)\\ 0.5\sin 2t\end{bmatrix}$$

10. (C) (A) is not a state-transition matrix, since $\Phi(0)\neq\mathbf{I}$

(B) is not a state-transition matrix since $\Phi(0)\neq\mathbf{I}$

(C) is a state-transition matrix since $\Phi(0)=\mathbf{I}$ and $[\Phi(t)]^{-1}=\Phi(-t)$

11. (C) $(s\mathbf{I}-\mathbf{A})=\begin{bmatrix} s & -1 & 0\\ 0 & s & -1\\ 1 & 2 & s+3\end{bmatrix}$

$|s\mathbf{I}-\mathbf{A}|=s^3+3s^2+2s+1,$

$\Rightarrow \quad s=-2.325,\ -0.338\pm j0.562$

12. (B) $\frac{X(s)}{U(s)}=(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$

$$=\frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} s^3+3s+2 & s+3 & 1\\ -1 & s(s+3) & s\\ -s & -2s-1 & s^2\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$=\frac{1}{s^3+3s^2+2s+1}\begin{bmatrix}1\\s\\s^2\end{bmatrix}$$

13. (C) $\frac{Y(s)}{U(s)}=\frac{CX(s)}{U(s)}$

$$=[1\ \ 0\ \ 0]\begin{bmatrix}\frac{1}{s^3+3s^2+2s+1}\\ \frac{s}{s^3+3s^2+2s+1}\\ \frac{s^2}{s^3+3s^2+2s+1}\end{bmatrix}=\frac{1}{s^3+3s^2+2s+1}$$

14. (D) $\frac{Y(s)}{U(s)}=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$

$$\mathbf{B}=[1\ \ 1]\frac{1}{\Delta s}\begin{bmatrix} s+1 & 1\\ 0 & s+1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\frac{s+2}{(s+1)^2}$$

15. (D) $T(s)=\begin{bmatrix}1\\1\end{bmatrix}(s\mathbf{I}-\mathbf{A})^{-1}\begin{bmatrix}1\\0\end{bmatrix}$

$$(s\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix}\frac{1}{s+1} & 0\\ 0 & \frac{1}{s+2}\end{bmatrix}$$

$$T(s)=\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{s+1} & 0\\ 0 & \frac{1}{s+2}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{s+1}$$

16. (C) $\dot{\mathbf{x}}=\mathbf{A}\cdot\mathbf{x}+\mathbf{B}\cdot\mathbf{u},\ y=\mathbf{C}\cdot\mathbf{x}+\mathbf{Du}$

$$\mathbf{A}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -2 & -3\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}10\\0\\0\end{bmatrix},\ \mathbf{C}=[1\ \ 0\ \ 0],\ \mathbf{D}=0$$

$$T(s)=\frac{Y(s)}{U(s)}=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}$$

$$(s\mathbf{I}-\mathbf{A})^{-1}=\frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} s^3+3s+2 & s+3 & 1\\ -1 & s(s+3) & s\\ -s & -2s-1 & s^2\end{bmatrix}$$

Substituting the all values,

$$T(s)=\frac{10(2s^2+3s+2)}{s^3+3s^2+2s+1}$$

17. (A) $\frac{C(s)}{R(s)}=\frac{b_0}{s^3+a_2s^2+a_1s+a_0}=\frac{24}{(s^3+9s^2+26s+24)}$

$(s^3+a_2s^2+a_1s+a_0)C(s)=b_0R(s)$

Taking the inverse Laplace transform assuming zero initial conditions

$\dddot{c}+a_2\ddot{c}+a_1\dot{c}+a_0c=b_0r$

$x_1=c=y,\ x_2=\dot{c}\ ,\ x_3=\ddot{c}$

$\dot{x}_1=\dot{c}=x_2\ ,\ \dot{x}_2=\ddot{c}=x_3$

$\dot{x}_3=\dddot{c}=b_0r-a_2\ddot{c}-a_1\dot{c}-a_0c$

$=-a_0x_1-a_1x_2-a_2x_3+b_0r,$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a_0 & -a_1 & -a_1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\b_0\end{bmatrix}\mathbf{r}$$

$a_0=24\ ,\ a_1=26,\ a_2=9,\ b_0=24$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\24\end{bmatrix}\mathbf{r}$$

$$y=[1\ \ 0\ \ 0]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

18. (B) Fourth order hence four state variable

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix}\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_0 \end{bmatrix}\mathbf{r},\ y = [1\ 0\ 0\ 0]\mathbf{x}$$

$a_0 = 100,\quad a_1 = 7,\quad a_2 = 10,\quad a_3 = 20,\quad b_0 = 100$

19. (B) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}, (s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^2+2}\begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}$

$$\Phi(t) = L^{-1}\{(s\mathbf{I}-\mathbf{A})\} = \begin{bmatrix} \cos\sqrt{2}t & \frac{\sin\sqrt{2}t}{\sqrt{2}} \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$$x(t) = \Phi(t)x(0) = \begin{bmatrix} \cos\sqrt{2}t + \frac{\sin\sqrt{2}t}{\sqrt{2}} \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$y = x_1 - x_2 = \frac{3}{\sqrt{2}}\sin\sqrt{2}t$$

20. (C) Find the transfer function of option

For (A) $,\frac{Y(s)}{U(s)} = \frac{1}{s-2},$

For (B) $,\frac{Y(s)}{U(s)} = \frac{1}{s-2}$

For (C), $\frac{Y(s)}{U(s)} = [0\ 1]\frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 0 \\ 2 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= [0\ 1]\frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 \\ s+3 \end{bmatrix} = \frac{s+3}{(s+1)(s+3)}$

So (C) is correct option.

21. (C) $\mathbf{A} = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix},$

$|s\mathbf{I} - \mathbf{A}| = s^2 + 7s + 7 \quad \Rightarrow \quad s = -5.79,\ -1.21$

22. (B) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 & -3 \\ 0 & s-6 & -5 \\ -1 & -4 & s-2 \end{bmatrix}$

$|s\mathbf{I} - \mathbf{A}| = s^3 - 8s^2 - 11s + 8 \quad \Rightarrow \quad s = 9.11,\ 0.53,\ -1.64$

23. (D) $X(s) = (s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(0) + \mathbf{B}\cdot\mathbf{u})$

$$= \begin{bmatrix} s-1 & -2 \\ 3 & s+1 \end{bmatrix}^{-1}\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{3}{s^2+9}\right)$$

$$\frac{1}{(s^2+5)(s^2+9)}\begin{bmatrix} 2s^3 + 4s^2 + 21s + 45 \\ s^3 - 7s^2 + 12s - 7 \end{bmatrix}$$

$$Y(s) = [1\ \ 2]X(s) = \frac{4s^3 - 10s^2 + 45s - 105}{(s^2+5)(s^2+9)}$$

24. (B) $X(s) = (s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(0) + \mathbf{B}\cdot\mathbf{u})$

$$= \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+1 & 0 \\ -1 & s+2 \end{bmatrix}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{1}{s}\right)$$

$$= \begin{bmatrix} \frac{(s+1)}{s(s+2)} \\ \frac{1}{s(s+1)(s+2)} \end{bmatrix}$$

$$Y(s) = [0\ \ 1]X(s) = \frac{1}{s(s+1)(s+2)}$$

$$\Rightarrow \quad y(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

25. (B) $X(s) = (s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(0) + \mathbf{B}\cdot\mathbf{u})$

$$= \begin{bmatrix} s+2 & -1 & 0 \\ 0 & s & -1 \\ 0 & 0 & s+1 \end{bmatrix}^{-1}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\frac{1}{s}\right) = \begin{bmatrix} \frac{1}{s(s+2)} \\ \frac{1}{s^2(s+2)} \\ \frac{1}{s^2(s+1)(s+2)} \end{bmatrix}$$

$$Y(s) = [1\ 0\ 0],\qquad X(s) = \frac{1}{s(s+2)}$$

$$y(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

26. (D) For a unit step input $e_{step}(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B}$

$$\mathbf{A} = \begin{bmatrix} -5 & 1 & 0 \\ 0 & -2 & 1 \\ 20 & -10 & 1 \end{bmatrix},\qquad \mathbf{A}^{-1} = \begin{bmatrix} -0.4 & 0.05 & -0.05 \\ -1 & -0.25 & -0.25 \\ -2 & 1.5 & -0.5 \end{bmatrix}$$

$$e_{step}(\infty) = 1 + [-1\ 1\ 0]\begin{bmatrix} -0.4 & 0.05 & -0.05 \\ -1 & 0.25 & -0.25 \\ -2 & 1.5 & -0.5 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$= 1 - 0.2 = 0.8$

27. (A) $e_{step}(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B},\ \mathbf{A}^{-1} = \begin{bmatrix} -2 & -\frac{1}{3} \\ 1 & 0 \end{bmatrix}$

$$e_{step}(\infty) = 1 + [1\ \ 1]\begin{bmatrix} -2 & -\frac{1}{3} \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 - \frac{1}{3} = \frac{2}{3}$$

28. (C) $e_{ramp}(\infty) = \lim_{t\to\infty}[(1 + \mathbf{CA}^{-1}\mathbf{B})t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}]$

$$1 + \mathbf{CA}^{-1}\mathbf{B} = \frac{2}{3},\qquad e_{ramp}(\infty) = \lim_{t\to\infty}\left[\frac{2}{3}t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}\right] = \infty$$

29. (B) $e_{step}(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B}$

$$\mathbf{A} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ \mathbf{C} = [-1\ \ 2\ \ 1]$$

$$\mathbf{A}^{-1} = \begin{bmatrix} -0.305 & 0.134 & 0.122 \\ 0.091 & -0.140 & -0.037 \\ -0.079 & -0.055 & -0.232 \end{bmatrix}$$

$$e_{step}(\infty) = 1 + 0.0976 = 1.0976$$

$$e_{ramp}(\infty) = \lim_{t\to\infty} [(1 + \mathbf{CA}^{-1}\mathbf{B})t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}] = \infty$$

30. (B) $\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1.286 & 0.143 & -0.714 \end{bmatrix}$

$$e_{step}(\infty) = 1 + [1 \;\; 0 \;\; 0]\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1.286 & 0.143 & -0.714 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$(\mathbf{A}^{-1})^2 = \begin{bmatrix} -1.286 & -0.143 & 0.714 \\ 0 & 0 & -1 \\ -0.776 & -0.102 & -0.776 \end{bmatrix}$$

$\mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B} = 0.714$, $e_{ramp}(\infty) = 0.714$

31. (B) $\dot{x}_1 = x_2 + u$, $\dot{x}_2 = -5x_2 - 6x_1 - 2u$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}u, \qquad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\mathbf{C}_M = [\mathbf{B} \;\; \mathbf{AB}] = \left[\begin{bmatrix} 1 \\ -2 \end{bmatrix} \;\; \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right] = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

32. (A) $y = x_1$, $y = [1 \;\; 0]\mathbf{x}$,

$$\mathbf{C} = [1 \;\; 0], \; \mathbf{CA} = [1 \;\; 0], \; \mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

33. (C) det $\mathbf{C}_M = 0$. Hence system is not controllable. det $\mathbf{O}_M = 1$. Hence system is observable.

34. (B) $\dot{x}_1 = -x_1 + x_2$, $\dot{x}_2 = -x_2 + u$, $\dot{x}_3 = -2x_3 + u$

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}u, \; \mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$\mathbf{A}^2\mathbf{B} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$$

$$\mathbf{C}_m = [\mathbf{B} \;\; \mathbf{AB} \;\; \mathbf{A}^2\mathbf{B}] = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$$

35. (A) $y = 10x_1 - 10x_2 + 10x_3$, $y = [10 \;\; -10 \;\; 10]\underline{\mathbf{x}}$

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \; \mathbf{C} = [10 \;\; -10 \;\; 10],$$

$$\mathbf{CA} = [10 \;\; -10 \;\; 10]\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = [10 \;\; 0 \;\; -20]$$

$$\mathbf{CA}^2 = [10 \;\; 0 \;\; -20]\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = [10 \;\; -10 \;\; 40]$$

$$\mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} = \begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -20 \\ 10 & -10 & 40 \end{bmatrix}$$

36. (A) $\det \mathbf{C}_m = \det\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix} = -1,$

Since the determinant is not zero, the 3×3 matrix is nonsingular and system is controllable

$$\det \mathbf{O}_M = \det\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -10 \\ 10 & -20 & 40 \end{bmatrix} = -3000$$

The rank of $\mathbf{O}_M$ is 3. Hence system is observable.

37. (B) $\dot{x}_2 = -5x_1 - \frac{21}{4}x_2 + u$, $\dot{x}_1 = x_2$, $y = 5x_1 + 4x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}, \; y = [5 \;\; 4]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

38. (B) $\mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -20 & 1 \end{bmatrix}$

det $\mathbf{O}_M = 0$. Thus system is not observable

$$\mathbf{C}_M = [\mathbf{B} \;\; \mathbf{AB}] = \begin{bmatrix} 0 & 1 \\ 1 & -\frac{21}{4} \end{bmatrix}$$

det $\mathbf{C}_M = -1$. Thus system is controllable.

39. (B) $\frac{dv_c}{dt} = i_c$, $\frac{di_L}{dt} = \frac{v_L}{4} = 0.25v_L$

v_C and i_L are state variable.

$$i_L = i_C + i_R \;, \; i_C = i_L - i_R = i_L - \frac{v_C}{2} \;, \; v_L = v_s - v_C$$

Hence equations are $\frac{dv_L}{dt} = i_L - \frac{v_C}{2} = -0.5v_C + i_L$

$$\frac{di_L}{dt} = 0.25(v_s - v_C) = -0.25v_C + 0.25v_s$$

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ -0.25 & 0 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 1 \end{bmatrix}v_s,$$

$$i_R = \frac{v_C}{2} = 0.5v_C, \qquad i_R = [0.5 \quad 0]\begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

40. (B) $\frac{dv_1}{dt} = i_2$, $\frac{di_3}{dt} = v_L$

Hence v_1 and i_3 are state variable.

$i_2 = i_1 - i_3 = (v_i - v_1) - i_3$, $i_2 = -v_1 - i_3 + v_i$

$v_L = v_1 - v_2 = v_1 - i_R$, $= v_1 - (i_3 + 4v_1) = -3v_1 - i_3$

$\frac{dv_1}{dt} = -v_1 - i_3 + v_i$, $\frac{di_3}{dt} = -3v_1 - i_3$, $\qquad y = i_R = 4v_1 + i_3$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i, \; i_R = [4 \quad 1]\begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$$

41. (C) Energy storage elements are capacitor and inductor. v_C and i_L are available in differential form and linearly independent. Hence v_C and i_L are suitable for state-variable.

42. (B) $\frac{1}{2}\frac{dv_C}{dt} = i_C \quad \Rightarrow \quad \frac{dv_C}{dt} = 2i_C$

$\frac{1}{2}\frac{di_L}{dt} = v_L \quad \Rightarrow \quad \frac{di_L}{dt} = 2v_L$

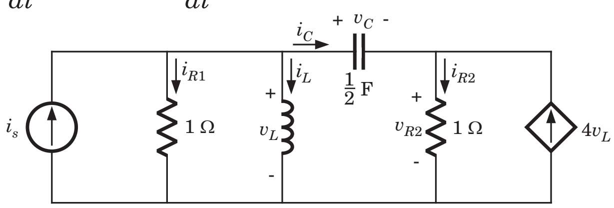


Fig. S6.7.42

$v_L = v_C + v_{R2} = v_C + i_{R2}$, $i_C + 4v_L = i_{R2}$

$v_L = v_C + i_C + 4v_L$, $-3v_L = v_C + i_C$ ...(i)

$i_C = i_s - i_{R1} - i_L$, $i_C = i_s - \frac{v_L}{1} - i_L$ ...(ii)

Solving equation (i) and (ii)

$-3(i_s - i_L - i_C) = v_C + i_C$, $2i_C = v_C - 3i_L + 3i_s$

$-3v_L = v_C + i_s - v_L - i_L$, $2v_L = -v_C + i_L - i_s$

$\frac{dv_C}{dt} = v_C - 3i_L + 3i_s$, $\frac{di_L}{dt} = -v_C + i_L - i_s$

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} i_s$$

43. (A) $\mathbf{B} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

44. (B) There are three energy storage elements, hence 3 variable. i_2 , i_4 and v_o are available in differentiated form hence these are state variable.

45. (A) $\frac{di_2}{dt} = v_2$, $\frac{di_4}{dt} = v_4$, $\frac{dv_o}{dt} = i_5$

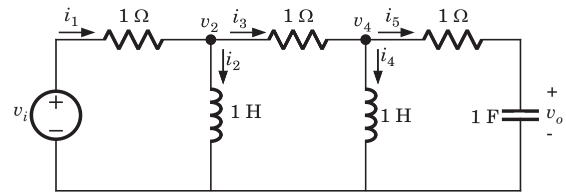


Fig. S6.7.45

Now obtain v_2 , v_4 and i_5 in terms of the state variable

$-v_i + i_1 + i_3 + i_5 + v_o = 0$

But $i_3 = i_1 - i_2$ and $i_5 = i_3 - i_4$

$-v_i + i_1 + (i_1 - i_2) + (i_3 - i_4) + v_o = 0$

$i_1 = \frac{2}{3}i_2 + \frac{1}{3}i_4 - \frac{1}{3}v_o + \frac{1}{3}v_i$

$v_2 = v_i - i_1 = -\frac{2}{3}i_2 - \frac{1}{3}i_4 + \frac{1}{3}v_o + \frac{2}{3}v_i$

$i_3 = i_1 - i_2 = -\frac{1}{3}i_2 + \frac{1}{3}i_4 - \frac{1}{3}v_o + \frac{1}{3}v_i$

$i_5 = i_3 - i_4 = -\frac{1}{3}i_2 - \frac{2}{3}i_4 - \frac{1}{3}v_o + \frac{1}{3}v_i$

$v_4 = i_5 + v_o = -\frac{1}{3}i_2 - \frac{2}{3}i_4 + \frac{2}{3}v_o + \frac{1}{3}v_i$

$$\begin{bmatrix} \dot{i}_2 \\ \dot{i}_4 \\ \dot{v}_o \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} v_i$$

$$\mathbf{A} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix},$$

46. (B) $\mathbf{B} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

47. (D) v_o is state variable

$$y = v_o , \qquad y = [0 \quad 0 \quad 1] = \begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix}$$

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10. If machine is not properly adjusted, the product resistance change to the case where $a_x = 1050\,\Omega$. Now the rejected fraction is

(A) 5046% (B) 10.57%
(C) 2.18% (D) 6.43%

11. Cannon shell impact position, as measured along the line of fire from the target point is described by a gaussian random variable X. It is found that 15.15% of shell falls 11.2 m or farther from the target in a direction toward the cannon, while 5.05% fall farther from the 95.6 m beyond the target. The value of a_x and σ_x for X is (Given that $F(1.03) = 0.8485$ and $F(1.64) = 0.9495$)

(A) $T + 40$ m and 50 m (B) $T + 40$ m and 30 m
(C) $T + 10$ m and 50 m (D) $T + 30$ m and 40 m

12. A gaussian random voltage X for which $a_X = 0$ and $\sigma_X = 4.2$ V appears across a $100\,\Omega$ resistor with a power rating of 0.25 W. The probability, that the voltage will cause an instantaneous power that exceeds the resistor's rating, is

(A) $2Q\left(\frac{5}{4.2}\right)$ (B) $Q\left(\frac{5}{4.2}\right)$
(C) $1 + Q\left(\frac{5}{4.2}\right)$ (D) $1 - Q\left(\frac{5}{4.2}\right)$

Statement for Question 13 -14 :

Assume that the time of arrival of bird at Bharatpur sanctuary on a migratory route, as measured in days from the first year (January 1 is the first day), is approximated as a gaussian random variable X with $a_X = 200$ and $\sigma_x = 20$ days. Given that : $F(0.5) = 0.6915$, $F(1.0) = 0.8413.$, $F(1.5) = 0.8531$, $F(1.55) = 0.9394$ and $F(2.0) = 0.9773$.

13. What is the probability that birds arrive after 160 days but on or before the 210^{th} day ?

(A) 0.6687 (B) 0.8413
(C) 0.8531 (D) 0.9773

14. What is the probability that bird will arrive after 231^{st} day ?

(A) 0.0432 (B) 0.1123
(C) 0.0606 (D) 0.0732

Statement for Question 15-16:

The life time of a system expressed in weeks is a Rayleigh random variable X for which

$$f_X(x) = \begin{cases} \frac{x}{200} e^{-\frac{x^2}{400}} & 0 \le x \\ 0 & x < 0 \end{cases}$$

15. The probability that the system will not last a full week is

(A) 0.01% (B) 0.25%
(C) 0.40% (D) 0.60%

16. The probability that the system lifetime will exceed in year is

(A) 0.01% (B) 0.05%
(C) 0.12% (D) 0.22%

17. The cauchy random variable has the following probability density function

$$f_X(x) = \frac{b/\pi}{b^2 + (x-a)^2}$$

For real numbers $0 < b$ and $-\infty < a < \infty$. The distribution function of X is

(A) $\frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right)$

(B) $\frac{1}{\pi}\cot^{-1}\left(\frac{x-a}{b}\right)$

(C) $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right)$

(D) $\frac{1}{2} + \frac{1}{\pi}\cot^{-1}\left(\frac{x-a}{b}\right)$

Statement for Question 18 - 19

The number of cars arriving at ICICI bank drive-in window during 10-min period is Poisson random variable X with $b = 2$.

18. The probability that more than 3 cars will arrive during any 10 min period is

(A) 0.249 (B) 0.143
(C) 0.346 (D) 0.543

19. The probability that no car will arrive is

(A) 0.516 (B) 0.459
(C) 0.246 (D) 0.135

20. The power reflected from an aircraft of complicated shape that is received by a radar can be described by an exponential random variable W. The density of W is

$$f_W(w) = \begin{cases} \frac{1}{W_0} e^{-w/W_0} & w > 0 \\ 0 & w < 0 \end{cases}$$

where W_0 is the average amount of received power. The probability that the received power is larger than the power received on the average is

(A) e^{-2} (B) e^{-1}

(C) $1 - e^{-1}$ (D) $1 - e^{-2}$

Statement for Question 21-23:

Delhi averages three murder per week and their occurrences follow a poission distribution.

21. The probability that there will be five or more murder in a given week is

(A) 0.1847 (B) 0.2461

(C) 0.3927 (D) 0.4167

22. On the average, how many weeks a year can Delhi expect to have no murders ?

(A) 1.4 (B) 1.9

(C) 2.6 (D) 3.4

23. How many weeds per year (average) can the Delhi expect the number of murders per week to equal or exceed the average number per week ?

(A) 15 (B) 20

(C) 25 (D) 30

24. A discrete random variable X has possible values $x_i = i^2$, $i = 1, 2, 3, 4$ which occur with probabilities 0.4, 0.25, 0.15, 0.1,. The mean value $\overline{X} = E[X]$ of X is

(A) 6.85 (B) 4.35
(C) 3.96 (D) 1.42

25. The random variable X is defined by the density

$$f_X(x) = \frac{1}{2} u(x) e^{-\frac{x}{2}}$$

The expected value of $g(X) = X^3$ is

(A) 48 (B) 192

(C) 36 (D) 72

26. The mean of random variable X is

(A) 1/4 (B) 1/6

(C) 1/3 (D) 1/5

27. The variance of random variable X is

(A) 1/10 (B) 3/80

(C) 5/16 (D) 3/16

28. A Random variable X is uniformly distributed on the interval (–5, 15). Another random variable $Y = e^{-\frac{X}{5}}$ is formed. The value of $E[Y]$ is

(A) 2 (B) 0.667

(C) 1.387 (D) 2.967

29. A random variable X has $\overline{X} = -3$, $\overline{x^2} = 11$ and $\sigma_X^2 = 2$ For a new random variable $Y = 2x - 3$, the $\overline{Y}$, $\overline{Y}^2$ and σ_Y^2 are

(A) 0, 81, 8 (B) – 6, 8, 89

(C) – 9, 89, 8 (D) None of the above

Statement for Question 31-32 :

A joint sample space for two random variable X and Y has four elements (1, 1), (2, 2), (3, 3) and (4, 4). Probabilities of these elements are 0.1, 0.35, 0.05 and 0.5 respectively.

30. The probability of the event$\{X \le 2.5, Y \le 6\}$ is

(A) 0.45 (B) 0.50

(C) 0.55 (D) 0.60

31. The probability of the event $\{X \le 3\}$ is

(A) 0.45 (B) 0.50

(C) 0.55 (D) 0.60

Statement for Question 32-34 :

Random variable X and Y have the joint distribution

$$F_{X,Y}(x, y) = \begin{cases} \frac{5}{4}\left(\frac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right) u(y), & 0 \le x \le 4 \\ 0 & x < 0 \text{ or } y < 0 \\ 1 + \frac{1}{4} e^{-5y^2} - \frac{5}{4} e^{-y^2}, & 4 \le x \text{ and any } y \ge 0 \end{cases}$$

32. The marginal distribution function $F_X(x)$ is

(A) $\begin{cases} 0, & x, > 0 \\ \dfrac{5x}{4(x+1)}, & -4 < x \le 4 \\ 1, & x \le -4 \end{cases}$ (B) $\begin{cases} 0, & x < 0 \\ \dfrac{5x}{4(x+1)}, & 0 \le x < 4 \\ 1, & x \ge 4 \end{cases}$

(C) $\begin{cases} 1, & x > 0 \\ \dfrac{5x}{4(x+1)}, & -4 < x \le 0 \\ 0, & x \le -4 \end{cases}$ (D) $\begin{cases} 1, & x < 0 \\ \dfrac{5x}{4(x+1)}, & 0 \le x < 4 \\ 0, & x \ge 4 \end{cases}$

33. The marginal distribution function $F_Y(y)$ is

(A) $\begin{cases} -\dfrac{5}{4}e^{-y^2}, & y > 0 \\ 1 + \dfrac{4}{4}e^{-5y^2}, & y \le 0 \end{cases}$

(B) $\begin{cases} 0, & y > 0 \\ 1 + \dfrac{1}{4}e^{-5y^2} - \dfrac{5}{4}e^{-y^2}, & y \le 0 \end{cases}$

(C) $\begin{cases} \dfrac{-5}{4}e^{-y^2}, & y < 0 \\ 1 + \dfrac{1}{4}e^{-5y^2} - \dfrac{5}{4}e^{-y^2}, & y \ge 0 \end{cases}$

(D) $\begin{cases} 0, & y < 0 \\ 1 + \dfrac{1}{4}e^{-5y^2} - \dfrac{5}{4}e^{-y^2}, & y \ge 0 \end{cases}$

34. The probability $P\{3 < X \le 5,\ 1 < y \le 2\}$ is

(A) 0.001 (B) 0.002

(C) 0.003 (D) 0.004

Statement for Question 35-39 :

Two random variable X and Y have a joint density

$$F_{X,Y}(x, y) = \frac{10}{4}[u(x) - u(x-4)]u(y)y^3e^{-(x+1)y^2}$$

35. The marginal density $f_X(x)$ is

(A) $5\dfrac{u(x) - u(x-4)}{(x+1)^2}$ (B) $5\dfrac{u(x) - u(x-4)}{(x+1)}$

(C) $\dfrac{5}{4}\dfrac{u(x) - u(x-4)}{(x+1)^2}$ (D) $\dfrac{5}{4}\dfrac{u(x) - u(x-4)}{(x+1)}$

36. The marginal density $f_Y(y)$ is

(A) $\frac{5}{4}y^2[e^{-y^2} - e^{-5y^2}]u(y)$

(B) $\frac{5}{2}y^2[e^{-y^2} - e^{-5y^2}]u(y)$

(C) $\frac{5}{4}y[e^{-y^2} - e^{-5y^2}]u(y)$

(D) $\frac{5}{2}y[e^{-y^2} - e^{-5y^2}]u(y)$

37. The marginal distribution function $F_X(x)$ is

(A) $\dfrac{5}{4}\dfrac{1}{(x+1)^2}[4(x) - 4(x-4)]$

(B) $\dfrac{5}{2}\dfrac{1}{(x+1)^2}[4(x) - 4(x-4)]$

(C) $\dfrac{5}{4}\dfrac{1}{(x+1)}[4(x) - 4(x-4)]$

(D) None of the above

38. The marginal distribution function $F_Y(y)$ is

(A) $[1 - e^{-y^2}]u(y)$ (B) $\frac{5}{2}[1 - e^{-y^2}]u(y)$

(C) $\frac{5}{4}[1 - e^{-y^2}]u(y)$ (D) None of the above

39. The joint distribution function is

(A) $\begin{cases} \dfrac{5}{4}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{4}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

(B) $\begin{cases} \dfrac{5}{8}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{2}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

(C) $\begin{cases} \dfrac{5}{8}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{4}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

(D) $\begin{cases} \dfrac{5}{4}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{2}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

40. The function

$$F_{X,Y}(x, y) = \frac{a}{2}\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{2}\right)\right]\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{y}{3}\right)\right]$$

is a valid joint distribution function for random variables X and Y if the constant a is

(A) $\dfrac{1}{\pi^2}$ (B) $\dfrac{2}{\pi^2}$

(C) $\dfrac{4}{\pi^2}$ (D) $\dfrac{8}{\pi^2}$

41. Random variable X and Y have the joint distribution function

$$F_{X,Y}(x,y)=\begin{cases}0, & x<0 \text{ or } y<0\\ \frac{27}{26}x\left(1-\frac{y^2}{27}\right), & 0\le x<1 \text{ and } 1\le y\\ \frac{27}{26}y\left(1-\frac{y^2}{27}\right), & 1\le x \text{ and } 0\le y<1\\ \frac{27}{26}xy\left(1-\frac{x^2y^2}{27}\right), & 0\le x<1 \text{ and } 0\le y<1\\ 1, & 1\le x \text{ and } 1\le y\end{cases}$$

The probability of the event $\{0<X\le 0.5,\ 0<Y\le 0.25\}$ is

(A) 0.13 (B) 0.24

(C) 0.69 (D) 1

Statement for question 42-43 :

The joint probability density function of random variable X and Y is given by

$$p_{XY}(x,y)=xye^{-\frac{(x^2+y^2)}{2}}u(x)u(y)$$

42. The $p_X(x)$ is

(A) $2xe^{-x^2}u(x)$ (B) $xe^{-\frac{x^2}{2}}u(x)$

(C) $xe^{-x^2}u(x)$ (D) $2xe^{\frac{x^2}{2}}u(x)$

43. The $p_{Y/X}(y/x)$ is

(A) $\frac{1}{2}ye^{-y^2}u(y)$ (B) $ye^{-y^2}u(y)$

(C) $ye^{-\frac{y^2}{2}}u(y)$ (D) $\frac{1}{2}ye^{-\frac{y^2}{2}}u(y)$

44. The probability density function of a random variable X is given as $f_X(x)$. A random variable Y is defined as $y=ax+b$ where $a<0$. The PDF of random variable Y is

(A) $bf_X\left(\frac{y-b}{a}\right)$ (B) $af_X\left(\frac{y-b}{a}\right)$

(C) $\frac{1}{a}f_X\left(\frac{y-b}{a}\right)$ (D) $\frac{1}{b}f_X\left(\frac{y-b}{a}\right)$

45. The function

$$f_{X,Y}(x,y)=\begin{cases}be^{-(x+y)} & 0<x<a \text{ and } 0<y<\infty\\ 0 & \text{else where}\end{cases}$$

is a valid joint density function if b is

(A) $\frac{a^2}{1-e^{-a}}$ (B) $\frac{a}{1-e^{-a}}$

(C) $\frac{1}{1-e^{-a}}$ (D) None of the above

Statement for Question 46-47 :

Random variable X and Y have the joint density $f_{X,Y}(x,y)=\frac{1}{2}u(x)u(y)e^{-\frac{x}{4}-\frac{y}{3}}$

46. The probability of the event $\{2<X\le 4,\ -1<Y\le 5\}$ is

(A) 0.1936 (B) 6.2964

(C) 0 (D) None of the above

47. The probability of the event $\{0<X<\infty,\ <y\le -2\}$ is

(A) 0.2349 (B) 0.3168

(C) 0.4946 (D) None of the above

48. Let X and Y be two statistically independent random variables uniformly distributed in the ranges (-1, 1) and (-2, 1) respectively. Let $Z=X+Y$. Then the probability that $(Z\le -2)$ is

(A) zero (B) 1/6

(C) 1/3 (D) 1/12

49. The probability density function of two statistically independent random variable X and Y are

$$f_X(x)=5u(x)e^{-5x}$$

$$f_Y(y)=24(y)e^{-2y}$$

The density of the sum $W=X+Y$ is

(A) $\frac{10}{6}[e^{-2\omega}-e^{5\omega}]u(w)$

((B) $\frac{10}{8}[e^{-2\omega}-e^{5\omega}]u(w)$

(C) $\frac{10}{13}[e^{-2\omega}-e^{-5\omega}]u(w)$

(D) $\frac{10}{2}[e^{-2\omega}-e^{-5\omega}]u(w)$

50. The density function of two random variable X and Y is

$$f_{X,Y}(x,y)=\begin{cases}\frac{1}{24} & 0<x<6 \text{ and } 0<y<4\\ 0 & \text{else where}\end{cases}$$

The expected value of the function $g(x,\ y)=(XY)$ is

(A) 64 (B) 96

(C) 32 (D) 48

51. The density function of two random variable X and Y is

$$f_{X,Y}(x,y) = \frac{e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}}{2\pi\sigma^2}$$

with σ^2 a constant. The mean value of the function $g(X,Y) = X^2 + Y^2$ is

(A) σ^2 (B) σ

(C) $2\sigma^2$ (D) 2σ

Statement for Question 52-54 :

The statistically independent random variable X and Y have mean values $\overline{X} = E[X] = 2$ and $\overline{Y} = E[Y] = Y$. They have second moments $\overline{X^2} = E[X^2] = 8$ and $Y^2 = E[Y^2] = 25$. Consider a random variable $W = 3X - Y$.

52. The mean value $E[W]$ is

(A) 2 (B) 4

(C) 8 (D) 25

53. The second moment of W is

(A) 145 (B) 49

(C) 97 (D) 0

54. The variance of the random variable is

(A) 4 (B) 45

(C) 49 (D) 54

55. Two random variable X and Y have the density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{xy}{9}, & 0 < x < 2 \text{ and } 0 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

The X and Y are

(A) Correlated but statistically independent

(B) uncorrelated but statistically independent

(C) Correlated but statistically dependent

(D) Uncorrelated but statistically dependent

56. The value of σ_X^2, σ_Y^2, R_{XY} and ρ are respectively

(A) $\frac{11}{4}\ \frac{27}{2}, \frac{1}{2}.\left(2 + \frac{1}{\sqrt{3}}\right)$, and $-3\sqrt{\frac{33}{2}}$

(B) $\frac{11}{4}\ \frac{11}{2}, \frac{1}{2}\left(2 + \frac{1}{\sqrt{3}}\right)$, and $-3\sqrt{\frac{33}{2}}$

(C) $\frac{9}{4}\ \frac{11}{2}, \frac{1}{2}\left(2 - \frac{1}{\sqrt{3}}\right)$, and $-\frac{1}{3}\sqrt{\frac{2}{33}}$

(D) $\frac{9}{4}, \frac{11}{2}, \frac{1}{2}\left(2 - \frac{1}{\sqrt{3}}\right)$, and $-\frac{1}{3}\sqrt{\frac{2}{33}}$

57. The mean value of the random variable

$W = (X + 3Y)^2 + 2X + 3$ is

(A) $98 + \sqrt{3}$ (B) $98 - \sqrt{3}$

(C) $49 - \sqrt{3}$ (D) $49 + \sqrt{3}$

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SOLUTION

1. (A) $P\{X>1\}=\int_1^\infty p_X(x)dx$

$=\int_1^\infty \frac{|x|}{2}e^{-|x|}dx=\frac{1}{2}\int_1^\infty x\,e^{-x}dx\ =0.368$

2. (C) $P\{-1<X\le 2\}=\int_{-1}^{0}-\frac{1}{2}xe^{x}dx+\int_0^2\frac{1}{2}xe^{-x}dx$

$=1-\frac{1}{e}-\frac{3}{2e^2}=0.429$

3. (A) Test 1: $f_X(x)\ge 0$ is true

Test 2: area must be 1 i.e. $\int_0^b\frac{\ell^{3x}}{4}dx=\frac{1}{4}\left[\frac{\ell^{3b}}{3}-\frac{1}{3}\right]=1$

Thus $b=\frac{1}{3}\ln 13$

4. (C) $\int_{-\infty}^{\infty}\rho(v)=1\Rightarrow\ \frac{1}{2}4k=1\ \Rightarrow k=\frac{1}{2}$

Thus $\rho(v)=\frac{kv}{4}=\frac{v}{8}$

Mean Square Value $=\int_{-\infty}^{\infty}x^2\rho(x)dx\ =\int_0^4 x^2\frac{x}{8}dx=8$

5. (B) For $x=0$, $F_X(x)=\int_0^x\frac{1}{2}e^{-\frac{x}{2}}dx=\left[1-e^{-\frac{x}{2}}\right]u(x)$

$P(A)=F_X(3)-F_X(1)=e^{-\frac{1}{2}}-e^{-\frac{3}{2}}=0.3834$

6. (D) $P(B)=F_X(2.5)\ =1-e^{-\frac{2.5}{2}}=0.7135$

7. (D) $C=A\cap B=\{1<X<2.5\}$

$P(C)=F_X(2.5)-F_X(1)=e^{-\frac{1}{2}}-e^{-\frac{2.5}{2}}=0.3200$

8. (A) $P(|X|>2)=P\{2<x\}+P\{x<-2\}$

$=1-P\{x\le 2\}+P\{x<-2\}=1-F(2)+F(-2)$

We know that for gaussian function $F(-x)=1-F(x)$

Thus $P(|X|>2)=1-F(2)+1-F(2)$

$=2-2F(2)=2-2(0.9772)=0.0456$

9. (C) Rejected resistor corresponds to $\{x<900\,\Omega\}$ and $\{x>1100\,\Omega\}$. Fraction rejected corresponds to probability of rejection.

P {resistor rejected} $=P\{X<900\}+P\{X<1100\}$

$=F_X(900)+[1-F_X(1100)]$

$=F\left(\frac{900-a_x}{\sigma_x}\right)+1-F\left(\frac{1100-a_x}{\sigma_x}\right)$

$=F\left(\frac{900-1000}{40}\right)+1-F\left(\frac{11000-1000}{40}\right)$

$=F(-2.5)+1-F(2.5)=1-F(2.5)+1-F(2.5)=2-2F(2.5)$

$=2-2(0.9938)=0.012$ or $1.2\ \%$

10. (B) P(resistor rejected) $=F\left(\frac{900-1050}{40}\right)+1$

$-F\left(\frac{1100-1050}{40}\right)=F(-3.75)+1-F(1.25)$

$=1-F(3.75)+1-F(1.25)$

$=2-0.9999-0.8944=0.1057$ or $20.57\ \%$

11. (D) $P\{x>T+95.6\}=0.0505$

$=1-F\frac{(T+95.6-a_x)}{\sigma_x}=F\left(\frac{T+95.6-a_x}{\sigma_x}\right)=0.9495$

This occurs when $\frac{T+95.6-a_x}{\sigma_x}=1.64$ ...(i)

$P\{x\le T-11.2\}=0.1515$

$=F\frac{(T-11.2-a_x)}{\sigma_x}=1-F\frac{(T-11.2-a_x)}{\sigma_x}=8485$

This occur when $-\frac{T-11.2-a_x}{\sigma_x}=1.03$ ...(ii)

Solving (i) and (ii) we get $a_x=T+30$ and $\sigma_x=40$

12. (A) 0.25 exceeds when $\frac{|x|^2}{100}>0.21$ or $|x|>5v$

$P(0.25\,\text{W exceeded})=P\{|x|>5\}$

$=P\{x>5\}+P\{x<-5\}=1-P(x\le 5)+P\{x<-5\}$

$=1-P\left(\frac{5-0}{4.2}\right)+P\left(\frac{-5-0}{4.2}\right)=1-F\left(\frac{5}{4.2}\right)+F\left(\frac{-5}{4.2}\right)$

$=1-F\left(\frac{5}{4.2}\right)+1-F\left(\frac{5}{4.2}\right)=2\left(1-F\left(\frac{5}{4.2}\right)\right)=2Q\left(\frac{5}{4.2}\right)$

13. (A) $P\{160<X\le 120\}=F_X(210)-F_X(160)$

$=F\left(\frac{210-200}{20}\right)-F\left(\frac{160-200}{20}\right)$

$=F(0.5)-F(-2)=F(0.5)+F(2)-1$

$=0.6915+0.9773-1=0.6687$

14. (C) $P\{X>231\}=1-P\{X\le 231\}=1-F_X(231)$

$=1-F\left(\frac{231-200}{20}\right)=1-F(1.55)=1-0.9394=0.0606$

15. (B) We use the Rayleigh distribution with $a=0$ and $b=400$

For probability $P\{X \le 1\} = F_X(1) = 1 - e^{-\frac{1}{400}}$

$=0.0025$ or 0.25 %

16. (C) $P\{X \ge 52\} = 1 - F_X(52)$

$$= 1 - \left[1 - e^{-\frac{52^2}{400}}\right] = 1 - e^{-\frac{52^2}{400}} = 0.00116 \text{ or } 0.12\ \%$$

17. (C) $F_X = \int_{-\infty}^{x} f_X(u)du = \int_{-\infty}^{x} \frac{(b/\pi)du}{b^2 + (u-a)^2}$

Let $v = u - a$ and $dv = du$ to get

$$F_X(x) = \frac{b}{\pi}\int_{-\infty}^{x-a} \frac{dv}{b^2 + v^2} = \frac{b}{\pi}\left[\frac{1}{b}\tan^{-1}\left(\frac{v}{b}\right)\right]_{-\infty}^{x-a}$$

$$= \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right)$$

18. (B) Here $f_X(x) = e^{-2}\sum_{k=0}^{\infty}\left(\frac{3^k}{k!}\right)\delta(x-k)$

$P\{x > 0\} = 1 - P\{x \le 3\}$

$= 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$

$$= 1 - e^{-2}\left\{\frac{2^0}{0!} + \frac{2^1}{2!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right\} = 1 - e^{-2}\left(\frac{19}{3}\right) = 0.1429$$

19. (D) $P(x=0) = e^{-2}\left|\frac{2^0}{0!}\right| = 0.135$

20. (B) $P\{W > W_0\} = 1 - P\{W \le W_0\} = 1 - F_W(W_0)$

$$= 1 - \left(1 - e^{-\frac{W_0}{W_0}}\right) = e^{-1}$$

21. (A) P {5 or more}$= 1 - P(0) - P(1) - P(2) - P(4)$

$$= 1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{11} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right] = 1 - \frac{131}{8}e^{-3} = 0.1847$$

22. (C) $P(0) = e^{-3} = 0.0498$

average number of week, per year with no murder $52e^{-3} = 2.5889$ week.

23. (D) P {3 or more}$= 1 - P(0) - P(1) - P(2)$

$$= 1 - e^{-3}\left[1 + 3 + \frac{3^2}{2}\right] = 1 - \frac{17}{2}e^{-3} = 0.5768$$

Average number of weeks per year that number of murder exceeds the average

$$= 52\left(1 - \frac{17}{2}e^{-3}\right) = 29.994 \text{ weeks}$$

24. (B) $E[X] = \overline{X} = \sum_{i=1}^{4} x_i P(x_i)$

$= 1.0(0.4) + 4(0.25) + 9(0.15) + 16(0.1) = 4.35$

25. (A) $E[g(X)] = E[X^3] = \int_0^{\infty}\frac{1}{2}x^3 e^{-\frac{x}{2}} = \frac{1}{2}\left[\frac{6}{(1/2)^4}\right] = 48$

26. (A) Mean of $X = \int_{-\infty}^{x} x f_X(x)dx = \int_0^1 x3(1-x)^2 dx = \frac{1}{4}$

27. (B) Variance of X is $\sigma_x^2 = E[X^2] - \mu_x^2$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^1 x^2 3(1-x)^2 dx = \frac{1}{10}$$

$$\sigma_x^2 = \frac{1}{10} - \left(\frac{1}{4}\right)^2 = \frac{3}{80}$$

Hence B is correct option

28. (B) Here $Y = g(X) = e^{-\frac{X}{3}}$

So $E[Y] = E[g(Y)] = \int_{-\infty}^{\infty} g(X) g_X(x)dx = \int_{-5}^{15} e^{-\frac{x}{5}} \frac{1}{15-(-5)} dx$

$$= \frac{1}{20}\left[-5e^{-\frac{x}{5}}\right]_{-5}^{15} = \frac{1}{5}[e^1 - e^{-3}] = 0.667$$

29. (C) $E[Y] = E[2X - 3] = 2\overline{X} - 3 = 2(-3) - 3 = -9$

$E[Y^2] = E[(2X-3)^2] = 4\overline{X^2} - 12\overline{X} - 9$

$= 4(11) - 12(-3) - 9 = 89$

$\sigma_Y^2 = \overline{Y^2} - \overline{Y}^2 = 89 - 9^2 = 8$

30. (A) $F_{XY}(x,y) = 0.1u(x-1)u(y-1) + 0.35u(x-2)u(y-2)$
$+ 0.05u(x-3)u(y-3) + 0.5u(x-y)u(y-4)$

$P\{X \le 2.5,\ Y \le 6.0\} = f_{XY}(2.5,\ 6.0) = 0.1 + 0.35 = 0.45$

31. (B) $P\{X \le 3.0\} = F_X(3.0) = F_{XY}(3.0, \infty)$

$= 0.1 + 0.35 + 0.05 = 0.5$

32. (B) $F_X(x) = F_{X,Y}(x,\ \infty)$

$$\lim_{y\to\infty}\frac{5}{4}\left(\frac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right)u(y) = \frac{5x}{4(x+1)}$$

$$\lim_{y\to\infty}\left(1 + \frac{1}{4}e^{-5y^2} - \frac{5}{4}e^{-y^2}\right) = 1$$

33. (B) $F_Y(y) = F_{X,Y}(\infty,\ y)$

$$= \int_{-\infty}^{\infty} 5e^{-(w-y)} u(w-y)u(y)2e^{-2y} dy = 10\int_0^w e^{-5w+3y} dy, \quad w > 0$$

$$= \frac{10}{3} u(w)[e^{-2w} - e^{-5w}]$$

50. (A) $E[(XY)^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 y^2 f_{X,Y}(x,y) dx\, dy$

$$= \int_{y=0}^{4}\int_{x=0}^{6} \frac{x^2y^2}{24} dx\ dy = 64$$

51. (C) $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x^2+y^2) \frac{e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}}{2\pi\sigma^2} dx\, dy$

$$= \int_{-\infty}^{\infty} \frac{x^2 e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dx \int_{-\infty}^{\infty} \frac{e^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dy + \int_{-\infty}^{\infty} \frac{y^2 e^{\frac{-y^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dy \int_{-\infty}^{\infty} \frac{e^{\frac{-x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dx$$

Both double integral are of the same form. the second factors equal 1 because they are area of a gaussian density. The first factor equal σ^2 because they are second moment of gaussian density with zero mean and variance σ^2.

Thus $E[g(x,y)] = E[(x^2+y^2)] 2\sigma^2$

52. (A) $E[W] = E[3X - Y] = 3\overline{X} - \overline{Y} = 6 - 4 = 2$

53. (B) $E[W^2] = E[(3X-Y)^2] = E[9X^2 - 6XY + Y^2]$

$= 9\overline{X^2} - 6\overline{XY} + \overline{Y^2} = 9\overline{X^2} - 6\overline{X}\,\overline{Y} + \overline{Y^2}$

$= 9(8) - 6(2)(4) + 25 = 49$

54. (B) $\sigma_W^2 = E[(W-\overline{W})^2] = E[W^2 - 2W\overline{W} + \overline{W}^2]$

$= \overline{W^2} - \overline{W}^2 = 49 - 4 = 45$

55. (B) $R_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\ f_{X,Y}(x,y)\ dxdy = \int_0^3\int_0^2 \frac{x^2y^2}{9} dx\, dy = \frac{8}{3}$

$E[X] = \int_0^3\int_0^2 \frac{x^2y}{9} dx\, dy = \frac{4}{3}$

$E[Y] = \int_0^3\int_0^2 \frac{x^2y}{9} dx\, dy = 2$

Since $R_{XY} = \frac{8}{3} = E[X]E[Y] = 2\left(\frac{4}{3}\right) = \frac{8}{3}$, we have X and Y uncorrelated form

From marginal densities $f_X(x) = \int_0^3 \frac{xy}{9} dy = \frac{x}{2}, \quad 0 < x < 2$

$f_Y(y) = \int_0^2 \frac{xy}{9} dy = \frac{2y}{9}, \quad 0 < y < 3$

we have $f_X(x)\ f_Y(y) = \frac{xy}{9}$, $0 < x < 2$ and $0 < y < 3$

Thus $f_{X,Y}(x,y) = f_X(x)\ f_Y(y)$ and X and Y are statistically independent.

56. (C) $\sigma_X^2 = \overline{X^2} - \overline{X}^2 = \frac{5}{2} - \left(\frac{1}{2}\right)^2 = \frac{9}{4}$

$\sigma_Y^2 = \overline{Y^2} - \overline{Y}^2 = \frac{11}{2} - (2)^2 = \frac{11}{2}$

$R_{XY} = \overline{XY} = C_{XY} + \overline{X}\,\overline{Y} = -\frac{1}{2\sqrt{3}} + \frac{1}{2}(2) = \frac{1}{2}\left(2 - \frac{1}{\sqrt{3}}\right)$

$\rho = \frac{C_{XY}}{\sigma_X\sigma_Y} = \frac{-1/2\sqrt{3}}{(\sqrt{914})(\sqrt{11/2})} = \frac{-1}{3}\sqrt{\frac{2}{33}}$

57. (B) $\overline{W} = \overline{(X+3Y)^2 + 2X + 3}$

$= 3 + 2\overline{X} + \overline{X^2} + 6\overline{XY} + 9\overline{Y^2}$

$= 3 + 2\left(\frac{1}{2}\right) + \frac{5}{2} + 6\left(\frac{1}{2}\right)\left(2 - \frac{1}{\sqrt{3}}\right) + 9\left(\frac{19}{2}\right) = 98 - \sqrt{3}$

CHAPTER

7.2

RANDOM PROCESS

Statement for Question 1 - 4 :

A random process $X(t)$ has periodic sample functions as shown in figure where A, T and $4t_0 \le T$ are constant but $\in$ is random variable uniformly distributed on the interval $(0, T)$.

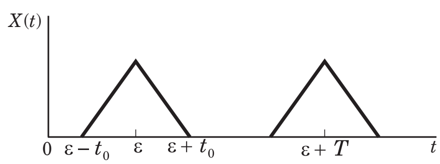

Fig. P7.2.1-4

1. The first order density function is

(A) $\begin{cases} \dfrac{T-2t_0}{T}\delta(x) + \dfrac{2t_0}{AT} & 0 \le x < A \\ 0 & \text{else where} \end{cases}$

(B) $\dfrac{T-2t_0}{T} + \dfrac{2t_0}{AT}[u(x) - u(x-A)]$

(C) $\begin{cases} \dfrac{T+2t_0}{T}\delta(x) + \dfrac{2t_0}{AT} & 0 \le x < A \\ 0 & \text{else where} \end{cases}$

(D) $\dfrac{T-2t_0}{T}\delta(x) + \dfrac{2T_0}{AT}$

2. The value of $E[X(t)]$ is

(A) $\dfrac{t_0 A}{2T}$ (B) $\dfrac{t_0 A}{T}$

(C) $\dfrac{t_0 A}{4T}$ (D) 0

3. The value of $E[X^2(t)]$ is

(A) $\dfrac{t_0 A^2}{T}$ (B) $\dfrac{t_0 A^2}{3T}$

(C) $\dfrac{2t_0 A^2}{3T}$ (D) 0

4. The value of σ_X^2 is

(A) $\dfrac{t_0 A}{T}\left[\dfrac{2}{3} - \dfrac{t_0}{T}\right]$ (B) $\dfrac{t_0 A^2}{T} - \dfrac{t_0}{T}$

(C) $\dfrac{t_0 A}{T}\left[\dfrac{2}{3} + \dfrac{t_0}{T}\right]$ (D) $\dfrac{t_0 A^2}{T}\left[\dfrac{2}{3} + \dfrac{t_0}{T}\right]$

5. An ergodic random power $x(t)$ has an auto-correlation function

$$R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2} 1 + 4\cos(12\tau)$$

The $|\bar{X}|$ is

(A) $\pm\sqrt{18}$ (B) $\pm\sqrt{13}$

(C) $\pm\sqrt{17}$ (D) $\pm\sqrt{18} \pm \sqrt{17}$

6. For random process $\overline{X} = 6$ and

$$R_{XX}(t,\ t+\tau) = 36 + 25e^{-|\tau|}.$$

Consider following statements :

1. $X(t)$ is first order stationary.
2. $X(t)$ has total average power of 36 W.
3. $X(t)$ is a wide sense stationary.
4. $X(t)$ has a periodic component.

The true statement is/are

(A) 1, 2, and 4 (B) 2, 3, and 4

(C) 2 and 3 (D) only 3

7. A random process is defined by $X(t) + A$ where A is continuous random variable uniformly distributed on (0,1). The auto correlation function and mean of the process is

(A) 1/2 & 1/3 (B) 1/3 & 1/2

(C) 1 & 1/2 (D) 1/2 & 1

Statement for Question 8 - 9 :

A random process is defined by $Y(t) = X(t)\cos(\omega_0 t + \theta)$ where $X(t)$ is a wide sense stationary random process that amplitude modulates a carrier of constant angular frequency ω_0 with a random phase θ independent of $X(t)$ and uniformly distributed on $(-\pi / \pi)$.

8. The $E[Y(t)]$ is

(A) $E[X(t)]$ (B) $-E[X(t)]$

(C) 1 (D) 0

9. The autocorrelation function of $Y(t)$ is

(A) $R_{XX}(\tau)\cos(\omega_0\tau)$ (B) $\frac{1}{2}R_{XX}(\tau)\cos(\omega_0\tau)$

(C) $2R_{XX}(\tau)\cos(\omega_0\tau)$ (D) None of the above

Statement for Question 10 - 11 :

Consider a low-pass random process with a white-noise power spectral density $S_X(\omega) = \mathcal{N}/2$ as shown in fig.P7.2.10-11.

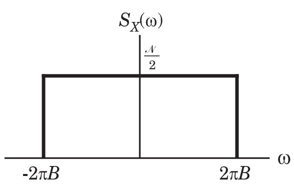

Fig.P7.2.10-11

10. The auto correlation function $R_X(\tau)$ is

(A) $2\mathcal{N}B$ sinc $(2\pi\beta\tau)$ (B) $\pi\mathcal{N}B$ sinc $(2\pi\beta\tau)$

(C) $\mathcal{N}B$ sinc $(2\pi\beta\tau)$ (D) None of the above

11. The power P_X is

(A) $2NB$ (B) πNB

(C) NB D) D$\frac{NB}{2\pi}$

12. If $X(t)$ is a stationary process having a mean value $E[X(t)] = 3$ and autocorrelation function $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$.

The variance of random variable $Y = \int_0^2 X(t)dt$ will be

(A) 1 (B) 2.31

(C) 4.54 (D) 0

13. A random process is defined by $X(t) = A\cos(\pi t)$ where A is a gaussian random variable with zero mean and variance σ_π^2. The density function of $X(0)$

(A) $\frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{x^2}{2\sigma_A^2}}$ (B) $\sqrt{2\pi}\sigma_A e^{-\frac{x^2}{2\sigma_A^2}}$

(C) 0 (D) 1

Statement for Question 14-15 :

The two-level semi-random binary process is defined by

$X(t) = A$ or $-A$

where $(n-1)T < t < nt$ and the levels A and $-A$ occur with equal probability. T is a positive constant and $n = 0, \pm 1, \pm 2$

14. The mean value $E[X(t)]$ is

(A) 1/2 (B) 1/4

(C) 1 (D) 0

15. The auto correlation $R_{XX}(t_1 = 0.5T,\ t_2 = 0.7T)$ will be

(A) 1 (B) 0

(C) A^2 (D) $A^2/2$

16. A random process consists of three samples function $X(t, s_1) = 2$, $X(t, s_2) = 2\cos t_1$ and $X(t, s_3) = 3\sin t$- each occurring with equal probability. The process is

(A) First order stationary

(B) Second order stationary

(C) Wide-sense stationary

(D) Not stationary in any sense

Statement for Question 17 - 19 :

The auto correlation function of a stationary ergodic random process is shown in fig.P.7.2.17-19

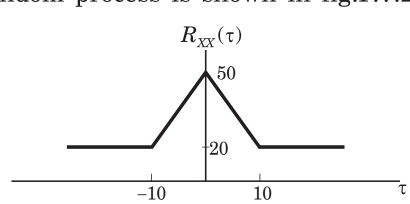

Fig. P7.2.17-19

17. The mean value $E[X(t)]$ is

(A) 50 (B) $\sqrt{50}$

(C) 20 (D) $\sqrt{20}$

18. The $E[X^2(t)]$ is

(A) 10 (B) $\sqrt{10}$

(C) 50 (D) $\sqrt{50}$

19. The variance σ_X^2 is

(A) 20 (B) 50

(C) 70 (D) 30

20. Two zero mean jointly wide sense stationary random process $X(t)$ and $Y(t)$ have no periodic components. It is know that $\sigma_X^2 = 5$ and $\sigma_Y^2 = 10$. The function, that can apply to the process is

(A) $R_{XX}(\tau) = 6u(\tau)e^{-3\tau}$ (B) $R_{YY}(\tau) = 5\left[\frac{\sin(3\tau)}{3\tau}\right]^2$

(C) $R_{XY}(\tau) = 9(1+2e^2)^{-1}$ (D) None of the above

21. A stationary zero mean random process $X(t)$ is ergodic has average power of 24 W and has no periodic component. The valid auto correlation function is

(A) $16 + 18\cos(3\tau)$ (B) $24\delta a^2(2\pi)$

(C) $\frac{e^{(-6\tau)}}{(1+3\tau^2)}$ (D) $24\delta(t-\tau)$

22. Air craft of Jet Airways at Ahmedabad airport arrive according to a poisson process at a rate of 12 per hour. All aircraft are handled by one air traffic controller. If the controller takes a 2 - minute coffee break, what is the probability that he will miss one or more arriving aircraft ?

(A) 0.33 (B) 0.44

(C) 0.55 (D) 0.66

23. Delhi airport has two check-out lanes that develop waiting lines if more than two passengers arrives in any one minute interval. Assume that a poission process describes the number of passengers that arrive for check-out. The probability of a waiting line if the average rate of passengers is 2 per minute, is

(A) 0.16 (B) 0.29

(C) 0.32 (D) 0.49

24. A complex random process $Z(t) = X(t) + jY(t)$ is defined by jointly stationary real process $X(t)$ and $Y(t)$. The $E[|Z(t)|^2]$ will be

(A) $2R_{XY}(0) + R_{XX}(0) + R_{YY}(0)$ (B) $R_{XX}(0) + R_{YY}(0)$

(C) $R_{XX}(0) - R_{YY}(0)$ (D) $R_{YY}(0) - R_{XX}(0)$

25. Consider random process $X(t) = A_0\cos(\omega_0 t + \theta)$ where A_0 and ω_0 are constant and θ is a random variable uniformly distributed on the interval $(0, \pi)$. The power in $X(t)$ is

(A) A^2 (B) $\frac{1}{2}A^2$

(C) $\frac{1}{4}A^2$ (D) 1

26. The non valid power spectral density function of a real random process is

(A) $\delta(\omega+\omega_0) - \delta(\omega-\omega_0)$ (B) $\frac{\omega^2}{\omega^2+25}$

(C) $\delta(\omega) + \frac{\omega^2}{\omega^2+16}$ (D) $\frac{\omega^2}{\omega^2+16}$

27. The valid power density spectrum is

(A) $\frac{\omega^2}{1+\omega^2+j\omega^2}$ (B) $\frac{\omega^2}{\omega^4+1} - \delta(\omega)$

(C) $e^{-(\omega-1)^2}$ (D) $\frac{\omega^2}{\omega^6+3\omega^2+3}$

28. A power spectrum is given as

$$\rho_{XX}(\omega) = \begin{cases} \frac{P}{1+(\omega/W)^2} & |\omega| < KW \\ 0 & |\omega| > KW \end{cases}$$

where $P, W,$ and K are real positive constants. The sums bandwidth of power spectrum is

(A) $W\sqrt{\frac{\tan + k}{k} - 1}$ (B) $W\sqrt{\frac{k}{\tan^{-1}k} - 1}$

(C) $W\sqrt{\frac{\tan^{-1}k}{k} + 1}$ (D) ∞

29. Consider the power spectrum given by

$$\rho_{XX}(\omega) = \begin{cases} P & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$

where P and W are real positive constants. The rms bandwidth of the power spectrum is

(A) $\frac{W}{\sqrt{2}}$ (B) $\frac{W^2}{3}$

(C) $\frac{W}{\sqrt{3}}$ (D) $\frac{W}{2}$

30. For a random process $R_{XX}(\tau) = P\cos^4(\omega_0\tau)$ where P and ω_0 are constants. The power in process is

(A) P (B) $2P$

(C) $3P$ (D) $4P$

31. A random process has the power density spectrum $\rho_{XX}(\omega) = \frac{6\omega^2}{[1+\omega^2]^3}$. The average power in process is

(A) 1/4 (B) 3/8

(C) 5/8 (D) 1/2

32. A deterministic signal $A\cos(\omega_0 t)$, where A and ω_0 are real constants is added to a noise process $N(t)$ for which $\rho_{NN}(\omega) = \frac{W^2}{W^2+\omega^2}$ and $W > 0$ is a constant. The ratio of average signal power to average noise power is

(A) 1 (B) $\frac{A}{W}$

(C) $\frac{2A}{W}$ (D) $\frac{A^2}{W}$

33. The autocorrelation function of a random process $X(t)$ is

$$R_{XX}(t, t+\tau) = 12e^{Y\tau^2}\cos^2(24t)$$

The $R_{XX}(\tau)$ is

(A) $6e^{-4\tau^2}$ (B) $12e^{-4\tau^2}$

(C) $48e^{-4\tau^2}$ (D) None of the above

34. If $X(t)$ and $Y(t)$ are real random process, the valid power density spectrum $f_{XX}(\omega)$ is

(A) $\frac{6}{6+7\omega^3}$ (B) $\frac{4e^{-3|\tau|}}{1+\omega^2}$

(C) $3 + j\omega^2$ (D) $18\delta(\omega)$

35. The cross correlation of jointly wide sense stationary process $X(t)$ and $Y(t)$ is $R_{XY}(\tau) = Au(\tau)e^{-W\tau}$ where $A > 0$ and $W > 0$ are constants. The $\rho_{XX}(\omega)$ is

(A) $\frac{A}{W^2-\omega^2}$ (B) $\frac{A}{W^2+\omega^2}$

(C) $\frac{A}{W+j\omega}$ (D) $\frac{A}{W-j\omega}$

36. A random process $X(t)$ is applied to a linear time invariant system. A response $Y(t) = X(t) - X(t-\tau)$ occurs when τ is a real constant. The system's transfer function is

(A) $1 - e^{j\omega\tau}$ (B) $2je^{-j\omega\tau/2}\sin\frac{\omega\tau}{2}$

(C) $2je^{-j\omega\tau/2}\cos\frac{\omega\tau}{2}$ (D) $1 + e^{-j\omega\tau}$

37. A random process $X(t)$ has an autocorrelation function $R_{XX}(\tau) = A^2 + Be^{-|\tau|}$ Where A and B are constants. A system have an input response

$$h(t) = \begin{cases} e^{-Wt} & 0 < t \\ 0 & t < 0 \end{cases}$$

where W is a real positive constant, which $X(t)$ is its input. The mean value of the response is

(A) $\frac{A}{W}$ (B) $\frac{A}{2W}$

(C) $\frac{2A}{W}$ (D) 0

38. In previous question if impulse response of system is

$$h(t) = \begin{cases} e^{-Wt}\sin(\omega_0 t) & 0 < t \\ 0 & t < 0 \end{cases}$$

where W and ω_0 are real positive constants, the mean value of response is

(A) $\frac{A\omega_0}{\omega_0^2+W^2}$ (B) $\frac{A}{2\omega_0}\left(\frac{1}{\omega_0^2+W^2}\right)$

(C) $\frac{2A}{\omega_0}\left(\frac{1}{\omega_0^2+W^2}\right)$ (D) $\frac{A}{2\omega_0}\left(\frac{1}{\omega_0^2+W^2}\right)$

39. A stationary random process $X(t)$ is applied to the input of a system for which $h(t) = 3u(t)t^2e^{-8t}$. If $E[X(t)] = 2$, the mean value of the system's response $Y(t)$ is

(A) $\frac{1}{128}$ (B) $\frac{1}{64}$

(C) $\frac{3}{128}$ (D) $\frac{1}{32}$

Statement for Question 40-41 :

A random process $X(t)$ is applied to a network with impulse response $h(t) = u(t)te^{-at}$ where $a > 0$ is a constant. The cross correlation of $X(t)$ with the output $Y(t)$ is known to have the same form $R_{XY}(\tau) = u(\tau)\tau e^{-a\tau}$

40. The auto correlation of $Y(t)$ is

(A) $\frac{4+a\tau}{4a^3}e^{-a|\tau|}$ (B) $\frac{1+a\tau}{3a^2}e^{-a|\tau|}$

(C) $\frac{4+a\tau}{8a^2}e^{-a|\tau|}$ (D) $\frac{1+a\tau}{4a^3}e^{-a|\tau|}$

41. The average power in $Y(t)$ is

(A) $\frac{1}{4a^3}$ (B) $\frac{1}{a^3}$

(C) $\frac{1}{3a^2}$ (D) None of the above

Statement for Question 42 - 43 :

A random noise $X(t)$ having a power spectrum $\rho_{XX}(\omega) = \frac{3}{49+\omega^2}$ is applied to a differentiator that has a transfer function $H(\omega) = j\omega$. The output is applied to a network for which $h(t) = u(t)t^2e^{-7t}$

42. The average power in $X(t)$ is

(A) 5/21 (B) 5/24

(C) 5/42 (D) 3/14

43. The power spectrum of $Y(t)$ is

(A) $\frac{4\omega^2}{(49+\omega^2)^3}$ (B) $\frac{12\omega^2}{(49+\omega^2)^4}$

(C) $\frac{42\omega^3}{(49+\omega^2)^2}$ (D) None of the above

44. White noise with power density $\mathcal{N}_0/2$ is applied to a lowpass network for which $|H(0)| = 2$. It has a noise bandwidth of 2 MHz. If the average output noise power is 0.1 W in a $1-\Omega$ resistor, the value of $\mathcal{N}_0$ is

(A) 12.5 nW/Hz (B) 12.5 μW/Hz

(C) 25 nW/Hz (D) 25 μW/Hz

45. An ideal filter with a mid-band power gain of 8 and bandwidth of 4 rad/s has noise $X(t)$ at its input with power spectrum $\rho_{XX}(\omega) = \frac{50}{\sqrt{8\pi}}e^{-\omega^2/8}$. The noise power at the network's output is $(F(2) = 0.9773)$

(A) 60.8 (B) 90.3

(C) 20.2 (D) 100.4

46. White noise with power density $\mathcal{N}_0/2 = 6$ μW/Hz is applied to an ideal filter of gain 1 and bandwidth W rad/s. If the output's average noise power is 15 watts, the bandwidth W is

(A) 2.5×10^{-6} (B) $2.5\pi\times10^{-6}$

(C) 5×10^{-6} (D) $\pi5\times10^{-6}$

47. A system have the transfer function $|H(\omega)|^2 = \frac{1}{1+(\omega/W)^4}$ where W is a real positive constant. The noise bandwidth of the system is

(A) $\frac{1}{3}\pi W\sqrt{2}$ (B) $\frac{1}{4}\pi W\sqrt{2}$

(C) $\frac{1}{6}\pi W\sqrt{2}$ (D) None of the above

****

SOLUTION

1. (A) Let $\in$ have value e. Now $P\{X \le x|\in = e\} = F_X(X|\in = e)$ and for any $\in$ must be zero for $x < 0$ because $x(t)$ is never negative. The event $\{X \le 0\}$ is satisfied whenever $x(t)$ is zero. This happens during the fraction of time $(T-2t_0)/T$. Hence $F_X(x|\in = e) = [(T-2T_0)/T\}u(x)$. For $0 \le x < A$ the additional time interval or fraction of time where $X \le x$ becomes 2 to $2t_0x/AT$.

Thus $F_X(x|\in = e) = \left(\frac{T-2t_0}{T}\right)u(x) + \frac{2t_0x}{AT}$, $0 \le x < A$

$= 1, A \le x$

$= 0, x < 0$

By differentiation

$$f_X(x|\in = e) = \left(\frac{T-2t_0}{T}\right)\delta(x) + \frac{2t_0}{AT},\ 0 \le x < A$$

$= 0$ else where

$$f_{X,e}(x,e) = f_X(x|\in = e)f_\in(e)$$

$$= \left(\frac{T-2t_0}{T^2}\right)\delta(x) + \frac{2T_0}{AT^2}, 0 \le x < A \text{ and } 0 < e < T$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,\in}(x,e)de$$

$$= \left(\frac{T-2t_0}{T}\right)\delta(x) + \frac{2t_0}{AT}. 0 \le x < A$$

$= 0$ elsewhere.

2. (B) $E[X(t)] = \int_{-\infty}^{\infty} xf_X(x)dx$

$$= \int_{-\infty}^{\infty} x\left(\frac{T-2t_0}{T}\right)\delta(x)dx + \int_0^A \frac{2t_0x}{AT}dx = \frac{t_0A}{T}$$

3. (C) $E[X^2(t)] = \int_{-\infty}^{\infty} x^2f_X(x)dx = \int_0^A \frac{2t_0x^2}{AT} = \frac{2t_0A^2}{3T}$

4. (D) $\sigma_X^2 = E[X^2(t)] - \{E[X(t)]\}^2$

$$= \frac{2t_0A^2}{3T} - \frac{t_0^2A^2}{T^2} = \frac{t_0A^2}{T}\left[\frac{2}{3} - \frac{t_0}{T}\right]$$

5. (A) We know that (i) if $X(t)$ has a periodic component then $R_{XX}(\tau)$ will have a periodic component with the same period. (ii) if $E[X(t)] = \bar{X} \mp 0$ and $X(t)$ is ergodic with no periodic components then $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \overline{X}^2$

Thus we get $|\overline{X}|^2 = 18$ or $\overline{X} = \pm\sqrt{18}$

6. (C) $\overline{X}$ = Constant and $R_{XX}(\tau)$ is not a function of t, so $X(t)$ is a wide sense stationary. So 1 is false & 3 is true.

$P_{XX} = R_{XX}(0) = 36 + 25 = 61$.

Thus 2 is false if $X(t)$ has a periodic component, then $R_{XX}(\tau)$ will have a periodic component with the same period. Thus 4 is false.

7. (B) $R_{XX}(t, t+\tau) = E[X(t)X(t+\tau)] = E[A^2] = \int_0^1 a^2 da = \frac{1}{3}$

$\overline{X} = E[X(t)] = E[A] = \int_0^1 a\, da = \frac{1}{2}$

8. (D) $E[Y(t)\} = E[X(t)\cos(\omega_0 t + \theta)]$

$= E_X[X(t)] \int_{-X}^{X} \cos(\omega_0 t + \theta) \frac{1}{2\pi} d\theta = 0$

where $E_X[\cdot]$ represent expectation with respect to X only

9. (B) $R_{YY}(t, t+\tau)$

$= E[X(t)\cos(\omega_0 \tau + \theta) X(t+\tau)\cos(\omega_0 t + \theta + \omega_0 \tau)]$

$= R_{XX}(\tau) \frac{1}{2}[\cos(\omega_0 \tau) + \cos(2\omega_0 t + 2\theta + \omega_0 \tau)]$

$= \frac{1}{2} R_{XX}(\tau)\cos(\omega_0 \tau)$

10. (C) $S_X(\omega) = \frac{\mathcal{N}}{2} \text{rect}\left(\frac{\omega}{4\pi b}\right)$

We know that $R_X(\tau) \longleftrightarrow S_X(\omega)$

$\frac{W}{\pi}\sin(Wt) \longleftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$

Here $W = 2\pi B$

Hence $R_X(\tau) = \frac{2\pi B}{\pi} \frac{\mathcal{N}}{2} \sin(2\pi B\tau) = \mathcal{N}B \text{ sinc } (2\pi B\tau)$

11. (C) $P_X = \overline{X^2} = R_X(0) = \text{N}B$ since $\text{sinc}(0) = 1$

12. (C) $E[Y] = E[\int_0^2 X(t)dt = \int_0^2 E[X(t)]dt = 3\int_0^2 dt = 6$

$E[Y^2] = E[\int_0^2 X(t)dt \int_0^2 X(u)du]] = \int_0^2\int_0^2 E[X(t)X(u)du\, dt]$

$= \int_0^2\int_0^2 R_{XX}(t-u)dt\, du = \int_0^2\int_0^2 [9 + 2e^{-|t-u|}]dt\, du$

$= 36 + 2\int_0^2\int_0^2 e^{-|t-u|}dt\, du = 4(10 + e^{-2})$

$\sigma_Y^2 = E[Y^2] - (E[Y])^2 = 4(1 + e^{-2}) = 4.541$

13. (A) For $t = 0, X(0) = A$, So $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{x^2}{2\sigma_A^2}}$

14. (D) $E[X(t)] = AP(A) + (-A)P(-A) = \frac{A}{2} - \frac{A}{2} = 0$

15. (C) Here $R_{XX}(t_1, t_2) = A^2$

If both t_1 and t_2 are in the same interval $(n-1)T < t, t_2 < nT, n = 0, \pm, \pm 2 ...$

and $R_{XX}(t_1, t_2) = 0$ otherwise

Hence $R_{XX}(0.5T, 0.7T) = A^2$

16. (D) Let $x_1 = 2$, $x_2 = 2\cos t$ and $x_3 = 3\sin(t)$

Then $f_X(x) = \frac{1}{2}\delta(x - x_1) + \frac{1}{3}\delta(x - x_2) + \frac{1}{3}\delta(x - x_3)$

and $E[X(t)] = \int_{-\infty}^{\infty} x f_X(x) dx$

$= \int_{-\infty}^{\infty} x\left[\frac{1}{3}\delta(x - x_1) + \frac{1}{3}\delta(x - x_2) + \frac{1}{3}\delta(x - x_2)\right]$

$= \frac{1}{3}[2 + 2\cos t + 3\sin t]$

The mean value is time dependent so $X(t)$ is not stationary in any sense.

17. (D) We know that for ergodic with no periodic component

$\lim_{|\tau| \to \infty} R_{XX}(\tau) = \overline{X}^2$, Thus $\overline{X}^2 = 20$ or $\overline{X} = \sqrt{20}$

18. (C) $R_{XX}(0) = E[X^2(t)] = R_{XX}(0) = 50 = \overline{X^2}$

19. (D) $\sigma_X^2 = \overline{X^2} - \overline{X}^2 = 50 - 20 = 30$

20. Here $\overline{X} = 0$, $\overline{Y} = 0$, $R_{XX}(0) = 5$, $\sigma_Y^2 = R_{YY}(0) = 10$

For (A) : Function does not have even symmetry

For (B) : Function does not satisfy $R_{YY}(0) = 10$

For (C) : Function does not satisfy $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)R_{YY}(0)} = \sqrt{50}$

21. (D) For (A) : It has a periodic component.

For (B) ; It is not even in τ, total power is also incorrect.

For (C) It depends on t not even in τ and average power is ∞.

22. (A) P (miss/or more aircraft) $= 1 - P$(miss 0)

$= 1 - P \text{ (0 arrive)} = 1 - \frac{(\lambda t)^0 e^{-\lambda t}}{0!}$

$$E[Y(t)] = A\int_{-\infty}^{\infty} h(\xi)d\xi = A\int_{0}^{\infty} e^{-Wt}dt = \frac{A}{W}$$

38. (A) $\overline{X} = A$

$$E[Y(t)] = \overline{Y} = \overline{X}\int_{-\infty}^{\infty} h(t)dt = A\int_{0}^{\infty} e^{-Wt}\sin(\omega_0 t)dt = \frac{A\omega_0}{\omega_0^2 + W^2}$$

39. (C) $\overline{Y} = \overline{X}\int_{-\infty}^{\infty} h(t)dt = 2\int_{0}^{\infty} 3t^2 e^{-8t}dt = \frac{3}{128}$

40. (D) $R_{YY}(\tau) = \int_{-\infty}^{\infty} R_{XY}(\tau + \xi)h(\xi)d\xi$

$$= e^{-a\tau}\int_{-\infty}^{\infty} u(\xi)u(\xi + \tau)(\tau\xi + \xi^2)e^{-2a\xi}d\xi$$

There are two cases of interest $\tau \geq 0$ and $\tau < 0$ Since $R_{YY}(\tau)$ is an even function we solve only the ease $\tau \geq 0$

$$R_{YY}(\tau) = e^{-a\tau}\int_{0}^{\infty}(\tau\xi + \xi^2)e^{-2a\xi}d\xi = \frac{1 + a\tau}{4a^3}e^{-a\xi}$$

41. (B) Power in $y(t) = R_{YY}(0) = \frac{1}{4a^3}$

42. (D) $P_{XX} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho_{XX}(\omega)d\omega = \frac{3}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{49 + \omega^2} = \frac{3}{14}$

43. (B) $h_2 = 49t)t^2e^{-7t} \xleftrightarrow{F} \frac{2}{(7 + j\omega)^3} = H_2(\omega)$

$$s_{YY}(\omega) = s_{XX}(\omega) = |H_1(\omega)H_2(\omega)|^2 = \frac{12\omega^2}{(49 + \omega^2)^4}$$

44. (A) $P_{YY} = \frac{N_0|H(0)|^2 W_n}{2\pi} = 0.1$

So $N_0 = \frac{2\pi(0.1)}{|H(0)|^2 W_n} = \frac{2\pi(0.1)}{(2)^2 2\pi \times 2 \times 10^6}$

$= 1.25 \times 10^{-8}$ W/Hz $= 12.5$ nW/Hz

45. (A) $P_{YY} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho_{XX}(\omega)|H(\omega)|^2 d\omega$

$$= \frac{1}{2\pi}\int_{4}^{4}\frac{50}{\sqrt{8\pi}}e^{-\frac{\omega^2}{8}(8)d\omega} = \frac{200}{\pi}\int_{4}^{4}\frac{e^{-\frac{\omega^2}{2(u)}}}{\sqrt{2\pi(4)}}$$

$$= \frac{200}{\pi}[F(2) - F(-2)] = \frac{200}{\pi}[2F(2) - 1)] = 60.8$$

46. (B) $P_{YY} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho_{XX}(\omega)|H(\omega)|^2 d\omega$

$$= \frac{1}{2\pi}\int_{-W}^{W} 6 \times 10^{-6}d\omega = \frac{6 \times 10^{-6}W}{\pi} = \frac{6 \times 10^{-6}W}{\pi} = 15$$

So $W = 2.5\pi \times 10^6$

47. (B) Noise bandwidth $W_n = \frac{\int_0^{\infty}|H(\omega)|^2 d\omega}{|H(0)|^2}$

$$W_n = \int_0^{\infty}|H(\omega)|^2 d\omega \text{since } H(0) = 1 = \int_0^{\infty}\frac{d\omega}{1 + (\omega/W)^4} = \frac{\pi W}{2\sqrt{2}}$$

****

CHAPTER

7.3

NOISE

1. The power spectral density of a bandpass white noise $n(t)$ is $\mathcal{N}/2$ as shown in fig.P.7.3.1. the value of $\overline{n^2}$ is

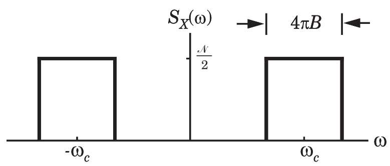

Fig. P7.3.1

(A) $\mathcal{N}B$ (B $2\mathcal{N}B$

(C) $2\pi\mathcal{N}B$ (D) $\frac{\mathcal{N}B}{\pi}$

2. In a receiver the input signal is 100 μV, while the internal noise at the input is 10 μV. With amplification the output signal is 2 V, while the output noise is 0.4 V. The noise figure of receiver is

(A) 2 (B) 0.5

(C) 0.2 (D) None of the above

3. A receiver is operated at a temperature of 300 K. The transistor used in the receiver have an average output resistance of 1 kΩ. The Johnson noise voltage for a receiver with a bandwidth of 200 kHz is

(A) 1.8 μV (B) 8.4 μV

(C) 4.3 μV (D) 12.6 μV

4. A resistor $R = 1$ kΩ is maintained at 17°C. The rms noise voltage generated in a bandwidth of 10 kHz is

(A) 16×10^{-14} V (B) 0.4 μ V

(C) 4 μ V (D) 16×10^{-18} V

5. A mixer stage has a noise figure of 20 dB. This mixer stage is preceded by an amplifier which has a noise figure of 9 dB and an available power gain of 15 dB. The overall noise figure referred to the input is

(A) 11.07 B) 18.23

(C) 56.48 (D) 97.38

6. A system has three stage cascaded amplifier each stage having a power gain of 10 dB and noise figure of 6 dB. the overall noise figure is

(A) 1.38 (B) 6.8

(C) 4.33 (D) 10.43

7. A signal process $m(t)$ is mixed with a channel noise $n(t)$. The power spectral density are as follows

$$S_m(\omega) = \frac{6}{9+\omega^2},\ S_n(\omega) = 6$$

The optimum Wiener-Hopf filter is

(A) $\frac{\omega^2+9}{\omega^2+10}$ (B) $\frac{1}{\omega^2+10}$

(C) $\frac{\omega^2+10}{\omega^2+9}$ (D) None of the above

Statement for Question 8-9

A sonar echo system on a sub marine transmits a random noise $n(t)$ to determine the distance to another targeted submarine. Distance R is given by $v\tau_R/2$ where v is the speed of the sound wave in water and τ_R is the time it takes the reflected version of $n(t)$ to return. Assume that $n(t)$ is a sample function of an ergodic random process $N(t)$ and T is very large.

8. The V will be

(A) $2R_{NN}(\tau_R - \tau_T)$ (B) $R_{NN}(\tau_R - 2\tau_T)$

(C) $R_{NN}(\tau_R - \tau_T)$ (D) $\frac{1}{2}R_{NN}(\tau_R - \tau_T)$

9. What value of the delay τ_T will cause v to be maximum ?

(A) τ_R (B) $2\tau_R$

(C) $3\tau_R$ (D) None of the above

10. Two resistor with resistance R_1 and R_2 are connected in parallel and have Physical temperatures T_1 and T_2 respectively. The effective noise temperature T_s of an equivalent resistor is

(A) $\frac{T_1R_1 + T_2R_2}{R_1 + R_2}$ (B) $\frac{T_1R_1 + T_2R_1}{R_1 + R_2}$

(C) $\frac{T_1T_2(R_1 + R_2)^2}{(T_1 + T_2)R_1R_2}$ (D) $\frac{(T_1 + T_2)R_1R_2}{T_1 + T_2(R_1 + R_2)^2}$

Statement for Question 11-12 :

An amplifier has a standard spot noise figure $F_0 = 6.31$ (8.0 dB). The amplifier, that is used to amplify the output of an antenna have antenna temperature of $T_a = 180$ K

11. The effective input noise temperature of this amplifier is

(A) 2520 K (B) 2120 K

(C) 2710 K (D) 1540 K

12. The operating spot noise figure is

(A) 3.2 dB (B) 6.4 dB

(C) 9.8 dB (D) 11.9 dB

13. An amplifier has three stages for which $T_{e1} = 200$ K (first stage), $T_{e2} = 450$ K, and $T_{e3} = 1000$K (last stage). If the available power gain of the second stage is 5, what gain must the first stage have to guarantee an effective input noise temperature of 250 K ?

(A) 10 (B) 13

(C) 16 (D) 19

Statement for Question 14-16

An amplifier has an operating spot noise figure of 10 dB when driven by a source of effective noise temperature 225 K.

14. The standard spot noise figure of amplifier is

(A) 4 dB (B) 5 dB

(C) 7 dB (D) 9 dB

15. If a matched attenuator with a loss of 3.2 dB is placed between the source and the amplifier's input, what is the operating spot noise figure of the attenuator amplifier cascade if the attenuator's physical temperature is 290 K ?

(A) 9 dB (B) 10.4 dB

(C) 11.3 dB (D) 13.3 dB

16. In previous question what is the standard spot noise figure of the cascade ?

(A) 10.3 dB (B) 12.2 dB

(C) 14.9 dB (D) 17.6 dB

17. Omega Electronics sells a microwave receiver (A) having an operating spot noise figure of 10 dB when driven by a source with effective noise temperature 130 K Digilink (B) sells a receiver with a standard spot noise figure of 6 dB. Microtronics (C) sells a receiver with standard spot noise figure of 8 dB when driven by a source with effective noise temperature 190 K. The best receiver to purchase is

(A) A (B) B

(C) C (D) all are equal

Statement for Question 18-20 :

An amplifier has three stages for which $T_{e1} = 150$ K (first stage), $T_{e2} = 350$ K, and $T_{e3} = 600$ K (output stage). Available power gain of the first stage is 10 and overall input effective noise temperature is 190 K

18. The available power gain of the second stage is

(A) 12 (B) 14

(C) 16 (D) 18

19. The cascade's standard spot noise figure is

(A) 1.3 dB (B) 2.2 dB

(C) 4.3 dB (D) 5.3 dB

20. What is the cascade's operating spot noise figure when used with a source of noise temperature $T_s = 50$ K

(A) 1.34 dB (B) 3.96 dB

(C) 6.81 dB (D) None of the above.

21. Three network are cascaded. Available power gains are $G_1 = 8$, $G_2 = 6$ and $G_3 = 20$. Respective input effective spot noise temperature are $T_{e1} = 40$ K, $T_{e2} = 100$ K and $T_{e3} = 180$ K.

(A) 58.33 K (B) 69.41 K

(C) 83.90 K (D) 98.39 K

22. Three identical amplifier, each having a spot effective input noise temperature of 125 K and available power G are cascaded. The overall spot effective input noise temperature of the cascade is 155 K. The G is

(A) 3 (B) 5

(C) 7 (D) 9

23. Three amplifier that may be connected in any order in a cascade are defined as follows:

Amplifier	Effective Input Noise Temperature	Available Power Gain
A	110 K	4
B	120 K	6
C	150 K	12

The sequence of connection that will give the lowest overall effective input noise temperature for the cascade is

(A) ABC (B) CBA

(C) ACB (D) BAC

24. What is the maximum average effective input noise temperature that an amplifier can have if its average standard noise figure is to not exceed 1.7 ?

(A) 203 K (B) 215 K

(C) 235 K (D) 255 K

25. An amplifier has an average standard noise figure of 2.0 dB and an average operating noise figure of 6.5 dB when used with a source of average effective source temperature $\overline{T}_s$. The $\overline{T}_s$ is

(A) 156.32 K (B) 100.81 K

(C) 48.93 K (D) None of the above

Statement for Question

An antenna with average noise temperature 60 K connects to a receiver through various microwave elements that can be modeled as an impedance matched attenuator with an overall loss of 2.4 dB and a physical temperatures of 275 K. The overall system noise temperature of the receiver $\overline{T}_{sys} = 820$ K.

26. The average effective input noise temperature of the receiver is

(A) 420.5 K (B) 320.5 K

(C) 220.5 K (D) 10.5 K

27. The average operating noise figure of the attenuator-receiver cascade is

(A) 13.67 d (B) 11.4 dB

(C) 1.4 dB (D) 1.367 dB

28. If receiver has an available power gain of 110 dB and a noise bandwidth of 10 MHz, the available output noise power of receiver is

(A) 6.5 mW (B) 8.9 mW

(C) 10.3 mV (D) 11.4 mV

29. If antenna attenuator cascade is considered as a noise source, its average effective noise temperature is

(A) 63 K (B) 149 K

(C) 263 K (D) 249 K

Statement for question 30-32 :

An amplifier when used with a source of average noise temperature 60 K, has an average operating noise figure of 5.

30. The $\overline{T}_e$ is

(A) 70 K (B) 110 K

(C) 149 K (D) 240 K

31. If the amplifier is sold to engineering public, the noise figure that would be quoted in a catalog is

(A) 0.46 (B) 0.94

(C) 1.83 (D) 2.93

32. What average operating noise figure results when the amplifier is used with an antenna of temperature 30 K ?

(A) 9.54 dB (B) 10.96 dB

(C) 11.23 dB (D) 12.96 dB

33. An engineer of RS communication purchase an amplifier with average operating noise figure of 1.8 when used with a $60\,\Omega$ broadband source having average source temperature of 80 K. When used with a different $60\,\Omega$ source the average operating noise figure is 1.25. The average noise temperature of the source is

(A) 125 K (B) 156 K

(C) 256 K (D) 292 K

34. The $\overline{T_e}$ for unit 1 and 2 unit are, respectively

(A) 126.4 K and 256.9 K

(B) 256.9 K and 126.4 K

(C) 527.8 K and 864.2 K

(D) 864.2 K and 527.8 K

35. The excess noise power of unit 1 and unit 2 are respectively

(A) 15.4 nW and 27.1 nW

(B) 23.8 nW and 21.1 nW

(C) 23.8 nW and 27.1 nW

(D) 15.4 nW and 21.1 nW

36. Consider following statement

S_1 : If the source noise temperature $\overline{T}_s$ is very small, unit-2 is the best to purchase

S_2 : If the source noise temperature $\overline{T}_s$ is very small unit - 1 is the best to purchase.

correct statement is

(A) S_1 (B) S_2

(C) both S_1 and S_2 (D) None

37. A source has an effective noise temperature of $T_s(\omega) = \frac{800}{100+\omega^2}$ and feeds an amplifier that has an available power gain of $G_a(\omega) = \left(\frac{8}{10+j\omega}\right)^2$. The $\overline{T}_s$ for this source is

(A) 10 K (B) 20 K

(C) 30 K (D) 40 K

38. A system have an impulse response

$$h = \begin{cases} e^{-Wt} & e < t \\ 0 & t < 0 \end{cases}$$

where W is a real positive constant. White noise with power density 5w/Hz is applied to this system. The mean-squared value of response is

(A) $1/W$ (B) $2.5/W$

(C) $4.5/W$ (D) $6/W$

39. White noise, for which $R_{XX}(\tau) = 10^{-2}8(\tau)$ is applied to a network with impulse response $h(t) = 4(t)3 - te^{-4t}$ The network's output noise power in a $1\,\Omega$ resistor is

(A) 0.15 mW (B) 0.35 mW

(C) 0.55 mW (D) 0.95 mW

40. White noise with power density $N_0/2 = 6(10^{-6})$ W/Hz is applied to an ideal fitter (gain= 1) with bandwidth W (rad/sec). For output's average noise power to be 15 W, the W must be

(A) $2.5\pi(10^{-6})$ (B) $-2.5\pi(10^{6})$

(C) $4.5\pi(10^{-2})$ (D) $4.5\pi(10^{6})$

41. An ideal filter with a mid-band power gain of 8 and bandwidth of 4 rad/s has noise $X(t)$ at its input with power spectrum $(F(2) = 0.9773)$

$$\rho_{XX}(\omega) = \left(\frac{50}{\sqrt{8\pi}}\right)e^{-\frac{\omega^2}{8}}$$

The noise power at the network's output is

(A) $\frac{164}{\pi}$ (B) $\frac{343}{\pi}$

(C) $\frac{211}{\pi}$ (D) $\frac{191}{\pi}$

42. A system has the power transfer function

$$|H(\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{W}\right)^4}$$

where W is a real positive constant. The noise bandwidth of the system is

(A) $\frac{\pi W}{2\sqrt{2}}$ (B) $\frac{\pi W}{\sqrt{2}}$

(C) $\frac{\pi W}{2}$ (D) None of the above

43. White noise with power density $\mathcal{N}_0/2$ is applied to a low pass network for which $|H(0)| = 2$. It has a noise bandwidth of 2 MHz. If the average output noise power is 8.1 W in a $1\,\Omega$ resistor, the $\mathcal{N}_0$ is

(A) 6.25×10^{8} W/Hz (B) 6.25×10^{-8} W/Hz

(C) 1.25×10^{8} W/Hz (D) 1.25×10^{-8} W/Hz

Statement for Question 44-46 :

An amplifier has a narrow bandwidth of 1 kHz and standard spot noise figure of 3.8 at its frequency of operation. The amplifier's available output noise power is 0.1 mW when its input is connected to a radio receiving antenna having an antenna temperature of 80 K.

44. The amplifier's input effective noise temperature T_e is

(A) 812 K (B) 600 K

(C) 421 K (D) 321 K

45. Its operating spot noise figure F_{op} is

(A) 5.16 (B) 7.98

(C) 11.15 (D) 16.23

46. Its available power gain G_a is

(A) 2×10^{12} (B) 4×10^{12}

(C) 8×10^{12} (D) 11×10^{12}

SOLUTION

1. (B) $\overline{n^2} = 2\int_{f_c+B}^{f_c+B} \frac{\mathcal{N}}{2} df = 2\mathcal{N}B$

2. (A) $\text{NF} = \frac{S_i/N_i}{S_o/N_o} = \frac{100/10}{2/0.4} = 2$

3. (A) $v_n^2 = 4kTBR$

$= 4 \times 1.38 \times 10^{-23} \times 300 \times 200 \times 10^3 \times 10^3 = 3.3 \times 10^{-12}$

$v_{nrms} = 1.8\mu$ V

4. (B) $\overline{v_n^2} = 4kTBR$, $T = (273 + 17)K = 290$ K,

$R = 1000\Omega$, $B = 10^4$ Hz, $k = 1.38 \times 10^{-23}$ J/K

$\overline{v_n^2} = 4 \times 1.38 \times 10^{-23} \times 290 \times 10^3 \times 10^4 = 16 \times 10^{-14}\ V^2$

$v_{nrms} = 0.4\mu$ V

5. (A) $F_1 = 9$ dB = 7.94, F_2 = 20 dB = 100

$A_1 = 15$ dB = 31.62,

$$F = F_1 + \frac{F_2 - 1}{A} = 7.94 + \frac{100 - 1}{31.62} = 11.07$$

6. (C) Gain of each stage $A_1 = A_2 = A_3 = 10$ dB

Noise figure of each stage

$F_1 = F_2 = F_3 = 6$ dB or $F_1 = F_2 = F_3 = 4$ db

$$F = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1 A_2} = 4 + \frac{4 - 1}{10} + \frac{4 - 1}{100} = 4.33$$

7. (B) $H_{op}(\omega) = \frac{S_m(\omega)}{S_m(\omega) + S_m(\omega)} = \frac{\frac{6}{9+\omega^2}}{\frac{6}{9+\omega^2} + 6} = \frac{1}{10 + \omega^2}$

8. (C) $V = \frac{1}{2T}\int_{-T}^{T} n(t - \tau_T)n(t - \tau_R)dt$

Since T is very large

$$V = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} n(t - \tau_T)n(t - \tau_R)dt = A[n(t - \tau_T)n(t - \tau_R)]$$

Since $N(t)$ is ergodic, $V \approx R_{NN}(\tau_R - \tau_T)$

9. (A) Because $|R_{NN}(\tau)| \le R_{NN}(0)$ for any auto correlation function, V will be maximum if $\tau_R = \tau_T$

10. (B) Use the current form of equivalent circuit

$i_n^2 = i_1^2 + i_2^2 = \frac{2kT_1 d\omega}{\pi R_1} + \frac{2kT_2 d\omega}{\pi R_2}$ where $i_n^2 = \frac{2kT_s d\omega}{\pi R}$,

Thus $T_s = \left(\frac{T_1}{R_1} + \frac{T_2}{R_2}\right)R = \frac{T_1 R_2 + T_2 R_1}{R_1 + R_2}$

11. (D) $T_e = T_0(F_0 - 1) = 290(6.31 - 1) = 1539.9$ K

12. (C) $F_{op} = 1 + \frac{T_e}{T_a} = 1 + \frac{1540}{180} = 9.56$ or 9.8 dB

13. (B) $T_e = 250 = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_2}$

$250 = 200 + \frac{450}{G_1} + \frac{1000}{5G_2}$ or $G_1 = 13$

14. (D) $F_0 = 1 + \frac{T_s}{T_0}(F_{op} - 1) = 1 + \frac{225}{290}(10 - 1)$

$= 7.98$ or 9.0 dB

15. (D) Here $L = 2.089$ or 3.2 dB, $T_L = 290$ K

$T_e = T_{e1} + \frac{T_{e2}}{G_1} = T_L(L-1) + \frac{T_0(F_0 - 1)}{1/L}$

$= 290[(2.089 - 1) + (2.089)(7.98 - 1)] = 4544.4$ K

$F_{op} = 1 + \frac{4544.4}{225} = 21.2$ or 13.3 dB

16. (B) $F_0 = 1 + \frac{4544.4}{290} = 16.67$ or 12.2 dB

17. (B) For A: $F_{op} = 10$(or 10 dB) when $T_s = 130$ K

$T_{eA} = 130(10 - 1) = 1170$ K

For B: $F_o = 3.98$(or 6 dB) when $T_s = 290$ K

$T_{eB} = 290(3.98 - 1) = 364.2$ K

For C: $F_o = 6.3$(or 8 dB) when $T_s = 190$ K

$T_{eC} = 190(6.3 - 1) = 1007$ K, (B) is better as T_{eB} is less.

18. (A) $T_e = T_{e1} + \frac{T_{e1}}{G_1} + \frac{T_{e3}}{G_1 G_2}$

$G_2 = \frac{T_{e3}}{G_1(T_e - T_{e1} - \frac{T_{e2}}{G_1})} = \frac{600}{10(190 - 150 - \frac{350}{10})} = 12$

19. (B) $F_0 = 1 + \frac{T_e}{T_0} = 1 + \frac{190}{290} = 1.655$ or 2.19 dB

20. (C) $F_{op} = 1 + \frac{T_e}{T_s} = 1 + \frac{190}{50} = 4.8$ or 6.81 dB

21. (A) $T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} = 40 + \frac{100}{8} + \frac{280}{8(6)} = 58.33$ K

22. (B) $T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} = T_{e1}\left[1 + \frac{1}{G} + \frac{1}{G^2}\right]$

or $(T_{e1} - T_e)G^2 + T_{e1}G + T_{e1} = 0$

$(125 - 155)G^2 + 125G + 125 = 0$

$6G^2 - 25G - 25 = 0$ or $G = 5$

23. (A) Sequence T_e

ABC$110 + \frac{120}{4} + \frac{150}{4(6)} = 146.25$

ACB$110 + \frac{150}{4} + \frac{120}{4(12)} = 150.00$

BAC $120 + \frac{110}{6} + \frac{150}{6(4)} = 144.583 \leftarrow$ Best

CBA$150 + \frac{120}{12} + \frac{110}{(12)(6)} = 161.528$

24. (A) $\overline{T}_e = T_0(\overline{F} - 1) \le 290(1.7 - 1) = 203$ K

25. (D) Here $\overline{F}_0 \approx 1.585$ (or 2.0 dB) and $\overline{F}_{OP} \approx 4.467$ (or 6.5 dB)

$\overline{T}_s = \frac{T_0(\overline{F}_0 - 1)}{\overline{F}_{op} - 1} = \frac{290(1.585 - 1)}{4.467 - 1} = 48.93$ K

26. (B) Here T_a = 60 K, $L = 1.738$ (or 2.4 dB), T_L = 275 K and $\overline{T}_{sys} = 820$ K. We know that

$\overline{T}_R = \frac{[\overline{T}_{sys} - T_a - T_L(L-1)]}{L} = \frac{820 - 60 - 275(1.738 - 1)}{1.738}$

$= 320.5$ K

27. (B) $\overline{F}_{op} = 1 + \frac{\overline{T}_e}{\overline{T}_s} = 1 + \frac{\overline{T}_{sys} - T_a}{\overline{T}_s} = 1 + \frac{820 - 60}{60}$

$= 13.67$ or 11.4 dB

28. (A) Here $G_R(\omega_0) = 10^{11}$ (or 110 dB) and W_{PV} = $2\pi(10^7)$ Hz

$N_{clo} = \frac{k\overline{T}_{sys}G_R(\omega)W_n}{2\pi L} = \frac{1.38(10^{-23})(820)(10^{11})(10^7)}{1.738}$

$= 651.110^{-5}$ or 6.51 mW

29. (C) $dN_{ao} = k[T_a + T_L(L-1)]\frac{d\omega}{2\pi} = kT_s\frac{d\omega}{2\pi}$

Thus $T_s = T_a + T_L(L-1)$

$= 60 + 275(1.738 - 1) = 263$ K

30. (D) $\overline{T}_e = \overline{T}_s(\overline{F}_{op} - 1) = 60(5 - 1) = 240$ K

31. (C) $\overline{F}_o = 1 + \frac{\overline{T}_e}{290} = 1 + \frac{240}{290} = 1.8276$

32. (A) $\overline{F}_{op} = 1 + \frac{\overline{T}_e}{\overline{T}_s} = 1 + \frac{240}{30} = 9$ or 9.54 dB

CHAPTER

7.4

AMPLITUDE MODULATION

Statement for Question 1 - 3

An AM signal is represented by

$$x(t) = (20 + 4\sin 500\pi t)\cos(2\pi \times 10^5 t)\ \text{V}$$

1. The modulation index is

(A) 20 (B) 4
(C) 0.2 (D) 10

2. The total signal power is

(A) 208 W (B) 204 W
(C) 408 W (D) 416 W

3. The total sideband power is

(A) 4 W (B) 8 W
(C) 16 W (D) 2 W

Statement for Question 4 - 5 :

An AM signal has the form $x(t) = [20 + 2\cos 3000\pi t + 10\cos 6000\pi t]\cos 2\pi f_c t$ where $f_c = 10^5$ Hz.

4. The modulation index is

(A) $\frac{201}{400}$ (B) $-\frac{201}{400}$
(C) $\frac{199}{400}$ (D) $-\frac{199}{400}$

5. The ratio of the sidebands power to the total power is

(A) $\frac{43}{226}$ (B) $\frac{26}{226}$
(C) $\frac{26}{226}$ (D) $\frac{43}{224}$

6. A 2 kW carrier is to be modulated to a 90% level. The total transmitted power would be

(A) 3.62 kW (B) 2.81 kW
(C) 1.4 kW (D) None of the above

7. An AM broadcast station operates at its maximum allowed total output of 50 kW with 80% modulation. The power in the intelligence part is

(A) 12.12 kW (B) 31.12 kW
(C) 6.42 kW (D) None of the above

8. The aerial current of an AM transmitter is 18 A when unmodulated but increases to 20 A when modulated.The modulation index is

(A) 0.68 (B) 0.73
(C) 0.89 (D) None fo the above

9. A modulating signal is amplified by a 80% efficiency amplifier before being combined with a 20 kW carrier to generate an AM signal. The required DC input power to the amplifier, for the system to operate at 100% modulation, would be

(A) 5 kW (B) 8.46 kW
(C) 12.5 kW (D) 6.25 kW

10. A 2 MHz carrier is amplitude modulated by a 500 Hz modulating signal to a depth of 70%. If the unmodulated carrier power is 2 kW, the power of the modulated signal is

(A) 2.23 kW (B) 2.36 kW
(C) 1.18 kW (D) 1.26 kW

11. A carrier is simultaneously modulated by two sine waves with modulation indices of 0.4 and 0.3. The resultant modulation index will be

(A) 1.0 (B) 0.7
(C) 0.5 (D) 0.35

12. In a DSB-SC system with 100% modulation, the power saving is

(A) 50% (B) 66%

(C) 75% (D) 100%

13. A 10 kW carrier is sinusoidally modulated by two carriers corresponding to a modulation index of 30% and 40% respectively. The total radiated power(is

(A) 11.25 kW (B) 12.5 kW

(C) 15 kW (D) 17 kW

14. In amplitude modulation, the modulation envelope has a peak value which is double the unmodulated carrier value. What is the value of the modulation index ?

(A) 25% (B) 50%

(C) 75% (D) 100%

15. If the modulation index of an AM wave is changed from 0 to 1, the transmitted power

(A) increases by 50% (B) increases by 75%

(C) increases by 100% (D) remains unaffected

16. A diode detector has a load of 1 kΩ shunted by a 10000 pF capacitor. The diode has a forward resistance of 1 Ω. The maximum permissible depth of modulation, so as to avoid diagonal clipping, with modulating signal frequency fo 10 kHz will be

(A) 0.847 (B) 0.628

(C) 0.734 (D) None of the above

17. An AM signal is detected using an envelop detector. The carrier frequency and modulating signal frequency are 1 MHz and 2 kHz respectively. An appropriate value for the time constant of the envelope detector is.

(A) 500 μ sec (B) 20 μ sec

(C) 0.2 μ sec (D) 1 μ sec

18. An AM voltage signal $s(t)$, with a carrier frequency of 1.15 GHz has a complex envelope $g(t) = A_C[1 + m(t)]$, where $A_c = 500$ V, and the modulation is a 1 kHz sinusoidal test tone described by $m(t) = 0.8\sin(2\pi \times 10^3 t)$ appears across a 50 Ω resistive load. What is the actual power dissipated in the load ?

