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OM

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ECE

PM 1-(B).

ACE

Control System
Part - II

All the Best
T...

Stability.

⇒ For an LTI System, the LTI System is said to be stable if ~~the~~ it satisfies the following Conditions.

1. If the input is bounded, the output must be bounded.
2. If the input signal to the system is zero, the output must be zero irrespective of all the initial conditions.

⇒ This Stabilities are classified into the two way based on operating Conditions.

1. <u>Conditional Stable System</u>:

⇒ Here, the system is stable for certain range of System. Components.

2. <u>Absolutely Stable System.</u>

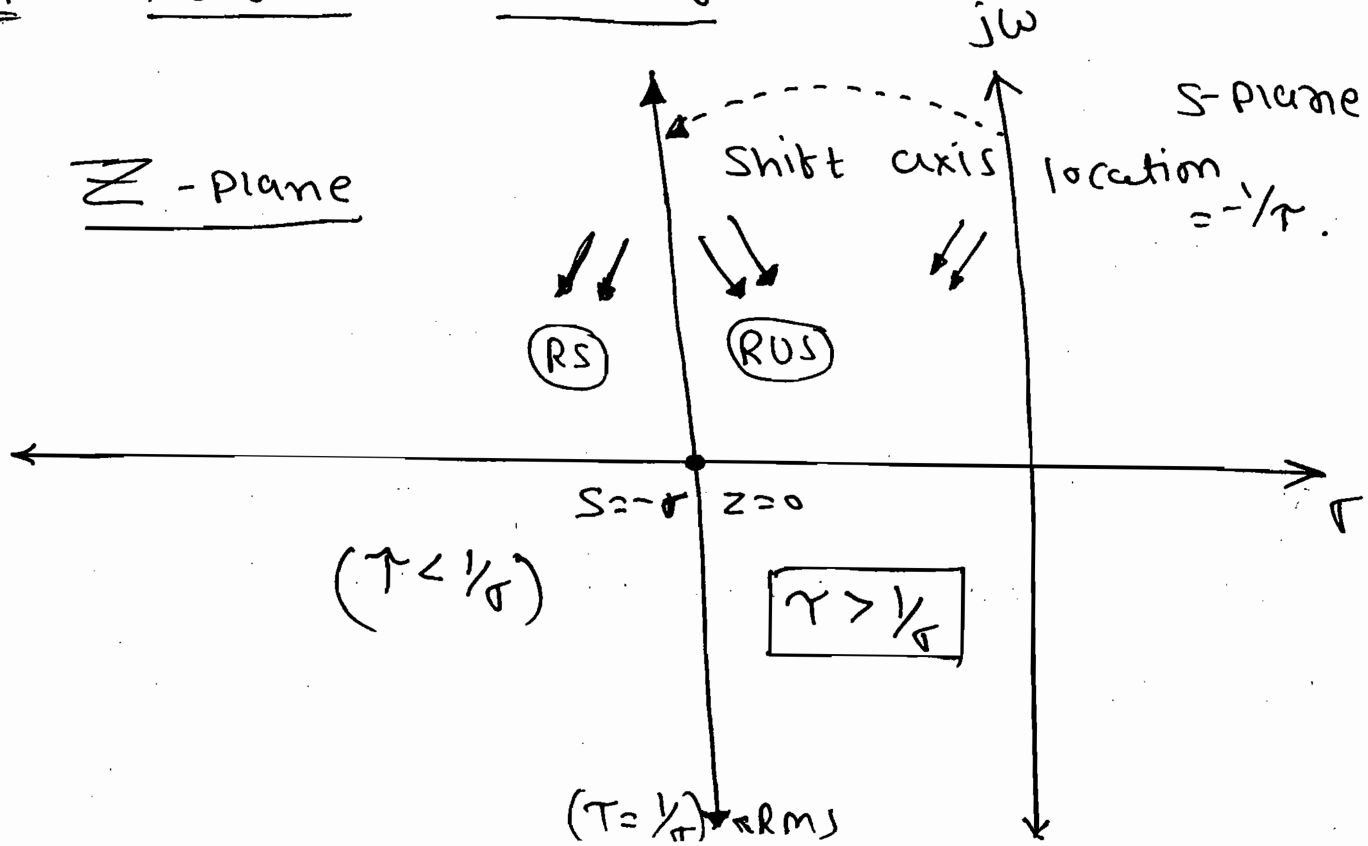
⇒ Here, the system is stable for all the values of System components.

③ Marginal (or) Critical (or) limitedly Stable System:

⇒ A linear time Invariant system is said to be marginal stable if for the bounded input, the output maintains the constant amplitude and freq. of oscillation.

→ The non-repeated pole on imaginary axis gives the constant amplitude and freq. of oscillation & the system is marginal stable.

* Relative Stability:



⇒ The relative Stability Concept is applicable only for Stable System.

⇒ By using relatively Stable Concept we can find System time constant, settling time and time required to reach steady state.

* Techniques used for calculate Stability are:

1) Routh - Hurwitz Criterion.
2) Root - Locus.
3) Bode plot.
4) Nyquist Plot.
5) Nicholas Chart.

* Routh - Hurwitz Criterion:

(RH Criterian):

* Purpose:

① To find the Closed Loop System Stability.

② To find the no. of Closed Loop Poles lies in the right, left, on imaginary axis of the s-plane.

③ ⇒ The main purpose of the RH- criteria is to find the no. of poles in right of s-plane only.

⇒ ④ To find the range of K-value for CL system stability.

⑤ To find the k value to become the system marginal stable. (or) Undamped system.

⑥ To find the natural freq. of oscillation (or) Undamped oscillation.

⑦ To find the relative stability.

→ By using relative stability concept we can find system time constant, setteling time t_s.

⑧ ⇒ To find a CL stability by using RH - criteria required char. eqn.

i.e. $\boxed{1 + GH = 0.}$

⇒ whereas in remain all the stability techniques required OLTF of a unity (or) Non-UFB system.

⇒ CL Stability

RH

$\xrightarrow{CE}$ $1 + G(s) \cdot H(s) = 0$

RL

BP

NP

⇓

$G(s) \cdot H(s)$.

→ OLTF of unity (or) Non UFB system.

⇒ The n^{th} order general form of Char. eqn is,

$$\xrightarrow{CE} \boxed{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s^1 + a_n s^0 = 0.}$$

$a_0, a_1, a_2 \ldots$ are coefficient.

$$
\begin{array}{c|cccc|c|c}
s^n & a_0 & a_2 & a_4 & \cdots & +/- & + \\
s^{n-1} & a_1 & a_3 & a_5 & \cdots & +/- & + \\
s^{n-2} & \dfrac{a_1 \cdot a_2 - a_0 a_3}{a_1} & \dfrac{a_1 a_4 - a_0 a_5}{a_1} & & \cdots & +/- & - \\
\vdots & \vdots & \vdots & & & \vdots & + \\
 & & & & & & - \\
s^0 & a_n & - - - & & & +/- & + \\
 & & & & & S & US
\end{array}
$$

⇒ The condition for the system stability stability, are

① All coefficient in the first column should have same sign and no-coefficient should be '0' in the first column.

② The no. of sign changes in the first column equal to no. of poles in the right plane and the system become unstable.

$\boxed{Q}$ Find the system stability to the following char. eqn's

① $s+10=0$.

② $s^2+25=0$.

③ $s^2+10s+10=0$.

④ $s^3+25s^2+8s+10=0$.

⑤ $s^3+7s^2+6s+100=0$.

⑥ $s^3+8s^2+4s+32=0$.

Soln:

① $s+a=0$.

$\therefore \xrightarrow{CE} as+b=0$.

$$\left.\begin{array}{c|c} s^1 & \underline{a} \\ s^0 & \underline{b} \end{array}\right\} \quad \begin{array}{l} b,a>0 \ \ \underline{(or)} \\ b,a<0 \Rightarrow S \end{array}$$

$\Rightarrow s+10=0$ is stable.

② $s^2+25=0$

$\xrightarrow{CE} as^2+bs+c=0$.

$$\begin{array}{c|cc} s^2 & a & c \\ s^1 & b & \\ s^0 & c & \end{array} \Rightarrow \text{if } \begin{array}{l} a,b,c>0 \ (or) \\ a,b,c<0 \\ \Downarrow \\ S \end{array}$$

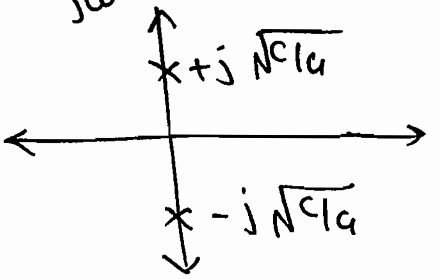
$\Rightarrow$ if $b=0$, $a,c>0$

$\Rightarrow$ MS

$\Rightarrow$

$\xrightarrow{CE} as^2 + c = 0$

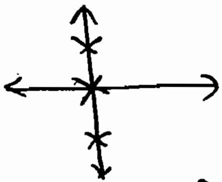
$\Rightarrow \quad s = \pm j\sqrt{c/a}$

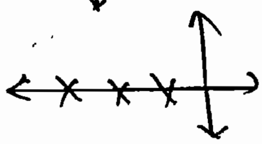


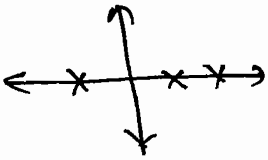
(MS).

$\xrightarrow{CE} as^3 + bs^2 + cs + d = 0.$

s^3	a	c
s^2	b	d
s^1	$\frac{bc-ad}{b}$	
s^0	d	

$bc = ad \rightarrow$  (MS)

$bc > ad \rightarrow$  (S)

$bc < ad \rightarrow$  $\rightarrow$ (US)

$\Rightarrow$ even powers of s- terms gives f.o.o.

$bs^2 + d = 0.$

$\therefore \quad s = \pm j\sqrt{\frac{d}{b}}$

$j\omega_n = \pm j\sqrt{d/b}$

$$\boxed{\omega_n = \sqrt{d/b} \text{ rad/sec.}}$$

$\Rightarrow \quad s^2 + 25 = 0 \Rightarrow$ (MS).

10

4. $s^3 + 25s^2 + 8s + 10 = 0.$

200

$\rightarrow (bc = 200) > (ad = 10) \Rightarrow$ S.

100

5. $s^3 + 7s^2 + 6s + 100 = 0.$

42

$\rightarrow (bc = 7 \times 6 = 42) < (ad = 100) \Rightarrow$ US.

32

6. $s^3 + 8s^2 + 4s + 32 = 0.$

32

$\rightarrow (bc = 32) = (ad = 32).$

$\Rightarrow$ MS.

F.O.O. $\quad 8s^2 + 32 = 0$

$$s^2 = 4$$

$$s = \pm j2$$

$$\therefore \boxed{\omega_n = 2 \text{ rad/sec}}$$

Q Find the no. of poles in the right of s-plane to the given char. eqn.

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0.$$

Soln:

s^4	1	3	5
s^3	2	4	2R, 1C
① s^2	1	5	
s^1	-6	2R, 1C.	
② s^0	5		

$\Rightarrow$ 2 sign changes:

hence 2 poles lies on RHS plane

System Unstable.

$\Rightarrow$ Total 4 pole and 2 left & 2 Right.

$\boxed{Q}$ Find the no. of Poles on Right to the given char. eqn:

① $s^4 + 2s^3 + 3s^2 + 2s + 1.$

Soln:

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 1 \\ s^3 & 2 & 2 & \\ s^2 & 2 & 1 & \\ s^1 & 1 & & \\ s^0 & 1 & & \end{array}$$

No sign Change.

→ Satable

→ No Poles on RH s-plane.

→ 4 Poles on LH s-plane.

② $s^4 + 2s^3 + 3s^2 + s + 2 = 0.$

Soln:

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 2 \\ s^3 & 2 & 1 & \\ s^2 & 5/2 & 2 & \\ s^1 & -3/5 & & \\ s^0 & 2 & & \end{array}$$

→ 2 sign changes

→ System ⇒ US

→ 2 poles on RH s-plane.

→ 2 Poles on LH s-plane.

③ $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0.$

Soln:

s^4	1	2	8
s^3	2	4	
s^2	$\cancel{0}\ \epsilon$	8	
s^1	$\frac{4\epsilon-16}{\epsilon}$		
s^0	8		

① $s^2 \to s^1$ ② $s^1 \to s^0$ $(-\infty)$

$$\lim_{\epsilon \to 0} \frac{4\epsilon - 16}{\epsilon} = -\infty.$$

→ 2 - sign changes, 2 - Poles on RH-s plane.
2 - poles on LH - s plane.

Difficulty - 1

⇒ Whenever any one element is '0', Replace '0' by smallest positive Constant 'ε' and continue the Routh Stability. Finally for $\epsilon = 0$ check the no. of sign Changes.

④ $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0.$

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0}\ \epsilon$	-12	0
s^2	$\frac{\overset{A}{2\epsilon+12}}{\epsilon}$	15	
s^1	$\frac{-12A - 15\epsilon}{A}$		
s^0	15		

① $s^3 \to s^2$ ② $s^1 \to s^0$ $(-\infty)$ (-12)

$$\lim_{\epsilon \to 0} \frac{2\epsilon + 12}{\epsilon} = \infty.$$

$$\lim_{\epsilon \to 0} -12 - \frac{15\epsilon}{A} = -12.$$

2 - sign changes

2 - Poles on RH-s plane

⑤ $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0.$

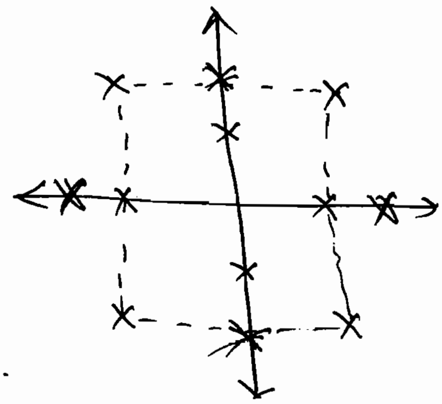
Sol^n:

s^5	1	3	2
s^4	1	3	2
s^3	0 (4)	$\cancel{0}$ 6	0
s^2	$3/2$	2	
s^1	$2/3$		
s^0	2		

AE: $1 \cdot s^4 + 3s^2 + 2 = 0.$

$\therefore \dfrac{dAE}{ds} = 4s^3 + 6s.$

AE: $1 \cdot s^4 + 3s^2 + 2 = 0. \rightarrow$



System $\rightarrow$ Marginaly Stable.

Difficulty - 2:

$\Rightarrow$ Whenever in the Routh tubular form Row of zero occurs then we required to form the Auxiliary eq^n by using the above row of zero coefficients and differentiate the Auxiliary equitation and replace zeros by the coefficients of differential auxiliary eq^n and Continue the Routh Stability.

$\rightarrow$ In the Routh tubular form row of zero occurs means the pole must lies

Symmetrical about the origin.

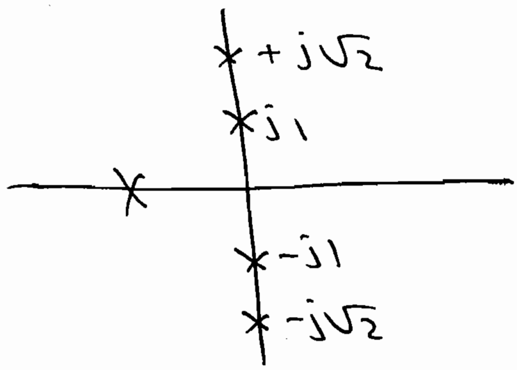
⇒ The Auxillary eq^n must consist only even power of s- terms because the Roots of Auxillary eq^{n}s must be symmetrical about origin.

→ The roots of auxillary eq^n are CL Poles which are symmetrical,

$s^4 + 3s^2 + 2 = 0.$

$\therefore (s^2+1)(s^2+2) = 0$

$s = \pm j1$, $s = \pm\sqrt{2}j$



⇒ Non- repeated poles on jω - axis hence MS.

⇒ Whenever in the Routh tabular form only once the Row of zero occurs and all the co-efficients in the 1st column are +ve then the system is Marginal stable because the poles must lies on the imaginary axis which are non- repeated.

→ 1 time Row of zero occurs means

the poles are symmetrical about the origin but not repeated.

⇒ The no. of time rows of zero indicates no. of poles are repeated.

⇒

s^5	−		
s^4	+		
s^3	+ ~~0~~	0	0
s^2	+		
s^1	+		
s^0	−		

① No sym.

① Sym.

AE (4^{th} order)
↓
4 poles symmetrical @ origin.

<u>Note:</u>

⇒ The sign change occurs below the AE there must be a symmetrical pole in the left to the pole placed in Right.

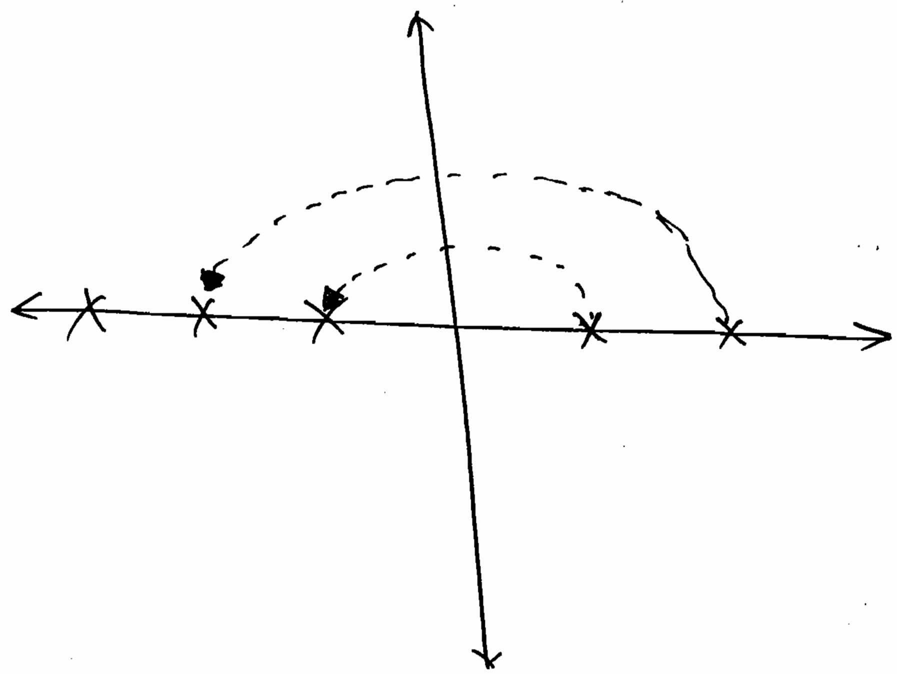
⇒ The sign change occur above the Auxiliary eqn there is no symmetrical pole in the left ~~to~~ the pole placed in the Right side.

⇒

s^5	+	No symm
s^4	+	→ AE → 4^{th} order (4 pole sym. about origin)
s^3	$\cancel{0}$ 0 0	1 Row
s^2	+	①
s^1	−	
s^0	+	② [2 poles in Real axis]

Symm

⇒



$\boxed{e}$ $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0.$

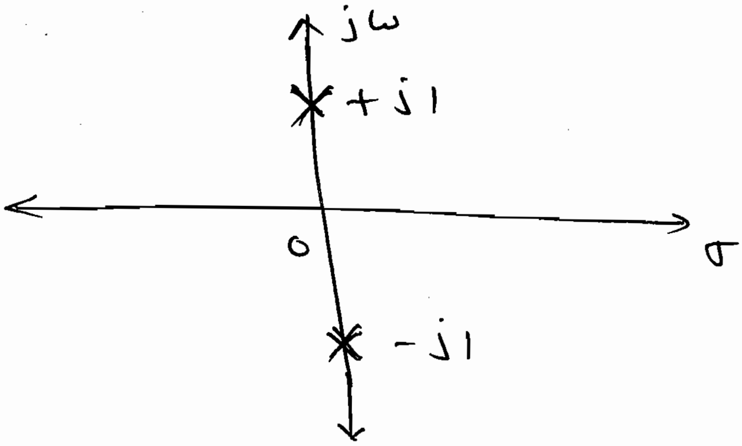
Soln:

s^6	1	4	5	2	
s^5	3	6	3		
s^4	2	4	2		→ $AE_1 \Rightarrow 2s^4 + 4s^2 + 2 = 0.$
s^3	$\cancel{0}$ 8	$\cancel{0}$ 8	0		→ 1^{st} row.
s^2	2	2			→ $AE_2 \Rightarrow 2s^2 + 2 = 0$
s^1	0	0			→ 2^{nd} row. → Repeated nature of pole.
s^0	2				

AE_2: $2s^2 + 2s^0 = 0$

$\boxed{s = \pm j1}$

(US)



⇒ Whenever many times Row of Zeros occurs and all the Co-efficients in the 1st Column are positive then the system is unstable the poles must lies on the imaginary axis which are repeated.

Q Find the no. of Poles in the Right the right of s-plane to the given char. eqn $s^4 + s^3 - s - 1 = 0$.

Soln:

s^4	1	0	-1
s^3	1	-1	
s^2	1	-1	
s^1	0	0	
s^0	-1		

AE: $s^2 - 1 = 0$

1 Row

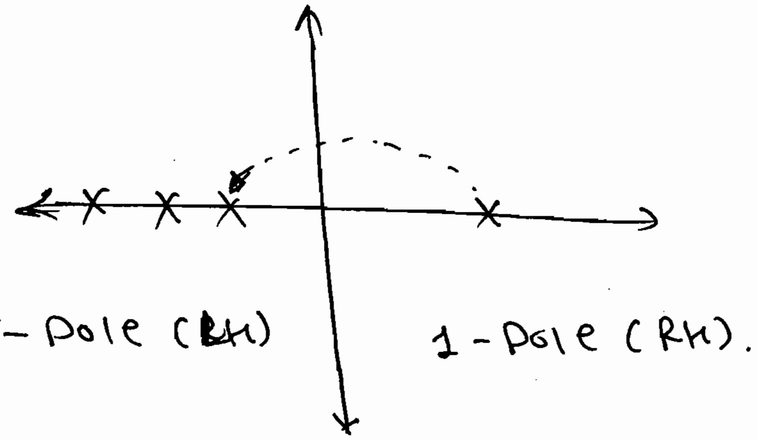
1

$s = \pm 1$

$\boxed{s = \pm 1}$

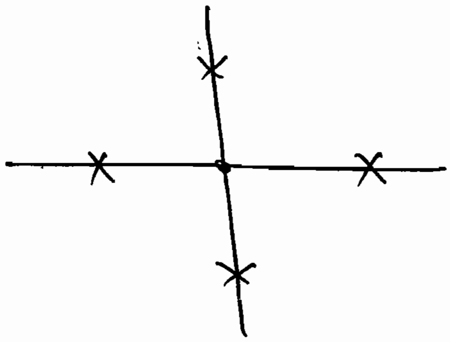
① - sign Change. below AE.

Hence, Symmetrical pole.



$\boxed{a}$ Identify the Routh tabular form to the given poles locations in the s-plane.

①



Soln: ⇒ 4 pole are Symmetrical → 1 Row of Zero.

⇒ 1 pole ~~below~~ ~~AE~~ RH → 1 sign change.

below AE because Symm. Pole in the left side.

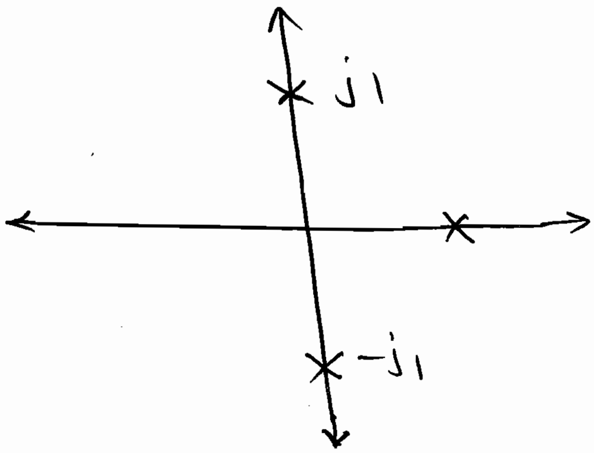
$\therefore \quad (s^2-1)(s^2+1)=0.$

$s^4-1=0$

s^4	1	0	-1	AE: $s^4-1=0$ (4th order).
s^3	$\cancel{0}$ 4	$\cancel{0}$	$\cancel{0}$	1 Row of zero.
s^2	$\cancel{0}$ ϵ	-1		
s^1	$-4/\epsilon$			$\lim_{\epsilon \to 0} -\frac{4}{\epsilon} = -\infty$
s^0	-1			

① (sign change between s^2 and s^1)

②



$\underline{\underline{\text{Sol}^n}}$:

$$(s^2+1)(s-1)=0.$$

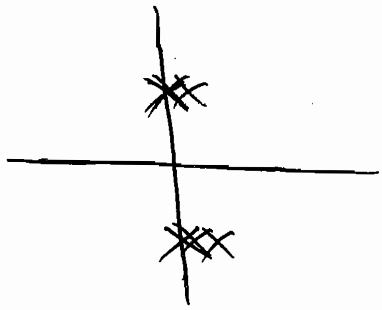
$\underline{CE} \rightarrow s^3 - s^2 + s - 1 = 0.$

s^3	1 ①	1
s^2	-1	-1
s^1	~~0~~ -2	~~0~~ → ROZ
s^0	-1	

AE: $s^2+1=0$

$\boxed{s = \pm j}$

③



$\underline{\underline{\text{Sol}^n}}$:

$$(s^2+1)^3 = 0.$$

$s = \pm j1, \pm j1, \pm j1$

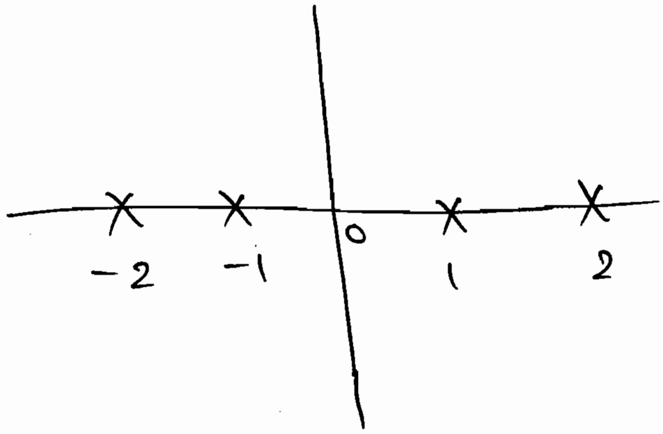
$= s^6 + 1 + 3s^4 + 3s^2$

$= s^6 + 3s^4 + 3s^2 + 1$

s^6	1	3	3	1
s^5	~~0~~ 6	~~0~~ 12	~~0~~ 6	~~0~~ → 1st row
s^4	1	2	1	
s^3	~~0~~ 4	~~0~~ 4	~~0~~ → 2nd row	
s^2	1	1	0	
s^1	~~0~~	~~0~~ → 3rd row		
s^0	1			

(a)

HW



Soln: 4 Poles are symmetrical and 1 Row of zero.

Two poles are RH splane and sym.

Hence 2 sign changes below the AE.

$\Rightarrow$ CE $\rightarrow$ $(s^2-1)(s^2-4)=0$.

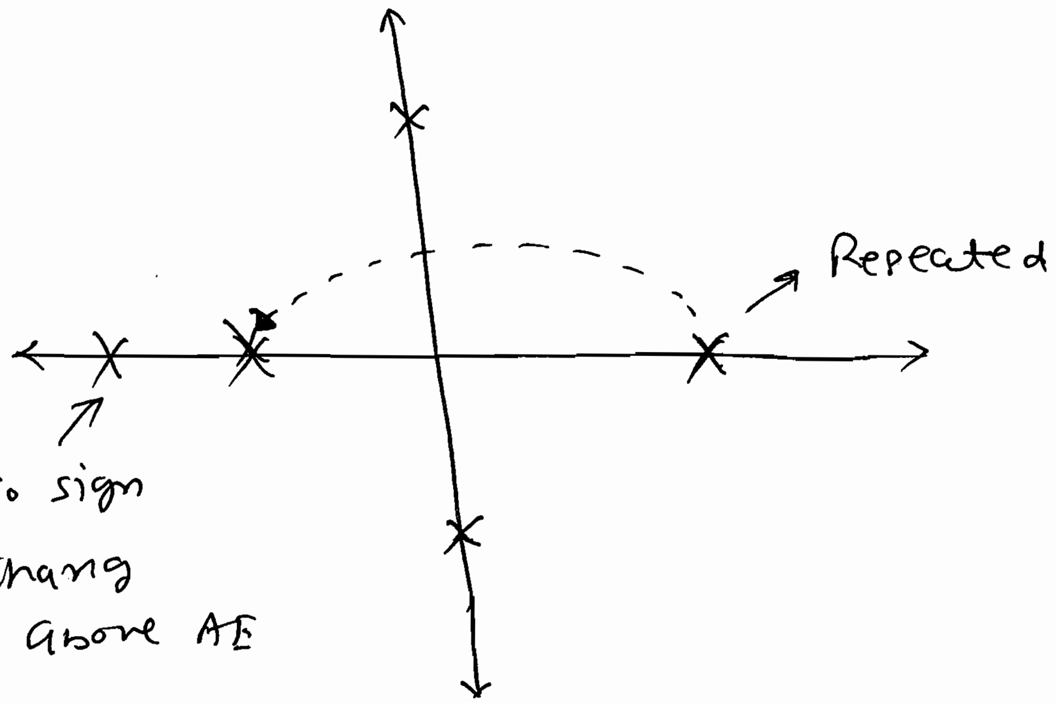
$\therefore\ s^4-5s^2+4=0.$

	s^4	1	-5	4 ← AE
	s^3	$\cancel{0}$ 4	$\cancel{0}$ -10	$\cancel{0}$ 0 → 1 RoZ
1	s^2	$-5/2$	4	
2	s^1	$18/5$		
	s^0	4		

Q Identify the no. of poles on the imaginary axis, in the left and Right plane of s to the given sample Routh tabular form:

s^7	+	
s^6	+	AE_1 6th order AE ⇒ 6 pole sym. @ origin
s^5	0 0 0	→ 1 ROZ
s^4	+	2 times ROZ ⇒ 2 poles are repeated.
s^3	0 0 0	→ 2 ROZ
s^2	+	
s^1	−	①
s^0	+	②

Sol:



(2)

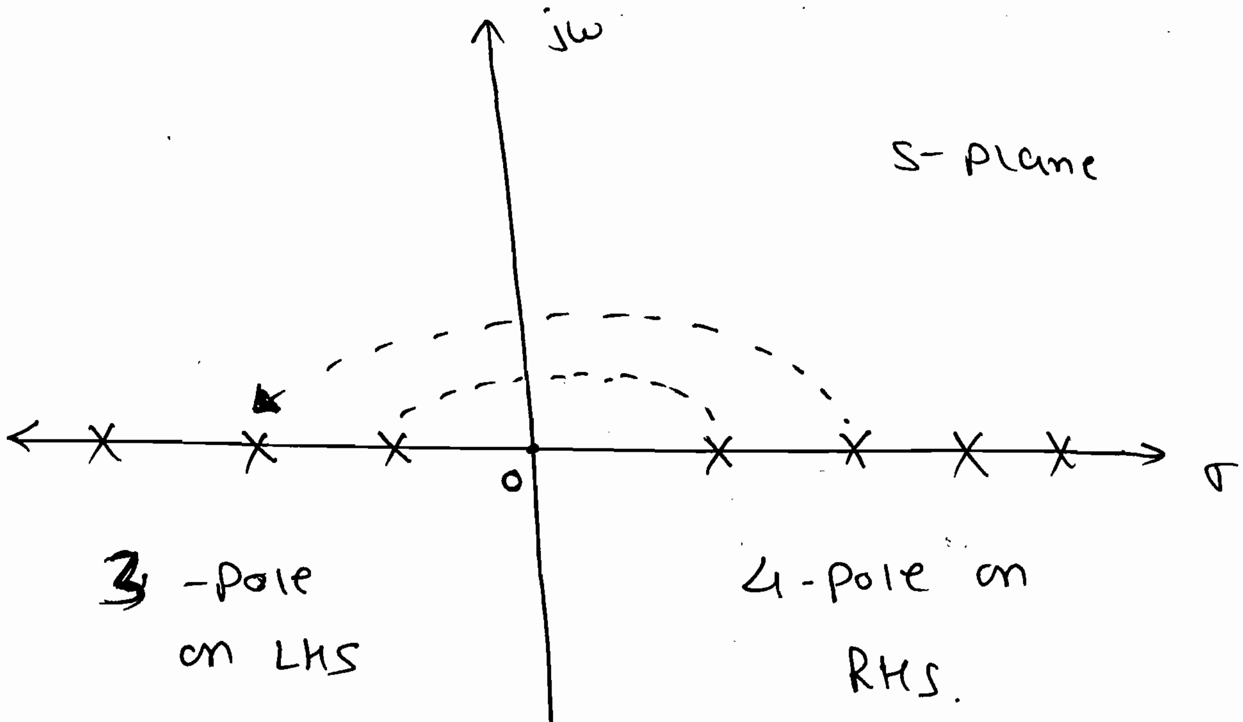
s^7	+
s^6	−
s^5	+
s^4	+
s^3	+ ~~0~~ 0 0 → I ROZ
s^2	+
s^1	−
s^0	+

(1) (2) Two sign change above AE hence two pole on RH-s-plane.

AE: 4th order 4 pole sym. (c) origin.

(1) (2) → 2 sign change below AE hence 2 sym. pole on RH-splane.

Soln:



* Conditional Stable System:-

a) Find the range of k value for system stability.

b) Find the k value to become the system marginal stable (or) Undamped system.

c) Find the natural freq^n of osc^n when the system is marginal stable to the char. eq^n.

$$s^3 + 8s^2 + 4s + K = 0.$$

Sol^n:

$$\begin{array}{c|cc}
s^3 & 1 & 4 \\
s^2 & 8 & K \\
\text{(MS)} \rightarrow s^1 & \frac{32-K}{8} > 0 \text{ (S)} & \\
\times\ s^0 \text{ (MS)} & K > 0 \text{ (S)} &
\end{array}$$

→ For stable,

$\frac{32-K}{8} > 0$ & $\boxed{K>0}$

$\Rightarrow 32-K>0$

$\boxed{K<32}$

so, $\boxed{0<K<32}$ ⇒ (S).

For, (ms),

$$\frac{32-K}{8} = 0$$

$\Rightarrow \boxed{K = 32} \Rightarrow \boxed{K_{mar.} = 32}$

Note:

$\Rightarrow$ For K marginal value consider only odd power of s power rows.

$$AE \rightarrow \quad 8s^2 + K = 0$$

$$8s^2 = -K_{marg.}$$

$$s^2 = -32/8$$

$$s = \pm j2.$$

$$j\omega_n = \pm j2$$

$$\Rightarrow \boxed{\omega_n = 2 \text{ rad/sec}}$$

$\boxed{a}$ $2s^3 + 5s^2 + 10s + (K+5) = 0.$

Soln:

s^3	2	10
s^2	5	$K+5$
s^1	$\frac{50-2(K+5)}{5}$	
s^0	$K+5$	

$\rightarrow$ For stable,

$$K+5 > 0$$

$$\boxed{K > -5}$$

$$\frac{40-2K}{5} > 0$$

$40 > 2K \Rightarrow \boxed{K < 20} \Rightarrow \boxed{-5 < K < 20} \Rightarrow$ S

$\Rightarrow$ For MS,

$$\frac{40-2K}{5} = 0$$

$$\boxed{K_{max} = 20}$$

$$AE \rightarrow \quad 5s^2 + K+5 = 0$$

$$5s^2 + 25 = 0$$

$$s^2 = -5$$

$$s = \pm j\sqrt{5}$$

$$\therefore \boxed{\omega_n = \sqrt{5} \text{ rad/sec}}$$

Another method:

$$2s^3 + 5s^2 + 10s + (K+5) = 0$$

$2K+10$

50

$$\therefore \quad 2K+10 = 50$$

$$K = 40/2$$

$$\Rightarrow \boxed{K_{max} = 20}$$

$\boxed{Q}$ Repeat the above problem for given

$$GH(s) = \frac{K}{s(s+2)(s+4)(s+6)}.$$

Sol^n: $CE \rightarrow \quad 1 + GH(s) = 0.$

$$s(s+2)(s+4)(s+6) + K = 0.$$

$$\therefore (s^2+2s)(s^2+10s+24) + K = 0.$$

$\Rightarrow \quad s^4 + 10s^3 + 24s^2 + 2s^3 + 20s^2 + 48s + K = 0.$

$\therefore \quad s^4 + 12s^3 + 44s^2 + 48s + K = 0.$

$$
\begin{array}{c|ccc}
s^4 & 1 & 44 & K \\
s^3 & \cancel{12}\ 1 & \cancel{48}\ 4 & \\
s^2 & 40 & K & \\
s^1 & \frac{160-K}{40} & & \\
s^0 & K & &
\end{array}
$$

$\Rightarrow$ For, Stable,

$$\boxed{K>0} \quad \& \quad \frac{160-K}{40} > 0 \Rightarrow \boxed{K<160}$$

$$\therefore \quad \boxed{0<K<160} \Rightarrow (S).$$

$\Rightarrow$ For (MS),

$$\therefore \quad \frac{160-K_{max}}{40} = 0$$

$$\therefore \quad \boxed{K_{max} = 160.}$$

$$AE \longrightarrow 40s^2 + K_{max} = 0$$

$$\therefore \quad 40s^2 + 160 = 0.$$

$$s^2 = -4.$$

$$\Rightarrow \boxed{s = \pm 2j.}$$

$$\Rightarrow \boxed{\omega_n = 2 \text{ rad/sec}}$$

$\boxed{Q}$ Find k and b values so that the $G(s) = \dfrac{k(s+1)}{s^3 + bs^2 + 3s + 1}$, $H(s) = 1$

oscillates with a freq. of 2 rad/sec.

Soln:

$$\omega_n = 2 \text{ rad/sec}$$

$\Rightarrow$ MS.

CE $\rightarrow$ $1 + GH(s) = 0.$

$$\therefore 1 + \frac{K(s+1)}{s^3 + bs^2 + 3s + 1} = 0.$$

$$\therefore s^3 + bs^2 + 3s + 1 + Ks + K = 0.$$

$$s^3 + bs^2 + (K+3)s + (K+1) = 0.$$

s^3	1	$K+3$
s^2	b	$(K+1)$
s^1	$\dfrac{b(K+3) - (K+1)}{b}$	
s^0	$K+1$	

$\Rightarrow$ For MS, $\dfrac{b(K+3) - (K+1)}{b} = 0.$

$$\therefore b(K+3) - (K+1) = 0$$

$$b = \frac{K+1}{K+3} \quad - (1).$$

$\therefore$ AE $\rightarrow$ $bs^2 + (K+1) = 0.$

$$\therefore \quad s^2 = -\left(\frac{K+1}{b}\right).$$

$$\therefore \quad s = \pm j\sqrt{\frac{K+1}{b}}.$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{K+1}{b}} = 2 \text{ rad/sec.}$$

$$\therefore \quad 4 = \frac{K+1}{b}.$$

from - eqn ①

$$4 = \frac{\cancel{K+1}}{\cancel{K+1}} \times K+3$$

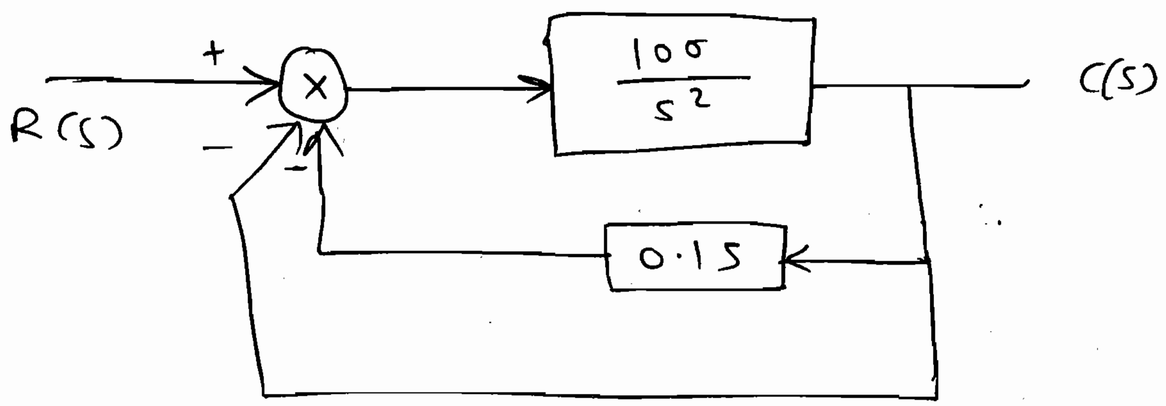
$$\boxed{K = 1}$$

$$\therefore \quad b = \frac{K+1}{4}$$

$$\Rightarrow \quad b = 1+1/4$$

$$\boxed{b = 0.5}$$

$\boxed{Q}$ Check the stability to the given BD.



Soln: Char. eqn $1 + GH(s) = 0.$

$$\therefore \quad 1 + \frac{100}{s^2}\left[1 + 0.1s\right] = 0.$$

$$\therefore \quad 1 + \frac{\overset{10}{\cancel{100}}}{s^2}\left[\frac{s+10}{\cancel{10}}\right] = 0$$

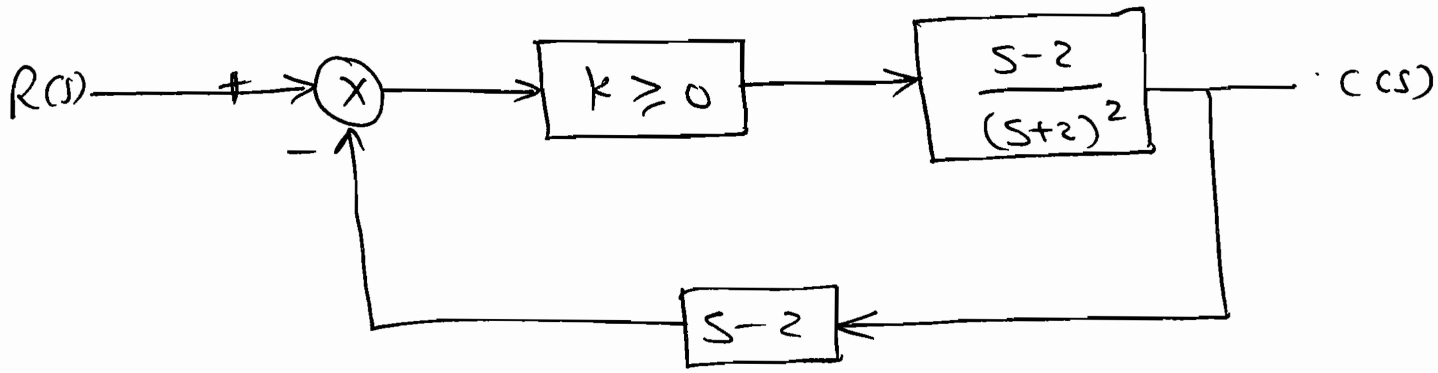
$$\Rightarrow s^2 + 10s + 100 = 0.$$

s^2	1	100
s^1	10	
s^0	100	

No sign change.

$\Rightarrow$ S.

$\boxed{Q}$ Find the Range of K value for system to be stable.



Soln: CE $\longrightarrow$ $1 + GH(s) = 0.$

$$\therefore \quad 1 + \frac{K(s-2)^2}{(s+2)^2} = 0.$$

$$\therefore \quad s^2 + 4s + 4 + Ks^2 - 4Ks + 4K = 0$$

$$\therefore \quad s^2(1+K) + s(4-4K) + 4K+4 = 0$$

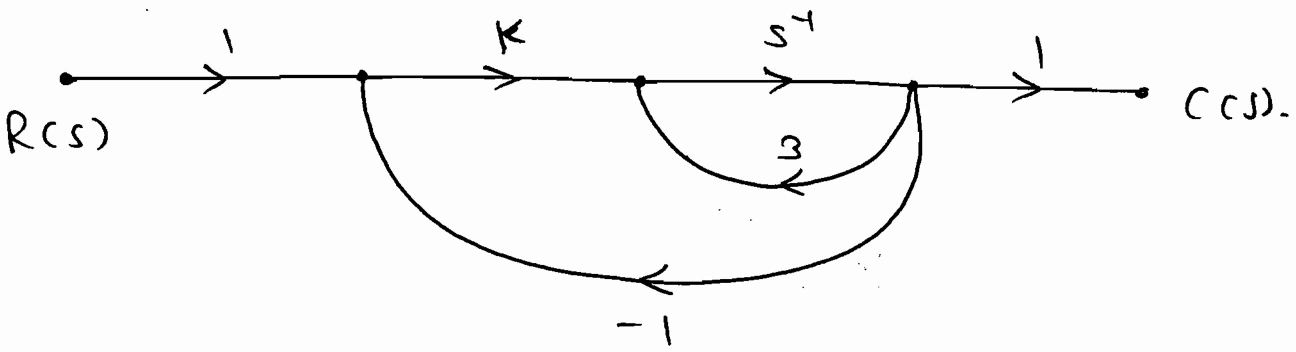
$$
\begin{array}{c|ll}
s^2 & 1+K>0 & 4+4K \\
s^1 & 4-4K>0 & \\
s^0 & 4+4K>0 &
\end{array}
$$

$\Rightarrow$

$K+1>0$	$4-4K>0$	$4+4K>0$
$K>-1$	$4>4K$	$4>-4K$
	$K<1$	$K>-1$

So, $\boxed{0 \le K < 1}$ ($\because$ $K \geq 0$ given).

Q Find the range of K value.



Soln:

CE $\rightarrow$ $1 + GH(s) = 0$.

$$\therefore \quad 1 - 3s^{-1} + Ks^{-1} = 0$$

$$1 - 3/s + \frac{K}{s} = 0.$$

$$\therefore \quad s + (K-3) = 0.$$

for (S), $K-3>0$

$$\boxed{K>3}$$

$\boxed{a}$ The Loop Gain of the system

$GH = \dfrac{K}{S(S+1)(S+2)}$, the value of K

for which the system just becomes

the unstable is, —?

Soln:

CE $\rightarrow$ $1 + GH(S) = 0$.

$$\therefore \quad 1 + \frac{K}{S(S+1)(S+2)} = 0.$$

$$\therefore \quad S(S^2 + 3S + 2) + K = 0.$$

$$\therefore \quad S^3 + 3S^2 + 2S + K = 0.$$

$\Rightarrow$ $\underbrace{\text{just becomes the Unstable}}$

$\Downarrow$

Marginal stable.

$$\therefore \quad \begin{array}{c|cc} S^3 & 1 & 2 \\ S^2 & 3 & K \\ S^1 & \frac{6-K}{3} & \\ S^0 & K & \end{array}$$

For, MS $\dfrac{6-K}{3} = 0$

$$\Rightarrow \boxed{K_{mar} = 6}$$

Q A system has $G(s) = \frac{k}{s^3+8s^2+4s}$, $H(s) = 1$. For what value of k the system will produce continuous oscn.

Soln:

$$1 + GH(s) = 0.$$

$$\therefore 1 + \frac{K}{s^3+8s^2+4s} = 0.$$

$$\therefore s^3 + 8s^2 + 4s + K = 0.$$

Contn oscillation → MS.

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 8 & K \\ s^1 & \frac{32-K}{8} & \\ s^0 & K & \end{array}$$

for MS, $\frac{32 - K_{mar}}{8} = 0.$

$$\Rightarrow \boxed{K_{mar} = 32}$$

Another method:

$$s^3 + 8s^2 + 4s + K = 0$$

$$\therefore \boxed{K = 32}$$

☆ Relative Stability

⇒ The Relative Stability Concept applicable for only Stable System.

