OM

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ECE

PMI(B)

~~[illegible]~~

Electromagnetic Theory

Sir: Koteswara Rao

Books

- → William Hayt
- → Sadiaku
- → Edminister
- → Mahapatra & Mahapatra
- → R.F. Harrington
- → Jordan & Balmain

✩ Topics

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✓ ⊙ Static fields (Electrostatics & steady magnetics).

→ The fields are independent of time. is called static fields.

✓ ⊙ Time Varying fields

→ Maxwell Eqn.

✓ ⊙ EM Waves.

– Defn:

→ A wave is a physical phenomena which reproduces After certain instant of time get some other place, the time delay betn the prior to the later locations is proportional to travelled distance. The whole phenomena consitutes a Wave. Therefore, an EM waves is not only fn of time but also a fn of distance. Insted of distance we use space co-ordinates.

✓ ⊙ Waveguide (Rectangular)

⊙ Basics of Antennas

✓ ⊙ Two wire transmission lines.

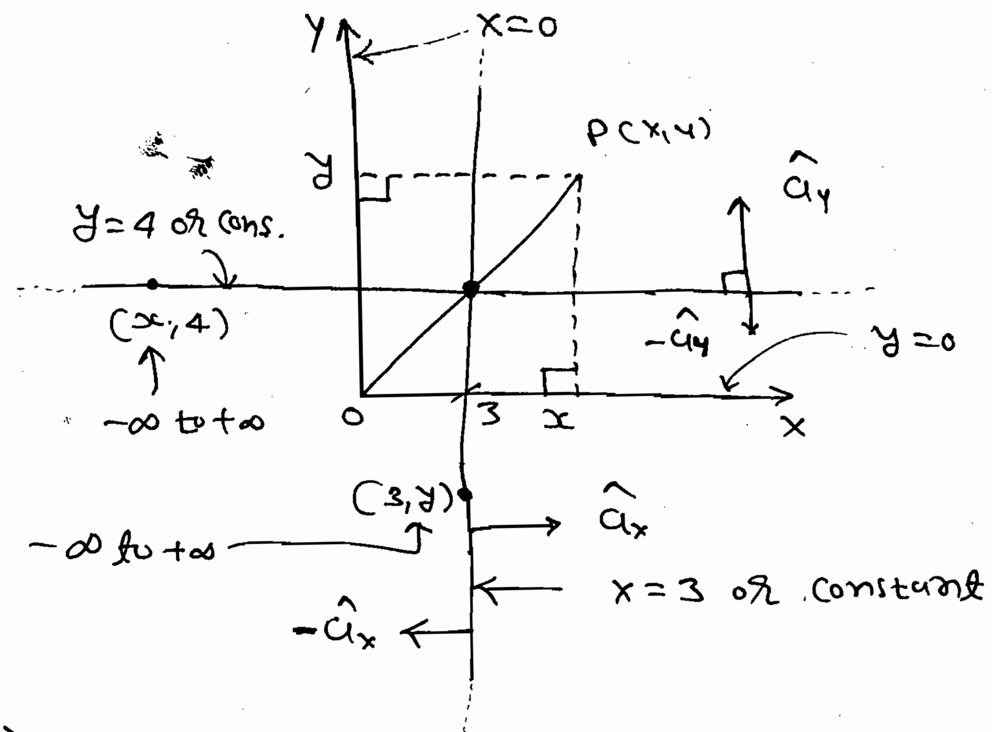
✓ ⊙ Scattering Parameters.

Coordinate Systems.

- 2-Dimensional
  - Rectangular (or) Cartesian
  - polar
- 3-dimensional
  - Rectangular or Cartesian
  - circular cylindrical
  - Spherical

* 2-Dimensional:

(i) Rectangular (or) Cartesian



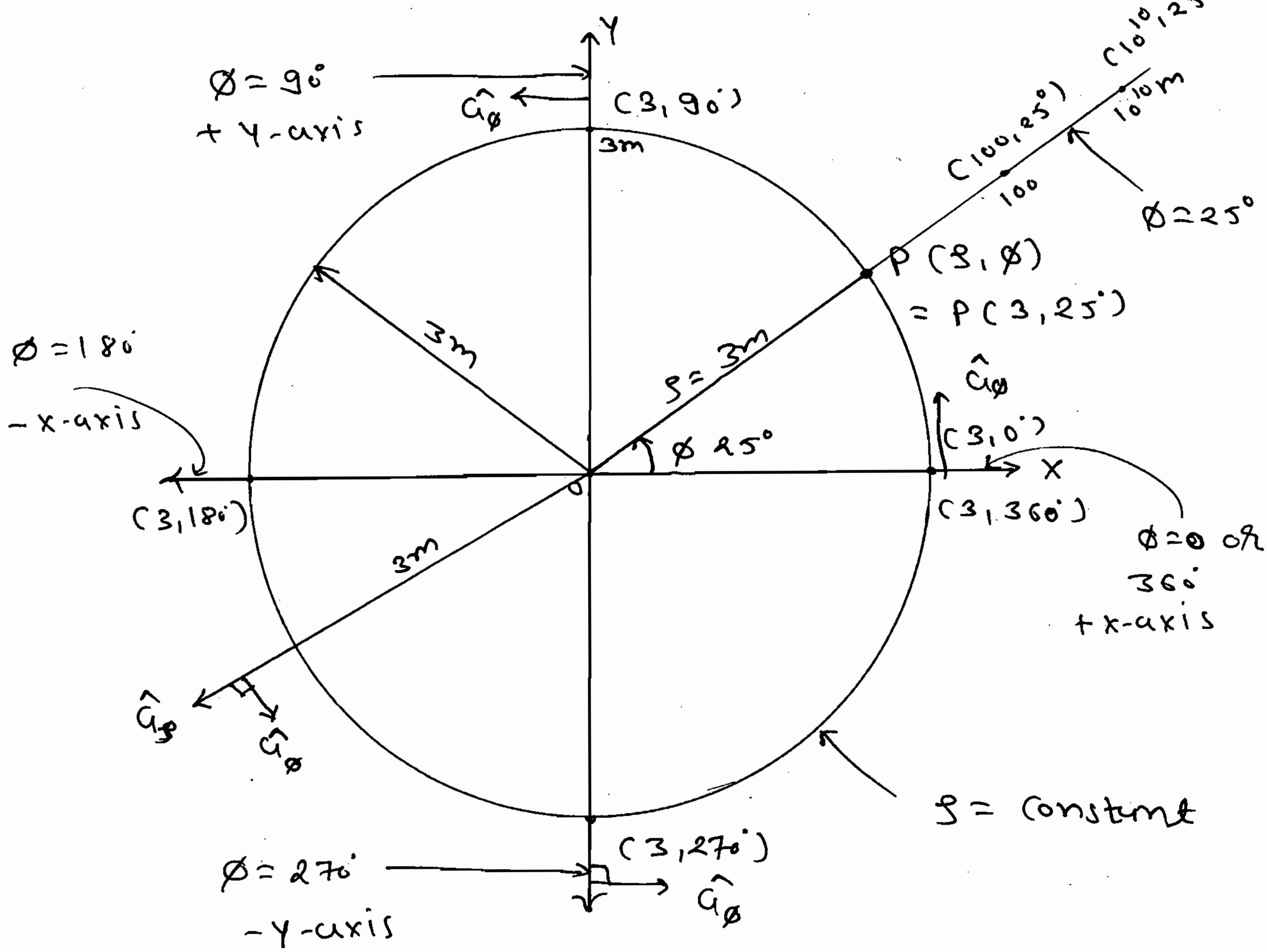
⇒ Integral	on $x = const.$	on $y = const.$
$x \to -\infty$ to $+\infty$ $y \to -\infty$ to $+\infty$	$y \to -\infty$ to $+\infty$	$x \to -\infty$ to $+\infty$.

→ $\hat{a}_x$, $\hat{a}_y$ are unit vectors and are orthogonal to each other. $\Rightarrow |\hat{a}_x| = |\hat{a}_y| = 1.$

→ They are represented along x-axis and along y-axis respectively.

→ They may be also represented as unit vectors normal to x = constant and y = constant respectively.

(ii) Polar



→ Locus of ρ = constant represents a circle whose centre coinsides should not origin. Therefore, ρ assumes all possible values ranging from 0 to ∞. All ρ = constant, φ assumess all possible values ranging from 0 to 2π.

Integrals	on $\rho = $ const.	on $\phi = $ const.
$\rho \to 0$ to ∞ $\phi \to 0$ to 2π	$\phi \to 0$ to 2π	$\rho \to 0$ to ∞.

→ Locus of $\phi = $ constant is a line emerging out from the origin. ϕ assumes all possible values ranging from 0 to 2π.

On $\phi = $ constant ρ assumes all possible values ranging from 0 to ∞.

→ $\hat{a}_\rho$ & $\hat{a}_\phi$ are unit vectors orthogonal to each other.

→ $\hat{a}_\rho$ is represented normal to $\rho = $ constant (or) normal to the circle.

→ Similarly, $\hat{a}_\phi$ is a unit vector projecting normal to $\phi = $ constant. and it is projecting in the counter clock wise direction as shown in figure.

→ $\hat{a}_\rho$ is normal to the circle and $\hat{a}_\phi$ is tangent to the circle.

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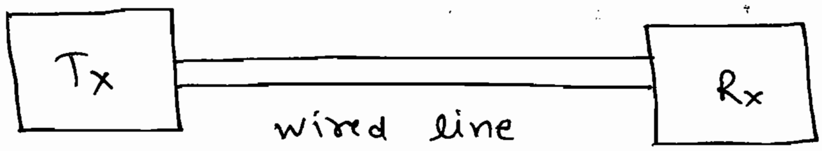

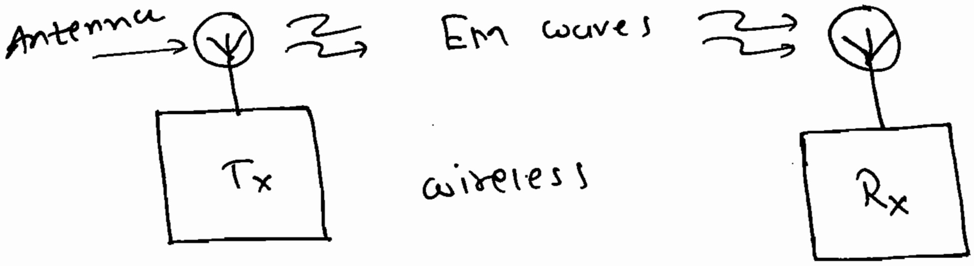

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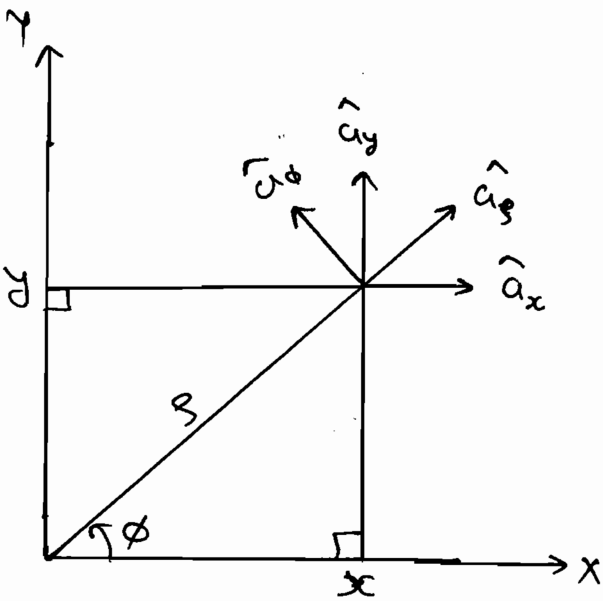
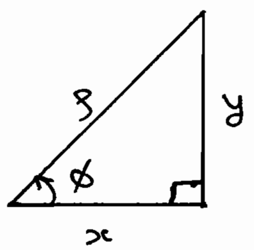
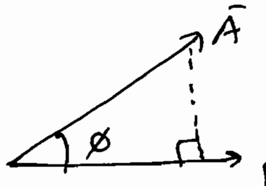






$\rightarrow y = \rho \sin\phi$

$x = \rho \cos\phi$

$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}(y/x)$

$\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}| \cdot \cos\phi$

$= AB$

if $\phi = 0^\circ$.

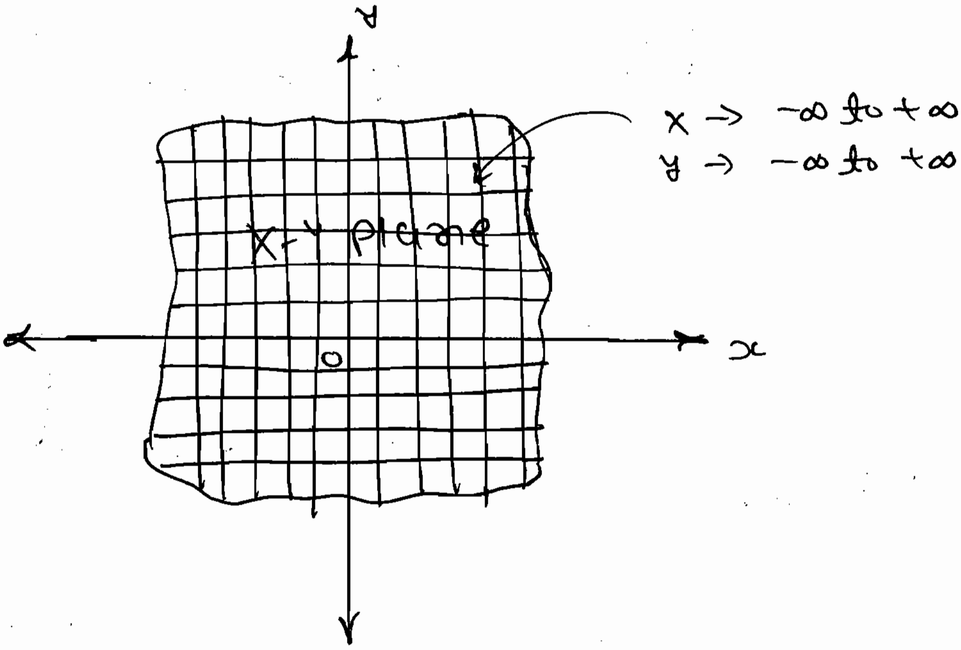
*

$\cdot$	$\hat{a}_\rho$	$\hat{a}_\phi$	$\hat{a}_z$
$\hat{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\hat{a}_y$	$\sin\phi$	$\cos\phi$	0
$\hat{a}_z$	0	0	1

*

* Plane:

→ Plane is a sheet like structure whose thickness is neglected.



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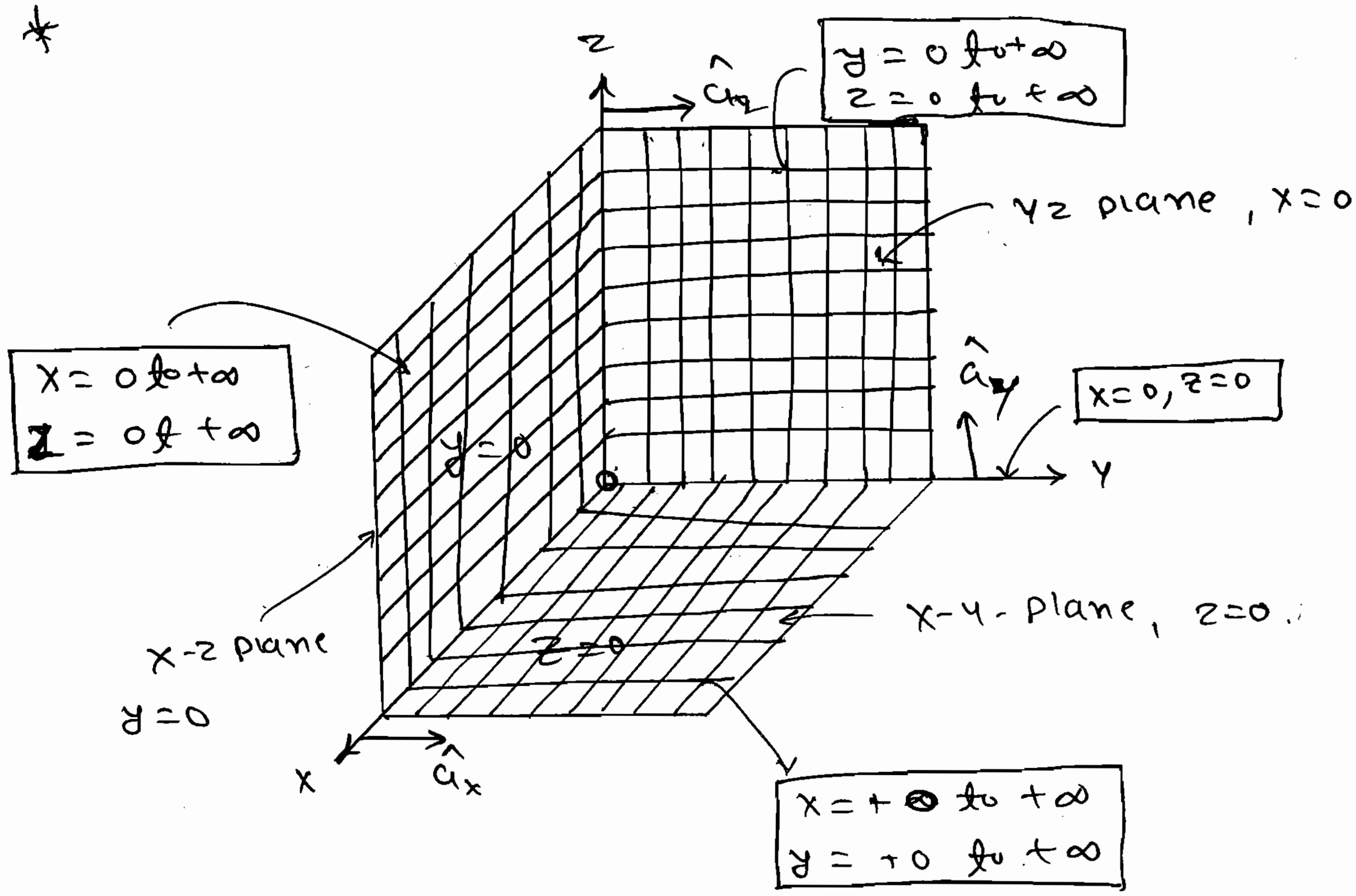
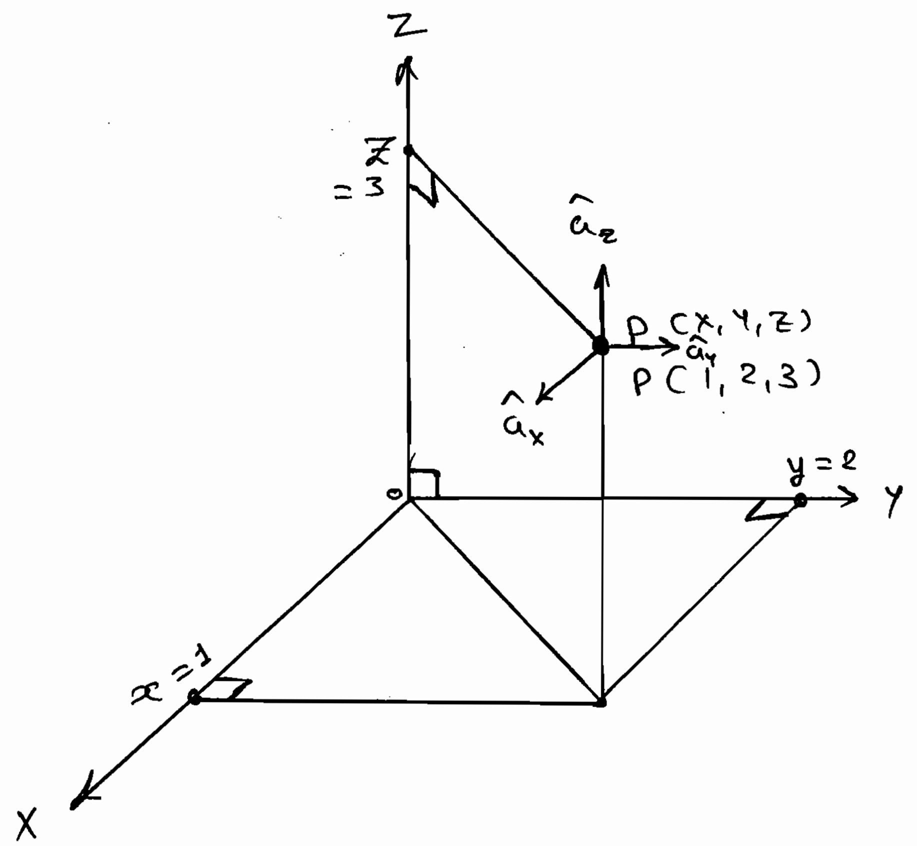


* Cartesian coordinates system



→ In general in a 3-D co-ordinates system fixing 3 coordinates that represents a point

→ fixing 2 coordinates that represents a line

→ fixing 1 co-ordinates that represents a plane.

⇒ In general

$x \rightarrow -\infty$ to $+\infty$

$y \rightarrow -\infty$ to $+\infty$

$z \rightarrow -\infty$ to $+\infty$

⇒ $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ are unit vectors and orthogonal to each other.

→ $|\hat{a}_x| = |\hat{a}_y| = |\hat{a}_z| = 1.$

⇒ $\hat{a}_x$, $\hat{a}_y$, and $\hat{a}_z$ are unit vectors and orthogonal to each other.

→ They are represented along x-axis, y-axis and z-axis.

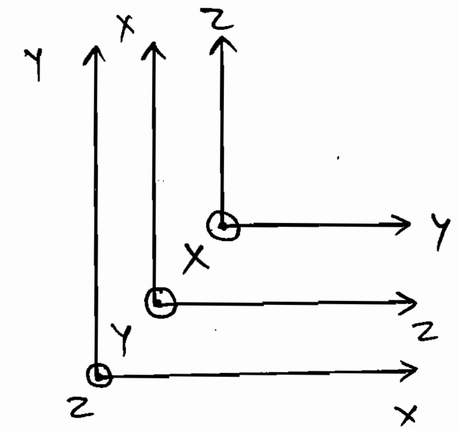
→ They may be also represented as unit vectors normal to $x =$ constant, $y =$ constant, $z =$ constant planes respectively.

* ⊙ vector/~~x~~-axis coming out of the paper

* $\boxed{\odot}$ vector/axis coming out of the paper.

* circular or cylindrical coordinate system.

⇒



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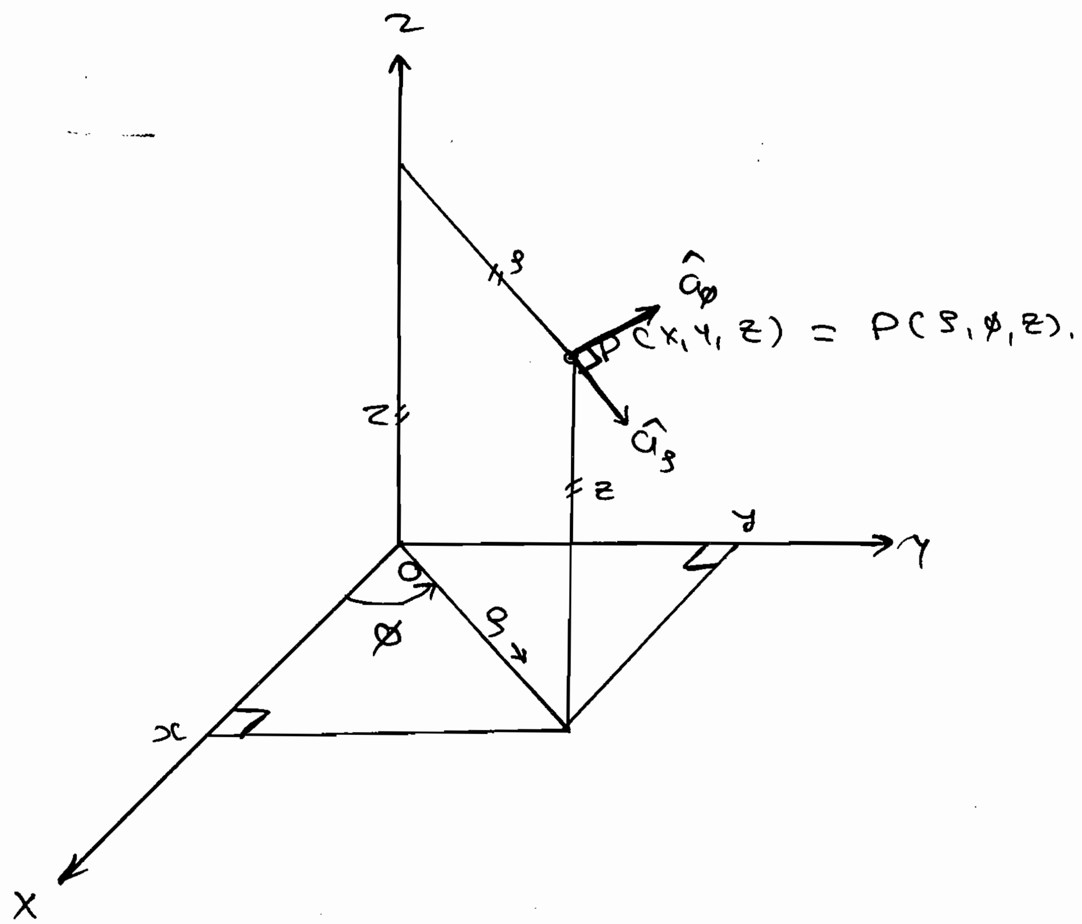
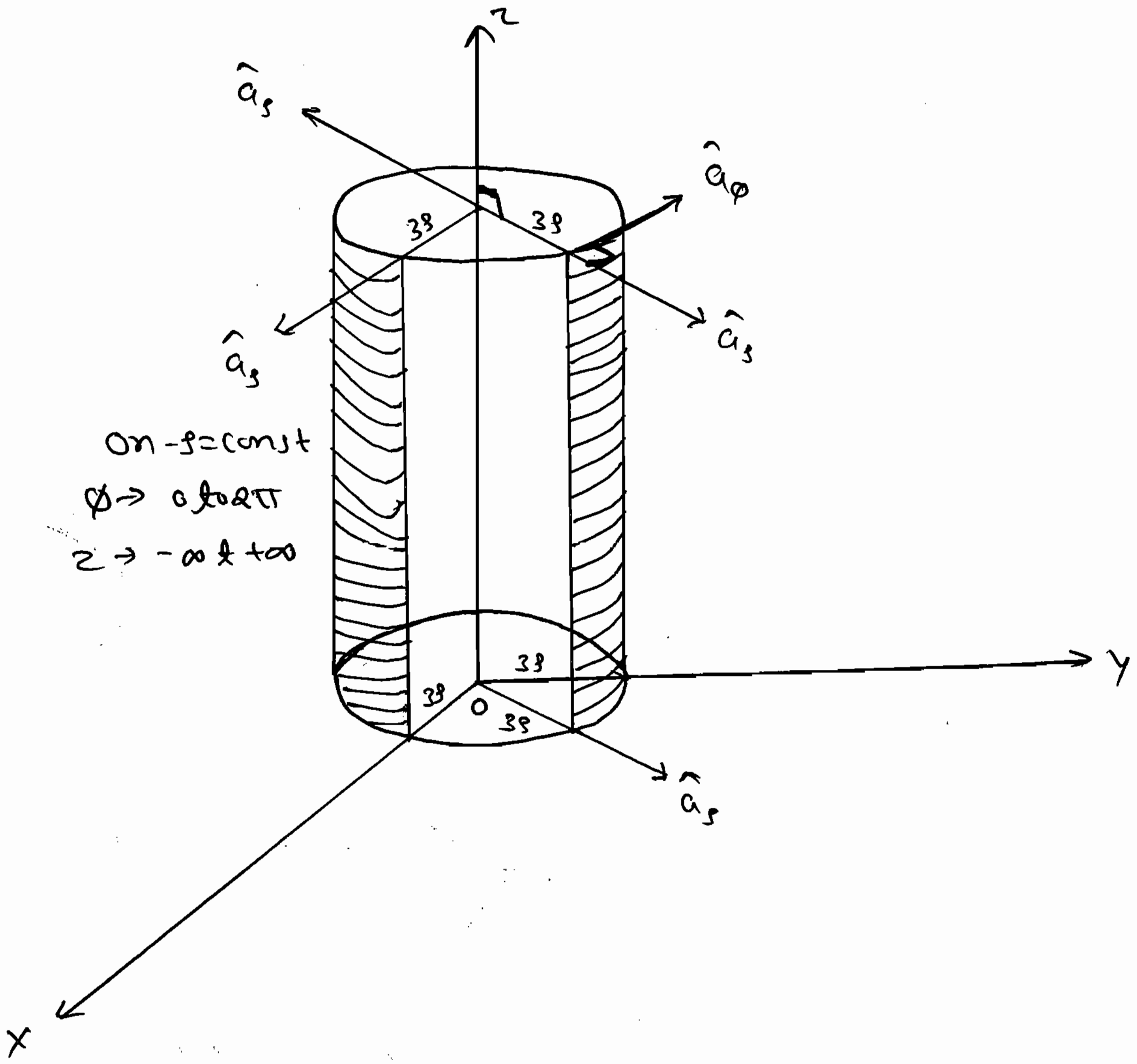


→ ρ = constant represents a cylindrical plane whose cross-section is circular and whose axis coincides with z-axis rather ρ is the distance measured normal to z-axis. ρ can assume all possible values ranging from 0 to +∞.

→ $\hat{a}_\rho$ is a unit vector projecting normal to ρ = constant plane.

⇛



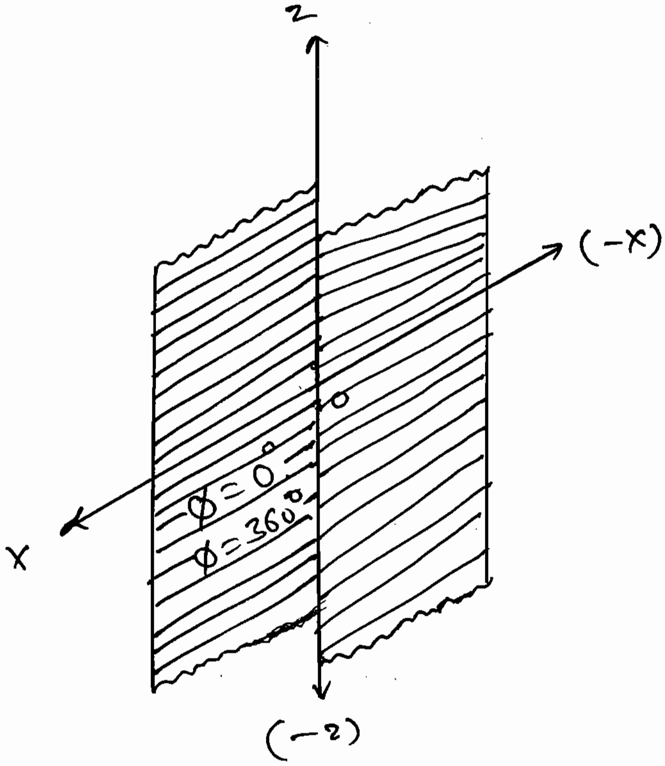
→ ϕ is called azimuth angle.

→ ϕ = constant plane is called elivation plane.

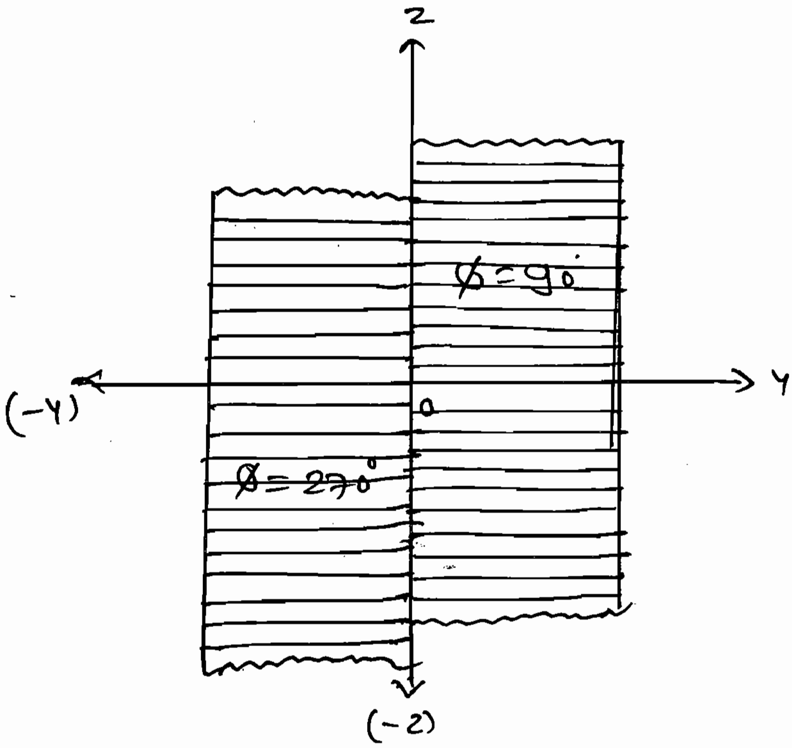
→ ϕ assumes all posibble values ranging from 0 to 2π.

→ $\hat{a}_\phi$ is a unit vector projecting normal to ϕ= constant plane.

$\Rightarrow$



$\Rightarrow$



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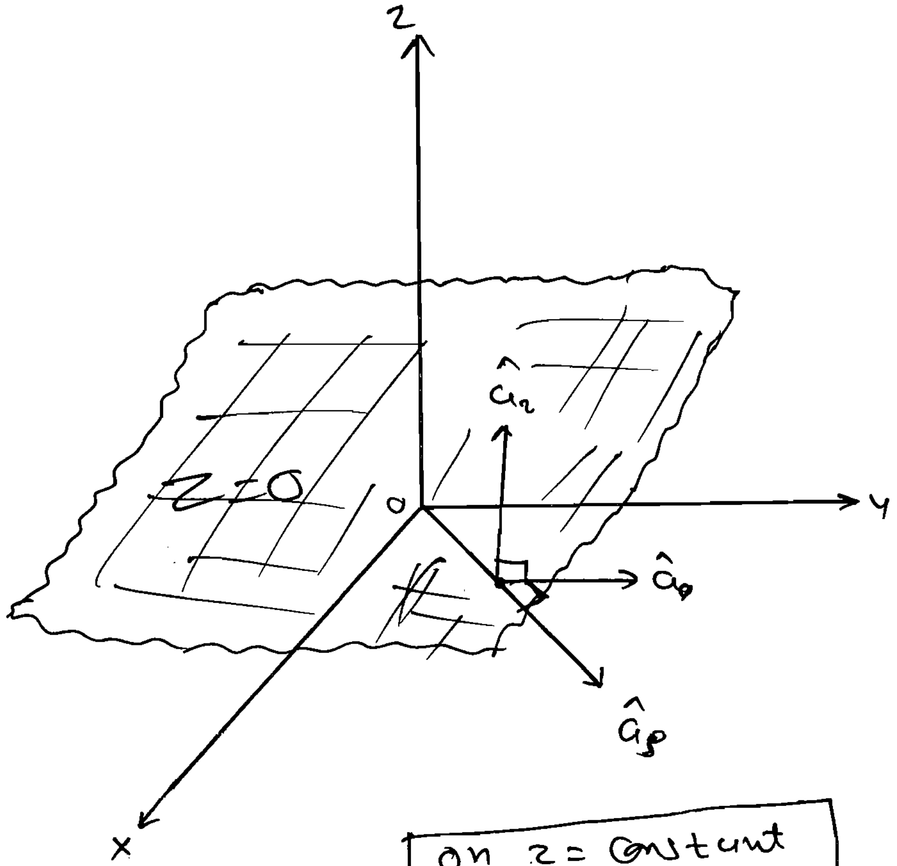


on $z =$ constant
$\rho \rightarrow 0$ to ∞
$\phi \rightarrow 0$ to 2π

→ on a perticular constant plane that perticular, unit vector would projecting normal to the plane the remaining 2 unit vectors could be projecting tangential to the plane.

for. e.g on $\rho =$ constant plane $\hat{a}_\rho$ could be Projecting normal to $\rho =$ constant and $\hat{a}_\phi$ & $\hat{a}_z$ could be projecting tangentical to the plane.

In general

$\rho \Rightarrow 0$ to ∞

$\phi \rightarrow 0$ to 2π

$z \rightarrow -\infty$ to $+\infty$.

→ $\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$ are unit vectors and orthogonal to each other.

⇒ In general,

$$\boxed{\begin{aligned} \vec{B} &= B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z \quad \rightarrow \text{cartesian} \\ \vec{B} &= B_\rho \hat{a}_\rho + B_\phi \hat{a}_\phi + B_z \hat{a}_z . \quad \rightarrow \text{cylindrical} \end{aligned}}$$

Ex-1 Let $\bar{B} = 2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z$ is define at a point $P(3,4,5)$ m. convert this vector in cylindrical system.

Ans:

$$\bar{B} = B_\rho \hat{a}_\rho + B_\phi \hat{a}_\phi + B_z \hat{a}_z$$

$$\therefore \bar{B} \cdot \hat{a}_\rho = B_\rho \cdot \underbrace{\hat{a}_\rho \cdot \hat{a}_\rho}_{1} + B_\phi \underbrace{\hat{a}_\phi \cdot \hat{a}_\rho}_{0} + B_z \underbrace{\hat{a}_z \cdot \hat{a}_\rho}_{0}.$$

$$\therefore \bar{B} \cdot \hat{a}_\rho = B_\rho .$$

$$\therefore B_\rho = \bar{B} \cdot \hat{a}_\rho$$

$$\therefore B_\rho = 2\hat{a}_x \cdot \hat{a}_\rho + 3\hat{a}_y \cdot \hat{a}_\rho + 4\hat{a}_z \cdot \hat{a}_\rho$$

$$B_\rho = 2\cos\phi + 3\sin\phi + 0$$

Now, $P(3,4,5)$.

$$\therefore \rho = \sqrt{3^2+4^2} = 5$$

$$\phi = \tan^{-1}\left(\frac{4}{3}\right) = 53.130^\circ$$

$\therefore B_\rho = 2\cos 53.13^\circ + 3\sin 53.13^\circ$

$\therefore \boxed{B_\rho = 3.6}$

similarly,

$\therefore B_\phi = \bar{B} \cdot \hat{a}_\phi , \quad B_z = \bar{B} \cdot \hat{a}_z$

$\therefore B_\phi = -2\sin\phi + 3\cos\phi.$

$\therefore \boxed{B_\phi = 0.2}$

$\therefore B_z = \bar{B} \cdot \hat{a}_z$

$\therefore B_z = B_z$

$\therefore \boxed{B_z = 4}$

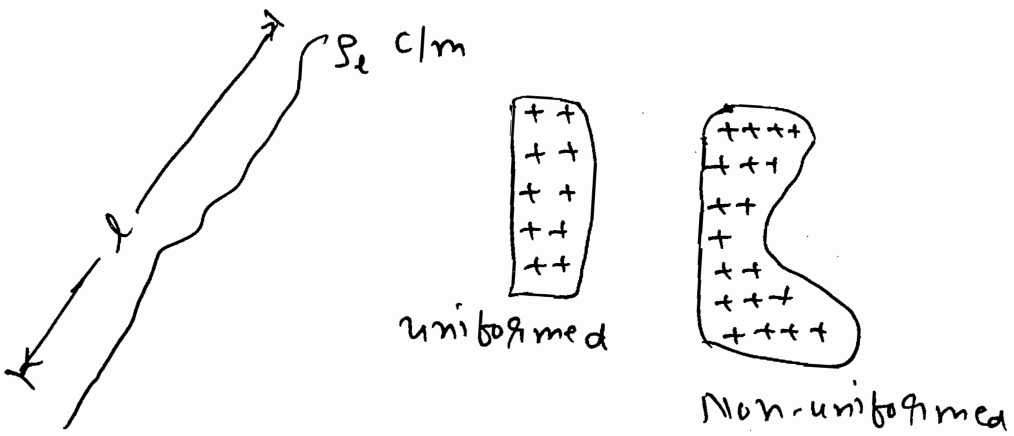
$\therefore \boxed{\bar{B} = 3.6\,\hat{a}_\rho + 0.2\,\hat{a}_\phi + 4\,\hat{a}_z}$

Charge Configuration/Classification

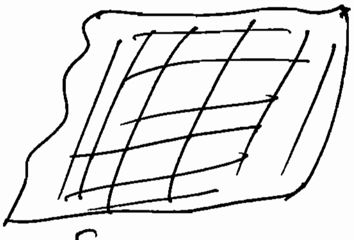
- Point charge 'Q' c.
  (The charge combined to a point is called point charge).
- Charge Distribution
  - Line ρ_ℓ
  - Surface ρ_s
  - Volume ρ_v
    - Uniform
    - Non-uniform

→ Distribution of charge per unit line is called line charge distribution it and is designated by ρ_ℓ. If ρ_ℓ is constant that may be called uniform otherwise non uniform.

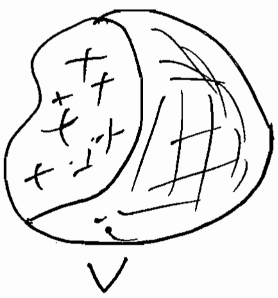
→ Having uniform charge distributions are impractical. The reason is due to mutual repulsion bet^n the like charges.



→ Distribution of charge per unit area is called surface charge density. and is designated by ρ_s C/m²



→ Distribution of charge per unit volume is called volume charge density and is designated by ρ_v C/m³.



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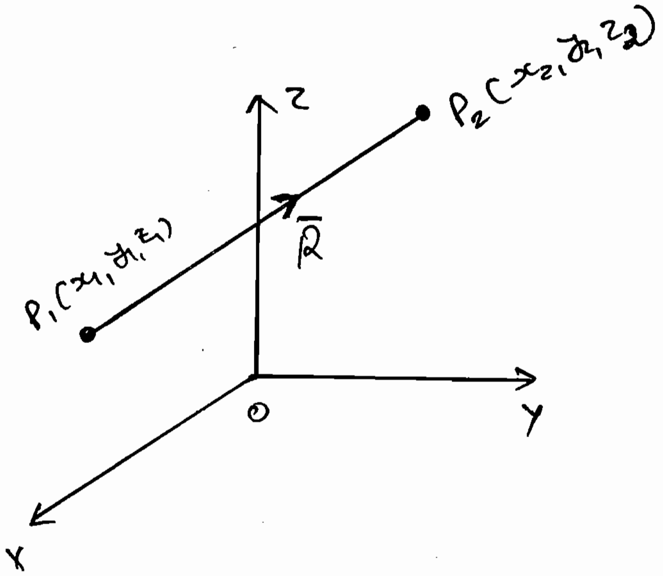


→ R is vector drawn from P_1 to P_2.

$$\therefore \bar{R} = (x_2-x_1)\hat{a}_x + (y_2-y_1)\hat{a}_y + (z_2-z_1)\hat{a}_z$$

$$\therefore |\bar{R}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|}$$ $\bar{a}_R$ = unit vector components of R.

$$|\hat{a}_R| = \frac{|\bar{R}|}{|\bar{R}|} = 1.$$

* ~~Newton's Force Law~~ Columb's ~~Force~~ Law:

→ it gives force of attraction (or) repulsion bet^n charge conducting bodies.

→ if the charges are like the force is repulsive otherwise attractive.

→ Capacitivity (or) permitivity specifies property of a medium and that indicates ability to store electrical energy.

$$F \propto Q_1 Q_2$$

$$\propto \frac{1}{|\bar{R}|^2}$$

$\epsilon = \epsilon_0 \epsilon_r$ F/m

$$\therefore \boxed{|\bar{F}| = \frac{Q_1 Q_2}{4\pi\epsilon |\bar{R}|^2} \text{ N}}$$

→ $\epsilon = \epsilon_0 \epsilon_r$ F/m

ϵ : Permitivity (or) capacitivity

ϵ_0 : Absolute permitivity → 8.854×10^{-12} F/m

$$= \frac{10^{-9}}{36\pi} \text{ F/m.}$$

ϵ_r = relative permitivity

(ⓔ) Dielectric constant (ϵ_r) has no unit.

→ The vector force acting on Q_2 due Q_1 along is given by.

$$\therefore \quad \bar{F} = \frac{Q_1 \cdot Q_2}{4\pi\epsilon|\bar{R}|^2} \cdot \hat{a}_R$$

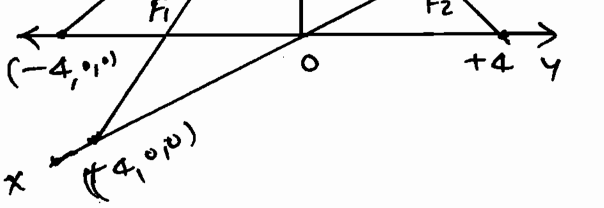
$$\boxed{\bar{F} = \frac{Q_1 \cdot Q_2}{4\pi\epsilon|\bar{R}|^2} \cdot \frac{\bar{R}}{|\bar{R}|} \cdot \text{'N'}}$$

→ The vector force acting on Q_1 due to Q_2 is $-F$.

further we write $\boxed{|\bar{F}| = |-\bar{F}|}$

Ex-1 Four point charges of 10 μC each one located on the x & y axis at ±4m. Find the vector force acting on 1 mC charge which is located on z-axis at z = 3 meters.

Ans:



$$\bar{F}_1 = \frac{10\times10^{-6}\times10^{-3}}{4\pi\times\frac{10^{-9}}{36\pi}\times(5)^2}\times\frac{-4\hat{a}_x+3\hat{a}_z}{(5)}$$

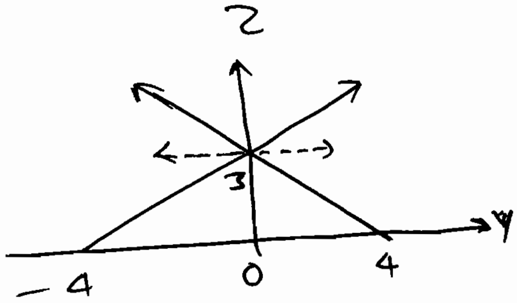
$$\bar{F}_2 = \frac{10\times10^{-6}\times10^{-3}}{4\pi\times\frac{10^{-9}}{36\pi}\times(5)^2}\times\frac{-4\hat{a}_y+3\bar{a}_z}{(5)}$$

$$\bar{F}_3 = \frac{10\times10^{-6}\times10^{-3}}{4\pi\times\frac{10^{-9}}{36\pi}\times(5)^2}\times\frac{+4\hat{a}_x+3\hat{a}_z}{(5)}$$

$$F_4 = \frac{10\times10^{-6}\times10^{-3}}{4\pi\times\frac{10^{-9}}{36\pi}\times(5)^2}\times\frac{+4\bar{a}_y+3\bar{a}_z}{(5)}$$

$$\therefore \bar{F} = \bar{F}_1+\bar{F}_2+\bar{F}_3+\bar{F}_4$$

$$\boxed{\bar{F} = 8.64\,\hat{a}_z\ N}$$

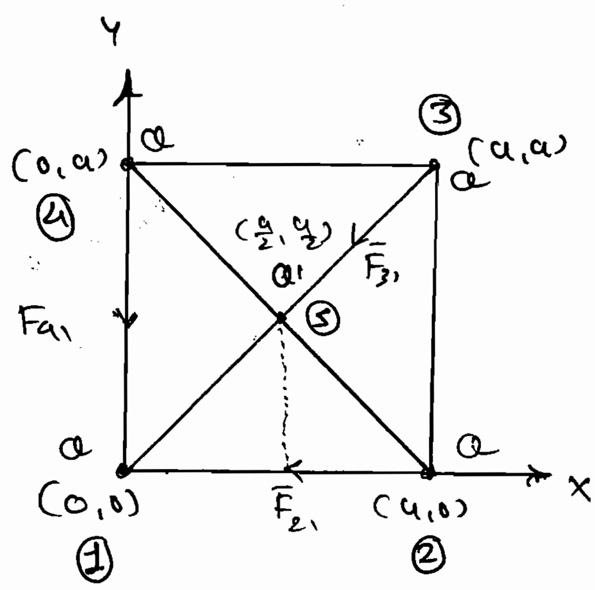


→ The 10 μC charges are located symmetrically on the x & y axis about z-axis which results in cancelation of horizontal force components and the resultant force would be along $\hat{a}_z$ direction only.