(A) 165 kW (B) 82.5 kW

(C) 3.3 kW (D) 6.6 kW

19. A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of period 100 μ sec. Which of the following frequencies will NOT be present in the modulated signal?

(A) 990 KHz (B) 1010 KHz

(C) 1020 KHz (D) 1030 KHz

20. For an AM signal, the bandwidth is 10 kHz and the highest frequency component present is 705 kHz. The carrier frequency used for this AM signal is

(A) 695 kHz (B) 700 kHz

(C) 705 kHz (D) 710 kHz

21. A message signal $m(t) = \text{sinc}\, t + \text{sinc}^2(t)$ modulates the carrier signal $(t) = A\cos 2\pi f_c t$. The bandwidth of the modulated signal is

(A) $2f_c$ (B) $\frac{1}{2}f_c$

(C) 2 (D) $\frac{1}{4}$

22. The signal $m(t) = \cos 2000\pi t + 2\cos 4000t$ is multiplied by the carrier $c(t) = 100\cos 2\pi f_c t$ where $f_c = 1$ MHz to produce the DSB signal. The expression for the upper side band (USB) signal is

(A) $100\cos(2\pi(f_c + 1000)t) + 200\cos(2\pi(f_c + 200)t)$

(B) $100\cos(2\pi(f_c - 1000)t) + 200\cos(2\pi(f_c - 2000)t)$

(C) $50\cos(2\pi(f_c + 1000)t) + 100\cos(2\pi(f_c + 2000)t)$

(D) $50\cos(2\pi(f_c - 1000)t) + 100\cos(2\pi(f_c - 100)t)$

Statement for Question 23-26 :

The Fourier transform $M(f)$ of a signal $m(t)$ is shown in figure. It is to be transmitted from a source to destination. It is known that the signal is normalized, meaning that $-1 \le m(t) \le 1$

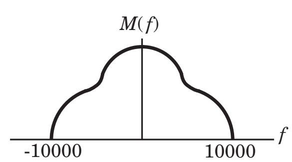

Fig.P7.4.23-26

23. If USSB is employed, the bandwidth of the modulated signal is

(A) 5 kHz (B) 20 kHz, 10 kHz

(C) 20 kHz (D) None of the above

24. If DSB is employed, the bandwidth of the modulated signal is

(A) 5 kHz (B) 10 kHz

(C) 20 kHz (D) None of the above

25. If an AM modulation scheme with $\alpha = 0.8$ is used, the bandwidth of the modulated signal is.

(A) 5 kHz (B) 10 kHz

(C) 20 kHz (D) None of the above

26. If an FM signal with $k_f = 60$ kHz is used, then the bandwidth of the modulated signal is

(A) 5 kHz (B) 10 kHz

(C) 20 kHz (D) None of the above

27. A DSB modulated signal $x(t) = Am(t)\cos 2\pi f_c t$ is mixed (multiplied) with a local carrier $x_L(t) = \cos(2\pi f_c t + \theta)$ and the output is passed through a LPF with a bandwidth equal to the bandwidth of the message $m(t)$. If the power of the signal at the output of the low pass filter is p_{out} and the power of the modulated signal by p_u, the $\frac{p_{out}}{p_u}$ is

(A) $0.5\cos\theta$ (B) $\cos^2\theta$

(C) $0.5\cos^2\theta$ (D) $\frac{1}{2}\cos^2\theta$

28. A DSB-SC signal is to be generated with a carrier frequency $f_c = 1$ MHz using a non-linear device with the input-output characteristic $v_o = a_0 v_i + a_1 v_i^3$ where a_0 and a_1 are constants. The output of the non-linear device can be filtered by an appropriate band-pass filter. Let $v_i = A'_c \cos(2\pi f'_c t) + m(t)$ where $m(t)$ is the message signal. Then the value of f'_c(in MHz) is

(A) 1.0 (B) 0.333

(C) 0.5 (D) 3.0

29. A non-linear device with a transfer characteristic given by $i = (10 + 2v_i + 0.2v_i^2)$ mA is supplied with a carrier of 1 V amplitude and a sinusoidal signal of 0.5 V amplitude in series. If at the output the frequency component of AM signal is considered, the depth of modulation is

(A) 18 % (B) 10 %

(C) 20 % (D) 33.33 %

Statement for Question 30-31

Consider the system shown in figP7.4.30-31. The modulating signal $m(t)$ has zero mean and its maximum (absolute) value is $A_m = \max|m(t)|$. It has bandwidth W_m. The nonlinear device has a input-output characteristic $y(t) = ax(t) + bx^2(t)$.

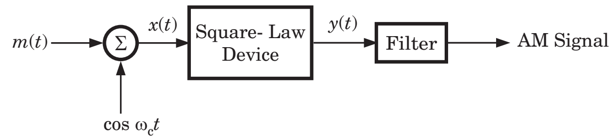

Fig.P7.4.30-31

30. The filter should be a

(A) LPP with bandwidth W

(B) LPF with bandwidth $2W$

(C) a BPF with center frequency f_0 and $BW = W$ such that $f_0 - W_m > f_0 - \frac{W}{2} > 2W_m$

(D) a BPF with center frequency f_0 and $BW = W$ such that $f_0 - W_m > f_0 - \frac{W}{2} > W_m$

31. The modulation index is

(A) $\frac{2b}{a}A_m$ (B) $\frac{2a}{b}A_m$

(C) $\frac{a}{b}A_m$ (D) $\frac{b}{a}A_m$

32. A message signal is periodic with period T, as shown in figure. This message signals is applied to an AM modulator with modulation index $\alpha = 0.4$. The modulation efficiency would be

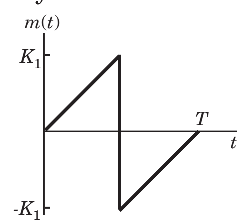

Fig.P7.4.32

(A) 51 % (B) 11.8 %

(C) 5.1 % (D) None of the above

Statement for Question 33-36

The figure 6.54-57 shows the positive portion of the envelope of the output of an AM modulator. The message signal is a waveform having zero DC value.

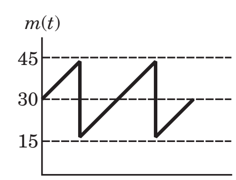

Fig.P7.4.33-36

33. The modulation index is

(A) 0.5 (B) 0.6

(C) 0.4 (D) 0.8

34. The modulation efficiency is

(A) 8.3 % (B) 14.28 %

(C) 7.69 % (D) None of the above

35. The carrier power is

(A) 60 W (B) 450 W

(C) 30 W (D) 900 W

36. The power in sidebands is

(A) 85 W (B) 42.5 W

(C) 56 W (D) 37.5 W

37. In a broadcast transmitter, the RF output is represented as

$$e(t) = 50[1 + 0.89\cos 5000t + 0.30\sin 9000t]\cos(6\times 10^6 t)\text{V}$$

What are the sidebands of the signals in radians ?

(A) 5×10^3 and 9×10^3

(B) 5.991×10^6, 5.995×10^6, 6.005×10^6 and 6.009×10^6

(C) 4×10^3, 1.4×10^4

(D) 1×10^6, 1.1×10^7, 3×10^6, and 1.5×10^7

38. An AM modulator has output

$$x(t) = 40\cos 400\pi t + 4\cos 360\pi t + 4\cos 440\pi t$$

The modulation efficiency is

(A) 0.01 (B) 0.02

(C) 0.03 (D) 0.04

39. An AM modulator has output

$$x(t) = A\cos 400\pi t + B\cos 380\pi t + B\cos 420\pi t$$

The carrier power is 100 W and the efficiency is 40%. The value of A and B are

(A) 14.14, 8.16 (B) 50, 10

(C) 22.36, 13.46 (D) None of the above

Statement for Question 40-41

A single side band signal is generated by modulating signal of 900-kHz carrier by the signal $m(t) = \cos 200\pi t + 2\sin 2000\pi t$. The amplitude of the carrier is $A_c = 100$.

40. The signal $\hat{m}(t)$ is

(A) $-\sin(2\pi 1000t) - 2\cos(2000\pi t)$

(B) $-\sin(2\pi 1000t) + 2\cos(2000\pi t)$

(C) $\sin(2\pi 1000t) + 2\cos(1000t)$

(D) $\sin(2\pi 1000t) - 2\cos(2\pi 1000t)$

41. The lower sideband of the SSB AM signal is

(A) $-100\cos(2\pi(f_c - 1000)t) + 200\sin(2\pi(f_c - 1000)t)$

(B) $-100\cos(2\pi(f_c - 1000)t) - 200\sin(2\pi(f_c - 1000)t)$

(C) $100\cos(2\pi(f_c - 1000)t) - 200\sin(2\pi(f_c - 1000)t)$

(D) $100\cos(2\pi(f_c - 1000)t) + 200\sin(2\pi(f_c - 1000)t)$

Statement for Question 42-43

Consider the system shown in figure 6.69-70. The average value of $m(t)$ is zero and maximum value of $|m(t)|$ is M. The square-law device is defined by $y(t) = 4x(t) + 10x(t)$.

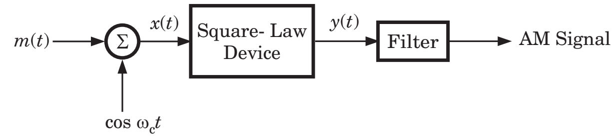

Fig. P7.4.42-43

42. The value of M, required to produce modulation index of 0.8, is

(A) 0.32 (B) 0.26

(C) 0.52 (D) 0.16

43. Let W be the bandwidth of message signal $m(t)$. AM signal would be recovered if

(A) $f_c > W$ (B) $f_c > 2W$

(C) $f_c \geq 3W$ (D) $f_c > 4W$

44. A super heterodyne receiver is designed to receive transmitted signals between 5 and 10 MHz. High-side tuning is to be used. The tuning range of the local oscillator for IF frequency 500 kHz would be

(A) 4.5 MHz - 9.5 MHz

(B) 5.5 MHz - 10.5 MHz

(C) 4.5 MHz - 10.5 MHz

(D) None of the above

45. A super heterodyne receiver uses an IF frequency of 455 kHz. The receiver is tuned to a transmitter having a carrier frequency of 2400 kHz. High-side tuning is to be used. The image frequency will be

(A) 2855 kHz (B) 3310 kHz

(C) 1845 kHz (D) 1490 kHz

46. In the circuit shown in fig.P7.4.46, the transformers are center tapped and the diodes are connected as shown in a bridge. Between the terminals 1 and 2 an a.c. voltage source of frequency 400 Hz is connected. Another a.c. voltage of 1.0 MHz is connected between 3 and 4. The output between 5 and 6 contains components at

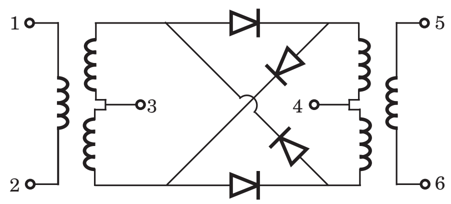

Fig.P7.4.46

(A) 400 Hz, 1.0 MHz, 1000.4 kHz, 999.6 kHz

(B) 400 Hz, 1000.4 kHz, 999.6 kHz

(C) 1 MHz, 1000.4 kHz, 999.6 kHz

(D) 1000.4 kHz, 999.6 kHz

47. A superheterodyne receiver is to operate in the frequency range 550 kHz-1650 kHz, with the intermediate frequency of 450 kHz. Let $R = \frac{C_{max}}{C_{min}}$ denote the required capacitance ratio of the local oscillator and I denote the image frequency (in kHz) of the incoming signal. If the receiver is tuned to 700 kHz, then

(A) $R = 4.41$, $I = 1600$ (B) $R = 2.10$, $I = 1150$

(C) $R = 3$, $I = 1600$ (D) $R = 9.0$, $I = 1150$

48. Consider a system shown in Figure . Let $X(f)$ and $Y(f)$ denote the Fourier transforms of $x(t)$ and $y(t)$ respectively. The ideal HPF has the cutoff frequency 10 kHz. The positive frequencies where $Y(f)$ has spectral peaks are

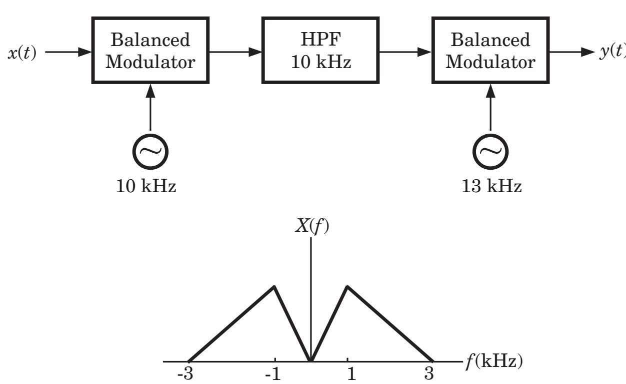

Fig.P7.4.48

(A) 1 kHz and 24 kHz

(B) 2 kHz and 24 kHz

(C) 1 kHz and 14 kHz

(D) 2 kHz and 14 kHz

49. In fig.P7.4.49

$$m(t) = \frac{2\sin 2\pi t}{t},\ s(t) = \cos 200\pi t \text{ and } n(t) = \frac{\sin 199\pi t}{t}$$

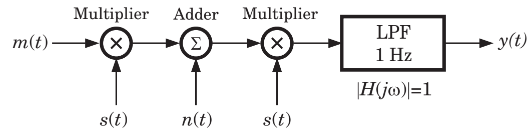

Fig.P7.4.49

The output $y(t)$ will be

(A) $\frac{\sin 2\pi t}{t}$

(B) $\frac{\sin 2\pi t}{t} + \frac{\sin \pi t}{t}\cos 3\pi t$

(C) $\frac{\sin 2\pi t}{t} + \frac{\sin 0.5\pi t}{t}\cos 1.5\pi t$

(D) $\frac{\sin 2\pi t}{t} + \frac{\sin \pi t}{t}\cos 0.75\pi t$

50. 12 signals each band-limited to 5 kHz are to be transmitted over a single channel by frequency division multiplexing. If AM -SSNB modulation guard band of 1 kHz is used, then the bandwidth of the multiplexed signal will be

(A) 51 kHz (B) 61 kHz

(C) 71 kHz (D) 81 kHz

51. Let $x(t)$ be a signal band-limited to 1 kHz. Amplitude modulation is performed to produce signal $g(t) = x(t)\sin 2000\pi t$. A proposed demodulation technique is illustrated in figure 6.83. The ideal low pass filter has cutoff frequency 1 kHz and pass band gain 2. The $y(t)$ would be

(A) $2y(t)$ (B) $y(t)$

(C) $\frac{1}{2}y(t)$ (D) 0

52. Suppose we wish to transmit the signal $x(t) = \sin 200\pi t + 2\sin 400\pi t$ using a modulation that create the signal $g(t) = x(t)\sin 400\pi t$. If the product $g(t)\sin 400\pi t$ is passed through an ideal LPF with cutoff frequency 400π and pass band gain of 2, the signal obtained at the output of the LPF is

(A) $\sin 200\pi t$ (B) $\frac{1}{2}\sin 200\pi t$

(C) $2\sin 200\pi t$ (D) 0

53. In a AM signal the received signal power is 10^{-10} W with a maximum modulating signal of 5 kHz. The noise spectral density at the receiver input is 10^{-18} W/Hz. If the noise power is restricted to the message signal bandwidth only, the signals-to-noise ratio at the input to the receiver is

(A) 43 dB (B) 66 dB

(C) 56 dB (D) 33 dB

Statement for Question 54-55

Consider the following Amplitude Modulated (AM) signal, where $f_m < B$

$$x_{AM}(t) = 10(1 + 0.5\sin 2\pi f_m t)\cos 2\pi f_c t.$$

54. The average side-band power for the AM signal given above is

(A) 25 (B) 12.5

(C) 6.25 (D) 3.125

55. The AM signal gets added to a noise with Power Spectra Density $S_n(f)$ given in the figure below. The ration of average sideband power to mean noise power would be

(A) $\frac{25}{8N_0B}$ (B) $\frac{25}{4N_0B}$

(C) $\frac{25}{2N_0B}$ (D) $\frac{25}{N_0B}$

Statement for Question 56-57

A certain communication channel is characterized by 80 dB attenuation and noise power-spectral density of 10^{-10}W/Hz. The transmitter power is 40 kW and the message signal has a bandwidth of 10 kHz.

56. In the case of conventional AM modulation, the predetecion SNR is

(A) 10^8 (B) 2×10^8

(C) 10^2 (D) 2×10^2

57. In case of SSB, the predetecion SNR is

(A) 2×10^2 (B) 4×10^2

(C) 2×10^3 (D) 4×10^3

****

SOLUTION

1. (C) $u(t) = (20 + 4\sin 500\pi t)\cos(2\pi\times10^5 t)$ V

$= 20(1 + 0.2\sin 500\pi t)\cos(2\pi\times10^5 t)$ V, $\alpha = 0.2$

2. (B) $P_c = \frac{20^2}{2} = 200$ W, $P_t = P_c\left(1 + \frac{(0.2)^2}{2}\right) = 204$ W

3. (A) $P_{sb} = P_t - P_c = 204 - 200 = 4$ W

4. (B) $x(t)$

$= [20 + 2\cos(2\pi1500t) + 10\cos(2\pi3000t)]\cos(2\pi f_c t)$

$= 20\left(1 + \frac{1}{10}\cos(2\pi1500t) + \frac{1}{2}\cos(2\pi3000t)\cos(2\pi f_c t)\right)$

This is the form of a conventional AM signal with message signal

$m(t) = \frac{1}{10}\cos(2\pi1500t) + \frac{1}{2}\cos(2\pi3000t)$

$= \cos^2(2\pi1500t) + \frac{1}{10}\cos(2\pi1500t) - \frac{1}{2}$

The minimum of $g(z) = z^2 + \frac{1}{10}z - \frac{1}{2}$ is achieved for $z = -\frac{1}{20}$ and it is $\min(g(z)) = -\frac{201}{400}$. Since $z = -\frac{1}{20}$ is in the range of cos $(2\pi1500t)$, we conclude that the minimum value of $m(t)$ is $-\frac{201}{400}$. Hence, the modulation index is $\alpha = -\frac{201}{400}$

5. (B) $x(t) = 20\cos(2\pi f_c t) + \cos(2\pi(f_c - 1500)t)$

$+ \cos(2\pi(f_c - 1500)t)$

$= 5\cos(2\pi(f_c 3000)t) + 5\cos(2\pi(f_c + 3000)t)$

The power in the sidebands is

$P_{\text{sidebands}} = \frac{1}{2} + \frac{1}{2} + \frac{25}{2} + \frac{25}{2} = 26$

The total power is $P_{\text{total}} = P_{\text{carrier}} + P_{\text{sidebands}} = 200 + 26 = 226$

The ratio of the sidebands power to the total power is

$\frac{\text{P}_{\text{sidebands}}}{\text{P}_{\text{total}}} = \frac{26}{226}$

6. (B) $P_t = P_c\left(1 + \frac{\alpha^2}{2}\right) = 2000\left(1 + \frac{0.9^2}{2}\right) = 2810$ W

7. (A) $P_t = P_c\left(1 + \frac{\alpha^2}{2}\right)$ or $50\times10^3 = P_c\left(1 + \frac{0.8^2}{2}\right)$

$P_c = 37.88$ kW, $P_i = (P_t - P_c) = (50 - 37.88) = 12.12$ kW

8. (A) $I_t = I_c\left(1+\frac{\alpha^2}{2}\right)^{\frac{1}{2}}$ or $20 = 18\left(1+\frac{\alpha^2}{2}\right)^{\frac{1}{2}}$ or $\alpha = 0.68$

9. (C) $P_t = 20000\left(1+\frac{1}{2}\right)$, $P_t = 30$ kW,

$P_i = 30 - 20 = 10$ kW

The DC input power $= \frac{10}{0.8} = 12.5$ kW.

10. (A) $P_c = 2$ kW, $\alpha = 70\% = 0.7$

$$P_t = P_c\left(1+\frac{\alpha^2}{2}\right)^{\frac{1}{2}} = 2\left(1+\frac{0.7^2}{2}\right) = 2.23 \text{ kW}$$

11. (C) $\alpha^2 = \alpha_1^a + \alpha_2^2 = 0.3^2 + 0.4^2 = 0.5^2$ or $\alpha = 0.5$

12. (B) In previous solution $P_c = \frac{2}{3}P$. If carrier is suppressed then $\frac{2}{3}P$ or 66% power will be saved.

13. (A) $P_t = P_c\left(1+\frac{\alpha_1^2}{2}+\frac{\alpha_2^2}{2}\right) = 10\left(1+\frac{0.3^2}{2}+\frac{0.4^2}{2}\right)$

$= 11.25$ kW

14. (D) $x(t) = A_c(1+\alpha\cos 2\pi f_m t)\cos 2\pi f_c t$

Here $A_c(1+\alpha) = 2A_c$, Thus $\alpha = 1$, therefor modulation index is 1 or 100% modulation.

15. (A) If modulation index α is 0, then

$$P_{t1} = \frac{A_c^2}{2}\left(1+\frac{0^2}{2}\right) = \frac{A_c^2}{2}$$

If modulation index is 1 then

$$P_{t2} = \frac{A_c^2}{2}\left(1+\frac{1^2}{2}\right) = \frac{3}{4}A_c^2, \quad \frac{P_{t2}}{P_{t1}} = \frac{3}{2}$$

Thus $P_{t2} = 1.5P_{t1}$ and P_{t2} is increases by 50%

16. (A) $f_m = 10$ kHz, $R = 1000\,\Omega$, $C = 10000$ pF

Hence $2\pi f_m RC = 2\pi\times 10^4 \times 10^3 \times 10^{-8} = 0.628$

$$\alpha_{max} = (1+(0.628)^2)^{-\frac{1}{2}} = 0.847$$

17. (B) $\frac{1}{f_c} \le RC \le \frac{1}{BW_m}$, Here $f_c = 1$ MHz

Signal Bandwidth $BW_m = 2f_m = 2\times 2\times 10^3 = 4$ kHz

Thus $\frac{1}{10^6} \le RC \le \frac{1}{4\times 10^3}$ or $10^{-6} \le RC \le 250\,\mu s$

Thus appropriate value is 20 μ sec

18. (A) $P_t = \frac{A_c^2}{2}\left[1+\frac{\alpha^2|m(t)|^2}{2}\right]$

Here modulation index $\alpha = 1$. Thus

$$P_t = \frac{500^2}{2}\left[1+\frac{0.8^2}{2}\right] = 165 \text{ kW}$$

19. (C) $c(t) = \sin 2\pi f_c t$, $f_c = 1000$ kHz, $x(t) = c(t)m(t)$

Expressing square wave as modulating signal $m(t)$

$$m(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_m(2n-1)]$$

The modulated output

$$x(t) = \left[\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_m(2n-1)]\right]\sin(2\pi 1000\times 10^3 t)$$

So frequency component $(10^6 \pm f_m(2n-1)$ will be present where $n = 1, 2, 3, \ldots$.

For $f_m = 10$ kHz and $n = 1$ & 2 frequency present is 990, 970, 1030 kHz. Thus 1020 kHz will be absent.

20. (B) $f_c + f_m = 705$ kHz,

$BW = 2f_m = 10$ kHz or $f_m = 5$ kHz

$f_c = 705 - 5 = 700$ kHz

21. (C) $x(t) = m(t)\,c(t) = A(\mathrm{sinc}(t) + \mathrm{sinc}^2(t)\cos(2\pi f_c t)$

Taking the Fourier transform of both sides, we obtain

$$X(f) = \frac{A}{2}[\Pi(f) + \Lambda(f)] * (\delta(f-f_c) + \delta(f+f_c))$$

$$= \frac{A}{2}[\Pi(f-f_c) + \Lambda(f-f_c) + \Pi(f+f_c) + \Lambda(f-f_c)]$$

Since $\Pi(f-f_c) \ne 0$ for $|f-f_c| < \frac{1}{2}$, whereas $\Lambda(f-f_c) \ne 0$ for $|f-f_c| < 1$. Hence, the bandwidth of the bandpass filter is 2.

22. (C) $x(t) = m(t)\,c(t)$

$= 100[\cos(2\pi 1000t) + 2\cos(2\pi 2000t)]\cos(2\pi f_c t)$

$= 100\cos(2\pi 1000t)\cos(2\pi f_c t) + 200\cos(2\pi 2000t)\cos(2\pi f_c t)$

$= \frac{100}{2}[\cos(2\pi(f_c+1000)t) + \cos(2\pi(f_c-1000)t)]$

$+ \frac{200}{2}[\cos(2\pi(f_c+2000)t) + \cos(2\pi(f_c-2000)t)]$

Thus, the upper sideband (USB) signal is

$x_u(t) = 50\cos[2\pi(f_c+1000)t] + 100(2\pi(f_c+2000)t)$

23. (B) When USSB is employed the bandwidth of the modulated signal is the same with the bandwidth of the message signal. Hence $W_{USSB} = W = 10^4$ Hz

37. (B) Sidebands are $(6\times10^6 \pm 5000)$ and $(6\times10^6 \pm 9000)$

Thus 6.005×10^6, 5.995×10^6, 5.991×10^6 or 5.991×10^6, 6.005×10^6 and 6.009×10^6

38. (B) $x(t)$ can be written as

$x(t) = (40 + 8\cos 40\pi t)\cos 400\pi t$

modulation index $\alpha = \frac{8}{40} = 0.2$

$P_c = \frac{1}{2}(40)^2 = 800$ W

The components at 180 Hz and 220 Hz are side band

$P_{sb} = \frac{1}{2}(4)^2 + \frac{1}{2}(4)^2 = 16$ W,

$$E_{eff} = \frac{P_{sb}}{P_c + P_{sb}} = \frac{16}{800+16}$$

39. (A) Carrier power $P_c = \frac{A^2}{2} = 100$ W, $A = 14.14$

$$E_{eff} = \frac{P_{sb}}{P_c + P_{sb}} = \frac{40}{100} \quad \text{or} \quad \frac{P_{sb}}{100 + P_{sb}} = 0.4$$

$P_{sb} = 66.67$ W, $\quad P_{sb} = \frac{1}{2}B^2 + \frac{1}{2}B^2 = 66.67 \quad$ or $B = 8.161$

40. (D) The Hilbert transform of $\cos(2\pi 1000t)$ is $\sin(2\pi 1000t)$, whereas the Hilbert transform of $\sin(2\pi 1000t)$ is $\cos(2\pi 1000t)$

Thus $\hat{m}(t) = \sin(2\pi 1000t) - \cos(2\pi 1000t)$

41. (D) The expression for the LSSB AM signal is.

$x_l(t) = A_c m(t)\cos(2\pi f_c t) + A_c m(t)\sin(2\pi f_c t)$

Substituting

$A_c = 100$, $m(t) = \cos(2\pi 1000t) + 2\sin(2\pi 1000t)$

and $\hat{m}(t) = \sin(2\pi 1000t) - 2\cos(2\pi 1000t)$

we obtain

$$x_l(t) = 100[\cos(2\pi 1000t) + 2\sin(2\pi 1000t)\cos(2\pi f_c t)]$$
$$+100[\sin(2\pi 1000t) - 2\cos(2\pi 1000t)\sin(2\pi f_c t)]$$
$$= 100[\cos(2\pi 1000t)\cos(2\pi f_c t) + \sin(2\pi 1000t)\sin(2\pi f_c t)]$$
$$+200[\cos(2\pi f_c t)\sin(2\pi 1000t) - \sin(2\pi f_c t)\cos(2\pi 1000t)]$$
$$= 100\cos(2\pi(f_c - 1000)t) - 200\sin(2\pi(f_c - 1000)t)$$

42. (D) $y(t) = 4(m(t) + \cos\omega_c t) + 10(m(t) + \cos\omega_c t)^2$

$= 4m(t) + 4\cos\omega_c t + 10m^2(t) + 20m(t)\cos\omega_c t + 5 + 5\cos 2\omega_c t$

$= 5 + 4m(t) + 10m^2(t) + 4[1 + 5m(t)]\cos\omega_c t + 5\cos 2\omega_c t$

The AM signal is, $\quad x_c(t) = 4[1 + 5m(t)]\cos\omega_c t$

$m(t) = Mm_n(t)$

$x_c(t) = 4[1 + 5Mm_n(t)]\cos\omega_c t$

$5M = 0.8 \quad$ or $\quad M = 0.16$

43. (C) The filter characteristic is shown in fig.S7.4.43

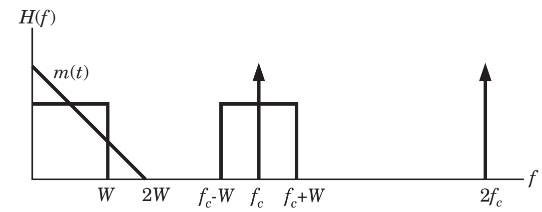

Fig.S7.4.43

$f_c - W > 2W$ or $f_c > 3W$,

$f_c + W < 2f$ or $f_c > W$

Therefore $f_c > 3W$

44. (B) Since High-side tuning is used

$f_{LO} = f_m + f_{IF} = 500$ kHz,

$f_{LOL} = 5 + 0.5 = 5.5$ MHz,

$f_{LOU} = 10 + 0.5 = 10.5$ MHz

45. (B) $f_{\text{image}} = f_L + 2f_{IF} = 2400 = 3310$ kHz

46. (D) The given circuit is a ring modulator. The output is DSB-SC signal. So it will contain $m(t)\cos(n\omega_c t)$ where $n = 1, 2, 3......$ Therefore there will be only (1 MHz ± 400 Hz) frequency component.

47. (A) $f_{\max} = 1650 + 450 = 2100$ kHz

$f_{\min} = 550 + 450 = 1000$ kHz. $\quad$ or $\quad f = \frac{1}{2\pi\sqrt{LC}}$

frequency is minimum, capacitance will be maximum

$R = \frac{C_{\max}}{C_{\min}} = \frac{f^2_{\max}}{f^2_{\min}} = (2.1)^2 \quad$ or $R = 4.41$

$f_i = f_c + 2f_{IF} = 700 + 2(455) = 1600$ kHz

48. (B) Since $X(f)$ has spectral peak at 1 kHz so at the output of first modulator spectral peak will be at (10 + 1) kHz and (10 − 1) kHz. After passing the HPF frequency component of 11 kHz will remain. The output of 2nd modulator will be (13 ± 11) kHz. So $Y(f)$ has spectral peak at 2 kHz and 24 kHz.

49. (C) $m(t)s(t) = y_1(t)$

$$= \frac{2\sin(2\pi t)\cos(200\pi t)}{t} = \frac{\sin(202\pi t) - \sin(198\pi t)}{t}$$

$$y_1(t) + n(t) = y_2(t) = \frac{\sin 202\pi t - \sin 198\pi t}{t} + \frac{\sin 198\pi t}{t}$$

$y_2(t)s(t) = y(t)$

$$= \frac{[\sin 202\pi t - \sin 198\pi t + \sin 199\pi t]\cos 200\pi t}{t}$$

$$= \frac{1}{2}[\sin(402\pi t) + \sin(2\pi t) - \{\sin(398\pi t) - \sin(2\pi t)\}$$

$$+ \sin(399\pi t) - \sin(\pi t)]$$

After filtering

$$y(t) = \frac{\sin(2\pi t) + \sin(2\pi t) - \sin(\pi t)}{2t}$$

$$= \frac{\sin(2\pi t) + 2\sin(0.5t)\cos(1.5\pi t)}{2t}$$

$$= \frac{\sin 2\pi t}{2t} + \frac{\sin 0.5\pi t}{t}\cos 1.5\pi t$$

50. (D) The total signal bandwidth = $5 \times 12 = 60$ kHz
There would be 11 guard band between 12 signal. So guard band width = 11 kHz
Total band width = 60 + 11 = 71 kHz

51. (D) $x_1(t) = g(t)\cos(2000\pi t)$

$$= x(t)\sin(2000\pi t)\cos(2000\pi t) = \frac{1}{2}x(t)\sin(4000\pi t)$$

$$X_1(j\omega) = \frac{1}{4j}X(j(\omega - 4000\pi)) - X(j(\omega + 4000\pi))$$

This implies that $X_1(j\omega)$ is zero for $|\omega| \le 2000\pi$ because $\omega < 2\pi f_m = 2\pi 1000$. When $x_1(t)$ is passed through a LPF with cutoff frequency 2000π, the output will be zero.

52. (A) $y(t) = g(t)\sin(400\pi t) = x(t)\sin^2(400\pi t)$

$$= (\sin(200\pi t) + 2\sin(400\pi t)\frac{(1 - \cos)(800\pi t)}{2}$$

$$= \frac{1}{2}[\sin(200\pi t) - \sin(200\pi t)\cos(800\pi t) + 2\sin(400\pi t)$$

$$-\sin(400\pi t)\cos(800\pi t)$$

$$= \frac{1}{2}\sin(200\pi t) - \frac{1}{4}[\sin(1000\pi t) - \sin(6000\pi t)]$$

$$+ \sin(400\pi t) - \frac{1}{4}[\sin(1200\pi t) - \sin(400\pi t)]$$

If this signal is passed through LPF with frequency 400π and gain 2, the output will be $\sin(200\pi t)$

53. (A) Message signal BW $f_m = 5$ kHz
Noise power density is 10^{-18} W/Hz
Total noise power is $10^{-18} \times 5 \times 10^3 = 5 \times 10^{-15}$ W
Input signal-to-noise ratio

$$\text{SNR} = \frac{10^{-10}}{5 \times 10^{-15}} = 2 \times 10^4 \text{ or } 43 \text{ dB}$$

54. (C) Average side band power is

$$\frac{A_c^2\alpha^2}{4} = \frac{10^2(0.5)^2}{4} = 6.25 \text{ W}$$

55. (D) Noise power = Area rendered by the spectrum $= N_0 B$
Ratio of average sideband power to mean noise

$$\text{Power} = \frac{6.25}{N_0 B} = \frac{25}{4N_0 B}$$

56. (C) Since the channel attenuation is 80 db, then

$$10\log\frac{P_T}{P_R} = 80$$

or $P_R = 10^{-8}P_T = 10^{-8} \times 40 \times 10^3 = 4 \times 10^{-4}$ Watts
If the noise limiting filter has bandwidth B, then the pre-detection noise power is

$$P_n = 2\int_{f_c - \frac{B}{2}}^{f_c + \frac{B}{2}} \frac{N_0}{2}\,df = N_0 B = 2 \times 10^{-10} \text{ B Watts}$$

In the case of DSB or conventional AM modulation, $B = 2W = 2 \times 10^4$ Hz, whereas in SSB modulation $B = W = 10^4$. Thus, the pre-detection signal to noise ratio in DSB and conventional AM is

$$\text{SNR}_{\text{DSB,AM}} = \frac{P_R}{P_n} = \frac{4 \times 10^{-4}}{2 \times 10^{-10} \times 2 \times 10^4} = 10^2$$

57. (A) In SSB modulation $B = W = 10^4$

$$\text{SNR}_{\text{SSB}} = \frac{4 \times 10^{-4}}{2 \times 10^{-10} \times 10^4} = 2 \times 10^2$$

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CHAPTER

7.6

DIGITAL TRANSMISSION

Statement for Question 1-5

Fig. P7.6.1-5 shows fourier spectra of signal $x(t)$ and y(t). Determine the Nyquist sampling rate for the given function in question.

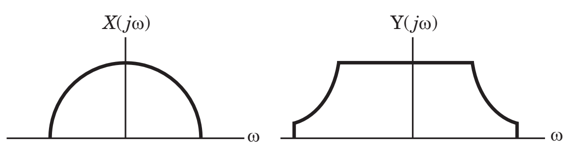

Fig.P7.6.1-5

1. $x(t)$

(A) 100 kHz (B) 200 kHz

(C) 300 kHz (D) 50 kHz

2. $y(t)$

(A) 50 kHz (B) 75 kHz

(C) 150 kHz (D) 300 kHz

3. $x^2(t)$

(A) 100 kHz (B) 150 kHz

(C) 250 kHz (D) 400 kHz

4. $y^3(t)$

(A) 100 kHz (B) 300 kHz

(C) 900 kHz (D) 120 kHz

5. $x(t)\,y(t)$

(A) 250 kHz (B) 500 kHz

(C) 50 kHz (D) 100 kHz

Statement for Question 6-7

A signal $x(t)$ is multiplied by rectangular pulse train $c(t)$ shown in fig.P7.6.6-7..

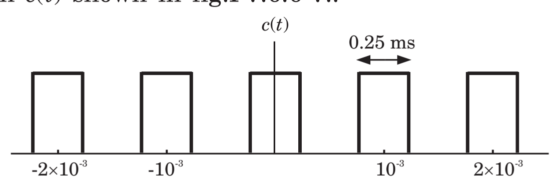

Fig.P7.6.6-7

6. $x(t)$ would be recovered form the product. $x(t)\ c(t)$ by using an ideal LPF if $X(j\omega) = 0$ for

(A) $\omega > 2000\pi$ (B) $\omega > 1000\pi$

(C) $\omega < 1000\pi$ (D) $\omega < 2000\pi$

7. If $X(j\omega)$ satisfies the constraints required, then the pass band gain A of the ideal lowpass filter needed to recover $x(t)$ from $e(t)\ x(t)$ is

(A) 1 (B) 2

(C) 4 (D) 8

8. Consider a set of 10 signals $x_i(t), i = 1, 2, 3, \ldots 10$.. Each signal is band limited to 1 kHz. All 10 signals are to be time-division multiplexed after each is multiplied by a carrier $e(t)$ shown in Figure. If the period T of $e(t)$ is chosen the have the maximum allowable value, the largest value of Δ would be

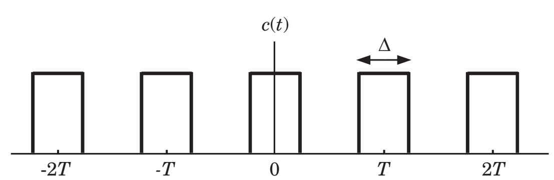

Fig.P7.6.8

(A) 5×10^{-3} sec (B) 5×10^{-4} sec
(C) 5×10^{-5} sec (D) 5×10^{-6} sec

9. A compact disc recording system samples a signals with a 16-bit analog-to-digital convertor at 44.1 kbits/s. The CD can record an hours worth of music. The approximate capacity of CD is
(A) 705.6 M Bytes (B) 317.5 M Bytes
(C) 2.54 M Bytes (D) 5.43 M Bytes

10. An analog signal is sampled at 36 kHz and quantized into 256 levels. The time duration of a bit of the binary coded signal is
(A) 5.78 μs (B) 3.47 μs
(C) 6.43 ms (D) 7.86 ms

11. An analog signal is quantized and transmitted using a PCM system. The tolerable error in sample amplitude is 0.5% of the peak-to-peak full scale value. The minimum binary digits required to encode a sample is
(A) 5 (B) 6
(C) 7 (D) 8

Statement for Question 12-13.

Ten telemetry signals, each of bandwidth 2kHz, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 1% extra bits.

12. The minimum possible data rate must be
(A) 272.64 k bits/sec (B) 436.32 k bits/sec
(C) 936.32 k bits/sec (D) None of the above

13. The minimum transmission bandwidth is
(A) 218.16 kHz (B) 468.32 kHz
(C) 136.32 kHz (D) None of the above

14. A Television signal is sampled at a rate of 20% above the Nyquist rate. The signal has a bandwidth of 6 MHz. The samples are quantized into 1024 levels. The minimum bandwidth required to transmit this signal would be
(A) 72 M bits/sec (B) 144 M bits/sec
(C) 72 k bits/sec (D) 144 k bits/sec

15. A CD record audio signals digitally using PCM. The audio signal bandwidth is 15 kHz. The Nyquist samples are quantized into 32678 levels and then binary coded. The minimum number of binary digits required to encode the audio signal
(A) 450 k bits/sec (B) 900 k bits/sec
(C) 980 340 k bits/sec (D) 490 170, k bits/sec

16. The American Standard Code for Information Interchange has 128 characters, which are binary coded. If a certain computer generates 1,000,000 character per second, the minimum bandwidth required to transmit this signal will be
(A) 1.4 M bits/sec (B) 14 M bits/sec
(C) 7 M bits/sec (D) 0.7 M bits/sec

17. A binary channel with capacity 36 k bits/sec is available for PCM voic transmission. If signal is band limited to 3.2 kHz, then the appropriate values of quantizing level L and the sampling frequency will be
(A) 32, 3.6 kHz (B) 64, 7.2 kHz
(C) 64, 3.6 kHz (D) 32, 7.2 kHz

18. Fig.P7.4.18 shows a PCM signals in which amplitude level of + 1 volt and - 1 volt are used to represent binary symbol 1 and 0 respectively. The code word used consists of three bits. The sampled version of analog signal from which this PCM signal is derived is

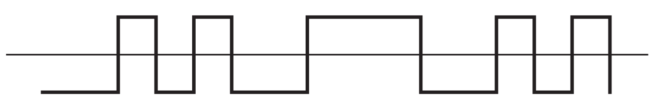

Fig.P7.4.18

(A) 4 5 1 2 1 3 (B) 8 4 3 1 2
(C) 6 4 3 1 7 (D) 1 2 3 4 5

19. A PCM system uses a uniform quantizer followed by a 8-bit encoder. The bit rate of the system is equal to 10^8 bits/s. The maximum message bandwidth for which the system operates satisfactorily is
(A) 25 MHz (B) 6.25 MHz
(C) 12.5 MHz (D) 50 MHz

20. Twenty-four voice signals are sampled uniformly at a rate of 8 kHz and then time-division multiplexed. The sampling process uses flat-top samples with 1 μs duration. The multiplexing operating includes provision for synchronization by adding and extra pulse of 1 μs duration. The spacing between successive pulses of the multiplexed signal is

(A) 4 μs (B) 6 μs

(C) 7.2 μs (D) 8.4 μs

21. A linear delta modulator is designed to operate on speech signals limited to 3.4 kHz. The sampling rate is 10 time the Nyquist rate of the speech signal. The step size δ is 100 m V. The modulator is tested with a this test signal required to avoid slope overload is

(A) 2.04 V (B) 1.08 V

(C) 4.08 V (D) 2.16 V

Statement fo Question 22-23 :

Consider a linear DM system designed to accommodate analog message signals limited to bandwidth of 3.5 kHz. A sinusoidal test signals of amplitude $A_{max} = 1$ V and frequency $f_m = 800$ Hz is applied to system. The sampling rate of the system is 64 kHz.

22. The minimum value of the step size to avoid overload is

(A) 240 mV (B) 120 mV

(C) 670 mV (D) 78.5 mV

23. The granular-noise power would be

(A) 1.68×10^{-3} W (B) 2.86×10^{-4} W

(C) 2.48×10^{-3} W (D) 1.12×10^{-4} W

24. The SNR will be

(A) 298 (B) 1.75×10^{-3}

(C) 4.46×10^{3} (D) 201

25. The output signal-to-quantization-noise ratio of a 10-bit PCM was found to be 30 dB. The desired SNR is 42 dB. It can be increased by increasing the number of quantization level.In this way the fractional increase in the transmission bandwidth would be (assume $\log_2 10 = 0.3$)

(A) 20% (B) 30%

(C) 40% (D) 50%

Statement for Question 26-27.

A signal has a bandwidth of 1 MHz. It is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 level using a μ-low quantizer with $\mu = 225$.

26. The signal-to-quantization-noise ratio is

(A) 34.91 dB (B) 38.06 dB

(C) 42.05 dB (D) 48.76 dB

27. It was found that a sampling rate 20% above the rate wou7ld be adequate. So the maximum SNR, that can be realized without increasing the transmission bandwidth, would be

(A) 60.4 dB (B) 70.3 dB

(C) 50.1 dB (D) None of the above

28. For a PCM signal the compression parameter $\mu = 100$ and the minimum signal to quantization-noise ratio is 50 dB. The number of bits per sample would be.

(A) 8 (B) 10

(C) 12 (D) 14

29. A sinusoid massage signal $m(t)$ is transmitted by binary PCM without compression. If the signal to-quantization-noise ratio is required to be at least 48 dB, the minimum number of bits per sample will be

(A) 8 (B) 10

(C) 12 (D) 14

30. A speech signal has a total duration of 20 sec. It is sampled at the rate of 8 kHz and then PCM encoded. The signal-to-quantization noise ratio is required to be 40 dB. The minimum storage capacity needed to accommodate this signal is

(A) 1.12 KBytes (B) 140 KBytes

(C) 168 KBytes (D) None of the above

31. The input to a linear delta modulator having fa step-size $\Delta = 0.628$ is a sine wave with frequency f_m and peak amplitude E_m. If the sampling frequency $f_s = 40$ kHz, the combination of the sinc-wave frequency and the peak amplitude, where slope overload will take piace is

	E_m	f_m
(A)	0.3 V	8 kHz
(B)	1.5 V	4 kHz
(C)	1.5 V	2 kHz
(D)	3.0 V	1 kHz

32. A sinusoidal signal with peak-to-peak amplitude of 1.536 V is quantized into 128 levels using a mid-rise uniform quantizer. The quantization-noise power is

(A) 0.768 V (B) $48 \times 10^{-6} V^2$

(C) $12 \times 10^{-6} V^2$ (D) 3.072 V

33. A signal is sampled at 8 kHz and is quantized using 8 bit uniform quantizer. Assuming SNR_q for a sinusoidal signal, the correct statement for PCM signal with a bit rate of R is

(A) $R = 32$ kbps, $\text{SNR}_q = 25.8$ dB

(B) $R = 64$ kbps, $\text{SNR}_q = 49.8$ dB

(C) $R = 64$ kbps, $\text{SNR}_q = 55.8$ dB

(D) $R = 32$ kbps, $\text{SNR}_q = 49.8$ dB

34. A 1.0 kHz signal is flat-top sampled at the rate of 180 samples sec and the samples are applied to an ideal rectangular LPF with cat-off frequency of 1100 Hz, then the output of the filter contains

(A) only 800 Hz component

(B) 800 and 900 Hz components

(C) 800 Hz and 1000 Hz components

(D) 800 Hz, 900 and 1000 Hz components

35. The Nyquist sampling interval, for the signal $\text{sinc}(700t) + \text{sinc}(500t)$ is

(A) $\frac{1}{350}$ sec (B) $\frac{\pi}{350}$ sec

(C) $\frac{1}{700}$ sec (D) $\frac{\pi}{175}$ sec

36. A signal $x(t) = 100\cos(24\pi \times 10^3)t$ is ideally sampled with a sampling period of 50 μsec and then passed through an ideal lowpass filter with cutoff frequency of 15 KHz. Which of the following frequencies is/are present at the filter output

(A) 12 KHz only (B) 8 KHz only

(C) 12 KHz and 9 KHz (D) 12 KHz and 8 KHz

37. In a PCM system, if the code word length is increased form 6 to 8 bits, the signal to quantization noise ratio improves by the factor.

(A) 8/6 (B) 12

(C) 16 (D) 8

38. Four signals $g_1(t)$, $g_2(t)$, $g_s(t)$ and $g_4(t)$ are to be multiplexed and transmitted. $g_1(t)$ and $g_4(t)$ have a bandwidth of 4 kHz, and the remaining two signals have bandwidth of 8 kHz,. Each sample requires 8 bit for encoding. What is the minimum transmission bit rate of the system.

(A) 512 kbps (B) 16 kbps

(C) 192 kbps (D) 384 kbps

39. Three analog signals, having bandwidths 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. The bit rate for the multiplexed signal is

(A) 115.2 kbps (B) 28.8 kbps

(C) 57.6 kbps (D) 38.4 kbps

40. The minimum sampling frequency (in samples/sec) required to reconstruct the following signal form its samples without distortion would be

$$x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$$

(A) 2×10^3 B) 4×10^3

(C) 6×10^3 (D) 8×10^3

41. The minimum step-size required for a Delta-Modulator operating at32 K samples/sec to track the signal (here $u(t)$ is the nuit function)

$x(t) = 125t(u(t) - u(t-1)) + (250 - 125t)(u(t-1) - u(t-2)\text{s}$

so that slope overload is avoided, would be

(A) 2^{-10} (B) 2^{-8}

(C) 2^{-6} (D) 2^{-4}

42. Four signals each band limited to 5 kHz are sampled at twice the Nyquist rate. The resulting PAM samples are transmitted over a single channel after time division multiplexing. The theoretical minimum transmissions bandwidth of the channel should be equal to.

(A) 5 kHz (B) 20 kHz

(C) 40 kHz (D) 80 kHz

43. Four independent messages have bandwidths of 100 Hz, 100 Hz, 200 Hz and 400 Hz respectively. Each is sampled at the Nyquist rate, time division multiplexed and transmitted. The transmitted sample rate, in Hz, is given by

(A) 200 (B) 400

(C) 800 (D) 1600

44. The Nyquist sampling rate for the signal $g(t) = 10\cos(50\pi t)\cos^2(150\pi t)$. Where '$t$' is in seconds, is

(A) 150 samples per second

(B) 200 samples per second

(C) 300 samples per second

(D) 350 samples per second

45. A TDM link has 20 signal channels and each channel is sampled 8000 times/sec. Each sample is represented by seven binary bits and contains an additional bit for synchronization. The total bit rate for the TDM link is

(A) 1180 K bits/sec (B) 1280 K bits/sec

(C) 1180 M bits/sec (D) 1280 M bits/sec

46. In a CD player, the sampling rate is 44.1 kHz and the samples are quantized using a 16-bit/sample quantizer. The resulting number of bits for a piece of music with a duration of 50 minutes is

(A) 1.39×10^9 (B) 4.23×10^9

(C) 8.46×10^9 (D) 12.23×10^9

47. Four voice signals. each limited to 4 kHz and sampled at Nyquist rate are converted into binary PCM signal using 256 quantization levels. The bit transmission rate for the time-division multiplexed signal will be

(A) 8 kbps (B) 64 kbps

(C) 256 kbps (D) 512 kbps

48. Analog data having highest harmonic at 30 kHz generated by a sensor has been digitized using 6 level PCM. What will be the rate of digital signal generated?

(A) 120 kbps (B) 200 kbps

(C) 240 kbps (D) 180 kbps

49. In a PCM system, the number of quantization levels is 16 and the maximum lsignal frequency is 4 kHz.; the bit transmission rate is

(A) 32 bits/s (B) 16 bits/s

(C) 32 kbits/s (D) 64 dbits/s

50. A speech signal occupying the bandwidth of 300 Hz to 3 kHz is converted into PCM format for use in digital communication. If the sampling frequency is8 kHz and each sample is quantized into 256 levels, then the output bit the rate will be

(A) 3 kb/s (B) 8 kb/s

(C) 64 kb/s (D) 256 kb/s

51. If the number of bits in a PCM system is increased from n to $n+1$, the signal-to-quantization noise ratio will increase by a factor.

(A) $\frac{(n+1)}{n}$ (B) $\frac{(n+1)^2}{n^2}$

(C) 2 (D) 4

52. In PCM system, if the quantization levels are increased form 2 to 8, the relative bandwidth requirement will.

(A) remain same (B) be doubled

(C) be tripled (D) become four times

53. Assuming that the signal is quantized to satisfy the condition of previous question and assuming the approximate bandwidth of the signal is W. The minimum required bandwidth for transmission of a binary PCM signal based on this quantization scheme will be.

(A) 5 W (B) 10 W

(C) 20 W (D) None of the above

****

16. (C) $128 = 2^7$,. We need 7 bits/character. For 1,000,000 character we need 7 Mbits/second. Thus minimum bandwidth = 7 Mbits/sec.

17. (D) $f_s > 2f_m = 6400$ Hz, $nf_s \le 63000$

$$n \le \frac{36000}{6400} = 5.63,\ n = 5,\ L = 2^n = 32,\ f_s = \frac{36000}{5} = 7.2 \text{ kHz.}$$

18. (D) The transmitted code word are

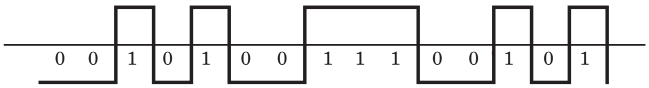

Fig.S.7.6.18

In 1st word 001(1)

In 2nd word 010(2)

In 3rd word 011(3)

In 4th word 100(4)

In 5th word 101 (5)

19. (B) Message bandwidth= W, Nyquist rate $= 2W$

Bandwidth $= 2W \times 8 = 16W$ bit/s

$$16W = 10^8, \quad \text{or} \quad W = \frac{10^8}{16} = 6.25 \text{ MHz}$$

20. (A) Sampling interval $T_s = \frac{1}{8k} = 125\ \mu s$. There are 24 channels and 1 sync pulse, so the time allotted to each channel is $T_c = \frac{T}{25} = 5\ \mu s$. The pulse duration is 1 μs. So the time between pulse is 4 μs.

21. (B) $A_{max} = \frac{\delta f_s}{\omega_m} = \frac{0.1 \times 68k}{2\pi \times 10^3} = 1.08$ V

22. (D) $A_{max} = \frac{\delta f_s}{\omega_m}$ or $\delta = \frac{A_{max}\omega_m}{f_s} = \frac{1 \times 2\pi \times 800}{50 \times 10^3} = 78.5$ mV

23. (D) $N_o = \frac{\delta^2 B}{3f_s} = \frac{(0.0785)^2 \times 3500}{3 \times 64000} = 1.122 \times 10^{-4}$ W

24. (C) $\frac{S_o}{N_o} = \frac{0.5}{1.12 \times 10^{-4}} = 4.46 \times 10^3$

25. (A) $\frac{S_o}{N_o} \propto L^2, \qquad L = 2^n \left(\frac{S_o}{N_o}\right)_{dB} = 10\log(C2^{2n})$

$= \log C + 20\, n \log 2 = \alpha + 6n$ dB. This equation shows that increasing n by one bits increase the by 6 dB. Hence an increase in the SNR by 12 dB can be accomplished by increasing 9is form 10 to 12, the transmission bandwidth would be increased by 20%

26. (B) $\frac{S_o}{N_o} = \frac{3L^2}{[\ln(\mu + 1)]^2} = 6394 = 38.06$ dB

27. (C) Nyquist Rate = 2 MHz

50% higher rate = 3 MHz, $\quad L = 256 = 2^8$

Thus transmission bandwidth is 3 MHz $\times$ 8 = 24 Mbits/s.

New sampling rate is at 20% above the Nyquist rate.

Sampling rate= $1.2 \times 2 = 2.4$ MHz.

bits per second= $\frac{24M\frac{bits}{sec}}{2.4MHz}$ = 10 bits

Level $= 2^{10} = 1024, \quad \frac{S_o}{N_o} = \frac{3(1024)^2}{(\ln 256)^2} = 102300 = 50.1$ dB

28. (B) $\frac{S_o}{N_o} = \frac{3L^2}{[\ln(\mu + 1)]^2} \ge 50$ dB, $\mu = 100$

$$\frac{3L^2}{[\ln 101]^2} = 100\,000 \quad \text{or} \quad L = 842.6$$

Because L is power of 2, we select $L = 1024 = 2^{10}$.

Thus 10 bits are required.

29. (A) $S_o = \overline{m^2(t)},\ N_o = \frac{3m_p^2}{3L^2},\ \frac{S_o}{N_o} = \frac{3L^2\overline{m^2(t)}}{m_p^2}$

since signal is sinusoidal $\frac{\overline{m^2(t)}}{m_p^2} = \frac{1}{2}$,

$\frac{3L^2}{2} = 48$ dB = 63096, $L = 205.09$

Since L is power of 2, so we select $L = 256$

Hence 256 = 2^8, So 8 bits per sample is required .

30. (B) $(SNR)_q = 1.76 + 6.02(n) = 40dB,\ n = 6.35$

We take the $n = 7$.

Capacity = $20 \times 8k \times 7 = 1.12$ Mbits = 140 Kbytes

31. (B) For slope overload to take place $E_m \ge \frac{\delta f_s}{2\pi f_m}$

This is satisfied with $E_m = 1.5$ V and $f_m = 4$ kHz.

32. (C) Step size $\delta = \frac{2m_p}{L} = \frac{1.536}{128} = 0.012$ V

quantization noise power

$$= \frac{\delta^2}{12} = \frac{(0.012)^2}{12} = 12 \times 10^{-6}\ V^2$$

33. (B) Bit Rate $= 8k \times 8$ = 64 kbps

$(SNR)_q = 1.76 + 6.02n$ dB $= 1.76 + 6.02 \times 8 = 49.8$ dB

34. (B) $f_s = 1800$ samples/sec, $\quad f_m = \frac{1800}{2} = 900$ Hz

Since the sampling rate is 1800 samples/sec the highest frequency that can be recovered is 900 Hz.

35. (C) $x(t) = \text{sinc } 700t + \text{sinc } 500t$

$= \frac{1}{\pi t}[\sin 700\pi t + \sin 500\pi t]$

$x(t)$ is band limited with $f_m = 350$ Hz, Thus Nyquist rate is $2f_m = 700$ Hz, Sampling interval $= \frac{1}{700}$ sec

36. (D) $f_s = \frac{1}{T} = \frac{1}{50 \times 10^{-6}} = 20$ kHz, $f_c = 12$ kHz

The frequency passed through LPF are f_c, $f_s - f_m$ or 12 kHz, 8 kHz

37. (C) $P = \frac{(\text{SNR})_1}{(\text{SNR})_2} = \frac{2^{2n_2}}{2^{2n_1}}$, Here n = code word length,

$n_1 = 6$, $n_2 = 8$, Thus rate $= \frac{2^{16}}{2^{12}} = 16$

38. (D) Signal $g_1(t)$, $g_2(t)$, $g_s(t)$ and $g_4(t)$ will have 8 k, 8 k, 16 k and 16 k sample/sec at Nyquist rate. Thus 48000 sample/sec bit rate $48000 \times 8 = 384$ kbps

39. (C) Analog signals, having bandwidth 1200 Hz, 600 Hz and 600 Hz have 2400, 1200 samples/sec at Nyquist rate. Hence 48000 sample/sec

bit rate = 48000 sample/sec $\times 12 = 57.6$ kbps

40. (C) $x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$

Maximum frequency component $= 3 \times 1000 = 3$ kHz

Sampling rate $= 2f_m = 6$ kHz

41. (B) Here f_s = 32 k sample/sec

$E_m = 125$, $f_m = \frac{1}{T} = \frac{1}{2}$

For slope-overload to be averted $E_m \geq \frac{f_s}{f_m}$

$\Delta \leq \frac{E_m f_m}{f_s}$ or $\Delta \leq \frac{125(\frac{1}{2})}{32 \times 10^3}$ or $\Delta \leq \frac{(128)(\frac{1}{2})}{32 \times 1024}$ or $\Delta \leq 2^{-8}$

42. (D) $f_m = 5$ kHz, Nyquist Rate $= 2 \times 5 = 10$ kHz

Since signal are sampled at twice the Nyquist rate so sampling rate $= 2 \times 10 = 20$ kHz.

Total transmission bandwidth $= 4 \times 20 = 80$ kHz

43. (D) Signal will be sampled 200, 200, 400 and 800 sample/sec thus 1600 sample per second,

44. (D) $g(t) = 10 \cos 50\pi t\left(\frac{1 + \cos 300\pi t}{2}\right)$

$= 5 \cos 50\pi t + 5 \cos 50\pi t \cos 300\pi t$

The maximum frequency component will be $150 + 25 = 175$ Hz.

Thus $f_s = 2 \times 175 = 350$ sample per second.

45. (B) Total sample $= 8000 \times 20 = 160$ k sample/sec

Bit for each sample $= 7 + 1 = 8$

Bit Rate $= 160\text{k} \times 8 = 1280 \times 10^3$ bits/sec

46. (B) The sampling rate is $f_s = 44100$ meaning that we take 44100 samples per second. Each sample is quantized using 16 bits so the total number of bits per second is 44100×16. For a music piece of duration 50 min = 3000- sec the resulting number of bits per channel (left and right) is $44100 \times 16 \times 3000 = 2.1168 \times 10^9$ and the overall number of bits is $2.1168 \times 10^9 \times 2 = 4.2336 \times 10^9$

47. (C) Nyquist Rate $= 2 \times 4\text{k} = 8$ kHz

Total sample $= 4 \times 8 = 32$ k sample/sec

$256 = 2^8$, so that 8 bits are required

Bit Rate $= 32\text{k} \times 8 = 256$ kbps

48. (D) Nyquist Rate $= 2 \times 30\text{k} = 60$ kHz

$2^n \geq 6$ Thus $n = 3$, Bit Rate $= 60 \times 3 = 18$ kHz

49. (C) Nyquist rate$= 2 \times 4 = 8$ kHz

$2^n = 16$ or $n = 4$, Bit Rate $= 4 \times 8 = 32$ kbits/sec

50. (C) $f_s = 8$ kHz, $2^n = 256 \Rightarrow n = 8$

Bit Rate $= 8 \times 8\text{k} = 64$ kb/x

51. (D) $\frac{S_o}{N_o} \alpha\ 2^{2n}$, If PCM is increased form n to $n + 1$, the ratio will increase by a factor 4. Which is independent of n.

52. (C) If $L = 2$, then $2 = 2^n$ or $n = 1$ ND If $L = 8$, then $8 = 2^n$ or $n = 3$. So relative bandwidth will be tripled.

53. (B) The minimum bandwidth requirement for transmission of a binary PCM signal is BW= vW. Since $v = 10$, we have BW = 10 W

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CHAPTER 7.8

SPREAD SPECTRUM

Statement for Question 1-3 :

A pseudo-noise (PN) sequance is generated using a feedback shift register of length $m = 4$. The chip rate is 10^7chips per second

1. The PN sequance length is

(A) 10 (B) 12

(C) 15 (D) 18

2. The chip duration is

(A) 1μs (B) 0.1 μs

(C) 0.1 ms (D) 1 ms

3. The period of PN sequance is

(A) 1.5 μs (B) 15 μs

(C) 6.67 ns (D) 0.67 ns

Statement for Question 4-5:

A direct sequence spread binary phase-shift-keying system uses a feedback shift register of Length 19 for the generation of PN sequence . The system is required to have an average probability of symbol error due to externally generated interfering signals that does not exceed 10^{-5}

4. The processing gain of system is

(A) 37 dB (B) 43 dB

(C) 57 dB (D) 93 dB

5. The Antijam margin is

(A) 47.5 dB (B) 93.8 dB

(C) 86.9 dB (D) 12.6 dB

6. A slow FH/MFSK system has the following parameters.

Number of bits per MFSK symbol = 4

Number of MFSK symbol per hop = 5

The processing gain of the system is

(A) 13.4 dB (B) 37.8 dB

(C) 6 dB (D) 26 dB

7. A fast FH/MFSK system has the following parameters.

Number of bits per MFSK symbol = 4

Number of pops per MFSK symbol = 4

The processing gain of the system is

(A) 0 dB (B) 7 dB

(C) 9 dB (D) 12 dB

Statement for Question 8-9:

A rate 1/2 convolution code with dfrec = 10 is used to encode a data requeence occurring at a rate of 1 kbps. The modulation is binary PSK. The DS spread spectrum sequence has a chip rate of 10 MHz

8. The coding gain is

(A) 7 dB (B) 12 dB

(C) 14 dB (D) 24 dB

9. The processing gain is

(A) 14 dB (B) 37 dB

(C) 58 dB (D) 104 dB

10. A total of 30 equal-power users are to share a common communication channel by CDM. Each user transmit information at a rate of 10 kbps via DS spread spectrum and binary PSK. The minimum chip rate to obtain a bit error probability of 10^{-5}

(A) 1.3×10^{6} chips/sec (B) 2.9×10^{5} chips/sec

(C) 1.9×10^{6} chips/sec (D) 1.3×10^{5} chips/sec

11. A CDMA system is designed based on DS spread spectrum with a processing gain of 1000 and BPSK modulation scheme. If user has equal power and the desired level of performance of an error probability of 10^{-6}, the number of user will be

(A) 89 (B) 117

(C) 147 (D) 216

12. In previous question if processing gain is changed to 500, then number of users will be

(A) 27 users (B) 38 users

(C) 42 users (D) 45 users

Statement for Question 13-15 :

A DS spread spectrum system transmit at a rate of 1 kbps in the presets of a tone jammer. The jammer power is 20 dB greater then the desired signal, and the required ϵ_b/J_0 to achieve satisfactory performance is 10 dB.

13. The spreading bandwidth required to meet the specifications is

(A) 10^{7} Hz (B) 10^{3} Hz

(C) 10^{5} Hz (D) 10^{6} Hz

14. If the jammer is a pulse jammer, then pulse duty cycle that results in worst case jamming is

(A) 0.14 (B) 0.05

(C) 0.07 (D) 0.10

15. The correspond probability of error is

(A) 4.9×10^{-3} (B) 6.3×10^{-3}

(C) 9.4×10^{-4} (D) 8.3×10^{-3}

Statement for question 16-18 :

A CDMA system consist of 15 equal power user that transmit information at a rate of 10 kbps, each using a DS spread spectrum signal operating at chip rate of 1 MHz. The modulation scheme is BPSK.

16. The Processing gain is

(A) 0.01 (B) 100

(C) 0.1 (D) 10

17. The value of ε_b/J_0 is

(A) 8.54 dB (B) 7.14 dB

(C) 17.08 dB (D) 14.28 dB

18. How much should the processing gain be increased to allow for doubling the number of users with affecting the autopad SNR

(A) 1.46 MHz (B) 2.07 MHz

(C) 4.93 MHz (D) 2.92 MHz

19. A DS/BPSK spread spectrum signal has a processing gain of 500. If the desired error probability is 10^{-5} and (ε_b/J_0) required to obtain an error probability of 10^{-5} for binary PSK is 9.5 dB, then the Jamming margin against a containers tone jammer is

(A) 23.6 dB (B) 17.5 dB

(C) 117.4 dB (D) 109.0 dB

Statement for Question 20-21 :

An $m = 10$ ML shift register is used to generate the pre hdarandlm sequence in a DS spread spectrum system. The chip duration is $T_c = l\,\mu s$ and the bit duration is $T_b = NT_c$, where N is the length (period of the m sequence).

20. The processing gain of the system is

(A) 10 dB (B) 20 dB

(C) 30 dB (D) 40 dB

21. If the required ε_b/J_0 is 10 and the jammer is a tone jammer with an average power J_{av}, then jamming margin is.

(A) 10 dB (B) 20 dB

(C) 30 dB (D) 40 dB

Statement for Question 22-23 :

An FH binary orthogonal FSK system employs an $m = 15$ stage liner feedback shift register that generates an ML sequence. Each state of the shift register selects one of L non over lapping frequency bands in the hopping pattern. The bit rate is 100 bits/s. The demodulator employ non coherent detection.

22. If the hop rate is one per bit, the hopping bandwidth for this channel is

(A) 6.5534 MHz (B) 9.4369 MHz

(C) 2.6943 MHz (D) None of the above

23. Suppose the hop rate is increased to 2 hops/bit and the receiver uses square law combining the signal over two hops. The hopping bandwidth for this channel is

(A) 3.2767 MHz (B) 13.1068 MHz

(C) 26.2136 MHz (D) 1.6384 MHz

Statement for Qquestion 24-25 :

In a fast FH spread spectrum system, the information is transmitted via FSK with non coherent detection. Suppose there are $N = 3$ hops/bit with hard decision decoding of the signal in each hop. The channel is AWGN with power spectral density $\frac{1}{2}\mathcal{N}_0$ and an SNR 20-13 dB (total SNR over the three hops)

24. The probability of error for this system is

(A) 0.013 (B) 0.0013

(C) 0.049 (D) 0.0049

25. In case of one hop per bit the probability of error is

(A) 1.96×10^{-5} (B) 1.96×10^{-7}

(C) 2.27×10^{-5} (D) 2.27×10^{-7}

Statement for Question 26-29 :

A slow FH binary FSK system with non coherent detection operates at $\varepsilon_b / J_0 = 10$, with hopping bandwidth of 2 GHz, and a bit rate of 10 kbps.

26. The processing gain of this system is

(A) 23 dB (B) 43 dB

(C) 43 dB (D) 53 dB

27. If the jammer operates as a partial band jammar, the bandwidth occupancy for worst case jamming is

(A) 0.4 GHz (B) 0.6 GHz

(C) 0.7 GHz (D) 0.9 GHz

28. The probability of error for the worst-case partial band jammer is

(A) 0.2996 (B) 0.1496

(C) 0.0368 (D) 0.0298

29. The minimum hop rate for a FH spread spectrum system that will prevent a jammer from operating five onives away from the receiver is

(A) 3.2 bHz (B) 3.2 MHz

(C) 18.6 MHz (D) 18.6 kHz

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SOLUTION

1. (C) The PN sequence length is

$N = 2^m - 1 = 2^4 - 1 = 15$

2. (B) The chip duration is

$T_C = \frac{1}{10^7} s = 0.1$ ms

3. (A) The period of the PN sequence is

$T = NT_C = 15 \times 0.1 = 1.5\ \mu s$

4. (C) $m = 19$

$n = 2^m - 1 = 2^{19} - 1 = 2^{19}$

The processing gain is $10\log_{10} N = 10\log_{10} 2^{19}$

$= 190 \times 0.3$ or 57 dB

5. (A) Antijam margin = (Processing gain) - $10\log_{10}\left(\frac{Eb}{N_0}\right)$

The probability of error is

$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

With $P_e = 10^{-5}$, we have $E_b / N_0 = 9$.

Hence, Antijam margin $= 57 - 10\log_{10} 9 = 57 - 9.5$

or =47.5 dB

6. (D) The precessing gain (PG) is

$\text{PG} = \frac{\text{FH Bandwidth}}{\text{Symbol Rate}} = \frac{W_c}{R_s} = 5 \times 4 = 20$

Hence, expressed in decibels, PG= $10\log_{10} 20 = 26$ db

7. (D) The processing gain is

$\text{PG} = 4 \times 4 = 16$

Hence, in decibels,

$\text{PG} = 10\log_{10} 16 = 12$ dB

8. (A) The coding gain is $R_{cd\,\min} = \frac{1}{2} \times 10 = 5$ or 7 dB

9. (B) The processing gain

$\frac{W}{R} = \frac{10^7}{2 \times 10^3} = 5 \times 10^3$ or 37 dB

10. (C) We assume that the interference is characterized as a zero-mean AWGN process with power spectral density J_0. To achieve an error probability of 10^{-5}, the required $\in_b / J_0 = 10$ we have

$$\frac{W/R}{J_{av}/P_{av}} = \frac{W/R}{N_u - 1} = \frac{\in_b}{J_0}$$

$$W/R = \left(\frac{\in_b}{J_0}\right)(N_u - 1)$$

$$W = R\left(\frac{\in_b}{J_0}\right)(N_u - 1)$$

where $R = 10^4$ bps, $N_u = 30$ and $\in_b / J_0 = 10$

Therefore, $W = 2.9 \times 10^6$ Hz

The minimum chip rate is $1/T_c = W = 2.9 \times 10^6$ chips/sec

11. (D) To achieve an error probability of 10^{-6}, we required $\left(\frac{\in_b}{J_0}\right)_{dB} = 10.5$ dB

Then, the number of users of the CDMA system is

$N_u = \frac{W/R}{\in_b / J_0} + 1 = \frac{1000}{11.3} + 1 = 89$ users

12. (D) If the processing gain is reduced to W/R = 500, then

$N_u = \frac{500}{11.3} + 1$ = 45 users

13. (D) We have a system where $(J_{av}/P_{av})_{dB} = 20$ dB, $R = 1000$ bps and $(\in_b / J_0)_{dB} = 10$ dB

Hence, we obtain $\left(\frac{W}{R}\right)_{dB} = \left(\frac{J_{av}}{P_{av}}\right)_{dB} + \left(\frac{\in_b}{J_0}\right)_{dB} = 30$ dB

$\frac{W}{R} = 1000$

$W = 1000R = 10^6$ Hz

14. (C) The duty cycle of a pulse jammer of worst-case jamming is $\alpha = \frac{0.71}{\in_b / J_0} = \frac{0.7}{10} = 0.07$

15. (D) The corresponding probability of error for this worst-case jamming is

$P_2 = \frac{0.083}{\varepsilon_b / J_0} = \frac{0.083}{10} = 8.3 \times 10^{-3}$

16. (B) Precessing gain is $\frac{W}{R} = \frac{10^6}{10^4} = 100$

17. (A) We have $N_u = 15$ users transmitting at a rate of 10,000 bps each, in a bandwidth of $W = 1MHz$.

The ε_b / J_0 is. $\frac{\varepsilon_b}{J_0} = \frac{W/R}{N_u - 1} = \frac{10^6 / 10^4}{14} = \frac{100}{14}$

$= 7.14$ or 8.54 dB

18. (B) With $N_u = 30$ and $\varepsilon_b/J_0 = 7.14$, the processing gain should be increased to

$W/R = (7.14)(29) = 207$

$W = 207 \times 104 = 2.07$ MHz

Hence the bandwidth must be increased to 2.07 MHz

19. (B) The processing gain is given as

$\frac{W}{R} = 500$ or 27 dB

The (ε_b/J_0) required to obtain an error probability of 10^{-5} for binary PSK is 9.5 dB. Hence, the jamming margin is

$$\left(\frac{J_{av}}{P_{av}}\right)_{dB} = \left(\frac{W}{R}\right)_{dB} - \left(\frac{\varepsilon_b}{J_0}\right)_{dB} = 27.95 \text{ or } 17.5 \text{ dB}$$

20. (C) The period of the maximum length shift register sequence is

$N = 2^{10} - 1 = 1023$

Since $T_b = NT_c$ then the processing gain is

$N\frac{T_b}{T_c} = 1023$ or 30 dB

21. (B) A Jamming margin is

$$\left(\frac{J_{av}}{P_{av}}\right)_{dB} = \left(\frac{W}{R_b}\right)_{dB} - \left(\frac{\varepsilon_b}{J_0}\right)_{dB} = 30 - 10 = 20 \text{ dB}$$

where $J_{av} = J_0 W \approx J_0/T_c = J_0 \times 10^6$

22. (A) The length of the shift-register sequence is

$L = 2^m - 12^{15} - 1 = 32767$ bits

For binary FSK modulation, the minimum frequency separation is $2/T$, where $1/T$ is the symbol (bit) rate. The hop rate is 100 hops/sec. Since the shift register has $L = 32767$ states and each state utilizes a bandwidth of $2/T = 200$ Hz, then the total bandwidth for the FH signal is 6.5534 MHz.

23. If the hopping rate is 2 hops/bit and the bit rate is 100 bits/sec, then, the hop rate is 200 hops/sec. The minimum frequency separation for orthogonality $2/T = 400$ Hz. Since there are $N = 32767$ states of the shift register and for each state we select one of two frequencies separated by 400 Hz, the hopping bandwidth is 13.1068 MHz.

24. (B) The total SNR for three hops is $20 \sim 13$ dB. Therefore the SNR per hop is 20/3. The probability of a chip error with non-coherent detection is

$$P = \frac{1}{2}e^{-\frac{\varepsilon_c}{2N_0}}$$

where $\varepsilon_c / N_0 = 20/3$. The probability of a bit error is

$P_b = 1 - (1-p)^2 = 1 - (1 - 2p + p^2) = 2p - p^2$

$$= e^{-\frac{\varepsilon_c}{2N_0}} - \frac{1}{2}e^{-\frac{\varepsilon_c}{2N_0}} = 0.0013$$

25. (C) In the case of one hop per bit, the SNR per bit is 20, Hence, $P_b = \frac{1}{2}e^{-\frac{\varepsilon_c}{2N_0}} = \frac{1}{2}e^{-10} = 2.27 \times 10^{-5}$

26. (D) We are given a hopping bandwidth of 2 GHz and a bit rate of 10 kbs.

Hence, $\frac{W}{R} = \frac{2 \times 10^9}{10^4} = 2 \times 10^5$ or 53 dB

27. (A) The bandwidth of the worst partial-band jammer is $\alpha * W$, where

$\alpha * W = 2/(\varepsilon_b/J_0) = 0.2$

Hence $\alpha * W = 0.4$ GHz

28. (C) The probability of error with worst-case partial-band jamming is $P_2 = \frac{e^{-1}}{(\varepsilon_b/J_0)} = \frac{e^{-1}}{10} = 3.68 \times 10^{-2}$

29. (D) $d = 5$ miles $= 8050$ meters

$\Delta d = 2 \times 8050 = 16100$

$\Delta d = x \times t$ or $t = \frac{\Delta d}{t}$

$\Rightarrow t = \frac{\Delta d}{x} = \frac{16100}{3 \times 10^8} = 5.367 \times 10^5$

$f = \frac{1}{t} = \frac{1}{5.367 \times 10^{-5}} = 18.63$ kHz

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10. A field is given as

$$\mathbf{G} = \frac{13}{x^2 + y^2}(y\mathbf{u}_x + 3\mathbf{u}_y + x\mathbf{u}_z)$$

The field at point (–2, 3, 4) is

(A) $13(-2\mathbf{u}_x + 3\mathbf{u}_y + 4\mathbf{u}_z)$ (B) $-2\mathbf{u}_x + 3\mathbf{u}_y + 4\mathbf{u}_z$

(C) $13(3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z)$ (D) $3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z$

11. A field is given as $\mathbf{F} = y\mathbf{u}_x + z\mathbf{u}_y + x\mathbf{u}_z$ The angle between $\mathbf{G}$ and $\mathbf{u}_x$ at point (2, 2, 0) is

(A) 45° (B) 30°

(C) 60° (D) 90°

12. A vector field is given as

$$\mathbf{G} = 12xy\mathbf{u}_x + 6(x^2 + 2)\mathbf{u}_y + 18z^2\mathbf{u}_z$$

The equation of the surface $\mathbf{M}$ on which $|\mathbf{G}| = 60$ is

(A) $4x^2y^2 + 4x^4 + 9z^4 + 2x^2 = 96$

(B) $2x^2y^2 + x^4 + 9z^4 + 2x^2 = 96$

(C) $2x^2y^2 + 4x^4 + 9z^4 + 2x^2 = 96$

(D) $4x^2y^2 + x^4 + 9z^4 + 2x^2 = 96$

13. A vector field is given by

$$\mathbf{E} = 4zy^2\mathbf{u}_z + 2y\sin 2x\,\mathbf{u}_y + y^2\sin 2x\,\mathbf{u}_z$$

The surface on which $\mathbf{E}_y = 0$ is

(A) Plane $y = 0$ (B) Plane $x = 0$

(C) Plane $x = \frac{3\pi}{2}$ (D) all

14. The vector field $\mathbf{E}$ is given by

$$\mathbf{E} = 6zy^2\cos 2x\,\mathbf{u}_x + 4xy\sin 2x\,\mathbf{u}_y + y^2\sin 2x\,\mathbf{u}_z$$

The region in which $\mathbf{E} = 0$ is

(A) $y = 0$ (B) $x = 0$

(C) $z = 0$ (D) $x = \frac{n\pi}{2}$

15. Two vector fields are $\mathbf{F} = -10\mathbf{u}_x + 20x(y-1)\mathbf{u}_y$ and $\mathbf{G} = 2x^2y\mathbf{u}_x - 4\mathbf{u}_y + 2\mathbf{u}_z$. At point A(2, 3, –4) a unit vector in the direction of $\mathbf{F} - \mathbf{G}$ is

(A) $0.18\mathbf{u}_x + 0.98\mathbf{u}_y - 0.05\mathbf{u}_z$

(B) $-0.18\mathbf{u}_x - 0.98\mathbf{u}_y + 0.05\mathbf{u}_z$

(C) $-0.37\mathbf{u}_x + 0.92\mathbf{u}_y + 0.02\mathbf{u}_z$

(D) $0.37\mathbf{u}_x - 0.92\mathbf{u}_y - 0.02\mathbf{u}_z$

16. A field is given as

$$\mathbf{G} = \frac{25}{x^2 + y^2}(x\mathbf{u}_x + y\mathbf{u}_y)$$

The unit vector in the direction of $\mathbf{G}$ at P(3, 4, –2) is

(A) $0.6\mathbf{u}_x + 0.8\mathbf{u}_y$ (B) $0.8\mathbf{u}_x + 0.6\mathbf{u}_y$

(C) $0.6\mathbf{u}_y + 0.8\mathbf{u}_z$ (D) $0.6\mathbf{u}_z + 0.6\mathbf{u}_x$

17. A field is given as $\mathbf{F} = xy\mathbf{u}_x + yz\mathbf{u}_y + zx\mathbf{u}_x$ The value of the double integral $I = \int_0^4\int_0^2 \mathbf{F}\cdot\mathbf{u}_y dzdx$ in the plane $y = 7$ is

(A) 128 (B) 56

(C) 190 (D) 0

18. Two vector extending from the origin are given as $\mathbf{R}_1 = 4\mathbf{u}_x + 3\mathbf{u}_y - 2\mathbf{u}_z$ and $\mathbf{R}_2 = 3\mathbf{u}_x - 4\mathbf{u}_y - 6\mathbf{u}_z$. The area of the triangle defined by $\mathbf{R}_1$ and $\mathbf{R}_2$ is

(A) 12.47 (B) 20.15

(C) 10.87 (D) 15.46

19. The four vertices of a regular tetrahedron are located at O (0, 0, 0), A(0, 1, 0), B($0.5\sqrt{3}$, 0.5, 0) and C ($\frac{0.5}{\sqrt{3}}$, 0.5, $\frac{\sqrt{2}}{3}$). The unit vector perpendicular (outward) to the face ABC is

(A) $0.41\mathbf{u}_x + 0.71\mathbf{u}_y + 0.29\mathbf{u}_z$

(B) $0.47\mathbf{u}_x + 0.82\mathbf{u}_y + 0.33\mathbf{u}_z$

(C) $-0.47\mathbf{u}_x - 0.82\mathbf{u}_y - 0.33\mathbf{u}_z$

(D) $-0.41\mathbf{u}_x - 0.71\mathbf{u}_y - 0.29\mathbf{u}_z$

20. The two vector are $\mathbf{R}_{AM} = 20\mathbf{u}_x + 18\mathbf{u}_y - 18\mathbf{u}_z$ and $\mathbf{R}_{AN} = -10\mathbf{u}_x + 8\mathbf{u}_y + 15\mathbf{u}_z$. The unit vector in the plane of the triangle that bisects the interior angle at A is

(A) $0.168\mathbf{u}_x + 0.915\mathbf{u}_y + 0.367\mathbf{u}_z$

(B) $0.729\mathbf{u}_x + 0.134\mathbf{u}_y - 0.672\mathbf{u}_z$

(C) $0.729\mathbf{u}_x + 0.134\mathbf{u}_y + 0.672\mathbf{u}_z$

(D) $0.168\mathbf{u}_x + 0.915\mathbf{u}_y - 0.367\mathbf{u}_z$

21. Two points in cylindrical coordinates are A($\rho = 5$, $\phi = 70°$, $z = -3$) and B($\rho = 2$, $\phi = 30°$, $z = 1$). A unit vector at A towards B is

(A) $0.03\mathbf{u}_x - 0.82\mathbf{u}_y + 0.57\mathbf{u}_z$

(B) $0.03\mathbf{u}_x + 0.82\mathbf{u}_y + 0.57\mathbf{u}_z$

(C) $-0.82\mathbf{u}_x + 0.003\mathbf{u}_y + 0.57\mathbf{u}_z$

(D) $0.003\mathbf{u}_x - 0.82\mathbf{u}_y + 0.57\mathbf{u}_z$

22. A field in cartesian form is given as

$$\mathbf{D} = x\mathbf{u}_x + \frac{y\mathbf{u}_y}{x^2 + y^2}$$

In cylindrical form it will be

(A) $\mathbf{D} = \frac{\mathbf{u}_\rho}{\rho}$ (B) $\mathbf{D} = \frac{\mathbf{u}_\rho}{\rho} + \frac{\mathbf{u}_\phi}{\cos\phi}$

(C) $\mathbf{D} = \rho\mathbf{u}_\rho$ (D) $\mathbf{D} = \rho\mathbf{u}_\rho + \cos\phi\,\mathbf{u}_\phi$

23. A vector extends from $A(\rho = 4,\ \phi = 40°, z = -2)$ to $B(\rho = 5,\ \phi = -110°, z = 1)$. The vector $\mathbf{R}_{AB}$ is

(A) $4.77\mathbf{u}_x + 7.30\mathbf{u}_y + 4\mathbf{u}_z$

(B) $-4.77\mathbf{u}_x - 7.30\mathbf{u}_y + 4\mathbf{u}_z$

(C) $-7.30\mathbf{u}_x - 4.77\mathbf{u}_y + 4\mathbf{u}_z$

(D) $7.30\mathbf{u}_x + 4.77\mathbf{u}_y + 4\mathbf{u}_z$

24. The surface $\rho = 3,\ \rho = 5,\ \phi = 100°,\ \phi = 130°,\ z = 3$ and $z = 4.5$ define a closed surface. The enclosed volume is

(A) 480 (B) 5.46

(C) 360 (D) 6.28

25. The surface $\rho = 2,\ \rho = 4,\ \phi = 45°,\ \phi = 135°,\ z = 3$ and $z = 4$ define a closed surface. The total area of the enclosing surface is

(A) 34.29 (B) 20.7

(C) 32.27 (D) 16.4

26. The surface $\rho = 3,\ \rho = 5,\ \phi = 100°,\ \phi = 130°,\ z = 3$ and $z = 4.5$ define a closed volume. The length of the longest straight line that lies entirely within the volume is

(A) 3.21 (B) 3.13

(C) 4.26 (D) 4.21

27. A vector field **H** is

$$\mathbf{H} = \rho z^2 \sin\phi\,\mathbf{u}_\rho + e^{-z}\sin\left(\frac{\phi}{2}\right)\mathbf{u}_\phi + \rho^3\mathbf{u}_z$$

At point $\left(2,\ \frac{\pi}{3},\ 0\right)$ the value of $\mathbf{H}\cdot\mathbf{u}_x$ is

(A) 0.25 (B) 0.433

(C) –0.433 (D) –0.25

28. A vector is $\mathbf{A} = y\mathbf{u}_x + (x + z)\mathbf{u}_y$. At point P(–2, 6, 3) **A** in cylindrical coordinate is

(A) $-0.949\mathbf{u}_\rho - 6.008\mathbf{u}_\phi$ (B) $0.949\mathbf{u}_\rho - 6.008\mathbf{u}_\phi$

(C) $-6.008\mathbf{u}_\rho - 0.949\mathbf{u}_\phi$ (D) $6.008\mathbf{u}_\rho + 0.949\mathbf{u}_\phi$

29. The vector

$$\mathbf{B} = \frac{10}{r}\mathbf{u}_r + r\cos\theta\,\mathbf{u}_\theta + \mathbf{u}_\phi$$

in cartesian coordinates at (-3, 4, 0) is

(A) $\mathbf{u}_x - 2\mathbf{u}_y$ (B) $-2\mathbf{u}_x + \mathbf{u}_y$

(C) $1.36\mathbf{u}_x + 2.72\mathbf{u}_y$ (D) $-2.72\mathbf{u}_x + 1.36\mathbf{u}_x$

30. The two point have been given $A(20,\ 30°,\ 45°)$ and $B(30,\ 115°,\ 160°)$. The $|\mathbf{R}_{AB}|$ is

(A) 22.2 (B) 44.4

(C) 11.1 (D) 33.3

31. The surface $r = 2$ and 4, $\theta = 30°$ and $60°$, $\phi = 20°$ and $80°$ identify a closed surface. The enclosed volume is

(A) 11.45 (B) 7.15

(C) 6.14 (D) 8.26

32. The surface $r = 2$ and 4, $\theta = 30°$ and $50°$ and $\phi = 20°$ and $60°$ identify a closed surface. The total area of the enclosing surface is

(A) 6.31 (B) 18.91

(C) 25.22 (D) 12.61

33. At point $P(r = 4,\ \theta = 0.2\pi,\ \ \phi = 0.8\pi)$, $\mathbf{u}_r$ in cartesian component is

(A) $0.48\mathbf{u}_x + 0.35\mathbf{u}_y + 0.81\mathbf{u}_z$

(B) $0.48\mathbf{u}_x - 0.35\mathbf{u}_y - 0.81\mathbf{u}_z$

(C) $-0.48\mathbf{u}_x + 0.35\mathbf{u}_y + 0.81\mathbf{u}_z$

(D) $0.48\mathbf{u}_x - 0.35\mathbf{u}_y - 0.81\mathbf{u}_z$

34. The expression for $\mathbf{u}_y$ in spherical coordinates at $P(r = 4,\ \theta = 0.2\pi,\ \phi = 0.8\pi)$ is

(A) $0.48\mathbf{u}_r + 0.35\mathbf{u}_\theta - 0.81\mathbf{u}_\phi$

(B) $0.35\mathbf{u}_r + 0.48\mathbf{u}_\theta - 0.81\mathbf{u}_\phi$

(C) $-0.48\mathbf{u}_r + 0.35\mathbf{u}_\theta - 0.81\mathbf{u}_\phi$

(D) $-0.35\mathbf{u}_r + 0.48\mathbf{u}_\theta - 0.81\mathbf{u}_\phi$

35. Given a vector field

$$\mathbf{D} = r\sin\phi\,\mathbf{u}_r - \frac{1}{r}\sin\theta\cos\phi\,\mathbf{u}_\theta + r^2\mathbf{u}_\phi$$

The component of **D** tangential to the spherical surface $r = 10$ at P(10, 150°, 330°) is

(A) $0.043\mathbf{u}_\theta + 100\mathbf{u}_\phi$

(B) $-0.043\mathbf{u}_\theta - 100\mathbf{u}_\phi$

(C) $110\mathbf{u}_\theta + 0.043\mathbf{u}_\phi$

(D) $0.043\mathbf{u}_\theta - 100\mathbf{u}_\phi$

36. The circulation of $\mathbf{F} = x^2\mathbf{u}_x - xz\mathbf{u}_y - y^2\mathbf{u}_z$ around the path shown in fig. P8.1.36 is

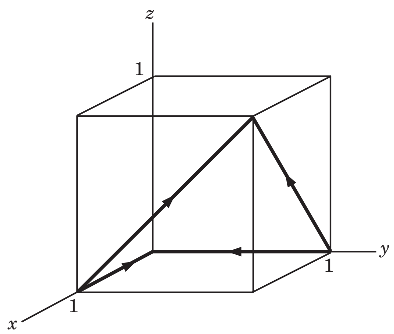

Fig. P8.1.36

(A) $-\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $-\frac{1}{6}$ (D) $\frac{1}{3}$

37. The circulation of $\mathbf{A} = \rho\cos\phi\,\mathbf{u}_\rho + z\sin\phi\,\mathbf{u}_z$ around the edge L of the wedge shown in Fig. P8.1.37 is

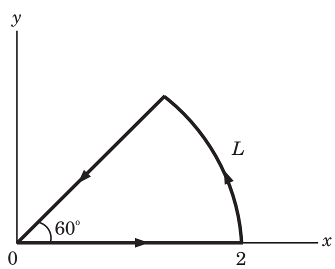

Fig. P8.1.37

(A) 1 (B) –1

(C) 0 (D) 3

38. The gradient of field $f = y^2x + xyz$ is

(A) $y(y+z)\mathbf{u}_x + x(2y+z)\mathbf{u}_y + xy\mathbf{u}_z$

(B) $y(2x+z)\mathbf{u}_x + x(x+z)\mathbf{u}_y + xy\mathbf{u}_z$

(C) $y^2\mathbf{u}_x + 2yx\mathbf{u}_y + xy\mathbf{u}_z$

(D) $y(2y+z)\mathbf{u}_x + x(2y+z)\mathbf{u}_y + xy\mathbf{u}_z$

39. The gradient of the field $f = \rho^2 z\cos 2\phi$ at point $(1, 45°, 2)$ is

(A) $4\mathbf{u}_\phi$ (B) $4\sqrt{2}\mathbf{u}_\phi$

(C) $-4\mathbf{u}_\phi$ (D) $-4\sqrt{2}\mathbf{u}_\phi$

40. The gradient of the function $G = r^3\sin 2\theta\sin 2\phi\sin\theta$ at point $P(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is

(A) $1.41\mathbf{u}_\rho + 3\mathbf{u}_z$ (B) $\mathbf{u}_x + \mathbf{u}_y + \mathbf{u}_z$

(C) $3.46\mathbf{u}_r + 9.3\mathbf{u}_\theta$ (D) All

41. The directional derivative of function $\Phi = xy + yz + zx$ at point P(3, –3, – 3) in the direction toward point Q(4, –1, –1) is

(A) –3 (B) 1

(C) –2 (D) 0

42. The temperature in a auditorium is given by $T = 2x^2 + y^2 - 2z^2$. A mosquito located at (2, 2, 1) in the auditorium desires to fly in such a direction that it will get warm as soon as possible. The direction, in that it must fly is

(A) $8\mathbf{u}_x + 8\mathbf{u}_y - 4\mathbf{u}_z$

(B) $2\mathbf{u}_x + 2\mathbf{u}_y - \mathbf{u}_z$

(C) $4\mathbf{u}_x + 4\mathbf{u}_y - 4\mathbf{u}_z$

(D) $-(2\mathbf{u}_x + 2\mathbf{u}_y - \mathbf{u}_z)$

43. The angle between the normal to the surface $x^2y + z = 3$ and $x\ln z - y^2 = -4$ at the point of intersection (–1, 2, 1) is

(A) 73.4° (B) 36.3°

(C) 16.6° (D) 53.7°

44. The divergence of vector $\mathbf{A} = yz\mathbf{u}_x + 4xy\mathbf{u}_y + y\mathbf{u}_z$ at point P(1, –2, 3) is

(A) 2 (B) -2

(C) 0 (D) 4

45. The divergence of the vector $\mathbf{A} = 2r\cos\theta\cos\phi\,\mathbf{u}_r + r^{1/2}\mathbf{u}_\phi$ at point P(1, 30°, 60°) is

(A) 2.6 (B) 1.5

(C) 4.5 (D) -4.5

46. The divergence of the vector

$$\mathbf{A} = \rho z^2\cos\phi\,\mathbf{u}_\rho + z\sin^2\phi\,\mathbf{u}_z \text{ is}$$

(A) $2\rho z^2\cos\phi\,\mathbf{u}_\rho + \sin^2\phi\,\mathbf{u}_z$

(B) $2\rho z^2\cos\phi\,\mathbf{u}_\rho + \sin^2\phi\,\mathbf{u}_z$

(C) $2z^2\cos\phi\,\mathbf{u}_\rho + \sin^2\phi\,\mathbf{u}_z$

(D) $z\sin 2\frac{\phi}{\rho}\mathbf{u}_\rho + 2\rho z\cos\phi\,\mathbf{u}_\phi + z^2\sin\phi\,\mathbf{u}_z$

47. The flux of $\mathbf{D} = \rho^2 \cos^2 \phi \, \mathbf{u}_\rho + 3 \sin \phi \, \mathbf{u}_\phi$ over the closed surface of the cylinder $0 \le z < 3$, $\rho = 3$ is

(A) 324 (B) 81π

(C) 81 (D) 64π

48. The curl of vector $\mathbf{A} = e^{xy}\mathbf{u}_x + \sin xy \, \mathbf{u}_y + \cos^2 xz \, \mathbf{u}_z$ is

(A) $y e^{xy}\mathbf{u}_x + x \cos xy \, \mathbf{u}_y - 2x \sin 2xz \, \mathbf{u}_z$

(B) $z \sin 2xy \, \mathbf{u}_y + (y \cos xy - xe^{xy})\mathbf{u}_z$

(C) $z \sin 2xy \, \mathbf{u}_y + (x \cos xy - xe^{xy})\mathbf{u}_z$

(D) $xy e^{xy}\mathbf{u}_x + xy \cos xy \, \mathbf{u}_y - 2xz \sin 2xz \, \mathbf{u}_z$

49. The curl of vector field

$$\mathbf{A} = \rho z \sin \phi \, \mathbf{u}_\rho + 3\rho z^2 \cos \phi \, \mathbf{u}_\rho \text{ at point } (5, 90°, 1) \text{ is}$$

(A) 0 (B) $12\mathbf{u}_\theta$

(C) $6\mathbf{u}_r$ (D) $5\mathbf{u}_\phi$

50. The curl of vector field

$$\mathbf{A} = r \cos \theta \, \mathbf{u}_r - \frac{1}{r} \sin \theta \, \mathbf{u}_\theta + 2r^2 \sin \theta \, \mathbf{u}_\phi \text{ is}$$

(A) $\cos \theta \, \mathbf{u}_r - \frac{1}{r} \cos \theta \, \mathbf{u}_\theta$

(B) $2r^2 \cos \theta \, \mathbf{u}_r - 4r \sin \theta \, \mathbf{u}_\theta + \left(\frac{1}{r^2} \sin \theta - r \sin \theta\right)\mathbf{u}_\phi$

(C) $4r \cos \theta \, \mathbf{u}_r - 6r \sin \theta \, \mathbf{u}_\theta + \sin \theta \, \mathbf{u}_\phi$

(D) 0

51. If $\mathbf{A} = (3y^2 - 2z)\mathbf{u}_x - 2x^2 z \, \mathbf{u}_y + (x + 2y)\mathbf{u}_z$, the value of $\nabla \times \nabla \times \mathbf{A}$ at P(–2, 3, –1) is

(A) $-(6\mathbf{u}_x + 4\mathbf{u}_y)$ (B) $8(\mathbf{u}_x + \mathbf{u}_y)$

(C) $-8(\mathbf{u}_x + \mathbf{u}_y)$ (D) 0

52. The $\mathbf{grad} \cdot \nabla \times \mathbf{A}$ of a vector field

$$\mathbf{A} = x^2 y \mathbf{u}_x + y^2 z \mathbf{u}_y - 2xz \mathbf{u}_z \text{ is}$$

(A) $2xy + 2yz - 2x$

(B) $x^2 y + y^2 z - 2xz$

(C) $2x^2 y + 2y^2 z - 2xz$

(D) 0

53. If $V = xy - x^2 y + y^2 z^2$, the value of the **div grad** V is

(A) 0

(B) $z + x^2 + 2y^2 z$

(C) $2y(z^2 - yz - x)$

(D) $2(z^2 - y^2 - y)$

54. If $V = x^2 y^2 z^2$, the laplacian of the field V is

(A) $2(xy^2 + yz^2 + zx^2)$

(B) $2(x^2 y^2 + y^2 z^2 + z^2 x^2)$

(C) $(x^2 y^2 + y^2 z^2 + z^2 x^2)$

(D) 0

55. The value of $\nabla^2 V$ at point P(3, 60°, –2) is if $V = \rho^2 z(\cos \phi + \sin \phi)$

(A) –8.2 (B) 12.3

(C) –12.3 (D) 0

56. If the scalar field $V = r^2 (1 + \cos \theta \sin \phi)$ then $\nabla^2 V$ is

(A) $1 + 2(1 - r^2) \cos \theta \sin \phi$

(B) $6 + 4 \cos \theta \sin \phi - \cot \theta \operatorname{cosec} \theta \sin \phi$

(C) $2 + 2(1 - r^2) \cos \theta \sin \phi$

(D) 0

57. $\nabla \ln \rho$ is equal to

(A) $\nabla \times (\phi \mathbf{u}_z)$ (B) $\nabla \times (z \mathbf{u}_\phi)$

(C) $\nabla \times (\rho \mathbf{u}_\phi)$ (D) $\nabla \times (\rho \mathbf{u}_z)$

58. If $\mathbf{r} = x\mathbf{u}_x + y\mathbf{u}_y + z\mathbf{u}_x$ then $(\mathbf{r} \cdot \nabla) r^2$ is equal to

(A) $2r^2$ (B) $3r^2$

(C) $4r^2$ (D) 0

59. If $\mathbf{r} = x\mathbf{u}_x + y\mathbf{u}_y + z\mathbf{u}_x$ is the position vector of point P(x, y, z) and $r = |\mathbf{r}|$ then $\nabla \cdot r^n \mathbf{r}$ is equal to

(A) nr^n (B) $(n + 3)r^n$

(C) $(n + 2)r^n$ (D) 0

60. If $\mathbf{F} = x^2 y \mathbf{u}_x - y \mathbf{u}_y$, the circulation of vector field $\mathbf{F}$ around closed path shown in fig. P8.1.60 is,

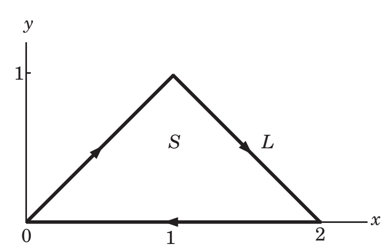

Fig. P8.1.60

(A) $\frac{7}{3}$ (B) $-\frac{7}{6}$

(C) $\frac{7}{6}$ (D) $-\frac{7}{3}$

61. If $\mathbf{A} = \rho \sin\phi\,\mathbf{u}_\rho + \rho^2\mathbf{u}_\rho$, and L is the contour of fig. P8.1.61, then circulation $\oint_L \mathbf{A}.d\mathbf{L}$ is

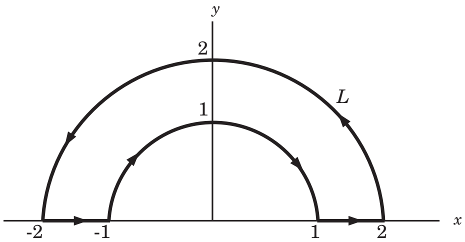

Fig. P8.1.61

(A) $7\pi + 2$ (B) $7\pi - 2$

(C) 7π (D) 0

62. The surface integral of vector

$$\mathbf{F} = 2\rho^2 z^2\,\mathbf{u}_\rho + \rho\cos^2\phi\,\mathbf{u}_z$$

over the region defined by $2 \le \rho \le 5$, $-1 < z < 1$, $0 < \phi < 2\pi$ is

(A) 44 (B) 176

(C) 88 (D) 352

63. If $\mathbf{D} = xy\mathbf{u}_x + yz\mathbf{u}_y + zx\mathbf{u}_z$, then the value of $\oiint \mathbf{A}\cdot d\mathbf{S}$ is, where S is the surface of the cube defined by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$

(A) 0.5 (B) 3

(C) 0 (D) 1.5

64. If $\mathbf{D} = 2\rho z\mathbf{u}_\rho + 3z\sin\phi\,\mathbf{u}_\rho - 4\rho\cos\phi\,\mathbf{u}_z$ and S is the surface of the wedge $0 < \rho < 2$, $\theta < \phi < 45°$, $0 < z < 5$, then the surface integral of **D** is

(A) 24.89 (B) 131.57

(C) 63.26 (D) 0

65. If the vector field

$$\mathbf{F} = (\alpha xy + \beta z^3)\mathbf{u}_x + (3x^2 - \gamma z)\mathbf{u}_y + (3\,xz^2 - y)\mathbf{u}_z$$

is irrotational, the value of α, β and γ is

(A) $\alpha = \beta = \gamma = 1$ (B) $\alpha = \beta = 1, \quad \gamma = 0$

(C) $\alpha = 0, \quad \beta = \gamma = 1$ (D) $\alpha = \beta = \gamma = 0$

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SOLUTIONS

1. (D) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$= \sqrt{(4-2)^2 + (-6-3)^2 + (3-(-1))^2} = \sqrt{4 + 81 + 16} = \sqrt{101}$

2. (A) $\mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A$

$= (3\mathbf{u}_x + 0\mathbf{u}_y + 2\mathbf{u}_z) - (5\mathbf{u}_x - \mathbf{u}_y + 0\mathbf{u}_z)$

$= -2\mathbf{u}_x + \mathbf{u}_y + 2\mathbf{u}_z$

$|\mathbf{R}_{AB}| = \sqrt{2^2 + 1 + 2^2} = 3$

$\mathbf{u}_R = -\frac{2}{3}\mathbf{u}_x + \frac{1}{3}\mathbf{u}_y + \frac{2}{3}\mathbf{u}_z$

3. (C) The component of **F** parallel to **G** is

$= \frac{\mathbf{F}\cdot\mathbf{G}}{G^2}\mathbf{G} = \frac{(10, -6, 5)\cdot(0.1, 0.2, 0.3)}{0.1^2 + 0.2^2 + 0.3^2}(0.1, 0.2, 0.3)$

$= 9.3(0.1, 0.2, 0.3) = (0.93, 1.86, 2.79)$

4. (C) The vector component of **F** perpendicular to **G** is

$= \mathbf{F} - \frac{\mathbf{F}\cdot\mathbf{G}}{G^2}\mathbf{G} = (3, 2, 1) - \frac{(3, 2, 1)\cdot(4, 4, -2)}{4^2 + 4^2 + 2^2}(4, 4, -2)$

$= (3, 2, 1) - (2, 2, -1) = (1, 0, 2) = \mathbf{u}_x + 2\mathbf{u}_z$

5. (C) $\mathbf{R} = 3\mathbf{u}_x + 4\mathbf{M} - \mathbf{N}$

$= 3\mathbf{u}_x + 4(2\mathbf{u}_x + 3\mathbf{u}_y - 4\mathbf{u}_z) - (-4\mathbf{u}_x + 4\mathbf{u}_y + 3\mathbf{u}_z)$

$= 15\mathbf{u}_x + 8\mathbf{u}_y - 19\mathbf{u}_z$

$|\mathbf{R}| = \sqrt{15^2 + 8^2 + 19^2} = 25.5 = 25.5$

6. (B) $\mathbf{R} = -\mathbf{M} + 2\mathbf{N}$

$= -(8\mathbf{u}_x + 4\mathbf{u}_y - 8\mathbf{u}_z) + 2(8\mathbf{u}_x + 6\mathbf{u}_y - 2\mathbf{u}_z)$

$= 8\mathbf{u}_x + 8\mathbf{u}_y + 4\mathbf{u}_z$

$\mathbf{u}_R = \frac{8\mathbf{u}_x + 8\mathbf{u}_y + 4\mathbf{u}_z}{\sqrt{8^2 + 8^2 + 4^2}}$

$= \frac{2}{3}\mathbf{u}_x + \frac{2}{3}\mathbf{u}_y + \frac{1}{3}\mathbf{u}_z = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

$-\mathbf{M} + 2\mathbf{N} = -(8, 4, -8) + 2(8, 6, -2) = (8, 8, 4)$

$\mathbf{u}_R = \frac{(8, 8, 4)}{\sqrt{8^2 + 8^2 + 4^2}} = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

$\mathbf{u}_R = \frac{2}{3}\mathbf{u}_x + \frac{2}{3}\mathbf{u}_y + \frac{1}{3}\mathbf{u}_z$

7. (C) Mid point is $\left(\frac{1+7}{2`}, \frac{-6-2}{2}, \frac{4+0}{2}\right) = (4, -4, 2)$

$\mathbf{u}_R = \frac{(4, -4, 2)}{\sqrt{4^2 + 4^2 + 2^2}} = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

$$= \frac{2}{3}\mathbf{u}_x - \frac{2}{3}\mathbf{u}_y + \frac{1}{3}\mathbf{u}_z$$

8. (A) $\mathbf{G} = 24(1)(2)\mathbf{u}_x + 12(1+2)\mathbf{u}_y + 18(-1)^2\mathbf{u}_z$

$$= 48\mathbf{u}_x + 36\mathbf{u}_y + 18\mathbf{u}_z$$

9. (A) $\mathbf{A} = (6, -2, -4)$, $\mathbf{B} = k\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

$|\mathbf{B} - \mathbf{A}| = 10$

$$\left(6 - \frac{2}{3}k\right)^2 + \left(-2 + \frac{2}{3}k\right)^2 + \left(-4 - \frac{1}{3}k\right)^2 = 100$$

$k^2 - 8k - 44 = 0 \quad \Rightarrow \quad k = 11.75,$

$$B = 11.75\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$$

$= (7.83, -7.83, -3.92)$

10. (D) $\mathbf{G} = \frac{13}{(-2)^2 + (3)^2}(3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z)$

$$= 3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z$$

11. (A) Let θ be the angle between $\mathbf{F}$ and $\mathbf{u}_x$,

Magnitude of $\mathbf{F}$ is $|\mathbf{F}| = \sqrt{y^2 + z^2 + x^2}$

$\mathbf{F} \cdot \mathbf{u}_x = (\mathbf{F})(1)\cos\theta = y$

$$\cos\theta = \frac{y}{\sqrt{y^2 + z^2 + x^2}} = \frac{2}{\sqrt{2^2 + 2^2}} = \frac{1}{\sqrt{2}}$$

$\theta = 45°$

12. (D) $|\mathbf{G}| = 60$

$$\sqrt{(12xy)^2 + (6(x^2 + 2))^2 + (18z^2)^2} = 60$$

$\Rightarrow \quad 144x^2y^2 + 36(x^4 + 4x^2 + 4) + 324z^4 = 3600$

$\Rightarrow \quad 4x^2y^2 + (x^4 + 4x^2 + 4) + 9z^4 = 100$

$\Rightarrow \quad 4x^2y^2 + x^4 + 9z^4 + 4x^2 = 96$

13. (D) For $E_y = 0$, $2y\sin 2x = 0 \quad \Rightarrow \quad y = 0$

$\sin 2x = 0, \Rightarrow \quad 2x = 0, \ \pi, 3\pi, \Rightarrow \quad x = 0, \ \frac{3\pi}{2}$

Hence (D) is correct.