[Q] A system has $G(s) = \dfrac{2}{s(s+1)(s+2)}$

$H(s) = 1$. With RH criteria determine its Relative Stability about the line $s = -1$. [or]

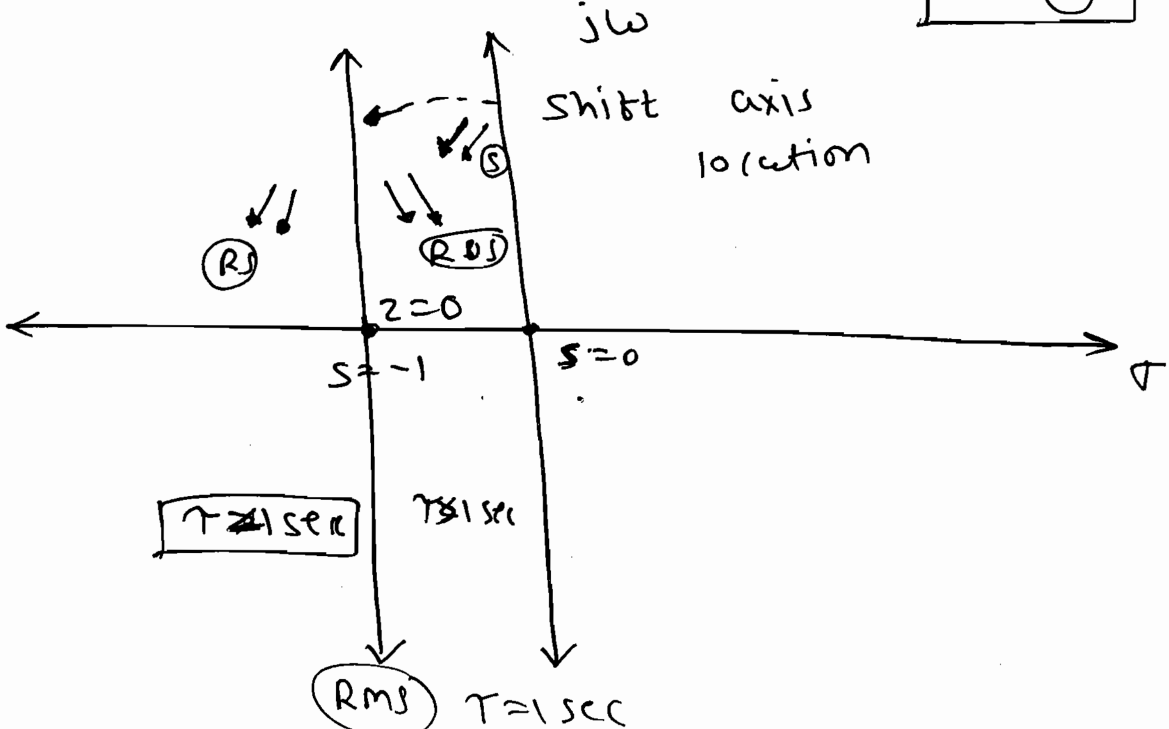
Check whether the time Const, greater (or) lesser (or) equal to 1 sec to the given system.

Sol^n:

CE → $1 + GH(s) = 0$

$$\Rightarrow s^3 + 3s^2 + 2s + 2 = 0.$$

$\boxed{6 > 2 \Rightarrow S}$



$\Rightarrow \quad s+1 = 0 = z$

$\therefore \boxed{s = z-1}$

$$\therefore G(z) = \frac{2}{(z-1)\,(z)\,(z+1)}.$$

$$\therefore 1 + G(z)\cdot H(z) = 1 + \frac{2}{z\,(z^2-1)}.$$

$$= z^3 - z + 2.$$

$$\begin{array}{c|cc}
z^3 & 1 & -1 \\
z^2 & \cancel{0}\ \epsilon & 2 \\
z^1 & \frac{-\epsilon - 2}{\epsilon} = -\infty & \\
z^0 & 2 &
\end{array}$$

① ②

Two sign change betn $s=0$ & $s=-1$.

2 poles betn $s=0$ & $s=-1$.

$\Rightarrow \quad S = z +$ Axis shift location

$s = (z - 1/\tau)$

RS $\Rightarrow (\tau) < ( ).$

RMS $\Rightarrow \tau = ( )!$

RUS $\Rightarrow \tau > ( ).$

* Limitation of RH criteria:

① The exact location of pole can not be determine.

② The RH criteria is not applicable for exponential sine (or) cosine terms because it gives the infinite series.

③ RH criteria is applicable to finite no. of terms.

Note: By using RH criteria we can get approximation soln to exponential term.

Q Find the value of K for stability.

$$G(s)\cdot H(s) = \frac{K\cdot e^{-sT}}{s(s+1)}.$$

Soln: CE ⇒ $1 + GH(s) = 0 \Rightarrow 1 + \frac{K\cdot e^{-sT}}{s(s+1)} = 0$

$$\Rightarrow 1 + \frac{K(1 - sT)}{s(s+1)} = 0$$

$$\Rightarrow s^2 + s + K - KsT = 0$$

$$\Rightarrow s^2 + (1 - KT)s + K = 0.$$

s^2	1	K
s^1	$1-K\tau$	
s^0	K.	

$1 - K\tau > 0$, $K > 0$

$\boxed{K < 1/\tau.}$

$\therefore$ $\Rightarrow$ $\boxed{0 < K < 1/\tau}$ for (5).

☆ Root Locus :-

Purpose:-

→ To find the CL system stability.

→ To find the range of K value for system stability.

→ To find the K value to become system marginal stable.

→ To find the natural freq. of oscillation (OR) Undamped oscillation when the system is marginal stable.

→ To find the K value to become the system undamped, underdamped, critical damped and overdamped system.

→ To find the relative stability. By using the relative stability concept we can find system time constant settling time.

→ If the Root locus branches moves towards the left then the system is more Relative stable.

→ If the Root locus branches moves towards the Right then the system is less Relative Stable.

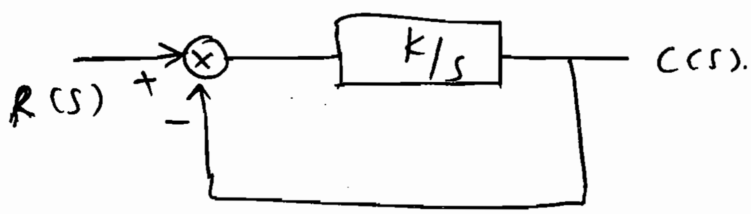
→ Best method to find the relative stability is Root Locus.

→ Best method to find absolute stability is RH - criteria.

* Definition of Root Locus :-

→ 'Root' means roots of chara. eqn which is CL Poles. 'Locus' means path. Hence, Root Locus means CL poles path by varying K value from 0 to ∞.

[Q] Draw the Root locus to the given system.



Sol^n: Char. eq^n $\Rightarrow$ $1 + GH(s) = 0$.

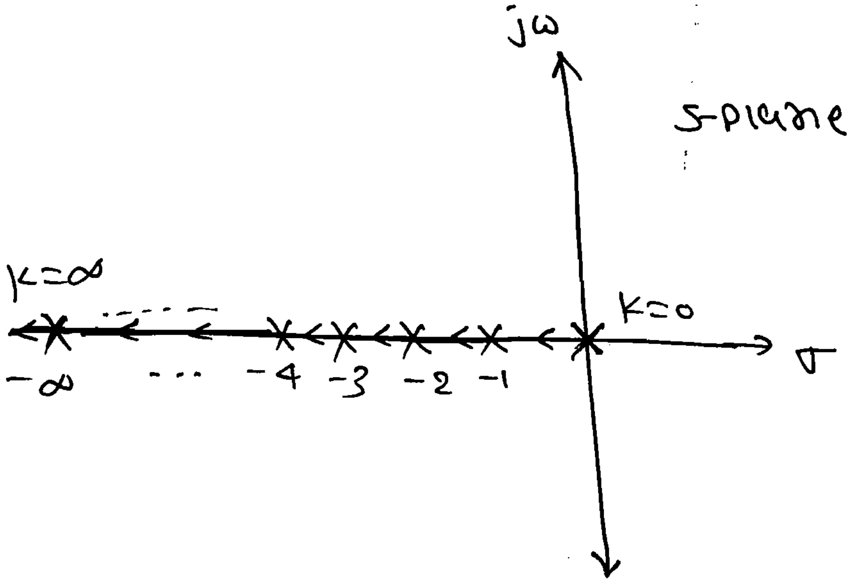
$$\Rightarrow 1 + \frac{K}{s} = 0$$

$$\boxed{s + k = 0.}$$

→ drawing a root locus means identifying the CL poles path.

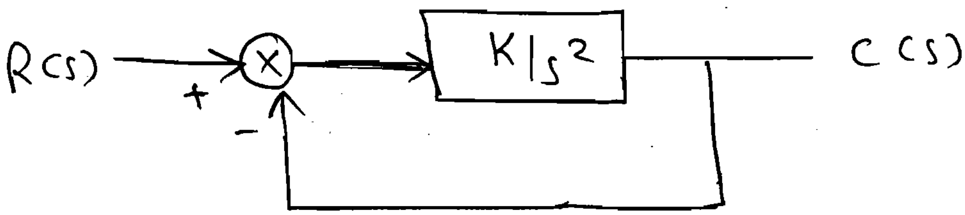
$$\underline{s = -k}$$

→ CL poles path is given by char. eq^n.



K value	Pole location $s = -k$
0 →	0
1	−1
2	−2
3	−3
⋮	
∞	−∞.

Q Draw Root Locus.



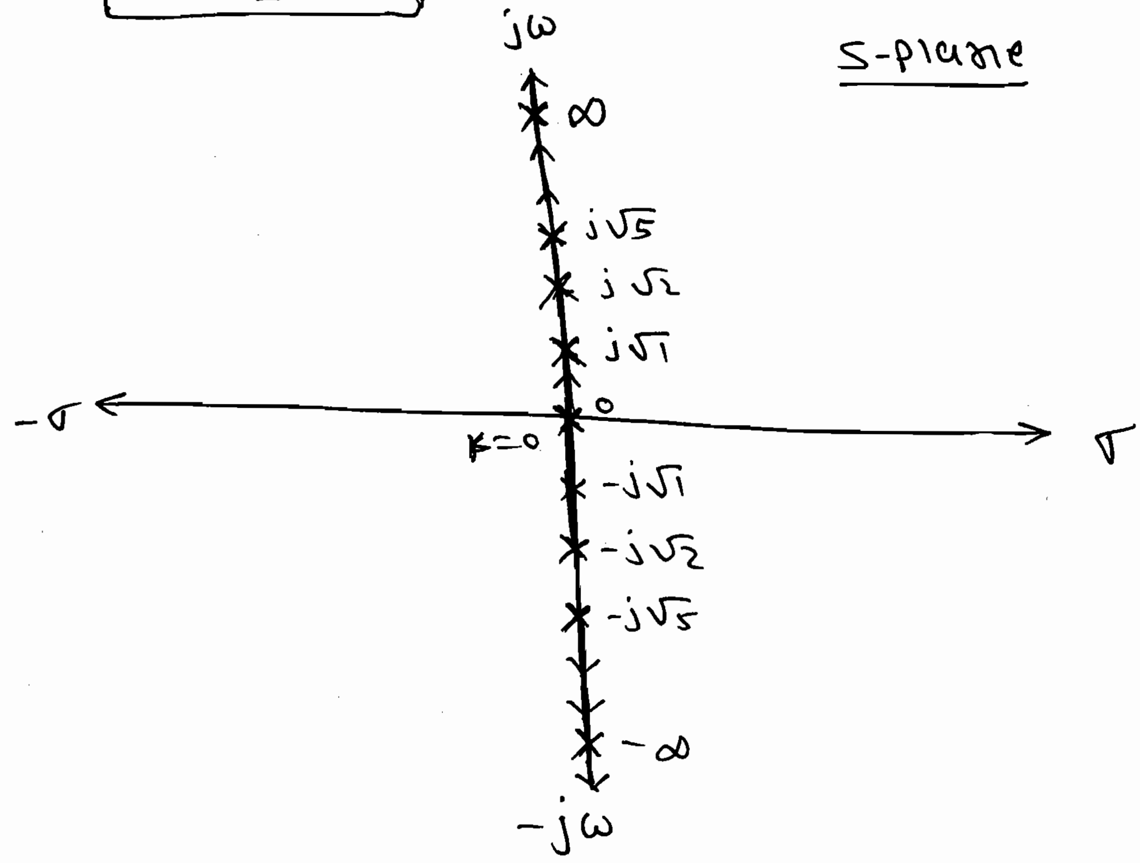
Sol^n: CE → $1 + GH(s) = 0$

$$\therefore 1 + \frac{K}{s^2} = 0.$$

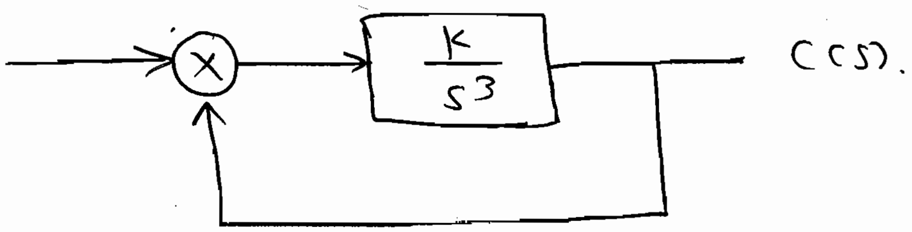
$\Rightarrow$ $s^2 + K = 0$

$$\boxed{s = \pm j\sqrt{K}}$$

$\Rightarrow$



a) Draw the Root Locus:



Soln:

$\xrightarrow{CE}$ $1 + \frac{K}{s^3} = 0$

$s^3 + K = 0.$

$s = ?$

$\Rightarrow$ As order increases finding the roots for the char. eqn is very difficult. Hence we can not draw the Root Locus diagram by using char. eqn.

→ To draw a Root locus diagram we use the open loop transfer function, But the Stability analysis is for CL system.

*** Relationship between OL transfer f^n & CLTF Poles & zeros:**

⇒ i] OLTF:

→ The CL Poles are given by char. eqn

$$1 + GH(s) = 0.$$

$$\Rightarrow 1 + K \cdot \frac{N(s)}{D(s)} = 0.$$

$$\xrightarrow{CE} D(s) + K \cdot N(s) = 0.$$

⇒ The CL Poles are nothing but the sum of OL poles, OL zeros with the f^n of system gain K.

$$\xrightarrow{OLTF} G(s) \cdot H(s) = K \frac{N(s)}{D(s)} \quad — (1).$$

$$\xrightarrow{OL\ Poles} D(s) = 0.$$

$$\xrightarrow{OL\ zeros} N(s) = 0.$$

Case–I: $K = 0$, $K = \left| - \frac{D(s)}{N(s)} \right|$

↓

$D(s) = 0$ ← CL Poles.

⇒ When $k = 0$, CL Poles = OL Poles.

⇒ Case - II:

$$N(s) = 0 \longrightarrow k = \pm\infty.$$

When $k = \infty$,	CL Poles = OL zeros.
When $k = -\infty$,	CL Poles = OL zeros.

⇒

When $k \Uparrow$ $\xrightarrow{0 \text{ to } \infty}$ $\underset{\text{OL Poles}}{\overset{k=0}{\times}} \longrightarrow\longrightarrow\longrightarrow \underset{\text{OL zeros.}}{\overset{k=\infty}{\bigcirc}}$

When $k \Uparrow$ $\xrightarrow{-\infty \text{ to } 0}$ $\underset{\text{OL zeros}}{\overset{k=-\infty}{\bigcirc}} \longrightarrow \underset{\text{OL poles.}}{\overset{k=0}{\times}}$

→ From above, we can conclude that when k increases from 0 to ∞ the direction of the root locus branch is from pole to zero because at OL poles k value is '0' and at OL zeros k value is '∞'.

⇒ If k increases from $-\infty$ to 0 then the direction of Root Locus Branch is from OL zero to OL poles.

because at OL zero, $K = -\infty$ and at OL Pole $K = 0$.

$\boxed{Q}$ Identify where the RL branch start and ends when K increased from 0 to ∞ for $G(s) \cdot H(s) = \frac{K(s+1)}{s(s+5)(s+10)}$.

Soln:

$$GH(s) = \frac{K(s+1)}{s(s+5)(s+10)}.$$

$\begin{bmatrix} \text{OL Poles} \\ K = 0 \end{bmatrix} \Rightarrow \quad s = 0, \; s = -5, \; s = -10. \leftarrow$ start

$\begin{bmatrix} \text{OL Zero} \\ K = \infty \end{bmatrix} \Rightarrow \quad s = -1, \; \underbrace{\infty, \infty}_{\text{angle of Asymptotes.}} \leftarrow$ end.

***⇒ To draw a RL diagram, no. of Poles is must equal to no. of zeros. If the zeros are less we assume zeros are at infinity. The direction of infinity is given by angle of asymptotes.

***⇒ To draw a RL diagram, for one Pole we need one zero, because the RL branch start at poles and ends at zeros.

* Angle & Magnitude Conditions:

→ The Construction Rules of Root Locus are formed by using the angle condition such that the Root Locus diagram gives the CL poles path and CL system stability.

→ The CL poles path given by Char. eqn,

(–ve FB)

CE →

$$1 + G(s)\cdot H(s) = 0.$$

$$GH(s) = -1 + j0$$

A.C. →

$$\angle G(s)\cdot H(s) = \angle -1 + j0$$

= odd multiples of $\pm 180°$.

$$\boxed{\angle GH(s) = \pm(2q+1)\,180°}$$

$$q = 0, 1, 2, \ldots$$

(+ve FB)

CE →

$$1 - G(s)\cdot H(s) = 0$$

$$G(s)\cdot H(s) = 1 + j0$$

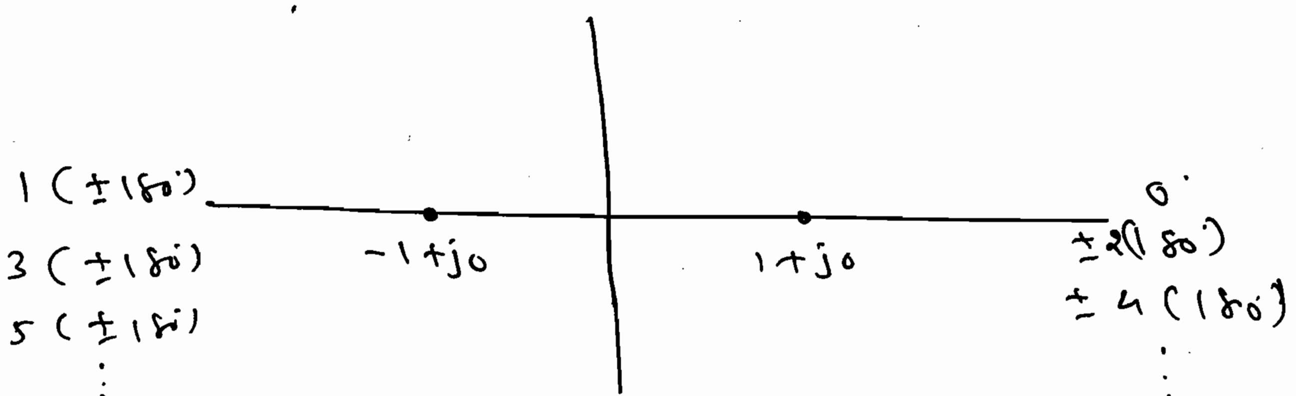
A.C. →

$$\angle G(s)\cdot H(s) = \angle 1 + j0$$

= even multiples of $\pm 180°$.

$$\boxed{\angle GH(s) = \pm 2q\,(180°)}$$

$$q = 0, 1, 2, \ldots$$



→ Roots which are formed by (-ve) FB Char. eqn are called Direct Root locus (DRL) (or) 180° Rules.

→ Roots which are formed by (+ve) FB Char. eqn are called Inverse Root locus (IRL) (or) Complementary Root locus (CRL) (or) 0° rules.

⇒

DRL	IRL
$R_3 \longrightarrow$ odd	even (statement).
$R_4 \longrightarrow 2q+1$	$2q$ (formula).
Case (iii) $R_6 \longrightarrow$ Left most	Right Most (statement)
$R_8 \longrightarrow 180°$	0° (formula).

* Purpose of angle Condn:

⇒ To check the any point lies on Root Locus (or) not that means all the points on the Root locus must satisfies the angle conditions.

Q Verify either the following points lies on the Root Locus ~~S.~~ or not to the following system,

$$G(s)\cdot H(s) = \frac{K}{s(s+5)(s+10)}$$ i) $s=-3$ ii) $s=-6$.

Soln: (i) $s=-3$.

$$\xrightarrow{AC} \angle GH\Big|_{s=-3} = \frac{\angle K}{\angle -3 \angle +2 \angle 7}$$

$$= \frac{0^\circ}{\pm 180^\circ + 0^\circ + 0^\circ}$$

$$\angle GH\Big|_{s=-3} = \mp 180^\circ \checkmark$$

Satisfies the odd multiple of 180°.

so, pole is on RL.

(ii) $s=-6$

$$\angle GH\Big|_{s=-6} = \frac{\angle K}{\angle -6 \angle -1 \angle 4}.$$

$$= \frac{0^\circ}{\pm 180^\circ \pm 180^\circ + 0^\circ}$$

$$= \mp 2(180^\circ) \times$$

→ Not satisfies the angle condition because even no. of 180°.

→ so, given root $s=-6$ do not lies on RL.

* Magnitude Condition:-

$\Rightarrow$ M.C. $\rightarrow$ $G(s) \cdot H(s) = -1 + j0.$

$$\boxed{|G(s) \cdot H(s)|_{\text{at any point which is on RL}} = 1.}$$

→ If given point is not on RL then M.C. is not valid.

→ so, the angle condⁿ must be satisfied to valid the Magnitude Condition (M.C.).

→ Magnitude Condition is valid when the given point is on RL. The given point on the RL is verified by angle Condition that means to apply a magnitude condition the A.C. must be satisfied.

Purpose:

$\Rightarrow$ To find the system gain at any point which is on RL.

$\boxed{a}$ Find the system gain at a point $s = -5 + j5$ to the following system

i.e. $\dfrac{K}{s(s+10)}$.

Sol^n:

$$\xrightarrow{AC} G(s)H(s)\Big|_{s=-5+j5} = \frac{\angle K}{\angle -5+j5 \ \angle 5+j5}$$

$$= \frac{0^\circ}{135^\circ + 45^\circ}$$

$$= -180^\circ$$

Satisfied the AC.

So, given Pole is on RL.

Now, $\xrightarrow{M.C.} \left| GH(s) \right|_{s=-5+j5} = 1.$

$$\therefore \left| \frac{K}{(-5+j5)(5+j5)} \right| = 1$$

$$\frac{K}{\sqrt{25+25} \cdot \sqrt{25+25}} = 1$$

$$\boxed{K = 50}$$

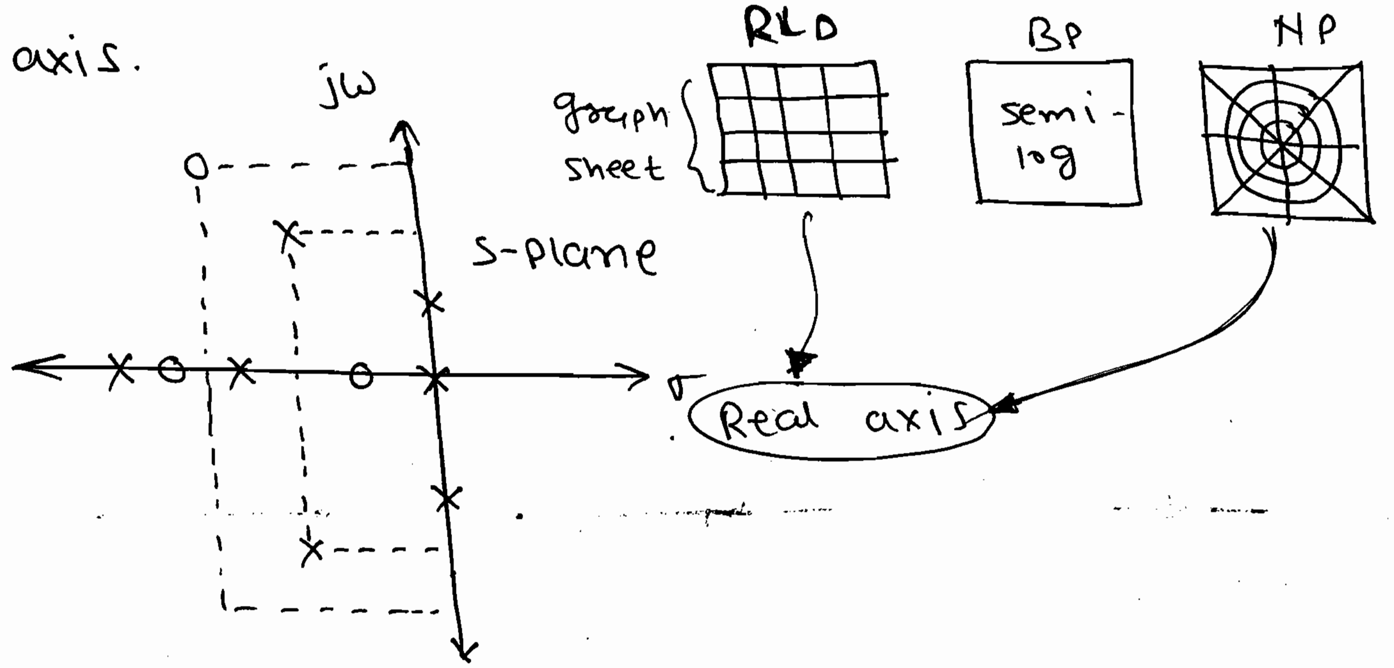
So, System gain at $s = -5 + j5$ is

$$\boxed{K = 50}$$

* Construction Rules for RL:-

Rule - 1 :- Symmetry:

⇒ The root locus diagram is Symm. about the real axis because the location of the poles and zeros in the S-plane Symm. about the real axis.



→ The Symm. not depends on poles and zeros location, it depends on the graph sheet and on which the plot is constructed the NP (Nyquist Plot) also Symm. about the real axis but not Bode plot because the bode plot drawn on non-linear graph sheet (semi-Log).

Rule - 2 :- No. of Loci (or) RL branches.

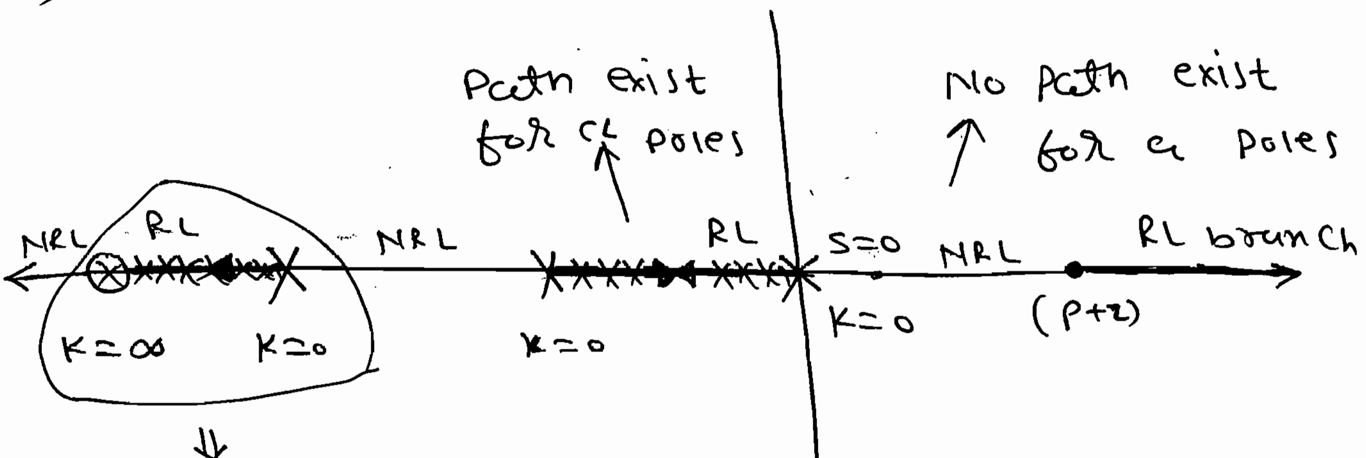
⇒ It depends on no. of Poles and zeros.

Case-(i) : Poles > Zeros : No of Loci = No of pole.

Case-(ii) : Zeros > Poles : No of Loci = No. of Zero.
(Poles < zeros)

Rule - 3 :- Real axis Loci.

⇒



⇓
RL branch
⟶ k↑ (0 to ∞).

⇒ A Point exist on Real axis root locus branches, the sum of the the Poles and Zeros to the Right hand side of that point should be odd.

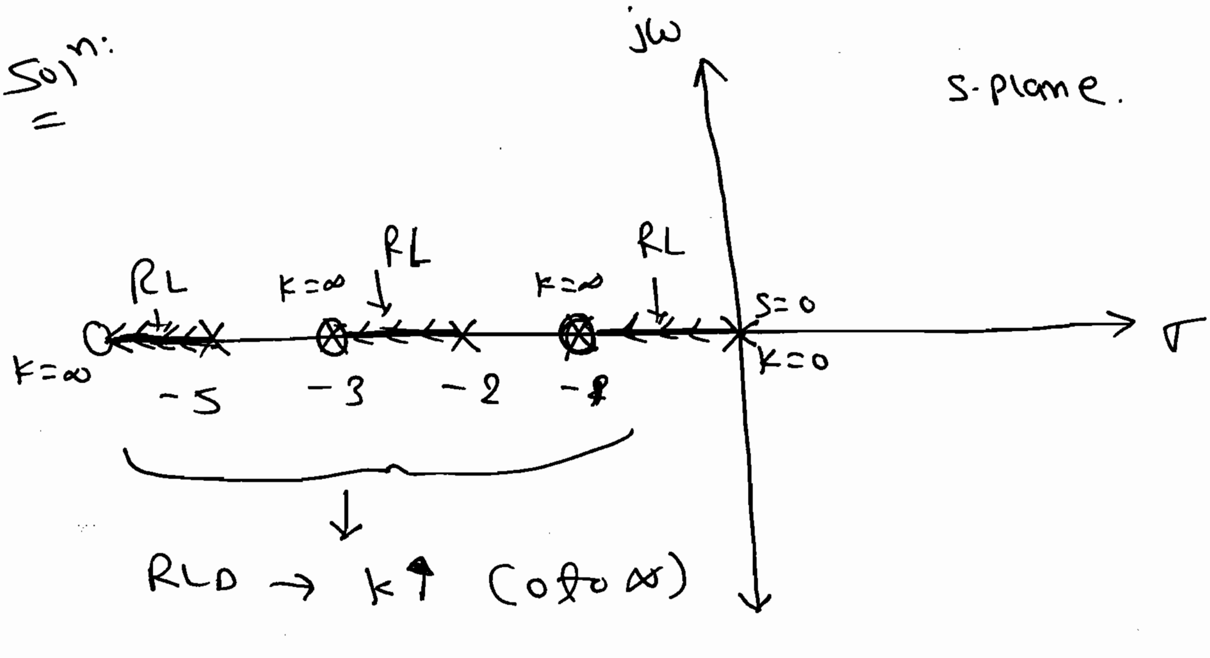
⇒ The Poles moves only on the RL branches. Once the pole reach the zero then it become the complete root locus branch for that perticular pole where k increased from 0 to ∞.

$\Rightarrow$ At the position of poles and zeros never apply the angle and magnitude conditions because all the poles and zeros must lies on the RL branches. because that are starting and ending points of RL branches and the k value at pole is zero. and k value at zero ∞. so never apply magnitude and condition.

$\boxed{Q}$ Identify the sections of Real axis which belongs to RL.

$$G(s)\cdot H(s) = \frac{k(s+1)(s+3)}{s(s+2)(s+5)}.$$

Soln:
=



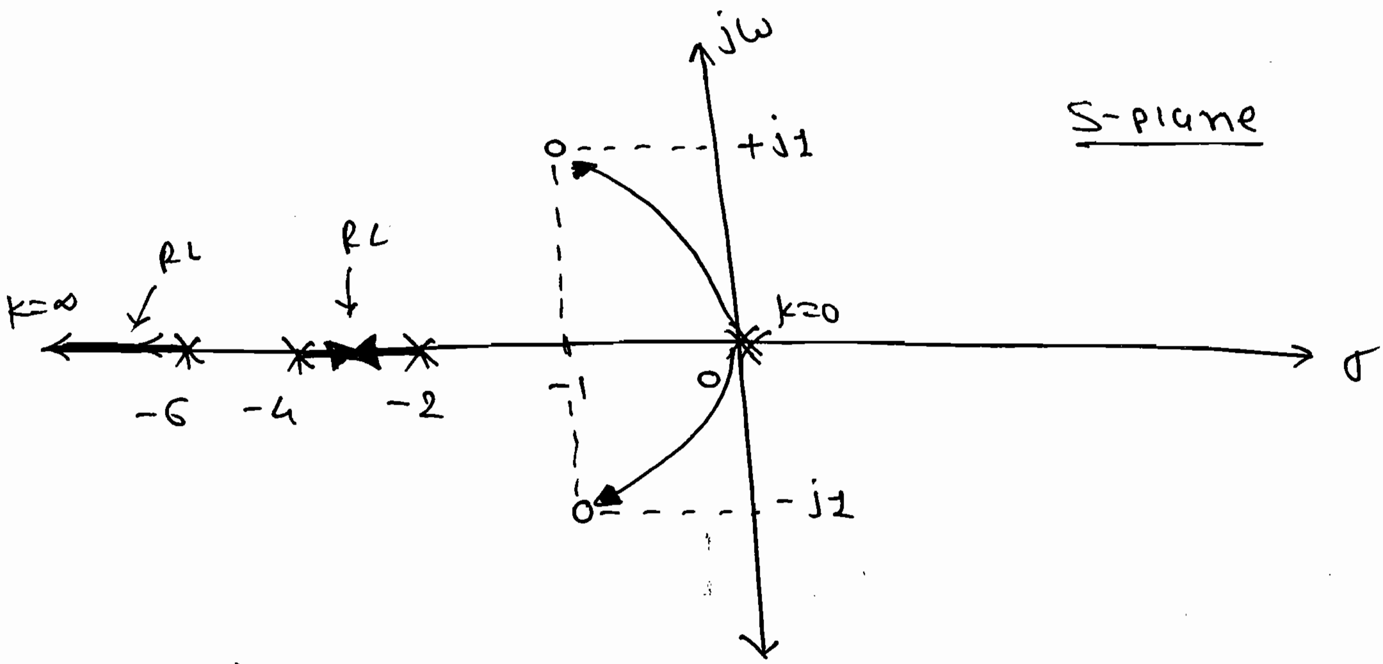
(a) Identify the following points which are on RL branches to the following:

$$G(s).H(s) = \frac{K(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)}.$$

Pole: $s=0$, $s=-2$, $s=-4$, $s=-6$, $s=-1$, $s=-3$, $s=-5$, $s=-6$, $s=-\infty$, $s=-1+j1$, $=-1-j1$.

Soln:

$$G(s).H(s) = \frac{K(s+1+j)(s+1-j)}{s^2(s+2)(s+4)(s+6)}.$$



So,

Valid.

$s=0$

$s=-2$

$s=-4$

$s=-6$

$s=-1+j$

$s=-1-j$

$s=-\infty$

Invalid

$s=-5$

$s=-1$

Rule - 4: Asymptotes:

⇒ Asymptotes are the RL branches which are approach to the infinity.

⇒ No. of Asymptotes $N = P - Z$.

Angle of Asymptotes $\theta = \frac{(2q+1)180^\circ}{(P-Z)}$,

$q = 0, 1, 2, \ldots (P-Z-1)$.

Note: The Asymptotes gives the directions of Zeros, when no. of Poles are greater than zeros.

Rule - 5 :- Centroid

⇒ The centroid is nothing but intersection point of asymptotes on the real axis.

⇒ (σ) Centroid $= \frac{\sum \text{Real part of Poles} - \sum \text{Real part of Zeros}}{(P-Z)}$

⇒ The centroid may be located anywhere on the real axis. It may (or) may not be on RL branch.

Q. Find the angle of Asymptotes and centroid $G(s)\cdot H(s) = \dfrac{K}{s(s+5)(s+10)}$.

Soln:

$P = 3, \quad Z = 0.$

$$\text{Centroid } (\sigma) = \frac{(-0-5-10) - (0)}{3-0}$$

$$= -15/3$$

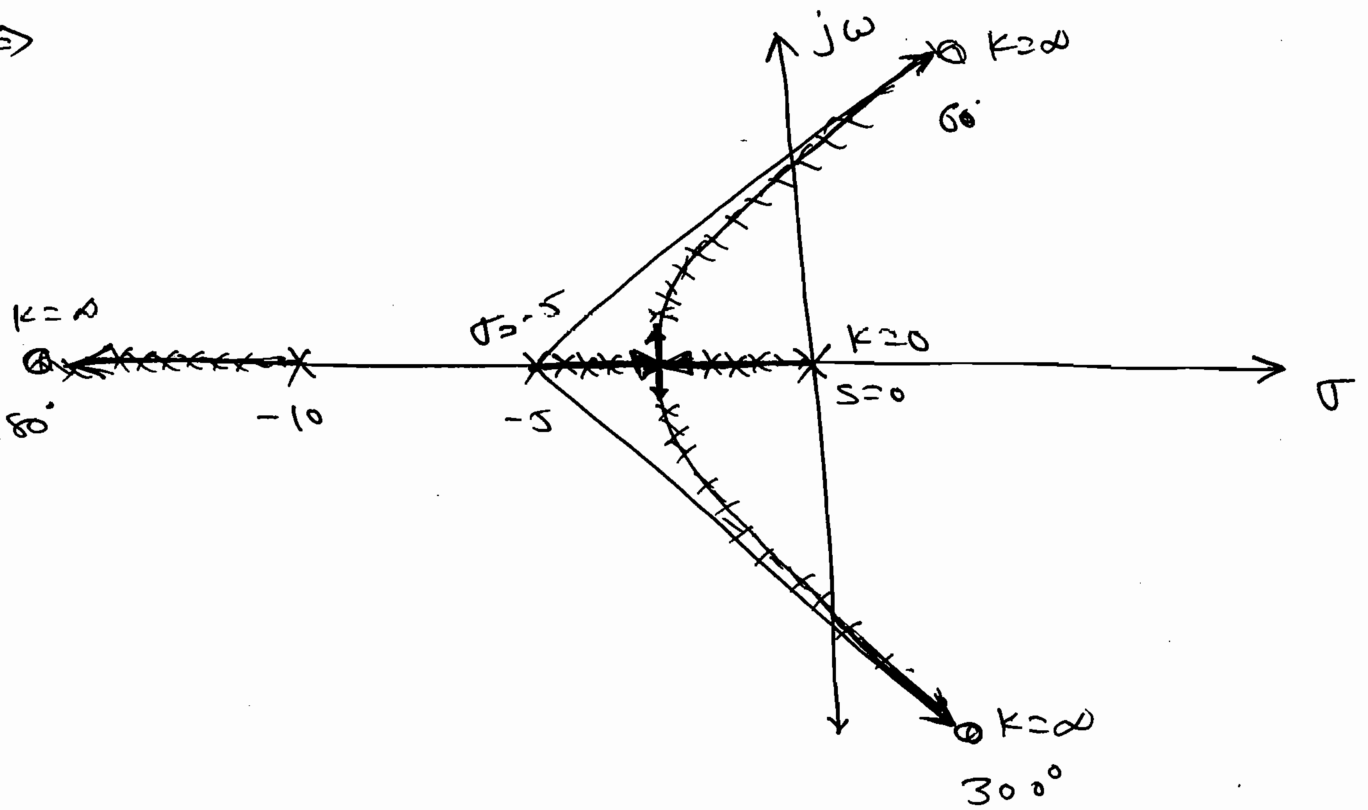
$$\boxed{\sigma = -5}$$

$$\therefore \quad \theta = \frac{(2q+1)\,180^\circ}{P-Z}$$

$$\therefore \quad \theta = \frac{(2q+1)\,\cancel{180^\circ}\,60^\circ}{\cancel{3}}$$

$$\theta = 60^\circ, 180^\circ, 300^\circ.$$

$\Rightarrow$



$\Rightarrow$ At Collision $\dfrac{\pm 180^\circ}{n} = \dfrac{\pm 180^\circ}{2} = \pm 90^\circ.$

$\Rightarrow$ The centroid is mainly required to draw the angle of asymptotes.

$\boxed{e}$ $G(s) \cdot H(s) = \dfrac{K(s+10)}{s^2(s+1)}$.

Soln:

angle $\theta = \dfrac{(2q+1)180°}{P-Z}$

$= \dfrac{(2q+1)180°}{3-1}$

$= \dfrac{180°}{2}$ 90 $(2q+1)$

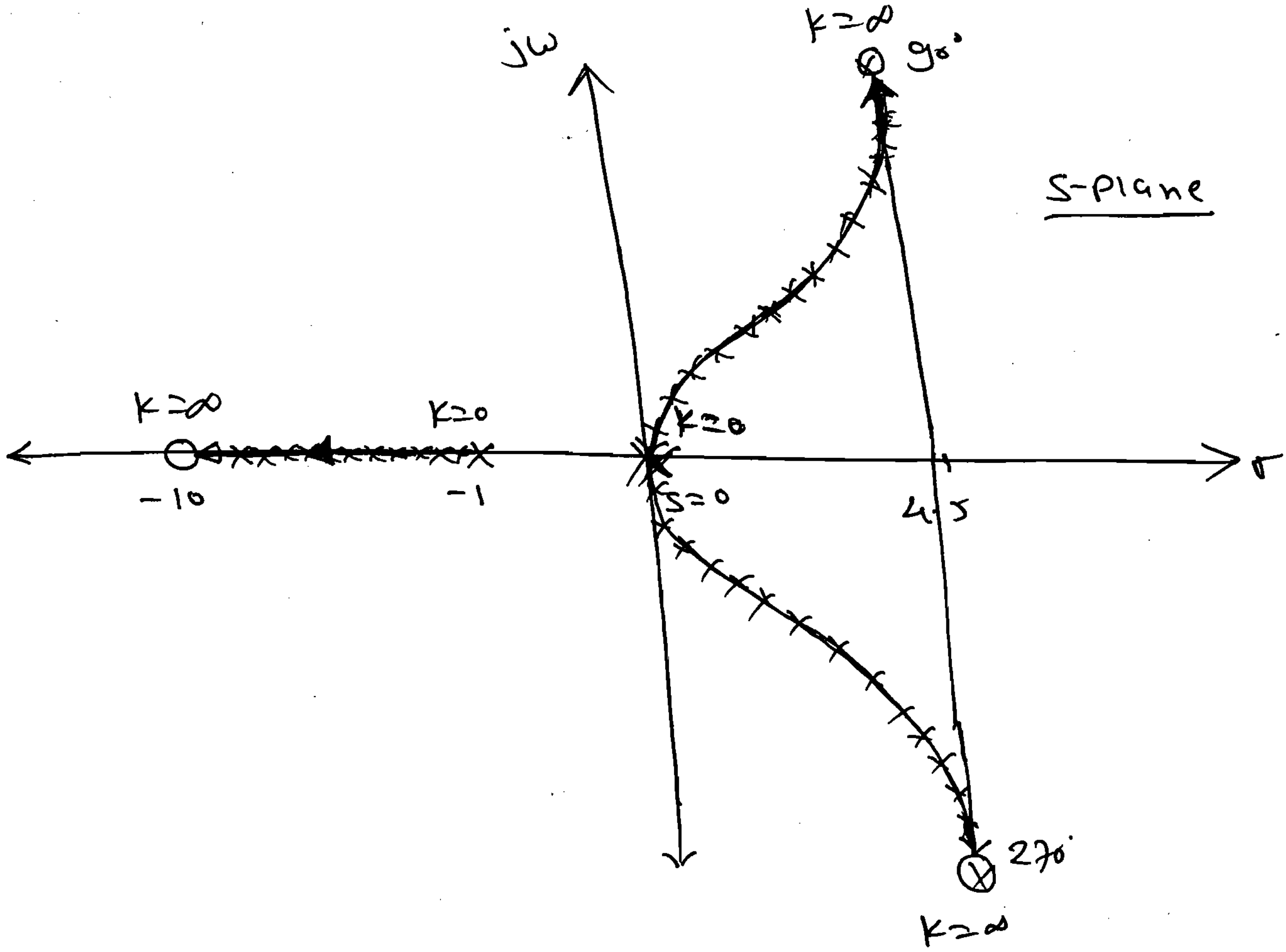
$\theta = 90°, 270°$

Centroid $= \dfrac{(-0-0-1)-(-10)}{3-1}$.

$= \dfrac{-1+10}{2}$

$\boxed{\sigma = 4.5}$

$\Rightarrow$



Rule - 6 :- Break Point [Junction of 2 (or) more Poles].

⇒ The point at which two (or) more poles meet (or) two or more poles directly located at any point then it is called Break point.

⇒ Breakaway Point :-

→ The point at which the Root Locus branches leaves the real axis is called breakaway point.

⇒ Breakin Point :-

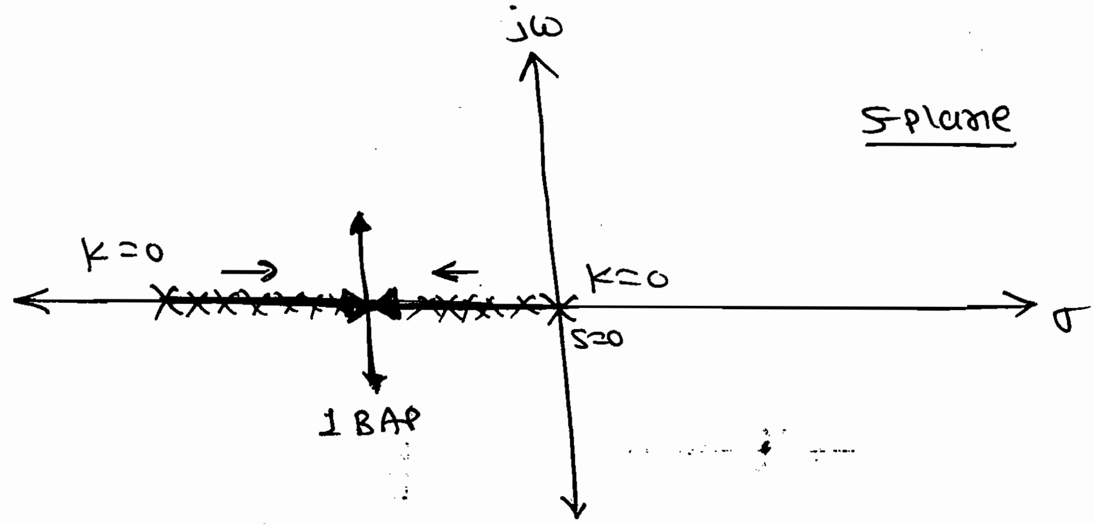
→ The point at which RL branches enters into the point on Real axis is called Breakin point.

* Finding the existence of the Break points:

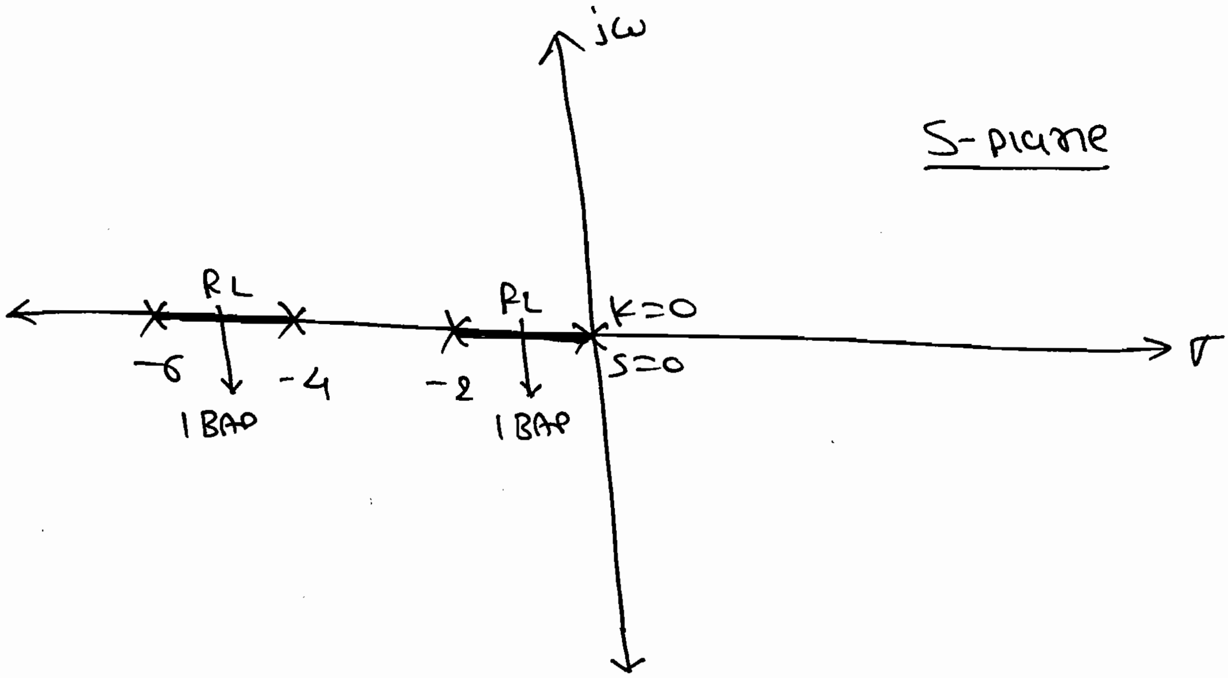
Case - I :

⇒ Whenever there are two adjacently placed poles in between there exist a RL branch then there should be the minimum one break away point (BAP) in between adjacently placed poles.