Ex-2 4 point charges of +Q each are located at the corners of a square. What point charge to be kept at a centre of a square so that the resultant force acting on an each charge which are located at a corners of a square is zero. (OR) it is required to hold 4 point charges of +Q each in equilibrium at corners of a square. What point charge to be kept at the centre of the square so that the charges would be in equilibrium

Hint: equilibrium means the resultant force acting on any charge which are located at the corners of the square is zero.

Ans:



→ We have to find value of Q' in terms of Q so that resultant force acting on any charge which are located at the corners of the square is zero.

for e.g.

Let's find force on 1

$\therefore \quad \overline{F_{21}} + \overline{F_{31}} + \overline{F_{41}} + \overline{F_{51}} = 0.$

$$\Rightarrow \quad \overline{F_{21}} = \frac{Q^2}{4\pi\epsilon(a^2)} \times \frac{-a\hat{a}_x}{a}$$

$$\overline{F_{41}} = \frac{Q^2}{4\pi\epsilon(a^2)} \times \frac{-a\hat{a}_y}{a}$$

$$\overline{F_{31}} = \frac{Q^2}{4\pi\epsilon(\sqrt{2}a)^2} \times \frac{-a\hat{a}_x - a\hat{a}_y}{\sqrt{2}a}$$

$$\therefore \quad \overline{F_{51}} = \frac{QQ'}{4\pi\epsilon\left(\frac{a}{\sqrt{2}}\right)^2} \times \frac{-\hat{a}_x - \hat{a}_y}{\sqrt{2}.}$$

→ Consider the sum ~~and~~ of $\hat{a}_x$ components of all forces and the same make equal to 0

$$\therefore \quad \frac{-Q^2}{4\pi\epsilon a^2} - \frac{Q^2}{4\pi\epsilon\sqrt{2}a^2} - \frac{QQ'}{4\pi\epsilon\frac{a^2}{\sqrt{2}}} = 0$$

$$\therefore \quad Q' = -0.956\,Q.$$

→ Even we consider the sum of $\hat{a}_y$ components the same ans is expected.

✓

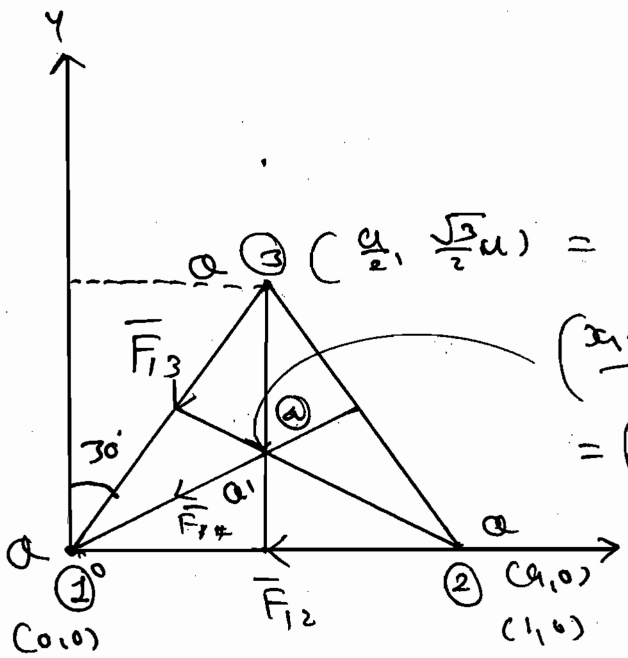
→

Ex-3 3 point charges of +Q each are located at the corners of a equilateral triangle. What point charge to be kept at the centre of the triangle so that the resultant force acting on on each charge which are located at the corners of the triangle is 0.

Ans: $Q' = -\frac{Q}{\sqrt{3}}$.

Ans:

let a = 1 unit



$$\left(\frac{a}{2}, \frac{\sqrt{3}}{2}a\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right).$$

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

$$= \left(\frac{0+\frac{a}{2}+a}{3}, \frac{0+\frac{\sqrt{3}}{2}a+0}{3}\right)$$

$$= \left(\frac{a}{2}, \frac{a}{2\sqrt{3}}\right)$$

$$\Rightarrow \bar{F}_{13} = \frac{kQ^2}{1}\left(-\frac{\hat{a}_x}{2} - \frac{\sqrt{3}}{2}\hat{a}_y\right) \quad \bar{R} = -\frac{a}{2}\hat{a}_x - \frac{\sqrt{3}}{2}a\,\hat{a}_y$$

$$\bar{R} = -\frac{\hat{a}_x}{2} - \frac{\sqrt{3}}{2}\hat{a}_y.$$

$$\Rightarrow \bar{F}_{12} = \frac{-kQ^2}{1}(\hat{a}_x). \quad \bar{R} = -\hat{a}_x$$

$$|\bar{R}| = 1.$$

$$\bar{R} = -\frac{\hat{a}_x}{2} - \frac{1}{2\sqrt{3}}\hat{a}_y$$

$$\Rightarrow \bar{F}_{14} = -\frac{kQQ'}{1/3} \times \left(\frac{+\frac{\hat{a}_x}{2} + \frac{1}{2\sqrt{3}}\hat{a}_y}{1/\sqrt{3}}\right).$$

$$|\bar{R}| = \sqrt{\frac{1}{4} + \frac{1}{12}} = \sqrt{\frac{1}{3}}$$

$$\Rightarrow \overline{F_{14}} = -kQQ' \times \frac{3}{2} \times \sqrt{3} \left(\hat{a}_x + \hat{a}_y \times \frac{1}{\sqrt{3}} \right)$$

Now, $\overline{F_{12}} + \overline{F_{13}} + \overline{F_{14}} = 0$.

let, only $\hat{a}_x$ - a component

$\therefore$

$$\therefore \; -\frac{kQ^2}{2} - kQ^2 - kQQ' \times \frac{3\sqrt{3}}{2} = 0.$$

$$\therefore \; -\frac{3}{2}Q = Q' \times \frac{3\sqrt{3}}{2}$$

$\therefore$ $\boxed{\cancel{Q' = -\frac{Q}{Q}}}$ $\boxed{Q' = \frac{-Q}{\sqrt{3}}\, C}$

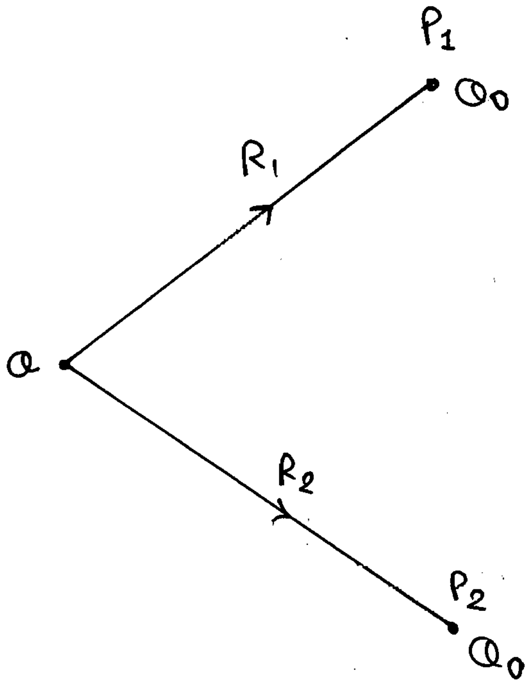
→ If we considered only y - component then we also get same answers.

* Electric Field ($\bar{E}$) :-

→ It is defined as force per unit charge.

→ Unit is N/C (or) V/m.

→



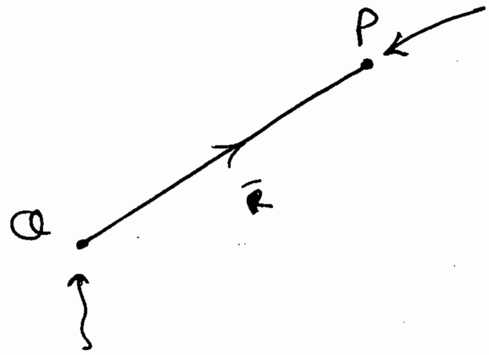
→ $$\bar{F_1} = \frac{Q \cdot Q_0}{4\pi\epsilon |\bar{R_1}|^2} \cdot \hat{a}_{R_1}$$

at P_1 ⟹ $$\frac{\bar{F_1}}{Q_0} = \frac{Q}{4\pi\epsilon |\bar{R_1}|^2} \cdot \hat{a}_{R_1} = \text{Electric field}$$

→ $$\bar{F_2} = \frac{Q \cdot Q_0}{4\pi\epsilon |\bar{R_2}|^2} \cdot \hat{a}_{R_2}$$

at P_2 ⟹ $$\frac{\bar{F_2}}{Q_0} = \frac{Q}{4\pi\epsilon |\bar{R_2}|^2} \cdot \hat{a}_{R_2} = \text{Electric field}$$

⇒ In general



Observation point where we want to find the electric field

Source point,

Source of electric field is electric charge.

$$\boxed{\begin{aligned} \bar{E} &= \frac{Q}{4\pi\epsilon|\bar{R}|^2} \cdot \hat{a}_R \\ \bar{E} &= \frac{Q}{4\pi\epsilon|\bar{R}|^2} \times \frac{\bar{R}}{|\bar{R}|} \end{aligned}}$$

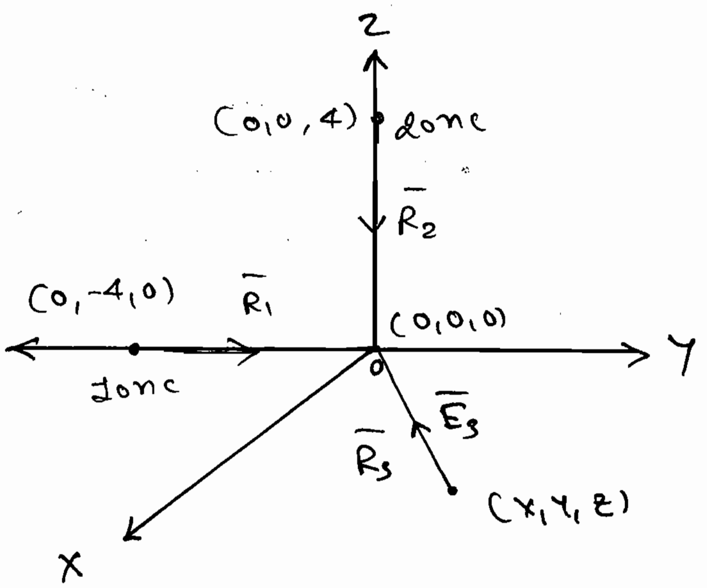
Ex-1 A point charge of 10 nC is located (0, −4, 0) m. An another charge of 20 nC is located at (0, 0, 4) m.

(i) find the electric field at the origin.

(ii) where should a 30 nC point charge to be located so that electric field at origin is 0.

Ans:

$\Rightarrow$



$$\bar{E}_1 = \frac{10\times10^{-9}}{4\pi\epsilon_0 \times (4)^2} \times \frac{4\hat{a}_y}{(4)} = 5.625\,\hat{a}_y \text{ V/m}$$

$$\bar{E}_2 = \frac{20\times10^{-9}}{4\pi \times \frac{10^{-9}}{36\pi}\,(4)^2} \times \frac{-4\hat{a}_z}{4} = -11.25\,\hat{a}_z \text{ V/m.}$$

① Electric field at the origin

$$= \bar{E}_1 + \bar{E}_2 = (5.625\hat{a}_y - 11.25\,\hat{a}_z) \text{ V/m.}$$

② $\bar{E}_1 + \bar{E}_2 + \bar{E}_3 = 0 \Rightarrow \bar{E}_3 = -(\bar{E}_1 + \bar{E}_2)$.

$$\therefore\ \bar{E}_3 = -[5.625\,\hat{a}_y - 11.25\,\hat{a}_z] \text{ V/m.}$$

$$\rightarrow \bar{E}_3 = \frac{30\times10^{-9}}{4\pi\times\frac{10^{-9}}{36\pi}\times(x^2+y^2+z^2)^{3/2}} \times -x\hat{a}_x + (-y\hat{a}_y) - z\hat{a}_z$$

$$\bar{E}_3 = \frac{270}{(x^2+y^2+z^2)^{3/2}}(-x\hat{a}_x - y\hat{a}_y - z\hat{a}_z)$$

$\Rightarrow$ Compare $\hat{a}_x \Rightarrow x = 0$

Compare $\hat{a}_y \Rightarrow -\frac{270y}{(y^2+z^2)^{3/2}} = -5.625$ — (A)

Compare $\hat{a}_z \Rightarrow \frac{-270z}{(y^2+z^2)^{3/2}} = 11.25$ — (B)

$\frac{(A)}{(B)} \Rightarrow -\frac{y}{z} = \frac{1}{2} \Rightarrow \boxed{\frac{z}{y} = -2}$

from (A) $\therefore \frac{270y}{y^3\left[1+(z/y)^2\right]^{3/2}} = 5.625.$

$\Rightarrow \frac{270y}{y^3\left[1+4\right]^{3/2}} = 5.625.$

$\therefore y = \pm 2.07 \text{ m}$

$z = -2y = \mp 4.14 \text{m}.$

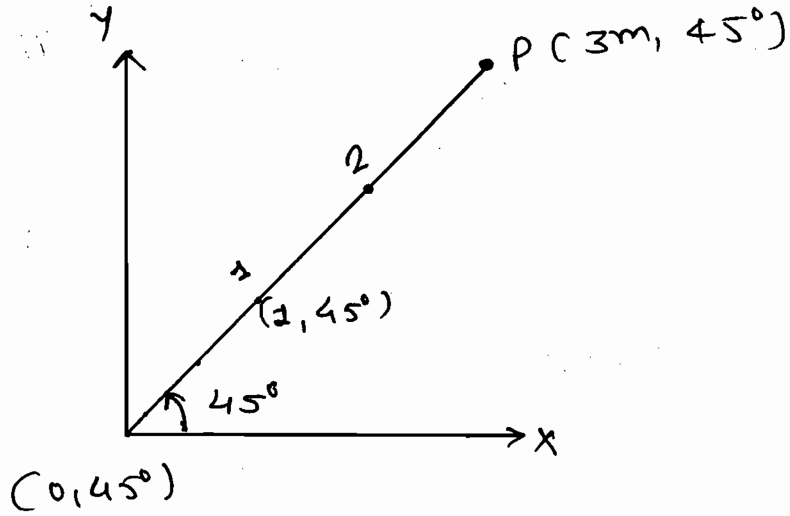
The posibilities.

$(0, 2.07, -4.14)$ ✓

$(0, -2.07, 4.14)$. ✗

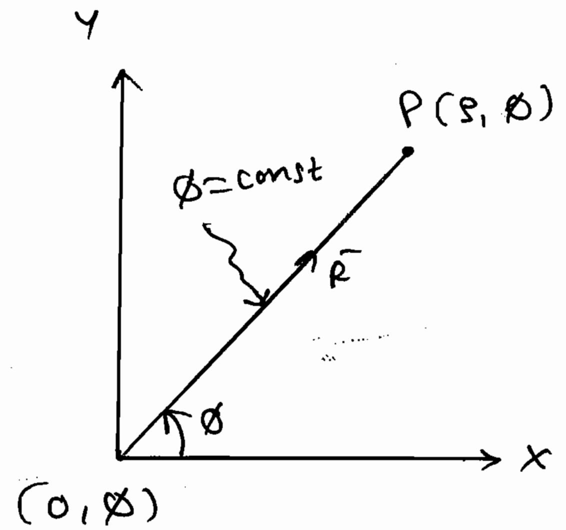
$\therefore$ Ans: $(0, 2.07, -4.14)$. m

*



→ We assume that the origin lies on $\phi = 45°$ line.

→ With ref. to P (3m, 45°), the origin is coordinated as (0m, 45°)



(origin lies on ϕ = const. lines)

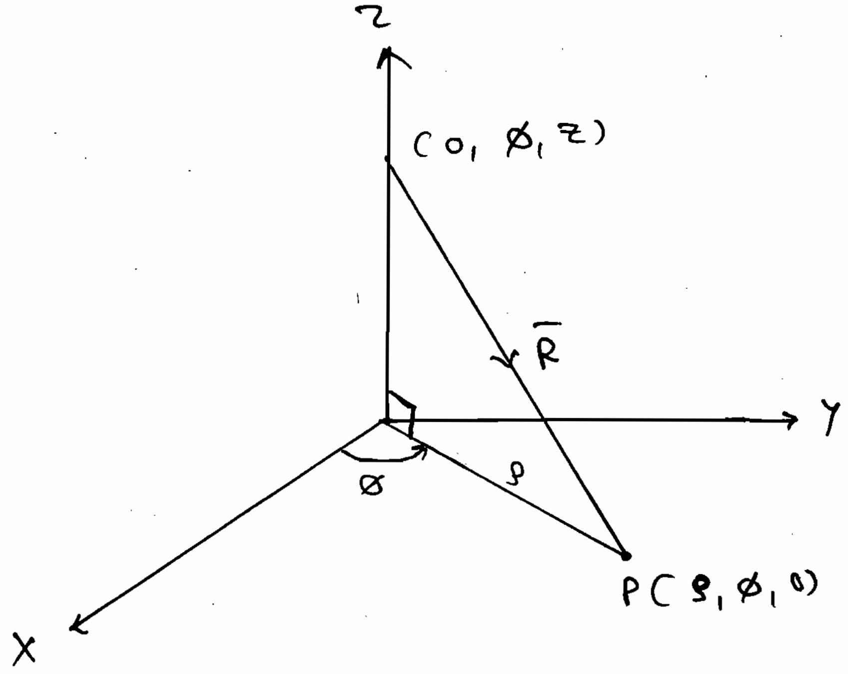
$$\bar{R} = (\rho - 0)\,\hat{a}_\rho + (\phi - \phi)\,\hat{a}_\phi$$

$$\therefore \bar{R} = \rho\,\hat{a}_\rho$$

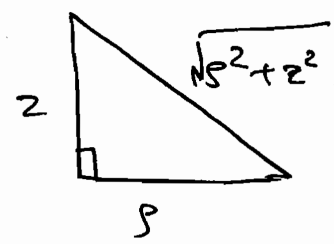
$$\therefore |\bar{R}| = \rho$$

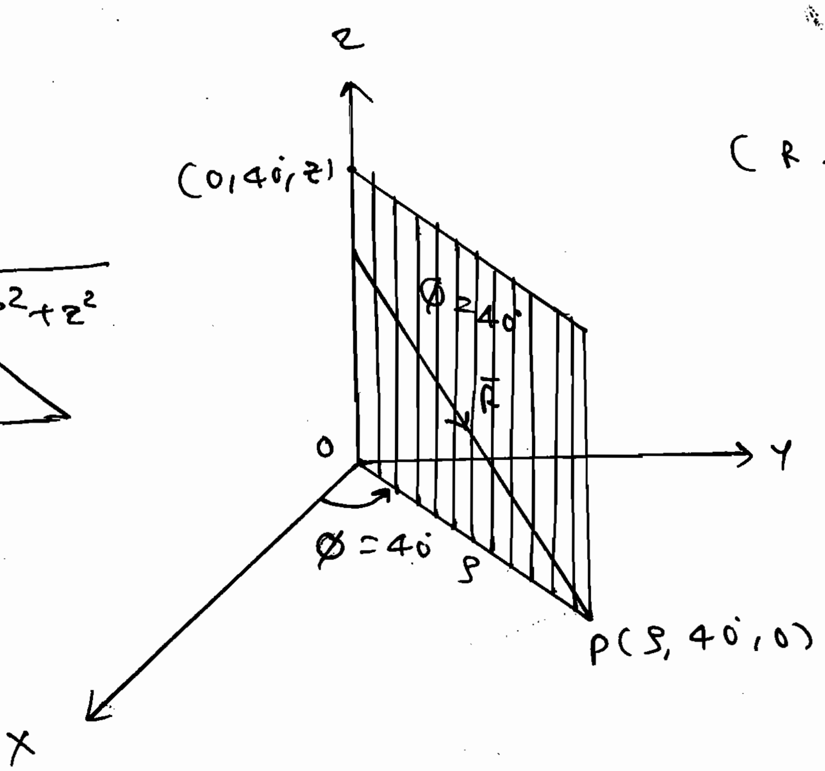
$$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|} = \hat{a}_\rho$$

*



→ With ref. to $P(\rho, \phi, 0)$ the point on the z-axis is co-ordinated as $(0, \phi, z)$.





(R lies on $\phi = 40°$)

$$\bar{R} = \rho \hat{a}_\rho - z \hat{a}_z$$

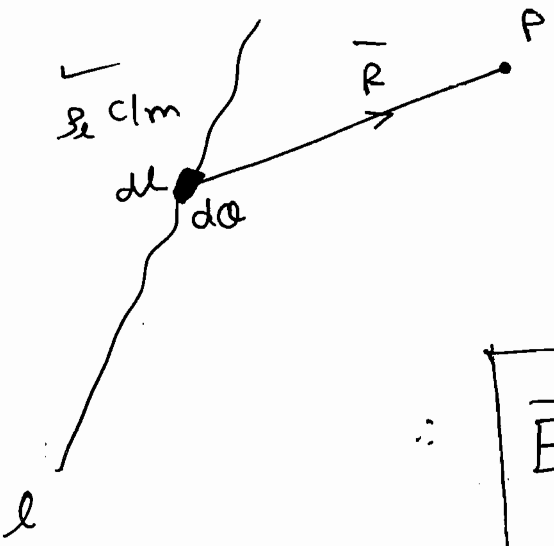
$$\therefore |\bar{R}| = \sqrt{\rho^2 + z^2}$$

→ The point on the z-axis also assumed to lying on $\phi = 40^\circ$ plane.

∴ with ref to $P(\rho, 40^\circ, 0)$, the point on the z-axis is co-ordinated as $(0, 40^\circ, z)$.

Ex-1 Expression for $\bar{E}$ due to a line with a Charge density of ρ_l C/m.

⇛



$$d\bar{E} = \frac{dQ}{4\pi\epsilon|\bar{R}|^2} \cdot \hat{a}_R$$

$$= \frac{\rho_l \, dl}{4\pi\epsilon|\bar{R}|^2} \times \hat{a}_R$$

∴ $$\boxed{\bar{E} = \int_l \frac{\rho_l \, dl}{4\pi\epsilon|\bar{R}|^2} \cdot \frac{\bar{R}}{|\bar{R}|} \ \text{V/m}}$$

→ $dQ = \rho_l \, dl$

→ We assume that 'dl' is so small such that it shrunking to the point. when we say, it is a point, ~~we~~ ~~consider~~ ~~that~~ ~~the~~ '~~dQ~~' ~~is~~ ~~located~~ then it can have coordinates.

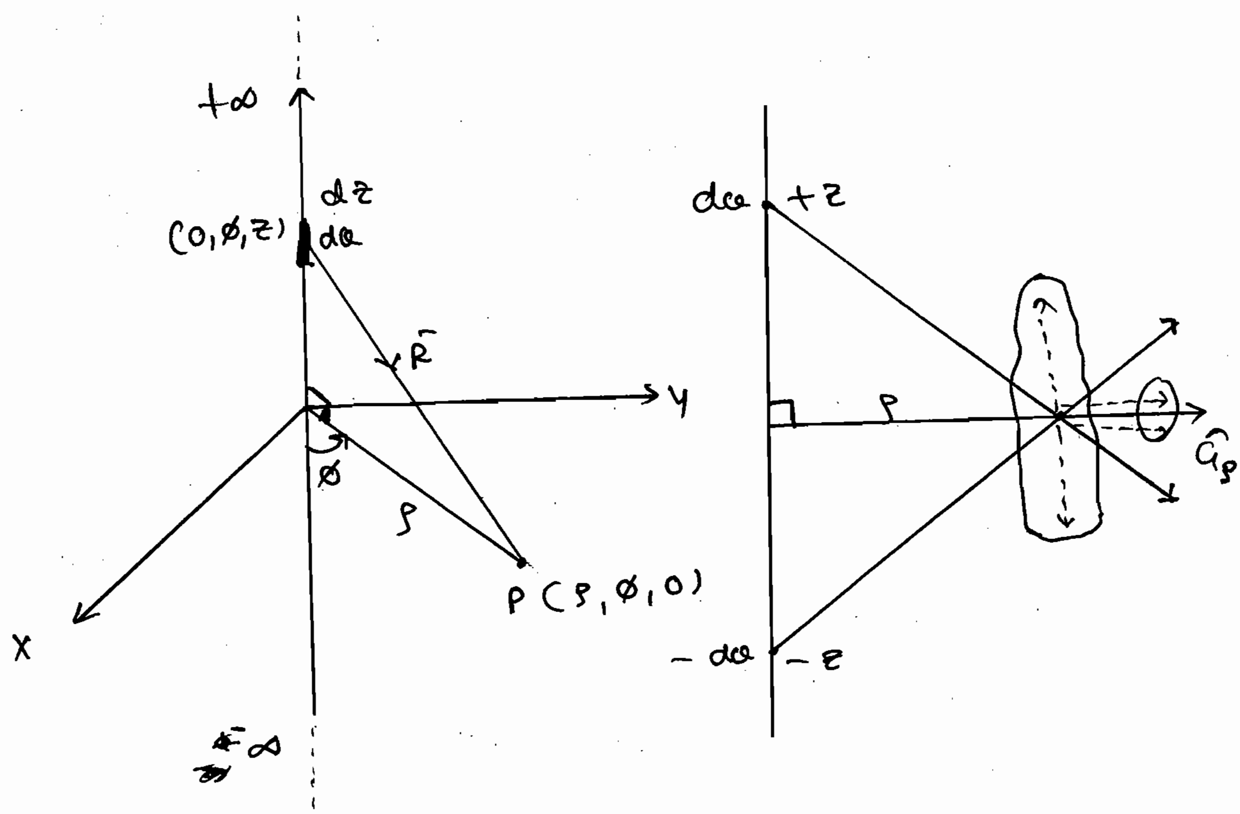
→ At this point, we consider that the 'dQ' is located.

<u>Ex-1</u> Find an expression for the electric field due to an infinite line with a uniform charge density of ρ_e c/m and show that

(i) Magnitude of the $\bar{E}$ is inversely proportional to the distance betn the infinite line and the observation ~~~~ point.

(ii) the direction of the $\bar{E}$ could be projecting in a direction normal to the infinite line.

<u>Ans:</u> we assume that the infinite line lies along z-axis. extending from $-\infty$ to $+\infty$ we find the electric field at some point on the x-y plane. for that convinience we ~~further~~ use circular cylindrical co-ordinates.



$\rightarrow \quad dQ = \rho_l \, dz$

$dz \rightarrow$ Shrunk to point

$\downarrow$

At this point, we assume that 'dQ' is located.

$\therefore$ with ref. $(\rho, \phi, 0)$, the point on the z-axis is Co-ordinated as $(0, \phi, z)$.

At this point dQ is located

$$\bar{R} = \rho \hat{a}_\rho - z \hat{a}_z$$

$$\therefore \quad d\bar{E} = \frac{\rho_l \, dz}{4\pi\epsilon\left(\sqrt{\rho^2+z^2}\right)^2} \cdot \frac{\rho \hat{a}_\rho - z\hat{a}_z}{\sqrt{\rho^2+z^2}}.$$

NOTE: for every dQ at +z on the +ve z-axis there exist an another dQ on the -ve z-axis at -z.

→ The charge configuration is symmetry about X-Y plane. which result in cancellation of $\hat{a}_z$ components and the resultant field would be along $\hat{a}_\rho$ direction only. i.e.

→ In general, no field component exist along the length of the line resulting $\bar{E}$ exists normal to line ignoring $\hat{a}_z$ Component ~~The~~ ~~total~~

→ The total field given by.

$$\bar{E} = \frac{\rho_l \rho}{4\pi\epsilon} \hat{a}_\rho \int_{-\infty}^{\infty} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

$z = \rho \tan\theta$

$dz = \rho \sec^2\theta \, d\theta$

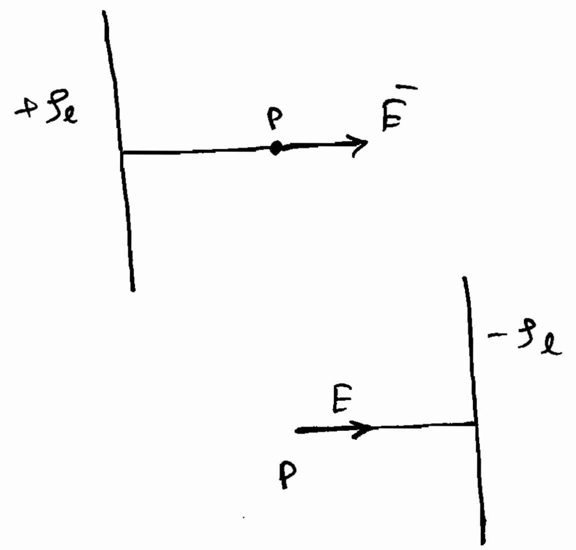
$(\rho^2+z^2)^{3/2} = \rho^3 \sec^3\theta.$

$$\therefore \bar{E} = \frac{\rho_l}{2\pi\epsilon\rho} \hat{a}_\rho$$

$$\therefore \boxed{\bar{E} = \frac{\rho_l}{2\pi\epsilon\rho} \hat{a}_\rho} \Rightarrow \boxed{|\bar{E}| \propto \frac{1}{\rho}.}$$

→ where, 'ρ' is the distance b/w the infinite line and the observation point.

⇒

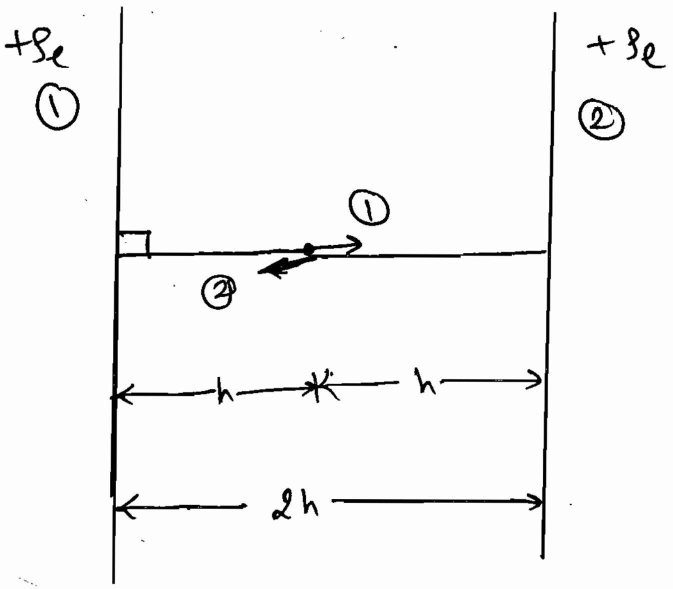


→ If ρe is +ve the direction of Ē would be away from the infinite line.

→ If ρe is -ve the direction of the Ē would be towards the infinite line.

Ex-1 Two infinite lines are parallel they are separated by $2h$ $(h>0)$. They are distributed with uniform line charge desity of $+\rho_e$ C/m each. Find mag. of the electric field. betn this infinite line and also find direction of the electric field.

Ans:

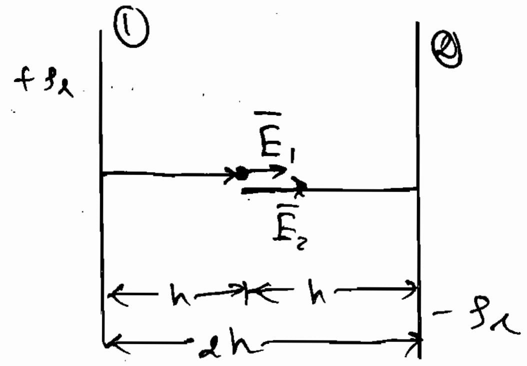


$\boxed{|\bar{E}|=0}$

∴ No point in defining the direction of $\bar{E}$.

Ex-2 Repeat the above problem if they are distributed with $+\rho_e$ C/m and $-\rho_e$ C/m.

Ans:



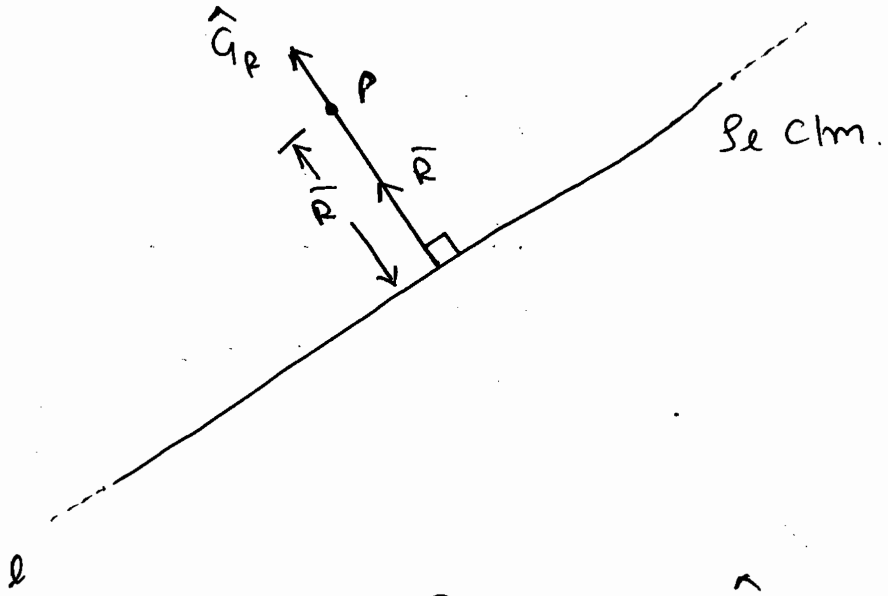
$$|\bar{E}| = \frac{\rho_l}{2\pi\epsilon h} + \frac{\rho_l}{2\pi\epsilon h}.$$

$$\boxed{|\bar{E}| = \frac{\rho_l}{\pi\epsilon h}}.$$

→ The direction of $\bar{E}$ would be towards the line which is having $\underline{\underline{-\rho_l}}$ C/m.

⊛ Expression for $\bar{E}$ due to an arbitrary oriented infinite line with a uniform Charge density of ρ_l C/m.

⇒

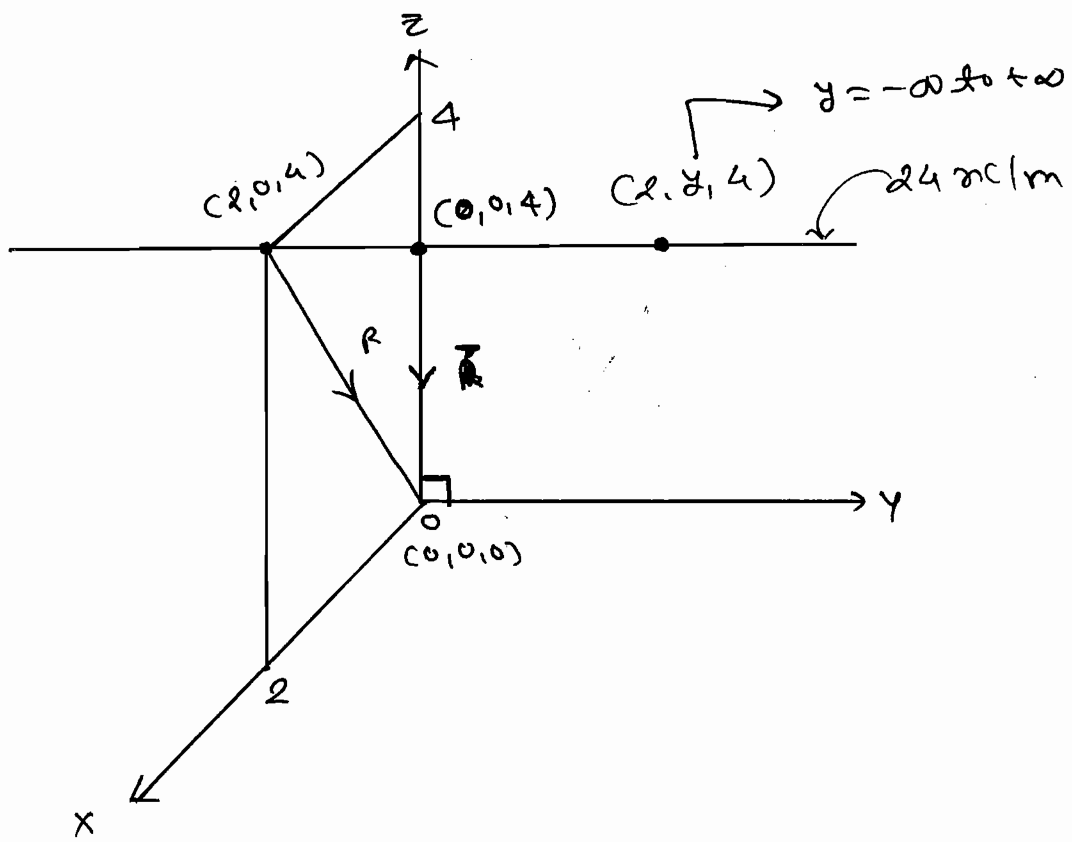


$$\bar{E} = \frac{\rho_l}{2\pi\epsilon|\bar{R}|} \cdot \hat{a}_R$$

$$\therefore \bar{E} = \frac{\rho_l}{2\pi\epsilon|\bar{R}|} \cdot \frac{\bar{R}}{|\bar{R}|} \text{ V/m.}$$

Ex-1 Find expression for the electric field at (a) origin (b) (4, 5, 6) m (c) (10, 10, 10) m. due to an infinite line with uniform charge density of 24 nC/m. which lies at $x = 2$, $z = 4$ m.

Ans:



① at origin

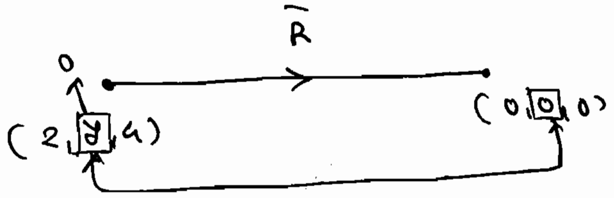
$$\therefore \quad \bar{R} = -2\hat{a}_x - 4\hat{a}_z$$

$$\therefore \quad \bar{E} = \frac{24\times 10^{-9}}{2\pi \times \frac{10^{-9}}{36\pi} \times \sqrt{20}} \times \frac{-2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}}$$

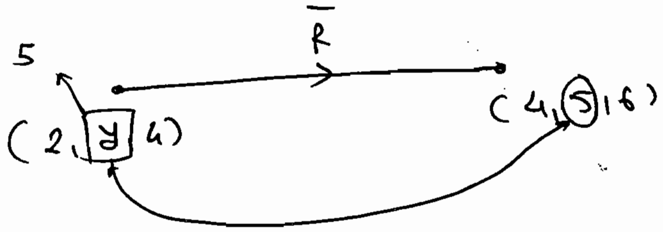
$$= \frac{216\times 2}{20} \times (-2\hat{a}_x - 4\hat{a}_z)$$

$$\therefore \boxed{\bar{E} = -43.2\,(\hat{a}_x + 2\hat{a}_z)\ \text{V/m}}$$

Short cut:



②



$$\bar{R} = 2\hat{a}_x + 2\hat{a}_z$$

$$\therefore \bar{E} = \frac{\rho_l}{2\pi\varepsilon|\bar{R}|} \times \frac{\bar{R}}{|\bar{R}|}.$$

$$= \frac{\cancel{24}^{3} \times \cancel{10^{-9}}}{\cancel{2\pi} \times \frac{\cancel{10^{-9}}}{18\,\cancel{36\pi}} \times \cancel{\sqrt{8}}} \times \frac{2\hat{a}_x + 2\hat{a}_z}{\sqrt{8}}$$

$$E = 54\,(2\hat{a}_x + 2\hat{a}_z)$$

$$\therefore \boxed{\bar{E} = 108(\hat{a}_x + \hat{a}_z).}\ \text{V/m}$$

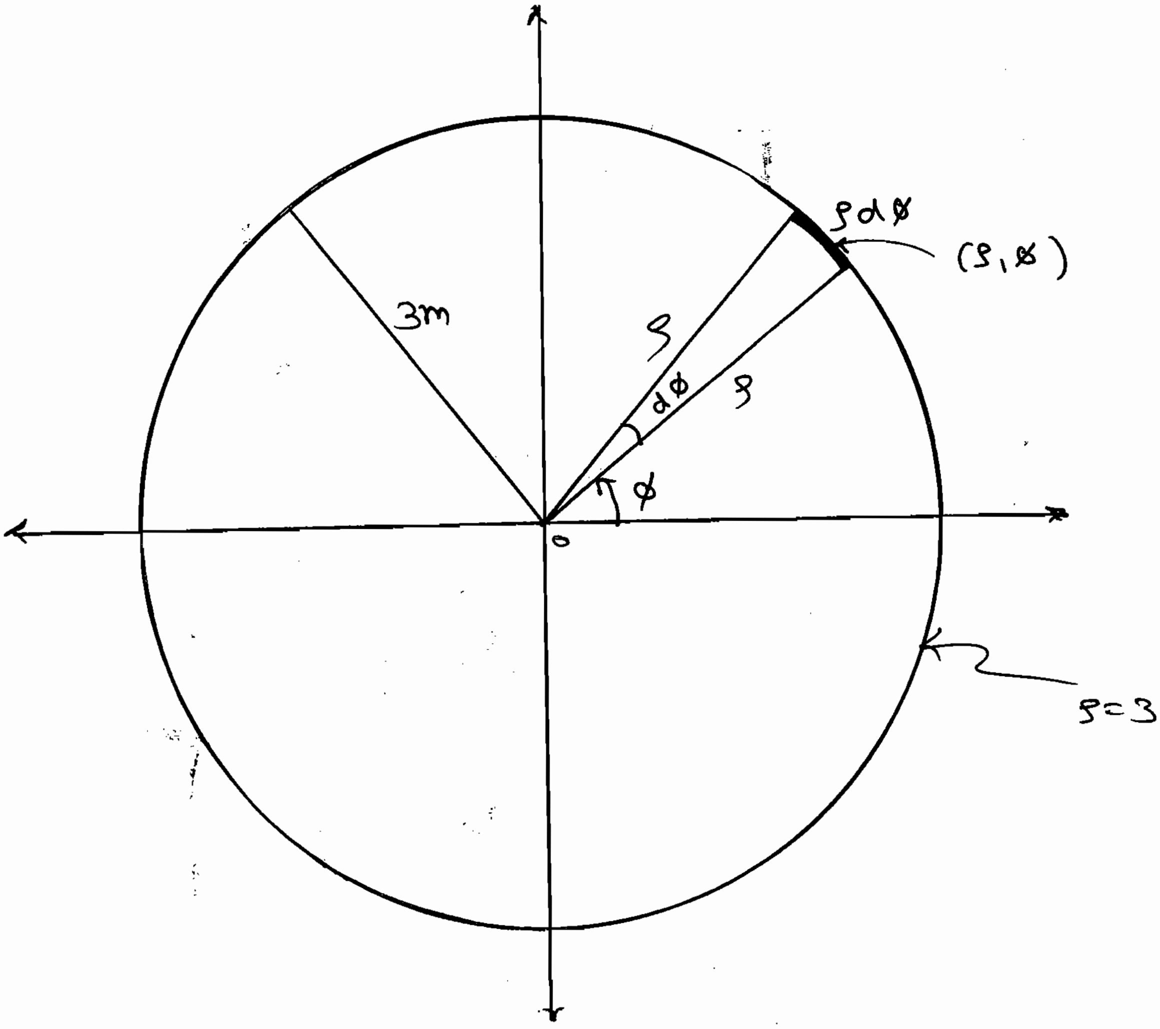
③

$$\bar{R} = 8\hat{a}_x + 6\hat{a}_z$$

$$\therefore \bar{E} = \frac{24 \times 10^{-9}}{\cancel{2\pi} \times \frac{10^{-9}}{18\,\cancel{36\pi}} \times 10} \times \frac{8\hat{a}_x + 6\hat{a}_z}{10}$$

$$\therefore \boxed{\bar{E} = 4.32\,(8\hat{a}_x + 6\hat{a}_z)\ \text{V/m}}$$

*



$\rightarrow$ $\rho=3,\ 0\leq \phi \leq 2\pi,\ z=0$

$\Rightarrow$ This represents, there exists a circle of radius 3m centered at origin and is located in $z=0$ plane.

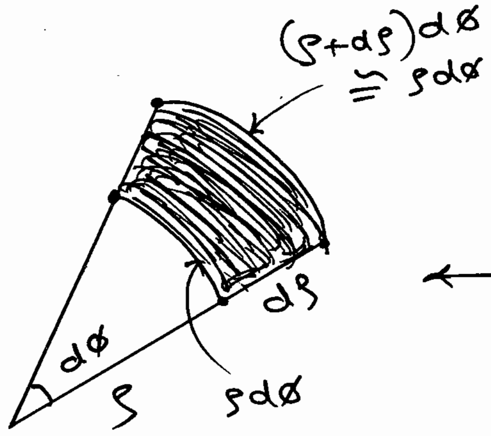
$$dl = \rho\, d\phi.$$

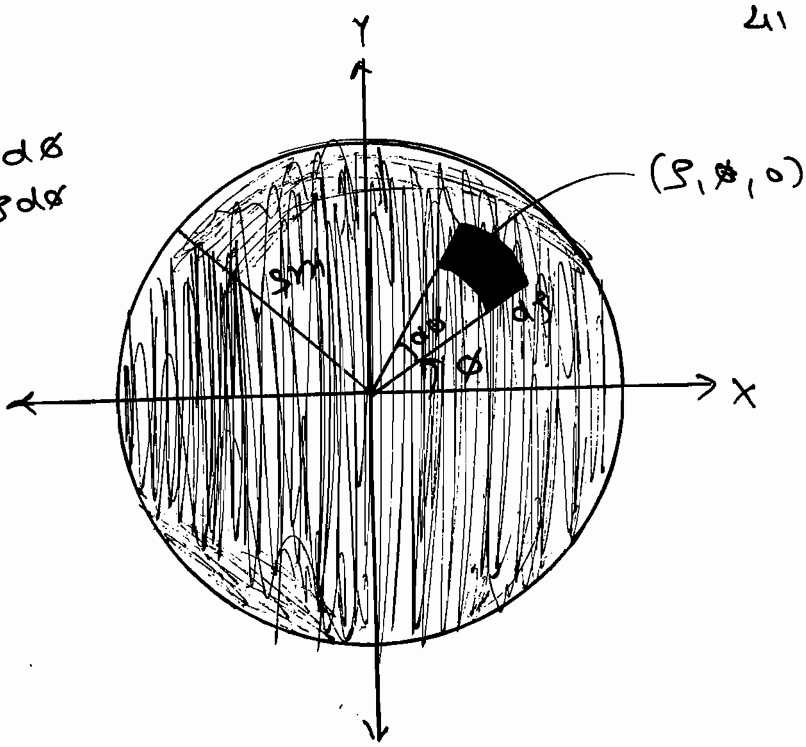
$$l = \int_{(z=0,\ \rho=3)}^{2\pi} \rho\, d\phi = 3\int_{0}^{2\pi} d\phi = 6\pi m.$$

$dl \rightarrow$ So small such that it is shrinking to a point (i.e) $d\phi \rightarrow 0$.

Then, that point is considered as (ρ,ϕ).