14. (A)

$\mathbf{E} = y(6zy\cos 2x\,\mathbf{u}_x + 4x\sin 2x\,\mathbf{u}_y + y\sin 2x\,\mathbf{u}_z)$

Hence in plane $y = 0$, $\quad E = 0$.

15. (C) $\mathbf{R} = \mathbf{F} - \mathbf{G}$

$= (-10\mathbf{u}_x + 20x(y-1)\mathbf{u}_y) - (2x^2y\mathbf{u}_x - 4\mathbf{u}_y + 2\mathbf{u}_z)$

At P(2, 3, − 4),

$\mathbf{R} = \mathbf{F} - \mathbf{G} = (-10\mathbf{u}_x + 80\mathbf{u}_y) - (24\mathbf{u}_x - 4\mathbf{u}_y + 2\mathbf{u}_z)$

$= -34\mathbf{u}_x + 84\mathbf{u}_y - 2\mathbf{u}_z$

$$\mathbf{u}_R = \frac{-34\mathbf{u}_x + 84\mathbf{u}_y - \mathbf{u}_z}{\sqrt{34^2 + 84^2 + 2^2}}$$

$= -0.37\mathbf{u}_x + 0.92\mathbf{u}_y - 0.02\mathbf{u}_z$

16. (A) At P(3, 4, −2)

$$\mathbf{G} = \frac{25}{3^2 + 4^2}(3\mathbf{u}_x + 4\mathbf{u}_y) = 3\mathbf{u}_x + 4\mathbf{u}_y$$

$$\mathbf{u}_G = \frac{3\mathbf{u}_x + 4\mathbf{u}_y}{\sqrt{3^2 + 4^2}} = 0.6\mathbf{u}_x + 0.8\mathbf{u}_y$$

17. (B) $\mathbf{F} \cdot \mathbf{u}_y = F_y = yz$

$$I = \int_0^4\int_0^2 yz\,dz\,dx = \int_0^4\left(\int_0^2 yz\,dz\right)dx = \int_0^4 2y\,dx = 2(4)y = 8y$$

At $y = 7$, $I = 8(7) = 56$

18. (B) Area $= \frac{1}{2}|\mathbf{R}_1 \times \mathbf{R}_2| = \frac{1}{2}\begin{bmatrix}\mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ 4 & 3 & -2 \\ 3 & -4 & -6\end{bmatrix}$

$= \mathbf{u}_x(-18 - 8) - \mathbf{u}_y(-24 + 6) + \mathbf{u}_z(-16 - 9)$

$= -26\mathbf{u}_x + 18\mathbf{u}_y - 25\mathbf{u}_z$

$|\mathbf{R}_1 \times \mathbf{R}_2| = \sqrt{26^2 + 18^2 + 25^2} = 40.31$

$$\text{area} = \frac{40.31}{2} = 20.15$$

19. (B) $\mathbf{R}_{BA} = (0, 1, 0) - (0.5\sqrt{3}, 0.5, 0) = (-0.5\sqrt{3}, 0.5, 0)$

$$\mathbf{R}_{BC} = \left(\frac{0.5}{\sqrt{3}}, 0.5, \frac{\sqrt{2}}{3}\right) - (0.5\sqrt{3}, 0.5, 0) = \left(-\frac{1}{\sqrt{3}}, 0, \frac{\sqrt{2}}{3}\right)$$

$$\mathbf{R}_{BA} \times \mathbf{R}_{BC} = \begin{bmatrix}\mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ 0.5\sqrt{3} & 0.5 & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{3}\end{bmatrix}$$

$$= \mathbf{u}_x 0.5\sqrt{\frac{2}{3}} - \mathbf{u}_y(0.5\sqrt{2}) + \mathbf{u}_z\frac{0.5}{\sqrt{3}}$$

$= 0.41\mathbf{u}_x - 0.71\mathbf{u}_y + 0.29\mathbf{u}_z$

The required unit vector is

$$= \frac{0.41\mathbf{u}_x + 0.71\mathbf{u}_y + 0.29\mathbf{u}_z}{\sqrt{0.41^2 + 0.71^2 + 0.29^2}}$$

$= 0.47\mathbf{u}_x + 0.81\mathbf{u}_y + 0.33\mathbf{u}_z$

20. (A) The non-unit vector in the required direction is

$$= \frac{1}{2}(\mathbf{u}_{AN} + \mathbf{u}_{AM})$$

$$\mathbf{u}_{AN} = \frac{(-10, 8, 15)}{\sqrt{100 + 64 + 225}} = (-0.507, 0.406, 0.761)$$

$$\mathbf{u}_{AM} = \frac{(20,\ 18,\ -10)}{\sqrt{400+324+100}} = (0.697,\ 0.627,\ -0.348)$$

$$\frac{1}{2}(\mathbf{u}_{AM} + \mathbf{u}_{AN})$$

$$= \frac{1}{2}[(0.697, 0.627, -0.348) + (-0.507, 0.406, 0.761)]$$

$$= (0.095,\ 0.516,\ 0.207)$$

$$\mathbf{u}_{bis} = \frac{(0.095,\ 0.516,\ 0.207)}{\sqrt{0.095^2 + 0.516^2 + 0.261^2}} = (0.168,\ \ 0.915,\ \ 0.367)$$

Hence (A) is correct.

21. (D) In cartesian coordinates

$$A(5\cos 70°,\ 5\sin 70°, -3) = A(1.71, 4.70, -3)$$

$$B(2\cos(-30°), 2\sin(-30°), -3) = B(1.73, -1, 1)$$

$$\mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A = (1.73, -1, 1) - (1.71, 4.70, -3)$$

$$= (0.02, -5.70, 4)$$

$$\mathbf{u}_{AB} = \frac{(0.02, -5.70, 4)}{\sqrt{0.02^2 + 5.70^2 + 4^2}} = (0.003,\ -0.82,\ 0.57)$$

22. (A) $x = \rho\cos\phi, \quad y = \rho\sin\phi$

$$\mathbf{D} = \frac{\rho\cos\phi\,\mathbf{u}_x + \rho\sin\phi\,\mathbf{u}_y}{\rho^2\cos^2\phi + \rho^2\sin^2\phi} = \frac{1}{\rho}(\cos\phi\,\mathbf{u}_x + \sin\phi\,\mathbf{u}_y)$$

$$D_\rho = \mathbf{D}\cdot\mathbf{u}_\rho = \frac{1}{\rho}[\cos\phi(\mathbf{u}_x\cdot\mathbf{u}_\rho) + \sin\phi(\mathbf{u}_y\cdot\mathbf{u}_\rho)]$$

$$= \frac{1}{\rho}[\cos^2\phi + \sin^2\phi] = \frac{1}{\rho}$$

$$D_\phi = \mathbf{D}\cdot\mathbf{u}_\phi = \frac{1}{\rho}[\cos\phi(\mathbf{u}_x\cdot\mathbf{u}_\phi) + \sin\phi(\mathbf{u}_y\cdot u_\phi)]$$

$$= \frac{1}{\rho}[\cos\phi(-\sin\phi) + \sin\phi(\cos\phi)] = 0$$

Therefore $\mathbf{D} = \frac{1}{\rho}\mathbf{u}_\rho$

23. (B) $A(4\cos 40°,\ 4\sin 40°, -2) = A(3.06, 2.57, -2)$

$$B(5\cos(-110°), 5\sin(-110°), 2) = B(-1.71, -4.7, 2)$$

$$\mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A = (-1.71, -4.7,\ 2) - (3.06, 2.57,\ -2)$$

$$= (-4.77, -7.3,\ 4)$$

24. (D) $Vol = \int_3^{4.5}\int_{100°}^{130°}\int_3^5 \rho\, d\rho\, d\phi\, dz = 6.28$

25. (C) Area is

$$= 2\int_{45°}^{135°}\int_2^4 \rho\, d\rho\, d\phi + \int_3^4\int_{45°}^{135°} 2\, d\phi\, dz + \int_3^4\int_{45°}^{135°} 4\, d\phi\, dz + 2\int_3^4\int_2^4 d\rho\, dz$$

$$= 2\left[\frac{\rho^2}{2}\right]_2^4\left(\frac{\pi}{2}\right) + (2)(1)\left(\frac{\pi}{2}\right) = 32.27$$

26. (A) $A(\rho = 3,\ \phi = 100°,\ z = 3) = A(-0.52, 2.95,\ 3)$

$$B(\rho = 5,\ \ \phi = 130°,\ \ z = 4.5) = B(-3.21, 3.83,\ 4.5)$$

length $= |\mathbf{B} - \mathbf{A}|$

$$\mathbf{B} - \mathbf{A} = (-3.21, 3.83, 4.5) - (-0.52, 2.95,\ 3)$$

$$= (-2.69,\ 0.88,\ 1.5)$$

$$|\mathbf{B} - \mathbf{A}| = |(-2.69,\ 0.88, 1.5)| = \sqrt{2.69^2 + 0.88^2 + 1.5^2} = 3.21$$

27. (C) At $P\left(2, \frac{\pi}{3}, 0\right)$, $\mathbf{H} = 0.5\mathbf{u}_\phi + 8\mathbf{u}_z$

$$\mathbf{u}_x = \cos\phi\,\mathbf{u}_\rho - \sin\phi\,\mathbf{u}_\phi = \frac{1}{2}(\mathbf{u}_\rho - \sqrt{3}\mathbf{u}_\phi)$$

$$\mathbf{H}\cdot\mathbf{u}_x = (0.5)\left(-\frac{\sqrt{3}}{2}\right) = -0.433$$

28. (A) $\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$

At P (–2, 6, 3)

$$\mathbf{A} = 6\mathbf{u}_x + \mathbf{u}_y\ ,\ \phi = \tan^{-1}\left(\frac{6}{-2}\right) = 108.43°$$

$$\cos\phi = -0.316,\ \sin\phi = 0.948$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} -0.316 & 0.948 & 0 \\ -0.948 & -0.316 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$$

$$A_\rho = 6(-0.316) + 0.948 = -0.949,$$

$$A_\phi = 6(-0.948) - 0.316 = -6.008,\ A_z = 0$$

Hence (A) is correct option.

29.(B) At P (–3, 4, 0)

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 0^2} = 5$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} = \frac{\pi}{2}$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{4}{-3} = 126.87°$$

$$\mathbf{B} = 2\mathbf{u}_r + \mathbf{u}_\phi$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\theta \\ \sin\theta\sin\phi & \cos\theta\cos\phi & \cos\phi \\ \cos\theta & -\sin\theta & 1 \end{bmatrix}\begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix}$$

$$= \begin{bmatrix} -0.6 & 0 & -0.8 \\ 0.8 & 0 & -0.6 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$B_x = 2(-0.6) + 0.8 = -2$$

$$B_y = 2(0.2) - 0.6 = 1$$

$$B_z = 0$$

Along 3, $C_3 = \int_0^2 \rho \cos\phi \, d\rho \Big|_{\phi=60^\circ} = \frac{\rho^2}{2} = -1$

$\int_L \mathbf{A} \cdot d\mathbf{L} = C_1 + C_2 + C_3 = 1$

38. (A) $\nabla f = \mathbf{u}_x \frac{\partial f}{\partial x} + \mathbf{u}_y \frac{\partial f}{\partial y} + \mathbf{u}_z \frac{\partial f}{\partial z}$

$= y(y+z)\mathbf{u}_x + x(2y+z)\mathbf{u}_y + xy\mathbf{u}_z$

39. (C) $\nabla f = \mathbf{u}_\rho \frac{\partial f}{\partial x} + \mathbf{u}_\phi \frac{1}{\rho}\frac{\partial f}{\partial y} + \mathbf{u}_z \frac{\partial f}{\partial z}$

$= 2\rho^2 z \cos 2\phi \, \mathbf{u}_\rho - 2\rho z \sin 2\phi \, \mathbf{u}_\phi + \rho^2 \cos 2\phi \, \mathbf{u}_z$

At P (1, 45°, 2), $\nabla f = -4\mathbf{u}_\phi$

40. (B) $r\sin\theta\cos\phi = x$, $r\sin\theta\sin\phi = y$, $r\cos\theta = z$

$G = r^3 \sin 2\theta \sin 2\phi \sin\theta$

$= r^3(2\sin\theta\cos\theta)(2\sin\phi\cos\phi)\sin\theta$

$= 4(r\sin\theta\cos\phi)(r\sin\theta\sin\phi)(r\cos\theta) = 4xyz$

$\nabla G = \mathbf{u}_x \frac{\partial(4xyz)}{\partial x} + \mathbf{u}_y \frac{\partial(4xyz)}{\partial x} + \mathbf{u}_z \frac{\partial(4xyz)}{\partial z}$

$= 4yz\mathbf{u}_x + 4xz\mathbf{u}_y + 4xy\mathbf{u}_z$

At P $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, $\nabla G = \mathbf{u}_x + \mathbf{u}_y + \mathbf{u}_z$

41. (C) $\nabla\Phi = (y+z)\mathbf{u}_x + (x+z)\mathbf{u}_y + (x+y)\mathbf{u}_z$

At P(3, –3, – 3),

$\nabla\Phi = -6\mathbf{u}_x$, $\mathbf{R}_{PQ} = (4, -1, -1) - (3, -3, -3) = (1, 2, 2)$

$\nabla\Phi \cdot \mathbf{u}_R = \frac{(-6\mathbf{u}_x)\cdot(\mathbf{u}_x + 2\mathbf{u}_y + 2\mathbf{u}_z)}{3} = -2$

42. (C) $\nabla T = \mathbf{u}_x \frac{\partial T}{\partial x} + \mathbf{u}_y \frac{\partial T}{\partial y} + \mathbf{u}_z \frac{\partial T}{\partial z}$

$= 2x\mathbf{u}_x + 2y\mathbf{u}_z - 4z\mathbf{u}_z$

At P(2, 2, 1), $\nabla T = 4\mathbf{u}_x + 4\mathbf{u}_y - 4\mathbf{u}_z$

43. (A) Let $f = x^2 y + z - 3$, $g = x\ln z - y^2 + 4$

$\nabla f = 2xy\mathbf{u}_x + x^2\mathbf{u}_y + \mathbf{u}_z$

$\nabla g = \ln z \, \mathbf{u}_x - 2y\mathbf{u}_y + \frac{x}{z}\mathbf{u}_z$

At point P (–1, 2, 1)

$\mathbf{u}_f = \frac{-4\mathbf{u}_x + \mathbf{u}_y + \mathbf{u}_z}{\sqrt{18}}$, $\mathbf{u}_g = \frac{-4\mathbf{u}_y - \mathbf{u}_z}{\sqrt{17}}$

$\cos\theta = \pm\,\mathbf{u}_f \cdot \mathbf{u}_g = \pm\frac{-5}{\sqrt{18\times 17}} = 0.286$

$\theta = \cos^{-1} 0.28 = 73.4^\circ$

44. (D) $\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0 + 4x + 0$

At P (1, –2, 3), $\nabla \cdot \mathbf{A} = 4$

45. (A)

$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, A_\phi)}{\partial\phi}$

$= \frac{1}{r^2}(6r^2\cos\theta\cos\phi) + 0 + 0$

At P (1, 30°, 60°), $\nabla\cdot\mathbf{A} = 6(1)(\cos 30^\circ)(\cos 60^\circ) = 2.6$

46. (C) $\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(A_\phi)}{\partial\theta} + \frac{\partial(A_z)}{\partial z}$

$= \frac{1}{\rho}\frac{\partial(\rho\rho z^2\cos\phi)}{\partial\rho} + \frac{\partial(z\sin^2\phi)}{\partial z} = 2z^2\cos\phi + \sin^2\phi$

47. (B) The flux is $\int_S \mathbf{D}\cdot d\mathbf{S}$, By divergence theorem

$\int_S \mathbf{D}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{D}\, dv$

$\nabla\cdot\mathbf{D} = \frac{1}{\rho}\frac{\partial(\rho\rho^2\cos^2\phi)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(z\sin\phi)}{\partial\rho} = 3\rho\cos^2\phi + \frac{3}{\rho}\sin\phi$

$\int_v \nabla\cdot\mathbf{D}\, dv = \int_v \left(3\rho\cos^2\phi + \frac{z}{\rho}\sin\phi\right)\rho\, d\phi\, dz\, d\rho$

$= \int_0^3 dz \int_0^3 z\rho^2 d\rho \int_0^{2\pi}\cos^2\phi\, d\phi + \int_0^3 z\,dz \int_0^3 d\rho \int_0^{2\pi}\sin\phi\, d\phi = 81\pi$

48. (B) $\nabla\times\mathbf{A} = \begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = \begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xy} & \sin xy & \cos^2 xz \end{bmatrix}$

$\mathbf{u}_x(0-0) - \mathbf{u}_y(2\cos xz\,(-\sin xz)z) + \mathbf{u}_z(y\cos xy - xe^{xy})$

$= z\sin 2xy\,\mathbf{u}_y + (y\cos xy - xe^{xy})\mathbf{u}_z$

49. (D) $\nabla\times\mathbf{A} = \frac{1}{\rho}\begin{bmatrix} \mathbf{u}_\rho & \mathbf{u}_\phi & \mathbf{u}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & A_\phi & A_z \end{bmatrix}$

$= \frac{1}{\rho}\begin{bmatrix} \mathbf{u}_\rho & \mathbf{u}_\phi & \mathbf{u}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \rho z\sin\phi & 2\rho^2 z^2\cos\phi & 0 \end{bmatrix}$

$= \frac{1}{\rho}\mathbf{u}_\rho(-6\rho^2 z\cos\phi) - \mathbf{u}_\phi(-\rho\sin\phi)\frac{1}{\rho}\mathbf{u}_z(6\rho z^2\cos\phi - \rho z\cos\phi)$

At point P(5, 90°, 1), $\nabla\times\mathbf{A} = 5\mathbf{u}_\phi$

50. (C) $\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta}\begin{bmatrix} \mathbf{u}_r & r\mathbf{u}_\theta & r\sin\theta \mathbf{u}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{bmatrix}$

$= \frac{1}{r^2 \sin\theta}\begin{bmatrix} \mathbf{u}_r & r\mathbf{u}_\theta & r\sin\theta \mathbf{u}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ r\cos\theta & -\sin\theta & 2r^3\sin^2\theta \end{bmatrix}$

$= \frac{1}{r^2\sin\theta}\mathbf{u}_r(2r^3\sin\theta - 0) - \frac{1}{r\sin\theta}\mathbf{u}_\theta(6r^2\sin^2\theta - 0) + \frac{1}{r}\mathbf{u}_\phi(0 + r\sin\theta)$

$= 4r\cos\theta\,\mathbf{u}_r - 6r\sin\theta\,\mathbf{u}_\theta + \sin\theta\,\mathbf{u}_\phi$

51. (A) $\nabla \times \mathbf{A} = \begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3y^2 - 2z) & (-2x^2 z) & (x + 2y) \end{bmatrix}$

$= \mathbf{u}_x(2 + 2x^2) - \mathbf{u}_y(1 - (-2)) + \mathbf{u}_z(-4xz - 6y)$

$\nabla \times \nabla \times \mathbf{A} = \begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2 + 2x^2) & 3 & -(4xz + 6y) \end{bmatrix}$

$= \mathbf{u}_x(-6) - \mathbf{u}_y(-4z) + \mathbf{u}_z(0) = -6\mathbf{u}_x + 4z\mathbf{u}_y$

At P(−2, 3, −1),

$\nabla \times \nabla \times \mathbf{A} = -6\mathbf{u}_x - 4\mathbf{u}_y = -(6\mathbf{u}_x + 4\mathbf{u}_y)$

52. (D) $\nabla \times \mathbf{A} = \begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & y^2 z & -2xz \end{bmatrix}$

$= -y^2\mathbf{u}_x + 2z\mathbf{u}_y - x^2\mathbf{u}_z$

$\nabla(\nabla \times \mathbf{A}) = 0$

53. (D) $\nabla V = \mathbf{u}_x\frac{\partial V}{\partial x} + \mathbf{u}_y\frac{\partial V}{\partial y} + \mathbf{u}_z\frac{\partial V}{\partial z}$

$= (z - 2xy)\mathbf{u}_x + (2yz^2 - x^2)\mathbf{u}_y + (x - 2y^2 z)\mathbf{u}_z$

$\nabla\cdot(\nabla V) = \frac{\partial(z - 2xy)}{\partial x} + \frac{\partial(2yz^2 - x^2)}{\partial y} + \frac{\partial(x - 2y^2 z)}{\partial z}$

$= -2y + 2z^2 - 2y^2 = 2(z^2 - y^2 - y)$

54. (B) $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$= 2(y^2z^2 + x^2z^2 + x^2y^2) = 2(x^2y^2 + y^2z^2 + z^2x^2)$

55. (A) $\nabla^2 V = \frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

$= 4z(\cos\phi + \sin\phi) - z(\cos\phi + \sin\phi) + 0 = 3z(\cos\phi + \sin\phi)$

At P(3, 60°, −2), $\nabla^2 V = 3(-2)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = -8.2$

56.(B)

$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

$= 6(1 + \cos\theta\sin\phi) - \frac{2\sin\theta}{\sin\theta}\cos\theta\sin\phi - \frac{\cos\theta\sin\phi}{\sin^2\theta}$

$= 6 + 4\cos\theta\sin\phi - \cot\theta\,\text{cosec}\,\theta\sin\phi$

57. (A) $\rho = \sqrt{x^2 + y^2}$

$\nabla\ln\rho = \mathbf{u}_x\frac{\partial\ln\rho}{\partial x} + \mathbf{u}_y\frac{\partial\ln\rho}{\partial y} + \mathbf{u}_z\frac{\partial\ln\rho}{\partial z} = \frac{x}{\rho^2}\mathbf{u}_x + \frac{y}{\rho^2}\mathbf{u}_y$

$\nabla \times \phi\mathbf{u}_z = \nabla \times \tan^{-1}\frac{y}{x}\mathbf{u}_z = \begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \tan^{-1}\frac{y}{x} \end{bmatrix}$

$= \frac{x}{x^2 + y^2}\mathbf{u}_x + \frac{y}{x^2 + y^2}\mathbf{u}_y = \frac{x}{\rho}\mathbf{u}_x + \frac{y}{\rho}\mathbf{u}_y$

58. (A) $(\mathbf{r}\cdot\nabla)r^2 = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}\right)(x^2 + y^2 + z^2)$

$= x(2x) + y(2y) + z(2z) = 2(x^2 + y^2 + z^2) = 2r^2$

59. (B) $\nabla\cdot r^n\,\mathbf{r} = \frac{\partial(xr^n)}{\partial x} + \frac{\partial(yr^n)}{\partial y} + \frac{\partial(zr^n)}{\partial z}$

where $r^n = (x^2 + y^2 + z^2)^{\frac{n}{2}}$

$\nabla\cdot r^n\,\mathbf{r} = 2x^2\left(\frac{n}{2}\right)(x^2 + y^2 + z^2)^{\frac{n}{2}-1}$

$+ 2y^2\left(\frac{n}{2}\right)(x^2 + y^2 + z^2)^{\frac{n}{2}-1} + 2z^2\left(\frac{n}{2}\right)(x^2 + y^2 + z^2)^{\frac{n}{2}-1}$

$+ r^n + r^n + r^n$

$= n(x^2 + y^2 + z^2)(x^2 + y^2 + z^2)^{\frac{n}{2}-1} + 3r^n$

$= nr^n + 3r^n = (n + 3)r^n$

60. (C) By Stokes theorem $\oint_L \mathbf{F}\cdot d\mathbf{L} = \oint_S (\nabla \times \mathbf{F})\cdot d\mathbf{S}$

$\nabla \times \mathbf{F} = -x^2\mathbf{u}_z$

$d\mathbf{S} = dxdy\,(-\mathbf{u}_z)$

$\oint_S (\nabla \times \mathbf{F})\cdot d\mathbf{S} = -\iint(-x^2)dxdy$

$= \int_0^1\int_0^x x^2 dydx + \int_1^2\int_0^{-x+2} x^2 dydx = \int_0^1 x^3 dx + \int_1^2 x^2(2 - x)dx$

$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_1^2$

$= \frac{1}{4} + \left[\frac{16}{3} - 4\right] - \left[\frac{2}{3} - \frac{1}{4}\right] = \frac{1}{4} + \frac{14}{3} - 4 + \frac{1}{4} = \frac{7}{6}$

61. (C) $\oint_C \mathbf{A}\cdot d\mathbf{L} = \left(\int_{ab} + \int_{bc} + \int_{cd} + \int_{da}\right)\mathbf{A}\cdot d\mathbf{L}$

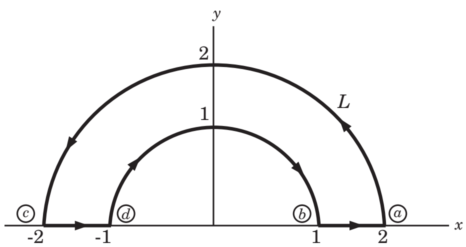

Fig. S8.1.61

Along ab, $d\phi = 0$, $\phi = 0$,

$$\mathbf{A}\cdot d\mathbf{L} = 0, \quad \int_a^b \mathbf{A}\cdot d\mathbf{L} = 0$$

Along bc, $d\rho = 0$, $\mathbf{A}\cdot d\mathbf{L} = \rho^3 d\phi$

$$\int_b^c \mathbf{A}\cdot d\mathbf{L} = \int_0^{\pi} \rho^3 d\phi = (2)^3\pi = 8\pi$$

Along cd, $d\phi = 0$, $\phi = \pi$, $\mathbf{A}\cdot d\mathbf{L} = 0$,

$$\int_c^d \mathbf{A}\cdot d\mathbf{L} = 0$$

Along da, $d\rho = 0$, $\mathbf{A}\cdot d\mathbf{L} = \rho^3 d\phi$

$$\int_d^a \mathbf{A}\cdot d\mathbf{L} = \rho^3\int_{\pi}^{0} d\phi = (1)^3(-\pi) = -\pi$$

$$\int \mathbf{A}\cdot d\mathbf{L} = 0 + 8\pi + 0 - \pi = 7\pi$$

62. (B) Using divergence theorem

$$\oint \mathbf{F}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{F}dv$$

$$\nabla\cdot\mathbf{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(2\rho^2 z^2) = 4z^2 = 4z^2$$

$$\int_v \nabla\cdot\mathbf{F}\,dv = \iiint 4z^2\rho d\rho d\phi dz = 4\int_{-1}^{1} z^2 dz\int_2^5 \rho d\rho\int_0^{2\pi} d\phi = 176$$

63. (D) $\oint_{\mathbf{S}} \mathbf{A}\cdot d\mathbf{S} = \oint_v (\nabla\cdot\mathbf{A})dv$

$$\nabla\cdot\mathbf{A} = \frac{\partial(xy)}{\partial x} + \frac{\partial(yz)}{\partial y} + \frac{\partial(zx)}{\partial z}x = y + z + x$$

$$\oint_{\mathbf{S}} \mathbf{A}\cdot d\mathbf{S} = \int_0^1\int_0^1\int_0^1 (y + z + x)dxdydz$$

$$= 3\left(\int_0^1 xdx\int_0^1 dy\int_0^1 dz\right) = 3\left(\frac{1}{2}\right) = 1.5$$

64. (B) $\nabla\cdot\mathbf{D} = \dfrac{1}{\rho}\dfrac{\partial(\rho D_\rho)}{\partial\rho} + \dfrac{1}{\rho}\dfrac{\partial(D_\phi)}{\partial\theta} + \dfrac{\partial(D_z)}{\partial z}$

$$= 4z + \frac{3}{\rho}z\cos\phi$$

$$\oint_{\mathbf{S}} \mathbf{D}\cdot d\mathbf{S} = \oint_v (\nabla\cdot\mathbf{D})dv$$

$$\oint \mathbf{D}\cdot d\mathbf{S} = \iiint\left(4z + \frac{3z}{\rho}\cos\phi\right)\rho d\rho d\phi dz$$

$$= 4\int_0^2 \rho d\rho\int_0^5 zdz\int_0^{45^\circ} d\phi + 3\int_0^2 d\rho\int_0^5 zdz\int_0^{45^\circ}\cos\phi\, d\phi$$

$$= 4\left(\frac{4}{2}\right)\left(\frac{25}{2}\right)\left(\frac{\pi}{4}\right) + 3(2)\left(\frac{25}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 131.57$$

65. (A) $\nabla\times\mathbf{F} = \begin{bmatrix}\mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \alpha x + \beta z^2 & 3x^2 - \gamma z & 3xz^2 - y\end{bmatrix}$

$$= (-1 + \gamma)\mathbf{u}_x + (3\beta z^2 - 3z^2)\mathbf{u}_y + (6x - \alpha x)\mathbf{u}_z$$

If $\mathbf{F}$ is irrotational, $\nabla\times\mathbf{F} = 0$

i.e. $\alpha = 1 = \beta = \gamma$.

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CHAPTER

8.2

ELECTROSTATICS

1. Let $Q_1 = 4\ \mu C$ be located at $P_1(3, 11, 8)$ while $Q_2 = -5\ \mu C$ is at $P_2(6, 15, 8)$. The force F_2 on Q_2 will be

(A) $-(4.32\mathbf{u}_x + 5.76\mathbf{u}_y)$ N (B) $4.32\mathbf{u}_x + 5.76\mathbf{u}_y$ N

(C) $-(4.32\mathbf{u}_x + 5.76\mathbf{u}_y)$ mN (D) $4.32\mathbf{u}_x + 5.76\mathbf{u}_y$ mN

2. Four 5 nC positive charge are located in the $z = 0$ plane at the corners of a square 8 mm on a side. A fifth 5 nC positive charge is located at a point 8 mm distant from each of the other charge. The magnitude of the total force on this fifth charge is

(A) 2×10^{-4} N (B) 4×10^{-4} N

(C) 0.014 N (D) 0.01 N

3. Four 40 nC are located at A(1, 0, 0), B(–1, 0, 0), C(0, 1, 0) and D(0, –1, 0) in free space. The total force on the charge at A is

(A) $24.6\mathbf{u}_x$ μN (B) $-24.6\mathbf{u}_x$ μN

(C) $-13.6\mathbf{u}_x$ μN (D) $13.76\mathbf{u}_x$ μN

4. Let a point charge 41 nC be located at $P_1(4, -2, 7)$ and a charge 45 nC be at $P_2(-3, 4, -2)$. The electric field **E** at $P_3(1, 2, 3)$ will be

(A) $0.13\mathbf{u}_x + 0.33\mathbf{u}_y + 0.12\mathbf{u}_z$

(B) $-0.13\mathbf{u}_x - 0.33\mathbf{u}_y - 0.12\mathbf{u}_z$

(C) $1.15\mathbf{u}_x + 2.93\mathbf{u}_y + 1.09\mathbf{u}_z$

(D) $-1.15\mathbf{u}_x - 2.93\mathbf{u}_y - 1.09\mathbf{u}_z$

5. Let a point charge 25 nC be located at $P_1(4, -2, 7)$ and a charge 60 nC be at $P_2(-3, 4, -2)$. The point, at which on the y axis, is $E_x = 0$, is

(A) –7.46 (B) –22.11

(C) –6.89 (D) (B) and (C)

6. A $2\ \mu C$ point charge is located at A (4, 3, 5) in free space. The electric field at P(8, 12, 2) is

(A) $131.1\mathbf{u}_\rho + 159.7\mathbf{u}_\phi - 49.4\mathbf{u}_z$

(B) $159.7\mathbf{u}_\rho + 27.4\mathbf{u}_\phi - 49.4\mathbf{u}_z$

(C) $131.1\mathbf{u}_\rho + 27.4\mathbf{u}_\phi - 49.4\mathbf{u}_z$

(D) $159.7\mathbf{u}_\rho + 137.1\mathbf{u}_\phi - 49.4\mathbf{u}_z$

7. A point charge of -10 nC is located at $P_1(0, 0, 0.5)$ and a charge of $2\ \mu C$ at the origin. The **E** at P(0, 2, 1) is

(A) $68.83\mathbf{u}_r + 14.85\ \mathbf{u}_\phi$ (B) $68.83\mathbf{u}_r + 64.01\mathbf{u}_\phi$

(C) $68.83\mathbf{u}_r - 14.85\ \mathbf{u}_\phi$ (D) $68.83\mathbf{u}_r - 64.01\mathbf{u}_\phi$

8. Charges of 20 nC and -20nC are located at (3, 0, 0) and (–3, 0, 0) and (–3, 0, 0), respectively. The magnitude of **E** at y axis is

(A) $\dfrac{1080}{(9 + y^2)^{3/2}}$ (B) $\dfrac{1080}{(9 + y^2)^3}$

(C) $\dfrac{108}{(9 + y^2)^{3/2}}$ (D) $\dfrac{108}{(9 + y^2)^3}$

9. A charge Q_0 located at the origin in free space produces a field for which $E_2 = 1$ kV/m at point P(–2, 2, –1). The charge Q_0 is

(A) $2\ \mu C$ (B) $-3\ \mu C$

(C) $3\ \mu C$ (D) $-2\ \mu C$

10. The volume charge density $\rho_v = \rho_o e^{-|x|-|y|-|z|}$ exist over all free space. The total charge present is

(A) $2\rho_o$ (B) $4\rho_o$

(C) $8\rho_o$ (D) $3\rho_o$

11. A uniform volume charge density of $0.2\ \mu C/m^2$ is present throughout the spherical shell extending from $r = 3$ cm to $r = 5$ cm. If $\rho = 0$ elsewhere, the total charge present throughout the shell will be

(A) 41.05 pC (B) 257.92 pC

(C) 82.1 pC (D) 129.0 pC

12. If $\rho_v = \frac{1}{z^2+10} 5e^{-0.1\rho}(\pi - |\phi|)\ \mu C/m^3$ in the region $0 \le \rho \le 10$, $-\pi < \phi < \pi$ and all z, and $\rho_v = 0$ elsewhere, the total charge present is

(A) 1.29 mC (B) 2.58 mC

(C) 0.645 mC (D) 0

13. The region in which $4 < r < 5$, $0 < \theta < 25°$, and $0.9\,\pi < \phi < 1.1\,\pi$ contains the volume charge density of $\rho_v = 10\,(r-4)(r-5)\ \sin\theta \sin\frac{\phi}{2}$. Outside the region, $\rho_v = 0$. The charge within the region is

(A) 0.57 C (B) 0.68 C

(C) 0.46 C (D) 0.23 C

14. A uniform line charge of 5 nC/m is located along the line defined by $y = -2$, $z = 5$. The electric field **E** at P(1, 2, 3) is

(A) $-9\mathbf{u}_y + 4.5\mathbf{u}_z$ (B) $9\mathbf{u}_y - 4.5\mathbf{u}_z$

(C) $-18\ \mathbf{u}_y + 9\mathbf{u}_z$ (D) $18\ \mathbf{u}_y - 9\mathbf{u}_z$

15. A uniform line charge of 6.25 nC/m is located along the line defined by $y = -2, z = 5$. The **E** at that point in the $z = 0$ plane where the direction of **E** is given by $\left(\frac{1}{3}\mathbf{u}_y - \frac{2}{3}\mathbf{u}_z\right)$, is

(A) $4.5\mathbf{u}_y + 9\mathbf{u}_z$ (B) $4.5\mathbf{u}_y - 9\mathbf{u}_z$

(C) $9\mathbf{u}_y - 18\mathbf{u}_z$ (D) $18\mathbf{u}_y - 36\mathbf{u}_z$

16. Uniform line charge of 20 nC/m and −20 nC/m are located in the $x = 0$ plane at $y = 3$ and $y = -3$ m respectively. The **E** at P(6, 0, 6) will be

(A) $-24\mathbf{u}_y$ V/m (B) $48\mathbf{u}_y$ V/m

(C) $-48\mathbf{u}_y$ V/m (D) $24\mathbf{u}_y$ V/m

17. Uniform line charges of 100 nC/m lie along the entire extent of the three coordinate axes. The **E** at P(−3, 2, −1) is

(A) $-1.92\mathbf{u}_x + 2\mathbf{u}_y - 1.08\mathbf{u}_z$ kV/m

(B) $-0.96\mathbf{u}_x + \mathbf{u}_y - 0.54\mathbf{u}_z$ kV/m

(C) $0.96\mathbf{u}_x - \mathbf{u}_y + 0.54\mathbf{u}_z$ kV/m

(D) $1.92\mathbf{u}_x - 2\mathbf{u}_y + 1.08\mathbf{u}_z$ kV/m

18. Two identical uniform charges with $\rho_l = 80$ nC/m are located in free space at $x = 0$, $y = \pm 3$ m. The force per unit length acting on the line at positive y arising from the charge at negative y is

(A) $9.375\mathbf{u}_y\ \mu N$ (B) $37.5\mathbf{u}_y\ \mu N$

(C) $19.17\mathbf{u}_y\ \mu N$ (D) $75\mathbf{u}_y\ \mu N$

19. A uniform surface charge density of $10\ nC/m^2$ is present in the region $x = 0$, $-2 < y < 2$ and all z if $\varepsilon = \varepsilon_o$, the electric field at P(3, 0, 0) has

(A) x component only

(B) y component only

(C) x and y component

(D) x, y and z component

20. The surface charge density is $\rho_s = 5\ nC/m^2$, in the region $\rho < 0.2$, $z = 0$, and is zero elsewhere. The electric field E at A($\rho = 0$, $z = 0.5$) is

(A) 5.4 V/m (B) 10.1 V/m

(C) 10.5 V/m (D) 20.2 V/m

21. Three infinite charge sheet are positioned as follows: $10\ nC/m^2$ at $x = -3$, $-40\ nC/m^2$ at $y = 4$ and $50\ nC/m^2$ at $z = 2$. The **E** at (4, 3, −2) is

(A) $0.56\mathbf{u}_x + 2.23\mathbf{u}_y - 2.8\mathbf{u}_z$ kV/m

(B) $0.56\mathbf{u}_x - 2.23\mathbf{u}_y + 2.8\mathbf{u}_z$ kV/m

(C) $0.56\mathbf{u}_x + 2.23\mathbf{u}_y + 2.8\mathbf{u}_z$ kV/m

(D) $-0.56\mathbf{u}_x - 2.23\mathbf{u}_y + 2.8\mathbf{u}_z$ kV/m

22. Let $\mathbf{E} = 5x^3\mathbf{u}_x - 15\ x^2y\mathbf{u}_y$. The equation of the stream line that passes through P(4, 2, 1) is

(A) $y = \frac{128}{x^3}$ (B) $x = \frac{128}{y^3}$

(C) $y = \frac{64}{x^2}$ (D) $x = \frac{64}{y^2}$

23. A point charge 10 nC is located at origin. Four uniform line charge are located in the $x = 0$ plane as follows : 40 nC/m at $y = 1$ and −5 m, −60 nC/m at $y = -2$ and −4 m. The **D** at P(0, −3, 4) is

(A) $-19.1\mathbf{u}_y + 25.5\mathbf{u}_z\ pC/m^2$

(B) $19.1\mathbf{u}_y - 25.5\mathbf{u}_z\ pC/m^2$

(C) $-16.4\mathbf{u}_y + 21.9\mathbf{u}_z\ pC/m^2$

(D) $16.4\mathbf{u}_y - 21.9\mathbf{u}_z\ pC/m^2$

24. A point charge 20 nC is located at origin. Four uniform line charge are located as follows 40 nC/m at $y = \pm 1$ and 50 nC/m at $y = \pm 2$. The electric flux that leaves the surface of a sphere, 4 m in radius, centered at origin is

(A) 1.33 nC (B) 1.89 μC

(C) 1.33 μC (D) 1.89 μC

25. The cylindrical surface $\rho = 8$ C contains the surface charge density, $\rho_s = 5e^{-20|z|}$ nC/m^2. The flux that leaves the surface $\rho = 8$ cm, $1\text{cm} < z < 5\text{cm}$ $30° < \phi < 90°$ is

(A) 270.7 nC (B) 9.45 nC

(C) 270.7 pC (D) 9.45 pC

26. Let $\mathbf{D} = 4xy\,\mathbf{u}_x + 2(x^2 + z^2)\mathbf{u}_y + 4yz\mathbf{u}_z$ C/m^2. The total charge enclosed in the rectangular parallelepiped $0 < x < 2$, $0 < y < 3$, $0 < z < 5$ m is

(A) 360 C (B) 180 C

(C) 100 C (D) 560 C

27. Volume charge density is located in free space as $\rho_v = 2e^{-1000r}$ nC/m^3 for $0 < r < 1$ mm and $\rho_v = 0$ elsewhere. The value of D_r on the surface $r = 1$ mm is

(A) 1.28 pC/m^2 (B) 0.28 pC/m^2

(C) 0.78 pC/m^2 (D) 0.32 pC/m^2

28. Spherical surfaces at $r = 2$ and 4 carry uniform charge densities of 20 nC/m^2 and –4 nC/m^2. The D_r at $2 < r < 4$ is

(A) $-\frac{16}{r^2}$ nC/m^2 (B) $\frac{16}{r^2}$ nC/m^2

(C) $\frac{80}{r^2}$ nC/m^2 (D) $-\frac{80}{r^2}$ nC/m^2

29. Given the electric flux density, $\mathbf{D} = 2xy\mathbf{u}_x + x^2\mathbf{u}_y + 6z^3\mathbf{u}_z$ C/m^2. The total charge enclosed in the volume $0 < x, y, z < a$ is

(A) $6a^5 + \frac{5}{3}a^4$ (B) $a^5 + 6a^4$

(C) $6a^5 + a^4$ (D) $\frac{5}{3}a^5 + 6a^4$

30. Let $\mathbf{D} = 5x^4y^4z^4\mathbf{u}_y$. The flux enclosed by volume $x = 3$ and 3.1, $y = 1$ and 1.1, and $z = 2$ and 2.1 is

(A) 49.6 (B) 24.8

(C) 35.4 (D) 36.4

31. A spherical surface of radius of 3 mm is centered at P(4, 1, 5) in free space. If $\mathbf{D} = x\mathbf{u}_x$ C/m^2 the net electric flux leaving the spherical surface is

(A) 113.1 μC (B) 339.3 nC

(C) 113.1 nC (D) 452.4 nC

32. The electric flux density is

$$\mathbf{D} = \frac{1}{z^2}[10xyz\mathbf{u}_x + 5x^2z\mathbf{u}_y + (2z^3 - 5x^2y)\mathbf{u}_z]$$

The volume charge density ρ_v at (–2, 3, 5) is

(A) 6.43 (B) 8.96

(C) 10.4 (D) 7.86

33. If $\mathbf{D} = 2r\mathbf{u}_r$ C/m^2, the total electric flux leaving the surface of the cube, $0 < x, y, z < 0.4$ is

(A) 0.32 (B) 0.34

(C) 0.38 (D) 0.36

34. If $\mathbf{E} = 4\mathbf{u}_x - 3\mathbf{u}_y + 5\mathbf{u}_z$ in the neighborhood of point P(6, 2, –3). The incremental work done in moving 5 C charge a distance of 2 m in the direction $\mathbf{u}_x + \mathbf{u}_y + \mathbf{u}_z$ is

(A) –60 J (B) 34.64 J

(C) –34.64 J (D) 60 JJ

35. If $\mathbf{E} = 100\mathbf{u}_\rho$ V/m , the incremental amount of work done in moving a 60 μC charge a distance of 2 mm from P(1, 2, 3) toward Q(2, 1, 4) is

(A) –5.4 μJ (B) 3.1 μJ

(C) –3.1 μJ (D) 0

36. A 10 C charge is moved from the origin to P(3, 1, –1) in the field $\mathbf{E} = 2x\mathbf{u}_x - 3y^2\mathbf{u}_y + 4\mathbf{u}_z$ V/m along the straight line path $x = -3y$, $y = x + 2z$. The amount of energy required is

(A) –40 J (B) 20 J

(C) –20 J (D) 40 J

37. A uniform surface charge density of 30 nC/m^2 is present on the spherical surface $r = 6$ mm in free space. The V_{AB} between A ($r = 2$cm, $\theta = 35°$, $\phi = 55°$) and B ($r = 3$cm, $\theta = 40°$, $\phi = 90°$) is

(A) 2.03 V (B) 10.17 V

(C) 4.07 mV (D) –10.17 V

38. A point charge is located at the origin in free space. The work done in carrying a charge 10 C from point A $(r = 4,\ \theta = \pi/6,\ \phi = \pi/4)$ to B$(r = 4,\ \theta = \pi/3,\ \pi/6)$ is

(A) 0.45 μ**J** (B) 0.32 μJ

(C) –0.45 μ**J** (D) 0

39. Let a uniform surface charge density of $5\ \text{nC/m}^2$ be present at the $z = 0$ plane, a uniform line charge density of 8 nC/m be located at $x = 0,\ \ z = 4$ and a point charge of 2 μC be present at P(2, 0, 0). If $V = 0$ at A(0, 0, 5), the V at B(1, 2, 3) is

(A) 10.46 kV (B) 1.98 kV

(C) 0.96 kV (D) 3.78 kV

40. A non uniform linear charge density, $\rho_L = 6/(z^2 + 1)$ nC/m lies along the z axis. The potential at P$(\rho = 1, 0, 0)$ in free space is $(V_\infty = 0)$

(A) 0 V (B) 216 V

(C) 144 V (D) 108 V

41. The annular surface, 1 cm $< \rho <$ 3 cm carries the nonuniform surface charge density $\rho_s = 5\rho\ \text{nC/m}^2$. The V at P(0, 0, 2 cm) is

(A) 81 mV (B) 90 mV

(C) 63 mV (D) 76 mV

42. If $V = 2xy^2z^3 + 3\ln(x^2 + 2y^2 + 3z^2)$ in free space the magnitude of electric field E at P (3, 2, –1) is

(A) 72.6 V/m (B) 79.6 V/m

(C) 75 V/m (D) 70.4 V/m

43. It is known that the potential is given by $V = 70r^{0.6}$ V. The volume charge density at $r = 0.6$ m is

(A) 1.79 nC/m^3 (B) –1.79 nC/m^3

(C) 1.22 nC/m^3 (D) –1.22 nC/m^3

44. The potential field $V = 80r^2 \cos\theta$ V. The volume charge density at point P(2.5, $\theta = 30°$, $\phi = 60°$) in free space is

(A) –2.45 nC/m^3 (B) 1.42 nC/m^3

(C) –1.42 nC/m^3 (D) 2.45 nC/m^3

45. Within the cylinder $\rho = 2,\ 0 < z < 1$ the potential is given by $V = 100 + 50\rho + 150\rho\sin\phi$ V. The charge lies within the cylinder is

(A) –4.94 nC (B) –4.86 nC

(C) –5.56 nC (D) –3.68 nC

46. A dipole having

$$\frac{Qd}{4\pi\varepsilon_o} = 100\ \text{V}\cdot\text{m}^2$$

is located at the origin in free space and aligned so that its moment is in the $\mathbf{u}_z$ direction. The E at point $(r = 1,\ 45°,\ \ \phi = 0)$ is

(A) 158.11 V/m (B) 194.21 V/m

(C) 146.21 V/m (D) 167.37 V/m

47. A dipole located at the origin in free space has a moment $p = 2 \times 10^{-9}\mathbf{u}_z$ C.m. The points at which $|E|_\theta = 1$ mV/m on line $y = z,\ \ x = 0$ are

(A) $y = z = \pm 23.35$ (B) $y = z = \pm 16.5,\ \ x = 0$

(C) $y = z = 16.5$ (D) $y = 0, z = 23.35,\ \ x = 0$

48. A dipole having a moment $\mathbf{p} = 3\mathbf{u}_x - 5\mathbf{u}_y + 10\mathbf{u}_z$ nC.m is located at P(1, 2, –4) in free space. The V at Q (2, 3, 4) is

(A) 1.31 V (B) 1.26 V

(C) 2.62 V (D) 2.52 V

49. A potential field in free space is expressed as $V = 40/xyz$. The total energy stored within the cube $1 < x, y, z < 2$ is

(A) 1548 pJ (B) 0

(C) 774 pJ (D) 387 pJ

50. Four 1.2 nC point charge are located in free space at the corners of a square 4 cm on a side. The total potential energy stored is

(A) 1.75 μJ (B) 2 μJ

(C) 3.5 μJ (D) 0

51. Given the current density

$$\mathbf{J} = 10^5[\sin(2x)\, e^{-2y}\mathbf{u}_x + \cos(2x)\, e^{-2y}\mathbf{u}_y]\ \text{kA/m}^2$$

The total current crossing the plane $y = 1$ in the $\mathbf{u}_y$ direction in the region $0 < x < 1,\ \ 0 < z < 2$ is

(A) 0 (B) 12.3 mA

(C) 24.6 mA (D) 6.15 mA

64. In a certain region where the relative permitivity is 2.4, $\mathbf{D} = 2\mathbf{u}_x - 4\mathbf{u}_y + 5\mathbf{u}_z$ nC/m^2. The polarization is

(A) $2.8\mathbf{u}_x - 5.6\mathbf{u}_y + 7\mathbf{u}_z$ nC/m^2

(B) $3.4\mathbf{u}_x - 6.9\mathbf{u}_y + 8.6\mathbf{u}_z$ nC/m^2

(C) $1.2\mathbf{u}_x - 2.3\mathbf{u}_y + 2.9\mathbf{u}_z$ nC/m^2

(D) $3.89\mathbf{u}_x - 6.43\mathbf{u}_y + 8.96\mathbf{u}_z$ nC/m^2

65. Medium 1 has the electrical permitivity $\varepsilon_1 = 1.5\varepsilon_o$ and occupies the region to the left of $x = 0$ plane. Medium 2 has the electrical permitivity $\varepsilon_2 = 2.5\varepsilon_o$ and occupies the region to the right of $x = 0$ plane. If $\mathbf{E}_1$ in medium 1 is $\mathbf{E}_1 = (2\mathbf{u}_x - 3\mathbf{u}_y + 1\mathbf{u}_z)$ V/m then $\mathbf{E}_2$ in medium 2 is

(A) $(2.0\mathbf{u}_x - 1.8\mathbf{u}_x + 0.6\mathbf{u}_z)$ V/m

(B) $(1.67\mathbf{u}_x - 3\mathbf{u}_y + \mathbf{u}_z)$ V/m

(C) $(1.2\mathbf{u}_x - 3\mathbf{u}_y + \mathbf{u}_z)$ V/m

(D) $(1.2\mathbf{u}_x - 1.8\mathbf{u}_y + 0.6\mathbf{u}_z)$ V/m

66. Two perfect dielectrics have relative permitivities $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 8$. The planner interface between them is the surface $x - y + 2z = 5$. The origin lies in region 1. If $\mathbf{E}_1 = 100\mathbf{u}_x + 200\mathbf{u}_y - 50\mathbf{u}_z$ V/m then $\mathbf{E}_2$ is

(A) $400\mathbf{u}_x + 800\mathbf{u}_y - 200\mathbf{u}_z$ V/m

(B) $400\mathbf{u}_x + 200\mathbf{u}_y - 50\mathbf{u}_z$ V/m

(C) $25\mathbf{u}_x + 200\mathbf{u}_y - 50\mathbf{u}_z$ V/m

(D) $125\mathbf{u}_x + 175\mathbf{u}_z$ V/m

67. The two spherical surfaces $r = 4$ cm and $r = 9$ cm are separated by two perfect dielectric shells, $\varepsilon_{r1} = 2$ for $4 < r < 6$ and $\varepsilon_{r2} = 5$ for $6 < r < 9$. If $\mathbf{E}_1 = \frac{1000}{r^2}\mathbf{u}_r$ then $\mathbf{E}_2$ is

(A) $\frac{5000}{r^2}\mathbf{u}_r$ V/m (B) $\frac{400}{r^2}\mathbf{u}_r$ V/m

(C) $\frac{2500}{r^2}\mathbf{u}_r$ V/m (D) $\frac{2000}{r^2}\mathbf{u}_r$ V/m

68. The surface $x = 0$ separate two perfect dielectric. For $x > 0$, let $\varepsilon_{r1} = 3$, while $\varepsilon_{r2} = 5$ where $x < 0$. If $\mathbf{E}_1 = 80\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z$ V/m then $\mathbf{E}_2$ is

(A) $(133.3\mathbf{u}_x - 100\mathbf{u}_z - 66.7\mathbf{u}_z)$ V/m

(B) $(133.3\mathbf{u}_x - 60\mathbf{u}_z - 40\mathbf{u}_z)$ V/m

(C) $(48\mathbf{u}_x - 36\mathbf{u}_y - 24\mathbf{u}_z)$ V/m

(D) $(48\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z)$ V/m

69. A potential field exists in a region where $\varepsilon = f(x)$. If $\rho_v = 0$, the $\nabla^2 V$ is

(A) $-\frac{1}{f(x)}\frac{dF}{dx}\frac{\partial V}{\partial x}$ (B) $f(x)\frac{df}{dx}\frac{\partial V}{\partial x}$

(C) $\frac{1}{f(x)}\frac{df}{dx}\frac{\partial V}{\partial x}$ (D) $-f(x)\frac{df}{dx}\frac{\partial V}{\partial x}$

70. If $V(x, y) = 4e^{2x} + f(x) - 3y^2$ in a region of free space where $\rho_v = 0$. It is know that both E_x and V are zero at the origin. The $V(x, y)$ is

(A) $3(x^2 - y^2)$ (B) $3(y^2 - x^2)$

(C) $4x^2 - 3y^2$ (D) $4y^2 - 3x^2$

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SOLUTIONS

1. (C) $\mathbf{F}_2 = \dfrac{Q_1 Q_2}{4\pi\varepsilon_o} \dfrac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|^3}$

$= \dfrac{(4\times10^{-6})(-5\times10^{-6})}{4\pi\varepsilon_o} \times \dfrac{(3\mathbf{u}_x + 4\mathbf{u}_y)}{5^3}$

$= (4.32\mathbf{u}_x + 5.76\mathbf{u}_y)$ mN

2. (D) Arranging the charge in the xy plane at location (4, 4), (–4, 4), (4, –4), (–4, 4), the fifth charge will be on the z-axis at location $z = 4\sqrt{2}$. By symmetry, the force on the fifth charge will be z directed, and will be four times the z component

$F = \dfrac{4}{\sqrt{2}} \times \dfrac{q^2}{4\pi\varepsilon_o d^2}$

$= \dfrac{4}{\sqrt{2}} \times \dfrac{(5\times10^{-9})^2}{4\pi\times(8.85\times10^{-12})(8\times10^{-3})^2} = 10^{-2}$ N

3. (D) The force will be

$F = \dfrac{(40\times10^{-9})^2}{4\pi\varepsilon_o}\left[\dfrac{\mathbf{R}_{CA}}{|\mathbf{R}_{CA}|^3} + \dfrac{\mathbf{R}_{DA}}{|\mathbf{R}_{DA}|^3} + \dfrac{\mathbf{R}_{BA}}{|\mathbf{R}_{BA}|^3}\right]$

where $\mathbf{R}_{CA} = \mathbf{u}_x - \mathbf{u}_y$, $\mathbf{R}_{DA} = \mathbf{u}_x + \mathbf{u}_y$, $\mathbf{R}_{BA} = 2\mathbf{u}_x$

$|\mathbf{R}_{CA}| = |\mathbf{R}_{DA}| = \sqrt{2}$, $|\mathbf{R}_{BA}| = 2$

$\mathbf{F} = \dfrac{(40\times10^{-9})^2}{4\pi\times(8.85\times10^{-9})}\left[\dfrac{\mathbf{u}_x - \mathbf{u}_y}{2\sqrt{2}} + \dfrac{\mathbf{u}_x + \mathbf{u}_y}{2\sqrt{2}} + \dfrac{2\mathbf{u}_x}{8}\right]$

$= 13.76\mathbf{u}_x$ μN

4. (C) $\mathbf{E} = \dfrac{10^{-9}}{4\pi\varepsilon_o}\left[\dfrac{41\mathbf{R}_{13}}{|\mathbf{R}_{13}|^2} + \dfrac{45\mathbf{R}_{23}}{|\mathbf{R}_{23}|^2}\right]$

$\mathbf{R}_{13} = -3\mathbf{u}_x + 4\mathbf{u}_y - 4\mathbf{u}_z$, $\mathbf{R}_{23} = 4\mathbf{u}_x - 2\mathbf{u}_y + 5\mathbf{u}_z$

$\mathbf{E} = 9\times10^9\times10^{-9}\times\left[\dfrac{41(-3\mathbf{u}_x + 4\mathbf{u}_y - 4\mathbf{u}_z}{(41)^{3/2}} + \dfrac{45(4\mathbf{u}_x + -2\mathbf{u}_y + 5\mathbf{u}_z}{(45)^{3/2}}\right]$

$= 1.152\mathbf{u}_x + 2.93\mathbf{u}_y + 1.089\mathbf{u}_z$

5. (D) The point is $P_3(0, y, 0)$

$\mathbf{R}_{13} = -4\mathbf{u}_x + (y+2)\mathbf{u}_y - 7\mathbf{u}_z$,

$\mathbf{R}_{12} = 3\mathbf{u}_x + (y-4)\mathbf{u}_y + 2\mathbf{u}_z$

$E_x = \dfrac{10^{-9}}{4\pi\varepsilon_o}\left[\dfrac{25\times(-4)}{[65+(y+2)^2]^{3/2}} + \dfrac{60\times3}{[13+(y-4)^2]^{3/2}}\right]$

To obtain $E_x = 0$, $0.48y^2 + 13.92y + 73.12 = 0$

which yields the two value $y = -6.89, -22.11$

6. (B) $\mathbf{E}_p = \dfrac{2\times10^{-6}}{4\pi\varepsilon_o} \dfrac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3}$

$= \dfrac{2\times10^{-6}}{4\pi\varepsilon_o}\left[\dfrac{4\mathbf{u}_x + 9\mathbf{u}_y - 3\mathbf{u}_z}{(106)^{3/2}}\right]$

$= 65.9\mathbf{u}_x + 148.3\mathbf{u}_y - 49.4\mathbf{u}_z$

Then at point P, $\rho = \sqrt{8^2 + 12^2} = 14.4$,

$\phi = \tan^{-1}\dfrac{12}{8} = 56.3°$, and $z = 2$

$E_\rho = \mathbf{E}_p \cdot \mathbf{u}_p = 65.9(\mathbf{u}_x \cdot \mathbf{u}_\rho) + 148.3(\mathbf{u}_y \cdot \mathbf{u}_\rho)$

$= 65.9\cos 56.3° + 148.3\sin 56.3 = 159.7$

$E_\phi = \mathbf{E}_\rho \cdot \mathbf{u}_\phi = 65.9(\mathbf{u}_x \cdot \mathbf{u}_\phi) + 148.3(\mathbf{u}_y \cdot \mathbf{u}_\phi)$

$= -65.9\sin 56.3° + 148.3\cos 56.3° = 27.4$,

$E_z = -49.4$

7. (C) $\mathbf{E}_p = \dfrac{2\times10^{-8}}{4\pi\varepsilon_o}\left[-\dfrac{\mathbf{R}_1}{|\mathbf{R}_1|^3} + \dfrac{2\mathbf{R}_2}{|\mathbf{R}_2|^3}\right]$

$\mathbf{R}_1 = (0, 2, 1) - (0, 0, 0.5) = (0, 2, 0.5)$

$\mathbf{R}_2 = (0, 2, 1) - (0, 0, 0) = (0, 2, 1)$

$\mathbf{E}_p = 9\times10^9\times10^{-8}\left[\dfrac{-(2\mathbf{u}_y + 0.5\mathbf{u}_z)}{(4.25)^{3/2}} + \dfrac{2(2\mathbf{u}_y + \mathbf{u}_z)}{(\sqrt{5})^{3/2}}\right]$

$\mathbf{E}_p = 54.9\mathbf{u}_y + 44.1\mathbf{u}_z$

At P, $r = \sqrt{5}$, $\theta = \cos^{-1}\dfrac{1}{\sqrt{5}} = 63.4°$ and $\phi = 90°$

So $E_r = \mathbf{E}_p \cdot \mathbf{u}_r = 54.9[\mathbf{u}_y \cdot \mathbf{u}_r] + 44.1[\mathbf{u}_z \cdot \mathbf{u}_r]$

$= 54.9\sin\theta\sin\phi + 44.1\cos\theta = 68.83$

$E_\theta = \mathbf{E}_\rho \cdot \mathbf{u}_\theta = 54.9[\mathbf{u}_y \cdot \mathbf{u}_\theta] + 44.1[\mathbf{u}_z \cdot \mathbf{u}_\theta]$

$= 54.9\cos\theta\sin\phi + 44.1(-\sin\theta) = -14.85$

$E_\phi = \mathbf{E}_\rho \cdot \mathbf{u}_\phi = 54.9(\mathbf{u}_y \cdot \mathbf{u}_\phi) + 44.1(\mathbf{u}_z \cdot \mathbf{u}_\phi) = 54.9\cos\phi = 0$

8. (A) Let a point on y axis be P(0, y, 0)

$\mathbf{E}_p = \dfrac{20\times10^{-8}}{4\pi\varepsilon_o}\left[-\dfrac{\mathbf{R}_1}{|\mathbf{R}_1|^3} - \dfrac{\mathbf{R}_2}{|\mathbf{R}_2|^3}\right]$

$\mathbf{R}_1 = (0, y, 0) - (3, 0, 0) = (-3, y, 0)$

$\mathbf{R}_2 = (0, y\ 0) - (-3, 0, 0) = (3, y, 0)$,

$|\mathbf{R}_1| = |\mathbf{R}_2| = \sqrt{9 + y^2}$

$\mathbf{E}_p = 20\times10^{-9}\times9\times10^9\left[\dfrac{-3\mathbf{u}_x + y\mathbf{u}_y}{(\sqrt{9+y^2})^3} + \dfrac{3\mathbf{u}_x + y\mathbf{u}_y}{(\sqrt{9+y^2})^3}\right]$

$= \dfrac{-1080\mathbf{u}_x}{(9+y^2)^{3/2}}$, $|\mathbf{E}| = = \dfrac{1080}{(9+y^2)^{3/2}}$

9. (B) The field at P will be

$\mathbf{E}_p = \dfrac{Q_0}{4\pi\varepsilon_o}\left[\dfrac{-2\mathbf{u}_x + 2\mathbf{u}_y - \mathbf{u}_z}{9^{3/2}}\right]$, $\mathbf{E}_z = 1$ kV/m

$Q_0 = -4\pi\varepsilon_o \times 9^{3/2} \times 10^3 = -3\,\mu$C

10. (C) This will be 8 times the integral of ρ_v over the first octant

$$Q = 8\int_0^\infty\int_0^\infty\int_0^\infty \rho_o e^{-x-y-z} dxdydz = 8\rho_o$$

11. (C) $Q = \int_0^{2\pi}\int_0^{\pi}\int_{0.03}^{0.05} 0.2r^2 \sin\theta\, drd\,\theta d\phi$

$$= \left[4\pi(0.2)\frac{r^3}{3}\right]_{0.03}^{0.05} = 82.1 \text{ pC}$$

12. (A) $Q = \int_\infty^{-\infty}\int_{-\pi}^{\pi}\int_0^{10} \frac{5e^{-0.1\rho}(\pi - |\phi|)}{z^2+10}$

$$= 5\left[\frac{e^{-0.1\rho}(-0.1-1)}{(0.1)^2}(-0.101)\right]_0^{10} \times \int_{-\infty}^{\infty} 2\int_0^{2\pi}\frac{(\pi-\phi)dz}{z^2+10}$$

$$Q = 5\times 26.4\int_{-\infty}^{\infty}\frac{\pi^2 dz}{z^2+10}$$

$$= 5\times 26.4\times\pi^2\left[\frac{1}{\sqrt{10}}\tan^{-1}\frac{z}{\sqrt{10}}\right]_{-\infty}^{\infty} = 1.29 \text{ mC}$$

13. (A)

$$\phi = 10\int_{0.9\pi}^{1.1\pi}\int_0^{25^\circ}\int_4^5 (r-4)(r-5)\sin\theta\cdot\sin\frac{\phi}{2}r^2\sin\theta\, d\rho d\theta d\phi$$

$$= 10\left[\frac{r^5}{5} - \frac{9r^4}{4} + \frac{20r^3}{3}\right]_4^5\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{25^\circ}\times\left[-2\cos\frac{\phi}{2}\right]_{0.9\pi}^{1.1\pi}$$

$$= 10\,[-3.39][0.0266][0.626] = 0.57 \text{ C}$$

14. (D) $\mathbf{E}_p = \frac{\rho_L}{2\pi\varepsilon_o}\frac{\mathbf{R}_p}{|\mathbf{R}_p|^2}$,

$\mathbf{R}_p = (1, 2, 3) - (1, -2, 5) = (0, 4, -2)$

$|\mathbf{R}_p|^2 = 20,$

$$\mathbf{E}_p = \frac{5\times 10^{-9}}{2\pi\varepsilon_o}\left[\frac{4\mathbf{u}_y - 2\mathbf{u}_z}{20}\right] = 18\mathbf{u}_y - 9\mathbf{u}_z$$

15. (C) With $z = 0$, the general field is

$$\mathbf{E}_{z=0} = \frac{\rho_L}{2\pi\varepsilon_o}\left[\frac{(y+2)\mathbf{u}_y - 5\mathbf{u}_z}{(y+2)^2+25}\right]$$

we require $|\mathbf{E}_2| = |2\mathbf{E}_y|$

So $2(y+2) = 5 \Rightarrow y = \frac{1}{2}$

$$\mathbf{E} = \frac{6.25}{2\pi\varepsilon_o}\left[\frac{2.5\mathbf{u}_y - 5\mathbf{u}_z}{6.25+25}\right] = 9\mathbf{u}_y - 18\mathbf{u}_z$$

16. (C) $E_p = \frac{\rho_L}{2\pi\varepsilon_o}\left[\frac{\mathbf{R}_{+Q}}{|\mathbf{R}_{+Q}|^2} - \frac{\mathbf{R}_{-Q}}{|\mathbf{R}_{-Q}|^2}\right]$

$R_{+Q} = (6, 0, 6) - (0, 3, 6) = (6, -3, 0)$

$R_{-Q} = (6, 0, 6) - (0, -3, 6) = (6, 3, 0)$

$$\mathbf{E}_P = 20\text{nC}\times 2\times 9\times 10^9\left[\frac{6\mathbf{u}_x - 3\mathbf{u}_y}{36+9} - \frac{6\mathbf{u}_x + 3\mathbf{u}_y}{36+9}\right]$$

$= -48\mathbf{u}_y$ V/m

17. (B) $\mathbf{E}_P = \frac{\rho_L}{2\pi\varepsilon_o}\left[\frac{\mathbf{R}_{xp}}{|\mathbf{R}_{xp}|^3} + \frac{\mathbf{R}_{yp}}{|\mathbf{R}_{yp}|^3} + \frac{\mathbf{R}_{zp}}{|\mathbf{R}_{zp}|^3}\right]$

$\mathbf{R}_{xp} = (-3, 0, -1,) - (-3, 0, 0,) = (0, 2, -1)$

Similarly $\mathbf{R}_{yp} = (-3, 0, -1)$, $\mathbf{R}_{zp} = (-3, 2, 0)$

$\mathbf{E}_p = 100\times 10^{-9}\times 2\times 9\times 10^{-9}$

$$\mathbf{E}_p = 100\times 10^{-9}\times 2\times 9\times 10^{-9}\times\left[\frac{2\mathbf{u}_y - \mathbf{u}_z}{5} + \frac{-3\mathbf{u}_x - \mathbf{u}_z}{10} + \frac{-3u_x + 2\mathbf{u}_y}{13}\right]$$

$= -0.96\mathbf{u}_x + \mathbf{u}_y - 0.54\mathbf{u}_z$ kV/m

18. (C) At $y = 4$, $\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o}\mathbf{u}_y$,

$d\mathbf{F} = dq\mathbf{E} = \rho_L dz\mathbf{E}$

$$\mathbf{F} = \int_0^1\frac{\rho_L^2 dz\mathbf{u}_y}{2\pi\varepsilon_o}(6) = 18.75\mathbf{u}_y\,\mu\text{N}$$

19. (A) $E = \int\int\frac{\rho_s dS}{4\pi\varepsilon_o}\frac{\mathbf{R}-\mathbf{R}'}{|\mathbf{R}-\mathbf{R}'|^3}$

where $\mathbf{R} = 3\mathbf{u}_x$ and $\mathbf{R}' = y\mathbf{u}_y + z\mathbf{u}_z$,

$$\mathbf{E}_{PA} = \frac{\rho_L}{4\pi\varepsilon_o}\int_{-\infty}^{\infty}\int_{-2}^{2}\frac{3\mathbf{u}_x - y\mathbf{u}_y - zu_z}{(9+y^2+z^2)^{3/2}}dydz$$

Due to odd function

$$\mathbf{E}_{PA} = \frac{\rho_L}{4\pi\varepsilon_o}\int_{-\infty}^{\infty}\int_{-2}^{2}\frac{3\mathbf{u}_x dydz}{(9+y^2+z^2)^{3/2}}$$

So there is only x component.

20. (D) There will be z component of E only

$\mathbf{R} = z\mathbf{u}_z$, $\mathbf{R}' = \rho\mathbf{u}_\rho$, $\mathbf{R} - \mathbf{R}' = z\mathbf{u}_z - \rho\mathbf{u}_z$

$$E_{z,P_a} = \frac{\rho_s}{4\pi\varepsilon_o}\int_0^{2\pi}\int_0^{0.2}\frac{z\rho d\rho d\phi}{(z^2+\rho^2)^{3/2}}$$

$$= \frac{2\pi\rho_s z}{4\pi\varepsilon_o}\left[\frac{1}{\sqrt{z^2+\rho^2}}\right]_0^{0.2} = \frac{\rho_s z}{\varepsilon_o}\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2+0.04}}\right]$$

$E_z = \frac{\rho_s}{\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2+0.04}}\right]$, at $z = 0.5$, $E_z = 20.2$ V/m

21. (A) Since charge sheet are infinite, the field magnitude associated with each one will be $\rho_s/2\varepsilon_o$, which is position independent. The field direction will depend on which side of a given sheet one is positioned.

$$E_A = \left[\frac{10\times 10^{-9}}{2\varepsilon_o}\mathbf{u}_x - \frac{40\times 10^{-9}}{2\varepsilon_o}\mathbf{u}_y - \frac{50\times 10^{-9}}{2\varepsilon_o}\mathbf{u}_z\right]$$

$= 0.56\mathbf{u}_x + 2.23\mathbf{u}_y - 2.8\mathbf{u}_z$

22. (A) $\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{-15x^2y}{5x^3} = \frac{-3y}{x}$

$\Rightarrow \quad \frac{dy}{y} = -\frac{3dx}{3} \quad \Rightarrow \quad \ln y = -3\ln x \ln Cy = \frac{C_1}{x^3},$

At P, $2 = \frac{C_1}{4^3} \quad \Rightarrow \quad C_1 = 128 \quad \Rightarrow \quad y = \frac{128}{x^3}$

23. (A) This point lies in the center of a symmetric arrangement of line charges, whose field will all cancel at that point. Thus **D** arises from the point charge alone

$\mathbf{D} = \frac{10\times10^{-9}}{4\pi}\frac{(-3\mathbf{u}_y + 4\mathbf{u}_z)}{(3^2+4^2)^{1.5}}$,

$\mathbf{D} = -19.1\mathbf{u}_y + 25.5\mathbf{u}_z \text{ pC/m}^2$

24. (C) $h_1 = 2\sqrt{4^2-1} = 7.75,$

$h_2 = 2\sqrt{4^2-1} = 6.93$

$Q_T = 2\times7.75\times40\text{n} + 2\times6.93\times50 + 20\text{n} = 1.33\ \mu\text{C}$

25. (D) $Q = \int_{0.01}^{0.05}\int_{30}^{90^\circ} 5e^{-20z}(0.08)d\phi dz \text{ nC}$

$= \left(\frac{\pi}{2}-\frac{\pi}{6}\right)(5)(0.08)\left(-\frac{1}{20}\right)e^{-20z}\Big|_{0.01}^{0.05}$

$= 9.45\times10^{-3} \text{ nC} = 9.45 \text{ pC}$

26. (A) Out of the 6 surface only 2 will contribute to the net outward flux. The y component of D will penetrate the surface $y=0$ and $y=z$ and net flux will be zero. At $x=0$ plane $D_x=0$ and at $z=0$ plane $D_z=0$.
This leaves the 2 remaining surfaces at $x=2$ and $z=5$.
The net outward flux become

$f = \int_0^5\int_0^3 \mathbf{D}|_{x=2}\cdot\mathbf{u}_x dydz + \int_0^3\int_0^2 \mathbf{D}|_{x=2}\cdot\mathbf{u}_z dzdy$

$= 5\int_0^3 4(2)ydy + 2\int_0^3 4(5)ydy = 360 \text{ C.}$

27. (D) $Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{0.001} 2e^{-1000r}r^2\sin\theta\, drd\theta d\phi$

$Q = 8\pi\left[\frac{-r^2e^{-1000r}}{1000}\Big|_0^{0.001} + \frac{2e^{-1000r}(-1000r-1)}{1000(1000)^2}\Big|_0^{0.001}\right]$

$Q = 4\times10^{-9}\text{nC},$

$D_r = \frac{Q}{4\pi r^2} = \frac{4.0\times10^{-9}}{4\pi\times(0.001)^2} = 0.32 \text{ C/m}^2$

28. (C) $4\pi r^2D_r = 4\pi(2^2)\,20\times10^{-9} \text{ C/m}^2$,

$D_r = \frac{80}{r^2} \text{ nC/m}^2$

29. (C) $Q = \oint_S \mathbf{D}\cdot\mathbf{n}\,dS = \underbrace{\int_0^a\int_0^a 2aydydz}_{\text{Front}} + \underbrace{\int_0^a\int_0^a -2(0)dydz}_{\text{Back}}$

$+ \underbrace{\int_0^a\int_0^a -x^2dxdz}_{\text{Left}} + \underbrace{\int_0^a\int_0^a x^2dxdz}_{\text{Right}} + \underbrace{\int_0^a\int_0^a -6(0)^3dxdy}_{\text{Botoom}} + \underbrace{\int_0^a\int_0^a 6a^3dxdy}_{\text{Top}}$

$= a^4 + 0 - \frac{a^4}{3} + \frac{a^4}{3} + 0 + 6a^5 = a^4 + 6a^5$

30. (C) $\nabla\cdot\mathbf{D} = \frac{\partial D_y}{\partial y} = 20x^4y^3z^4$

Center of cube =(3.05 1.05 2.05) and Volume $\nabla V = (0.1)^3 = 0.001$

$\phi = 20\,(3.05)^4\,(1.05)^3\,(2.05)^3\,(2.05)^4\ 0.001 = 35.4$

31. (C) $\Phi = (\nabla\cdot\mathbf{D})\,\Delta v$

$= \frac{\partial(x)}{\partial x}\left(\frac{4}{3}\pi(0.003)^3\right) = 113.1 \text{ nC}$

32. (B) $\rho_v = \nabla\cdot\mathbf{D} = \left[\frac{10y}{z} + 0 + \left(\frac{2+10x^2y}{z^3}\right)\right]_{(-2,3,5)} = 8.96$

33. (C) $\nabla\cdot\mathbf{D} = \frac{1}{r^2}\frac{d}{dr}(r^2\times2r) = 6$,

$\int_V \nabla\cdot\mathbf{D}\,dv = 6\times(0.4)^3 = 0.38$

34. (C) $dW = -q\mathbf{E}\cdot d\mathbf{L}$

$= -5(4\mathbf{u}_x - 3\mathbf{u}_y + 5\mathbf{u}_z)\cdot\frac{(\mathbf{u}_x+\mathbf{u}_y+\mathbf{u}_z)(2)}{\sqrt{3}}$

$= -\frac{10}{\sqrt{3}}(4-3+5) = -34.64 \text{ J}$

35. (B) The vector in this direction is
$(2,\ 1,\ 4) - (7,\ 2,\ 3) = (1,\ -1,\ 1)$

$\mathbf{u}_{PQ} = \frac{\mathbf{u}_x - \mathbf{u}_y + \mathbf{u}_z}{\sqrt{3}}$, $dW = -q\mathbf{E}\cdot d\mathbf{L}$

$= -(60\times10^{-6})\left(100\mathbf{u}_\rho\cdot\frac{(\mathbf{u}_x-\mathbf{u}_y+\mathbf{u}_z)(2\times10^{-3})}{\sqrt{3}}\right)$

$= -12\times\frac{10^{-6}}{\sqrt{3}}(\mathbf{u}_\rho\cdot\mathbf{u}_x - \mathbf{u}_\rho\cdot\mathbf{u}_y)$

At ρ, $\phi = \tan^{-1}\left(\frac{2}{1}\right) = 63.4^\circ$

$\mathbf{u}_\rho\cdot\mathbf{u}_x = \cos 63.4^\circ = 0.447,$

$\mathbf{u}_\rho\cdot\mathbf{u}_y = \sin 63.4^\circ = 0.894$

$dW = 3.1\ \mu\text{J}$

36. (A) $W = -q\int\mathbf{E}\cdot d\mathbf{L}$

$$= -10 \int (2x\mathbf{u}_x - 3y^2\mathbf{u}_y + 4\mathbf{u}_z)(dx\mathbf{u}_x + dy\mathbf{u}_y + dz\mathbf{u}_z)$$

$$= -10 \int_0^3 2x dx + 10 \int_0^1 3y^2 dy - 10 \int_0^{-1} 4dz = -40 \text{ J}$$

37. (D) $r_A = 2$ cm, $r_B = 3$ cm

$$V(r) = \frac{Q}{4\pi\varepsilon_o r} = \frac{4\pi a^2 \rho_s}{4\pi\varepsilon_o r} = \frac{(6\times10^{-3})^2 \times 30\times10^{-9}}{8.85\times10^{-12}\times r}$$

$$V(r) = \frac{0.122}{r}, \; V_{AB} = V_A - V_B = \frac{0.122}{0.02} = \frac{0.122}{0.03} = 2.03$$

38. (D) $W = -\dfrac{Q_1Q_2}{4\pi\varepsilon_o}\displaystyle\int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{Q_1Q_2}{4\pi\varepsilon_o}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$

$r_A = r_B$, $W = 0$

39. (B) $V_P(s) = \dfrac{Q}{4\pi\varepsilon_o r}$,

$$V_l(\rho) = -\int \frac{\rho_L d\rho}{2\pi\varepsilon_o \rho} + C_1 = -\frac{\rho_L}{2\pi\varepsilon_o}\ln\rho + C_1$$

$$V_s(z) = -\int \frac{\rho_s}{2\varepsilon_o} dz + C_2 = \frac{\rho_s z}{2\varepsilon_o} + C_2$$

$$V = \frac{Q}{4\pi\varepsilon_o r} - \frac{\rho_L}{2\pi\varepsilon_o}\ln\rho - \frac{\rho_s}{2\pi\varepsilon_o} z + C$$

Here r, ρ, z are the scalar distance from the charge.

$r = \sqrt{2^2 + 5^2} = \sqrt{29}$, $\rho = \sqrt{(5-4)^2} = 1$, $z = 5$

By putting these value. $C = -1.93\times10^3$

At point N, $r = \sqrt{(2-1)^2 + 2^2 + z^2} = \sqrt{14}$

$\rho = \sqrt{1^2 + 1^2} = \sqrt{2}$, $z = 3$, $V_B = 1.98$ kV

40. (D) $V_\rho = \displaystyle\int_{-\infty}^{\infty} \frac{\rho_L dz}{4\pi\varepsilon_o R} = \int_{-\infty}^{\infty} \frac{6\times10^{-9} dz}{4\pi\varepsilon_o (z^2+1)^{3/2}} = 108$ V

41. (A) $V_\rho = \displaystyle\iint \frac{\rho_s dS}{4\pi\varepsilon_o |\mathbf{R} - \mathbf{R}'|}$,

$\mathbf{R} = z\mathbf{u}_z, \mathbf{R}' = \rho\mathbf{u}_\rho, \; dS = \rho d\rho d\phi,$

$$V_\rho = \int_0^{2\pi}\int_{0.01}^{0.03} \frac{(5\times10^{-9})\rho^2 d\rho d\phi}{4\pi\varepsilon_o\sqrt{\rho^2 + z^2}} ,$$

$$V_p = \frac{5\times10^{-9}}{2\varepsilon_o}\left[\frac{\rho}{2}\sqrt{\rho^2+z^2} - \frac{z^2}{2}\ln(\rho + \sqrt{\rho^2+z^2})\right]_{0.01}^{0.03}$$

At $z = 0.02$, $V_p = 0.081$ V

42. (C) $\mathbf{E} = -\nabla V$

$$= -\left[2y^2z^2 + \frac{6x}{x^2+2y^2+3z^2}\right]\mathbf{u}_x - \left[4xyz^3 + \frac{12y}{x^2+2y^2+3z^2}\right]\mathbf{u}_y - \left[6xy^2z^2 + \frac{18z}{x^2+2y^2+3z^2}\right]\mathbf{u}_z$$

$\mathbf{E}_p = 7.1\mathbf{u}_x + 22.8\mathbf{u}_y - 71.1\mathbf{u}_z$ V/m,

$|\mathbf{E}| = 75$ V/m

43. (D) $\mathbf{E} = -\nabla V = -\dfrac{dV}{dr}\mathbf{u}_r = -(0.6)(70)r^{-0.4}\mathbf{u}_r$

$= -42r^{-0.4}\mathbf{u}_r$ V/m,

$\mathbf{D} = \varepsilon_o\mathbf{E} = -42r^{-0.4}\varepsilon_o\mathbf{u}_r$ C/m^2

$$\rho_v = \nabla\cdot\mathbf{D} = \frac{1}{r^2}\frac{d}{dr}(r^2D_r) = \frac{1}{r^2}\frac{d}{dr}(-42\varepsilon_o r^{1.6}) = -67.2\frac{\varepsilon_o}{r^{1.4}}$$

At $r = 0.6$ m,

$$\rho_v = -\frac{67.2\times8.85\times10^{-12}}{(0.6)^{1.4}} = -1.22 \text{ nC/m}^3$$

44. (A) $\mathbf{E} = -\nabla V = -\dfrac{\partial V}{\partial r}\mathbf{u}_r - \dfrac{1}{r}\dfrac{\partial V}{\partial\theta}\mathbf{u}_\theta$

$= -160r\cos\theta\,\mathbf{u}_r + 80r\sin\theta\,\mathbf{u}_\theta$ V/m

$\mathbf{D} = \varepsilon_o\mathbf{E} = -80\varepsilon_o(2r\cos\theta\,\mathbf{u}_r - r\sin\theta\,\mathbf{u}_\theta)$

$$\rho_v = \nabla\cdot\mathbf{D} = \left[\frac{1}{r^2}\frac{2}{\partial r}(r^2D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_r\sin\theta)\right]$$

$$\rho_v = -80\varepsilon_o\left[\frac{1}{r}2\times3r^2\cos\theta - \frac{12\sin\theta\cos\theta}{r\sin\theta}\right]$$

$\rho_V = -320\varepsilon_o\cos\theta = -2.45$ nC/m^3

45. (C) $\mathbf{E} = -\nabla V = -\dfrac{\partial V}{\partial\rho}\mathbf{u}_\rho - \dfrac{1}{\rho}\dfrac{\partial V}{\partial\phi}\mathbf{u}_\phi$

$= -(50 + 150\sin\phi)\mathbf{u}_\rho - (150\cos\phi)\mathbf{u}_\phi$,

$\mathbf{D} = \varepsilon_o E$, $\rho_v = \nabla\cdot\mathbf{D} = \varepsilon_o\nabla\cdot\mathbf{E}$

$$\rho_v = \varepsilon_o\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}E_\phi\right]$$

$$= \varepsilon_o\left[-\frac{(50+150\sin\phi)}{\rho} + \frac{150\sin\phi}{\rho}\right] = -\frac{50\varepsilon_o}{\rho} \text{ C/m}^2$$

$$Q = \int_0^1\int_0^{2\pi}\int_0^2 \frac{50\varepsilon_o}{\rho}\rho d\rho d\phi dz = -2\pi(50)\varepsilon_o 2 = -5.56 \text{ nC}$$

46. (A) $V = \dfrac{Qd\cos\theta}{4\pi\varepsilon_o r^2} = \dfrac{100\cos\theta}{r^2}$,

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial r}\mathbf{u}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\mathbf{u}_\theta\right)$$

$$= \frac{100}{r^3}(2\cos\theta\,\mathbf{u}_r + \sin\theta\,\mathbf{u}_\theta),$$

$$|\mathbf{E}| = 100(4\cos^2\theta + \sin^2\theta)^{1/2} = 100\times\sqrt{\frac{5}{2}} = 158.11 \text{ V/m}$$

47. (A) $\mathbf{E} = \dfrac{2\times10^{-9}}{4\pi\varepsilon_o r^3}[2\cos\theta\,\mathbf{u}_r + \sin\theta\,\mathbf{u}_\theta]$,

$y = z$ lies at $\theta = 45°$

$$E_\theta = \frac{2\times10^{-9}}{4\pi\varepsilon_o r^3}\frac{1}{\sqrt{2}} = 10^{-3} \quad \text{(required)}$$

$r^3 = 12.73\times10^3, \quad r = 23.35$

48. (A) $V = \dfrac{\mathbf{P}\cdot(\mathbf{R}-\mathbf{R}')}{4\pi\varepsilon_o|\mathbf{R}-\mathbf{R}'|^3}$

where $\mathbf{R}-\mathbf{R}' = \mathbf{Q}-\mathbf{P} = (1,\ 1,\ \ 8)$

So $V_P = \dfrac{(3\mathbf{u}_x - 5\mathbf{u}_y + 10\mathbf{u}_z)\cdot(\mathbf{u}_x + \mathbf{u}_y + 8\mathbf{u}_z)\times10^{-9}}{4\pi\varepsilon_o(1+1+8^2)^{1.5}}$

=1.31 V

49. (A) $\mathbf{E} = -\nabla V = 40\left[\dfrac{1}{x^2yz}\mathbf{u}_x + \dfrac{1}{xy^2z}\mathbf{u}_y + \dfrac{1}{xyz^2}\mathbf{u}_z\right]$

$$W_e = \frac{\varepsilon_o}{2}\iiint \mathbf{E}\cdot\mathbf{E}dv$$

$$W_e = 800\varepsilon_o\int_1^2\int_1^2\int_1^2\left[\frac{1}{x^4y^2z^2} + \frac{1}{x^2y^4z^2} + \frac{1}{x^2y^2z^4}\right]dxdydz$$

$= 1548$ pJ

50. (A) $W = \dfrac{1}{2}\sum_{n=1}^{4} q_n V_n$

$$V_1 = V_{21} + V_{31} + V_{41} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{0.04} + \frac{1}{0.04} + \frac{1}{0.04\sqrt{2}}\right]$$

$V_1 = V_2 = V_3 = V_4$

$$W = \frac{1}{2}(4)q_1V_1 = \frac{2\times(1.2\times10^{-9})^2}{4\pi\varepsilon_o(0.04)}\left[2 + \frac{1}{\sqrt{2}}\right] = 1.75\ \mu\text{J}$$

51. (B) $I = \iint_S \mathbf{J}\cdot\mathbf{n}\big|_{y=1} dS = \int_0^2\int_0^1 \mathbf{J}\cdot\mathbf{u}_y\big|_{y=1} dxdz$

$$= \int_0^2\int_0^1 10^5\cos(2x)\,2^{-2y}\,dxdz = 12307\text{ kA} = 12.3\text{ MA}$$

52. (C) $I = \iint_S \mathbf{J}\cdot\mathbf{n}\,dS$

$$= \int_0^{2\pi}\int_{0.1\pi}^{0.3\pi}\frac{800\sin\theta}{(0.80)^2+4}(0.80)^2\sin\theta\,d\theta d\phi = 154.8\text{ A}$$

53. (D) $F = ma = qE,$

$$a = \frac{qE}{m} = \frac{(-1.602\times10^{-19})(-4\times10^6)}{9.11\times10^{-3}}\mathbf{u}_z = 7.0\times10^{17}\mathbf{u}_z\text{ m/s}^2,$$

$v = at = 7.0\times10^{17}t\mathbf{u}_z$ m/s

54. (D) $\dfrac{\partial\rho_V}{\partial t} = -\nabla\cdot\mathbf{J} = \dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}(\rho J_\rho) + \dfrac{\partial J_z}{\partial z}$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(25) + \frac{\partial}{\partial z}\left(\frac{-20}{\rho^2+0.01}\right) = 0$$

55. (A) $I = \iint_S \mathbf{J}\cdot\mathbf{n}\big|_{z=0.2} dS$

$$= \int_0^{2\pi}\int_0^{0.4}\frac{40}{\rho^2+(0.1)^2}\rho d\rho d\phi = \frac{40}{2}\log[\rho^2+(0.1)^2]\Big|_0^{0.4}(2\pi)$$

$= 40\pi\log17 = 356$ A

56. (B) So $E_{al} = E_{st} = \dfrac{J_{al}}{\sigma_{al}} = \dfrac{J_{st}}{\sigma_{st}} \Rightarrow J_{al} = \dfrac{\sigma_{ac}}{\sigma_{st}}J_{st}$

$I = \pi(2\times10^{-3})J_{st} + \pi[(4\times10^{-3})^2 - (2\times10^{-3})^2]J_{al} = 80$

Solving $\quad J_{st} = 3.2\times10^5\text{ A/m}^2$

57. (B) $J = \dfrac{4}{2\pi\rho l}\mathbf{u}_\rho\text{ A/m}^2,$

$$E = \frac{J}{\sigma} = \frac{4}{2\pi\rho l\sigma} = \frac{12.73}{\rho l}\mathbf{u}_\rho\text{ V/m}$$

$$V = -\int_5^3\mathbf{E}\cdot d\mathbf{L} = \int_3^5\frac{12.73}{\rho l}\mathbf{u}_\rho\cdot\mathbf{u}_\rho d\rho = \frac{12.73}{l}\ln\frac{5}{3} = \frac{6.51}{l}\text{ V}$$

$$R = \frac{V}{I} = \frac{6.51}{4l} = \frac{1.63}{\rho}\ \Omega$$

58. (D) $\mathbf{E} = -\nabla V = -\left(\dfrac{\partial V}{\partial\rho}\mathbf{u}_\rho + \dfrac{1}{\rho}\dfrac{\partial V}{\partial\phi}\mathbf{u}_\phi + \dfrac{\partial V}{\partial z}\mathbf{u}_z\right)$

$$= -z^2\cos\phi\,\mathbf{u}_\rho + \frac{(\rho+1)}{\rho}z^2\sin\phi\,\mathbf{u}_\phi - 2(\rho+1)z\cos\phi\,\mathbf{u}_z$$

$\mathbf{E} = -1.82\mathbf{u}_\phi + 14.5\mathbf{u}_\phi - 2.67\mathbf{u}_z$ V/m

$$\rho_s = \varepsilon_o\mathbf{E}\cdot\mathbf{n}\big|_s = \varepsilon_o\frac{\mathbf{E}\cdot\mathbf{E}}{|\mathbf{E}|}$$

$$\rho_s = \varepsilon_o\sqrt{1.82^2 + 14.5^2 + 2.67^2} = 131.5\text{ pC/m}^2$$

59. (C) $V = \dfrac{40\cos\frac{\pi}{3}\sin\frac{\pi}{2}}{2^3} = 2.5$ V

So the equation of the surface is

$$\frac{40\cos\theta\sin\phi}{r^2} = 2.5, \quad 16\cos\theta\sin\phi = r^3$$

60. (A) $\mathbf{E} = -\nabla V,\ \ \mathbf{D} = \varepsilon\mathbf{E} = -\varepsilon_o\nabla V$

$$= -\varepsilon_o\left[200z\frac{\partial}{\partial x}\frac{x}{(x^2+4)}\mathbf{u}_x + \frac{200x}{x^2+4}\mathbf{u}_z\right]\text{ C/m}^2$$

$$\mathbf{D}_{(z=0)} = -\frac{200\varepsilon_o x}{x^2+4}\mathbf{u}_z\text{ C/m}^2,$$

$$\rho_s = \mathbf{D}\cdot\mathbf{u}_z\big|_{z=0} = \frac{-200\varepsilon_o x}{x^2+4}\text{ C/m}^2,$$

$$Q = \int_{-3}^{0}\int_0^2\frac{-200\varepsilon_o x}{x^2+4}dxdy = -(3)(200)\varepsilon_o\frac{1}{2}\ln[x^2+4]\Big|_0^2$$

$= -300\ln2 = -1.84$ nC

61. (B) $\mathbf{E} = -\nabla V$, $-3xy^2z^3$ satisfy this equation.

62. (D) The plane can be replaced by –60 nC at Q (2, 5, –6).

$\mathbf{R} = (5, 3, 1) - (2, 4, 6) = (3, -1, -5)$

$\mathbf{R}' = (5, 3, 1) - (2, 4, -6) = (3, -1, 7)$,

$|\mathbf{R}| = \sqrt{35}$, $|\mathbf{R}'| = \sqrt{59}$

$$V_P = \frac{q}{4\pi\varepsilon_o R} - \frac{q}{4\pi\varepsilon_o R'} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{\sqrt{35}} - \frac{1}{\sqrt{59}}\right]$$

$$V_p = 60\times10^{-9}\times9\times10^{-9}\left[\frac{1}{\sqrt{35}} - \frac{1}{\sqrt{59}}\right] = 21 \text{ V}$$

63. (A) Using method of images

$$V_p - V_0 = -\frac{\rho_L}{2\pi\varepsilon_o}\ln\left[\frac{\text{Final Distance from the charge}}{\text{Initial Distance from the charge}}\right]$$

$V_0 = 0$,

$$V_p = -\frac{\rho_L}{2\pi\varepsilon_o}\left[\ln\frac{1}{2} + \ln\frac{\sqrt{1^2+(2-1)^2}}{1} - \ln\frac{\sqrt{1^2+(2-(-2))^2}}{1} - \ln\frac{\sqrt{1^2+3^2}}{1}\right]$$

= 2.40 kV

64. (C) $\mathbf{P} = \mathbf{D} - \varepsilon_o\mathbf{E} = \mathbf{D} - \frac{\mathbf{D}}{\varepsilon_r} = \frac{\mathbf{D}}{\varepsilon_r}(\varepsilon_r - 1)$

$$\mathbf{P} = (2\mathbf{u}_x - 4\mathbf{u}_y + 5\mathbf{u}_z)\frac{(2.4-1)\times10^{-9}}{2.4}$$

$\mathbf{P} = 1.2\mathbf{u}_x - 2.3\mathbf{u}_y + 2.9\mathbf{u}_z \text{ nC/m}^2$

65. (C) $\mathbf{D}_{n1} = \mathbf{D}_{n2} \Rightarrow \varepsilon_1\mathbf{E}_{N1} = \varepsilon_2\mathbf{E}_{N2}$

$= 2\times1.5\varepsilon_o = 2.5\varepsilon_o E_{n2} \Rightarrow E_{n2} = 1.2$

$E_{t1} = E_{t2}$, $\mathbf{E}_2 = 1.2\mathbf{u}_x - 3\mathbf{u}_y + 1\mathbf{u}_z$

66. (D) The unit vector that is normal to the surface is

$$\mathbf{u}_N = \frac{\nabla F}{|\nabla F|} = \frac{\mathbf{u}_x - \mathbf{u}_y + 2\mathbf{u}_z}{\sqrt{6}},$$

$$E_{n1} = \mathbf{E}_1\cdot\mathbf{u}_N = \frac{1}{\sqrt{6}}[100 - 200 - 100] = -81.7 \text{ V/m}$$

$$\mathbf{E}_{n1} = -81.7\frac{1}{\sqrt{6}}[\mathbf{u}_x - \mathbf{u}_y + 2\mathbf{u}_z]$$

$= -33.33\mathbf{u}_x + 33.33\mathbf{u}_y - 66.67\mathbf{u}_z$ V/m

$\mathbf{E}_{t1} = \mathbf{E}_1 - \mathbf{E}_{n1} = 133.3\mathbf{u}_x + 166.7\mathbf{u}_z + 16.67\mathbf{u}_z$

$\mathbf{E}_{t1} = \mathbf{E}_{t2}$ and $\mathbf{D}_{n1} = \mathbf{D}_{n2}$

$\Rightarrow \varepsilon_{r1}\varepsilon_o\mathbf{E}_{n1} = \varepsilon_{r2}\varepsilon_o\mathbf{E}_{n2}$

$$\mathbf{E}_{n2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\mathbf{E}_{n1} = \frac{1}{4}\mathbf{E}_{n1}, \quad \mathbf{E}_2 = \mathbf{E}_{t2} + \frac{1}{4}\mathbf{E}_{n1}$$

$= 133.3\mathbf{u}_x + 166.7\mathbf{u}_y + 16.67\mathbf{u}_z - 8.33\mathbf{u}_x + 8.33\mathbf{u}_y - 16.67\mathbf{u}_z$

$= 125\mathbf{u}_x + 175\mathbf{u}_y$ V/m

67. (B) $\mathbf{D}_{n1} = \mathbf{D}_{n2}$, $\varepsilon_{r1}\varepsilon_o\mathbf{E}_{n1} = \varepsilon_{r2}\varepsilon_o\mathbf{E}_{n2}$

Since E is normal to the surface

$$\mathbf{E}_{n2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\mathbf{E}_{n1} = \frac{2}{5}\times\frac{1000}{r^2}\mathbf{u}_r = \frac{400}{r^2}\mathbf{u}_r \text{ V/m}$$

68. (D) $\mathbf{D}_{n1} = \mathbf{D}_{n2}$ and $\mathbf{E}_{t1} = \mathbf{E}_{t2}$, $\mathbf{D} = \varepsilon\mathbf{E}$

$\mathbf{D}_{n1} = 80\times3\varepsilon_o = \mathbf{D}_{n2} = 5\varepsilon_o\mathbf{E}_{n1}$, $\mathbf{E}_{n1} = 48$

$\mathbf{E}_2 = 48\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z$

69. (A) $\mathbf{D} = \varepsilon\mathbf{E} = -f(x)\nabla V$,

$\nabla\cdot D = \rho_v = 0 = \nabla\cdot(-f(x)\nabla V)$

$0 = \nabla\cdot(-f(x)\nabla V)$

$$= -\left[\frac{dF}{dx}\frac{\partial V}{\partial x} + f(x)\frac{\partial^2V}{\partial x^2} + f(x)\frac{\partial^2V}{\partial y^2} + f(x)\frac{\partial^2V}{\partial z^2}\right]$$

$$= -\left[\frac{dF}{dx}\frac{\partial V}{\partial x} + f(x)\nabla^2V\right]$$

$$\Rightarrow \quad \nabla^2V = -\frac{1}{f(x)}\frac{dF}{dx}\frac{\partial V}{\partial x}$$

70. (A) $\rho_v = 0 \Rightarrow \nabla^2V = 0$,

$$\nabla^2V = 16e^{2x} + \frac{\partial^2 f}{\partial x^2} - 6 = 0$$

$$\frac{\partial f}{\partial x} = -16e^{2x} + 6 \Rightarrow \frac{\partial f}{\partial x} = -8e^{2x} + 6x + C_1$$

$$E_x = \frac{\partial V}{\partial x} = 8e^{2x} + \frac{\partial f}{\partial x}$$

$$E_x(0) = 8 + \left.\frac{\partial f}{\partial x}\right|_{x=0} = 0 \Rightarrow \left.\frac{\partial f}{\partial x}\right|_{x=0} = -8$$

It follow that $C_1 = 0$

Integrating again

$f(x_1) = -4e^{2x} + 3x^2 + C$, $f(0, 1) = -4 + C_2$

$V(0, 0) = 0 = 4 + f(0) \Rightarrow C_2 = 0.\ f(0) = -4$

$f(x) = -4e^{2x} + 3x^2$,

$V(x, y) = 4e^{2x} - 4e^{2x} + 3x^2 - 3y^2 = 3(x^2 - y^2)$

****

Statement for Q.9–11:

An infinite filament on the z-axis carries 10 mA in the $\mathbf{u}_z$ direction. Three uniform cylindrical current sheets are also present at 400 mA/m at $\rho = 1$ cm, –250 mA/m at $\rho = 2$ cm and 300 mA/m at $\rho = 3$ cm.

9. The magnetic field H_ϕ at $\rho = 0.5$ cm is

(A) 0.32 A/m (B) 0.64 A/m

(C) 1.36 mA/m (D) 0

10. The magnetic field H_ϕ at $\rho = 1.5$ cm is

(A) 1.63 A/m (B) 0.37 A/m

(C) 2.64 A/m (D) 0

11. The magnetic field H_ϕ at $\rho = 3.5$ cm is

(A) 0.14 A/m (B) 0.56 A/m

(C) 0.27 A/m (D) 0.96 A/m

Statement for Q.12–14:

In the fig. P8.3.12–14 The region $0 \le z \le 2$ is filled with an infinite slab of magnetic material ($\mu_r = 2.5$). The surface of the slab at $z = 0$ and $z = 2$, respectively, carry surface current $30\mathbf{u}_x$ A/m and $-40\mathbf{u}_x$ as shown in fig.