⇒



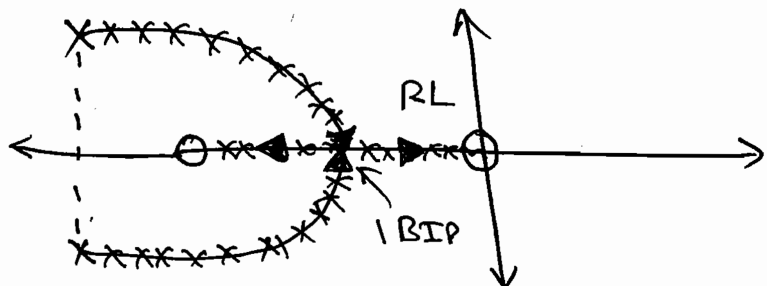
e.g. $G(s) = \dfrac{K}{s(s+2)(s+4)(s+6)}$.



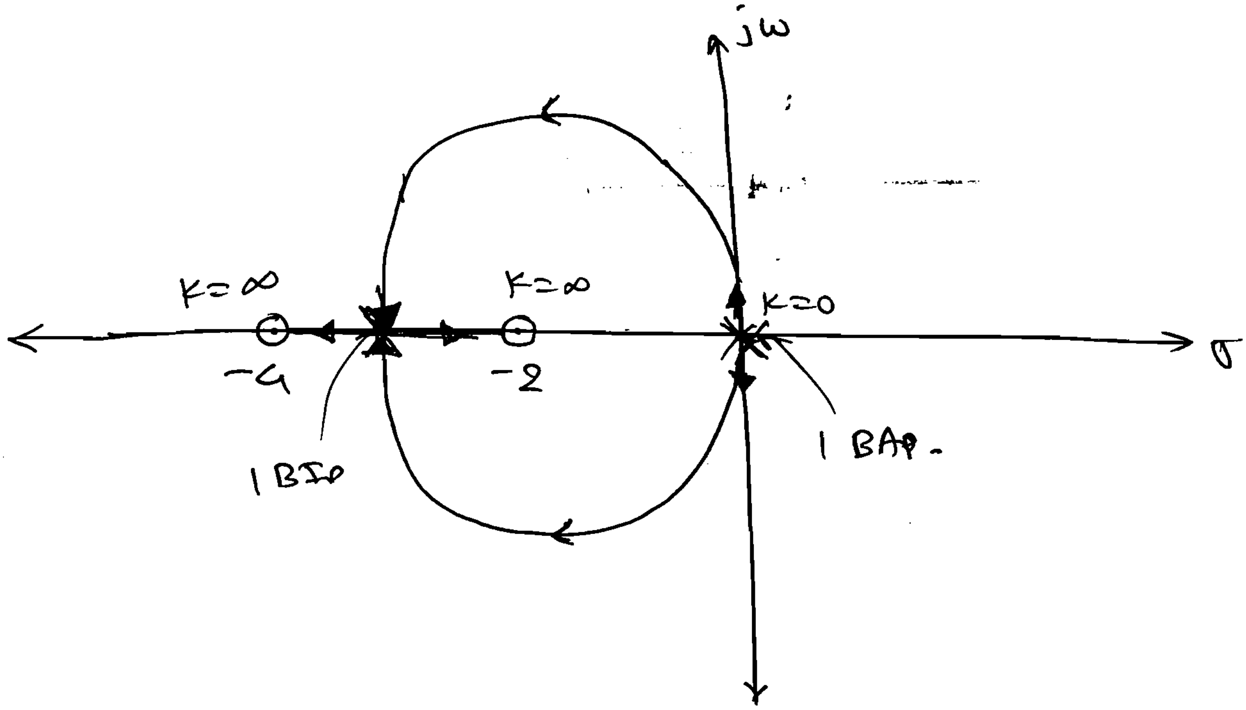
Case-II:-

⇒ Whenever there exist a two adjecently placed zero in betn there exist a RL branch then there should be the minimum one break in point in betn adjecently placed zero.

⇒



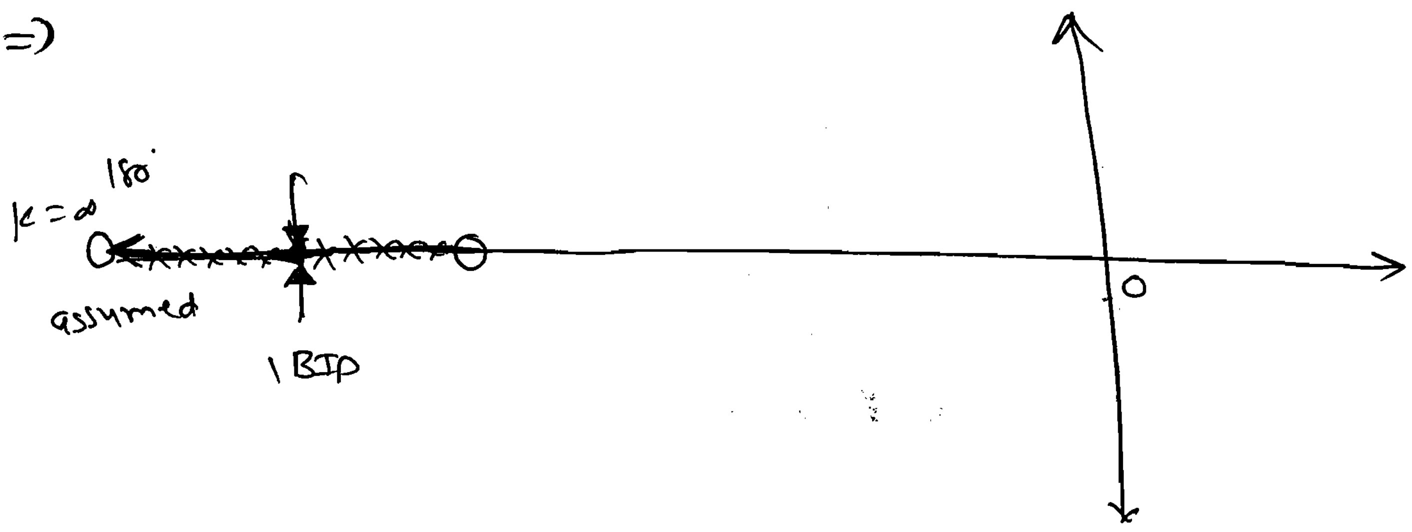
e.g. $G(s)H(s) = \frac{K(s+2)(s+4)}{s^2}$.



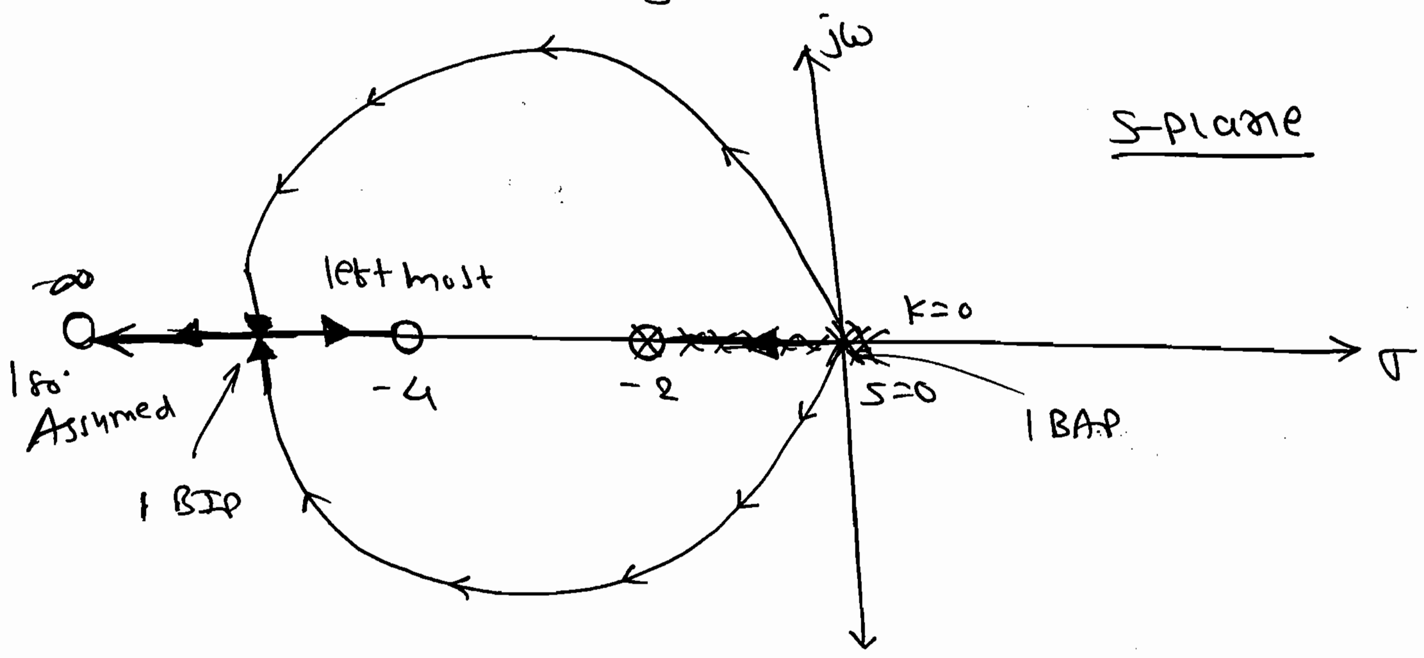
Case - III :- $\boxed{P>Z}$

⇒ Whenever there exist left most side zero to the left most side of that zero there exist a RL branch then there should be the minimum one breaking point to the left most side of the zero when no. of poles are greater than no. of zeros.

⇒



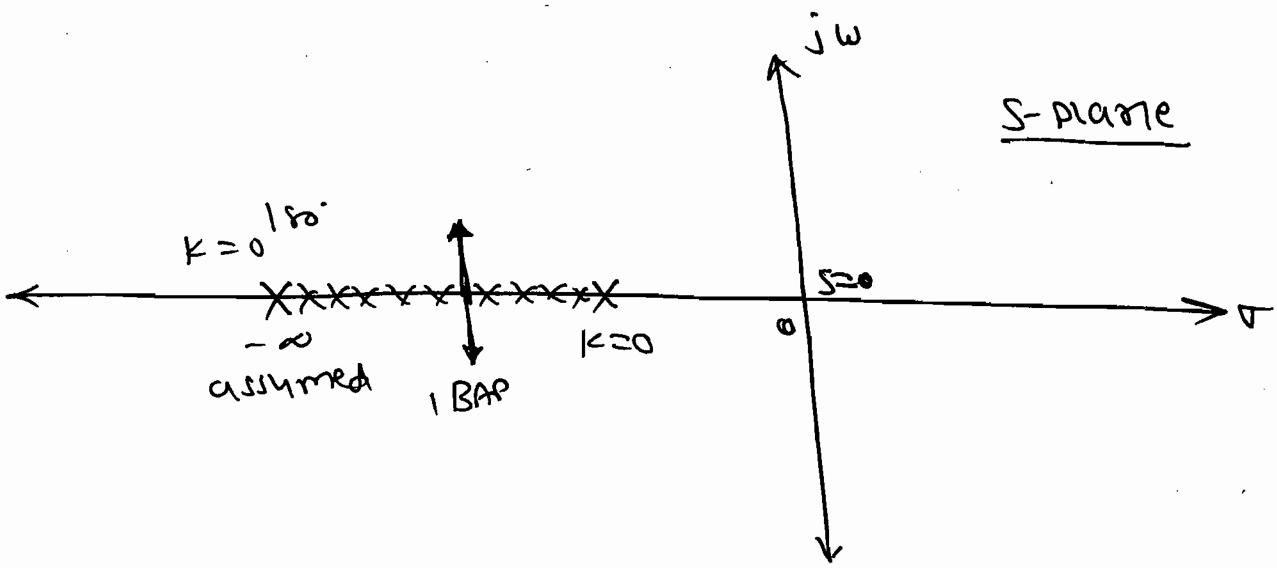
e.g. $G(s) \cdot H(s) = \dfrac{K(s+2)(s+4)}{s^3}$.



2 BPS

Case-IV: P<Z

⇒



⇒ whenever there exist a left most side pole to the left most side of that pole there exist a RL branch then there should be a minimum one breakaway point to the left most side of the zero. when the no. of poles less than zeros only.

⇒ This case is practically not exist because the control systems are LPF that means the no. of poles must be greater than zeros only.

* Finding the location of Break points:

Step - 1: Form the char. equation.

Step - 2: Rewrite the above eqn in the form of $K = F(s)$.

Step - 3: Differentiate k with respect to 's' and make equal to '0'. The roots of $\frac{dK}{ds} = 0$ gives the valid and invalid Breakpoint.

→ The valid BP is the one which must be on RL branch (or), for valid B.P. K value in step-2 should be (+ve).

Q Find the location of BP.

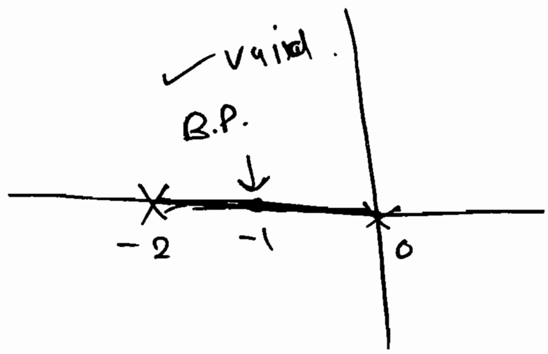
① $GH = \frac{K}{s(s+2)}$.

Soln: CE → $1 + GH = 0$

$$1 + \frac{K}{s(s+2)} = 0$$

$$\Rightarrow K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2 = 0 \Rightarrow \boxed{s = -1}$$



Q ② $GH = \dfrac{K}{S(S+2)(S+4)}$.

Soln,

$$1 + GH = 0$$

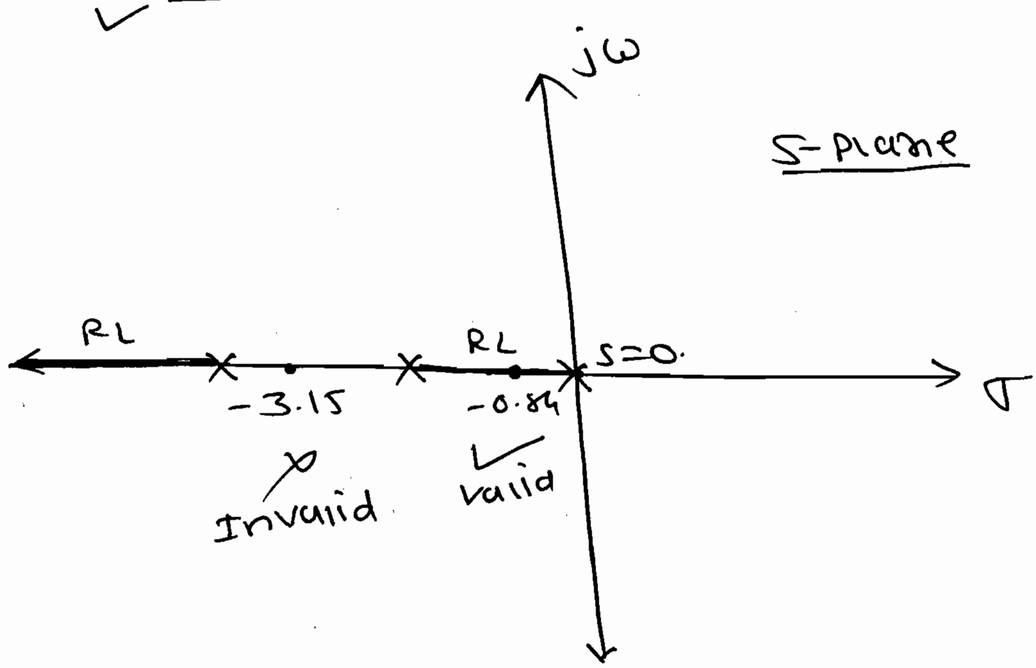
$$\therefore \quad 1 + \frac{K}{S(S+2)(S+4)} = 0.$$

$$K = -S^3 - 6S^2 - 8S.$$

$$\therefore \frac{dK}{dS} = -3S^2 - 12S - 8 = 0.$$

$$\boxed{S = -0.84} \checkmark, \quad \boxed{S = -3.15} \times$$

⇒



③ $GH = \dfrac{K(S+4)}{S(S+2)}$.

Soln

$$1 + GH = 0.$$

$$1 + \frac{K(S+4)}{S(S+2)} = 0.$$

$$\therefore \quad K = -\left[\frac{S^2+2S}{S+4}\right].$$

$$\therefore \frac{dK}{dS} = -\left[\frac{(S+4)(2S+2) - (S^2+2S)(1)}{(S+4)^2}\right] = 0.$$

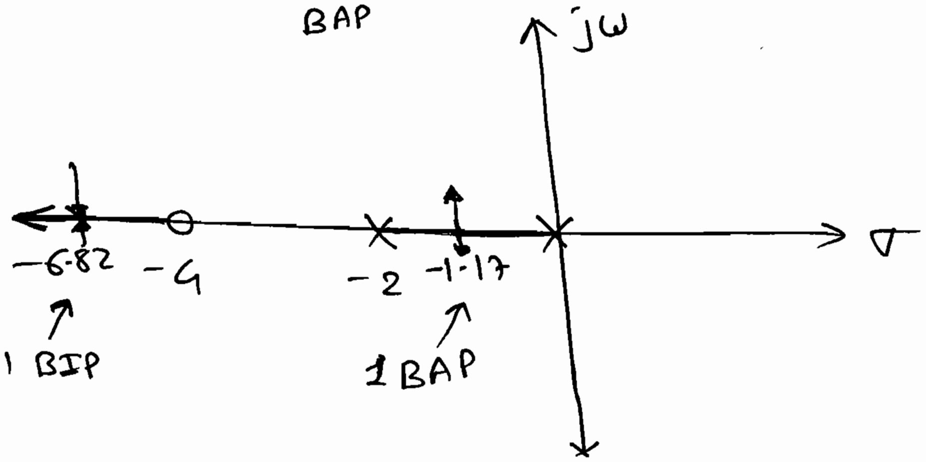
$$\Rightarrow \quad -2s^2 - 8s - 2s - 8 + s^2 + 2s = 0.$$

$$\therefore \quad s^2 + 8s + 8 = 0.$$

$$s = -1.17, \ -6.82.$$

BAP   BIP.

$\Rightarrow$



Rule - 7 : Intersection Point with Imaginary axis:

$\Rightarrow$ The intersection point with Imaginary axis is obtained by RH criteria.

(i) Form the Char. equation.

(ii) Write the Routh - tabular form.

(iii) Find the K marginal value.

(iv) Form the Auxillary equation. The roots of auxillary equation gives the valid or invalid intersection point with imaginary.

$\Rightarrow$ For valid intersection point with imaginary the K marginal be +ve.

Q Find the intersection point with imaginary axis $G(s) \cdot H(s) = \dfrac{K}{s(s+2)(s+4)}$.

Sol^n:

CE $\rightarrow$ $1 + GH(s) = 0$

$$1 + \frac{K}{s(s^2+6s+8)} = 0.$$

$$\Rightarrow s^3 + 6s^2 + 8s + K = 0.$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	
s^0	K	

for MS,

$$48 - K = 0$$

$$\Rightarrow \boxed{K_{mar} = 48.}$$

$\therefore$ AE $\rightarrow$ $6s^2 + K_{mar} = 0$

$$6s^2 + 48 = 0$$

$$s^2 = -8$$

$$\boxed{s = \pm j\sqrt{8}.}$$

Rule-8 :- Angle of Departure / Arrival :-

$\Rightarrow$ The angle of departure calculated at a complex conjugate poles and angle of Arrival calculated at a complex conjugate zero.

⇒ <u>Angle of Departure</u>:-

⇒ It gives that with what angle the pole depart or leaves from the initial position given by angle of departure.

→ $\phi_d = 180^\circ + \angle GH\big|_{\text{at a (+ve) imag. Complex pole.}}$
→ $\phi_d = 180^\circ - \phi$ where, $\phi = \sum \phi_p - \sum \phi_z$.

⇒ <u>Angle of Arrival</u>:-

⇒ It gives that with what angle the poles arrives, terminates at the complex zero given by angle of arrival.

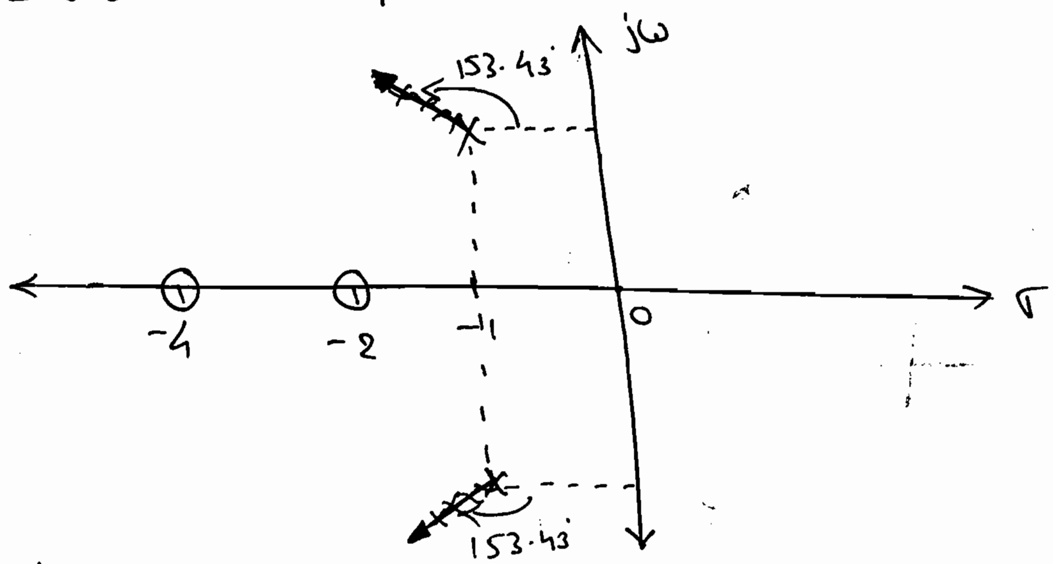
→ $\phi_a = 180 - \angle GH\big|_{\text{at a (+ve) img. Complex Zero.}}$
→ $\phi_a = 180 + \phi$ where, $\phi = \sum \phi_p - \sum \phi_z$.

$\boxed{Q}$ Calculate the angle of departure at a complex pole $G(s)\cdot H(s) = \dfrac{K(s+2)(s+4)}{s^2+2s+2}$.

Soln:

Poles : $s = -1 \pm j1$.

zeros : $s = -2, -4$.



$$\angle GH \Big|_{s=-1+j1} = \frac{\angle K \cdot \angle 1+j1 \cdot \angle 3+j1}{\angle 0^\circ \cdot \angle j2}$$

$$= \frac{0^\circ + 45^\circ + 18.43^\circ}{0^\circ + 90^\circ}$$

$$\angle GH = -26.27^\circ$$

$$\therefore \quad \phi_d = 180^\circ + \angle GH = 180^\circ + (-26.27^\circ)$$

$$\boxed{\phi_d = 153.43^\circ}$$

$\boxed{Q}$ Calculate Angle angle $G(s)\cdot H(s) = \dfrac{K(s^2+2s+2)}{(s+2)(s+4)}$.

Soln:

$$\angle GH \Big|_{s=-1+j} = \frac{\angle K \; \angle 0 \; \angle 2j}{\angle 1+1j \cdot \angle 3+1j}$$

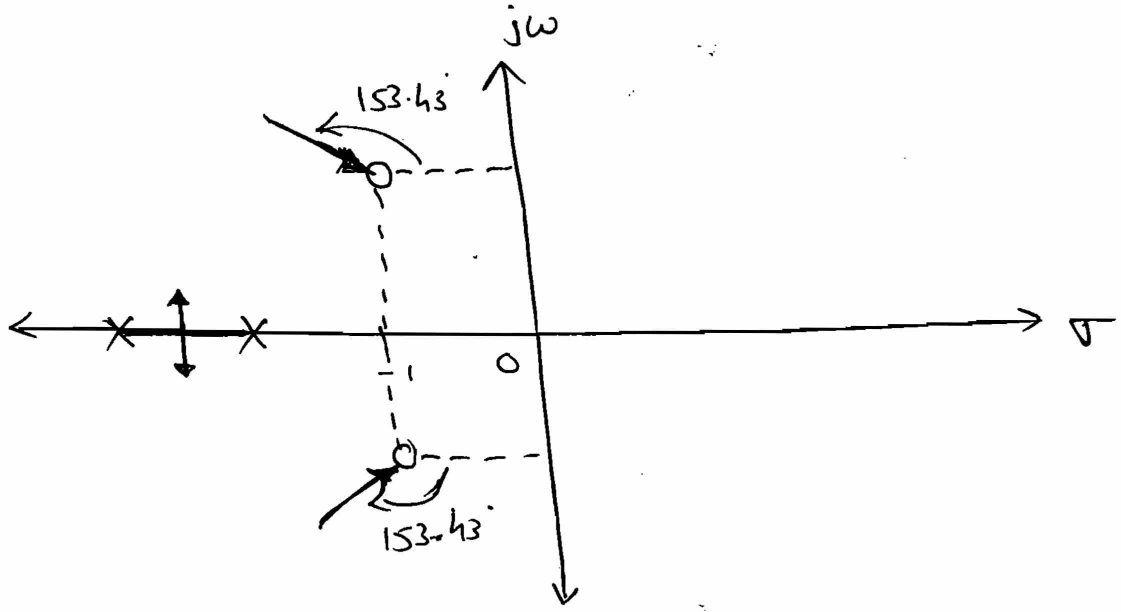
$$= \frac{0 + 0 + 90^\circ}{45^\circ + 18.43^\circ}$$

$\angle GH = +26.27^\circ$

$\therefore \phi_a = 180^\circ - \angle GH.$

$= 180^\circ - 26.27^\circ$

$\boxed{\phi_a = 153.43^\circ}$



Note:

→ Whenever all the zeros and poles are interchanged the angle of departure equal to angle of arrival. the break in point is equal to Breakaway point.

→ The shape of the RL diagram is also same except the direction.

* Procedure:

① Identify the RL branches and B.P.

② Find the centroid and angle of Asymptotes. (If required).

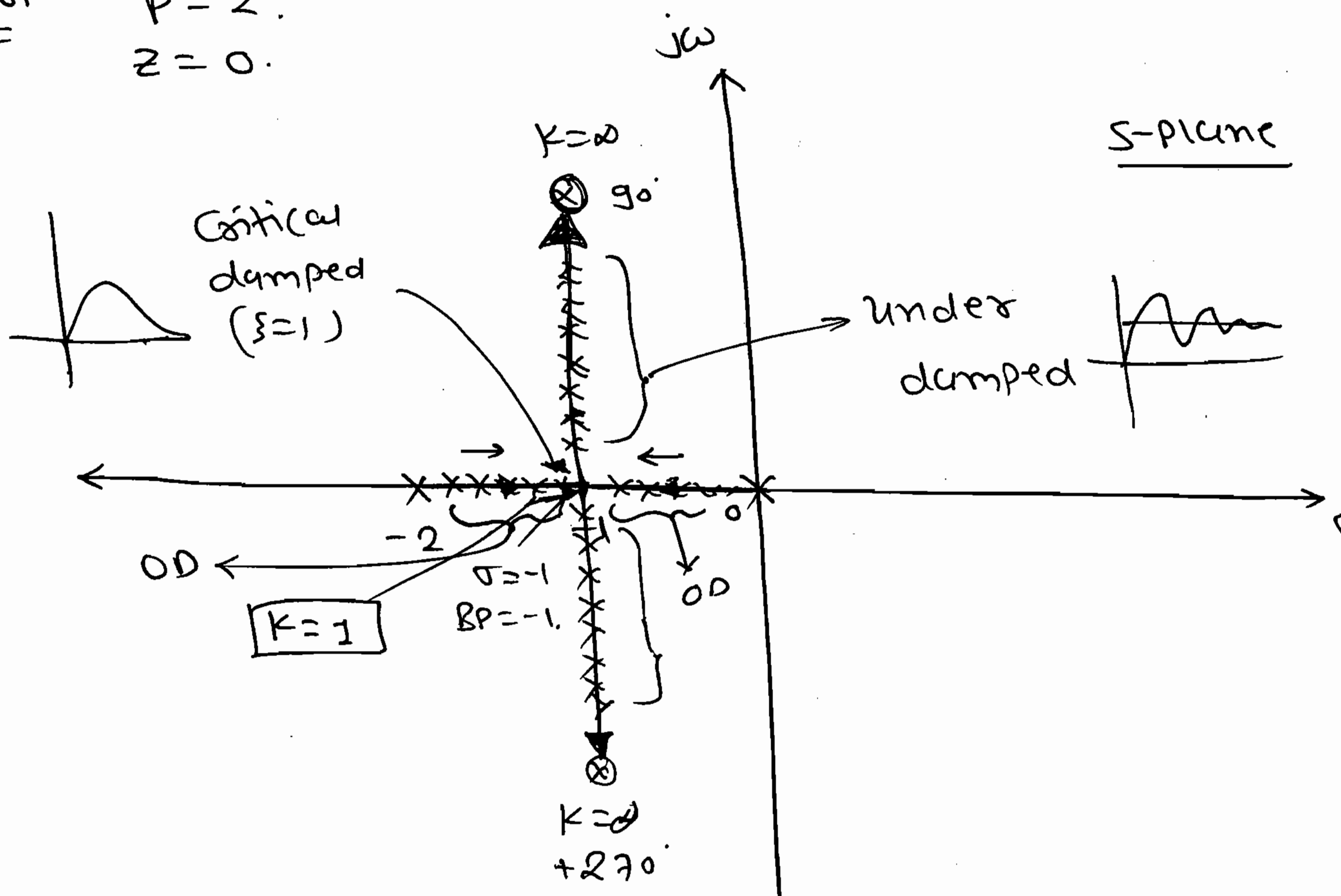
③ Find angle of departure and arrival.

(iv) Vary the K value from 0 to ∞ identify the path from pole to zero such that the Root locus diagram pole must reaches the zero.

Q Draw the RL diagram to the following systems and find the CL system stability.

① $G(s) \cdot H(s) = \dfrac{K}{s(s+2)}$.

Soln: $P = 2$.
$Z = 0$.



⇒ Centroid $\sigma = \dfrac{(0-2)-(0)}{2-0}$

$\boxed{\sigma = -1}$

$\Rightarrow \underrightarrow{A.A.}$ $\theta = \frac{(2q+1)}{P-2} \times 180^\circ$

$= \frac{(2q+1)}{2} \times 180^\circ$

$\theta = 90^\circ, 270^\circ$

$\Rightarrow \underrightarrow{B.P.}$ $s^2 + 2s = 0.$

$2s + 2 = 0.$

$\boxed{s = -1}$

$\Rightarrow$ The above system having over damped, critical damped and under damped nature but not undamped. To set the K values for different nature of the systems are required to find K value at the BP.

Method - I:

$\underrightarrow{M.C.}$ $\left| \frac{K}{s(s+2)} \right|_{s=-1} = 1.$

$\Rightarrow \left| \frac{K}{(-1)(1)} \right| = 1.$

$\Rightarrow \boxed{K = 1}$

Method - II:

$K = \frac{\text{Product of lengths from the pt to poles}}{\text{Product of lengths from the pt to zeros.}}$

$$= \frac{C(1)\,C(U)}{\text{No zero}}$$

$\boxed{K = 1}$

$\Rightarrow$ $0 < K < 1 \longrightarrow$ OD.

$K = 1 \longrightarrow$ CD.

$K > 1 \longrightarrow$ underdamped.

$\Rightarrow$ The Root Locus diagram gives the CL poles path. But not the OL poles path.

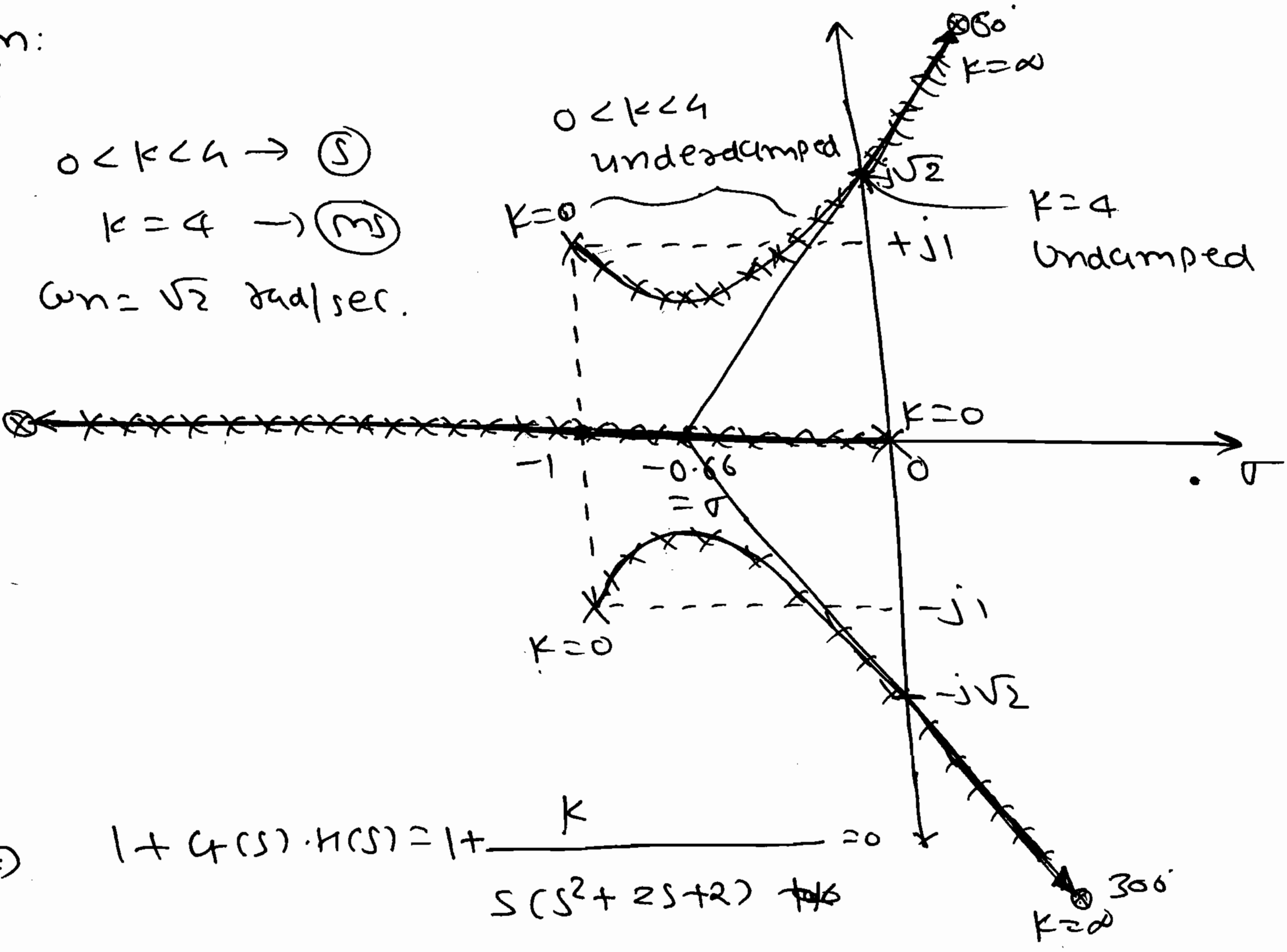
(2) $G(s) \cdot H(s) = \dfrac{K}{s(s^2 + 2s + 2)}$.

Soln:

$0 < K < 4 \rightarrow$ (S)

$K = 4 \rightarrow$ (MS)

$\omega_n = \sqrt{2}$ rad/sec.



$\Rightarrow$ $1 + G(s) \cdot H(s) = 1 + \dfrac{K}{s(s^2 + 2s + 2)} = 0$

$= s^3 + 2s^2 + 2s + K = 0.$

$\Rightarrow$ centroid $\sigma = + \dfrac{(-1-1) - 0}{P-2}$

$= -2/3$

$\boxed{\sigma = -0.66}$

$\rightarrow$ BP $s^3 + 2s^2 + 2s = 0$

$3s^2 + 4s + 2 = 0.$

$s = -\frac{2}{3} + j2/3 \quad , -2/3 - j4/3 \quad \times$ (Not valid)

$\rightarrow$ A.A. $\theta = \dfrac{(2q+1)180^\circ}{P-2}$

$= \dfrac{(2q+1)\cancel{180^\circ}60^\circ}{3}$

$\theta = 60^\circ, 180^\circ, 300^\circ.$

$\rightarrow$ I.P $s^3 + 2s^2 + 2s + K = 0$

$\boxed{K = 4}$

AE $\rightarrow$ $2s^2 + \overset{K}{\cancel{2s}} = 0$

$\cancel{4s + 2 = 0}$

$2s^2 = -4$

$\boxed{s = \pm j\sqrt{2}.}$

$\therefore \quad j\omega_n = \pm j\sqrt{2}$

$\boxed{\omega_n = \sqrt{2} \text{ rad/sec}}$

$\Rightarrow$ Angle of Departure:

$\angle GH = \dfrac{\angle K}{\angle 1+j \quad \angle 2j} = \dfrac{0^\circ}{135^\circ + 90^\circ} = -225^\circ$

$$\therefore \quad \phi_d = 180^\circ + \angle GH'$$

$$= 180^\circ - 225^\circ$$

$$\boxed{\phi_d = -45^\circ}$$

Q $\quad GH = \dfrac{KS}{S^2+4}$.

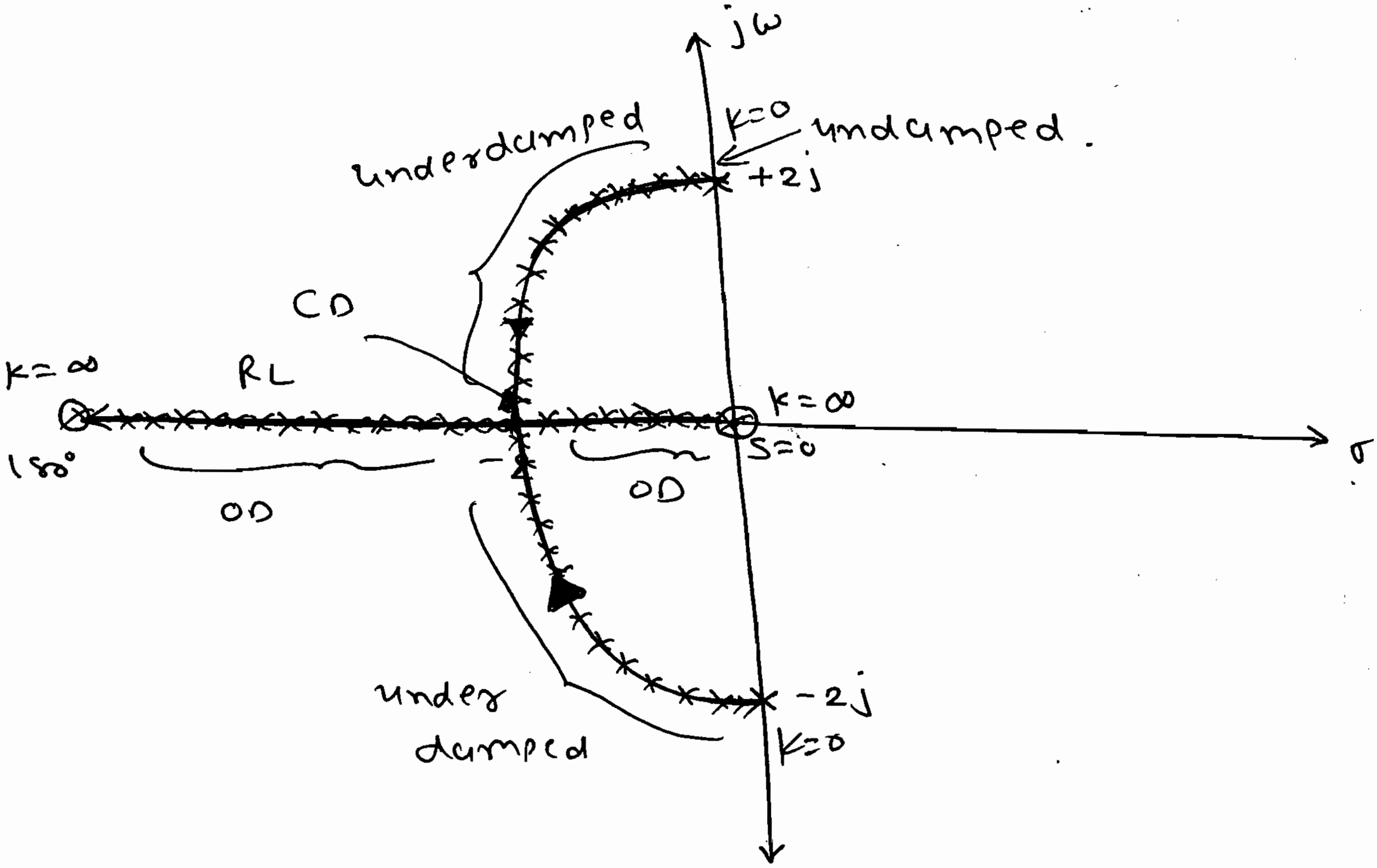
Sol^n:

$P = 2, \quad Z = 1.$

$P - Z = N = 1.$

$\xrightarrow{A.A}$ $\dfrac{(2q+1)180^\circ}{P-Z} = \dfrac{(2q+1)180^\circ}{1} = 180^\circ.$

Note: The centroid is required when the no. of asymptotes are more than 1.



$\xrightarrow{BP.}$ $\quad 1 + GH = 0.$

$$1 + \frac{KS}{S^2+4} = 0 \qquad \Rightarrow \quad \frac{dK}{dS} = -\left[\frac{S(2S) - S^2 - 4}{S^2}\right] = 0.$$

$$K = -\left[\frac{S^2+4}{S}\right] \qquad \Rightarrow \quad 2S^2 - S^2 = 4$$

$$S^2 = 4 \quad \Rightarrow \boxed{S = -2}, \quad \cancel{S = +2}$$

$$\angle GH\Big|_{s=2j} = \frac{\angle K \ \angle 2j}{\angle 0 \ \angle 4j} = 0^\circ$$

$$\phi_d = 180^\circ + \angle GH = 180^\circ$$

$\Rightarrow \xrightarrow{M.C.}$ $$\Big|GH\Big|_{s=-2} = 1.$$

$$\therefore \left|\frac{K(-2)}{4+4}\right| = 1.$$

$$\boxed{K=4}$$

$\Rightarrow$ $K=0 \rightarrow$ undamped.

$0 < K < 4 \rightarrow$ underdamped.

$K = 4 \rightarrow$ critical damped.

$\infty > K > 4 \longrightarrow$ OD.

$\boxed{a}$ $$GH(s) = \frac{K}{s^4-1}.$$

Soln.: Poles: $s^4 - 1 = 0$

$(s^2-1)(s^2+1) = 0$

$\Rightarrow s = \pm 1, \ \pm j.$

$\Rightarrow P - Z = 4 - 0 = 4 = N.$

$\xrightarrow{A.A.}$ $$\theta = \frac{(2q+1)}{4} \times \cancel{180}\ 45^\circ$$

$\theta = 45^\circ, \cancel{90}^\circ, 135^\circ, \cancel{225}, 315^\circ$

Centroid $$\sigma = \frac{(-1+1) - 0}{4} = 0. \Rightarrow \boxed{\sigma = 0}$$

BP $\longrightarrow$ $4s^3 = 0$

$$\boxed{B.P = 0}$$

$\xrightarrow{A.C.}$ $\quad \angle GH \Big|_{s=+j1} = \dfrac{\angle K}{\angle 1+j \; \angle -1+j \; \angle 0 \; \angle 2j}$

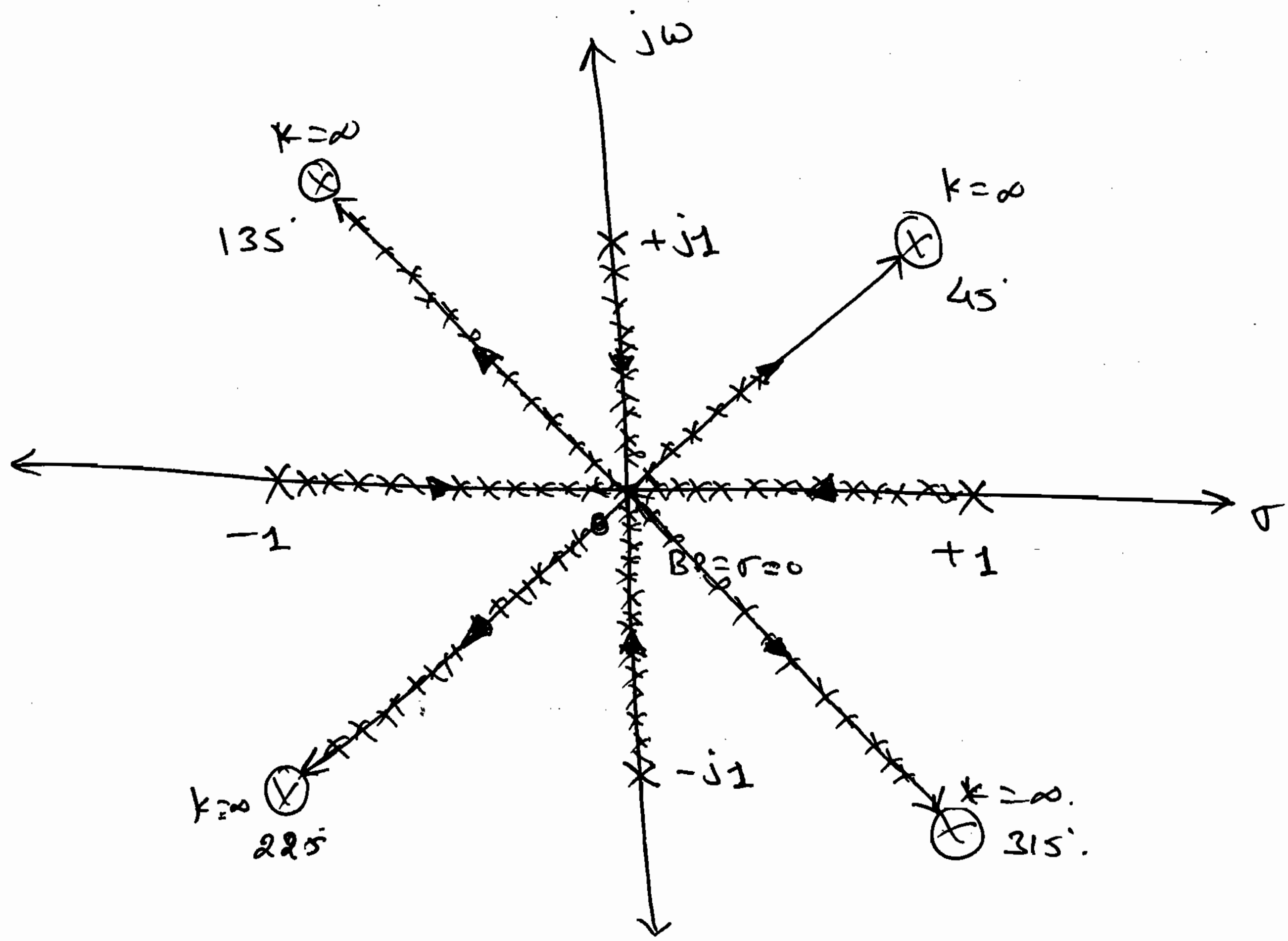
$$= \frac{0^\circ}{45^\circ + 135^\circ + 90^\circ}$$

$$= -270^\circ$$

$\therefore \quad \phi_d = 180^\circ - 270^\circ = -90^\circ.$

$\Rightarrow \quad \boxed{B_P = \sigma \Rightarrow \phi_d = \mp 90^\circ.}$

$\Rightarrow$



$\Rightarrow \quad \xrightarrow{M.C.} \quad \left| \dfrac{K}{0-1} \right| = 1 \Rightarrow \boxed{K=1}$

$\Rightarrow$ Whenever BP = centroid and the RL diagram having complex conjugate poles then the angle of depurture at complex conjugate pole is $\mp 90^\circ$.