→*





→ ⊙ $0 \le \rho \le 3,\ 0 \le \phi \le 2\pi,\ z=0$

⊙ $\rho \le 3,\ 0 \le \phi \le 2\pi,\ z=0$

⊙ $\rho \le 3,\ \underline{z=0}$

This represents a circular disk of radius 3m, centered at origin and is located in $Z=0$ plane.

$$ds = \rho\, d\rho\, d\phi$$

$$S = \int_0^3 \int_0^{2\pi} \rho\, d\rho\, d\phi$$

$$= \int_0^3 \rho\, [\phi]_0^{2\pi}\, d\rho.$$

$$= \int_0^3 (2\pi)\rho\, d\rho$$

$$= \left[\frac{\rho^2}{2}\right]_0^3 \times 2\pi$$

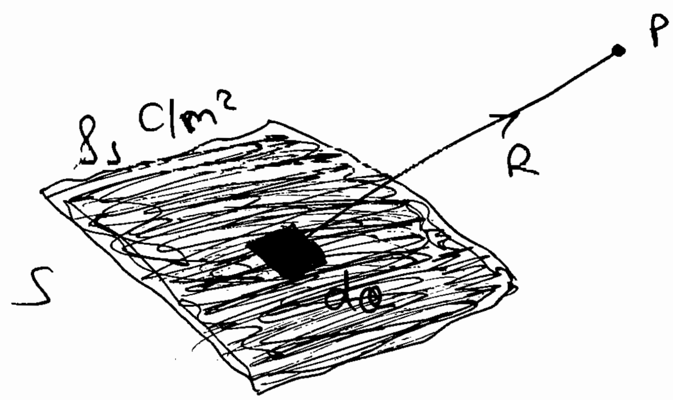
$$\therefore \boxed{S = 9\pi\ m^2}$$

$ds \to$ so small such that it shrinks to a point (i.e) $d\rho \to 0,\ d\phi \to 0.$

$\therefore$ That point is coordinated as (ρ,ϕ) or $(\rho,\phi,0)$.

* Electric field ($\bar{E}$) due to a sheet with a uniform charge density of ρ_s C/m^2.

⇒



→ $dQ = \rho_s \, ds$

ds → shrink to a point.

At this point, we consider that 'dQ' is located. When we say, it is a point, then it can have co-ordinates.

$$dE = \frac{dQ}{4\pi\epsilon |\bar{R}|^2} \hat{a}_R$$

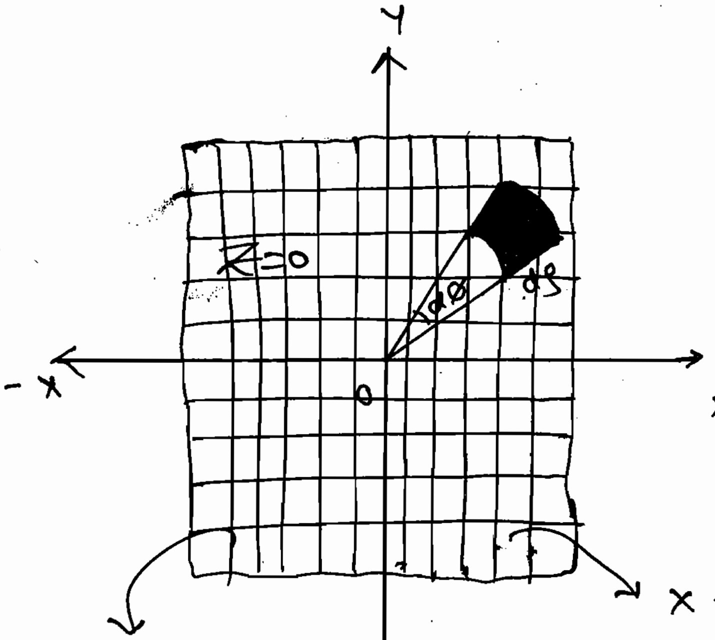
$$\therefore \boxed{\bar{E} = \int_S \frac{\rho_s \, ds}{4\pi\epsilon |\bar{R}|^2} \times \frac{\bar{R}}{|\bar{R}|} \ \text{V/m}}$$

double integral.

Ex-1 Find an expression for the $\bar{E}$ due to an infinite sheet whith the uniform Charge density of ρ_s C/m².

Ans: → We assume that the infinite sheet is located in the $Z=0$ plane. We find the $\bar{E}$ at some point on the z-axis. For the convinece we use cylindrical coordinates.

⇒



$x \to -\infty$ to $+\infty$

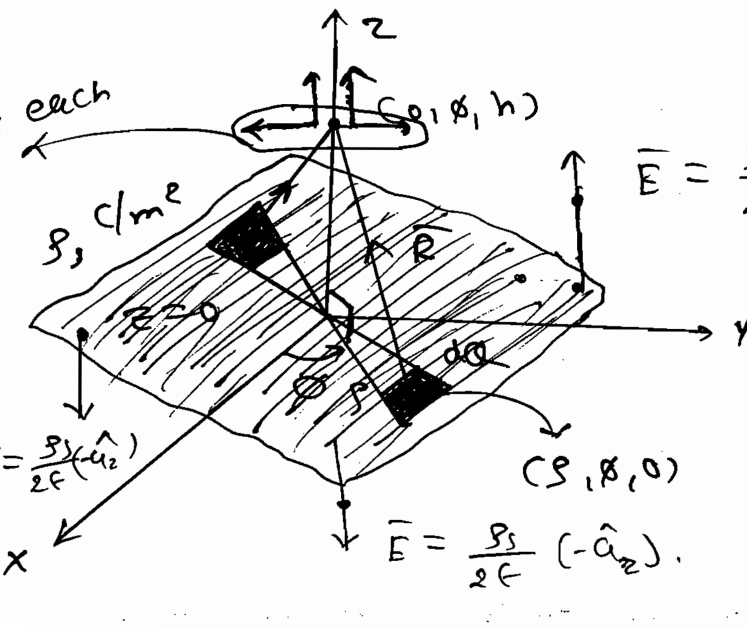
$y \to -\infty$ to $+\infty$.

$\rho \to 0$ to ∞

$\phi \to 0$ to 2π

OR

⇒



$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_z$

$\bar{E} = \frac{\rho_s}{2\epsilon}(-\hat{a}_z)$

$\bar{E} = \frac{\rho_s}{2\epsilon}(-\hat{a}_z)$.

$\rightarrow \quad ds = \rho \, d\rho \, d\phi.$

$dQ = \rho_s \, ds$

$dQ = \rho_s \, \rho \, d\rho \, d\phi$

$ds \rightarrow$ Shrinks to a point

$\downarrow$

This point is coordinated as $(\rho, \phi, 0)$

[i.e $d\rho \rightarrow 0, d\phi \rightarrow 0$].

At this point 'dQ' is located,

$$d\bar{E} = \frac{\rho_s \cdot \rho \, d\rho \, d\phi}{4\pi\epsilon\left(\sqrt{\rho^2 + h^2}\right)^2} \times \frac{-\rho \hat{a}_\rho + h \hat{a}_z}{\sqrt{\rho^2 + h^2}}.$$

→ As shown in figure for every dQ on the sheet there exists an another dQ diametrically oposite side. Therefore, the charge configuration is symmetry about z-axis. which results in cancellation of horizontal field components. And the resultant $\bar{E}$ would be along $\hat{a}_z$ direction only.

i.e. No field component exists $\parallel^{el}$ to the infinite sheet.

→ The resultant field exists in the direction to the normal to the sheet. Here, ignoring $\hat{a}_\rho$ components. The total field is given by,

→ Ignoring $\hat{a}_\rho$ component, the total field is given by,

$$\bar{E} = \frac{\rho_s h}{4\pi\epsilon}\hat{a}_z \int_{0\,(\rho)}^{\infty}\int_{0\,(\phi)}^{2\pi} \frac{\rho\, d\rho\, d\phi}{(\rho^2+h^2)^{3/2}}$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$\int_0^{\infty} \frac{\rho\, d\rho}{(\rho^2+h^2)^{3/2}} = \frac{1}{h}.$$

put $\rho^2+h^2 = t$.

$\therefore\ 2\rho\, d\rho = dt$

$\therefore\ \rho\, d\rho = \frac{1}{2}dt$

$$\therefore\ \boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_z \text{ V/m}}$$

→ In, general $\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_n \text{ V/m}}$

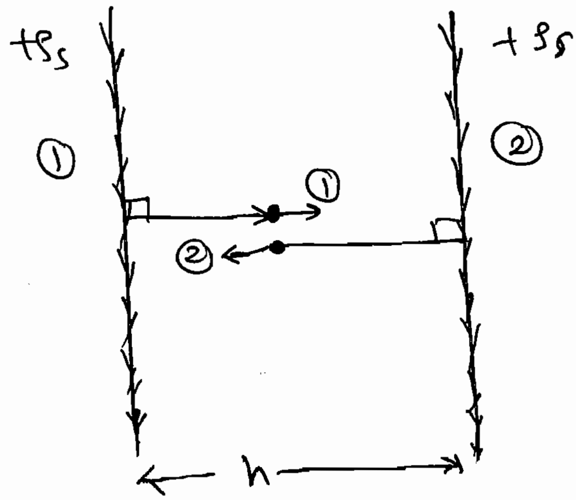
where, $\hat{a}_n$ is the normal unit vector at the observation point with ref. to infinite sheet.

→ If ρ_s is +ve, the direction of $\bar{E}$ would be away from the infinite sheet.

→ If ρ_s is −ve, the direction of $\bar{E}$ would be towards to the infinite sheet.

Ex-1 Two infinite sheets are $\parallel^{el}$, they are separated by 2h m. they are distributed with uniform charge density of ρ_s C/m^2 each find the electric field at any point betn this two infinite sheet.

Ans:

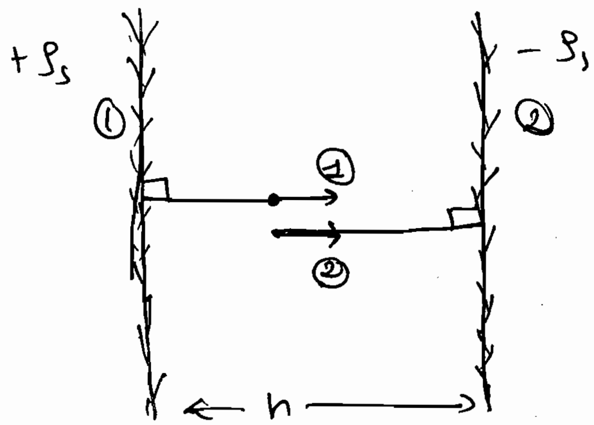


$|\bar{E}| = 0$,

The fields add in out of phase.

Ex-2 Repeate the above example If they are distributed with uniform charge density of $+\rho_s$ C/m^2 and $-\rho_s$ C/m^2. The

Ans:

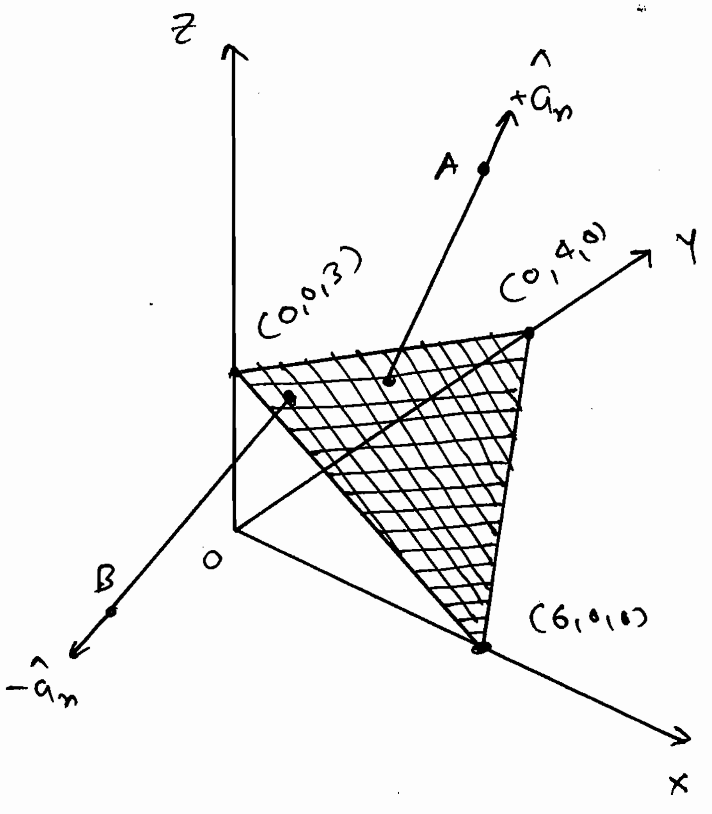


$$|\bar{E}| = \frac{\rho_s}{2\epsilon} + \frac{\rho_s}{2\epsilon} = \frac{\rho_s}{\epsilon}.$$

→ The fields add in in-phase.
The direction of $\bar{E}$ would be towards the sheet which is having $-\rho_s$ C/m^2.

Ex-3 An Infinite Sheet with a uniform charge density of 24 nC/m^2 is lies in a plane define by $2x + 3y + 4z = 12$. Find the $\bar{E}$ in all the regions.

Ans:



→ $E\ (\text{at } A) = \dfrac{\rho_s}{2\epsilon}\,\hat{a}_n$

$E\ (\text{at } B) = \dfrac{\rho_s}{2\epsilon}\,(-\hat{a}_n)$

$$\hat{a}_n = \frac{2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z}{\sqrt{2^2+3^2+4^2}}$$

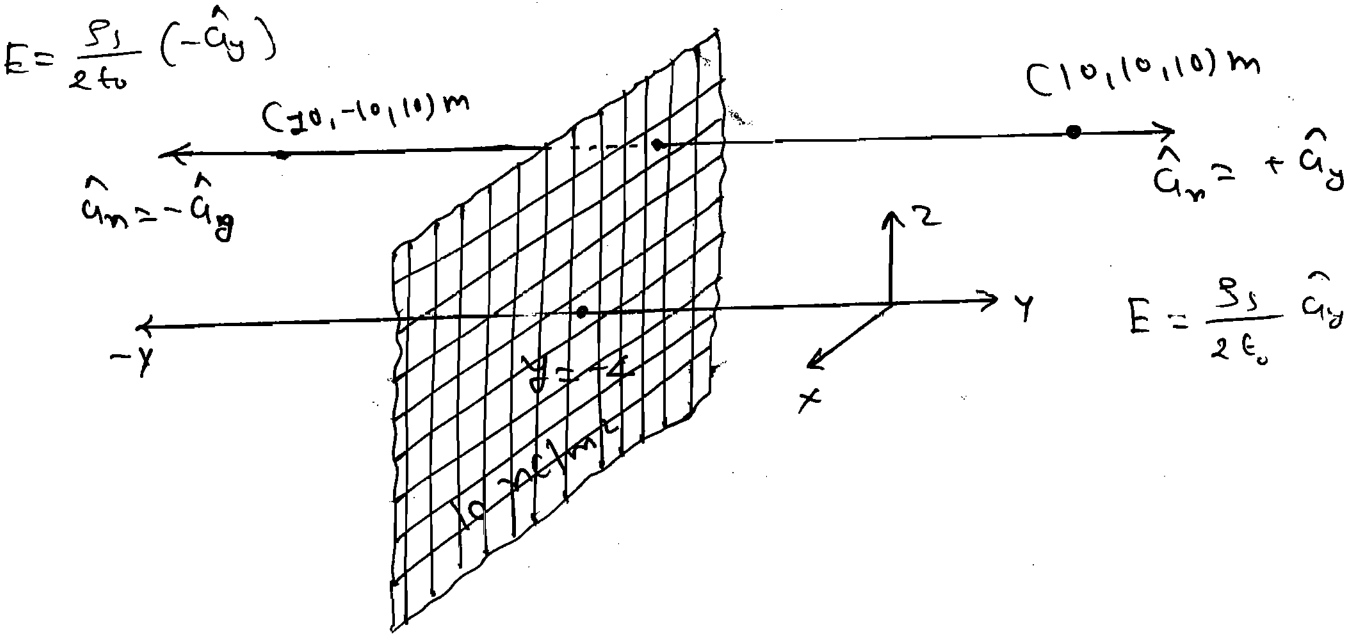
Ex-4 An Infinite Sheet with a uniform charge density of 10 nC/m² is lies at $y = -4$ m. Find the $\bar{E}$ at

(i) (10, 10, 10) m

(ii) (10, -10, 10) m.

Ans:

The infinite sheet is ||el to z-x plane.



(i) at (10, 10, 10)

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_y \quad \text{V/m}$$

(ii) at (10, -10, 10)

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}(-\hat{a}_y). \quad \text{V/m.}$$

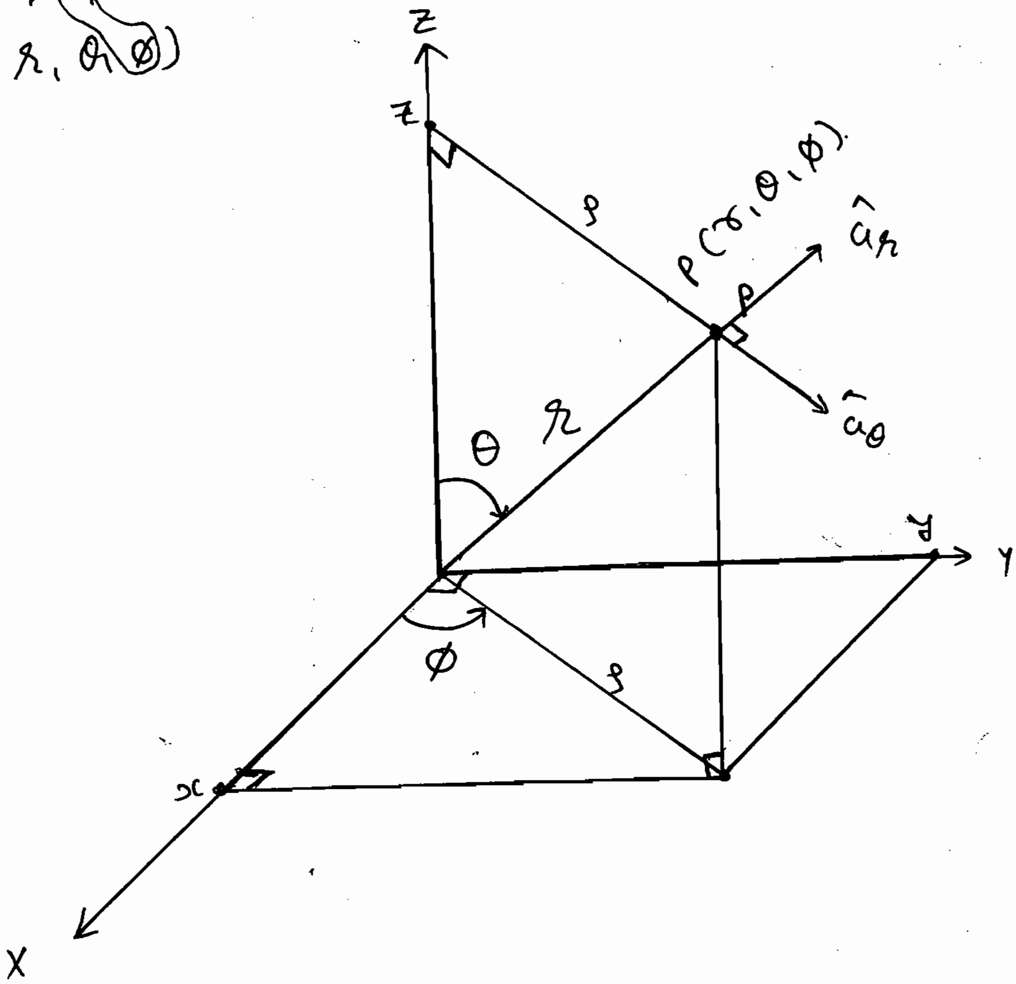
*

P (x, y, z)

P (ρ, φ, z)

P (r, θ, φ)

→



⟹ $\rho = r\sin\theta$

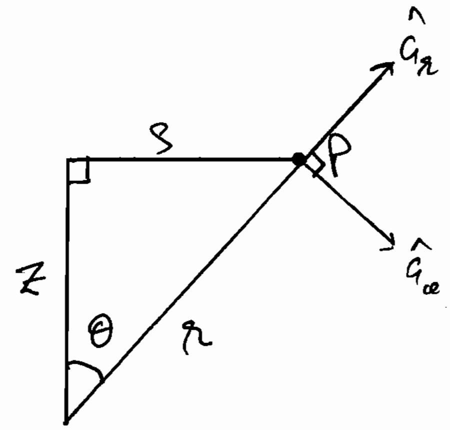
$z = r\cos\theta$

$x = \rho\cos\phi$

$\therefore x = r\sin\theta\cdot\cos\phi$

$\therefore y = \rho\sin\phi$

$\therefore y = r\sin\theta\cdot\sin\phi$



$r = \sqrt{x^2+y^2+z^2}$ $\quad 0 \le r \le \infty$

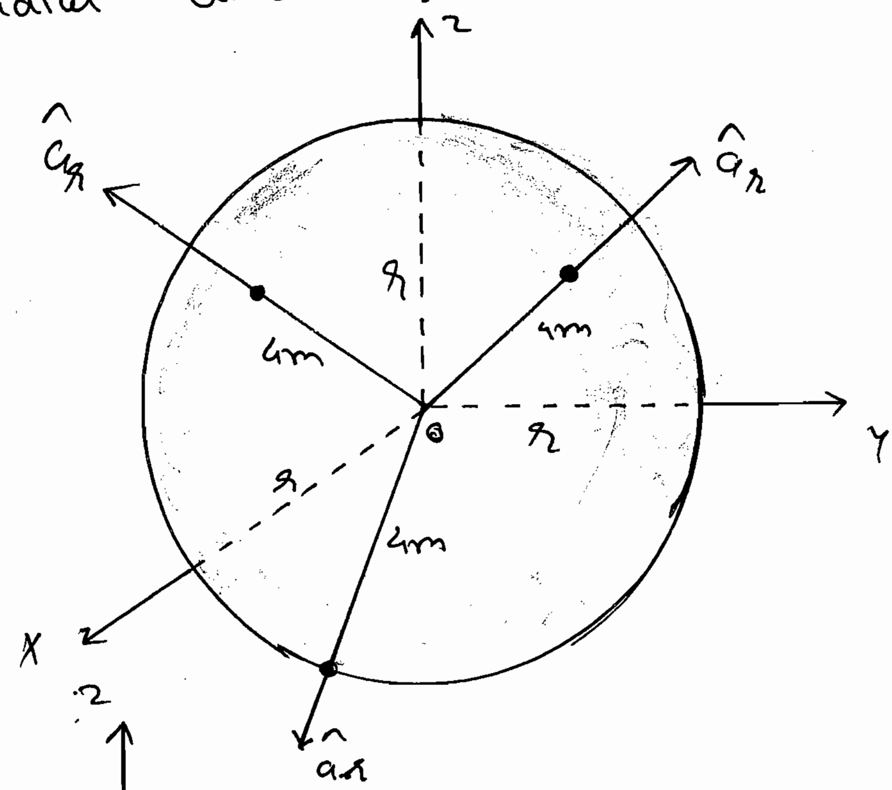
$\theta = \cos^{-1}\left(\frac{z}{r}\right)$ $\quad 0 \le \theta \le \pi$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$ $\quad 0 \le \phi \le 2\pi$

→ Locus of r = Constant represents a Sphere (or) a sphericereical whoes centre coinsides with the origin. Therefore, r assume all possible values ranging from 0 to ∞.

→ $\hat{a}_r$ is a unit vector projecting normal to r = constant (or) normal to the Sphere. also called radial direction.

→

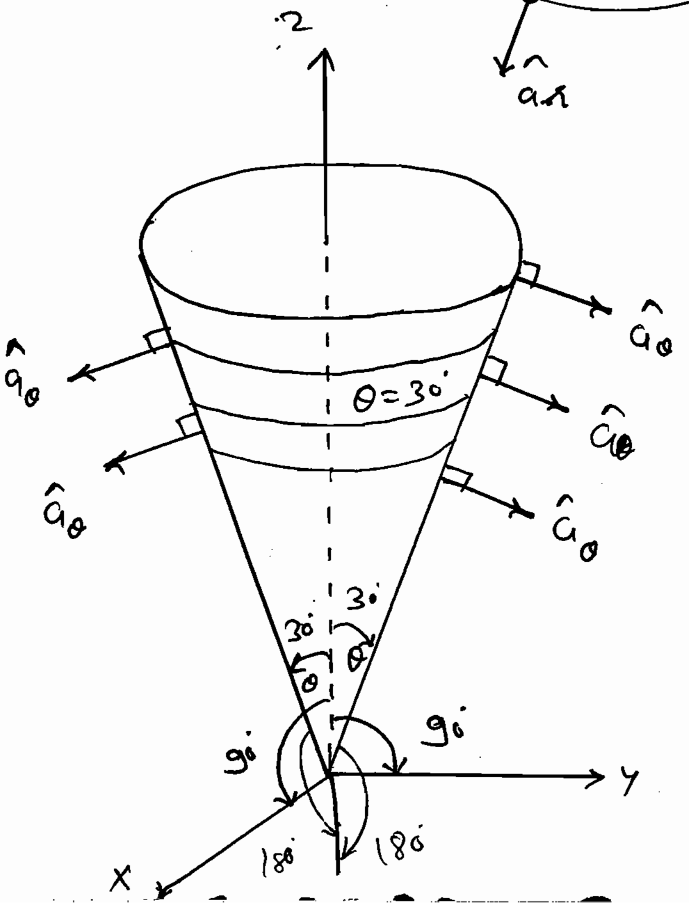


on r-const.

$\theta \rightarrow 0$ to π

$\phi \rightarrow 0$ to 2π

→ Si



on θ = const.

$r \rightarrow 0$ to ∞

$\phi \rightarrow 0$ to 2π

→ θ = constant for $\theta < 90°$ represents a conical plane, $\theta = 90°$ represents x-y plane, $\theta = 90°$ is called azimuthal plane.

→ As shown in the figure θ assumes all possible values ranging from 0 to π.

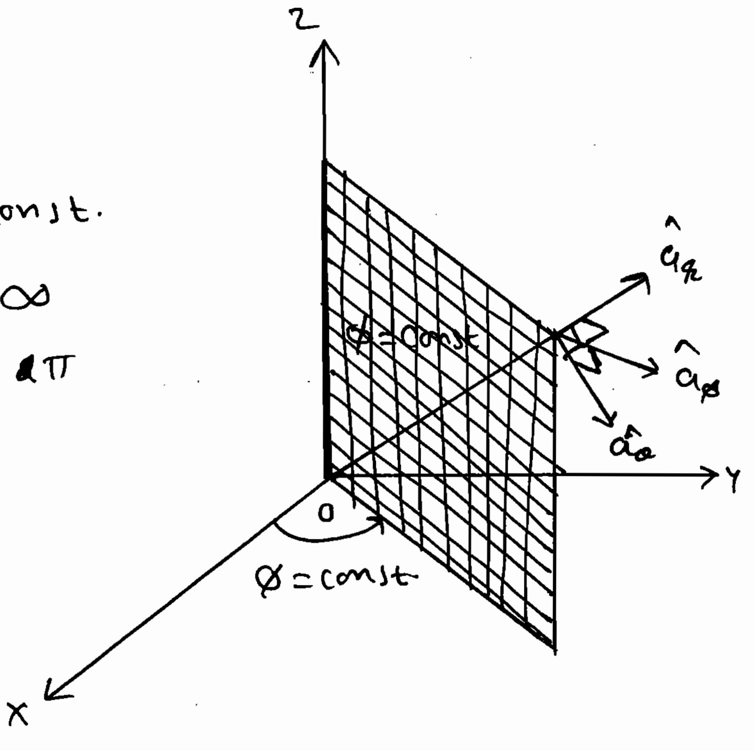
→ $\hat{a}_\theta$ is a unit vector projecting normal to θ = constant plane. Further, $\hat{a}_r$, $\hat{a}_\theta$, $\hat{a}_\phi$ orthogonal to each other.

→

on ϕ = const.

$r \to 0$ to ∞

$\theta \to 0$ to 2π

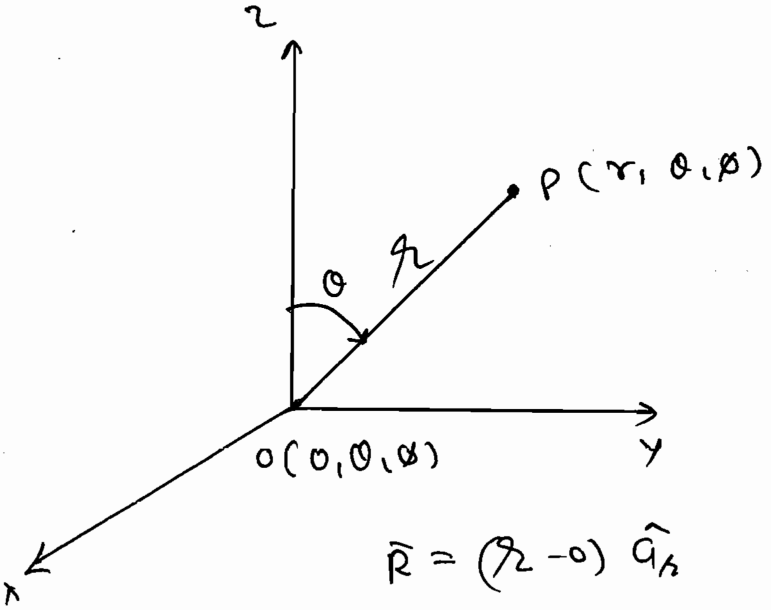


→ ϕ assumes all possible values ranging from 0 to 2π. $\hat{a}_\phi$ is a unit vector projecting normal to ϕ = const. plane. further we write $\hat{a}_r$, $\hat{a}_\theta$ and $\hat{a}_\phi$ are orthogonal to each other.

→ $\phi =$ const. plane is also called Elevation plane.

(Q)- with ref. to $P(r, \theta, \phi)$ what are the co-ordinates of the origin. ?

(A) = $O(0, \theta, \phi)$.



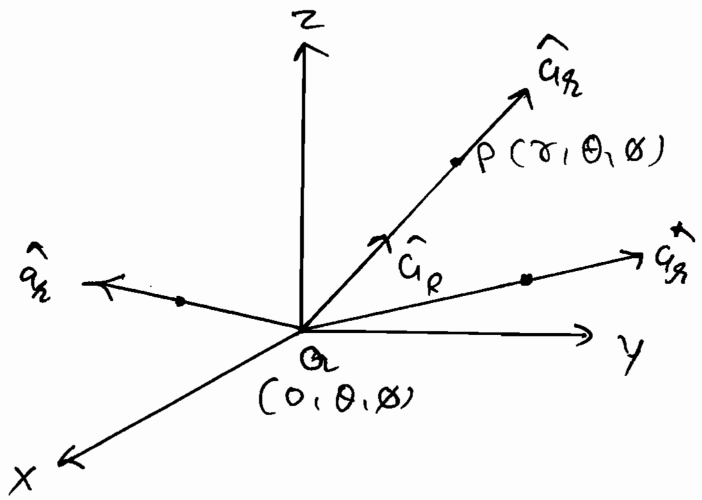
$$\bar{R} = (r - 0)\,\hat{a}_r$$

$$\therefore \bar{R} = r\,\hat{a}_r$$

$$\Rightarrow \hat{a}_R = \hat{a}_r .$$

Q-1 A point charge of Q columbs located at the origin. find $\bar{E}$ at a distant point P in Spherical co-ordinates.

Ans:



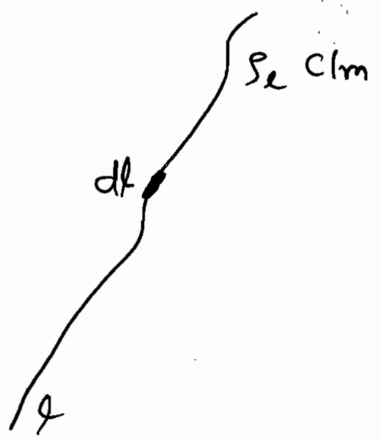
$$\bar{E} = \frac{Q}{4\pi\epsilon|\bar{R}|^2} \cdot \hat{a}_R$$

$$= \frac{Q}{4\pi\epsilon r^2}\,\hat{a}_r .$$

→ Thus, the Direction of $\bar{E}$ would be along radial (or) $\hat{a}_r$ direction.

* Total charge Calculation:

(1) Line Charge:

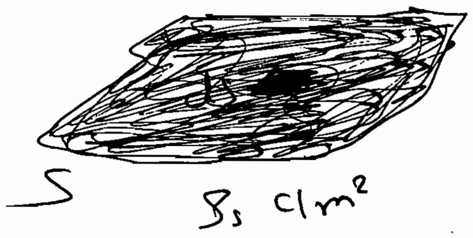


$dQ = \rho_l dl.$

$$\therefore \boxed{Q = \int_l \rho_l \, dl.}$$

single I

(2) Surface Charge:

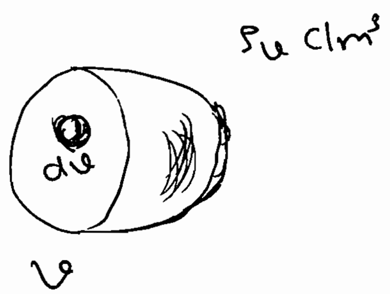


$dQ = \rho_s ds$

$$\therefore \boxed{Q = \int_s \rho_s \, ds}.$$

double I

(3) Volume Charge:

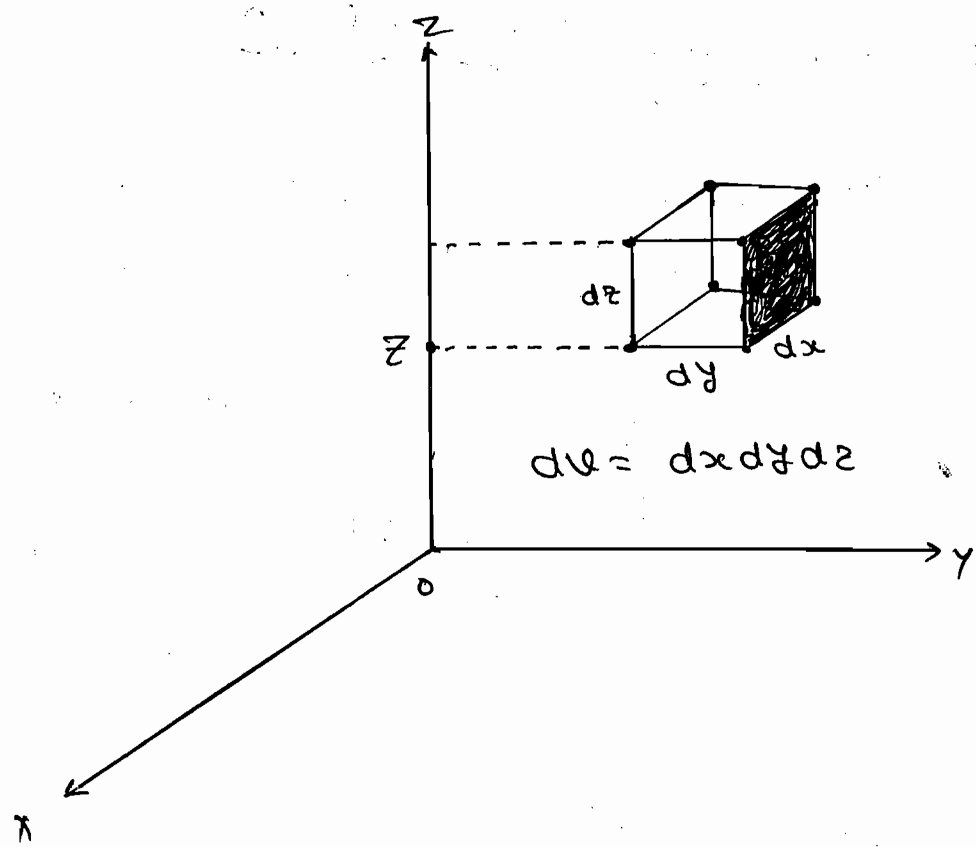


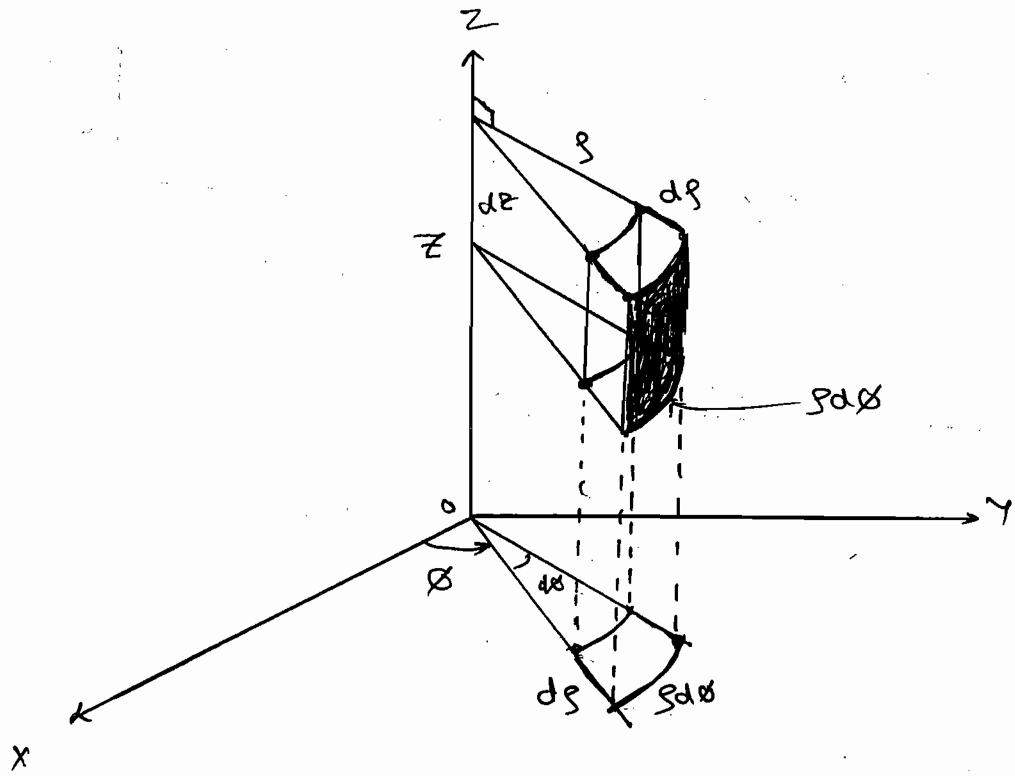
$\therefore \; dQ = \rho_v dv$

$$\therefore \boxed{Q = \int_v \rho_v \, dv.}$$

Triple I

* Construction of $d\upsilon$.

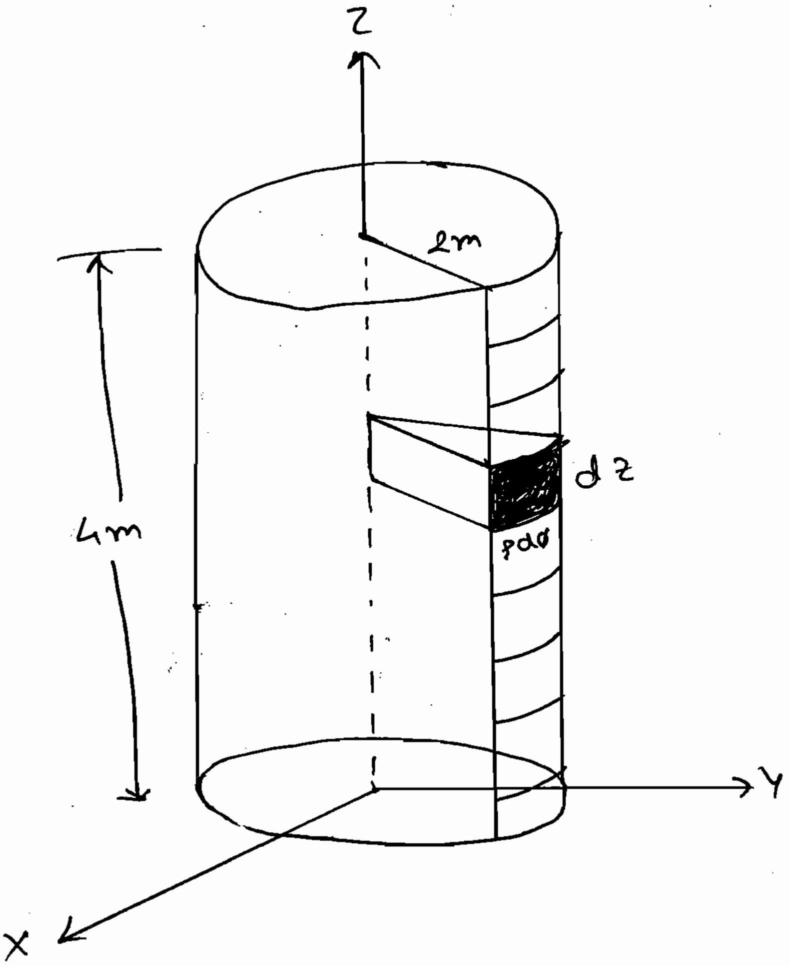




$$d\upsilon = d\rho \cdot \rho \cdot d\phi \, dz$$

$$d\upsilon = \rho \, d\rho \, d\phi \, dz$$

*



→ $\boxed{\rho = 2m, \quad 0 \le \phi \le 2\pi, \quad 0 \le z \le 4}$

This represents a cylindrical sheet with radius 2m and height 4m.

$$ds = \rho \, d\phi \, dz$$

$$\therefore S = \int_{0\,(\phi)}^{2\pi} \int_{0\,(z)}^{4} \rho \, d\phi \, dz.$$

$$\therefore S = 2 \, (\phi)_0^{2\pi} \, (z)_0^4$$

$$\therefore S = 2(2\pi)(4)\, m.$$

$$\boxed{S = 16\pi \ m^2}$$

Ex-1 Find the total charge with in each of the volume indicate below:

(1) $\rho_v = 10 z^2 e^{-0.1x} \sin \pi y \ nC/m^3$

$0 \le x \le 1, \quad 1 \le y \le 2, \quad 2.5 \le z \le 4.5.$

(2) $\rho_v = 2xyz^2 \ nC/m^3.$

$0 \le \rho \le 2, \quad 0 \le \phi \le \frac{\pi}{2}, \quad 0 \le z \le 1.$

(3) $\rho_v = 10 e^{-10\rho} \cdot e^{-z} \ nC/m^3 \ ; \ 1^{st}$ octant.

Ans: $dv = dx\,dy\,dz.$

$dQ = \rho_v \, dv$

$\therefore$

(1) $$Q = \int_{x=0}^{1} \int_{y=1}^{2} \int_{z=2.5}^{4.5} 10 z^2 \cdot e^{-0.1x} \cdot \sin \pi y \cdot dx\,dy\,dz$$

$$\therefore Q = 10 \times \left[\frac{e^{-0.1x}}{-0.1} \right]_0^1 \times \left[-\frac{\cos \pi y}{\pi} \right]_1^2 \times \left[\frac{z^3}{3} \right]_{2.5}^{4.5}$$

$$\therefore Q = 10 \times \left[\frac{e^{-0.1} - 1}{+0.1} \right] \times \left[+ \frac{1+1}{\pi} \right] \times \left[\frac{(4.5)^3 - (2.5)}{3} \right]$$

$\therefore Q =$ ______ .

(2) $x = \rho \cos \phi, \quad y = \rho \sin \phi$

$\therefore \rho_v = \rho^2 \sin 2\phi \cdot z^2 \ nC/m^3.$

$\therefore \quad dv = \rho \, d\rho \, d\phi \, dz.$

$\therefore \quad Q = \int_v \rho_v \, dv.$

$$\therefore \quad Q = \int_{\rho=0}^{2} \int_{\phi=0}^{\pi/2} \int_{z=0}^{1} \rho \cdot \rho^2 \cdot \sin 2\phi \cdot z^2 \cdot d\rho \, d\phi \, dz$$

$$\therefore \quad Q = \left[\frac{\rho^4}{4}\right]_0^2 \times \left[\frac{-\cos 2\phi}{2}\right]_0^{\pi/2} \times \left[\frac{z^3}{3}\right]_0^1$$

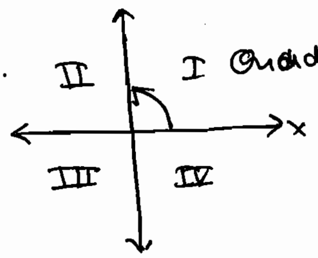
$$= \left[\frac{\cancel{8}\times 2^{2}}{\cancel{4}}\right] \times \left[\frac{\cancel{1}+1}{\cancel{2}}\right] \times \left[\frac{1}{3}\right].$$

$$\therefore \quad \boxed{Q = \frac{4}{3} \ nC}$$

(3) $\rho_v = 10 e^{-10\rho} \cdot e^{-z}$ nC/m³.

~~1st octant.~~

$\therefore$



I- Quadrant : x, y are +ve

i.e $\left.\begin{matrix} 0 \le x \le \infty \\ 0 \le y \le \infty \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} \rho \to 0 \text{ to } \infty \\ \phi \to 0 \text{ to } \frac{\pi}{2} \end{matrix}\right.$

I - Octant $\to$ x, y, z are +ve

$\Downarrow$

$\left.\begin{matrix} x \to 0 \text{ to } \infty \\ y \to 0 \text{ to } \infty \\ z \to 0 \text{ to } \infty \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} \rho \to 0 \text{ to } \infty \\ \phi \to 0 \text{ to } \pi/2 \\ z \to 0 \text{ to } \infty. \end{matrix}\right.$

$dv = \rho \, d\rho \, d\phi \, dz.$

$$\therefore \quad Q = 10 \int_{0\,(\rho)}^{\infty} \int_{0\,(\phi)}^{\pi/2} \int_{0}^{\infty} e^{-10\rho} \cdot e^{-z} \cdot \rho \, d\rho \, d\phi \, dz.$$

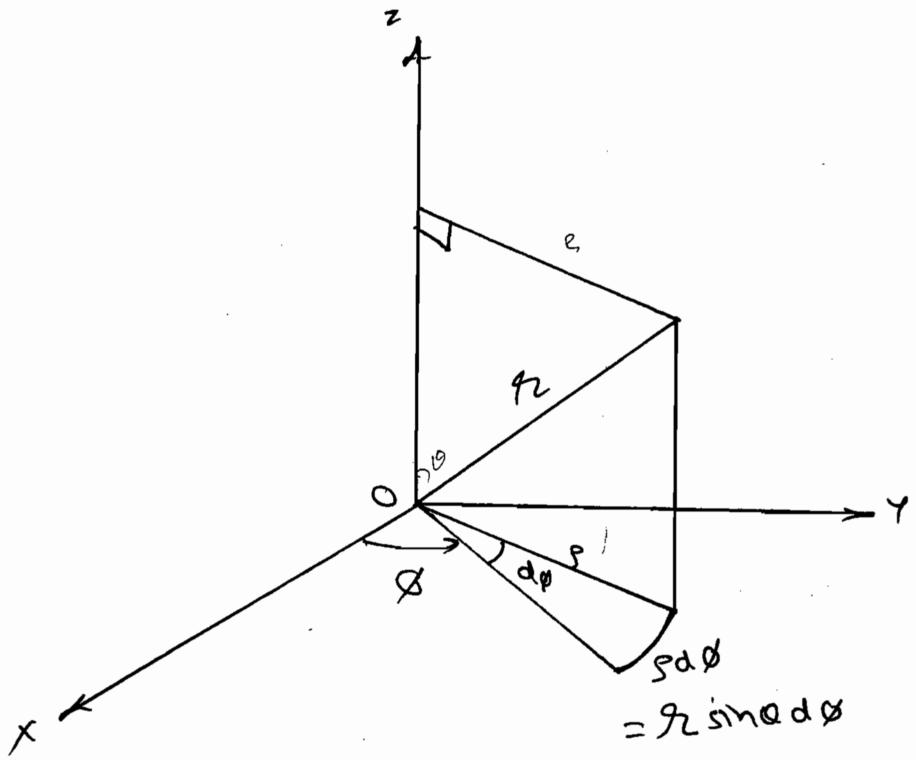
$$\therefore Q = 10\left(\int_0^\infty \rho\, e^{-10\rho}\, d\rho \times \int_0^{\pi/2} d\phi \times \int_0^\infty e^{-z}\, dz\right) nC.$$

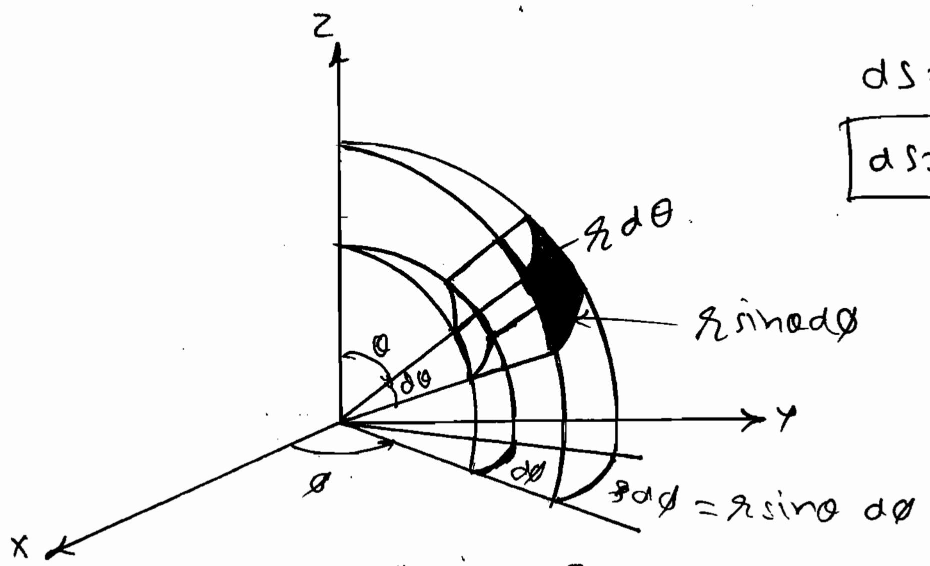
$$\therefore Q = 10\left[\rho\cdot\frac{e^{-10\rho}}{-10} + (1)\frac{e^{-10\rho}}{(10)^2}\right]_0^\infty \times \frac{\pi}{2} \times \left[\frac{e^{-z}}{-1}\right]_0^\infty$$

$$\therefore Q = \cancel{10}\left[0 + \frac{1}{\underset{+10}{100}}\right] \times \frac{\pi}{2} \times 0+1.$$

$$\therefore \boxed{Q = +\frac{\pi}{20}.\ nC}$$

*





$$ds = r\,d\theta \cdot r\sin\theta\, d\phi$$

$$\boxed{ds = r^2\sin\theta\, d\theta\, d\phi.}$$

$\Rightarrow r = 2m \;,\; 0 \le \theta \le \pi \;,\; 0 \le \phi \le 2\pi$

$\Rightarrow$ This represents a sphere of 2m centered at origin.