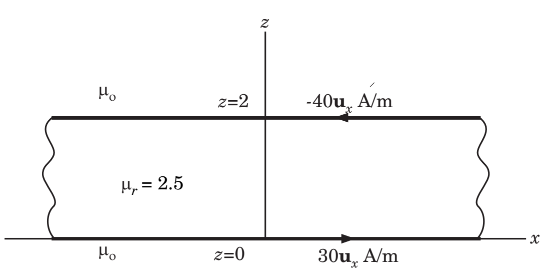

Fig. P8.3.12–14

12. In the region $0 < z < 2$ the **H** is

(A) $-35\mathbf{u}_y$ A/m (B) $35\mathbf{u}_y$ A/m

(C) $-5\mathbf{u}_y$ A/m (D) $5\mathbf{u}_y$ A/m

13. In the region $z < 0$ the **H** is

(A) $5\mathbf{u}_y$ A/m (B) $-5\mathbf{u}_y$ A/m

(C) $10\mathbf{u}_y$ A/m (D) $-10\mathbf{u}_y$ A/m

14. In the region $z > 2$ the **H** is

(A) $5\mathbf{u}_y$ A/m (B) $-5\mathbf{u}_y$ A/m

(C) $35\mathbf{u}_y$ A/m (D) $-35\mathbf{u}_y$ A/m

Statement for Q.15–16:

In the cylindrical region

$$H_\phi = \frac{2}{\rho} + \frac{\rho}{2} \quad \text{for } \rho \le 0.6$$

$$H_\phi = \frac{3}{\rho} \quad \text{for } \rho > 0.6$$

15. The current density **J** for $\rho < 0.6$ mm is

(A) $2\mathbf{u}_z$ A/m (B) $-2\mathbf{u}_z$ A/m

(C) $\mathbf{u}_z$ A/m (D) 0

16. The current density **J** for $\rho > 0.6$ mm is

(A) $2\mathbf{u}_z$ A/m (B) $-3\mathbf{u}_z$ A/m

(C) $3\mathbf{u}_z$ A/m (D) 0

17. An electron with velocity $\mathbf{v} = (3\mathbf{u}_x + 12\mathbf{u}_y - 4\mathbf{u}_z) \times 10^5$ m/s experiences no net forces at a point in a magnetic field $\mathbf{B} = \mathbf{u}_x + 2\mathbf{u}_y + 3\mathbf{u}_z$ mWb/m^2. The electric field E at that point is

(A) $-4.4\mathbf{u}_x + 1.3\mathbf{u}_y + 0.6\mathbf{u}_z$ kV/m

(B) $4.4\mathbf{u}_x - 1.3\mathbf{u}_y - 0.6\mathbf{u}_z$ kV/m

(C) $-4.4\mathbf{u}_x + 1.3\mathbf{u}_y + 0.6\mathbf{u}_z$ kV/m

(D) $4.4\mathbf{u}_x - 1.3\mathbf{u}_y - 0.6\mathbf{u}_z$ kV/m

18. A point charge of 2×10^{-16} C and 5×10^{-26} kg is moving in the combined fields $\mathbf{B} = -3\mathbf{u}_x + 2\mathbf{u}_y - \mathbf{u}_z$ mT and $\mathbf{E} = 100\mathbf{u}_x - 200\mathbf{u}_y + 300\mathbf{u}_z$ V/m. If the charge velocity at $t = 0$ is $\mathbf{v}(0) = (2\mathbf{u}_x - 3\mathbf{u}_y - 4\mathbf{u}_z)\, 10^5$ m/s, the acceleration of charge at $t = 0$ is

(A) $600[3\mathbf{u}_x + 2\mathbf{u}_y - 3\mathbf{u}_z]10^9$ m/s^2

(B) $400[6\mathbf{u}_x + 6\mathbf{u}_y - 3\mathbf{u}_z]10^9$ m/s^2

(C) $400[6\mathbf{u}_x - 6\mathbf{u}_y + 3\mathbf{u}_z]10^9$ m/s^2

(D) $800[6\mathbf{u}_x + 6\mathbf{u}_y - \mathbf{u}_z]10^9$ m/s^2

19. An electron is moving at velocity $\mathbf{v} = 4.5 \times 10^7 \mathbf{u}_y$ m/s along the negative y-axis. At the origin, it encounters the uniform magnetic field $\mathbf{B} = 2.5\mathbf{u}_z$ mT, and remains in it up to $y = 2.5$ cm. If we assume that the electron remains on the y–axis while it is in the magnetic field, at $y = 50$ cm the x and z coordinate are respectively

(A) 1.23 m, 0.23 m (B) –1.23 m, –0.23 m

(C) –11.7 cm, 0 (D) 11.7 cm, 0

Statement for Q.20–22:

A rectangular loop of wire in free space joins points A(1, 0, 1) to B(3, 0, 1) to C(3, 0, 4) to D(1, 0, 4) to A. The wire carries a current of 6 mA flowing in the $\mathbf{u}_z$ direction from B to C. A filamentary current of 15 A flows along the entire z, axis in the $\mathbf{u}_z$ directions.

20. The force on side BC is

(A) $-18\mathbf{u}_x$ nN (B) $18\mathbf{u}_x$ nN

(C) $3.6\mathbf{u}_x$ nN (D) $-3.6\mathbf{u}_x$ nN

21. The force on side AB is

(A) $23.4\mathbf{u}_z$ μN (B) $16.4\mathbf{u}_z$ μN

(C) $19.8\mathbf{u}_z$ nN (D) $26.3\mathbf{u}_z$nN

22. The total force on the loop is

(A) $36\mathbf{u}_x$ nN (B) $-36\mathbf{u}_x$ nN

(C) $54\mathbf{u}_x$ nN (D) $-54\mathbf{u}_x$ nN

23. Consider the rectangular loop on $z = 0$ plane shown in fig. P8.3.23. The magnetic flux density is $\mathbf{B} = 6x\mathbf{u}_x - 9y\mathbf{u}_y + 3z\mathbf{u}_z$ Wb/m^2. The total force experienced by the rectangular loop is

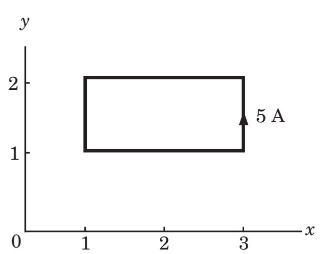

Fig. P8.3.23

(A) $30\mathbf{u}_z$ N (B) $-30\mathbf{u}_z$ N

(C) $36\mathbf{u}_z$ N (D) $-36\mathbf{u}_z$ N

Statement for Q.24–25:

Three uniform current sheets are located in free space as follows: $8\mathbf{u}_z$ A/m at $y = 0$, $-4\mathbf{u}_z$ A/m at $y = 1$ and $-4\mathbf{u}_z$ A/m at $y = -1$. Let $\mathbf{F}$ be the vector force per meter length exerted on a current filament carrying 7 mA in the $\mathbf{u}_L$ direction.

24. If the current filament is located at $x = 0$, $y = 0.5$ and $\mathbf{u}_L = \mathbf{u}_z$, then $\mathbf{F}$ is

(A) $35.2\mathbf{u}_y$ nN/m (B) $-35.2\mathbf{u}_y$ nN/m

(C) $105.6\mathbf{u}_y$ nN/m (D) 0

25. If the current filament is located at $y = 0.5$, $z = 0$, and $\mathbf{u}_L = \mathbf{u}_x$, then $\mathbf{F}$ is

(A) $35.2\mathbf{u}_y$ nN/m (B) $68.3\mathbf{u}_x$ nN/m

(C) $105.6\mathbf{u}_z$ nN/m (D) 0

26. Two infinitely long parallel filaments each carry 100 A in the $\mathbf{u}_z$ direction. If the filaments lie in the plane $y = 0$ at $x = 0$ and $x = 5$ mm, the force on the filament passing through the origin is

(A) $0.4\mathbf{u}_x$ N/m (B) $-0.4\mathbf{u}_x$ N/m

(C) $4\mathbf{u}_x$ mN/m (D) $-4\mathbf{u}_x$ mN/m

Statement for Q.27–28:

A conducting current strip carrying $\mathbf{K} = 6\mathbf{u}_z$ A/m lies in the $x = 0$ plane between $y = 0.5$ and $y = 1.5$ m. There is also a current filament of $I = 5$ A in the $\mathbf{u}_z$ direction on the z –axis.

27. The force exerted on the filament by the current strip is

(A) $12.2\mathbf{u}_y$ μN/m (B) $6.6\mathbf{u}_y$ μN/m

(C) $-12.2\mathbf{u}_y$ μN/m (D) $-6.6\mathbf{u}_y$ μN/m

28. The force exerted on the strip by the filament is

(A) $-6.6\mathbf{u}_y$ μN/m (B) $6.6\mathbf{u}_y$ μN/m

(C) $2.4\mathbf{u}_x$ μN/m (D) $-2.4\mathbf{u}_x$ μN/m

Statement for Q.29–32:

In a certain material for which $\mu_r = 6.5$,

$$\mathbf{H} = 10\mathbf{u}_x + 25\mathbf{u}_y - 40\mathbf{u}_z \text{ A/m}$$

29. The magnetic susceptibility χ_m of the material is

(A) 5.5 (B) 6.5

(C) 7.5 (D) None of the above

30. The magnetic flux density $\mathbf{B}$ is

(A) $82\mathbf{u}_x + 204\mathbf{u}_y - 327\mathbf{u}_z$ μWb/m^2

(B) $82\mathbf{u}_x + 204\mathbf{u}_y - 327\mathbf{u}_z$ μA/m

(C) $82\mathbf{u}_x + 204\mathbf{u}_y - 327\mathbf{u}_z$ mT

(D) $82\mathbf{u}_x + 204\mathbf{u}_y - 327\mathbf{u}_z$ mA/m

31. The magnetization $\mathbf{M}$ is

(A) $75\mathbf{u}_x + 187.5\mathbf{u}_y - 300\mathbf{u}_z$ A/m^2

(B) $75\mathbf{u}_x + 187.5\mathbf{u}_y - 300\mathbf{u}_z$ A/m^2

(C) $55\mathbf{u}_x + 137.5\mathbf{u}_y - 220\mathbf{u}_z$ A/m^2

(D) $55\mathbf{u}_x + 137.5\mathbf{u}_y - 220\mathbf{u}_z$ A/m^2

32. The magnetic energy density is

(A) 19 mJ/m^2 (B) 9.5 mJ/m^2

(C) 16.3 mJ/m^2 (D) 32.6 mJ/m^2

Statement for Q.33–34:

For a given material magnetic susceptibility $\chi_m = 3.1$ and within which $\mathbf{B} = 0.4y\mathbf{u}_z$ T.

33. The magnetic field **H** is

(A) $986.8y\mathbf{u}_z$ kA/m (B) $151.6y\mathbf{u}_z$ kA/m

(C) $102.7y\mathbf{u}_z$ kA/m (D) $77.6y\mathbf{u}_z$ kA/m

34. The magnetization **M** is

(A) $241y\mathbf{u}_z$ kA/m (B) $318.2y\mathbf{u}_z$ kA/m

(C) $163y\mathbf{u}_z$ kA/m (D) None of the above

35. In a material the magnetic field intensity is $H = 1200$ A/m when $B = 2\ Wb/m^2$. When H is reduced to 400 A/m, $B = 1.4\ Wb/m^2$. The change in the magnetization M is

(A) 164 kA/m (B) 326 kA/m

(C) 476 kA/m (D) 238 kA/m

36. A particular material has 2.7×10^{29} atoms/m^3 and each atom has a dipole moment of $2.6 \times 10^{-30}\mathbf{u}_y\ A \cdot m^2$. The **H** in material is ($\mu_r = 4.2$)

(A) $2.94\mathbf{u}_y$ A/m (B) $0.22\mathbf{u}_y$ A/m

(C) $0.17\mathbf{u}_y$ A/m (D) $2.24\mathbf{u}_y$ A/m

37. In a material magnetic flux density is 0.02 Wb/m^2 and the magnetic susceptibility is 0.003. The magnitude of the magnetization is

(A) 47.6 A/m (B) 23.4 A/m

(C) 16.3 A/m (D) 8.4 A/m

38. A uniform field $\mathbf{H} = -600\mathbf{u}_y$ A/m exist in free space. The total energy stored in spherical region 1 cm in radius centered at the origin in free space is

(A) 0.226 J/m^3 (B) 1.452 J/m^3

(C) 1.68 J/m^3 (D) 0.84 J/m^3

39. The magnetization curve for an iron alloy is approximately given by

$$B = \frac{1}{3}H + H^2\ \mu Wb/m^2$$

If H increases from 0 to 210 A/m, the energy stored per unit volume in the alloy is

(A) 6.2 MJ/m^3 (B) 1.3 MJ/m^3

(C) 2.3 kJ/m^3 (D) 2.9 kJ/m^3

40. If magnetization is given by $\mathbf{H} = \frac{6}{a}(-y\mathbf{u}_x + x\mathbf{u}_y)$ in a cube of size a, the magnetization volume current density is

(A) $\frac{12}{a}\mathbf{u}_z$ (B) $\frac{6}{a}(x - y)$

(C) $\frac{6}{a}\mathbf{u}_z$ (D) $\frac{3}{a}(x - y)$

41. The point P(2, 3, 1) lies on the planner boundary separating region 1 from region 2. The unit vector $\mathbf{u}_{N12} = 0.6\mathbf{u}_x + 0.48\mathbf{u}_y + 0.64\mathbf{u}_z$ is directed from region 1 to region 2. If $\mu_{r1} = 2$, $\mu_{r2} = 8$ and $\mathbf{H}_1 = 100\mathbf{u}_x - 300\mathbf{u}_y + 200\mathbf{u}_z$ A/m, then $\mathbf{H}_2$ is

(A) $40.3\mathbf{u}_x + 48.3\mathbf{u}_y - 178.9\mathbf{u}_z$ A/m

(B) $80.2\mathbf{u}_x - 315.8\mathbf{u}_y + 178.9\mathbf{u}_z$ A/m

(C) $40.3\mathbf{u}_x - 315.8\mathbf{u}_y - 178.9\mathbf{u}_z$ A/m

(D) $80.2\mathbf{u}_x + 48.3\mathbf{u}_y + 178.9\mathbf{u}_z$ A/m

42. The plane separates air ($z > 0, \mu_r = 1$) from iron ($z \le 0$, $\mu_r = 20$). In air magnetic field intensity is $\mathbf{H} = 10\mathbf{u}_x + 15\mathbf{u}_y - 3\mathbf{u}_z$ A/m. The magnetic flux density in iron is

(A) $5.02\mathbf{u}_x + 7.5\mathbf{u}_y - 0.076\mathbf{u}_z$ mWb/m^2

(B) $12.6\mathbf{u}_x + 18.9\mathbf{u}_y - 75.4\mathbf{u}_z$ $\mu Wb/m^2$

(C) $251\mathbf{u}_x + 377\mathbf{u}_y - 3.77\mathbf{u}_z$ $\mu Wb/m^2$

(D) $251\mathbf{u}_x + 377\mathbf{u}_y - 1508\mathbf{u}_z$ $\mu Wb/m^2$

43. The plane $2x + 3y - 4z = 1$ separates two regions. Let $\mu_{r1} = 2$ in region 1 defined by $2x + 3y - 4z > 1$, while $\mu_{r2} = 5$ in region 2 where $2x + 3y - 4z < 1$. In region $\mathbf{H}_1 = 50\mathbf{u}_x - 30\mathbf{u}_y + 20\mathbf{u}_z$ A/m. In region 2, $\mathbf{H}_2$ will be

(A) $63.4\mathbf{u}_x + 43.18\mathbf{u}_y - 19.4\mathbf{u}_z$ A/m

(B) $52.9\mathbf{u}_x - 25.66\mathbf{u}_y + 14.2\mathbf{u}_z$ A/m

(C) $48.6\mathbf{u}_x - 16.4\mathbf{u}_y - 46.3\mathbf{u}_z$ A/m

(D) None of the above

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SOLUTIONS

1. (C) $\mathbf{H} = \int_{-\infty}^{\infty} \frac{Id\mathbf{L} \times \mathbf{u}_R}{4\pi R^2}$

$= \frac{I}{4\pi} \int_0^{\infty} \frac{-\mathbf{u}_y dy[2\mathbf{u}_x + (3-y)\mathbf{u}_y]}{[2^2 + (3-y)^2]^{3/2}} =$

$= \frac{I}{4\pi} \int_0^{\infty} \frac{2\mathbf{u}_z dy}{[2^2 + (3-y)^2]^{3/2}}$

$3 - y = 2\tan\theta,\ -dy = 2\sec^2\theta,$

$\theta_1 = 56.31°,\ \theta_2 = -90°$

$\mathbf{H} = \frac{I}{4\pi} \int_{-90°}^{56.31°} \frac{2\,\mathbf{u}_z d\theta}{2\sec\theta}$

$= \frac{I}{4\pi}\mathbf{u}_z [\sin\theta]_{-90°}^{56.31°} = 145.8\mathbf{u}_z$ mA/m

2. (A) $\mathbf{H} = \mathbf{H}_y + \mathbf{H}_z$, $\mathbf{H}_z = \frac{I_z}{2\pi\rho}\mathbf{u}_\phi$

$\rho = \sqrt{(-3)^2 + (4)^2} = 5$

$\mathbf{u}_\phi = \frac{-\mathbf{u}_z \times (-3\mathbf{u}_x + 4\mathbf{u}_y)}{5} = \frac{3\mathbf{u}_y + 4\mathbf{u}_x}{5}$

$\mathbf{H}_z = \frac{24}{2\pi(5)} \frac{(4\mathbf{u}_x + 3\mathbf{u}_y)}{5} = 0.611\mathbf{u}_x + 0.458\mathbf{u}_y$ mA/m

$\mathbf{H}_y = \frac{I_y}{2\pi\rho}\mathbf{u}_\phi$, $\rho = \sqrt{(-3)^2 + (5)^2} = \sqrt{34}$

$\mathbf{u}_\phi = \mathbf{u}_y \times \frac{(-3\mathbf{u}_x + 5\mathbf{u}_z)}{\sqrt{34}} = \frac{3\mathbf{u}_z - 5\mathbf{u}_x}{\sqrt{34}}$

$\mathbf{H}_y = \frac{12}{2\pi\sqrt{34}} \frac{(-5\mathbf{u}_x + 3\mathbf{u}_z)}{\sqrt{34}}$

$= -0.281\mathbf{u}_x + 0.168\mathbf{u}_z$ mA/m

$\mathbf{H} = \mathbf{H}_y + \mathbf{H}_z = 0.331\mathbf{u}_x + 0.458\mathbf{u}_y + 0.168\mathbf{u}_z$ mA/m

3. (A) $\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{u}_\phi - \int_{-4}^{4} \frac{Idz\mathbf{u}_z \times (\rho\mathbf{u}_\rho - z\mathbf{u}_z)}{4\pi(\rho^2 + z^2)^{3/2}}$

$\frac{I}{2\pi\rho}\mathbf{u}_\phi - \frac{I}{4\pi}\int_{-4}^{4} \frac{\rho dz}{4\pi(\rho^2 + z^2)^{3/2}}\mathbf{u}_\phi$

$= \frac{I}{2\pi\rho}\mathbf{u}_\phi - \frac{I}{4\pi}\frac{8}{\rho\sqrt{(\rho^2 + 16)}}\mathbf{u}_\phi$

$= \frac{I}{2\pi\rho}\left(1 - \frac{4}{\sqrt{(\rho^2 + 16)}}\right)\mathbf{u}_\phi$

At $\rho = -3$, $\phi = 60°$, $I = 3\pi$,

$\mathbf{H} = 0.1\mathbf{u}_\phi$ A/m

4. (B) $\mathbf{H} = \int \frac{Id\mathbf{L} \times \mathbf{u}_R}{4\pi R^2}$, $Id\mathbf{L} = 4dx\mathbf{u}_x$

$\mathbf{u}_R = \frac{-(1+x)\mathbf{u}_x + 3\mathbf{u}_y + 2\mathbf{u}_z}{R}$

$R = \sqrt{(1+x)^2 + 3^2 + 2^2} = \sqrt{x^2 + 2x + 14}$

$\mathbf{H} = \int_{-\infty}^{\infty} \frac{4dx\mathbf{u}_x \times [-(1+x)\mathbf{u}_x + 3\mathbf{u}_y + 2\mathbf{u}_z]}{4\pi(x^2 + 2x + 14)^{3/2}}$

$= \int_{-\infty}^{\infty} \frac{(12\mathbf{u}_z - 8\mathbf{u}_y)dx}{4\pi(x^2 + 2x + 14)^{3/2}} = \frac{2(12\mathbf{u}_z - 8\mathbf{u}_y)}{4\pi 13}$

$= 0.147\mathbf{u}_z - 0.098\mathbf{u}_y$ A/m

5. (B) $\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{u}_\phi - \int_{-a}^{a} \frac{Idz\mathbf{u}_z \times (\rho\mathbf{u}_\rho - z\mathbf{u}_z)}{4\pi(\rho^2 + z^2)^{3/2}}$

$= \frac{I}{2\pi\rho}\mathbf{u}_\phi - \int_{-a}^{a} \frac{\rho Idz\mathbf{u}_\phi}{4\pi(\rho^2 + z^2)^{3/2}}$

$\int_{-a}^{a} \frac{\rho Idz\mathbf{u}_\phi}{4\pi(\rho^2 + z^2)^{3/2}} = \frac{\rho I\mathbf{u}_\phi}{4\pi} \left.\frac{z}{\rho^2(\rho^2 + z^2)^{3/2}}\right|_{-a}^{a} = \frac{Ia\mathbf{u}_\phi}{2\pi\rho(\rho^2 + z^2)^{3/2}}$

$= \frac{I}{2\pi\rho}\left(1 - \frac{a}{\sqrt{(\rho^2 + a^2)}}\right)\mathbf{u}_\phi$ A/m

At $\rho = 1$, $H = \frac{I}{2\pi\rho}$

$\Rightarrow\quad 1 - \frac{a}{\sqrt{1 + a^2}} = \frac{1}{2}$

$\Rightarrow\quad a = \frac{1}{\sqrt{3}} = 0.577$ m

6. (A) $\mathbf{H} = \int \frac{Id\mathbf{L} \times \mathbf{u}_R}{4\pi R^2}$

$= \int_0^{2\pi} \frac{Id\phi\mathbf{u}_\phi \times (-\mathbf{u}_\rho)}{4\pi a} = \frac{I}{2a}\mathbf{u}_z$ A/m

$I = 3$ A, $a = 0.5$ m, $\mathbf{H} = 3\mathbf{u}_z$ A/m

7. (D) $\mathbf{H} = \iint \frac{\mathbf{K} \times \mathbf{u}_R dxdy}{4\pi R^2}$

$= \int_{-2}^{2}\int_{-\infty}^{\infty} \frac{4\mathbf{u}_x \times (-x\mathbf{u}_x - y\mathbf{u}_y - 3\mathbf{u}_z)dx\,dy}{4\pi(x^2 + y^2 + 9)^{3/2}}$

$= \int_{-2}^{2}\int_{-\infty}^{\infty} \frac{4(-y\mathbf{u}_z - 3\mathbf{u}_y)dx\,dy}{4\pi(x^2 + y^2 + 9)^{3/2}}$

$= \int_{-2}^{2}\int_{-\infty}^{\infty} \frac{12\,\mathbf{u}_y dx\,dy}{4\pi(x^2 + y^2 + 9)^{3/2}}$

$= -\frac{3}{\pi}\mathbf{u}_y \int_{-2}^{2} \frac{2}{y^2 + 9}dy$

$-\frac{6}{\pi}\mathbf{u}_y \frac{1}{3}\tan^{-1}\left.\left(\frac{y}{3}\right)\right|_{-2}^{2} = -0.75\mathbf{u}_y$ A/m

8. (D) $\mathbf{H} = \int \frac{Id\mathbf{L} \times \mathbf{u}_R}{4\pi R^2}$

$= \int_0^\infty \frac{-Idz\mathbf{u}_z \times (-z\mathbf{u}_z + \mathbf{u}_y)}{4\pi(1+z^2)^{3/2}} + \int_0^\infty \frac{Idx\mathbf{u}_x \times (-x\mathbf{u}_x + \mathbf{u}_y)}{4\pi(1+x^2)^{3/2}}$

$= \int_0^\infty \frac{Idz\mathbf{u}_x}{4\pi(1+z^2)^{3/2}} + \int_0^\infty \frac{Idx\mathbf{u}_z}{4\pi(1+x^2)^{3/2}}$

$= \frac{I}{4\pi}\left(\left. \frac{z\mathbf{u}_x}{\sqrt{(1+z^2)}} \right|_\infty^0 + \left. \frac{x\mathbf{u}_z}{\sqrt{(1+x^2)}} \right|_\infty^0 \right)$

$= \frac{I}{4\pi}(\mathbf{u}_x + \mathbf{u}_z) = 0.8(\mathbf{u}_x + \mathbf{u}_z)$ mA/m

9. (A) Using Ampere's circuital law

$\oint \mathbf{H} \cdot d\mathbf{L} = 2\pi\rho H_\phi = I_{encl}$

At $\rho = 0.5$ cm, $I_{encl} = 10$ mA

$2\pi(5 \times 10^{-3})H_\phi = 10,\ H_\phi = 0.32$ A/m

10. (B) At $\rho = 1.5$ cm enclosed current

$I_{encl} = 10 + 2\pi(0.01)(400) = 35.13$ mA

$2\pi(0.015)H_\phi = 35.13 \times 10^{-3} \Rightarrow H_\phi = 0.37$ A/m

11. (C) The enclosed current is

$I_{encl} = 10 + 2\pi(0.01)400 - 2\pi(0.02)250 + 2\pi(0.03)300$

$= 60.3$ mA/m

$2\pi(0.035)H_\phi = 60.3$ M $\Rightarrow H_\phi = 0.27$ A/m

12. (A) $\mathbf{H} = \frac{1}{2}(-30 - 40)\mathbf{u}_x \times (-\mathbf{u}_z) = -35\mathbf{u}_y$ A/m

13. (B) $\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{u}_n$

$= \frac{1}{2}(30 - 40)\mathbf{u}_x \times (-\mathbf{u}_z) = -5\mathbf{u}_y$ A/m

14. (A) $\mathbf{H} = \frac{1}{2}(-30 + 40)\mathbf{u}_x + (-\mathbf{u}_z) = 5\mathbf{u}_y$ A/m

15. (C) $\mathbf{J} = \nabla \times \mathbf{H} = \frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial\rho}\mathbf{u}_z$

$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(2 + \frac{\rho^2}{2}\right)\mathbf{u}_z = \mathbf{u}_z$ A/m

16. (D) $\mathbf{J} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{3}{\rho}\right) = 0$

17. (A) $\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

If $\mathbf{F} = 0$, $\mathbf{E} = -\mathbf{v} \times \mathbf{B} = \mathbf{B} \times \mathbf{v}$

$\begin{bmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ 1 & 2 & 3 \\ 3 & 12 & -4 \end{bmatrix} \times 10^5 \times 10^{-3}$

$= [\mathbf{u}_x(-8 - 36) - \mathbf{u}_y(-4 - 9) + \mathbf{u}_z(12 - 6)] \times 10^2$ V/m

$= -4.4\mathbf{u}_x + 1.3\mathbf{u}_y + 0.6\mathbf{u}_z$ kV/m

18. (D) $\mathbf{v}(0) \times \mathbf{B} = (2\mathbf{u}_x - 3\mathbf{u}_y - 4\mathbf{u}_z)10^5$

$\times (-3\mathbf{u}_x + 2\mathbf{u}_y - \mathbf{u}_z)10^{-3}$

$= 1100\mathbf{u}_x + 1400\mathbf{u}_y - 500\mathbf{u}_z$

$\mathbf{F}(0) = Q\,[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$

$= 2 \times 10^{-16}[1200\mathbf{u}_x + 1200\mathbf{u}_y - 200\mathbf{u}_z]$

$= 4 \times 10^{-14}[6\mathbf{u}_x + 6\mathbf{u}_y - \mathbf{u}_z]$

$\mathbf{F} = m\mathbf{a} \Rightarrow \mathbf{a} = \frac{\mathbf{F}}{m} = \frac{4 \times 10^{-14}}{5 \times 10^{-26}}[6\mathbf{u}_x + 6\mathbf{u}_y - \mathbf{u}_z]$

$= 800[6\mathbf{u}_x + 6\mathbf{u}_y - \mathbf{u}_z]10^9$ m/s^2

19. (C) $\mathbf{F} = e\,\mathbf{v} \times \mathbf{B}$

$= -(1.6 \times 10^{-19})(4.5 \times 10^7\mathbf{u}_y)(2.5 \times 10^{-3}\mathbf{u}_z)$

$= -1.8 \times 10^{-14}\mathbf{u}_x$ N

This force will be constant during the time the electron travels the field. It establishes a negative x–directed velocity as it leaves the field, given by the acceleration times the transit time t_t ,

$v_x = \frac{Ft_t}{m} = \left(\frac{-1.8 \times 10^{-14}}{9.1 \times 10^{-31}}\right)\left(\frac{2.5 \times 10^{-2}}{4.5 \times 10^7}\right) = -1.1 \times 10^7$ m/s

$t_{50} = \frac{0.5 - 0.025}{4.5 \times 10^7} = 1.06 \times 10^{-8}$ s

In that time, the electron moves to an x coordinate given by

$x = v_x t_{50} = -(1.1 \times 10^7)(1.06 \times 10^{-8}) = -0.117$ m

$x = -11.7$ cm, $z = 0$

20. (A) $\mathbf{F}_{BC} = \int_B^C I_{\text{loop}} d\mathbf{L} \times \mathbf{B}_{\text{from wire at BC}}$

$= \int_1^4 (6 \times 10^{-3})dz\mathbf{u}_z \times \frac{15\mu_o}{2\pi(3)}\mathbf{u}_y = -1.8 \times 10^{-8}\mathbf{u}_x = -18\mathbf{u}_x$ nN

21. (C) The field from the long wire now varies with position along the loop segment.

$\mathbf{F}_{AB} = \int_1^3 (6 \times 10^{-3})dx\mathbf{u}_x \times \frac{15\mu_o}{2\pi x}\mathbf{u}_y$

$= \frac{45 \times 10^{-3}}{\pi}\mu_o \ln 3\mathbf{u}_z = 19.8\mathbf{u}_z$ nN

22. (A) This will be the vector sum of the forces on the four sides. By symmetry, the forces on sides AB and CD will be equal and opposite, and so will cancel. This leaves the sum of forces on side BC and DA

$$\mathbf{F}_{DA} = \int_1^4 -(6\times10^{-3})dx\mathbf{u}_z \times \frac{15\mu_o}{2\pi(1)}\mathbf{u}_y = 54\mathbf{u}_x \text{ nN}$$

$$\mathbf{F}_{total} = \mathbf{F}_{DA} + \mathbf{F}_{BC} = (54-18)\mathbf{u}_x = 36\mathbf{u}_x \text{ nN}$$

23. (A) $\mathbf{F} = \int Id\mathbf{L}\times\mathbf{R}$

$$= I\int_1^2 dx\mathbf{u}_x \times \mathbf{B} + I\int_1^2 dy\mathbf{u}_y \times \mathbf{B} + I\int_1^3 dx\mathbf{u}_x \times \mathbf{B} + I\int_2^1 dy\mathbf{u}_y \times \mathbf{B}$$

$$\mathbf{u}_x \times \mathbf{B} = \mathbf{u}_x[6x\mathbf{u}_x - 9y\mathbf{u}_y + 3z\mathbf{u}_z] = 3z\mathbf{u}_y - 9y\mathbf{u}_z$$

$$\mathbf{u}_y \times \mathbf{B} = \mathbf{u}_y[6x\mathbf{u}_x - 9y\mathbf{u}_y + 3z\mathbf{u}_z] = 3z\mathbf{u}_x - 6x\mathbf{u}_z$$

$z = 0$ for all element

$$\mathbf{F} = I\int_1^3 dx(-9y\mathbf{u}_z)_{y=1} + I\int_1^2 dy(-6x\mathbf{u}_z)_{x=3} + I\int_3^1 dx(-9y\mathbf{u}_z)_{y=2} + I\int_2^1 dy(-6x\mathbf{u}_z)_{x=1}$$

$$= I(-18 - 18 + 36 + 6)\mathbf{u}_z = 5\times 6\mathbf{u}_z = 30\mathbf{u}_z \text{ N}$$

24. (B) Within the region $-1 < y < 1$, the magnetic fields from the two outer sheets (carrying $-4\mathbf{u}_z$ A/m) cancel, leaving only the field from the center sheet. Therefore $\mathbf{H} = -4\mathbf{u}_x$ A/m $(0 < y < 1)$ and $\mathbf{H} = 4\mathbf{u}_x$ A/m $(-1 < y < 0)$. Outside ($y > 1$ and $y < -1$) the fields from all three sheet cancel, leaving $H = 0$ $(y > 1,\ y < -1)$. So at $x = 0,\ y = 0.5$

$$\frac{\mathbf{F}}{m} = I\mathbf{u}_z \times \mathbf{B} = (7\times10^{-3})\mathbf{u}_z \times -4\mu_o\mathbf{u}_x = -35.2\mathbf{u}_y \text{ nN/m}$$

25. (D) $\frac{\mathbf{F}}{m} = I\mathbf{u}_x \times (-4\mu_o\mathbf{u}_x) = 0$

26. (A) $\mathbf{F} = \int_0^1 Id\mathbf{L}\times\mathbf{B}$

$$= \int_0^1 100dz\mathbf{u}_z \times \frac{-100\,\mu_o\mathbf{u}_y}{2\pi(5\times10^{-3})} = 0.4\mathbf{u}_x \text{ N/m}$$

27. (B) The field from the current strip at the filament location

$$\mathbf{B} = \int_{0.5}^{1.5} \frac{6\mu_o\mathbf{u}_x}{2\pi y}dy = \frac{3\mu_o}{\pi}\ln\left(\frac{1.5}{0.5}\right)\mathbf{u}_x$$

$$= 1.32\times10^{-6}\mathbf{u}_x \text{ Wb/m}^2$$

$$\mathbf{F} = \int_0^1 Id\mathbf{L}\times\mathbf{B}$$

$$= \int_0^1 5dz\mathbf{u}_z \times 1.32\times10^{-6}\mathbf{u}_x dz = 6.6\mathbf{u}_y\ \mu\text{N/m}$$

28. (A) $\mathbf{F} = \int_{area} \mathbf{K}\times\mathbf{B}dS = \int_0^1\int_{0.5}^{1.5} 6\mathbf{u}_z \times \frac{-5\mu_o\mathbf{u}_x}{2\pi y}dy$

$$= -\frac{15\mu_o}{\pi}\ln\left(\frac{1.5}{0.5}\right)\mathbf{u}_y = -6.6\mathbf{u}_y\ \mu\text{N/m}$$

29. (A) $\chi_m + 1 = \mu_r$, $\chi_m + 1 = 6.5$, $\chi_m = 5.5$

30. (A) $\mathbf{B} = \mu\mathbf{H} = \mu_o\mu_r\mathbf{H}$

$$= 4\pi\times10^{-7}\times 6.5(10\mathbf{u}_x + 25\mathbf{u}_y - 40\mathbf{u}_z)$$

$$= 82\mathbf{u}_x + 204\mathbf{u}_y - 327\mathbf{u}_z\ \mu\text{Wb/m}^2$$

31. (C) $\mathbf{M} = \chi_m\mathbf{H} = 55\mathbf{u}_x + 137.5\mathbf{u}_y - 220\mathbf{u}_z$ A/m

32. (B) $W = \frac{1}{2}\mathbf{H}\cdot\mathbf{B} = \frac{1}{2}\mu H^2$

$$= \frac{1}{2}\times 6.5\times 4\pi\times10^{-7}(100 + 625 + 1600) = 9.5 \text{ mJ/m}^2$$

33. (D) $\mu_r = \chi_m + 1 = 3.1 + 1 = 4.1,\quad \mu = \mu_o\mu_r = 4.1\mu_o$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{0.4y\mathbf{u}_z}{4.1\times4\pi\times10^{-7}} = 77.6y\mathbf{u}_z \text{ kA/m}$$

34. (A) $\mathbf{M} = \chi_m\mathbf{H} = (3.1)(77.6)y\mathbf{u}_z = 241y\mathbf{u}_z$ kA/m

35. (C) For case 1, $\mu = \frac{B_1}{H} = \frac{2}{1200}$

$$\mu_{r1} = \frac{\mu}{\mu_o} = \frac{1}{600}\times\frac{1}{4\pi\times10^{-7}} = 1326.3$$

$$\chi_m = \mu_r - 1 = 1325.3$$

$$M_1 = \chi_m H_1 = 1.590\times10^6 \text{ A/m}$$

For case 2, $\mu = \frac{B_2}{H_2} = \frac{1.4}{400}$

$$\mu_{r1} = \frac{1.4}{400\times4\pi\times10^{-7}} = 2785.2$$

$$\chi_m = 2784.2\ 2784.2$$

$$M_2 = \chi_m H_2 = 1.114\times10^6 \text{ A/m}$$

$$\Delta M = (1.590 - 1.114)\times10^6 = 476 \text{ kA/m}$$

36. (B) $\mathbf{M} = N\mathbf{m} = (2.7\times10^{29})(2.6\times10^{-30}\mathbf{u}_y)$

$$= 0.7\mathbf{u}_y \text{ A/m}$$

$$\mathbf{H} = \frac{\mathbf{M}}{\mu_r - 1} = \frac{0.7\mathbf{u}_y}{4.2 - 1} = 0.22\mathbf{u}_y \text{ A/m}$$

37. (A) $\mathbf{M} = \frac{\mathbf{B}}{\mu_o}\left(\frac{1}{\chi_m} + 1\right)^{-1}$

$$=\frac{0.02}{4\pi\times10^{-7}}\left(\frac{1}{0.003}+1\right)^{-1}=47.6\ \text{A/m}$$

38. (A) $W=\frac{1}{2}\mathbf{H}\cdot\mathbf{B}=\frac{1}{2}\mu_o H^2$

$$=\frac{1}{2}(4\pi\times10^{-7})(600)^2=0.226\ \text{J/m}^3$$

39. (A) $W=\int_0^{H_o}H.dB=\int_0^{H_o}H\left(\frac{1}{3}+2H\right)dH$

$$=\frac{H_o^2}{6}+\frac{2H_o^3}{3}=6.2\ \text{MJ/m}^3$$

40. (A) $\mathbf{J}_b=\nabla\times\mathbf{M}=\frac{12}{a}\mathbf{u}_z$

41. (B) $\mathbf{B}_1=200\mu_o\mathbf{u}_x-600\mu_o\mathbf{u}_y+400\mu_o\mathbf{u}_z$

Its normal component at the boundary is

$\mathbf{B}_{1N}=(\mathbf{B}_1\cdot\mathbf{u}_{N12})\mathbf{u}_{N12}$

$=(52.8\mathbf{u}_x+42.24\mathbf{u}_y+56.32\mathbf{u}_z)\mu_o=\mathbf{B}_{2N}$

$\Rightarrow\quad \mathbf{H}_{2N}=\frac{\mathbf{B}_{2N}}{8\mu_o}=6.60\mathbf{u}_x+5.28\mathbf{u}_y+7.04\mathbf{u}_z$

$\mathbf{H}_{1N}=\frac{\mathbf{B}_{1N}}{12\mu_o}=26.40\mathbf{u}_x+21.12\mathbf{u}_y+28.16\mathbf{u}_z$

$\mathbf{H}_{1T}=\mathbf{H}_1-\mathbf{H}_{1N}=(100\mathbf{u}_x-300\mathbf{u}_y+200\mathbf{u}_z)$
$-(26.4\mathbf{u}_x+21.12\mathbf{u}_y+28.16\mathbf{u}_z)$

$=73.6\mathbf{u}_x-321.12\mathbf{u}_y+171.84\mathbf{u}_z$

$\mathbf{H}_{1T}=\mathbf{H}_{2T}$

$\mathbf{H}_2=\mathbf{H}_{2N}+\mathbf{H}_{2T}=80.2\mathbf{u}_x-315.8\mathbf{u}_y+178.9\mathbf{u}_z$ A/m

42. (C) $\mathbf{H}_{N1}=-3\mathbf{u}_z$, $\mathbf{H}_{T1}=10\mathbf{u}_x+15\mathbf{u}_y$

$\mathbf{H}_{T2}=\mathbf{H}_{T1}=10\mathbf{u}_x+15\mathbf{u}_y$

$\mathbf{H}_{N2}=\frac{\mu_1}{\mu_2}\mathbf{H}_{N1}=\frac{1}{20}(-3\mathbf{u}_z)=0.15\mathbf{u}_z$

$\mathbf{H}_2=\mathbf{H}_{N2}+\mathbf{H}_{T2}=10\mathbf{u}_x+15\mathbf{u}_y-0.15\mathbf{u}_z$

$\mathbf{B}_2=\mu_2\mathbf{H}_2=20\times4\pi\times10^{-7}(10\mathbf{u}_x+15\mathbf{u}_y-0.15\mathbf{u}_z)$

$=251\mathbf{u}_x+377\mathbf{u}_y-3.77\mathbf{u}_z\ \mu\text{Wb/m}^2$

43. (B) At the boundary normal unit vector

$$\mathbf{u}_n=\frac{\nabla(2x+3y-4z)}{|\nabla(2x+3y-4z)|}=\frac{2\mathbf{u}_x+3\mathbf{u}_y-4\mathbf{u}_z}{\sqrt{29}}$$

$=0.37\mathbf{u}_x+0.56\mathbf{u}_y-0.74\mathbf{u}_z$

Since this vector is found through the gradient, it will point in the direction of increasing values of $2x+3y-4z$, and so will be directed into region 1. Thus $\mathbf{u}_n=\mathbf{u}_{n21}$.

The normal component of $\mathbf{H}_1$ is

$\mathbf{H}_{N1}=(\mathbf{H}_1\cdot\mathbf{u}_{N21})\mathbf{u}_{N21}$

$\mathbf{H}_1\cdot\mathbf{u}_{N21}=(50\mathbf{u}_x-30\mathbf{u}_y+20\mathbf{u}_z)\cdot(0.37\mathbf{u}_x+0.56\mathbf{u}_y-0.74\mathbf{u}_z)$

$=18.5-16.8-14.8=-13.1$

$(\mathbf{H}_1\cdot\mathbf{u}_{N21})\mathbf{u}_{N21}=(-13.1)(0.37\mathbf{u}_x+0.56\mathbf{u}_y-0.74\mathbf{u}_z)$

$=-4.83\mathbf{u}_x-7.24\mathbf{u}_y+9.66\mathbf{u}_z$ A/m

Tangential component of $\mathbf{H}_1$ at the boundary

$\mathbf{H}_{T1}=\mathbf{H}_1-\mathbf{H}_{N1}$

$=(50\mathbf{u}_x-30\mathbf{u}_y+20\mathbf{u}_z)-(-4.83\mathbf{u}_x-7.24\mathbf{u}_y+9.66\mathbf{u}_z)$

$=54.83\mathbf{u}_x-22.76\mathbf{u}_y+10.34\mathbf{u}_z$ A/m

$\mathbf{H}_{T2}=\mathbf{H}_{T1}$

$\mathbf{H}_{N2}=\frac{\mu_{r1}}{\mu_{r2}}\mathbf{H}_{N1}=\frac{2}{5}(-4.83\mathbf{u}_x-7.24\mathbf{u}_y+9.66\mathbf{u}_z)$

$=-1.93\mathbf{u}_x-2.90\mathbf{u}_y+3.86\mathbf{u}_z$ A/m

$\mathbf{H}_2=\mathbf{H}_{T2}+\mathbf{H}_{N2}=(54.83\mathbf{u}_x-22.76\mathbf{u}_y+10.34\mathbf{u}_z)$
$+(-1.93\mathbf{u}_x-2.9\mathbf{u}_y+3.86\mathbf{u}_z)$

$=52.9\mathbf{u}_x-25.66\mathbf{u}_y+14.2\mathbf{u}_z$

****

Statement for Q.8–9:

The location of the sliding bar in fig. P8.4.8–9 is given by $x = 5t + 4t^3$. The separation of the two rails is 30 cm. Let $\mathbf{B} = x^2\mathbf{u}_z$ T.

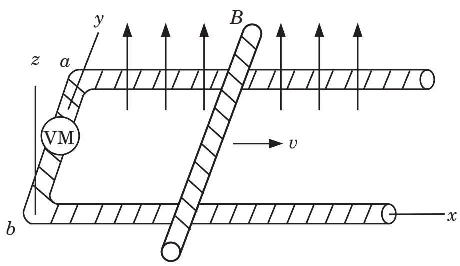

Fig. P8.4.8–9.

8. The voltmeter reading at $t = 0.5$ s is

(A) –21.6 V (B) 21.6 V

(C) –6.3 V (D) 6.3 V

9. The voltmeter reading at $x = 0.6$ m is

(A) –1.68 V (B) 1.68 V

(C) –0.933 V (D) 0.933 V

Statement for Q.10–11:

A perfectly conducting filament containing a 250Ω resistor is formed into a square as shown in fig. P8.4.10-11.

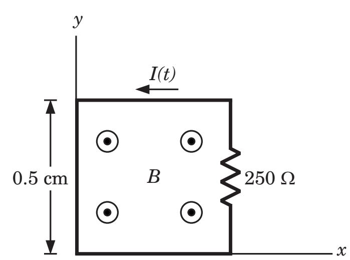

Fig. P8.4.10–11

10. If $\mathbf{B} = 6\cos(120\pi t - 30°)\mathbf{u}_z$ T, then the value of $I(t)$ is

(A) $2.26\sin(120\pi t - 30°)$ A

(B) $2.26\cos(120\pi t - 30°)$ A

(C) $-2.26\sin(120\pi t - 30°)$ A

(D) $-2.26\cos(120\pi t - 30°)$ A

11. If $\mathbf{B} = 2\cos\pi(ct - y)\,\mathbf{u}_z$ μT, where c is the velocity of light, then $I(t)$ is

(A) $1.2(\cos\pi ct - \sin\pi ct)$ μA

(B) $1.2(\cos\pi ct - \sin\pi ct)$ mA

(C) $1.2(\sin\pi ct - \sin\pi ct)$ μA

(D) $1.2(\sin\pi ct - \sin\pi ct)$ mA

Statement for Q.12–13:

Consider the fig. P8.4.12–13. The rails have a resistance of 2 Ω/m. The bar moves to the right at a constant speed of 9 m/s in a uniform magnetic field of 0.8 T. The bar is at $x = 2$ m at $t = 0$.

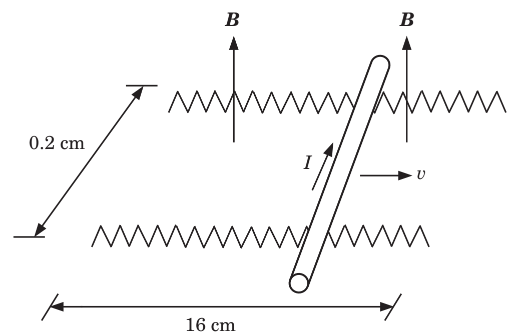

Fig. P8.4–12–14

12. If 6 Ω resistor is present across the left-end with the right end open-circuited, then at $t = 0.5$ sec the current I is

(A) –45 mA (B) 45 mA

(C) –60 mA (D)60 mA

13. If 6 Ω resistor is present across each end, then I at 0.5 sec is

(A) –12.3 mA (B) 12.3 mA

(C) –7.77 mA (D) 77.7 mA

Statement for Q.14–15:

The internal dimension of a coaxial capacitor is $a = 1.2$ cm, $b = 4$ cm and $c = 40$ cm. The homogeneous material inside the capacitor has the parameter $\varepsilon = 10^{-11}$ F/m, $\mu = 10^{-5}$ H/m and $\sigma = 10^{-5}$ S/m.The electric field intensity is $\mathbf{E} = \frac{10^7}{\rho}\cos(10^5 t)\,\mathbf{u}_p$ V/m.

14. The current density $\mathbf{J}$ is

(A) $\frac{200}{\rho}\sin(10^5 t)\,\mathbf{u}_\rho$ A/m^2

(B) $\frac{400}{\rho}\sin(10^5 t)\,\mathbf{u}_\rho$ A/m^2

(C) $\frac{100}{\rho}\cos(10^5 t)\,\mathbf{u}_\rho$ A/m^2

(D) None of the above

15. The quality factor of the capacitor is

(A) 0.1 (B) 10

(C) 0.2 (D) 20

16. The following fields exist in charge free regions

$\mathbf{P} = 60 \sin(\omega t + 10x)\, \mathbf{u}_z$

$\mathbf{Q} = \frac{10}{\rho} \cos(\omega t - 2\rho)\, \mathbf{u}_\phi$

$\mathbf{R} = 3\rho^2 \cot\phi\, \mathbf{u}_\rho + \frac{1}{\rho}\cos\phi\, \mathbf{u}_\phi$

$\mathbf{S} = \frac{1}{r} \sin\theta \sin(\omega t - 6r)\, \mathbf{u}_\theta$

The possible electromagnetic fields are

(A) **P**, **Q** (B) **R**, **S**

(C) **P**, **R** (D) **Q**, **S**

17. A parallel-plate capacitor with plate area of 5 cm^2 and plate separation of 3 mm has a voltage $50 \sin(10^3 t)$ V applied to its plates. If $\varepsilon_r = 2$, the displacement current is

(A) $148 \cos(10^{10} t)$ nA (B) $261 \cos(10^{10} t)$ μA

(C) $261 \cos(10^{10} t)$ nA (D) $148 \cos(10^{10} t)$ μA

18. In a coaxial transmission line ($\varepsilon_r = 1$), the electric field intensity is given by

$$\mathbf{E} = \frac{100}{\rho} \cos(10^9 t - 6z)\, \mathbf{u}_\rho \ \text{V/m}.$$

The displacement current density is

(A) $-\frac{100}{\rho} \sin(10^9 t - 6z)\, \mathbf{u}_\rho$ A/m^2

(B) $\frac{116}{\rho} \sin(10^9 t - 6z)\, \mathbf{u}_\rho$ A/m^2

(C) $-\frac{0.9}{\rho} \sin(10^9 t - 6z)\, \mathbf{u}_\rho$ A/m^2

(D) $-\frac{216}{\rho} \cos(10^9 t - 6z)\, \mathbf{u}_\rho$ A/m^2

Statement for Q.19–21:

Consider the region defined by $|x|, |y|$ and $|z| < 1$. Let $\varepsilon = 5\varepsilon_o$, $\mu = 4\mu_o$, and $\sigma = 0$ the displacement current density $\mathbf{J}_d = 20 \cos(1.5 \times 10^8 t - ax)\, \mathbf{u}_y$ μA/m^2. Assume no DC fields are present.

19. The electric field intensity **E** is

(A) $6 \sin(1.5 \times 10^8 t - ax)\, \mathbf{u}_y$ mV/m

(B) $6 \cos(1.5 \times 10^8 t - ax)\, \mathbf{u}_y$ mV/m

(C) $3 \cos(1.5 \times 10^8 t - ax)\, \mathbf{u}_y$ mV/m

(D) $3 \sin(1.5 \times 10^8 t - ax)\, \mathbf{u}_y$ mV/m

20. The magnetic field intensity is

(A) $-4a \sin(1.5 \times 10^8 t - ax)\, \mathbf{u}_z$ μA/m

(B) $-4a \sin(1.5 \times 10^8 t - ax)\, \mathbf{u}_z$ mA/m

(C) $4a \sin(1.5 \times 10^8 t - ax)\, \mathbf{u}_z$ μA/m

(D) $4a \sin(1.5 \times 10^8 t - ax)\, \mathbf{u}_z$ mA/m

21. The value of a is

(A) 4.3 (B) 2.25

(C) 5 (D) 6

Statement for Q.22–23:

Let $\mathbf{H} = 2\cos(10^{10} t - \beta x)\, \mathbf{u}_z$ A/m, $\mu = 3 \times 10^{-5}$ H/m, $\varepsilon = 1.2 \times 10^{-10}$ F/m and $\sigma = 0$ everywhere.

22. The electric flux density **D** is

(A) $120 \cos(10^{10} t - \beta x)$ nC/m^2

(B) $-120 \cos(10^{10} t - \beta x)$ nC/m^2

(C) $120 \cos(10^{10} t + \beta x)$ nC/m^2

(D) None of the above

23. The magnetic flux density **B** is

(A) $6.67 \times 10^4 \cos(10^{10} t + \beta x)$ T

(B) $6.67 \times 10^4 \cos(10^{10} t - \beta x)$

(C) $6 \times 10^{-5} \cos(10^{10} t + \beta x)$ T

(D) $6 \times 10^{-5} \cos(10^{10} t - \beta x)$ T

Statement for Q.24–25:

A material has $\sigma = 0$ and $\varepsilon_r = 1$. The magnetic field intensity is $\mathbf{H} = 4\cos(10^6 t - 0.01z)\, \mathbf{u}_y$ A/m.

24. The electric field intensity E is

(A) $4.52 \sin(10^6 t - 0.01z)$ kV/m

(B) $4.52 \sin(10^6 t - 0.01z)$ V/m

(C) $4.52 \cos(10^6 t - 0.01z)$ V/m

(D) $4.52 \cos(10^6 t - 0.01z)$ kV/m

25. The value of μ_r is

(A) 2 (B) 3

(C) 4 (D) 16

26. The surface $\rho = 3$ and 10 mm, and $z = 0$ and 25 cm are perfect conductors. The region enclosed by these surface has $\mu = 2.5 \times 10^{-6}$ H/m, $\varepsilon = 4 \times 10^{-11}$ F/m and $\sigma = 0$. If $\mathbf{H} = \frac{2}{\rho} \cos 8\pi z \cos \omega t\, \mathbf{u}_\phi$ A/m, then the value of ω is

(A) $2\pi \times 10^6$ rad/s (B) $8\pi \times 10^6$ rad/s

(C) $2\pi \times 10^8$ rad/s (D) $8\pi \times 10^8$ rad/s

27. For distilled water $\mu = \mu_o$, $\varepsilon = 81\varepsilon_o$, and $\sigma = 2 \times 10^{-3}$ S/m, the ratio of conduction current density to displacement current density at 1 GHz is

(A) 1.11×10^{-5} (B) 4.44×10^{-4}

(C) 2.68×10^{-6} (D) 1.68×10^{-7}

28. A conductor with cross-sectional area of 10 cm^2 carrier a conductor current $2 \sin(10^9 t)$ mA. If $\sigma = 2.5 \times 10^6$ S/m and $\varepsilon_r = 4.6$, the magnitude of the displacement current density is

(A) 48.4 μA/m^2 (B) 8.11 nA/m^2

(C) 32.6 nA/m^2 (D) 16.4 μA/m^2

29. In a certain region

$$\mathbf{J} = (4y\mathbf{u}_x + 2xz\mathbf{u}_y + z^3\mathbf{u}_z)\sin(10^4 t) \text{ A/m}$$

If volume charge density ρ_v in $z = 0$ plane is zero, then ρ_v is

(A) $3z^2 \cos(10^4 t)$ mC/m^3

(B) $0.3z^2 \cos(10^4 t)$ mC/m^3

(C) $-3z^2 \cos(10^4 t)$ mC/m^3

(D) $-0.3z^2 \cos(10^4 t)$ mC/m^3

30. In a charge-free region ($\sigma = 0$, $\varepsilon = \varepsilon_o\varepsilon_r$, $\mu = \mu_o$) magnetic field intensity is $\mathbf{H} = 10\cos(10^{11} t - 4y)\,\mathbf{u}_z$ A/m. The displacement current density is

(A) $-40 \sin(10^9 t - 4y)\,\mathbf{u}_y$ A/m

(B) $40 \sin(10^9 t - 4y)\,\mathbf{u}_y$ A/m

(C) $-40 \sin(10^3 t - 4y)\,\mathbf{u}_x$ A/m

(D) $40 \sin(10^9 t - 4y)\,\mathbf{u}_x$ A/m

31. In a nonmagnetic medium ($\varepsilon_r = 6.25$) the magnetic field of an EM wave is $\mathbf{H} = 6\cos\beta x \cos(10^8 t)\,\mathbf{u}_z$ A/m. The corresponding electric field is

(A) $903 \sin(0.83x)\sin(10^8 t)$ V/m

(B) $903 \sin(1.2x)\sin(10^8 t)$ V/m

(C) $903 \sin(0.83x)\cos(10^8 t)$ V/m

(D) $903 \sin(1.2x)\cos(10^8 t)$ V/m

32. In a nonmagnetic medium

$$\mathbf{E} = 5\cos(10^9 t - 8x)\,\mathbf{u}_x + 4\sin(10^9 t - 8x)\,\mathbf{u}_z \text{ V/m.}$$

The dielectric constant of the medium is

(A) 3.39 (B) 1.84

(C) 5.76 (D) 2.4

****

SOLUTIONS

1. (B) emf $= -\dfrac{d\Phi}{dt} = -\dfrac{d}{dt}\int \mathbf{B}\cdot d\mathbf{S}$

$= 2\pi(0.2)^2(20)(377)\sin 377t \text{ mV} = 0.95\cos 377t$ V

2. (A) emf $= \dfrac{1}{2}B_o\omega L^2 = \dfrac{1}{2}(4)(2)(2)^2 = 16$ V

3. (C) Since B is constant over the loop area, the flux is

$\Phi = \pi(0.1)^2 B = 0.31\cos(120\pi t)$

emf $= V_{ba}(t) = -\dfrac{d\Phi}{dt}$

$= 0.31(120\pi)\sin(120\pi t) = 118.43\sin(120\pi t)$

$V_{ab} = -118.43\sin(120\pi t)$

4. (D) $I = \dfrac{V_{ab}}{R} = \dfrac{118.43\sin(120\pi t)}{250} = 0.47\sin(120\pi t)$

5. (A) emf $= -\dfrac{d\Phi}{dt} = -\dfrac{d}{dt}\iint_{loop\ area} \mathbf{B}\cdot\mathbf{u}_z dz$

$= \dfrac{d}{dt}(3)(4)(6)\cos 5000t = -360000\sin 5000t$

$I = \dfrac{\text{emf}}{R} = -\dfrac{360000\sin 5000t}{900\times 10^3} = -0.4\sin 5000t$ A

6. (C) $\Phi = \int_0^1\int_0^1 20\mu_o\cos(3\times 10^8 t - y)\,dx\,dy$

$= [20\mu_o\sin(3\times 10^8 t - y)]_0^1$

$= 20\,\mu_o[\sin(3\times 10^8 t - 1) - \sin(3\times 10^8 t)]$ Wb

Emf $= -\dfrac{d\Phi}{dt}$

$= -20\times(4\pi\times 10^{-7})(3\times 10^8)\times[\cos(3\times 10^8 t - 1) - \cos(3\times 10^8 t)]$

$= 7540[\cos(3\times 10^8 t) - \cos(3\times 10^8 t - 1)]$ V

7. (D) In this case $\Phi = [20\mu_o(2\pi)\sin(3\times 10^8 t - y)]_0^{2\pi} = 0$

8. (A) $\Phi = \iint_{area}\mathbf{B}\cdot d\mathbf{S} = \int_0^{0.3}\int_0^x \tau^2 d\tau dy$

$= 0.1x^3 = 0.1(5t + 4t^3)^3$ Wb

emf $= -\dfrac{d\Phi}{dt} = -0.1(3)(5t + 4t^3)^2(5 + 12t^2)$

At $t = 0.5$ s, emf $= -0.1(3)(2.5 + 0.5)^2(5 + 3) = -21.6$ V

9. (C) At $x = 0.6$ m, $0.6 = 5t + 4t^3 \Rightarrow t = 0.119$ s

At $t = 0.119$ s, emf $= -0.933$ V

10. (A) $\Phi = \iint_{area} \mathbf{B}\cdot d\mathbf{S} = 6(0.5)^2 \cos(120\pi t - 30°)$ Wb

$$\text{emf} = -\frac{d\Phi}{dt} = 6(0.5)^2(120\pi)\sin(120\pi t - 30°)$$

The current is $\frac{\text{emf}}{R} = \frac{6(0.5)^2(120\pi)}{250}\sin(120\pi t - 30°)$ A

$= 2.26\sin(120\pi t - 30°)$ A

11. (D) $\Phi = \iint_{area} \mathbf{B}\cdot d\mathbf{S} = (0.5)(2)\int_0^{0.5}\cos(\pi y - \pi ct)\,dy$

$$= \frac{1}{\pi}\left[\sin\left(\pi ct - \frac{\pi}{2}\right) - \sin \pi ct\right] = \frac{1}{\pi}[-\cos\pi ct - \sin\pi ct]\ \mu\text{Wb}$$

$$\text{emf} = -\frac{d\Phi}{dt} = c[\cos\pi ct - \sin\pi ct]\ \mu\text{V}$$

$$I(t) = \frac{\text{emf}}{R} = \frac{3\times 10^8}{250}[\cos\pi ct - \sin\pi ct]\ \mu\text{A}$$

$= 1.2[\cos\pi ct - \sin\pi ct]$ A

12. (A) The flux in the left-hand closed loop is

$\Phi_l = B\times\text{area} = (0.8)(0.2)(2+9t)$

$$\text{emf}_l = -\frac{d\Phi_L}{dt} = -(0.16)(9) = -1.44\ \text{V}$$

While the bar in motion, the loop resistance is increasing with time,

$R_l = 6 + 2[2(2+9t)]\Omega$, At $t = 0.5$, $R_l = 32\,\Omega$

$$I_l = -\frac{1.44}{32} = -45\ \text{mA}$$

13. (C) In this case, there will be contribution to the current from the right loop, which is now closed. The flux in the right loop, whose area decreases with time, is $\Phi_r = (0.8)(0.2)(16-2-9t)$

$$\text{emf}_R = -\frac{d\Phi_R}{dt} = 1.44\ \text{V}$$

$R_r = 6 + 2(2(14-9t))$, At 0.5 s, $R_r = 44\,\Omega$

The contribution to the current from the right loop

$$I_r = \frac{-144}{44} = 032.7\ \text{mA}$$

The total current $= -32.7 - 45 = -77.7$ mA

14. (C) $\mathbf{J} = \sigma\mathbf{E} = \frac{100}{\rho}\cos(10^5 t)\,\mathbf{u}_\rho$ A/m^2

15. (A) Total conduction current

$$I_C = \iint \mathbf{J}\cdot d\mathbf{S} = 2\pi\rho l J = 2\pi\rho l\,\frac{100}{\rho}\cos(10^5 t)\,\mathbf{u}_\rho\ \text{A/m}^2$$

$= 80\pi\cos(10^5 t)$ A

$$\mathbf{J}_d = \frac{\partial\mathbf{D}}{\partial t} = \frac{\partial\varepsilon\mathbf{E}}{\partial t} = -\frac{10}{\rho}\sin(10^5 t)\ \text{A/m}^2$$

$$I_d = 2\pi\rho l\mathbf{J}_d = -2\pi l(10)\sin(10^5 t) = -8\pi\sin(10^5 t)\ \text{A}$$

Quality factor $\frac{|I_d|}{|I_c|} = \frac{8}{80} = 0.1$

16. (A) $\nabla\cdot\mathbf{P} = 0$, $\nabla\times\mathbf{P} = -\frac{\partial P_z}{\partial x}\mathbf{u}_y \neq 0$

$\mathbf{P}$ is a possible EM field

$\nabla\cdot\mathbf{Q} = 0$, $\nabla\times\mathbf{Q} = \frac{1}{\rho}\frac{\partial}{\partial\rho}[10\cos(\omega t - 2\rho)]\mathbf{u}_z \neq 0$

$\mathbf{Q}$ is a possible EM field

$\nabla\cdot\mathbf{R} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(3\rho^2\cot\phi)\cdot\frac{\sin\phi}{\rho} \neq 0$, $\mathbf{R}$ is not an EM field.

$$\nabla\cdot\mathbf{S} = \frac{1}{r^2\sin\theta}\sin(\omega t - 6r)\frac{\partial(\sin^2\phi)}{\partial r} \neq 0$$

$\mathbf{S}$ is not an EM field. Hence (A) is correct.

17. (A) $D = \varepsilon E = \varepsilon\frac{V}{d} \Rightarrow J_d = \frac{dD}{dt} = \frac{\varepsilon}{d}\frac{dV}{dt}$

$$I_d = J\cdot S = \frac{\varepsilon S}{d}\frac{dV}{dt} = \frac{2\varepsilon_o 5\times 10^{-4}}{3\times 10^{-3}}10^3\times 50\cos(10^3 t)$$

$= 148\cos(10^{10}t)$ nA

18. (C)

$$\mathbf{J}_d = \frac{\partial\mathbf{D}}{\partial t} = \varepsilon\frac{\partial\mathbf{E}}{\partial t} = \varepsilon_o\frac{100}{\rho}[-\sin(10^9 t - 6z)]10^9\,\mathbf{u}_\rho\ \text{A/m}^2$$

$$= -\frac{0.9}{\rho}\sin(10^9 t - 6z)\,\mathbf{u}_\rho\ \text{A/m}^2$$

19. (D) $\mathbf{D} = \int \mathbf{J}_d dt + C_1 = \frac{20\times 10^{-6}}{1.5\times 10^8}\sin(1.5\times 10^8 - ax)\,\mathbf{u}_y$

$= 1.33\times 10^{-13}\sin(1.5\times 10^8 t - ax)\,\mathbf{u}_y$ C/m^3

C_1 is set to zero since no DC fields are present.

$$\mathbf{E} = \frac{\mathbf{D}}{\varepsilon} = \frac{1.33\times 10^{-13}}{5\varepsilon_o}\sin(1.5\times 10^8 - ax)\,\mathbf{u}_y$$

$= 3\times 10^{-3}\sin(1.5\times 10^8 t - ax)$ V/m

20. (D) $\nabla\times\mathbf{E} = \frac{\partial E_y}{\partial x}\mathbf{u}_z = -\frac{\partial\mathbf{B}}{\partial t}$

$$= -a(3\times 10^{-3})\cos(1.5\times 10^8 t - ax)\,\mathbf{u}_z = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\mathbf{B} = \frac{a(3\times 10^{-3})}{1.5\times 10^8}\sin(1.5\times 10^8 t - ax)\,\mathbf{u}_z$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{2\times 10^{-11}}{4\times 4\pi\times 10^{-7}}\sin(1.5\times 10^8 t - ax)\,\mathbf{u}_z\ \text{A/m}$$

$= 4\times 10^{-6}a\sin(1.5\times 10^8 t - ax)\,\mathbf{u}_z$ mA/m

21. (B) $\nabla\times\mathbf{H} = -\frac{\partial H_z}{\partial x}\mathbf{u}_y = \mathbf{J}_d$

$= a^2(4\times10^{-6})\cos(1.5\times10^8 t - ax) = J_D$

Comparing the result

$a^2 4\times10^{-6} = 20\times10^{-6}, \quad a=\sqrt{5}=2.25$

22. (B) $\nabla\times\mathbf{H} = -\frac{\partial H_z}{\partial x}\mathbf{u}_y = \frac{\partial \mathbf{D}}{\partial t}$

$\frac{\partial \mathbf{D}}{\partial t} = 2\beta\sin(10^{10}t-\beta x)\,\mathbf{u}_y$

$\mathbf{D} = -\frac{2\beta}{10^{10}}\cos(10^{10}t-\beta x)\,\mathbf{u}_y\ \text{C/m}^2$

$\beta = \frac{w}{v} = 10^{10}\sqrt{\mu\varepsilon} = 10^{10}\sqrt{3\times10^{-5}\times1.2\times10^{-10}} = 600$

$\mathbf{D} = -120\cos(10^{10}t-\beta x)\,\mathbf{u}_y\ \text{nC/m}^2$

23. (D) $\mathbf{B} = \mu\mathbf{H} = 6\times10^{-5}\cos(10^{10}t-\beta x)\,\mathbf{u}_z\ \text{T}$

24. (A) $\nabla\times\mathbf{H} = -\frac{\partial H_y}{\partial z}\mathbf{u}_x$

$\Rightarrow \quad \nabla\times\mathbf{H} = 0.04\cos(10^6t-0.01z)\,\mathbf{u}_x = \varepsilon_o\frac{\partial\mathbf{E}}{\partial t}$

$\mathbf{E} = \frac{0.04\sin(10^6t-0.01z)\,\mathbf{u}_x}{10^6\varepsilon_o}$

$= 4.52\sin(10^6t-0.01z)\,\mathbf{u}_x\ \text{kV/m}$

25. (B) $\nabla\times\mathbf{E} = \frac{\partial E_x}{\partial z}\mathbf{u}_y = -\mu\frac{\partial\mathbf{H}}{\partial t}$

$\frac{-0.04(0.01)}{10^6\varepsilon_o}\cos(10^6t-0.01z)\,\mathbf{u}_y = -\mu_r\mu_o\frac{\partial\mathbf{H}}{\partial t}$

$\mathbf{H} = \frac{0.04(0.01)}{(10^6)(10^6)\mu_r\mu_o\varepsilon_o}\sin(10^6t-0.013)\,\mathbf{u}_y$

$\frac{0.04(0.01)}{10^{12}\mu_r\mu_o\varepsilon_o} = 4 \quad\Rightarrow\quad \mu_r = \frac{(0.04)(0.01)}{4(10^{12})}\times(3\times10^8)^2 = 9$

26. (D) $\nabla\times\mathbf{H} = -\frac{\partial H_\phi}{\partial z}\mathbf{u}_\rho$

$= \frac{16\pi}{\rho}\sin(8\pi z)\cos(\omega t)\,\mathbf{u}_\rho = \varepsilon\frac{\partial\mathbf{E}}{\partial t}$

$\mathbf{E} = \frac{16\pi}{\rho\varepsilon\omega}\sin(8\pi z)\sin(\omega t)\,\mathbf{u}_\rho$

$\nabla\times\mathbf{E} = \frac{\partial E_\rho}{\partial z}\mathbf{u}_\phi = \frac{(16\pi)(8\pi)}{\rho\varepsilon\omega}\cos(8\pi z)\sin(\omega t)\,\mathbf{u}_\phi = -\mu\frac{\partial\mathbf{H}}{\partial t}$

$\mathbf{H} = \frac{128\pi^2}{\rho\varepsilon\omega^2}\cos(8\pi z)\cos(\omega t)\,\mathbf{u}_\phi$

This result must be equal to the given **H** field. Thus

$\frac{128\pi^2}{\rho\varepsilon\mu\omega^2} = \frac{2}{\rho} \quad\Rightarrow\quad \omega = \frac{8\pi}{\sqrt{\mu\varepsilon}} = \frac{8\pi}{\sqrt{4\times10^{-11}\times2.5\times10^{-6}}}$

$= 8\pi\times10^8\ \text{rad/s}$

27.(B) At high frequency $\frac{J_c}{J_d} = \frac{\sigma E}{\omega\varepsilon E} = \frac{\sigma}{\omega\varepsilon}$

$= \frac{2\times10^{-3}}{2\pi\times10^9\times81\times\varepsilon_o} = 4.44\times10^{-4}$

28. (C) $J_c = \frac{I_c}{S} = \sigma E \quad\Rightarrow\quad E = \frac{I_c}{\sigma S}$

$\Rightarrow\quad J_d = \varepsilon\frac{\partial E}{\partial t} = \frac{\varepsilon}{\sigma S}\frac{\partial I_c}{\partial t}$

$\Rightarrow\quad J_d = \frac{4.6\varepsilon_o(10^9)}{2.5\times10^6\times10\times10^{-4}}2\cos(10^9t)\,10^{-3}$

$|J_d| = 32.6\ \text{nA/m}^2$

29. (B) $\nabla\cdot\mathbf{J} = (0+0+3z^2)\sin(10^4t) = -\frac{8\rho}{\partial t}$

$\rho_v = \frac{3z^2\cos(10^4t)}{10^4} + C_1$

At $z=0,\ \rho_v=0,\ C_1=0$

$\rho_v = 0.3z^2\cos(10^4t)\ \text{mC/m}^3$

30. (D) $\mathbf{J}_d = \nabla\times\mathbf{H} = \frac{\partial H_z}{\partial y}\mathbf{u}_x$

$= 40\sin(10^9t-4y)\,\mathbf{u}_y\ \text{A/m}$

31. (A) $\nabla\times\mathbf{H} = -\frac{\partial H_z}{\partial y}\mathbf{u}_z = \varepsilon\frac{\partial\mathbf{E}}{\partial t}$

$= 6\beta\sin(\beta x)\cos(10^8t)\,\mathbf{u}_y$

$\mathbf{E} = \frac{1}{\varepsilon}\int 6\beta\sin(\beta x)\cos(10^8t)\,\mathbf{u}_y\,dt$

$= \frac{6\beta}{\varepsilon10^{10}}\sin(\beta x)\sin(10^8t)\,\mathbf{u}_y$

$\varepsilon = 6.25\varepsilon_o\ ,\quad \beta = \frac{\omega}{v} = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{10^8}{3\times10^8}\sqrt{6.25} = 0.833$

$\mathbf{E} = \frac{6(0.833)}{6.25\varepsilon_o\times10^8}\sin(\beta x)\sin(10^8t)\,\mathbf{u}_y\ \text{V/m}$

$= 903\sin(0.83x)\ \sin(10^8t)\,\mathbf{u}_y\ \text{V/m}$

32. (C) For nonmagnetic medium $\mu_r = 1$

$\beta = \frac{\omega}{v} = \frac{\omega}{c}\sqrt{\varepsilon_r}\ ,\quad \omega = 10^9,\ \beta = 8,$

$8 = \frac{10^9}{3}\times10^8\sqrt{\varepsilon_r} \quad\Rightarrow\quad \varepsilon_r = 5.76$

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CHAPTER 8.5

ELECTROMAGNETIC WAVE PROPAGATION

Statement for Q.1–3:

A y-polarized uniform plane wave with a frequency of 100 MHz propagates in air in the $+x$ direction and impinges normally on a perfectly conducting plane at $x = 0$. The amplitude of incident **E**-field is 6 mV/m.

1. The phasor $\mathbf{H}_s$ of the incident wave in air is

(A) $16e^{-j\frac{2\pi}{3}x}\mathbf{u}_z$ μA/m (B) $-16e^{-j\frac{2\pi}{3}x}\mathbf{u}_z$ μA/m

(C) $16e^{-j\frac{2\pi}{3}x}\mathbf{u}_x$ μA/m (D) $-16e^{-j\frac{2\pi}{3}x}\mathbf{u}_x$ μA/m

2. The **E**-field of total wave in air is

(A) $j12\sin\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

(B) $-j12\sin\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

(C) $12\cos\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

(D) $-12\cos\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

3. The location in air nearest to the conducting plane, where total **E**-field is zero, is

(A) $x = 1.5$ m (B) $x = -1.5$ m

(C) $x = 3$ m (D) $x = -3$ m

4. The phasor magnetic field intensity for a 400 MHz uniform plane wave propagating in a certain lossless material is $(6\mathbf{u}_y - j5\mathbf{u}_z)e^{-j18x}$ A/m . The phase velocity v_p is

(A) 6.43×10^6 m/s (B) 2.2×10^7 m/s

(C) 1.4×10^8 m/s (D) None of the above

Statement for Q.5–6:

A uniform plane wave in free space has electric field $\mathbf{E}_s = (2\mathbf{u}_z + 3\mathbf{u}_y)e^{-j\beta x}$ V/m.

5. The magnetic field phasor $\mathbf{H}_s$ is

(A) $(-5.3\mathbf{u}_y - 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

(B) $(5.3\mathbf{u}_y - 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

(C) $(-5.3\mathbf{u}_y + 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

(D) $(5.3\mathbf{u}_y + 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

6. The average power density in the wave is

(A) 34 mW/m^2 (B) 17 mW/m^2

(C) 22 mW/m^2 (D) 44 mW/m^2

7. The electric field of a uniform plane wave in free space is given by $\mathbf{E}_s = 12\pi(\mathbf{u}_y + j\mathbf{u}_z)e^{-j15x}$. The magnetic field phasor $\mathbf{H}_s$ is

(A) $\frac{12}{\eta_o}(-\mathbf{u}_z + j\mathbf{u}_y)e^{-j15x}$ (B) $\frac{12}{\eta_o}(\mathbf{u}_z + j\mathbf{u}_y)e^{-j15x}$

(C) $\frac{12}{\eta_o}(-\mathbf{u}_z - j\mathbf{u}_y)e^{-j15x}$ (D) $\frac{12}{\eta_o}(\mathbf{u}_z - j\mathbf{u}_y)e^{-j15x}$

Statement for Q.8–9:

A lossy material has $\mu = 5\mu_o$, $\varepsilon = 2\varepsilon_o$. The phase constant is 10 rad/m at 5 MHz.

8. The loss tangent is

(A) 2913 (B) 1823

(C) 2468 (D) 1374

9. The attenuation constant α is

(A) 4.43 (B) 9.99

(C) 5.57 (D) None of the above

Statement for Q.10–11:

At 50 MHz a lossy dielectric material is characterized by $\mu = 2.1\mu_o$, $\varepsilon = 3.6\varepsilon_o$ and $\sigma = 0.08$ S/m. The electric field is $\mathbf{E}_s = 6e^{-j\gamma x}\mathbf{u}_z$ V/m.

10. The propagation constant γ is

(A) $7.43 + j2.46$ per meter

(B) $2.46 + j7.43$ per meter

(C) $6.13 + j5.41$ per meter

(D) $5.41 + j6.13$ per meter

11. The impedance η is

(A) 101.4 Ω (B) 167.4 Ω

(C) 98.3 Ω (D) 67.3 Ω

Statement for Q.12–13:

A non magnetic medium has an intrinsic impedance $360\angle 30°\,\Omega$.

12. The loss tangent is

(A) 0.866 (B) 0.5

(C) 1.732 (D) 0.577

13. The Dielectric constant is

(A) 1.634 (B) 1.234

(C) 0.936 (D) 0.548

Statement for Q.14–15:

The amplitude of a wave traveling through a lossy nonmagnetic medium reduces by 18% every meter. The wave operates at 10 MHz and the electric field leads the magnetic field by 24°.

14. The propagation constant is

(A) $0.198 + j0.448$ per meter

(B) $0.346 + j0.713$ per meter

(C) $0.448 + j0.198$ per meter

(D) $0.713 + j0.346$ per meter

15. The skin depth is

(A) 2.52 m (B) 5.05 m

(C) 8.46 m (D) 4.23 m

16. A 60 m long aluminium ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_2 = 1$) pipe with inner and outer radii 9 mm and 12 mm carries a total current of $16\sin(10^6\pi t)$ A. The effective resistance of the pipe is

(A) 0.19 Ω (B) 3.48 Ω

(C) 1.46 Ω (D) 2.43 Ω

17. Silver plated brass wave guide is operating at 12 GHz. If at least the thickness of silver ($\sigma = 6.1 \times 10^7$ S/m, $\mu_r = \varepsilon_r = 1$) is 5δ, the minimum thickness required for wave-guide is

(A) 6.41 μm (B) 3.86 μm

(C) 5.21 μm (D) 2.94 μm

Statement for Q.18–19:

A uniform plane wave in a lossy nonmagnetic media has

$$\mathbf{E}_s = (5\mathbf{u}_x + 12\mathbf{u}_y)e^{-\gamma z}, \quad \gamma = 0.2 + j3.4 \text{ m}^{-1}$$

18. The magnitude of the wave at $z = 4$ m and $t = T/8$ is

(A) 10.34 (B) 5.66

(C) 4.36 (D) 12.60

19. The loss suffered by the wave in the interval $0 < z < 3$ m is

(A) 4.12 dB (B) 8.24 dB

(C) 10.42 dB (D) 5.21 dB

Statement for Q.20–22:

The plane wave $\mathbf{E} = 42\cos(\omega t - z)\,\mathbf{u}_x$ V/m in air normally hits a lossless medium ($\mu_r = 1$, $\varepsilon_r = 4$) at $z = 0$.

20. The SWR s is

(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) None of the above

21. The transmission coefficient τ is

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$

(C) $\frac{1}{3}$ (D) 3

22. The reflected electric field is

(A) $-14\cos(\omega t - z)\,\mathbf{u}_x$ V/m

(B) $-14\cos(\omega t + z)\,\mathbf{u}_x$ V/m

33. The region $z < 0$ is characterized by $\varepsilon_r = \mu_r = 1$ and $\sigma = 0$. The total electric field here is given $\mathbf{E}_s = 150 e^{-j10z}\,\mathbf{u}_x + 50\angle 20^\circ e^{j10z}\,\mathbf{u}_x$ V/m. The intrinsic impedance of the region $z > 0$ is

(A) $692 + j176\,\Omega$ (B) $193 - j49\,\Omega$

(C) $176 + j692\,\Omega$ (D) $49 - j193\,\Omega$

Statement for Q.34–35:

Region 1, $z < 0$ and region 2, $z > 0$, are both perfect dielectrics. A uniform plane wave traveling in the $\mathbf{u}_z$ direction has a frequency of 3×10^{10} rad/s. Its wavelength in the two region are $\lambda_1 = 5$ cm and $\lambda_2 = 3$ cm.

34. On the boundary the reflected energy is

(A) 6.25% (B) 12.5%

(C) 25% (D) 50%

35. The SWR is

(A) 1.67 (B) 0.6

(C) 2 (D) 1.16

36. A uniform plane wave is incident from region 1 ($\mu_r = 1, \quad \sigma = 0$) to free space. If the amplitude of incident wave is one-half that of reflected wave in region, then the value of ε_r is

(A) 4 (B) 3

(C) 16 (D) 9

37. A 150 MHz uniform plane wave is normally incident from air onto a material. Measurements yield a SWR of 3 and the appearance of an electric field minimum at 0.3λ in front of the interface. The impedance of material is

(A) $502 - j641\,\Omega$ (B) $641 - j502\,\Omega$

(C) $641 + j502\,\Omega$ (D) $502 + j641\,\Omega$

38. A plane wave is normally incident from air onto a semi-infinite slab of perfect dielectric ($\varepsilon_r = 3.45$). The fraction of transmitted power is

(A) 0.91 (B) 0.3

(C) 0.7 (D) 0.49

Statement for Q.39–40:

Consider three lossless region :

Region 1 ($z < 0$): $\mu_1 = 4\,\mu$H/m, $\varepsilon_1 = 10$ pF/m

Region 2 ($0 < z < 6$ cm): $\mu_2 = 2\,\mu$H/m, $\varepsilon_2 = 25$ pF/m

Region 3 ($z > 6$ cm): $\mu_3 = 4\,\mu$H/m, $\varepsilon_3 = 10$ pF/m

39. The lowest frequency, at which a uniform plane wave incident from region 1 onto the boundary at $z = 0$ will have no reflection, is

(A) 2.96 GHz (B) 4.38 GHz

(C) 1.18 GHz (D) 590 MHz

40. If frequency is 50 MHz, the SWR in region 1 is

(A) 0.64 (B) 1.27

(C) 2.38 (D) 4.16

41. A uniform plane wave in air is normally incident onto a lossless dielectric plate of thickness $\lambda/8$, and of intrinsic impedance $\eta = 260\,\Omega$. The SWR in front of the plate is

(A) 1.12 (B) 1.34

(C) 1.70 (D) 1.93

42. The **E**-field of a uniform plane wave propagating in a dielectric medium is given by

$$\mathbf{E} = 2\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right)\mathbf{u}_x - \sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)\mathbf{u}_y \text{ V/m}$$

The dielectric constant of medium is

(A) 3 (B) 9

(C) 6 (D) $\sqrt{6}$

43. An electromagnetic wave from an under water source with perpendicular polarization is incident on a water-air interface at angle 20° with normal to surface. For water assume $\varepsilon_r = 81, \mu_r = 1$. The critical angle θ_c is

(A) 83.62° (B) 6.38°

(C) 42.6° (D) None of the above

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SOLUTIONS

1. (A) $\omega = 2\pi \times 10^8$ rad/s

$$\beta = \frac{\omega}{c} = \frac{2\pi \times 10^8}{3 \times 10^8} = \frac{2\pi}{3} \text{ rad/m}$$

$$\mathbf{E}_s = 6e^{-j\frac{2\pi}{3}x}\mathbf{u}_y \text{ mV/m}$$

$$\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_x, \quad \mathbf{u}_y \times \mathbf{u}_H = \mathbf{u}_x, \quad \mathbf{u}_H = \mathbf{u}_z$$

$$\mathbf{H}_s = \frac{6}{120\pi}e^{-j\frac{2\pi}{3}x}\mathbf{u}_z = 16e^{-j\frac{2\pi}{3}x}\mathbf{u}_z \ \mu\text{A/m}$$

2. (B) For conducting plane $\Gamma = -1$,

$$\mathbf{E}_r = -6e^{j\frac{2\pi}{3}x}\mathbf{u}_y \text{ mV/m},$$

$$\mathbf{E} = \mathbf{E}_i + \mathbf{E}_r = \left(6e^{-j\frac{2\pi}{3}x}\mathbf{u}_y - 6e^{-j\frac{2\pi}{3}x}\mathbf{u}_y\right) \text{mV/m}$$

$$= -j12\sin\left(\frac{2\pi}{3}x\right)\mathbf{u}_y \text{ mV/m}$$

3. (B) The electric field vanish at the surface of the conducting plane at $x = 0$. In air the first null occur at

$$x = -\frac{\lambda_1}{2} = -\frac{\pi}{\beta_1} = -\frac{3}{2} \text{ m}$$

4. (C) $v_p = \frac{\omega}{\beta} = \frac{2\pi \times 400 \times 10^6}{18} = 1.4 \times 10^8$ m/s

5. (C) The wave is propagating in forward x direction. Therefore $\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_x$.

For $\mathbf{u}_E = \mathbf{u}_z$, $\mathbf{u}_z \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = -\mathbf{u}_y$

For $\mathbf{u}_E = \mathbf{u}_y$, $\mathbf{u}_y \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = \mathbf{u}_z$

$$\mathbf{H}_s = \frac{1}{120\pi}(-2\mathbf{u}_y + 3\mathbf{u}_z)e^{-j\beta x} = (-5.3\mathbf{u}_y + 8\mathbf{u}_z)e^{-j\beta x} \text{ mA/m}$$

6. (B) $\mathbf{P}_{avg} = \frac{1}{2}\text{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\}$

$$\frac{1}{2}\{(5.3)\mathbf{u}_x + 3(8)\mathbf{u}_x\} \times 10^{-3} = 17.3\mathbf{u}_x \text{ mW/m}^2$$

7. (D) Since Pointing vector is in the positive x direction, therefore $\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_x$.

For $\mathbf{u}_E = \mathbf{u}_y$, $\mathbf{u}_y \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = \mathbf{u}_z$

For $\mathbf{u}_E = \mathbf{u}_z$, $\mathbf{u}_z \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = -\mathbf{u}_y$,

$$\mathbf{H}_s = \frac{12}{\eta_o}(\mathbf{u}_z - j\mathbf{u}_y)e^{-j15x}$$

8. (B) Loss tangent $\frac{\sigma}{\omega\varepsilon} = x$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$\Rightarrow \quad 10 = \frac{2\pi \times 5 \times 10^6}{3 \times 10^8}\sqrt{\frac{5 \times 2}{2}\left[\sqrt{1 + x^2} + 1\right]}$$

$$\Rightarrow \quad x = \frac{\sigma}{\omega\varepsilon} = 1823$$

9. (B) $\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1 + x^2} - 1}}{\sqrt{\sqrt{1 + x^2} + 1}}$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{\sqrt{1822}}{\sqrt{1824}}$$

$$\alpha = 10 \times 0.999 = 9.99$$

10. (D) $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.08}{3.6 \times 50 \times 10^6 \times 2\pi\varepsilon_o} = 8$$

$$\alpha = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8}\sqrt{\frac{(2.1)(3.6)}{2}(\sqrt{65} - 1)} = 5.41$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$= \frac{2\pi \times 50 \times 10^6}{3 \times 10^8}\sqrt{\frac{(2.1)(3.6)}{2}(\sqrt{65} + 1)} = 6.13$$

$\gamma = \alpha + j\beta = 5.41 + j6.13$ per meter.

11. (A) $|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left(1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{\frac{1}{4}}} = \frac{120\pi\sqrt{\frac{2.1}{3.6}}}{64^{\frac{1}{4}}} = 101.4$

12. (C) $\frac{\sigma}{\omega\varepsilon} = \tan 2\theta_n = \tan 60° = 1.732$

13. (D) $|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left(1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{\frac{1}{4}}}$

$$\Rightarrow \quad 360 = \frac{\frac{120\pi}{\varepsilon_r}}{(1 + 1.732^2)^{\frac{1}{4}}} \quad \Rightarrow \quad \varepsilon_r = 0.548$$

14. (A) $|\mathbf{E}| = E_o e^{-\alpha z}$

$E_o e^{-\alpha 1} = (1 - 0.18)E_o$

$e^{-\alpha 1} = 0.82 \Rightarrow \alpha = \ln \frac{1}{0.82} = 0.198$

$\theta_n = 24° \Rightarrow \tan 2\theta_n = \frac{\sigma}{\omega\varepsilon} = 1.111$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}}$$

$$\frac{0.198}{\beta} = \frac{\sqrt{\sqrt{234} - 1}}{\sqrt{\sqrt{234} + 1}} \Rightarrow \beta = 0.448$$

$\gamma = \alpha + j\beta = 0.198 + j0.448$

15. (B) $\delta = \frac{1}{\alpha} = \frac{1}{0.198} = 5.05$

16. (A) $\omega = \pi 10^6 \Rightarrow f = 5 \times 10^5$ Hz,

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \frac{1}{\sqrt{\pi \times 5 \times 10^5 \times 3.5 \times 10^7 \times \mu_o}} = 120\ \mu\text{m}$$

$R_{ac} = \frac{l}{\sigma \delta w}$

Since δ is very small, $w = 2\pi\rho_{outer}$

$$R_{ac} = \frac{60}{3.5 \times 10^7 \times 120 \times 10^{-6} \times 2\pi \times 12 \times 10^{-3}} = 0.19\ \Omega$$

17. (D) $t = 5\delta = \frac{5}{\sqrt{\pi f \mu \sigma}}$

$$= \frac{5}{\sqrt{\pi \times 12 \times 10^9 \times \mu_o \times 6.1 \times 10^7}} = 2.94\ \mu\text{m}$$

18. (B) $\mathbf{E} = \text{Re}\{\mathbf{E}_s e^{j\omega t}\} = (5\mathbf{u}_x + 12\mathbf{u}_y)e^{-0.2z} \cos(\omega t - 3.4z)$

At $z = 4$ m, $t = \frac{T}{8}$

$$\mathbf{E} = (5\mathbf{u}_x + 12\mathbf{u}_y)e^{-0.8} \cos\left(\frac{\pi}{4} - 13.6\right)$$

$$|\mathbf{E}| = 13e^{-0.8} \cos\left(\frac{\pi}{4} - 13.6\right) = 5.66$$

19. (D) Loss $= \alpha\Delta z = 0.2 \times 3 = 0.6 Np$

$1Np = 8.686$ DB, $0.6Np = 5.21$ dB.

20. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{2}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{\eta_o}{2} - \eta_o}{\frac{\eta_o}{2} + \eta_o} = -\frac{1}{3}$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$$

21. (A) $\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\frac{\eta_o}{2}}{\frac{\eta_o}{2} + 2} = \frac{2}{3}$

22. (A) $E_{or} = \Gamma E_{oi} = -\frac{1}{3}(42) = -14$

$E_r = -14 \cos(\omega t - z)\, \mathbf{u}_x$ V/m

23. (C) $\eta_1 = \eta_o$, $\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_o}{\varepsilon_r} = \frac{\eta_o}{2}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_{1\,1}} = \frac{\frac{\eta_o}{2} - \eta_o}{\frac{\eta_o}{2} + \eta_o} = -\frac{1}{3}$$

24. (D) $\eta_1 = \eta_o$, $\eta_2 = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_o \sqrt{\frac{\mu_r}{12.5}}$

$\frac{E_{or}}{E_{oi}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

But $E_{or} = \eta_1 H_{or} = \Gamma E_{oi}$

$$\eta_1 H_{or} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_{oi} \Rightarrow \eta_1 = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \frac{18}{6 \times 10^{-3}}$$

$$\eta_1 = \eta_o \Rightarrow \eta_o = \left(\frac{\eta_2 - \eta_o}{\eta_2 + \eta_o}\right) 3000$$

$$\frac{377}{3000} = \frac{\eta_2 - 377}{\eta_2 + 377} \Rightarrow \eta_2 = 485.37 = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$\Rightarrow \varepsilon_r = 12.5$, $\mu_r = 20.75$

25. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{2}$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\frac{1}{3}$

$E_{or} = -\frac{1}{3}(10) = -\frac{10}{3}$

$H_{or} = \frac{E_{or}}{\eta_o} = \frac{10}{3 \times 377} = 8.8 \times 10^{-3}$

$\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_k$, $-\mathbf{u}_y \times \mathbf{u}_H = -\mathbf{u}_z \Rightarrow \mathbf{u}_H = -\mathbf{u}_x$

$\mathbf{H}_r = -8.8 \cos(\omega t - z)\, \mathbf{u}_x$ mA/m

$\beta = 1 = \frac{\omega}{v} = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r}$

$$\omega = \frac{3\times 10^8}{\sqrt{12\times 3}} = 0.5\times 10^8 \text{ rad/s.}$$

26. (D) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{3}} = 0.58\eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0.58\eta_o - \eta_o}{0.58\eta_o + \eta_o} = -0.266$$

$\tau = 1 + \Gamma = 0.734$, $\quad E_{ot} = \tau E_{oi} = 7.34$

$\mathbf{E}_t = 7.34\cos(\omega t - z)\,\mathbf{u}_y$ V/m

27. (B) $\mathbf{E}_{Total} = \mathbf{E}_i + \mathbf{E}_r$, $\quad E_{or} = \Gamma E_{oi} = -2.66$

$\mathbf{E}_{Total} = 10\cos(\omega t - z)\,\mathbf{u}_y - 2.66\cos(\omega t + z)\,\mathbf{u}_y$ V/m

28. (B) $\mu_o = \mu_1 = \mu_2$

$\sin\theta_{t1} = \sqrt{\frac{\varepsilon_o}{\varepsilon_1}}\sin\theta_i \Rightarrow \sin\theta_{t1} = \sqrt{\frac{1}{4.5}}\sin 45° = 0.333$

$\Rightarrow \theta_{t1} = 19.47°$

29. (B) $\sin\theta_{t2} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_{t1} = \sqrt{\frac{4.5}{2.25}}(0.333) = 0.47$

$\Rightarrow \theta_{t2} = \sin^{-1} 0.47 = 28°$

30. (A) Since both media are non magnetic

$\tan\theta_B = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \sqrt{\frac{2.6\varepsilon_o}{\varepsilon_o}} = \sqrt{2.6}$

But $\cos\theta_t = \frac{\eta_1}{\eta_2}\cos\theta_B = \frac{\eta_o}{\frac{\eta_o}{\sqrt{2.6}}}\cos 58.2° = \sqrt{2.6}\cos 58.2°$

$\Rightarrow \theta_t = 31.8°$

31. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{5}} = 0.447\eta_o$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.38$, $\tau = 1 + \Gamma = 0.62$

$E_t = \tau E_i = 92.7\cos(\omega t - 8y)\,\mathbf{u}_z$ V/m

32. (B) $|\Gamma|^2 = 0.2$, $\quad \Gamma = \pm 0.447$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} - \eta_o\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}}{\eta_o\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} + \eta_o\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}} = \frac{\sqrt{\frac{\mu_{r2}}{\mu_{r2}^3}} - \sqrt{\frac{\mu_{r1}}{\mu_{r1}^3}}}{\sqrt{\frac{\mu_{r2}}{\mu_{r2}^3}} + \sqrt{\frac{\mu_{r1}}{\mu_{r1}^3}}} = \frac{\mu_{r1} - \mu_{r2}}{\mu_{r1} + \mu_{r2}}$$

$\Rightarrow \frac{\mu_{r2}}{\mu_{r1}} = \frac{1 \mp 0.447}{1 \pm 0.447} = 0.382,\ 2.62$

$\Rightarrow \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \left(\frac{\mu_{r2}}{\mu_{r1}}\right)^3 = 0.056,\ 17.9$

33.(A) $\Gamma = \frac{E_r}{E_i} = \frac{50\angle 20°}{150} = \frac{e^{j20}}{3}$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $\eta_1 = \eta_o$,

$$\eta_2 = \eta_o\left(\frac{1+\Gamma}{1-\Gamma}\right) = 377\left(\frac{1+\frac{e^{j20}}{3}}{1-\frac{e^{j20}}{3}}\right) = 692 + j176\,\Omega$$

34. (A) $\varepsilon_{r1} = \left(\frac{2\pi c}{\lambda_1\omega}\right)^2$, $\varepsilon_{r2} = \left(\frac{2\pi c}{\lambda_2\omega}\right)^2 \Rightarrow \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \left(\frac{\lambda_2}{\lambda_1}\right)^2$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{\eta_o}{\sqrt{\varepsilon_{r2}}} - \frac{\eta_o}{\sqrt{\varepsilon_{r1}}}}{\frac{\eta_o}{\sqrt{\varepsilon_{r2}}} + \frac{\eta_o}{\sqrt{\varepsilon_{r1}}}} = \frac{\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} - 1}{\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} + 1} = \frac{\frac{\lambda_2}{\lambda_1} - 1}{\frac{\lambda_2}{\lambda_1} + 1}$$

$\Rightarrow \Gamma = \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1} = \frac{3-5}{3+5} = -\frac{1}{4}$

The fraction of the incident energy that is reflected is $\Gamma^2 = \frac{1}{16} = 6.25\%$.

35. (A) $s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{1}{4}}{1-\frac{1}{4}} = \frac{5}{3}$

36. (D) $\eta_2 = \eta_o$, $\eta_1 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{\varepsilon_r}}$

$\Gamma = \frac{|\boldsymbol{E_i}|}{|\boldsymbol{E_r}|} = \frac{\mathbf{1}}{\mathbf{2}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$\Rightarrow \frac{\eta_o - \frac{\eta_o}{\sqrt{\varepsilon_r}}}{\eta_o + \frac{\eta_o}{\sqrt{\varepsilon_r}}} = \frac{1}{2} \Rightarrow \varepsilon_r = 9$

37. (C) At minimum $\frac{(\phi+\pi)}{2\beta} = 0.3\lambda$,

$\beta = \frac{2\pi}{\lambda} \Rightarrow \phi = 0.2\pi$

$|\Gamma| = \frac{s-1}{s+1} = \frac{3-1}{3+1} = \frac{1}{2}$

$\Gamma = 0.5e^{j0.2\pi} = \frac{\eta_2 - \eta_o}{\eta_2 + \eta_o}$

$\Rightarrow \eta_2 = \eta_o\left(\frac{1+0.5e^{j0.2\pi}}{1-0.5e^{j0.2\pi}}\right) = 641 + j502\ \Omega$

38. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\dfrac{\mu_r}{\varepsilon_r}} = \dfrac{\eta_o}{\sqrt{3.45}}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\dfrac{\eta_o}{\sqrt{3.45}} - \eta_o}{\dfrac{\eta_o}{\sqrt{3.45}} + \eta_o} = -0.3$$

The transmitted fraction is $1 - |\Gamma|^2 = 1 - 0.09 = 0.91$.

39. (C) This frequency gives the condition $\beta_2 d = \pi$

Where $d = 6$ cm, $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$

$$\Rightarrow \quad \omega\sqrt{\mu_2\varepsilon_2} = \frac{\pi}{0.06}$$

$$\Rightarrow \quad f = \frac{1}{2 \times 0.06\sqrt{2 \times 10^{-6} \times 25 \times 10^{-12}}} = 1.18 \text{ GHz}$$

40. (B) At 50 MHz,

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \mathbf{2\pi \times 50 \times 10^6\sqrt{2 \times 10^{-6} \times 25 \times 10^{-12}} = 2.2}$$

$$\beta_2 d = 2.22(0.06) = 0.133$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{4 \times 10^{-6}}{10^{-11}}} = 632\ \Omega$$

$$\eta_3 = 632\ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{2 \times 10^{-6}}{25 \times 10^{-12}}} = 283\ \Omega$$

The input impedance at the first interface is

$$\eta_{in} = \eta_2\left(\frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}\right) = 283\left(\frac{632 + j283(0.134)}{283 + j632(0.134)}\right)$$

$$= 590 - j138$$

$$\Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1} = \frac{590 - j138 - 632}{590 - j138 + 632} = 0.12\angle -100.5^\circ$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.12}{1 - 0.12} = 1.27$$

41. (C) $\beta d = \dfrac{2\pi}{\lambda} \cdot \dfrac{\lambda}{8} = \dfrac{\pi}{4}$, $\tan\dfrac{\pi}{4} = 1$

$\eta_2 = 260$, $\eta_1 = \eta_3 = \eta_o$

$$\eta_{in} = \eta_2\left(\frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}\right) = 260\left(\frac{377 + j260}{260 + j377}\right)$$

$$= 243 - j92\ \Omega$$

$$\Gamma = \frac{\eta_{in} - \eta_o}{\eta_{in} + \eta_o} = \frac{243 - j92 - 377}{243 - j92 + 377} = 0.26\angle -137^\circ$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1.26}{0.74} = 1.70$$

42. (A) $\omega = 10^8$ rad/s, $\beta = \dfrac{1}{\sqrt{3}}$ rad/m, $v = \dfrac{c}{\sqrt{\varepsilon_r}} = \dfrac{\omega}{\beta}$

$$\Rightarrow \quad \frac{3 \times 10^8}{\sqrt{\varepsilon_r}} = \frac{10^8}{1/\sqrt{3}} \quad \Rightarrow \quad \varepsilon_r = 3.$$

43. (B) $\theta_c = \sin^{-1}\sqrt{\dfrac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \sin^{-1}\sqrt{\dfrac{1}{81}} = 6.38^\circ$

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CHAPTER 8.7

WAVEGUIDES

Statement for Q.1–3:

A 2 cm by 3 cm rectangular waveguide is filled with a dielectric material with $\varepsilon_r = 6$. The waveguide is operating at 20 GHz with TM_{11} mode.

1. The cutoff frequency is

(A) 3.68 GHz (B) 22.09 GHz

(C) 9.02 GHz (D) 16.04 GHz

2. The phase constant is

(A) 816 rad/m (B) 412 rad/m

(C) 1009 rad/m (D) 168 rad/m

3. The phase velocity is

(A) 1.24×10^8 m/s (B) 1.54×10^6 m/s

(C) 3.05×10^8 m/s (D) 7.48×10^8 m/s

4. In an an-filled rectangular wave guide, the cutoff frequency of a TE_{10} mode is 5 GHz where as that of TE_{01} mode is 12 GHz. The dimensions of the guide is

(A) 3 cm by 1.25 cm (B) 1.25 cm by 3 cm

(C) 6 cm by 2.5 cm (D) 2.5 cm by 6 cm

5. Consider a 150 m long air-filled hollow rectangular waveguide with cutoff frequency 6.5 GHz. If a short pulse of 7.2 GHz is introduced into the input end of the guide, the time taken by the pulse to return the input end is

(A) 920 ns (B) 460 ns

(C) 230 ns (D) 430 ns

Statement for Q.6–7:

In an air-filled rectangular waveguide the cutoff frequencies for TM_{11} and TE_{03} modes are both equal to 12 GHz.

6. The dominant mode is

(A) TM_{10} (B) TM_{01}

(C) TE_{01} (D) TE_{10}

7. At dominant mode the cutoff frequency is

(A) 11.4 GHz (B) 4 GHz

(C) 5 GHz (D) 8 GHz

8. For an air-filled rectangular waveguide given that

$$E_z = 10 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12} t - \beta z) \text{ V/m}$$

If the waveguide has cross-sectional dimension $a = 6$ cm and $b = 3$ cm, then the intrinsic impedance of this mode is

(A) 373.2 Ω (B) 378.9 Ω

(C) 375.1 Ω (D) 380.0 Ω

Statement for Q.9–10:

In an air-filled waveguide, a TE mode operating at 6 GHz has

$$E_y = 15 \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \sin(\omega t - 12z) \text{ V/m}$$

9. The cutoff frequency is

(A) 4.189 GHz (B) 5.973 GHz

(C) 8.438 GHz (D) 7.946 GHz

10. The intrinsic impedance is

(A) 35.72 Ω (B) 3978 Ω

(C) 1989 Ω (D) 71.44 Ω

Statement for Q.11–12.

Consider an air-filled rectangular wave guide with $a = 2.286$ cm and $b = 1.016$ cm. The y-component of the TE mode is

$$E_y = 12 \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin\left(10\pi \times 10^{10} t - \beta z\right) \text{ V/m}$$

11. The propagation constant γ is

(A) $j4094.2$ (B) $j400.7$

(C) $j2733.3$ (D) $j276.4$

12. The intrinsic impedance is

(A) 743 Ω (B) 168 Ω

(C) 986 Ω (D) 144 Ω

Statement for Q.13–14:

Consider a air-filled waveguide operating in the TE_{12} mode at a frequency 20% higher than the cutoff frequency.

13. The phase velocity is

(A) 1.66×10^8 m/s (B) 5.42×10^8 m/s

(C) 2.46×10^8 m/s (D) 9.43×10^8 m/s

14. The group velocity is

(A) 1.66×10^8 m/s (B) 4.42×10^8 m/s

(C) 2.46×10^8 m/s (D) 9.43×10^8 m/s

15. A rectangular waveguide is filled with a polyethylene ($\varepsilon_r = 2.25$) and operates at 24 GHz. The cutoff frequency of a certain mode is 16 GHz. The intrinsic impedance of this mode is

(A) 2248 Ω (B) 337.2 Ω

(C) 421.4 Ω (D) 632.2 Ω

16. The cross section of a waveguide is shown in fig. P8.7.16. It has dielectric discontinuity as shown in fig. P8.7.16. If the guide operate at 8 GHz in the dominant mode, the standing wave ratio is

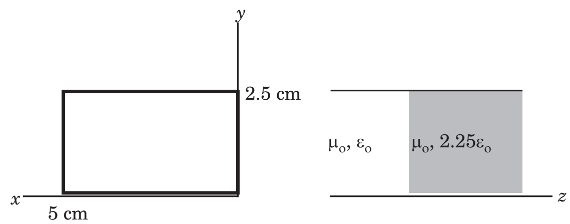

Fig. P8.7.16

(A) –3.911 (B) 2.468

(C) 1.564 (D) 4.389

Statement for Q.17–19:

Consider the rectangular cavity as shown in fig. P8.7.17–19.