$\Rightarrow$ Whenever BP = σ and all the poles symmetrical about BP then all the poles meet at the BP.

$\Rightarrow$ In the above root locus diagram four poles meet at the break point then K value at the BP is $K = 1$.

$\Rightarrow$ The CL TF at the BP is $\frac{C(s)}{R(s)} = \frac{1}{s^4}$.

$\boxed{a}$ $G_H = \dfrac{K(s+2)(s+4)}{(s^2+2s+2)}$.

Soln:

Pole: $s = -1 \pm j$

zero: $s = -2, -4$.

$\Rightarrow$ $N = P - 2 = 0 \Rightarrow \boxed{P = 2}$

Note: whenever no. poles $\leq$ no. of zeros then centroid is not required and angle of asymptotes not exist.

BP. $\longrightarrow$ $K = -\dfrac{(s^2+2s+2)}{(s^2+6s+8)}$

$$\frac{dK}{ds} = -\left[\frac{(s^2+6s+8)(2s+2) - (s^2+2s+2)(2s+6)}{(s^2+6s+8)^2}\right] = 0.$$

$\Rightarrow$ $\cancel{2s^3} + \cancel{12s^2} + \cancel{16s} + \cancel{2s^2} + \cancel{12s} + \cancel{16} - \cancel{2s^3} - \cancel{6s^2} - \cancel{4s^2}$

$\cancel{-12s} \cancel{-12} = 0.$

~~$4s^2 + 16s + 4 = 0$~~

~~$8s^2 + 28s + 4 = 0$~~

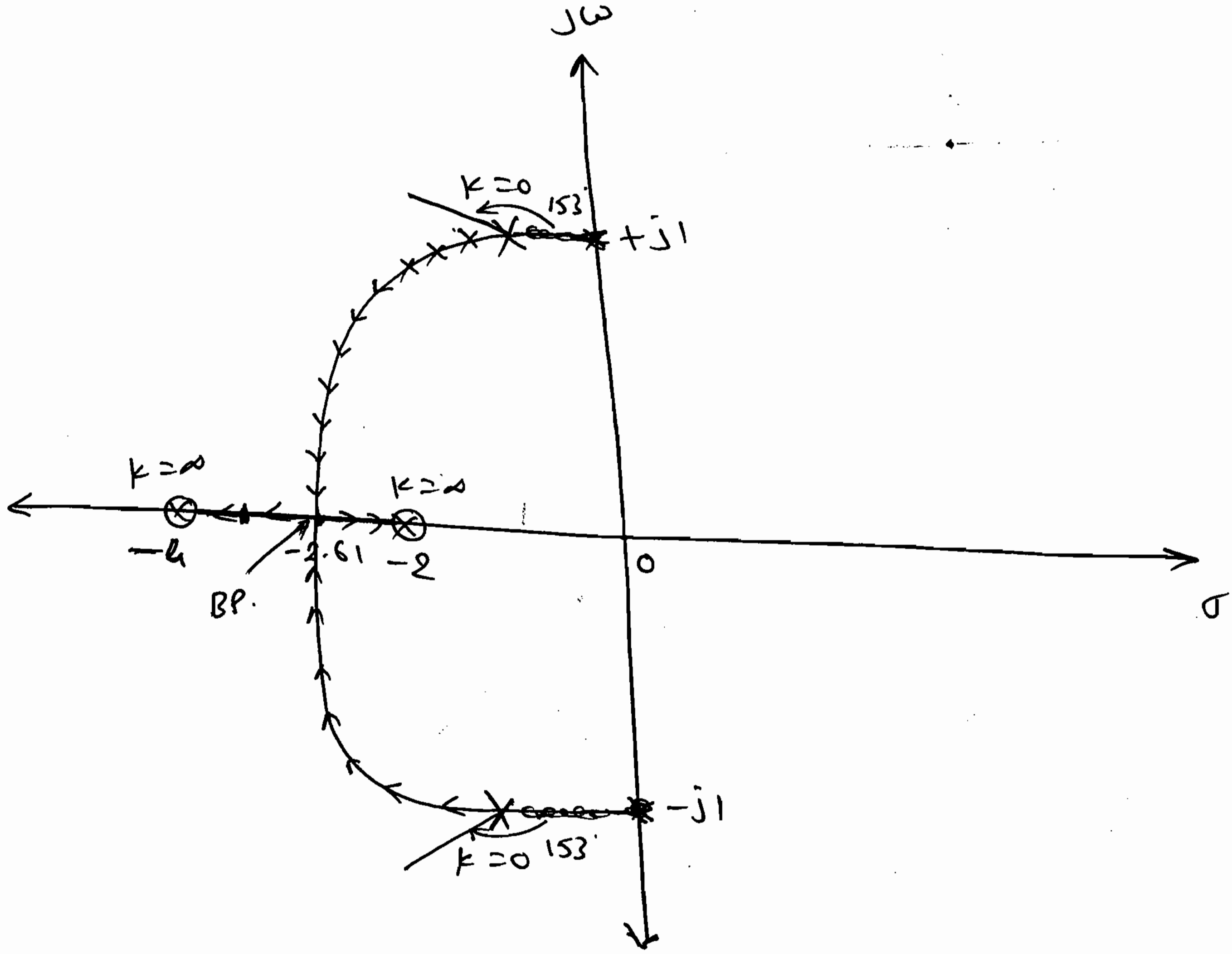
$\Rightarrow \quad \cancel{2}s^3 + 2s^2 + 12s^2 + 1\cancel{2}s + 16s + 16 - 2\cancel{s^3} - 6s^2$
$- 4s^2 - 1\cancel{2}s - 4s - 12 = 0.$

$4s^2 + 12s + 4 = 0$

$s = -0.38, \times$

$s = -2.61$ ✓

$\Rightarrow$



$$\Rightarrow \quad \angle GH\Big|_{s=-1+j} = \frac{\angle K \angle 1+j \angle 3+j}{\angle 0^\circ \angle 2j}$$

$$= \frac{\angle 0^\circ + 45^\circ + 18.43^\circ}{90^\circ}$$

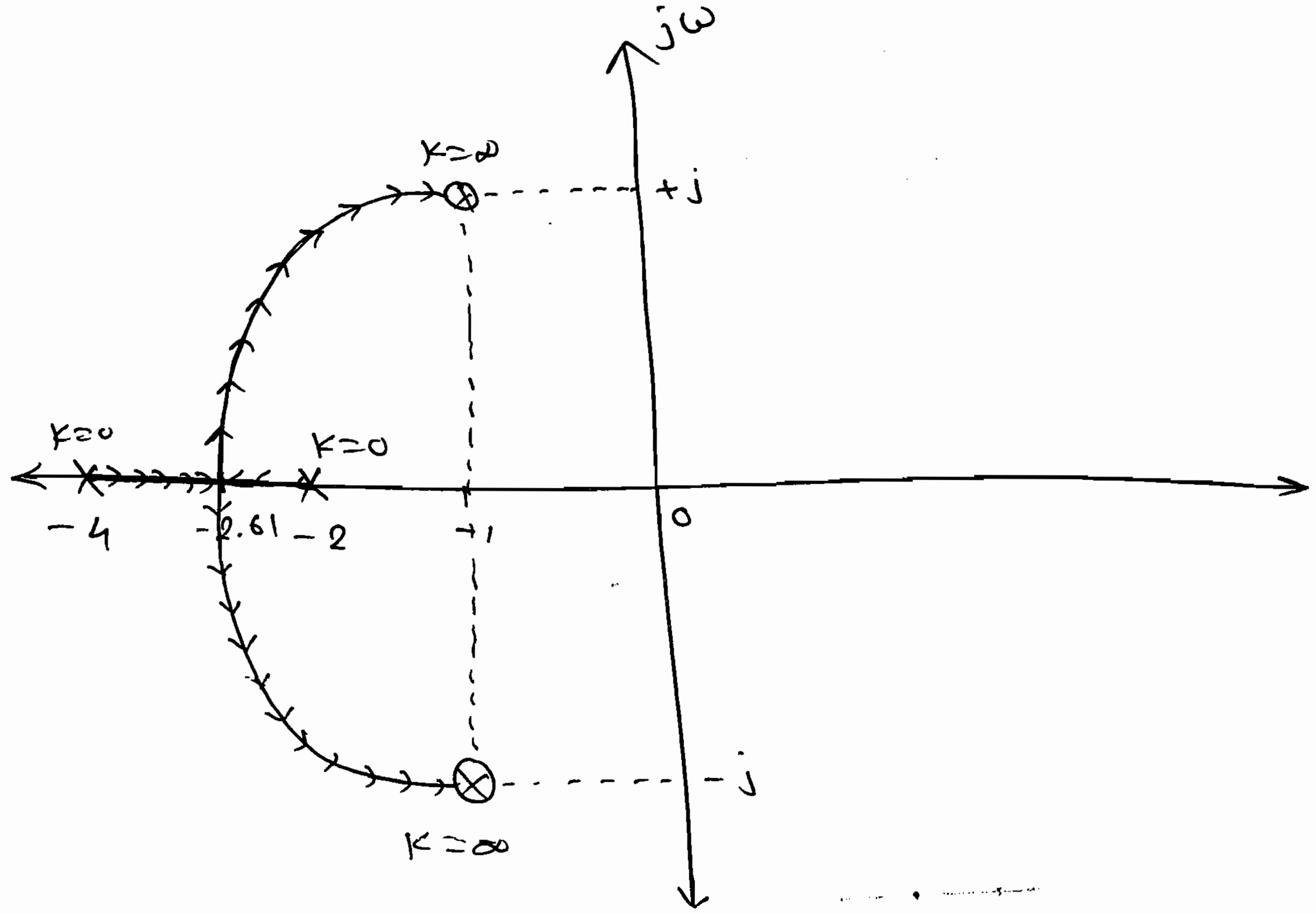
$$= -26.27^\circ$$

$\phi_d = 180^\circ + \angle GH$

$\phi_d = 180 - 26.27^\circ \quad \Rightarrow \quad \boxed{\phi_d = 153.87^\circ}$

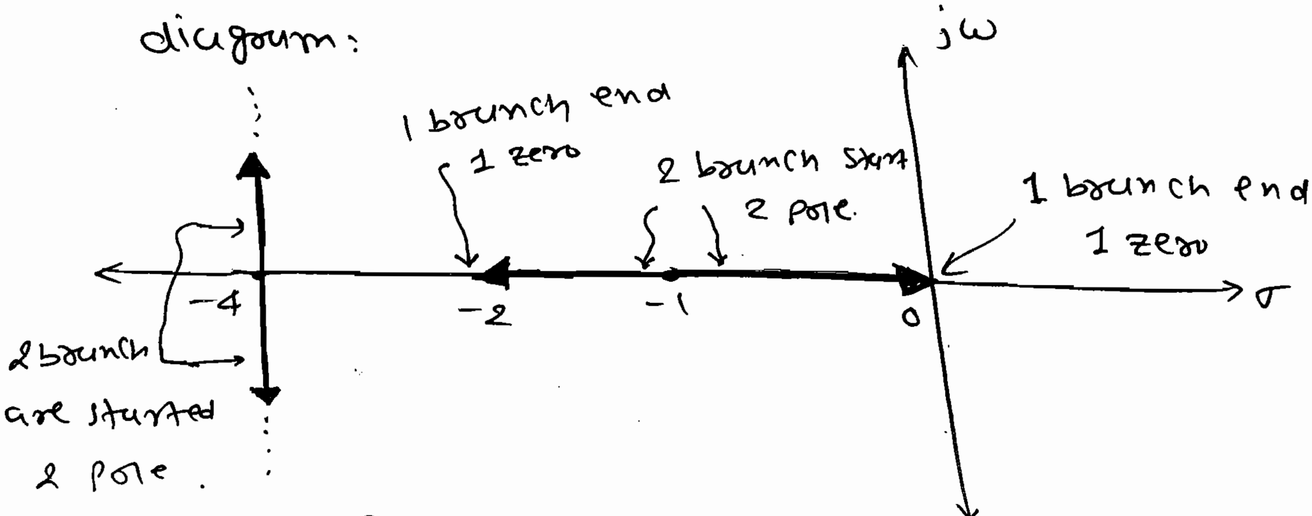
$\boxed{Q}$ $$GH = \frac{K(s^2+2s+2)}{(s+2)(s+4)}.$$

Soln:



Note: The correct Root locus diagram is the one that it should be symmetrical about the real axis and the direction of the root locus branch is from pole to zero when K is increase from 0 to ∞.

$\boxed{Q}$ Find the TF to the given root locus diagram:



$$\Rightarrow \quad \boxed{GH = \frac{K s (s+2)}{(s+1)^2 (s+4)^2}}.$$

Q Draw the Root Locus of the following:

① $$GH = \frac{K}{s(s+1)^2 (s+2)}$$

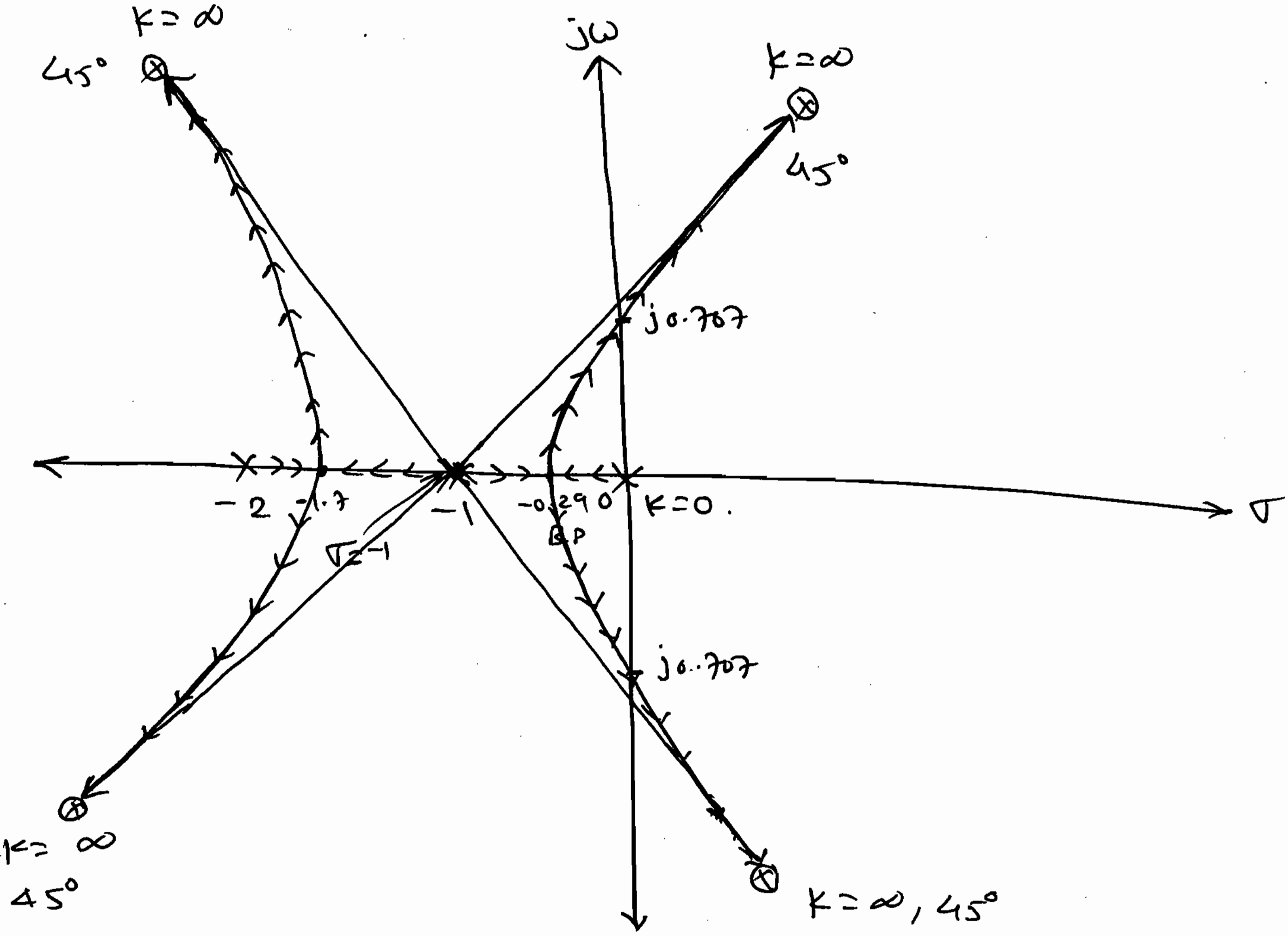
Soln: Poles: $P = 4$, $s=0$, $s=-1,-1$, $s=-2$

Zeros: $Z = 0$

$\Rightarrow \quad P - Z = 4 - 0 = 4, \quad N = 4,$

$\xrightarrow{A.A.} \quad \theta = \frac{(2q+1)}{(P-Z)} \times 180^\circ = \frac{(2q+1)}{4} \times 180$ [45°]

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ.$



$\Rightarrow$ Centroid $$\sigma = \frac{0-1-1-2-0}{4} = -1.$$

B.P → $K = -s(s+2)(s+1)^2.$

$\therefore K = -[(s^2+2s)(s^2+2s+1)].$

$\therefore K = -[s^4 + 2s^3 + s^2 + 2s^3 + 4s^2 + 2s].$

$$\frac{dK}{ds} = -[4s^3 + 12s^2 + 10s + 2] = 0.$$

$\therefore$ B.P → $s = -0.292,\ -1.71,\ -1.$

$\therefore$

$$\begin{array}{c|ccc} s^4 & 1 & 5 & K \\ s^3 & 4 & 2 & \\ s^2 & 9/2 & K & \\ s^1 & \frac{9-4K}{9/2} & & \\ s^0 & K & & \end{array}$$

$\Rightarrow$ $9 - 4K_{mar} = 0$

$K_{mar} = 9/4.$

$\therefore$ AE → $9s^2 + K = 0$

$$\frac{9s^2}{2} = -9/4$$

$$\boxed{s = \pm j1/\sqrt{2}}$$

2) $$GH = \frac{K(s+1)^2}{s(s+2)}.$$

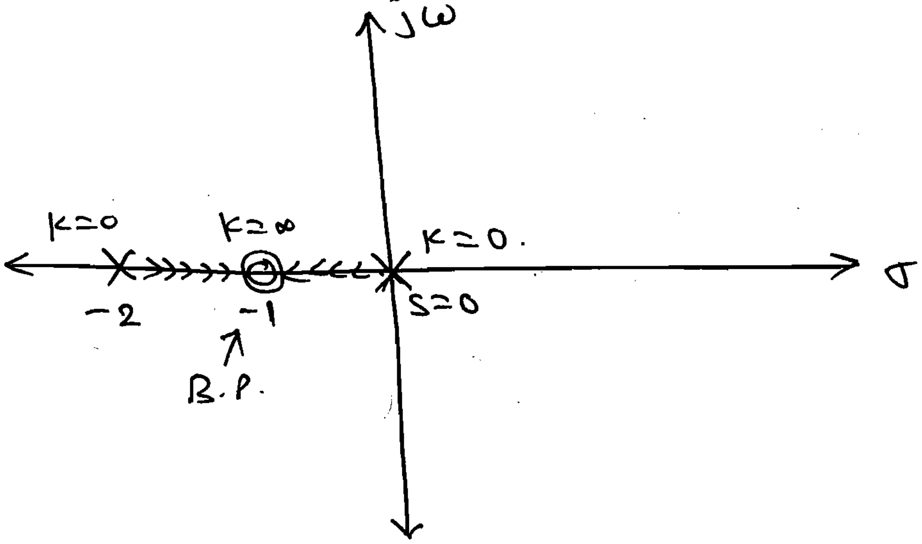
Soln: Poles: $s = -1, -1, 0, -2 \Rightarrow P = 4$

zeros: $s = -1, -1, \Rightarrow Z = 2.$

$P - Z = N = 2 - 2 = 0 \Rightarrow$ No asymptotes.

No centroid.

⇒



⇒ B.P $\longrightarrow$ $K = -\dfrac{(s^2+2s)}{(s^2+2s+1)}$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s^2+2s+1)(2s+2) - (s^2+2s)(2s+2)}{0}\right] = 0.$$

$$\Rightarrow 2s+2=0$$

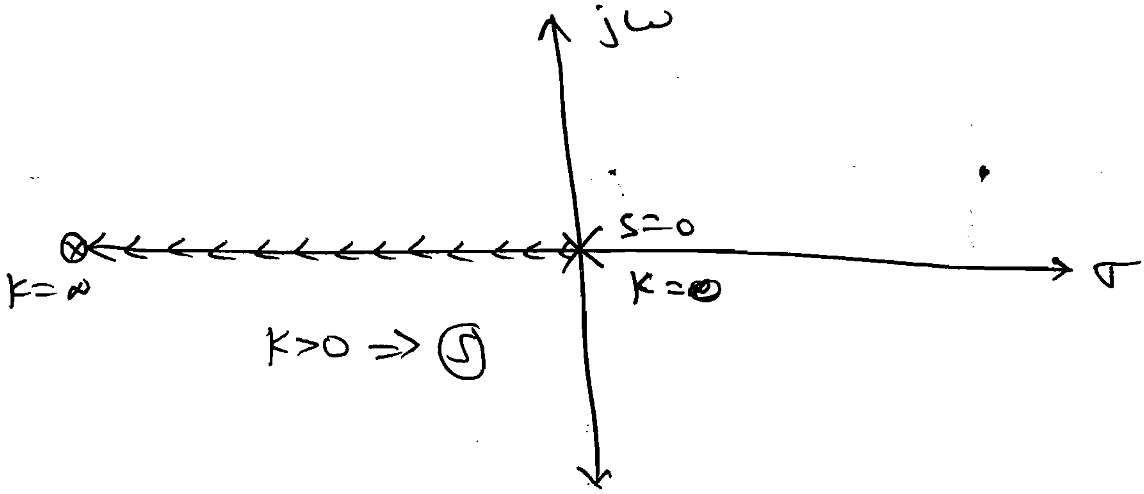
$\boxed{s=-1}$ ← B.P.

(3) $GH = \dfrac{K}{s}$.

<u>Note</u>: → Whenever the transfer function consist only pole at origin then the Root Locus diagram is nothing but angle of asymptotes.

⇒ $P \Rightarrow 1 \Rightarrow s=0.$, $Z \Rightarrow 0.$

$P-Z = N = 1 \Rightarrow \theta = 180^\circ$

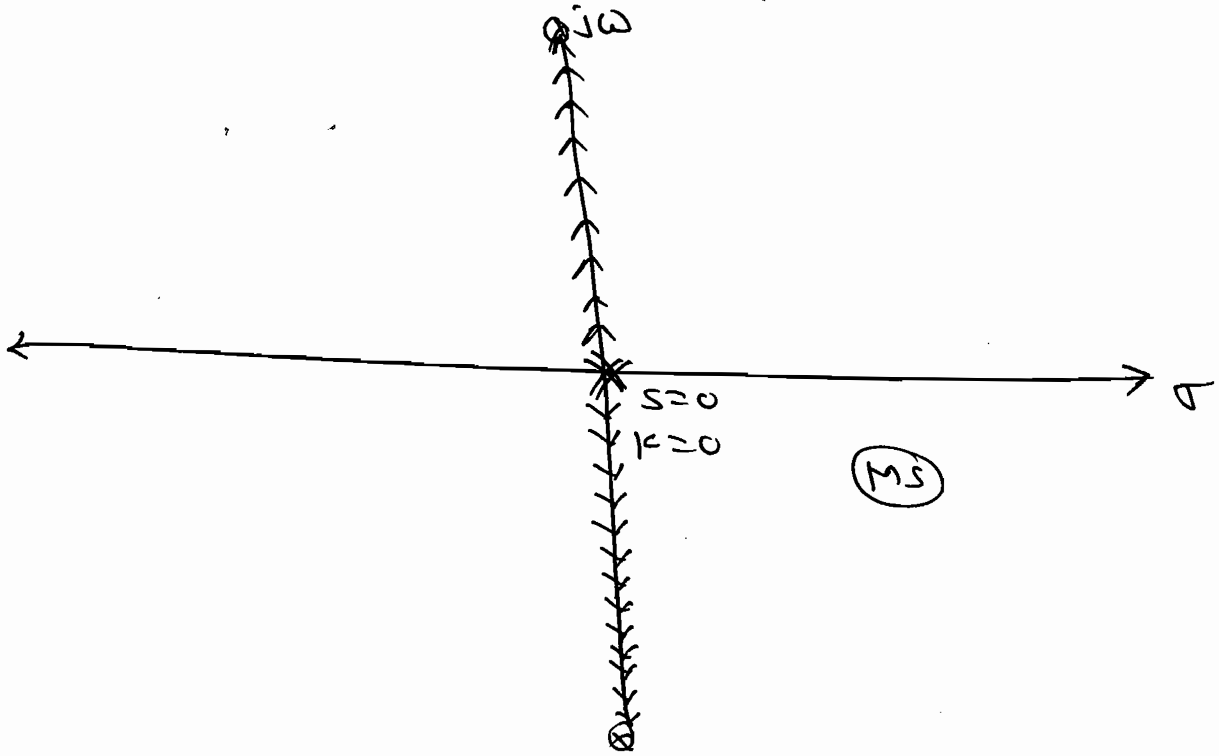


④ $GH = K/s^2$.

Soln: $1 + GH = 0 \Rightarrow s^2 + K = 0 \Rightarrow s = \pm j\sqrt{K}$.

$P = 2,\ Z = 0 \Rightarrow N = P - Z = 2$

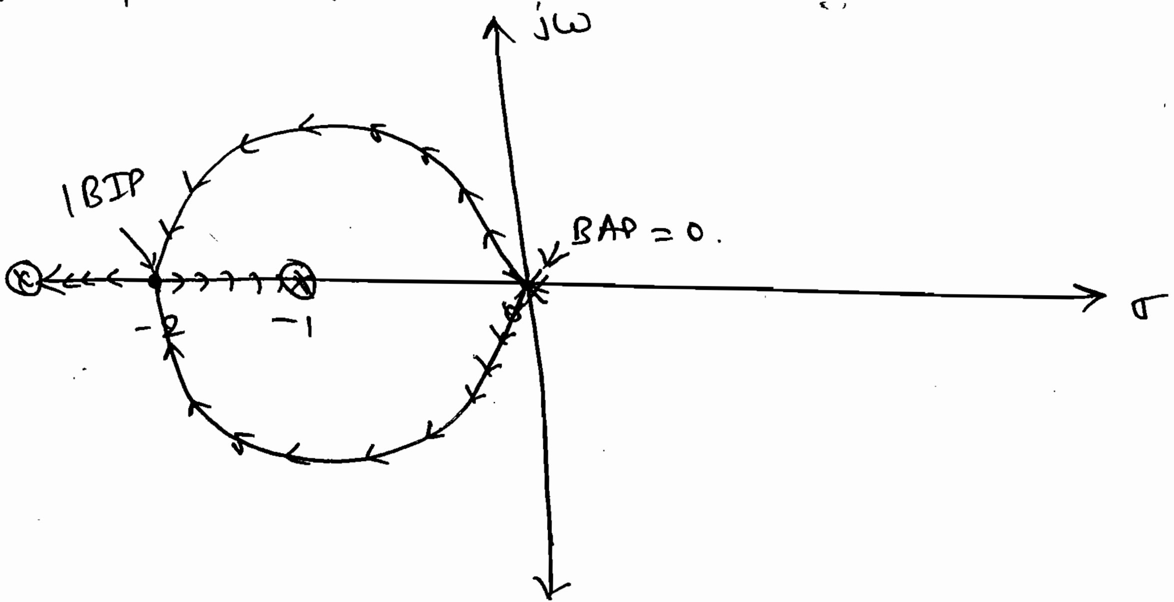
$\xrightarrow{A.A} \theta = 90°, 270°$



Note: The above system is Marginal stable for all values of $k>0$. To make it stable we required to add a finite zero in the left side.

⑤ $\dfrac{K(s+1)}{s^2}$.

Soln: $P = 2,\ Z = 1,\ P - Z = N = 1 \Rightarrow \theta = 180°$.



$\rightarrow$ B.P. $\quad K = -\dfrac{s^2}{(s+1)}$.

$$\frac{dK}{ds} = -\left[\frac{(s+1)(2s) - s^2}{(s+1)^2}\right] = 0.$$

$$\Rightarrow \quad 2s^2 + 2s - s^2 = 0$$

$$s^2 + 2s = 0$$

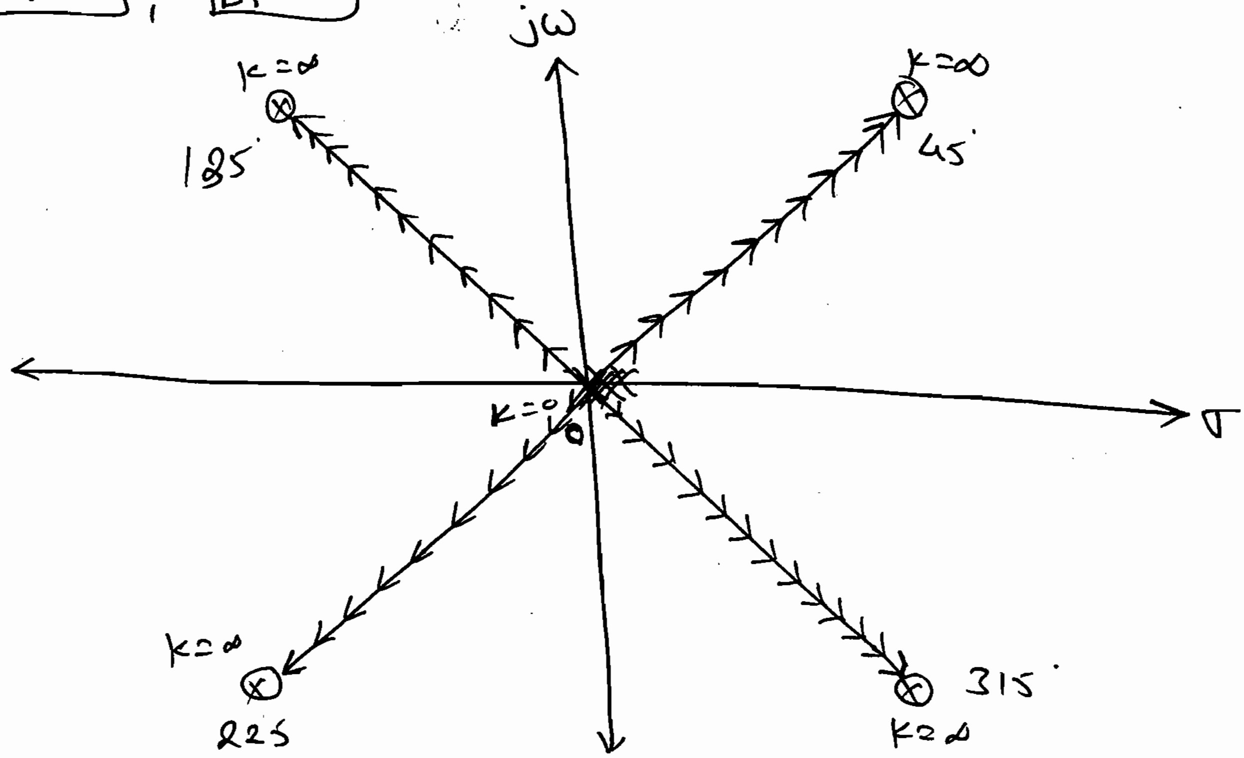
$$\Rightarrow \quad s = 0, -2$$

(6) $GH = \dfrac{K}{s^4}$.

Soln: $P = 4, \quad Z = 0 \quad \Rightarrow \quad P - Z = 4 = N \Rightarrow$

$\xrightarrow{A.A} \theta = 45°, 135°, 225°, 315°.$

$\boxed{\sigma = 0}$, $\boxed{BP = 0}$

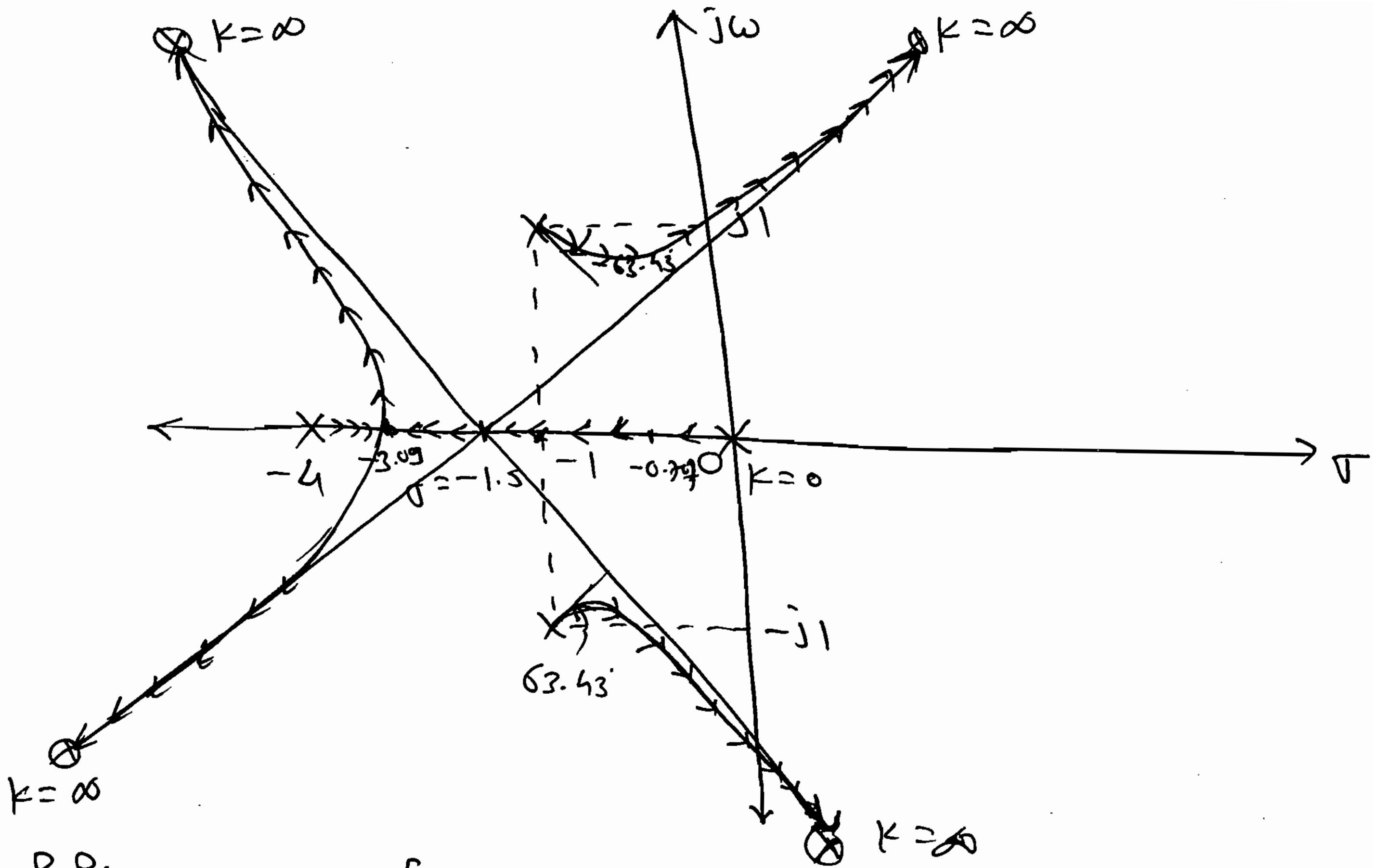


(7) $GH(s) = \dfrac{K}{s(s^2+2s+2)\ (s+4)}$.

Soln: $P = 4, \quad Z = 0 \quad \Rightarrow \quad P - Z = N = 4 \Rightarrow \theta = 45°, 135°, 225°, 315°.$

$$\sigma = \frac{-0 - 1 - 1 - 4}{4}.$$

$$\boxed{\sigma = -1.5}$$



B.P. →

$$K = -\left[(s^2+4s)(s^2+2s+2)\right].$$

$$\therefore K = -\left[s^4 + 2s^3 + 2s^2 + 4s^3 + 8s^2 + 8s\right].$$

$$\therefore K = -\left[s^4 + 6s^3 + 10s^2 + 8s\right].$$

$$\frac{dK}{dK} = -\left[4s^3 + 18s^2 + 20s + 8\right] = 0.$$

$$B.P. = -3.09,\ -0.703,\ -0.703.$$

A.C. →

$$\angle GH\Big|_{s=-1+j} = \frac{\angle K}{\angle -1+j \ \angle 2j \ \angle 3+j}$$

$$= \frac{0^\circ}{135^\circ + 90^\circ + 18.43}$$

$$= -243.43.$$

$$\therefore \ \phi_d = 180 + \angle GH$$

$$\boxed{\phi_d = -63.435^\circ}$$

* Effect of addition of Poles and zeros:

=> The addition of poles and zeros only in the left of s-plane.

① Addition of poles:

=> The root locus branches shifted towards the right of s-plane.

=> The relative stability of system decreases.

=> The range of K value for system stability decreases.

=> The system becomes more oscillatory.

=> The addition of poles decreases the BW (higher cut of freq. decreases).

=> As BW decreases, the rise time increases ~~increases~~ and the system becomes has slow response.

=> The noise is eleminated.

=> The system gives the more accurate o/p.

=> As the root locus moving towards the right side time constant increases and setteling time also increases.

⇒ The damping ratio decreases.

⇒ As ξ decreases, % Mp increases.

② **Addition of Zeros:**

⇒ The root locus branches shifted towards the left side.

⇒ The system become more relative stable.

⇒ The range K value for system stability increases.

⇒ The system become less oscillatory.

⇒ If BW of the system increases (higher cut off freq. increases).

⇒ As BW increases, rise time decreases the system gives the quick response the noise signal enter into the system (less accurate).

⇒ As the root locus branches shifted toward left side, the time constant decreases and settling time also decreases.

⇒ The damping ratio ξ increases, % Mp decreases the system become more relative stable.

⇒ The addition of Pole makes the system

more accurate.

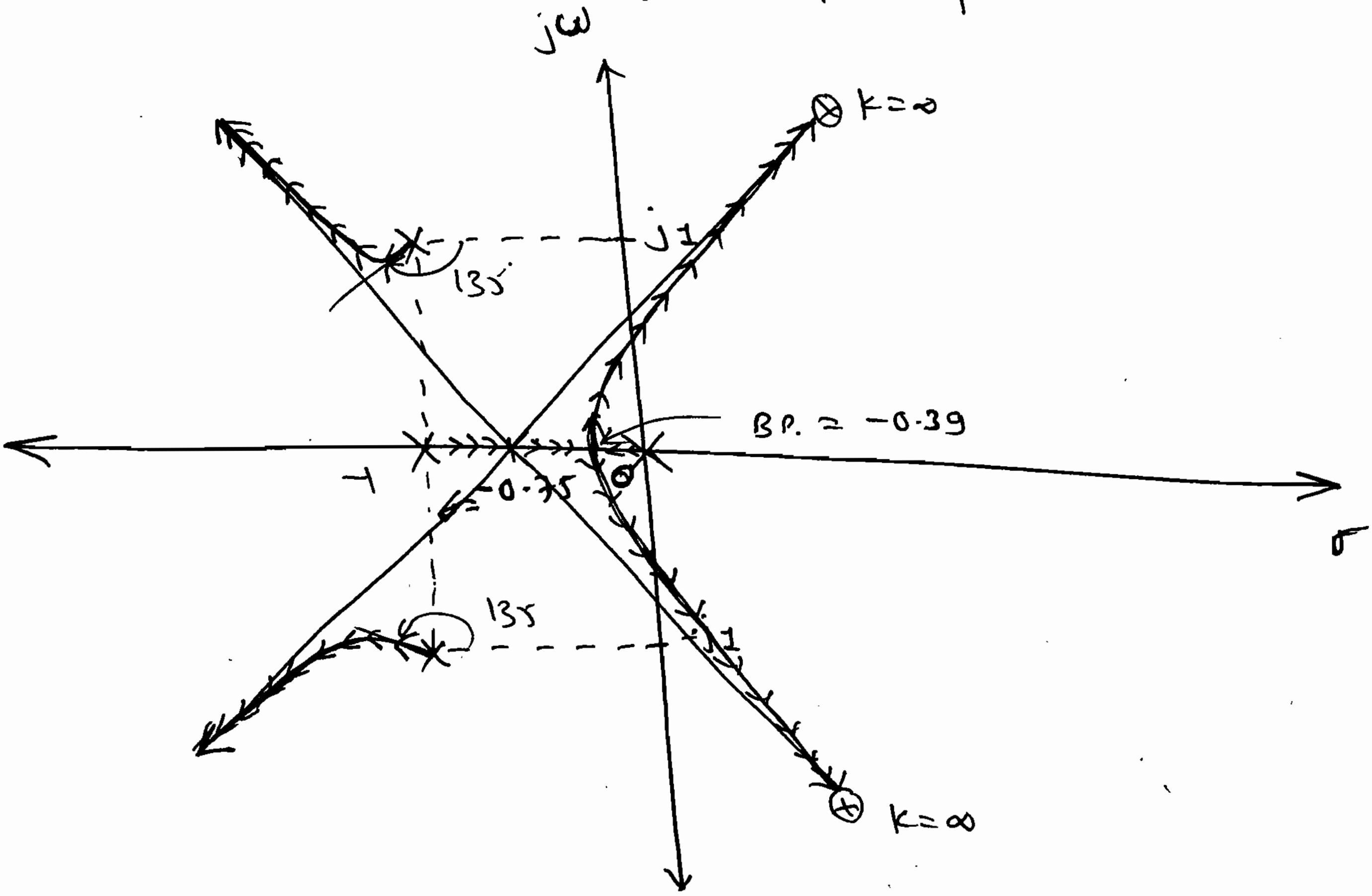
⇒ Addition of zero makes the system quick Response.

$\boxed{8}$ $GH(s) = \dfrac{K}{s(s^2+2s+2)(s+1)}$.

Soln:

$P = 4, \quad z = 0, \quad P - z = 4 - 0 = 4 = N.$

$\theta = 45°, 135°, 225°, 315°.$



$\Rightarrow \sigma = \dfrac{0-1-1-1}{4} = -3/4 = -0.75$

B.P. →

$K = -(s^2+s)(s^2+2s+2).$

$K = -[s^4 + 2s^3 + 2s^2 + s^3 + 2s^2 + 2s].$

$K = -[s^4 + 3s^3 + 4s^2 + 2s].$

$\Rightarrow \dfrac{dK}{ds} = -[4s^3 + 9s^2 + 8s + 2] = 0.$

$\rightarrow$ B.P. $= -0.39, -0.93, -0.93$

$\Rightarrow \xrightarrow{A.C.} \angle GH\Big|_{s=-1+j} = \dfrac{\angle K}{\angle -1+j \cdot \angle 2j \; \angle j}$

$$\angle GH = \frac{0^\circ}{135^\circ + 90^\circ + 90^\circ}$$

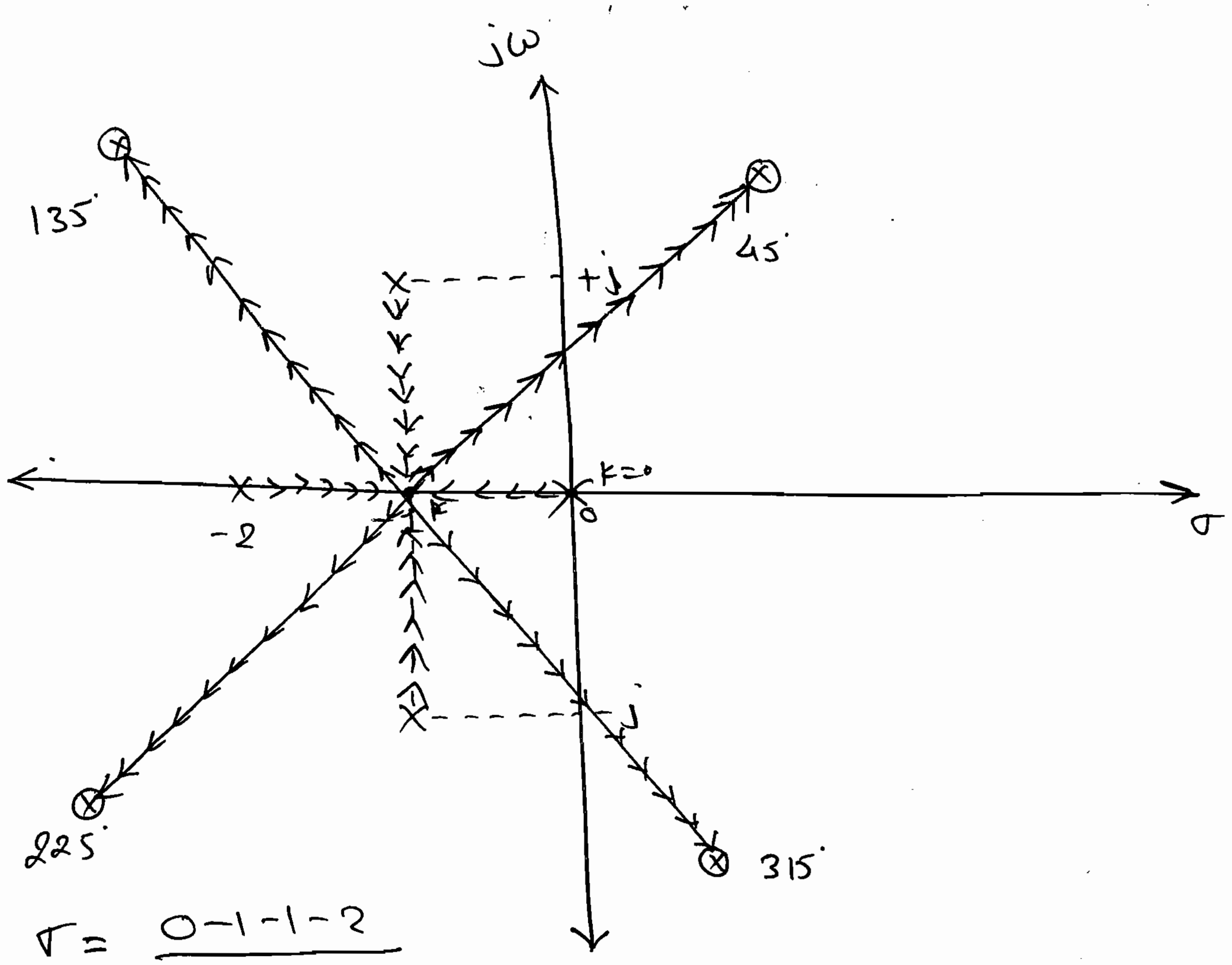
$$= -315^\circ$$

$\therefore \phi_d = 180 + \angle GH = 180 - 315$

$$\boxed{\phi_d = -135^\circ}$$

$\boxed{9}$ $GH(s) = \dfrac{K}{s(s^2+2s+2)(s+2)}$

Soln: $P=4$, $z=0$, $p-z = N = 4 \Rightarrow \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$.



$\Rightarrow \sigma = \dfrac{0-1-1-2}{4}$

$$\boxed{\sigma = -1}$$

Note: If $BP = \sigma$ & all the poles symmetrical about BP then all the poles meet at the BP.