$ds = r^2 \sin\theta \, d\theta \, d\phi$

$$\therefore S = \int_{0\,(\theta)}^{\pi} \int_{0\,(\phi)}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$r = 2m$

$$\therefore S = (2)^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$\therefore S = 4\pi(2)^2 \, m^2.$$

$$\therefore \boxed{S = 16\pi \; m^2}$$

Ex-1 Let, $\rho_v = \frac{4}{3} \frac{\cos^2\theta \cdot \sin^2\phi}{r^2(r^2+1)}$ define for universe.

Find the total charge.

Ans: $\boxed{\text{Universe}} \longrightarrow$ $r \rightarrow 0$ to ∞
$\phi \rightarrow 0$ to 2π
$\theta \rightarrow 0$ to π.

$\therefore dv = r^2 \sin\theta \, d\theta \, d\phi.$

$$\therefore Q = \int_v \rho_v \cdot dv$$

$$\therefore Q = \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \frac{4}{3} \cdot \frac{\cos^2\theta \cdot \sin^2\phi}{\cancel{r^2}(r^2+1)} \times \cancel{r^2} \sin\theta \cdot d\theta \, d\phi.$$

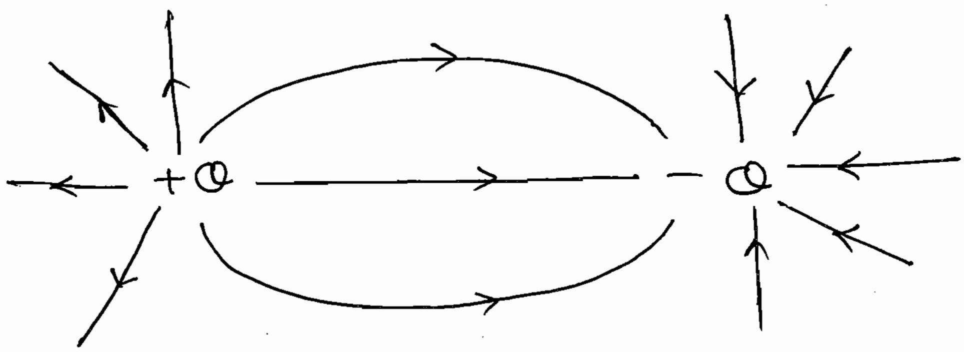
$$= \frac{4}{3}\left[\tan^{-1} r\right]_0^{\infty} \times \left[-\frac{\cos^3\theta}{3}\right]_0^{\pi} \times \left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]_0^{2\pi}.$$

$$= \frac{4}{3}\left[\frac{\pi}{2}\right] \times \left[\frac{1}{3} + \frac{1}{3}\right] \times \left[\pi\right].$$

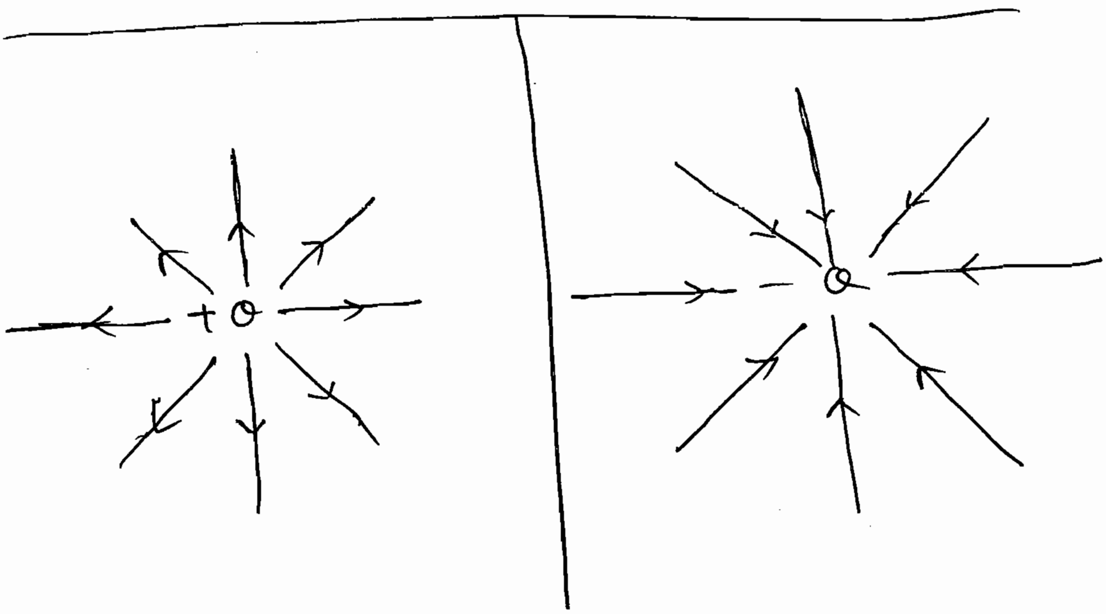
$$\therefore \boxed{Q = \frac{4}{9}\pi^2}$$

* Electric Flux (ψ).

⇒



⇒



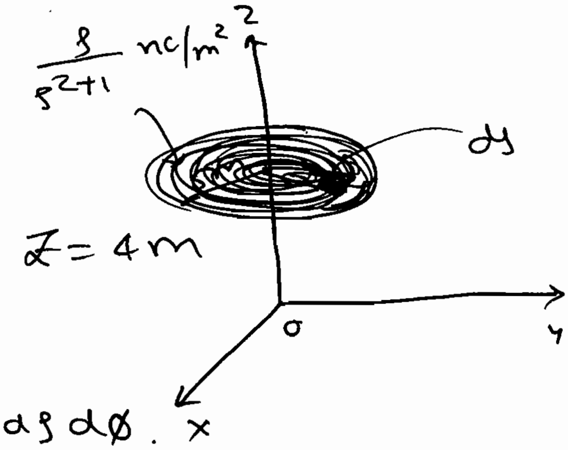
→ An Electric flux originates from a +ve charge and ends with a negative charge. In the absence of -ve charge electric flux terminates at infinity.

→ 1C of electric charge would result 1C of electric flux. rather ~~Q C of~~

→ Q C of electric charge would result Q C of electric flux.

$$\boxed{\psi = Q \ C}$$

Ex-1 How much Electric flux would result from a non-uniform surface charge density $\frac{\rho}{\rho^2+1}$ nC/m² define for $\rho \le 5m$, $z = 4m$.



Ans:

$$ds = \rho \, d\rho \, d\phi.$$

$$\therefore \quad Q = \int_S \rho_s \cdot ds.$$

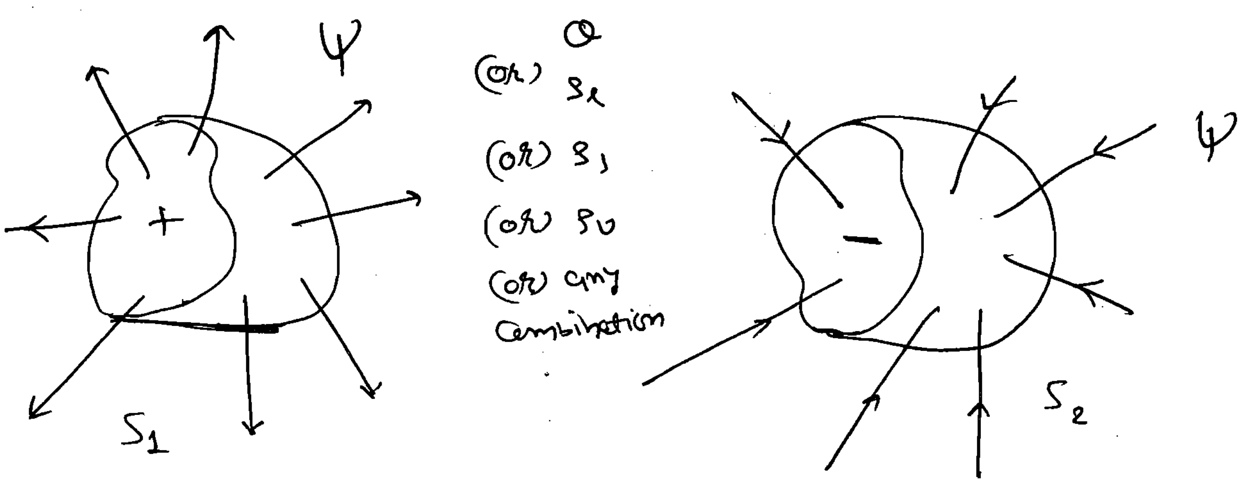
$$\therefore \quad Q = \int_{\rho=0}^{5} \int_{\phi=0}^{2\pi} \frac{\rho}{\rho^2+1} \rho \, d\rho \, d\phi. \quad z = 4m$$

$$\therefore \quad \psi = Q = \int_0^5 \frac{(\rho^2+1)-1}{(\rho^2+1)} d\rho \times [\phi]_0^{2\pi}$$

$$\therefore \quad \psi = \left[[5] - \left[\tan^{-1}\rho\right]_0^5 \right] \times 2\pi$$

$$\therefore \boxed{\psi = [5 - \tan^{-1} 5] \times 2\pi \ nC.}$$

*



S_1 & S_2 = arbitary closed surfaces.

$$\psi_{net} = Q_{enc.}$$

→ S_1 & S_2 are two arbitary closed surfaces. We assume that they enclosed some charge configuration i.e. either Q ~~(are~~ ρ_L (or) ρ_S (or) ρ_v (or) any combination. Some how we ~~are~~ have calculated the total charge within them.

→ Further, we assume that S_1 encloses +ve charge which would results flux living surface. The amount of electric flux living surface is equals to the charge enclosed within it.

→ Further we assume that S_2 encloses -ve charge. ~~which~~ ~~would~~ Therefore, the flux enter the closed surface.

→ Neither the electric flux living the surface or entering the surface, the electric flux passing through the closed surfaces.

→ Gauss's Law's state that the net electric flux passing through any closed surface is equals to the charge enclosed by that surface.

→ $\boxed{\Psi_{net} = Q_{enc}}$

Ex-1 What net electric flux passes through a sphere of radius 2.5m centered at the origin. Given the charge configuration.

(1) point charges of $Q = 2^{-x^2}$ nC. which are located ~~at~~ on the x-axis at $x = 0, \pm 1, \pm 2, \pm 3$ m

Ans: 2.125 nC

(2) An infinite line with a uniform charge density of $1/z^2+1 = \frac{1}{z^2+1}$ nC/m lies along z-axis.

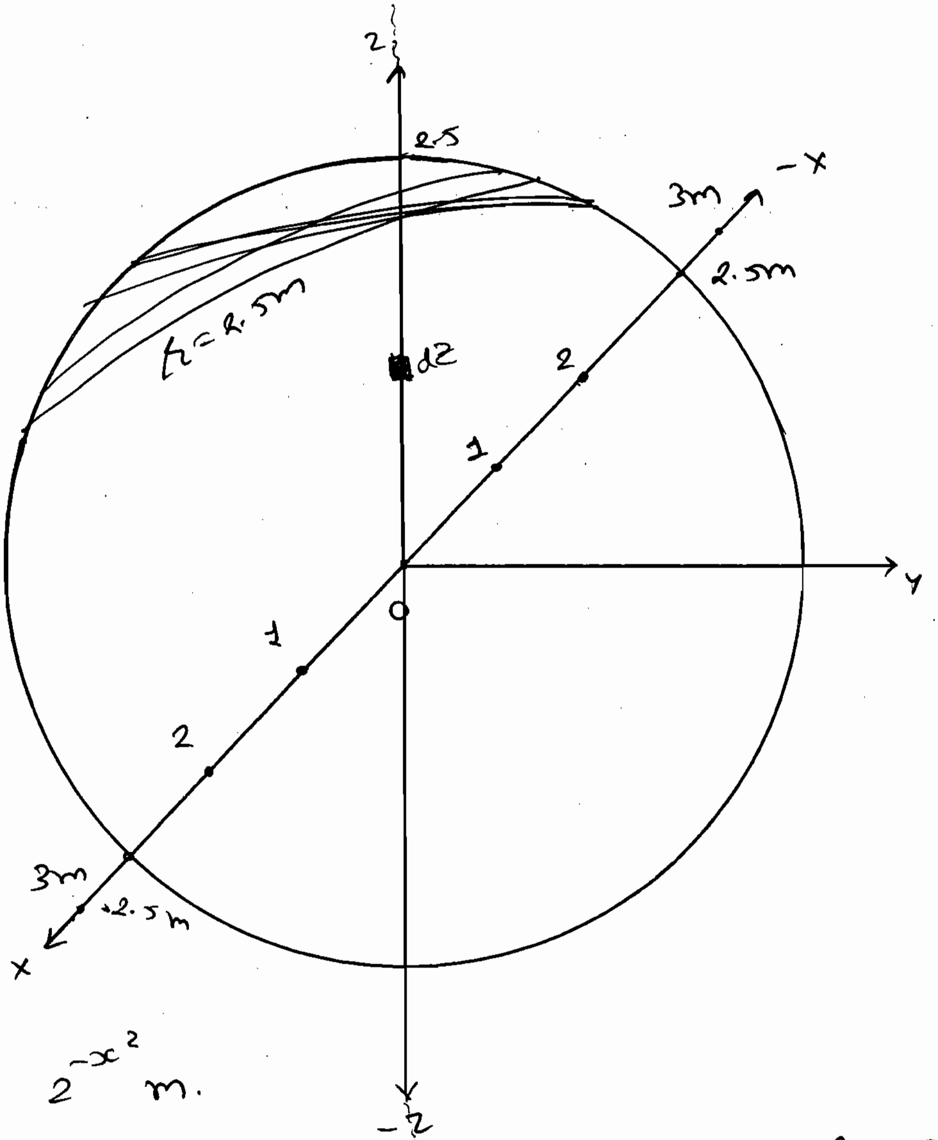
Ans: 2.38 nC

(3) A non-uniform surface charge density of $\frac{1}{x^2+y^2+4}$ nC/m². lies in z = 0 plane.

Ans: 2.95 nC.

(4) Uniform line charge density 20 nC/m, lies in z = 0 plane and are located at $y = 0, \pm 1, \pm 2, \pm 3$ m.

Ans: 403 nC.



① $Q = 2^{-x^2}$ m.

→ The charges at $x = \pm 3m$ are not enclosed by the sphere

$$\psi_{net} = Q_{enc}.$$

$$\therefore \psi_{net} = 2^{-(2)^2} + 2^{-(1)^2} + 2^{-(0)^2} + 2^{-(-1)^2} + 2^{-(-2)^2} + 0$$

$$\therefore \psi_{net} = \frac{1}{16} + \frac{1}{2} + 1 + \frac{1}{16} + \frac{1}{2}$$

$$\boxed{\psi_{net} = 2.125 \text{ nC.}}$$

② Part of the infinite line is enclosed by sphere.

For $|z| \leq 2.5$ (or) $-2.5 \leq z \leq 2.5$.

$$\therefore dQ = \rho_e dz.$$

$$\therefore \psi_{net} = Q_{enc.}$$

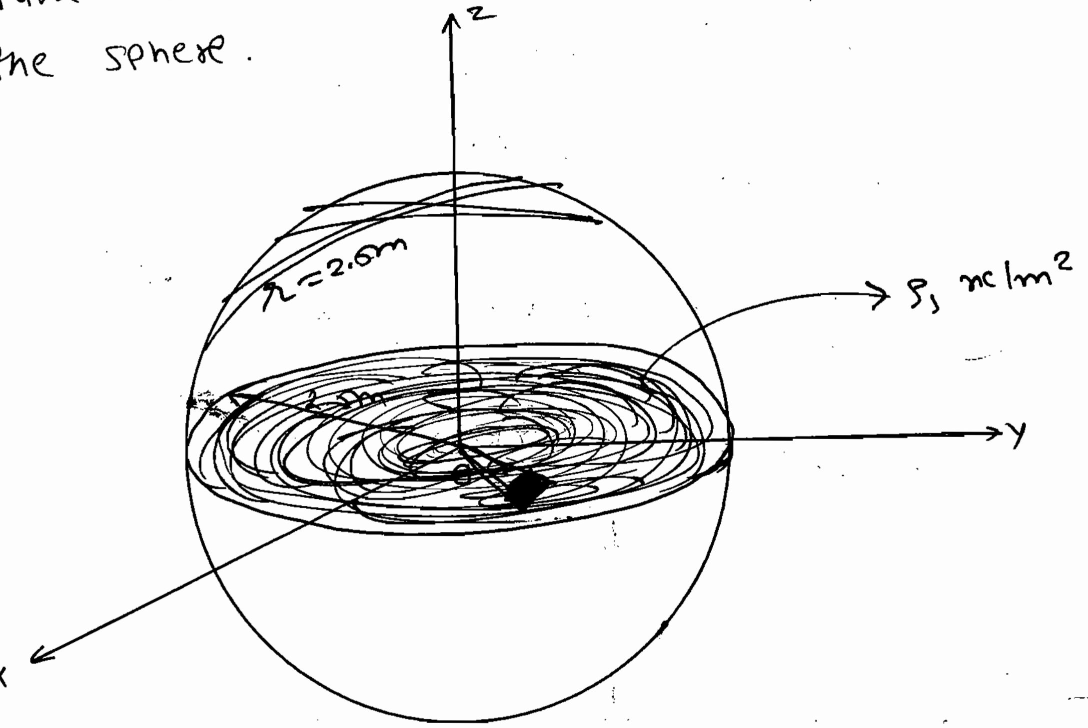
$$\therefore \quad Q_{enc} = \int_{-2.5}^{2.5} \frac{1}{z^2+1}\, dz.$$

$$= \left[\tan^{-1} z\right]_{-2.5}^{2.5}$$

keep the calci in Radiance.

$$\therefore \quad Q_{enc} = 2.38 \text{ nC}$$

(3) Part of the infinite sheet is enclosed by the sphere.



→ Sphere encloses a circular disk of radius 2.5m centered at origin and it located in z=0 plane.

$$\rho_s = \frac{1}{x^2+y^2+z^2} \text{ nc/m}^2.$$

here, $\rho \leq 2.5$ m ; $z=0$

Put $x = \rho\cos\phi$, $y = \rho\sin\phi$

$$\therefore \rho_s = \frac{1}{\rho^2+4} \text{ nc/m}^2$$

& $\quad ds = \rho\, d\rho\, d\phi$

$\therefore dQ = \rho_s\, ds$

$\therefore \psi_{net} = Q_{enclosed}$

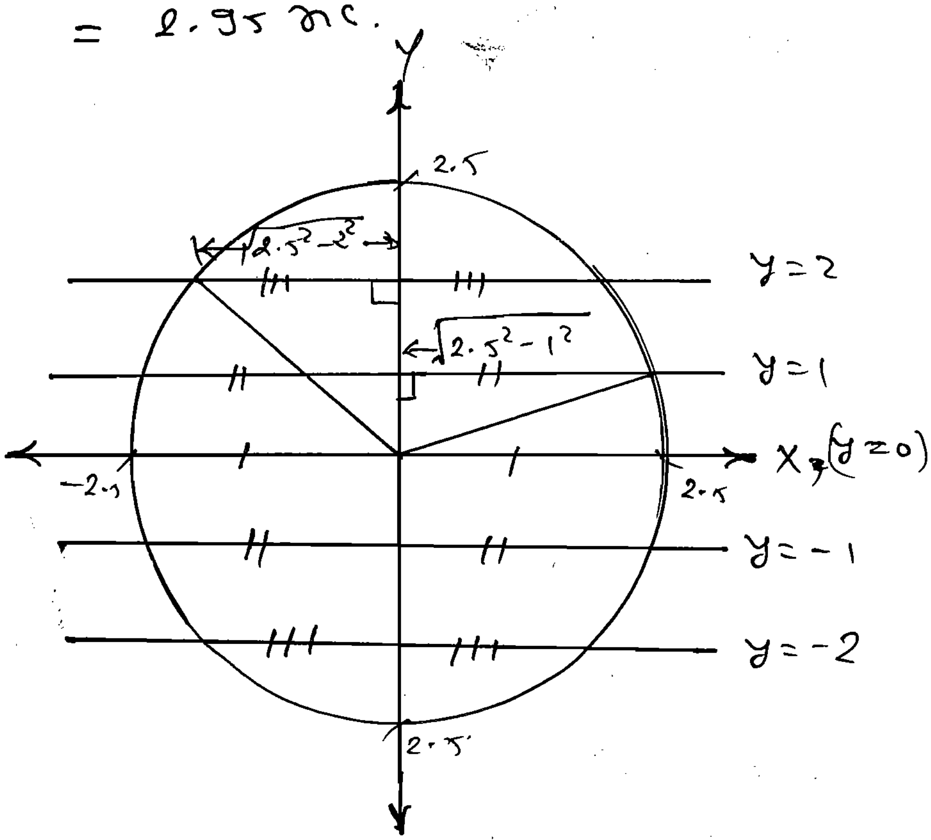
$= \int_s dQ$

$= \int_{\phi=0}^{2\pi} \int_{\rho=0}^{2.5} \frac{\rho\, d\rho\, d\phi}{(\rho^2+4)}$.

$= \frac{1}{2} \left[\ln(\rho^2+4) \right]_0^{2.5} \times [\phi]_0^{2\pi}$

$= 2.95$ nC.

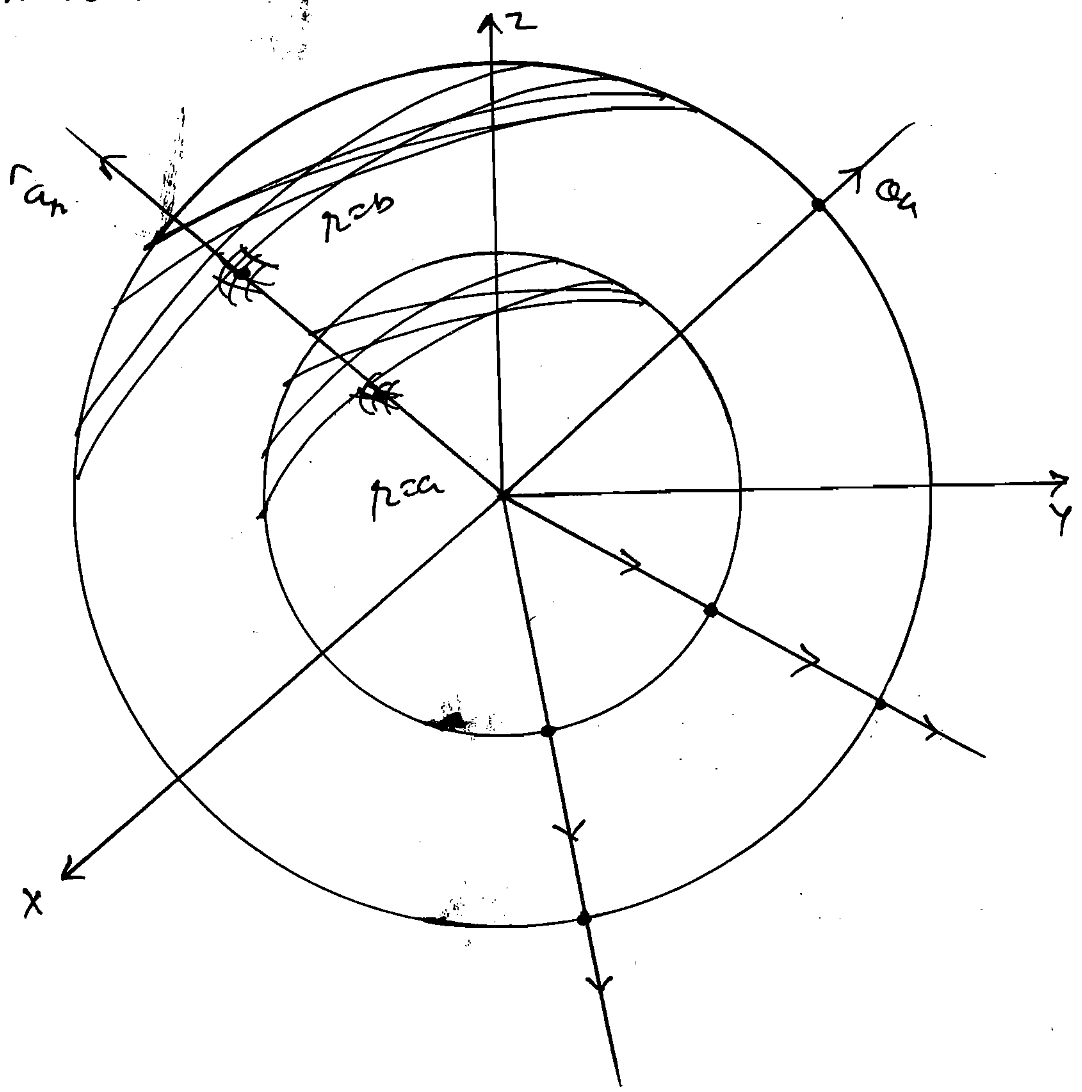
(4)



$\rightarrow \psi_{net} = Q_{enc.} = 20\times10^{-9} \left[4\sqrt{2.5^2-2^2} + 4\sqrt{2.5^2-1^2} \right] + 5 \times 20 \times 10^{-9}$

$= 403$ nC.

* Electric Flux Density [$\bar{D}$ C/m²]



→ Flux per unit Area = Electric Flux Density

→ Through $r=a$

$$\psi_{net} = Q_{enc} = Q$$

Surface area of Sphere $= 4\pi a^2$

$\therefore$ Flux Density $= \dfrac{Q}{4\pi a^2}$ C/m².

Through $r=b$.

Flux density $= \dfrac{Q}{4\pi b^2}$ C/m².

→ In general, the magnitude of the flux density through sphere of radius 'r' m $(r>0)$

$$|\bar{D}| = \frac{Q}{4\pi r^2} \quad C/m^2$$

→ As shown, the flux density is changing its value along radial direction $(\hat{a}_r)$

∴ we write

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\hat{a}_r}$$

$\bar{E}$ due to 'Q' located at the origin is given by

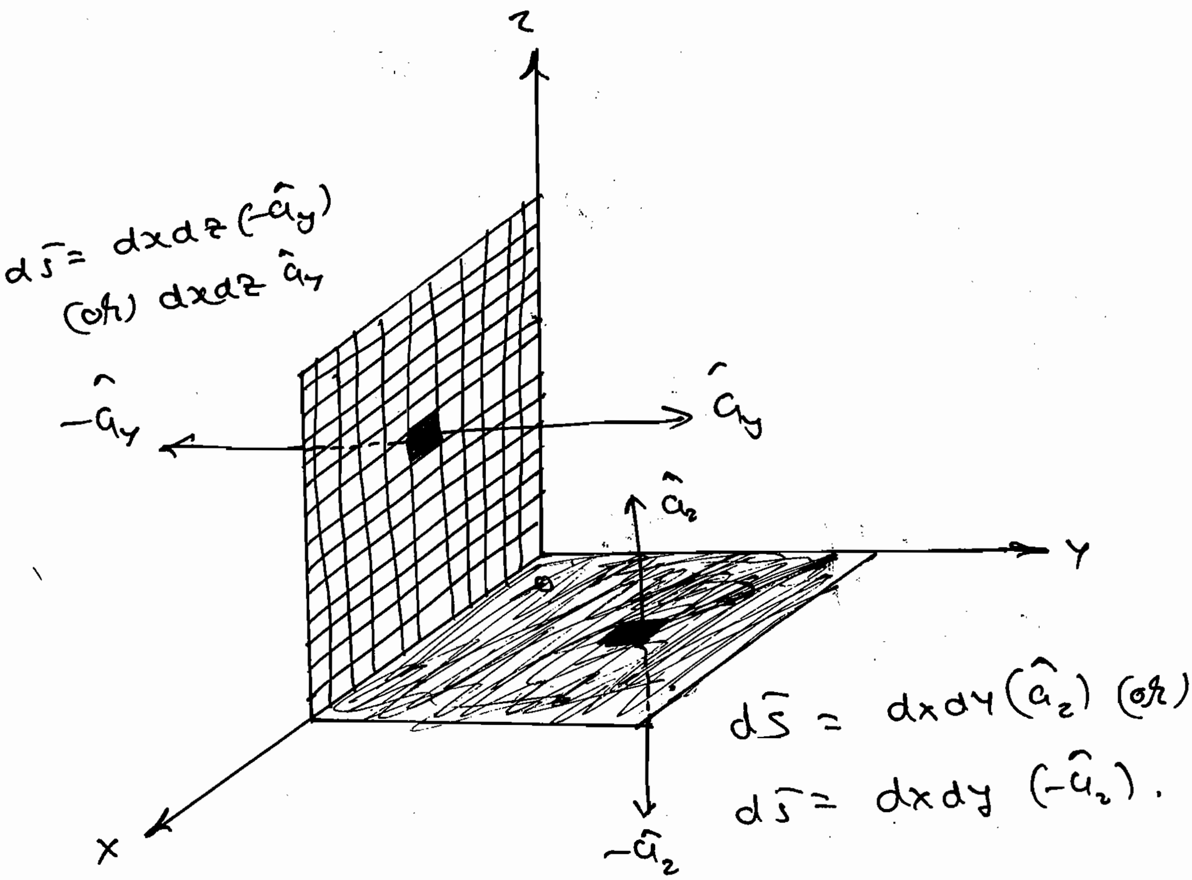
$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon r^2}\hat{a}_r}$$

$$\therefore \boxed{\bar{D} = \epsilon\bar{E}}$$

Procedure for the calculation of $\bar{D}$ and $\bar{E}$ are identically same.

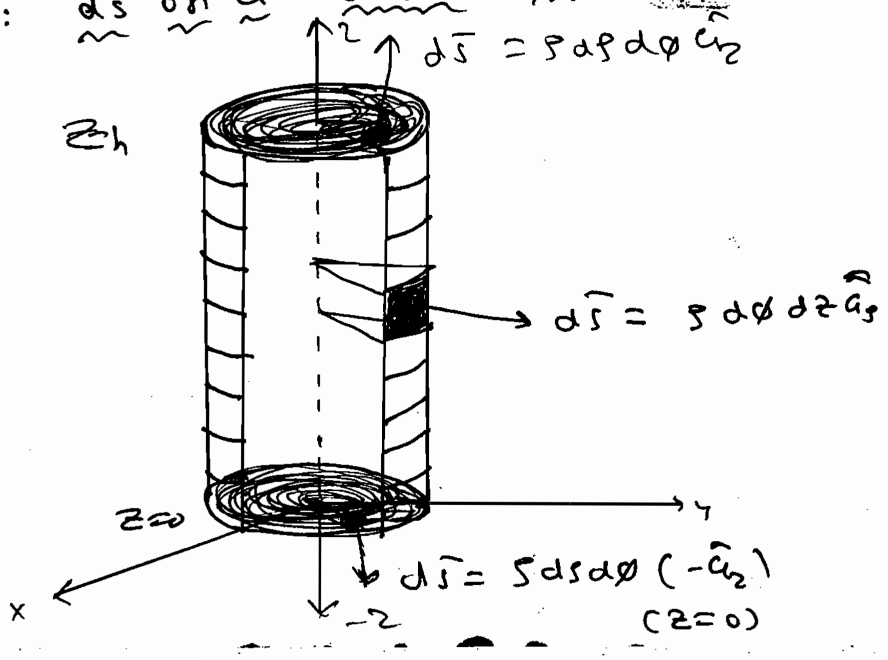
* Vector differential surface element: $d\bar{s}$

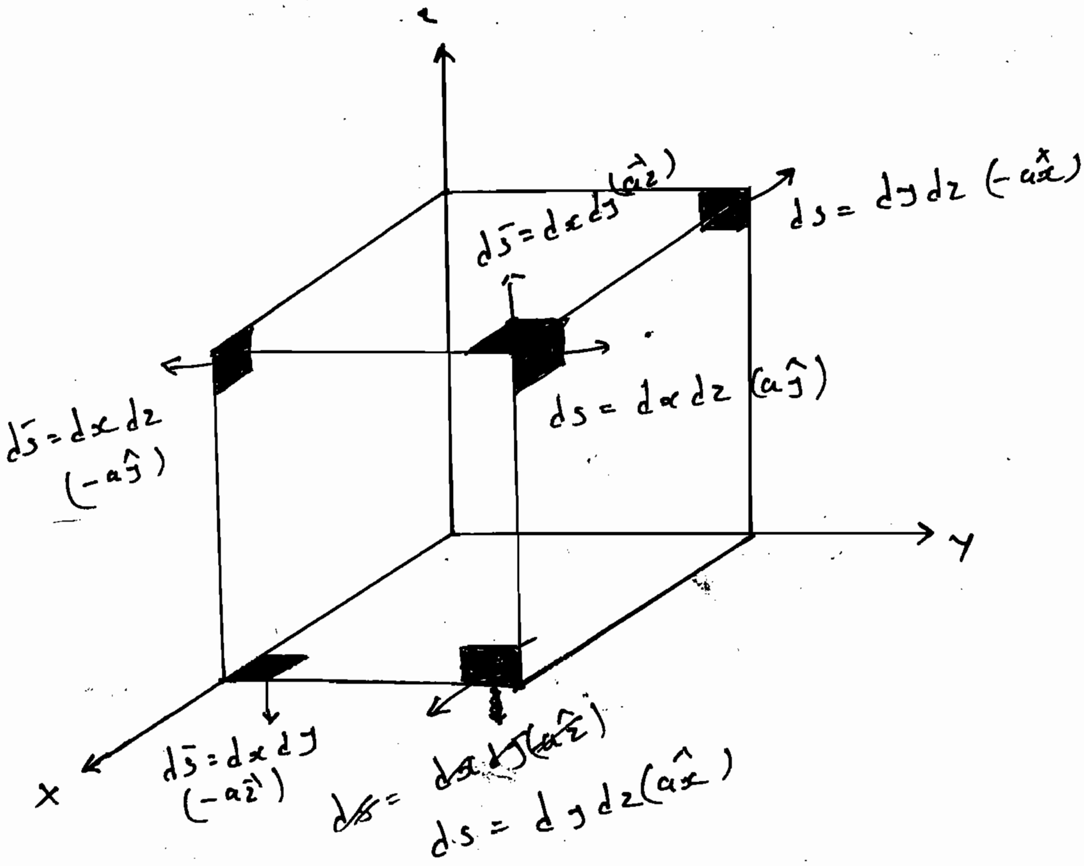
→ Case-1: open plane (or) open surface.



→ The vector differential surface element $d\bar{s}$ at any point on the open plane would be projecting normal to the surface.

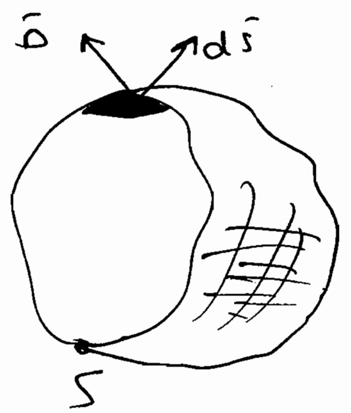
*** Case-2:** $d\bar{s}$ on a closed surface:





→ At any point on the closed surface $d\bar{s}$ could be projecting outward normal to the surface.

* Integral form of Gauss's Law:



→ Figure shows an arbitrary closed surface (S) $d\bar{s}$ would at any point on this surface be projecting outward normal to the surface.

surface is enclosing some charge configuration. Somehow we have calculated $\bar{D}$ at any point on the closed surface

for e.g. $\bar{D}$ makes an angle α with $d\bar{s}$

[the direction of $\bar{D}$ at any point on the closed surface will freely depends upon the charge configuration within the closed surface. whereas direction of $d\bar{s}$ at any point on the closed surface would be projecting outward normal to the surface.].

→ α making any possible value betn 0 to 180°.

→ The differential amount of flux passing through $d\bar{s}$ i.e. in a direction normal to the surface at that point is the projection of $\bar{D}$ on to the $d\bar{s}$.

→ Mathematically,

$$d\varphi = |\bar{D}|.|d\bar{s}| \cos\alpha.$$
$$= \bar{D}.d\bar{s} \quad \text{if } \alpha$$

if $\alpha = 0°$ or $180°$ max. amount of flux passes through $d\bar{s}$

→ If $\alpha = 90°$, zero flux passes through $d\bar{s}$.

→ We write Gauss Law's Integral form.

$$\therefore \boxed{\psi_{net} = \oint_S \bar{D} \cdot d\bar{s} = Q_{enc.}}$$

→ We assume that the closed surface is enclosing a volume charge density of ρ_v C/m³.

$$\therefore Q_{enc} = \int_v \rho_v \, dv$$

$$\therefore \oint_S \bar{D} \cdot d\bar{s} = \int_v \rho_v \, dv. \quad — ①$$

Now, using Divergence theorem.

$$\therefore \oint_S \bar{D} \cdot d\bar{s} = \int_V \nabla \cdot \bar{D} \, dV. \quad — ②$$

$\therefore$ Comparing ① & ②

$$\therefore \boxed{\nabla \cdot \bar{D} = \rho_v}$$

→ This is a point form of Gauss Law.

(or) Gauss Divergence theorem.

$$\rightarrow \nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}.$$

$\rightarrow \nabla \cdot \bar{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}.$

$\rightarrow \nabla \cdot \bar{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta D_\theta).$

$+ \frac{1}{r \sin\theta} \frac{\partial D_\phi}{\partial \phi}.$

Ex-1 In a region the electric flux density is given by $\bar{D} = (2x\,\hat{a}_x + 3y\,\hat{a}_y - kz\,\hat{a}_z)$ C/m². assume Charge free region then find the value of k.

Ans: Charge free region $\rho_v = 0$

$\therefore \nabla \cdot \bar{D} = \rho_v = 0$

$\therefore \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \partial/\partial z\, D_z = 0.$

$\therefore 2 + 3 - k = 0$

$\boxed{k = 5}$

$\therefore -kz = $ C/m²

$\therefore +kz = 5$ C/m²

$\boxed{k = 5 \text{ C/m}^3}$

(a) 5 C/m²

(b) 5 C/m

(c) 5 C-m.

(d) 5 C/m³ ✓

$\underline{\underline{Ex-2}}$ The magnitude of the Electric flux density is proportional to r^K where k is constant. $r \rightarrow$ spherical coordinate. The $\bar{D}$ is projecting in the radial direction. Choose the value of K such that electric flux density has zero divergence.

Ans: (a) -2 (b) -4 (c) -8 (d) -16.

$\rightarrow$

$$|\bar{D}| \propto r^K$$

$$\therefore \bar{D} = C_1 r^K \cdot \hat{a}_r = D_r \hat{e}_r.$$

$$\therefore \nabla \cdot \bar{D} = 0.$$

$$\therefore \cancel{\frac{1}{r^2}\left(r^2 \frac{\partial D_r}{\partial r}\right) = 0}.$$

$$\therefore \cancel{\frac{1}{r^2} r^2 \times C_1 r^k}$$

$$\therefore \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) = 0.$$

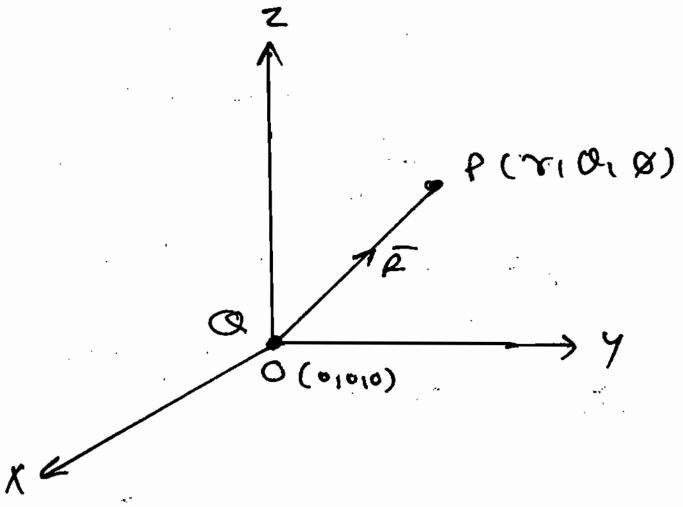
$$\therefore \frac{\partial}{\partial r}\left(C_1 r^{K+2}\right) = 0$$

$$\therefore (K+2)\, r^{K+1} = 0.$$

$$\boxed{K = -2}$$

Ex-3 A point charge of Q c is located at origin. If no other charge is present what is the value $\nabla\cdot\bar{D}$ at any point other than the origin.

Ans: $\boxed{\nabla\cdot\bar{D}=0}$



at P $\bar{D}=\frac{Q}{4\pi r^2}\hat{a}_r$

$\bar{D}=D_r\hat{a}_r$

$\therefore\ \nabla\cdot\bar{D}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r)+\ldots$

$=\frac{1}{r^2}\,\partial/\partial r\left(\cancel{r^2}\cdot\frac{Q}{4\pi\cancel{r^2}}\right)$

$\therefore\ \boxed{\nabla\cdot\bar{D}=0.}$

Ex-4 Let $\bar{D}=(4x^3\hat{a}_x+x^2z\,\hat{a}_y+2xy\,\hat{a}_z)$ nC/m² $0\le x,y,z\le 1$. Find the amount of ψ passing through closed surface defined by $0\le x,y,z\le 1$. and also find the amount of charge enclosed within it. also indicate weither the flux entering a close surface or

Ans:

$$\psi_{net}=\oint_S\bar{D}\cdot d\bar{s}=\oiiint \rho_v\,dv=Q_{enclosed}.$$

$\rightarrow \Psi_{net} = Q_{enc} = \oint_{S} \bar{D}\cdot d\bar{s} = \int_{x=0} \bar{D}\cdot d\bar{s} + \int_{x=1} \bar{D}\cdot d\bar{s}$

$$+ \int_{y=0} \bar{D}\cdot d\bar{s} + \int_{y=1} \bar{D}\cdot d\bar{s} + \int_{z=0} \bar{D}\cdot d\bar{s} + \int_{z=1} \bar{D}\cdot d\bar{s}$$

Surface is at	$d\bar{s}$	surface limit	$\bar{D}\cdot d\bar{s}$	$\bar{D}\cdot d\bar{s}$ on the surface	$\int_s \bar{D}\cdot d\bar{s}$.
$x=0$	$dydz(-\hat{a}_x)$	$0\le y,z\le 1$	$-4x^3\,dydz$	$-4(0)^3\,dydz$	$-4\int_0^1\int_0^1 (0)^3 dydz = 0.$
$x=1$	$dydz\,(\hat{a}_x)$	$0\le y,z\le 1$	$+4x^3\,dydz$	$+4\,dydz$	$-4\int_0^1\int_0^1 dydz = 4.$
$y=0$	$dxdz\,(-\hat{a}_y)$	$0\le x,z\le 1$	$-x^2z$	$-x^2z\,dxdz$ ~~$-1dxdz$~~	$-\int_0^1\int_0^1 -x^2z\,dxdz = -1/6$
$y=1$	$dxdz\,(\hat{a}_y)$	$0\le x,z\le 1$	$+x^2z$	$+x^2z\,dxdz$	$+\int_0^1\int_0^1 x^2z\,dxdz = 1/6.$
$z=0$	$dxdy\,(-\hat{a}_z)$	$0\le x,y\le 1$	$-2xy$	$-2xy\,dxdy$	$-2\int_0^1\int_0^1 = xydxdy = -1/2.$
$z=1$	$dxdy\,(\hat{a}_z)$	$0\le x,y\le 1$	$2xy$	$2xy\,dxdy$	$+2\int_0^1\int_0^1 xydxdy = 1/2$

$Q_{enc} = 4nC.$

$\rightarrow Q_{enc} = +4\,nC.$

$\rightarrow$ The enclosed charge is +ve therefore, the electric flux ~~[illegible]~~ leaving the closed surface.

$\rightarrow \nabla\cdot\bar{D} = 12x^2 + 0 + 0 = \rho_v$

$\rho_v = 12x^2$

$\therefore Q_{enclosed} = \int_v \rho_v dv = \int_0^1\int_0^1\int_0^1 12x^2\,dydydz$

$\therefore Q_{enc} = 12 \times \left[\frac{x^3}{3}\right]_0^1 [y]_0^1 [z]_0^1$

$= \frac{12}{3}$

$\therefore \boxed{Q_{enc} = 4\ nC.}$

Ex-5 → Let, $\bar{D} = 12x^2yz\,\hat{a}_x + 2xy\,\hat{a}_y + 3x^2z\,\hat{a}_z$ nC/m^2. find the amount of Electric flux passing through a surface define by $x = 1$, $0 \le y, z \le 2$. $\hat{a}_x$ direction.

Ans: $\bar{D} = \underline{12x^2yz\,\hat{a}_x} + 2xy\,\hat{a}_y + 3x^2z\,\hat{a}_z$

$\therefore Q_{enc} = \int_0^2\int_0^2 12yz\,dy\,dz.$

$d\bar{S} = dy\,dz\,\hat{a}_x$

$\bar{D}\cdot d\bar{S} = 12x^2yz.$

at $x = 1$

$\bar{D}\cdot d\bar{S} = 12yz.$

$= \frac{12}{4}[y^2]_0^2[z^2]_0^2$

$= \frac{12}{4} \times 4 \times 4$

$\therefore \boxed{Q_{enc} = 48\ nC}$

$\therefore \boxed{\psi_{net} = Q_{enc} = 48\ nC}$

Ex-6 Let, $\bar{D} = \frac{r}{3}\,\hat{a}_r$ nC/m^2 where r is a unit vector in the Spherical Co-ordinat, $\hat{a}_r$ is a unit vector in the radial direction. find ρ_v, amount of electric flux passing through a sphere of radius 1m. centered at the origin. also indicate the electric flux leaving the surface or entering the surface.