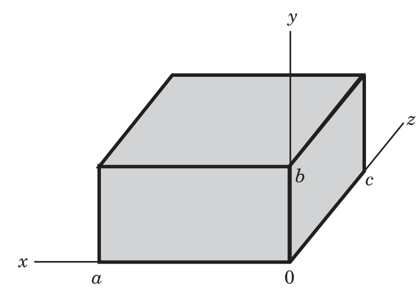

Fig. P8.7.17–19

17. If $a < b < c$, the dominant mode is

(A) TE_{011} (B) TM_{110}

(C) TE_{101} (D) TM_{101}

18. If $a > b > c$, then the dominant mode is

(A) TE_{011} (B) TM_{110}

(C) TE_{101} (D) TM_{101}

19. If $a = c > b$, then the dominant mode is

(A) TE_{011} (B) TM_{110}

(C) TE_{101} (D) TM_{101}

20. The air filled cavity resonator has dimension $a = 3$ cm, $b = 2$ cm, $c = 4$ cm. The resonant frequency for the TM_{110} mode is

(A) 5 GHz (B) 6.4 GHz

(C) 16.2 GHz (D) 9 GHz

frequency, the TM_1 mode propagates through the guide without suffering any reflective loss at the dielectric interface. This frequency is

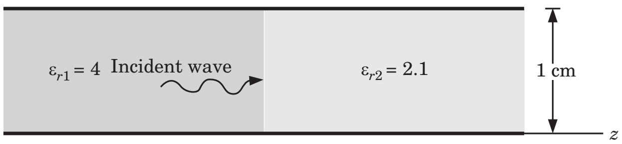

Fig. P8.7.34

(A) 8.6 GHz (B) 12.8 GHz

(C) 4.3 GHz (D) 7.5 GHz

Statement for Q.35–36:

A 6 cm × 4 cm rectangular wave guide is filled with dielectric of refractive index 1.25.

35. The range of frequencies over which single mode operation will occur is

(A) 2.24 GHz $< f <$ 3.33 GHz

(B) 2 GHz $< f <$ 3 GHz

(C) 4.48 GHz $< f <$7.70 GHz

(D) 4 GHz $< f <$ 6 GHz

36. The range of frequencies, over which guide support both TE_{10} and TE_{01} modes and no other, is

(A) 3.35 GHz $< f <$ GHz

(B) 2.5 GHz $< f <$ 3.6 GHz

(C) 3 GHz $< f <$ 3.6 GHz

(D) 2.5 GHz $< f <$ 4.02 GHz

37. Two identical rectangular waveguide are joined end to end where $a = 2b$. One guide is air filled and other is filled with a lossless dielectric of ε_r . it is found that up to a certain frequency single mode operation can be simultaneously ensured in both guide. For this frequency range, the maximum allowable value of ε_r is

(A) 4 (B) 2

(C) 1 (D) 6

38. A parallel-plate guide operates in the TEM mode only over the frequency range $0 < f < 3\,\text{GHz}$. The dielectric between the plates is teflon ($\varepsilon_r = 2.1$). The maximum allowable plate separation b is

(A) 3.4 cm (B) 6.8 cm

(C) 4.3 cm (D) 8.6 cm

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SOLUTIONS

1. (A) $f_c = \dfrac{v}{2}\sqrt{\left(\dfrac{m}{a}\right)^2 + \left(\dfrac{n}{b}\right)^2}$

$= \dfrac{3\times 10^8}{2\sqrt{6\times 10^{-2}}}\sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{3}\right)^2} = 3.68 \text{ GHz}$

2. (C) $\beta_p = \beta\sqrt{1-\left(\dfrac{f_c}{f}\right)^2} = \dfrac{\omega}{v}\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}$

$\Rightarrow \quad \beta_p = \dfrac{2\pi\times 20\times 10^9\sqrt{6}}{3\times 10^8}\sqrt{1-\left(\dfrac{3.68}{20}\right)^2} = 1009 \text{ rad/m}$

3. (A) $v_p = \dfrac{\omega}{\beta_p} = \dfrac{2\pi\times 20\times 10^9}{1009} = 1.24\times 10^8 \text{ m/s}$

4. (A) For TE_{10} mode $f_c = \dfrac{v}{2a}$,

$a = \dfrac{v}{2f_c} = \dfrac{3\times 10^8}{2\times 5\times 10^9} = 3 \text{ cm}$

For TE_{01} mode $f_c = \dfrac{v}{2b}$,

$b = \dfrac{v}{2f_c} = \dfrac{3\times 10^8}{2\times 12\times 10^9} = 1.25 \text{ cm}$

5. (D) $v = \dfrac{c}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}} = \dfrac{3\times 10^8}{\sqrt{1-\left(\dfrac{6.5}{7.2}\right)^2}} = 6.975\times 10^8 \text{ ms}$

$t = \dfrac{2l}{v} = \dfrac{2\times 150}{6.975\times 10^8} = 430 \text{ ns}$

6. (C) $12\times 10^9 = \dfrac{3\times 10^8}{2}\sqrt{0+\left(\dfrac{3}{b}\right)^2} \quad\Rightarrow\quad b = 3.75 \text{ cm}$

$1.2\times 10^9 = \dfrac{3\times 10^8}{2}\sqrt{\left(\dfrac{1}{a}\right)^2 + \left(\dfrac{1}{3.75\times 10^{-2}}\right)^2} \Rightarrow a = 1.32 \text{ cm}$

Since $a < b$, the dominant mode is TE_{01} .

7. (B) $f_{c01} = \dfrac{v}{2b} = \dfrac{3\times 10^8}{2\times 3.75\times 10^{-2}} = 4 \text{ GHz}$

8. (C) $E_z \neq 0$, this must be TM_{23} mode $(m = 2,\ n = 3)$

$f_c = \dfrac{3\times 10^8}{2\times 10^{-2}}\sqrt{\left(\dfrac{2}{6}\right)^2 + \left(\dfrac{3}{3}\right)^2} = 15.81 \text{ GHz}$

$f = \dfrac{\omega}{2\pi} = \dfrac{10^{12}}{2\pi} = 159.2 \text{ GHz}$

$$\eta_{TM} = 377\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 377\sqrt{1-\left(\frac{15.81}{159.2}\right)^2} = 375.1\,\Omega$$

9. (B) $m = 2,\ n = 1,\ \beta_p = 12,\ f = 6$ GHz

$$\beta_p = \frac{\omega}{v}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \Rightarrow 12 = \frac{2\pi\times 6\times 10^9}{3\times 10^8}\sqrt{1-\left(\frac{f_c}{6}\right)^2}$$

$\Rightarrow\ f_c = 5.973$ GHz

10. (B) $\eta_{TE} = \dfrac{377}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{377}{\sqrt{1-\left(\frac{5.973}{6}\right)^2}} = 3978\ \Omega$

11. (B) $m = 2,\ n = 3,$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times 10^8}{2\times 10^{-2}}\sqrt{\left(\frac{2}{2.286}\right)^2+\left(\frac{3}{1.016}\right)^2}$$

$= 46.2$ GHz

$$f = \frac{10\pi\times 10^{10}}{2\pi} = 50\ \text{GHz}$$

$$\beta_p = \frac{\omega}{v}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi\times 50\times 10^9}{3\times 10^8}\sqrt{1-\left(\frac{46.2}{50}\right)^2}$$

$= 400.7\ \text{m}^{-1},\ \ \gamma = j\beta_p = j\,400.7$

12. (C) $\eta_{TE} = \dfrac{377}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{377}{\sqrt{1-\left(\frac{46.2}{50}\right)^2}} = 986\ \Omega$

13. (A) $v = c,\ f = 1.2 f_c$

$$\boldsymbol{v_p} = \frac{\boldsymbol{v}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\mathbf{3\times 10^8}}{\sqrt{1-\left(\frac{f_c}{1.2 f_c}\right)^2}} = \mathbf{5.42\times 10^8}\ \text{m/s}$$

14. (A) $v_g = v\sqrt{1-\left(\frac{f_c}{f}\right)^2} = c\sqrt{1-\left(\frac{f_c}{1.2 f_c}\right)^2} = 1.66\times 10^8$ m/s

15. (B) $\eta = \dfrac{377}{\sqrt{\varepsilon_r}} = \dfrac{377}{1.5} = 251.33\ \Omega$

$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{251.33}{\sqrt{1-\left(\frac{16}{24}\right)^2}} = 337.2\ \Omega$$

16. (C) Since $a > b$, the dominant mode is TE_{10} .

In free space $f_c = \dfrac{c}{2a} = \dfrac{3\times 10^8}{2\times 0.05} = 3$ GHz

$$\eta_1 = \frac{\eta_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{3}{8}\right)^2}} = 406.7\ \Omega$$

In dielectric medium

$$f_c = \frac{c}{2a\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{2\times 0.05\sqrt{2.25}} = 2\ \text{GHz}$$

$$\eta = \frac{\eta_o}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.25}} = 251.33\,\Omega,\ \ \eta_2 = \frac{251.33}{\sqrt{1-\left(\frac{2}{8}\right)^2}} = 259.23\,\Omega$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{259.23-406.7}{259.23+406.7} = -0.22$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.22}{1-0.22} = 1.564$$

17. (A) $f_r = \dfrac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}$

where for TM mode to z

$m = 1,\ 2,\ 3......$,

$n = 1,\ 2,\ 3......$,

$p = 0,\ 1,\ 2$ ......

For TE mode to z

$m = 1,\ 2,\ 3.....,$

$n = 1,\ 2,\ 3.....$

$p = 1,\ 2,\ 3......$,

if $a < b < c$, then $\dfrac{1}{a} > \dfrac{1}{b} > \dfrac{1}{c}$

The lowest TM mode is TM_{110} with

$$f_{r1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{011} with

$$f_{r2} = \frac{v}{2}\sqrt{\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$$

$f_{r2} > f_{r1}$, Hence the dominant mode is TE_{011}

18. (B) If $a > b > c$ then $\dfrac{1}{a} < \dfrac{1}{b} < \dfrac{1}{c}$

The lowest TM mode is TM_{110} with

$$f_{r1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{101} with

$$f_{r2} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{c}\right)^2}$$

$f_{r2} > f_{r1}$ Hence the dominant mode is TM_{110} .

19. (C) If $a = c > b$, then $\frac{1}{a} = \frac{1}{c} < \frac{1}{b}$

The lowest TM mode is TM_{110} with

$$f_{r1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{101} with

$$f_{r2} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}$$

$f_{r_2} < f_{r_1}$ Hence the dominant mode is TE_{101}.

20. (D) $f_r = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$

$$= \frac{3\times 10^8}{2\times 0.01}\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{2}\right)^2} = 9 \text{ GHz}$$

21. (A) $m = n = 1$, $p = 0$, $a = b = c$, $f_r = 2$ Ghz,

$$f_r = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$2\times 10^9 = \frac{3\times 10^8}{2}\frac{\sqrt{2}}{a} \Rightarrow a = 10.6 \text{ cm}$$

22. (A) $f_c = \frac{mc}{2b\sqrt{\varepsilon_r}} = \frac{2\times 3\times 10^8}{2\times 0.01\sqrt{\varepsilon_r}} = 10\times 10^9 \Rightarrow \varepsilon_r = 9$

23. (A) For a propagating mode $f > f_{cm}$,

$$f_{cm} = \frac{mc}{2b\sqrt{\varepsilon_r}}, \quad f > \frac{mc}{2b\sqrt{\varepsilon_r}} \Rightarrow m < \frac{2fb\sqrt{\varepsilon_r}}{c}$$

$$m < \frac{2\times 30\times 10^9 \times 0.01\sqrt{2.5}}{3\times 10^8} \Rightarrow m < 3.16$$

The maximum allowed m is 3. The propagating mode will be TM_1, TE_1, TM_2, TE_2, TM_3, TE_3 and TEM Thus total 7 modes.

24. (B) $f_{cm} = \frac{mc}{2b\sqrt{\varepsilon_r}}$, $f_{c2} = 2f_{c1} = 15$ GHz

$$f = \frac{c}{\lambda} = \frac{3\times 10^8}{0.015} = 20 \text{ GHz}$$

$$v_{g2} = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 3\times 10^8\sqrt{1 - \left(\frac{15}{20}\right)^2} = 2\times 10^8 \text{ m/s}$$

s

25. (A) $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$, $\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$

Mode	TE_{10}	TE_{01}	TE_{11}	TE_{20}
λ_c(cm)	14.4	6.8	6.15	7.21

$\lambda > \lambda_c$. Hence TE_{10} mode can be used.

26. (C) Let $a = kb$, $1 < k < 2$

$$f_{cmn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{1.5\times 10^8}{a}\sqrt{m^2 + k^2n^2}$$

Dominant mode is TE_{10}, $f_{c10} = \frac{1.5\times 10^8}{a}$

$$3 \text{ GHz} > 1.2f_c \Rightarrow 3\times 10^9 > \frac{1.2\times 1.5\times 10^8}{a}$$

$\Rightarrow a > 6$ cm

The next higher mode is TE_{01}, $f_{c01} = \frac{1.5\times 10^8}{b}$,

$$3 \text{ GHz} < 0.8f_{c01} \Rightarrow 3\times 10^9 < \frac{0.8\times 1.5\times 10^8}{b}$$

$\Rightarrow b < 4$ cm, Thus (C) is correct option.

27. (C) $f_c = \frac{c}{2a} = \frac{3\times 10^8}{2\times 0.065} = 2.3$ GHz

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{3\times 10^8}{\sqrt{1 - \left(\frac{2.3}{3}\right)^2}} = 4.7\times 10^8 \text{ m/s}$$

28. (B) For TE_{10} mode

$$R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}} = \sqrt{\frac{\pi\times 9\times 10^9\times 4\pi\times 10^{-7}}{1.1\times 10^7}} = 0.0568$$

$$\alpha_c = \frac{R_s\left(1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right)}{b\eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{0.0568\left(1 + \frac{2\times 1.5}{2.4}\left(\frac{3.876}{9}\right)^2\right)}{1.5\times 10^{-2}\times 233.8\sqrt{1 - \left(\frac{3.876}{9}\right)^2}}$$

$= 0.022$

29. (B) $\frac{\sigma_d}{\omega\varepsilon} = \frac{10^{-15}}{2\pi\times 9\times 10^9\times 2.6\times 8.85\times 10^{-12}} = \frac{10^{-15}}{1.3}$

$\frac{\sigma_d}{\omega\varepsilon} << 1$, hence $v \approx \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{2.6}}$, $\eta \approx \frac{377}{\sqrt{2.6}} = 233.8$

$$f_c = \frac{3\times 10^8}{2\times 2.4\times 10^{-2}\sqrt{2.6}} = 3.876 \text{ GHz}$$

$$\alpha_d = \frac{\sigma_d\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{10^{-15}\times 233.8}{2\sqrt{1 - \left(\frac{3.876}{9}\right)^2}} = 1.3\times 10^{-13} \text{ Np/m}$$

30. (D) Dominant mode is TE_{10} mode

$$f_c = \frac{c}{2a} = \frac{3\times 10^8}{2\times 0.072} = 2.08 \text{ GHz}$$

$$R_s = \sqrt{\frac{\pi f \mu_o}{\sigma_c}} = \sqrt{\frac{\pi\times 3\times 10^9\times 4\pi\times 10^{-7}}{5.8\times 10^7}} = 1.429\times 10^{-2}\ \Omega$$

For TE_{10} mode

$$\alpha_c = \frac{R_s\left(1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right)}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{1.429\times 10^{-2}\left(1+\frac{2\times 3.4}{7.2}\left(\frac{2.08}{3}\right)\right)}{377\times 0.034\sqrt{1-\left(\frac{2.08}{3}\right)^2}}$$

$$= 2.25\times 10^{-3}\ \text{Np/m}$$

$$e^{-\alpha_c z} = \frac{1}{2} \Rightarrow z = \frac{1}{\alpha_c}\ln 2 = 308 \text{ m}$$

31. (B)

$$f_{cmn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times 10^8}{2\times 0.01}\sqrt{\left(\frac{m}{8}\right)^2+\left(\frac{n}{10}\right)^2}$$

$$= 15\sqrt{\left(\frac{m}{8}\right)^2+\left(\frac{n}{10}\right)^2}\ \text{GHz}$$

$f_{c10} = 1.875$ GHz

$f_{c01} = 1.5$ GHz, $f_{c11} = 2.4$ GHz

$f_{c20} = 3.75$GHz , $f_{c02} = 3$ GHz,

$f_{c21} = 4.04$ GHz, $f_{c12} = 3.54$ GHz,

$f_{c30} = 5.625$ GHz , $f_{c03} = 4.5$ GHz

If $f_c < f$, then mode will be transmit. Hence six mode will be transmitted.

32. (C) For dominant mode $(m = 1, n = 0)$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times 10^8}{2\times 0.04} = 3.75 \text{ GHz}$$

Since given frequency is below the cutoff frequency, 3 GHz will not be propagated and get attenuated

$$\gamma = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\left(\frac{\omega}{\nu}\right)^2}$$

$\beta = 0$, Since wave is attenuated,

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2-\left(\frac{\omega}{c}\right)^2} = \sqrt{\left(\frac{\pi}{0.04}\right)^2-\left(\frac{2\pi\times 3\times 10^9}{3\times 10^8}\right)^2} = 47.1$$

33. (B) $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$$f_{c10} = \frac{c}{2a} = \frac{3\times 10^8}{2\times 0.08} = 1.875 \text{ GHz}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\left(\frac{\omega}{\nu}\right)^2} = \sqrt{\left(\frac{\pi}{0.08}\right)^2-\left(\frac{2\pi\times 1.5\times 1.875}{3\times 10^8}\right)^2}$$

$$= j43.9$$

34. (B) The ray angle is such that the wave is interface at Brewster's angle $\theta_B = \tan^{-1}\sqrt{\frac{2.1}{4}} = 35.9°$.

The ray angle $\theta = 90° - 35.9° = 54.1°$

$$f_{c1} = \frac{c}{2b\sqrt{\varepsilon_{r1}}} = \frac{3\times 10^{10}}{2\times 1\times 2} = 7.5 \text{ GHz}$$

$$f = \frac{f_{c1}}{\cos\theta} = \frac{7.5}{\cos 54.1°} = 12.8 \text{ GHz}$$

35. (A) $f_c = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$$f_{c10} = \frac{c}{a2\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{2\times 1.25\times 0.06} = 2 \text{ GHz}$$

$$f_{c01} = \frac{c}{b2\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{2\times 1.25\times 0.04} = 3 \text{ GHz}$$

2 GHz $< f <$ 3 GHz

36. (C) $f_{c11} = \frac{3\times 10^8}{2\times 1.25\times 10^{-2}}\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{4}\right)^2} = 3.6$ GHz

3 GHz $< f <$ 3.6 GHz

37. (A) $f_c = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{2b}\right)^2+\left(\frac{n}{b}\right)^2}$, In guide 1 $\varepsilon_r = 1$

lowest cutoff frequency $f_{c10} = \frac{c}{2(2b)}$

Next lowest cutoff frequency $f_{c20} = \frac{c}{2b}$

In guide 2 lowest cutoff frequency $f'_{c10} = \frac{c}{2\sqrt{\varepsilon_{r2}}(2b)}$

Next lowest cutoff frequency $f'_{c20} = \frac{c}{2\sqrt{\varepsilon_{r2}}(b)}$

For single mode $f'_{c10} < f < f'_{c10}$

$$\Rightarrow \frac{c}{2(2b)} < f < \frac{c}{2\sqrt{\varepsilon_r}(b)} \Rightarrow \varepsilon_r < 4$$

38. (A) $f < f_c \Rightarrow f < \frac{v}{2b} = \frac{3\times 10^8}{2\times b\times\sqrt{2.1}}$

$$\Rightarrow 3\times 10^9 < \frac{3\times 10^8}{2\times b\times\sqrt{2.1}} \Rightarrow b < 3.4 \text{ cm}$$

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CHAPTER

8.8

ANTENNAS

1. A Hertizian dipole at the origin in free space has $dl = 10$ cm and $I = 20\cos(2\pi\times 10^7 t)$ A. The $|E|$ at the distant point (100, 0, 0) is

(A) 0.252 V/m (B) 0.126 V/m

(C) 0.04 V/m (D) 0.08 V/m

Statement for Q.2–3:

A 25 A source operating at 300 MHz feeds a Hertizian dipole of length 4 mm situated at the origin. Consider the point P(10, 30°, 90°).

2. The H at point P is

(A) $j0.25$ mA/m (B) 94.25 mA/m

(C) $j0.5$ mA/m (D) 188.5 mA/m

3. The E at point P is

(A) $j0.25$ mV/m (B) $j0.5$ mV/m

(C) $j94.25$ mV/m (D) $j188.5$ mV/m

4. An antenna can be modeled as an electric dipole of length 4 m at 3 MHz. If current is uniform over its length, then radiation resistance of the antenna is

(A) 1.974 Ω (B) 1.263 Ω

(C) 2.186 Ω (D) 2.693 Ω

Statement for Q.5–6:

A antenna located on the surface of a flat earth transmit an average power of 150 kW. Assume that all the power is radiated uniformly over the surface of hemisphere with the antenna at the center.

5. The time-average poynting vector at 50 km is

(A) $6.36\mathbf{u}_r\ \mu\text{W/m}^2$ (B) $4.78\mathbf{u}_r\ \mu\text{W/m}^2$

(C) $9.55\mathbf{u}_r\ \mu\text{W/m}^2$ (D) $12.73\mathbf{u}_r\ \mu\text{W/m}^2$

6. The maximum electric field at that location is

(A) 24 mV/m (B) 85 mV/m

(C) 109 mV/m (D) 12 mV/m

7. In free space, an antenna has a far-zone field given by $\mathbf{E} = \frac{1}{r}10\sin 2\theta\, e^{-j\beta r}\mathbf{u}_\theta$ V/m. The radiated power is

(A) 0.23 W (B) 0.89 W

(C) 1.68 W (D) 1.23 W

8. At the far field, an antenna produces $\mathbf{P}_{ave} = \frac{1}{r^2}\cos\theta\cos\phi\,\mathbf{u}_r$ W/m², where $0 < \theta < \pi$ and $0 < \phi < \frac{\pi}{2}$. The directive gain of the antenna is

(A) $\cos\theta\cos\phi$ (B) $2\sin\theta\cos\phi$

(C) $8\cos\theta\sin\phi$ (D) $8\sin\theta\cos\phi$

Statement for Q.9–10:

The radiation intensity of antennas has been given. Determine the directivity of antenna.

9. $U(\theta,\phi) = \sin^2\theta,\ \ 0 < \theta < \pi,\ \ 0 < \phi < 2\pi$

(A) 1.875 (B) 2.468

(C) 3.943 (D) 6.743

10. $U(\theta,\phi) = 4\sin^2\theta\sin^2\phi,\ \ 0 < \theta < \pi,\ \ 0 < \phi < \pi$

(A) 15 (B) 12

(C) 3 (D) 6

11. The radiation intensity of a antenna is given by $U(\theta, \phi) = 8\sin^2\theta\cos^2\phi$, where $0 < \theta < \pi$ and $0 < \phi < \pi$. The directive gain is

(A) $6\sin^2\theta\cos^2\phi$ (B) $3\sin^2\theta\cos^2\phi$
(C) $3\sin^2\phi\cos^2\theta$ (D) $6\sin^2\phi\cos^2\theta$

Statement for Q.12–13:

At the far field, an antenna radiates a field

$$E_\phi = \frac{0.4\cos^2\theta}{4\pi r} e^{-j\beta r} \text{ kV/m}$$

12. The total radiated power is

(A) 1.36 W (B) 2.14 W
(C) 0.844 W (D) 3.38 W

13. The directive gain at $\theta = \pi/3$ is

(A) 0.3125 (B) 0.625
(C) 1.963 (D) 3.927

14. An antenna has directivity of 100 and operates at 150 MHz. The maximum effective aperture is

(A) 31.8 m^2 (B) 62.4 m^2
(C) 26.4 m^2 (D) 13.2 m^2

15. Two half wave dipole antenna are operated at 100 MHz and separated by 1 km. If 100 W is transmitted by one, the power received by the other is $(D = 1.68)$

(A) 12 μW (B) 10 mW
(C) 18 mW (D) 16 μW

16. The electric field strength impressed on a half wave dipole is 6 mV/m at 60 MHz. The maximum power received by the antenna is $(D = 1.68)$

(A) 159 nW (B) 230 nW
(C) 196 μW (D) 318 μW

17. The power transmitted by a synchronous orbit satellite antenna is 480 W. The antenna has a gain of 40 dB at 15 GHz. The earth station is located at distance of 24, 567 km. If the antenna of earth station has a gain of 32 dB, the power received is

(A) 32 pW (B) 3.2 fW
(C) 10.2 pW (D) 1.3 fW

18. The directive gain of an antenna is 36 dB. If the antenna radiates 15 kW at a distance of 60 km, the time average power density at that distance is

(A) 9.42 $\mu W/m^2$ (B) 6.83 mW/m^2
(C) 1.32 mW/m^2 (D) 10.46 mW/m^2

19. Two identical antenna separated by 12 m are oriented for maximum directive gain. At a frequency of 5 GHz, the power received by one is 30 dB down from the transmitted by the other. The gain of antenna is

(A) 22 dB (B) 16 dB
(C) 19 dB (D) 13 dB

Statement for Q.20–21:

An L-band pulse radar has common transmitting and receiving antenna. The antenna having directive gain of 36 dB operates at 1.5 GHz and transmits 200 kW. The object is 120 km from the radar and its scattering cross section is 8 m^2.

20. The magnitude of the incident electric field intensity of the object is

(A) 1.82 V/m (B) 2.46 V/m
(C) 0.34 V/m (D) 0.17 V/m

21. The magnitude of the scattered electric field at the radar is

(A) 18 μW (B) 12 μW
(C) 17 mW (D) 126 mW

22. A transmitting antenna with a 300 MHz carrier frequency produces 2 kW of power. If both antennas has unity power gain, the power received by another antenna at a distance of 1 km is

(A) 11.8 mW (B) 18.4 mW
(C) 18.4 μW (D) 12.7 μW

23. A bistatic radar system shown in fig. P8.7.23 has following parameters: $f = 5$ GHz, $G_{dt} = 34$ dB, $G_{dr} = 22$ dB. To obtain a return power of 8 pW the minimum necessary radiated power is

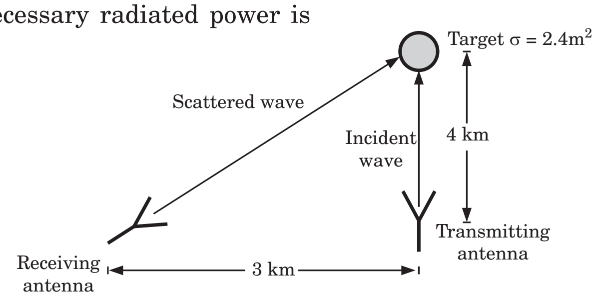

Fig. P8.7.23

(A) 1.394 kW (B) 2.046 kW
(C) 1.038 kW (D) 3.46 kW

24. The radiation resistance of an antenna is 63 Ω and loss resistance 7 Ω. If antenna has power gain of 16, then directivity is

(A) 48.26 dB (B) 12.5 dB
(C) 38.96 dB (D) 24.7 dB

25. An antenna is desired to operate on a frequency of 40 MHz whose quality factor is 50. The bandwidth of antenna is

(A) 5.03 MHz (B) 800 kHz
(C) 127 kHz (D) None of the above

26. A thin dipole antenna is $\lambda/15$ long. If its loss resistance is 1.2 Ω, the efficiency is

(A) 41.1% (B) 59%
(C) 74.5% (D) 25.5%

Statement for Q.27–29:

An array comprises of two dipoles that are separated by the wavelength. The dipoles are fed by currents of the same magnitude and phase.

27. The array factor is

(A) $2\cos(\pi\cos\theta + 45°)$ (B) $2\cos(\pi\sin\theta)$
(C) $2\cos(\pi\sin\theta + 45°)$ (D) $2\cos(\pi\cos\theta)$

28. The nulls of the pattern occur when θ is

(A) 30°, 150° (B) 60°, 120°
(C) 45°, 135° (D) 0, 180°

29. The maximum of the pattern occur at

(A) $\theta = 45°,\ 135°$ (B) $\theta = 0,\ 90°,\ 180°$
(C) $\theta = 30°,\ 150°$ (D) $\theta = 60°,\ 150°$

30. An array comprises two dipoles that are separated by half wavelength. If the dipoles are fed by currents, that are 180° out of phase with each other, then array factor is

(A) $\sin\left(\frac{\pi}{4}\cos\theta + \frac{\pi}{4}\right)$ (B) $\cos\left(\frac{\pi}{4}\cos\theta + \frac{\pi}{2}\right)$
(C) $\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)$ (D) $\sin\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)$

31. An antenna consists of 4 identical Hertizian dipoles uniformly located along the z–axis and polarized in the z-direction. The spacing between the dipole is $\lambda/4$. The group pattern function is

(A) $4\cos\left(\frac{\pi}{4}\cos\theta\right)\cos\left(\frac{\pi}{2}\cos\theta\right)$
(B) $4\cos\left(\frac{\pi}{4}\cos\theta\right)\cos\left(\frac{\pi}{8}\cos\theta\right)$
(C) $4\cos\left(\frac{\pi}{4}\cos\theta\right)\sin\left(\frac{\pi}{2}\cos\theta\right)$
(D) $4\cos\left(\frac{\pi}{4}\cos\theta\right)\sin\left(\frac{\pi}{8}\cos\theta\right)$

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SOLUTIONS

1. (B) $\beta = \frac{\omega}{c} = \frac{2\pi \times 10^7}{3\times 10^8} = \frac{2\pi}{30}$

At far field $|E_\theta| = \frac{\eta I_o \beta dl}{4\pi r}\sin\theta$

$\eta = 120\pi = 377,\ I_o = 20,\ dl = 10$ cm

At (100 cm, 0, 0), $\theta = \frac{\pi}{2}$

$$|E_\theta| = \frac{120\pi \times 20 \times 0.1}{4\pi \times 100} \times \frac{2\pi}{30} = 0.126 \text{ V/m}$$

2. (A) $\beta = \frac{\omega}{c} = \frac{2\pi \times 300 \times 10^6}{3\times 10^8} = 2\pi$

$r = 10\,\text{m},\ \theta = 30°,\ \phi = 90°$

At far field $H = H_\phi = \frac{jI_o\beta dl}{4\pi r}\sin\theta\, e^{-j\beta r}$

$$\Rightarrow\quad H_\phi = \frac{j(2.5)(2\pi)(4\times 10^{-3})}{4\pi(10)} e^{-j2\pi 10} = j0.25 \text{ mA/m}$$

3. (C) $E = E_\theta = \eta H_\phi = 377 H_\phi = j94.25$ mV/m

4. (B) $\lambda = \frac{c}{f} = \frac{3\times 10^8}{3\times 10^6} = 100$

$\frac{dl}{\lambda} = \frac{4}{100} = \frac{1}{25} < \frac{1}{10}$

$$R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 = \frac{80\pi^2}{625} = 1.263\ \Omega$$

5. (C) $P_{rad} = \int P_{ave}\cdot dS = P_{ave}\cdot 2\pi r^2$

$$P_{ave} = \frac{P_{rad}}{2\pi r^2} = \frac{150\times 10^3}{2\pi(50\times 10^3)^2} = 9.55\ \mu\text{W/m}^2$$

$\mathbf{P} = 9.55\mathbf{u}_r\ \mu\text{W/m}^2$

6. (B) $P_{ave} = \frac{(E_{max})^2}{2\eta} \quad\Rightarrow\quad E_{max} = \sqrt{2\eta P_{ave}}$

$$\Rightarrow\quad E_{max} = \sqrt{2\times 377\times 9.55\times 10^{-6}} = 85 \text{ mV/m}$$

7. (B) $P_{ave} = \frac{|E|^2}{2\eta}$

$$P_{rad} = \iint \frac{100\sin^2 2\theta}{2\eta}\sin\theta\, d\theta\, d\phi$$

$$= \frac{100}{2\times 120\pi}\int_0^\pi (2\pi)(2\sin\theta\cos\theta)^2\sin\theta\, d\theta$$

$$= \frac{10}{3}\int_0^\pi \sin^3\theta\cos^2\theta d\theta = 0.89 \text{ W}$$

$$G_d = \frac{4\pi(12)}{0.06}\sqrt{\frac{1}{10^3}} = 79.48 = 19 \text{ dB}$$

20. (A) $G_d = 36 \text{ dB} = 3981$

$$G_d = \frac{4\pi r^2 P_i}{P_{rad}} \quad \Rightarrow \quad P_i = \frac{G_d P_{rad}}{4\pi r^2} = \frac{|E|^2}{2\eta}$$

$$\Rightarrow \quad |E| = \sqrt{\frac{(240\pi)\, G_d P_{rad}}{4\pi r^2}}$$

$$= \sqrt{\frac{(60)(3981)(200\times 10^3)}{(120\times 10^3)^2}}$$

$$= 1.82 \text{ V/m}$$

21. (B) $|E_s| = \sqrt{\frac{|E_r|^2 \sigma}{4\pi r^2}} = \frac{|E_i|}{r}\sqrt{\frac{\sigma}{4\pi}}$

$$= \frac{1.82}{120\times 10^3}\sqrt{\frac{8}{4\pi}} = 12 \ \mu\text{W}$$

22. (D) $\lambda = \frac{c}{f} = \frac{3\times 10^8}{300\times 10^6} = 1 \text{ m}$

$$P_r = G_{dr}G_{dt}\left(\frac{\lambda}{4\pi r}\right)^2 P = (1)(1)\left(\frac{1}{4\pi 10^3}\right)^2 2000 = 12.7 \ \mu\text{W}$$

23. (C) $P_r = \frac{G_{dr}G_{dt}}{4\pi}\left(\frac{\lambda}{4\pi r_1 r_2}\right)^2 \sigma P_{rad}$

$G_{dt} = 34 \text{ dB} = 2512,\ G_{dr} = 22 \text{ dB} = 158.5$

$r_1 = 3 \text{ km},\ r_2 = \sqrt{3^2 + 4^2} = 5 \text{ km}$

$$\lambda = \frac{3\times 10^8}{5\times 10^9} = 0.06 \text{ m},\ P_r = 8 \text{ pW}$$

$$8\times 10^{-12} = \frac{(2512)(158.5)}{4\pi}\left(\frac{0.06}{4\pi(3\text{k})(5\text{k})}\right)^2 (2.4) P_{rad}$$

$$\Rightarrow \quad P_{rad} = 1.038 \text{ kW}$$

24. (B) Efficiency $= \frac{63}{63+7} = 0.9$

$$D = \frac{Gain}{Efficiency} = \frac{16}{0.9} = 17.78 = 12.5 \text{ dB}$$

25. (B) $BW = \frac{f}{Q} = \frac{40\times 10^6}{50} = 800 \text{ kHz}$

26. (C) Radiation resistance $R_r = 80\pi^2\left(\frac{dl}{l}\right)^2$

$$= 80\times\pi^2\times\left(\frac{1}{15}\right)^2 = 3.51\ \Omega$$

$$\text{Efficiency} = \frac{R_r}{R_r + R_L} = \frac{3.51}{3.51 + 1.2} = 74.5\ \%$$

27. (D) $\beta d = \frac{2\pi}{\lambda}\lambda = 2\pi,\ \alpha = 0$

$$AF = 2\cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right) = 2\cos(\pi\cos\theta) = 2\cos(\pi\cos\theta)$$

28. (B) $\cos(\pi\cos\theta) = 0$

$$\Rightarrow \qquad \pi\cos\theta = \pm\frac{\pi}{2},\ \pm\frac{3\pi}{2}$$

$$\cos\theta = \pm\frac{1}{2} \qquad \Rightarrow \qquad \theta = 60°,\ 120°$$

29. (B) Maxima occur when $\frac{d(AF)}{d\theta} = 0$

$$\sin(\pi\cos\theta)\,\pi\sin\theta = 0 \quad \Rightarrow \quad \theta = 0, 90°,\ 180°$$

30. (B) $\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi,\quad \alpha = \pi$

$$AF = 2\cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right) = \cos\left(\frac{\pi}{4}\cos\theta + \frac{\pi}{2}\right)$$

31. (A) $(AF)_N = \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$

$\psi = \beta d\cos\theta + \alpha,\ N = 4$

$$\frac{\sin 4x}{\sin x} = \frac{2\sin 2x\cos 2x}{\sin x} = 4\cos x\cos 2x$$

$$\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2},\ \alpha = 0,\ \frac{\psi}{2} = \frac{\pi}{4}\cos\theta$$

$$AF = 4\cos\left(\frac{\pi}{4}\cos\theta\right)\cos\left(\frac{\pi}{2}\cos\theta\right).$$

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CHAPTER 9.1

LINEAR ALGEBRA

1. If $\mathbf{A} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -2 & \lambda \end{bmatrix}$ is a singular matrix, then λ is

(A) 0 (B) –2

(C) 2 (D) –1

2. If **A** and **B** are square matrices of order 4×4 such that $\mathbf{A} = 5\mathbf{B}$ and $|\mathbf{A}| = \alpha \cdot |\mathbf{B}|$, then α is

(A) 5 (B) 25

(C) 625 (D) None of these

3. If **A** and **B** are square matrices of the same order such that $\mathbf{AB} = \mathbf{A}$ and $\mathbf{BA} = \mathbf{A}$, then **A** and **B** are both

(A) Singular (B) Idempotent

(C) Involutory (D) None of these

4. The matrix, $\mathbf{A} = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is

(A) Idempotent (B) Involutory

(C) Singular (D) None of these

5. Every diagonal element of a skew–symmetric matrix is

(A) 1 (B) 0

(C) Purely real (D) None of these

6. The matrix, $\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

(A) Orthogonal (B) Idempotent

(C) Unitary (D) None of these

7. Every diagonal elements of a Hermitian matrix is

(A) Purely real (B) 0

(C) Purely imaginary (D) 1

8. Every diagonal element of a Skew–Hermitian matrix is

(A) Purely real (B) 0

(C) Purely imaginary (D) 1

9. If **A** is Hermitian, then $i\mathbf{A}$ is

(A) Symmetric (B) Skew–symmetric

(C) Hermitian (D) Skew–Hermitian

10. If **A** is Skew–Hermitian, then $i\mathbf{A}$ is

(A) Symmetric (B) Skew–symmetric

(C) Hermitian (D) Skew–Hermitian.

11. If $\mathbf{A} = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then adj. **A** is equal to

(A) **A** (B) $\mathbf{c}^t$

(C) $3\mathbf{A}^t$ (D) $3\mathbf{A}$

12. The inverse of the matrix $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$ is

(A) $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}$ (D) None of these

13. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix}$, then $\mathbf{A}^{-1}$ is equal to

(A) $\frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ (B) $\frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ (D) None of these

14. If the rank of the matrix, $\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{bmatrix}$ is 2, then the value of λ is

(A) –13 (B) 13

(C) 3 (D) None of these

15. Let **A** and **B** be non–singular square matrices of the same order. Consider the following statements.

(I) $(\mathbf{AB})^T = \mathbf{A}^T\mathbf{B}^T$ (II) $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

(III) $\text{adj}(\mathbf{AB}) = (\text{adj}.\mathbf{A})(\text{adj}.\mathbf{B})$ (IV) $\rho(\mathbf{AB}) = \rho(\mathbf{A})\rho(\mathbf{B})$

(V) $|\mathbf{AB}| = |\mathbf{A}| \cdot |\mathbf{B}|$

Which of the above statements are false ?

(A) I, III & IV (B) IV & V

(C) I & II (D) All the above

16. The rank of the matrix $\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$ is

(A) 3 (B) 2

(C) 1 (D) None of these

17. The system of equations $3x - y + z = 0$, $15x - 6y + 5z = 0$, $\lambda x - 2y + 2z = 0$ has a non–zero solution, if λ is

(A) 6 (B) -6

(C) 2 (D) -2

18. The system of equation $x - 2y + z = 0$, $2x - y + 3z = 0$, $\lambda x + y - z = 0$ has the trivial solution as the only solution, if λ is

(A) $\lambda \neq -\frac{4}{5}$ (B) $\lambda = \frac{4}{3}$

(C) $\lambda \neq 2$ (D) None of these

19. The system equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = 12$ is inconsistent, if λ is

(A) 3 (B) –3

(C) 0 (D) None of these.

20. The system of equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ has

(A) a unique solution

(B) no solution

(C) an infinite number of solutions

(D) none of these

21. If **A** is an n–row square matrix of rank $(n-1)$, then

(A) $\text{adj}\,\mathbf{A} = 0$ (B) $\text{adj}\,\mathbf{A} \neq 0$

(C) $\text{adj}\,\mathbf{A} = I_n$ (D) None of these

22. The system of equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ has

(A) a unique solution

(B) no solution

(C) an infinite number of solution

(D) none of these

23. The eigen values of $\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 9 & -5 \end{bmatrix}$ are

(A) ± 1 (B) 1, 1

(C) –1, –1 (D) None of these

24. The eigen values of $\mathbf{A} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are

(A) 0, 3, –15 (B) 0, – 3 , – 15

(C) 0, 3, 15 (D) 0, – 3, 15

25. If the eigen values of a square matrix be 1, –2 and 3, then the eigen values of the matrix 2**A** are

(A) $\frac{1}{2}, -1, \frac{3}{2}$ (B) 2 , – 4 , 6

(C) 1 , –2, 3 (D) None of these.

26. If **A** is a non–singular matrix and the eigen values of **A** are 2 , 3 , – 3 then the eigen values of $\mathbf{A}^{-1}$ are

(A) 2 , 3 , – 3 (B) $\frac{1}{2}, \frac{1}{3}, \frac{-1}{3}$

(C) $2|\mathbf{A}|, 3|\mathbf{A}|, -3|\mathbf{A}|$ (D) None of these

27. If $-1, 2, 3$ are the eigen values of a square matrix $\mathbf{A}$ then the eigen values of $\mathbf{A}^2$ are

(A) $-1, 2, 3$ (B) 1, 4, 9

(C) 1, 2, 3 (D) None of these

28. If $2, -4$ are the eigen values of a non–singular matrix $\mathbf{A}$ and $|\mathbf{A}| = 4$, then the eigen values of $\operatorname{adj}\mathbf{A}$ are

(A) $\frac{1}{2}, -1$ (B) $2, -1$

(C) $2, -4$ (D) $8, -16$

29. If 2 and 4 are the eigen values of $\mathbf{A}$ then the eigenvalues of $\mathbf{A}^T$ are

(A) $\frac{1}{2}, \frac{1}{4}$ (B) 2, 4

(C) 4, 16 (D) None of these

30. If 1 and 3 are the eigenvalues of a square matrix $\mathbf{A}$ then $\mathbf{A}^3$ is equal to

(A) $13(\mathbf{A} - \mathbf{I}_2)$ (B) $13\mathbf{A} - 12\mathbf{I}_2$

(C) $12(\mathbf{A} - \mathbf{I}_2)$ (D) None of these

31. If $\mathbf{A}$ is a square matrix of order 3 and $|\mathbf{A}| = 2$ then $\mathbf{A}(\operatorname{adj}\mathbf{A})$ is equal to

(A) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) None of these

32. The sum of the eigenvalues of $\mathbf{A} = \begin{bmatrix} 8 & 2 & 3 \\ 4 & 5 & 9 \\ 2 & 0 & 5 \end{bmatrix}$ is equal to

(A) 18 (B) 15

(C) 10 (D) None of these

33. If 1, 2 and 5 are the eigen values of the matrix $\mathbf{A}$ then $|\mathbf{A}|$ is equal to

(A) 8 (B) 10

(C) 9 (D) None of these

34. If the product of matrices

$$\mathbf{A} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and}$$

$$\mathbf{B} = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is a null matrix, then θ and ϕ differ by

(A) an odd multiple of π

(B) an even multiple of π

(C) an odd multiple of $\frac{\pi}{2}$

(D) an even multiple $\frac{\pi}{2}$

35. If $\mathbf{A}$ and $\mathbf{B}$ are two matrices such that $\mathbf{A} + \mathbf{B}$ and $\mathbf{AB}$ are both defined, then $\mathbf{A}$ and $\mathbf{B}$ are

(A) both null matrices

(B) both identity matrices

(C) both square matrices of the same order

(D) None of these

36. If $\mathbf{A} = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$

then $(\mathbf{I} - \mathbf{A})\cdot\begin{bmatrix} \cos\alpha & -\sin\frac{\alpha}{2} \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is equal to

(A) $\mathbf{I} + \mathbf{A}$ (B) $\mathbf{I} - \mathbf{A}$

(C) $\mathbf{I} + 2\mathbf{A}$ (D) $\mathbf{I} - 2\mathbf{A}$

37. If $\mathbf{A} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for every positive integer n, $\mathbf{A}^n$ is equal to

(A) $\begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$ (B) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

(C) $\begin{bmatrix} 1-2n & 4n \\ n & 1+2n \end{bmatrix}$ (D) None of these

38. If $\mathbf{A}_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then consider the following statements :

I. $\mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \mathbf{A}_{\alpha\beta}$ II. $\mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \mathbf{A}_{(\alpha+\beta)}$

III. $(\mathbf{A}_\alpha)^n = \begin{bmatrix} \cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha \end{bmatrix}$

IV. $(\mathbf{A}_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

Which of the above statements are true ?

(A) I and II (B) I and IV

(C) II and III (D) II and IV

39. If **A** is a 3-rowed square matrix such that $|\mathbf{A}| = 3$, then adj(adj **A**) is equal to :

(A) 3A (B) 9A

(C) 27A (D) none of these

40. If **A** is a 3-rowed square matrix, then $|\text{adj}(\text{adj}\,\mathbf{A})|$ is equal to

(A) $|\mathbf{A}|^6$ (B) $|\mathbf{A}|^3$

(C) $|\mathbf{A}|^4$ (D) $|\mathbf{A}|^2$

41. If **A** is a 3-rowed square matrix such that $|\mathbf{A}| = 2$, then $|\text{adj}(\text{adj}\,\mathbf{A}^2)|$ is equal to

(A) 2^4 (B) 2^8

(C) 2^{16} (D) None of these

42. If $\mathbf{A} = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) None of these

43. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ then $\mathbf{A}^{-1}$ is

(A) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \\ 1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 \\ 3 & 1 \\ 2 & 7 \end{bmatrix}$ (D) Undefined

44. If $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$ then **AB** is

(A) $\begin{bmatrix} -1 & -8 & -10 \\ -1 & -2 & 5 \\ 9 & 22 & 15 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & -10 \\ -1 & -2 & -5 \\ 0 & 21 & -15 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 21 & 15 \end{bmatrix}$

45. If $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$, then $\mathbf{AA}^T$ is

(A) $\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ 1 & 26 \end{bmatrix}$ (D) Undefined

46. The matrix, that has an inverse is

(A) $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix}$

47. The skew symmetric matrix is

(A) $\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & 6 \\ -5 & -6 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 5 & 2 \\ 6 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 5 \\ 3 & 5 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 3 & 3 \\ 2 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

48. If $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, the product of **A** and **B** is

(A) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

49. Matrix **D** is an orthogonal matrix $\mathbf{D} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$. The value of $|\mathbf{B}|$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) 1 (D) 0

50. If $\mathbf{A}_{n\times n}$ is a triangular matrix then det **A** is

(A) $\prod_{i=1}^{n}(-1)a_{ii}$ (B) $\prod_{i=1}^{n} a_{ii}$

(C) $\sum_{i=1}^{n}(-1)a_{ii}$ (D) $\sum_{i=1}^{n} a_{ii}$

51. If $\mathbf{A} = \begin{bmatrix} t^2 & \cos t \\ e^t & \sin t \end{bmatrix}$, then $\frac{d\mathbf{A}}{dt}$ will be

(A) $\begin{bmatrix} t^2 & \sin t \\ e^t & \sin t \end{bmatrix}$

(B) $\begin{bmatrix} 2t & \cos t \\ e^t & \sin t \end{bmatrix}$

(C) $\begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix}$

(D) Undefined

52. If $\mathbf{A} \in \mathbf{R}_{n \times n}$, $\det \mathbf{A} \neq 0$, then

(A) **A** is non singular and the rows and columns of **A** are linearly independent.

(B) **A** is non singular and the rows **A** are linearly dependent.

(C) **A** is non singular and the **A** has one zero rows.

(D) **A** is singular.

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SOLUTIONS

1. (B) **A** is singular if $|\mathbf{A}| = 0$

$$\Rightarrow \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -(-1)\begin{vmatrix} 1 & -2 \\ -2 & \lambda \end{vmatrix} + 2\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ -2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 4) + 2(3) = 0 \quad \Rightarrow \lambda - 4 + 6 = 0 \quad \Rightarrow \lambda = -2$

2. (C) If k is a constant and **A** is a square matrix of order $n \times n$ then $|k\mathbf{A}| = k^n |\mathbf{A}|$.

$\mathbf{A} = 5\mathbf{B} \Rightarrow |\mathbf{A}| = |5\mathbf{B}| = 5^4 |\mathbf{B}| = 625 |\mathbf{B}|$

$\Rightarrow \alpha = 625$

3. (B) **A** is singular, if $|\mathbf{A}| = 0$,

A is Idempotent, if $\mathbf{A}^2 = \mathbf{A}$

A is Involutory, if $\mathbf{A}^2 = I$

Now, $\mathbf{A}^2 = \mathbf{AA} = (\mathbf{A}B)\,\mathbf{A} = \mathbf{A}(B\mathbf{A}) = \mathbf{A}B = \mathbf{A}$

and $\mathbf{B}^2 = \mathbf{BB} = (\mathbf{BA})\mathbf{B} = \mathbf{B}(\mathbf{AB}) = \mathbf{BA} = \mathbf{B}$

$\Rightarrow \quad \mathbf{A}^2 = \mathbf{A}$ and $\mathbf{B}^2 = \mathbf{B}$,

Thus **A** & **B** both are Idempotent.

4. (B) Since, $\mathbf{A}^2 = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{I}, \quad \mathbf{A}^2 = \mathbf{I} \quad \Rightarrow \quad \mathbf{A}$ is involutory.

5. (B) Let $\mathbf{A} = [a_{ij}]$ be a skew–symmetric matrix, then

$\mathbf{A}^T = -\mathbf{A}$, $\quad \Rightarrow \quad a_{ij} = -a_{ij}$,

if $i = j$ then $a_{ii} = -a_{ii} \quad \Rightarrow \quad 2a_{ii} = 0 \quad \Rightarrow \quad a_{ii} = 0$

Thus diagonal elements are zero.

6. (C) **A** is orthogonal if $\mathbf{AA}^T = \mathbf{I}$

A is unitary if $\mathbf{AA}^Q = \mathbf{I}$, where $\mathbf{A}^Q$ is the conjugate transpose of **A** i.e., $\mathbf{A}^Q = (\overline{\mathbf{A}})^T$.

Here,

$$\mathbf{AA}^Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}_2$$

Thus **A** is unitary.

7. (A) A square matrix $\mathbf{A}$ is said to be Hermitian if $\mathbf{A}^Q = \mathbf{A}$. So $a_{ij} = \bar{a}_{ji}$. If $i = j$ then $a_{ii} = \bar{a}_{ii}$ i.e. conjugate of an element is the element itself and a_{ii} is purely real.

8. (C) A square matrix $\mathbf{A}$ is said to be Skew-Hermitian if $\mathbf{A}^Q = -\mathbf{A}$. If $\mathbf{A}$ is Skew–Hermitian then $\mathbf{A}^Q = -\mathbf{A}$

$\Rightarrow \quad \bar{a}_{ji} = -a_{ij}$,

if $i = j$ then $\bar{a}_{ii} = -a_{ii} \Rightarrow \quad a_{ii} + \bar{a}_{ii} = 0$

it is only possible when a_{ii} is purely imaginary.

9. (D) $\mathbf{A}$ is Hermitian then $\mathbf{A}^Q = \mathbf{A}$

Now, $(i\mathbf{A})^Q = \bar{i}\ \mathbf{A}^Q = -i\mathbf{A}^Q = -i\mathbf{A}, \Rightarrow \quad (i\mathbf{A})^Q = -(i\mathbf{A})$

Thus $i\mathbf{A}$ is Skew–Hermitian.

10. (C) $\mathbf{A}$ is Skew–Hermitian then $\mathbf{A}^Q = -\mathbf{A}$

Now, $(i\mathbf{A})^Q = \bar{i}\mathbf{A}^Q = -(-\mathbf{A}) = i\mathbf{A}$ then $i\mathbf{A}$ is Hermitian.

11. (C) If $\mathbf{A} = [a_{ij}]_{n\times n}$ then $\det \mathbf{A} = [c_{ij}]^T_{n\times n}$

Where c_{ij} is the cofactor of a_{ij}

Also $c_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor of a_{ij} , obtained by leaving the row and the column corresponding to a_{ij} and then take the determinant of the remaining matrix.

Now, $M_{11} =$ minor of a_{11} i.e. $-1 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = -3$

Similarly

$$M_{12} = \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = 6 \ ; \ M_{13} = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -6$$

$$M_{21} = \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -6 \ ; \ M_{22} = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 3 \ ;$$

$$M_{23} = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 6 \ ; \ M_{31} = \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 6 \ ;$$

$$M_{32} = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 6 \ ; \ M_{33} = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 3$$

$C_{11} = (-1)^{1+1} M_{11} = -3;\ C_{12} = (-1)^{1+2} M_{12} = -6$;

$C_{13} = (-1)^{1+3} M_{13} = -6;\ C_{21} = (-1)^{2+1} M_{21} = 6;$

$C_{22} = (-1)^{2+2} M_{22} = 3;\ C_{23} = (-1)^{2+3} M_{23} = -6;$

$C_{31} = (-1)^{3+1} M_{31} = 6;\ C_{32} = (-1)^{3+2} M_{32} = -6$;

$C_{33} = (-1)^{3+3} M_{33} = 3$

$$\det \mathbf{A} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = 3\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}^T = 3\mathbf{A}^T$$

12. (A) Since $\mathbf{A}^{-1} = \dfrac{1}{|\mathbf{A}|}$ adj $\mathbf{A}$

Now, Here $|\mathbf{A}| = \begin{vmatrix} -1 & 2 \\ 3 & -5 \end{vmatrix} = -1$

Also, adj $\mathbf{A} = \begin{bmatrix} -5 & -3 \\ -2 & -1 \end{bmatrix}^T \Rightarrow$ adj $\mathbf{A} = \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}$

$$\mathbf{A}^{-1} = \frac{1}{-1}\begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$$

13. (A) Since, $\mathbf{A}^{-1} = \dfrac{1}{|\mathbf{A}|}$adj $\mathbf{A}$

$$|\mathbf{A}| = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{vmatrix} = 4 \neq 0,$$

$$\text{adj}\,\mathbf{A} == \begin{bmatrix} 4 & 10 & -10 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

14. (B) A matrix $\mathbf{A}_{(m\times n)}$ is said to be of rank r if

(i) it has at least one non–zero minor of order r, and

(ii) all other minors of order greater than r, if any; are zero. The rank of $\mathbf{A}$ is denoted by $\rho(\mathbf{A})$. Now, given that $\rho(\mathbf{A}) = 2 \rightarrow$ minor of order greater than 2 i.e., 3 is zero.

Thus $|\mathbf{A}| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{vmatrix} = 0$

$\Rightarrow \quad 2(35 - 4\lambda) + 1(20 - \lambda) + 3(16 - 7) = 0,$

$\Rightarrow \quad 70 - 8\lambda + 20 - \lambda + 27 = 0,$

$\Rightarrow \quad 9\lambda = 117 \quad \Rightarrow \quad \lambda = 13$

15. (A) The correct statements are

$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$,

adj $(\mathbf{AB}) =$ adj $(\mathbf{B})$ adj $(\mathbf{A})$

$\rho(\mathbf{AB}) \neq \rho(\mathbf{A})\,\rho(\mathbf{B})$, $|\mathbf{A}B| = |\mathbf{A}| \cdot |\mathbf{B}|$

Thus statements I, II, and IV are wrong.

16. (B) Since

$|\mathbf{A}| = 2(-9 + 8) + 2(-2 + 3) = -2 + 2 = 0$

$\Rightarrow \quad \rho(\mathbf{A}) < 3$

Again, one minor of order 2 is $\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6 \neq 0$

$\Rightarrow \quad \rho(\mathbf{A}) = 2$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 5 \\ -4 & -5-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (3-\lambda)(-5-\lambda)+16=0 \Rightarrow \quad -15+\lambda^2+2\lambda+16=0$

$\Rightarrow \quad \lambda^2+2\lambda+1=0 \quad \Rightarrow \quad (\lambda+1)^2=0 \quad \Rightarrow \quad \lambda=-1,-1$

Thus eigen values are -1, -1

24. (C) Characteristic equation is $|\mathbf{A}-\lambda\mathbf{I}|=0$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda^2-18\lambda^2+45\lambda=0$

$\Rightarrow \quad \lambda(\lambda-3)(\lambda-15)=0 \quad \Rightarrow \quad \lambda=0,3,15$

25. (B) If eigen values of **A** are λ_1, λ_2, λ_3 then the eigen values of $k\mathbf{A}$ are $k\lambda_1$, $k\lambda_2$, $k\lambda_3$. So the eigen values of $2\mathbf{A}$ are $2, -4$ and 6

26. (B) If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are the eigen values of a non–singular matrix **A**, then $\mathbf{A}^{-1}$ has the eigen values $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots\ldots, \frac{1}{\lambda_n}$. Thus eigen values of $\mathbf{A}^{-1}$ are $\frac{1}{2}, \frac{1}{3}, \frac{-1}{3}$.

27. (B) If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are the eigen values of a matrix **A**, then $\mathbf{A}^2$ has the eigen values $\lambda_1^2, \lambda_2^2, \ldots\ldots, \lambda_n^2$. So, eigen values of $\mathbf{A}^2$ are 1, 4, 9.

28. (B) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of **A** then the eigen values adj **A** are $\frac{|\mathbf{A}|}{\lambda_1}, \frac{|\mathbf{A}|}{\lambda_2}, \ldots\ldots, \frac{|\mathbf{A}|}{\lambda_n}$; $|\mathbf{A}| \neq 0$. Thus eigenvalues of adj **A** are $\frac{4}{2}, \frac{-4}{4}$ i.e. 2 and–1.

29. (B) Since, the eigenvalues of **A** and $\mathbf{A}^T$ are square so the eigenvalues of $\mathbf{A}^T$ are 2 and 4.

30. (B) Since 1 and 3 are the eigenvalues of **A** so the characteristic equation of **A** is

$(\lambda-1)(\lambda-3)=0 \Rightarrow \lambda^2-4\lambda+3=0$

Also, by Cayley–Hamilton theorem, every square matrix satisfies its own characteristic equation so

$\mathbf{A}^2-4\mathbf{A}+3\mathbf{I}_2=0$

$\Rightarrow \quad \mathbf{A}^2=4\mathbf{A}-3\mathbf{I}_2$

$\Rightarrow \mathbf{A}^3=4\mathbf{A}^2-3\mathbf{A}=4(4\mathbf{A}-3\mathbf{I})-3\mathbf{A}$

$\Rightarrow \mathbf{A}^3=13\mathbf{A}-12\mathbf{I}_2$

31. (A) Since $\mathbf{A}(\text{adj } \mathbf{A})=|\mathbf{A}|\mathbf{I}_3$

$$\Rightarrow \quad \mathbf{A}(\text{adj } \mathbf{A})=2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

32. (A) Since the sum of the eigenvalues of an n–square matrix is equal to the trace of the matrix (i.e. sum of the diagonal elements)

so, required sum $=8+5+5=18$

33. (B) Since the product of the eigenvalues is equal to the determinant of the matrix so $|\mathbf{A}|=1\times2\times5=10$

34. (C)

$$\mathbf{AB}=\begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}=\mathbf{A}$$

null matrix when $\cos(\theta-\phi)=0$

This happens when $(\theta-\phi)$ is an odd multiple of $\frac{\pi}{2}$.

35. (C) Since $\mathbf{A}+\mathbf{B}$ is defined, **A** and **B** are matrices of the same type, say $m\times n$. Also, **AB** is defined. So, the number of columns in **A** must be equal to the number of rows in **B** i.e. $n=m$. Hence, **A** and **B** are square matrices of the same order.

36. (A) Let $\tan\frac{\alpha}{2}=t$, then, $\cos\alpha=\dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\dfrac{1-t^2}{t+t^2}$

and $\sin\alpha=\dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\dfrac{2t}{1+t^2}$

$$(\mathbf{I}-\mathbf{A})\cdot\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$=\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\times\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$=\begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\times\begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{(1+t^2)} \\ \frac{2t}{(1+t^2)} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}=\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}=(\mathbf{I}+\mathbf{A})$$

37. (B) $\mathbf{A}^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where $n = 2$.

38. (D) $\mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = \mathbf{A}_{\alpha+\beta}$

Also, it is easy to prove by induction that

$(\mathbf{A}_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

39. (A) We know that $\text{adj}(\text{adj}\,\mathbf{A}) = |\mathbf{A}|^{n-2} \cdot \mathbf{A}$.

Here $n = 3$ and $|\mathbf{A}| = 3$.

So, $\text{adj}(\text{adj}\,\mathbf{A}) = 3^{(3-2)} \cdot \mathbf{A} = 3\mathbf{A}$.

40. (C) We have $|\text{adj}(\text{adj}\,\mathbf{A})| = |\mathbf{A}|^{(n-1)^2}$

Putting $n = 3$, we get $|\text{adj}(\text{adj}\,\mathbf{A})| = |\mathbf{A}|^4$.

41. (C) Let $\mathbf{B} = \text{adj}(\text{adj}\,\mathbf{A}^2)$.

Then, $\mathbf{B}$ is also a 3×3 matrix.

$|\text{adj}\{\text{adj}(\text{adj}\,\mathbf{A}^2)\}| = |\text{adj}\,\mathbf{B}| = |\mathbf{B}^3|^{3-1} = |\mathbf{B}|^2$

$= |\text{adj}(\text{adj}\,\mathbf{A}^2)|^2 = \left[\left|\mathbf{A}^2\right|^{(3-1)^2}\right]^2 = |\mathbf{A}|^{16} = 2^{16}$

$\left[\because\ |\mathbf{A}^2| = |\mathbf{A}|^2\right]$

42. (C) $\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, So, $2x = 1 \Rightarrow x = \frac{1}{2}$.

43. (D) Inverse matrix is defined for square matrix only.

44. (C) $\mathbf{AB} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} (2)(1)+(-1)(3) & (2)(-2)+(-1)(4) & (2)(-5)+(-1)(0) \\ (1)(1)+(0)(3) & (1)(-2)+(0)(4) & (1)(-5)+(0)(0) \\ (-3)(1)+(4)(3) & (-3)(-2)+(4)(4) & (-3)(-5)+(4)(0) \end{bmatrix}$

$= \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & -22 & 15 \end{bmatrix}$

45. (C) $\mathbf{AA}^T = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} (1)(1)+(2)(2)+(0)(0) & (1)(3)+(2)(-1)+(0)(4) \\ (3)(1)+(-1)(2)+(4)(0) & (3)(3)+(-1)(-1)+(4)(4) \end{bmatrix}$

$= \begin{bmatrix} 5 & 1 \\ 1 & 26 \end{bmatrix}$

46. (B) if $|\mathbf{A}|$ is zero, $\mathbf{A}^{-1}$ does not exist and the matrix $\mathbf{A}$ is said to be singular. Only (B) satisfy this condition.

$|\mathbf{A}| = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = (5)(1) - (2)(2) = 1$

47. (A) A skew symmetric matrix $\mathbf{A}_{n\times n}$ is a matrix with $\mathbf{A}^T = -\mathbf{A}$. The matrix of (A) satisfy this condition.

48. (C) $\mathbf{AB} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} (1)(1)+(1)(0)+(0)(1) \\ (1)(1)+(0)(0)+(1)(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

49. (C) For orthogonal matrix

$\det \mathbf{M} = 1$ And $\mathbf{M}^{-1} = \mathbf{M}^T$, therefore Hence $\mathbf{D}^{-1} = \mathbf{D}^T$

$\mathbf{D}^T = \begin{bmatrix} A & C \\ B & 0 \end{bmatrix} = \mathbf{D}^{-1} = \frac{1}{-BC}\begin{bmatrix} 0 & -B \\ -C & A \end{bmatrix}$

This implies $B = \frac{-C}{-BC} \Rightarrow B = \frac{1}{B} \Rightarrow B = \pm 1$

Hence $B = 1$

50. (B) From linear algebra for $\mathbf{A}_{n\times n}$ triangular matrix

$\det \mathbf{A} = \prod_{i=1}^{n} a_{ii}$, The product of the diagonal entries of $\mathbf{A}$

51. (C) $\frac{d\mathbf{A}}{dt} = \begin{bmatrix} \frac{d(t^2)}{dt} & \frac{d(\cos t)}{dt} \\ \frac{d(e^t)}{dt} & \frac{d(\sin t)}{dt} \end{bmatrix} = \begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix}$

52. (A) If $\det \mathbf{A} \neq 0$, then $\mathbf{A}_{n\times n}$ is non-singular, but if $\mathbf{A}_{n\times n}$ is non-singular, then no row can be expressed as a linear combination of any other. Otherwise $\det \mathbf{A} = 0$

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CHAPTER

9.2

DIFFERENTIAL CALCULUS

1. If $f(x) = x^3 - 6x^2 + 11x - 6$ is on [1, 3], then the point $c \leftrightarrow]1, 3[$ such that $f'(c) = 0$ is given by

(A) $c = 2 \pm \frac{1}{\sqrt{2}}$ (B) $c = 2 \pm \frac{1}{\sqrt{3}}$

(C) $c = 2 \pm \frac{1}{2}$ (D) None of these

2. Let $f(x) = \sin 2x, 0 \le x \le \frac{\pi}{2}$ and $f'(c) = 0$ for $c \leftrightarrow]0, \frac{\pi}{2}[$. Then, c is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) None

3. Let $f(x) = x(x+3)e^{-\frac{x}{2}}, -3 \le x \le 0$. Let $c \leftrightarrow]-3, 0[$ such that $f'(c) = 0$. Then, the value of c is

(A) 3 (B) –3

(C) –2 (D) $-\frac{1}{2}$

4. If Rolle's theorem holds for $f(x) = x^3 - 6x^2 + kx + 5$ on [1, 3] with $c = 2 + \frac{1}{\sqrt{3}}$, the value of k is

(A) –3 (B) 3

(C) 7 (D) 11

5. A point on the parabola $y = (x-3)^2$, where the tangent is parallel to the chord joining A (3, 0) and B (4, 1) is

(A) (7, 1) (B) $\left(\frac{3}{2}, \frac{1}{4}\right)$

(C) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (D) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

6. A point on the curve $y = \sqrt{x-2}$ on [2, 3], where the tangent is parallel to the chord joining the end points of the curve is

(A) $\left(\frac{9}{4}, \frac{1}{2}\right)$ (B) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(C) $\left(\frac{7}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{9}{2}, \frac{1}{4}\right)$

7. Let $f(x) = x(x-1)(x-2)$ be defined in $[0, \frac{1}{2}]$. Then, the value of c of the mean value theorem is

(A) 0.16 (B) 0.20

(C) 0.24 (D) None

8. Let $f(x) = \sqrt{x^2 - 4}$ be defined in [2, 4]. Then, the value of c of the mean value theorem is

(A) $-\sqrt{6}$ (B) $\sqrt{6}$

(C) $\sqrt{3}$ (D) $2\sqrt{3}$

9. Let $f(x) = e^x$ in [0, 1]. Then, the value of c of the mean-value theorem is

(A) 0.5 (B) $(e-1)$

(C) $\log(e-1)$ (D) None

10. At what point on the curve $y = (\cos x - 1)$ in $]0, 2\pi[$, is the tangent parallel to x–axis ?

(A) $\left(\frac{\pi}{2}, -1\right)$ (B) $(\pi, -2)$

(C) $\left(\frac{2\pi}{3}, \frac{-3}{2}\right)$ (D) None of these

11. log sin $(x+h)$ when expanded in Taylor's series, is equal to

(A) $\log \sin x + h\cot x - \frac{1}{2}h^2 \operatorname{cosec}^2 x + \ldots$

(B) $\log \sin x + h\cot x + \frac{1}{2}h^2 \sec^2 x + \ldots$

(C) $\log \sin x - h\cot x + \frac{1}{2}h^2 \operatorname{cosec}^2 x + \ldots$

(D) None of these

12. sin x when expanded in powers of $\left(x-\frac{\pi}{2}\right)$ is

(A) $1+\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{2}\right)^2}{4!}+\ldots$

(B) $1-\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^2}{4!}-\ldots$

(C) $\left(x-\frac{\pi}{2}\right)^2+\frac{\left(x-\frac{\pi}{2}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{2}\right)^5}{5!}+\ldots$

(D) None of these

13. $\tan\left(\frac{\pi}{4}+x\right)$ when expanded in Taylor's series, gives

(A) $1+x+x^2+\frac{4}{3}x^3+\ldots$

(B) $1+2x+2x^2+\frac{8}{3}x^3+\ldots$

(C) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots$

(D) None of these

14. If $u=e^{xyz}$, then $\frac{\partial^3 u}{\partial x\partial y\partial z}$ is equal to

(A) $e^{xyz}[1+xyz+3x^2y^2z^2]$

(B) $e^{xyz}[1+xyz+x^3y^3z^3]$

(C) $e^{xyz}[1+3xyz+x^2y^2z^2]$

(D) $e^{xyz}[1+3xyz+x^3y^3z^3]$

15. If $z=f(x+ay)+\phi(x-ay)$, then

(A) $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$

(B) $\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}$

(C) $\frac{\partial^2 z}{\partial y^2}=-\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2}$

(D) $\frac{\partial^2 z}{\partial x^2}=-a^2\frac{\partial^2 z}{\partial y^2}$

16. If $u=\tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ equals

(A) $2\cos 2u$

(B) $\frac{1}{4}\sin 2u$

(C) $\frac{1}{4}\tan u$

(D) $2\tan 2u$

17. If $u=\tan^{-1}\frac{x^3+y^3+x^2y-xy^2}{x^2-xy+y^2}$, then the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

(A) $\frac{1}{2}\sin 2u$

(B) $\sin 2u$

(C) $\sin u$

(D) 0

18. If $u=\phi\left(\frac{y}{x}\right)+x\psi\left(\frac{y}{x}\right)$, then the value of

$$x^2\frac{\partial^2 u}{dx^2}+2xy\frac{\partial^2 u}{dx\,dy}+y^2\frac{\partial^2 u}{\partial y^2}\text{, is}$$

(A) 0

(B) u

(C) $2u$

(D) $-u$

19. If $z=e^x\sin y$, $x=\log_e t$ and $y=t^2$, then $\frac{dz}{dt}$ is given by the expression

(A) $\frac{e^x}{t}(\sin y-2t^2\cos y)$

(B) $\frac{e^x}{t}(\sin y+2t^2\cos y)$

(C) $\frac{e^x}{t}(\cos y+2t^2\sin y)$

(D) $\frac{e^x}{t}(\cos y-2t^2\sin y)$

20. If $z=z(u,v)$, $u=x^2-2xy-y^2$, $v=a$, then

(A) $(x+y)\frac{\partial z}{\partial x}=(x-y)\frac{\partial z}{\partial y}$

(B) $(x-y)\frac{\partial z}{\partial x}=(x+y)\frac{\partial z}{\partial y}$

(C) $(x+y)\frac{\partial z}{\partial x}=(y-x)\frac{\partial z}{\partial y}$

(D) $(y-x)\frac{\partial z}{\partial x}=(x+y)\frac{\partial z}{\partial y}$

21. If $f(x,y)=0$, $\phi(y,z)=0$, then

(A) $\frac{\partial f}{\partial y}\cdot\frac{\partial\phi}{\partial z}=\frac{\partial f}{\partial x}\cdot\frac{\partial\phi}{\partial y}\cdot\frac{dz}{dx}$

(B) $\frac{\partial f}{\partial y}\cdot\frac{\partial\phi}{\partial z}\cdot\frac{\partial f}{\partial x}=\frac{\partial f}{\partial x}\cdot\frac{dz}{dx}$

(C) $\frac{\partial f}{\partial y}\cdot\frac{\partial\phi}{\partial z}\cdot\frac{dz}{dx}=\frac{\partial f}{\partial x}\cdot\frac{\partial\phi}{\partial y}$

(D) None of these

22. If $z=\sqrt{x^2+y^2}$ and $x^3+y^3+3axy=5a^2$, then at $x=a$, $y=a$, $\frac{dz}{dx}$ is equal to

(A) $2a$

(B) 0

(C) $2a^2$

(D) a^3

23. If $x = r\cos\theta$, $y = r\sin\theta$ where r and θ are the functions of x, then $\frac{dx}{dt}$ is equal to

(A) $r\cos\theta \frac{dr}{dt} - r\sin\theta \frac{d\theta}{dt}$ (B) $\cos\theta \frac{dr}{dt} - r\sin\theta \frac{d\theta}{dt}$

(C) $r\cos\theta \frac{dr}{dt} + \sin\theta \frac{d\theta}{dt}$ (D) $r\cos\theta \frac{dr}{dt} - \sin\theta \frac{d\theta}{dt}$

24. If $r^2 = x^2 + y^2$, then $\frac{\partial^2 r}{dx^2} + \frac{\partial^2 r}{\partial y^2}$ is equal to

(A) $r^2\left\{\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right\}$ (B) $2r^2\left\{\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right\}$

(C) $\frac{1}{r^2}\left\{\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right\}$ (D) None of these

25. If $x = r\cos\theta$, $y = r\sin\theta$, then the value of $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}$ is

(A) 0 (B) 1

(C) $\frac{\partial r}{\partial x}$ (D) $\frac{\partial r}{\partial y}$

26. If $u = x^m y^n$, then

(A) $du = mx^{m-1}y^n + nx^m y^{n-1}$ (B) $du = mdx + ndy$

(C) $udu = mxdx + nydy$ (D) $\frac{du}{u} = m\frac{dx}{x} + n\frac{dy}{y}$

27. If $y^3 - 3ax^2 + x^3 = 0$, then the value of $\frac{d^2y}{dx^2}$ is equal to

(A) $-\frac{a^2x^2}{y^5}$ (B) $\frac{2a^2x^2}{y^5}$

(C) $-\frac{2a^2x^4}{y^5}$ (D) $-\frac{2a^2x^2}{y^5}$

28. $z = \tan^{-1}\frac{y}{x}$, then

(A) $dz = \frac{xdy - ydx}{x^2 + y^2}$ (B) $dz = \frac{xdy + ydx}{x^2 + y^2}$

(C) $dz = \frac{xdx - ydy}{x^2 + y^2}$ (D) $dz = \frac{xdx - ydy}{x^2 + y^2}$

29. If $u = \log\frac{x^2 + y^2}{x + y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(A) 0 (B) 1

(C) u (D) eu

30. If $u = x^{n-1}yf\left(\frac{y}{x}\right)$, then $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 y}{\partial y\,\partial x}$ is equal to

(A) nu (B) $n(n-1)u$

(C) $(n-1)\frac{\partial u}{\partial x}$ (D) $(n-1)\frac{\partial u}{\partial y}$

31. Match the List–I with List–II.

List–I

(i) If $u = \frac{x^2y}{x+y}$ then $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x\,\partial y}$

(ii) If $u = \frac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{x^{\frac{1}{4}} + y^{\frac{1}{4}}}$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

(iii) If $u = x^{\frac{1}{2}} + y^{\frac{1}{2}}$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

(iv) If $u = f\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

List–II

(1) $-\frac{3}{16}u$ (2) $\frac{\partial u}{\partial x}$

(3) 0 (4) $-\frac{1}{4}u$

Correct match is—

	(I)	(II)	(III)	(IV)
(A)	1	2	3	4
(B)	2	1	4	3
(C)	2	1	3	4
(D)	1	2	4	3

32. If an error of 1% is made in measuring the major and minor axes of an ellipse, then the percentage error in the area is approximately equal to

(A) 1% (B) 2%

(C) π% (D) 4%

33. Consider the Assertion (A) and Reason (R) given below:

Assertion (A): If $u = xyf\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$

Reason (R): Given function u is homogeneous of degree 2 in x and y.

Of these statements

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true and R is not a correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

34. If $u = x \log xy$, where $x^3 + y^3 + 3xy = 1$, then $\frac{du}{dx}$ is equal to

(A) $(1 + \log xy) - \frac{x}{y}\left(\frac{x^2 + y}{y^2 + x}\right)$

(B) $(1 + \log xy) - \frac{y}{x}\left(\frac{y^2 + x}{x^2 + y}\right)$

(C) $(1 - \log xy) - \frac{x}{y}\left(\frac{x^2 + y}{y^2 + x}\right)$

(D) $(1 - \log xy) - \frac{y}{x}\left(\frac{y^2 + x}{x^2 + y}\right)$

35. If $z = xyf\left(\frac{y}{x}\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) z (B) $2z$

(C) xz (D) yz

36. $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing in the interval

(A) $]\,2, 3\,[$ (B) $]\,-\infty, 3\,[$

(C) $]\,-\infty, 2\,[\cup]\,3, \infty$ (D) None of these

37. $f(x) = \frac{x}{(x^2 + 1)}$ is increasing in the interval

(A) $]\,-\infty, -1\,[\, \cup\,]\,1, \infty\,[$ (B) $]\,-1, 1\,[$

(C) $]\,-1, \infty\,[$ (D) None of these

38. $f(x) = x^4 - 2x^2$ is decreasing in the interval

(A) $]\,-\infty, -1\,[\, \cup\,]\,0, 1\,[$ (B) $]\,-1, 1\,[$

(C) $]\,-\infty, -1\,[\, \cup\,]\,1, \infty\,[$ (D) None of these

39. $f(x) = x^9 + 3x^7 + 6$ is increasing for

(A) all positive real values of x

(B) all negative real values of x

(C) all non-zero real values of x

(D) None of these

40. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in each interval, then

(A) $k < 3$ (B) $k \le 3$

(C) $k > 3$ (D) $k \ge 3$

41. If $a < 0$, then $f(x) = e^{ax} + e^{-ax}$ is decreasing for

(A) $x > 0$ (B) $x < 0$

(C) $x > 1$ (D) $x < 1$

42. $f(x) = x^2 e^{-x}$ is increasing in the interval

(A) $]\,-\infty, \infty\,[$ (B) $]\,-2, 0\,[$

(C) $]\,2, \infty\,[$ (D) $]\,0, 2\,[$

43. The least value of a for which $f(x) = x^2 + ax + 1$ is increasing on $]\,1, 2, [$ is

(A) 2 (B) -2

(C) 1 (D) -1

44. The minimum distance from the point (4, 2) to the parabola $y^2 = 8x$, is

(A) $\sqrt{2}$ (B) $2\sqrt{2}$

(C) 2 (D) $3\sqrt{2}$

45. The co-ordinates of the point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$, are

(A) $(-2, -8)$ (B) $(2, -8)$

(C) $(-2, 0)$ (D) None of these

46. The shortest distance of the point $(0, c)$, where $0 \le c < 5$, from the parabola $y = x^2$ is

(A) $\sqrt{4c + 1}$ (B) $\frac{\sqrt{4c + 1}}{2}$

(C) $\frac{\sqrt{4c - 1}}{2}$ (D) None of these

47. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(A) e (B) $e^{-\frac{1}{e}}$

(C) $\left(\frac{1}{e}\right)^e$ (D) None of these

48. The minimum value of $\left(x^2 + \frac{250}{x}\right)$ is

(A) 75 (B) 50

(C) 25 (D) 0

49. The maximum value of $f(x) = (1 + \cos x)\sin x$ is

(A) 3 (B) $3\sqrt{3}$

(C) 4 (D) $\frac{3\sqrt{3}}{4}$

50. The greatest value of

$$f(x)=\frac{\sin 2x}{\sin\left(x+\frac{\pi}{4}\right)}$$

on the interval $[0, \frac{\pi}{2}]$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

(C) 1 (D) $-\sqrt{2}$

51. If $y=a\log x+bx^2+x$ has its extremum values at $x=-1$ and $x=2$, then

(A) $a=-\frac{1}{2},\ b=2$ (B) $a=2,\quad b=-1$

(C) $a=2,\quad b=-\frac{1}{2}$ (D) None of these

52. The co-ordinates of the point on the curve $4x^2+5y^2=20$ that is farthest from the point (0, –2) are

(A) $(\sqrt{5}\,,\ 0)$ (B) $(\sqrt{6}\,,\ 0)$

(C) (0, 2) (D) None of these

53. For what value of $x\left(0\le x\le\frac{\pi}{2}\right)$, the function $y=\frac{x}{(1+\tan x)}$ has a maxima ?

(A) tan x (B) 0

(C) cot x (D) cos x

****

SOLUTIONS

1. (B) A polynomial function is continuous as well as differentiable. So, the given function is continuous and differentiable.

$f(1)=0$ and $f(3)=0$. So, $f(1)=f(3)$.

By Rolle's theorem Ec such that $f'(c)=0$.

Now, $f'(x)=3x^2-12x+11$

$\Rightarrow\quad f'(c)=3c^2-12c+11.$

Now, $f'(c)=0\quad\Rightarrow\quad 3c^2-12c+11=0$

$\Rightarrow\quad c=\left(2\pm\frac{1}{\sqrt{3}}\right).$

2. (A) Since the sine function is continuous at each $x\leftrightarrow R$, so $f(x)=\sin 2x$ is continuous in $\left[0,\frac{\pi}{2}\right]$.

Also, $f'(x)=2\cos 2x$, which clearly exists for all $x\leftrightarrow\ ]0,\frac{\pi}{2}[$ .So, $f(x)$ is differentiable in $x\leftrightarrow\ ]0,\frac{\pi}{2}[$.

Also, $f(0)=f\left(\frac{\pi}{2}\right)=0$. By Rolle's theorem, there exists $c\leftrightarrow\ ]0,\frac{\pi}{2}[$ such that $f'(c)=0$.

$2\cos 2c=0\quad\Rightarrow\quad 2c=\frac{\pi}{2}\quad\Rightarrow\quad c=\frac{\pi}{4}$.

3. (C) Since a polynomial function as well as an exponential function is continuous and the product of two continuous functions is continuous, so $f(x)$ is continuous in [–3, 0].

$$f'(x)=(2x+3)\cdot e^{-\frac{x}{2}}-\frac{1}{2}e^{-\frac{x}{2}}(x^2+3x)=e^{-\frac{x}{2}}\left[\frac{x+6-x^2}{2}\right]$$

which clearly exists for all $x\leftrightarrow\ ]-3,0[$.

$f(x)$ is differentiable in] –3, 0 [.

Also, $f(-3)=f(0)=0$.

By Rolle's theorem $c\leftrightarrow\ ]-3,\ 0\ [$ such that $f'(c)=0$.

Now, $f'(c)=0\quad\Rightarrow\quad e^{-\frac{c}{2}}\left[\frac{c+6-c^2}{2}\right]=0$

$c+6-c^2=0$ i.e. $c^2-c-6=0$

$\Rightarrow\quad (c+2)(3-c)=0\quad\Rightarrow\quad c=-2,\ c=3.$

Hence, $c=-2\leftrightarrow\ ]-3,\ 0\ [$.

4. (D) $f'(x)=3c^2-12x+k$

$f'(c)=0\quad\Rightarrow\quad 3c^2-12c+k=0$

$f\left(\frac{\pi}{2}\right)=1,\ f'\left(\frac{\pi}{2}\right)=0,\ f''\left(\frac{\pi}{2}\right)=-1,$

$f'''\left(\frac{\pi}{2}\right)=0,\ f''''\left(\frac{\pi}{2}\right)=1,\ \ldots.$

13. (B) Let $f(x)=\tan x$ Then,

$$f\left(\frac{\pi}{4}+x\right)=f\left(\frac{\pi}{4}\right)+xf'\left(\frac{\pi}{4}\right)+\frac{x^2}{2!}\cdot f''\left(\frac{\pi}{4}\right)+\frac{x^3}{3!}f'''\left(\frac{\pi}{4}\right)+\ldots$$

$f'(x)=\sec^2,\ f''(x)=2\sec^2 x\tan x,$

$f'''(x)=2\sec^4 x+4\sec^2 x\tan^2 x$ etc.

Now,

$f\left(\frac{\pi}{4}\right)=1,\ f'\left(\frac{\pi}{4}\right)=2,\ \ f''\left(\frac{\pi}{4}\right)=4,\ f'''\left(\frac{\pi}{4}\right)=16,\ \ldots$

Thus $\tan\left(\frac{\pi}{4}+x\right)=1+2x+\frac{x^2}{2}\cdot 4+\frac{x^3}{6}\cdot 16+\ldots$

$=1+2x+2x^2+\frac{8}{3}x^3+\ldots$

14. (C) Here $u=e^{xyz} \quad\Rightarrow\quad \frac{\partial u}{\partial x}=e^{xyz}\cdot yz$

$$\frac{\partial^2 u}{\partial x\partial y}=ze^{xyz}+yze^{xyz}\cdot xz=e^{xyz}(z+xyz^2)$$

$$\frac{\partial^3 u}{\partial x\partial y\partial z}=e^{xyz}\cdot(1+2xyz)+(z+xyz^2)\,e^{xyz}\cdot xy$$

$=e^{xyz}(1+3xyz+x^2y^2z^2)$

15. (B) $z=f(x+ay)+\phi(x-ay)$

$$\frac{\partial z}{\partial x}=f'(x+ay)+\phi'(x-ay)$$

$$\frac{\partial^2 z}{dx^2}=f''(x+ay)+\phi''(x-ay)\ldots.(1)$$

$$\frac{\partial z}{\partial y}=af'(x+ay)-a\phi'(x-ay)$$

$$\frac{\partial^2 z}{\partial y^2}=a^2f''(x+ay)+a^2\phi''(x-ay)\ldots.(2)$$

Hence from (1) and (2), we get $\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}$

16. (B) $u=\tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$$\Rightarrow\quad \tan u=\frac{x+y}{\sqrt{x}+\sqrt{y}}=f \text{ (say)}$$

Which is a homogeneous equation of degree 1/2

By Euler's theorem. $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=\frac{1}{2}f$

$$\Rightarrow\quad x\frac{\partial(\tan u)}{\partial x}+y\frac{\partial(\tan u)}{\partial y}=\frac{1}{2}\tan u$$

$$x\sec^2 u\frac{\partial u}{\partial x}+y\sec^2 u\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$$

$$\Rightarrow\quad x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin u\cos u=\frac{1}{4}\sin 2u$$

17. (A) Here $\tan u=\frac{x^3+y^3+x^2y-xy^2}{x^2-xy+y^2}=f$ (say)

Which is homogeneous of degree 1

Thus $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=f$

As above question number 16 $x\frac{\partial f}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$

18. (A) Let $v=\phi\left(\frac{y}{x}\right)$ and $w=x\Psi\left(\frac{y}{x}\right)$

Then $u=v+w$

Now v is homogeneous of degree zero and w is homogeneous of degree one

$$\Rightarrow\quad x^2\frac{\partial^2 v}{\partial x^2}+2xy\frac{\partial^2 v}{\partial x\,\partial y}+y^2\frac{\partial^2 v}{\partial y^2}=0\ldots.(1)$$

and $x^2\frac{\partial^2 w}{\partial x^2}+2xy\frac{\partial^2 w}{\partial x\,\partial y}+y^2\frac{\partial^2 w}{\partial y^2}=0\ldots.(2)$

Adding (1) and (2), we get

$$x^2\frac{\partial^2}{\partial x^2}(v+w)+2xy\frac{\partial^2}{\partial x\,\partial y}(v+w)+y^2\frac{\partial^2}{\partial y^2}(v+w)=0$$

$$\Rightarrow\quad x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$$

19. (B) $z=e^x\sin y \quad\Rightarrow\quad \frac{\partial z}{\partial x}=e^x\sin y$

And $\frac{\partial z}{\partial y}=e^x\cos y,\ x=\log_e t \quad\Rightarrow\quad \frac{dx}{dt}=\frac{1}{t}$

And $y=t^2 \quad\Rightarrow\quad \frac{dy}{dt}=2t$

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

$$=e^x\sin y\cdot\frac{1}{t}+e^x\cos y\cdot 2t=\frac{e^x}{t}(\sin y+2t^2\cos y)$$

20. (C) Given that

$z=z(u,v),\ u=x^2-2xy-y^2,\ v=a\ldots.(i)$

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\ldots.(ii)$$

and $\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\ldots.(iii)$

From (i),

$$\frac{\partial u}{\partial x}=2x-2y,\ \frac{\partial u}{\partial y}=-2x-2y,\ \frac{\partial v}{\partial x}=0,\ \frac{\partial v}{\partial y}=0$$

Substituting these values in (ii) and (iii)

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}(2x-2y) + \frac{\partial z}{\partial v}\cdot 0....(iv)$$

and $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot(-2x-2y) + \frac{\partial z}{\partial v}\cdot 0....(v)$

From (iv) and (v), we get

$$(x+y)\frac{\partial z}{\partial x} = (y-x)\frac{\partial z}{\partial y}$$

21. (C) Given that $f(x, y) = 0,\ \phi(y, z) = 0$

These are implicit functions

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}},\quad \frac{dz}{dy} = -\frac{\frac{\partial \phi}{\partial y}}{\frac{\partial \phi}{\partial z}}$$

$$\frac{dy}{dx}\cdot\frac{dz}{dy} = \left(\frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}\right)\times\left(\frac{-\frac{\partial \phi}{\partial y}}{\frac{\partial \phi}{\partial z}}\right)$$

or, $\frac{\partial f}{\partial y}\cdot\frac{\partial \phi}{\partial z}\cdot\frac{dz}{dx} = \frac{\partial f}{\partial x}\cdot\frac{\partial \phi}{\partial y}$

22. (B) Given that $z = \sqrt{x^2+y^2}$

and $x^3 + y^3 + 3axy = 5a^2...(i)$

$$\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dx}....(ii)$$

from (i), $\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2+y^2}}\cdot 2x,\ \frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x^2+y^2}}\cdot 2y$

and $3x^2 + 3y^2\frac{dy}{dx} + 3ax\frac{dy}{dx} + 3ay.1 = 0$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{x^2+ay}{y^2+ax}\right)$$

Substituting these value in (ii), we get

$$\frac{dz}{dx} = \frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}}\left(-\frac{x^2+ay}{y^2+ax}\right)$$

$$\left(\frac{dz}{dx}\right)_{(a,a)} = \frac{a}{\sqrt{a^2+a^2}} + \frac{a}{\sqrt{a^2+a^2}}\left(-\frac{a^2+aa}{a^2+a.a}\right) = 0$$

23. (B) Given that $x = r\cos\theta,\ y = r\sin\theta....(i)$

$$\frac{dx}{dt} = \frac{\partial x}{\partial r}\cdot\frac{dr}{dt} + \frac{\partial x}{\partial \theta}\cdot\frac{d\theta}{dt}....(ii)$$

From (i), $\frac{\partial x}{\partial r} = \cos\theta,\ \frac{\partial x}{\partial \theta} = -r\sin\theta$

Substituting these values in (ii), we get

$$\frac{dx}{dt} = \cos\theta\frac{dr}{dt} - r\sin\theta\cdot\frac{d\theta}{dt}$$

24. (C) $r^2 = x^2 + y^2 \Rightarrow \frac{\partial r}{\partial x} = 2x$ and $\frac{\partial r}{\partial y} = 2y$

and $\frac{\partial^2 r}{\partial x^2} = 2$ and $\frac{\partial^2 r}{\partial y^2} = 2 \Rightarrow \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = 2 + 2 + 4$

and $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 4x^2 + 4y^2 = 4r^2$

$$\Rightarrow \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = \frac{1}{r^2}\left\{\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right\}$$

25. (A) $x = r\cos\theta,\quad y = r\sin\theta$

$$\Rightarrow \tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{\partial\theta}{\partial x} = \frac{1}{1+(y/x)^2}\left(\frac{-y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

and $\frac{\partial^2\theta}{\partial x^2} = \frac{-2xy}{(x^2+y^2)^2}$

Similarly $\frac{\partial^2\theta}{\partial y^2} = \frac{2xy}{(x^2+y^2)^2}$ and $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$

26. (D) Given that $u = x^m y^n$

Taking logarithm of both sides, we get

$\log u = m\log x + n\log y$

Differentiating with respect to x, we get

$$\frac{1}{u}\frac{du}{dx} = m\cdot\frac{1}{x} + n\cdot\frac{1}{y}\frac{dy}{dx} \text{ or, } \frac{du}{u} = m\frac{dx}{x} + n\cdot\frac{dy}{y}$$

27. (D) Given that $f(x, y) = y^3 - 3ax^2 + x^3 = 0$

$f_x = -6ax + 3x^2,\ f_y = 3y^2,\ f_{xx} = -6a + 6x,$

$f_{yy} = 6y,\ f_{xy} = 0$

$$\frac{d^2y}{dx^2} = -\left[\frac{f_{xx}(f_y)^2 - 2f_xf_yf_{xy} + f_{yy}(f_x)^2}{(f_y)^3}\right]$$

$$= -\left[\frac{(6x-6a(3y^2)^2 - 0 + 6y(3x^2-6ax)^2}{(3y^2)^3}\right]$$

$$= -\frac{2}{y^5}(-ax^3 - ay^3 + 4a^2x^2)$$

$$= -\frac{2}{y^5}[-a(a^3+y^3) + 4a^2x^2]$$

$$= -\frac{2}{y^5}[-a(3ax^2) + 4a^2x^2]\ [\therefore x^3 + y^3 - 3ax^2 = 0]$$

$$= -\frac{2a^2x^2}{y^5}$$

28. (A) Given that $z = \tan^{-1}\frac{y}{x}....(i)$

$$\frac{dz}{dx}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dx}....(ii)$$

From (i) $\frac{\partial z}{\partial x}=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{-y}{x^2}\right)=\frac{-y}{x^2+y^2}$

$$\frac{\partial z}{\partial y}=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{1}{x}\right)=\frac{x}{x^2+y^2}$$

Substituting these in (ii), we get

$$\frac{dz}{dx}=\frac{-y}{x^2+y^2}+\frac{x}{x^2+y^2}\cdot\frac{dy}{dx},\ dz=\frac{xdy-ydx}{x^2+y^2}$$

29. (B) $u=\log\frac{x^2+y^2}{x+y}$, $e^u=\frac{x^2+y^2}{x+y}=f$ (say)

f is a homogeneous function of degree one

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=f\ \Rightarrow\ x\frac{\partial e^u}{\partial x}+y\frac{\partial e^u}{\partial y}=e^u$$

or $xe^u\frac{\partial u}{\partial x}+ye^u\frac{\partial u}{\partial y}=e^u$

or, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$

30. (C) Given that $u=x^{n-1}yf\left(\frac{y}{x}\right)$.

It is a homogeneous function of degree n

Euler's theorem $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$

Differentiating partially w.r.t. x, we get

$$x\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}+y\frac{\partial^2 u}{\partial y\,\partial x}=\frac{n\partial u}{\partial x}$$

$$\Rightarrow\quad x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y\,\partial x}=(n-1)\frac{\partial u}{\partial x}$$

31. (B) In (a) $u=\frac{x^2y}{x+y}$ It is a homogeneous function of degree 2.

$$x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\,\partial y}=(n-1)\frac{\partial u}{\partial x}=\frac{\partial u}{\partial x}\ \text{(as in question 30)}$$

In (b) $u=\frac{x^{1/2}-y^{1/2}}{x^{1/4}+y^{1/4}}$. It is a homogeneous function of degree $\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{1}{4}$

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$$

$$=\frac{1}{4}\left(\frac{1}{4}-1\right)u=-\frac{3}{16}u$$

In (c) $u=x^{1/2}+y^{1/2}$ It is a homogeneous function of degree $\frac{1}{2}$.

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,dy}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$$

$$=\frac{1}{2}\left(\frac{1}{2}-1\right)u=-\frac{1}{4}u$$

In (d)$u=f\left(\frac{y}{x}\right)$ It is a homogeneous function of degree zero.

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0.u=0$$

Hence correct match is

a	b	c	d
2	1	3	4

32. (B) Let $2a$ and $2b$ be the major and minor axes of the ellipse

Area $A=\pi ab$

$\Rightarrow\quad \log A=\log\pi+\log a+\log b$

$\Rightarrow\quad \partial(\log A)=\partial(\log\pi)+\partial(\log a)+\partial(\log b)$

$\Rightarrow\quad \frac{\partial A}{A}=0+\frac{\partial a}{a}+\frac{\partial b}{b}$

$\Rightarrow\quad \frac{100}{A}\partial A=\frac{100}{a}\partial a+\frac{100}{b}\cdot\partial b$

But it is given that $\frac{100}{a}\partial a=1$, and $\frac{100}{b}\partial b=1$

$$\frac{100}{A}\partial A=1+1=2$$

Thus percentage error in A =2%

33. (A) Given that $u=xyf\left(\frac{y}{x}\right)$. Since it is a homogeneous function of degree 2.

By Euler's theorem $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$ (where $n=2$)

Thus $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$

34. (A) Given that $u=x\log xy$....(i)

$x^3+y^3+3xy=1$....(ii)

we know that $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\frac{dy}{dx}$....(ii)

From (i) $\frac{\partial u}{\partial x}=x\cdot\frac{1}{xy}\cdot y+\log xy=1+\log xy$

and $\frac{\partial u}{\partial y}=x\cdot\frac{1}{xy}\cdot x=\frac{x}{y}$

From (ii), we get

$$3x^2 + 3y^2 \frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y \cdot 1\right) = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x^2+y}{y^2+x}\right)$$

Substituting these in (A), we get

$$\frac{du}{dx} = (1 + \log xy) + \frac{x}{y}\left\{-\left(\frac{x^2+y}{y^2+x}\right)\right\}$$

35. (B) The given function is homogeneous of degree 2.

Euler's theorem $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$

36. (C) $f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$

Clearly, $f'(x) > 0$ when $x < 2$ and also when $x > 3$.

$f(x)$ is increasing in $]-\infty, 2[\cup]3, \infty[$.

37. (B) $f'(x) = \frac{(x^2+1) - 2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

Clearly, $(x^2+1)^2 > 0$ for all x.

So, $f'(x) > 0 \quad \Rightarrow \quad (1-x^2) > 0$

$\Rightarrow \quad (1-x)(1+x) > 0$

This happens when $-1 < x < 1$.

So, $f(x)$ is increasing in $]-1, 1[$.

38. (A) $f'(x) = 4x^3 - 4x = 4x(x-1)(x+1)$.

Clearly, $f'(x) < 0$ when $x < -1$ and also when $x > 1$.

Sol. $f(x)$ is decreasing in $]-\infty, -1[\cup]1, \infty[$.

39.(C) $f'(x) = 9x^8 + 21x^6 > 0$ for all non-zero real values of x.

40. (C) $f'(x) = 3kx^2 - 18x + 9 = 3[kx^2 - 6x + 3]$

This is positive when $k > 0$ and $36 - 12k < 0$ or $k > 3$.

41. (A) $f(x) = (e^{ax} + e^{-ax}) = 2\cosh ax$.

$f'(x) = 2a \sinh ax < 0$ When $x > 0$ because $a < 0$

42. (D) $f'(x) = -x^2e^{-x} + 2xe^{-x} = e^{-x}x(2-x)$.

Clearly, $f'(x) > 0$ when $x > 0$ and $x < 2$.

43. (B) $f'(x) = (2x + a)$

$1 < x < 2 \quad \Rightarrow \quad 2 < 2x < 4 \quad \Rightarrow \quad 2 + a < 2x + a < 4 + a$

$\Rightarrow \quad (2+a) < f'(x) < (4+a)$.

For $f(x)$ increasing, we have $f'(x) > 0$.

$\therefore 2 + a \geq 0$ or $a \geq -2$. So, least value of a is -2.

44. (B) Let the point closest to (4, 2) be $(2t^2, 4)$.

Now, $D = \sqrt{(2t^2-4)^2 + (4t-2)^2}$ is minimum when

$E = (2t^2-4)^2 + (4t-2)^2$ is minimum.

Now, $E = 4t^4 - 16t + 20$

$\Rightarrow \quad \frac{dE}{dt} = 16t^3 - 16 = 16(t-1)(t^2+t+1)$

$\frac{dE}{dt} = 0 \quad \Rightarrow \quad t = 1$

$\frac{d^2E}{dt^2} = 48t^2$. So, $\left[\frac{d^2E}{dt^2}\right]_{(t=1)} = 48 > 0$.

So, $t = 1$ is a point of minima.

Thus Minimum distance $= \sqrt{(2-4)^2 + (4-2)^2} = 2\sqrt{2}$.

45. (A) Let the required point be $P(x, y)$. Then, perpendicular distance of $P(x, y)$ from $y - 3x + 3 = 0$ is

$$p = \frac{|y - 3x + 3|}{\sqrt{10}} = \frac{|x^2 + 7x + 2 - 3x + 3|}{\sqrt{10}}$$

$$= \frac{|x^2 + 4x + 5|}{\sqrt{10}} = \frac{|(x+2)^2 + 1|}{\sqrt{10}} \text{ or } p = \frac{(x+2)^2+1}{\sqrt{10}}$$

So, $\frac{dp}{dx} = \frac{2(x+2)}{\sqrt{10}}$ and $\frac{d^2p}{dx^2} = \frac{2}{\sqrt{10}}$

$\frac{dp}{dx} = 0 \quad \Rightarrow \quad x = -2$, Also, $\left(\frac{d^2p}{dx^2}\right)_{x=-2} > 0$.

So, $x = -2$ is a point of minima.

When $x = -2$, we get $y = (-2)^2 + 7 \times (-2) + 2 = -8$.

The required point is $(-2, -8)$.

46. (C) Let $A(0, c)$ be the given point and $P(x, y)$ be any point on $y = x^2$.

$D = \sqrt{x^2 + (y-c)^2}$ is shortest when $E = x^2 + (y-c)^2$ is shortest.

Now,

$E = x^2 + (y-c)^2 = y + (y-c)^2 \Rightarrow E = y^2 + y - 2cy + c^2$

$\frac{dE}{dy} = 2y + 1 - 2c$ and $\frac{d^2E}{dy^2} = 2 > 0$.

$\frac{dE}{dy} = 0 \quad \Rightarrow \quad y = \left(c - \frac{1}{2}\right)$

Thus E minimum, when $y = \left(c - \frac{1}{2}\right)$

Also, $D = \sqrt{\left(c - \frac{1}{2}\right) + \left(c - \frac{1}{2} - c\right)^2} \quad \left[\because x^2 = y = \left(c - \frac{1}{2}\right)\right]$

$= \sqrt{c - \frac{1}{4}} = \frac{\sqrt{4c-1}}{2}$

47. (B) Let $y = \left(\frac{1}{x}\right)^x$ then, $y = x^{-x}$

$\Rightarrow \quad \frac{dy}{dx} = -x^{-x}(1 + \log x)$

$\frac{d^2y}{dx^2} = x^{-x}(1 + \log x)^2 - x^{-x} \cdot \frac{1}{x}$

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad 1 + \log x = 0 \quad \Rightarrow \quad x = \frac{1}{e}$

$\left[\frac{d^2y}{dx^2}\right]_{\left(x=\frac{1}{e}\right)} = -\left(\frac{1}{e}\right)^{-\frac{1}{e}-1} < 0.$

So, $x = \frac{1}{e}$ is a point of maxima. Maximum value $= e^{1/e}$.

48. (A) $f'(x) = 2x - \frac{250}{x^2}$ and $f''(x) = \left(2 + \frac{500}{x^3}\right)$

$f'(x) = 0 \quad \Rightarrow \quad 2x - \frac{250}{x^2} = 0 \quad \Rightarrow \quad x = 5.$

$f''(5) = 6 > 0$. So, $x = 5$ is a point of minima.

Thus minimum value $= \left(25 + \frac{250}{5}\right) = 75.$

49. (D) $f'(x) = (2\cos x - 1)(\cos x + 1)$ and

$f''(x) = -\sin x(1 + 4\cos x)$.

$f'(x) = 0 \quad \Rightarrow \quad \cos x = \frac{1}{2}$ or $\cos x = -1 \quad \Rightarrow \quad x = \pi/3$ or $x = \pi$.

$f''\left(\frac{\pi}{3}\right) = \frac{-3\sqrt{3}}{2} < 0$. So, $x = \pi/3$ is a point of maxima.

Maximum value $= \left(\sin\frac{\pi}{3}\right)\left(1 + \cos\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{4}$.

50. (C) $f(x) = \frac{2\sin x \cos x}{\frac{\sin x + \cos x}{\sqrt{2}}}$

$= \frac{2\sqrt{2}}{(\sec x + \operatorname{cosec} x)} = \frac{2\sqrt{2}}{z}$ (say),

where $z = (\sec x + \operatorname{cosec} x)$.