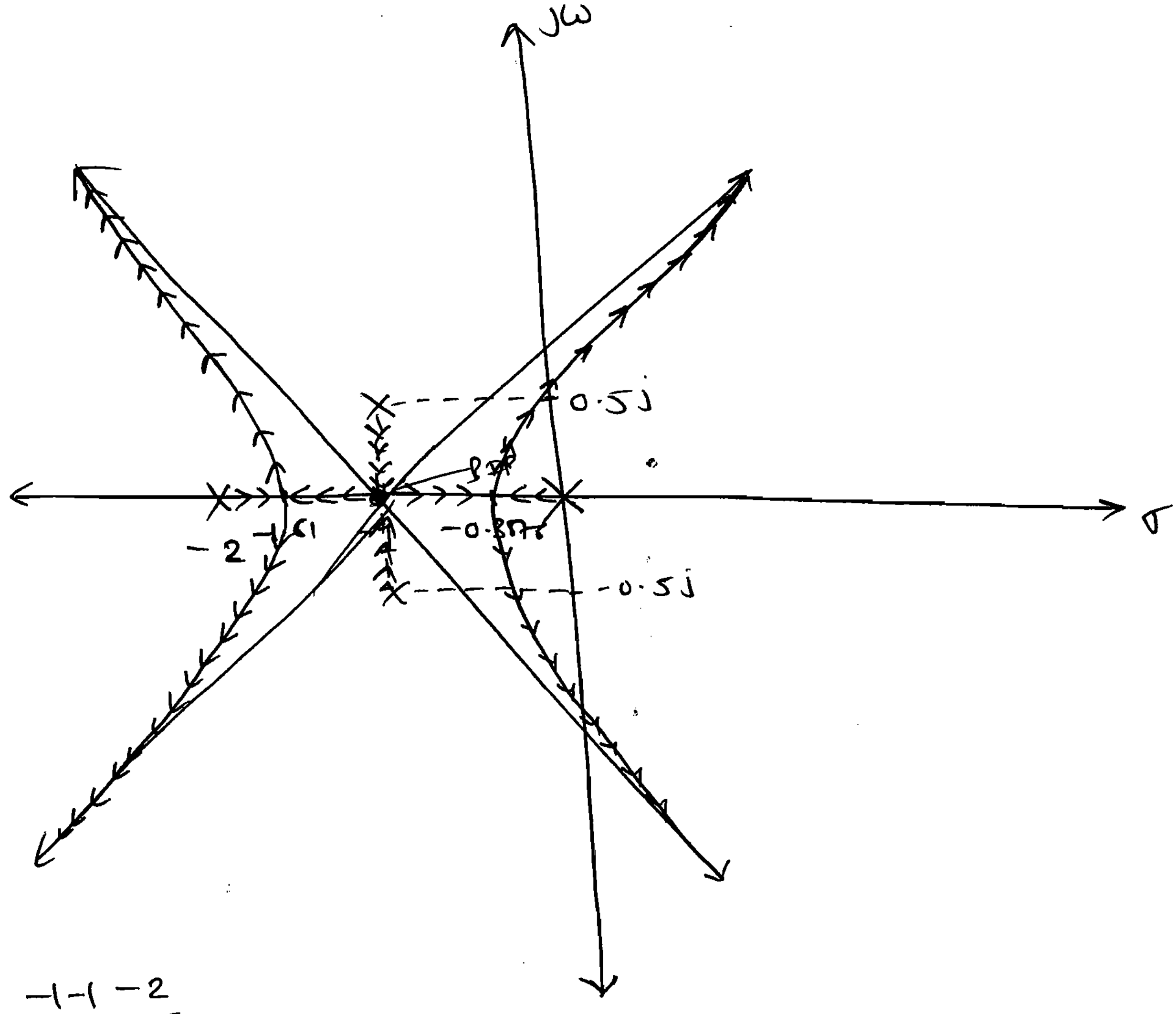
$\Rightarrow$ In the above diagram 4 poles meet at the B.P.

$\Rightarrow$ The K value at the B.P. is $1 \times 1 \times 1 \times 1 = 1$.

The CLTF at the BP is $\frac{1}{(s+1)^4}$.

$\boxed{10}$ $GH = \dfrac{K}{s(s^2+2s+1.25)(s+2)}$.

Sol^n: $P = 4$, $z = 0$, $N = P - z = 4 \Rightarrow \theta = 45°, 135°, 225°, 315°$.



$\sigma = \dfrac{-1-1-2}{4}$

$\boxed{\sigma = -1}$

$$K = -\left[(s^2+2s)(s^2+2s+1.25)\right].$$

$$\therefore K = -\left[s^4 + 2s^3 + 1.25s^2 + 2s^3 + 4s^2 + 2.5s\right].$$

$$K = -\left[s^4 + 4s^3 + 5.25s^2 + 2.5s\right]$$

$$\therefore \frac{dK}{ds} = -\left[4s^3 + 12s^2 + 10.5s^2 + 2.5\right] = 0.$$

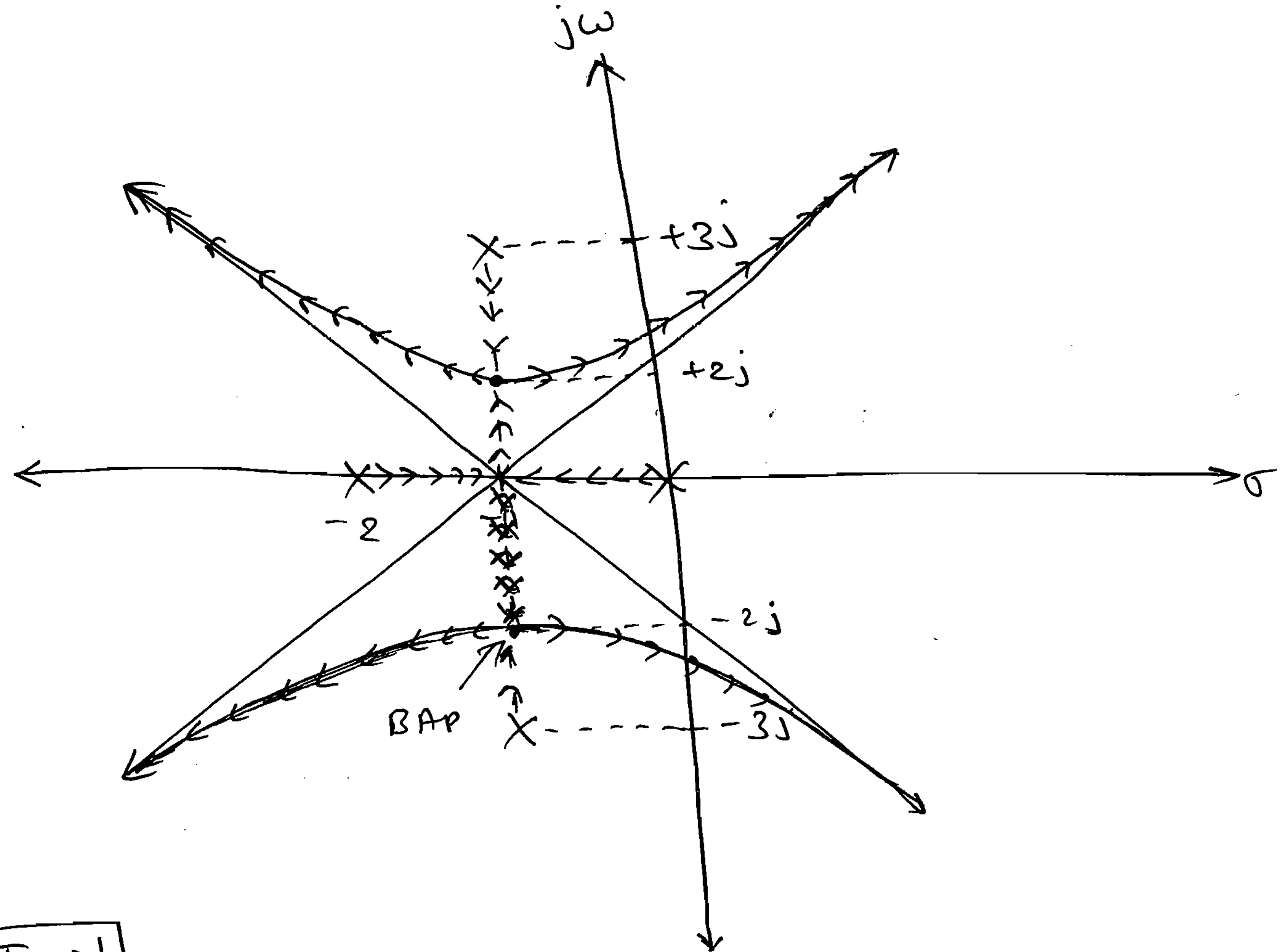
B.P. = −0.3876, −1.61, −1.

↑ BAP (−0.3876), ↑ BAP (−1.61), ↑ BIP (−1)

$\boxed{11}$ $$GH(s) = \frac{K}{s(s^2+2s+10)(s+2)}.$$

Sol^n:

$P=4$, $Z=0$, $P-Z=N=4$, $\theta = 45°, 135°, 225°, 315°$.



$\therefore \boxed{\sigma = -1}$

$$K = -\left[(s^2+2s)(s^2+2s+10)\right].$$

$$\therefore\ k = -\left[s^4 + 2s^3 + 10s^2 + 2s^3 + 4s^2 + 20s \right].$$

$$\therefore\ \frac{dk}{ds} = -\left[4s^3 + 12s^2 + 28s + 20 \right] = 0.$$

B.P. $\longrightarrow$ $s = -1,\ -1 \pm 2i$.

Note: σ = Real part of Complex Pole.

- Symmetric: $\underbrace{\text{Symmetric}}_{\text{1 BP}}$
- Complex poles are very close to Real axis
  - 1 BIP & 2 BAP
  - ||
  - 3 BP's
- Complex poles are far away from Real axis
  - 3 BP's
    - 1 BIP
    - 1 pair of Complex BAP's.

$\boxed{12}$ $$GH(s) = \frac{K}{s(s^2+6s+10)}$$

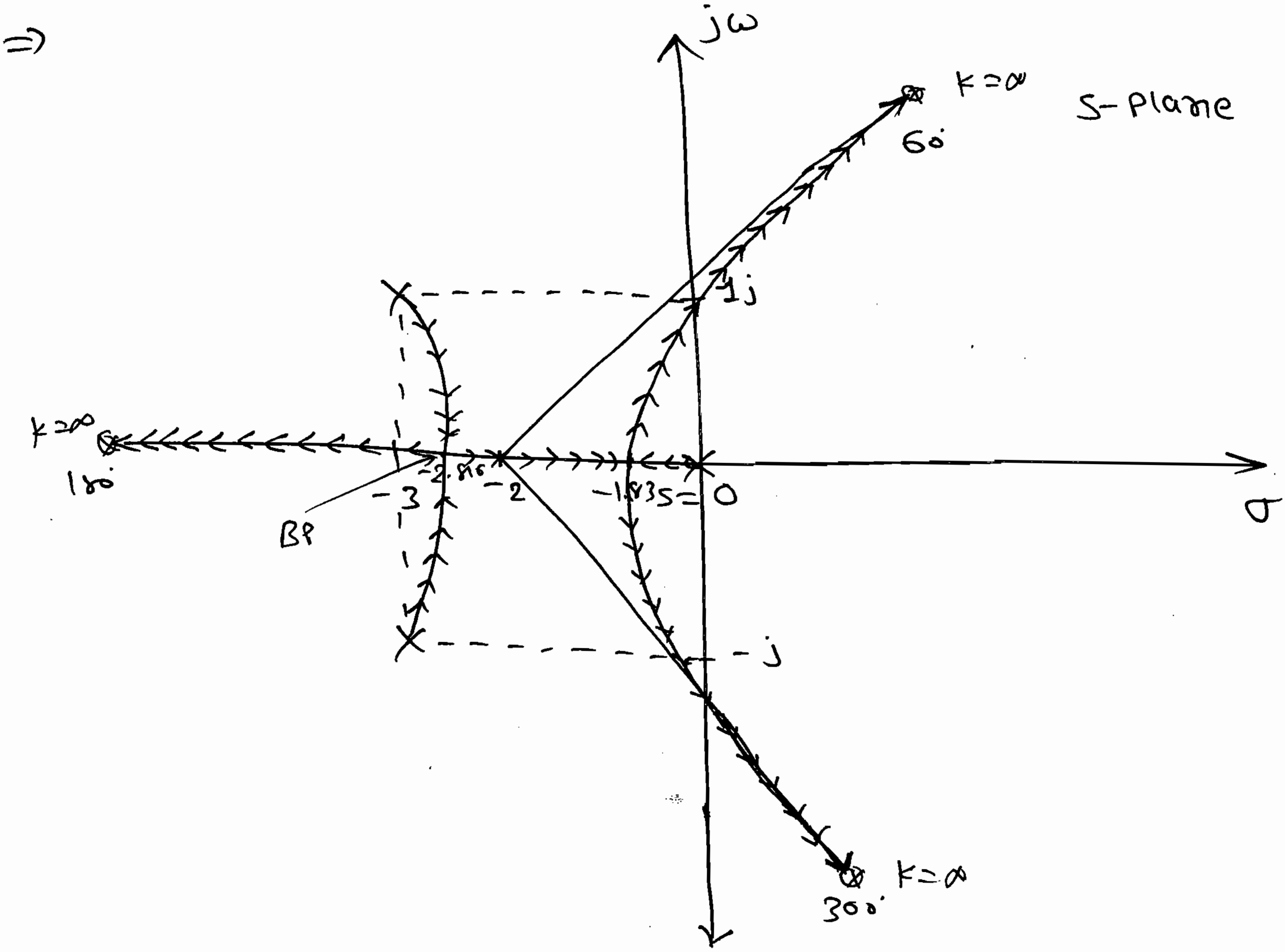
Sol^n: Pole: 3
Zero: 0 $\Rightarrow$ $P - Z = 3$ $\Rightarrow$ $\theta = 60^\circ, 180^\circ, 300^\circ$.

$\rightarrow$ Poles: $s = -3 \pm j1$
$s = 0$.

$\rightarrow$ Centroid: $$\sigma = \frac{-3-3-0-0}{3} = -2$$

$$\boxed{\sigma = -2}$$

$\Rightarrow$



$\Rightarrow \xrightarrow{B.P.}$ $K = -[s^3 + 6s^2 + 10s]$.

$$\frac{dK}{ds} = -[3s^2 + 12s + 10] = 0.$$

$\Rightarrow$ B.P.: $s = -1.183, \; -2.816,$

$\boxed{12}$ $GH = \dfrac{K(s+1)}{s^2(s+K_1)}$.

① $K_1 = 20$. ② $K_1 = 9$.

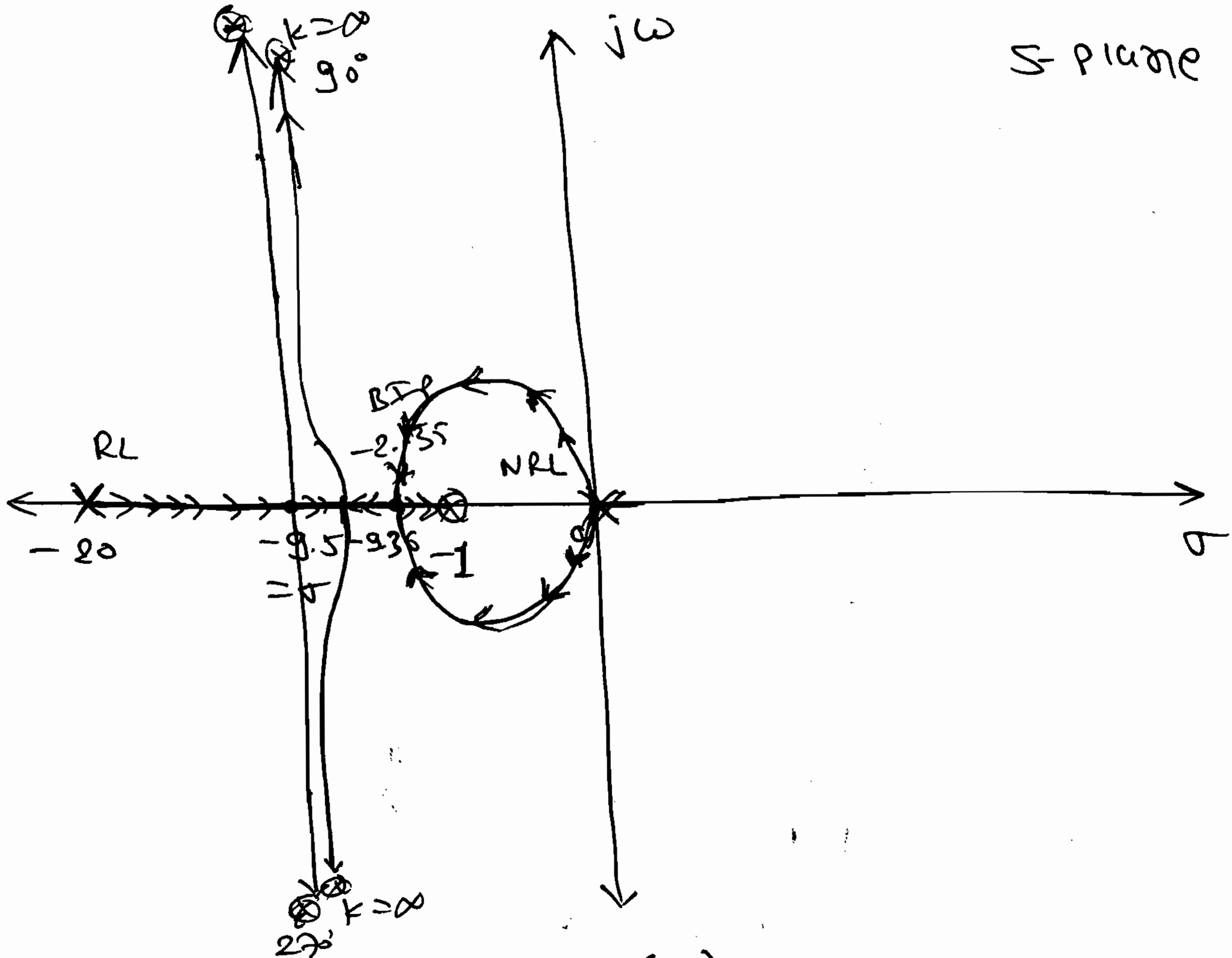
③ $K_1 = 2$. ④ $K_1 = 0.1$.

Sol^n: ① $K_1 = 20$

$\Rightarrow$ $GH = \dfrac{K(s+1)}{s^2(s+20)}$.

$\Rightarrow$ Poles: 3 $\Rightarrow$ $P - Z = N = 3 - 1 = 2$ $\Rightarrow$ $\xrightarrow{A.A}$ $\theta = 90°, 270°$

zeros: 1



$\Rightarrow$ Centroid $\sigma = \dfrac{-20-(-1)}{2} = \dfrac{-19}{2} = -9.5$

$\rightarrow$ B.P $\quad K = -\dfrac{s^2(s+20)}{(s+1)}$

$$K = -\left(\frac{s^3+20s^2}{s+1}\right).$$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+1)(3s^2+40s)-(s^3+20s^2)(1)}{(s+1)^2}\right] = 0$$

$$\Rightarrow 3s^3+40s^2+3s^2+40s-s^3-20s^2=0.$$

$$2s^3+23s^2+40s=0$$

$$s=0,\ -2.135,\ -9.36.$$

So, $\quad s=0 \Rightarrow$ BIP

$s=-9.36 \Rightarrow$ BAP.

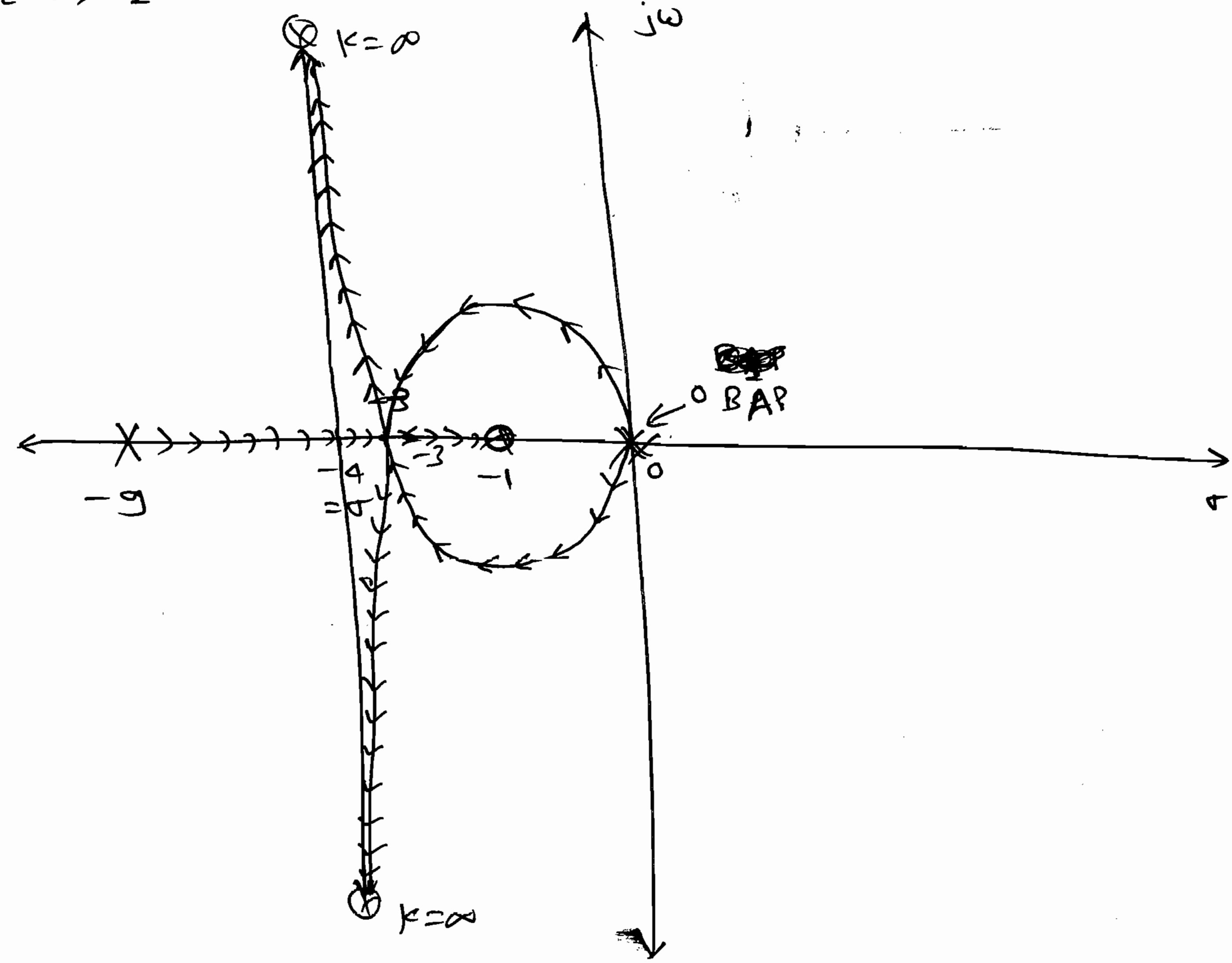
$s=-2.135 \Rightarrow$ BAP.

② $K_1 = 9$.

$$\Rightarrow \quad GH = \frac{K(s+1)}{s^2(s+9)}.$$

$P \Rightarrow 3$

$Z \Rightarrow 1$ $\quad \Rightarrow \quad P - Z = N = 2 \quad \Rightarrow \quad \xrightarrow{A.A} \quad \theta = 90^\circ, 270^\circ.$



$$\Rightarrow \quad \sigma = \frac{-9-0+1}{2} = -4.$$

$$\xrightarrow{B.P.} \quad K = -\frac{(s^3+9s^2)}{(s+1)}.$$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+1)(3s^2+18s) - (s^3+9s^2)(1)}{(s+1)^2} \right] = 0$$

$$\Rightarrow \quad 3s^3 + 18s^2 + 3s^2 + 18s - s^3 + 9s^2 = 0.$$

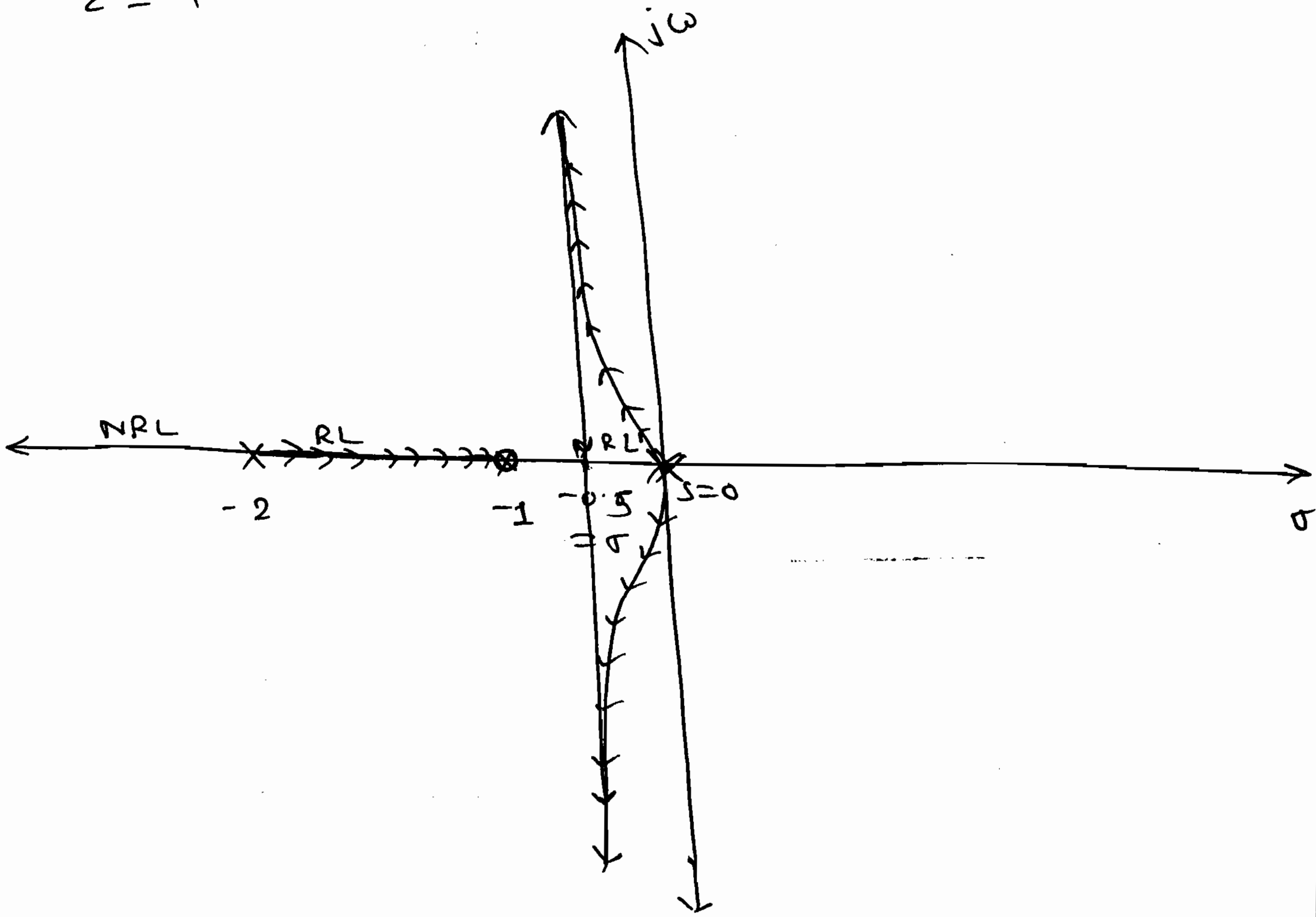
$$\Rightarrow \quad 2s^3 + 12s^2 + 18s = 0.$$

$\xrightarrow{B.P.}$ $s = 0, -3, -3$

(3) $K_1 = 2$

$$\Rightarrow GH = \frac{K(s+1)}{s^2(s+2)}.$$

$\Rightarrow P = 3$, $Z = 1$ $\Rightarrow$ $P - Z = N = 2$ $\Rightarrow$ $\xrightarrow{A.A}$ $\theta = 90°, 270°$.



$$\Rightarrow \sigma = \frac{(-0-0-2)-(-1)}{2} = \frac{-2+1}{2} = -0.5.$$

$\Rightarrow$ $\xrightarrow{BP}$ $K = -\dfrac{(s^3+2s^2)}{(s+1)}.$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+1)(3s^2+4s) - (s^3+2s^2)(1)}{(s+1)^2}\right] = 0.$$

$$\Rightarrow 3s^3 + 4s^2 + 3s^2 + 4s - s^3 - 2s^2 = 0.$$

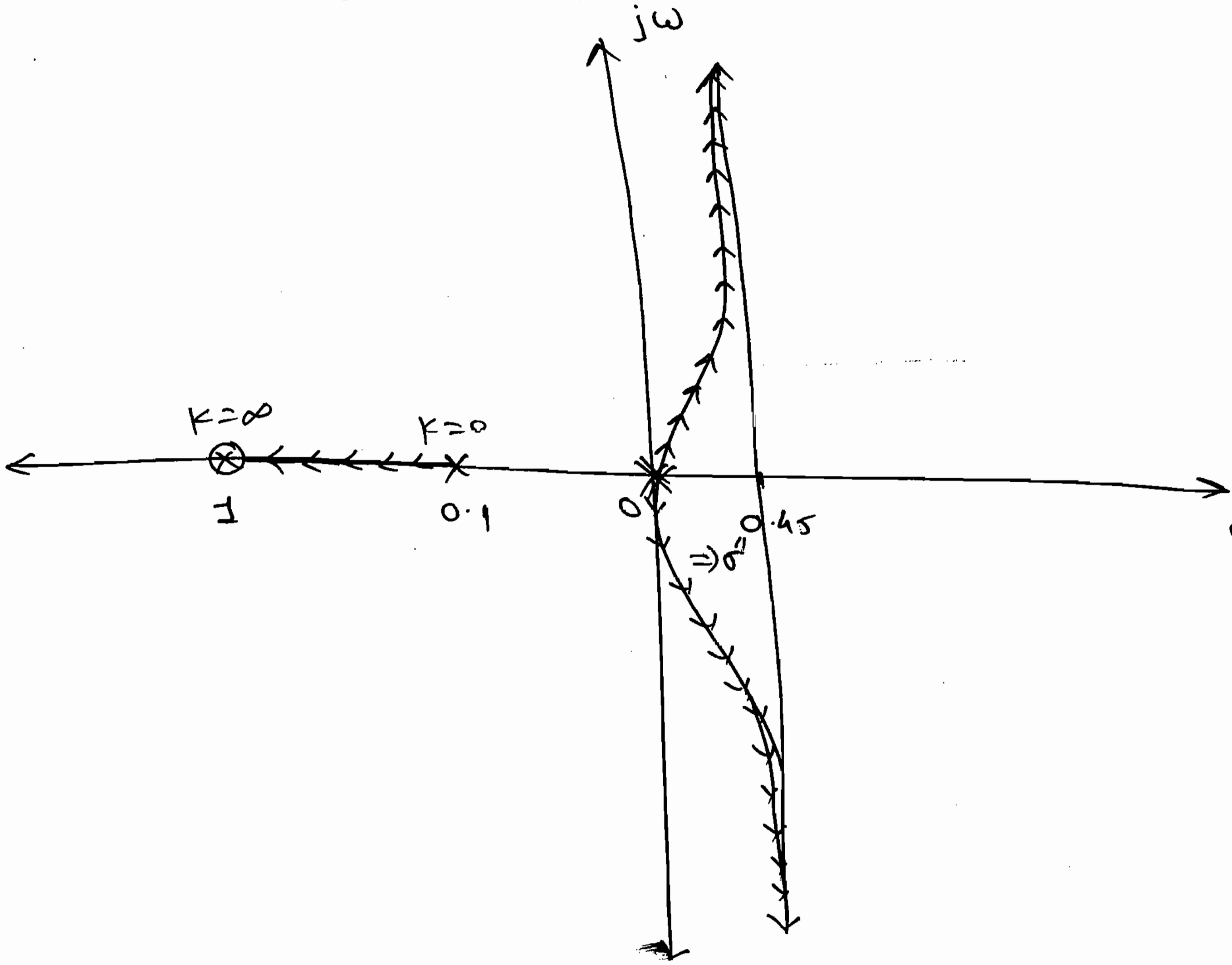
$$2s^3 + 5s^2 + 4s = 0. \Rightarrow s = 0, \cancel{\ \ }, -1.25 + 0.66i$$

$\boxed{4}$ $K_1 = 0.1$

$$\Rightarrow \quad GH = \frac{K(s+1)}{s^2(s+0.1)}.$$

$$\Rightarrow \left.\begin{array}{l} P = 3 \\ z = 1 \end{array}\right\} \Rightarrow P - z = N = 2 \Rightarrow \xrightarrow{A.A.} \theta = 90°, 270°.$$

$$\Rightarrow \sigma = \frac{(-0.1-0-0)-(-1)}{2} = \frac{-0.1+1}{2} = 0.45.$$



$\underrightarrow{B.P.}$ $$K = -\frac{(s^3+0.1s^2)}{(s+1)}.$$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+1)(3s^2+0.2s) - (s^3+0.1s^2)}{(s+1)^2}\right] = 0.$$

$$\Rightarrow 3s^3 + 0.2s^2 + 3s^2 + 0.2s - s^3 - 0.1s^2 = 0.$$

$$2.9s^3 + 3.1s^2 + 0.2s = 0. \Rightarrow s = 0, -1, -0.068$$

Note: Whenever the Complex Poles are very close to real axis the no. of break points on the real axis increases.

$\boxed{Q}$ Draw the root Locus Diagram to the given char. eqn by Considering

1. K as a system gain.
2. a as a system gain.

Soln: $\xrightarrow{CE}$ $s^2 + as + K = 0.$

Note: To draw a root locus diagram in the OLTF the system gain and its product term must be in numerator and remain all should be in denominator.

e.g. $$G_H = \frac{K}{s^2 + s(K+2)+2}.$$

Not in standard form.

$\xrightarrow{CE}$ $1 + G_H(s) = 0.$

$$1 + \frac{K}{s^2 + s(K+2)+2} = 0 \qquad G_H = \frac{K\,N(s)}{D(s)}.$$

$\Rightarrow s^2 + Ks + 2s + 2 + K = 0.$

$\Rightarrow s^2 + K(s+2) + 2s + 2 = 0$ ~~⇒ GH=~~

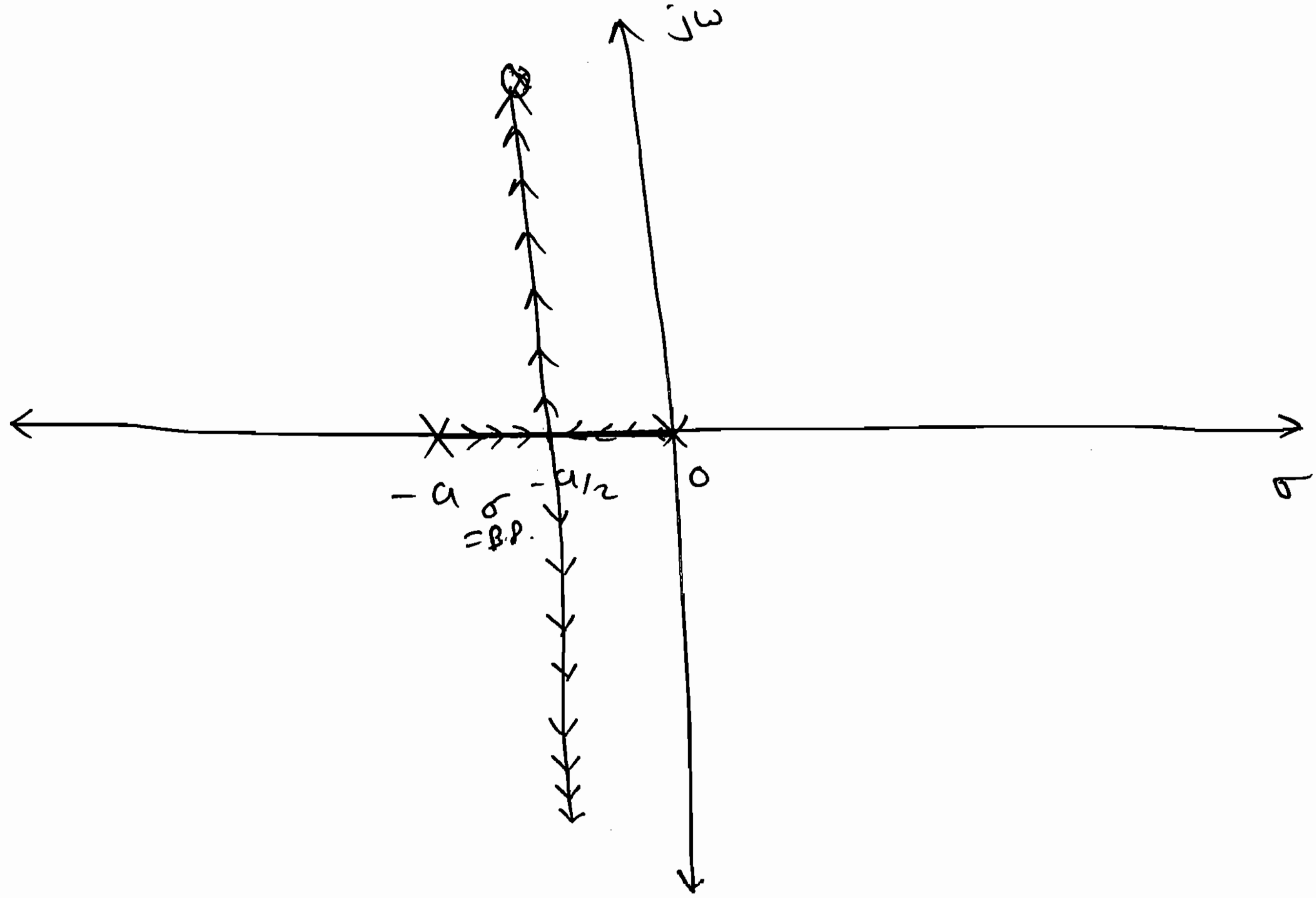
$$\Rightarrow G_H = \frac{K(s+2)}{s^2 + 2s + 2}.$$

case - (i) given

$\xrightarrow{CE}$ $s^2 + as + k = 0.$

$\Rightarrow$ $GH = \dfrac{K}{s^2+as}.$ "k" is system gain.

Poles: 2
zeros: 0 $\Rightarrow$ $N = P - Z = 2 - 0 = 2$ $= \cancel{180°}.\ 90°, 270°$



$\Rightarrow$ $\sigma = \dfrac{-0+a-0}{2} = -a/2$

$\xrightarrow{B.P.}$ $K = -(s^2+as)$

$\dfrac{dK}{ds} = -[2s+a] = 0$

$\xrightarrow{BP}$ $s = -a/2$

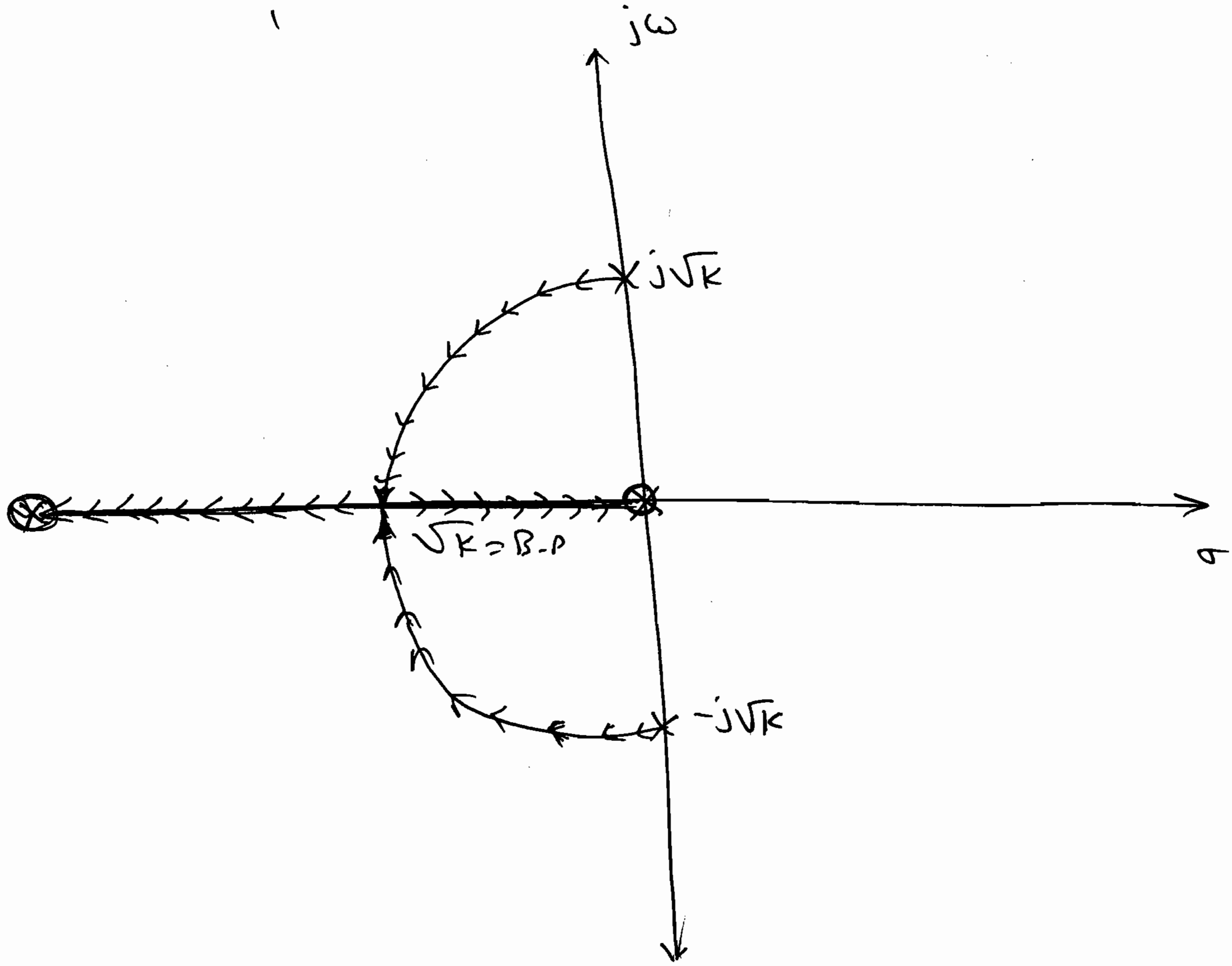
case-(ii) : "a" is system gain.

$\xrightarrow{CE}$ $s^2 + as + k = 0.$

$\Rightarrow$ $GH = \dfrac{as}{s^2+k}.$

$\left.\begin{array}{l} P:\ 2 \\ Z = 1 \end{array}\right\} \Rightarrow \quad P - Z = N = 2 - 1 = 1 \quad \Rightarrow \quad \theta = 180°.$

$\Rightarrow \sigma = \dfrac{0 - (-a)}{1} = a.$



$\xrightarrow{B.P.} \quad a = -\left[\dfrac{s^2 + K}{s}\right].$

$\Rightarrow \quad \dfrac{da}{ds} = -\left[\dfrac{s(2s) - s^2 - K}{s^2}\right] = 0.$

$\Rightarrow \quad 2s^2 - s^2 - K = 0$

$s^2 = K$

$s = \pm \sqrt{K}$

$\xrightarrow{B.P} \quad s = -\sqrt{K}.$ ✓

$s = +\sqrt{K}$ ✗

$\boxed{Q}$ Draw the root-locus for $GH(s) = \frac{Ke^{-s}}{s(s+1)}$.

Sol^n:

Note: To draw a root locus diagram in the transfer f^n the s-term should not have the negative sign.

$$\Rightarrow GH(s) = \frac{Ke^{-s}}{s(s+1)} = \frac{K(1-s)}{s(s+1)} = \frac{-K(s-1)}{s(s+1)}.$$

By default -ve FB. | +ve FB

CE $\rightarrow$ $1 + GH(s) = 0$

$$1 - \frac{K(s-1)}{s(s+1)} = 0.$$

IRL

$$\angle \frac{K(s-1)}{s(s+1)} = \angle(1+j0)$$

$$= 0^\circ \text{ (IRL)}$$

$\longrightarrow$ CE $1 - GH(s) = 0$

$$1 + \frac{K(s-1)}{s(s+1)} = 0$$

DRL

$$\angle \frac{K(s-1)}{s(s+1)} = \angle -1 + j0$$

$$= 180^\circ \text{ (DRL).}$$

$\Rightarrow$ By default,

$K\uparrow$ (0 to ∞).

$\times \longrightarrow \bigcirc$

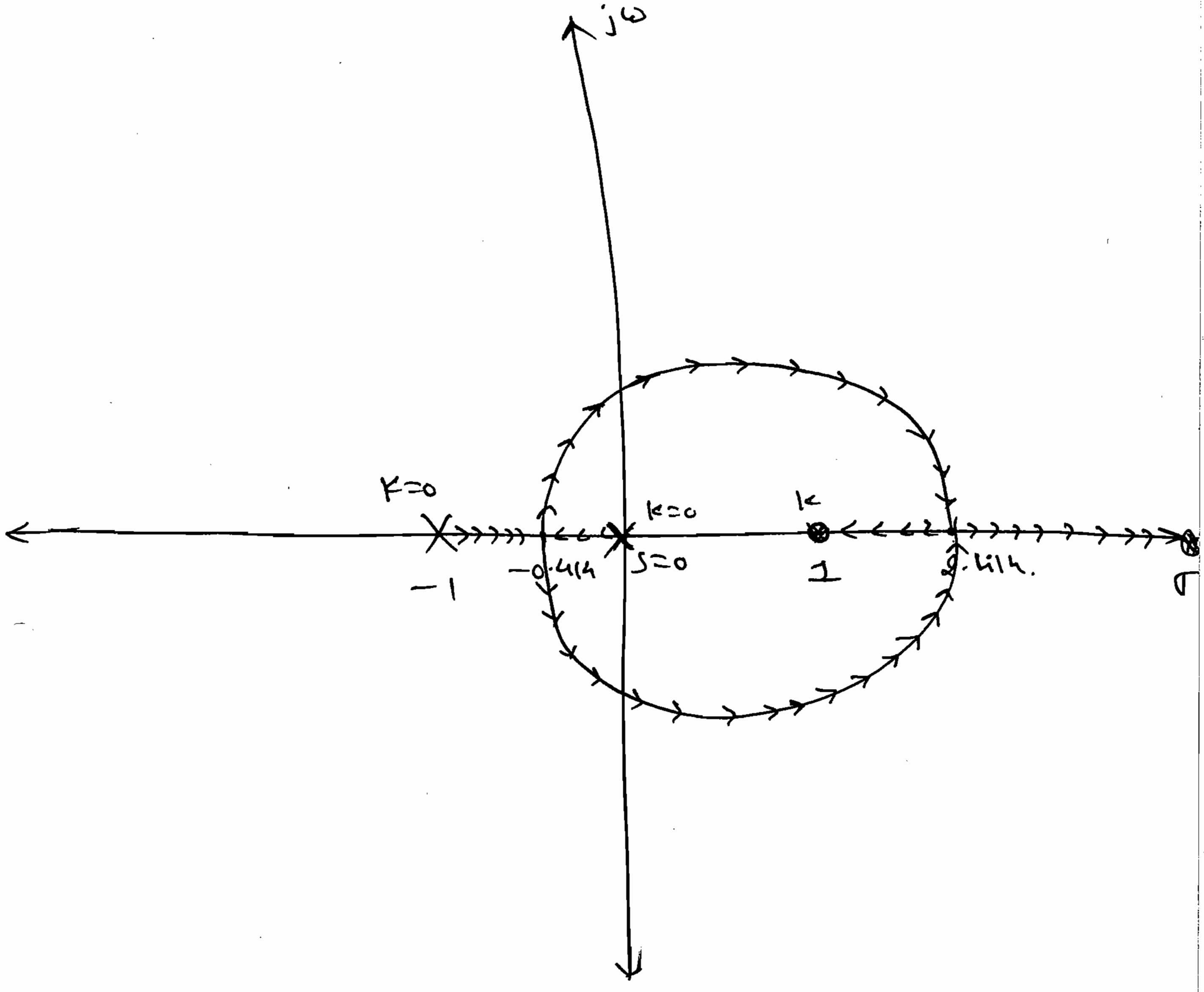
$K = 0$ $\quad$ $K = \infty$.

Now, Poles : 2, Zeros : 1 $\Big\} \Rightarrow N = P - Z = 2 - 1 = 1$, $\Rightarrow \theta = \cancel{180}\ 0^\circ$.

$$\frac{2q}{(P-Z)} \times 180^\circ.$$

$$\sigma = \frac{0 - 1 - (1)}{1}.$$

$$\sigma = -2.$$



$\Rightarrow \underrightarrow{BP} \quad k = \dfrac{s(s+1)}{+(s-1)}$.

$$\frac{dk}{ds} = \frac{(s-1)(2s+1) - s^2 - s}{(s-1)^2} = 0.$$

$\therefore +2s^2 \cancel{\pm s} \mp 2s \mp 1 - s^2 - \cancel{s} = 0.$

$\cancel{2s^2 + s - 2 = 0}$. $\qquad s^2 - 2s - 1 = 0$.