Ans: $\rho_v = \nabla\cdot\bar{D}$

$$\therefore \rho_v = \nabla \cdot \bar{D}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot D_r).$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot \frac{r}{3} \right).$$

$$= \frac{3r^2}{3r^2}$$

$$\boxed{\rho_v = 1 \ nC/m^3}$$

$$\therefore Q_{enclose} = \int_v \rho_v \, dv.$$

$$= \int_0^1 \int_0^{2\pi} \int_0^{\pi} \rho_v \cdot r^2 \sin\theta \cdot dr \, d\theta \, d\phi.$$

$$= \int_0^1 \int_0^{2\pi} \int_0^{\pi} r^2 \cdot \sin\theta \cdot dr \, d\theta \, d\phi.$$

$$= \frac{1}{3} \times 2\pi \times [-\cos\theta]_0^{\pi}$$

$$\boxed{Q_{enclosed} = \frac{4\pi}{3} = \psi_{net} \ nC.}$$

$$\therefore Q_{enclosed} = \oint_S \bar{D} \cdot d\bar{S}$$

$$d\bar{S} = r^2 \sin\theta \cdot d\theta \, d\phi \cdot \hat{a}_r.$$

$$\therefore \bar{D} \cdot d\bar{S} = \frac{r^3}{3} \sin\theta \cdot d\theta \, d\phi.$$

$$\therefore Q_{enclos} = \int_0^{\pi} \int_0^{2\pi} \frac{r^3}{3} \sin\theta \cdot d\theta \, d\phi.$$

$$r = 1 \, m$$

$$= \frac{1}{3}[2\pi][-\cos\theta]_0^{\pi}$$

$$= \frac{1}{3} \times 2\pi \times 2$$

$$\therefore \boxed{Q_{enclosed} = \frac{4\pi}{3}\, nC = \psi_{net}}$$

→ flux Leaving the sphere. as Q is +ve.

Ex-6 Let, $\bar{D} = -\frac{20}{\rho^2}[\sin^2\phi\, \hat{a}_\rho + \sin 2\phi\, \hat{a}_\phi]\ nC/m^2$ find the amount of electric flux passing through a closed region define by $1 \le \rho \le 2$, $0 \le \phi \le \frac{\pi}{2}$, $0 \le z \le 1$.

(a)

Ans:

$$\bar{D} = -\frac{20}{\rho^2}[\sin^2\phi\, \hat{a}_\rho + \sin 2\phi\, \hat{a}_\phi]\ nC/m^3.$$

$$\therefore \nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(D_\phi).$$

$$= \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(-\frac{20}{\rho}\sin^2\phi\right) + \frac{1}{\rho}\,\partial/\partial\phi\left[-\frac{20}{\rho^2}\sin 2\phi\right]$$

$$\nabla\cdot\bar{D} = \frac{+20\sin^2\phi}{\rho^3} + \left(-\frac{40}{\rho^3}\cos 2\phi\right]$$

$$\therefore \rho_v = \frac{20\sin^2\phi}{\rho^3} - \frac{40}{\rho^3}\cos 2\phi = \frac{20}{\rho^3}\left[\frac{1}{2} - \frac{\cos 2\phi}{2} - 2\cos 2\phi\right]$$

$$= \frac{20}{\rho^3}\left[\frac{1}{2} - \frac{5}{2}\cos 2\phi\right].$$

$$\boxed{\rho_v = 20/\rho^3\left[\frac{1}{2} - \frac{5}{2}\cos 2\phi\right]}$$

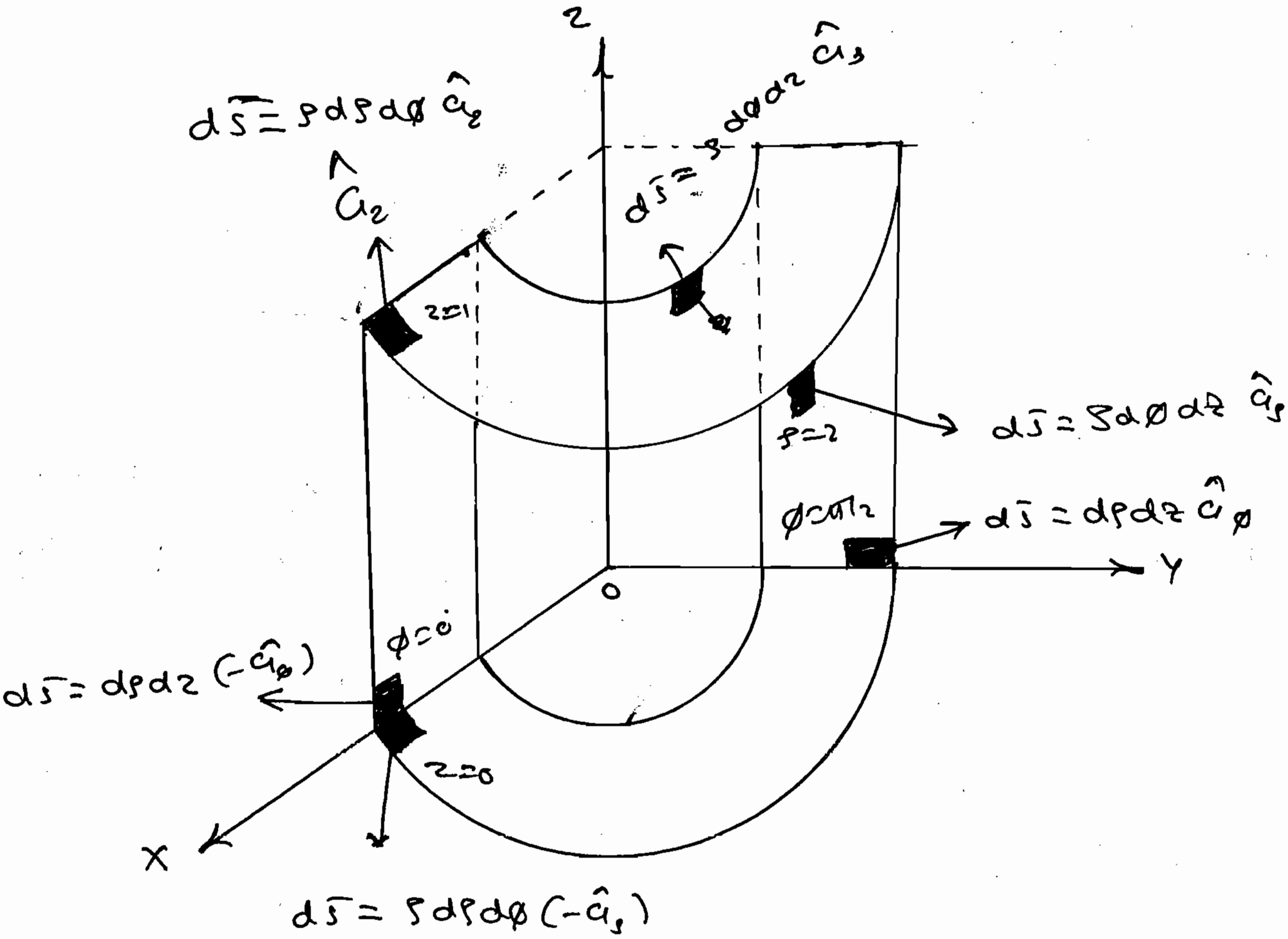
$$\therefore Q_{enclosed} = \psi_{net} = \int_v \rho_v\, dv$$

$$= \int_1^2\int_0^{\pi/2}\int_0^1 \frac{20\sin^2\phi}{\rho^3}\cdot \rho\, d\rho\, d\phi\, dz.$$

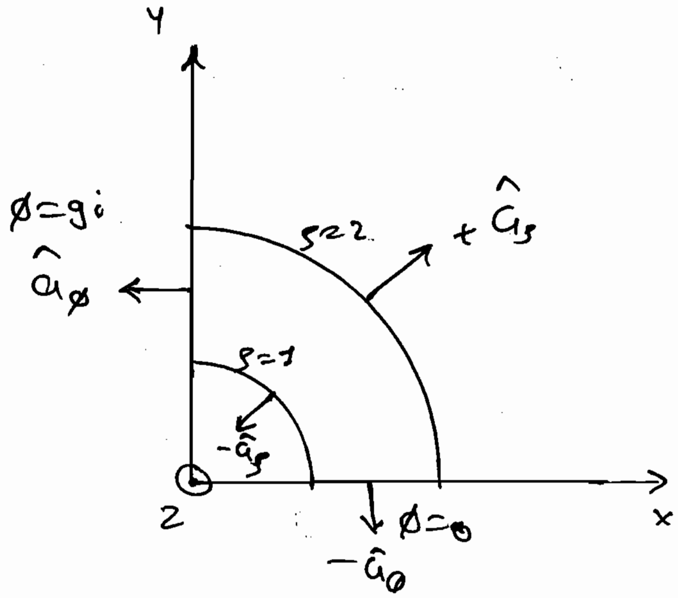
$$= 20\left[-\frac{1}{\rho}\right]_1^2 \times [z]_0^1 \left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]_0^{\pi/2} + 9.$$

$$= +10 \times \frac{\pi}{4} = +2.5\pi\ nC.$$

→



z
$d\bar{s} = \rho\, d\rho\, d\phi\, \hat{a}_z$
$\hat{a}_z$
$d\bar{s} = \rho\, d\phi\, dz\, \hat{a}_\rho$
z=1
$\rho=2$
$d\bar{s} = \rho\, d\phi\, dz\, \hat{a}_\rho$
$\phi=\pi/2$
$d\bar{s} = d\rho\, dz\, \hat{a}_\phi$
Y
0
$\phi=0$
$d\bar{s} = d\rho\, dz\, (-\hat{a}_\phi)$
z=0
X
$d\bar{s} = \rho\, d\rho\, d\phi\, (-\hat{a}_z)$



Y
$\phi=90$
$\hat{a}_\phi$
$\rho=2$
$+\hat{a}_\rho$
$\rho=1$
$-\hat{a}_\rho$
z
$\phi=0$
$-\hat{a}_\phi$
x

* Work:

→ Work is defined as a force acting over distance.

→ Work done in moving a charge of Q Columbs from an initial point to the final point in the vicinity of electric field is given by

$$\boxed{W = -Q \int_{\text{initial}}^{\text{final}} \bar{E} \cdot d\bar{l} \quad \text{Joules.}}$$

Ex-1 find a W.D. in moving a 5 μC Charge from the origin to (2,-1,4) m through the field $(2xyz\,\hat{a}_x + x^2z\,\hat{a}_y + x^2y\,\hat{a}_z)$ V/m. via the path (0,0,0) to (2,0,0) to (2,-1,0) to (2,-1,4).

Ans: $d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$.

$$\bar{E} \cdot d\bar{l} = 2xyz\,dx + x^2z\,dy + x^2y\,dz.$$

$$W.D. = -Q \int_{\text{initial}}^{\text{final}} 2xyz\,dx + x^2z\,dy + x^2y\,dz.$$

$$d\bar{l} = d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

→ $d\bar{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$.

$$W.D. = -q\left[\int_1 \bar{E}\cdot d\bar{l} + \int_2 \bar{E}\cdot d\bar{l} + \int_3 \bar{E}\cdot d\bar{l}\right].$$

$\therefore$ ① Path-1 $(0,0,0)$ to $(2,0,0)$.

$\therefore$ $x \to 0$ to 2

$\therefore$ $x = 2$.

$y = 0 \Rightarrow dy$

$z = 0 \quad = dz = 0.$

$$\therefore \quad I_1 = \int_0^l \bar{E}\cdot d\bar{l} = \int_0^2 0. = 0.$$

② Path-2: $(2,0,0)$ to $(2,-1,0)$

$\therefore$ $x = 2 \Rightarrow dx = dz = 0$

$y = 0$ to -1.

$z = 0$.

$$\therefore \quad I_2 = \int_0^{-1} 4y\,(0) = 0.$$

③ Path-3: $(2,-1,0)$ to $(2,-1,4)$.

$x = 2 \Rightarrow dx = 0$

$y = -1 \Rightarrow dy = 0$.

$z = 0$ to 4,

$$I_3 = \int_0^4 -4.\,dz.$$

$$I_3 = -16$$

$$\therefore \quad W.D = -q[I_1 + I_2 + I_3]$$

$$= -5\times10^{-6}[-16]$$

$$\therefore \boxed{W.D. = 80\,\mu J}$$

Ex. 2 Repeate the above example ~~[illegible]~~ the path $x = -2y$, $z = 2x$. $(0,0,0)$ to $(2,-1,4)$.

Ans: $d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$.

$\therefore\ \bar{E}\cdot d\bar{l} = 2xyz\,dx + x^2 z\,dy + x^2 y\,dz$.

$$W = -Q\int_{\text{initial}}^{\text{final}} \bar{E}\cdot d\bar{l} = -5\times10^{-6}\left[2\int_0^2 xyz\,dx + \int_0^{-1} x^2 z\,dy + \int_0^4 x^2 y\,dz\right]$$

$$= \cancel{-5\times10^{-6}\left[2\int_0^2 -x^3\,dx + \int_0^{-1} 2x^3\,dx + \int_0^4 -x^3\,dx\right]}.$$

$$= \cancel{-5\times10^{-6}\left[-2\left[\tfrac{1}{16}\right] - \left[\tfrac{1}{1}\right] - \left[\tfrac{1}{256}\right]\right]}.$$

$$= \cancel{5\times10^{-6}\left[\tfrac{1}{8} + 1 + \tfrac{1}{256}\right]}.$$

$\cancel{= 5\times10^{-6}}$

$$W = -5\times10^{-6}\left[2\int_0^2 -x^3\,dx + \int_0^{-1} 8y^3\,dy + \int_0^4 -\frac{z^3}{8}\,dz\right].$$

$$\therefore W = -5\times10^{-6}\left[-\frac{1}{8} + \frac{8}{4}^{2} - \frac{64\ 8}{8}\right]$$

$$= -5\times10^{-6}\left[-6 - \tfrac{1}{8}\right].$$

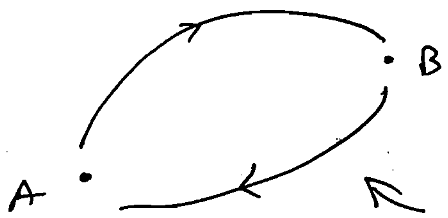
$$W = \frac{49\times5}{8}\times10^{-6}\ \text{J}.$$

→ (x_1, y_1, z_1) to (x_2, y_2, z_2).

$\therefore$ (x_1, y_1) to (x_2, y_2) $\quad$ (x_1, z_1), (x_2, z_2).

$$\therefore \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}, \quad \frac{z - z_1}{z_2 - z_1} = \frac{x - x_1}{x_2 - x_1}.$$

*



Closed path.

→ $-Q \oint_l \bar{E} \cdot d\bar{l} = 0 \Rightarrow$ 'Q' can't be zero.

$$\therefore \boxed{\oint_l \bar{E} \cdot d\bar{l} = 0} \Rightarrow$$ <u>Work done over a closed path is ZERO.</u>

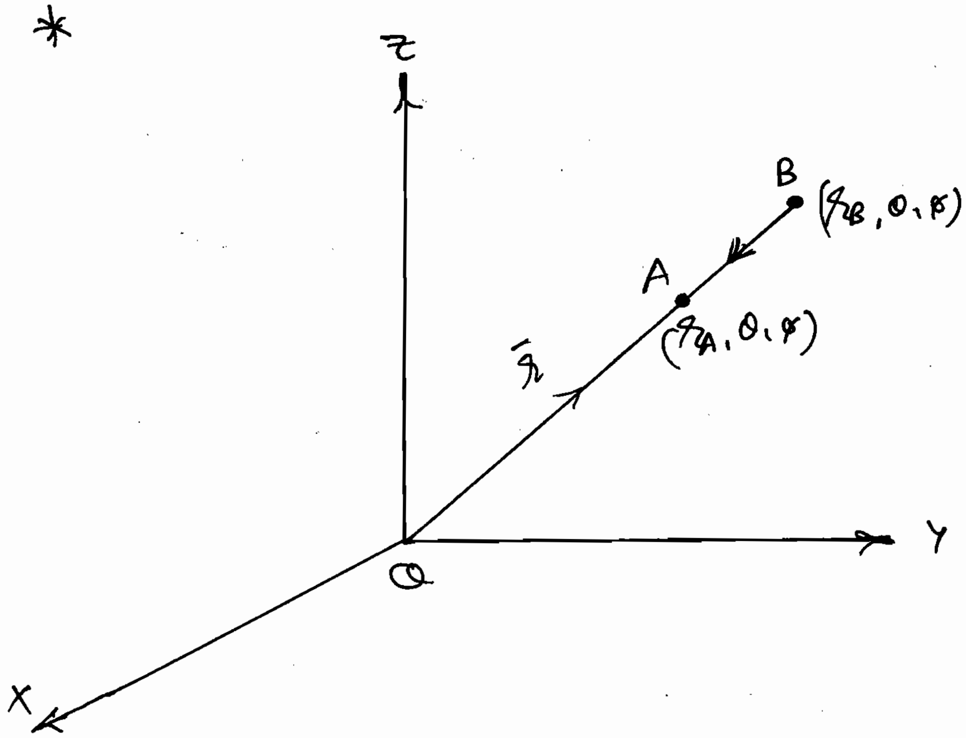
* Potential:

$\rightarrow$ $$W = -Q \int_{B}^{A} \bar{E} \cdot d\bar{l} \quad J$$

We define the potential at 'A' with ref. to 'B' is given by,

$$V_{AB} = \frac{W}{Q} = -\int_{B}^{A} \bar{E} \cdot d\bar{l} \quad J/C \text{ (or) Volts.}$$

*



$\rightarrow$ $$\bar{E} = \frac{Q}{4\pi\epsilon r^2} \hat{a}_r$$

From B to A $\quad d\bar{l} = dr\, \hat{a}_r$.

$$\therefore \bar{E} \cdot d\bar{l} = \frac{Q}{4\pi\epsilon r^2} \cdot dr.$$

$$V_{AB} = -\int_{r_B}^{r_A} \frac{Q}{4\pi r^2 \epsilon} dr.$$

$$\therefore \quad V_{AB} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_A} - \frac{1}{r_B}\right].$$

if r_B is choosen at infinity,

$$r_B \to \infty$$

$$\frac{1}{r_B} = 0.$$

Then,

$$V_{AB} = \frac{Q}{4\pi\epsilon r_A} = V_A.$$

In general

$$V_P = \frac{Q}{4\pi\epsilon|\bar{R}|} + C.$$

V_P is the potential at P due to 'Q'. $|\bar{R}|$ is the distance b/w the charge 'Q' and the observation point 'P'.

C = 0 if the ref. point is choosen at infinity.

* Potential Function:

→ Potentials is a functions of space co-ordinates V (x, y, z) (or)

V (ρ, φ, z) (or)

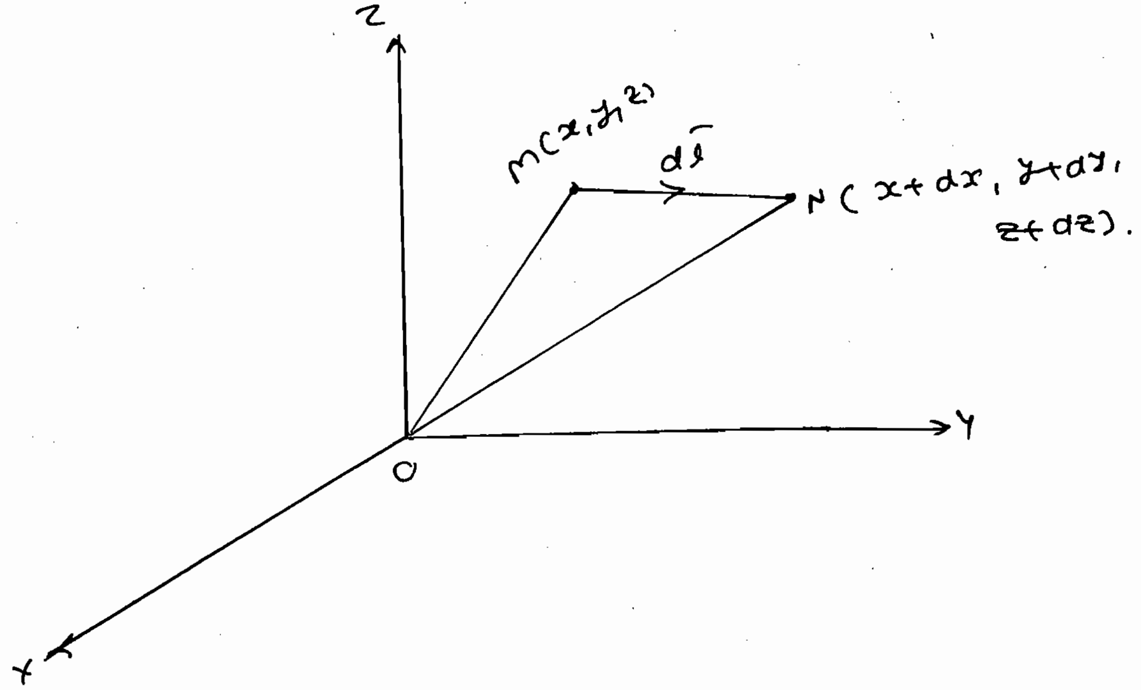
V (r, θ, φ).

* Relation b/w Potential Gradient and Electric field:

→ We assume that two neighbourhood points M, N because of some charge configuration we have known the potential function $V(x,y,z)$.

→ Further, we assume that potential at M is different from potential at N and there exist a potential difference of dV volts.

→



→(1) $d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z.$

$V(x,y,z)$ is known.

(2) $dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}.dz.$

• We introduce ∇ (or) Del (or) Gradient operator

→ Gradient of scalar function in vector.

(3) $\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z.$

② interms of ① & ③.

$$dV = \nabla V \cdot d\bar{l} \quad -④$$

$$\therefore V = -\int \bar{E} \cdot d\bar{l}$$

$$\therefore dV = -\bar{E} \cdot d\bar{l} \quad -⑤$$

From ④ & ⑤

$$\therefore -\bar{E} \cdot d\bar{l} = \nabla V \cdot d\bar{l}$$

suppressing $d\bar{l}$ on both sides.

$$\boxed{\bar{E} = -\nabla V}$$

(I) Electric field is the gradient of the scalar electric potential functions.

(II) (a) From eq-⑤ we can conclude that electric field projects normal to an equipotential surface

(b) $\bar{E}$ would be project from a higher potential surface to towards lower potential surface.

* Equipotential surface:

→ It is that surface on which the potential difference betn any two points is '0'.

→ we assume that the point m and n lies on equipotential surface.

(c) From eqn ④ we can conclude that potential can vary its value normal to an equipotential surface.

*
$\rightarrow \nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \rightarrow V(x,y,z)$

$\rightarrow \nabla . V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z \rightarrow V(\rho,\phi,z)$

$\rightarrow \nabla . V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta} . \frac{\partial V}{\partial \phi} . \hat{a}_\phi . \rightarrow V(r,\theta,\phi).$

Ex:- ~~ØØ~~
$\Rightarrow$ In a certain region the potential field distribution is given by $V(r) = 100\sqrt{r}$ volts where, r is spherical coordinates assume medium to be free space. Find $\bar{E}$, $\bar{D}$ & amount of flux passing through a sphere of radius 5m. centered at origin. also ~~indicate~~ find charge enclosed and also indicate the flux leaving the surface or entering the surface.

Ans:
$V(r) = 100(r)^{1/2}$

$\therefore \bar{E} = -\nabla . V$

$\therefore \bar{E} = -\frac{d}{dr}(100 r^{1/2}).$

$= -100 \times \frac{1}{d\sqrt{r}}\hat{a}_r$

$\therefore \bar{E} = \frac{-50}{\sqrt{r}}\hat{a}_r \text{ V/m.}$

$\therefore \bar{D} = \epsilon \bar{E}$

$\therefore \bar{D} = -\frac{50\epsilon_0}{\sqrt{r}}\hat{a}_r \text{ C/m}^2$

$\therefore Q = \oint_S \bar{D} \cdot d\bar{S} = \int \cancel{\rho_v} \psi.$

$\therefore Q = \int_v \rho_v \, dv$

$\therefore \quad \nabla \cdot \bar{D} = +\frac{50}{2} \epsilon_0 \, r^{-3/2}.$

$$\therefore \quad Q_{enclosed} = \int_0^5 \int_0^{\pi} \int_0^{2\pi} \rho_v \, r^2 \sin\theta \cdot dr\, d\theta\, d\phi.$$

$$= [2\pi] \left[-\cos\theta\right]_0^{\pi} \times 25 \times \int_0^5 (r)^{\frac{1}{2}} \cdot dr.$$

$$= 100\pi \times \frac{2}{3} (5)^{3/2}.$$

$$\bar{D} \cdot d\bar{s} = -50 \epsilon_0 r^{3/2} \sin\theta . d\theta d\phi.$$

at $r = 5m$

$$\bar{D} \cdot d\bar{s} = -50 \epsilon_0 (5)^{3/2} \sin\theta \, d\theta d\phi.$$

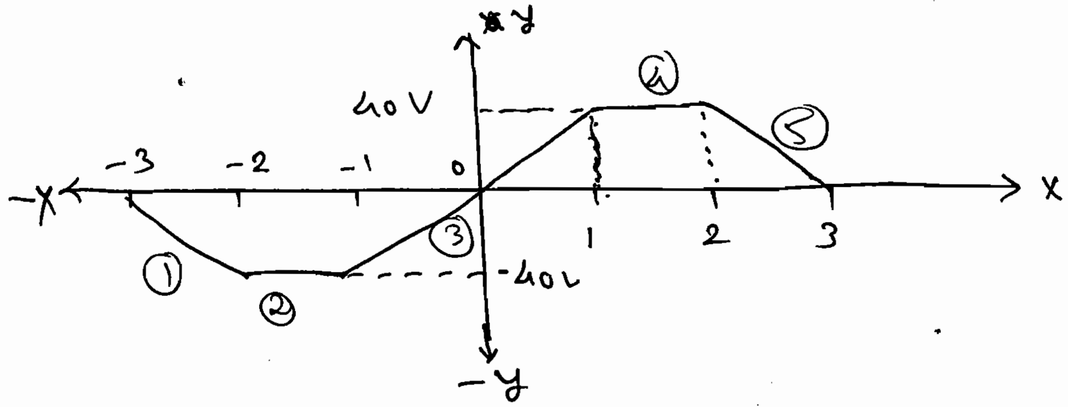
$$\therefore \quad \psi_{net} = Q_{enc} = \oint_s \bar{D} \cdot d\bar{s}$$

$$= -50 \epsilon_0 (5)^{3/2} \int_0^{\pi} \int_0^{2\pi} \sin\theta . \, d\theta \, d\phi$$

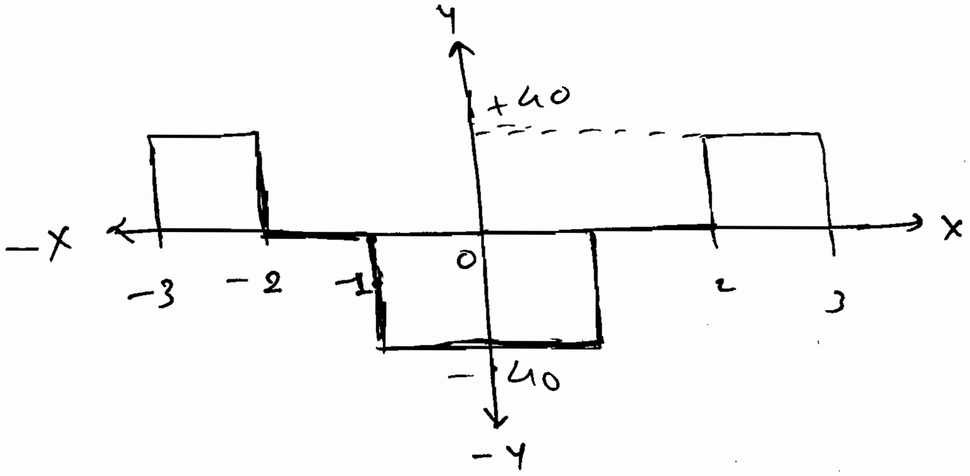
$$\psi_{net} = Q_{enc} = -50 \epsilon_0 (5)^{3/2} (4\pi) \; C$$

$\therefore$ '$-\psi$' is entering to the closed surface.

Ex-2 In certain region the potential field distribution is given by the following sketch plot the corresponding electric field.



Ans:



→ seg-2

$-2 < x < -1.$

$V(x) = -40V$

$E_x = -\frac{dV}{dx} = 0$

seg-3 $-1 < x < 1.$

$(-1, -40), (1, 40)$

$m = 40$

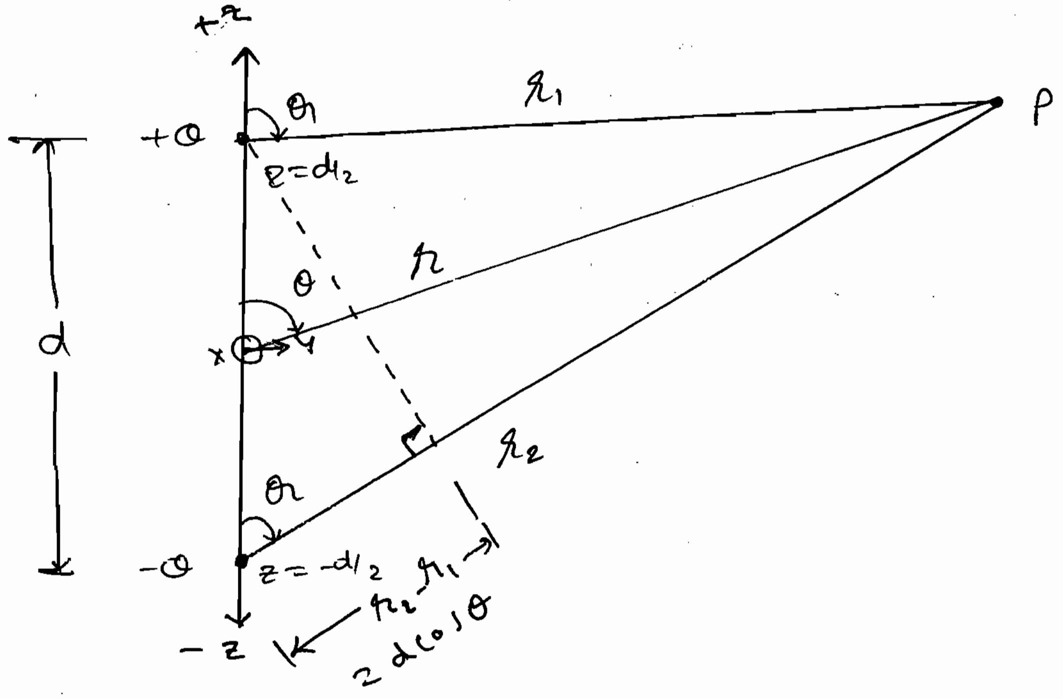
$V(x) + 40 = 40(x+1).$

$\therefore V(x) = 40x.$

$\therefore E_x = -\partial V/\partial x.$

$\therefore \boxed{E_x = -40.\ V/m}$

* Dipole:



$$\rightarrow \quad V_P = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right].$$

$$V_P = \frac{Q}{4\pi\epsilon}\left[\frac{r_2 - r_1}{r_1 \cdot r_2}\right].$$

$\therefore \; r >> d.$

$\Rightarrow \theta_1 \approx \theta_2 \approx \theta$

$\therefore r_2 - r_1 = d\cos\theta$

$$\frac{1}{r_1 \cdot r_2} = \frac{1}{r^2}$$

$$\therefore \; V_P = \frac{Q}{4\pi\epsilon} \times \frac{d\cos\theta}{r^2}$$

$$\therefore \; \boxed{V_P = \frac{Q\,d\cos\theta}{4\pi\epsilon r^2}}$$

$$\therefore \; \boxed{V_P \propto \frac{1}{r^2}}$$

Ans:

$$\bar{E} = -\nabla \cdot V$$

$$\therefore \bar{E} = -\frac{\partial V}{\partial r}\hat{a}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta$$

$$\bar{E} = \left(\frac{Qd\cos\theta}{2\pi\epsilon r}\hat{a}_r + \frac{Qd\sin\theta}{4\pi\epsilon r^3}\hat{a}_\theta\right) \text{ V/m.}$$

$$\therefore \boxed{\bar{E} = \left(\frac{Qd\cos\theta}{2\pi\epsilon r^3}\hat{a}_r + \frac{Qd\sin\theta}{4\pi\epsilon r^3}\hat{a}_\theta\right) \text{ V/m.}}$$

So, far the quantities $\bar{E}$, ψ, $\bar{D}$ & V have been analyzed. From the knowledge of given charge configuration there are no procedure available for the measurement of this charge configuration but there are procedures available for the measurement of potentials at the given points.

→ From the known potentials if we are able to develope potential function i.e. $V(x, y, z)$ or $V(\rho, \phi, z)$ or $V(r, \theta, \phi)$ then

$$\bar{E} = -\nabla V$$

$$\bar{D} = \epsilon \bar{E}$$

$$\oint_s \bar{D} \cdot d\bar{s} = Q_{enc} = \psi_{net}$$

$$\nabla \cdot \bar{D} = \rho_v.$$

→ For developing this potential fn Poissions eqn and laplace eqn are used.

* Poissions and Lepiacian's Eq^n:

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot \epsilon \bar{E} = \rho_v.$$

(Homogenius medium).

$$\epsilon \nabla \cdot \bar{E} = \rho_v.$$

$$\therefore \nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon}.$$

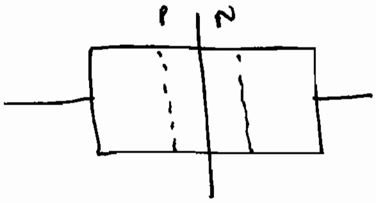
$$\therefore \nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}.$$

$$\therefore \boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}} \longrightarrow \text{poission eq}^n.$$

in a region of interent if $\rho_v = 0$

then $\boxed{\nabla^2 V = 0.} \longrightarrow$ Leplacian eq^n.

→ For analysing junction characterstics of a PN diode one dimensional possions eq^{n}s is used because junction has a charge i.e. it is an ionic region.

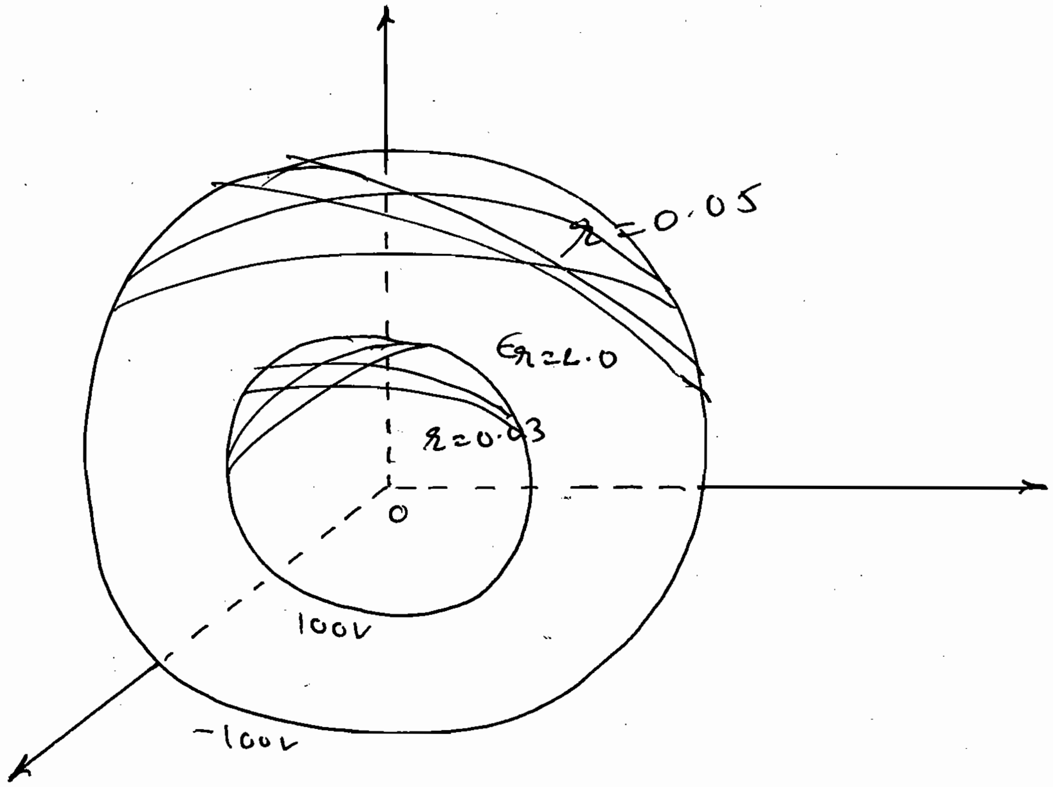


$$\frac{d^2 V}{dx^2} = -\rho/\epsilon \quad (PE).$$

Ex-1 Two concentric conducting spheres having radii 3 cm and 5 cm are centered at origin. The potential on the inner sphere is 100 V while the outer sphere is yet −100 V. The region betn them is filled with a homogeneity dielectric having relative permitivity 2.0. Find

1. Potential function.
2. potential mid way betn the conducting sphere.
3. The value of r at which $V = 0$.
4. Find the expression for electric field.

Ans:



→ as shown in figure, there exists equipotential surfaces at r = constant we know that potential varies normal to equipotential surface.

→ Therefore, Potential f^n must be a r alone. since ρ_v is not mention bet^n the sphere. we assume $\rho_v = 0$. Therefore, Laplacian eq^n reduces to

$$\nabla^2 V = 0$$

$$\therefore \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0.$$

cross mutiply and intigree,

$\therefore$

$$\therefore r^2 \frac{\partial V}{\partial r} = C_1$$

$$\therefore \frac{dV}{dr} = \frac{C_1}{r^2}.$$

$\therefore$ integrate again

$$\therefore V = -\frac{C_1}{r} + C_2.$$

$$V(\text{at } r = 0.03) = 100 = -\frac{C_1}{0.03} + C_2.$$

$$V(\text{at } r = 0.005) = -100 = -\frac{C_1}{0.05} + C_2.$$

solve C_1 and C_2.

$$\therefore C_1 = -15, \quad C_2 = -400.$$

$$\therefore \boxed{V(r) = \left(\frac{15}{r} - 400\right) \text{ Volts.}}$$

(1) $V(r) = \left(\frac{15}{r} - 400\right)$ V.

(2) $V(\text{at } r = 0.04\,m)$

$$V(0.04) = \left(\frac{15}{0.04} - 400\right)$$

(3) $0 = \frac{15}{r} - 400$

$\therefore V = 0$ at $r = \frac{15}{400}$ m.

$\therefore \boxed{r = 3.75\text{cm}}$

(4) $\bar{E} = -\nabla . V.$

$= -\frac{\partial V}{\partial r} \hat{a}_r$

$\therefore \boxed{\bar{E} = +\frac{15}{r^2} \hat{a}_r . \quad V/m.}$

→ The obtained $\bar{E}$ is projecting along $\hat{a}_r$ direction and it is projecting from a higher potential surface to towards lower potential.

* Current: (I).

→ it is a rate of charge.

→ $J = \rho v. = \sigma E$

$\bar{J}_c = \rho_v \bar{v}_d \quad A/m^2$

$\bar{J}_c = \sigma \bar{E} \quad A/m^2$

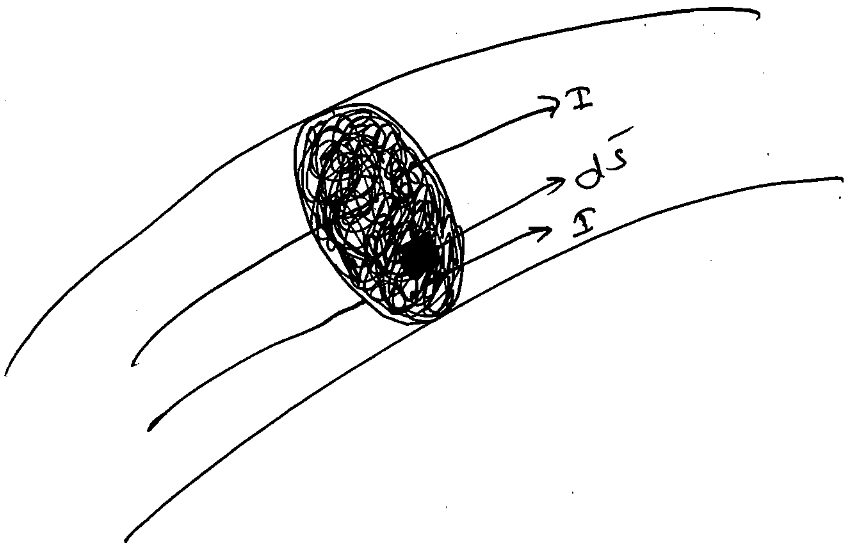
$\bar{J}_c$: Conduction current Density (A/m²)

ρ_v : volume charge density (C/m³).

σ = Conductivity (℧/m).

$\bar{v}_d$: drift velocity (m/s).

$\bar{E}$ = Applied Electric field.



→ Across, 's', we now the Conduction current density ($\bar{J}_c$ A/m²).

→ The diff. amount of current dI passing length $d\bar{s}$ is given by

$$dI = \bar{J}_c \cdot d\bar{s}$$

$$\therefore I = \int_S \bar{J}_c \cdot d\bar{s}.$$

Ex-1 In certain region the Conduction current density is given by $-10^5 \nabla V$ A/m² where $V = 10e^{-x}\sin y$ volts. scalar electric potential function. Find

① Conductivity of medium.

② Amount of current passing through $x=1$, $0 \le y, z \le 1$ in $\hat{a}_x$ direction.

Ans: ① $\bar{J}_c = \sigma \bar{E}$.

$\bar{J}_c = \sigma(-\nabla V)$.

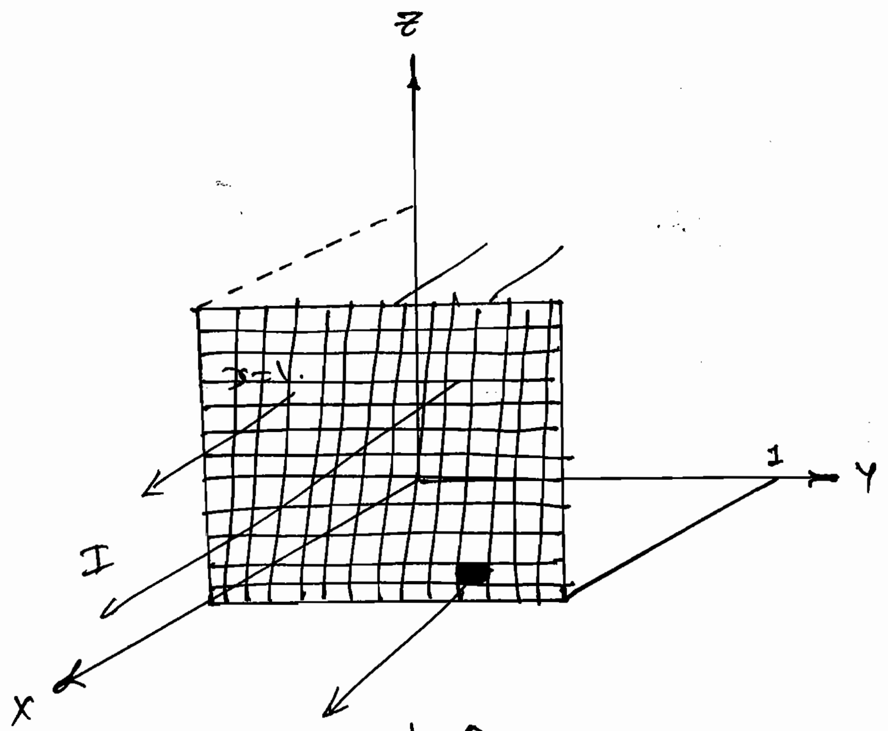
$\therefore \bar{J}_c = 10^5 (-\nabla V)$.

$\therefore \sigma = 10^5$ ℧/m.

② $\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y$.

$\nabla V = (-10e^{-x}\sin y\,\hat{a}_x + 10e^{-x}\cos y\,\hat{a}_y)$

$\therefore \bar{J}_c = 10e^{-x}\sin y\,\hat{a}_x - 10e^{-x}\cos y\,\hat{a}_y$.



$\rightarrow \bar{J}_c \cdot d\bar{S} = 10^6 e^{-x}\sin y\,dy\,dz$.

$\bar{J}_c \cdot d\bar{S} = 10^6 e^{-1}\sin y\,dy\,dz$.

at $x=1$

$$I = \int_S \bar{J}_c \cdot d\bar{S} = 10^6 e^{-1}\int_0^1\int_0^1 \sin y\,dy\,dz$$

$= 10^6 e^{-1} [1 - \cos 1] A.$

keep the calci in Rad.

$\therefore \boxed{I = 169 \, kA}$

* Continuity of a Current Equation:

$\rightarrow \quad \oint_S \bar{J_c} \cdot d\bar{s} = -\frac{dQ}{dt} = -\frac{d}{dt} \int_v \rho_v \, dv.$

(or) $\quad \oint_S \bar{J_c} \cdot d\bar{s} = -\frac{\partial}{\partial t} \int_v \rho_v \, dV. \quad -①$

Using divergence theorem.

$\oint_S \bar{J_c} \cdot d\bar{s} = \int_v \nabla \cdot \bar{J} \cdot dv. \quad -②$

$\therefore \quad \therefore \quad \boxed{\nabla \cdot \bar{J} = -\frac{\partial}{\partial t} \rho_v.}$

point form.

$\rightarrow \quad \bar{J} = \sigma \cdot \bar{E}.$

$\therefore \nabla \cdot \bar{J} = \sigma \cdot \nabla \cdot \bar{E}$

$\nabla \cdot \bar{J} = \sigma \cdot \nabla \cdot (\bar{D}/\epsilon)$

$\therefore \nabla \cdot \bar{J} = \frac{\sigma}{\epsilon} \times \nabla \cdot \bar{D}.$

$\therefore \nabla \cdot \bar{J} = \frac{\sigma}{\epsilon} \rho_v$

$\therefore \frac{\sigma}{\epsilon} \rho_v = -\frac{d\rho_v}{dt}.$

$$\therefore \boxed{\frac{d\rho_v}{dt} + \frac{\sigma}{\epsilon}\rho_v = 0}$$

$$\therefore m + \frac{\sigma}{\epsilon} = 0$$

$$\therefore m = -\frac{\sigma}{\epsilon}$$

$$\therefore \rho_v = C_1 e^{-\frac{\sigma}{\epsilon}t}$$

$$\therefore \boxed{\rho_v = C_1 e^{-t/\tau}}$$

where, $\tau = \epsilon/\sigma$.

τ = Relaxation time.

→ We conclude that as the time progresses the charge density inside a conductor decays exponetially. The rate at which it decays exponetially depends upon conductivity the conductor. If a conductor having infinite conductivity the density inside a conductor tends to zero within no time. In other words, for a transient time they may be some non zero charge inside a conductor.

→ Further we can conclude that if any charge is present in any conductor it resides on the surface of the conductor only.

Ex-1 Find the relaxation time for a copper conductor whose conductivity is 58 M℧/m. Assume $\epsilon = \epsilon_0$. Also find % of charge density after 1 relaxation time and after 5 relaxation time.

Ans:

→ $\tau = \dfrac{\epsilon_0}{\sigma}$.