$\frac{dz}{dx} = \sec x \tan x - \operatorname{cosec} x \cot x = \frac{\cos x}{\sin^2 x}(\tan^3 x - 1).$

$\frac{dz}{dx} = 0 \quad \Rightarrow \quad \tan x = 1 \quad \Rightarrow \quad x = \frac{\pi}{4}$ in $\left[0, \frac{\pi}{2}\right]$.

Sign of $\frac{dz}{dx}$ changes from $-ve$ to $+ve$ when x passes through the point $\pi/4$. So, z is minimum at $x = \pi/4$ and therefore, $f(x)$ is maximum at $x = \pi/4$.

Maximum value $= \frac{2\sqrt{2}}{[\sec(\pi/4) + \operatorname{cosec}(\pi/4)]} = 1.$

51. (C) $\frac{dy}{dx} = \frac{a}{x} + 2bx + 1$

$\left[\frac{dy}{dx}\right]_{(x=-1)} = 0 \quad \Rightarrow \quad -a - 2b + 1 = 0 \quad \Rightarrow \quad a + 2b = 1$....(i)

$\left[\frac{dy}{dx}\right]_{(x=2)} = 0 \quad \Rightarrow \quad \frac{a}{2} + 4b + 1 = 0$

$\Rightarrow \quad a + 8b = -2$....(ii)

Solving (i) and (ii) we get $b = -\frac{1}{2}$ and $a = 2$.

52. (C) The given curve is $\frac{x^2}{5} + \frac{y^2}{4} = 1$ which is an ellipse.

Let the required point be $(\sqrt{5}\cos\phi,\ 2\sin\phi)$. Then,

$D = \sqrt{(\sqrt{5}\cos\phi - 0)^2 + (2\sin\phi + 2)^2}$ is maximum

when $z = D^2$ is maximum

$z = 5\cos^2\phi + 4(1 + \sin\phi)^2$

$\Rightarrow \quad \frac{dz}{d\phi} = -10\cos\phi\sin\phi + 8(1 + \sin\phi)\cos\phi$

$\frac{dz}{d\phi} = 0 \quad \Rightarrow \quad 2\cos\phi(4 - \sin\phi) = 0$

$\Rightarrow \quad \cos\phi = 0 \quad \Rightarrow \quad \phi = \frac{\pi}{2}.$

$\frac{dz}{d\phi} = -\sin 2\phi + 8\cos\phi \quad \Rightarrow \quad \frac{d^2z}{d\phi^2} = -2\cos 2\phi - 8\sin\phi$

when $\phi = \frac{\pi}{2}$, $\frac{d^2z}{d\phi^2} < 0$.

z is maximum when $\phi = \frac{\pi}{2}$. So, the required point is $\left(\sqrt{5}\cos\frac{\pi}{2},\ \sin\frac{\pi}{2}\right)$ i.e. (0, 2).

53. (D) Let $z = \frac{1 + \tan x}{x} = \frac{1}{x} + \frac{\tan x}{x}$

Then, $\frac{dz}{dx} = -\frac{1}{x^2} + \sec^2 x$ and $\frac{d^2z}{dx^2} = \frac{2}{x^3} + 2\sec^2 x \tan x$

$\frac{dz}{dx} = 0 \quad \Rightarrow \quad -\frac{1}{x^2} + \sec^2 x = 0 \quad \Rightarrow \quad x = \cos x.$

$\left[\frac{d^2z}{dx^2}\right]_{x=\cos x} = 2\cos^3 x + 2\sec^2 x \tan x > 0.$

Thus z has a minima and therefore y has a maxima at $x = \cos x$.

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CHAPTER

9.3

INTEGRAL CALCULUS

1. $\int \frac{x}{x^2+1}\,dx$ is equal to

(A) $\frac{1}{2}\log(x^2+1)$ (B) $\log(x^2+1)$

(C) $\tan^{-1}\frac{x}{2}$ (D) $2\tan^{-1}x$

2. If $F(a)=\frac{1}{\log a}$, $a>1$ and $F(x)=\int a^2dx+K$ is equal to

(A) $\frac{1}{\log a}(a^x-a^a+1)$ (B) $\frac{1}{\log a}(a^x-a^a)$

(C) $\frac{1}{\log a}(a^x+a^a+1)$ (D) $\frac{1}{\log a}(a^x+a^a-1)$

3. $\int \frac{dx}{1+\sin x}$ is equal to

(A) $-\cot x+\operatorname{cosec} x+c$ (B) $\cot x+\operatorname{cosec} x+c$

(C) $\tan x-\sec x+c$ (D) $\tan x+\sec x+c$

4. $\int \frac{(3x+1)}{2x^2-2x+3}\,dx$ is equal to

(A) $\frac{3}{4}\log(2x^2-2x+3)+\frac{\sqrt{5}}{2}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)$

(B) $\frac{4}{3}\log(2x^2-2x+3)+\sqrt{5}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)$

(C) $\frac{4}{3}\log(2x^2-2x+3)+\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)$

(D) $\frac{3}{4}\log(2x^2-2x+3)+\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)$

5. $\int \frac{dx}{1+3\sin^2 x}$ is equal to

(A) $\frac{1}{2}\tan^{-1}(\tan x)$ (B) $2\tan^{-1}(\tan x)$

(C) $\frac{1}{2}\tan^{-1}(2\tan x)$ (D) $2\tan^{-1}\left(\frac{1}{2}\tan x\right)$

6. $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}\,dx$ is equal to

(A) $\frac{9}{25}x+\frac{1}{25}\log(3\sin x+4\cos x)$

(B) $\frac{18}{25}x+\frac{2}{25}\log(3\sin x+4\cos x)$

(C) $\frac{18}{25}x+\frac{1}{25}\log(3\sin x+4\cos x)$

(D) None of these

7. $\int\sqrt{3+8x-3x^2}\,dx$ is equal to

(A) $\frac{3x-4}{3\sqrt{3}}\sqrt{3+8x-3x^2}-\frac{25}{18\sqrt{3}}\sin^{-1}\left(\frac{3x-4}{5}\right)$

(B) $\frac{3x-4}{6}\sqrt{3+8x-3x^2}+\frac{25\sqrt{3}}{18}\sin^{-1}\left(\frac{3x-4}{5}\right)$

(C) $\frac{3x-4}{6\sqrt{3}}\sqrt{3+8x-3x^2}-\frac{25}{18\sqrt{3}}\sin^{-1}\left(\frac{3x-4}{5}\right)$

(D) None of these

8. $\int \frac{dx}{\sqrt{2x^2+3x+4}}$ is equal to

(A) $\frac{1}{\sqrt{2}}\sin^{-1}\frac{4x+3}{\sqrt{23}}$ (B) $\frac{1}{\sqrt{2}}\sinh^{-1}\frac{4x+3}{\sqrt{23}}$

(C) $\frac{1}{\sqrt{2}}\cosh^{-1}\frac{4x+3}{\sqrt{23}}$ (D) None of these

9. $\int \frac{2x+3}{\sqrt{x^2+x+1}}\,dx$ is equal to

(A) $2\sqrt{x^2+x+1}+2\sinh^{-1}\frac{2x+1}{\sqrt{3}}$

(B) $\sqrt{x^2+x+1}+2\sinh^{-1}\frac{2x+1}{\sqrt{3}}$

(C) $2\sqrt{x^2+x+1}+\sinh^{-1}\frac{2x+1}{\sqrt{3}}$

(D) $2\sqrt{x^2+x+1}-\sinh^{-1}\frac{2x+1}{\sqrt{3}}$

10. $\int \frac{dx}{\sqrt{x-x^2}}$ is equal to

(A) $\sqrt{x-x^2}+c$ (B) $\sin^{-1}(2x-1)+c$

(C) $\log(2x-1)+c$ (D) $\tan^{-1}(2x-1)+c$

11. $\int \frac{1}{(x+1)\sqrt{1-2x-x^2}}\,dx$ is equal to

(A) $\sqrt{2}\cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$ (B) $\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$

(C) $-\sqrt{2}\cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$ (D) $-\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$

12. $\int \frac{dx}{\sin x+\cos x}$ is equal to

(A) $\frac{1}{\sqrt{2}}\log\tan\left(x+\frac{\pi}{4}\right)$ (B) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)$

(C) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)$ (D) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{4}+\frac{\pi}{4}\right)$

13. $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ is equal to

(A) $\sin(x-a)\log\sin(x-b)$

(B) $\log\sin\left(\frac{x-a}{x-b}\right)$

(C) $\sin(a-b)\log\left\{\frac{\sin(x-a)}{\sin(x-b)}\right\}$

(D) $\frac{1}{\sin(a-b)}\log\left\{\frac{\sin(x-a)}{\sin(x-b)}\right\}$

14. $\int \frac{dx}{e^x-1}$ is equal to

(A) $\log(e^x-1)$ (B) $\log(1-e^x)$

(C) $\log(e^{-x}-1)$ (D) $\log(1-e^x)$

15. $\int \frac{dx}{1+x+x^2+x^3}$ is equal to

(A) $\frac{1}{2}\left[\log\frac{(x+1)^2}{x^2+1}+\tan^{-1}x\right]$

(B) $\frac{1}{4}\left[\log\frac{(x+1)^2}{x^2+1}+2\tan^{-1}x\right]$

(C) $\frac{1}{2}\left[\log\frac{(x+1)^2}{x^2+1}-2\tan^{-1}x\right]$

(D) None of these

16. $\int \frac{\sin x}{1-\sin x}\,dx$ is equal to

(A) $-x+\sec x+\tan x+k$ (B) $-x+\sec x+\tan x$

(C) $-x+\sec x-\tan x$ (D) $-x-\sec x-\tan x$

17. $\int e^x\{f(x)+f'(x)\}dx$ is equal to

(A) $e^x f'(x)$ (B) $e^x f(x)$

(C) $e^x+f(x)$ (D) None of these

18. The value of $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ is

(A) $e^x\tan\frac{x}{2}+c$ (B) $e^x\cot\frac{x}{2}+c$

(C) $e^x\tan x+c$ (D) $e^x\cot x+c$

19. $\int \frac{x^3}{x^2+1}\,dx$ is equal to

(A) $x^2+\log(x^2+1)+c$

(B) $\log(x^2+1)-x^2+c$

(C) $\frac{1}{2}x^2-\frac{1}{2}\log(x^2+1)+c$

(D) $\frac{1}{2}x^2+\frac{1}{2}\log(x^2+1)+c$

20. $\int \sin^{-1}x\,dx$ is equal to

(A) $x\sin^{-1}x+\sqrt{1-x^2}+c$ (B) $x\sin^{-1}x-\sqrt{1-x^2}+c$

(C) $x\sin^{-1}x+\sqrt{1+x^2}+c$ (D) $x\sin^{-1}x-\sqrt{1-x^2}+c$

21. $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\,dx$ is equal to

(A) $\sin x$ (B) x

(C) $\cos x$ (D) $\tan x$

22. The value of $\int_0^1 |5x-3|\,dx$ is

(A) –1/2 (B) 13/10

(C) 1/2 (D) 23/10

39. $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\, dydx$ is equal to

(A) $\frac{7}{60}$ (B) $\frac{3}{35}$

(C) $\frac{4}{49}$ (D) None of these

40. The value of $\int_0^1 \int_0^{\sqrt{1+x^2}} dy\, dx$ is

(A) $\frac{\pi}{4} \log(\sqrt{2}+1)$ (B) $\frac{\pi}{4} \log(\sqrt{2}-1)$

(C) $\frac{\pi}{2} \log(\sqrt{2}+1)$ (D) None of these

41. If A is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$, then $\iint_A ydxdy$ is equal to

(A) $\frac{48}{5}$ (B) $\frac{36}{5}$

(C) $\frac{32}{5}$ (D) None of these

42. The area of the region bounded by the curves $x^2 + y^2 = a^2$ and $x + y = a$ in the first quadrant is given by

(A) $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} dxdy$ (B) $\int_0^a \int_0^{\sqrt{a^2-x^2}} dxdy$

(C) $\int_{a-x}^{\sqrt{a^2-y^2}} \int_0^a dxdy$ (D) None of these

43. The area bounded by the curves $y = 2\sqrt{x}$, $y = -x$, $x = 1$ and $x = 4$ is given by

(A) 25 (B) $\frac{33}{2}$

(C) $\frac{47}{4}$ (D) $\frac{101}{6}$

44. The area bounded by the curves $y^2 = 9x$, $x - y + 2 = 0$ is given by

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{3}{2}$ (D) $\frac{5}{4}$

45. The area of the cardioid $r = a(1 + \cos\theta)$ is given by

(A) $2\int_{\theta=0}^{\pi} \int_{r=0}^{a(1+\cos\theta)} rdrd\theta$ (B) $2\int_0^{\pi} \int_{r=a}^{a(1+\cos\theta)} rdrd\theta$

(C) $2\int_0^{\pi/2} \int_{r=0}^{a(1+\cos\theta)} rdrd\theta$ (D) $2\int_0^{\pi/4} \int_{r=0}^{a(1+\cos\theta)} rdrd\theta$

46. The area bounded by the curve $r = \theta\cos\theta$ and the lines $\theta = 0$ and $\theta = \frac{\pi}{2}$ is given by

(A) $\frac{\pi}{4}\left(\frac{\pi^2}{16} - 1\right)$ (B) $\frac{\pi}{16}\left(\frac{\pi^2}{6} - 1\right)$

(C) $\frac{\pi}{16}\left(\frac{\pi^2}{16} - 1\right)$ (D) None of these

47. The area of the lemniscate $r^2 = a^2\cos 2\theta$ is given by

(A) $4\int_0^{\pi/4} \int_0^{a\sqrt{\cos 2\theta}} rdrd\theta$ (B) $2\int_0^{\pi/2} \int_0^{a\sqrt{\cos 2\theta}} rdrd\theta$

(C) $4\int_0^{\pi/2} \int_0^{a\sqrt{\cos 2\theta}} rdrd\theta$ (D) $2\int_0^{\pi} \int_0^{a\sqrt{\cos 2\theta}} rdrd\theta$

48. The area of the region bounded by the curve $y(x^2 + 2) = 3x$ and $4y = x^2$ is given by

(A) $\int_0^1 \int_{y=0}^{x^2/4} dxdy$ (B) $\int_0^1 \int_{y=0}^{x^2/4} dydx$

(C) $\int_0^2 \int_{y=x^2/4}^{3x/(x^2+2)} dydx$ (D) $\int_{y=0}^1 \int_{y=x^2/4}^{3x/(x^2+2)} dxdy$

49. The volume of the cylinder $x^2 + y^2 = a^2$ bounded below by $z = 0$ and bounded above by $z = h$ is given by

(A) πah (B) $\pi a^2 h$

(C) $\frac{1}{3}\pi a^3 h$ (D) None of these

50. $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dxdydz$ is equal to

(A) $(e-1)^3$ (B) $\frac{3}{2}(e-1)$

(C) $(e-1)^2$ (D) None of these

51. $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$ is equal to

(A) 4 (B) –4

(C) 0 (D) None of these

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SOLUTIONS

1. (A) $\int \frac{x}{x^2+1}dx$

Put $x^2+1=t \Rightarrow 2xdx=dt$

$\int \frac{x}{x^2+1}dx = \int \frac{1}{2}\cdot\frac{1}{t}dt$

$=\frac{1}{2}\log t = \frac{1}{2}\log(x^2+1)$

2. (A) $F(x)=\int a^x dx + K = \frac{a^x}{\log a}+K$

$\Rightarrow \quad F(a)=\frac{a^a}{\log a}+K$

$K=\frac{1}{\log a}-\frac{a^a}{\log a}=\frac{1-a^a}{\log a}$

$F(x)=\frac{a^x}{\log a}+\frac{1-a^a}{\log a}=\frac{1}{\log a}[a^x-a^a+1]$

3. (C) $\int \frac{dx}{1+\sin x}$

$=\int \frac{dx}{\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)+2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=\int \frac{dx}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}=\int \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx$

Put $1+\tan\frac{x}{2}=t$

$\Rightarrow \quad \sec^2\frac{x}{2}dx=2dt \quad \Rightarrow \quad \int\frac{2dt}{t^2}dt=-\frac{2}{t}+K$

$=\frac{-2}{1+\tan\frac{x}{2}}+K=\frac{-2\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}+K$

$=\frac{-2\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\times\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}+K$

$=\frac{-2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}+K$

$=\frac{-(1+\cos x)+\sin x}{\cos x}+k=\tan x-\sec x-1+K$

$=\tan x-\sec x+c$

4. (A) Let $I=\int\frac{3x+1}{2x^2-2x+3}dx$

Let $3x+1=p(4x-2)+q \quad \Rightarrow \quad p=\frac{3}{4},\ q=\frac{5}{2}$

$I=\frac{3}{4}\int\frac{4x-2}{2x^2-2x+3}dx+\frac{5}{2}\int\frac{dx}{2x^2-2x+3}$

$=\frac{3}{4}\log(2x^2-2x+3)+\frac{5}{4}\int\frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2}$

$=\frac{3}{4}\log(2x^2-2x+3)+\frac{5}{4}\frac{1}{\left(\frac{\sqrt{5}}{2}\right)}\tan^{-1}\frac{x-\frac{1}{2}}{\frac{\sqrt{5}}{2}}$

5. (C) Let $I=\int\frac{dx}{1+3\sin^2 x}$

$=\int\frac{\text{cosec}^2 x\,dx}{\text{cosec}^2 x+3}=\int\frac{\text{cosec}^2 x\,dx}{(1+\cot^2 x)+3}$

Put $\cot x=t \quad \Rightarrow \quad -\text{cosec}^2 x\,dx=dt$

$I=\int\frac{-dt}{4+t^2}=\frac{1}{2}\cot^{-1}\frac{t}{2}=\frac{1}{2}\cot^{-1}\left(\frac{\cot x}{2}\right)$

$=\frac{1}{2}\tan^{-1}(2\tan x)$

6. (C) Let $I=\int\frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$

Let $(2\sin x+3\cos x)=p(3\cos x-4\sin x)$
$+q(3\sin x+4\cos x)$

$p=\frac{1}{25}\ ,\ q=\frac{18}{25}$

$I=\frac{1}{25}\int\frac{3\cos x-4\sin x}{3\sin x+4\cos x}dx+\frac{18}{25}\int\frac{3\sin x+4\cos x}{3\sin x+4\cos x}dx$

$=\frac{1}{25}\log(3\sin x+4\cos x)+\frac{18}{25}x$

7. (B) $\int\sqrt{3+8x-3x^2}dx=\sqrt{3}\int\sqrt{\left(\frac{5}{3}\right)^2-\left(x-\frac{4}{3}\right)^2}dx$

$=\sqrt{3}\frac{1}{2}\left\{\left(x-\frac{4}{3}\right)\sqrt{\left(\frac{5}{3}\right)^2-\left(x-\frac{4}{3}\right)^2}+\left(\frac{5}{3}\right)^2\sin^{-1}\left(\frac{x-\frac{4}{3}}{\frac{5}{3}}\right)\right\}$

$=\frac{3x-4}{6}\sqrt{3+8x-3x^2}+\frac{25\sqrt{3}}{18}\sin^{-1}\frac{3x-4}{5}$

8. (B) $\int\frac{dx}{\sqrt{2x^2+3x+4}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2}}$

$$= \frac{1}{\sqrt{2}} \sinh^{-1} \frac{x + \frac{3}{4}}{\left(\frac{\sqrt{23}}{4}\right)} = \frac{1}{\sqrt{2}} \sinh^{-1} \frac{4x+3}{\sqrt{23}}$$

9. (B) $\int \frac{2x+3}{\sqrt{x^2+x+1}}\, dx$

$$= \int \frac{2x+1}{\sqrt{x^2+x+1}}\, dx + \int \frac{2dx}{\sqrt{x^2+x+1}}$$

$$= \int \frac{2x+1}{\sqrt{x^2+x+1}}\, dx + 2\int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$= \frac{(x^2+x+1)^{1/2}}{\frac{1}{2}} + 2\sinh^{-1} \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= 2\sqrt{x^2+x+1} + 2\sinh^{-1} \frac{2x+1}{\sqrt{3}}$$

10. (B) $\int \frac{dx}{\sqrt{x}\sqrt{1-x}} = I$

Put $x = \sin^2\theta \;\Rightarrow\; dx = 2\sin\theta\cos\theta\, d\theta$

$$I = \int \frac{2\sin\theta\cos\theta}{\sin\theta\sqrt{1-\sin^2\theta}}\, d\theta = \int \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}\, d\theta$$

$$I = \int 2d\theta = 2\theta + c = 2\sin^{-1}\sqrt{x} + c$$

$$I = \sin^{-1}(2x-1) + c$$

11. (D) Let $I = \int \frac{1}{(x+1)\sqrt{1-2x-x^2}}\, dx$

Put $x + 1 = \frac{1}{t} \;\Rightarrow\; dx = -\frac{1}{t^2}\, dt$

$$I = \int \frac{-\frac{1}{t^2}\, dt}{\frac{1}{t}\sqrt{1 - 2\left(\frac{1}{t}-1\right) - \left(\frac{1}{t}-1\right)^2}} = -\int \frac{dt}{\sqrt{2t^2-1}}$$

$$= -\frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{t^2 - \left(\frac{1}{\sqrt{2}}\right)^2}} = -\frac{1}{\sqrt{2}} \cosh^{-1} \frac{t}{1/\sqrt{2}}$$

$$= -\frac{1}{\sqrt{2}} \cosh^{-1}\left(\frac{\sqrt{2}}{x+1}\right)$$

12. (C) $\int \frac{dx}{\sin x + \cos x}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sin\left(x+\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(x+\frac{\pi}{4}\right) dx$$

$$= \frac{1}{\sqrt{2}} \left[-\log\cot\frac{1}{2}\left(x+\frac{\pi}{4}\right)\right] = \frac{1}{\sqrt{2}} \log\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)$$

13. (D) $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)dx}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \times \int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int [\cot(x-a) - \cot(x-b)]dx$$

$$= \frac{1}{\sin(a-b)} [\log\sin(x-a) - \log\sin(x-b)]\, dx$$

$$= \frac{1}{\sin(a-b)} \log\left\{\frac{\sin(x-a)}{\sin(x-b)}\right\}$$

14. (D) Let $I = \int \frac{dx}{e^x - 1} = \int \frac{e^{-x}\, dx}{1-e^{-x}}$

Put $1 - e^{-x} = t \;\Rightarrow\; e^{-x}dx = dt$

$$I = \int \frac{dt}{t} = \log t = \log(1-e^{-x})$$

15. (B) Let $I = \int \frac{dx}{1+x+x^2+x^3}$

$$= \int \frac{dx}{(1+x)(1+x^2)}$$

Let $\frac{1}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$

$1 = A(1+x^2) + (Bx+C)(1+x)$

Comparing the coefficients of x^2, x and constant terms,

$A + B = 0,\; B + C = 0,\; C + A = 1$

Solving these equations, we get

$A = \frac{1}{2},\; B = -\frac{1}{2},\; C = \frac{1}{2}$

$$I = \frac{1}{2}\int \frac{1}{1+x}\, dx - \frac{1}{2}\int \frac{x-1}{x^2+1}\, dx$$

$$= \frac{1}{2}\log(1+x) - \frac{1}{2}\log(x^2+1) + \frac{1}{2}\tan^{-1} x$$

$$= \frac{1}{4}\left[\log\frac{(x+1)^2}{x^2+1} + 2\tan^{-1} x\right]$$

16. (B) Let $I = \int \frac{\sin x}{1-\sin x} dx$

$= \int \frac{1-(1-\sin x)}{1-\sin x} dx$

$= \int \frac{1}{1-\sin x} dx - \int dx = \int \frac{1+\sin x}{1-\sin^2 x} dx - x$

$= \int \frac{1+\sin x}{\cos^2 x} dx - x = \int (\sec^2 x + \sec x \tan x) dx - x$

$= \tan x + \sec x - x$

17. (B) Let $I = \int e^x \{f(x) + f'(x)\} dx$

$= \int e^x f(x) dx + \int e^x f'(x) dx$

$= \{f(x)e^x - \int f'(x) e^x dx\} + \int e^x f'(x)\, dx = f(x) \cdot e^x$

18. (A) Let $I = \int e^x \left(\frac{1+\sin x}{1+\cos x} \right) dx$

$= \int e^x \left(\frac{1 + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} \right) dx$

$= \frac{1}{2} \int e^x \sec^2 \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx$

$= \frac{1}{2} \left\{ e^x \cdot 2\tan\frac{x}{2} - \int e^x \cdot 2\tan\frac{x}{2} dx \right\} + \int e^x \tan \frac{x}{2} dx$

$= e^x \tan \frac{x}{2} + c$

19. (C) $I = \int \frac{x^3}{x^2+1} dx = \int \frac{x \cdot x^2}{x^2+1} dx$

$= \int \frac{x(x^2+1-1)}{x^2+1} dx = \int x dx - \int \frac{x}{x^2+1} dx$

$= \frac{1}{2} x^2 - \frac{1}{2} \log(x^2+1) + c$

20. (A) Let $I = \int \sin^{-1} x\, dx = \int \sin^{-1} x \cdot 1 \cdot dx$

$= \sin^{-1} x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x\, dx$

$= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$

In second part put $1 - x^2 = t^2$

$x dx = -t dt = x\sin^{-1} x + \int dt$

$= x\sin^{-1} x + t = x \sin^{-1} x + \sqrt{1-x^2} + c$

21. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx$

$= \int \frac{\sin x + \cos x}{\sqrt{(\sin^2 x + \cos^2 x) + 2\sin x \cos x}} dx$

$= \int \frac{\sin x + \cos x}{\sqrt{(\cos x + \cos x)^2}} dx$

$= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int dx = x$

22. (D) $\int_0^{3/5} |5x-3|\, dx = -\int_0^{3/5} |5x-3|\, dx + \int_{3/5}^{1} |5x-3|\, dx$

$= \left(-\frac{5}{2} x^2 + 3x \right)_0^{3/5} + \left(\frac{5x^2}{2} - 3x \right)_{3/5}^{1}$

$= \left(-\frac{9}{10} + \frac{9}{5} \right) + \left[\left(\frac{5}{2} - 3 \right) - \left(\frac{9}{10} - \frac{9}{5} \right) \right]$

$= \frac{9}{10} + \left(-\frac{1}{2} + \frac{9}{10} \right) = \frac{13}{10}$

23. (B) $\int_0^1 \frac{dx}{e^x + e^{-x}} = \int_0^1 \frac{e^x\, dx}{e^{2x}+1}$

Put $e^x = t \quad \Rightarrow \quad e^x dx = dt = \int_1^e \frac{dt}{t^2+1} = [\tan^{-1} t]_1^e$

$= \tan^{-1} e - \tan^{-1} 1 = \tan^{-1} e - \frac{\pi}{4}$

24. (D) $\int_0^c x(1-x)\, dx = \int_0^c (x - x^2) dx$

$= \left(\frac{1}{2} x^2 - \frac{1}{3} x^3 \right)_0^c = \frac{1}{6} c^2 (3 - 2c)$

$\int_0^c x(1-x) dx = 0 \quad \Rightarrow \quad \frac{1}{6} c^2 (3-2c) = 0$

$\Rightarrow \quad c = \frac{3}{2}$

25. (D) Put $x^2 + x = t \quad \Rightarrow \quad (2x+1) dx = dt$

$\int_0^1 \frac{2x+1}{\sqrt{x+x^2}} dx = \int_0^2 \frac{dt}{\sqrt{t}} = 2(t^{1/2})_0^2 = 2\sqrt{2}$

26. (A) $\int_{\pi}^{\pi} x^4 \sin^5 x dx$

Since, $f(-x) = (-x)^4 \sin^5 (-x) = -x^4 \sin^5 x$

$f(x)$ is odd function thus

$\int_{\pi}^{\pi} x^4 \sin^5 x\, dx = 0$

27. (A) $\int_0^{\pi/2} \cos^2 x\, dx = \int_0^{\pi/2} \frac{1}{2} (\cos 2x + 1)\, dx$

$$= \frac{1}{2}\left(\frac{1}{2}\sin 2x + x\right)_0^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{1}{2}(\sin \pi - \sin 0) + \left(\frac{\pi}{2} - 0\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}(0 - 0) - 0 + \frac{\pi}{2}\right] = \frac{\pi}{4}$$

Aliter 1. $\int_0^{\pi/2} \cos^2 x\, dx = \dfrac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\,\Gamma\left(\frac{4}{2}\right)} = \dfrac{\frac{1}{2}\pi}{2} = \dfrac{\pi}{4}$

Aliter 2. Use Walli's Rule $\int_0^{\pi/2} \cos^2 x = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$

28. (B) Let $I = \int_0^a \sqrt{a^2 - x^2}\, dx$

Put $x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$ when $x = 0, \theta = 0$, when $x = a, \theta = \frac{\pi}{2}$

$$I = \int_0^{\pi/2} \sqrt{a^2 - a^2\sin^2\theta}\; a\cos\theta\, d\theta$$

$$= a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta = a^2 \cdot \frac{1}{2}\cdot\frac{\pi}{2} \quad \text{(By Walli's Formula)}$$

$$= \frac{\pi a^2}{4}$$

Aliter: $\int_0^a \sqrt{a^2 - x^2}\,dx$

$$= \left[\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \sin^{-1}\frac{x}{a}\right]_0^a = \left[0 + \frac{\pi a^2}{4}\right] = \frac{\pi a^2}{4}$$

29. (D) Let $I = \int_0^{\pi/2} \log(\tan x)\, dx$ ....(1)

$$I = \int_0^{\pi/2} \log\tan\left(\frac{\pi}{2} - x\right) dx$$

$$I = \int_0^{\pi/2} \log(\cot x) \text{....(2)}$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi/2} [\log(\tan x) + \log(\cot x)]dx$$

$$= \int_0^{\pi/2} \log(\tan x \cdot \cot x)\, dx$$

$$= \int_0^{\pi/2} \log 1\, dx = 0 \Rightarrow I = 0$$

30. (D) Let $I = \int_0^1 2\sin\left(\frac{\pi t}{2} - \frac{\pi}{4}\right)dt$ ....(i)

$$= \int_0^1 2\sin\left(\frac{\pi}{2}(1-t) - \frac{\pi}{4}\right)dt = \int_0^1 2\sin\left(\frac{\pi}{4} - \frac{\pi}{2}t\right)dt$$

$$= -\int_0^1 2\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)dt = -1$$

$$2I = 0 \Rightarrow I = 0$$

31. (C) Let $I = \int_0^{2a} \dfrac{f(x)}{f(x) + f(2a - x)}\, dx$ ....(1)

$$I = \int_0^{2a} \frac{f(2a - x)}{f(2a - x) + f(x)}\, dx \text{....(2)}$$

Adding (1) and (2), we get

$$2I = \int_0^{2a} \frac{f(x) + f(2a - x)}{f(x) + f(2a - x)}\, dx = \int_0^{2a} 1\cdot dx = [x]_0^{2a} = 2a$$

$$\Rightarrow I = a$$

32. (C) Let $I = \int_0^1 \dfrac{e^{\sqrt{1-x^2}}}{\sqrt{1 - x^2}}\, xdx$

Put $\sqrt{1 - x^2} = t$

$$\Rightarrow \frac{1}{2\sqrt{1 - x^2}}(-2x)\, dx = dt$$

when $x = 0, t = 1$, when $x = 1, t = 0$

$$I = \int_1^0 -e^t dt = -[e^t]_1^0 = -[e^0 - e^1] = e - 1$$

33. (B) Let $I = \int_0^1 \dfrac{dx}{1 - x + x^2}$

$$= \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\frac{\sqrt{3}}{2}}\left[\tan^{-1}\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]_0^1$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{1}{\sqrt{3}} - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right] = \frac{2}{\sqrt{3}}\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$$

$$= \frac{2\pi}{3\sqrt{3}} = \frac{2\pi\sqrt{3}}{9}$$

34. (B) Let $I = \int_{-1}^1 \dfrac{|x|}{x}\, dx = \int_{-1}^0 \dfrac{-x}{x}\, dx + \int_0^1 \dfrac{x}{x}\, dx$

$$= \int_{-1}^0 -1dx + \int_0^1 1\cdot dx = -[x]_{-1}^0 + [x]_0^1$$

$$= -[0 - (-1)] + [1 - 0] = 0$$

35. (C) $\int_0^{100\pi} |\sin x| dx = 100\int_0^{\pi} |\sin x| dx$

[... sin x is periodic with period π]

$$=100\int_0^{\pi}\sin x\,dx=100(-\cos x)_0^{\pi}$$

$$=100(-\cos\pi+\cos 0)=100(1+1)=200.$$

36. (C) Let $I=\int_0^{\pi}\cos^m x\sin nx\,dx=\int_0^{\pi}f(x)\,dx$

Where $f(x)=\cos^m x\sin^n x$

$f(\pi-x)=\cos^m(\pi-x)\sin^n(\pi-x)$

$=(-\cos x)^m(\sin x)^n$

$=-\cos^m x\sin^n x$, if m is odd

$I=\int_0^{\pi}\cos^m x\sin^n x\,dx=0$, if m is odd

37. (A) Let $I=\int_0^{\pi}xF(\sin x)\,dx$ ....(1)

$$=\int_0^{\pi}(x-\pi)F[\sin(\pi-x)]dx$$

$$I=\int_0^{\pi}(\pi-x)F(\sin x)dx \text{ ....(2)}$$

Adding (1) and (2), we get

$$2I=\int_0^{\pi}\pi F(\sin x)dx$$

$$\Rightarrow\quad I=\frac{1}{2}\int_0^{\pi}\pi F(\sin x)\,dx$$

38. (B) Let $I=\int_0^{\pi/2}\frac{e^x}{2}\left(\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right)dx$

$$=\int_0^{\pi/2}\frac{1}{2}e^x\sec^2\frac{x}{2}\,dx+\int_0^{\pi/2}e^x\tan\frac{x}{2}\,dx=I_1+I_2$$

Here, $I_1=\int_0^{\pi/2}\frac{1}{2}e^x\sec^2\frac{x}{2}\,dx$

$$=\left[\frac{1}{2}e^x\cdot 2\tan\frac{x}{2}\right]_0^{\pi/2}-\frac{1}{2}\int_0^{\pi/2}e^x\cdot 2\tan\frac{x}{2}\,dx$$

$$=\left(e^{\pi/2}\tan\frac{\pi}{4}-0\right)-\int_0^{\pi/2}e^x\tan\frac{x}{2}\,dx$$

$=e^{\pi/2}-I_2$, $I_1+I_2=e^{\pi/2}$

$I=I_1+I_2=e^{\pi/2}$

39. (B) $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dy\,dx=\int_0^1\left[x^2y+\frac{1}{3}y^3\right]_x^{\sqrt{x}}dx$

$$=\int_0^1\left[x^{5/2}+\frac{1}{3}x^{3/2}-x^3-\frac{1}{3}x^3\right]dx$$

$$=\left[\frac{2}{7}x^{7/2}+\frac{2}{15}x^{5/2}-\frac{1}{3}x^4\right]_0^1=\frac{3}{35}$$

40. (D) $\int_0^1\int_0^{\sqrt{1+x^2}}dydx=\int_0^1[y]_0^{\sqrt{1+x^2}}\,dx$

$$=\int_0^1\sqrt{1+x^2}dx$$

$$=\frac{1}{2}[x\sqrt{1+x^2}+\log(x+\sqrt{1+x^2})]_0^1$$

$$=\frac{1}{2}[\sqrt{2}+\log(1+\sqrt{2})]$$

41. (A) Let $I=\iint_A ydxdy$,

Solving the given equations $y^2=4x$ and $x^2=4y$, we get $x=0$, $x=4$. The region of integration A is given by

$$A=\int_0^4\int_{x^2/4}^{2\sqrt{x}}ydydx=\int_0^4\left[\frac{y^2}{2}\right]_{x^2/4}^{2\sqrt{x}}dx$$

$$=\int_0^4\frac{1}{2}\left(4x-\frac{x^4}{10}\right)dx=\left[x^2-\frac{x^5}{160}\right]_0^4=\frac{48}{5}$$

42. (A) The curves are

$x^2+y^2=a^2$... ....(i)

$x+y=a$... ....(ii)

The curves (i) and (ii) intersect at A (a, 0) and B (0,a)

The required area $A=\int_{x=0}^{a}\int_{y=a-x}^{\sqrt{a^2-x^2}}dydx$

43. (D) The given equations of the curves are

$y=2\sqrt{x}$ i.e., $y^2=4x$....(i) $\quad y=-x$....(ii)

If a figure is drawn then from fig. the required area is

$$A=\int_1^4\int_{-x}^{2\sqrt{x}}dydx=\int_1^4[y]_{-x}^{2\sqrt{x}}=\int_1^4[2\sqrt{x}+x]dx$$

$$=\left(\frac{32}{3}+8\right)-\left(\frac{4}{3}+\frac{1}{2}\right)=\frac{101}{6}$$

44. (B) The equations of the given curves are

$y^2=9x$....(i) $\quad x-y+2=0$....(ii)

The curves (i) and (ii) intersect at

A(1, 3) and B(4, 6)

If a figure is drawn then from fig. the required area is

$$A=\int_1^4\int_{x+2}^{3\sqrt{x}}dydx=\int_1^4[y]_{x+2}^{3\sqrt{x}}dx$$

$$=\int_1^4[3\sqrt{x}-(x+2)]dx=\left[2x^{3/2}-\frac{1}{2}x^2-2x\right]_1^4$$

$$=(16-8-8)-\left(2-\frac{1}{2}-2\right)=\frac{1}{2}$$

45. (A) The equation of the cardioid is

$r = a\,(1+\cos\theta)$ ....(i)

If a figure is drawn then from fig. the required area is

Required area $A = 2\int_{\theta=0}^{\pi}\int_{r=0}^{a(1+\cos\theta)} r\,dr\,d\theta$

46. (C) The equation of the given curve is

$r = \theta\cos\theta$....(i)

The required area

$$A = \int_{\theta=0}^{\pi/2}\int_{r=0}^{\theta\cos\theta} r\,dr\,d\theta = \int_0^{\pi/2}\left[\frac{1}{2}r^2\right]_o^{\theta\cos\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2}\theta^2\cos^2\theta\,d\theta = \frac{1}{4}\int_0^{\pi/2}\theta^2(1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{4}\int_0^{\pi/2}\theta^2 d\theta + \frac{1}{4}\int_0^{\pi/2}\theta^2\cos 2\theta\,d\theta$$

$$= \frac{1}{4}\left[\frac{1}{3}\theta^3\right]_0^{\pi/2} + \frac{1}{4}\left[\left(\theta^2\frac{\sin 2\theta}{2}\right)_0^{\pi/2} - \int_0^{\pi/2} 2\theta\frac{\sin 2\theta}{2}\,d\theta\right]$$

$$= \frac{\pi^3}{96} + \frac{1}{4}\left[-\int_0^{\pi/2}\theta\sin 2\theta\,d\theta\right]$$

$$= \frac{\pi^3}{96} - \frac{1}{4}\left[\left(-\theta\frac{\cos 2\theta}{2}\right)_0^{\pi/2} - \int_0^{\pi/2}\left(-\frac{\cos 2\theta}{2}\right)d\theta\right]$$

$$= \frac{\pi^3}{96} + \frac{1}{4}\left(\frac{-\pi}{4} - 0\right) - \frac{1}{8}\int_0^{\pi/2}\cos 2\theta\,d\theta$$

$$= \frac{\pi^3}{96} - \frac{\pi}{16} - \frac{1}{8}\left(\frac{1}{2}\sin 2\theta\right)_0^{\pi/2} = \frac{\pi}{16}\left(\frac{\pi^2}{16} - 1\right)$$

47. (A) The curve is $r^2 = a^2\cos 2\theta$

If a figure is drawn then from fig. the required area is

$$A = 4\int_{\theta=0}^{\pi/4}\int_{r=0}^{a\sqrt{\cos 2\theta}} r\,dr\,d\theta = 4\int_0^{\pi/4}\left[\frac{1}{2}r^2\right]_0^{a\sqrt{\cos 2\theta}} d\theta$$

$$= 2\int_0^{\pi/4} a^2\cos 2\theta\,d\theta = 2a^2\left[\frac{\sin 2\theta}{2}\right]_0^{\pi/4} = a^2$$

48. (C) The equations of given curves are

$y(x^2+2) = 3x$....(i) and $4y = x^2$....(ii)

The curve (i) and (ii) intersect at A (2, 1).

If a figure is drawn then from fig. the required area is

The required area $A = \int_{x=0}^{2}\int_{y=x^2/4}^{3x/(x^2+2)} dx\,dy$

49. (B) The equation of the cylinder is $x^2 + y^2 = a^2$

The equation of surface CDE is $z = h$.

If a figure is drawn then from fig. the required area is

Thus the equation volume is $V = 4\int_A z\,dx\,dy$

$$= 4\int_0^a\int_0^{\sqrt{a^2-x^2}} h\,dy\,dx = 4h\int_0^a [y]_0^{\sqrt{a^2-x^2}} dx = 4h\int_0^a\sqrt{a^2-x^2}\,dx$$

Let $x = a\sin\theta$, $\Rightarrow$ $dx = a\cos\theta\,d\theta$,

Volume $V = 4h\int_0^{\pi/2}\sqrt{a^2 - a^2\sin^2\theta}\cdot a\cos\theta\,d\theta$

$$= 4ha^2\int_0^{\pi/2}\cos^2\theta\,d\theta = 4ha^2\cdot\frac{1}{2}\cdot\frac{\pi}{2} = \pi a^2 h.$$

50. (A) $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dx\,dy\,dz$

$$= \int_0^1\int_0^1 [e^{x+y+z}]_0^1\,dy\,dz = \int_0^1\int_0^1 [e^{1+y+z} - e^{y+z}]\,dy\,dz$$

$$= \int_0^1 [e^{1+y+z} - e^{y+z}]_0^1\,dz$$

$$= \int_0^1 [(e^{2+z} - e^{1+z}) - (e^{1+z} - e^z)]\,dz$$

$$= \int_0^1 (e^{2+z} - 2e^{1+z} + e^z)\,dz = [e^{2+z} - 2e^{1+z} + e^z]_0^1$$

$$= (e^3 - 2e^2 + e) - (e^2 - 2e + 1)$$

$$= e^3 - 3e^2 + 3e - 1 = (e-1)^3$$

51. (C) $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$

$$= \int_{-1}^{1}\int_0^z\left[\frac{(x+y+z)^2}{2}\right]_{x-y}^{x+z} dx\,dz$$

$$= \int_{-1}^{1}\int_0^z\left[\frac{(2x+2z)^2}{2} - \left(\frac{2x}{2}\right)^2\right] dx\,dz$$

$$= 2\int_{-1}^{1}\left[\int_0^3((x+z)^2 - x^2)\,dx\right]dz = 2\int_{-1}^{1}\left[\frac{(x+z)^3}{3} - \frac{x^3}{3}\right]_0^z dz$$

$$= \frac{2}{3}\int_{-1}^{1}[(2z)^3 - z^3 - z]\,dz = \frac{2}{3}\int_{-1}^{1} 6z^3\,dz = 4\left[\frac{z^4}{4}\right]_{-1}^{1}$$

$$= 4\left(\frac{1}{4} - \frac{1}{4}\right) = 0$$

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10. The integration of $f(z) = x^2 + ixy$ from A(1, 1) to B(2, 4) along the straight line AB joining the two points is

(A) $\frac{-29}{3} + i11$ (B) $\frac{29}{3} - i11$

(C) $\frac{23}{5} + i6$ (D) $\frac{23}{5} - i6$

11. $\int_c \frac{e^{2z}}{(z+1)^4} dz = ?$ where c is the circle of $|z| = 3$

(A) $\frac{4\pi i}{9} e^{-3}$ (B) $\frac{4\pi i}{9} e^{3}$

(C) $\frac{4\pi i}{3} e^{-1}$ (D) $\frac{8\pi i}{3} e^{-2}$

12. $\int_c \frac{1-2z}{z(z-1)(z-2)} dz = ?$ where c is the circle $|z| = 1.5$

(A) $2 + i6\pi$ (B) $4 + i3\pi$

(C) $1 + i\pi$ (D) $i3\pi$

13. $\int_c (z - z^2) dz = ?$ where c is the upper half of the circle $z = 1$

(A) $\frac{-2}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{-3}{2}$

14. $\int_c \frac{\cos \pi z}{z-1} dz = ?$ where c is the circle $|z| = 3$

(A) $i2\pi$ (B) $-i2\pi$

(C) $i6\pi^2$ (D) $-i6\pi^2$

15. $\int_c \frac{\sin \pi z^2}{(z-2)(z-1)} dz = ?$ where c is the circle $|z| = 3$

(A) $i6\pi$ (B) $i2\pi$

(C) $i4\pi$ (D) 0

16. The value of $\frac{1}{2\pi i}\int_c \frac{\cos \pi z}{z^2-1} dz$ around a rectangle with vertices at $2 \pm i$, $-2 \pm i$ is

(A) 6 (B) $i2e$

(C) 8 (D) 0

Statement for Q. 17–18:

$$f(z_0) = \int_c \frac{3z^2 + 7z + 1}{(z - z_0)} dz$$, where c is the circle $x^2 + y^2 = 4$.

17. The value of $f(3)$ is

(A) 6 (B) $4i$

(C) $-4i$ (D) 0

18. The value of $f'(1-i)$ is

(A) $7(\pi + i2)$ (B) $6(2 + i\pi)$

(C) $2\pi(5 + i13)$ (D) 0

Statement for 19–21:

Expand the given function in Taylor's series.

19. $f(z) = \frac{z-1}{z+1}$ about the points $z = 0$

(A) $1 + 2(z + z^2 + z^3 ......)$ (B) $-1 - 2(z - z^2 + z^3 ......)$

(C) $-1 + 2(z - z^2 + z^3 ......)$ (D) None of the above

20. $f(z) = \frac{1}{z+1}$ about $z = 1$

(A) $\frac{-1}{2}\left[1 - \frac{1}{2}(z-1) + \frac{1}{2^2}(z-1)^2 ....\right]$

(B) $\frac{1}{2}\left[1 - \frac{1}{2}(z-1) + \frac{1}{2^2}(z-1)^2 ....\right]$

(C) $\frac{1}{2}\left[1 + \frac{1}{2}(z-1) + \frac{1}{2^2}(z-1)^2 ....\right]$

(D) None of the above

21. $f(z) = \sin z$ about $z = \frac{\pi}{4}$

(A) $\frac{1}{\sqrt{2}}\left[1 + \left(z - \frac{\pi}{4}\right) - \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 - ....\right]$

(B) $\frac{1}{\sqrt{2}}\left[1 + \left(z - \frac{\pi}{4}\right) + \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 + ...\right]$

(C) $\frac{1}{\sqrt{2}}\left[1 - \left(z - \frac{\pi}{4}\right) - \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 - ....\right]$

(D) None of the above

22. If $|z + 1| < 1$, then z^{-2} is equal to

(A) $1 + \sum_{n=1}^{\infty}(n+1)(z+1)^{n-1}$

(B) $1 + \sum_{n=1}^{\infty}(n+1)(z+1)^{n+1}$

(C) $1 + \sum_{n=1}^{\infty} n(z+1)^{n}$

(D) $1 + \sum_{n=1}^{\infty}(n+1)(z+1)^{n}$

Statement for Q. 23–25.

Expand the function $\frac{1}{(z-1)(z-2)}$ in Laurent's series for the condition given in question.

23. $1<|z|<2$

(A) $\frac{1}{z}+\frac{2}{z^2}+\frac{3}{z^3}+.......$

(B) $...-z^{-3}-z^{-2}-z^{-1}-\frac{1}{2}-\frac{1}{4}z-\frac{1}{8}z^2-\frac{1}{18}z^3-...$

(C) $\frac{1}{z^2}+\frac{3}{z^2}+\frac{7}{z^4}...........$

(D) None of the above

24. $|z|>2$

(A) $\frac{6}{z}+\frac{13}{z^2}+\frac{20}{z^3}+........$

(B) $\frac{1}{z}+\frac{8}{z^2}+\frac{13}{z^3}+.........$

(C) $\frac{1}{z^2}+\frac{3}{z^3}+\frac{7}{z^4}+.........$

(D) $\frac{2}{z^2}-\frac{3}{z^3}+\frac{4}{z^4}-........$

25. $|z|<1$

(A) $1+3z\frac{+7}{2}z^2+\frac{15}{4}z^2.....$

(B) $\frac{1}{2}+\frac{3}{4}z+\frac{7}{8}z^2+\frac{15}{16}z^3...$

(C) $\frac{1}{4}+\frac{3}{4}+\frac{z^2}{8}+\frac{z^3}{16}.......$

(D) None of the above

26. If $|z-1|<1$, the Laurent's series for $\frac{1}{z(z-1)(z-2)}$ is

(A) $-(z-1)-\frac{(z-1)^3}{2!}-\frac{(z-1)^5}{5!}-..........$

(B) $-(z-1)^{-1}-\frac{(z-1)^3}{2!}-\frac{(z-1)^5}{5!}-.........$

(C) $-(z-1)-(z-1)^3-(z-1)^5-..........$

(D) $-(z-1)^{-1}-(z-1)-(z-1)^3-(z-1)^5-.........$

27. The Laurent's series of $\frac{1}{z(e^z-1)}$ for $|z|<2$ is

(A) $\frac{1}{z^2}+\frac{1}{2z}+\frac{1}{12}+6z+\frac{1}{720}z^2+..........$

(B) $\frac{1}{z^2}-\frac{1}{2z}+\frac{1}{12}-\frac{1}{720}z^2+..........$

(C) $\frac{1}{z}+\frac{1}{12}+\frac{1}{634}z^2+\frac{1}{720}z^2+..........$

(D) None of the above

28. The Laurent's series of $f(z)=\frac{z}{(z^2+1)(z^2+4)}$ is, where $|z|<1$

(A) $\frac{1}{4}z-\frac{5}{16}z^3+\frac{21}{64}z^5..........$

(B) $\frac{1}{2}+\frac{1}{4}z^2+\frac{5}{16}z^4+\frac{21}{64}z^6..........$

(C) $\frac{1}{2}z-\frac{3}{4}z^3+\frac{15}{8}z^5..........$

(D) $\frac{1}{2}+\frac{1}{2}z^2+\frac{3}{4}z^4+\frac{15}{8}z^6..........$

29. The residue of the function $\frac{1-e^{Zz}}{z^4}$ at its pole is

(A) $\frac{4}{3}$

(B) $\frac{-4}{3}$

(C) $\frac{-2}{3}$

(D) $\frac{2}{3}$

30. The residue of $z\cos\frac{1}{z}$ at $z=0$ is

(A) $\frac{1}{2}$

(B) $\frac{-1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{-1}{3}$

31. $\int_c \frac{1-2z}{z(1-z)(z-2)}dz=?$ where c is $|z|=1.5$

(A) $-i3\pi$

(B) $i3\pi$

(C) 2

(D) –2

32. $\int_c \frac{z\ \cos z}{\left(z-\frac{\pi}{2}\right)}dz=?$ where c is $|z-1|=1$

(A) 6π

(B) -6π

(C) $i2\pi$

(D) None of the above

33. $\int_c z^2e^{\frac{1}{z}}dz=?$ where c is $|z|=1$

(A) $i3\pi$

(B) $-i3\pi$

C) $\frac{i\pi}{3}$

(D) None of the above

34. $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}=?$

(A) $\frac{-2\pi}{\sqrt{2}}$

(B) $\frac{2\pi}{\sqrt{3}}$

(C) $2\pi\sqrt{2}$

(D) $-2\pi\sqrt{3}$

35. $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = ?$

(A) $\frac{\pi ab}{a+b}$

(B) $\frac{\pi(a+b)}{ab}$

(C) $\frac{\pi}{a+b}$

(D) $\pi(a+b)$

36. $\int_0^{\infty} \frac{dx}{1+x^6} = ?$

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{3}$

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SOLUTIONS

1. (C) Since, $f(z) = u + iv = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$; $z \neq 0$

$\Rightarrow \quad u = \frac{x^3 - y^3}{x^2+y^2}$; $\quad v = \frac{x^3+y^3}{x^2+y^2}$

Cauchy Riemann equations are

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

By differentiation the value of $\frac{\partial u}{\partial x}, \frac{\partial y}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ at$(0,0)$

we get $\frac{0}{0}$, so we apply first principle method.

At the origin,

$$\frac{\partial u}{\partial x} = \lim_{h\to 0} \frac{u(0+h,\ 0) - u(0,0)}{h} = \lim_{h\to 0} \frac{h^3/h^2}{h} = 1$$

$$\frac{\partial u}{\partial v} = \lim_{h\to 0} \frac{u(0,0+k) - u(0,0)}{k} = \lim_{k\to 0} \frac{-k^3/k^2}{k} = -1$$

$$\frac{\partial v}{\partial x} = \lim_{h\to 0} \frac{v(0+h,0) - v(0,0)}{h} = \lim_{h\to 0} \frac{h^3/h^2}{h} = 1$$

$$\frac{\partial v}{\partial y} = \lim_{k\to 0} \frac{v(0,\ 0+k), v(0,0)}{k} = \lim_{k\to 0} \frac{k^3/k^2}{k} = 1$$

Thus, we see that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Hence, Cauchy-Riemann equations are satisfied at $z = 0$.

Again, $f'(0) = \lim_{z\to 0} \frac{f(z) - f(0)}{z}$

$$= \lim_{z\to 0} \left[\frac{(x^3-y^3) + i(x^3+y^3)}{(x^2+y^2)} \frac{1}{(x+iy)}\right]$$

Now let $z \to 0$ along $y = x$, then

$$f'(0) = \lim_{z\to 0}\left[\frac{(x^3-y^3) + i(x^3+y^3)}{(x^2+y^2)} \frac{1}{(x+iy)}\right] = \frac{2i}{2(1+i)} = \frac{1+i}{2}$$

Again let $z \to 0$ along $y = 0$, then

$$f'(0) = \lim_{x\to 0}\left[\frac{x^3 + i(x^3)}{(x^2)} \frac{1}{x}\right] = 1 + i$$

So we see that $f'(0)$ is not unique. Hence $f'(0)$ does not exist.

2. (A) Since, $f'(z) = \frac{df}{dz} = \lim_{\Delta z\to 0} \frac{\Delta f}{\Delta z}$

or $f'(z) = \lim_{\Delta z \to 0} \frac{\Delta u + i\Delta v}{\Delta x + i\Delta y}$ ....(1)

Now, the derivative $f'(z)$ exits of the limit in equation (1) is unique i.e. it does not depends on the path along which $\Delta z \to 0$.

Let $\Delta z \to 0$ along a path parallel to real axis

$\Rightarrow \quad \Delta y = 0 \therefore \Delta z \to 0 \quad \Rightarrow \quad \Delta x \to 0$

Now equation (1)

$$f'(z) = \lim_{\Delta x \to 0} \frac{\Delta u + i\Delta v}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} + i \lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x}$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad ....(2)$$

Again, let $\Delta z \to 0$ along a path parallel to imaginary axis, then $\Delta x \to 0$ and $\Delta z \to 0 \to \Delta y \to 0$

Thus from equation (1)

$$\phi'(z) = \lim_{\Delta y \to 0} \frac{\Delta z + i\Delta v}{i\Delta y} = \lim_{\Delta y \to 0} \frac{\Delta u}{i\Delta y} + i \lim_{\Delta y \to 0} \frac{\Delta v}{i\Delta z} = \frac{\partial u}{i\partial y} + \frac{\partial v}{\partial y}$$

$$f'(z) = \frac{-i\partial u}{\partial y} + \frac{\partial v}{\partial y} \quad ....(3)$$

Now, for existence of $f'(z)$ R.H.S. of equation (2) and (3) must be same i.e.,

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x} = \frac{-\partial u}{\partial y}$$

$$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$$

3. (A) Given $f(z) = x^2 + iy^2$ since, $f(z) = u + iv$

Here $u = x^2$ and $v = y^2$

Now, $u = x^2 \quad \Rightarrow \quad \frac{\partial u}{\partial x} = 2x$ and $\frac{\partial u}{\partial y} = 0$

and $v = y^2 \quad \Rightarrow \quad \frac{\partial v}{\partial x} = 0$ and $\frac{\partial v}{\partial y} = 2y$

we know that $\quad f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} \quad ....(1)$

and $f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}....(2)$

Now, equation (1) gives $f'(z) = 2x \quad ....(3)$

and equation (2) gives $f'(z) = 2y \quad ....(4)$

Now, for existence of $f'(z)$ at any point is necessary that the value of $f'(z)$ most be unique at that point, whatever be the path of reaching at that point

From equation (3) and (4) $2x = 2y$

Hence, $f'(z)$ exists for all points lie on the line $x = y$.

4. (B) $\frac{\partial u}{\partial x} = 2(1 - y)$; $\frac{\partial^2 u}{\partial x^2} = 0 \quad ....(1)$

$\frac{\partial u}{\partial y} = -2x$; $\frac{\partial^2 u}{\partial y^2} = 0 \quad ....(2)$

$\Rightarrow \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, Thus u is harmonic.

Now let v be the conjugate of u then

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

(by Cauchy-Riemann equation)

$\Rightarrow \quad dv = 2x\,dx + 2(1 - y)dy$

On integrating $v = x^2 - y^2 + 2y + C$

5. (C) Given $f(z) = u + i\,v \quad ....(1)$

$\Rightarrow \quad if(z) = -v + iu \quad ....(2)$

add equation (1) and (2)

$\Rightarrow \quad (1 + i)f(z) = (u - v) + i(u + v)$

$\Rightarrow \quad F(z) = U + iV$

where, $F(z) = (1 + i)f(z)$; $U = (u - v)$; $V = u + v$

Let $F(z)$ be an analytic function.

Now, $U = u - v = e^x(\cos y - \sin y)$

$\frac{\partial U}{\partial x} = e^x(\cos y - \sin y) \quad$ and $\quad \frac{\partial U}{\partial y} = e^x(-\sin y - \cos y)$

Now, $dV = \frac{-\partial U}{\partial y}dx + \frac{\partial U}{\partial x}dy....(3)$

$= e^x(\sin y + \cos y)dx + e^x(\cos y - \sin y)dy$

$= d[e^x(\sin y + \cos y)]$

on integrating $V = e^x(\sin y + \cos y) + c_1$

$F(z) = U + iV = e^x(\cos y - \sin y) + ie^x(\sin y + \cos y) + ic_1$

$= e^x(\cos y + i\sin y) + ie^x(\cos y + i\sin y) + ic_1$

$F(z) = (1 + i)e^{x+iy} + ic_1 = (1 + i)e^z + ic_1$

$(1 + i)f(z) = (1 + i)e^z + ic_1$

$$\Rightarrow \quad f(z) = e^z + \frac{i}{1 + i}c_1 = e^z + c_1\frac{i(1 - i)}{(1 + i)(1 - i)} = e^z + \frac{(i + 1)}{2}c_1$$

$\Rightarrow \quad f(z) = e^z + (1 + i)c$

6. (C) $u = \sinh x \cos y$

$\frac{\partial u}{\partial x} = \cosh x \cos y = \phi(x, y)$

and $\frac{\partial u}{\partial y} = -\sinh x \sin y = \psi(x, y)$

by Milne's Method

$f'(z) = \phi(z, 0) - i\psi(z, 0) = \cosh z - i \cdot 0 = \cosh z$

On integrating $f(z) = \sinh z +$ constant

$\Rightarrow \quad f(z) = w = \sinh z + ic$

(As u does not contain any constant, the constant c is in the function x and hence i.e. in w).

7. (A) $\frac{\partial v}{\partial x} = 2y = h(x, y)$, $\frac{\partial v}{\partial y} = 2x = g(x, y)$

by Milne's Method $f'(z) = g(z, 0) + ih(z, 0) = 2z + i\,0 = 2z$

On integrating $f(z) = z^2 + c$

8. (D) $\frac{\partial v}{\partial y} = \frac{-(x^2+y^2)-(x-y)2y}{(x^2+y^2)^2}$

$= \frac{y^2 - x^2 - 2xy}{(x^2+y^2)^2} = g(x, y)$

$\frac{\partial v}{\partial x} = \frac{(x^2+y^2)-(x-y)2x}{(x^2+y^2)^2} = \frac{y^2 - x^2 + 2xy}{(x^2+y^2)^2} = h(x, y)$

By Milne's Method

$f'(z) = g(z,0) + ih(z,0) = -\frac{1}{z^2} + i\left(-\frac{1}{z^2}\right) = -(1+i)\frac{1}{z^2}$

On integrating

$f(z) = (1+i)\int \frac{1}{z^2}dz + c = (1+i)\frac{1}{z} + c$

9. (A) $\frac{\partial u}{\partial x} = \frac{2\cos 2x(\cosh 2y - \cos 2x) - 2\sin^2 2x}{(\cosh 2y - \cos 2x)^2}$

$= \frac{2\cos 2x \cosh 2y - 2}{(\cosh 2y - \cos 2y)^2} = \phi(x, y)$

$\frac{\partial u}{\partial y} = \frac{2\sin 2x \sinh 2y}{(\cosh 2y - \cos 2x)^2} = \psi(x, y)$

By Milne's Method

$f'(z) = \phi(z,0) - i\psi(z,0)$

$= \frac{2\cos 2z - 2}{(1-\cos 2z)^2} - i(0) = \frac{-2}{1-\cos 2z} = -\text{cosec}^2 z$

On integrating

$f(z) = -\int \text{cosec}^2 z\, dz + ic = \cot z + ic$

10. $x = at + b,\ y = ct + d$

On A, $z = 1 + i$ and On B, $z = 2 + 4i$

Let $z = 1 + i$ corresponds to $t = 0$

and $z = 2 + 4i$ corresponding to $t = 1$

then, $t = 0 \quad \Rightarrow \quad x = b,\ y = d$

$\Rightarrow \quad b = 1,\ d = 1$

and $t = 1 \quad \Rightarrow \quad x = a + b,\ y = c + d$

$\Rightarrow \quad 2 = a + 1,\ 4 = c + 1 \Rightarrow \quad a = 1,\ c = 3$

AB is , $y = 3t + 1 \Rightarrow \quad dx = dt$; $dy = 3\,dt$

$\int_c f(z)dz = \int_c (x^2 + ixy)(dx + idy)$

$= \int_{t=0}^{1} [(t+1)^2 + i(t+1)(3t+1)][dt + 3i\,dt]$

$= \int_0^1 [(t^2 + 2t + 1) + i(3t^2 + 4t + 1)](1 + 3i)dt$

$= (1+3i)\left[\frac{t^3}{3} + t^2 + t + i(t^3 + 2t^2 + t)\right]_0^1 = -\frac{29}{3} + 11i$

11. (D) We know by the derivative of an analytic function that

$f''(z_o) = \frac{n!}{2\pi i}\int_c \frac{f(z)\,dz}{(z-z_o)^{n+1}}$ or $\int_c \frac{f(z)\,dz}{(z-z_o)^{n+1}} = \frac{2\pi i}{n!} f^n(z_o)$

Taking $n = 3$, $\int_c \frac{f(z)\,dz}{(z-z_o)^4} = \frac{\pi i}{3} f''(z_o)$ ....(1)

Given $f_c \frac{e^{2z}dz}{(z+1)^4} = \int_c \frac{e^{2z}dz}{[z-(-1)]^4}$

Taking $f(z) = e^{2z}$, and $z_o = -1$ in (1), we have

$\int_c \frac{e^{2z}dz}{(z+1)^4} = \frac{\pi i}{3} f'''(-1)$....(2)

Now, $f(z) = e^{2z} \quad \Rightarrow \quad f'''(z) = 8e^{2z}$

$\Rightarrow \quad f'''(-1) = 8e^{-2}$

equation (2) have

$\Rightarrow \quad \int_c \frac{e^{2z}dz}{(z+1)^4} = \frac{8\pi i}{3} e^{-2}$ ....(3)

If is the circle $|z| = 3$

Since, $f(z)$ is analytic within and on $|z| = 3$

$\int_{|z|=3} \frac{e^{2z}dz}{(z+1)^4} = \frac{8\pi i}{3} e^{-z}$

12. (D) Since, $\frac{1-2z}{z(z-1)(z-2)} = \frac{1}{2z} + \frac{1}{z-1} - \frac{3}{2(z-2)}$

$\int_c \frac{1-2z}{z(z-1)(z-2)} dz = \frac{1}{2} I_1 + I_2 - \frac{3}{2} I_3$....(1)

Since, $z = 0$ is the only singularity for $I_1 = \int_c \frac{1}{z} dz$ and it lies inside $|z| = 1.5$, therefore by Cauchy's integral Formula

$I_1 = \int_c \frac{1}{z} dz = 2\pi i$ ....(2)

$\left[f(z_o) = \frac{1}{2\pi i}\int_c \frac{f(z)\,dz}{z - z_o}\right]$ [Here $f(z) = 1 = f(z_o)$ and $z_o = 0$]

Similarly, for $I_2 = \int_c \frac{1}{z-1} dz$, the singular point $z = 1$ lies inside $|z| = 1.5$, therefore $I_2 = 2\pi i$....(3)

For $I_3 = \int_c \frac{1}{z-2} dz$, the singular point $z = 2$ lies outside the circle $|z| = 1.5$, so the function $f(z)$ is analytic everywhere in c i.e. $|z| = 1.5$, hence by Cauchy's integral theorem

$I_3 = \int_c \frac{1}{z-2} dz = 0$....(4)

using equations (2), (3), (4) in (1), we get

$\int_c \frac{1-2z}{z(z-1)(z-2)} dz = \frac{1}{2}(2\pi i) + 2\pi i - \frac{3}{2}(0) = 3\pi i$

13. (B) Given contour c is the circle $|z| = 1$

$\Rightarrow \quad z = e^{i\theta} \quad \Rightarrow \quad dz = ie^{i\theta} d\theta$

Now, for upper half of the circle, $0 \le \theta \le \pi$

$$\int_c (z - z^2) dz = \int_{\theta=0}^{\pi} (e^{i\theta} - e^{2i\theta}) ie^{i\theta} d\theta$$

$$= i\int_0^{\pi} (e^{2i\theta} - e^{3i\theta}) d\theta = i\left[\frac{e^{2i\theta}}{2i} - \frac{e^{3i\theta}}{3i}\right]_0^{\pi}$$

$$= i \cdot \frac{1}{i}\left[\frac{1}{2} \cdot (e^{2\pi i} - 1) - \frac{1}{3}(e^{3\pi x} - 1)\right] = \frac{2}{3}$$

14. (B) Let $f(z) = \cos \pi z$ then $f(z)$ is analytic within and on $|z| = 3$, now by Cauchy's integral formula

$$f(z_o) = \frac{1}{2\pi i}\int_c \frac{f(z)}{z - z_o} dz \Rightarrow \int_c \frac{f(z)\, dz}{z - z_o} = 2\pi i f(z_o)$$

take $f(z) = \cos \pi z$, $z_o = 1$, we have

$$\int_{|z|=3} \frac{\cos \pi z}{z - 1} dz = 2\pi i f(1) = 2\pi i \cos \pi = -2\pi i$$

15. (D) $\int_c \frac{\sin \pi z^2}{(z-1)(z-2)} dz$

$$= \int_c \frac{\sin \pi z^2}{z - 2} dz - \int_c \frac{\sin \pi z^2}{z - 1} dz$$

$= 2\pi i f(2) - 2\pi i f(1)$ since, $f(z) = \sin \pi z^2$

$\Rightarrow \quad f(2) = \sin 4\pi = 0$ and $f(1) = \sin \pi = 0$

16. (D) Let, $I = \frac{1}{2\pi i}\int_c \frac{1}{z^2 - 1} \cos \pi z\, dz$

$$= \frac{1}{2 \cdot 2\pi i}\int_c \left(\frac{1}{z-1} - \frac{1}{z+1}\right)\cos \pi z\, dz$$

Or $I = \frac{1}{4\pi i}\int_c \left(\frac{\cos nz}{z-1} - \frac{\cos nz}{z+1}\right) dz$

17. (D) $f(3) = \int_c \frac{3z^2 + 7z + 1}{z - 3} dz$, since $z_o = 3$ is the only singular point of $\frac{3z^2 + 7z + 1}{z - 3}$ and it lies outside the circle $x^2 + y^2 = 4$ i.e., $|z| = 2$, therefore $\frac{3z^2 + 7z + 1}{z - 3}$ is analytic everywhere within c.

Hence by Cauchy's theorem—

$$f(3) = \int_c \frac{3z^2 + 7z + 1}{z - 3} dz = 0$$

18. (C) The point $(1 - i)$ lies within circle $|z| = 2$ ($\because$ the distance of $1 - i$ i.e., (1, 1) from the origin is $\sqrt{2}$ which is less than 2, the radius of the circle).

Let $\phi(z) = 3z^2 + 7z + 1$ then by Cauchy's integral formula

$$\int_c \frac{3z^2 + 7z + 1}{z - z_o} dz = 2\pi i \phi(z_o)$$

$\Rightarrow \quad f(z_o) = 2\pi i \phi(z_o) \quad \Rightarrow \quad f'(z_o) = 2\pi i \phi'(z_o)$

and $f''(z_o) = 2\pi i\ \phi''(z_o)$

since, $\phi(z) = 3z^2 + 7z + 1$

$\Rightarrow \quad \phi'(z) = 6z + 7$ and $\phi''(z) = 6$

$f'(1 - i) = 2\pi i[6(1 - i) + 7] = 2\pi(5 + 13i)$

19. (C) $f(z) = \frac{z - 1}{z + 1} = 1 - \frac{2}{z + 1}$

$\Rightarrow \quad f(0) = -1,\ f(1) = 0$

$\Rightarrow \quad f'(z) = \frac{2}{(z+1)^2} \quad \Rightarrow \quad f'(0) = 2;$

$f''(z) = \frac{-4}{(z-1)^3} \quad \Rightarrow \quad f''(0) = -4;$

$f'''(z) = \frac{12}{(z+1)^4} \quad \Rightarrow \quad f'''(0) = 12;$ and so on.

Now, Taylor series is given by

$$f(z) = f(z_0) + (z - z_0)f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \frac{(z - z_0)^3}{3!} f'''(z_0) + \ldots..$$

about $z = 0$

$$f(z) = -1 + z(2) + \frac{z^2}{2!}(-4) + \frac{z^3}{3!}(12) + \ldots.$$

$= -1 + 2z - 2z^2 + 2z^3 \ldots.$

$f(z) = -1 + 2(z - z^2 + z^3 \ldots.)$

20. (B) $f(z) = \frac{1}{z + 1} \quad \Rightarrow \quad f(1) = \frac{1}{2}$

$f'(z) = \frac{-1}{(z+1)^2} \quad \Rightarrow \quad f'(1) = \frac{-1}{4}$

$f''(z) = \frac{2}{(z+1)^3} \quad \Rightarrow \quad f''(1) = \frac{1}{4}$

$f'''(z) = \frac{-6}{(z+1)^4} \quad \Rightarrow \quad f'''(1) = -\frac{3}{8}$ and so on.

Taylor series is

$$f(z) = f(z_0) + (z - z_0)f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \frac{(z - z_0)^3}{3!} f'''(z_0) + \ldots$$

about $z = 1$

$$f(z) = \frac{1}{2} + (z - 1)\left(\frac{-1}{4}\right) + \frac{(z-1)^2}{2!}\left(\frac{1}{4}\right) + \frac{(z-1)^3}{3!}\left(-\frac{3}{8}\right) + \ldots$$

$$= \frac{1}{2} - \frac{1}{2^2}(z - 1) + \frac{1}{2^3}(z - 1)^2 - \frac{1}{2^4}(z - 1)^3 + \ldots.$$

or $f(z) = \frac{1}{2}\left[1 - \frac{1}{2}(z - 1) + \frac{1}{2^2}(z - 1)^2 - \frac{1}{2^3}(z - 1)^3 + \ldots.\right]$

21. (A) $f(z)=\sin z \Rightarrow f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$f'(z)=\cos z \Rightarrow f'\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$f''(z)=-\sin z \Rightarrow f''\left(\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

$f'''(z)=-\cos z \Rightarrow f'''\left(\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$ and so on.

Taylor series is given by

$$f(z)=f(z_0)+(z-z_0)f'(z_0)+\frac{(z-z_0)^2}{2!}f''(z_0)+\frac{(z-z_0)^3}{3!}f'''(z_0)+....$$

about $z=\frac{\pi}{4}$

$$f(z)=\frac{1}{\sqrt{2}}+\left(z-\frac{\pi}{4}\right)\frac{1}{\sqrt{2}}+\frac{\left(z-\frac{\pi}{4}\right)^2}{2!}\left(-\frac{1}{\sqrt{2}}\right)+\frac{\left(z-\frac{\pi}{4}\right)^3}{3!}\left(-\frac{1}{\sqrt{2}}\right)+\dots$$

$$f(z)=\frac{1}{\sqrt{2}}\left[1+\left(z-\frac{\pi}{4}\right)-\frac{1}{2!}\left(z-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(z-\frac{\pi}{4}\right)^3-\dots\right]$$

22. (D) Let $f(z)=z^{-2}=\frac{1}{z^2}=\frac{1}{[1-(1+z)]^2}$

$f(z)=[1-(1+z)]^{-2}$

Since, $|1+z|<1$, so by expanding R.H.S. by binomial theorem, we get

$$f(z)=1+2(1+z)+3(1+z)^2+4(1+z)^3+\dots+(n+1)(1+z)^n+\dots$$

or $f(z)=z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(z+1)^n$

23. (B) Here $f(z)=\frac{1}{(z-1)(z-2)}=\frac{1}{z-2}-\frac{1}{z-1}$....(1)

Since, $|z|>1 \Rightarrow \frac{1}{|z|}<1$ and $|z|<2 \Rightarrow \frac{|z|}{2}<1$

$$\frac{1}{z-1}=\frac{1}{z\left(1-\frac{1}{z}\right)}=\frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}$$

$$=\frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\dots\right)$$

and $\frac{1}{z-2}=\frac{-1}{2}\left(1-\frac{z}{2}\right)^{-1}=-\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{9}+\dots\right]$

equation (1) gives—

$$f(z)=-\frac{1}{2}\left(1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{9}+..\right)-\frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\dots\right)$$

or $f(z)=\dots-z^{-4}-z^{-2}-z^{-1}-\frac{1}{2}-\frac{1}{4}z-\frac{1}{8}z^2-\frac{1}{18}z^3-\dots$

24. (C) $\frac{2}{|z|}<1 \Rightarrow \frac{1}{|z|}<\frac{1}{2}<1 \Rightarrow \frac{1}{|z|}<1$

$$\frac{1}{z-1}=\frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}=\frac{1}{2}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\dots\right)$$

and $\frac{1}{z-2}=\frac{1}{z}\left(1-\frac{2}{z}\right)^{-1}=\frac{1}{z}\left(1+\frac{2}{z}+\frac{4}{z^2}+\frac{8}{z^3}+....\right)$

Laurent's series is given by

$$f(z)=\frac{1}{z}\left(1+\frac{2}{z}+\frac{4}{z^2}+\frac{98}{z^3}+..\right)-\frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+..\right)$$

$$=\frac{1}{z}\left(\frac{1}{z}+\frac{3}{z^2}+\frac{7}{z^3}+\dots\right)$$

$$\Rightarrow \quad f(z)=\frac{1}{z^2}+\frac{3}{z^3}+\frac{7}{z^4}+\dots$$

25. (B) $|z|<1$, $\frac{1}{z-2}-\frac{1}{z-1}=-\frac{1}{2}\left(1-\frac{z}{2}\right)^{-1}+(1-z)^{-1}$

$$=-\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\dots\right]+(1+z+z^2+z^3+\dots)$$

$$f(z)=\frac{1}{2}+\frac{3}{4}z+\frac{7}{8}z^2+\frac{15}{16}z^3+\dots$$

26. (D) Since, $\frac{1}{z(z-1)(z-2)}=\frac{1}{2z}-\frac{1}{z-1}+\frac{1}{2(z-2)}$

For $|z-1|<1$ Let $z-1=u$

$\Rightarrow \quad z=u+1$ and $|u|<1$

$$\frac{1}{z(z-1)(z-2)}=\frac{1}{2z}-\frac{1}{z-1}+\frac{1}{2(z-2)}$$

$$=\frac{1}{2(u+1)}-\frac{1}{u}+\frac{1}{2(u-1)}=\frac{1}{2}(1+u)^{-1}-u^{-1}-\frac{1}{2}(1-u)^{-1}$$

$$=\frac{1}{2}[1-u+u^2-u^3+\dots]-u^{-1}-\frac{1}{2}(1+u+u^2+u^3+\dots)$$

$$=\frac{1}{2}(-2u-2u^3-\dots)-u^{-1}=-u-u^3-u^5-\dots-u^{-1}$$

Required Laurent's series is

$$f(z)=-(z-1)^{-1}-(z-1)-(z-1)^3-(z-1)^5-\dots$$

27. (B) Let $f(z)=\frac{1}{z(e^z-1)}$

$$=\frac{1}{z\left[1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+\frac{z^4}{4!}+\dots-1\right]}$$

$$\int_c f(z)dz = 2\pi i \times \frac{1}{6} = \frac{1}{3}\pi i$$

34. (B) Let $z = e^{i\theta} \quad \Rightarrow \quad d\theta = \frac{-idz}{z} \quad ; \quad z \le \theta \le 2\pi$

and $\cos\theta = \frac{1}{2}\left(z + \frac{1}{z}\right)$

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \int_c \frac{\frac{-idz}{z}}{2 + \frac{1}{2}\left(z+\frac{1}{z}\right)}; \quad c: |z| = 1$$

$$= -2i\int_c \frac{dz}{z^2 + 4z + 1}$$

Let $f(z) = \frac{1}{z^2 + 4z + 1}$

$f(z)$ has poles at $z = -2 + \sqrt{3}, -2 - \sqrt{3}$ out of these only $z = -2 + \sqrt{3}$ lies inside the circle $c: |z| = 1$

$$\int_c f(z)dz = 2\pi i(\text{Residue at } z = -2 + \sqrt{3})$$

Now, residue at $z = -2 + \sqrt{3}$

$$= \lim_{z \to -2+\sqrt{3}} (z + 2 - \sqrt{3})f(z) = \lim_{z\to -2+\sqrt{3}} \frac{1}{(z + 2 + \sqrt{3})} = \frac{1}{2\sqrt{3}}$$

$$\int_c f(z)dz = 2\pi i \times \frac{1}{2\sqrt{3}} = \frac{\pi i}{\sqrt{3}}$$

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = -2i \times \frac{\pi i}{\sqrt{3}} = \frac{2\pi}{\sqrt{3}}$$

35. (C) $I = \int_c \frac{z^2}{(z^2+a^2)(z^2+b^2)}dz = \int_c f(z)dz$

where c is be semi circle r with segment on real axis from $-R$ to R.

The poles are $z = \pm ia$, $z = \pm ib$. Here only $z = ia$ and $z = ib$ lie within the contour c

$$\int_c f(z)dz = 2\pi i$$

(sum of residues at $z = ia$ and $z = ib$)

Residue at $z = ia$,

$$= \lim_{z\to ia}(z - ia)\frac{z^2}{(z-ia)(z-ia)(z^2+b^2)} = \frac{a}{2i(a^2-b^2)}$$

Residue at $z = ib$

$$= \lim_{z\to ib}(z-ib)\frac{z^2}{(z-ia)(z+ia)(z+ib)(z-ib)} = \frac{-b}{2i(a^2-b^2)}$$

$$\int_c f(z)dz = \int_r f(z)dz + \int_{-R}^{R} f(z)dz$$

$$= \frac{2\pi i}{2i(a^2-b^2)}(a-b) = \frac{\pi}{a+b}$$

Now $\int_r f(z)dz = \int_0^{\pi} \frac{ie^{2i\theta} iRe^{i\theta}d\theta}{(R^2e^{2i\theta}+a^2)(R^2e^{2i\theta}+b^2)}$

$$= \int_0^{\pi} \frac{\frac{e^{3i\theta}}{R}d\theta}{\left(e^{2i\theta}+\frac{a^2}{R^2}\right)\left(e^{2i\theta}+\frac{b^2}{R^2}\right)}$$

Now when $R \to \infty$, $\int_r b(z)dz = 0$

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}dz = \frac{\pi}{a+b}$$

36. (C) Let $I = \int_c \frac{dz}{1+z^6} = \int_c f(z)dz$

c is the contour containing semi circle r of radius R and segment from $-R$ to R.

For poles of $f(z)$, $1 + z^6 = 0$

$\Rightarrow \quad z = (-1)^{\pi/6} = e^{i(2n+1)\pi/6}$

where $n = 0, 1, 2, 3, 4, 5, 6$

Only poles $z = \frac{-\sqrt{3}+i}{2}, i, \frac{\sqrt{3}+i}{2}$ lie in the contour

Residue at $z = \frac{+\sqrt{3}+i}{2}$

$$= \frac{1}{(z_1-z_2)(z_1-z_3)(z_1-z_4)(z_1-z_5)(z_1-z_6)}$$

$$= \frac{1}{3i(1+\sqrt{3}i)} = \frac{1-\sqrt{3}i}{12i}$$

Residue at $z = i$ is $\frac{1}{6i}$

Residue at $z = \frac{1+\sqrt{3}i}{12i}$ is $= \frac{1}{3i(1-\sqrt{3}i)} = \frac{1+\sqrt{3}i}{12i}$

$$\int_c f(z)dz = \int_r f(z)dz + \int_{-R}^{R} f(z)dz$$

$$= \frac{2\pi i}{12i}(1 - \sqrt{3}i + 1 + \sqrt{3}i + 2i) = \frac{2\pi}{3}$$

or $\int_r f(z)dz + \int_{-R}^{R} f(z)dz = \frac{2\pi}{3}$....(1)

Now $\int_c f(z)dz = \int_0^{\pi} \frac{iRe^{i\theta}d\theta}{1+R^6e^{6i\theta}} = \int_0^{\pi} \frac{\frac{ie^{i\theta}d\theta}{R^5}}{\frac{1}{R^6}+e^{6i\theta}}$

where $R \to \infty$, $\int_r f(z)dz \to 0$

$$(1) \to \int_0^{\infty} \frac{ax}{1+x^6} = \frac{2\pi}{3}$$

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CHAPTER

9.6

PROBABILITY AND STATISTICS

1. In a frequency distribution, the mid value of a class is 15 and the class interval is 4. The lower limit of the class is

(A) 14 (B) 13
(C) 12 (D) 10

2. The mid value of a class interval is 42. If the class size is 10, then the upper and lower limits of the class are

(A) 47 and 37 (B) 37 and 47
(C) 37.5 and 47.5 (D) 47.5 and 37.5

3. The following marks were obtained by the students in a test: 81, 72, 90, 90, 86, 85, 92, 70, 71, 83, 89, 95, 85,79, 62. The range of the marks is

(A) 9 (B) 17
(C) 27 (D) 33

4. The width of each of nine classes in a frequency distribution is 2.5 and the lower class boundary of the lowest class is 10.6. The upper class boundary of the highest class is

(A) 35.6 (B) 33.1
(C) 30.6 (D) 28.1

5. In a monthly test, the marks obtained in mathematics by 16 students of a class are as follows:
0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8

The arithmetic mean of the marks obtained is

(A) 3 (B) 4
(C) 5 (D) 6

6. A distribution consists of three components with frequencies 45, 40 and 15 having their means 2, 2.5 and 2 respectively. The mean of the combined distribution is

(A) 2.1 (B) 2.2
(C) 2.3 (D) 2.4

7. Consider the table given below

Marks	Number of Students
0 – 10	12
10 – 20	18
20 – 30	27
30 – 40	20
40 – 50	17
50 – 60	6

The arithmetic mean of the marks given above, is

(A) 18 (B) 28
(C) 27 (D) 6

8. The following is the data of wages per day: 5, 4, 7, 5, 8, 8, 8, 5, 7, 9, 5, 7, 9, 10, 8 The mode of the data is

(A) 5 (B) 7
(C) 8 (D) 10

9. The mode of the given distribution is

Weight (in kg)	40	43	46	49	52	55
Number of Children	5	8	16	9	7	3

(A) 55 (B) 46
(C) 40 (D) None

10. If the geometric mean of x, 16, 50, be 20, then the value of x is

(A) 4 (B) 10
(C) 20 (D) 40

11. If the arithmetic mean of two numbers is 10 and their geometric mean is 8, the numbers are

(A) 12, 18 (B) 16, 4
(C) 15, 5 (D) 20, 5

12. The median of
0, 2, 2, 2, –3, 5, –1, 5, 5, –3, 6, 6, 5, 6 is

(A) 0 (B) –1.5
(C) 2 (D) 3.5

13. Consider the following table

Diameter of heart (in mm)	Number of persons
120	5
121	9
122	14
123	8
124	5
125	9

The median of the above frequency distribution is

(A) 122 mm (B) 123 mm
(C) 122.5 mm (D) 122.75 mm

14. The mode of the following frequency distribution, is

Class interval	Frequency
3–6	2
6–9	5
9–12	21
12–15	23
15–18	10
18–21	12
21–24	3

(A) 11.5 (B) 11.8
(C) 12 (D) 12.4

15. The mean-deviation of the data 3, 5, 6, 7, 8, 10, 11, 14 is

(A) 4 (B) 3.25
(C) 2.75 (D) 2.4

16. The mean deviation of the following distribution is

x	10	11	12	13	14
f	3	12	18	12	3

(A) 12 (B) 0.75
(C) 1.25 (D) 26

17. The standard deviation for the data 7, 9, 11, 13, 15 is

(A) 2.4 (B) 2.5
(C) 2.7 (D) 2.8

18. The standard deviation of 6, 8, 10, 12, 14 is

(A) 1 (B) 0
(C) 2.83 (D) 2.73

19. The probability that an event A occurs in one trial of an experiment is 0.4. Three independent trials of experiment are performed. The probability that A occurs at least once is

(A) 0.936 (B) 0.784
(C) 0.964 (D) None

20. Eight coins are tossed simultaneously. The probability of getting at least 6 heads is

(A) $\frac{7}{64}$ (B) $\frac{37}{256}$
(C) $\frac{57}{64}$ (D) $\frac{249}{256}$

21. A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve a problem, selected at random from the book?

(A) 0.16 (B) 0.63
(C) 0.97 (D) 0.20

22. A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident ?

(A) 5% (B) 45%
(C) 35% (D) 15%

23. The odds against a husband who is 45 years old, living till he is 70 are 7:5 and the odds against his wife who is 36, living till she is 61 are 5:3. The probability that at least one of them will be alive 25 years hence, is

(A) $\frac{61}{96}$ (B) $\frac{5}{32}$
(C) $\frac{13}{64}$ (D) None

24. The probability that a man who is x years old will die in a year is p. Then amongst n persons $A_1, A_2, \ldots, A_n$ each x years old now, the probability that A_1 will die in one year is

(A) $\frac{1}{n^2}$ (B) $1-(1-p)^n$

(C) $\frac{1}{n^2}[1-(1-p)^n]$ (D) $\frac{1}{n}[1-(1-p)^n]$

25. A bag contains 4 white and 2 black balls. Another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, the probability that both are white is

(A) $\frac{1}{24}$ (B) $\frac{1}{4}$

(C) $\frac{5}{24}$ (D) None

26. A bag contains 5 white and 4 red balls. Another bag contains 4 white and 2 red balls. If one ball is drawn from each bag, the probability that one is white and one is red, is

(A) $\frac{13}{27}$ (B) $\frac{5}{27}$

(C) $\frac{8}{27}$ (D) None

27. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. The probability that the gun hits the plane is

(A) 0.76 (B) 0.4096

(C) 0.6976 (D) None of these

28. If the probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

(A) pq (B) $p(1-q)$

(C) $q(1-p)$ (D) $p+1-2pq$

29. In a binomial distribution, the mean is 4 and variance is 3. Then, its mode is

(A) 5 (B) 6

(C) 4 (D) None

30. If 3 is the mean and (3/2) is the standard deviation of a binomial distribution, then the distribution is

(A) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$ (B) $\left(\frac{1}{2}+\frac{3}{2}\right)^{12}$

(C) $\left(\frac{4}{5}+\frac{1}{5}\right)^{60}$ (D) $\left(\frac{1}{5}+\frac{4}{5}\right)^{5}$

31. The sum and product of the mean and variance of a binomial distribution are 24 and 18 respectively. Then, the distribution is

(A) $\left(\frac{1}{7}+\frac{1}{8}\right)^{12}$ (B) $\left(\frac{1}{4}+\frac{3}{4}\right)^{16}$

(C) $\left(\frac{1}{6}+\frac{5}{6}\right)^{24}$ (D) $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$

32. A die is thrown 100 times. Getting an even number is considered a success. The variance of the number of successes is

(A) 50 (B) 25

(C) 10 (D) None

33. A die is thrown thrice. Getting 1 or 6 is taken as a success. The mean of the number of successes is

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$

(C) 1 (D) None

34. If the sum of mean and variance of a binomial distribution is 4.8 for five trials, the distribution is

(A) $\left(\frac{1}{5}+\frac{4}{5}\right)^{5}$ (B) $\left(\frac{1}{3}+\frac{2}{3}\right)^{5}$

(C) $\left(\frac{2}{5}+\frac{3}{5}\right)^{5}$ (D) None of these

35. A variable has Poission distribution with mean m. The probability that the variable takes any of the values 0 or 2 is

(A) $e^{-m}\left(1+m+\frac{m^2}{2!}\right)$ (B) $e^{m}(1+m)^{-3/2}$

(C) $e^{3/2}(1+m^2)^{-1/2}$ (D) $e^{-m}\left(1+\frac{m^2}{2!}\right)$

36. If X is a Poission variate such that $P(2)=9P(4)+90P(6)$, then the mean of X is

(A) ± 1 (B) ± 2

(C) ± 3 (D) None

37. When the correlation coefficient $r = \pm 1$, then the two regression lines
(A) are perpendicular to each other
(B) coincide
(C) are parallel to each other
(D) do not exist

38. If $r = 0$, then
(A) there is a perfect correlation between x and y
(B) x and y are not correlated.
(C) there is a positive correlation between x and y
(D) there is a negative correlation between x and y

39. If $\Sigma x_i = 15$, $\Sigma y_i = 36$, $\Sigma x_i y_i = 110$ and $n = 5$, then cov (x, y) is equal to
(A) 0.6 (B) 0.5
(C) 0.4 (D) 0.225

40. If cov $(x, y) = -16.5$, var $(x) = 2.89$ and var $(y) = 100$, then the coefficient of correlation r is equal to
(A) 0.36 (B) –0.64
(C) 0.97 (D) –0.97

41. The ranks obtained by 10 students in Mathematics and Physics in a class test are as follows

Rank in Maths	Rank in Chem.
1	3
2	10
3	5
4	1
5	2
6	9
7	4
8	8
9	7
10	6

The coefficient of correlation between their ranks is
(A) 0.15 (B) 0.224
(C) 0.625 (D) None

42. If $\Sigma x_i = 24$, $\Sigma y_i = 44$, $\Sigma x_i y_i = 306$, $\Sigma x_i^2 = 164$, $\Sigma y_i^2 = 574$ and $n = 4$, then the regression coefficient b_{yx} is equal to
(A) 2.1 (B) 1.6
(C) 1.225 (D) 1.75

43. If $\Sigma x_i = 30$, $\Sigma y_i = 42$, $\Sigma x_i y_i = 199$, $\Sigma x_i^2 = 184$, $\Sigma y_i^2 = 318$ and $n = 6$, then the regression coefficient b_{xy} is
(A) –0.36 (B) –0.46
(C) 0.26 (D) None

44. Let r be the correlation coefficient between x and y and b_{yx}, b_{xy} be the regression coefficients of y on x and x on y respectively then
(A) $r = b_{xy} + b_{yx}$ (B) $r = b_{xy} \times b_{yx}$
(C) $r = \sqrt{b_{xy} \times b_{yx}}$ (D) $r = \frac{1}{2}(b_{xy} + b_{yx})$

45. Which one of the following is a true statement.
(A) $\frac{1}{2}(b_{xy} + b_{yx}) = r$ (B) $\frac{1}{2}(b_{xy} + b_{yx}) < r$
(C) $\frac{1}{2}(b_{xy} + b_{yx}) > r$ (D) None of these

46. If $b_{yx} = 1.6$ and $b_{xy} = 0.4$ and θ is the angle between two regression lines, then $\tan\theta$ is equal to
(A) 0.18 (B) 0.24
(C) 0.16 (D) 0.3

47. The equations of the two lines of regression are : $4x + 3y + 7 = 0$ and $3x + 4y = 8 = 0$. The correlation coefficient between x and y is
(A) 1.25 (B) 0.25
(C) –0.75 (D) 0.92

48. If cov$(X, Y) = 10$, var $(X) = 6.25$ and var$(Y) = 31.36$, then $\rho(X, Y)$ is
(A) $\frac{5}{7}$ (B) $\frac{4}{5}$
(C) $\frac{3}{4}$ (D) 0.256

49. If $\Sigma x = \Sigma y = 15$, $\Sigma x^2 = \Sigma y^2 = 49$, $\Sigma xy = 44$ and $n = 5$, then $b_{xy} = ?$
(A) $-\frac{1}{3}$ (B) $-\frac{2}{3}$
(C) $-\frac{1}{4}$ (D) $-\frac{1}{2}$

50. If $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 1650$, $\Sigma y^2 = 1500$, $\Sigma xy = 50$ and $n = 25$, then the line of regression of x on y is
(A) $22x + 9y = 146$ (B) $22x - 9y = 74$
(C) $22x - 9y = 146$ (D) $22x + 9y = 74$

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SOLUTION

1. (B) Let the lower limit be x. Then, upper limit is $x+4$. $\frac{x+(x+4)}{2}=15 \Rightarrow x=13.$

2. (A) Let the lower limit be x. Then, upper limit $x+10$.

$\frac{x+(x+10)}{2}=42 \Rightarrow x=37.$

Lower limit $=37$ and upper limit $=47$.

3. (D) Range = Difference between the largest value $=(95-62)=33.$

4. (B) Upper class boundary $=10.6+(2.5\times 9)=33.1.$

5. (B)

Marks	Frequency f	$f\times 1$
0	2	0
2	2	4
3	3	9
4	1	4
5	4	20
6	2	12
7	1	7
8	1	8
	$\Sigma f=16$	$\Sigma(f\times x)=64$

A.M. $=\frac{\Sigma(f\times x)}{\Sigma f}=\frac{64}{16}=4.$

6. (B) Mean $=\frac{45\times 2+40\times 2.5+15\times 2}{100}=\frac{220}{100}=2.2.$

7. (B)

Class	Mid value x	Frequency f	Deviation $d=x-A$	$f\times d$
0–10	5	12	–20	–240
10–20	15	18	–10	–180
20–30	$25=A$	27	0	0
30–40	35	20	10	200
40–50	45	17	20	320
50–60	55	6	30	180
		$\Sigma f=100$		$\Sigma(f\times d)=390$

A.M. $=A+\frac{\Sigma(fd)}{\Sigma f}=\left(25+\frac{300}{100}\right)=28.$

8. (C) Since 8 occurs most often, mode =8.

9. (B) Clearly, 46 occurs most often. So, mode =46.

10. (B) $(x\times 16\times 50)^{1/3}=20 \Rightarrow x\times 16\times 50=(20)^3$

$\Rightarrow x=\left(\frac{20\times 20\times 20}{16\times 50}\right)=10.$

11. (B) Let the numbers be a and b Then,

$\frac{a+b}{2}=10 \Rightarrow (a+b)=20$ and

$\sqrt{ab}=8 \Rightarrow ab=64$

$a-b=\sqrt{(a+b)^2-4ab}=\sqrt{44-256}=\sqrt{144}=12.$

Solving $a+b=20$ and $a-b=12$ we get $a=16$ and $b=4$.

12. (D) Observations in ascending order are

–3, –3, –1, 0, 2, 2, 2, 5, 5, 5, 5 6, 6, 6

Number of observations is 14, which is even.

Median $=\frac{1}{2}$[7 the term +8 the term] $=\frac{1}{2}(2+5)=3.5.$

13. (A) The given Table may be presented as

Diameter of heart (in mm)	Number of persons	Cumulative frequency
120	5	5
121	9	14
122	14	28
123	8	36
124	5	41
125	9	50

Here $n=50$. So, $\frac{n}{2}=25$ and $\frac{n}{2}+1=26.$

Medium $=\frac{1}{2}$(25th term +26 th term) $=\frac{122+122}{2}=122.$

[… Both lie in that column whose c.f. is 28]

14. (B) Maximum frequency is 23. So, modal class is 12–15.

$L_1=12,\ L_2=15,\ f=23,\ f_1=21$ and $f_2=10.$

Thus Mode $=L_1+\frac{f-f_1}{2f-f_1-f_2}(L_2-L_1)$

$= 12 + \dfrac{(23-21)}{(46-21-10)}(15-12) = 12.4.$

15. (C) Mean $= \left(\dfrac{3+5+6+7+8+10+11+14}{8}\right) = 8.$

$\Sigma\delta = |3-8| + |5-8| + |8-8| + |10-8| + |11-8| + |14-8|$

$= 22$

Thus Mean deviation $= \dfrac{\Sigma\delta}{n} = \dfrac{22}{8} = 2.75.$

16. (B)

x	f	$f \times x$	$\delta = \lvert x - M \rvert$	$f \times \delta$
10	3	30	2	6
11	12	132	1	12
12	18	216	0	0
13	12	156	1	12
14	3	42	2	6
	$\Sigma f = 48$	$\Sigma fx = 576$		$\Sigma f\delta = 36$

Thus $M = \dfrac{576}{48} = 12.$

So, Mean deviation $= \dfrac{\Sigma f\delta}{n} = \dfrac{36}{48} = 0.75$

17. (D) $m = \dfrac{7+9+11+13+15}{5} = \dfrac{55}{5} = 11.$

$\Sigma\delta^2 = |7-11|^2 + |9-11|^2 + |11-11|^2 + |13-11|^2 + |15-11|^2 = 40$

$\sigma = \sqrt{\dfrac{\Sigma\delta^2}{n}} = \sqrt{\dfrac{40}{5}} = \sqrt{8} = 2\sqrt{2} = 2 \times 1.41 = 2.8.$

18. (C) $M = \dfrac{6+8+10+12+14}{5} = \dfrac{50}{5} = 10.$

$\Sigma\delta^2 = |6-10|^2 + |8-10|^2 + |10-10|^2 + |12-10|^2 + |14-10|^2 = 40$

$6 = \sqrt{\dfrac{\Sigma\delta^2}{n}} = \sqrt{\dfrac{40}{5}}$

$= \sqrt{8} = 2\sqrt{2} = 2 \times 1.414 = 2.83$ (app.)

19. (B) Here $p = 0.4$, $q = 0.6$ and $n = 3$.

Required probability $= P$(A occurring at least once)

$= {}^3C_1 \cdot (0.4) \times (0.6)^2 + {}^3C_2 \cdot (0.4)^2 \times (0.6) + {}^3C_3 \cdot (0.4)^3$

$= \left(3 \times \dfrac{4}{10} \times \dfrac{36}{100} + 3 \times \dfrac{16}{100} \times \dfrac{6}{10} + \dfrac{64}{1000}\right) = \dfrac{784}{1000} = 0.784.$

20. (B) $p = \dfrac{1}{2}$, $q = \dfrac{1}{2}$, $n = 8$. Required probability

$= P$ (6 heads or 7 heads or 8 heads)

$= {}^8C_6 \cdot \left(\dfrac{1}{2}\right)^6 \cdot \left(\dfrac{1}{2}\right)^2 + {}^8C_7 \cdot \left(\dfrac{1}{2}\right)^7 \cdot \dfrac{1}{2} + {}^8C_8 \cdot \left(\dfrac{1}{2}\right)^8$

$= \dfrac{8 \times 7}{2 \times 1} \times \dfrac{1}{256} + 8 \times \dfrac{1}{256} + \dfrac{1}{256} = \dfrac{37}{256}$

21. (C) Let E = the event that A solves the problem. and F = the event that B solves the problem.

Clearly E and F are independent events.

$P(E) = \dfrac{90}{100} = 0.9, \quad P(F) = \dfrac{70}{100} = 0.7,$

$P(E \equiv F) = P(E) \cdot P(F) = 0.9 \times 0.7 = 0.63$

Required probability $= P(E \cup F)$

$= P(E) + P(F) - P(E \equiv F) = (0.9 + 0.7 - 0.63) = 0.97.$

22. (C) Let E =event that A speaks the truth.

F =event that B speaks the truth.

Then, $P(E) = \dfrac{75}{100} = \dfrac{3}{4}$, $\quad P(F) = \dfrac{80}{100} = \dfrac{4}{5}$

$P(\overline{E}) = \left(1 - \dfrac{3}{4}\right) = \dfrac{1}{4}$, $\quad P(\overline{F}) = \left(1 - \dfrac{4}{5}\right) = \dfrac{1}{5}$

P (A and B contradict each other).

$= P$[(A speaks truth and B tells a lie) or (A tells a lie and B speaks the truth)]

$= P\,(E$ and $\overline{F}) + P\,(\overline{E}$ and $F)$

$= P(E) \cdot P(\overline{F}) + P(\overline{E}) \cdot P(F)$

$= \dfrac{3}{4} \times \dfrac{1}{5} + \dfrac{1}{4} \times \dfrac{4}{5} = \dfrac{3}{20} + \dfrac{1}{5} = \dfrac{7}{20} = \left(\dfrac{7}{20} \times 100\right)\% = 35\%.$

23. (A) Let E = event that the husband will be alive 25 years hence and F =event that the wife will be alive 25 years hence.

Then, $P(E) = \dfrac{5}{12}$ and $P(F) = \dfrac{3}{8}$

Thus $P(\overline{E}) = \left(1 - \dfrac{5}{12}\right) = \dfrac{7}{12}$ and $P(F) = \left(1 - \dfrac{3}{8}\right) = \dfrac{5}{8}$.

Clearly, E and F are independent events.

So, $\overline{E}$ and $\overline{F}$ are independent events.

P(at least one of them will be alive 25 years hence)

$= 1 - P$(none will be alive 24 years hence)

$= 1 - P(\overline{E} \equiv \overline{F}) = 1 - P(\overline{E}) \cdot P(\overline{F}) = \left(1 - \dfrac{7}{12} \times \dfrac{5}{8}\right) = \dfrac{61}{96}$

24. (D) P(none dies)

$= (1-p)(1-p)....n$ times $= (1-p)^n$

P(at least one dies) $= 1 - (1-p)^n$.

$P(A_1$ dies$) = \dfrac{1}{n}\{1 - (1-p)^n\}.$

39. (C) $\bar{x} = \frac{\Sigma x_i}{n} = \frac{15}{5} = 3, \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{36}{5} = 7.2$

$$\text{cov}(x, y) = \left(\frac{\Sigma x_i y_i}{n} - \bar{x}\,\bar{y}\right) = \left(\frac{110}{5} - 3 \times 7.2\right) = 0.4$$

40. (D) $r = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \cdot var(y)}} = \frac{-16.5}{\sqrt{2.89 \times 100}} = -0.97.$

41. (B) $D_i = -2, -8, -2, 3, 3, -3, 3, 0, 2, 4.$

$\Sigma D_i^2 = (4 + 64 + 4 + 9 + 9 + 9 + 9 + 0 + 4 + 16) = 128.$

$$R = \left[1 - \frac{6(\Sigma D_i^2)}{n(n^2 - 1)}\right] = \left(1 - \frac{6 \times 128}{10 \times 99}\right) = \frac{37}{165} = 0.224.$$

42. (A) $b_{yx} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]}$

$$= \frac{\left(306 - \frac{24 \times 44}{4}\right)}{\left[164 - \frac{(24)^2}{4}\right]} = \frac{(306 - 264)}{(164 - 144)} = \frac{42}{20} = 2.1$$

43. (B) $b_{yx} = \frac{\left[\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}\right]}{\left[\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right]} = \frac{\left(199 - \frac{30 \times 42}{6}\right)}{\left[318 - \frac{42 \times 42}{6}\right]}$

$$= \frac{(199 - 210)}{(318 - 294)} = \frac{-11}{24} = -0.46.$$

44. (C) $b_{yx} = r \cdot \frac{\sigma y}{\sigma x}$ and $b_{xy} = r \cdot \frac{\sigma x}{\sigma y}$

$r^2 = b_{xy} \times b_{yx} \quad \Rightarrow \quad r = \sqrt{b_{xy} \times b_{yx}}.$

45. (C) $\frac{1}{2}(b_{xy} + b_{yx}) > r$ is true if $\frac{1}{2}\left[r \cdot \frac{\sigma y}{\sigma x} + r \cdot \frac{\sigma x}{\sigma y}\right] > r$

i.e. if $\sigma_y^2 + \sigma_x^2 > 2\,\sigma_x\,\sigma_y$

i.e. if $(\sigma_y - \sigma_x)^2 > 0$, which is true.

46. (A) $r = \sqrt{1.6 \times 0.4} = \sqrt{.64} = 0.8$

$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \quad \Rightarrow \quad \frac{\sigma_y}{\sigma_x} = \frac{b_{yx}}{r} = \frac{1.6}{0.8} = 2$

$m_1 = \frac{1}{r} \cdot \frac{\sigma_y}{\sigma_x} = \frac{1}{0.8} \times 2 = \frac{5}{2}, \qquad m_2 = r \cdot \frac{\sigma_y}{\sigma_x} = 0.8 \times 2 = 1.6.$

$$\tan\theta = \left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) = \left(\frac{2.5 - 1.6}{1 + 2.5 \times 1.6}\right) = \frac{0.9}{5} = 0.18.$$

47. (C) Given lines are : $y = -2 - \frac{3}{4}x$

and $x = \left(-\frac{7}{4} - \frac{3}{4}y\right)$

$b_{yx} = \frac{-3}{4}$ and $b_{xy} = \frac{-3}{4}$.