$\cancel{s = -1, 2}$. $\qquad s = 2.414, \; -0.4142$

$\qquad\qquad\qquad \uparrow \qquad\qquad \uparrow$

$\qquad\qquad\qquad$ BEP $\qquad\quad$ BAP.

* <u>Verification Process to Select Ans.</u>

$\xrightarrow{CE} 1 + GH = 0$

$$1 + \frac{K(1-s)}{s(s+1)} = 0.$$

$$CE \rightarrow s^2+s+K-Ks=0.$$

$$K=0 \rightarrow s^2+s=0 \Rightarrow s=0,-1.$$

$$K=1 \rightarrow s^2+s+1-s=0 \Rightarrow s=\pm j1.$$

$$K=2 \rightarrow s^2+s+2-2s=0.$$

$$s^2-s+2=0 \Rightarrow s=1\pm j1.3.$$

Note: For a complete Root locus the range of K value is from $-\infty$ to $+\infty$.

Q. Draw the complete root locus the range of $K=-\infty$ to $+\infty$.

$$GH(s)=\frac{K}{s(s+2)}.$$

Soln:

By default

$$CE \rightarrow 1+GH=0$$

IRL $\rightarrow$ $-\infty<K<0 \Rightarrow$ O→→→→X ($K=-\infty$, $K=0$)

$$1-\frac{K}{s(s+2)}=0$$

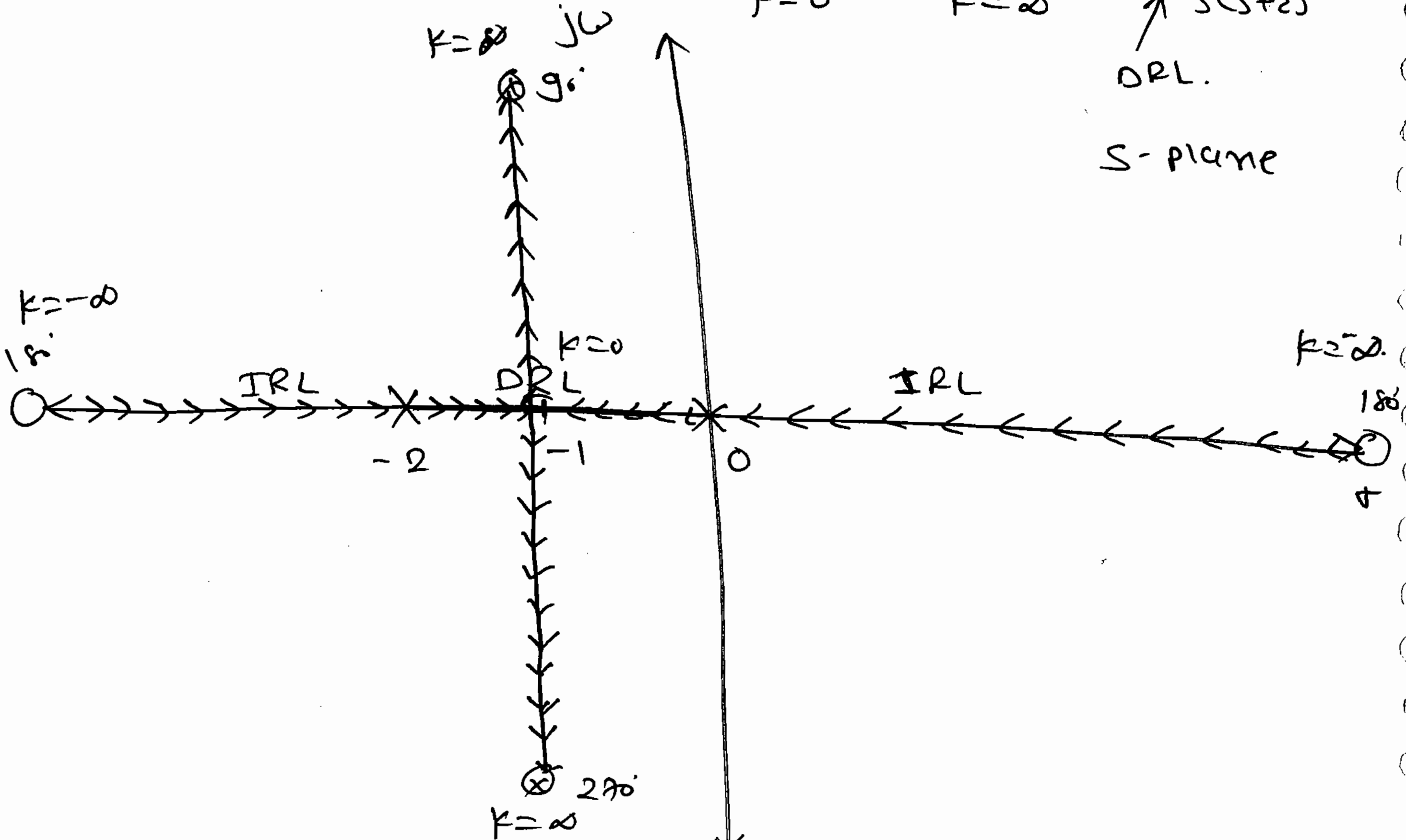
IRL

DRL $\rightarrow$ $0<K<\infty \Rightarrow$ X→→→→O ($K=0$, $K=\infty$)

$$1+\frac{K}{s(s+2)}=0$$

DRL.

S-plane



$$\Rightarrow \quad \sigma = \frac{-2-0-0}{P-Z}$$

$$= \frac{-2-0-0}{2-0}$$

$$\boxed{\sigma = -1}$$

$$\left.\begin{array}{l} P: 2 \\ Z = 0 \end{array}\right\} \Rightarrow \quad N = P-Z = 2.$$

$$\theta = \frac{(2q)180^\circ}{P-Z}$$

$$= \frac{\cancel{2}q \times 180^\circ}{\cancel{2}}$$

$$\therefore \boxed{\theta = 0^\circ, 180^\circ}$$

$$BP \longrightarrow \quad K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2$$

$$\Rightarrow \boxed{s = -1}$$

☆ Bode Plot:

⇒ Purpose:

⇒ To draw the frequency response of OLTF.

⇒ To find the closed loop system stability.

⇒ To find the Gain margin, phase margin, gain cross over frequency, phase cross over frequency.

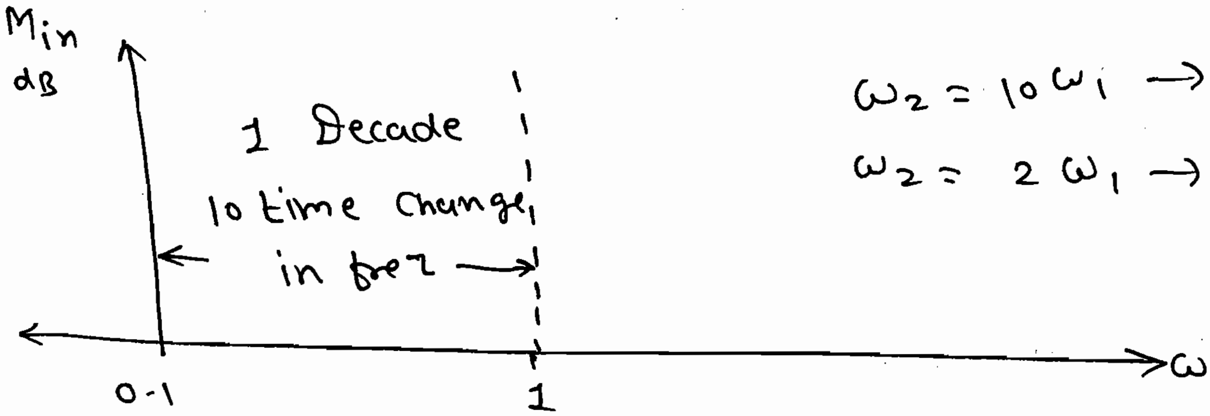
⇒ To find the relative stability by using GM & PM. If the GM & PM is very very large then the system is more relative stable. but the system response is slow. If GM & PM is very small, ~~and more osc~~ then the system become less relative stable and more oscillatory.

⇒ The optimum range of GM is 5 to 10 dB & PM is 30° to 40°.

⇒ The Bode plot consist the two plots. one is the magnitude plot and other is phase plot.

⇒ Bode Plot → (i) Magnitude plot.
→ (ii) Phase plot.

⇒ Magnitude Plot:



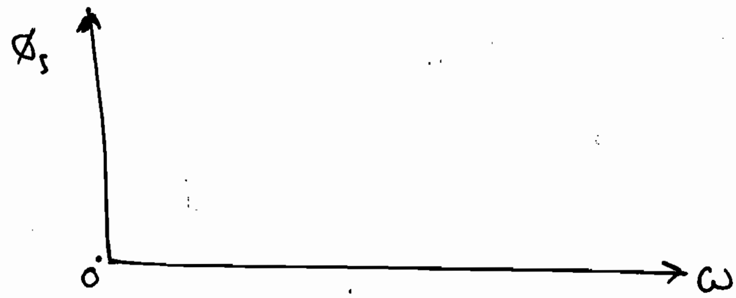
$\omega_2 = 10\,\omega_1 \rightarrow$ decade

$\omega_2 = 2\,\omega_1 \rightarrow$ octave.

⇒ $20\log 10 \longleftrightarrow 20\log 2$

20 dB | decade ⟷ 6 dB | octave.

⇒ Phase plot:



* Procedure to draw the Bode Plot:

1) S is replaced by jω to convert it to freq. domain.

2) Write the magnitude and convert into (dB).

⇒ The Mag. in dB $M_{dB} = 20\log|GH(j\omega)|$.

3) Find the phase angle using;

$$\phi = \tan^{-1}\left(\frac{I_P}{R_P}\right).$$

4) Vary the 'ω' from min to max and draw the magnitude and phase plot approximately.

Q Draw the Bode plot for $G(s)\cdot H(s) = K$.

Soln:

$$G(s).H(s) = K$$

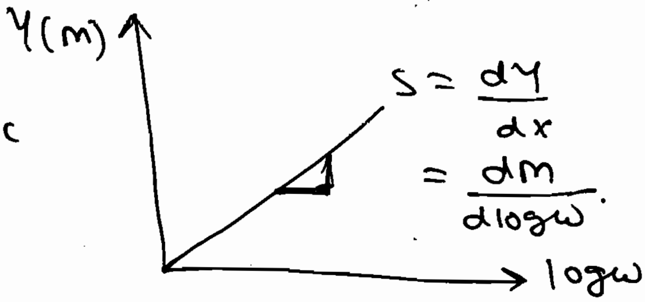
$$\Rightarrow \quad s \rightarrow j\omega$$

$$GH(j\omega) = K.$$

$$|GH(j\omega)| = K$$

$$M_{in\ dB} = 20\log|GH(j\omega)| = \boxed{20\log K}$$

$$\Rightarrow \text{Slope} = \frac{dm}{d\log\omega} = 0\ \text{dB/dec}$$



$$S = \frac{dY}{dx} = \frac{dm}{d\log\omega}.$$

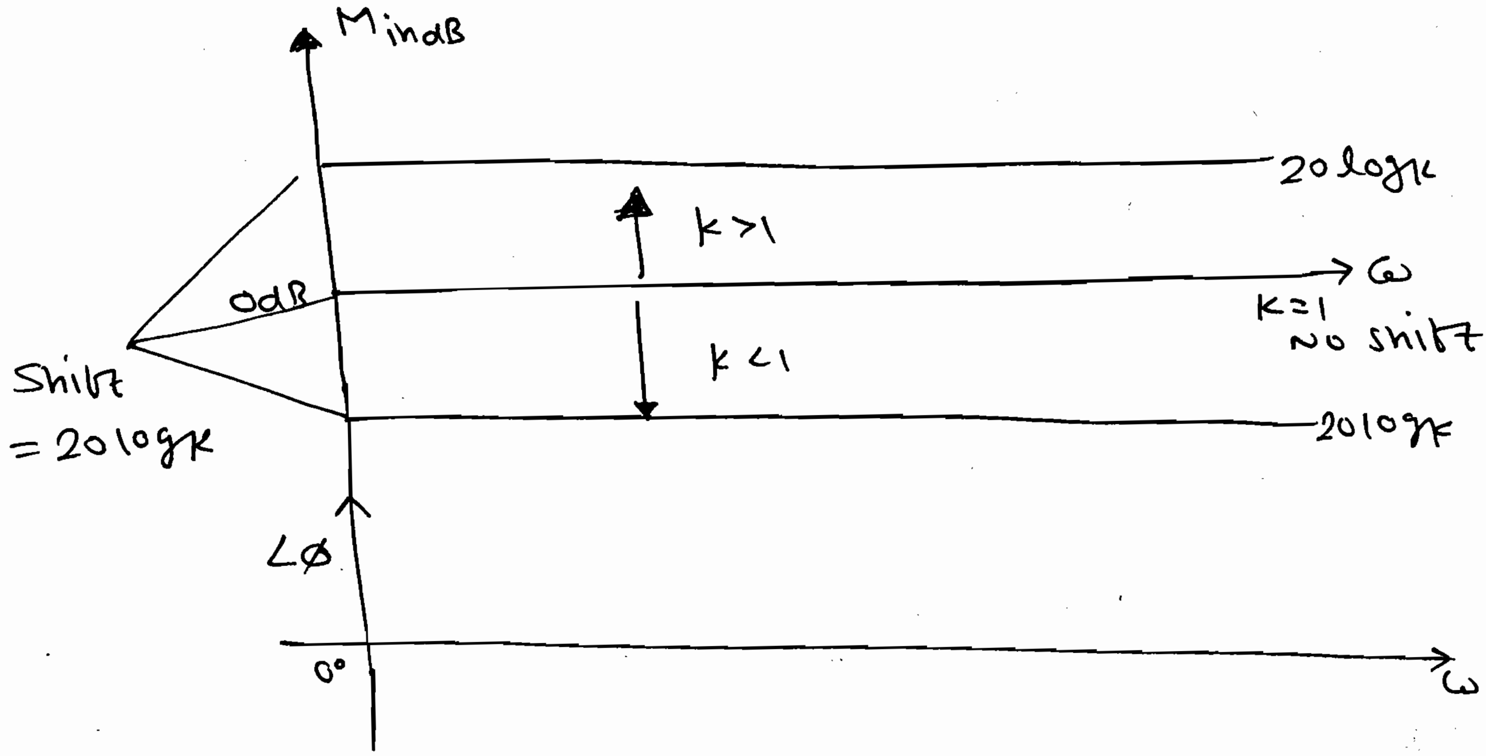
$M_{in\,dB} = 20\log K$

- $K = 1 \Rightarrow M_{dB} = 0$ dB
- $K > 1$ (10) $\Rightarrow M_{dB} = 20$ dB
- $K < 1$ (0.1) $\Rightarrow M_{dB} = -20$ dB.

$$\Rightarrow \angle\phi = \angle GH(j\omega) = \angle(K + j0)$$

$$= \tan^{-1}(0/K)$$

$$\boxed{\angle\phi = 0^\circ}$$



Note: The phase plot is independent of K value where as the shift in the magnitude plot depends on K value.

* n-Poles at origin — n-zeros at origin.

$GH(s) = 1/s^n$ — $GH(s) = s^n$

→ $s \to j\omega$ — $s \to j\omega$

$GH(j\omega) = \frac{1}{(j\omega)^n}$ — $GH(j\omega) = (j\omega)^n$

→ $M = \frac{1}{\omega^n}$ — → $M = (\omega)^n$

⇒ $M_{in\,dB} = 20 \log\left(\frac{1}{\omega^n}\right)$ — → $M_{in\,dB} = 20n \log(\omega)$

$= -20n \log(\omega)$ — ⇒ Slope

⇒ Slope $\frac{dM}{d\log\omega} = -20n$ — $\frac{dM}{d\log\omega} = 20n$

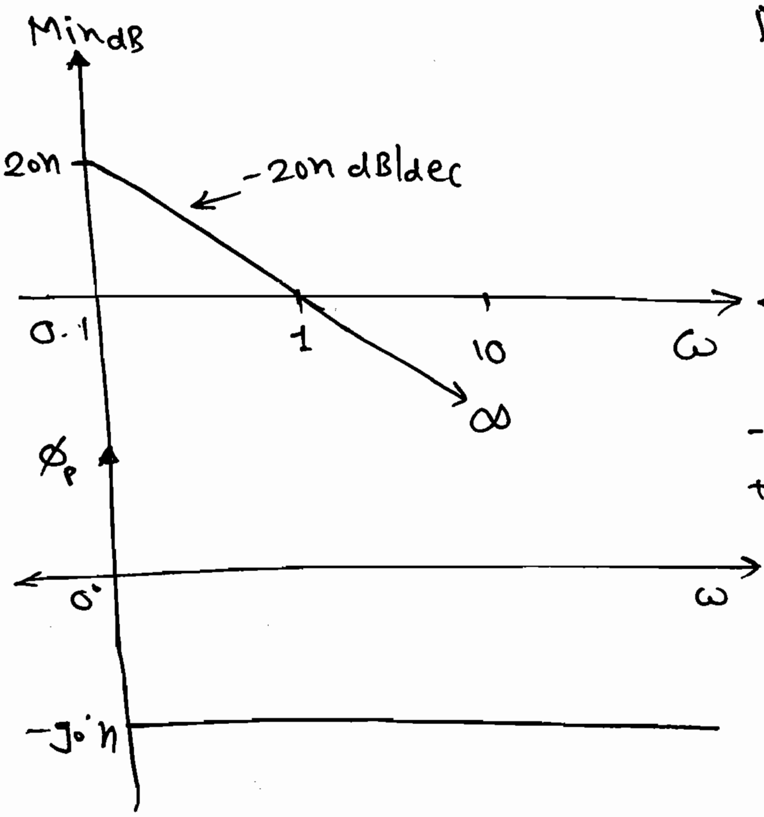
⇒

$\Rightarrow \angle\phi = \dfrac{1}{\angle j\omega \ldots n\,times}$

$\Rightarrow \angle\phi = -90^\circ n$

*** Conclusion:**

$$\boxed{\begin{aligned} S &= -20n\ dB/dec \\ \phi &= -90^\circ n \end{aligned}}$$

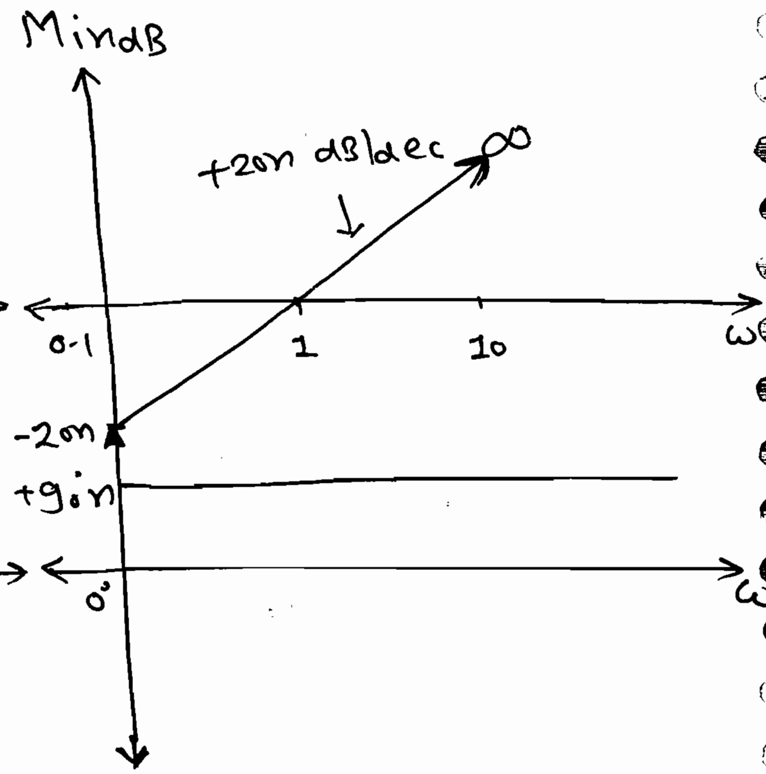


$\Rightarrow \angle\phi = \angle j\omega \ldots n\,times$

$\Rightarrow \angle\phi = +90^\circ n.$

*** Conclusion:**

$$\boxed{\begin{aligned} S &= +20n\ dB/dec \\ \phi &= +90^\circ n. \end{aligned}}$$



$\Rightarrow$ Whenever the transfer f^n Consist a Poles and zeros at origin then the Plot starts with a magnitude of opposite sign of slop at a freq of 0.1 and it should be passes through 0 db line intersect at $\omega = 1$ and extended upto ∞ if no corner freq exist when $K = 1$.

Q Draw the Bode Plot $G(s).H(s) = \frac{100}{s^8}$.

Soln:

$$G(s).H(s) = \frac{100}{s^8}.$$

$$\therefore M = \frac{100}{\omega^8}.$$

$$M_{in\,dB} = 20 \log \left(\frac{100}{\omega^8} \right).$$

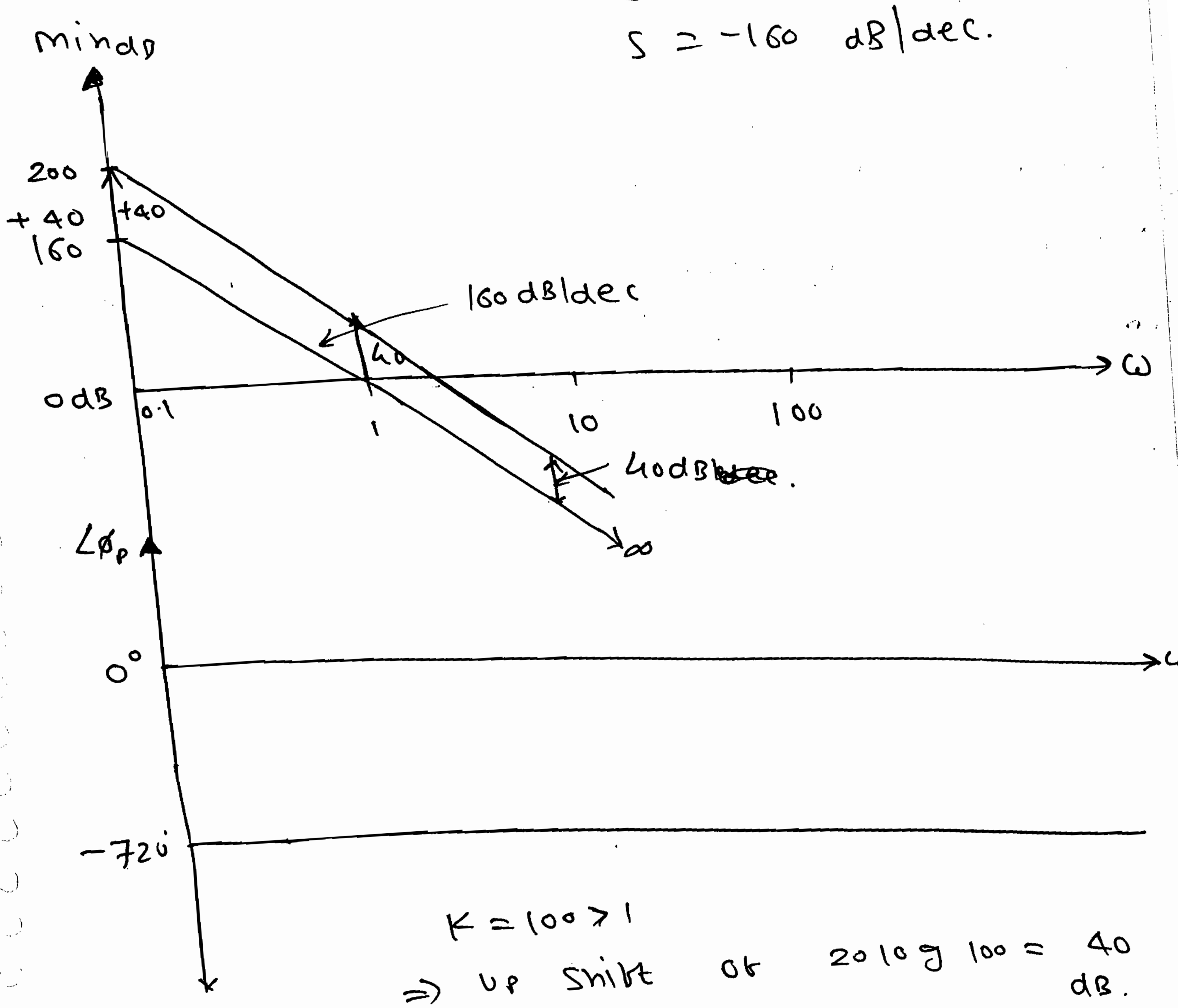
$$\phi = -8 \times 90^\circ = -720^\circ$$

$\Rightarrow$ 8 pole at origin

$s = -20n$

$s = -20 \times 8$

$s = -160$ dB/dec.



$K = 100 > 1$

$\Rightarrow$ up shift of $20 \log 100 = 40$ dB.

$\Rightarrow$ n- finite Poles

$$GH(s) = \frac{1}{(s\tau+1)^n}$$

$s \to j\omega$

$$GH(j\omega) = \frac{1}{(j\omega\tau+1)^n}$$

$$M = \left(\frac{1}{\sqrt{(\omega\tau)^2+1}}\right)^n$$

$$M_{dB\,actual} = -20n \log\sqrt{(\omega\tau)^2+1}$$

$$\Rightarrow \phi_{actual} = \frac{\angle 1+j0}{\angle(1+j\omega\tau)\ldots n\,times}$$

$$\Rightarrow \phi_{actual} = -n\tan^{-1}(\omega\tau).$$

* Asymptotic / Approximate

Case - 1: $\omega\tau < 1$

neglect $\omega\tau$

$$M_{asy} = -20\log 1 = 0\,dB/dec$$

$$S = \frac{dM}{d\log\omega} = 0 \quad dB/dec$$

$$\phi_{asm} = \frac{\angle 1+j0}{\angle 1+j0\ldots n\,times}$$

$$\phi_{asy} = 0^{\circ}.$$

Case - 2: $\omega\tau > 1$

n- finite zeros.

$$GH(s) = (s\tau+1)^n.$$

$$GH(j\omega) = (j\omega\tau+1)^n.$$

$$M = \left(\sqrt{(\omega\tau)^2+1}\right)^n.$$

$$M_{dB\,actual} = +20\log\sqrt{(\omega\tau)^2+1}.$$

$$\phi_{actual} = \angle(1+j\omega\tau)\ldots n\,times.$$

$$\phi_{actual} = +n\tan^{-1}(\omega\tau).$$

* Asymptotic | Approximate:

Case - 1: $\omega\tau < 1$

neglect $\omega\tau$.

$$M_{asy} = 20\log 1 = 0\,dB/dec.$$

$$S = \frac{dM}{d\log\omega} = 0 \quad dB/dec.$$

$$\phi_{asm} = \angle 1+j0\ldots n\,times$$

$$\phi_{asm} = 0^{\circ}.$$

Case - 2: $\omega\tau > 1$.

Neg - 1

$$M_{asym} = -20n \log(\omega\tau)$$

$$M_{asym} = -20n\log\omega - 20n\log\tau$$

$$S = \frac{dM}{d\log\omega} = -20n \text{ db/dec}$$

$$\phi_{asm} = \frac{\angle 1 + j0}{\angle j\omega\tau \cdots n\text{ times}}$$

$$\phi_{asm} = -90^\circ n$$

Neg - 1

$$M_{asym} = +20n\log(\omega\tau).$$

$$M_{asym} = +20n\log\omega - 20\log\tau$$

$$S = \frac{dM}{d\log\omega} = +20n \text{ db/dec.}$$

$$\phi_{asm} = \angle j\omega\tau \cdots n\text{ times}$$

$$\phi_{asm} = +90^\circ n.$$

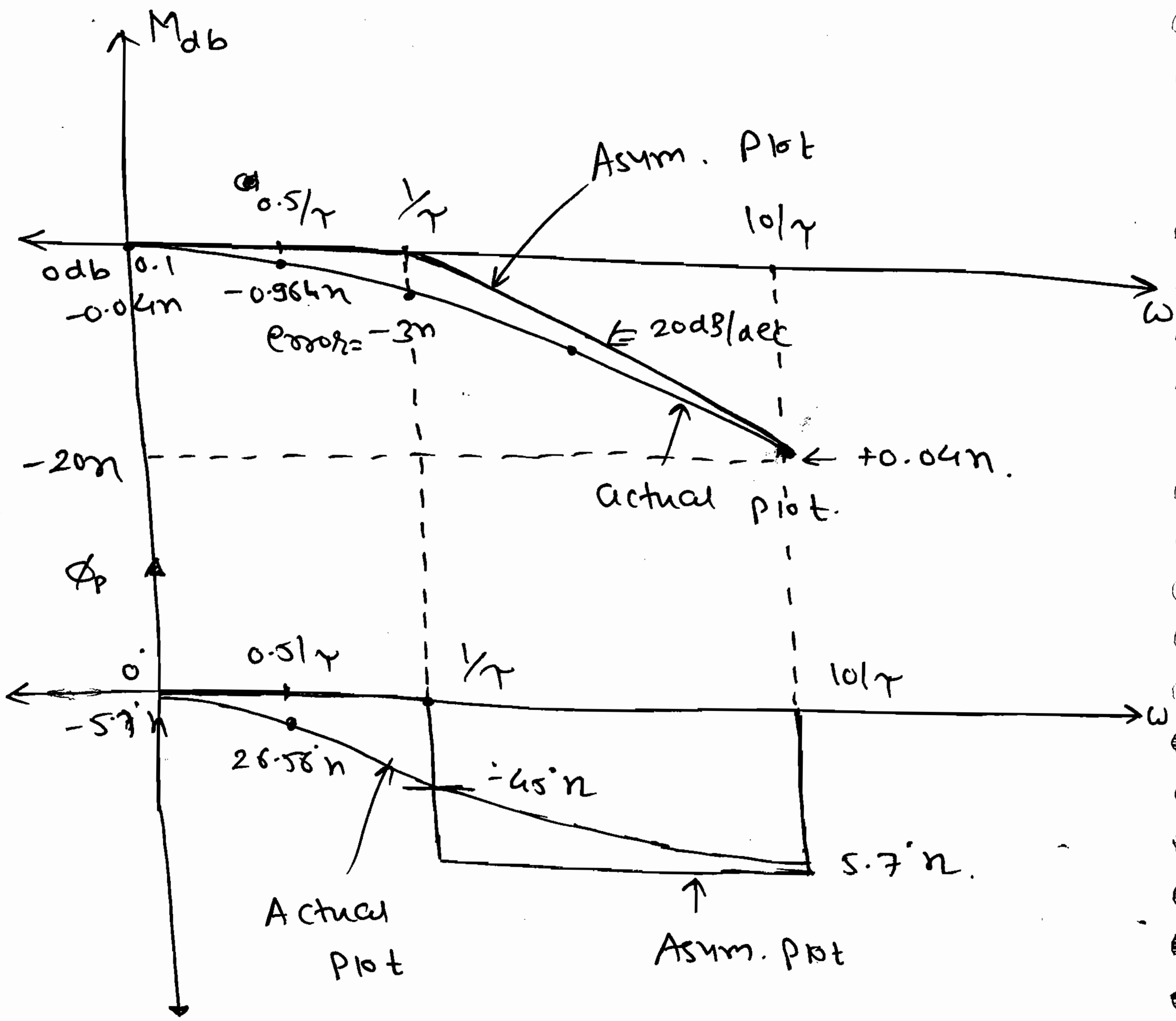
* Corner Frequency:

⇒ The freq. at which Slope changing from one level to another level is called corner freqs.

⇒ The corner frez. is nothing but the finite Poles and zeros location in the form of Magnitude.

	S	Ø
< CF	0 db/dec	0
> CF	−20°n dB/dec	−90°n.

⇒



⇒ $$\text{Error} = \underbrace{\text{Actual value}}_{\text{TF}} - \underbrace{\text{Asym. value}}_{\text{Plot}}$$

⇒ To get the errors in the plot the Actual value is obtained from the transfer f^n and the asymptotic value is obtain from the Plot.

* Error at Corner Freq. :-

* Magnitude plot:

$\Rightarrow$ Error at CF $\omega = 1/\tau$ = $\begin{Bmatrix} M_{actual} \\ \omega = 1/\tau \end{Bmatrix} - \begin{Bmatrix} M_{asym} \\ \omega = 1/\tau \end{Bmatrix}$.

$$= -20^{n} \log \sqrt{\left(\frac{1}{\tau} \times \tau\right)^2 + 1} \quad - 0$$

$$= -20 \log \sqrt{2} \cdot n \quad - 0$$

$$= -3n \quad - \quad 0$$

$$= -3n \text{ dB.}$$

$\Rightarrow$ error is Symm. about the Corner freq.

* Phase plot:

$\Rightarrow$ Error at CF $\omega = 1/\tau$ = $\begin{Bmatrix} \phi_{Actual} \\ \omega = 1/\tau \end{Bmatrix} - \begin{Bmatrix} \phi_{Asymptotic} \\ \omega = 1/\tau \end{Bmatrix}$.

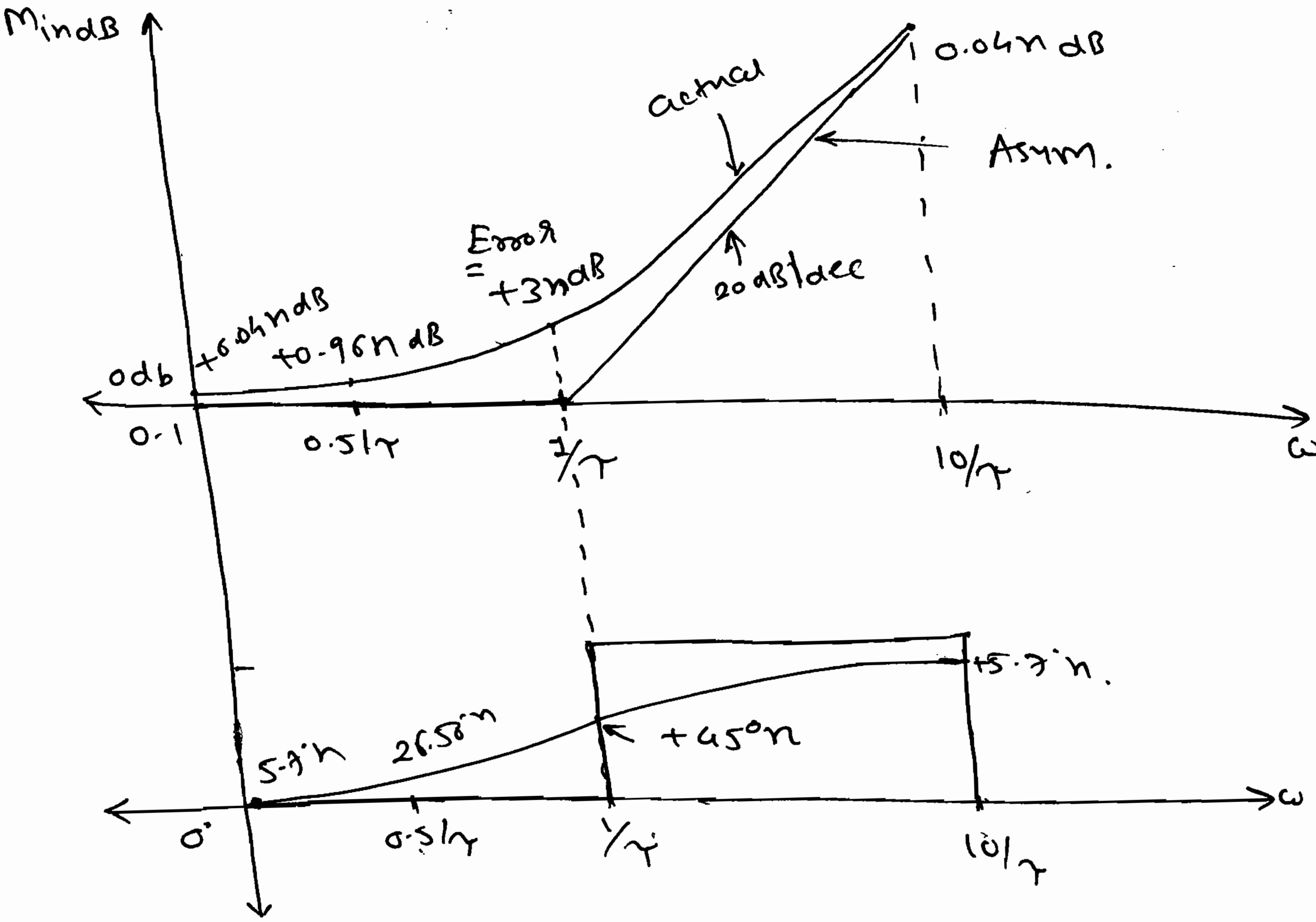
$$= -n \tan^{-1}(\omega\tau) \quad - 0$$

$$= -n \tan^{-1}(1) \quad - 0$$

$$= -45^{\circ} n \quad - 0$$

$$= -45^{\circ} n.$$

$\Rightarrow$ The error is max. at Corner freq., the error is Symm. about corner freq. whatever the error exist below 1 decade the same error exist after one decade also.



$\boxed{a}$ Draw the Bode plot for

$$GH(s) = \frac{10(s+5)^2}{s(s+2)(s+10)}$$

Soln:

$$GH(j\omega) = \frac{10(j\omega+5)^2}{(j\omega)(j\omega+2)(j\omega+10)}$$

$$M = |GH(j\omega)| = \frac{10\cdot(\omega^2+5)}{\omega\cdot\sqrt{\omega^2+4}\cdot\sqrt{\omega^2+100}}$$

$$\angle GH = \phi = -90^\circ - \tan^{-1}(\omega/2) + 2\tan^{-1}(\omega/5) - \tan^{-1}(\omega/10).$$

$$M_{in\,dB} = 20\log_{10}\left[\frac{10\cdot(\omega^2+5)}{\omega\cdot\sqrt{\omega^2+4}\cdot\sqrt{\omega^2+100}}\right]$$

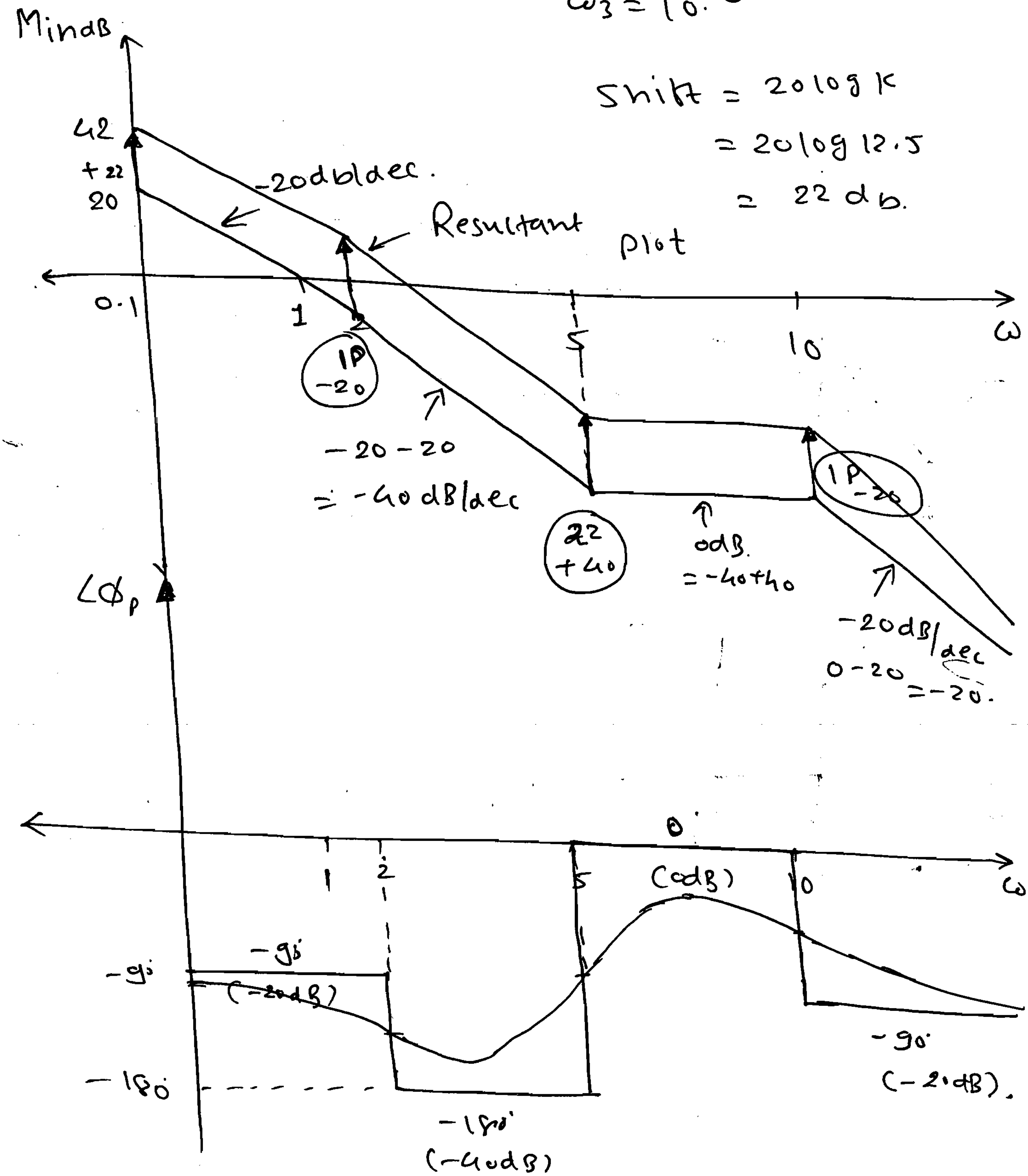
⇒ Time constant form:

$$GH(s) = \frac{\cancel{10} \times \overset{12.5}{\cancel{25}}\ (1+ s/5)^2}{\cancel{2} \times \cancel{10}\ s\ (1+ s/2)(1+ s/10)} \rightarrow CF.$$

$I P_o$ $s \Rightarrow -20dB/dec.$

$$\left.\begin{array}{l} \omega_1 = 2, \\ \omega_2 = 5. \\ \omega_3 = 10. \end{array}\right\} \rightarrow CF.$$

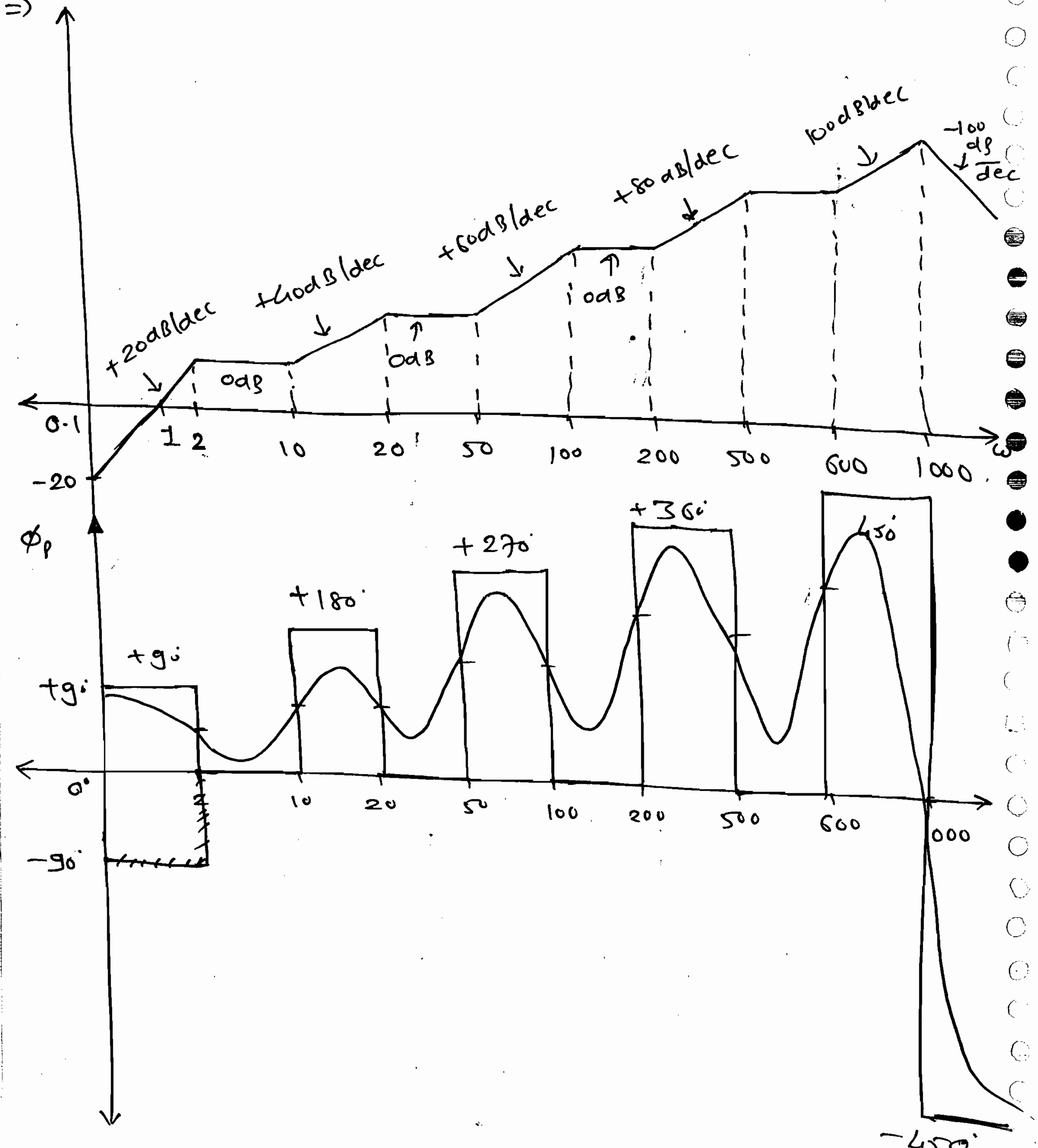
Shift = 20 log K
= 20 log 12.5
= 22 db.



⇒ The Initial slope of the plot is given by poles and zeros located at origin.