$\therefore \tau = \dfrac{8.85 \times 10^{-12}}{56 \times 10^{6}}$

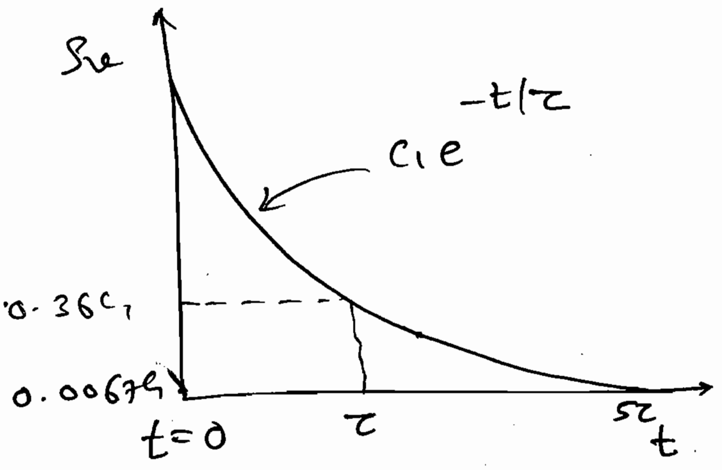
$\therefore \tau = 1.6 \times 10^{-19}$ s.

→ $\rho_v \; (\text{at } t = \tau) = c_1 e^{-\tau/\tau} = c_1/e = 0.36\,c_1$

$\rho_v \; (\text{at } t = \tau) = 36\ \%$ of c_1.

$\rho_v \; (\text{at } t = 5\tau) = c_1 e^{-5\tau/\tau} = c_1/e^5 = 0.0067\,c_1$.

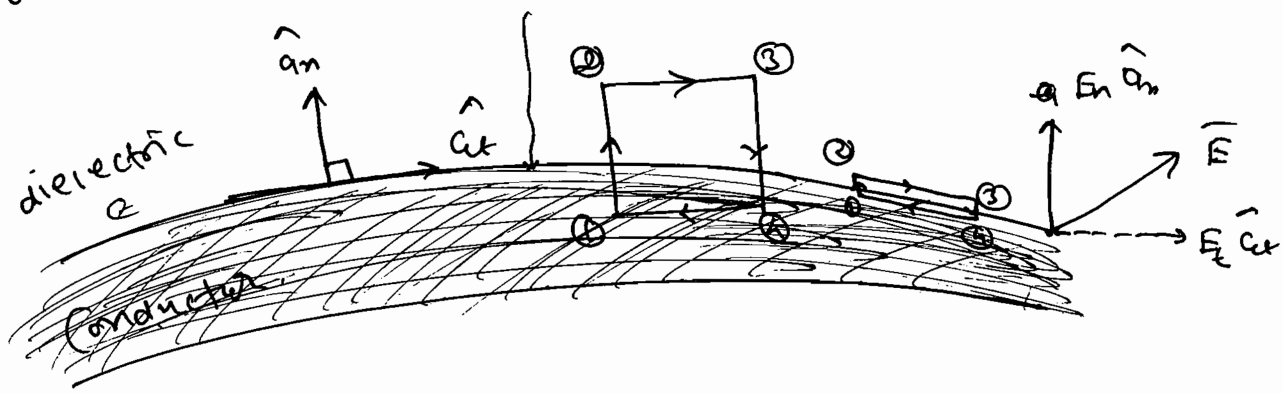
$\therefore \rho_v \; (\text{at } t = 5\tau) = 0.67\ \%$ of c_1.



* Boundary Conditions:

* **Case-1:** Conductor interface (or) conductor to dielectric interface.

→ An interface is a plane like structure where two mediums are interacting. In this case we assume conductor interface (or) conductor to dielectric interface. We have to investigate behaviour of electric field and electric flux density across the interface.



→ $\hat{a}_n$: Normal unit vector directed from conductor to dielectric.

→ $\hat{a}_t$: Unit vector tangential to the Interface.

→ $\oint_l \bar{E} \cdot d\bar{l} = 0.$

$$\Rightarrow \int_{1-2} \bar{E} \cdot d\bar{l} + \int_{2-3} \bar{E} \cdot d\bar{l} + \int_{3-4} \bar{E} \cdot d\bar{l} + \underbrace{\int_{4-1} \underbrace{\bar{E}}_{0} \cdot d\bar{l}}_{0} = 0.$$

→ The $\int_{4-1}$ has to be computed inside a conductor.

→ The charge is zero inside a conductor. Therefore, Electric field is zero inside a Conductor and hence this integral vanishes.

→ We are interested to investigate behaviour of electric field across interface. ~~to~~ To accomplishe this we choose path like 1-2 and 3-4 so small such that the path 2-3 is grazing (touching) the interface, which would result the $\int_{1-2}$ and $\int_{3-4}$ vanishing.

→ $$\int_{2-3} \bar{E}\cdot d\bar{l} = 0.$$

→ For path (2-3) $\Rightarrow d\bar{l} = dl\,\hat{a}_t$

Let $\bar{E} = E_n \hat{a}_n + E_t \hat{a}_t$

This is assumed across the interface.

$\therefore \quad \bar{E}\cdot d\bar{l} = E_n \cdot \hat{a}_n \cdot \cancel{dl}\,\hat{a}_t + E_t\,\hat{a}_t \cdot dl\,\hat{a}_t$

$\therefore \quad \bar{E}\cdot d\bar{l} = E_t\,dl$

$\therefore \quad \Rightarrow \int E_t \cdot dl = 0.$

$\because$ 'dl' cant be ZERO.

$\therefore \quad \boxed{E_t = 0}$

→ Tangential components of electric field across a conductor to dielectric interface vanishing.

Ex: Across a conductor interface identify the correct one from the following: where $\bar{E}$ is the electric field across the interface, $\hat{a}_t$: unit vector tangential to interface. $\hat{a}_n$: the unit vector normal to interface.

(i) $\bar{E} \cdot \hat{a}_t = 0$

(ii) $\bar{E} \times \hat{a}_n = 0$

(iii) $\hat{a}_n \times \bar{E} = 0$.

(iv) All the above. ✓

(v) $\bar{D} \cdot \hat{a}_n = \rho_s$

(vi) $\bar{D} \cdot \hat{a}_t = 0$

(vi) $\hat{a}_n \cdot \bar{D} = \rho_s$

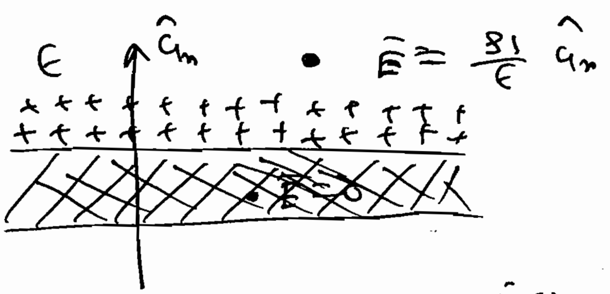
⇒ We assume that the Interface has a non-zero surface charge density of ρ_s C/m². By using Gauss Law we can show that normal components of Electric flux densities are equal to surface charge density. By expression

$$D_n = \rho_s \text{ (or)}$$

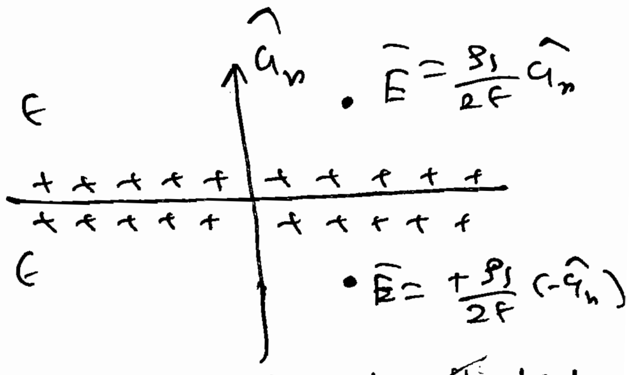
$$\bar{D} = \rho_s \hat{a}_n \quad \text{and}$$

$$\therefore \bar{E} = \frac{\rho_s}{\epsilon} \hat{a}_n .$$

Imp



→ The entire charge lies on the top of the conductor surface.



→ The charge is distributed on both sides of conductor sheet.

Ex-1 A Charge density of 1 nC/m² is placed on a Conductor Surface. Assume interface is free Space. Find the magnitude of the electric field.

Ans:

$$\bar{E} = \frac{\rho_s}{\epsilon}\hat{a}_n = \frac{1\times 10^{-9}}{\frac{1}{36\pi}\times 10^{-9}}\hat{a}_n$$

$$\therefore \boxed{|\bar{E}| = 36\pi \text{ V/m.}}$$

Ex-2 A Positive Charge is distributed on a Conductor Surface. Assume the Interface is free space. Given that $\bar{D}$ ~~at~~ across interface is equal to $\bar{D} = 2(\hat{a}_x + \sqrt{3}\hat{a}_y)$ nC/m². Find the value of charge density across the interface.

Ans:

$$\bar{E} = \frac{\rho_s}{\epsilon}\hat{a}_n.$$

$$\hat{a}_n = \frac{2\hat{a}_x + 2\sqrt{3}\hat{a}_y}{\sqrt{4+12}}$$

$$\therefore \bar{D} = \rho_s\hat{a}_n$$

$$\therefore \cancel{|\bar{D}| = \rho_s\hat{a}_n}$$

$$\hat{a}_n = \frac{\hat{a}_x}{4} + \sqrt{3}/_2\,\hat{a}_y.$$

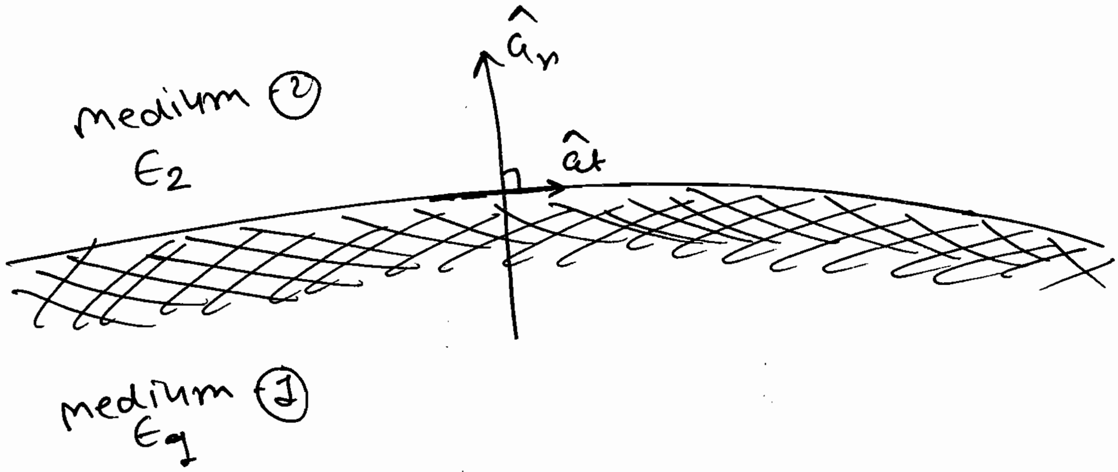
$$|\bar{D}| = \sqrt{2^2 + 2^2(3)} = 4.$$

$$\therefore \bar{D} = 4\left\{\frac{2\hat{a}_x + 2\sqrt{3}\hat{a}_y}{4}\right\}$$

$$\bar{D} = \rho_s\hat{a}_n$$

$$\therefore \rho_s = 4 \text{ nC/m}^2.$$

Case-2: Dielectric to Dielectric interface:



→ $\hat{a}_n$: Normal unit vector directed in from (1) to (2)

→ $\hat{a}_t$: Tan unit vector tangential to the interface.

We can show that

$$\boxed{(1)\ E_{t1} = E_{t2}.}$$

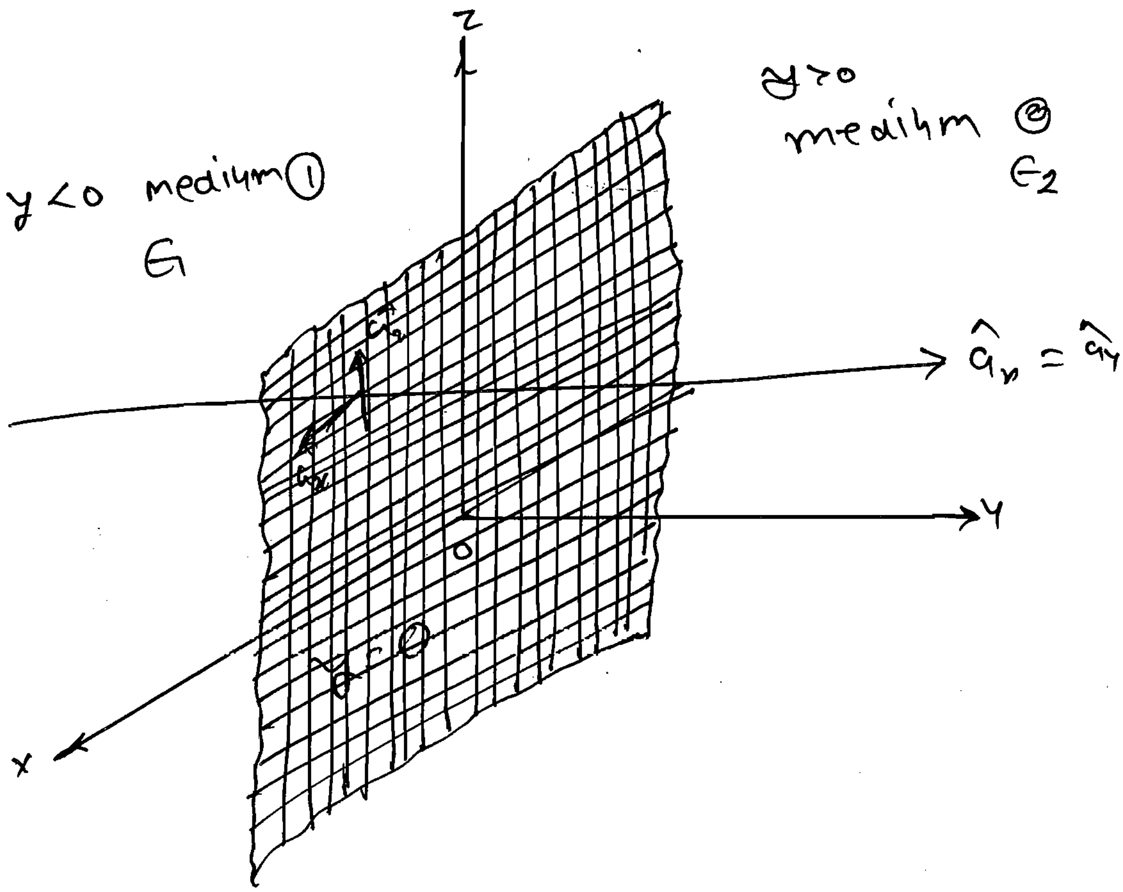
Tangential components of E-fields are continuous across a dielectric to dielectric interface.

(2) (a) $\boxed{D_{n2} - D_{n1} = \rho_s}$

→ The normal components of electric flux densities are discontinued by an amount of surface charge density.

(b) if $\rho_s = 0$ (charge free interface).

→ The normal components of electric flux densities are continuous across a charge free interface.



→ figure shows that interface is define by $y = 0$. Medium-1 is define d for $y < 0$ and is characterised by ϵ_1.

→ Medium-2 is defined for $y > 0$ and is characterised by ϵ_2. To this interface the normal unit vector is $\hat{a}_n = \hat{a}_y$ and the unit vectors tangential to interface are $\hat{a}_x$ and $\hat{a}_z$

Ex-1 → with reference to the figure shown above and given that let, $\epsilon_1 = 2\epsilon_0$, $\epsilon_2 = 3\epsilon_0$.

$$\bar{E}_1 = (4\hat{a}_x + 5\hat{a}_y + 6\hat{a}_z) \text{ V/m.}$$

Find $\bar{D}_1$, $\bar{D}_2$, and E_2. and assume that the interface is charge free. (i.e) $\rho_s = 0$ across $y=0$.

Ans:

$$\bar{E}_1 = \underbrace{4\hat{a}_x + 6\hat{a}_z}_{E_{t1}} + \underbrace{5\hat{a}_y}_{E_{n1}}$$

$$\therefore \bar{E}_1 = E_{t1}\hat{a}_t + E_{n1}\hat{a}_n$$

→ (i.e) Any field vector across the interface can be represented as a vectorical sum of tangential and normal components.

$$\bar{E}_1 = 4\hat{a}_x + 6\hat{a}_z + 5\hat{a}_y.$$

$$\bar{D}_1 = \bar{E}_1 \epsilon_1$$

$$\therefore \bar{D}_1 = 4\epsilon_1\hat{a}_x + 6\epsilon_1\hat{a}_z + 5\epsilon_1\hat{a}_y$$

$$E_{t1} = E_{t2}$$

$$\therefore \bar{E}_2 = 4\hat{a}_x + 6\hat{a}_z + E_{y2}\hat{a}_y.$$

$$\therefore D_{n1} = D_{n2} \quad (\because \rho_s = 0.$$

$$\therefore \bar{D}_2 = D_{x2}\hat{a}_x + D_{z2}\hat{a}_z + 5\epsilon_1\hat{a}_y.$$

$$\therefore \bar{D}_2 = \epsilon_2\bar{E}_2$$

$$\therefore D_{x2}\hat{a}_x + D_{z2}\hat{a}_z + 5\epsilon_1\hat{a}_y = 4\epsilon_2\hat{a}_x + 6\epsilon_2\hat{a}_z + E_{y2}\epsilon_2\hat{a}_y.$$

$$\therefore D_{x2} = 4\epsilon_2 \ C/m^2$$

$$D_{z2} = 6\epsilon_2 \ C/m^2$$

$$E_{y2} = \frac{5\epsilon_1}{\epsilon_2} \ V/m.$$

Ex-2 Repeat the above problem if the interface has a non-zero surface charge density of ρ_s C/m^2.

Ans: let, $\bar{D}_2 = D_{x_2}\hat{a}_x + D_{z_2}\hat{a}_z + D_{y_2}\hat{a}_z$

$\therefore D_{n_2} - D_{n_1} = \rho_s$

$D_{y_2} - D_{y_1} = \rho_s$

$\therefore D_{y_2} = \rho_s + 5\epsilon_1.$

$\therefore D_2 = D_{x_2}\hat{a}_x + D_{z_2}\hat{a}_z + (\rho_s + 5\epsilon_1)\hat{a}_y.$

$\therefore D_2 = \epsilon_2 \bar{E}_2$

$\therefore$

$D_{x_2} = 4\epsilon_2 \ C/m^2$

$D_{z_2} = 6\epsilon_2 \ C/m^2.$

$\therefore E_{y_2} = \dfrac{\rho_s + 5\epsilon_1}{\epsilon_2} \ V/m.$

Ex-3 Repeat above 2 example by assuming the Interface as $z=0$. $z<0$ is medium 1 and is characterized by ϵ_1 whereas $z>0$ is medium 2 and is characterized by ϵ_2.

Ans: $\bar{E} = \underbrace{4\hat{a}_x + 5\hat{a}_y}_{E\hat{a}_t} + \underbrace{6\hat{a}_z}_{E\hat{a}_n}.$

$\therefore \bar{E} = \bar{E}_{n_1} + \bar{E}_{t_1}.$

$\therefore \bar{E}_{t_1} = 4\hat{a}_x + 5\hat{a}_y$

$\bar{E}_{n_1} = 6\hat{a}_z$

is directed in from medium unit vector in 1 to 2. is $\hat{a}_z$. & unit vector tangential to interface are $\hat{a}_x$ and $\hat{a}_y$.

Ans. → $\bar{E}_{t_2} = \bar{E}_{t_1}$

$$\therefore \boxed{\bar{E}_{t_2} = 4\hat{a}_x + 5\hat{a}_y.}$$

$$\therefore \bar{E}_2 = \bar{E}_{t_2} + \bar{E}_{n_2}.$$

$$\therefore \bar{D}_{n_2} = \bar{D}_{n_1}.$$

$$\therefore \epsilon_2 \bar{E}_{n_2} = \epsilon_1 \bar{E}_{n_1}.$$

$$\therefore \bar{E}_{n_2} = \frac{\epsilon_1}{\epsilon_2} \bar{E}_{n_1}.$$

$$\therefore \bar{E}_{n_2} = \frac{2}{3} \times 6\hat{a}_z = 4\hat{a}_z.$$

$$\therefore \boxed{\bar{E}_2 = 4\hat{a}_x + 5\hat{a}_y + 4\hat{a}_z.}$$

$$\therefore \bar{D}_1 = \epsilon_1 \bar{E}_1$$

$$\boxed{\bar{D}_1 = \epsilon_0 (8\hat{a}_x + 10\hat{a}_y + 12\hat{a}_z)}$$

$$\therefore \bar{D}_2 = \epsilon_2 \bar{E}_2$$

$$\boxed{\bar{D}_2 = \epsilon_0 [12\hat{a}_x + 15\hat{a}_y + 12\hat{a}_z).}$$

Now, there ~~is~~ are normal components of $\bar{D}$ that are discontinued by surface charge density ρ_s C/m².

$$\therefore D_{n_2} - D_{n_1} = \rho_s \text{ C/m}^2.$$

$$\therefore \epsilon_2 E_{n_2} - \epsilon_1 E_{n_1} = \rho_s.$$

$\therefore$

$$\therefore \quad \epsilon_2 E_{n2} = \epsilon_1 E_{n1} + \rho_s.$$

$$\therefore \quad E_{n2} = \frac{\epsilon_1 E_{n1} + \rho_s}{\epsilon_2}.$$

$$\therefore \quad E_{n2} = \frac{6\epsilon_1 + \rho_s}{\epsilon_2}$$

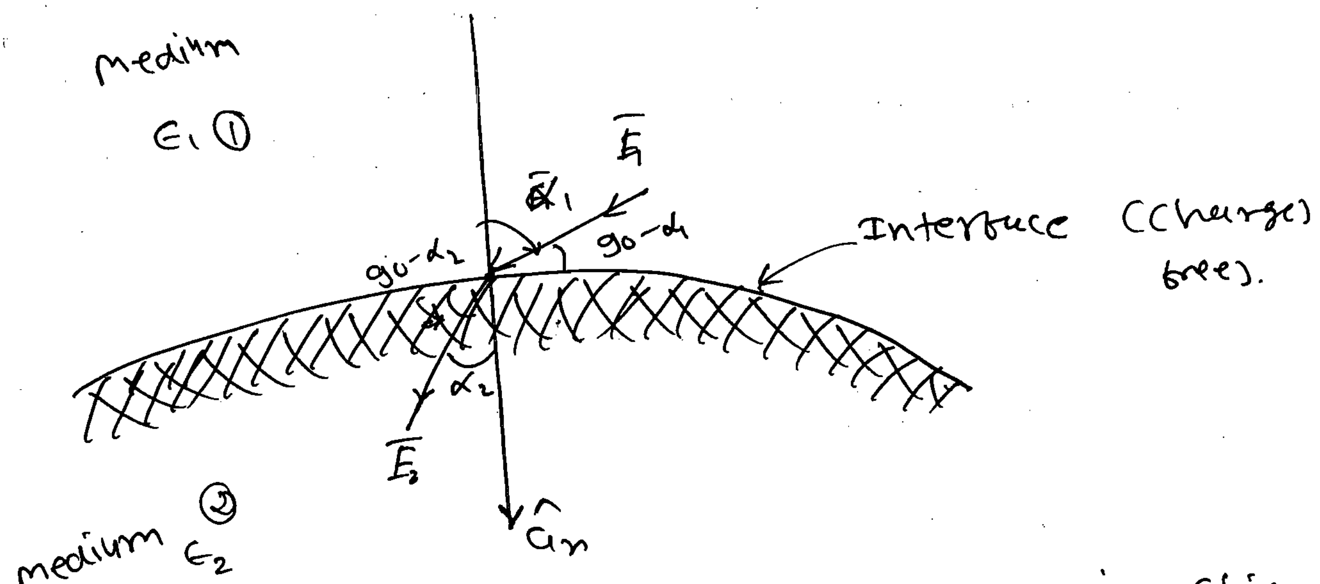
$$\therefore \quad \boxed{\bar{E}_2 = 4\hat{a}_x + 5\hat{a}_y + \left(\frac{6\epsilon_1 + \rho_s}{\epsilon_2}\right)\hat{a}_z.}$$

$$\therefore \quad \boxed{\bar{D}_2 = 4\epsilon_2\hat{a}_x + 5\epsilon_2\hat{a}_y + (6\epsilon_1 + \rho_s)\hat{a}_z}$$

$$\bar{D}_1 = \epsilon_1 \bar{E}_1$$

$$\therefore \quad \boxed{\bar{D}_1 = \epsilon_0 (8\hat{a}_x + 10\hat{a}_y + 12\hat{a}_z).}$$

Ex-3



→ Fig. shows charge free dielectric to dielectric interface. Further it is shown normal unit vector directed in from medium ① to medium ② of P. Relate an expression $\alpha_1, \alpha_2, \epsilon_1, \epsilon_2$.

Ans ::

$$|\bar{E}_{t1}| = |\bar{E}_{t2}|.$$

$$\therefore \quad E_2 \cos(90-\alpha_2) = E_1 \cos(90-\alpha_1).$$

$$\therefore \quad E_2 \sin\alpha_2 = E_1 \sin\alpha_1.$$

$$\therefore \quad \cancel{\epsilon_1 |\bar{E}_{n1}| = |\bar{E}_{n2}| \epsilon_2}.$$

$$\therefore \quad \cancel{E_2 \cos\alpha_2 = E_1 \sin\alpha_1 \cos\alpha_2}.$$

$$\therefore \quad \boxed{\cancel{\epsilon_1 \tan\alpha_1 = \epsilon_2 \tan\alpha_2}}$$

$$\therefore \quad D_{n1} - D_{n2} = \rho_s$$

But $\rho_s = 0$

$$D_{n1} = D_{n2}.$$

$$\therefore \quad \epsilon_1 |E_1| \cos\alpha_1 = \epsilon_2 |E_2| . \cos\alpha_2$$

$$\therefore \quad \frac{\tan\alpha_1}{\epsilon_1} = \frac{\tan\alpha_2}{\epsilon_2}.$$

$$\therefore \quad \boxed{\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\epsilon_1}{\epsilon_2}}$$

Ex-1 An interface is define by $2x+3y+4z=12$ origin side of the interface is medium ① and is characterised by $\epsilon_1 = 2\epsilon_0$ otherside of the interface is medium-② and is characterised by free space. Let, the electric field in the medium-① is given by $\bar{E}_1 = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z$ V/m. Assume charge free interface. Find $\bar{D}_1$, $\bar{E}_2$, $\bar{D}_2$.

Ans:

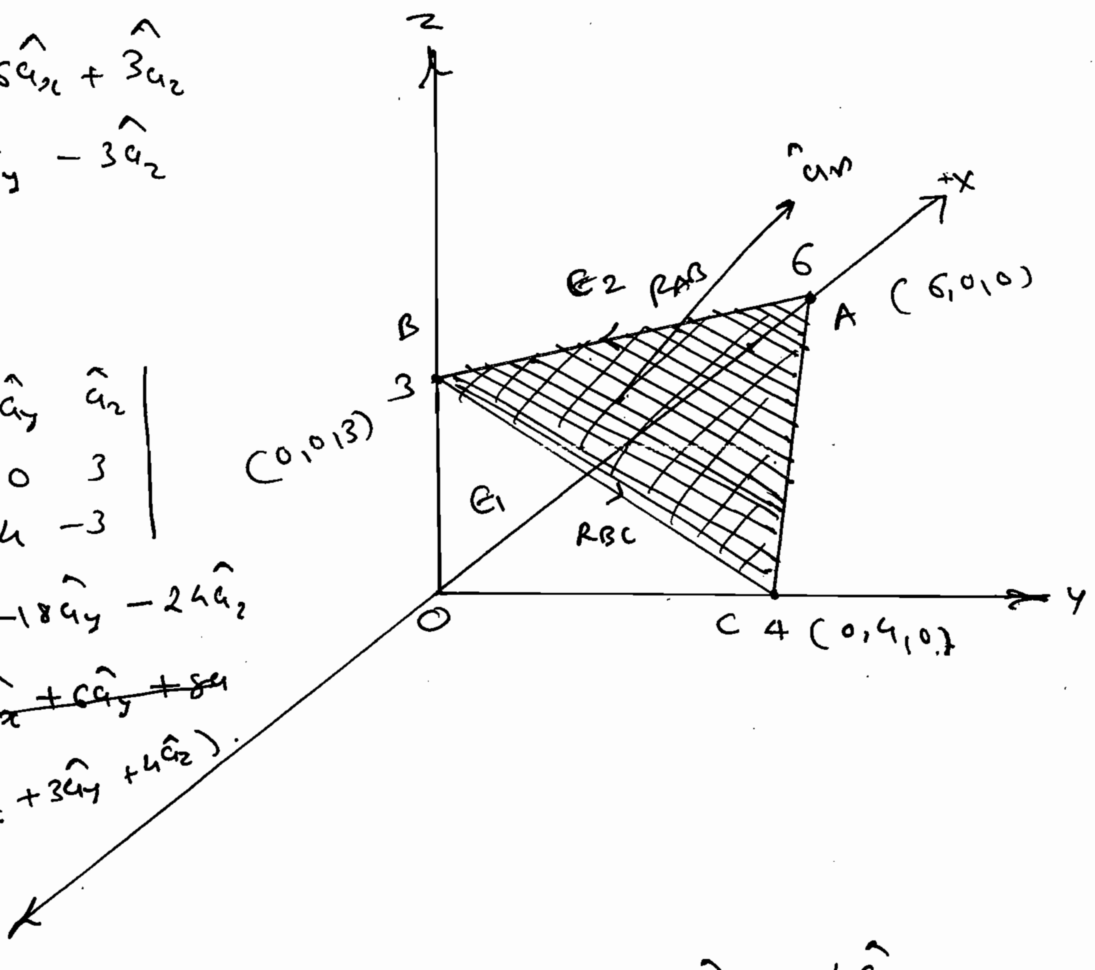
$$2x+3y+4z=12.$$

$$\therefore \frac{x}{6} + \frac{y}{4} + \frac{z}{3} = 1.$$

$R_{AB} \times R_{BC}$

$$\bar{R}_{AB} = -6\hat{a}_x + 3\hat{a}_z$$

$$\bar{R}_{BC} = 4\hat{a}_y - 3\hat{a}_z$$



$$\therefore \bar{R}_{AB} \times \bar{R}_{BC} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -6 & 0 & 3 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= -12\hat{a}_x - 18\hat{a}_y - 24\hat{a}_z$$

~~$\hat{a}_n = -3[4\hat{a}_x + 6\hat{a}_y + 8a$~~

$$\bar{R}_n = -6(2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z).$$

$$\therefore \hat{a}_n = \frac{-6(2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z)}{32.31} \qquad \hat{a}_n = \frac{2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z}{\sqrt{2^2+3^2+4^2}}.$$

$$\therefore \hat{a}_n = -\frac{(2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z)}{5.385}$$

$\rightarrow \quad \bar{E}_1 = E_{n1}\,\hat{a}_n + E_{t2}\,\hat{a}_t.$

$\bar{E}_2 = E_{n2}\,\hat{a}_n + E_{t2}\,\hat{a}_t.$

$\rightarrow \quad \bar{E}_1 \cdot \hat{a}_n = E_{n1} \cdot \underbrace{\hat{a}_n \cdot \hat{a}_n}_{1} + E_{t2} \underbrace{\hat{a}_t \cdot \hat{a}_n}_{0}.$

$\therefore \quad \boxed{E_{n1} = \bar{E}_1 \cdot \hat{a}_n.} \rightarrow (1)$

$\therefore \quad \boxed{\bar{E}_{n1} = E_{n1} \cdot \hat{a}_n.} \rightarrow (2)$

$\therefore \quad \boxed{\bar{E}_{t1} = \bar{E} - \bar{E}_{n1}} \rightarrow (3)$

∴ Now, tangential components of $\bar{E}$ are continueous.

So, $\bar{E}_{t1} = \bar{E}_{t2}$

$\therefore \quad \boxed{\bar{E}_{t2} = \bar{E}_{t1}.} \rightarrow (4)$

∴ Now, $D_{n1} = \epsilon_1 E_{n1}.$

$\therefore \quad \bar{E}_{n2} = \frac{\bar{D}_{n2}}{\epsilon_2} =$

normal components of $\bar{D}$ are continueous (∵ charge free so $\rho_s = 0$).

$\boxed{\bar{D}_{n1} = \bar{D}_{n2}.} \rightarrow (5)$

$\therefore \quad \bar{D}_{n1} = \epsilon_1 \bar{E}_{n1}.$

$\therefore \quad \boxed{\bar{D}_{n2} = \epsilon_1 \bar{E}_{n1}} \rightarrow (6)$

$\therefore \quad \bar{E}_{n2} = \frac{\bar{D}_{n2}}{\epsilon_2} = \frac{\epsilon_1}{\epsilon_2} \bar{E}_{n1}$

$\therefore \quad \boxed{\bar{E}_{n2} = \frac{\epsilon_1}{\epsilon_2} \bar{E}_{n1}} \rightarrow (7)$

So, $\boxed{\bar{E}_2 = \bar{E}_{t_2} + \bar{E}_{n_2}} \rightarrow (8)$

$\therefore \cancel{\bar{D}_2} = \bar{D}_{t_1} + \bar{D}_{n_2}$

$\boxed{\bar{D}_1 = \epsilon_1 \bar{E}_1} \rightarrow (10)$

$\boxed{\bar{D}_2 = \epsilon_2 \bar{E}_2} \rightarrow (9)$

$\rightarrow \bar{E}_1 = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z.$

$\therefore \hat{a}_n = \dfrac{5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z}{10.49}.$

$\hat{a}_n \; \cancel{\bar{E}_{n1}} = \cancel{E_{n1}\,\hat{a}_n}$

$E_{n1} = \bar{E}_1 \cdot \hat{a}_n$

$\therefore E_{n1} = \dfrac{25 + 36 + 49}{10.49}.$

$\boxed{E_{n1} = 10.49}$

$\therefore \bar{E}_{n1} = E_{n1}\,\hat{a}_n$

$\therefore \bar{E}_{n1} = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z.$

$\therefore \bar{E}_{t1} = 0$

$\therefore \bar{E}_{t2} = 0.$

$\therefore \bar{D}_{n1} = \bar{D}_{n2}$

$\therefore \epsilon\,\bar{E}_{n2} = \epsilon_1 \bar{E}_{n1}$

$\therefore \bar{E}_{n2} = \frac{2}{3}(5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z).$

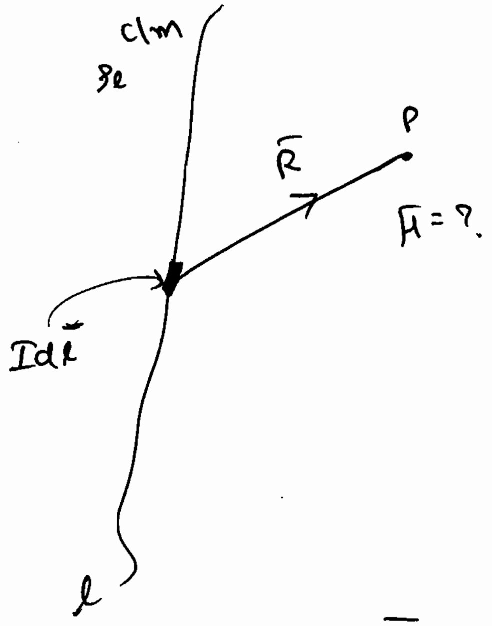
$\therefore \bar{E}_{n2} = \frac{10}{3}\hat{a}_x + 4\hat{a}_y + \frac{14}{3}\hat{a}_z.$

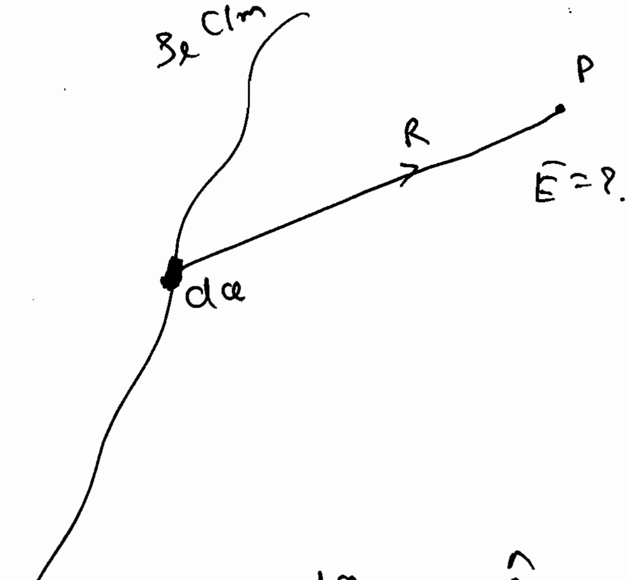
$\therefore \bar{E}_2 = \bar{E}_{n2} + \bar{E}_{t2}$

$\therefore \bar{E}_2 = \frac{10}{3}$

☆ Magnetics Fields (Steady)

Fields R Independent of time.





$$d\bar{E} = \frac{dQ}{4\pi\epsilon|\bar{R}|^2} \cdot \hat{a}_R .$$

$$d\bar{H} = \frac{I d\bar{l} \times \bar{a}_R}{4\pi|\bar{R}|^2}$$

Current Element $= Id\bar{l}$

- → Current multiplied vector diff. length
- → This is vector quantity.
- → Source of magnetic field.

→ There exists a similarity betn electric and magnetic fields.

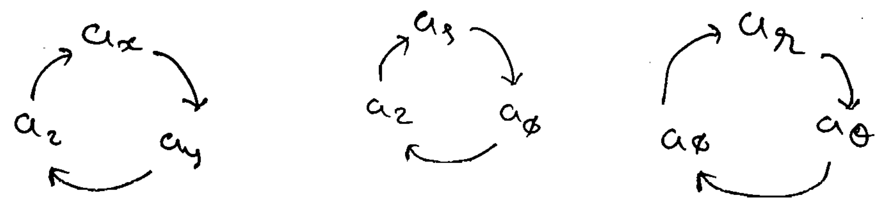
→ Both fields are proportional to the corresponding sources.

→ Both fields are inversely proportional to square of distance from their corresponding sources.

→ Both fields are vector fields.

⇒ Bio Savart's Law :

$$\boxed{\bar{H} = \int_{l} \frac{I d\bar{l} \times \hat{a}_R}{4\pi |\bar{R}|^2} \quad A/m.}$$

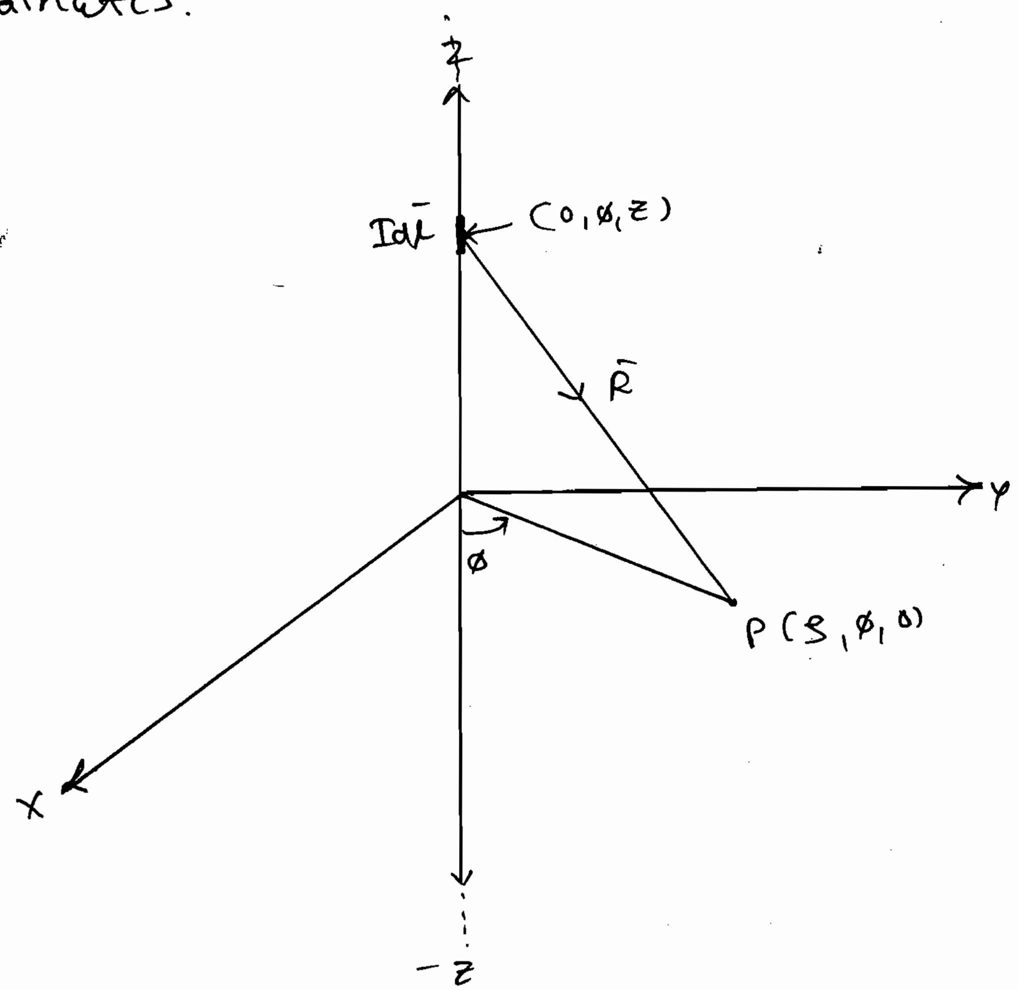


Ex-1 Find an expression for the magnetic field intensity due to a long straight infinite filamentary conductor which carries a direct current of I A. Show that the magnetude of the $\bar{H}$ magnetic ~~is~~ filed ~~inversely~~ intensity is inversly proportional to the distance betn infinite current filament and the observation point.

Ans: We assume that the infinite current filament lies along z axis. and is extending from $-\infty$ to $+\infty$. We find the magnetic field intensity at some point on the x-y plane.

→ Say at ρ point $P(\rho, \phi, 0)$.

for the convinience we circular cylindrical co-ordinates.



→ $d\bar{l} = dz\,\hat{a}_z$.

$\therefore\; I d\bar{l} = I dz\,\hat{a}_z$.

→ $\bar{R} = \rho\,\hat{a}_\rho - z\,\hat{a}_z$ $\hat{a}_R = \dfrac{\rho\,\hat{a}_\rho - z\,\hat{a}_z}{\sqrt{\rho^2+z^2}}$

$\therefore\; I d\bar{l} \times \hat{a}_R = \dfrac{I\rho\, dz}{\sqrt{\rho^2+z^2}} \cdot \hat{a}_\phi$

so, $d\bar{H} = \dfrac{I d\bar{l} \times \hat{a}_R}{4\pi |\bar{R}|^2}$.

$\therefore$

$$d\bar{H} = \frac{I\,dz\,\rho}{4\pi\,(\rho^2+z^2)^{3/2}} \cdot \hat{a}_\phi$$

$$\therefore \quad \bar{H} = \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{\rho}{(\rho^2+z^2)^{3/2}}\,dz \cdot \hat{a}_\phi$$

Now, let $z = \rho\tan\theta$

$\therefore dz = \rho\sec^2\theta.$

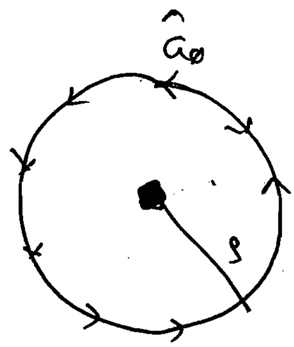
$\therefore (-\infty, \infty) \Rightarrow (-\pi/2, \pi/2).$

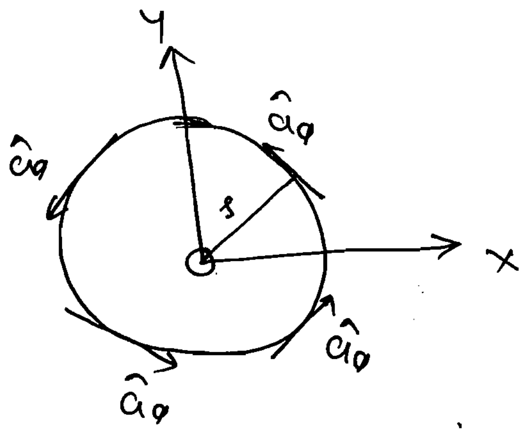
$$\bar{H} = \frac{I}{4\pi}\int_{-\pi/2}^{\pi/2} \frac{\rho^2\sec^2\theta}{\rho^3\sec^3\theta} \cdot d\theta \quad \hat{a}_\phi$$

$$\therefore \boxed{\bar{H} = \frac{I}{2\pi\rho} \cdot \hat{a}_\phi.}$$

$$\therefore \boxed{|\bar{H}| \propto \frac{1}{\rho}.}$$

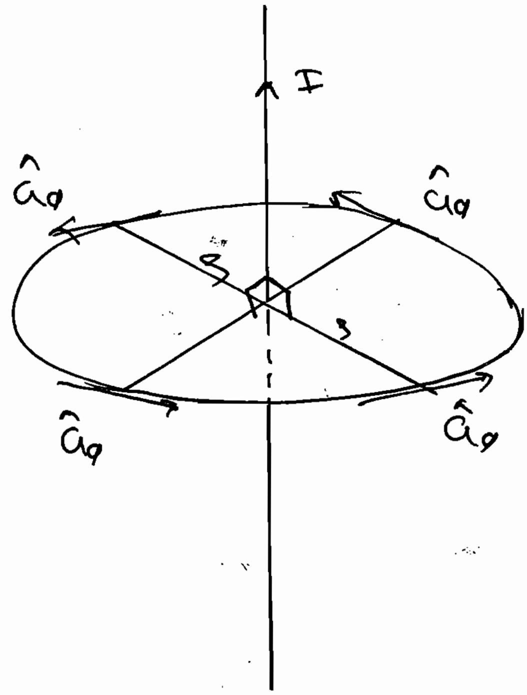
→





→ Magnitude of magnetic field intensity is inversely proportional to the distance bet^n the infinite current filament and the observation point. The direction of the magnetic field is around the conductor.

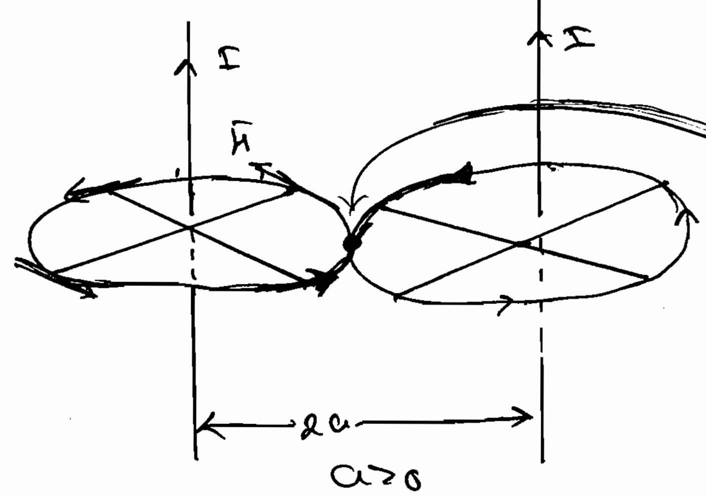
it encircles the conductor.



⇒ Two infinite current filament are parallel: case-1: currents are in same direction:

→ They are separated by 2a m. (a>0). They carry equal current of I amp. in same direction. find the magnitude of the magnetic field intensity at the middle point betⁿ this two infinite current filaments. Assume that this conductor's carry equal currents of I amps. in the same direction.