So, $r^2 = \left(\frac{-3}{4} \times \frac{-3}{4}\right) = \frac{9}{16}$ or $r = -\frac{3}{4} = -0.75.$

[$\because$ b_{yx} and b_{xy} are both negative $\rightarrow$ r is negative]

48. (A) $\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\,\text{var}(Y)}} = \frac{10}{\sqrt{6.25 \times 31.36}} = \frac{5}{7}$

49. (C) $b_{yx} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$

$$= \left(\frac{5 \times 44 - 15 \times 15}{5 \times 49 - 15 \times 15}\right) = -\frac{1}{4}$$

50. (B) $b_{xy} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2}$

$$= \frac{25 \times 50 - 125 \times 100}{25 \times 1500 - 100 \times 100} = \frac{9}{22}$$

Also, $\bar{x} = \frac{125}{25} = 5, \qquad \bar{y} = \frac{100}{25} = 4.$

Required line is $x = \bar{x} + b_{xy}(y - \bar{y})$

$\Rightarrow \quad x = 5 + \frac{9}{22}(y - 4) \quad \Rightarrow \quad 22x - 9y = 74.$

(B) $\frac{x^2}{2}+\frac{x^5}{20}+\frac{x^8}{160}+\frac{x^{11}}{4400}$

(C) $\frac{x^2}{2}+\frac{x^5}{20}+\frac{x^8}{160}+\frac{x^{11}}{2400}$

(D) $\frac{x^2}{2}+\frac{x^5}{40}+\frac{x^8}{480}+\frac{x^{11}}{2400}$

12. For $dy/dx = xy$ given that $y = 1$ at $x = 0$. Using Euler method taking the step size 0.1, the y at $x = 0.4$ is

(A) 1.0611 (B) 2.4680

(C) 1.6321 (D) 2.4189

Statement for Q. 13–15.

For $dy/dx = x^2 + y^2$ given that $y = 1$ at $x = 0$. Determine the value of y at given x in question using modified method of Euler. Take the step size 0.02.

13. y at $x = 0.02$ is

(A) 1.0468 (B) 1.0204

(C) 1.0346 (D) 1.0348

14. y at $x = 0.04$ is

(A) 1.0316 (B) 1.0301

(C) 1.403 (D) 1.0416

15. y at $x = 0.06$ is

(A) 1.0348 (B) 1.0539

(C) 1.0638 (D) 1.0796

16. For $dy/dx = x + y$ given that $y = 1$ at $x = 0$. Using modified Euler's method taking step size 0.2, the value of y at $x = 1$ is

(A) 3.401638 (B) 3.405417

(C) 9.164396 (D) 9.168238

17. For the differential equation $dy/dx = x - y^2$ given that

x:	0	0.2	0.4	0.6
y:	0	0.02	0.0795	0.1762

Using Milne predictor–correction method, the y at next value of x is

(A) 0.2498 (B) 0.3046

(C) 0.4648 (D) 0.5114

Statement for Q. 18–19:

For $\frac{dy}{dx} = 1 + y^2$ given that

x:	0	0.2	0.4	0.6
y:	0	0.2027	0.4228	0.6841

Using Milne's method determine the value of y for x given in question.

18. $y(0.8) = ?$

(A) 1.0293 (B) 0.4228

(C) 0.6065 (D) 1.4396

19. $y(1.0) = ?$

(A) 1.9428 (B) 1.3428

(C) 1.5555 (D) 2.168

Statement for Q.20–22:

Apply Runge Kutta fourth order method to obtain $y(0.2)$, $y(0.4)$ and $y(0.6)$ from $dy/dx = 1 + y^2$, with $y = 0$ at $x = 0$. Take step size $h = 0.2$.

20. $y(0.2) = ?$

(A) 0.2027 (B) 0.4396

(C) 0.3846 (D) 0.9341

21. $y(0.4) = ?$

(A) 0.1649 (B) 0.8397

(C) 0.4227 (D) 0.1934

22. $y(0.6) = ?$

(A) 0.9348 (B) 0.2935

(C) 0.6841 (D) 0.563

23. For $dy/dx = x + y^2$, given that $y = 1$ at $x = 0$. Using Runge Kutta fourth order method the value of y at $x = 0.2$ is $(h = 0.2)$

(A) 1.2735 (B) 2.1635

(C) 1.9356 (D) 2.9468

24. For $dy/dx = x + y$ given that $y = 1$ at $x = 0$. Using Runge Kutta fourth order method the value of y at $x = 0.2$ is $(h = 0.2)$

(A) 1.1384 (B) 1.9438

(C) 1.2428 (D) 1.6389

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SOLUTIONS

1. (B) Let $f(x) = x^3 - 4x - 9$

Since $f(2)$ is negative and $f(3)$ is positive, a root lies between 2 and 3.

First approximation to the root is

$$x_1 = \frac{1}{2}(2+3) = 2.5.$$

Then $f(x_1) = 2.5^3 - 4(2.5) - 9 = -3.375$

i.e. negative∴ The root lies between x_1 and 3. Thus the second approximation to the root is

$$x_2 = \frac{1}{2}(x_1 + 3) = 2.75.$$

Then $f(x_2) = (2.75)^3 - 4(2.75) - 9 = 0.7969$ i.e. positive.

The root lies between x_1 and x_2. Thus the third approximation to the root is $x_3 = \frac{1}{2}(x_1 + x_2) = 2.625.$

Then $f(x_3) = (2.625)^3 - 4(2.625) - 9 = -1.4121$ i.e. negative.

The root lies between x_2 and x_3. Thus the fourth approximation to the root is $x_4 = \frac{1}{2}(x_2 + x_3) = 2.6875.$

Hence the root is 2.6875 approximately.

2. (B) Let $f(x) = x^3 - 2x - 5$

So that $f(2) = -1$ and $f(3) = 16$

i.e. a root lies between 2 and 3.

Taking $x_0 = 2, x_1 = 3, f(x_0) = -1, f(x_1) = 16,$ in the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) = 2 + \frac{1}{17} = 2.0588$$

Now, $f(x_2) = f(2.0588) = -0.3908$ i.e., that root lies between 2.0588 and 3.

Taking $x_0 = 2.0588, x_1 = 3, f(x_0)$

$= -0.3908,\ f(x_1) = 16$ in (i), we get

$$x_3 = 2.0588 - \frac{0.9412}{16.3908}(-0.3908) = 2.0813$$

Repeating this process, the successive approxima- tions are

$x_4 = 2.0862, x_5 = 2.0915, x_6 = 2.0934, x_7 = 2.0941,$

$x_8 = 2.0943$ etc.

Hence the root is 2.094 correct to 3 decimal places.

3. (C) Let $f(x) 2x - \log_{10} x - 7$

Taking $x_0 = 3.5,\ x_1 = 4,$ in the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

$$= 3.5 - \frac{0.5}{0.3979 + 0.5441}(-0.5441) = 3.7888$$

Since $f(3.7888) = -0.0009$ and $f(4) = 0.3979,$ therefore the root lies between 3.7888 and 4.

Taking $x_0 = 3.7888, x_1 = 4,$ we obtain

$$x_3 = 3.7888 - \frac{0.2112}{0.3988}(-.009) = 3.7893$$

Hence the required root correct to three places of decimal is 3.789.

4. (D) Let $f(x) = xe^x - 2,$ Then $f(0) = -2,$ and

$f(1) = e - 2 = 0.7183$

So a root of (i) lies between 0 and 1. It is nearer to 1. Let us take $x_0 = 1.$

Also $f'(x) = xe^x + e^x$ and $f'(1) = e + e = 5.4366$

By Newton's rule, the first approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{0.7183}{5.4366} = 0.8679$$

$f(x_1) = 0.0672,\quad f'(x_1) = 4.4491.$

Thus the second approximation x_2 is

$$x_2 = x_1 - \frac{f(x_1)}{f(x_1)} = 0.8679 - \frac{0.0672}{4.4491} = 0.8528$$

Hence the required root is 0.853 correct to 3 decimal places.

5. (B) Let $y = x + \log_{10} x - 3.375$

To obtain a rough estimate of its root, we draw the graph of (i) with the help of the following table :

x	1	2	3	4
y	-2.375	-1.074	0.102	1.227

Taking 1 unit along either axis $= 0.1$, The curve crosses the x–axis at $x_0 = 2.9$, which we take as the initial approximation to the root.

Now let us apply Newton–Raphson method to

$f(x) = x + \log_{10} x - 3.375$

$$f'(x) = 1 + \frac{1}{x}\log_{10} e$$

$f(2.9) = 2.9 + \log_{10} 2.9 - 3.375 = -0.0126$

$$f'(2.9) = 1 + \frac{1}{2.9}\log_{10} e = 1.1497$$

The first approximation x_1 to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.9 + \frac{0.0126}{1.1497} = 2.9109$$

$f(x_1) = -0.0001, f'(x_1) = 1.1492$

Thus the second approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.9109 + \frac{0.0001}{1.1492} = 2.91099$$

Hence the desired root, correct to four significant figures, is 2.911

6. (B) Let $x = \sqrt{28}$ so that $x^2 - 28 = 0$

Taking $f(x) = x^2 - 28$, Newton's iterative method gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 28}{2x_n} = \frac{1}{2}\left(x_n + \frac{28}{x_n}\right)$$

Now since $f(5) = -3, f(6) = 8$, a root lies between 5 and 6.

Taking $x_0 = 5.5$,

$$x_1 = \frac{1}{2}\left(x_0 + \frac{28}{x_0}\right) = \frac{1}{2}\left(5.5 + \frac{28}{5.5}\right) = 5.29545$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{28}{x_1}\right) = \frac{1}{2}\left(5.29545 + \frac{28}{5.29545}\right) = 5.2915$$

$$x_3 = \frac{1}{2}\left(x_2 + \frac{28}{x_2}\right) = \frac{1}{2}\left(5.2915 + \frac{28}{5.2915}\right) = 5.2915$$

Since $x_2 = x_3$ upto 4 decimal places, so we take $\sqrt{28} = 5.2915$.

7. (B) Let $h = 0.1$, given $x_0 = 0$, $x_1 = x_0 + h = 0.1$

$$\frac{dy}{dx} = 1 + xy \Rightarrow \frac{d^2y}{dx^2} = x\frac{dy}{dx} + y$$

$$\frac{d^3y}{dx^3} = x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}, \quad \frac{d^4y}{dx^4} = x\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2}$$

given that $x = 0$, $y = 1$

$$\Rightarrow \frac{dy}{dx} = 1; \frac{d^2y}{dx^2} = 1, \frac{d^3y}{dx^3} = 2, \frac{d^4y}{dx^4} = 3 \text{ and so on}$$

The Taylor series expression gives :

$$y(x+h) = y(x) + h\frac{dy}{dx} + \frac{h^2}{2!}\frac{d^2y}{dx^2} + \frac{h^3}{3!}\frac{d^3y}{dx^3} +$$

$$\Rightarrow y(0.1) = 1 + 0.1 \times 1 + \frac{(0.1)^2}{2!} \cdot 1 + \frac{(0.1)^3}{3!} 2 + \dots$$

$$\Rightarrow y(0.1) = 1 + 0.1 + \frac{0.01}{2} + \frac{0.001}{3} + \dots$$

$$= 1 + 0.1 + 0.005 + 0.000033 \dots\dots\dots = 1.1053$$

8. (B) Let $h = 0.1$, given $x_0 = 0$, $y_0 = 1$

$$x_1 = x_0 + h = 0.1, \quad \frac{dy}{dx} = x - y^2$$

at $x = 0$, $y = 1$, $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$$

at $x = 0$, $y = 1$, $\frac{d^2y}{dx^2} = 1 + 2 = 3$

$$\frac{d^3y}{dx^3} = -2\left(\frac{dy}{dx}\right)^2 - 2y\frac{d^2y}{dx^2}$$

at $x = 0$, $y = 1$, $\frac{d^3y}{dx^3} = -8$

$$\frac{d^4y}{dx^4} = -2\left[3\frac{dy}{dx}\frac{d^2y}{dx^2} + y\frac{d^3y}{dx^3}\right]$$

at $x = 0$, $y = 1$ $\frac{d^4y}{dx^4} = 34$

The Taylor series expression gives

$$y(x+h) = y(x) + h\frac{dy}{dx} + \frac{h^2}{2!}\frac{d^2y}{dx^2} + \frac{h^3}{3!}\frac{d^3y}{dx^3} + \frac{h^4}{4!}\frac{d^4y}{dx^4} + \dots$$

$$y(0.1) = 1 + 0.1(-1) + \frac{(0.1)^2}{2!}3 + \frac{(0.1)^3}{3!}(-8) + \frac{(0.1)^4}{4!}34 + \dots\dots$$

$$= 1 - 0.1 + 0.015 - 0.001333 + 0.0001417 = 0.9138$$

9. (C) Here $f(x, y) = x^2 + y^2$, $x_0 = 0$ $y_0 = 0$

We have, by Picard's method

$$y = y_0 + \int_{x_0}^{x} f(x, y)dx \quad \dots\dots(1)$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

Where $y_0 = 0 + \int_0^x f(x, 0)dx = \int_0^x x^2 dx.$ ...(2)

The second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})dx = 0 + \int_0^x f\left(x, \frac{x^3}{3}\right)dx$$

$$= 0 + \int_0^x \left(x^2 + \frac{x^6}{9}\right)dx = \frac{x^3}{3} + \frac{x^7}{63}$$

Now, $y(0.4) = \frac{(0.4)^3}{3} + \frac{(0.4)^7}{63} = 0.02135$

10. (C) Here $f(x, y) = y - x$; $x_0 = 0$, $y_0 = 2$

We have by Picard's method

$$y = y_0 + \int_{x0}^{x} f(x, y)\,dx$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)dx = 2 + \int_0^x f(x, 2)\,dx$$

$$= 2 + \int_0^x (2 - x)dx \quad = 2 + 2x - \frac{x^2}{2} \qquad ....(1)$$

The second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^x f(x, y^{(1)})dx$$

$$= 2 + \int_{x_0}^x f\left(x, 2 + 2x - \frac{x^2}{2}\right)dx$$

$$= 2 + \int_0^2 (2 + 2x - \frac{x^2}{2} - x)dx$$

$$= 2 + 2x + \frac{x^2}{2} - \frac{x^3}{6} \qquad ....(2)$$

The third approximation to y is given by

$$y^{(3)} = y_0 + \int_{x_0}^x f(x, y^{(2)})dx$$

$$= 2 + \int_{x_0}^x f\left(x, 2 + 2x + \frac{x^2}{2} - \frac{x^3}{6}\right)dx$$

$$= 2 + \int_0^x \left(2 + 2x + \frac{x^2}{2} - \frac{x^3}{6} - \right)dx$$

$$= 2 + 2x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24}$$

11. (B) Here $f(x, y) = x + y^2, \quad x_0 = 0 \quad y_0 = 0$

We have, by Picard's method

$$y = y_0 + \int_{x_0}^x f(x, y_0)dx$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^x f(x, y_0)dx \quad = 0 + \int_0^x f(x, 0)dx$$

$$= 0 + \int_0^x x dx \quad = \frac{x^2}{2}$$

The second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^x f(x, y^{(1)})dx \quad = 0 + \int_0^x f\left(x, \frac{x^2}{2}\right)dx$$

$$= \int_0^x \left(x + \frac{x^4}{4}\right)dx \quad = \frac{x^2}{2} + \frac{x^5}{50}$$

The third approximation is given by

$$y^{(3)} = y_0 + \int_{x_0}^x f(x, y^{(2)})dx$$

$$= 0 + \int_0^x f\left(x, \frac{x^2}{2} + \frac{x^5}{20}\right)dx$$

$$= \int_0^x \left(x + \frac{x^4}{4} + \frac{x^{10}}{400} + \frac{2x^7}{40}\right)dx = \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400}$$

12. (A) x: 0 0.1 0.2 0.3 0.4

Euler's method gives

$$y_{n+1} = y_n + h(x_n, y_n) \qquad ....(1)$$

$n = 0$ in (1) gives

$y_1 = y_0 + hf(x_0, y_0)$

Here $x_0 = 0, \quad y_0 = 1, \quad h = 0.1$

$y_1 = 1 + 0.1\ f(0, 1) \ = 1 + 0 \ = 1$

$n = 0$ in (1) gives $y_2 = y_1 + h\ f(x_1, y_1)$

$= 1 + 0.1\ f(0.1, 1) \ = 1 + 0.1(0.1) \ = 1 + 0.01$

Thus $y_2 = y_{(0.2)} = 1.01$

$n = 2$ in (1) gives

$y_3 = y_2 + hf(x_2, y_2) \ = 1.01 + 0.1 f(0.2, 1.01)$

$y_3 = y_{(0.3)} = 1.01 + 0.0202 \ = \ 1.0302$

$n = 3$ in (1) gives

$y_4 = y_3 + hf(x_3, y_3) \ = 1.0302 + 0.1f(0.3, 1.0302)$

$= 1.0302 + 0.03090$

$y_4 = y_{(0.4)} = 1.0611$

Hence $y_{(0.4)} = 1.0611$

13. (B) The Euler's modified method gives

$y_1^* = y_0 + hf(x_0, y_0),$

$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^*)]$

Now, here $h = 0.02, \quad y_0 = 1, \quad x_0 = 0$

$y_1^* = 1 + 0.02f(0, 1), \quad y_1^* = 1 + 0.02 \ = 1.02$

Next $y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x, y_1^*)]$

$= 1 + \frac{0.02}{2}[f(0, 1) + f(0.02, \ 1.02)]$

$= 1 + 0.01[1 + 1.0204] = 1.0202$

So, $y_1 = y(0.02) = 1.0202$

14. (D) $y_2^* = y_1 + h\ f(x_1, y_1)$

$= 1.0202 + 0.02[f(0.02, \ 1.0202)]$

$= 1.0202 + 0.0204 \quad = 1.0406$

Next $y_2 = y_1 + \frac{h}{2}[f(x, y) + f(x_2, y_2^*)]$

$y_2 = 1.0202 + \frac{0.02}{2}[f(0.02, 1.0202) + f(0.04, 1.0406)]$

$= 1.0202 + 0.01[1.0206 + 1.0422] = 1.0408$

$y_2 = y_{(0.04)} = 1.0408$

15. (C) $y_3^* = y_2 + hf(x_2, y_2)$

$= 1.0416 + 0.02f(0.04, 1.0416)$

$= 1.0416 + 0.0217 = 1.0633$

Next $y_3 = y_2 + \frac{h}{2}[f(x_2, y_2) + f(x_3, y_3^*)]$

$k_2 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1\right) = (0.2)f(0.1, 0.1) = 0.202$

$k_3 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2\right) = (0.2)f(0.1, 0.101) \quad = 0.2020$

$k_4 = hf(x_0 + h, \ y_0 + k_3) \quad = 0.2f(0.2, 0.2020) = 0.20816$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2\,(.202) + 2\,(.20204) + 0.20816],$

$k = 0.2027$

such that $\quad y_1 = y(0.2) = y_0 + k \quad = 0 + 0.2027 \quad = 0.2027$

21. (C) We now to find $\ y_2 = y(0.4), \quad k_1 = hf(x_1, y_1)$

$= (0.2)f(0.2, 0.2027) \quad = 0.2\,(1.0410) \quad = .2082$

$k_2 = hf\left(x_1 + \frac{1}{2}h\,, y_1 + \frac{1}{2}k_1\right)$

$= (0.2)f(0.3, 0.3068) \quad = 0.2188$

$k_3 = hf\left(x_1 + \frac{1}{2}h\,, y_1 + \frac{1}{2}k_2\right)$

$= 0.2f(0.3, 0.3121) \quad = .2194$

$k_4 = hf(x_1 + h, y_1 + k_3) \quad = 0.2f(0.4, .4221) \quad = 0.2356$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2082 + 2(.2188) + 2(.2194) + 0.356] = 0.2200$

$y_2 = y_{(0.4)} = y_1 + k \quad = 0.2200 + .2027 = 0.4227$

22. (C) We now to find $y_3 = y_{(0.6)}\ , \quad k_1 = hf(x_2, y_2)$

$= (0.2)f(0.4, 0.4228) \quad = 0.2357$

$k_2 = hf\left(x_2 + \frac{1}{2}h, y_2 + \frac{1}{2}k_1\right)$

$= (0.2)f(0.5, 0.5406) \quad = 0.2584$

$k_3 = hf\left(x_2 + \frac{1}{2}h, y_2 + \frac{1}{2}k_2\right)$

$= 0.2f(0.5, .5520) \quad = 0.2609$

$k_4 = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2357 + 2(.2584) + 2(0.2609) + 0.2935]$

$= \frac{1}{6}[0.2357 + 0.5168 + 0.5218 + 0.2935] = 0.2613$

$y_3 = y_{(0.6)} = y_2 + k \quad = .4228 + 0.2613 \quad = 0.6841$

23. (A) Here given $\quad x_0 = 0 \qquad y_0 = 1, \quad h = 0.2$

$f(x, y) = x + y^2$

To find $\ y_1 = y_{(0.2)}\ ,$

$k_1 = hf(x_0, y_0) \quad = (0.2)f(0, 1) = (0.2) \times 1 = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.2)f(0.1, 1.1) \quad = 0.2(1.31) \quad = 0.262$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2f(0.1, 1.131) \quad = 0.2758$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= (0.2)\ f\ (0.2, 1.2758) = 0.3655$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + 2k_4]$

$= \frac{1}{6}[0.2 + 2\,(0.262) + 2\,(0.2758) + 0.3655] = 0.2735$

Here $\quad y_1 = y_{(0.2)} = y_0 + k \quad = 1 + 0.2735 \quad \Rightarrow \quad 1.2735$

24. (C) Here $\ f\,(x, y) \ = \ x + y \quad h = 0.2$

To find $\ y_1 = y_{(0.2)}\ ,$

$k_1 = hf\,(x_0, y_0) \quad = 0.2f(0, 1) \quad = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.2)f(0.1, 1.1) \quad = 0.24$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.2)f(0.1, 1.12) = 0.244$

$k_4 = hf\,(x_0 + h, y_0 + k_3) \quad = (0.2)f(0.2, 1.244) \quad = 0.2888$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2(0.24) + 2(0.244) + 0.2888] \quad = 0.2428$

$y_1 = y_{(0.2)} = y_0 + k \quad = 1 + 0.2428 \quad = 1.2428$

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CHAPTER

10.5

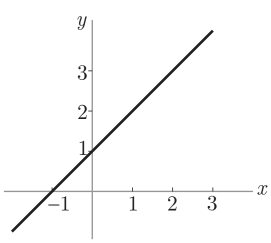
EC-07

1. If E Denotes expectation, the variance of a random variable X is given by

(A) $E[X^2] - E^2[X]$ (B) $E[X^2] + E^2[X]$

(C) $E[X^2]$ (D) $E^2[X]$

2. The following plot shows a function y which varies linearly with X. The value of the integral $I = \int_1^2 y\,dx$ is



(A) 1.0 (B) 2.5

(C) 4.0 (D) 5.0

3. For $|x| << 1$, $\coth(x)$ can be approximated as

(A) x (B) x^2

(C) $\frac{1}{x}$ (D) $\frac{1}{x^2}$

4. $\lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta}$ is

(A) 0.5 (B) 1

(C) 2 (D) not defined

5. Which of the following functions is strictly bounded ?

(A) $\frac{1}{x^2}$ (B) e^x

(C) x^2 (D) e^{-x^2}

6. For the function e^{-x}, the linear approximation around $x = 2$ is

(A) $(3 - x)e^{-2}$ (B) $1 - x$

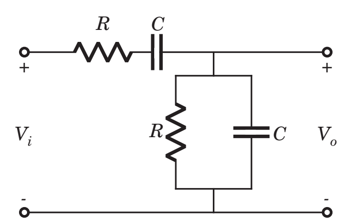
(C) $[3 + 2\sqrt{2} - 1(1 + \sqrt{2}x]e^{-2}$ (D) e^{-2}

7. An independent voltage source in series with an impedance $Z_s = R_s + jX_s$ delivers a maximum average power to a load impedance Z_L when

(A) $Z_L = R_S + jX_S$ (B) $Z_L = R_S$

(C) $Z_L = jX_S$ (D) $Z_L = R_S - jX_S$

8. The RC circuit shown in the figure is



(A) a low-pas filter (B) a high-pass filter

(C) a band-pass filter (D) a band-reject filter

9. The electron and hole concentrations in an intrinsic semiconductor are n_i per cm^3 at 300 K. Now, if acceptor impurities are introduced with a concentration of N_A per cm^3(where $N_A >> n_i$) the electron concentration per cm^3 at 300 K will be

(A) n_i (B) $n_i + N_A$

(C) $N_A - n_i$ (D) $\frac{n_i^2}{N_A}$

10. In a $p^{+}n$ junction diode under reverse biased the magnitude of electric field is maximum at

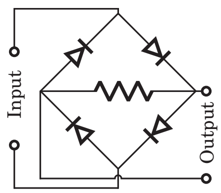
(A) the edge of the depletion region on the p –side

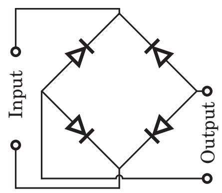
(B) the edge of the depletion region on the n –side

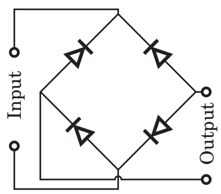
(C) the $p^{+}n$ junction

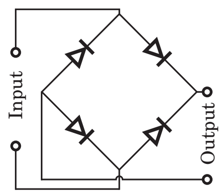
(D) the center of the depletion region on the n-side

11. The correct full wave rectifier circuit is

(A)


(B)


(C)


(D)


12. In a trans-conductance amplifier, it is desirable to have

(A) a large input resistance and a large output resistance

(B) a large input resistance and a small output resistance

(C) a small input resistance and a large output resistance

(D) a small input resistance and a small output resistance

13. $X = 01110$ and Y = 11001 are two 5-bit binary numbers represented in two's complement format. The sum of X and Y represented in two's complement format using 6 bits is

(A) 100111 (B) 0010000

(C) 000111 (D) 101001

14. The Boolean function Y = AB + CD is to be realized using only 2-input NAND gates. The minimum number of gates required is

(A) 2 (B) 3

(C) 4 (D) 5

15. If closed-loop transfer function of a control system is given as $T(s) = \frac{s-5}{(s+2)(s+3)}$ then It is

(A) an unstable system

(B) an uncontrollable system

(C) a minimum phase system

(D) a non-minimum phase system

16. If the Laplace transform of a signal $y(t)$ is $Y(s) = \frac{1}{s(s-1)}$, then its final value is

(A) -1 (B) 0

(C) 1 (D) unbounded

17. If $R(\tau)$ is the auto correlation function of a real, wide-sense stationary random process, then which of the following is NOT true

(A) $R(\tau) = R(-\tau)$

(B) $|R(\tau)| \le R(0)$

(C) $R(\tau) = -R(-\tau)$

(D) The mean square value of the process is $R(0)$

18. If $S(f)$is the power spectral density of a real, wide-sense stationary random process, then which of the following is ALWAYS true?

(A) $S(0) \le S(f)$ (B) $S(f) \ge 0$

(C) $S(-f) = -S(f)$ (D) $\int_{-\infty}^{\infty} S(f)df = 0$

19. A plane wave of wavelength λ is traveling in a direction making an angle 30° with positive x –axis. The $\vec{E}$ field of the plane wave can be represented as ($E_0$is constant)

(A) $\vec{E} = yE_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x \frac{\pi}{\lambda}z\right)}$ (B) $\vec{E} = yE_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x \frac{\sqrt{3}\pi}{\lambda}z\right)}$

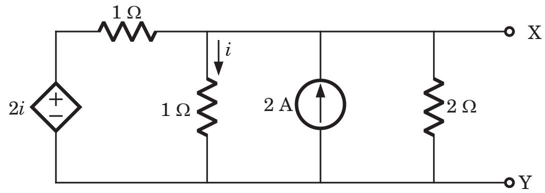
(C) $\vec{E} = yE_0 e^{j\left(\omega t + \frac{\sqrt{3}\pi}{\lambda}x \frac{\pi}{\lambda}z\right)}$ (D) $\vec{E} = yE_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x + \frac{\sqrt{3}\pi}{\lambda}z\right)}$

20. If C is close curve enclosing a surface S, then the magnetic field intensity $\vec{H}$, the current density $\vec{j}$ and the electric flux density $\vec{D}$ are related by

(A) $\int_s\int \vec{H}.d\vec{s} = \oint_c \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{l}$

(B) $\int_s \vec{H}.d\vec{l} = \oiint_s \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} d\right).d\vec{s}$

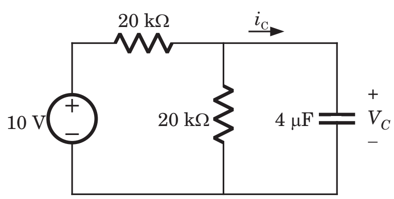
29. For the circuit shown in the figure, the Thevenin voltage and resistance looking into $X-Y$ are



(A) $\frac{4}{3}V, 2\Omega$ (B) $4V, \frac{2}{3}\Omega$

(C) $\frac{4}{3}V, \frac{2}{3}\Omega$ (D) $4V, 2\Omega$

30. In the circuit shown, V_c is 0 volts at $t=0$ sec. for $t>0$, the capacitor current $i_c(t)$, where t is in seconds, is given by



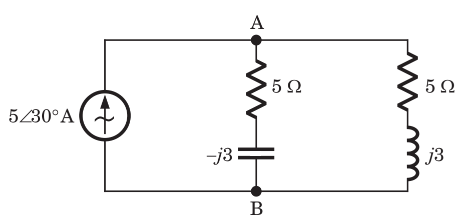
(A) $0.50 \exp(-25t)$ mA

(B) $0.25 \exp(-25t)$ mA

(C) $0.50 \exp(-25t)$ mA

(D) $0.25 \exp(-25t)$ mA

31. In the AC network shown in the figure, the phasor voltage V_{AB} (in volts) is



(A) 0 (B) $5\angle 30^\circ$
(C) $12.5\angle 30^\circ$ (D) $17\angle 30^\circ$

32. A p^+n junction has a built-in potential of 0.8 V. The depletion layer width at reverse bias of 1.2V is 2 μm. For a reverse bias of 7.2 V, the depletion layer width will be

(A) 4 µm (B) 4.9 µm

(C) 8 µm (D) 12 µm

33. Group I lists four types of $p-n$ junction diodes. match each device in Group I with one of the option in Group II to indicate the bias condition of the device in its normal mode of operation.

Group-I	Group-II
(P) Zener Diode	(1) Forward bias
(Q) Solar cell	(2) Reverse bias
(R) LASER diode	
(S) Avalanche Photodiode	

(A) P - 1 Q - 2 R - 1 S - 2

(B) P - 2 Q - 1 R - 1 S - 2

(C) P - 2 Q - 2 R - 1 S - 2

(D) P - 2 Q - 1 R - 2 S - 2

34. The DC current gain (β) of a BJT is 50. Assuming that the emitter injection efficiency is 0.995, the base transport factor is

(A) 0.980 (B) 0.985

(C) 0.990 (D) 0.995

35. group I lists four different semiconductor devices. match each device in Group I with its characteristic property in Group II.

Group-I	Group-II
(P)BJT	(1) Population inversion
(Q)MOS capacitor	(2)Pinch-off voltage
(R) LASER diode	(3) Early effect
(S) JFET	(4) Fat-band voltage

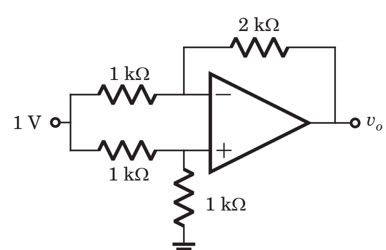
(A) P - 3 Q - 1 R - 4 S - 2

(B) P - 1 Q - 4 R - 3 S - 2

(C) P - 3 Q - 4 R - 1 S - 2

(D) P - 3 Q - 2 R - 1 S - 4

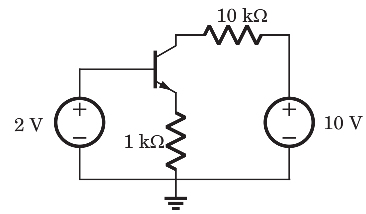
36. For the Op-Amp circuit shown in the figure, V_o is



(A) -2 V (B) -1 V

(C) -0.5 V (D) 0.5 V

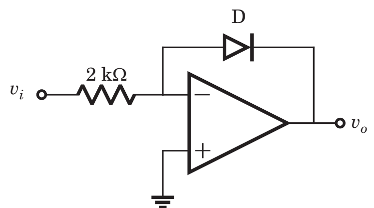
37. For the BJT circuit shown, assume that the β of the transistor is very large and $V_{BE}=0.7V$. The mode of operation of the BJT is



(A) cut-off (B) saturation

(C) normal active (D) reverse active

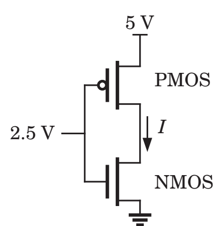
38. In the Op-Amp circuit shown, assume that the diode current follows the equation $I = I_s \exp(V/V_T)$. For $V_i = 2V, V_0 = V_{01}$, and for $V_i = 4V, V_0 = V_{02}$. The relationship between V_{01} and V_{02} is



(A) $V_{02} = \sqrt{2} V_{01}$ (B) $V_{02} = e^2 V_{01}$

(C) $V_{02} = V_{01} \ln 2$ (D) $V_{01} - V_{02} = V_T \ln 2$

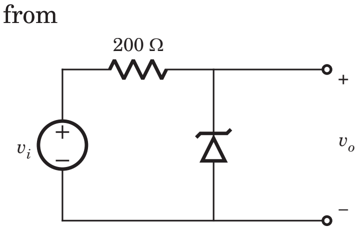
39. In the CMOS inverter circuit shown, if the transconductance parameters of the NMOS and PMOS transistors are $k_n = k_p = \mu_n C_{ox} \frac{W_n}{L_n} = \mu_p C_{ox} \frac{W_p}{L_p} = 40\,\mu A/V^2$ and their threshold voltages are $V_{THn} = |V_{THp}| = 1V$, the current I is



(A) 0 A (B) $25\,\mu A$

(C) $45\,\mu A$ (D) $90\,\mu A$

40. For the Zener diode shown in the figure, the Zener voltage at knee is 7V, the knee current is negligible and the Zener dynamic resistance is 10Ω. if the input voltage (V_i) range is from 10 to $16V$, the output voltage (V_0) ranges from



(A) 7.00 to 7.29 V (B) 7.14 to 7.29 V

(C) 7.14 to 7.43 V (D) 7.29 to 7.43 V

41. The Boolean expression $Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A} B C \overline{D} + A \overline{B}\,\overline{C} D + A B \overline{C}\,\overline{D}$ can be minimized to

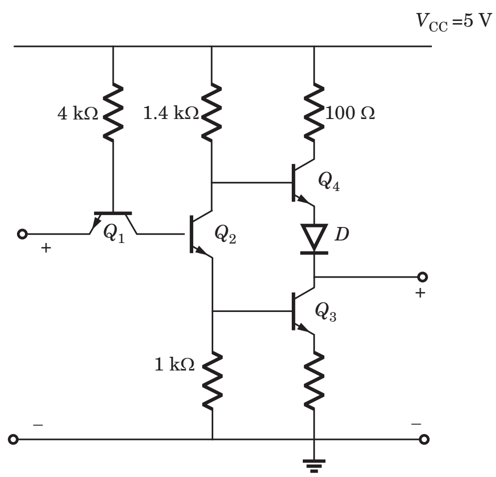
(A) $Y = \overline{A}\,\overline{B}\,\overline{C} D + \overline{A} B \overline{C} + A \overline{C} D$

(B) $Y = \overline{A}\,\overline{B}\,\overline{C} D + B C \overline{D} + A \overline{B}\,\overline{C} D$

(C) $Y = \overline{A} B C \overline{D} + \overline{B}\,\overline{C} D + A \overline{B}\,\overline{C} D$

(D) $Y = \overline{A} B C \overline{D} + \overline{B}\,\overline{C} D + A B \overline{C}\,\overline{D}$

42. The circuit diagram of a standard TTL NOT gate is shown in the figure. $V_i = 2.5V$, the modes of operation of the transistors will be



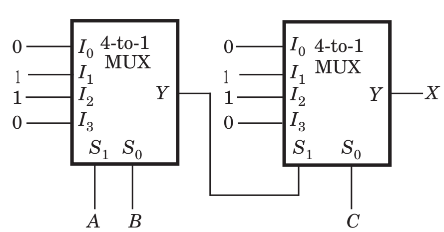
(A) Q_1: revere active; Q_2: normal active; Q_3: saturation; Q_4: cut-off

(B) Q_1: revere active; Q_2: saturation; Q_3: saturation; Q_4: cut-off

(C) Q_1: normal active; Q_2: cut-off; Q_3: cut-off; Q_4: saturation

(D) Q_1: saturation; Q_2: saturation; Q_3: saturation; Q_4: normal active

43. In the following circuit, X is given by



(A) $X = A \overline{B}\,\overline{C} + \overline{A} B \overline{C} + \overline{A}\,\overline{B} C + ABC$

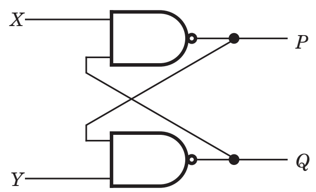
(B) $X = \overline{A} BC + A \overline{B} C + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

(C) $X = AB + BC + AC$

(D) $X = \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{C}$

44. The following binary values were applied to the X and Y inputs of NAND latch shown in the figure in the sequence indicated below

$X = 0, Y = 1;\ X = 0, Y = 0;\ X = 1,\ Y = 1.$



The corresponding stable P, Q outputs will be

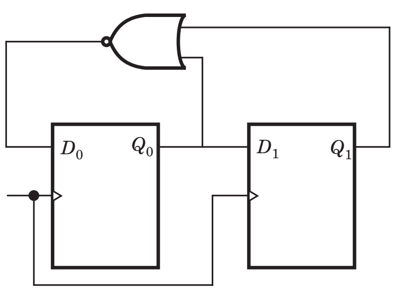
(A) $P = 1, Q = 0;\ \ P = 1, Q = 0;\ \ P = 1, Q = 0$ or $P = 0, Q = 1$

(B) $P = 1, Q = 0;\ \ P = 0, Q = 1;$ or $P = 0\ Q = 1;\ \ P = 0, Q = 1$

(C) $P = 1, Q = 0;\ \ P = 1, Q = 1;\ \ P = 1, Q = 0$ or $P = 0, Q = 1$

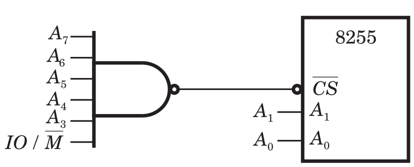
(D) $P = 1, Q = 0;\ \ P = 1, Q = 1;\ \ P = 1, Q = 1$

45. For the circuit shown, the counter state (Q_1Q_0) follows the sequence



(A) 00, 01, 10, 11, 00
(B) 00, 01, 10, 00, 01
(C) 00, 01, 11, 00, 01
(D) 00, 10, 11, 00, 10

46. An 8255 chip is interfaced to an 8085 microprocessor system as an I/O mapped I/O as show in the figure. The address lines A_0 and A_1 of the 8085 are used by the 8255 chip to decode internally its thee ports and the Control register. The address lines A_3 to A_7 as well as the $IO/\overline{M}$ signal are used for address decoding. The range of addresses for which the 8255 chip would get selected is



(A) F8H - FBH
(B) F8H - FCH
(C) F8H - FFH
(D) F0H - F7H

47. (A) The 3-dB bandwidth of the low-pas signal $e^{-t}u(t)$, where $u(t)$ is the unit step function, is given by

(A) $\frac{1}{2\pi}$ Hz
(B) $\frac{1}{2\pi}\sqrt{\sqrt{2}-1}$ Hz
(C) ∞
(D) 1 Hz

48. A Hilbert transformer is a

(A) non-linear system
(B) non-causal system
(C) time-varying system
(D) low-pass system

49. The frequency response of a linear, time-invariant system is given by $H(f) = \frac{5}{1+j10\pi f}$. The step response of the system is

(A) $5\left(1-e^{-5t}\right)u(t)$
(B) $5\left(1-e^{-\frac{t}{5}}\right)u(t)$
(C) $\frac{1}{2}\left(1-e^{-5t}\right)u(t)$
(C) $\frac{1}{5}\left(1-e^{-\frac{t}{5}}\right)u(t)$

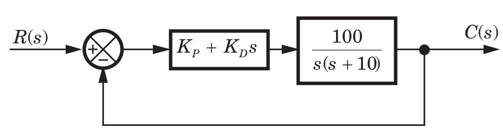
50. A 5-point sequence $x[n]$ is given as $x[-3]=1, x[-2]=1, x[-1]=0, x[0]=5, x[1]=1$. Let $X(e^{j\omega})$ denote the discrete-time Fourier transform of $x[n]$. The value of $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$ is

(A) 5
(B) 10π
(C) 16π
(D) $5 + j10\pi$

51. The z-transform $x[z]$ of a sequence $x[n]$ is given by $X(z) = \frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of $x[n]$ includes the unit circle. The value of x[0] is

(A) -0.5
(B) 0
(C) 0.25
(D) 0.5

52. A Control system with PD controller is shown in the figure If the velocity error constant $K_V = 1000$ and the damping ration $\zeta = 0.5$, then the value of K_P and K_D are



(A) $K_P = 100, K_D = 0.09$
(B) $K_P = 100, K_D = 0.9$
(C) $K_P = 10, K_D = 0.09$
(D) $K_P = 10, K_D = 0.9$

53. The transfer function of a plant is

$$T(s) = \frac{5}{(s+5)\left(s^2+s+1\right)}$$

The second-order approximation of $T(s)$ using dominant pole concept is

(A) $\frac{1}{(s+5)(s+1)}$ (B) $\frac{5}{(s+5)(s+1)}$

(C) $\frac{5}{s^2+s+1}$ (D) $\frac{1}{s^2+s+1}$

54. The open-loop transfer function of a plant is given as $G(s)=\frac{1}{s^2-1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that an stabilize this control system is

(A) $\frac{10(s-1)}{s+2}$ (B) $\frac{10(s+4)}{s+2}$

(C) $\frac{10(s+2)}{s+10}$ (D) $\frac{2(s+2)}{s+10}$

55. A unity feedback control system has an open-loop transfer function

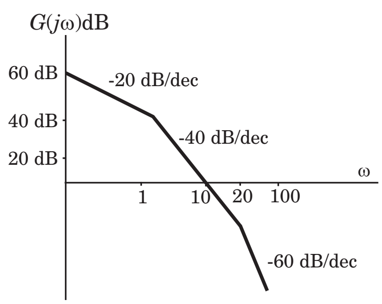
$$G(s)=\frac{K}{s(s^2+7s+12)}.$$

The gain K for which $s=1+j1$ will lie on the root locus of this system is

(A) 4 (B) 5.5

(C) 6.5 (D) 10

56. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is



(A) $\frac{1}{(s+1)(s+20)}$ (B) $\frac{1}{s(s+1)(s+20)}$

(C) $\frac{100}{s(s+1)(s+20)}$ (D) $\frac{100}{s(s+1)(1+0.05s)}$

57. The state space representation of a separately excited DC servo motor dynamics is given as

$$\begin{bmatrix}\frac{d\omega}{dt}\\ \frac{di_a}{dt}\end{bmatrix}=\begin{bmatrix}-1 & 1\\ -1 & -10\end{bmatrix}\begin{bmatrix}\omega\\ i_a\end{bmatrix}+\begin{bmatrix}0\\ 10\end{bmatrix}u$$

(A) $\frac{10}{s^2+11s+11}$ (B) $\frac{1}{s^2+11s+11}$

(C) $\frac{10s+10}{s^2+11s+11}$ (D) $\frac{1}{s^2+s+11}$

58. In delta modulation, the slope overload distortion can be reduced by

(A) decreasing the step size
(B) decreasing the granular noise
(C) decreasing the sampling rate
(D) increasing the step size

59. The raised cosine pulse $p(t)$ is used for zero ISI in digital communications. The expression for $p(t)$ with unity roll-off facto is given by

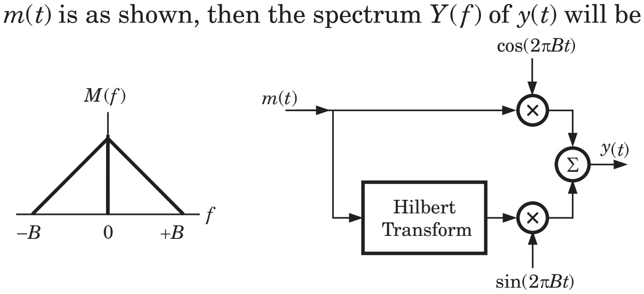
$$p(t)=\frac{\sin 4\pi Wt}{4\pi Wt(1-16W^2t^2)}$$

The value of $p(t)$ at $t=\frac{1}{4W}$ is

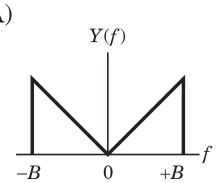
(A) -0.5 (B) 0

(C) 0.5 (D) ∞

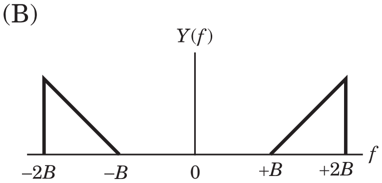
60. In the following scheme, if the spectrum $M(f)$ of $m(t)$ is as shown, then the spectrum $Y(f)$ of $y(t)$ will be



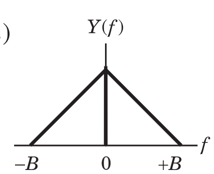
(A)



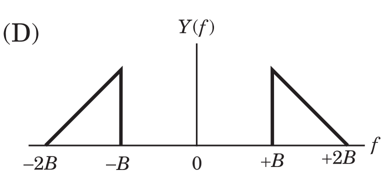
(B)



(C)



(D)



61. During transmission over a certain binary communication channel, bit errors occurs independently with probability p. The probability of AT MOST one bit in error in a block of n bits is given by

(A) p^n (B) $1-p^n$
(C) $np(1-p)^{n-1}+(1-p)^n$ (D) $1-(1-p)^n$

62. In a GSM system, 8 channels can co-exist in 200 KHz bandwidth using TDMA. A GSM based cellular operator is allocated 5 MHz bandwidth. Assuming a frequency reuse factor of $\frac{1}{5}$, i.e. a five-cell repeat pattern, the maximum number of simultaneous channels that can exist in one cell is

(A) 200 (B) 40
(C) 25 (D) 5

63. In a Direct Sequence CDMA system the chip rate is 1.2288×10^6 chips per second. If the processing gain is desired to be at Least 100, the data rate

(A) must be less than or equal to 12.288×10^3 bits/sec

(B) must be greater than 12.288×10^3 bits per sec

(C) must be exactly equal to 12.288×10^3 bits per sec

(D) can take any value less than 122.88×10^3 bits/sec

64. An air-filled rectangular waveguide has inner dimensions of 3 cm × 2 cm. The wave impedance of the TE_{20} mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance $\eta_0 = 377\,\Omega$)

(A) 308Ω (B) 355Ω
(C) 400Ω (D) 461Ω

65. The $\vec{H}$ field (in A/m) of a plane wave propagating in free space is given by

$$\vec{H} = x\frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + y\frac{5}{\eta_0}\sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

The time average power flow density in Watts is

(A) $\frac{\eta_0}{100}$ (B) $\frac{100}{\eta_0}$

(C) $50\eta_0^2$ (D) $\frac{50}{\eta_0}$

66. The $\vec{E}$ field in a rectangular waveguide of inner dimensions $a \times b$ is given by

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{2}\right)H_0\sin\left(\frac{2\pi x}{a}\right)^2\sin(\omega t - \beta z)y$$

Where H_0 is a constant, and a and b are the dimensions along the x-axis and the y-axis respectively. The mode of propagation in the waveguide is

(A) TE_{20} (B) TM_{11}

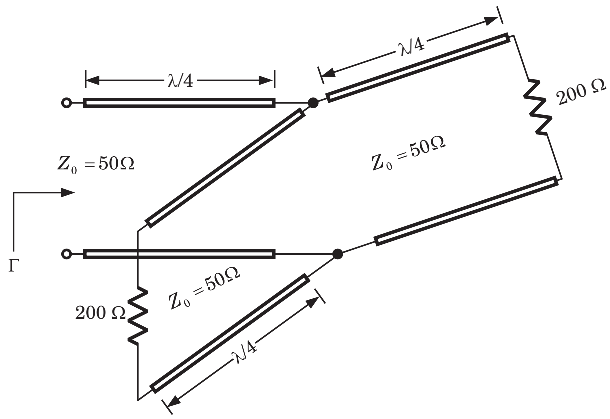
(C) TM_{20} (D) TE_{10}

67. A load of 50Ω is connected in shunt in a 2-wire transmission line of $Z_0 = 50\Omega$ as shown in the figure. The 2-port scattering parameter matrix (s-matrix) of the shunt element is

(A) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$

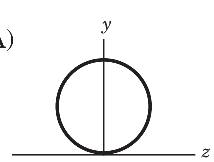
68. The parallel branches of a 2-wire transmission line are terminated in 100 Ω and 200 Ω resistors as shown in the figure. The characteristic impedance of the line is $Z_0 = 50\Omega$ and each section has a length of $\frac{\lambda}{4}$. The voltage reflection coefficient Γ at the input is

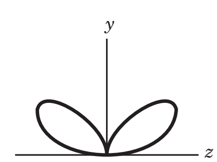


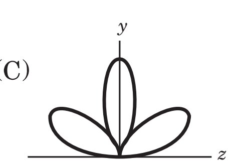
(A) $-j\frac{7}{5}$ (B) $\frac{-5}{7}$

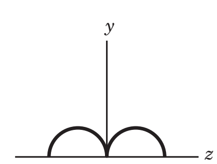
(C) $j\frac{5}{7}$ (D) $\frac{5}{7}$

69. A $\frac{\lambda}{2}$ dipole is kept horizontally at a height of $\frac{\lambda_0}{2}$ above a perfectly conducting infinite ground plane. The radiation pattern in the lane of the dipole ($\vec{E}$ plane) looks approximately as

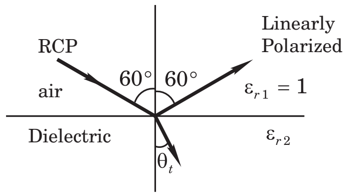
(A)


(B)


(C)


(D)


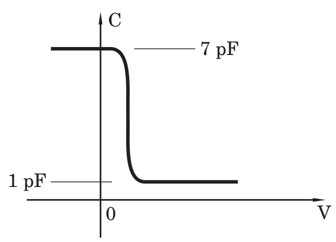
70. A right circularly polarized (RCP) plane wave is incident at an angle of 60° to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant ξ_{r2} is



(A) $\sqrt{2}$ (B) $\sqrt{3}$
(C) 2 (D) 3

Common Data for Questions 71, 72, 73:

The figure shows the high-frequency capacitance-voltage(C-V) characteristics of a Metal/ SiO_2/silicon (MOS) capacitor having an area of $1\times10^{-4}\,cm^2$. Assume that the perimitivities $(\varepsilon_0\varepsilon_r)$ of silicon and S_iO_2 are 1×10^{-12} F/cm and 3.5×10^{-13} F/cm respectively.



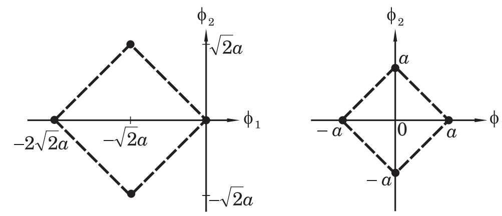
71. The gate oxide thickness in the MOS capacitor is
(A) 50 nm (B) 143 nm
(C) 350 nm (D) 1 μm

72. The maximum depletion layer width in silicon is
(A) 0.143 μm (B) 0.857 μm
(C) 1 μm (D) 1.143 μm

73. Consider the following statements about the C-V characteristics plot:
S1: The MOS capacitor has an n-type substrate.
S2: If positive charges are introduced in the oxide, the C-V plot will shift to the left.
Then which of the following is true?
(A) Both S1 and S2 are true
(B) S1 is true and Se is false
(C) S1 is false and S2 is true
(D) Both S1 and S2 are false

Common Data for Questions 74, 75 :

Two 4-ray signal constellations are shown. It is given that ϕ_1 and ϕ_2 constitute an orthonormal basis for the two constellations. Assume that the four symbols in both the constellations are equiprobable. Let $\mathcal{N}_0/2$ denote the power spectral density of white Gaussian noise.



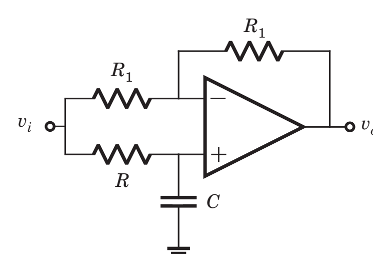
74. The ratio of the average energy of constellation 1 to the average energy of constellation 2 is
(A) $4a^2$ (B) 4
(C) 2 (D) 8

75. If these constellations are used for digital communications over an AWGN channel, then which of the following statements is true ?
(A) Probability of symbol error for Constellation 1 is lower
(B) Probability of symbol error for Constellation 1 is higher
(C) Probability of symbol error is equal for both the constellations
(D) The value of N_0 will determine which of the two constellations has a lower probability of symbol error,

Linked Answer Questions: Q. 76 to Q. 85 Carry Two marks Each.

Statement for Linked Answer Questions 76 & 77:

Consider the Op-Amp circuit shown in the figure.



76. The transfer function $V_0(s)/V_i(s)$ is

(A) $\dfrac{1-sRC}{1+sRC}$ (B) $\dfrac{1+sRC}{1-sRC}$

(C) $\dfrac{1}{1-sRC}$ (D) $\dfrac{1}{1+sRC}$

77. If $V_i = V_1 \sin(\omega t)$ and $V_0 = V_2 \sin(\omega t - \phi)$, then the minimum and maximum values of ϕ (in radians) are respectively

(A) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (B) 0 and $\frac{\pi}{2}$

(C) $-\pi$ and 0 (D) $-\frac{\pi}{2}$ and 0

Statement for Linked Answer Questions 78 & 79.

An 8085 assembly language program is given below.

```
Line 1:  MVI A, B5H
     2:  MVI B, OEH
     3:  XRI 69H
     4:  ADD B
     5:  ANI 9BH
     6:  CPI 9FH
     7:  STA 3010H
     8:  HLT
```

78. The contents of the accumulator just after execution of the ADD instruction in line 4 will be

(A) C3H (B) EAH

(C) DCH (D) 69H

79. After execution of line 7 of the program, the status of the CY and Z flags will be

(A) CY = 0, Z = 0 (B) CY = 0, Z = 1

(C) CY = A, Z = 0 (D) CY = 1, Z = 1

Statement for Linked Answer Questions 80 & 81.

Consider a linear system whose state space representation is $x(t) = Ax(t)$. If the initial state vector of the system is $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response is $x(t) = \begin{bmatrix} e^{-2x} \\ -2e^{-2t} \end{bmatrix}$. If the itial state vector of the system changes to $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response becomes $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

80. The eigenvalue and eigenvector pairs $(\lambda_i V_i)$ for the system are

(A) $\left(-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(B) $\left(-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(C) $\left(-1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(D) $\left(-2, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(1, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

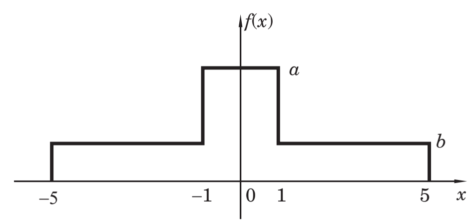
81. The system matrix A is

(A) $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

Statement fo Linked Answer Questions 82 & 83:

An input to a 6-level quantizer has the probability density function $f(x)$ as shown in the figure. Decision boundaries of the quantizer are chosen so as t maximize the entropy of the quantizer output. It is given that 3 consecutive decision boundaries are '–1','0' and '1'.



82. The values of a and b are

(A) $a = \dfrac{1}{6}$ and $b = \dfrac{1}{12}$ (B) $a = \dfrac{1}{5}$ and $b = \dfrac{3}{40}$

(C) $a = \dfrac{1}{4}$ and $b = \dfrac{1}{16}$ (D) $a = \dfrac{1}{3}$ and $b = \dfrac{1}{24}$

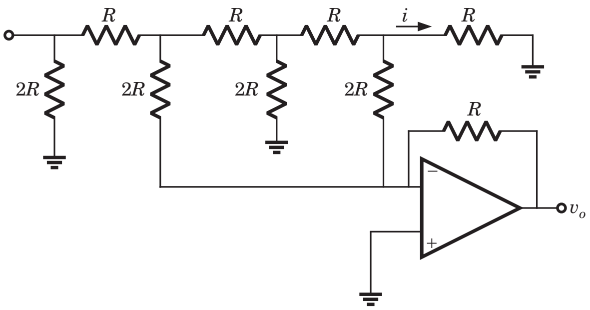
83. Assuming that the reconstruction levels of the quantizer are the mid-points of the decision boundaries, the ratio of signal power to quantization noise power is

(A) $\dfrac{152}{9}$ (B) $\dfrac{64}{3}$

(C) $\dfrac{76}{3}$ (D) 28

Statement for Linked Answer Questions 84 & 85.

In the digital-to Analog converter circuit shown in the figure below, $V_R = 10$ V and $R = 10$ kΩ.



84. The current is

(A) $31.25\mu A$ (B) $62.5\mu A$

(C) $125\mu A$ (D) $250\mu A$

85. The voltage V_0 is

(A) -0.781 V (B) -1.562 V

(C) -3.125 V (D) -6.250 V

****

ANSWER

1. A	2. B	3. C	4. A	5. D
6. A	7. D	8. C	9. D	10 .C
11. C	12. A	13. C	14. B	15. D
16. A	17. C	18. B	19. A	20. D
21. C	22. A	23. C	24. D	25. B
26. B	27. A	28. D	29. D	30. A
31. D	32. A	33. B	34. B	35. C
36. C	37. B	38. D	39. D	40. C
41. D	42. B	43. A	44. C	45. B
46. C	47. A	48. A	49. B	50. B
51. D	52. B	53. C	54. A	55. D
56. D	57. A	58. D	59. C	60. A
61. C	62. B	63. A	64. C	65. D
66. A	67. B	68. D	69. C	70. D
71. A	72. B	73. C	74. B	75. B
76. A	77. C	78. B	79. C	80. A
81. D	82. A	83.	84. B	85. C

CHAPTER

10.1

EC-03

Duration : Three Hours Maximum Marks : 150

Q.1—30 carry one mark each

1. The minimum number of equations required to analyze the circuit shown in Fig. Q. 1 is

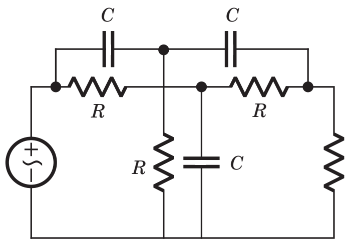


Fig. Q1

(A) 3 (B) 4

(C) 6 (D) 7

2. A source of angular frequency 1 rad/sec has a source impedance consisting of 1Ω resistance in series with 1 H inductance. The load that will obtain the maximum power transfer is

(A) 1 Ω resistance

(B) 1 Ω resistance in parallel with 1 H inductance

(C) 1 Ω resistance in series with 1 F capacitor

(D) 1 Ω resistance in parallel with 1 F capacitor

3. A series RLC circuit has a resonance frequency of 1 kHz and a quality factor $Q = 100$. If each of R, L and C is doubled from its original value, the new Q of the circuit is

(A) 25 (B) 50

(C) 100 (D) 200

4. The Laplace transform of $i(t)$ is given by

$$I(s) = \frac{2}{s(1+s)}$$

As $t \to \infty$, The value of $i(t)$ tends to

(A) 0 (B) 1

(C) 2 (D) ∞

5. The differential equation for the current $i(t)$ in the circuit of Fig. Q.5 is

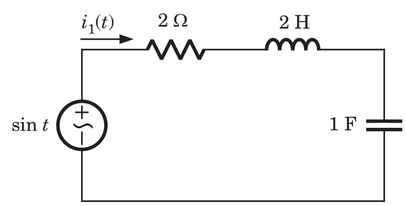


Fig. Q5

(A) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) = \sin t$

(B) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i(t) = \cos t$

(C) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) = \cos t$

(D) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i(t) = \sin t$

6. n-type silicon is obtained by doping silicon with

(A) Germanium (B) Aluminium

(C) Boron (D) Phosphorus

7. The bandgap of silicon at 300 K is

(A) 1.36 eV (B) 1.10 eV

(C) 0.80 eV (D) 0.67 eV

20. A 0 to 6 counter consists of 3 flip flops and a combination circuit of 2 input gate(s). The combination circuit consists of

(A) one AND gate

(B) one OR gate

(C) one AND gate and one OR gate

(D) two AND gates

21. The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by $g_p(t) = \sum_{n=-\infty} c_n e^{j2\pi f_0 t}$. It is given that $c_3 = 3 + j5$. Then c_{-3} is

(A) $5 + j3$ (B) $-3 - j5$

(C) $-5 + j3$ (D) $3 - j5$

22. Let $x(t)$ be the input to a linear, time-invariant system. The required output is $4x(t-2)$. The transfer function of the system should be

(A) $4e^{j4\pi f}$ (B) $2e^{-j8\pi f}$

(C) $4e^{-j4\pi f}$ (D) $2e^{j8\pi f}$

23. A sequence $x(n)$ with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The output at $n = 4$ is

(A) –6 (B) zero

(C) 2 (D) –4

24. Fig. Q.24 shows the Nyquist plot of the open-loop transfer function $G(s)H(s)$ of a system. If $G(s)H(s)$ has one right-hand pole, the closed-loop system is

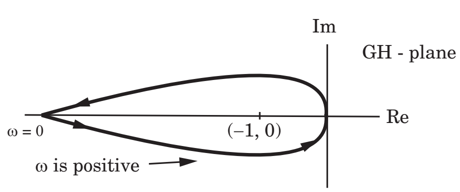


Fig. Q24

(A) always stable

(B) unstable with one closed-loop right hand pole

(C) unstable with two closed-loop right hand poles

(D) unstable with three closed-loop right hand poles

25. A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has

(A) a higher type number

(B) reduced damping

(C) higher noise amplification

(D) larger transient overshoot

26. The input to a coherent detector is DSB-SC signal plus noise. The noise at the detector output is

(A) the in-phase component

(B) the quadrature component

(C) zero

(D) the envelope

27. The noise at the input to an ideal frequency detector is white. The detector is operating above threshold. The power spectral density of the noise at the output is

(A) raised-cosine (B) flat

(C) parabolic (D) Gaussian

28. At a given probability of error, binary coherent FSK is inferior to binary coherent PSK by

(A) 6 dB (B) 3 dB

(C) 2 dB (D) 0 dB

29. The unit of $\nabla \times H$ is

(A) Ampere (B) Ampere/meter

(C) Ampere/meter^2 (D) Ampere-meter

30. The depth of penetration of electromagnetic wave in a medium having conductivity σ at a frequency of 1 MHz is 25 cm. The depth of penetration at a frequency of 4 MHz will be

(A) 6.25 cm (B) 12.50 cm

(C) 50.00 cm (D) 100.00 cm

Q.31—90 carry two marks each.

31. Twelve 1 Ω resistance are used as edges to form a cube. The resistance between two diagonally opposite corners of the cube is

(A) $\frac{5}{6}$ Ω (B) 1 Ω

(C) $\frac{6}{5}$ (D) $\frac{3}{2}$ Ω

32. The current flowing through the resistance R in the circuit in Fig. Q.32 has the form $P\cos 4t$, where P is

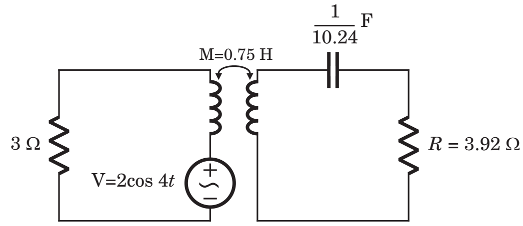


Fig. Q32

(A) $(0.18 + j\,0.72)$ (B) $(0.46 + j\,1.90)$

(C) $-(0.18 + j\,1.90)$ (D) $-(0.192 + j\,0.144)$

The circuit for Q.33–34 are given in Fig. Q.33–34. For both the questions, assume that the switch S is in position 1 for a long time and thrown to position 2 at $t = 0$.

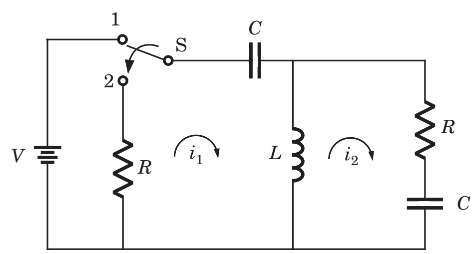


Fig. Q33-34

33. At $t = 0^+$, the current i_1 is

(A) $\frac{-V}{2R}$ (B) $\frac{-V}{R}$

(C) $\frac{-V}{4R}$ (D) zero

34. $I_1(s)$ and $I_2(s)$ are the Laplace transforms of $i_1(t)$ and $i_2(t)$ respectively. The equations for the loop currents $I_1(s)$ and $I_2(s)$ for the circuit shown in Fig. Q.33–34, after the switch is brought from position 1 to position 2 at $t = 0$, are

(A) $\begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1 s \\ I_2 s \end{bmatrix} = \begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1 s \\ I_2 s \end{bmatrix} = \begin{bmatrix} -\frac{V}{s} \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + Ls + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1 s \\ I_2 s \end{bmatrix} = \begin{bmatrix} -\frac{V}{s} \\ 0 \end{bmatrix}$

(D) $\begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + Ls + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1 s \\ I_2 s \end{bmatrix} = \begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix}$

35. An input voltage

$$v(t) = 10\sqrt{2}\cos(t + 10) + 10\sqrt{3}\cos(2 + 10°)\ \text{V}$$

is applied to a series combination of resistance $R = 1\Omega$ and an inductance $L = 1$ H. The resulting steady state current $i(t)$ in ampere is

(A) $10\cos(t + 55°) + 10\cos(2t + 10° + \tan^{-1} 2)$

(B) $1 - \cos(t + 55°) + 10\sqrt{\frac{3}{2}}\cos(2t + 55°)$

(C) $10\cos(t - 55°) + 10\cos(2t + 10° - \tan^{-1} 2)$

(D) $1 - \cos(t - 55°) + 10\sqrt{\frac{3}{2}}\cos(2t - 35°)$

36. The driving-point impedance $Z(s)$ of a network has the pole-zero locations as shown in Fig. Q.36. If $Z(0) = 3$, then $Z(s)$ is

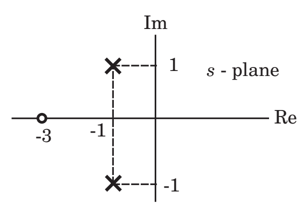


Fig. Q36

(A) $\frac{3(s+3)}{s^2 + 2s + 3}$ (B) $\frac{2(s+3)}{s^2 + 2s + 2}$

(C) $\frac{3(s-3)}{s^2 - 2s - 2}$ (D) $\frac{2(s-3)}{s^2 - 2s - 3}$

37. The impedance parameters Z_{11} and Z_{12} of the two-port network in Fig. Q.37 are

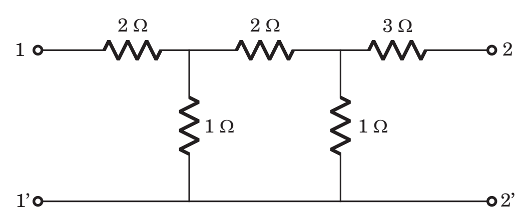


Fig. Q37

(A) $Z_{11} = 2.75\Omega$ and $Z_{12} = 0.25\ \Omega$

(B) $Z_{11} = 3\Omega$ and $Z_{12} = 0.5\ \Omega$

(C) $Z_{11} = 3\Omega$ and $Z_{12} = 0.25\ \Omega$

(D) $Z_{11} = 2.25\ \Omega$ and $Z_{12} = 0.5\ \Omega$

38. An n-type silicon bar 0.1 cm long and 100 μm^2 in cross-sectional area has a majority carrier concentration of $5 \times 10^{20} / m^3$ and the carrier mobility is 0.13 m^2/V-s at 300 K. If the charge of an electron is 1.5×10^{-19} coulomb, then the resistance of the bar is

(A) 10^6 Ohm (B) 10^4 Ohm

(C) 10^{-1} Ohm (D) 10^{-4} Ohm

39. The electron concentration in a sample of uniformly doped n-type silicon at 300 K varies linearly from 10^{17} /cm^3 at $x = 0$ to 6×10^{16} /cm^3 at $x = 2\ \mu m$. Assume a situation that electrons are supplied to keep this concentration gradient constant with time. If electronic charge is 1.6×10^{-19} coulomb and the diffusion constant $D_n = 35$ cm^2/s, the current density in the silicon, if no electric field is present, is

(A) zero (B) –112 A/cm^2

(C) +1120 A/cm^2 (D) –1120 A/cm^2

40. Match items in Group 1 with items in Group 2, most suitably.

Group 1	**Group 2**
P. LED	1. Heavy doping
Q. Avalanche photo diode	2. Coherent radiation
R.Tunnel diode	3.Spontaneous emission
S. LASER	4. Current gain

(A)	(B)	(C)	(D)
P–1	P–2	P–3	P–2
Q–2	Q–3	Q–4	Q–1
R–4	R–1	R–1	R–4
S–3	S–4	S–2	S–3

41. At 300 K, for a diode current of 1 mA, a certain germanium diode requires a forward bias of 0.1435 V, whereas a certain silicon diode requires a forward bias of 0.718 V. Under the conditions stated above, the closest approximation of the ratio of reverse saturation current in germanium diode to that in silicon diode is

(A) 1 (B) 5

(C) 4×10^3 (D) 8×10^3

42. A particular green LED emits light of wavelength 5490 A°. The energy bandgap of the semiconductor material used there is (Plank's constant $= 6.626 \times 10^{-34}$ J – s)

(A) 2.26 eV (B) 1.98 eV

(C) 1.17 eV (D) 0.74 eV

43..When the gate-to-source voltage (V_{GS}) of a MOSFET with threshold voltage of 400 mV, working in saturation is 900 mV, the drain current is observed to be 1 mA. Neglecting the channel width modulation effect and assuming that the MOSFET is operating at saturation, the drain current for an applied V_{GS} of 1400 mV is

(A) 0.5 mA (B) 2.0 mA

(C) 3.5 mA (D) 4.0 mA

44. If P is Passivation, Q is n-well implant, R is metallization and S is source/drain diffusion, then the order in which they are carried out in a standard n-well CMOS fabrication process, is

(A) P–Q–R–S (B) Q–S–R–P

(C) R–P–S–Q (D) S–R–Q–P

45. An amplifier without feedback has a voltage gain of 50, input resistance of 1 kΩ and output resistance of 2.5 kΩ. The input resistance of the current-shunt negative feedback amplifier using the above amplifier with a feedback factor of 0.2, is

(A) 1/11 kΩ (B) 1/5 kΩ

(C) 5 kΩ (D) 11 kΩ

46. In the amplifier circuit shown in Fig. Q.46, the values of R_1 and R_2 are such that the transistor is operating at $V_{CE} = 3$ V and $I_C = 1.5$ mA when its β is 150. For a transistor with β of 200, the operating point (V_{CE}, I_C) is

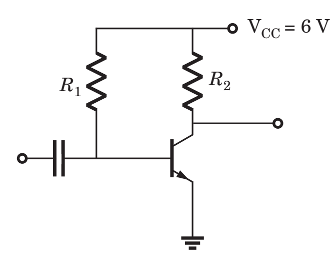


Fig. Q46

(A) (2 V, 2 mA) (B) (3 V, 2 mA)

(C) (4 V, 2 mA) (D) (4 V, 1 mA)

47. The oscillator circuit shown in Fig. Q.47 has an ideal inverting amplifier. its frequency of oscillation (in Hz) is

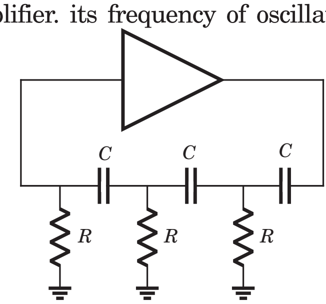


Fig. Q47

(A) $\dfrac{1}{(2\pi\sqrt{6}RC)}$ (B) $\dfrac{1}{(2\pi RC)}$

(C) $\dfrac{1}{(\sqrt{6}RC)}$ (D) $\dfrac{\sqrt{6}}{(2\pi RC)}$

48. The output voltage of the regulated power supply shown in Fig. Q.48 is

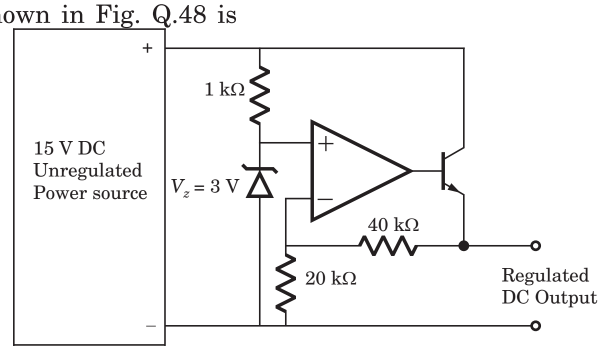


Fig. Q48

(A) 3 V (B) 6 V
(C) 9 V (D) 12 V

49. The action of a JFET in its equivalent circuit can best be represented as a
(A) Current Controlled Current Source
(B) Current Controlled Voltage Source
(C) Voltage Controlled Voltage Source
(D) Voltage Controlled Current Source

50. If the op-amp in Fig. Q.50 is ideal, the output voltage V_{out} will be equal to

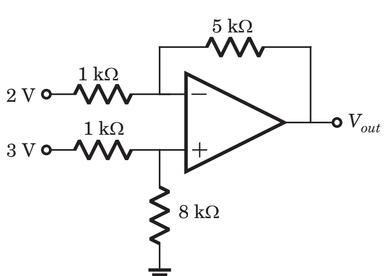


Fig. Q50

(A) 1 V (B) 6 V

(C) 14 V (D) 17 V

51. Three identical amplifiers with each one having a voltage gain of 50, input resistance of 1 kΩ and output resistance of 250 Ω, are cascaded. The open circuit voltage gain of the combined amplifier is
(A) 49 dB (B) 51 dB
(C) 98 dB (D) 102 dB

52. An ideal sawtooth voltage waveform of frequency 500 Hz and amplitude 3 V is generated by charging a capacitor of 2 μF in every cycle. The charging requires
(A) constant voltage source of 3 V for 1 ms
(B) constant voltage source of 3 V for 2 ms
(C) constant current source of 3 mA for 1 ms
(D) constant current source of 3 mA for 2 ms

53. The circuit shown in Fig. Q.53 has 4 boxes each described by inputs, P, Q, R and outputs Y, Z with $Y = P \oplus Q \oplus R$, $Z = RQ + \overline{P}R + Q\overline{P}$. The circuit acts as a

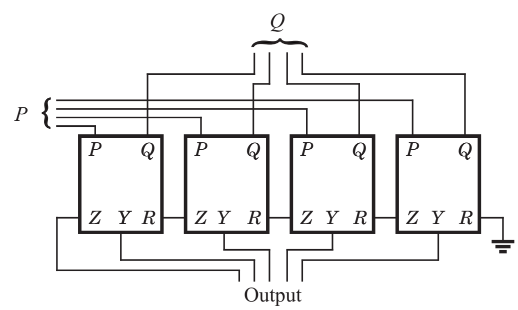


Fig. Q53

(A) 4 bit adder giving $P + Q$
(B) 4 bit substracter giving $P - Q$
(C) 4 bit substracter giving $Q - R$
(D) 4 bit adder giving $P + Q + R$

54. If the functions W, X, Y and Z are as follows

$W = R + \overline{P}Q + \overline{R}S$

$X = PQ\overline{R}\,\overline{S} + \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S} + P\overline{Q}\,\overline{R}\,\overline{S}$

$Y = RS + \overline{PR + P\overline{Q} + \overline{P}\,\overline{Q}}$

$Z = R + S + \overline{PQ + \overline{P}\cdot\overline{Q}\cdot\overline{R} + P\overline{Q}\cdot\overline{S}}$ Then

(A) $W = Z$, $X = \overline{Z}$ (B) $W = Z$, $X = Y$

(C) $W = Y$ (D) $W = Y = \overline{Z}$

55. A 4 bit ripple counter and a 4 bit synchronous counter are made using flip flops having a propagation delay of 10 ns each. If the worst case delay in the ripple counter and the synchronous counter be R and S respectively, then

(A) R =10 n, S =40 ns (B) R =40 ns, S =10 ns

(C) R =10 ns, S =30 ns (D) R =30 ns, S =10 ns

56. The DTL, TTL, ECL and CMOS families of digital ICs are compared in the following 4 columns

	P	Q	R	S
Fanout is minimum	DTL	DTL	TTL	CMOS
Power consumption is minimum	TTL	CMOS	ECL	DTL
Propagation delay is minimum	CMOS	ECL	TTL	TTL

The correct column is

(A) P (B) Q

(C) R (D) S

57. The circuit shown in Fig. Q.57 is a 4 bit DAC. The input bits 0 and 1 are represented by 0 and 5 V respectively. The OP AMP is ideal, but all the resistance and the 5 V inputs have a tolerance of ±10%. The specification (rounded to the nearest multiple of 5%) for the tolerance of the DAC is

(A) ±35% (B) ±20%

(C) ±10% (D) ±5%

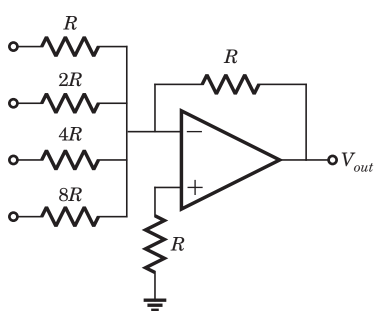


Fig. Q57

58. The circuit shown in Fig. Q.58 converts

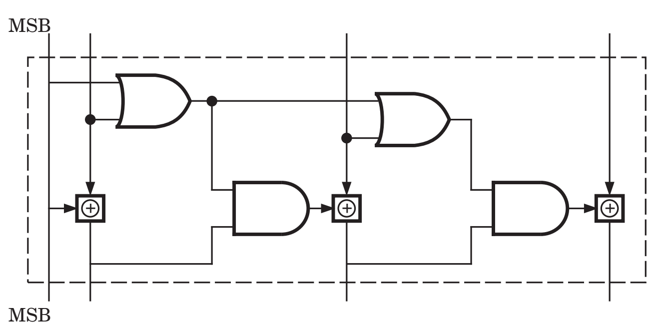


Fig. Q58

(A) BCD to binary code
(B) Binary to excess –3 code
(C) Excess –3 to Gray code
(D) Gray to Binary code

59. In the circuit shown in Fig. Q.59, A is a parallel-in, parallel-out 4 bit register, which loads at the rising edge of the clock C. The input lines are connected to a 4 bit bus, W. Its output acts as the input to a 16 × 4 ROM whose output is floating when the enable input E is 0. A partial table of the contents of the ROM is as follows

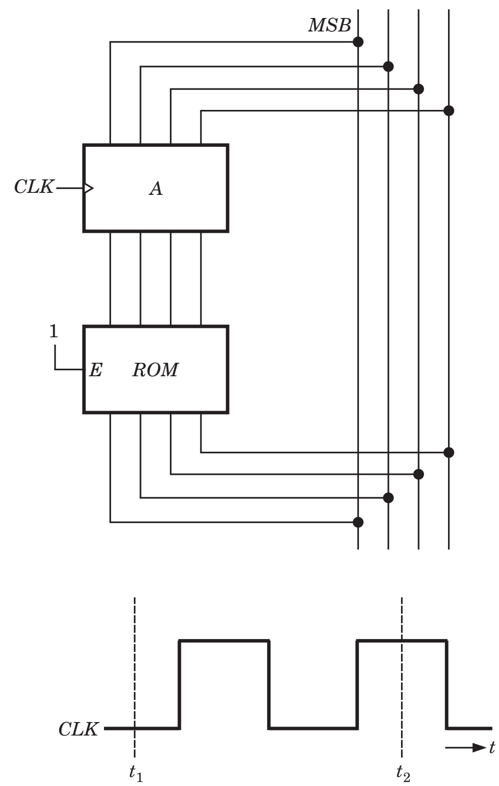


Fig. Q59

Address	Data
0	0011
2	1111
4	0100
6	1010
8	1011
10	1000
12	0010
14	1000

The clock to the register is shown, and the data on the W bus at time t_1 is 0110. The data on the bus at time t_2 is

(A) 1111 (B) 1011

(C) 1000 (D) 0010

60. In an 8085 microprocessor, the instruction CMP B has been executed while the content of the accumulator is less than that of register B. As a result

(A) Carry flag will be set but Zero flag will be reset

(B) Carry flag will be reset but Zero flag will be set

(C) Both Carry flag and Zero flag will be reset

(D) Both Carry flag and Zero flag will be set

61. Let X and Y be two statistically independent random variables uniformly distributed in the ranges $(-1, 1)$ and $((-2, 1)$ respectively. Let $Z = X + Y$. Then the probability that $(Z \leq -2)$ is

(A) zero (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{12}$

62. Let P be linearity, Q be time-invariance, R be causality and S be stability. A discrete time system has the input-output relationship,

$$y(n) = \begin{cases} x(n) & n \geq 1 \\ 0, & n = 0 \\ x(n+1) & n \leq -1 \end{cases}$$

where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties

(A) P, S but not Q, R (B) P, Q, S but not R

(C) P, Q, R, S (D) Q, R, S but not P

Data for Q.63–64 are given below. Solve the problems and choose the correct answers.

The system under consideration is an RC low-pass filter (RC-LPF) with $R = 1\ \text{k}\Omega$ and $C = 1.0\ \mu\text{F}$.

63. Let $H(f)$ denote the frequency response of the RC-LPF. Let f_1 be the highest frequency such that $0 \leq |f| \leq f_1 \frac{|H(f_1)|}{H(0)}$ 0.95. Then f_1 (in Hz) is

(A) 327.8 (B) 163.9

(C) 52.2 (D) 104.4

64. Let $t_g(f)$ be the group delay function of the given RC-LPF and $f_2 = 100$ Hz. Then $t_g(f_2)$ in ms, is

(A) 0.717 (B) 7.17

(C) 71.7 (D) 4.505

Data for Q.65–66 are given below. Solve the problems and choose the correct answers.

$X(t)$ is a random process with a constant mean value of 2 and the autocorrelation function

$$R_X(\tau) = 4[e^{-0.2|\tau|} + 1].$$

65. Let X be the Gaussian random variable obtained by sampling the process at $t = t_i$ and let

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

The probability that $[x \leq 1]$ is

(A) $1 - Q(0.5)$ (B) $Q(0.5)$

(C) $Q(\frac{1}{2\sqrt{2}})$ (D) $1 - Q(\frac{1}{2\sqrt{2}})$

66. Let Y and Z be the random variables obtained by sampling $X(t)$ at $t = 2$ and $t = 4$ respectively. Let $W = Y - Z$. The variance of W is

(A) 13.36 (B) 9.36

(C) 2.64 (D) 8.00

67. Let $x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$. $x(t)$ is sampled with the rectangular pulse train shown in Fig. Q.67. The only spectral components (in kHz) present in the sampled signal in the frequency range 2.5 kHz to 3.5 kHz are

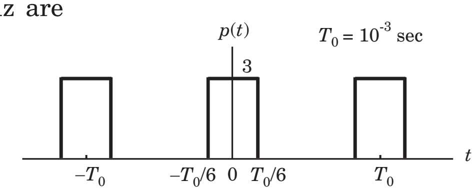


Fig. Q67

(A) 2.7, 3.4 (B) 3.3, 3.6

(C) 2.6, 2.7, 3.3, 3.4, 3.6 (D) 2.7, 3.3

68. The signal flow graph of a system is shown in Fig. Q.68. The transfer function $\frac{C(s)}{R(s)}$ of the system is

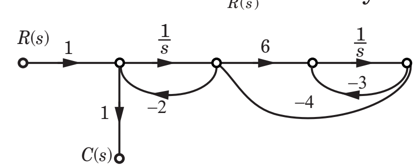


Fig. Q68

(A) $\dfrac{6}{s^2 + 29s + 6}$ (B) $\dfrac{6s}{s^2 + 29s + 6}$

(C) $\dfrac{s(s+2)}{s^2 + 29s + 6}$ (D) $\dfrac{s(s+27)}{s^2 + 29s + 6}$

69 The root locus of the system

$$G(s)H(s) = \frac{K}{s(s+2)(s+3)}$$

has the break-away point located at

(A) (–0.5, 0) (B) (–2.548, 0)

(C) (–4, 0) (D) (–0.784, 0)

70. The approximate Bode magnitude plot of a minimum phase system is shown in Fig. Q.70. The transfer function of the system is

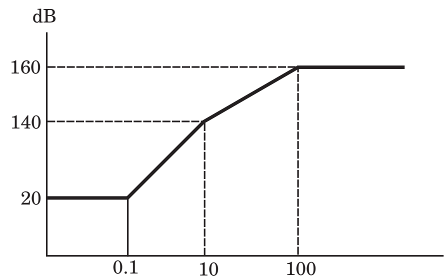


Fig. Q70

(A) $10^8 \frac{(s+0.1)^3}{(s+10)^2(s+100)}$ (B) $10^7 \frac{(s+0.1)^3}{(s+10)(s+100)}$

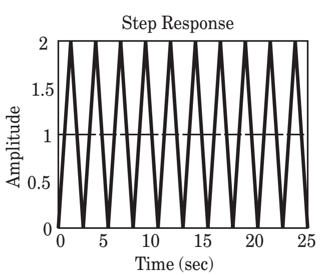
(C) $10^8 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$ (D) $10^9 \frac{(s+0.1)^3}{(s+10)(s+100)^2}$

71. A second-order system has the transfer function

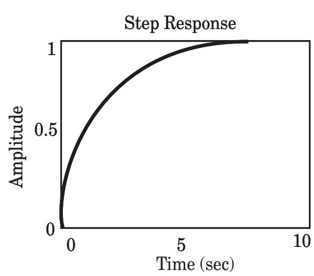
$$\frac{C(s)}{R(s)} = \frac{4}{s^2+4s+4}$$

With $r(t)$ as the unit-step function, the response $c(t)$ of the system is represented by

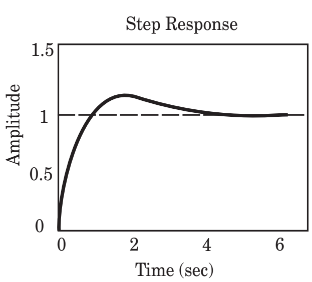
(A)



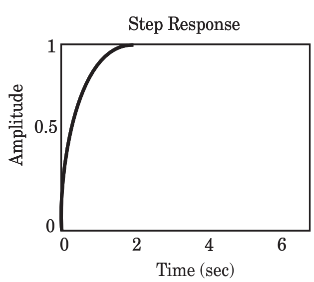
(B)



(C)



(D)



72. The gain margin and the phase margin of a feedback system with

$$G(s)H(s) = \frac{s}{(s+100)^3} \text{ are}$$

(A) - dB, 0° (B) ∞, ∞

(C) ∞, 0° (D) 88.5 dB, ∞

73. The zero-input response of a system given by the state-space equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ is}$$

(A) $\begin{bmatrix} te^t \\ t \end{bmatrix}$ (B) $\begin{bmatrix} e^t \\ t \end{bmatrix}$

(C) $\begin{bmatrix} e^t \\ te^t \end{bmatrix}$ (D) $\begin{bmatrix} t \\ te^t \end{bmatrix}$

74. A DSB-SC signal is to be generated with a carrier frequency $f_c = 1$ MHz using a nonlinear device with the input-output characteristic $v_0 = a_0 v_1 + a_1 v_i^3$ where a_0 and a_1 are constants. The output of the nonlinear device can be filtered by an appropriate band-pass filter. Let $v_i = A_c^l \cos(2\pi f_c^l t) + m(t)$ where $m(t)$ is the message signal. Then the value of f_c^l (in MHz) is

(A) 1.0 (B) 0.333

(C) 0.5 (D) 3.0

The data for Q.75-76 are given below. Solve the problems and choose the correct answers.

Let $m(t) = \cos[(4\pi \times 10^3)t]$ be the message signal and $c(t) = 5\cos[(2\pi \times 10^6)t]$ be the carrier.

75. $c(t)$ and $m(t)$ are used to generate an AM signal. The modulation index of the generated AM signal is 0.5. Then the quantity $\frac{\text{Total side band power}}{\text{Carrier power}}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{3}$ (D) $\frac{1}{8}$

76. $c(t)$ and $m(t)$ are used to generate an FM signal. If the peak frequency deviation of the generated FM is three times the transmission bandwidth of the AM signal, then the coefficient of the term $\cos[2\pi(1008 \times 10^3 t)]$ in the FM signal (in terms of the Bessel coefficients) is

(A) $5J_4(3)$ (B) $\frac{5}{2}J_8(3)$

86. A uniform plane wave traveling in air is incident on the plane boundary between air and another dielectric medium with $\varepsilon_r = 4$. The reflection coefficient for the normal incidence, is

(A) zero (B) $0.5 \angle 180°$

(C) $0.333 \angle 0°$ (D) $0.333 \angle 180°$

87. If the electric field intensity associated with a uniform plane electromagnetic wave traveling in a perfect dielectric medium is given by $E(z, t) = 10 \cos(2\pi \times 10^7 t - 0.1\pi z)$ volt/m, then the velocity of the traveling wave is

(A) 3.00×10^8 m/sec (B) 2.00×10^8 m/sec

(C) 6.28×10^7 m/sec (D) 2.00×10^7 m/sec

88. A short-circuited stub is shunt connected to a transmission line as shown in Fig. Q.88. If $Z_0 = 50$ ohm, the admittance Y seen at the junction of the stub and the transmission line is

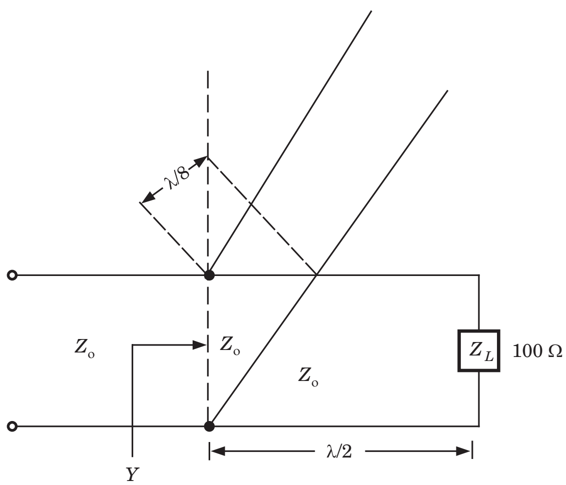


Fig. Q.88

(A) $(0.01 - j0.02)$ mho (B) $(0.02 - j0.01)$ mho

(C) $(0.04 - j0.02)$ mho (D) $(0.02 + j0)$ mho

89. A rectangular metal wave guide filled with a dielectric material of relative permitivity $\varepsilon_r = 4$ has the inside dimensions 3.0 cm × 1.2 cm. The cut-off frequency for the dominant mode is

(A) 2.5 GHz (B) 5.0 GHz

(C) 10.0 GHz (D) 12.5 GHz

90. Two identical antennas are placed in the $\theta = \pi/2$ plane as shown in Fig. Q.90. The elements have equal amplitude excitation with 180° polarity difference, operating at wavelength λ. The correct value of the magnitude of the far-zone resultant electric field strength normalized with that of a single element, both computed for $\phi = 0$, is

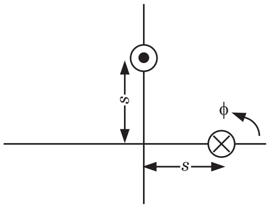


Fig. Q.90

(A) $2\cos\left(\frac{2\pi s}{\lambda}\right)$ (B) $2\sin\left(\frac{2\pi s}{\lambda}\right)$

(C) $2\cos\left(\frac{\pi s}{\lambda}\right)$ (D) $2\sin\left(\frac{\pi s}{\lambda}\right)$

****

ANSWER SHEET

1.	(B)	2.	(C)	3.	(B)	4.	(C)	5.	(C)
6.	(D)	7.	(B)	8.	(A)	9.	(C)	10.	(A)
11.	(B)	12.	(D)	13.	(B)	14.	(C)	15.	(A)
16.	(D)	17.	(C)	18.	(B)	19.	(B)	20.	(D)
21.	(D)	22.	(C)	23.	(B)	24.	(A)	25.	(C)
26.	(A)	27.	(A)	28.	(D)	29.	(B)	30.	(B)
31.	(A)	32.	(*)	33.	(D)	34.	(D)	35.	(C)
36.	(B)	37.	(A)	38.	(C)	39.	(C)	40.	(C)
41.	(C)	42.	(A)	43.	(D)	44.	(B)	45.	(A)
46.	(A)	47.	(A)	48.	(C)	49.	(D)	50.	(B)
51.	(D)	52.	(D)	53.	(B)	54.	(A)	55.	(B)
56.	(C)	57.	(A)	58.	(D)	59.	(C)	60.	(A)
61.	(A)	62.	(A)	63.	(C)	64.	(B)	65.	(A)
66.	(C)	67.	(A)	68.	(A)	69	(D)	70	(A)
71.	(B)	72.	(D)	73.	(C)	74	(A)	75	(D)
76.	(D)	77.	(B)	78.	(A)	79	(C)	80	(D)
81.	(B)	82.	(D)	83.	(B)	84.	(C)	85.	(C)
86.	(D)	87.	(B)	88.	(A)	89	(B)	90.	(D)

5. For the R-L circuit shown in Fig. Q.5, the input voltage $v_i(t) = u(t)$. The current $i(t)$ is

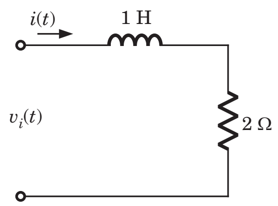
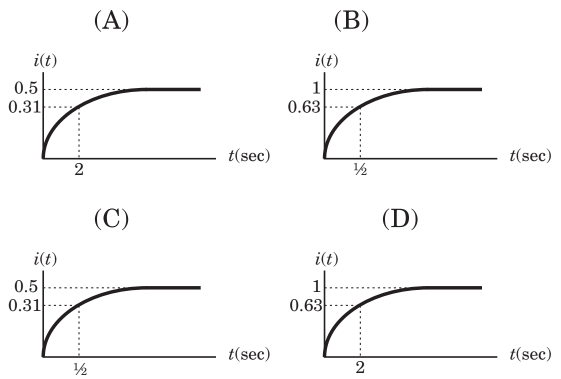


Fig Q.5



6. The impurity commonly used for realizing the base region of a silicon n-p-n transistor is

(A) Gallium (B) Indium

(C) Boron (D) Phosphorus

7. If for a silicon n-p-n transistor, the base-to-emitter voltage (V_{BE}) is 0.7 V and the collector-to-base voltage (V_{CB}) is 0.2 V, then the transistor is operating in the

(A) normal active mode (B) saturation mode

(C) inverse active mode (D) cutoff mode

8. Consider the following statements S1 and S2.

S1 : The β of a bipolar transistor reduces if the base width is increased.

S2 : The β of a bipolar transistor increases if the doping concentration in the base is increased.

Which one of the following is correct ?

(A) S1 is FALSE and S2 is TRUE

(B) Both S1 and S2 are TRUE

(C) Both S1 and S2 are FALSE

(D) S1 is TRUE and S2 is FALSE

9. An ideal op-amp is an ideal

(A) voltage controlled current source

(B) voltage controlled voltage source

(C) current controlled current source

(D) current controlled voltage source

10. Voltage series feedback (also called series-shunt feedback) results in

(A) increase in both input and output impedances

(B) decrease in both input and output impedances

(C) increase in input impedance and decrease in output impedance

(D) decrease in input impedance and increase in output impedance

11. The circuit in Fig. Q.11 is a

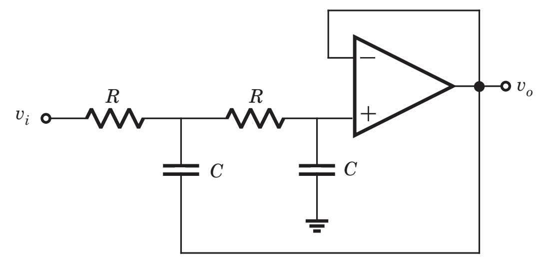


Fig Q.11

(A) low-pass filter (B) high-pass filter

(C) band-pass filter (D) band-reject filter

12. Assuming $V_{CEsat} = 0.2$ V and $\beta = 50$, the minimum base current (I_B) required to drive the transistor in Fig. Q.12 to saturation is

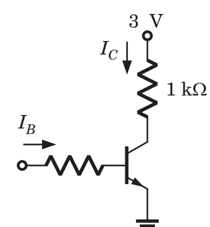


Fig Q12.

(A) 56 μA (B) 140 mA

(C) 60 μA (D) 3 mA

13. A master-slave flip-flop has the characteristic that

(A) change in the input is immediately reflected in the output

(B) change in the output occurs when the state of the master is affected

(C) change in the output occurs when the state of the slave is affected

(D) both the master and the slave states are affected at the same time

14. The range of signed decimal numbers that can be represented by 6-bit 1's complement numbers is

(A) –31 to +31 (B) –63 to +64

(C) –64 to +63 (D) –32 to +31

15. A digital system is required to amplify a binary-encoded audio signal. The user should be able to control the gain of the amplifier from a minimum to a maximum in 100 increments. The minimum number of bits required to encode, in straight binary, is

(A) 8 (B) 6

(C) 5 (D) 7

16. Choose the correct one from among the alternatives A, B, C, D after matching an item from Group 1 with the most appropriate item in Group 2.

Group 1	Group 2
P: Shift register	1: Frequency division
Q: Counter	2: Addressing in memory chips
R: Decoder	3: Serial to parallel data conversion

(A)	(B)	(C)	(D)
P–3	P–3	P–2	P–1
Q–2	Q–1	Q–1	Q–3
R–1	R–2	R–3	R–2

17. Fig. Q.17 shows the internal schematic of a TTL AND-OR-Invert (AOI) gate. For the inputs shown in Fig. Q.17, the output Y is

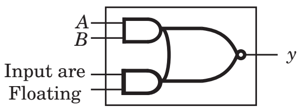


Fig Q.17

(A) 0 (B) 1

(C) AB (D) $\overline{AB}$

18. Fig. Q.18 is the voltage transfer characteristic of

(A) an NMOS inverter with enhancement mode transistor as load

(B) an NMOS inverter with depletion mode transistor as load

(C) a CMOS inverter

(D) a BJT inverter

19. The impulse response $h[n]$ of a linear time-invariant system is given by

$$h[n] = u[n+3] + u[n-2] - 2u[n-7]$$

where $u[n]$ is the unit step sequence. The above system is

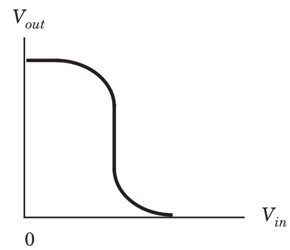


Fig Q.18

(A) stable but not causal

(B) stable and causal

(C) causal but unstable

(D) unstable and not causal

20. The distribution function $F_X(x)$ of a random variable X is shown in Fig. Q.20. The probability that $X = 1$ is

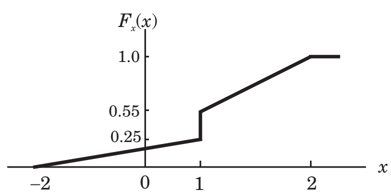


Fig Q.20

(A) zero (B) 0.25

(C) 0.55 (D) 0.30

21. The z-transform of a system is

$$H(z) = \frac{z}{z-0.2}$$

If the ROC is $|z| < 0.2$, then the impulse response of the system is

(A) $(0.2)^n u[n]$ (B) $(0.2)^n u[-n-1]$

(C) $-(0.2)^n u[n]$ (D) $-(0.2)^n u[-n-1]$

22. The Fourier transform of a conjugate symmetric function is always

(A) imaginary (B) conjugate anti-symmetric

(C) real (D) conjugate symmetric

33. Consider the Bode magnitude plot shown in Fig. Q.33. The transfer function $H(s)$ is

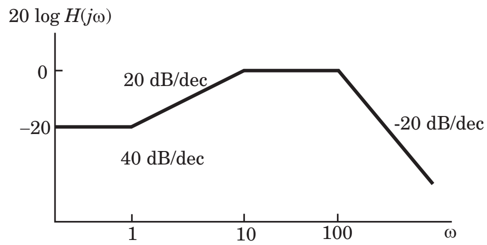


Fig Q.33

(A) $\frac{(s+10)}{(s+1)(s+100)}$ (B) $\frac{10(s+1)}{(s+10)(s+100)}$

(C) $\frac{10^2(s+1)}{(s+10)(s+100)}$ (D) $\frac{10^3(s+100)}{(s+1)(s+10)}$

34. The transfer function $H(s)=\frac{V_o(s)}{V_i(s)}$ of an R–L–C circuit is given by

$$H(s)=\frac{10^6}{s^2+20s+10^6}$$

The Quality factor (Q-factor) of this circuit is

(A) 25 (B) 50

(C) 100 (D) 5000

35. For the circuit shown in Fig. Q.35, the initial conditions are zero. Its transfer function $H(s)=V_C(s)/V_i(s)$ is

(A) $\frac{1}{s^2+10^3s+10^6}$ (B) $\frac{10^6}{s^2+10^3s+10^6}$

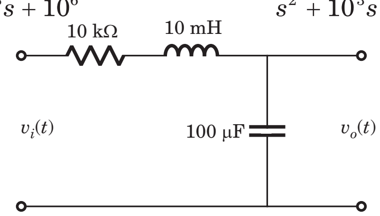


Fig Q35.

(C) $\frac{10^3}{s^2+10^3s+10^6}$ (D) $\frac{10^6}{s^2+10^6s+10^6}$

36. A system described by the following differential equation

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=x(t)$$

is initially at rest. For input $x(t)=2u(t)$, the output $y(t)$ is

(A) $(1-2e^{-t}+e^{-2t})u(t)$ (B) $(1+2e^{-t}-e^{-2t})u(t)$

(C) $(0.5+e^{-t}+1.5e^{-2t})u(t)$ (D) $(0.5+2e^{-t}+2e^{-2t})u(t)$

37. Consider the following statements S1 and S2.

S1 : At the resonant frequency the impedance of a series R–L–C circuit is zero.

S2 : In a parallel G–L–C circuit, increasing the conductance G results in increase in its Q factor.

Which one of the following is correct ?

(A) S1 is FALSE and S2 is TRUE

(B) Both S1 and S2 are TRUE

(C) S1 is TRUE and S2 is FALSE

(D) Both S1 and S2 are FALSE

38. In an abrupt p–n junction, the doping concentrations on the p-side and n-side are $N_A=9\times10^{16}/\text{cm}^3$ respectively. The p-n junction is reverse biased and the total depletion width is 3 μm. The depletion width on the p-side is

(A) 2.7 μm (B) 0.3 μm

(C) 2.25 μm (D) 0.75 μm

39. The resistivity of a uniformly doped n-type silicon sample is 0.5 Ω-cm. If the electron mobility (μ_n) is 1250 cm^2/V-sec and the charge of an electron is 1.6×10^{-19} Coulomb, the donor impurity concentration (N_D) in the sample is

(A) $2\times10^{16}/\text{cm}^3$ (B) $1\times10^{16}/\text{cm}^3$

(C) $2.5\times10^{15}/\text{cm}^3$ (D) $5\times10^{15}/\text{cm}^3$

40. Consider an abrupt p-n junction. Let V_{bi} be the built-in potential of this junction and V_R be the applied reverse bias. If the junction capacitance (C_j) is 1 pF for $V_{bi}+V_R=1$ V, then for $V_{bi}+V_R=4$ V, C_j will be

(A) 4 pF (B) 2 pF

(C) 0.25 pF (D) 0.5 pF

41. Consider the following statements S1 and S2.

S1 : The threshold voltage (V_T) of a MOS capacitor decreases with increase in gate oxide thickness.

S2 : The threshold voltage (V_T) of a MOS capacitor decreases with increase in substrate doping concentration.

Which one of the following is correct ?