$\boxed{a}$ $$G_H(s) = \frac{s(1+s/10)^2 (1+s/50)^3 (1+s/200)^4 (1+s/600)^5}{(1+s/2)(1+s/20)^2(1+s/100)^3(1+s/500)^4 (1+s/1k)^{10}}$$

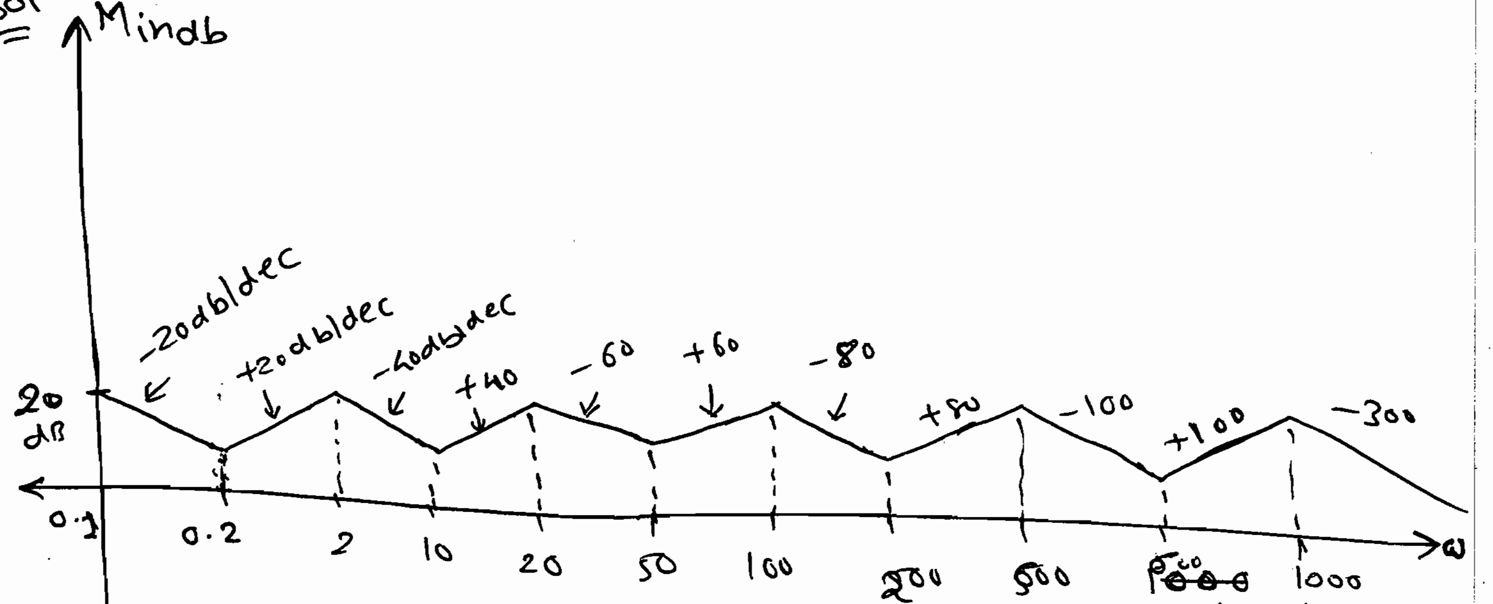
M in dB
⇒

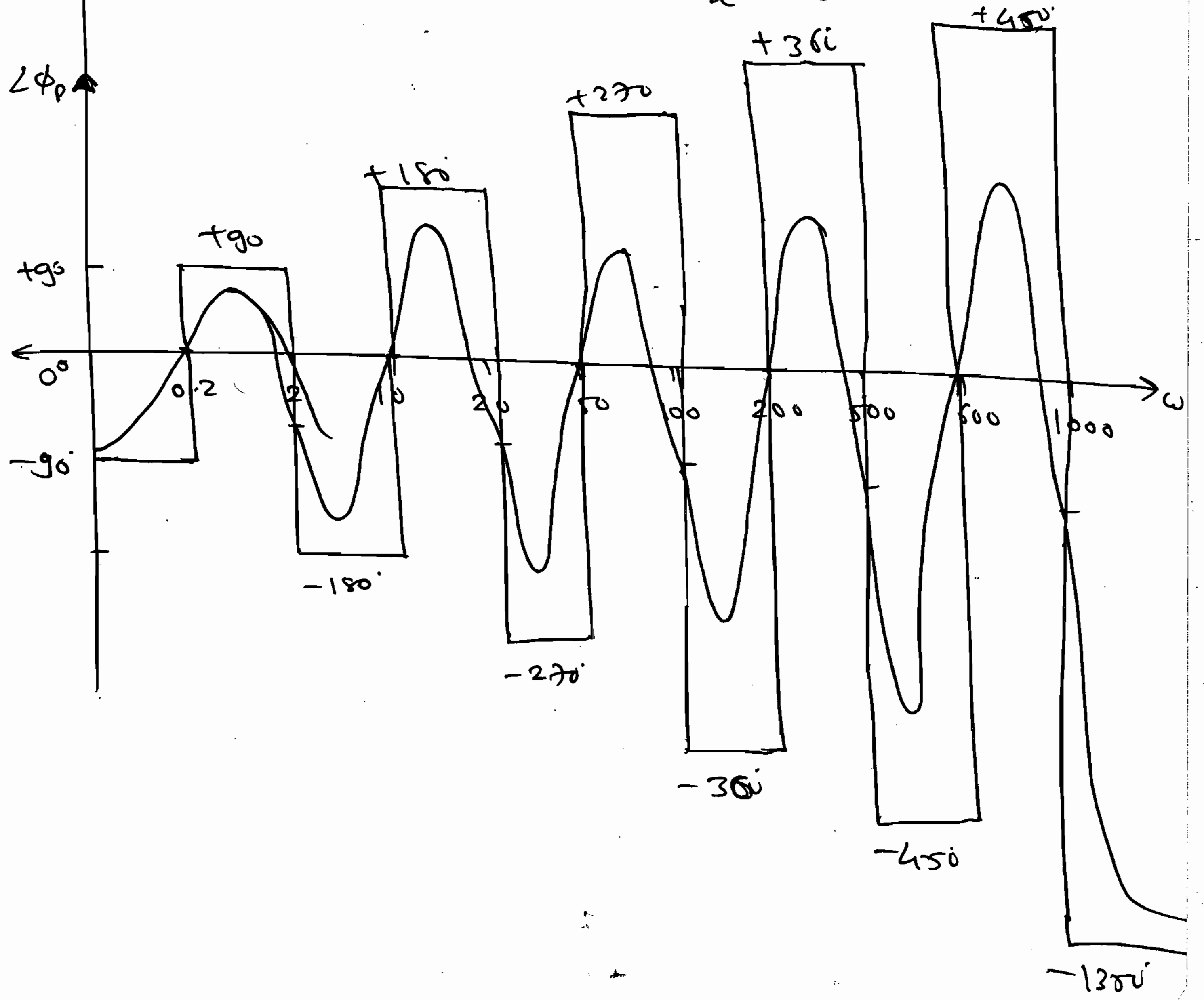


Q

$$GH(s) = \frac{(1+s/0.2)^2 (1+s/10)^4 (1+s/50)^6 (1+s/200)^8 \times (1+s/600)^{10}}{s(1+s/2)^3 (1+s/20)^5 (1+s/100)^7 (1+s/500)^9 \times (1+s/1000)^{20}}$$

Sol^n:





Q Find the change in slope at the following corner freqs

1) $\omega = 2$ 4) 50   7) $\omega = 500$

2) $\omega = 10$ 5) $\omega = 100$ 8) $\omega = 1K$.

3) $\omega = 20$ 6) $\omega = 200$

→ Find the slope of the line betn two corner freqs.

1) $\omega = 2$ to 10.   4) High freq asymptote

2) $\omega = 20$ to 50.

3) $\omega = 200$ to 500

→ Find the slopes around the corner freq.

1) $\omega = 2$, 20, 200, 1K. for.

$$G(s).H(s) = \frac{s^5 (1+s/10)^{20} (1+s/50)^{50} (1+s/200)^{200} (1+s/600)^{600}}{(1+s/2)^{10} (1+s/20)^{30} (1+s/100)^{100} (1+s/500)^{500} (1+s/1K)^{1K}}$$

Soln:

$$\boxed{\text{Change in slope} = \text{New slope} - \text{previous slope}.}$$

CF	P | Z (-20 | +20)	CS
2	10P	-200
10	20Z	+400
20	30P	-600
50	50Z	+1000
100	-100P	-2000
200	200Z	+4000
500	500P	-20000
1000	1000P	~~+2~~ -20000

$\Rightarrow$ Slope- betn ω_1 & ω_2:

Note :$\rightarrow$ To get a slope of line betn two freqs from ω_1 to ω_2 then consider all the terms in TF up to ω_1 only, get the no. of poles and zeros. and resultant slope.

1) $\omega = 2$ to 10

$\Rightarrow$ >2 ✓ IN $\quad$ <10 ✗ out

$\therefore$ $P = 10$, $Z = 5$

$\Rightarrow$ $5P \Rightarrow -100.$

2) $\omega = 20$ to $50.$

$\Rightarrow$ >20 IN $\quad$ <50 out $\Rightarrow$ $P = \cancel{50}\ 40$, $Z = 25$

$\Rightarrow$ $40 - 25 = 15P \Rightarrow -300.$

3) $\omega = 200$ to 500

$\Rightarrow$ >200 IN $\quad$ <500 out. $\Rightarrow$ $P = 140$, $Z = 275$

$\Rightarrow$ $P - Z = 40 - 275 = \cancel{235}Z$

235 zero $\Rightarrow +2700.$

~~(4)~~ ~~ω~~ 4) ∞

$$\left.\begin{array}{l} P = 1640 \\ Z = 875 \end{array}\right\} \Rightarrow P = 765 \Rightarrow -15300.$$

(3) Find ~~fer~~ Slopes around the frez.

(i) $\omega = 2$

>2	<2
IN	out
$p = 10$	$z = 5$
$z = 5$	$p = 0$
$p = 5$	$z = 5$
$\Rightarrow -100$	$\Rightarrow +100$

(ii) $\omega = 200$

>200	<200
IN	out
$p = 140$	$p = 140$
$z = 275$	$z = 75$
$z = 135$	$p = +65$
$\Rightarrow 2700$	$\Rightarrow -1300$

(iii) $\omega = 20$.

>20	<20
IN	out
$p = 40$	$p = 10$
$z = 25$	$z = 25$
$p = 15$	$z = 15$
$\Rightarrow -300$	$\Rightarrow +300$

(iv) $\omega = 1K$.

>1000	<1000
IN	out
$p = 1640$	$p = 640$
$z = 875$	$z = 875$
$p = 765$	$z = 235$
$\Rightarrow -15{,}300$	$+4700$

* Transfer Function from the magnitude plot:

⇒ **Procedure:**

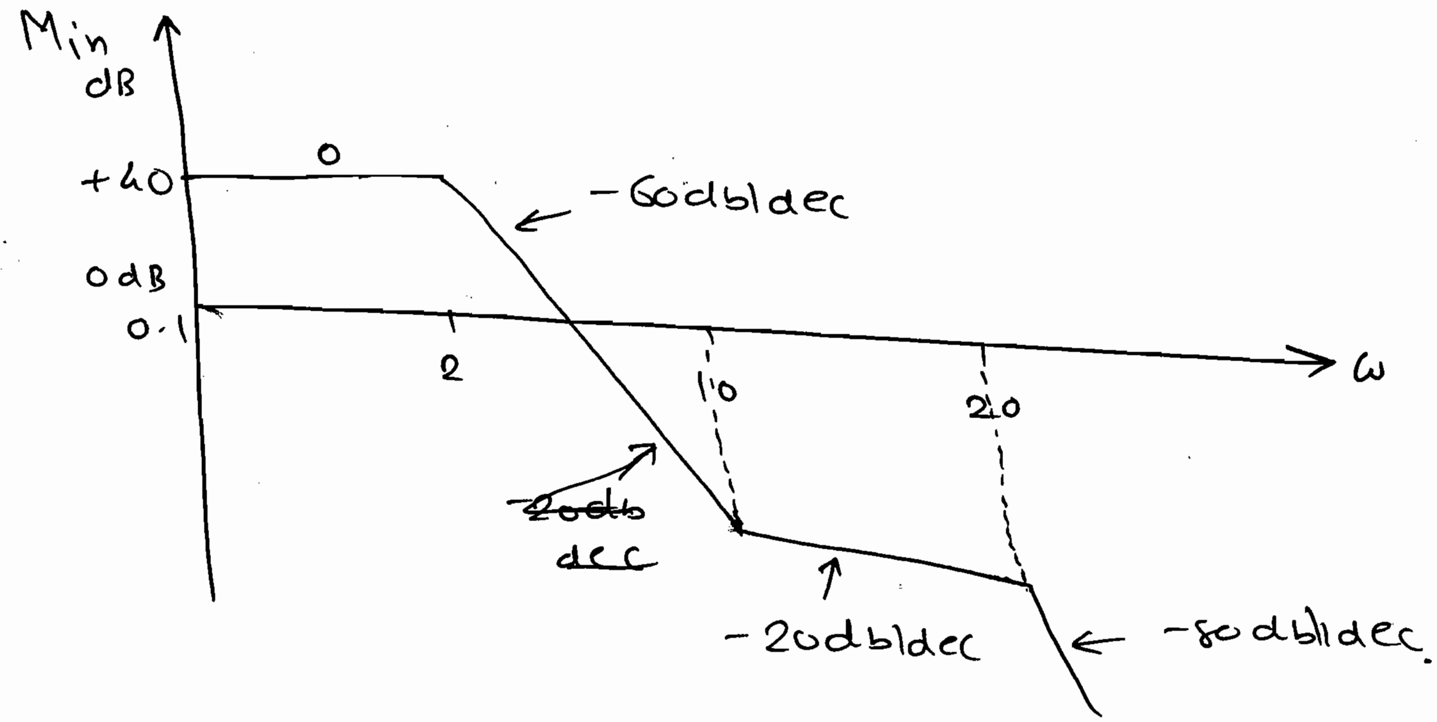
1) Observe the initial slope it gives the no. of zeros and poles at origin.

2) Find the change slope at each and every corner frez. The change

in slope is (+ve), consider finite zero.
If change in slope is (-ve) consider the finite poles.

3) Find the K value by using known magnitude at known freq.

$\boxed{Q}$ Find the TF to the given asymptotic magnitude plot of a minimum phase system.



Soln: The initial slope is zero so no poles and zeros at origin.

$\Rightarrow$ CS = $-60 - 0 = -60$

poles $20 \times 3 \rightarrow$ 3 poles at $\omega = 2$.

$\Rightarrow$ CS = $-20 - (-60) = +40 \rightarrow 2 \times 20$ at $\omega = 10$

zero $\rightarrow$ 2 zeros

$\Rightarrow \quad Cs = -80 - (-20) = -60$
$\Rightarrow$ 3 poles at $\omega = 20$.

So, $GH(s) = \dfrac{K\left(\frac{s}{10}+1\right)^2}{\left(\frac{s}{2}+1\right)^3\left(\frac{s}{20}+1\right)^3}$.

at $\omega = 0.1$ $\quad M_{indB} = 0.1$.

$$M = |GH(j\omega)| = \frac{K\left[\left(\frac{\omega}{10}\right)^2+1\right]}{\left[\left(\frac{\omega}{2}\right)^2+1\right]^{3/2}\left[\left(\omega/20\right)^2+1\right]^{3/2}}$$

$$\Rightarrow M_{indB} = 20\log K + 40\log\sqrt{1+\cancel{(0.1/10)^2}} - 60\log\sqrt{\cancel{(0.1/2)^2}+1} - 60\log\sqrt{\cancel{(0.1/20)^2}+1}$$

$$M_{db}\Big|_{\omega=0.1} = 40$$

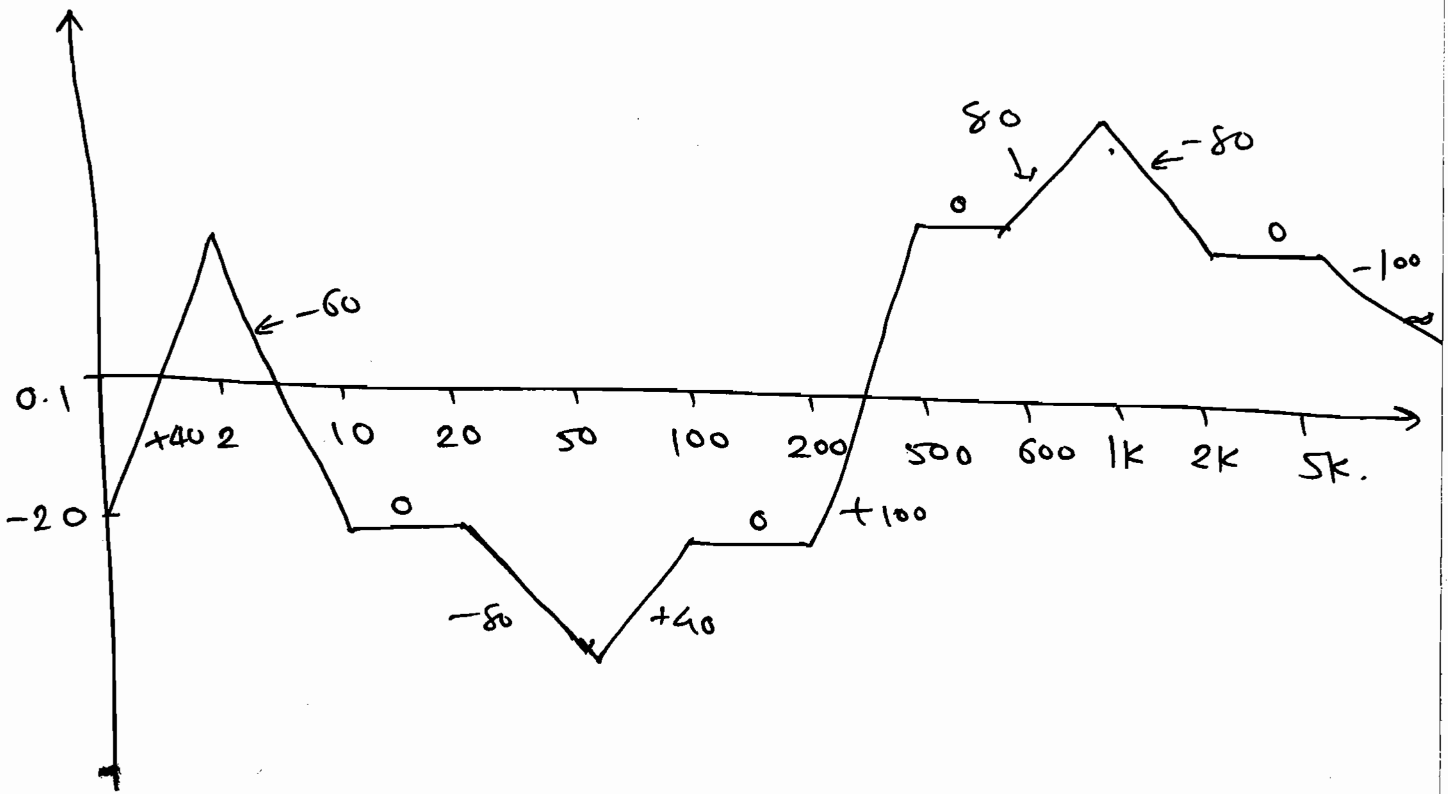
$\therefore \cancel{40}^{\,2} = \cancel{20}\log_{10}K$.

$\Rightarrow \log_{10}K = 2$

$K = 10^2 \Rightarrow \boxed{K=100}$

<u>Note</u>: To get the K value compare the corner freq. with freq. where the magnitude is known if the corner freq. is greater than (or) equal to one (CF ≥ 1). then neglect the corner freq.

Q. Find the TF:



Soln:

$$G H(s) = \frac{K s^2 \left(\frac{s}{10}+1\right)^3 \left(\frac{s}{50}+1\right)^6 \left(\frac{s}{200}+1\right)^5 \left(\frac{s}{600}+1\right)^4 \left(\frac{s}{2k}+1\right)^4}{\left(\frac{s}{2}+1\right)^3 \left(\frac{s}{20}+1\right)^4 \left(\frac{s}{100}+1\right)^2 \left(\frac{s}{500}+1\right)^5 \left(\frac{s}{1k}+1\right)^8 \times \left(\frac{s}{5k}+1\right)^5}$$

$$M_{in\,dB}\Big|_{\omega=0.1} = -20.$$

$$\therefore -20 = 20\log K + 40 \log \omega.$$

$$\Rightarrow -20 = 20\log K + 40 \log 0.1.$$

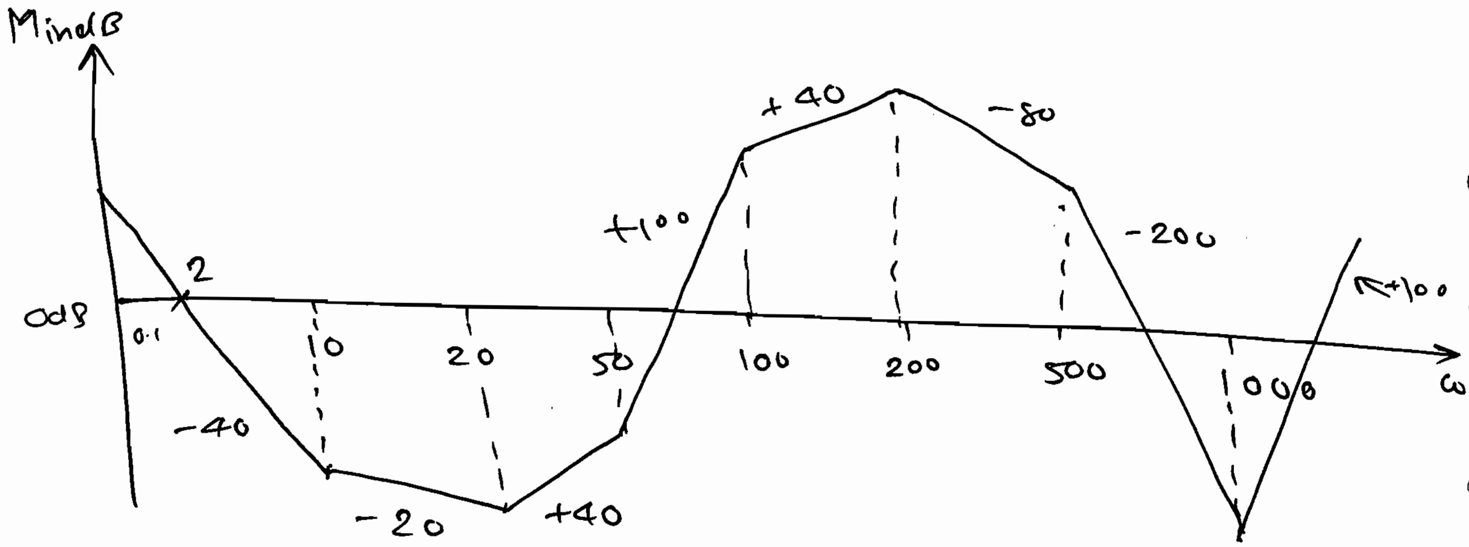
$$-20 = 20\log K - 40.$$

$$20 \log K = 20$$

$$\Rightarrow \log K = 1$$

$$\Rightarrow \boxed{K = 10}$$

Q Find K



Sol^n:

$$GH(s) = \frac{K\left(1+\frac{s}{10}\right)^1\left(1+\frac{s}{20}\right)^3\left(1+s/50\right)^3\left(1+s/1K\right)^{15}}{s^2\left(1+\frac{s}{100}\right)^3\left(1+\frac{s}{200}\right)^6\left(1+s/500\right)^6}$$

Now, at $\omega = 2$, $M_{db} = 0\,dB$.

$$\therefore \quad 0\,dB = 20\log_{10}\left(\frac{K}{\omega^2}\right).$$

$$\Rightarrow \quad 0\,dB = 20\log_{10}K - 40\log\omega \Big|_{\omega=2}$$

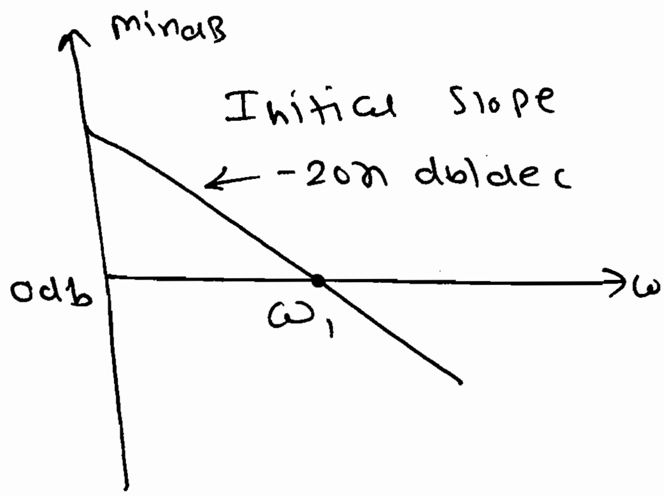
$$\cancel{20}\log K = \overset{2}{\cancel{40}}\log\omega$$

$$\log K = 2\log 2$$

$$\log K = \log 2^2$$

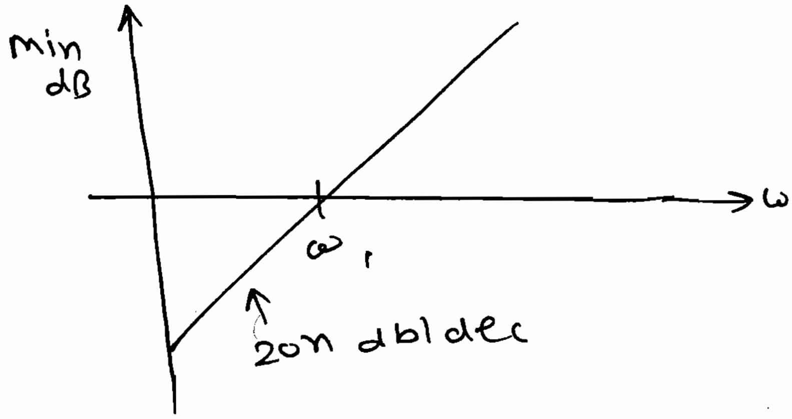
$$\boxed{K = 4}$$

*



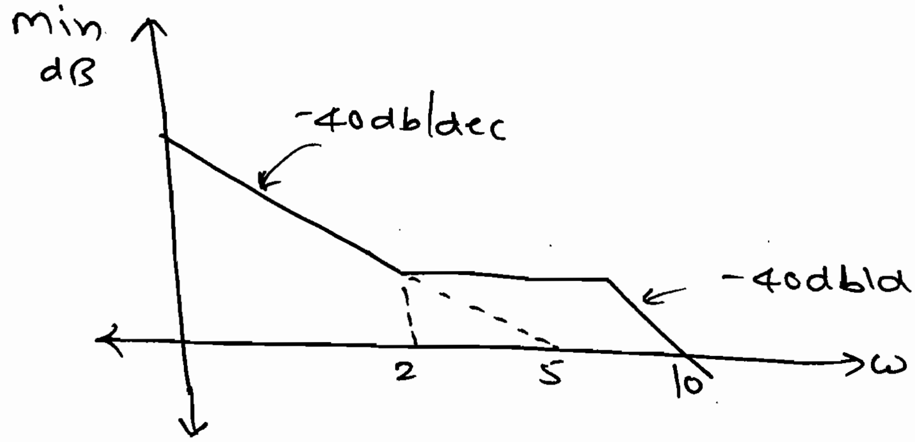
$\Rightarrow \quad K = (\omega_1)^n.$

*



$\Rightarrow \quad K = (\omega_1)^{-n}.$

Q Find TF.



Sol^n: Initial slope $= -40$ dB/dec $= -2 \times 20$ dB/dec

$\boxed{n=2}$

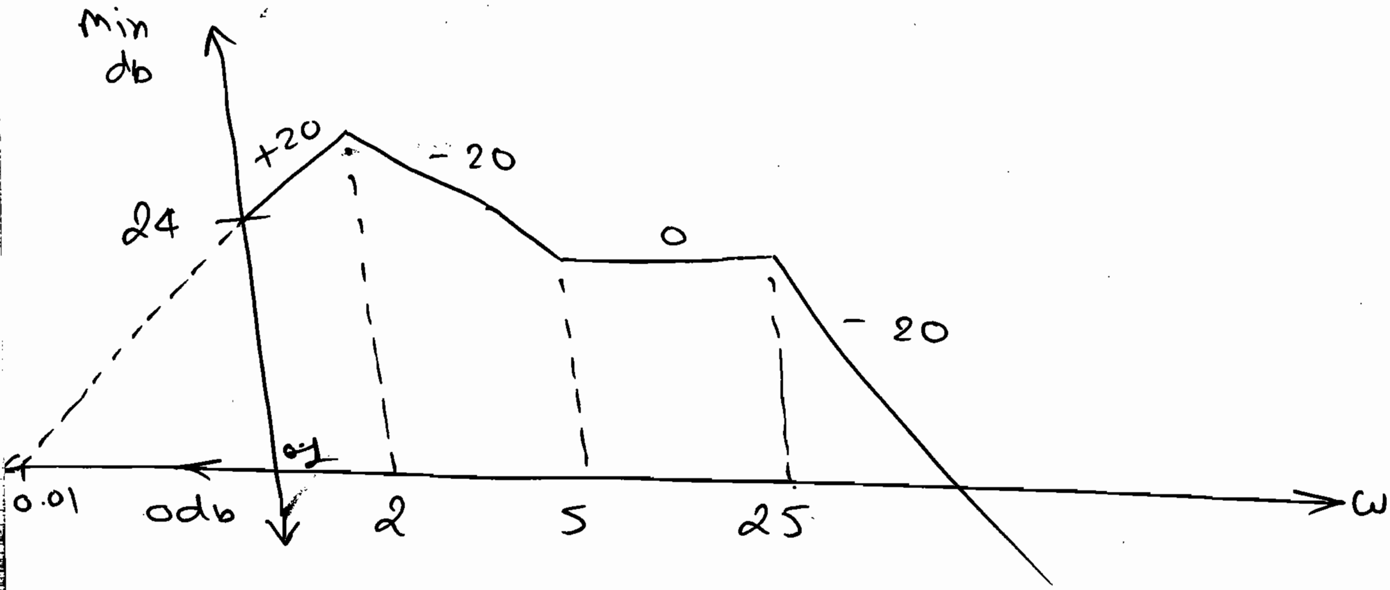
$\Rightarrow$ Initial slope crosses the ω axis at $\omega = 5$ rad/sec. & $n = 2$

$\Rightarrow \quad K = (\omega)^n$

$K = (5)^2 = 25.$

$$\Rightarrow \boxed{TF = \frac{25\,(1+s/2)^2}{s^2\,(1+s/10)^2}}.$$

Q Find the TF.



Soln:

$$GH(s) = \frac{K\,s^2 \times (1+s/5)}{(1+s/2)^2\,(1+s/25)}.$$

Now $\quad M\,in\,db\big|_{\omega=0.1} = 24.\,db$

$\therefore \quad 24 = 20\log(K\cancel{s}\cdot\omega).$

$\Rightarrow 24 = 20\log_{10}K + 20\log(0.1).$

$24 + 1/20 = 20\log_{10}K.$

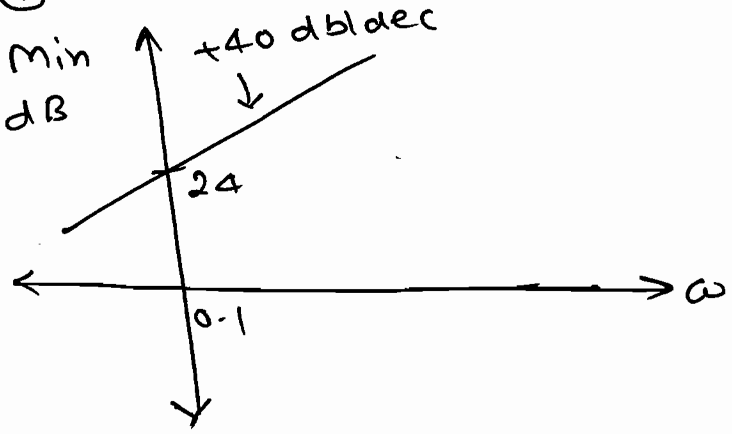
$\log_{10}K = 1.2$

$\Rightarrow \boxed{K = 15.85}$

Q

Find K value for the following Bode plots:

①



Sol^n:

$$GH(s) = K.s^2.$$

at $\omega = 0.1$, $M_{db} = 24$ dB.

$$\Rightarrow \quad 24 = 20\log(K.\omega^2).$$

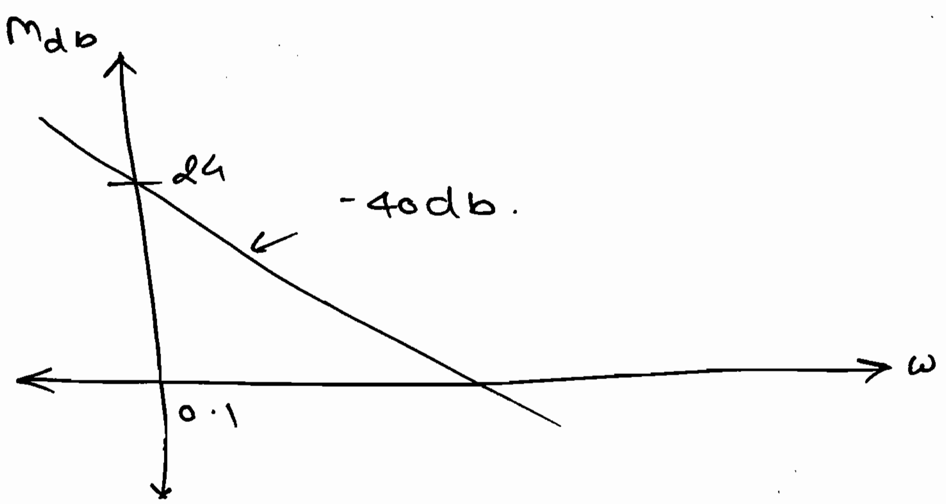
$$\Rightarrow \quad 24 = 20\log K + 40\log(0.1).$$

$$24 = 20\log K - 40.$$

$$64 = 20\log K$$

$$\Rightarrow \quad \boxed{K = 1584.89}$$

②



Sol^n:

$$GH(s) = \frac{K}{s^2}.$$

$$M_{db}\Big|_{\omega=0.1} = 24 \text{ dB}.$$

$\Rightarrow \quad 24 = 20\log K - 40\log(\omega).$

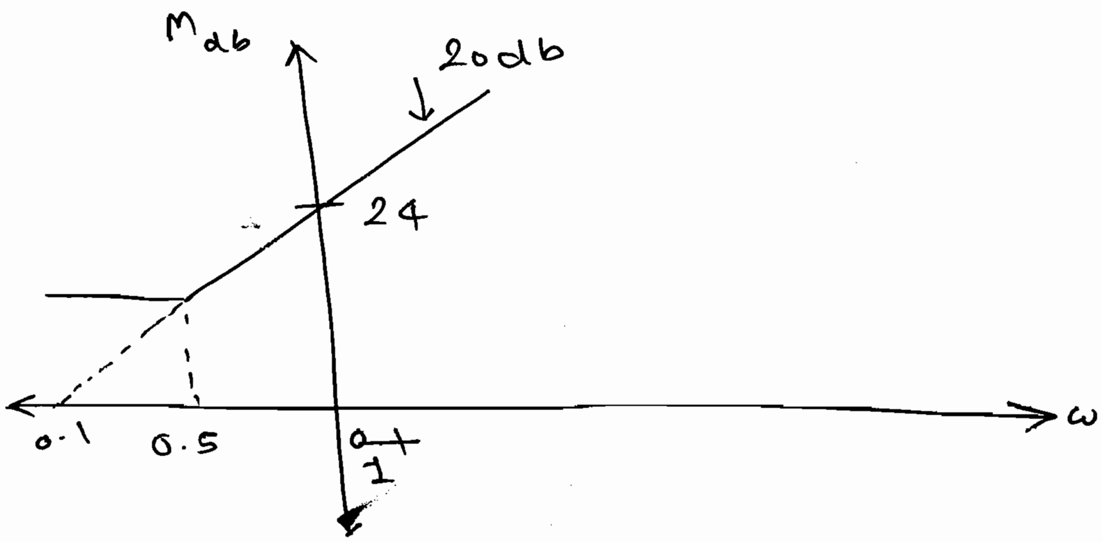
$\therefore \quad 24 = 20\log K - 40\log(0.1).$

$\therefore \quad 24 - 40 = 20\log K$

$\therefore \quad \log K = -\frac{16}{20}.$

$\Rightarrow \quad \boxed{K = 0.15848}$

3



$\underline{\text{Sol}^n:}$

$$G H(s) = \frac{K \times (1 + s/0.5)}{1}.$$

at $\rightarrow$ $M_{db}\Big|_{\omega = 1} = 24$ db.

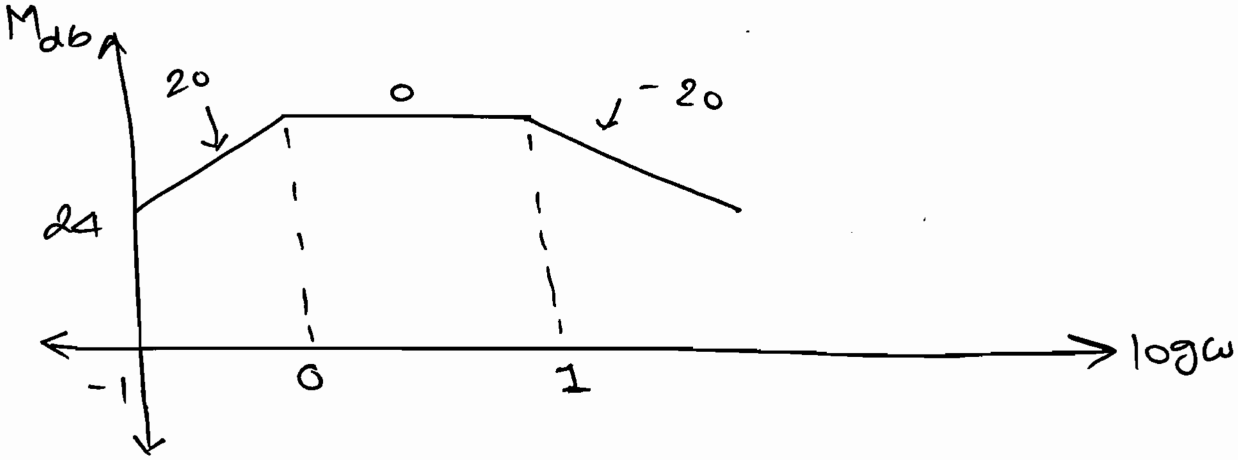
$\therefore \quad 24 = 20\log K + 20\log\left(\sqrt{1 + (\omega/0.5)^2}\right).$

$\therefore \quad 24 = 20\log_{10} K + 20\log\sqrt{1+4}$

$\therefore \quad 24 - 6.989 = 20\log_{10} K$

$\Rightarrow \quad \boxed{K = 7.0876}$

④



Soln:

$\log\omega = -1 \Rightarrow \omega = 0.1$

$\log\omega = 0 \Rightarrow \omega = 1.$

$\log\omega = 1 \Rightarrow \omega = 10.$

So, $$GH(s) = \frac{ks}{\left(\frac{s}{1}+1\right)^1\left(\frac{s}{10}+1\right)^1}$$

Now, $$M_{db}\Big|_{\omega=0.1} = 24.$$

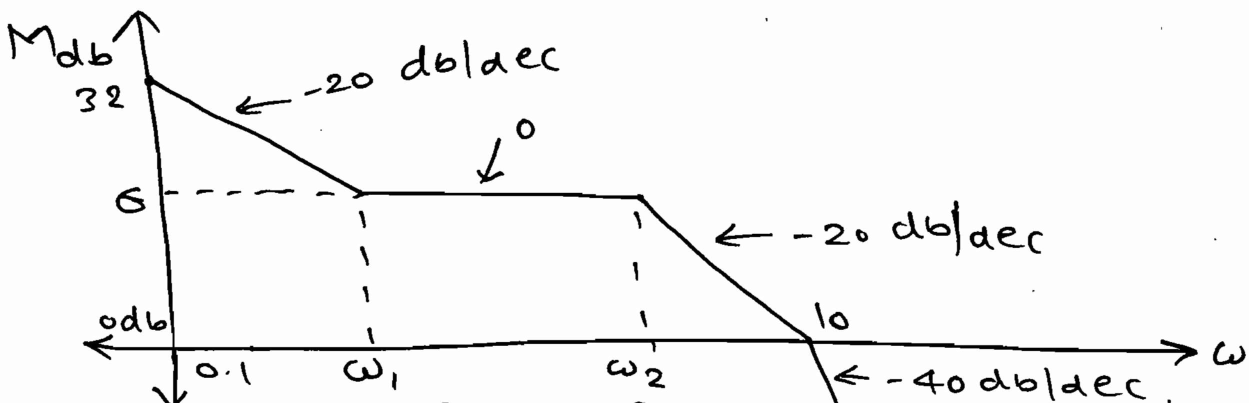
$$\therefore\ 24 = 20\log K + 20\log\omega - 20\log\sqrt{\cancel{\omega^2}^{\,0}+1} - 20\log\sqrt{\cancel{\frac{\omega^2}{100}}^{\,0}+1}.$$

$$\therefore\ 24 = 20\log K - 20$$

$$\frac{24+20}{20} = \log_{10} K$$

$$\Rightarrow\ \boxed{K = 158.489}$$

$\boxed{Q}$ Find the ω_1, ω_2, TF



Solⁿ: → Slope = $\dfrac{dM}{d\log\omega}$.

$\therefore \quad -20\, db/dec = \dfrac{6-0}{\cancel{\log\omega_1} \;\; \log\omega_2 - \log 10}$.

$\therefore \quad -20 = \dfrac{6}{\log\omega_2 - 1}$

$\therefore \quad -20\log\omega_2 + 20 = 6$

$20\log\omega_2 = +14$

$\log\omega_2 = 14/20$

$\Rightarrow \quad \boxed{\omega_2 = 5 \text{ rad/sec}}$

→ Slope = $\dfrac{dM}{d\log\omega}$

$\therefore \quad -20 = \dfrac{32-6}{\log 0.1 - \log\omega_1}$.

$\therefore \quad -20 = \dfrac{26}{-1 - \log\omega_1}$.

$\therefore \quad 20 + 20\log\omega_1 = 26$.

$20\log\omega_1 = 6$.

$\boxed{\omega_1 = 2 \text{ rad/sec}}$

$\Rightarrow$ TF $GH(s) = \dfrac{K\,(1+s/2)^1}{s\,(1+s/5)^1\,(1)\,(1+s/10)}$

$M_{db}\big|_{\omega=0.1} = 32$.

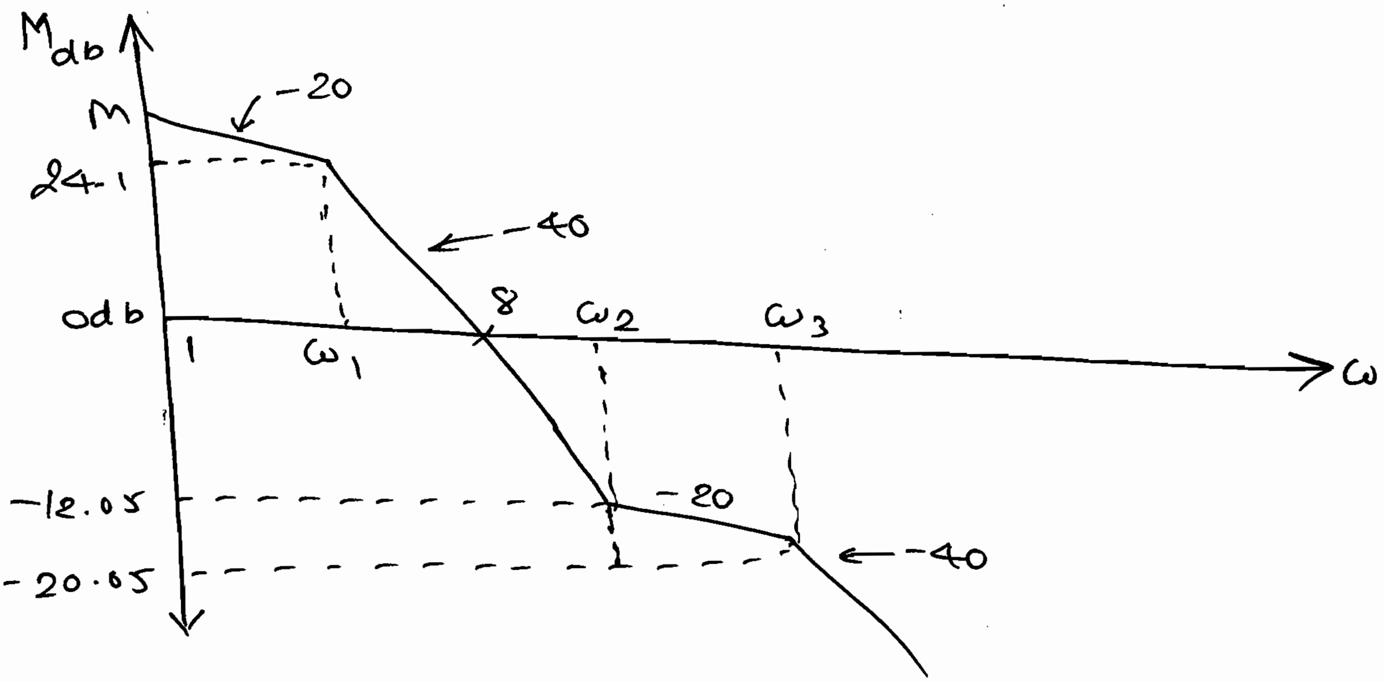
$\Rightarrow \quad 32 = 20\log k - 20\log(0.1)$.

$$\therefore \quad \frac{32-20}{20} = \log_{10} K$$

$$\Rightarrow \quad \boxed{K = 3.98 \simeq 4}$$

So, TF $\boxed{GH(s) = \dfrac{4\,(1+s/2)}{s\,(1+s/5)\,(1+s/10)}}$.

* (a) Find the Magnitude M, ω_1, ω_2 and ω_3.



Solⁿ:

$$\text{Slope} = \frac{dM}{d\log_{10}\omega}.$$

$$\therefore \quad +40 = \frac{+12.05 - 0}{\log\omega_2 - \log 8}.$$

$$\therefore \quad +40\log\omega_2 - 40\log 8 = 12.05.$$

$$40\log\omega_2 = 48.17$$

$$\Rightarrow \quad \omega_2 = 15.848$$

$$\boxed{\omega_2 \simeq 16 \text{ rad/sec}}$$

$$\Rightarrow \quad \text{Slope} = \frac{dM}{d\log\omega}$$

$$\Rightarrow \quad -40 = \frac{24.1 - 0}{\log\omega_1 - \log 8}.$$

$$\therefore \quad -40\log\omega_1 + 40\log 8 = 24.1.$$

$$40\log\omega_1 = 12.12$$

$$\Rightarrow \quad \boxed{\omega_1 = 2 \text{ rad/sec}}$$

$$\Rightarrow \quad \text{Slope} = \frac{dM}{d\log\omega}.$$

$$\Rightarrow \quad -20 = \frac{M - 24.1}{\log\omega_1 - \log\omega_1}$$

$$2\cancel{0} + 20\log\omega_1 = M - 24.1$$

$$2\cancel{0} + 20\log 2 = M - 24.1$$

$$\Rightarrow \quad \cancel{M}/2 \quad \boxed{M = 30.1 \text{ dB}}$$

$$\Rightarrow \quad \text{Slope} = \frac{dM}{d\log\omega}.$$

$$\therefore \quad -20 = \frac{-12.05 + 20.05}{\log\omega_2 - \log\omega_3}.$$

$$\therefore \quad -20\log 6 + 20\log\omega_3 = 8.$$

$$20\log\omega_3 = 34.02/1.$$

$$\Rightarrow \boxed{\omega_3 \simeq 40 \text{ rad/sec}}$$

Now,

TF $\quad GH(s) = \dfrac{K\,(1+s/16)}{s\left(1+\frac{s}{2}\right)(1+s/40)}$.

$$M_{db}\Big|_{\omega=1} = 30.1 \text{ db.}$$

$$\therefore \quad 30.1 \text{ db} = 20\log k - 20\log(1).$$

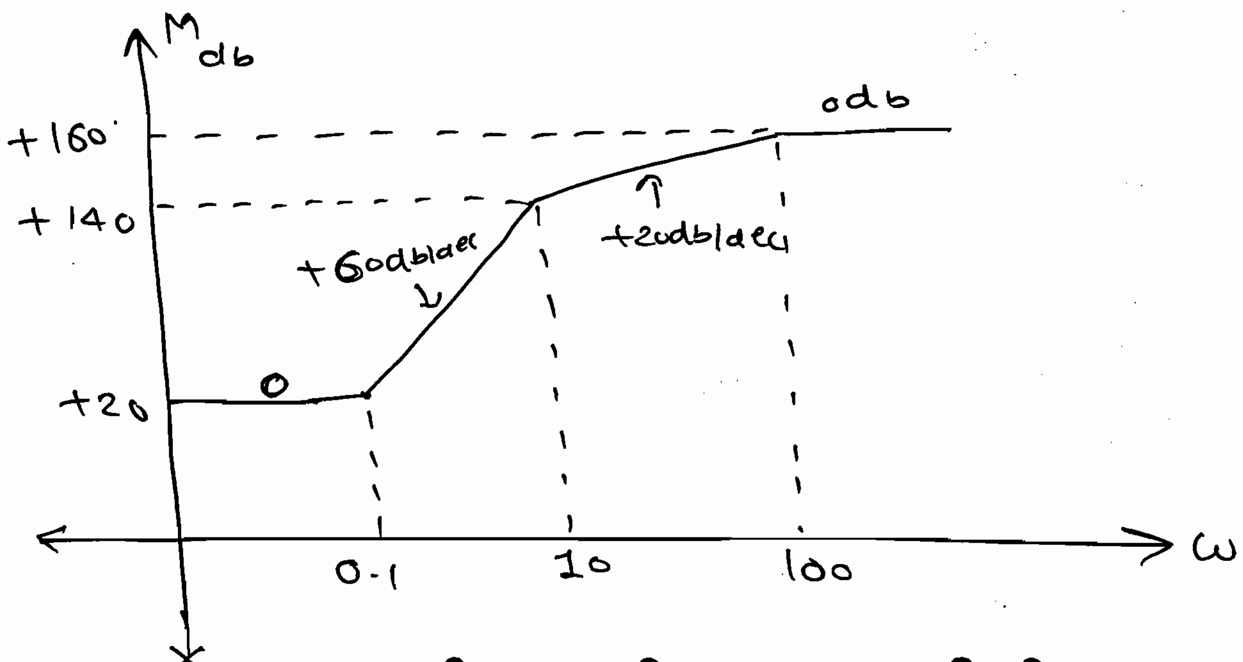
$$\Rightarrow \quad 20\log K = 30.1.$$

$$\log K = \frac{30.1}{20}$$

$$\Rightarrow \boxed{K \simeq 32}$$

$$\Rightarrow \boxed{GH(s) = \frac{32\left(1+\frac{s}{16}\right)}{s(1+s/2)(1+s/40)}}$$

Q The Asymptotic Aproximation of the logM vs freq plot of a minimum phase system is shown in figure its TF is ___ ?