*

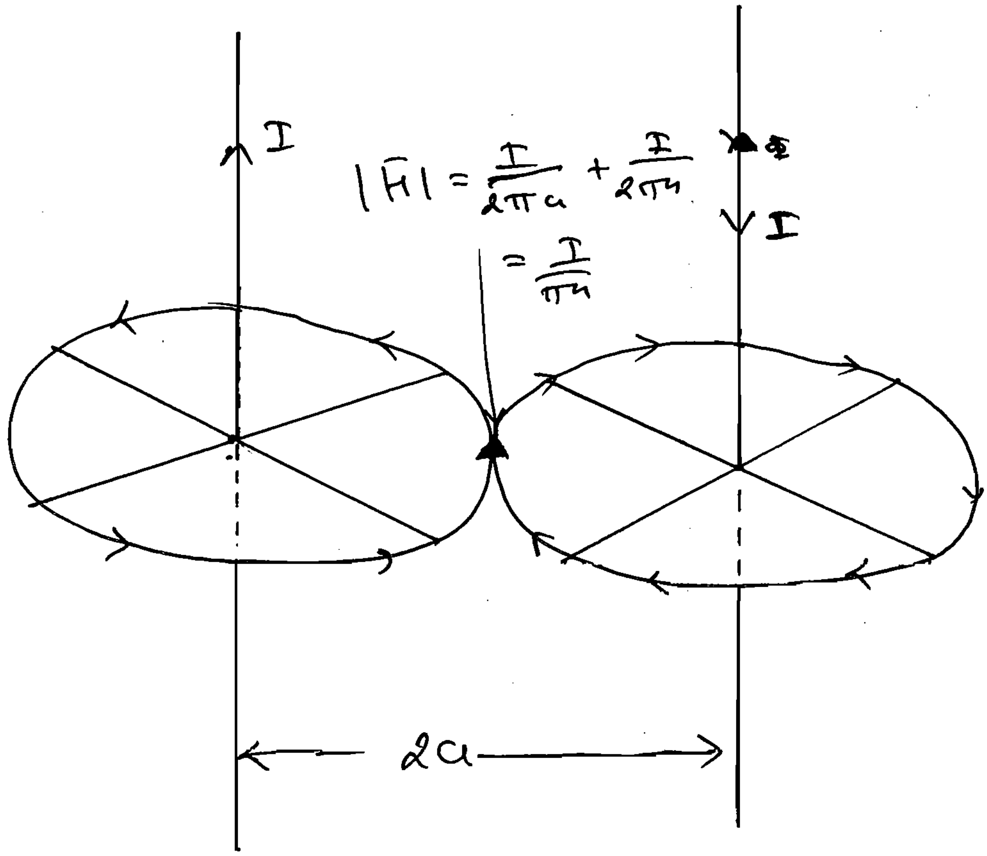


$|\bar{H}| = 0$

The fields add in out of phase

$\therefore |\bar{H}| = 0$.

Case-2: Currents are in opposite direction.

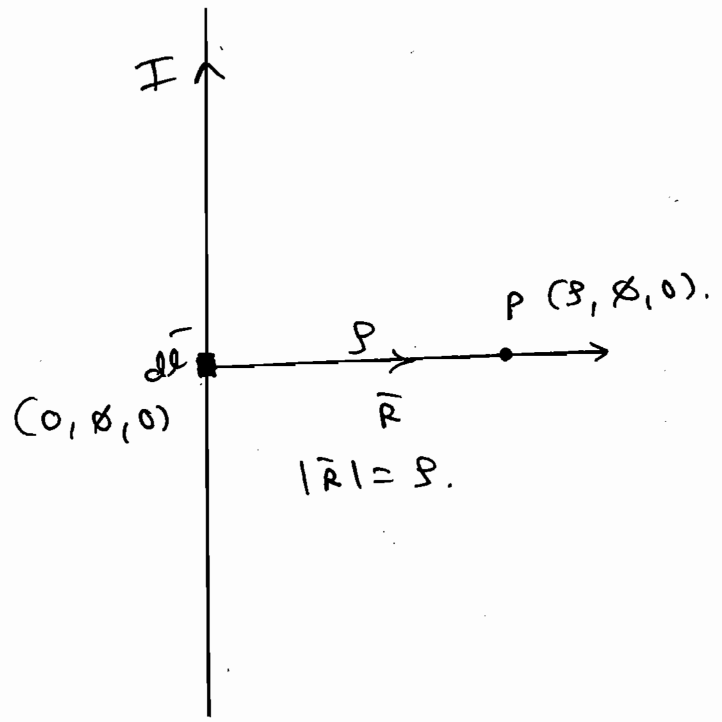


$\rightarrow$ $|\bar{H}| = \frac{I}{2\pi a} + \frac{I}{2\pi a} = \frac{I}{\pi a}$.

The field are add ~~and~~ in phase.

* General Expression for the Magnetic Field Intensity due to an infinite current filament.

⇒



→ $d\bar{l} = dz\,\hat{a}_z$.

∴ $\bar{R} = \rho\,\hat{a}_\rho$.

$\hat{a}_R = \hat{a}_\rho$.

∴ $d\bar{l} \times \hat{a}_R = \hat{a}_\phi\,dz$.

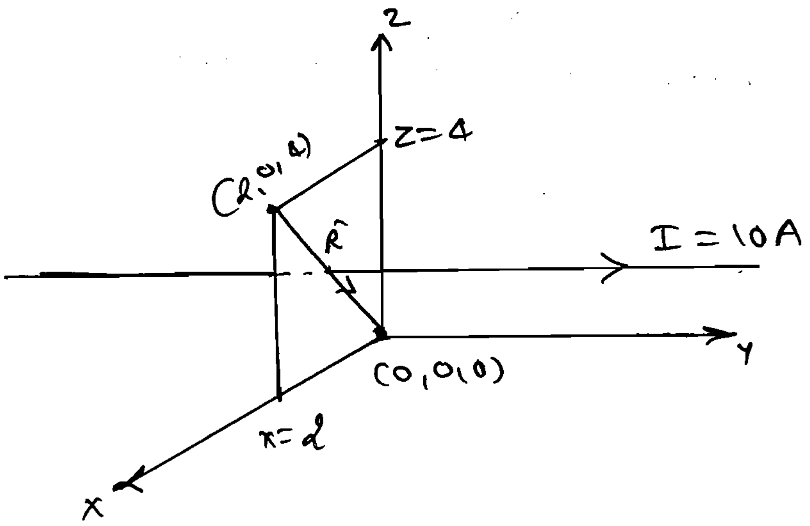
The unit vector of $d\bar{l} \times \bar{a}_R = \hat{a}_\phi$

→ ∴ $\boxed{\bar{H} = \dfrac{I}{2\pi|\bar{R}|} \cdot \text{Unit vector of } (d\bar{l} \times \hat{a}_R)}$.

Short-cut formula.

Ex-1 ⇒ An infinite current filament is lies at $x = 2$, $z = 4m$ it carries a current of 20 A along +ve y direction. Find $\bar{H}$ at the origin?

Ans:



$$\Rightarrow \quad \bar{R} = -2\hat{a}_x - 4\hat{a}_z \qquad , \; d\bar{l} = dy\,\hat{a}_y$$

$$|\bar{R}| = \cancel{\frac{-2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}}} \quad \sqrt{20}.$$

$$\therefore \quad \hat{a}_R = \frac{-2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}} \quad , \; d\bar{l} = dy\,\hat{a}_y$$

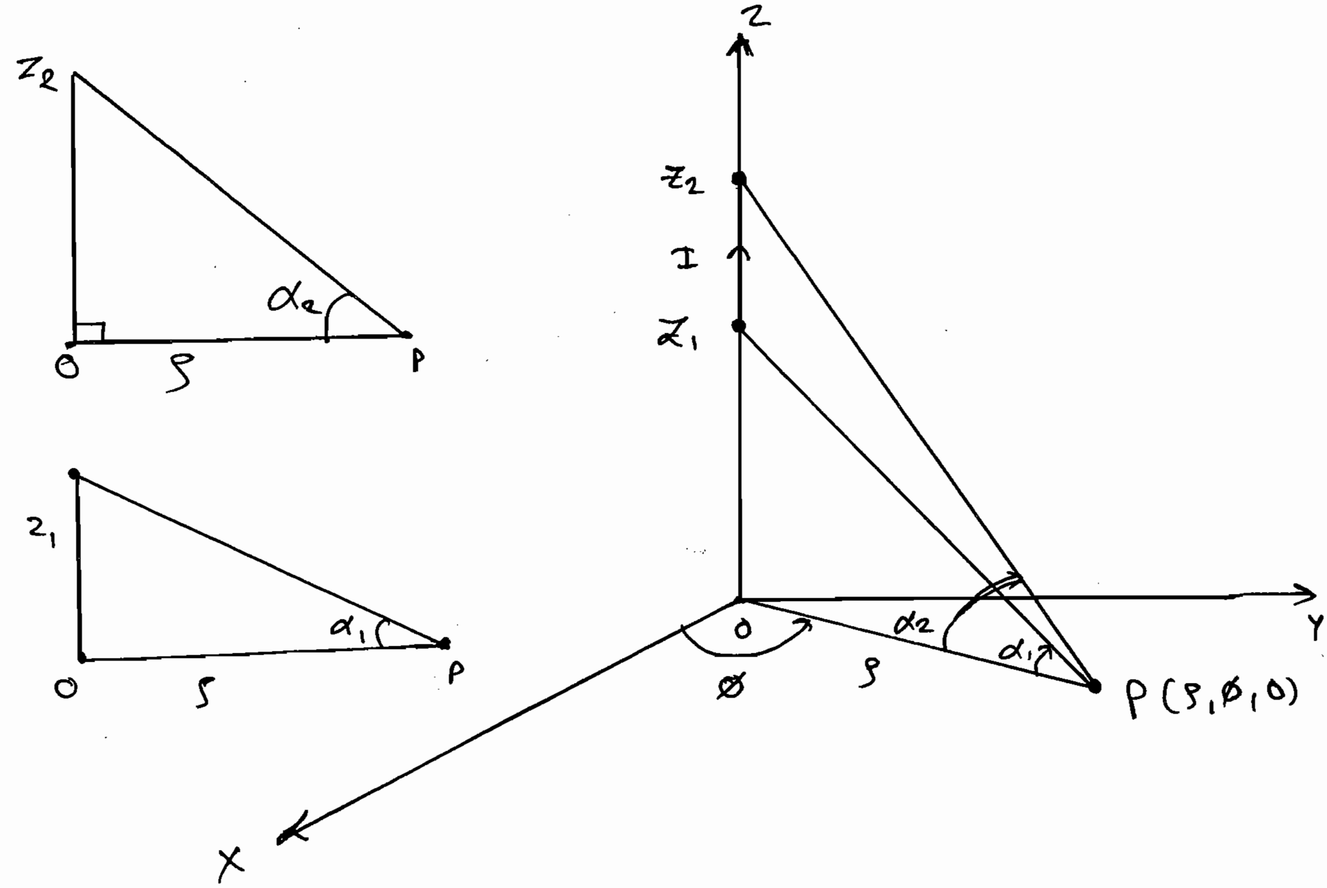
$$\therefore \quad d\bar{l} \times \bar{a}_R = \frac{2\hat{a}_z - 4\hat{a}_x}{\sqrt{20}}\, dy$$

Unit vector of $(d\bar{l} \times \bar{a}_R) = \dfrac{2\hat{a}_z - 4\hat{a}_x}{\sqrt{20}}$.

$$\therefore \quad \bar{H} = \frac{I}{2\pi\sqrt{20}} \cdot \frac{2\hat{a}_z - 4\hat{a}_x}{\sqrt{20}}$$

$$\therefore \quad \boxed{\bar{H} = \frac{I(\hat{a}_z - 2\hat{a}_x)}{20\pi} \; \text{A/m.}}$$

*



→ Figure shows a finite length current filament lies along z-axis. It carries a current of I A.

→ Find the Magnetic field Intensity at $P(\rho, \phi, 0)$ in terms of α_1 & α_2.

→ We know that ~~for~~ ~~infi~~

$$d\bar{H} = \frac{I \rho \, dz}{4\pi (\rho^2 + z^2)^{3/2}} \cdot \hat{a}_\phi .$$

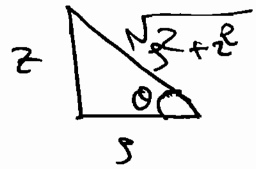
Now, for finite length (z_1 to z_2).

$$\therefore \bar{H} = \int_{z_1}^{z_2} \frac{I \rho \, dz}{4\pi (\rho^2 + z^2)^{3/2}} \hat{a}_\phi$$

$$\therefore \bar{H} = \frac{I}{4\pi} \hat{a}_\phi \int_{z_1}^{z_2} \frac{\rho \, dz}{(\rho^2 + z^2)} .$$

Put $z = \rho \tan\theta$

$\therefore dz = \rho \sec^2\theta$



$$\therefore \bar{H} = \frac{I}{4\pi} \int_{z_1}^{z_2} \frac{\rho^2 \sec^2\theta \cdot d\theta}{\rho^3 \sec^3\theta} \hat{a}_\phi$$

$$\therefore \sin\theta = \frac{z}{\sqrt{z^2+\rho^2}}$$

$$\cos\theta = \frac{\rho}{\sqrt{\rho^2+z^2}}$$

$$\bar{H} = \frac{I}{4\pi\rho} \cdot \left[\sin\theta\right]_{z_1}^{z_2}$$

$$\bar{H} = \frac{I}{4\pi\rho} \left[\frac{z}{(z^2+\rho^2)^{1/2}}\right]_{z_1}^{z_2} \hat{a}_\phi$$

$$\bar{H} = \frac{I}{4\pi\rho} \left[\frac{z_2}{(z^2+\rho^2)^{3/2}} - \frac{z_1}{\sqrt{z^2+\rho^2}}\right] \cdot \hat{a}_\phi$$

$$\therefore \boxed{\bar{H} = \frac{I}{4\pi\rho} \left[\sin\alpha_2 - \sin\alpha_1\right] \hat{a}_\phi} \quad \text{A/m.}$$

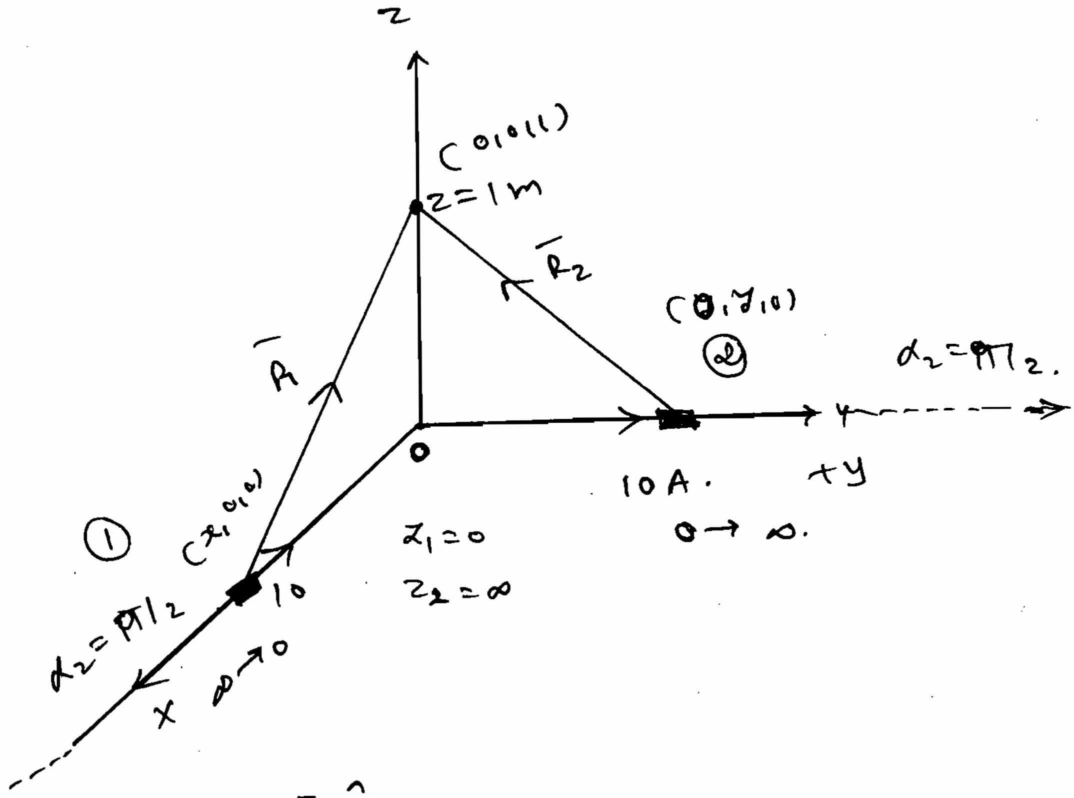
→ If $z_2 \to \infty \Rightarrow \alpha_2 = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1$.

→ If $z_1 \to -\infty \Rightarrow \alpha_2 = -\frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = -1$.

$$\therefore \bar{H} = \frac{I}{4\pi\rho} \left[1 - (-1)\right] \hat{a}_\phi$$

$$\therefore \boxed{\bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

* A current of 10A is directed in from infinity towards origin on the positive x-axis and then and then map to ∞ on the +ve y-axis find magnitude of magnetic field intensity on the ~~z~~-axis at $z = 1\,m$.



$$\bar{H} = \frac{I d\bar{l} \times \hat{a}_R}{4\pi |\bar{R}|}.$$

$\therefore$

$$\bar{H} = \frac{I}{2\pi |\bar{R}|} \text{. Unit vector of } (\hat{a}_R \times d\bar{l})$$

$\therefore$

①

$$d\bar{l}_1 = dx\, \hat{a}_x$$

$$\bar{R}_1 = -x\,\hat{a}_x + \hat{a}_z$$

$$\therefore |\bar{R}_1| = \sqrt{x^2+1}.$$

$$\therefore \hat{a}_{R_1} = \frac{-x\,\hat{a}_x + \hat{a}_z}{\sqrt{x^2+1}}$$

$$\therefore \hat{a}_{R_1} \times d\bar{l}_1 = \frac{-dx}{\sqrt{x^2+1}}\,\hat{a}_y$$

②

$$d\bar{l}_2 = dy\,\hat{a}_y.$$

$$\bar{R}_2 = \frac{-y\,\hat{a}_y + \hat{a}_z}{\sqrt{1+y^2}}$$

$$|\bar{R}_2| = \sqrt{1+y^2}$$

$$\hat{a}_{R_2} = \frac{-y\,\hat{a}_y + \hat{a}_z}{\sqrt{1+y^2}}.$$

$$\hat{a}_{R_2} \times d\bar{l}_2 = -\frac{dy}{\sqrt{1+y^2}}.\,\hat{a}_x.$$

$$\therefore d\bar{H} = \frac{I\,dy}{4\pi(y^2+1)^{3/2}}\hat{a}_x - \frac{I\,dx}{4\pi(x^2+1)^{3/2}}\cdot\hat{a}_y.$$

$$\therefore \bar{H} = \frac{I}{4\pi}\left[\int_0^\infty \hat{a}_x \frac{dy}{(y^2+1)^{3/2}} - \hat{a}_y \int_\infty^0 \frac{dx}{(x^2+1)^{3/2}}\cdot\hat{a}_y.\right]$$

$$\therefore \bar{H} = \frac{I}{4\pi}\left[\hat{a}_x + \hat{a}_y\right].$$

$$\left(\because \int_0^\infty \frac{1}{\sqrt{x^2+1}}dx = 1 \qquad \int_0^\infty \frac{dy}{\sqrt{y^2+1}} = 1.\right)$$

$$\therefore \boxed{\bar{H} = \frac{I}{4\pi}\left[\hat{a}_x + \hat{a}_y\right).}$$

Ex-2 Find the magnetic field intensity on the axis of a circular current loop of radius a which carries a direct current of I A also find the magnetic field Intensity at the centre of the circular current loop.

→ we assume that the circular current loop is located in z=0 plane and centred at origin. Therefore z axis would become the axis of the circular current loop on the z.

$$\therefore I\,d\bar{l} = I a\,d\phi\,\hat{a}_\phi$$

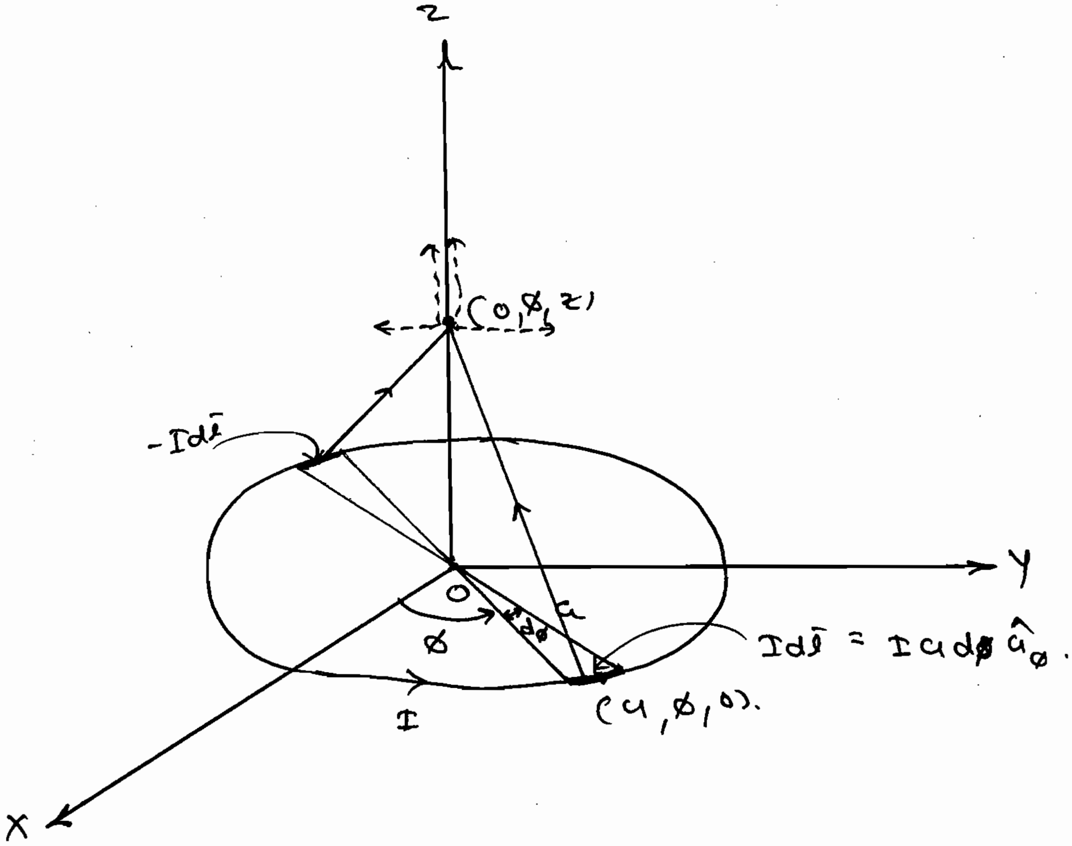
$$\therefore \bar{R} = -a\hat{a}_\rho + \cancel{b}\,\hat{a}_z.$$

$$\therefore \hat{a}_R = \frac{-a\hat{a}_\rho + \cancel{b}\,\hat{a}_z}{\sqrt{a^2+z^2}}.$$

$$I\,d\bar{l} \times \bar{a}_R$$

$$= \frac{I a^2\,d\phi\,\hat{a}_z + I a b\,\hat{a}_\rho}{\sqrt{a^2+\cancel{b}^2}}$$

$\Rightarrow$



→ As shown in the figure for every differential current filament on the circular current loop there exist an another differential current filament diameterically opposite side which results in cancellation of horizontal field components and the resultant field would be along $\hat{a}_z$ direction only.

→ Ignoring $\hat{a}_\rho$ components the total field is given by

$$d\bar{H} = \frac{I d\bar{l} \times \hat{a}_R}{4\pi |\bar{R}|^2}.$$

$$\therefore \quad d\bar{H} = \frac{I a^2 \hat{a}_z + I a b \hat{a}_\rho}{4\pi (a^2+b^2)^{3/2}}.$$

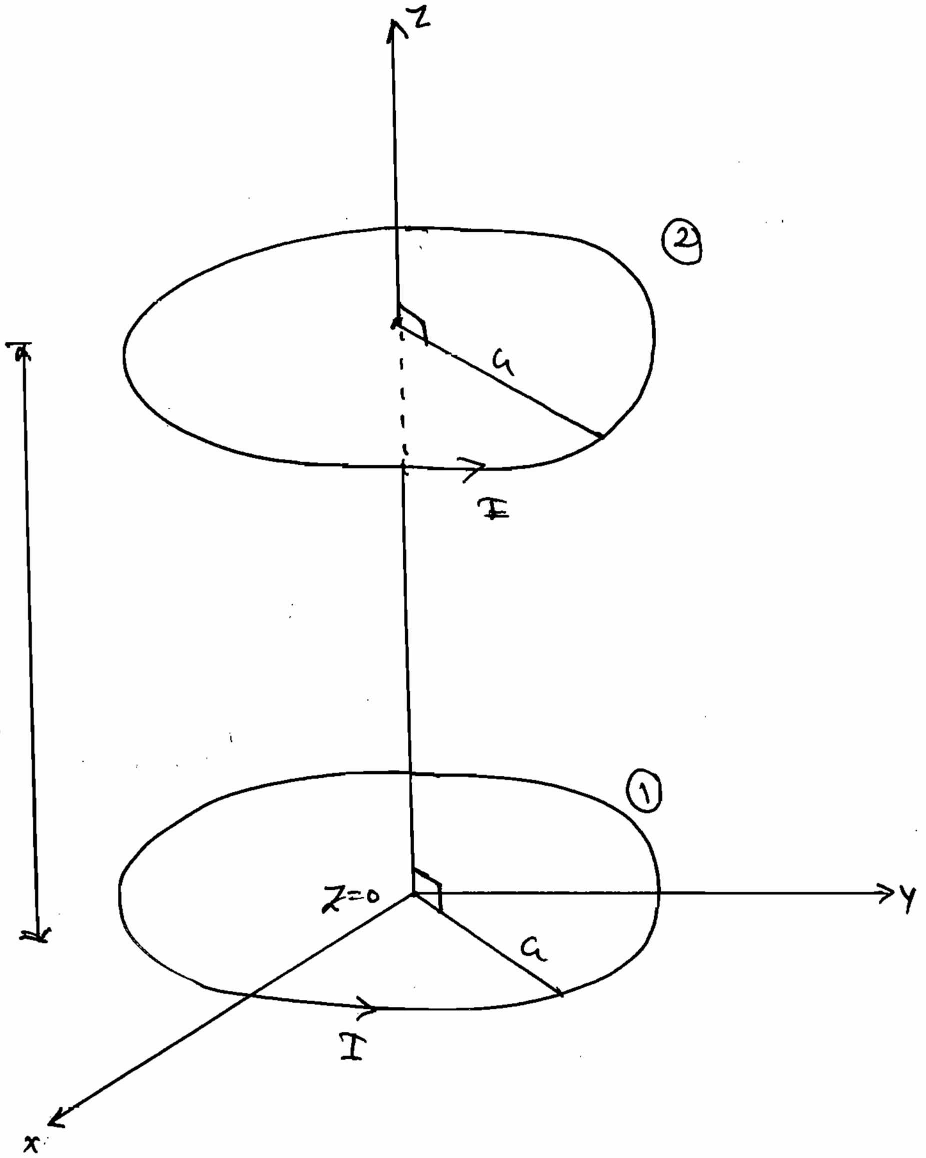
$$\bar{H} = \frac{Ia^2}{4\pi\,(a^2+b^2)^{3/2}} \int_0^{2\pi} d\phi.$$

$$\therefore \boxed{\bar{H} = \frac{Ia^2}{2\,(a^2+b^2)^{3/2}}\,\hat{a}_z \quad A/m.}$$

→ At the centre of the loop (put $b=0$), the expression for magnetic field intensity is given by

$$\boxed{\bar{H} = \frac{I}{2a}\,\hat{a}_z \quad A/m.}$$

*



→ Figure shows parallel circular current loops ① & ② find $\bar{H}$ at $z=b$ at the centre of loop -②.

$$\bar{H}_1 = \frac{Ia^2}{2(a^2+b^2)^{3/2}} \hat{a}_z$$

$$\bar{H}_2 = \frac{I}{2a} \hat{a}_z$$

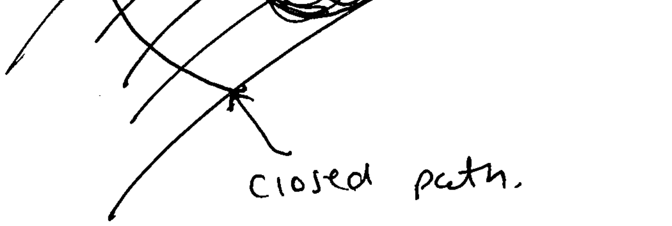
$$\therefore \bar{H} = \bar{H}_1 + \bar{H}_2$$

$$\therefore \boxed{\bar{H} = \left(\frac{Ia^2}{2(a^2+b^2)^{3/2}} + \frac{I}{2a} \right) \hat{a}_z}$$

* Ampere's Law:

→ The line Integral of Magnetic field Intensity around a closed path is equal to current enclosed by the path.

$$\boxed{\oint_l \bar{H} \cdot d\bar{l} = I_{enc.}}$$



$$\therefore I = \int_s \bar{J}_c \cdot d\bar{s}$$

$$\therefore \oint_l \bar{H} \cdot d\bar{l} = \int_s \bar{J}_c \cdot d\bar{s}$$

Now, By stoke's' theorem.

$$\therefore \oint_l \bar{H} \cdot d\bar{l} = \int_s \underline{\underline{(\nabla \times \bar{H})}} \, d\bar{s} = \int_s \underline{\underline{\bar{J}_c}} \cdot d\bar{s}$$

$$\therefore \boxed{\nabla \times \bar{H} = \bar{J}_c}$$

point form of Ampere's Law.

→ The closed path is touching the conductor.

$\therefore$ The total current in enclosed by the path $I_{enc} = I$.

$$\rightarrow \nabla \times \bar{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$\rightarrow \nabla \times \bar{H} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho\,\hat{a}_\phi & \hat{a}_z \\ \partial/\partial \rho & \partial/\partial \phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$\rightarrow \nabla \times \bar{H} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\,\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ H_r & r H_\theta & r\sin\theta\, H_\phi \end{vmatrix}$$

Ex-1 Let, $\bar{H} = -y(x^2+y^2)\hat{a}_x + x(x^2+y^2)\hat{a}_y$ Alm. 133

Find the amount of current passing through $z=0$, $-1 \le x \le 1$, $-2 \le y \le 2$ in $\hat{a}_z$ direction.

Ans:

$$\nabla \times \bar{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y(x^2+y^2) & x(x^2+y^2) & 0 \end{vmatrix}$$

$$\therefore \nabla \times \bar{H} = 0 - 0 + \hat{a}_z\left(3x^2+y^2) + (x^2+3y^2\right).$$

$$\therefore \nabla \times \bar{H} = 4(x^2+y^2)\,\hat{a}_z \quad \text{Alm.}$$

$$\bar{J}_c \cdot d\bar{s} = \left(4(x^2+y^2)\,\hat{a}_z\right)(dx\,dy\,\hat{a}_z).$$

$$\therefore \bar{J}_c \cdot d\bar{s} = 4(x^2+y^2)\,dx\,dy.$$

$$\therefore I = \int_s \bar{J}_c \cdot d\bar{s} = 4\int_{-1}^{1}\int_{-2}^{2}(x^2+y^2)\,dx\,dy.$$

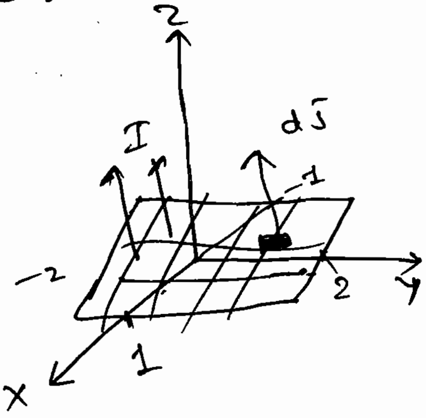
$$= 4\left[\int_{-2}^{2}\int_{-1}^{1} x^2\,dx\,dy + \int_{-1}^{1}\int_{-2}^{2} y^2\,dx\,dy\right]$$

$$= 4\left[\left[\frac{x^3}{3}\right]_{-1}^{1}[y]_{-2}^{2} + \left[\frac{y^3}{3}\right]_{-2}^{2}[x]_{-1}^{1}\right].$$

$$I = 4\left[\left(\frac{2}{3}\times 4\right) + \frac{16}{3}\times 2\right].$$

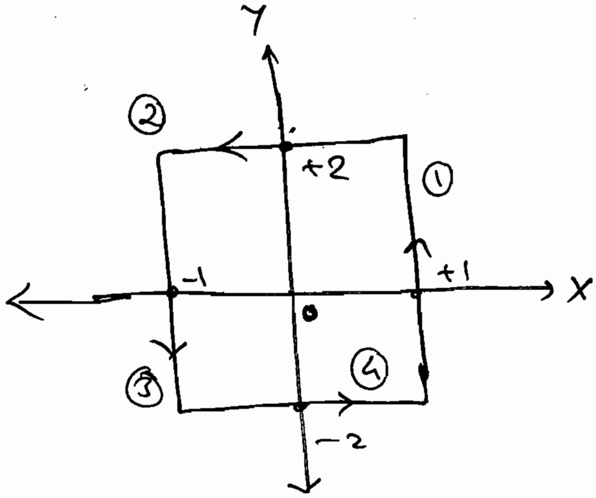
$$= 4\left[\frac{8}{3} + \frac{32}{3}\right].$$

$$\therefore \boxed{I = 160/3 \text{ A}}$$



→ Method-3 : by Integral form

$$I_{enc} = \oint_l \bar{H}\cdot d\bar{l}$$



$$\therefore I_{enc} = \int_{①} \bar{H}\cdot d\bar{l} + \int_{②} \bar{H}\cdot d\bar{l} + \int_{③} \bar{H}\cdot d\bar{l} + \int_{④} \bar{H}\cdot d\bar{l}.$$

∴ Path No	$d\bar{l}$	Path is at	Path limit	$\bar{H}\cdot d\bar{l}$	$\bar{H}\cdot d\bar{l}$ at $x=1$
①	$dy\,\hat{a}_y$	$x=1$	$-2 \le y \le 2$	$x(x^2+y^2)dy$	$(1+y^2)\,dy$
②	$dx\,\hat{a}_x$	$y=2$	$+1 \le y \le -1$	$-y(x^2+y^2)dx$	$-2(x^2+4)\,dx$
③	$dy\,\hat{a}_y$	$x=-1$	$2 \le y \le -2$	$x(x^2+y^2)dy$	$-(1+y^2).dy$
④	$dx\,\hat{a}_x$	$y=-2$	$-1 \le x \le 1$	$-y(x^2+y^2)dx$	$2(x^2+4).dx$

$$\oint_{①} \bar{H}\cdot d\bar{l} = \int_{-2}^{2} (y^2+1)\,dy = \left[\frac{y^3}{3}+y\right]_{-2}^{2} = \frac{8}{3}+2+8/3-2$$
$$= 16/3 + 4$$

$$\int_{②} \bar{H}\cdot d\bar{l} = \int_{1}^{-1} -2(x^2+4)\,dx = 2\left[\frac{x^3}{3}+4x\right]_{-1}^{1} = \frac{4}{3} + 16$$

$$\int_{③} \bar{H}\cdot d\bar{l} = \int_{2}^{-2} -(1+y^2)\,dy = \left[\frac{y^3}{3}+y\right]_{-2}^{2} = 16/3 + 4$$

$$\int_{④} \bar{H}\cdot d\bar{l} = \int_{-1}^{1} 2(x^2+4) = 2\left[x^3/3 + 4x\right]_{-1}^{1} = 16$$
$$= \frac{4}{3} + 16.$$

$$\therefore I = \oint_l \bar{H} \cdot d\bar{l}$$

$$= \frac{16}{3} + \frac{16}{3} + \frac{4}{3} + \frac{4}{3} \cdot t_{10}$$

$$\therefore I = \frac{40}{3} \cdot t_{10}$$

$$\boxed{I = \frac{160}{3}\ A}$$

* Application of Ampere's law:

⇒ For symmetrical current distribution where we have an idea about the direction of magnetic field Intensity then one can find out magnitude of the magnetic field intensity. by using the following procedure.

① one has to choose a suitable appropriate closed path which is enclosing partially (or) fully the given current distribution

② $d\bar{l}$ always lies along the path ✓

③ The closed path is so choosen in such a way that $\bar{H}$ may lie along the path or normal to the path

$\bar{H} \cdot d\bar{l} = |\bar{H}||d\bar{l}|$ if H lies along the path,

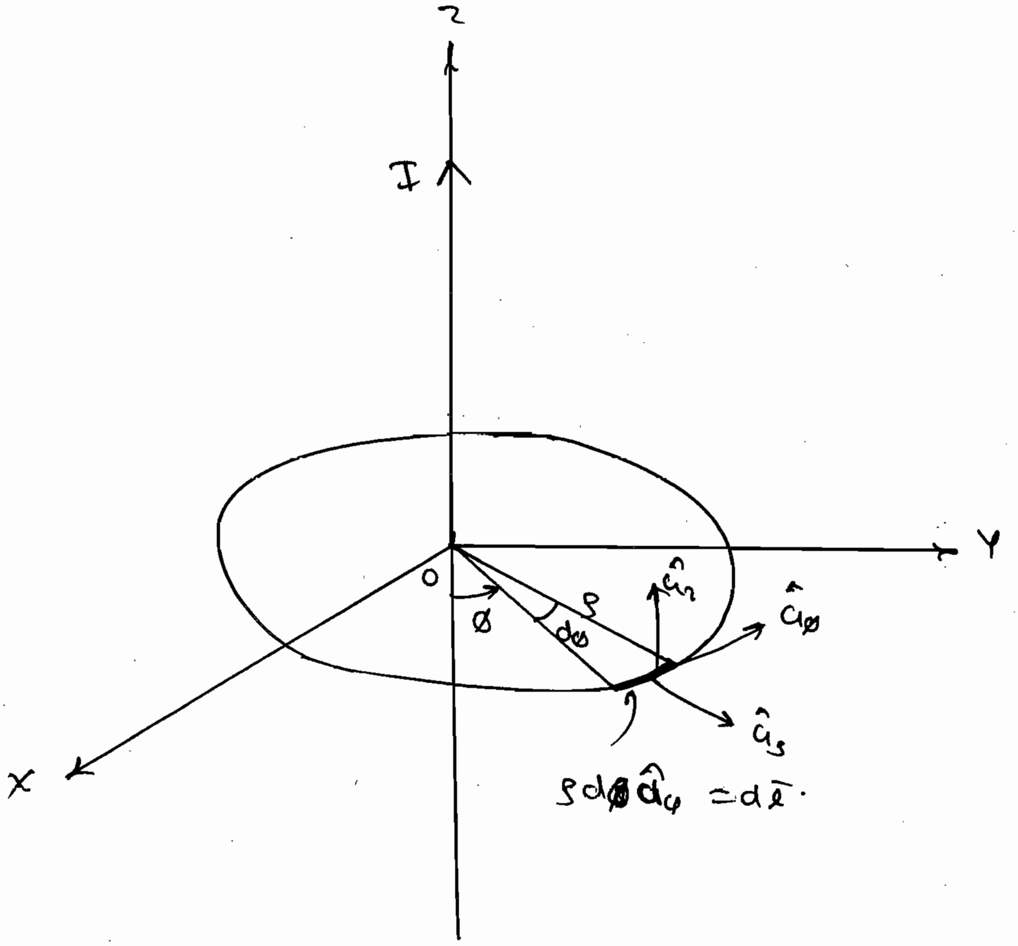
$\bar{H} \cdot d\bar{l} = 0$ if $\bar{H}$ normal to the path.

④ over that part of the path where $\bar{H}$ lies along the path, on that part of the path

$\bar{H}$ is constant.

Ex-$\frac{1}{2}$ Find $\bar{H}$ due to a long straight infinite filamentary conductor which carries a direct current of I A.

Ans: We assume that the infinite current filament lies along z axis



① A closed circular path is choosen of $\rho = \text{const.}$ ($\rho > 0$) is choosen.

② $d\bar{l} = \rho d\phi \hat{a}_\phi$.

③ $\bar{H} = H_\phi \boxed{\hat{a}_\phi}$ only → $\bar{H}$ would be around the conductor.

$\bar{H} \cdot d\bar{l} = \rho H_\phi d\phi$.

④ $|\bar{H}| = H_\phi$ must be const. on $\rho = \text{const.}$

$\because$ $\bar{H}$ lies along the path.

$I_{enc} = I$ (The total current is enclosed).

$\therefore \oint \bar{H} \cdot d\bar{l} = I_{enc}.$

$\therefore I_{enc} = H_\phi \rho \int_0^{2\pi} d\phi.$

$\therefore I = H_\phi \rho \, 2\pi.$

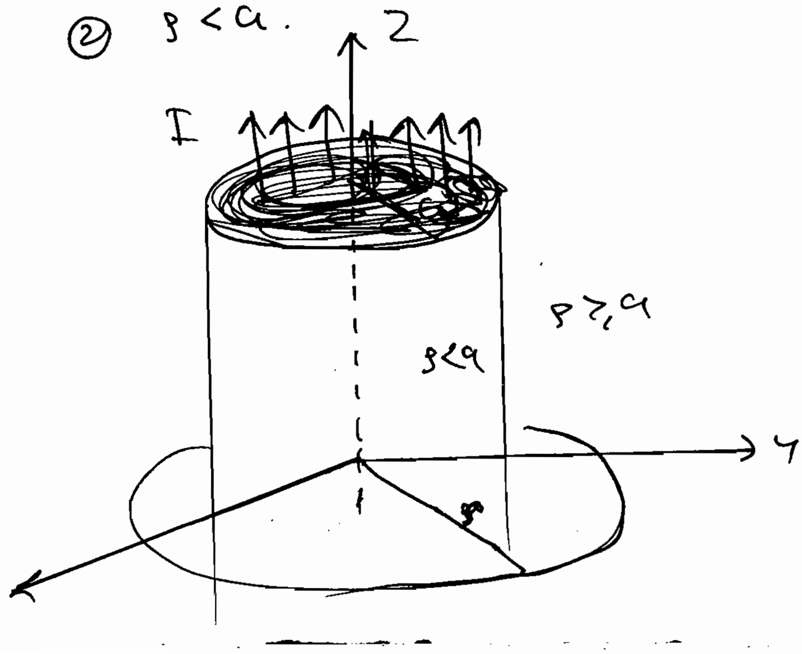
$\therefore H_\phi = \frac{I}{2\pi\rho}.$

$$\therefore \boxed{\bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

Ex 2 Find $\bar{H}$ due to a ~~along~~ ~~along~~ infinite solid cylindrical conductor of radius a where current I uniformally distributed throughout the cross section.

Ans 2 we assume that the solid cylindrical conductor possion along z-axis as shown in ~~in~~ the figure. where the current I is uniformly distributed throughout the cross section one can find $\bar{H}$ for

(1) $\rho \geq a$ (2) $\rho < a$.



① The circular closed path of $\rho =$ constant $(\rho > 0)$ is choosen.

② $d\bar{l} = a\,d\phi\,\hat{a}_\phi$.

③ $\bar{H} = H_\phi\,\hat{a}_\phi$.

$\bar{H}\cdot d\bar{l} = a H_\phi\,d\phi$.

④ $|\bar{H}| = H_\phi$ must be const. on $\rho =$ const.

$\because$ $\bar{H}$ lies along the path.

$I_{enc} = I$ (The total current is enclosed).

$\therefore \oint_l \bar{H}\cdot d\bar{l} = I_{enc}$.

$\therefore a H_\phi \int_0^{2\pi} d\phi = I$.

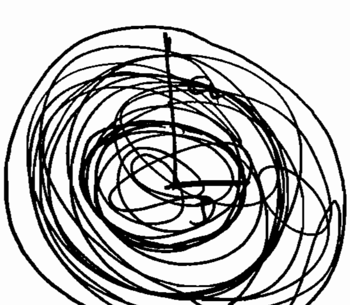
$\therefore 2\pi a H_\phi = I$.

$\therefore H_\phi = \dfrac{I}{2\pi a}$.

$\therefore \boxed{\bar{H} = \dfrac{I}{2\pi a}\,\hat{a}_\phi}$

② $\rho < a$.

$\therefore$



$\pi a^2 \rightarrow I$

$\pi \rho^2 \rightarrow I'$

$\therefore I_{enc} = \dfrac{\pi\rho^2}{\pi a^2} \times I$

$\therefore I_{enc} = \dfrac{I\rho^2}{a^2}$.

$\oint \bar{H}\cdot d\bar{l} = I_{enc} \Rightarrow H_\phi\,(2\pi\rho) = \dfrac{I\rho^2}{a^2}$.

$$\therefore \quad H_\phi = \frac{I\rho}{2\pi a^2}.$$

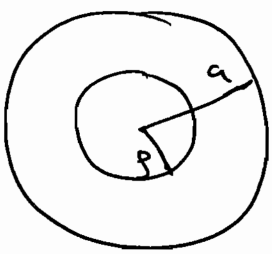
$$\therefore \quad \boxed{\bar{H} = \frac{I\rho}{2\pi a^2}\,\hat{a}_\phi.} \quad \text{for } \rho < a.$$

$$\boxed{\begin{array}{llll} \text{(i)} & \rho \geq a & \bar{H} = \dfrac{I}{2\pi\rho}\,\hat{a}_\phi & \Rightarrow |\bar{H}| \propto 1/\rho. \\ \text{(ii)} & \rho < a & \bar{H} = \dfrac{I\rho}{2\pi a^2}\,\hat{a}_\phi & \Rightarrow |\bar{H}| \propto \rho. \end{array}}$$

Ex-2 Repeat the above problem if it is a hollow cylindrical conductor of radius a.

Ans: (i) $\rho \geq a$ $\quad \bar{H} = \dfrac{I}{2\pi\rho}\,\hat{a}_\phi$

(ii) $\rho < a$ $\quad \bar{H} = 0$



$\rho < a$

$I_{enc} = 0.$

* Magnetic Flux Density ($\bar{B}$)

→ Unit :→ T (or) wb/m².

∴ $\boxed{\bar{B} = \mu \bar{H}.}$

$\mu = \mu_0 \mu_r$ H/m.

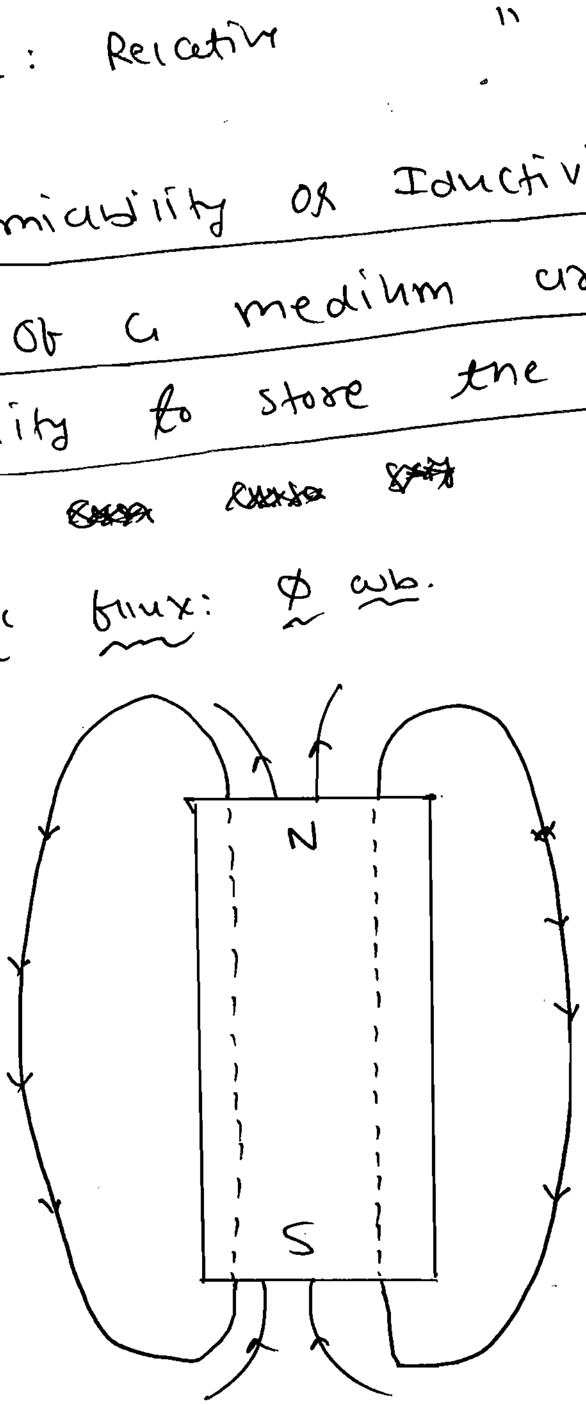
∴ μ : permiabiiitity | Inductivity (H/m).