(A) S1 is FALSE and S2 is TRUE

(B) Both S1 and S2 are TRUE

(C) Both S1 and S2 are FALSE

(D) S1 is TRUE and S2 is FALSE

42. The drain of an n-channel MOSFET is shorted to the gate so that $V_{GS} = V_{DS}$. The threshold voltage (V_T) of the MOSFET is 1 V. If the drain current (I_D) is 1 mA for $V_{GS} = 2$ V, then for $V_{GS} = 3$ V, I_D is

(A) 2 mA (B) 3 mA

(C) 9 mA (D) 4 mA

43. The longest wavelength that can be absorbed by silicon, which has the bandgap of 1.12 eV, is 1.1 μm. If the longest wavelength that can be absorbed by another material is 0.87 μm, then the bandgap of this material is

(A) 1.416 eV (B) 0.886 eV

(C) 0.854 eV (D) 0.706 eV

44. The neutral base width of a bipolar transistor, biased in the active region, is 0.5 μm. The maximum electron concentration and the diffusion constant in the base are $10^{14}/\text{cm}^3$ and $D_n = 25\,\text{cm}^2/\text{sec}$ respectively. Assuming negligible recombination in the base, the collector current density is (the electron charge is 1.6×10^{-19} Coulomb)

(A) 800 A/cm^2 (B) 9 A/cm^2

(C) 200 A/cm^2 (D) 2 A/cm^2

45. Assume that the β of the transistor is extremely large and $V_{BE} = 0.7$ V, I_C and V_{CE} in the circuit shown in Fig. Q.45 are

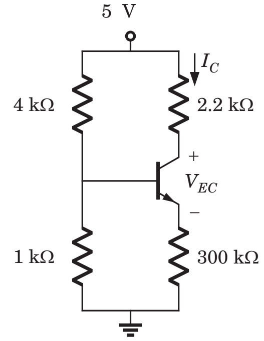


Fig Q.45

(A) $I_C = 1\,\text{mA}$, $V_{CE} = 4.7$ V

(B) $I_C = 0.5\,\text{mA}$, $V_{CE} = 3.75$ V

(C) $I_C = 1\,\text{mA}$, $V_{CE} = 2.5$ V

(D) $I_C = 0.5\,\text{mA}$, $V_{CE} = 3.9$ V

46. A bipolar transistor is operating in the active region with a collector current of 1 mA. Assuming that the β of the transistor is 100 and the thermal voltage (V_T) is 25 mV, the transconductance (g_m) and the input resistance (r_π) of the transistor in the common emitter configuration, are

(A) $g_m = 25$ mA/V and $r_\pi = 15.625$ kΩ

(B) $g_m = 40$ mA/V and $r_\pi = 4.0$ kΩ

(C) $g_m = 25$ mA/V and $r_\pi = 2.5$ kΩ

(D) $g_m = 40$ mA/V and $r_\pi = 2.5$ kΩ

47. The value of C required for sinusoidal oscillations of frequency 1 kHz in the circuit of Fig. Q.47 is

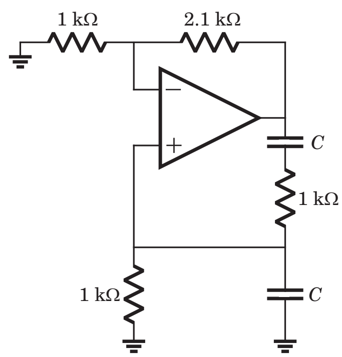


Fig Q.47

(A) $\frac{1}{2\pi}$ μF (B) 2π μF

(C) $\frac{1}{2\pi\sqrt{6}}$ μF (D) $2\pi\sqrt{6}$ μF

48. In the op-amp circuit given in Fig. Q.48, the load current i_L is

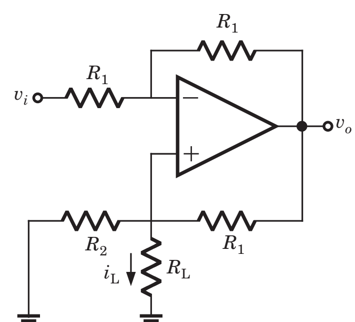


Fig Q.48

(A) $-\frac{v_s}{R_2}$ (B) $\frac{v_s}{R_2}$

(C) $-\frac{v_s}{R_L}$ (D) $\frac{v_s}{R_1}$

57. Consider the sequence of 8085 instructions given below

LXI H, 9258

MOV A, M

CMA

MOV M , A

Which one of the following is performed by this sequence?

(A) Contents of location 9258 are moved to the accumulator

(B) Contents of location 9258 are compared with the contents of the accumulator

(C) Contents of location 8529 are complemented and stored in location 8529

(D) Contents of location 5892 are complemented and stored in location 5892

58. A Boolean function f of two variables x and y is defined as follows :

$$f(0,0)=f(0,1)=f(1,1)=1;\quad f(1,0)=0$$

Assuming complements of x and y are not available, a minimum cost solution for realizing f using only 2-input NOR gates and 2-input OR gates (each having unit cost) would have a total cost of

(A) 1 unit (B) 4 units

(C) 3 units (D) 2 units

59. It is desired to multiply the numbers 0AH by 0BH and store the result in the accumulator. The numbers are available in registers B and C respectively. A part of the 8085 program for this purpose is given below:

```
         MVI A, 00H
LOOP:    __________
         __________
         __________
         HLT
         END
```

The sequence of instructions to complete the program would be

(A) JNZ LOOP, ADD B, DCR C

(B) ADD B, JNZ LOOP, DCR C

(C) DCR C, JNZ LOOP, ADD B

(D) ADD B, DCR C, JNZ LOOP

60. A 1 kHz sinusoidal signal is ideally sampled at 1500 samples /sec and the sampled signal is passed through an ideal low-pass filter with cut-off frequency 800 Hz. The output signal has the frequency

(A) zero Hz (B) 0.75 kHz

(C) 0.5 kHz (D) 0.25 kHz

61. A rectangular pulse train $s(t)$ as shown in Fig. Q.61 is convolved with the signal $\cos^2(4\pi\times10^3 t)$. The convolved signal will be a

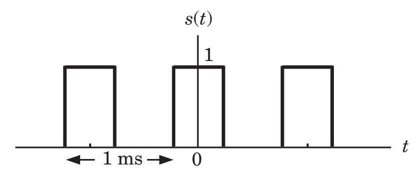


Fig Q.61

(A) DC (B) 12 kHz sinusoid

(C) 8 kHz sinusoid (D) 14 kHz sinusoid

62. Consider the sequence

$$x[n]=[-4-j5\quad 1+j2\quad 5]$$
$$\uparrow$$

The conjugate anti-symmetric part of the sequence is

(A) $[-4-j2.5\quad j2\quad 4-j2.5]$

(B) $[-j2.5\quad 1\quad j2.5]$

(C) $[-j2.5\quad j2\quad 0]$

(D) $[-4\quad 1\quad 4]$

63. A causal LTI system is described by the difference equation

$$2y[n]=\alpha y[n-2]-2x[n]+\beta x[n-1]$$

The system is stable only if

(A) $|\alpha|=2,\ |\beta|<2$

(B) $|\alpha|>2,\ |\beta|>2$

(C) $|\alpha|<2$, any value of β

(D) $|\beta|<2$, any value of α

64. A causal system having the transfer function

$$H(s)=\frac{1}{s+2}$$

is excited with $10u(t)$. The time at which the output reaches 99% of its steady state value is

(A) 2.7 sec (B) 2.5 sec

(C) 2.3 sec (D) 2.1 sec

65. The impulse response $h[n]$ of a linear time invariant system is given as

$$h[n] = \begin{cases} -2\sqrt{2} & n = 1, -1 \\ 4\sqrt{2} & n = 2, -2 \\ 0 & \text{otherwise} \end{cases}$$

If the input to the above system is the sequence $e^{j\pi n/4}$, then the output is

(A) $4\sqrt{2}e^{j\pi n/4}$ (B) $4\sqrt{2}e^{-j\pi n/4}$

(C) $4e^{j\pi n/4}$ (D) $-4e^{j\pi n/4}$

66. Let $x(t)$ and $y(t)$ with Fourier transforms $F(f)$ and $Y(f)$ respectively be related as shown in Fig. Q.66. Then $Y(f)$ is

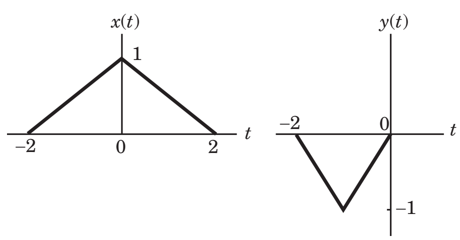


Fig Q.66

(A) $-\frac{1}{2}X(f/2)e^{-j2\pi f}$ (B) $-\frac{1}{2}X(f/2)e^{j2\pi f}$

(C) $-X(f/2)\,e^{j2\pi f}$ (D) $-X(f/2)\,e^{-j2\pi f}$

67. A system has poles at 0.01 Hz, 1 Hz and 80 Hz; zeros at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is

(A) –90° (B) 0°

(C) 90° (D) –180°

68. Consider the signal flow graph shown in Fig. Q.68. The gain $\frac{x_5}{x_1}$ is

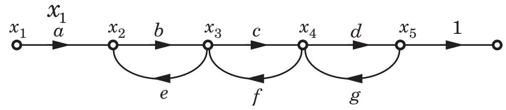


Fig Q.68

(A) $\dfrac{1-(be+cf+dg)}{abcd}$

(B) $\dfrac{bedg}{1-(be+cf+dg)}$

(C) $\dfrac{abcd}{1-(be+cf+dg)+bedg}$

(D) $\dfrac{1-(be+cf+dg)+bedg}{abcd}$

69. If $\mathbf{A} = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix}$, then $\sin At$ is

(A) $\begin{bmatrix} \sin(-4t)+2\sin(-t) & -\sin(-4t)+2\sin(-t) \\ -\sin(-4t)+\sin(-t) & 2\sin(-4t)+\sin(-t) \end{bmatrix}$

(B) $\begin{bmatrix} \sin(-2t) & \sin(2t) \\ \sin(t) & \sin(-3t) \end{bmatrix}$

(C) $\begin{bmatrix} \sin(4t)+2\sin(t) & 2\sin(-4t)-2\sin(-t) \\ -\sin(-4t)+\sin(t) & 2\sin(4t)+\sin(t) \end{bmatrix}$

(D) $\begin{bmatrix} \cos(-t)+2\cos(t) & 2\cos(-4t)-2\sin(-t) \\ -\cos(-4t)+\sin(-t) & -2\cos(4t)+\cos(t) \end{bmatrix}$

70. The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s^2+s+2)(s+3)}$$

The range of K for which the system is stable is

(A) $\frac{21}{4} > K > 0$ (B) $13 > K > 0$

(C) $\frac{21}{4} < K < \infty$ (D) $-6 < K < \infty$

71. For the polynomial

$$P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$$

the number of roots which lie in the right half of the s-plane is

(A) 4 (B) 2

(C) 3 (D) 1

72. The state variable equations of a system are :

$$\dot{x}_1 = -3x_1 - x_2 = u,\ \dot{x}_2 = 2x_1 \quad , \quad y = x_1 + u$$

The system is

(A) controllable but not observable

(B) observable but not controllable

(C) neither controllable nor observable

(D) controllable and observable

73. Given $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the state transition matrix e^{At} is given by

(A) $\begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & e^{t} \\ e^{t} & 0 \end{bmatrix}$

(C) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ (D) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

74. Consider the signal $x(t)$ shown in Fig. Q.74. Let $h(t)$ denote the impulse response of the filter matched to $x(t)$, with $h(t)$ being non-zero only in the interval 0 to 4 sec. The slope of $h(t)$ in the interval $3 < t < 4$ sec is

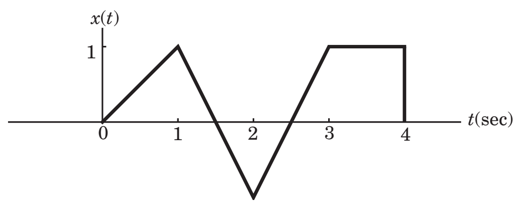


Fig. Q.74

(A) ½ sec^{-1} (B) -1 sec^{-1}

(C) $-1/2$ sec^{-1} (D) 1 sec^{-1}

75. A 1 mW video signal having a bandwidth of 100 MHz is transmitted to a receiver through a cable that has 40 dB loss. If the effective one-sided noise spectral density at the receiver is 10^{-20} Watt/Hz, then the signal-to-noise ratio at the receiver is

(A) 50 dB (B) 30 dB

(C) 40 dB (D) 60 dB

76. A 100 MHz carrier of 1V amplitude and a 1 MHz modulating signal of 1V amplitude are fed to a balanced modulator. The output of the modulator is passed through an ideal high-pass filter with cut-off frequency of 100 MHz. The output of the filter is added with 100 MHz signal of 1V amplitude and 90° phase shift as shown in Fig. Q.76. The envelope of the resultant signal is

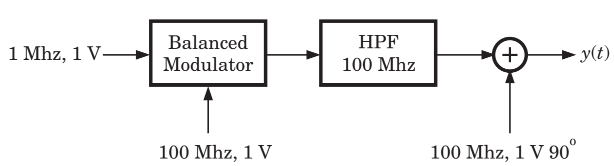


Fig Q.76

(A) constant (B) $\sqrt{1+\sin(2\pi\times 10^6 t)}$

(C) $\sqrt{5/4-\sin(2\pi\times 10^6 t)}$ (D) $\sqrt{5/4+\cos(2\pi\times 10^6 t)}$

77. Two sinusoidal signals of same amplitude and frequencies 10 kHz and 10.1 kHz are added together. The combined signal is given to an ideal frequency detector. The output of the detector is

(A) 0.1 kHz sinusoid (B) 20.1 kHz sinusoid

(C) a linear function of time (D) a constant

78. Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the voltage at the detector input can lie between the levels –0.25 V and +0.25 V with equal probability; when binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1 V with equal probability. If the detector has a threshold of 0.2V (i.e. if the received signal is greater than 0.2V, the bit is taken as 1), the average bit error probability is

(A) 0.15 (B) 0.2

(C) 0.05 (D) 0.5

79. A random variable X with uniform density in the interval 0 to 1 is quantized as follows:

$$\text{if } 0 \le X \le 0.3, \quad x_q = 0$$
$$\text{if } 0.3 \le X \le 1, \quad x_q = 0.7$$

where x_q is the quantized value of X. The root-mean square value of the quantization noise is

(A) 0.573 (B) 0.198

(C) 2.205 (D) 0.266

80. Choose the correct one from among the alternatives A, B, C, D after matching an item from Group 1 with the most appropriate item in Group 2.

Group 1	**Group 2**
1 : FM	P : Slope overload
2 : DM	Q : μ-law
3 : PSK	R : Envelope detector
4 : PCM	S : Capture effect
	T : Hilbert transfer
	U : Matched filter

(A)	(B)	(C)	(D)
1–T	1–S	1–S	1–U
2–P	2–U	2–P	2–R
3–U	3–P	3–U	3–S
4–S	4–T	4–Q	4–Q

81. Three analog signals, having bandwidth 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. The bit rate for the multiplexed signal is

(A) 1, 15.2 kbps (B) 28.8 kbps

(C) 27.6 kbps (D) 38.4 kbps

82. Consider a system shown in Fig. Q.82. Let $X(f)$ and $Y(f)$ denote the Fourier transforms of $x(t)$ and $y(t)$ respectively. The ideal HPF has the cutoff frequency 10 kHz.

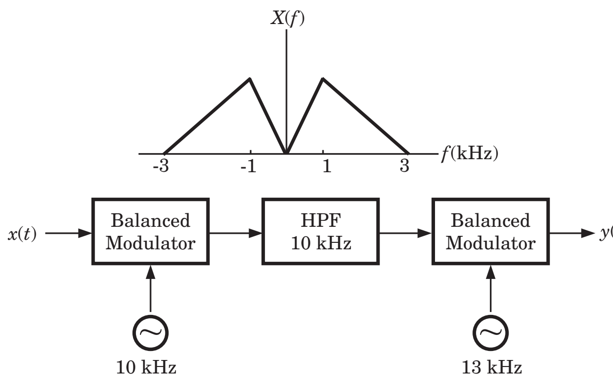


Fig Q.82

The positive frequencies where $Y(f)$ has spectral peaks are

(A) 1 kHz and 24 kHz (B) 2 kHz and 24 kHz

(C) 1 kHz and 14 kHz (D) 2 kHz and 14 kHz

83. A parallel plate air-filled capacitor has plate area of 10^{-4} m^2 and plate separation of 10^{-3} m. It is connect- ed to a 0.5 V, 3.6 GHz source. The magnitude of the displacement current is ($\varepsilon_o = 1/36\pi \times 10^{-9}$ F/m)

(A) 10 mA (B) 100 mA

(C) 10 A (D) 1.59 mA

84. A source produces binary data at the rate of 10 kbps. The binary symbols are represented as shown in Fig.Q.84

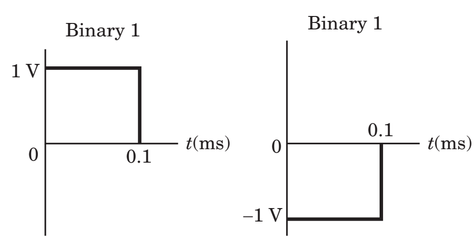


Fig Q.84

The source output is transmitted using two modulation schemes, namely Binary PSK (BPSK) and Quadrature PSK (QPSK). Let B_1 and B_2 be the bandwidth requirements of BPSK respectively. Assuming that the bandwidth of the above rectangular pulses is 10 kHz, B_1 and B_2 are

(A) B_1 = 20 kHz, B_2 = kHz

(B) B_1 = 10 kHz, B_2 = 10 kHz

(C) B_1 = 20 kHz, B_2 = 10 kHz

(D) B_1 = 10 kHz, B_2 = 10 kHz

85. Consider a 300 Ω, quarter-wave long (at 1 GHz) transmission line as shown in Fig. Q.85. It is connected to a 10 V, 50Ω source at one end and is left open circuited at the other end. The magnitude of the voltage at the open circuit end of the line is

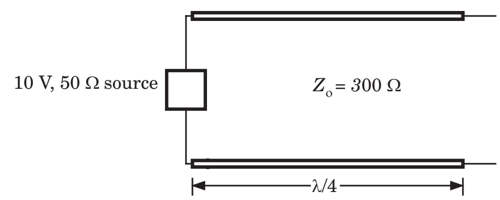


Fig Q.85

(A) 10 V (B) 5 V

(C) 60 V (D) 60/7 V

86. In a microwave test bench, why is the microwave signal amplitude modulated at 1 kHz ?

(A) To increase the sensitivity of measurement

(B) To transmit the signal to a far-off place

(C) To study amplitude modulation

(D) Because crystal detector fails at microwave frequencies

87. If $\vec{\mathbf{E}} = (\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y)e^{jkz-j\omega t}$ and $\vec{\mathbf{H}} = (k/\omega\mu)(\hat{\mathbf{a}}_y + j\hat{\mathbf{a}}_x)e^{jkz-j\omega t}$,the time-averaged Poynting vector is

(A) null vector (B) $(k/\omega\mu)\hat{\mathbf{a}}_z$

(C) $(2k/\omega\mu)\hat{\mathbf{a}}_z$ (D) $(k/2\omega\mu)\hat{\mathbf{a}}_z$

88. Consider an impedance $Z = R + jX$ marked with point P in an impedance Smith chart as shown in Fig. Q.88. The movement from point P along a constant resistance circle in the clockwise direction by an angle 45° is equivalent to

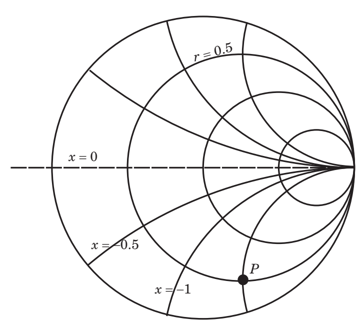


Fig. Q.88

CHAPTER

10.3

EC-05

Duration : Three Hours Maximum Marks : 150

Question 1- 30 Carry one Mark each.

1. The following differential equation has

$$3\frac{d^2y}{dt^2} + 4\left(\frac{dy}{dt}\right)^3 + y^2 + 2 = x$$

(A) degree = 2, order = 1

(B) degree = 3, order = 2

(C) degree = 4, order = 3

(D) degree = 2, order =3

2. Choose the function $f(t); -\infty < t < \infty$ for which a Fourier series cannot be defined.

(A) $3\sin(25t)$ (B) $4\cos(20t + 3) + 2\sin(710t)$

(C) $e^{-|t|}\sin(25t)$ (D) 1

3. A fair dice is rolled twice. The probability that an odd number will follow on even number is

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

4. A solution of the following differential equation is given by

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$$

(A) $y = e^{2x} + e^{-3x}$ (B) $y = e^{2x} + e^{3x}$

(C) $y = e^{-2x} + e^{-3x}$ (D) $y = e^{-2x} + e^{-3x}$

5. The function $x(t)$ is shown in the figure. Even and odd parts of a unit step function $u(t)$ are respectively,

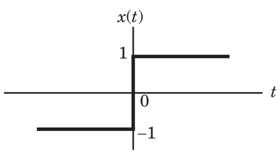


Fig. Q5

(A) $\frac{1}{2}, \frac{1}{2}x(t)$ (B) $-\frac{1}{2}, \frac{1}{2}x(t)$

(C) $\frac{1}{2}, -\frac{1}{2}x(t)$ (D) $-\frac{1}{2}, -\frac{1}{2}x(t)$

6. The region of convergence of z - transform of the sequence $\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be

(A) $|z| < \frac{5}{6}$ (B) $|z| > \frac{5}{6}$

(C) $\frac{5}{6} < |z| < \frac{6}{5}$ (D) $\frac{6}{5} < |z| < \infty$

7. The condition on R, L and C such that the step response $y(t)$ in the figure has no oscillations, is

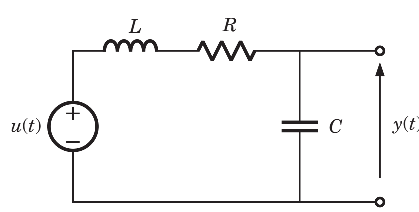


Fig. Q7

(A) $R \geq \frac{1}{2}\sqrt{\frac{L}{C}}$ (B) $R \geq \sqrt{\frac{L}{C}}$

(C) $R \geq 2\sqrt{\frac{L}{C}}$ (D) $R = \sqrt{\frac{1}{LC}}$

8. The *ABCD* parameters of an ideal n:1 transformer shown in the figure are $\begin{bmatrix} n & 0 \\ 0 & X \end{bmatrix}$. The value of x will be

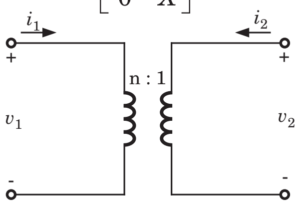


Fig. Q8

(A) n (B) $\frac{1}{n}$

(C) n^2 (D) $\frac{1}{n^2}$

9. In a series *RLC* circuit, $R = 2$ kΩ, $L = 1$ H and $C = \frac{1}{400}$ μ. The resonant frequency is

(A) 2×10^4 Hz (B) $\frac{1}{\pi} \times 10^4$ Hz

(C) 10^4 Hz (D) $2\pi \times 10^4$ Hz

10. The maximum power that can be transferred to the load resistor R_L from the voltage source in the figure is

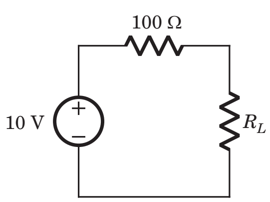


Fig. Q10

(A) 1 W (B) 10 W

(C) 0.25 W (D) 0.5 W

11. The bandgap of Silicon at room temperature is

(A) 1.3 eV (B) 0.7 eV

(C) 1.1 eV (D) 1.4 eV

12. A Silicon PN junction at a temperature of 20° C has a reverse saturation current of 10 pico - Amperes (pA). The reserve saturation current at 40° C for the same bias is approximately

(A) 30 pA (B) 40 pA

(C) 50 pA (D) 60 pA

13. The primary reason for the widespread use of Silicon in semiconductor device technology is

(A) abundance of Silicon on the surface of the Earth.

(B) larger bandgap of Silicon in comparison to Germanium.

(C) favorable properties of Silicon - dioxide (SiO2)

(D) lower melting point.

14. The effect of current shunt feedback in an amplifier is to

(A) increase the input resistance and decrease the output resistance.

(B) increase both input and output resistance

(C) decrease both input and output resistance.

(D) decrease the input resistance and increase the output resistance.

15. The input resistance of the amplifier shown in the figure is

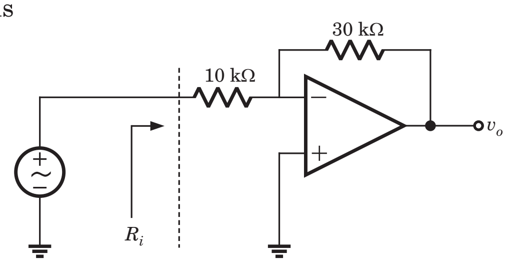


Fig. Q15

(A) $\frac{30}{4}$ kΩ (B) 10 kΩ

(C) 40 kΩ (D) infinite

16. The first and the last critical frequency of an *RC* - driving point impedance function must respectively be

(A) a zero and a pole

(B) a zero and a zero

(C) a pole and a pole

(D) a pole and a zero

17. The cascode amplifier is a multistage configuration of

(A) CC - CB (B) CE - CB

(C) CB - CC (D) CE - CC

18. Decimal 43 in Hexadecimal and BCD number system is respectively

(A) B2, 0100 011

(B) 2B, 0100 0011

(C) 2B, 0011 0100

(D) B2, 0100 0100

19. The Boolean function f implemented in the figure using two input multiplexes is

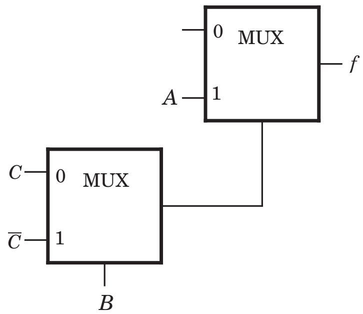


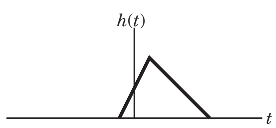
Fig. Q19

(A) $A\overline{B}C + AB\overline{C}$ (B) $ABC + A\overline{B}\overline{C}$

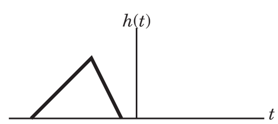
(C) $\overline{A}BC + \overline{A}\overline{B}\overline{C}$ (D) $\overline{A}\overline{B}C + \overline{A}B\overline{C}$

20. Which of the following can be impulse response of a causal system?

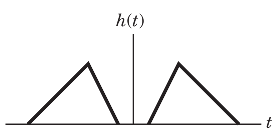
(A)



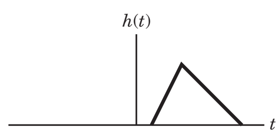
(B)



(C)



(D)



21. Let $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $y(n) = x^2(n)$ and $Y(e^{j\omega})$ be the Fourier transform of $y(n)$ then $Y(e^{j0})$ is

(A) $\frac{1}{4}$ (B) 2

(C) 4 (D) $\frac{4}{3}$

22. Find the correct match between group 1 and group 2

Group 1	Group II
P. $\{1 + km(t)\}A\sin(\omega_c t)$	W. Phase Modulation
Q. $km(t)A\sin(\omega_c t)$	X.Frequency Modulation
R. $A\sin(\omega_c t + km(t))$	Y. Amplitude Modulation

(A) P-Z, Q-Y, R-X, S-W (B) P-W, Q-X, R-Y, S-Z

(C) P-X, Q-W, R-Z, S-Y (D) P-Y, Q-Z, R-W, S-X

23. The power in the signal $s(t) = 8\cos\left(20\pi - \frac{\pi}{2}\right) + 4\sin(15\pi t)$ is

(A) 40 (B) 41

(C) 42 (D) 82

24. Which of the following analog modulation scheme requires the minimum transmitted power and minimum channel bandwidth?

(A) VSB (B) DSB - SC

(C) SSB (D) AM

25. A linear system is equivalently represented by two sets of state equations:

$$\dot{X} = AX + BU \text{ And } \dot{W} = CW + DU$$

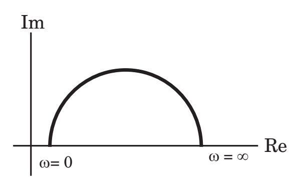
The eigenvalues of the representations are also computed as $[\lambda]$ and $[\mu]$. Which one of the following statements is true?

(A) $[\lambda] = [\mu]$ and $X = W$ (B) $[\lambda] = [\mu]$ and $X \neq W$

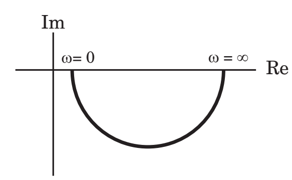
(C) $[\lambda] \neq [\mu]$ and $X = W$ (D) $[\lambda] = [\mu]$ and $X \neq W$

26. Which one of the following polar diagrams corresponds to a lag network?

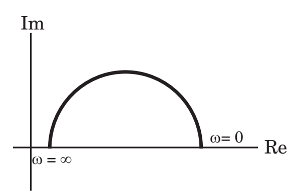
(A)



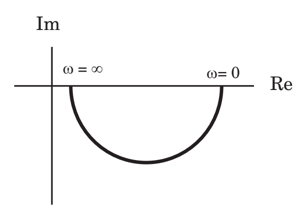
(B)



(C)



(D)



27. Despite the presence of negative feedback, control systems still have problems of instability because the

(A) Components used have non-linearities

(B) Dynamic equations of the subsystem are not known exactly.

(C) Mathematical analysis involves approximations.

(D) System has large negative phase angle at high frequencies.

37. Given an orthogonal matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, [AA^T]^{-1} \text{ is}$$

(A) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$

38. For the circuit show in the figure, the instantaneous current $i_i(t)$ is

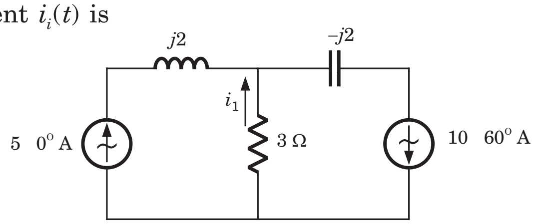


Fig. Q38

(A) $\frac{10\sqrt{3}}{2}\angle 90°$ Amps. (B) $\frac{10\sqrt{3}}{2}\angle -90°$ Amps.

(C) $5\angle 60°$ Amps (D) $5\angle -60°$ Amps

39. Impedance Z as shown in the given figure is

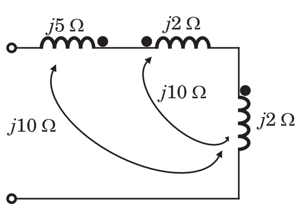


Fig. Q39

(A) $j29\ \Omega$ (B) $j9\ \Omega$

(C) $j19\ \Omega$ (D) $j39\ \Omega$

40. For the circuit shown in figure, Thevenin's voltage and Thevenin's equivalent resistance at terminals a - b is

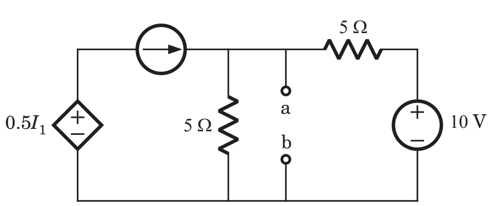


Fig. Q40

(A) 5 V and 2 Ω (B) 7.5 V and 2.5 Ω

(C) 4 V and 2 Ω (D) 3 V and 2.5 Ω

41. If $R_1 = R_2 = R_3 = R$ and $R_3 = 1.1R$ in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is

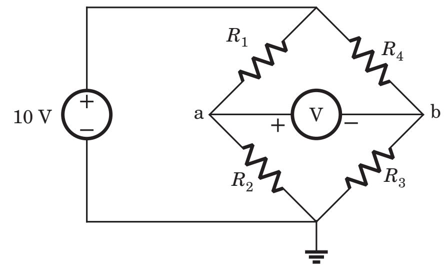


Fig. Q41

(A) 0.238 V (B) 0.138 V

(C) –0.238 V (D) 1 V.

42. The h parameters of the circuit shown in the figure are

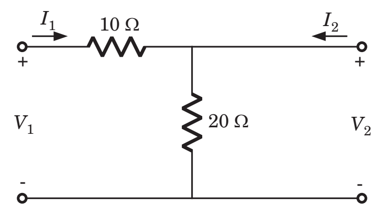


Fig. Q42

(A) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

(C) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

43. A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage V_O at time $t = 2$ sec is

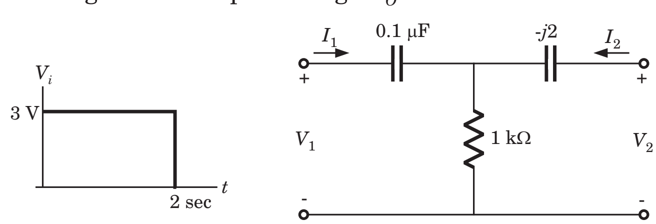


Fig. Q43

(A) 3 V (B) –3 V

(C) 4 V (D) –4 V

44. A Silicon sample A is doped with 10^{18} atoms/cm^3 of boron. Another sample b of identical dimension is doped with 10^{18} atoms/cm^3 phosphorus. The ratio of electron to hole mobility is 3. The ratio of conductivity of the sample A to B is

(A) 3 (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

45. A Silicon PN junction diode under reverse bias has depletion region of width 10 μm. The relative permitivity of Silicon, $\varepsilon_r = 11.7$ and the preemptively of free space $\varepsilon_o = 8.85 \times 10^{-12}$ F/m. The depletion capacitance of the diode per square meter is

(A) 100 μF (B) 10 μF

(C) 1 μF (D) 20 μF

46. For an npn transistor connected as shown in figure $V_{BE} = 0.7$ volts. Given that reverse saturation current of the junction at room temperature 300 K is 10^{-13} A, the emitter current is

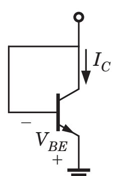


Fig. Q46

(A) 30 mA (B) 39 mA

(C) 49 mA (D) 20 mA

47. The voltage e_o is indicated in the figure has been measured by an ideal voltmeter. Which of the following can be calculated ?

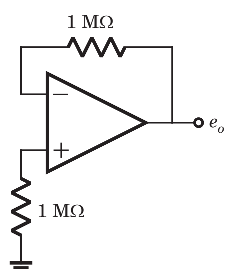


Fig. Q47

(A) Bias current of the inverting input only

(B) Bias current of the inverting and non-inverting inputs only

(C) Input offset current only

(D) Both the bias currents and the input offset current.

48. The OP-amp circuit shown in the figure is filter. The type of filter and its cut. Off frequency are respectively.

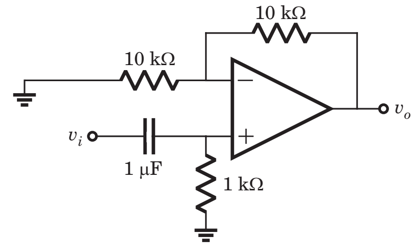


Fig. Q48

(A) high pass, 1000 rad/sec.

(B) Low pass, 1000 rad/sec.

(C) high pass, 1000 rad/sec.

(D) low pass, 10000 rad/sec.

49. In an ideal differential amplifier shown in the figure, a large value of (R_E)

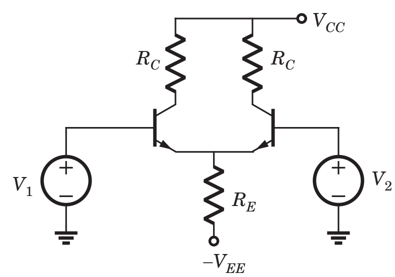


Fig. Q49

(A) increase both the differential and common - mode gains

(B) increases the common mode gain only

(C) decreases the differential mode gain only

(D) decreases the common mode gain only.

50. For an n-channel MOSFET and its transfer curve shown in the figure, the threshold voltage is

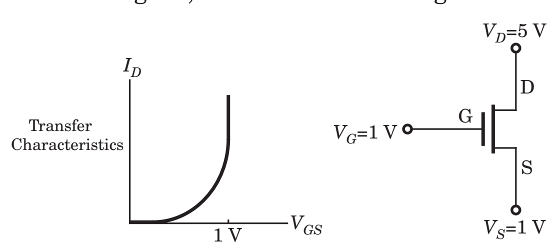


Fig. Q50

(A) 1 V and the device is in active region

(B) -1 V and the device is in saturation region

(C) 1 V and the device is in saturation region

(D) -1 V and the device is in active region.

57. The given figure shows a ripple counter using positive edge triggered flip-flops. If the present state of the counter is $Q_2Q_1Q_0 = 001$ then its next state Q_2Q_1Q will be

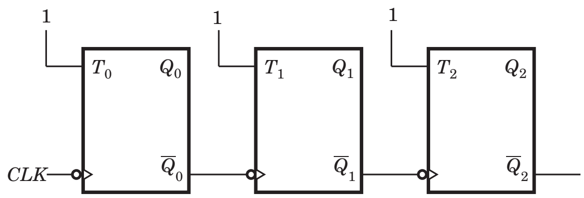


Fig. Q57

(A) 010
(D) 100
(B) 111
(D) 101

58. What memory address range is NOT represents by chip # 1 and chip # 2 in the figure A_0 to A_{15} in this figure are the address lines and CS means chip select.

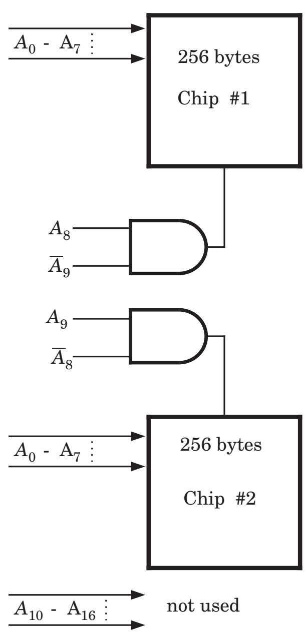


Fig. Q58

(A) 0100 - 02FF.
(B) 1500 - 16FF
(C) F900-FAFF
(D) F800 - F9FF

59. The output $y(t)$ of a linear time invariant system is related to its input $x(t)$ by the following equation

$$y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d + T)$$

The filter transfer function $H(\omega)$ of such a system is given by

(A) $(1 + \cos \omega T)e^{-j\omega t_d}$
(B) $(1 + 0.5\cos \omega T)e^{-j\omega t_d}$
(C) $(1 - \cos \omega T)e^{-j\omega t_d}$
(D) $(1 - 0.5\cos \omega T)e^{-j\omega t_d}$

60. Match the following and choose the correct combination.

Group 1

E. Continuos and periodic signal
F. Continuous and periodic signal
G. Discrete and aperiodic signal
H. Discrete and periodic signal

Group 2

1. Fourier representation is continuous and aperiodic
2. Fourier representation is discrete and aperiodic
3. Fourier representation is continuous
4. Fourier representation is discrete and periodic

(A) E-3, F-2, G-4, H-1
(A) E-1, F-3, G-2, H-4
(C) E-1, F-2, G-3, H-4
(D) E-2, F-1, G-4, H-3

61. A signal $x(n) = \sin(\omega_0 n + \phi)$ is the input to a linear time- invariant system having a frequency response $H(e^{j\omega})$. If the output of the system $Ax(n - n_0)$ then the most general form of will be

(A) $-n_0 w_0 + \beta$ for any arbitrary real
(B) $-n_0 w_0 + 2\pi k$ for any arbitrary integer k
(C) $n_0 w_0 + 2\pi k$ for any arbitrary integer k
(D) $-n_0 w_0 \phi$

62. For a signal the Fourier transform is $X(f)$). Then the inverse Fourier transform of $X(3f + 2)$ is given by

(A) $\frac{1}{2}x\left(\frac{t}{2}\right)e^{j3\pi t}$
(B) $\frac{1}{3}x\left(\frac{t}{3}\right)e^{-\frac{j4\pi t}{3}}$
(C) $3x(3t)e^{-j4\pi t}$
(D) $x(3t + 2)$

63. The polar diagram of a conditionally stable system for open loop gain $K = 1$ is shown in the figure. The open loop transfer function of the system is known to be stable. The closed loop system is stable for

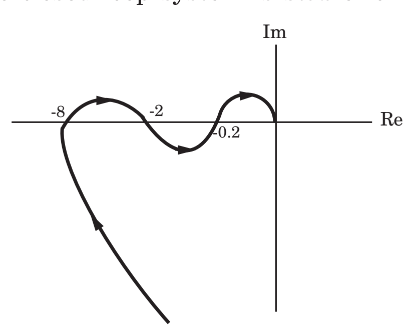


Fig. Q63

(A) $K < 5$ and $\frac{1}{2} < K < \frac{1}{8}$ (B) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

(C) $K < \frac{1}{8}$ and $5 < K$ (D) $K > \frac{1}{8}$ and $5 > K$

64. In the derivation of expression for peak percent overshoot

$$M_p = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right) \times 100\%$$

Which one of the following conditions is NOT required?

(A) System is linear and time invariant

(B) The system transfer function has a pair of complex conjugate poles and no zeroes.

(C) There is no transportation delay in the system.

(D) The system has zero initial conditions.

65. Given the ideal operational amplifier circuit shown in the figure indicate the correct transfer characteristics assuming ideal diodes with zero cut-in voltage.

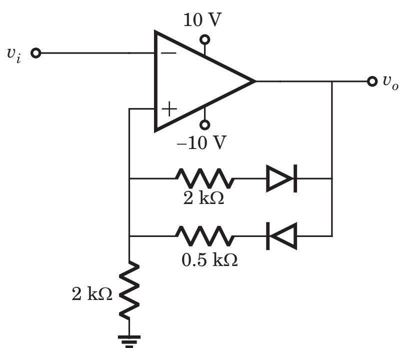
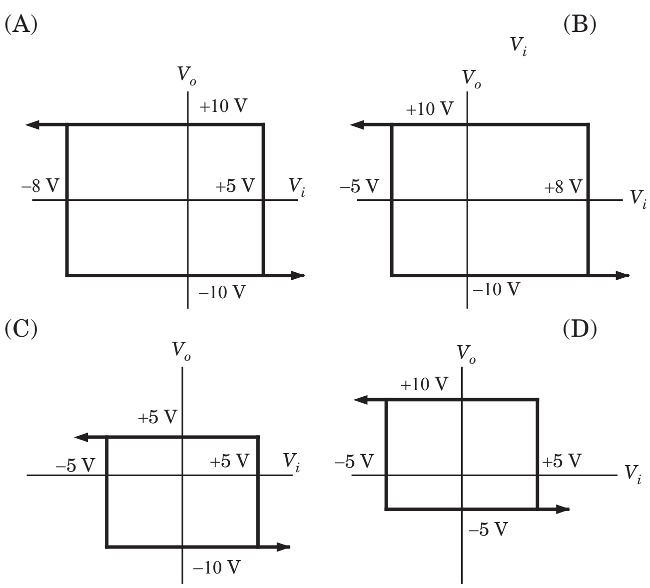


Fig. Q65



66. A ramp input applied to an unity feedback system results in 5% steady state error. The type number and zero frequency gain of the system are respectively

(A) 1 and 20 (B)0 and 20

(C) 0 and $\frac{1}{20}$ (D) 1 and $\frac{1}{20}$

67. A double integrator plant $G(s) = K/s^2$, $H(s) = 1$ is to be compensated to achieve the damping ratio   and and undamped natural frequency, $\omega = 5$ rad/s which one of the following compensator $G_e(s)$ will be suitable ?

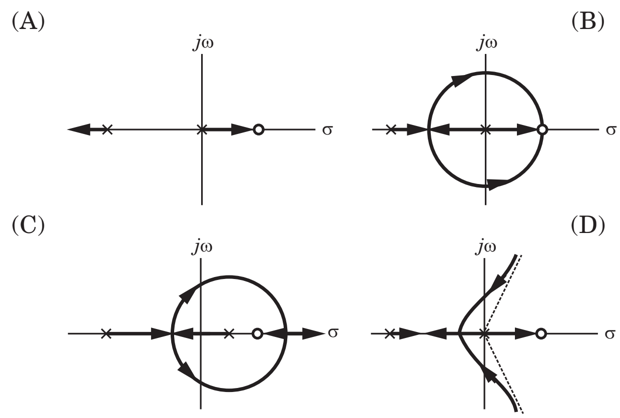
(A) $\frac{s+3}{s+9.9}$ (B) $\frac{s+9.9}{s+3}$

(C) $\frac{s-6}{s+8.33}$ (D) $\frac{s-6}{s}$

68. An unity feedback system is given as

$$G(s) = \frac{K(1-s)}{s(s+3)}.$$

Indicate the correct root locus diagram.



69. A MOS capacitor made using P type substrate is in the accumulation mode. The dominant charge in the channel is due to the presence of

(A) holes

(B) electrons

(C) positively charged ions

(D) negatively charged ions

70. A device with input $x(t)$ and output $y(t)$ is characterized $y(t) = x^2(t)$. An FM signal with frequency deviation of 90 kHz and modulating signal bandwidth of 5 kHz is applied to this device. The bandwidth of the output signal is

(A) 370 kHz (B) 190 kHz

(C) 380 kHz (C) 95 kHz

COMMON DATA QUESTION 78, 79, 80:

Given, $r_d = 20$ kΩ, $I_{DSS} = 10$ mA, $V_p = -8$ V

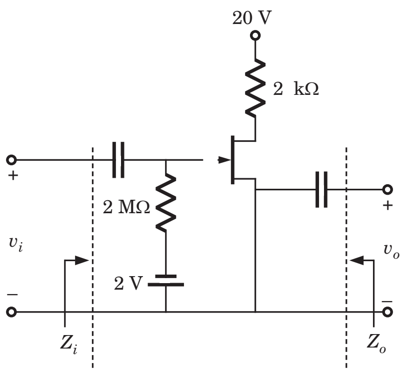


Fig. Q78

78. Z_i and Z_O of the circuit are respectively

(A) 2 MΩ and 2 kΩ (B) 2 MΩ and $\frac{20}{11}$ kΩ

(C) ∞ and 2 kΩ (D) ∞ and $\frac{20}{11}$ kΩ

79. I_D and V_{DS} under DC conditions are respectively

(A) 5.625 mA and 8.75 V (B) 7.500 mA and 5.00 V

(C) 4.500 mA and 11.00 V (D) 6.250 mA and 7.50 V

80. Transconductance in milli-Siemens (mS) and voltage gain of the amplifier are respectively

(A) 1.875 mS and 3.41 (B) 1.875 mS and –3.41

(C) 3.3 mS and –6 (D) 3.3 mS and 6

Linked Answer Questions : Q.81a to 85b Carry Two Marks Each

Statement For Linked Answer Questions 81a and 81b:

Consider an 8085 microprocessor system.

81a. The following program starts at location 0100H.

LXI SP, OOFF

LXI H, 0701

MVI A, 20H

SUB M

The content of accumulator when the program counter reaches 0109 H is

(A) 20 H (B) 02 H

(C) 00 H (D) FF H

81b. If in addition following code exists from 019H onwards,

ORI 40 H

ADD M

What will be the result in the accumulator after the last instruction is executed?

(A) 40 H (B) 20 H

(C) 60 H (D) 42 H

Statement for Linked Answer Question 82a and 82b:

The dopen loop transfer function of a unity feedback system is given by

82a. The gain and phase crossover frequencies in rad/sec are, respectively

(A) 0.632 and 1.26 (B) 0.632 and 0.485

(C) 0.485 and 0.632 (D) 1.26 and 0.632

82b. Based on the above results, the gain and phase margins of the system will be

(A) –7.09 dB and 87.5° (B) 7.09 dBand 87.5°

(C) 7.09 dB and –87.5° (D) –7.09 and –87.5°

Statement for linked answer question 83a and 83b

Asymmetric three - level midtread quantizer is to be designed assuming equiprobable occurrence of all quantization levels.

83a. If the probability density function is divided into three regions as shown in the figure, the value of a in the figure is

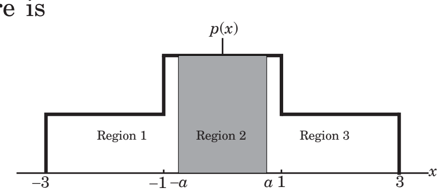


Fig. Q83

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

83b. The quantization noise power for the quantization region between - a and + a in the figure is

(A) $\frac{4}{81}$ (B) $\frac{1}{9}$

(C) $\frac{5}{81}$ (D) $\frac{2}{81}$

Statement of Linked Answer Questions 84a and 84b

Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 and a resistive load is shown in the figure.

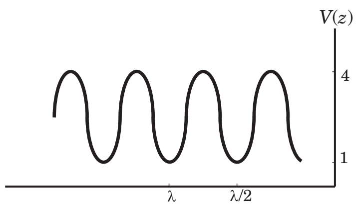


Fig. Q84a and Q84b

84a. The value of the load resistance is

(A) 50 Ω (B) 200 Ω

(C) 12.5 Ω (D) 0

84b. The reflection coefficient is given by

(A) –0.6 (B) –1

(C) 0.6 (D) 0

Statement of Linked Answer Question 85a and 85b:

A sequence x (n) has non-zero values as shown in the figure.(A)

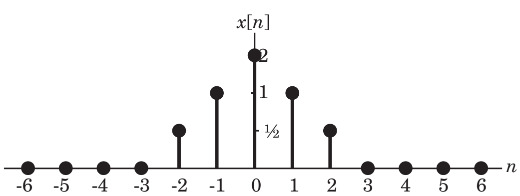
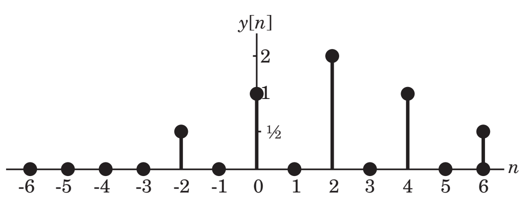


Fig. Q85

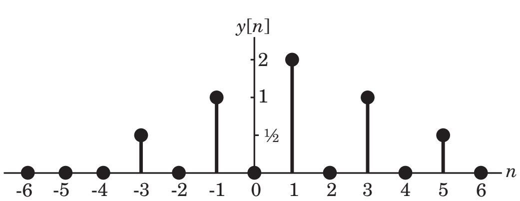
85a. The sequence

$$y[n] = \begin{cases} x\left(\frac{n}{2}-1\right) & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases} \quad \text{will be}$$

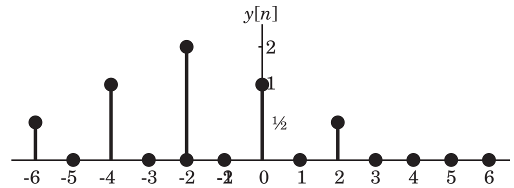
(A)



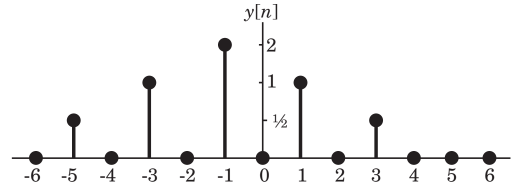
(B)



(C)



(D)



85b. The Fourier transform of $y[2n]$ will be

(A) $e^{-2j\omega}[\cos 4\omega + 2\cos 2\omega + 2]$

(B) $[\cos 2\omega + 2\cos \omega + 2]$

(C) $e^{-j\omega}[\cos 2\omega + 2\cos \omega + 2]$

(D) $e^{-2j\omega}[\cos 2\omega + 2\cos \omega + 2]$

****

CHAPTER

10.4

EC-06

Duration : Three Hours Maximum Marks : 150

Q.1 to carry Q.20 one marks each and Q.21 to Q.85 carry two marks each.

1. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) 3

2. $\nabla \times \nabla \times \mathbf{P}$ Where **P** is a vector, is equal to

(A) $\mathbf{P} \times \nabla \times \mathbf{P} - \nabla^2 \mathbf{P}$ (B) $\nabla^2 \mathbf{P} + \nabla(\nabla \cdot \mathbf{P})$

(C) $\nabla^2 \mathbf{P} + \nabla \times \mathbf{P}$ (D) $\nabla(\nabla \cdot \mathbf{P}) - \nabla^2 \mathbf{P}$

3. $\iint (\nabla \times \mathbf{P}) \cdot ds$ where **P** is a vector, is equal to

(A) $\oint \mathbf{P} \cdot \mathbf{dl}$ (B) $\oint \nabla \times \nabla \times \mathbf{P} \cdot \mathbf{dl}$

(C) $\oint \nabla \times \mathbf{P} \cdot \mathbf{dl}$ (D) $\oint \nabla \cdot \mathbf{P} dv$

4. A probability density function is of the form

$$p(x) = Ke^{-\alpha|x|},\ x \in (-\infty,\ \infty)$$

The value of K is

(A) 0.5 (B) 1

(C) 0.5 (D) α

5. A solution for the differential equation

$$x(t) + 2x(t) = \delta(t)$$

With initial condition $x(0-) = 0$

(A) $e^{-2t}u(t)$ (B) $e^{2t}u(t)$

(C) $e^{-t}u(t)$ (D) $e^{t}u(t)$

6. A low-pass filter having a frequency response $H(j\omega) = A(\omega)e^{j\phi(\omega)}$ does not produce any phase distortions if

(A) $A(\omega) = C\omega^2, \phi(\omega) = k\omega^3$ (B) $A(\omega) = C\omega^2, \phi(\omega) = k\omega$

(C) $A(\omega) = C\omega, \phi(\omega) = k\omega^2$ (D) $A(\omega) = C, \phi(\omega) = k\omega^{-1}$

7. The values of voltage (V_D) across a tunnel-diode corresponding to peak and valley currents are V_p, V_D respectively. The range of tunnel-diode voltage for $V_D$which the slope of its $I - V_D$ characteristics is negative would be

(A) $V_D < 0$ (B) $0 \le V_D < V_p$

(C) $V_p \le V_D < V_v$ (D) $V_D \ge V_v$

8. The concentration of minority carriers in an extrinsic semiconductor under equilibrium is

(A) Directly proportional to the doping concentration

(B) Inversely proportional to the doping concentration

(C) Directly proportional to the intrinsic concentration

(D) Inversely proportional to the intrinsic concentration

9. Under low level injection assumption, the injected minority carrier current for an extrinsic semiconductor is essentially the

(A) Diffusion current (B) Drift current

(C) Recombination current (D) Induced current

10. The phenomenon known as "Early Effect" in a bipolar transistor refers to a reduction of the effective base-width caused by

(A) Electron – Hole recombination at the base

(B) The reverse biasing of the base – collector junction

(C) The forward biasing of emitter-base junction

(D) The early removal of stored base charge during saturation-to-cut off switching

11. The input impedance (Z_i) and the output impedance (Z_0) of an ideal trans-conductance (voltage controlled current source) amplifier are

(A) $Z_i = 0,\ Z_0 = 0$ (B) $Z_i = 0,\ Z_0 = \infty$

(C) $Z_i = \infty,\ Z_0 = 0$ (D) $Z_i = \infty,\ Z_0 = \infty$

12. An n-channel depletion MOSFET has following two points on its $I_D - V_{GS}$ curve :

(i) $V_{GS} = 0$ at $I_D = 12$ mA and
(ii) $V_{GS} = -6$ Volts at $I_D = 0$ mA

Which of the following Q – point will give the highest trans – conductance gain for small signals?

(A) $V_{GS} = -6$ Volts (B) $V_{GS} = -3$ Volts

(C) $V_{GS} = 0$ Volts (D) $V_{GS} = 3$ Volts

13. The number of product terms in the minimized sum-of-product expression obtained through the following K – map is (where, "d" denotes don't care states)

1	0	0	1
0	d	0	0
0	0	d	1
1	0	0	1

(A) 2 (B) 3

(C) 4 (D) 5

14. Let $x(t) \leftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal $x(5t-3)$ in terms of $X(j\omega)$ is given as

(A) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$ (B) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(C) $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$ (D) $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

15. The Dirac delta function is defined as

(A) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(B) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(C) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt = 1$

(D) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt = 1$

16. If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$ then the region of convergence of $x_1[n] - x_2[n]$ includes

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{2}{3} < |z| < 3$

(C) $\frac{3}{2} < |z| < 3$ (D) $\frac{1}{3} < |z| < \frac{2}{3}$

17. The open-loop function of a unity-gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system in dB is given by

(A) 0 (B) 1

(C) 20 (D)

18. In the system shown below, $x(t) = (\sin t)u(t)$ In steady-state, the response $y(t)$ will be

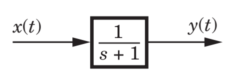


Fig Q. 18

(A) $\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$ (B) $\frac{1}{\sqrt{2}}\sin\left(t + \frac{\pi}{4}\right)$

(C) $\frac{1}{\sqrt{2}}e^{-t}\sin t$ (D) $\sin t - \cos t$

19. The electric field of an electromagnetic wave propagation in the positive direction is given by

$$E = \hat{a}_x \sin(\omega t - \beta z) + \hat{a}_y \sin(\omega t - \beta z + \pi/2)$$

The wave is

(A) Linearly polarized in the z–direction

(B) Elliptically polarized

(C) Left-hand circularly polarized

(D) Right-hand circularly polarized

30. A two-port network is represented by ABCD, parameters given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If port – 2 is terminated by the input impedance seen at port – 1 is given by

(A) $\frac{A + BR_L}{C + DR_L}$ (B) $\frac{AR_L + C}{BR_L + D}$

(C) $\frac{DR_L + A}{C + BR_L}$ (D) $\frac{AR_L + B}{D + CR_L}$

31. In the two port network shown in the figure below z_{12} and z_{21} are respectively

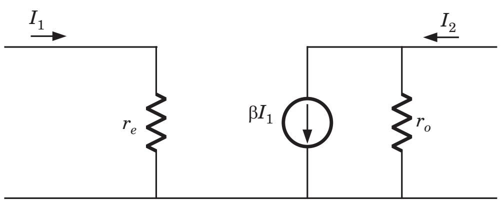


Fig Q.31

(A) r_e and βr_o (B) 0 and $-\beta r_o$

(C) 0 and βr_o (D) r_e and $-\beta r_o$

32. The first and the last critical frequencies (singularities) of a driving point impedance function of a passive network having two kinds of elements, are a pole and a zero respectively. The above property will be satisfied by

(A) RL network only

(B) RC network only

(C) LC network only

(D) RC as well as RL networks

33. A 2 mH inductor with some initial current can be represented as shown below, where s is the Lap lace Transform variable. The value of initial current is

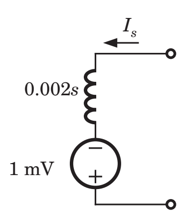


Fig Q.33

(A) 0.5A (B) 2.0A

(C) 1.0 A (D) 0.0 A

34. In the figures shown below, assume that all the capacitors are initially uncharged. If $v_i(t) = 10u(t)$ Volts, $v_o(t)$is given by

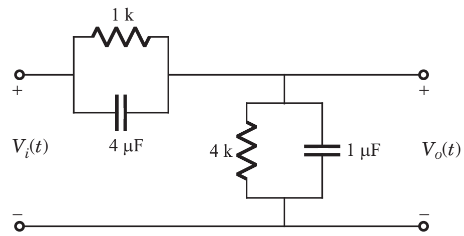


Fig. Q.34

(A) $8e^{-0.004t}$ Volts (B) $8(1 - e^{-0.004t})$ Volts

(C) $8u(t)$ Volts (D) 8 Volts

35. Consider two transfer functions

$$G_1(s) = \frac{1}{s^2 + as + b} \text{ And } G_2(s) = \frac{s}{s^2 + as + b}$$

The 3–dB bandwidths of their frequency responses are, respectively

(A) $\sqrt{a^2 - 4b}, \sqrt{a^2 + 4b}$ (B) $\sqrt{a^2 + 4b}, \sqrt{a^2 - 4b}$

(C) $\sqrt{a^2 - 4b}, \sqrt{a^2 - 4b}$ (D) $\sqrt{a^2 + 4b}, \sqrt{a^2 + 4b}$

36. A negative resistance R_{neg} is connected to a passive network N having driving point impedance $Z_1(s)$ as shown below. For $Z_2(s)$ to be positive real,

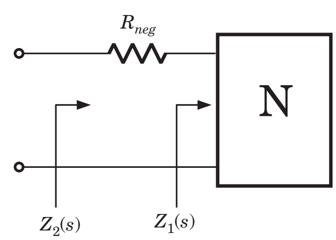


Fig Q.36

(A) $|R_{neg}| \le \text{Re}\, Z_1(j\omega),\ \forall\omega$ (B) $|R_{neg}| \le |Z_1(j\omega)|,\ \forall\omega$

(C) $|R_{neg}| \le \text{Im}\, Z_1(j\omega),\ \forall\omega$ (D) $|R_{neg}| \le \angle Z_1(j\omega),\ \forall\omega$

37. In the circuit shown below, the switch was connected to position 1 at $t < 0$ and at $t = 0$, it is changed to position 2. Assume that the diode has zero voltage drop and a storage time t_s. For $0 < t \le t_s$, v_R is given by (all in Volts)

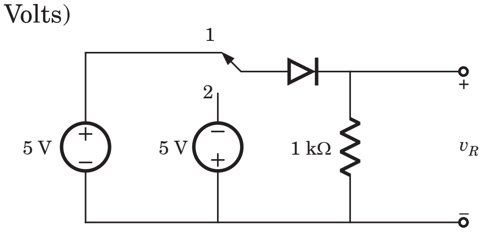


Fig Q.37

(A) $v_R = -5$ (B) $v_R = +5$

(C) $0 \le v_R < 5$ (D) $-5 \le v_R < 0$

38. The majority carriers in an n–type semiconductor have an average drift velocity v in a direction perpendicular to a uniform magnetic field **B**. The electric field **E** induced due to Hall effect acts in the direction.

(A) $\mathbf{v} \times \mathbf{B}$ (B) $\mathbf{B} \times \mathbf{v}$

(C) along **v** (D) opposite to **v**

39. Find the correct match between Group 1 and Group 2.

Group 1	Group 2
E-Varactor diode	1-Voltage reference
F-PIN diode	2-High frequency switch
G-Zener diode	3-Tuned circuits
H-Schottky diode	4-Current controlled attenuator

(A) E-4, F-2, G-1, H-3 (B) E-2, F-4, G-1, H-3

(C) E-3, F-4, G-1, H-2 (D) E-1, F-3, G-2, H-4

40. A heavily doped n- type semiconductor has the following data:

Hole-electron ratio :0.4

Doping concentration :4.2×10^8 atoms/m^3

Intrinsic concentration :1.5×10^4 atoms/m^3

The ratio of conductance of the n-type semiconductor to that of the intrinsic semiconductor of same material and ate same temperature is given by

(A) 0.00005 (B) 2,000

(C) 10,000 (D) 20,000

41. For the circuit shown in the following figure, the capacitor C is initially uncharged. At $t = 0$ the switch S is closed. In the figures shown the OP AMP is supplied with and the ground has been shown by the symbol

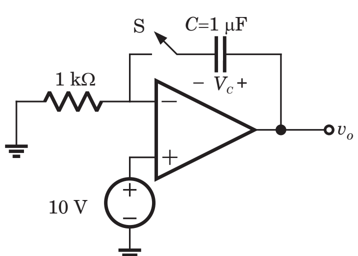


Fig Q.41

The voltage V_C across the capacitor at $t = 1$ is

(A) 0 Volt (B) 6.3 Volts

(C) 9.45 Volts (D) 10 Volts

42. For the circuit shown below, assume that the zener diode is ideal with a breakdown voltage of 6 volts. The waveform observed across R is

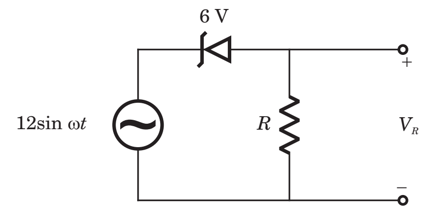
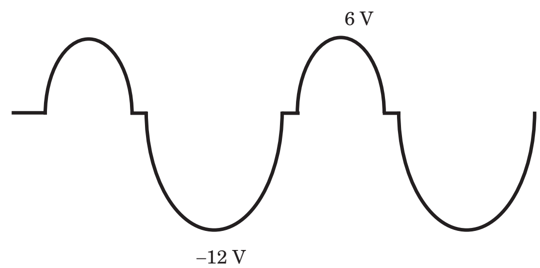


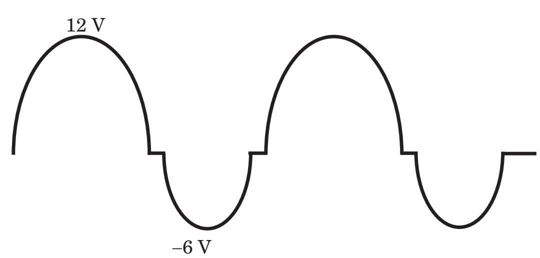
Fig Q.42

(A)

(B)



(D)



(C)

Q. 43 A new Binary Coded Pentary (BCP) number system is proposed in which every digit of a base–5 number is represented by its corresponding 3–bit binary code. For example, the base–5 number 24 will be represented by its BCP code 010100. In this numbering system, the BCP code 10001001101 corresponds of the following number is base–5 system

(A) 423 (B) 1324

(C) 2201 (D) 4231

44. An I / O peripheral device shown in Fig.(b) below is to be interfaced to an 8085 microprocessor. To select the I/O device in the I/O address range D4 H – D7 H, its chip–select ($\overline{CS}$) should be connected to the output of the decoder shown in as below:

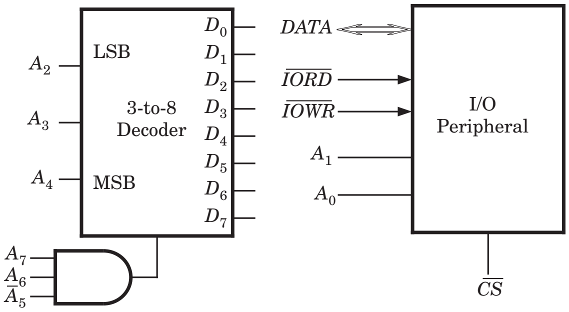


Fig Q.44

(A) output 7 (B) output 5

(C) output 2 (D) output 0

45. For the circuit shown in figures below, two 4 – bit parallel – in serial – out shift registers loaded with the data shown are used to feed the data to a full adder. Initially, all the flip – flops are in clear state. After applying two clock pulses, the outputs of the full-adder should be

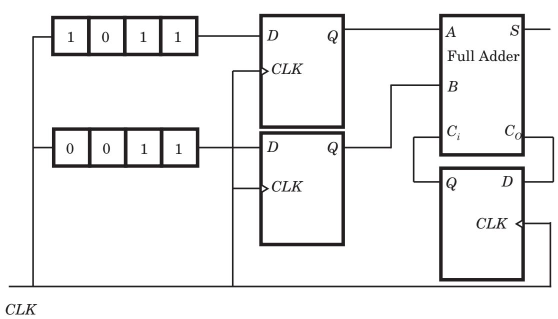


Fig Q.45

(A) $S = 0$ $C_0 = 0$ (B) $S = 0$ $C_0 = 1$

(C) $S = 1$ $C_0 = 0$ (D) $S = 1$ $C_0 = 1$

46. A 4 – bit D / A converter is connected to a free – running 3 – big UP counter, as shown in the following figure. Which of the following waveforms will be observed at V_O ?

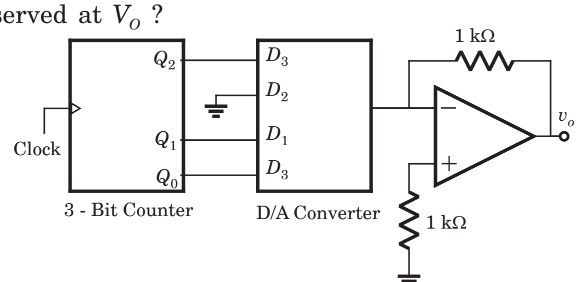
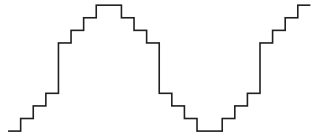
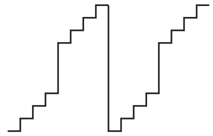


Fig Q.46

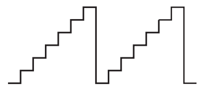
(A)



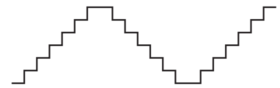
(B)



(C)



(D)



47. Two D – flip – flops, as shown below, are to be connected as a synchronous counter that goes through the following sequence

$$00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow \dots$$

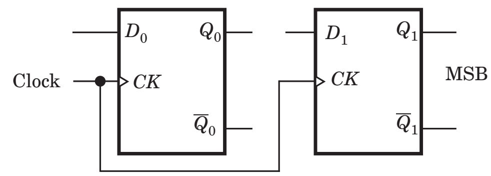


Fig Q.47

The inputs D_0 and D_1 respectively should be connected as,

(A) $\overline{Q}_1$ and Q_0 (B) $\overline{Q}_0$ and Q_1

(C) $\overline{Q}_1 Q_0$ and $\overline{Q}_1 Q_0$ (D) $\overline{Q}_1\,\overline{Q}_0$ and $Q_1 Q_0$

48. Following is the segment of a 8085 assembly language program

```
            LXI SP, EFFF H
            CALL 3000 H
            :
            :
            :
3000 H      LXI H, 3CF4
            PUSH PSW
            SPHL
            POP PSW
            RET
```

On completion of RET execution, the contents of SP is

(A) 3CF0 H (B) 3CF8 H

(C) EFFD H (D) EFFF H

49. The point P in the following figure is stuck at 1. The output f will be

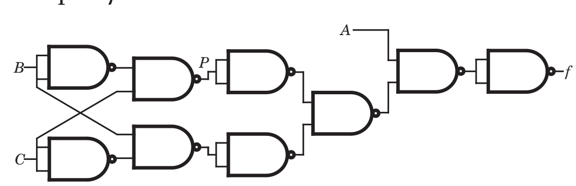


Fig Q.49

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

59. A linear system is described by the following state equation

$$\dot{X}(t) = AX(t) + BU(t), \quad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The state transition matrix of the system is

(A) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ (B) $\begin{bmatrix} -\cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix}$

(C) $\begin{bmatrix} -\cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$ (D) $\begin{bmatrix} -\cos t & -\sin t \\ \cos t & \sin t \end{bmatrix}$

60. The minimum step- size required for a Delta – Modulator operating at 32 K , samples/sec to track the signal (here $u(t)$ is the unit function)

$$x(t) = 125t(u(t) - u(t-1)(250 - 125t)(u(t-1) - u(t-2))$$

So that slope overload is avoided, would be

(A) 2^{-10} (B) 2^{-8}

(C) 2^{-6} (D) 2^{-4}

61. A zero mean white Gaussian noise is passed through an ideal lowpass filter of bandwidth 10 kHz. The output is then uniformly sampled with sampling period $t_s = 0.03$ msec. The samples so obtained would be

(A) correlated (B) statistically independent

(C) uncorrelated (E) orthogonal

62. A source generates three symbols with probabilities 0.25, 0.25, 0.50 at a rate of 3000 symbols per second. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate as

(A) 6000 bits/sec (B) 4500 bits/sec

(C) 3000 bits/sec (D) 1500 bits/sec

63. The diagonal clipping in Amplitude Demodulation (using envelope detector) can be avoided if RC time – constant of the envelope detector satisfies the following condition, (here W is message bandwidth and ω_c is carrier frequency both in rad /sec)

(A) $RC < \frac{1}{W}$ (B) $RC > \frac{1}{W}$

(C) $RC < \frac{1}{\omega_c}$ (D) $RC > \frac{1}{\omega_c}$

64. In the following figure the minimum value of the constant "C" , which is to be added to $y_1(t)$ and $y_2(t)$ such that $y_1(t)$ and $y_2(t)$ and are different, is

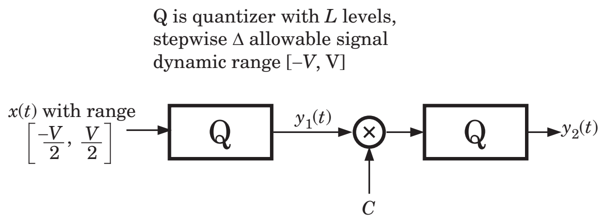


Fig Q.64

(A) Δ (B) $\frac{\Delta}{2}$

(C) $\frac{\Delta^2}{12}$ (D) $\frac{\Delta}{L}$

65. A message signal with 10 kHz bandwidth is lower side Band SSB modulated with carrier $f_{c1} = 10^6$ Hz frequency the resulting signal is then passed through a Narow Band Frequency Modulator with carrier frequency $f_{c2} = 10^9$ Hz. The bandwidth of the output would be

(A) 4×10^4 Hz (B) 2×10^6 Hz

(C) 2×10^9 Hz (D) 2×10^{10} Hz

66. A medium of relative permitivity $\varepsilon_{r2} = 2$ forms an interface with free – space. A point source of electromagnetic energy is located in the medium at a depth of 1 meter from the interface. Due to the total internal reflection, the transmitted beam has a circular cross-section over the interface. The area of the beam cross-section at the interface is given by

(A) 2π m^2 (B) π^2 m^2

(C) $\frac{\pi}{2}$ m^2 (D) π m^2

67. A medium is divide into regions I and II about $x = 0$ plane, as shown in the figure below. An electromagnetic wave with electric field $E_1 = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ is incident normally on the interface from region I. The electric file E_2 in region II at the interface is

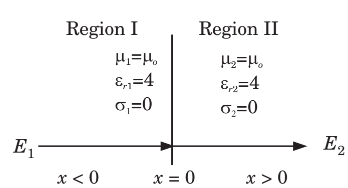


Fig Q.67

(A) $E_2 = E_1$ (B) $4\hat{a}_x + 0.75\hat{a}_y - 1.25\hat{a}_z$

(C) $3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ (D) $-3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$

68. When a planes wave traveling in free-space is incident normally on a medium having the fraction of power transmitted into the medium is given by

(A) 8/9 (B) 1/2

(C) 1/3 (D) 5/6

69. A rectangular wave guide having TE_{10} mode as dominant mode is having a cut off frequency 18-GHz for the mode TE_{30}. The inner broad – wall dimension of the rectangular wave guide is

(A) 5/3cms (B) 5 cms

(C) 5/2 cms (D) 10 cms

70. A mast antenna consisting of a 50 meter long vertical conductor operates over a perfectly conducting ground plane. It is base-fed at a frequency of 600 kHz. The radiation resistance of the antenna in Ohms is

(A) $\frac{2\pi^2}{5}$ (B) $\frac{\pi^2}{5}$

(C) $\frac{4\pi^2}{5}$ (D) $20\pi^2$

Common Data for Question 71,72,73:

In the transistor amplifier circuit show in the figure below, the transistor has the following parameters:

$$\beta_{DC} = 60,\ V_{BE} = 0.7\ \text{V},\ h_{ie} \to \infty,\ h_{fe} \to \infty$$

The capacitance can be assumed to be infinite.

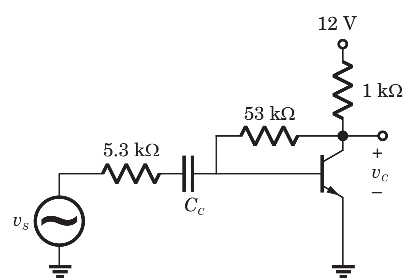


Fig Q.70

71. Under the DC conditions, the collector – to- emitter voltage drop is

(A) 4.8 Volts (B) 5.3 Volts

(C) 6.0 Volts (D) 6.6 Volts

72. If β_{DC} is increase by 10%, the collector – to- emitter voltage drop

(A) increases by less than or equal to 10%

(B) decreases by less than or equal to 10%

(C) increases by more than 10%

(D) decreases by more than 10%

73. The small signal gain of the amplifier v_c/v_s is

(A) 10 (B) –5.3

(C) 5.3 (D) 10

Common Data for Question 74, 75 :

Let $g(t) = p(t) * p(t)$ where $*$ denotes convolution and $p(t) = u(t) - u(t-1)$ with $u(t)$ being the unit step function.

74. The impulse response of filter matched to the signal $s(t) = g(t) - \delta(t-2) * g(t)$ is given as:

(A) $s(1-t)$ (B) $-s(1-t)$

(C) $-s(t)$ (D) $s(t)$

75. An Amplitude Modulated signal is given as

$$x_{AM} = 100(p(t) + 0.5g(t))\cos\omega_c t$$

In the interval. One set of possible values of the modulating signal and modulation index would be

(A) t, 0.5 (B) t, 1.0

(C) t, 2.0 (D) t^2, 0.5

Linked Answer Question : Q.75 to Q.85 carry two marks each.

Statement of Linked Answer Question 76 & 77:

A regulated power supply, shown in figure below, has an unregulated input (UR) of 15 volts and generates a regulated output Use the component values shown in the figure.

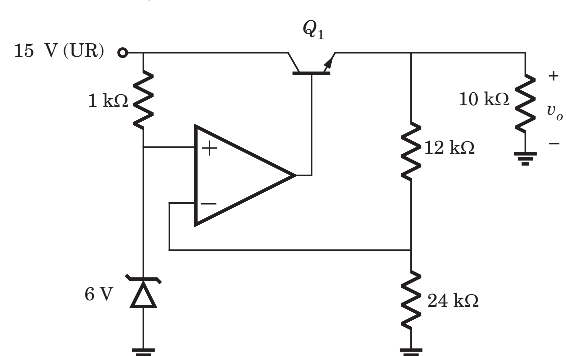


Fig Q.76

take 400 μs for an electromagnetic wave to travel from source end to load end and vice – versa. At $t = 400$ μs, the voltage at the load end is found to be 40 volts.

84. The load resistance is

(A) 25 Ohms (B) 50 Ohms

(C) 75 Ohms (D) 100 Ohms

85. The steady state current through the load resistance is

(A) 1.2 Amps (B) 0.3 Amps

(C) 0.6 Amps (D) 0.4 Amps

****

ANSWER

1. C	2. D	3. A	4. C	5. A
6. B	7. C	8. B	9. A	10 .B
11. D	12. D	13. A	14. A	15. D
16. D	17.D	18. A	19. C	20. A
21. A	22. B	23. D	24. C	25. D
26. C	27. A	28. C	29. A	30. D
31. B	32. B	33. A	34. B	35. B
36. B	37. D	38. A	39. C	40. D
41. D	42. B	43. D	44. B	45. D
46. B	47. A	48. B	49. D	50. B
51. C	52. B	53. C	54. B	55. D
56. B	57. C	58. D	59.A	60. B
61. B	62. B	63. D	64. C	65. B
66. D	67. C	68. A	69. C	70. A
71. C	72. B	73. A	74. D	75. A
76. C	77. B	78. A	79.	80. C
81. B	82. C	83.C	84.	85.

CHAPTER

10.7

EC-09

Q.1 - Q. 20 carry one mark each.

1. The order of the differential equation

$$\frac{d^2y}{dr^2} + \left(\frac{dy}{dt}\right)^3 + y^4 = e^{-1} \text{ is}$$

(A) 1 (B) 2

(C) 3 (D) 4

2. The Fourier series of a real periodic function has only

P. Cosine terms if it is even
Q. sine terms if it is even
R. cosine terms if it is odd
S. sine terms if it is odd

Which of the above statements are correct?

(A) P and S (B) P and R

(C) Q and S (D) Q and R

3. A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is true ?

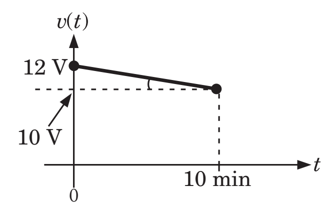
(A) f has frequency components at 0 and $1/2\pi$ Hz

(B) f has frequency components at 0 and $1/\pi$ Hz

(C) f has frequency components at $1/2\pi$ and $1/\pi$ Hz

(D) f has frequency components at $0.1/2\pi$ and $1/\pi$ Hz

4. A fully charged mobile phone with a 12 V battery is good for a 10 minute talk-time. Assume that, during the talk-time, the battery delivers a constant current of 2 A and its voltage drops linearly from 12 V to 10 V as shown in the figure. How much energy does the battery deliver during this talk-time ?



(A) 220 J (B) 12 kJ

(C) 13.2 kJ (D) 14.4 kJ

5. In an n-type silicon crystal at room temperature, which of the following can have a concentration of 4×10^{19} cm^{-3}?

(A) Silicon atoms (B) Holes

(C) Dopant atoms (D) Valence electrons

6. The full forms of the abbreviations TTL and CMOS in reference to logic families are

(A) Triple Transistor Logic and Chip Metal Oxide Semiconductor

(B) Tristate Transistor Logic and Chip Metal Oxide Semiconductor

(C) Transistor Transistor Logic and Complementary Metal Oxide Semiconductor

(D) Tristate Transistor Logic and Complementary Metal Oxide Silicon

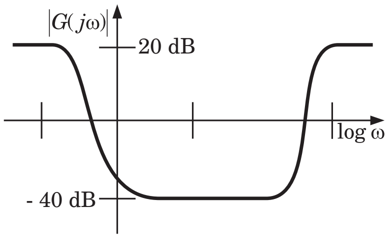
7. The ROC of Z-transform of the discrete time sequence

$$x(n) = \left(\frac{1}{3}\right)^{\pi} u(n) - \left(\frac{1}{2}\right)^{n} u(-n-1) \text{ is}$$

(A) $\frac{1}{3} < |z| < \frac{1}{2}$ (B) $|z| > \frac{1}{2}$

(C) $|z| < \frac{1}{3}$ (D) $2 < |z| < 3$

8. The magnitude plot of a rational transfer function $G(s)$ with real coefficients is shown below. Which of the following compensators has such a magnitude plot ?



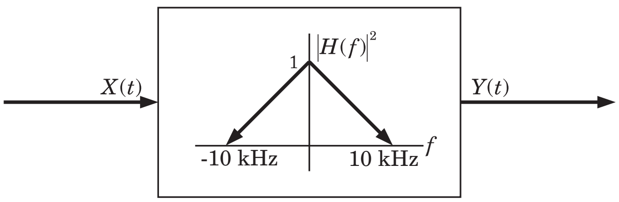
(A) Lead compensator

(B) Lag compensator

(C) PID compensator

(D) Lead-lag compensator

9. A white noise process $X(t)$ with two-sided power spectral density 1×10^{-10} W/Hz is input to a filter whose magnitude squared response is shown below.

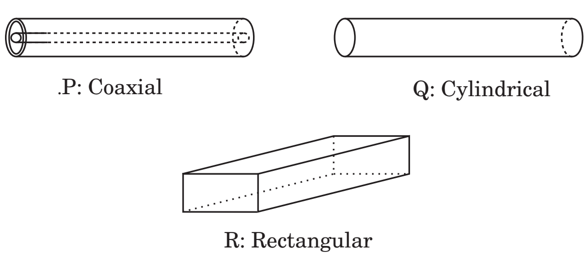


The power of the output process $Y(t)$ is given by

(A) 5×10^{-7} W (B) 1×10^{-6} W

(C) 2×10^{-6} W (D) 1×10^{-5} W

10. Which of the following statements is true regarding the fundamental mode of the metallic waveguides shown ?



(A) Only P has no cutoff-frequency

(B) Only Q has no cutoff-frequency

(C) Only R has no cutoff-frequency

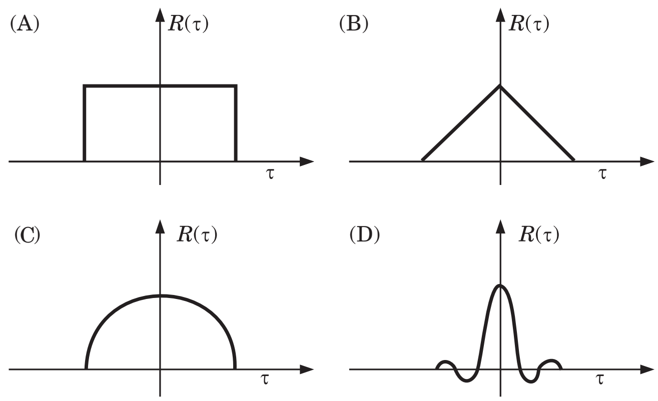
(D) All three have cut-off frequencies

11. A fair coin is tossed 10 times. What is the probability that ONLY the first two tosses will yield heads ?

(A) $\left(\frac{1}{2}\right)^2$ (B) ${}^{10}C_2\left(\frac{1}{2}\right)^2$

(C) $\left(\frac{1}{2}\right)^{10}$ (D) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

12. If the power spectral density of stationary random process is a sinc-squared function of frequency, the shape of its autocorrelation is



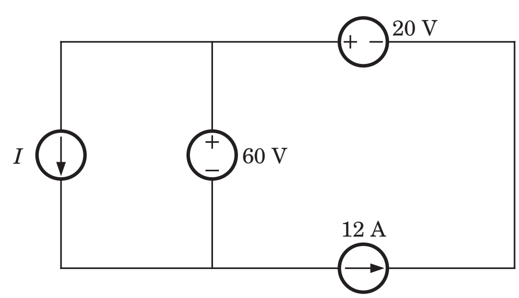
13. If $f(z) = c_0 + c_1 z^{-1}$, then $\oint_{\text{unit circle}} \frac{1 + f(z)}{z} dz$ is given by

(A) $2\pi c_1$ (B) $2\pi(1 + c_0)$

(C) $2\pi j c_1$ (D) $2\pi j(1 + c_0)$

14. In the interconnection of ideal sources shown in the figure, it is known that the 60 V source is absorbing power.

Which of the following can be the value of the current source I ?



(A) 10 A (B) 13 A

(C) 15 A (D) 18 A

15. The ratio of the mobility to the diffusion coefficient in a semiconductor has the units

(A) V^{-1} (B) cm.V^{-1}

(C) V.cm^{-1} (D) V.s

16. In a microprocessor, the service routine for a certain interrupt starts from a fixed location of memory which cannot be externally set, but the interrupt can be delayed or rejected Such an interrupt is

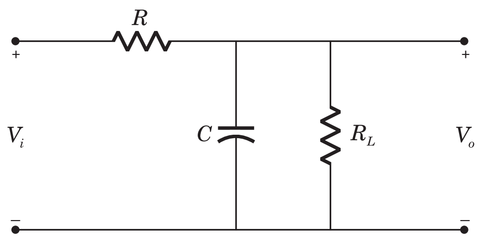
(A) non-maskable and non-vectored

(B) maskable and non-vectored

(C) non-maskable and vectored

(D) maskable and vectored

17. If the transfer function of the following network is $\frac{V_0(s)}{V_i(s)} = \frac{1}{2 + sCR}$ the value of the load resistance R_L is



(A) R/4 (B) R/2

(C) R (D) 2 R

18. Consider the system $\frac{dx}{dt} = Ax + Bu$ with $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix}$ where p and q are arbitrary real numbers. Which of the following statements about the controllability of the system is true ?

(A) The system is completely state controllable for any nonzero values of p and q

(B) Only $p = 0$ and $q = 0$ result in controllability

(C) The system is uncontrollable for all values of p and q

(D) We cannot conclude about controllability from the given data

19. For a message signal $m(t) = \cos(2\pi f_m t)$ and carrier of frequency f_c, which of the following represents a single side-band (SSB) signal ?

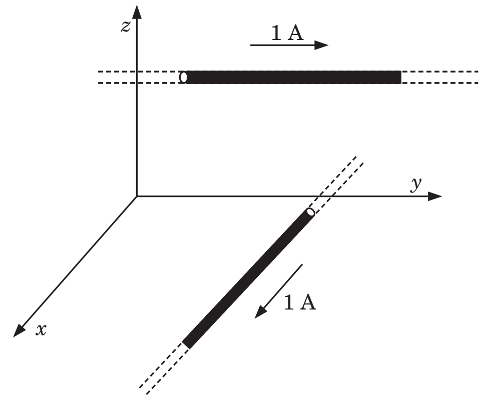
(A) $\cos(2\pi f_m t)\cos(2\pi f_c t)$

(B) $\cos(2\pi f_c t)$

(C) $\cos[2\pi(f_c + f_m)t]$

(D) $[1 + \cos(2\pi f_m t)\cos(2\pi f_c t)]$

20. Two infinitely long wires carrying current are as shown in the figure below. One wire is in the $y - z$ plane and parallel to the y –axis. The other wire is in the $x - y$ plane and parallel to the x –axis. Which components of the resulting magnetic field are non-zero at the origin ?



(A) x, y, z components (B) x, y components

(C) y, z components (D) x, z components

Q. 21 to Q. 60 carry two marks each.

21. Consider two independent random variables X and Y with identical distributions. The variables X and Y take values 0, 1 and 2 with probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$ respectively. What is the conditional probability $P(X + Y = 2 | X - Y = 0)$?

(A) 0 (B) $\frac{1}{16}$

(C) $\frac{1}{6}$ (D) 1

22. The Taylor series expansion of $\frac{\sin x}{x - \pi}$ at $x = \pi$ is given by

(A) $1 + \frac{(x-\pi)^2}{3!} + \ldots$ (B) $-1 - \frac{(x-\pi)^2}{3!} + \ldots$

(C) $1 - \frac{(x-\pi)^2}{3!} + \ldots$ (D) $-1 + \frac{(x-\pi)^2}{3!} + \ldots$

23. If a vector field $\vec{V}$ is related to another vector field $\vec{A}$ through $\vec{V} = \nabla \times \vec{A}$, which of the following is true? Note : C and S_C refer to any closed contour and any surface whose boundary is C.

(A) $\oint_C \vec{V}.\vec{dl} = \iint_{S_C} \vec{A}.\vec{dS}$ (B) $\oint_C \vec{A}.\vec{dl} = \iint_{S_C} \vec{V}.\vec{dS}$

(C) $\oint_C \nabla \times \vec{V}.\vec{dl} = \iint_{S_C} \nabla \times \vec{A}.\vec{dS}$ (D) $\oint_C \nabla \times \vec{A}.\vec{dl} = \iint_{S_C} \vec{V}.\vec{dS}$

24. Given that $F(s)$ is the one-sided Laplace transform of $f(t)$, the Laplace transform of $\int_0^t f(\tau)d\tau$ is

(A) $sF(s) - f(0)$ (B) $\frac{1}{s}F(s)$

(C) $\int_0^s F(\tau)d\tau$ (D) $\frac{1}{s}[F(s) - f(0)]$

25. Match each differential equation in Group I to its family of solution curves from Group II.

Group 1	Group II
P. $\frac{dy}{dx} = \frac{y}{x}$	1. Circles
Q. $\frac{dy}{dx} = -\frac{y}{x}$	2. Straight lines
R. $\frac{dy}{dx} = \frac{x}{y}$	3. Hyperbolas
S. $\frac{dy}{dx} = -\frac{x}{y}$	

(A) $P-2, Q-3, R-3, S-1$

(B) $P-1, Q-3, R-2, S-1$

(C) $P-2, Q-1, R-3, S-3$

(D) $P-3, Q-2, R-1, S-2$

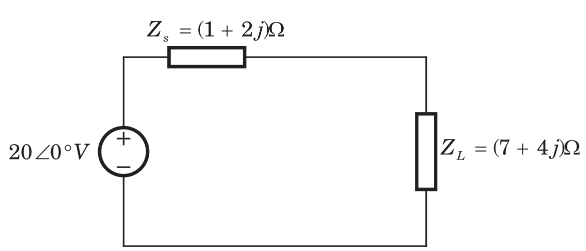
26. The eigen values of the following matrix are

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(A) $3,\ 3+5j,\ 6-j$ (B) $-6+5j,\ 3+j,\ 3-j$

(C) $3+j,\ 3-j,\ 5+j$ (D) $3,\ -1+3j,\ -1-3j$

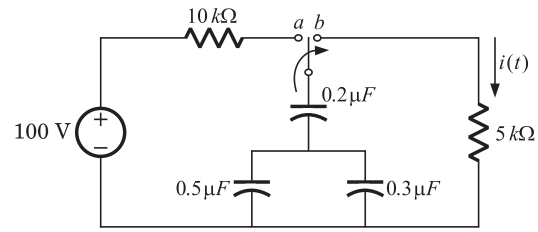
27. An AC source of RMS voltage 20 V with internal impedance $Z_s = (1+2j)\Omega$ feeds a load of impedance $Z_L = (7+4j)\Omega$ in the figure below. The reactive power consumed by the load is



(A) 8 VAR (B) 16 VAR

(C) 28 VAR (D) 32 VAR

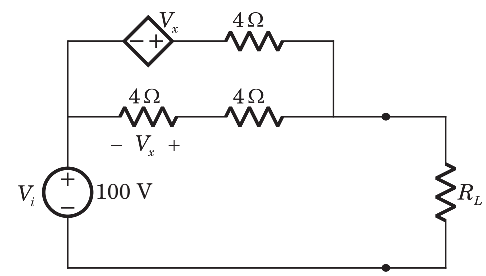
28. The switch in the circuit shown was on position a for a long time, and is move to position b at time $t = 0$. The current $i(t)$ for $t > 0$ is given by



(A) $0.2e^{-125t}u(t)$ mA (B) $20e^{-1250t}u(t)$ mA

(C) $0.2e^{-1250t}u(t)$ mA (D) $20e^{-1000t}u(t)$ mA

29. In the circuit shown, what value of R_L maximizes the power delivered to R_L ?



(A) 2.4 Ω (B) $\frac{8}{3}\Omega$

(C) 4 Ω (D) 6 Ω

30. The time domain behavior of an RL circuit is represented by

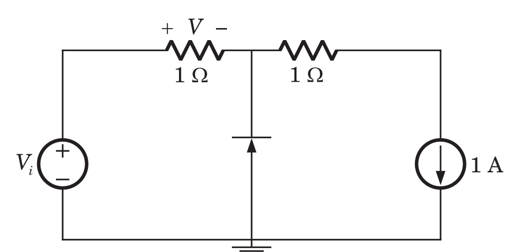
$$L\frac{di}{dt} + Ri = V_0(1 + Be^{-Rt/L}\sin t)u(t).$$

For an initial current of $i(0) = \frac{V_0}{R}$, the steady state value of the current is given by

(A) $i(t) \to \frac{V_0}{R}$ (B) $i(t) \to \frac{2V_0}{R}$

(C) $i(t) \to \frac{V_0}{R}(1+B)$ (D) $i(t) \to \frac{2V_0}{R}(1+B)$

31. In the circuit below, the diode is ideal. The voltage V is given by



(A) $\min(V_i, 1)$ (B) $\max(V_i, 1)$

(C) $\min(-V_i, 1)$ (C) $\max(-V_i, 1)$

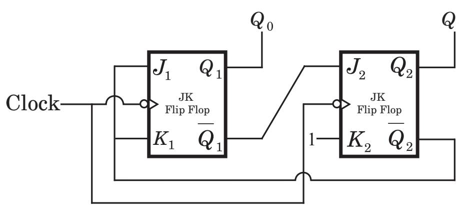
(A) NAND: first (0,1) then (0,1) NOR: first (1,0) then (0,0)

(B) NAND: first (1,0) then (1,0) NOR: first (1,0) then (1,0)

(C) NAND: first (1,0) then (1,0) NOR: first (1,0) then (0,0)

(D) NAND: first (1,0) then (1,1) NOR: first (0,1) then (0,1)

39. What are the counting states (Q_1, Q_2) for the counter shown in the figure below?



(A) 11, 10, 00, 11, 10,...   (B) 01, 10, 11, 00, 01,...

(C) 00, 11, 01, 10, 00,...   (D) 01, 10, 00, 01, 10,...

40. A system with transfer function $H(z)$ has impulse response $h(.)$ defined as $h(2) = 1$, $h(3) = -1$ and $h(k) = 0$ otherwise. Consider the following statements.

S1: $H(z)$ is a low-pass filter.

S2: $H(z)$ is an FIR filter.

Which of the following is correct?

(A) Only S2 is true

(B) Both S1 and S2 are false

(C) Both S1 and S2 are true, and S2 is a reason for S1

(D) Both S1 and S2 are true, but S2 is not a reason for S1

41. Consider a system whose input x and output y are related by the equation

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)d\tau$$

where $h(t)$ is shown in the graph.

Which of the following four properties are possessed by the system ?

BIBO: Bounded input gives a bounded output.

Causal: The system is causal,

LP: The system is low pass.

LTI: The system is linear and time-invariant.

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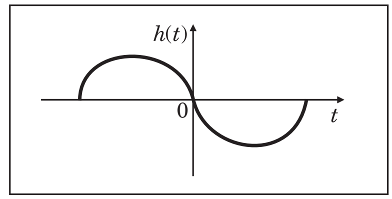


A) Causal, LP   (B) BIBO, LTI

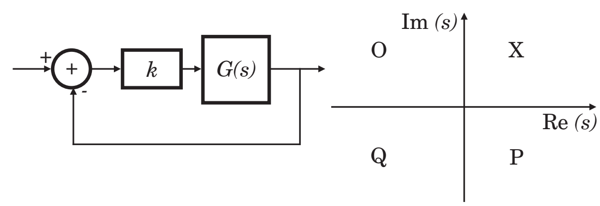
(C) BIBO, Causal, LTI   (D) LP, LTI

42. The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence {1,0,2,3} is

(A) [0,-2+2j,2,-2-2j]   (B) [2,2+2j,6,2-2j]

(C) [6,1-3j,2,1+3j]   (D) [6-1+3j,0,-1,-3j]

43. The feedback configuration and the pole-zero locations of $G(s) = \frac{s^2 - 2s + 2}{s^2 + 2s + 2}$ are shown below. The root locus for negative values of k, i.e. for $-\infty < k < 0$, has breakaway/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to



(A) $\pm\sqrt{2}$ and $0°$ (B) $\pm\sqrt{2}$ and $45°$

(C) $\pm\sqrt{3}$ and $0°$ (D) $\pm\sqrt{3}$ and $45°$

44. An LTI system having transfer function $\frac{s^2 + 1}{s^2 + 2s + 1}$ and input $x(t) = \sin(t + 1)$ is in steady state. The output is sampled at a rate ω_s rad/s to obtain the final output $\{x(k)\}$. Which of the following is true?

(A) $y(.)$ is zero for all sampling frequencies ω_s

(B) $y(.)$ is nonzero for all sampling frequencies ω_s

(C) $y(.)$ is nonzero for $\omega_s > 2$, but zero for $\omega_s < 2$

(D) $y(.)$ is zero for $\omega_s > 2$, but nonzero for $\omega_2 < 2$

45. The unit step response of an under-damped second order system has steady state value of -2. Which one of the following transfer functions has these properties ?

(A) $\frac{-2.24}{s^2 + 2.59s + 1.12}$ (B) $\frac{-3.82}{s^2 + 1.91s + 1.91}$

(C) $\frac{-2.24}{s^2 - 2.59s + 1.12}$ (D) $\frac{-382}{s^2 - 1.91s + 1.91}$

46. A discrete random variable X takes values from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance of X as 1.5. Which of the following statements is true ?

k	1	2	3	4	5
$P(X=k)$	0.1	0.2	0.4	0.2	0.1

(A) Both the student and the teacher are right

(B) Both the student and the teacher are wrong

(C) The student is wrong but the teacher is right

(D) The student is right but the teacher is wrong

Q. 47 A message signal given by

$$1n(t)=\left(\frac{1}{2}\right)\cos \omega_1 t-\left(\frac{1}{2}\right)\sin \omega_2 t$$

is amplitude-modulated with a carrier of frequency ω_c to generate

$$s(t)=[1+m(t)]\cos \omega_c t$$

What is the power efficiency achieved by this modulation scheme ?

(A) 8.33% (B) 11.11%

(C) 20% (D) 25%

48. A communication channel with AWGN operating at a signal to noise ratio SNR >> 1 and bandwidth B has capacity C_1. If the SNR is doubled keeping B constant, the resulting capacity C_2 is given by

(A) $C_2=2C_1$ (B) $C_2=C_1+B$

(C) $C_2=C_1+2B$ (D) $C_2=C_1+0.3B$

49. A magnetic field in air is measured to be

$$\vec{B}=B_0\left(\frac{x}{x^2+y^2}\hat{y}-\frac{y}{x^2+y^2}\hat{x}\right)$$

What current distribution leads to this field ?

[Hint : The algebra is trivial in cylindrical coordinates.]

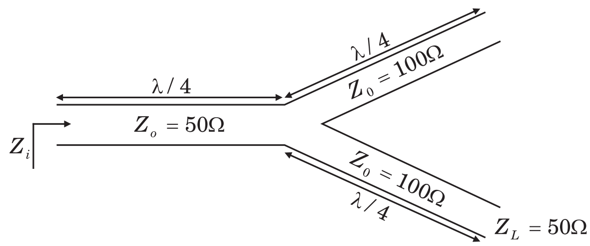
(A) $\vec{j}=\frac{B_0\hat{z}}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r\neq 0$

(B) $\vec{j}=-\frac{B_0\hat{z}}{\mu_0}\left(\frac{2}{x^2+y^2}\right), r\neq 0$

(C) $\vec{j}=0, r\neq 0$

(D) $\vec{j}=\frac{B_0\hat{z}}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r\neq 0$

50. A transmission line terminates in two branches, each of length $\lambda/4$, as shown. The branches are terminated by 50Ω loads. The lines are lossless and have the characteristic impedances shown. Determine the impedance Z_i as seen by the source.



(A) 200Ω (B) 100Ω

(C) 50Ω (D) 25Ω

Common Date Questions

Common Date for Questions 51 and 52:

Consider a silicon p-n junction at room temperature having the following parameters:

Doping on the n-side $=1\times 10^{17}$ cm^{-3}

Depletion width on the n-side $=0.1\,\mu m$

Depletion width on the p-side $=1.0\,\mu m$

Intrinsic carrier concentration $=1.4\times 10^{10}$ cm^{-3}

Thermal voltage $=26$ mV

Permittivity of free space $=8.85\times 10^{-14}$ F.cm^{-1}

Dielectric constant of silicon $=12$

51 The built-in potential of the junction

(A) is 0.70 V

(B) is 0.76 V

(C) is 0.82

(D) cannot be estimated from the data given

52. The peak electric field in the device is

(A) 0.15 MV . cm^{-1}, directed from p-region to n-region

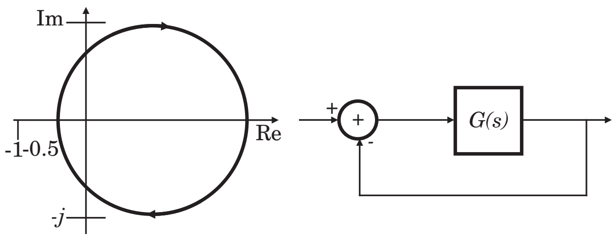
(B) 0.15 MV . cm^{-1}, directed from n-region to p-region

(C) 1.80 MV . cm^{-1}, directed from p-region to n-region

(D) 1.80 MV . cm^{-1}, directed from n-region to p-region

Common Data for Questions 53 and 54:

The Nyquist plot of a stable transfer function $G(s)$ is shown in the figure. We are interested in the stability of the closed loop system in the feedback configuration shown.



53. Which of the following statements is true?

(A) $G(s)$ is an all-pass filter

(B) $G(s)$ has a zero in the right-half plane

(C) $G(s)$ is the impedance of a passive network

(D) $G(s)$ is marginally stable

54. The gain and phase margins of $G(s)$ for closed loop stability are

(A) 6 dB and 180° (B) 3 dB and 180°

(C) 6 dB and 90° (D) 3 dB and 90°

Common Data for Questions 55 and 56:

The amplitude of a random signal is uniformly distributed between -5 V and 5 V.

55. If the signal to quantization noise ratio required in uniformly quantizing the signal is 43.5 dB, the step size of the quantization is approximately

(A) 0.0333 V (B) 0.05 V

(C) 0.0667 V (D) 0.10 V

56. If the positive values of the signal are uniformly quantized with a step size of 0.05 V, and the negative values are uniformly quantized with a step size of 0.1 V, the resulting signal to quantization noise ratio is approximately

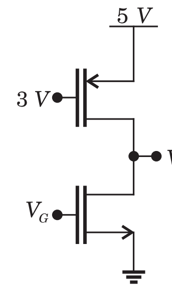
(A) 46 dB (B) 43.8 dB

(C) 42 dB (D) 40 dB

Linked Answer Questions

Statement for Linked Answer Questions 57 and 58 :

Consider the CMOS circuit shown, where the gate voltage V_G of the n-MOSFET is increased from zero, while the gate voltage of the p-MOSFET is kept constant at 3 V. Assume that, for both transistors, the magnitude of the threshold voltage is 1 V and the product of the trans-conductance parameter and the (W/L) ratio, i.e. the quantity $\mu C_{ax}(W/L)$, is 1 mA . V^{-2}.



57. For small increase in V_G beyond 1 V, which of the following gives the correct description of the region of operation of each MOSFET ?

(A) Both the MOSFETs are in saturation region

(B) Both the MOSFETs are in triode region

(C) n-MOSFET is in triode and p-MOSFET is in saturation region

(D) n-MOSFET is in saturation and p-MOSFET is in triode region.

58. Estimate the output voltage V_0 for $V_G = 1.5\,V$.
[Hint : Use the appropriate current-voltage equation for each MOSFET, based on the anser to Q. 57.]

Statement for Linked Answer Questions 59 and 60:

Two products are sold from a vending machine, which has two push buttons P_1 and P_2. When a button is pressed, the price of the corresponding product is displayed in a 7-segment display.

If no buttons are pressed, '0' is displayed, signifying 'Rs. 0'.

If only P_1 is pressed, '2' is displayed, signifying 'Rs. 2'.

If only P_2 is pressed, '5' is displayed, signifying 'Rs. 5'.

If both P_1 and P_2 are pressed, 'E' is displayed, signifying 'Error'.