μ_0 : Absolute permiability | Inductivity (H/m).

μ_r : Relative " "

→ ® Permiability or Inductivity specifies property of a medium and that indicates the ability to store the magnetic energy.

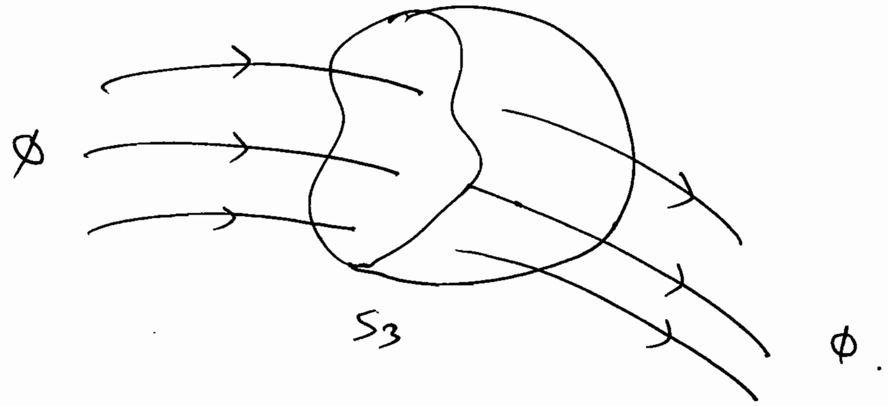
* Magnetic flux: ϕ wb.



→ The amount of magnetic flux passing through a cross sectional surface 's' is given by,

$$\phi = \int_S \bar{B} \cdot d\bar{s}$$

↖ cross sectional



S_3 = arbitrary closed surface.

→ $$\oint_S \bar{B} \cdot d\bar{s} = 0.$$

$$\therefore \int_v \nabla \cdot \bar{B} \cdot dv = 0.$$

$$\therefore \boxed{\nabla \cdot \bar{B} = 0.}$$

Gauss law for H-fields.

→ for electric field

$$\psi_{net} = Q_{enc} = \oint_s \bar{D} \cdot d\bar{s} = \int_v \nabla \cdot \bar{D}\, dv.$$

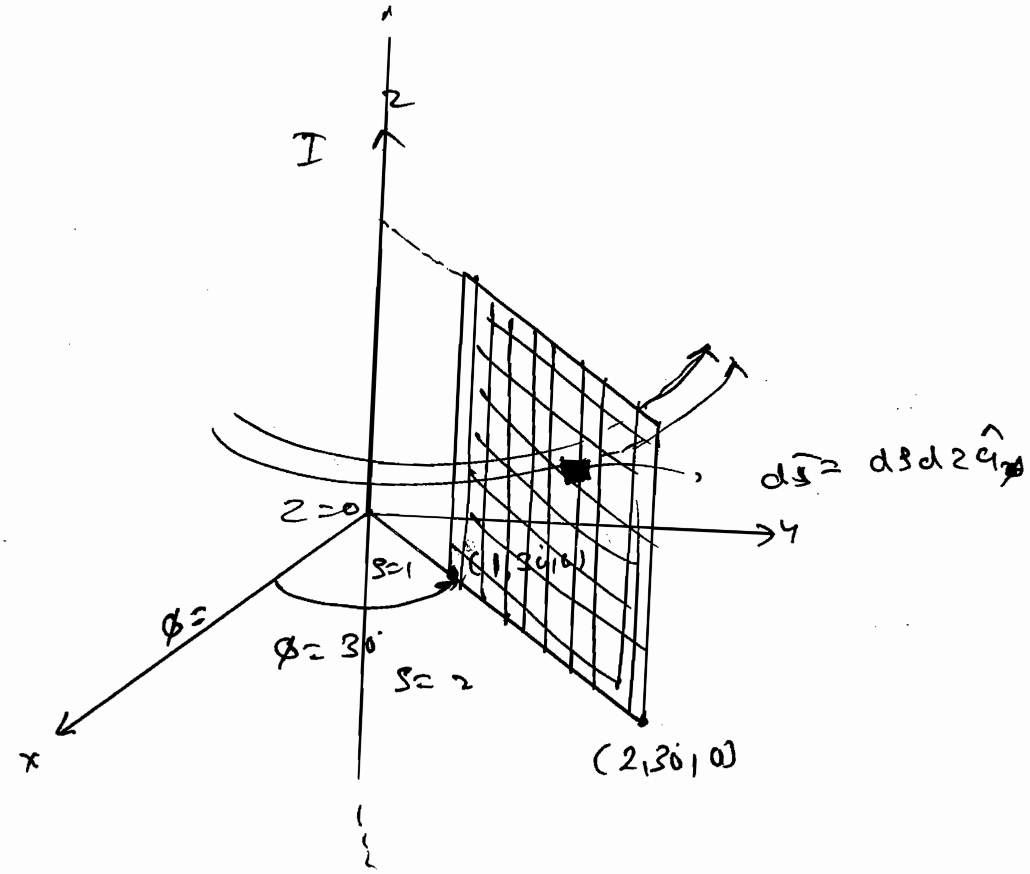
$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$

Gauss field for E-fields.

→ Unlike a electric flux the magnetic flux would not have starting point and an ending point it enters the closed surface and leaves the same closed surface as shown above. one can find out the amount of magnetic flux passing through a cross sectional surface.

Ex-1 Find the amount of magnetic flux passing through a cross-sectional surface define by $\phi = 30°$, $1 \le \rho \le 2$, $0 \le z \le 3$. due to an infinite current filament lies along z-axis. which carries a direct current of 2.5 A along +ve z-direction. Assume $\mu = \mu_0$.

Ans:



$\rightarrow \; d\bar{s} = d\rho \, dz \, \hat{a}_\phi .$

$\bar{B} = \mu \bar{H}$

$\therefore \bar{B} = \mu_0 \bar{H}$

$$\therefore \bar{B} = \frac{\mu_0 I}{2\pi\rho} \hat{a}_\phi .$$

$$\therefore \quad \bar{B} \cdot d\bar{s} = \frac{\mu_0 I}{2\pi\rho} \, d\rho \, dz .$$

$$\therefore \quad \phi = \int_S \bar{B} \cdot d\bar{s}$$

$$= \int_1^2 \int_0^3 \frac{\mu_0 I}{2\pi\rho} \, d\rho \, dz .$$

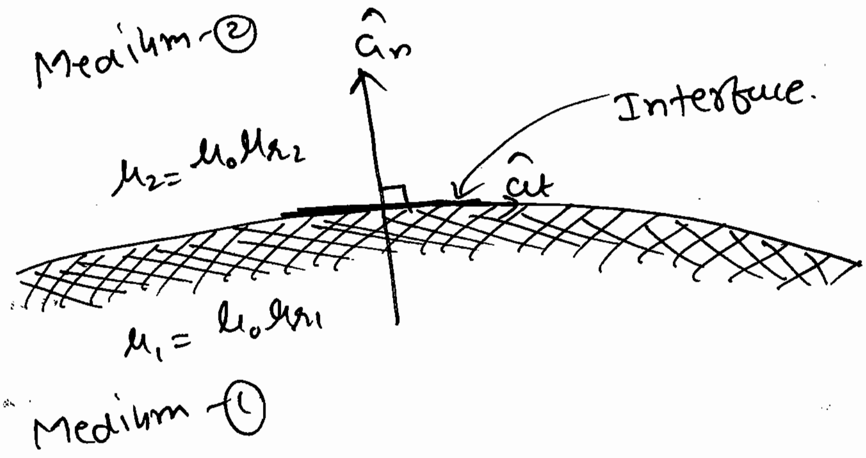
$$= \frac{\mu_0 I}{2\pi\rho} \times [\ln 2 - 1] \times [3] .$$

$$\therefore \boxed{\phi = \frac{3\mu_0 I}{2\pi\rho} \ln 2 \text{ wb}}$$

$$\therefore \boxed{\phi = \frac{3\mu_0 I}{2\pi} \ln 2 \quad \text{wb}}$$

$$\phi = \frac{7.5 \, \mu_0 \ln 2}{2\pi} . \quad \text{wb.}$$

* Boundary Condition:



(I) Using Amperes Law, one can show that

(a) $\boxed{H_{t1} = H_{t2}.}$

Tangential components of H-fields are continuous across a current free interface.

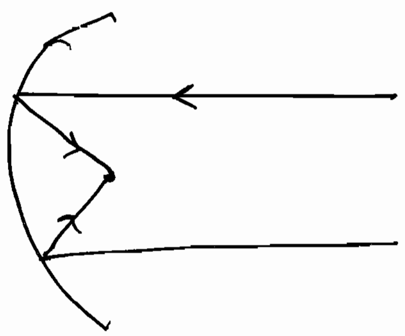
(b) $\boxed{(\bar{H}_1 - \bar{H}_2) \times \hat{a}_n = \bar{J}_s}$ (A/m)

→ Tangential components of H-fields are discontinued by an amount of surface current densities.

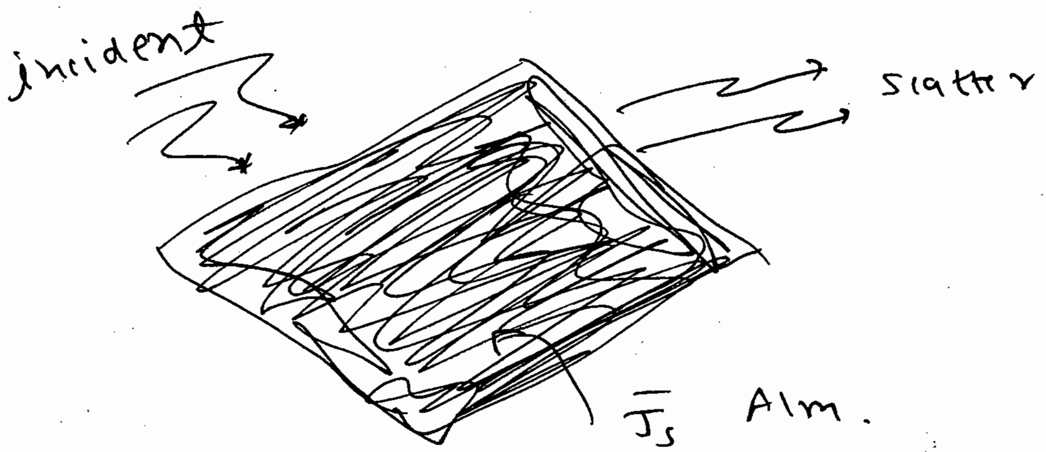
(II) Using Gauss Law for H-field,

$\boxed{B_{n1} = B_{n2}.}$

i.e. Normal component of magnetic flux densities are continuous across the interface.



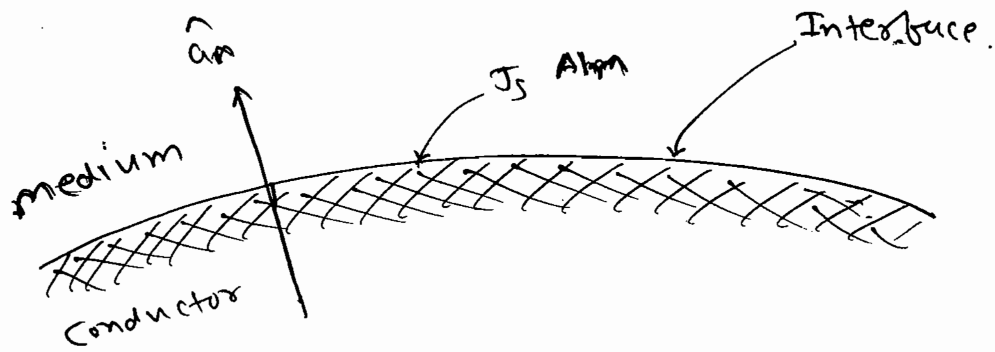
Parabolic antenna.



PEC: Perfect electric Conductor

$\bar{J}_s$: current per unit width (A/m)

* → Behaviour of Magnetic field intensities across a conductor interface.



① $\hat{a}_n \times \bar{H} = \bar{J}_s$

→ Tangential component of magnetic fields are equal to surface current density.

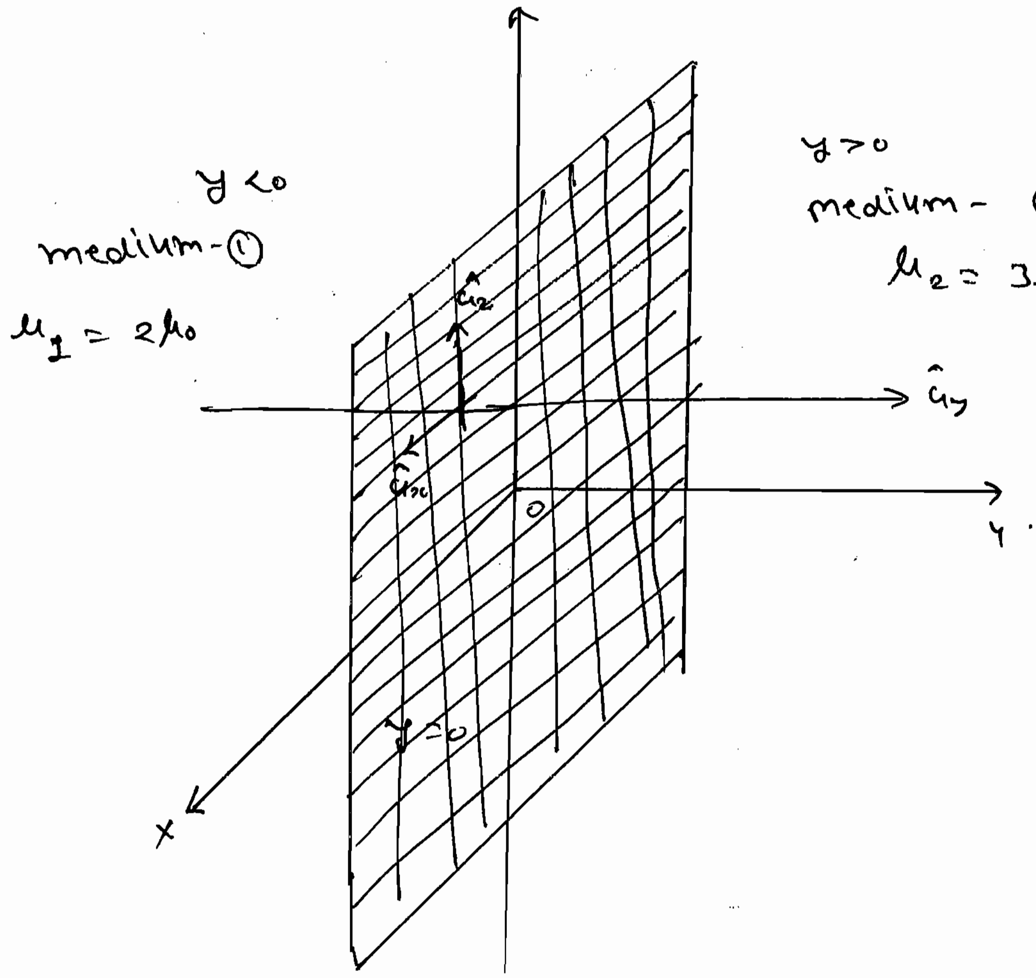
② $B_n = 0$ (or) $H_n = 0$.

→ Normal components of magnetic ~~fixed~~ field ~~flux~~ ~~densities~~ intensities are vanishes across the conductor interface.

J_s : Surface current density across the conductor interface.

Ex-1 The plane $y=0$ separates two mediums $y<0$ is medium-1 and is characterised by $2\mu_0$ and $y>0$ is medium-2 and is characterised by $3\mu_0$ given that $\bar{H}_1 = 10\hat{a}_z$ and $\bar{H}_2 = 10\hat{a}_x$. Find the surface current density across the interface.

Ans:



$\bar{H}_1 = 10\hat{a}_z$, $\bar{H}_2 = 10\hat{a}_x$.

→ $(\bar{H}_1 - \bar{H}_2) \times \hat{a}_n = \bar{J}_s$

$\therefore \bar{J}_s = (10\hat{a}_z - 10\hat{a}_x)\,\hat{a}_y$

$$\boxed{\bar{J}_s = -10\hat{a}_x - 10\hat{a}_z \text{ A/m.}}$$

Ex-2 Refering to the above figure let, $\bar{H}_1 = (3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z)$ A/m &

$\bar{H}_2$

assume current free interface find $\bar{B}_1$, $\bar{H}_2$ and $\bar{B}_2$

Ans:

$$\bar{B}_1 = \mu_1 [\bar{H}_1].$$

$$\therefore \boxed{\bar{B}_1 = \mu_0 [6\hat{a}_x + 8\hat{a}_y + 10\hat{a}_z).}$$

$$\therefore \bar{H}_1 = 3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z.$$

$$\therefore \bar{H}_1 = H_{t1}\hat{a}_t + H_{n1}\hat{a}_n.$$

$$\therefore \bar{H}_{t1} = 3\hat{a}_x + 5\hat{a}_z, \quad \bar{H}_{n1} = 4\hat{a}_y.$$

$\therefore$ Now, $\bar{H}_{t1} = \bar{H}_{t2}$ $(\because J_s = 0)$

$(\because$ tangential components are equal).

$$\therefore \boxed{\bar{H}_{t2} = 3\hat{a}_x + 5\hat{a}_z.}$$

now, $\bar{H}_2 = H_{t2}\hat{a}_t + H_{n2}\hat{a}_n.$

$$\therefore B_{n1} = B_{n2} \quad (\because J_s = 0).$$

$$\therefore B_{n2} = B_{n1}.$$

$$\therefore \mu_2 H_{n2} = \mu_1 H_{n1}.$$

$$\therefore \bar{H}_{n2} = \frac{\mu_1}{\mu_2} \times 4\hat{a}_y.$$

$$\therefore \bar{H}_{n2} = \frac{2}{3} \times 4\hat{a}_y$$

$$\therefore \boxed{\bar{H}_{n2} = \frac{8}{3}\hat{a}_y}$$

$$\therefore \boxed{\bar{H}_2 = 3\hat{a}_x + \frac{8}{3}\hat{a}_y + 5\hat{a}_z.}$$

$\therefore \quad \bar{B}_2 = \mu_2 \bar{H}_2$

$$\therefore \quad \boxed{\bar{B}_2 = \mu_0 [\, 9\hat{a}_x + 8\hat{a}_y + 15\hat{a}_z].}$$

Ex-3 In the above problem assume that the interface has non zero surface current density of $\bar{J}_s = (5\hat{a}_x + 10\hat{a}_z)$ A/m. Find $\bar{B}_1$, $\bar{H}_2$ and $\bar{B}_2$.

Ans:

$\bar{H}_1 = 3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$.

$\bar{B}_1 = \mu_1 \bar{H}_1$

$$\therefore \quad \boxed{\bar{B}_1 = \mu_0 [\, 6\hat{a}_x + 8\hat{a}_y + 10\hat{a}_z]}$$

Now, $(\bar{H}_1 - \bar{H}_2) \times \hat{a}_n = \bar{J}_s$

$\therefore \quad \cancel{H_{t1}\hat{a}_t}$

$[(H_{t1} - H_{t2})\hat{a}_t + (H_{n1} - H_{n2})\hat{a}_n] \times \hat{a}_n = \bar{J}_s$.

$\therefore \quad (H_{t1} - H_{t2})(\hat{a}_t \times \hat{a}_n) = \bar{J}_s$.

$\therefore \quad \hat{a}_t = \dfrac{3}{\sqrt{34}}\hat{a}_x + \dfrac{5}{\sqrt{34}}\hat{a}_z$.

$\therefore \quad \hat{a}_n = \hat{a}_y$.

$\therefore \quad \hat{a}_t \times \hat{a}_n = \dfrac{12}{\sqrt{34}}\hat{a}_z - \dfrac{20}{\sqrt{34}}\hat{a}_x$.

$\therefore \quad (H_{t1} - H_{t2})\left(\dfrac{12}{\sqrt{34}}\hat{a}_z - \dfrac{20}{\sqrt{34}}\hat{a}_x\right) = 5\hat{a}_x + 10\hat{a}_z$.

$\therefore \dfrac{12}{\sqrt{34}}(H_{t1} - H_{t2}) = 10, \qquad \dfrac{-20}{\sqrt{34}}(H_{t1} - H_{t2}) = 5.$

$$(H_{t1} - H_{t2}) = \frac{5}{(\sqrt{3}n)} \sqrt{3n}$$

$$H_{t2} - H_{t1} = \frac{\sqrt{34}}{4}.$$

$$H_{t1} - H_{t2} = \frac{\sqrt{34}}{4}.$$

$$\therefore \quad \bar{H}_{t1} = H_{t1} \cdot \hat{a}_t$$

$$\therefore \quad H_{t1} = \bar{H}_{t1} \cdot \hat{a}_{t1}$$

$$= \cancel{3\hat{a}_x} + \cancel{5}$$

$$(\bar{H}_1 - \bar{H}_2) \times \hat{a}_n = \bar{J}_s.$$

$$\therefore \left[(3 - H_{x2})\hat{a}_x + (4 - H_{y2})\hat{a}_y + (5 - H_{z2})\hat{a}_z\right] \times \hat{a}_y$$

$$= 5\hat{a}_x + 10\hat{a}_z.$$

$$\therefore \quad \cancel{7} - \quad 3 - H_{x2} = 10 \quad \Rightarrow \quad H_{x2} = -7.$$

$$\therefore \quad -(5 - H_{z2}) = 5.$$

$$\therefore \quad H_{z2} = 10.$$

$$\therefore \quad \bar{H}_2 = -7\hat{a}_x + H_{y2}\hat{a}_y + 10\hat{a}_z.$$

$$\therefore \quad B_2 = \mu_2 \bar{H}_2$$

$$\therefore \quad \cancel{B_2 =} \quad \cancel{\mu_2 [-21\hat{a}_x +}$$

$$-7\mu_2\hat{a}_x + \mu_2 H_{y2}\hat{a}_y + 10\mu_2\hat{a}_z = B_{x2}\hat{a}_x + 4\mu_1\hat{a}_y + B_{z2}\hat{a}_z.$$

$$\therefore \quad B_{x2} = -7\mu_2$$

$$B_{z2} = 10\mu_2$$

$$\bar{B}_1 = \mu_1 \bar{H}_1$$

$$\boxed{\bar{B}_1 = (6\hat{a}_x + 8\hat{a}_y + 10\hat{a}_z)\mu_1}$$

$$H_{y2} = 4\frac{\mu_1}{\mu_2} \quad \text{A/m}.$$

$$\therefore \quad H_{y2} = \frac{8}{3}\hat{a}_y.$$

$$\therefore \boxed{\bar{H}_2 = -7\hat{a}_x + \frac{8}{3}\hat{a}_y + 10\hat{a}_z}$$

$$\bar{B}_2 = (-21\hat{a}_x + 8\hat{a}_y + 30\hat{a}_z)\mu_2.$$

☆ Time Varying Fields:

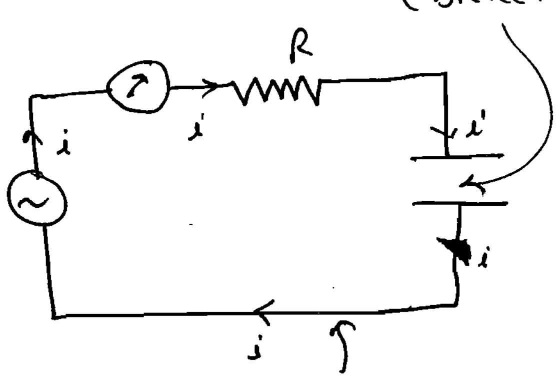
→ The existing Ampere's Law, when it is applied to the time varying fields in a non-conducting medium the Law is having some inconsitency or unsatisfaction. This inconsitency is been eliminated by adding a new term $\bar{J}_D$ as follows:

$$\boxed{\nabla \times \bar{H} = \bar{J}_c + \bar{J}_D} \rightarrow \text{Modified Amperes Law}$$

∴ take ∇ on both the side.

$$\nabla \cdot \nabla \times \bar{H} = \nabla \cdot \bar{J}_c + \nabla \cdot \bar{J}_D.$$

∴ Divergence of curl is zero.



$$\therefore \nabla \cdot \bar{J}_c = -\nabla \cdot \bar{J}_D.$$

$$\therefore \nabla \cdot \bar{J}_c = -\frac{\partial \rho_v}{\partial t}.$$

$$\therefore \nabla \cdot \bar{J}_D = +\frac{\partial \rho_v}{\partial t}.$$

$$\therefore \nabla \cdot \bar{J}_D = \partial/\partial t\,(\nabla \cdot \bar{D}).$$

$$\therefore \nabla \cdot \bar{J}_D = \nabla \cdot \frac{\partial \bar{D}}{\partial t}.$$

∴ Suppressing ∇ both the side.

$$\therefore \boxed{\bar{J}_D = \frac{\partial \bar{D}}{\partial t}}$$

$\bar{J}_D$ = Displacement current density.

→ $\bar{J}_D$ is defined as time rate of change of electric flux Density.

→ $\bar{J}_c$ dominates in a conducting medium and is zero in perfect Dielectric.

→ $\bar{J}_D$ dominates in a dielectric medium and is zero in perfect conductor.

→ The modified Ampere's Law is written as $\boxed{\bar{J}_c + \bar{J}_D = \nabla \times \bar{H}}$.

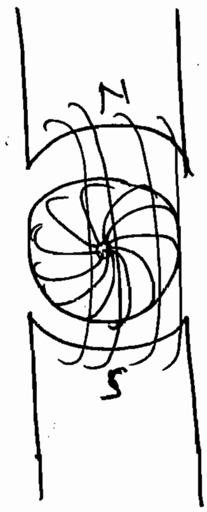
$$\therefore \nabla \times \bar{H} = \bar{J}_c + \bar{J}_D.$$

$$\therefore \boxed{\oint_l H \cdot d\bar{l} = \int_s (\bar{J}_c + \bar{J}_D)\, d\bar{S}.}$$

$$\therefore \boxed{\begin{aligned} \bar{J}_c &= \sigma \bar{E} \quad A/m^2. \\ \bar{J}_D &= \frac{\partial D}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad A/m^2. \end{aligned}}$$

* Faraday's Law of Electromagnetic Conduction:

→ When a stationary conductor acts by a moving magnetic flux then or vice versa then emf will be induced. This induced emf will inturn produces a magnetic flux which opposes original flux [Lenz's law). Mathematically we write



$$\text{emf} = -\frac{\partial \Phi}{\partial t}.$$

$$\therefore \oint_l \bar{E}\cdot d\bar{l} = -\frac{\partial \phi}{\partial t}.$$

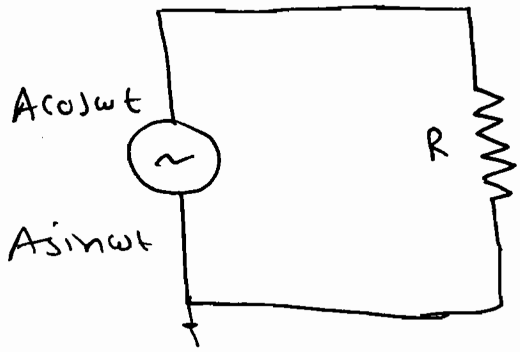
$$\Downarrow$$

$$\therefore \oint_l \bar{E}\cdot d\bar{l} = -\frac{\partial}{\partial t}\int_s \bar{B}\cdot d\bar{s}.$$

$$\therefore \int_s (\nabla \times \bar{E})\, ds = -\partial/\partial t \int_s \bar{B}\cdot d\bar{s}$$

$$\therefore \boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

→ Weither Source is $A\cos\omega t$ or $A\sin\omega t$ the average power deliver to $1\,\Omega$ Resistor is identically same which is equal to $A^2/2$.



$P = A^2/2$

$P = A^2/2$

$0.1\angle 45°$ Volts → phasor form.

→ $0.1\cos(\omega t + 45°)$ (or)

→ $0.1\sin(\omega t + 45°)$.

→ or $\text{Im}\left[0.1e^{j(\omega t+45°)}\right]$ (or)

→ $\text{Re}\left[0.1e^{j(\omega t+45°)}\right]$.

→ When the quantities are represented in the phasor form we suppresses the time variation for the mathematical convinience this time variations are approximated as cosine or sine or $e^{j\omega t}$.

→ $\bar{E} \to \bar{E}(x,y,z,t)$ (or) $\bar{E}(\rho,\phi,z,t)$ or $\bar{E}(r,\theta,\phi,t)$.

↳ It is a function of time and space coordinates.

$$\bar{E} = \text{Re}\left[\bar{E}_s \cdot e^{j\omega t}\right].$$

$\bar{E}_r$ is called phasor form of $\bar{E}$.

$\bar{E}_r \rightarrow \bar{E}_r(x,y,z)$ (or) $\bar{E}_r(\rho,\phi,z)$ (or) $\bar{E}_r(r,\theta,\phi)$.

$\hookrightarrow$ It is a f^n of space coordinates only.

$$\rightarrow \quad \nabla \times \bar{E} = -\frac{\partial B}{\partial t}.$$

$$\therefore \nabla \times [\,Re\{\bar{E}_r \cdot e^{j\omega t}\}] = -\partial/\partial t\,[Re\{\bar{B}_r \cdot e^{j\omega t}\}].$$

$$\therefore \nabla \times [Re\{\bar{E}_r \cdot e^{j\omega t}\}] = -j\omega\,[Re\{\bar{B}_r \cdot e^{j\omega t}\}].$$

$\therefore$ Suppressing time variation bothe the side.

$$\therefore \boxed{\nabla \times \bar{E}_r = -j\omega \bar{B}_r}$$

This is Faraday's law in phasor form.

So, $\partial/\partial t = j\omega = s$

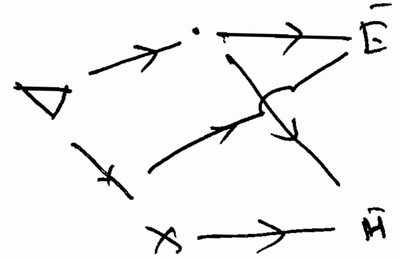
$\int dt = \frac{1}{j\omega} = 1/s.$

Name of the Law	Integral form	Point form	Phasor form.
1. Faraday's Law	$\oint \bar{E}\cdot d\bar{l} = -\frac{\partial}{\partial t}\int \bar{B}\, d\bar{s}$	$\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$. $= -\mu\frac{\partial \bar{H}}{\partial t}$	$\nabla\times\bar{E}_s = -j\omega \bar{B}_s$.
2. Modified Ampere's Law	$\oint \bar{H}\cdot d\bar{l} = \int_s (\bar{J}_c + \bar{J}_D)\, d\bar{s}$	$\nabla\times\bar{H} = \bar{J}_c + \bar{J}_D$ $= \sigma\bar{E} + \epsilon\frac{\partial \bar{E}}{\partial t}$	$\nabla\times\bar{H}_s = \bar{J}_{cs} + \bar{J}_{Ds}$ $= \sigma\bar{E}_s + \epsilon\frac{\partial \bar{E}_s}{\partial t}$.
3. Gauss Law for E-field	$\oint \bar{D}\cdot d\bar{s} = Q_{enc}$ $= \int_v \rho_v dV$	$\nabla\cdot\bar{D} = \rho_v$	$\nabla\cdot\bar{D}_s = \rho_s$.
Gauss Law for H-field	$\oint \bar{B}\cdot d\bar{s} = 0$	$\nabla\cdot\bar{B} = 0$	$\nabla\cdot\bar{B}_s = 0$.

Maxwell's equation

$$\bar{J}_D = \frac{\partial \bar{D}}{\partial t} = \epsilon\frac{\partial \bar{E}}{\partial t}$$

$$\bar{J}_c = \sigma\bar{E}$$



→ Maxwell had proved that any Electromagnetic problem can be solved by using above four eqns.

Ex-1 Let $\bar{D} = (3x\,\hat{a}_x + ky\,\hat{a}_y + 7z\,\hat{a}_z)$ nc/m²

Assume $\underbrace{\text{Charge free region.}}_{\rho_v = 0}$

Ans:

$\therefore \nabla \cdot \bar{D} = \rho_v$

But, $\rho_v = 0$.

$\therefore 3 + k + 7 = 0$

$\therefore k = -11$ nc/m³

$k(y) = 11$ nc/m²

m $\quad k = 11/y$ nc/m²

$\therefore \boxed{k = 11 \text{ nc/m}^3}$

Ex-2 Let, $\bar{E} = (kx - 100t)\,\hat{a}_y$ V/m, $\bar{H} = (x + 20t)\,\hat{a}_z$

Aim. Assume $\mu = 0.25$ H/m.

→ $\nabla \times \bar{E} = -\mu \dfrac{\partial \bar{H}}{\partial t}$.

$$\therefore \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & Kx-100t & 0 \end{vmatrix} = -\mu\,(0 + 20)\,\hat{a}_z .$$

$\therefore 0 - 0 + (K(1) - 0)\,\hat{a}_z = -20\mu\,\hat{a}_z$.

$\therefore k = -20\mu$

$\therefore k = -20 \times \frac{1}{4}$,

$\therefore \boxed{k = -5 \text{ V/m}^2}$

unit is meter

m

$k(x) = -50$ V/m.

$\therefore \boxed{k = -50 \text{ V/m}^2}$

* EM Waves:

⇒ Linear Medium :-

→ A medium is said to be linear in that medium $\bar{D}, \bar{E}$ must have same direction (or) $\bar{B}, \bar{H}$ must have same direction. It does not mean that all are having same direction.

⇒ Homogenius Medium :-

→ Usually at high frequency medium properties is characterised by μ and ϵ. If these are constant throughout the medium then the medium is said to be a homogenius medium.

⇒ Isotropic Medium:

→ In this medium μ and ϵ scalar constant.

In general,

real part

$$\mu = \mu' - j\mu''$$

$$\epsilon = \epsilon' - j\epsilon''$$

Img. part

$$\mu = \mu_0 \mu_r$$

$$\epsilon = \epsilon_0 \epsilon_r$$

①	②	③
$\mu_1 = \mu_0 \mu_{r1}$	$\mu_2 = \mu_2' - j\mu_2''$	$\mu_3 = \mu_0 \mu_{r3}$
$\epsilon_1 = \epsilon_0 \epsilon_{r1}$	$\epsilon_2 = \epsilon_2' - j\epsilon_2''$	$\epsilon_3 = \epsilon_0 \epsilon_{r3}$
Homo 1	Homo 2	Homo 3

Isotropic

→ Real part of μ & ϵ indicates storage property of medium.

→ Imaginary part of μ & ϵ indicates disipiate property of medium.

→ An isotropic medium is homogenius whereas homogenius medium need not be isotropic.

→ Charge free medium : $\rho_v = 0$.

→ Non-Conducting medium : $\sigma = 0$.

* Unbounded medium:

→ There are no boundaries to meet in any direction.

→ We assume that the wave is propogating through a Linear homogenius isotropic charge free non-conducting and unbounded medium.

⇒ Writting the maxwell's eqn for the above assumed medium.

(1) $\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$

(2) $\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t}$ (Non conducting medium is assumed $\sigma = 0$).

(3) $\nabla \cdot \bar{D} = 0$ ($\because$ Charge free medium is assumed $\rho_v = 0$).

$\Rightarrow \nabla \cdot \epsilon \bar{E} = 0$

$\Rightarrow \nabla \cdot \bar{E} = 0$ ($\because$ homogeneous medium is assumed).

(4) $\nabla \cdot \bar{H} = 0.$

$\nabla \cdot \bar{B} = 0.$

→ Taking curl on ① both the sides.

$$\nabla \times \nabla \times \bar{E} = -\mu \nabla \times \frac{\partial \bar{H}}{\partial t}.$$

$$\therefore \underbrace{\nabla(\nabla \cdot \bar{E})}_{0} - \nabla^2 \bar{E} = -\mu \, \partial/\partial t \, \underbrace{(\nabla \times \bar{H})}_{+\frac{\partial \bar{E}}{\partial t} \times \epsilon}.$$

$$\therefore \boxed{\nabla^2 \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.}$$ → Vector wave eqⁿs.

Similarly taking curl on ② both sides.

$$\therefore \boxed{\nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}.}$$

→ For simplicity Let us solved the problem in Cartesian coordinate.

→ Expanding wave eqⁿ in cartesian coordinate.

$$\therefore \frac{\partial^2 \bar{E}_x}{\partial x^2} + \frac{\partial^2 \bar{E}_y}{\partial y^2} + \frac{\partial^2 \bar{E}_z}{\partial z^2} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.$$

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \frac{\partial^2 \bar{H}}{\partial y^2} + \frac{\partial^2 \bar{H}}{\partial z^2} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}.$$

These are 2nd order 4 dim. PDE's.

→ $\bar{E}$ and $\bar{H}$ are f^ns of time and space coordinates.

→ For simplicity Let us assumed that wave is propogating along z - direction in unbounded medium.

→ since there are no boundaries to meet along X & Y directions. Then we conclude the partical variation of any field component with respect to x and y i.e.

$\partial/\partial y\,(\;) = 0$, $\partial/\partial x\,(\;) = 0$.

$\longleftrightarrow (z)$.

→ The eqn reduced to

$$\frac{\partial^2 \bar{E}}{\partial z^2} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.$$

Similarly

$$\frac{\partial^2 \bar{H}}{\partial z^2} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}.$$

These are 2nd order 2 - Dim PDES.

→ We assumed Charge free region:

$\nabla \cdot \bar{E} = 0.$

$$\therefore \frac{\partial (E_x)}{\partial x} + \frac{\partial (E_y)}{\partial y} + \frac{\partial (E_z)}{\partial z} = 0.$$

$$0 + 0 + \frac{dE_z}{dt} = 0.$$

$\therefore$ E_z may be zero (or) constant.

$\rightarrow$ Max. value of D.D. $= |\nabla \phi|$.

(OR)

Greatest rate of increase.

* Angle betn the two surfaces:

$\rightarrow$ Let, $\phi_1 (x,y,z) = c_1$ &

$\phi_2 (x,y,z) = c_2$

$$\therefore \cos\theta = \frac{\nabla \phi_1 \cdot \nabla \phi_2}{|\nabla \phi_1| \cdot |\nabla \phi_2|}.$$

* For solenoidal vector $\nabla \cdot \bar{F} = 0$.
* For Irrotational vector $\nabla \times \bar{F} = 0$.
* Green Theorem in a plane.

$$\rightarrow \oint m(x,y)\, dx + N^{(x,y)} dy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial m}{\partial y}\right] dx\, dy.$$

* Del operator (∇).

$\rightarrow$ $\nabla = \partial/\partial x \, \bar{a}_x + \partial/\partial y \, \bar{a}_y + \partial/\partial z \, \bar{a}_z$.

* gradient of a scalar Field:

$\rightarrow$ $\text{grad } V = \nabla V = \frac{dV}{dN}\Big|_{max} \bar{a}_N$.

* ~~Divergence of a vector:~~

$\therefore$ $\text{grad } V = \nabla V = \frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z$.

* Divergence of a vector:

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

*

	Coordinate			Multiplier			Unit vector		
General	u	v	w	h_1	h_2	h_3	$\bar{a}_u$	$\bar{a}_v$	$\bar{a}_w$
Cartesian	x	y	z	1	1	1	$\bar{a}_x$	$\bar{a}_y$	$\bar{a}_z$
Cylindrical	ρ	ϕ	z	1	ρ	1	$\bar{a}_\rho$	$\bar{a}_\phi$	$\bar{a}_z$
Spherical	r	θ	ϕ	1	r	$r\sin\theta$	$\bar{a}_r$	$\bar{a}_\theta$	$\bar{a}_\phi$

$$\rightarrow \nabla \phi = \frac{1}{h_1}\frac{\partial \phi}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial \phi}{\partial v}\bar{a}_v + \frac{1}{h_3}\frac{\partial \phi}{\partial w}\bar{a}_w$$

$$\rightarrow \nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3 A_u) + \frac{\partial}{\partial v}(h_3 h_1 A_v) + \frac{\partial}{\partial w}(h_1 h_2 A_w)\right].$$

$$\rightarrow \nabla \times \bar{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1 \bar{a}_u & h_2 \bar{a}_v & h_3 \bar{a}_w \\ \partial/\partial u & \partial/\partial v & \partial/\partial w \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

$$\rightarrow \nabla^2 \phi = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial \phi}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_1 h_3}{h_2}\frac{\partial \phi}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial \phi}{\partial w}\right)\right].$$

$$\rightarrow \nabla^2 \bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial}{\partial u}\bar{A}_1\right) + \frac{\partial}{\partial v}\left(\frac{h_1 h_3}{h_2}\frac{\partial}{\partial v}\bar{A}_1\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial}{\partial w}\bar{A}\right)\right].$$

* Divergence Theorem:

$$\int_V (\nabla \cdot \bar{A})\, dv = \oint_S \bar{A} \cdot d\bar{s}$$

surface to volume

$$\therefore \boxed{\oint_S \bar{A} \cdot d\bar{s} = \int_V \nabla \cdot \bar{A}\, dv.}$$

* Stoke's Theorem:

$$\int_S (\nabla \times \bar{A}) \cdot d\bar{s} = \oint_C \bar{A} \cdot d\bar{l}$$

line to surface

$$\therefore \boxed{\oint_C \bar{A} \cdot d\bar{l} = \int_S \nabla \times \bar{A} \cdot d\bar{s}.}$$

* Directional derivative (D.D):

→ The Directional derivative of a diffn scalar function in the direction of vector $\bar{a}$ is

$$D.D = \nabla \phi \cdot \frac{\bar{a}}{|\bar{a}|}.$$

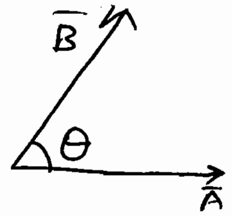
* Unit Vector, Vector and its magnitude.

→ $\bar{a}_{AB} = \dfrac{\overline{AB}}{|\overline{AB}|}$.

→ $\overline{AB} = (x_2 - x_1)\,\hat{a}_x + (y_2 - y_1)\,\hat{a}_y + (z_2 - z_1)\,\bar{a}_z$.

→ $|\overline{AB}|$ or $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

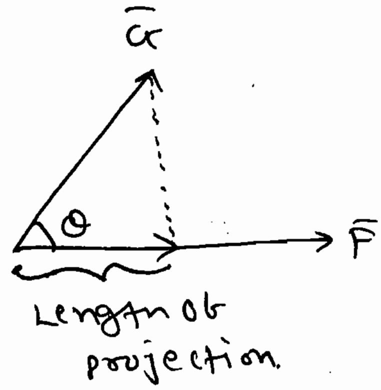
* Scalar or Dot Product:



→ $\boxed{\bar{A} \cdot \bar{B} = AB\cos\theta.}$

where θ = angle betn $\bar{A}$ & $\bar{B}$

* Length of Projection:



PCA

→ Length of projection

$= \bar{G} \cdot \bar{a}_F$

→ Vector projection. $= (\bar{G} \cdot \bar{a}_F)\,\bar{a}_F$

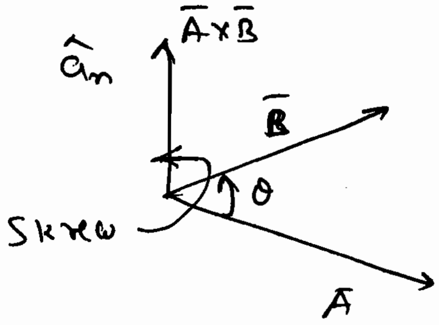
$= \dfrac{\bar{G} \cdot \bar{F}}{F^2} \times \bar{F}$.

* Cross Product:

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\rightarrow \quad \bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

$\rightarrow$



$$\therefore \quad \bar{A} \times \bar{B} = AB \sin\theta \, \hat{a}_n$$

* Application of cross product:

$\rightarrow$ Area of parallelogram $= |\bar{AB} \times \bar{AC}|$.

$\rightarrow$ Area of the triangle ABC $= \frac{1}{2} |\bar{AB} \times \bar{AC}|$

* Scalar triple Product:

$$\rightarrow \quad \bar{A} \times (\bar{B} \times \bar{C}) = \bar{A}\cdot\bar{C}\,(\bar{B}) - \bar{A}\cdot\bar{B}\,(\bar{C}).$$

$$\rightarrow \quad \bar{A}\cdot\bar{B} \times \bar{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

* Coordinates systems:

1. Cartesian (or) rectangular coordinate system
2. Cylindrical coordinate system
3. Spherical coordinate system.

① Cartesian (or) Rectangular Coordinate system.

→ Any point in cartesian system is the intersection of $x =$ constant, $y =$ constant and the $z =$ constant planes.

→ point in cartesian system = $P(x, y, z)$

Unit vectors are $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$.

$$d\bar{l} = dx, dy, dz$$

→ $d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$.

→ $d\bar{s} = dy\,dz\,\hat{a}_x$ ($x =$ constant)

$d\bar{s} = dz\,dx\,\hat{a}_y$ ($y =$ constant).

$d\bar{s} = dx\,dy\,\hat{a}_z$ ($z =$ constant).

→ $dv = dx\,dy\,dz$

② Cylindrical system:

→ Point is (ρ, ϕ, z).

→ Unit vectors $\bar{a}_\rho$, $\bar{a}_\phi$, $\bar{a}_z$.

→ Differential lengths are $d\rho$, $\rho\,d\phi$, dz.

→ $d\bar{l} = d\rho\,\bar{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$.

$d\bar{s} = \rho\,d\phi\,dz\,\hat{a}_\rho$ ($\rho =$ constant).

$d\bar{s} = d\rho\,dz\,\hat{a}_\phi$ ($\phi =$ constant).

$d\bar{s} = \rho\,d\rho\,d\phi\,\hat{a}_z$ ($z =$ constant).

③ Spherical coordinate system:

→ $P(r, \theta, \phi)$.

→ $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$

→ $dr, r\,d\theta, r\sin\theta\,d\phi$.

→ $d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$. ($r$ = constant)

→ $d\bar{s} = r\sin\theta \, dr \, d\phi \, \hat{a}_\theta$. ($\theta$ = constant).

→ $d\bar{s} = r \, dr \, d\theta \, \hat{a}_\phi$ (ϕ = constant).

→ $d\bar{l} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$.

* Transformation from cartesian to cylindrical vector and vice versa:

	$\bar{a}_\rho$	$\bar{a}_\phi$	$\bar{a}_z$
$\bar{a}_x$.	$\cos\phi$	$-\sin\phi$	0
$\bar{a}_y$.	$\sin\phi$	$\cos\phi$	0
$\bar{a}_z$.	0	0	1

$[B]_\rho = [A^T][B]_x$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

$\rho = \sqrt{x^2+y^2}$

$\phi = \tan^{-1}(y/x)$

$z = z$

* Transformation of vector from cartesian to spherical or vice versa:

	$\bar{a}_r$	$\bar{a}_\theta$	$\bar{a}_\phi$
$\bar{a}_x$.	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\bar{a}_y$.	$\sin\theta\cdot\sin\phi$	$\cos\theta \, \sin\phi$	$\cos\phi$
$\bar{a}_z$.	$\cos\theta$	$-\sin\theta$	1

$x = r\sin\theta\cdot\cos\phi$

$y = r\sin\theta\cdot\sin\phi$

$z = r\cos\theta$

$r = \sqrt{x^2+y^2+z^2}$

$\theta = \cos^{-1}(z/r)$

$\phi = \tan^{-1}(y/x)$.