Soln: Slope-1 $\Rightarrow$ 0 . db/dec.

Slope-2 $\Rightarrow \dfrac{140 - 20}{-\log 0.1 + \log 10} = \dfrac{120}{+1+1} = +60$ db/dec

Slope-3 $\Rightarrow \dfrac{dM}{d\log\omega} = \dfrac{160-140}{\log 100 - \log 10} = \dfrac{20}{2-1}$

$= 20$ db/dec.

So, TF $G_H(s) = \dfrac{K\left(1+\frac{s}{0.1}\right)^3}{(1+s/10)^2\,(1+s/100)^1}$.

$M_{in\,dB}\Big|_{\omega=0.1} = 20.$

$\therefore\ 20 = 20\log K + 60\log\sqrt{1+\left(\frac{0.1}{0.1}\right)^2}.$

$\therefore\ 20 = 20\log K + 60\times\cancel{\log}^{\,0}\sqrt{2}.$

~~20 - 9.03 = 20 log K~~

$20 = 20\log K$

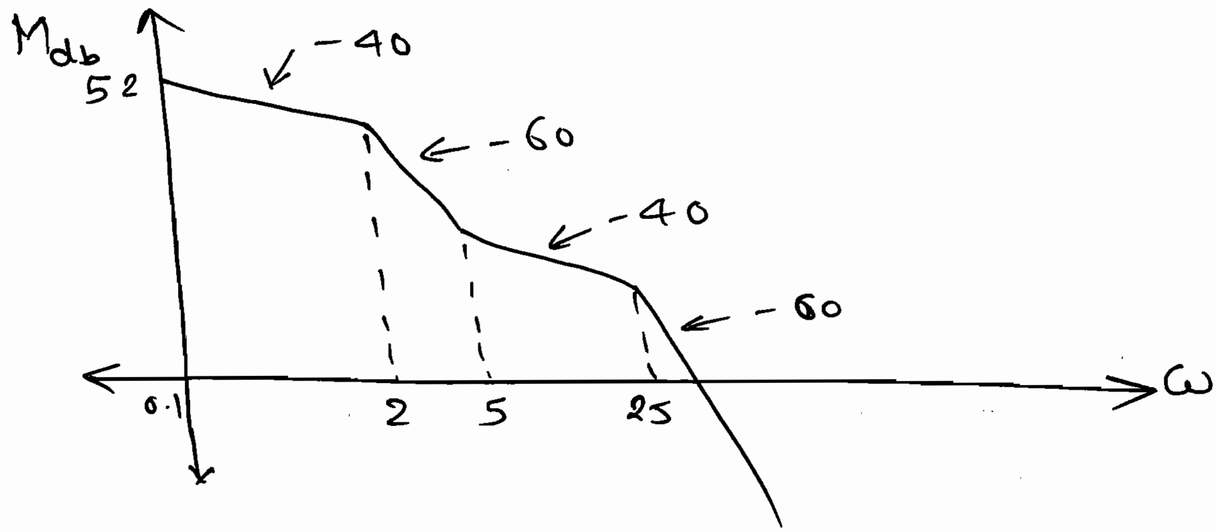
$\therefore \Rightarrow \boxed{K=10}$

$\therefore\ G_H(s) = \dfrac{10\,(1+s/0.1)^3}{\left(1+\frac{s}{10}\right)^2\,(1+s/100)^1}.$

$\therefore\ G_H(s) = \dfrac{10\times\left(\frac{1}{0.1}\right)^3\,(s+0.1)^3\times 10^2\times 100}{(s+10)\,(s+100)}.$

$$\Rightarrow \boxed{GH(s) = \frac{10^8 (s+0.1)^3}{(s+10)^2 (s+100)}}$$

Q



Soln:

$$TF = GH(s) = \frac{K(1+s/5)^1}{s^2 (1+s/2)^1 (1+s/25)^1}$$

Now, $M_{db}\Big|_{\omega=0.1} = 52\ db$

$$\Rightarrow 52 = 20\log K - 40\log\omega.$$

$$\therefore 52 = 20\log K - 40\log(0.1).$$

$$\frac{\cancel{102}\ 12}{20} = \log K.$$

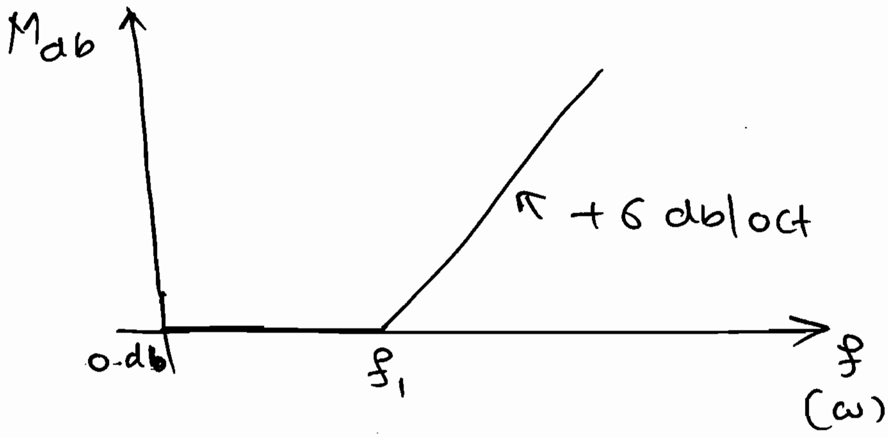
$$\therefore \Rightarrow \boxed{K \simeq 4.}$$

$$\Rightarrow TF = \frac{4(1+s/5)}{s^2 (1+s/2)(1+s/25)}.$$

$$= \frac{4}{\cancel{8}} \times 2 \times \frac{\cancel{5}}{\cancel{25}} \frac{(s+5)}{s^2 (s+2)(s+25)}.$$

$$\Rightarrow \boxed{TF = \frac{40(s+5)}{s^2(s+2)(s+25)}}.$$

Q Find the TF



<u>Soln:</u>

$$GH(s) = K\left(1+\frac{s}{\tau_1}\right).$$

$$GH(j\omega) = K\left(1+\frac{j\cancel{2\pi} f}{\cancel{2\pi} f_1}\right)$$

$$GH(f). = K\left(1 + j\frac{f}{f_1}\right).$$

Here, $M_{in\,dB}\big|_{\omega=0.1} = 0$ db

$$\Rightarrow K = 1.$$

$$\therefore \boxed{GH(f) = \left(1 + j\frac{f}{f_1}\right)}.$$

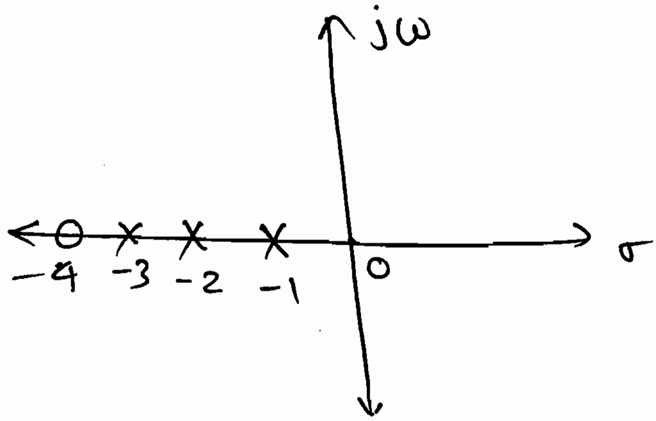
* Classification of System :-

1) Minimum phase system :-

⇒ A System in which all the finite poles and finite zeros lies in the left of s-plane then it is called minimum phase system.

⇒ The minimum phase ~~sm~~ system gives the angle $\leq 90^\circ$.

e.g :



$$MPS = \frac{(s+4)}{(s+2)(s+3)(s+1)}.$$

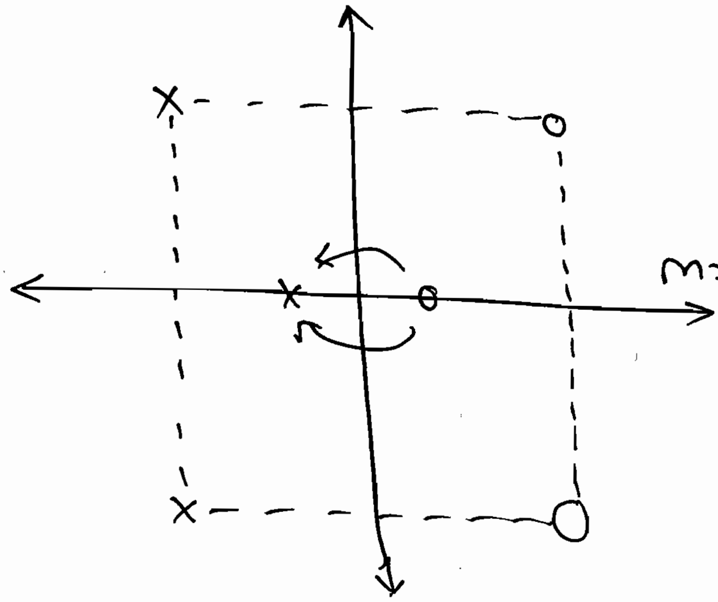
2) All Pass System:-

⇒ A system in which all poles are lies in the left of s-plane and all the zeros are lies in Right of the s-plane which are summetrical about imaginary axis then it is called All pass system.

⇒ All pass system gives Magnitude of

'1', and the Phase angle varies b/w $\pm 180°$.

⇒



e.g.

$$\boxed{ALPS = \frac{s-1}{s+1}}$$

$m=1$, $\phi = \angle \pm 180°$.

3) **Non-Minimum Phase System :-**

⇒ A system in which one (or) more zero (or) one (or) more pole (or) Both poles and zeros lies in the Right of the s-plane then it is called Non-minimum phase system.

⇒ The NMP gives the more -ve angle at $\omega = \infty$.

⇒ The NMP system may be Unstable.

e.g. $NMPS = \dfrac{(s+1)(s-2)}{(s+3)(s+5)}$.

$$NMPS = \left[\frac{(S+1)(S+2)}{(S+3)(S+5)}\right] \times \left(\frac{S-2}{S+2}\right)$$

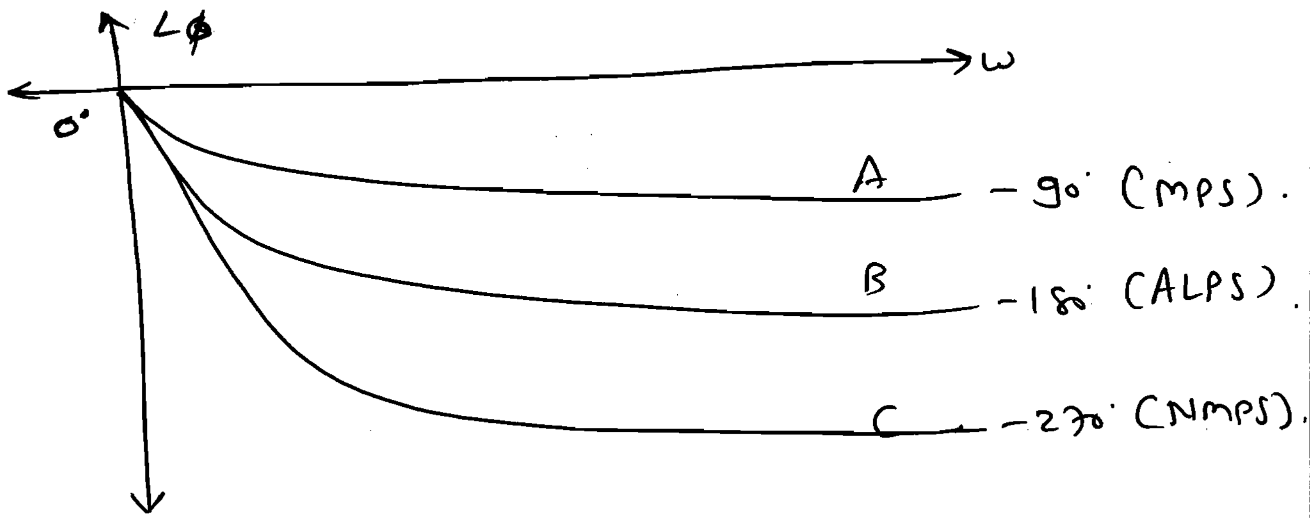
$\downarrow$ MPS $\quad\quad \downarrow$ APS

$\Rightarrow$ $\boxed{NMPS = MPS \times APS.}$

$\Rightarrow$ NMPS is nothing but the product of MPS and ALPS.

$$\Rightarrow \quad \phi_{NMPS} = \phi_{MPS} + \phi_{ALPS}.$$

a Identify the curves A, B, C in the given phase plot.



* Stability Condition :-

→ The stability conditions are to find the CL sys stability.

→ The CL system stability is given by char. eq^n i.e. $1 + G(s) \cdot H(s) = 0$.

then s is replaced by $j\omega$.

$\rightarrow \quad 1 + G(j\omega) \cdot H(j\omega) = 0.$

$G(j\omega) \cdot H(j\omega) = -1 + j0.$

Mg $\rightarrow$ $|G(j\omega) \cdot H(j\omega)| = 1$ in linear

$M_{in\,dB} = 20\log 1 = 0\,dB.$

$$\boxed{M_{in\,dB} = 0\,dB}$$

$\Rightarrow$ Gain Crossover frequency: (ω_{gc}):

$\Rightarrow$ The freq. at which the magnitude equal to '1' in linear and '0' in db is called Gain cross over freq.

$\Rightarrow$ Write the phase angle.

$$\boxed{\angle GH(j\omega) = \phi = \angle -1 + j0 = -180^\circ.}$$

$\Rightarrow$ Phase Cross over frequency: (ω_{pc}):

$\Rightarrow$ The freq. at which the ~~magnitude~~ phase angle equal to -180°, is called phase cross over frequency. (ω_{pc}).

⇒ **Gain Margin:-**

⇒ Inverse of Magnitude at ω_{pc} gives the Gain Margin.

⇒ The Gain margin is the factor by which the system gain is increased to bring the system verge of the stability.

$$\therefore \boxed{GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}}$$

$$\Rightarrow \boxed{GM_{db} = -20\log|GH(j\omega)|_{\omega=\omega_{pc}}}$$

⇒ **Phase - Margin:**

⇒ It is a additional phase lag required to add to the system to bring the system at verge of the stability.

$$\boxed{PM = 180^\circ + \angle GH(j\omega)\Big|_{\omega=\omega_{gc}}}$$

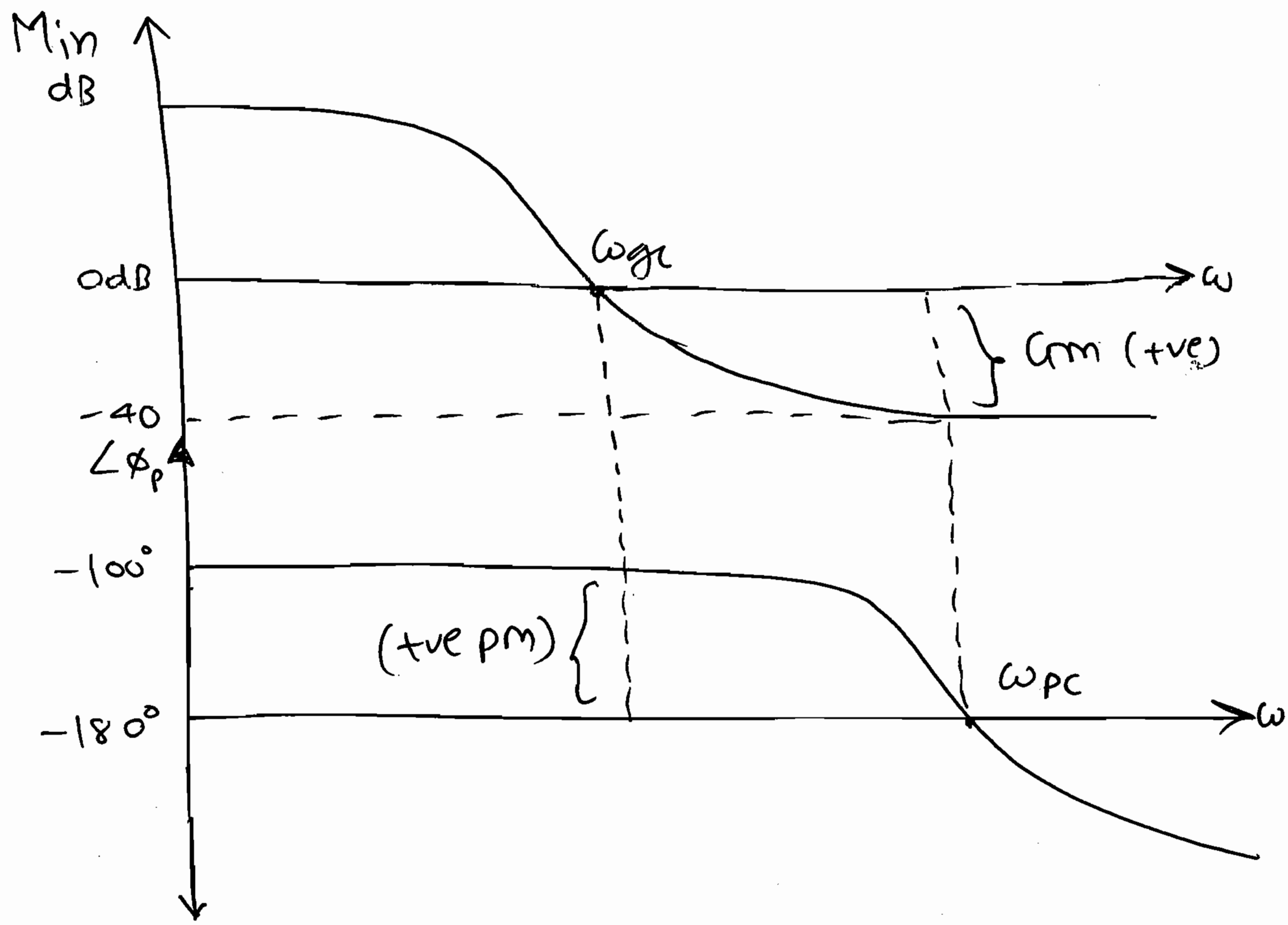
*

$\Rightarrow$ $\omega_{pc} > \omega_{gc} \longrightarrow$ S $GM \begin{cases} \rightarrow +ve \text{ in dB} \\ \rightarrow >1 \text{ in L} \end{cases}$ & $\{PM +ve\}$

$\Rightarrow$ $\omega_{pc} = \omega_{gc} \longrightarrow$ MS $GM \begin{cases} \rightarrow 0 \text{ db} \\ \rightarrow 1 \text{ L} \end{cases}$ & $\{PM = 0°\}$

$\Rightarrow$ $\omega_{pc} < \omega_{gc} \longrightarrow$ US $GM \begin{cases} \rightarrow -ve \text{ in dB} \\ \rightarrow <1 \text{ in L} \end{cases}$ & $\{PM -ve\}$

Q Identify the Stability to the given Bode plot:



$\Rightarrow$ $\omega_{pc} > \omega_{gc} \Rightarrow$ S

$\Rightarrow$ $GM = -(M_{in} dB) \Big|_{\omega = \omega_{pc}}$

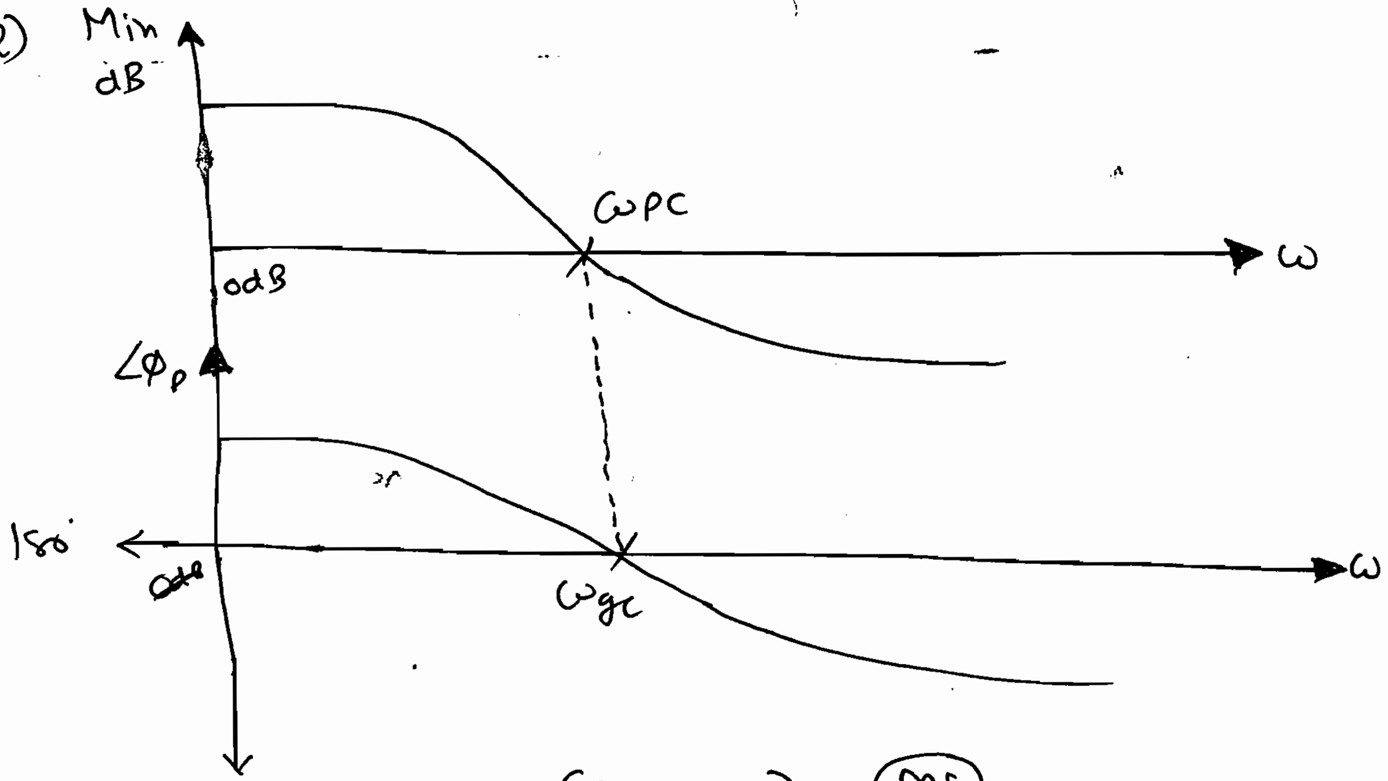
$= -(-40 \, db)$.

$= +40 \, db$

$\Rightarrow$ Phase margin $PM = 180^\circ + \angle GH \Big|_{\omega = \omega_{gc}}$

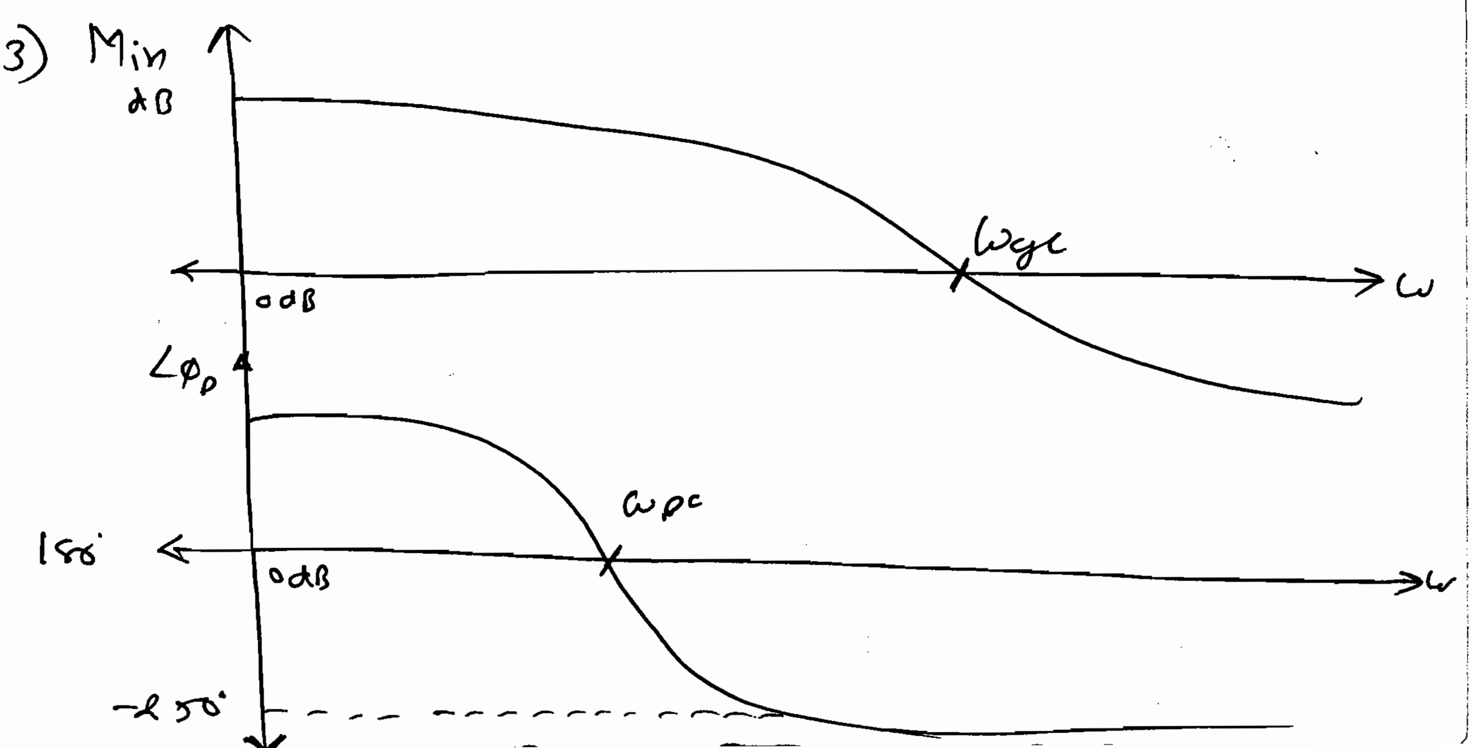
$\therefore PM = 180^\circ - 100 = +80^\circ > 0 \Rightarrow +ve$.

2)



$\omega_{pc} = \omega_{gc} \Rightarrow$ MS.

3)



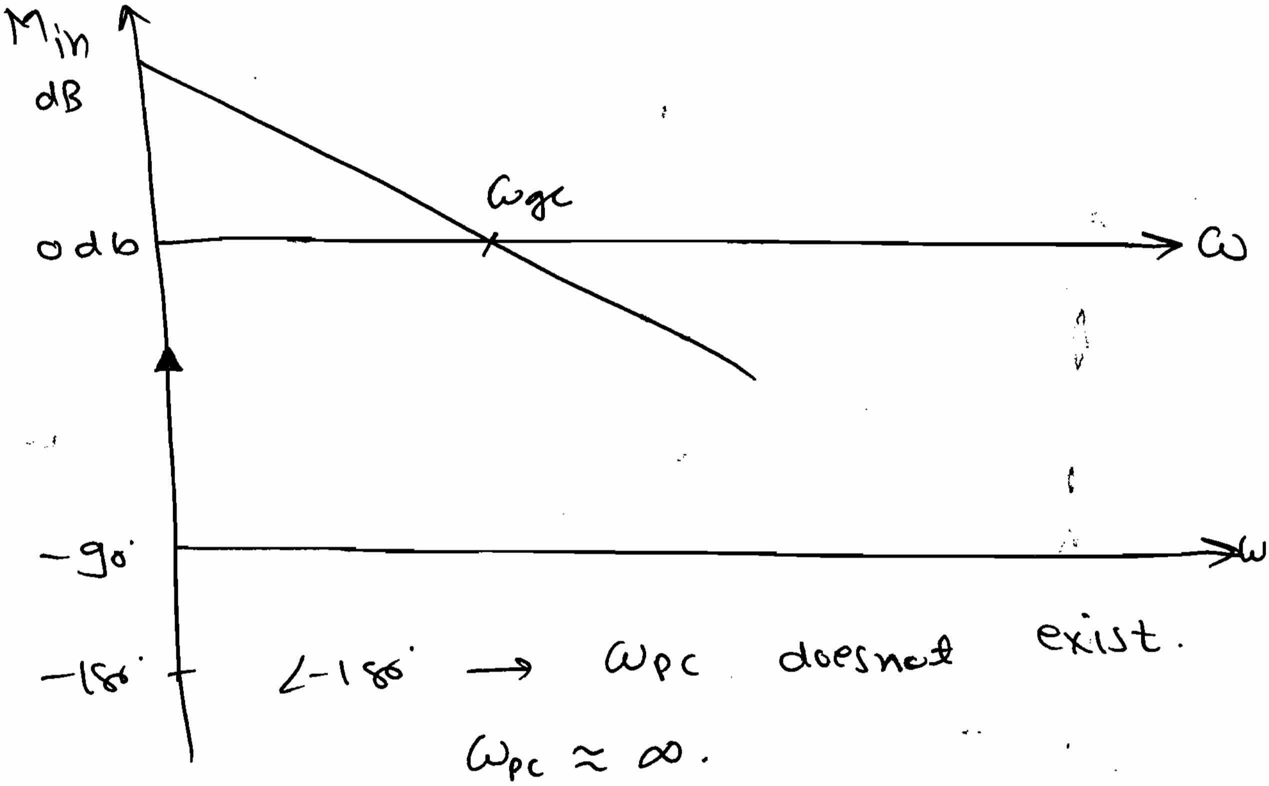
$\Rightarrow \quad \omega_{pc} < \omega_{gc} \longrightarrow$ US

$GM = -50\ (dB)$

$PM = 180^\circ - 250^\circ$

$PM = -70^\circ \quad \Rightarrow$ US.

4)



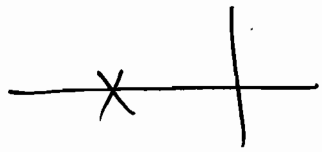
$\angle -180^\circ \longrightarrow \omega_{pc}$ doesnot exist.

$\omega_{pc} \approx \infty$.

$\omega_{pc} \gg \omega_{gc} \Rightarrow$ S.

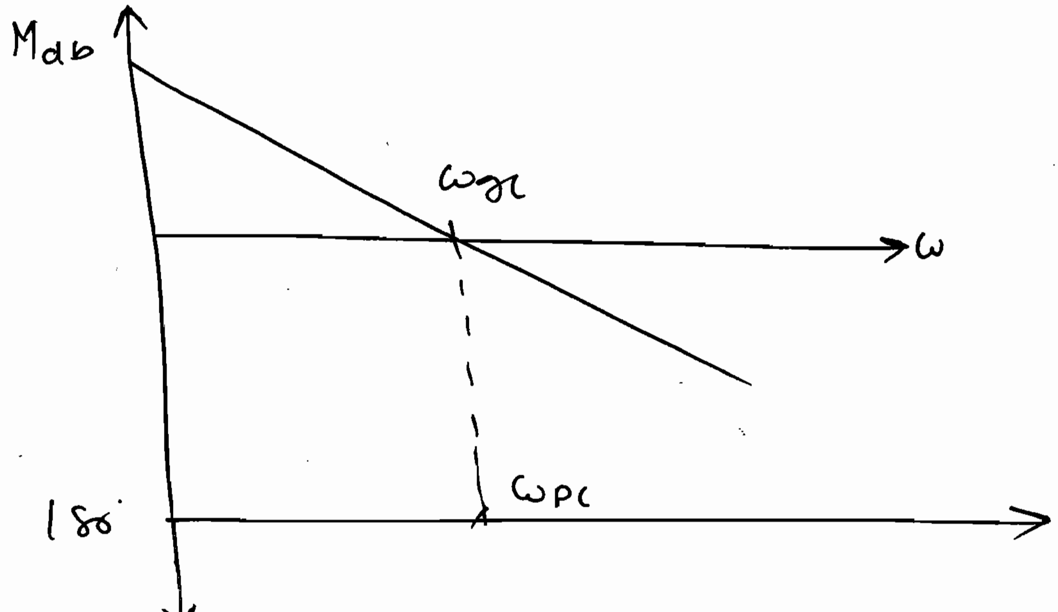
$G(s) = 1/s, \quad H(s) = 1.$

$CLTF = \dfrac{1}{s+1}.$



CL $\longrightarrow$ S.

5)



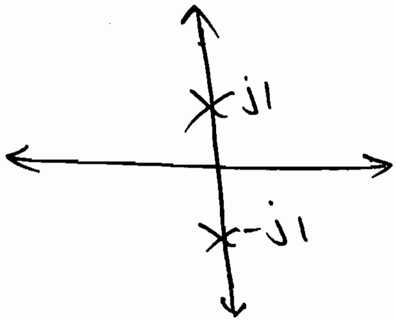
$\Rightarrow \omega_{pc} = 0$ to ∞

$\omega_{pc} = \omega_{gc} \Rightarrow$ MS.

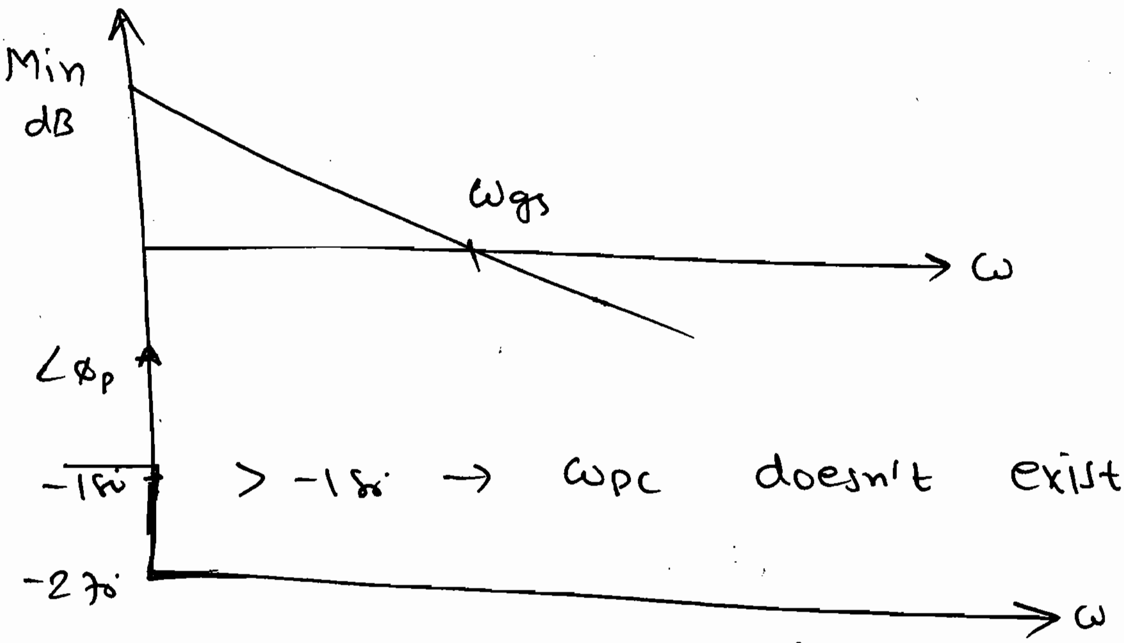
e.g.

$GH(s) = 1/s^2 \Rightarrow CLTF = \dfrac{1}{s^2+1}$.

CL $\Rightarrow$ MS



5)



$> -180° \rightarrow \omega_{pc}$ doesn't exist

$\omega_{pc} << \omega_{gc}$ $\quad \omega_{pc} \approx 0$.

$\longrightarrow$ US.

eg

$\rightarrow G(s) = \dfrac{1}{s^3}, \quad H(s) = 1$.

$CLTF = \dfrac{1}{s^3+1}$ terms missing $\Rightarrow$ US.

Note: Whenever the plot ~~are~~ (or) TF maintains less (-ve) than 180° at all the freq. range then the system is stable because here

$\omega_{pc} >> \omega_{gc}$. (Actually ω_{pc} does not exist but approximatelly infinity.)

$\Rightarrow$ Whenever the plot (or) TF maintains exactly equal to $-180°$ at all the freq. range then the system is marginal stable because here $\omega_{pc} = \omega_{gc}$.

$\Rightarrow$ Whenever the plot (or) TF maintains more (-ve) than $180°$ at all the freq. range then the system is unstable because here $\omega_{pc} < \omega_{gc}$. (Actually ω_{pc} does not exist but Approximatelly 0).

* Complex Bode Plot:

① n-Poles (complex):

$$\Rightarrow \quad G(s) \cdot H(s) = \left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \right)^n.$$

$s \to j\omega$

$$GH(j\omega) = \left(\frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n j\omega + \omega_n^2} \right)^n.$$

$$G_H(j\omega) = \left(\frac{\omega_n^2}{-\omega^2 + j2\xi\omega\omega_n + \omega_n^2} \right)^n$$

$$G_H(j\omega) = \left(\frac{1}{1-(\omega/\omega_n)^2 + j2\xi(\omega/\omega_n)} \right)^n$$

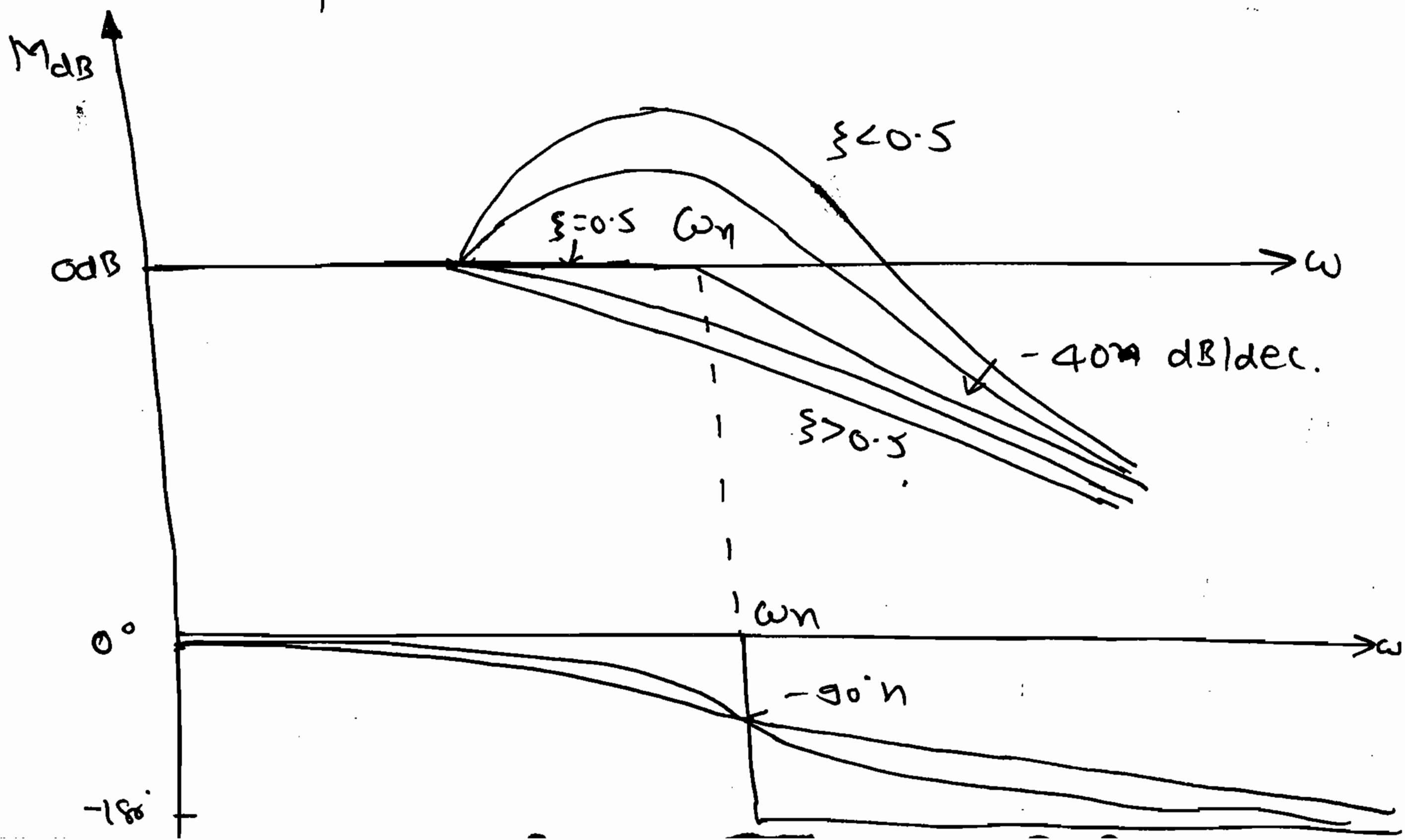
$$\therefore M = |G_H(j\omega)| = \left(\frac{1}{\sqrt{\{1-(\omega/\omega_n)^2\}^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \right)^n$$

$$\therefore M_{\text{in dB actual}} = -20^n \log \sqrt{\{1-(\omega/\omega_n)^2\}^2 + (2\xi\omega/\omega_n)^2}$$

$$\phi_{\text{actual}} = -n \tan^{-1}\left[\frac{2\xi\,\omega/\omega_n}{1-(\omega/\omega_n)^2}\right].$$

$\omega_n \longrightarrow$ Corner freq.

	S	ϕ
$<$ CF	0 dB	0°
$>$ CF	-40^n dB/dec	$-180^\circ n$



$\Rightarrow$ Correction at Corner freq.

$$M_{\substack{\text{correction} \\ (\omega=\omega_n)}} = -20n\log 2\xi.$$

$$\phi_{\substack{\text{correction} \\ (\omega=\omega_n)}} = -90^\circ n.$$

$\Rightarrow$ The correction at corner freq. depends on ξ in the Magnitude plot where in the phase plot the correction at corner freq. is other than ~~the~~ ~~correction~~ ~~depends~~ ~~on~~ ~~$\xi\omega_n$~~ is constant other than corner freq. the correction depends on $\xi\omega_n$.

② **n- Complex Zeros:**

$$\Rightarrow \quad G_H(s) = \left(\frac{s^2 + 2\xi\omega_n s + \omega_n^2}{\omega_n^2}\right)^n$$

$s \to j\omega$

$$G_H(j\omega) = \left(1 - (\omega/\omega_n)^2 + j\,2\xi(\omega/\omega_n)\right)^n.$$

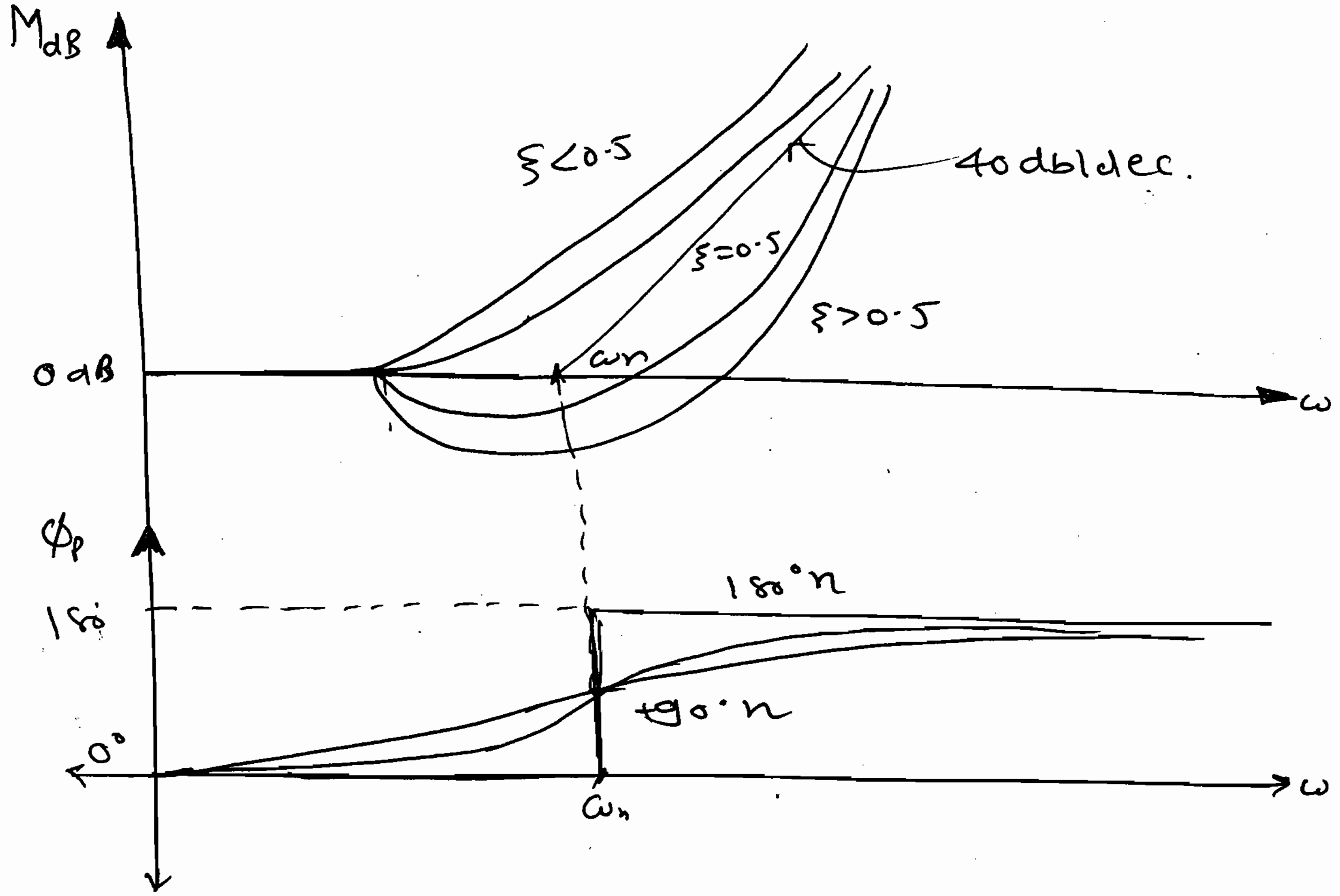
$$\therefore M = |G_H(j\omega)| = \left(\sqrt{\{1-(\omega/\omega_n)^2\}^2 + (2\xi\,\omega/\omega_n)^2}\right)^n$$

$$M_{dB} = 20n\log\sqrt{\{1-(\omega/\omega_n)^2\}^2 + (2\xi\omega/\omega_n)^2}.$$

$$\Rightarrow \quad \phi_{actual} = n \tan^{-1}\left[\frac{2\xi\omega/\omega_n}{1-(\omega/\omega_n)^2}\right].$$

$\omega_n \longrightarrow$ corn freq.

	M	ϕ
$<$CF	0 dB/dec	$0°$
$>$CF	$+40n$	$+180°n$



$\Rightarrow$ <u>Correction at Corner freq.</u> :-

$$M_{correction \text{ at CF } (\omega=\omega_n)} = +20n\log(2\xi).$$

$$\phi_{correction \text{ at CF } (\omega=\omega_n)} = +90°n.$$

Q Draw the Bode plot to the given system.

$$GH(s) = \frac{s^2 \left(1 + s/20 + \frac{s^2}{100}\right)^4}{\left(1 + s/3 + s^2/9\right)^3 \left(1 + s/50\right)^4}.$$

Soln:

$\omega_{n_1}^2 = 9$

$\omega_{n_1} = 3$ rad/sec.

$2\xi_1 \frac{\cancel{\omega}}{\omega_n} = \cancel{\omega}/3$

$\frac{2\xi_1}{3} = 1/3$

$\boxed{\xi_1 = 0.5}$

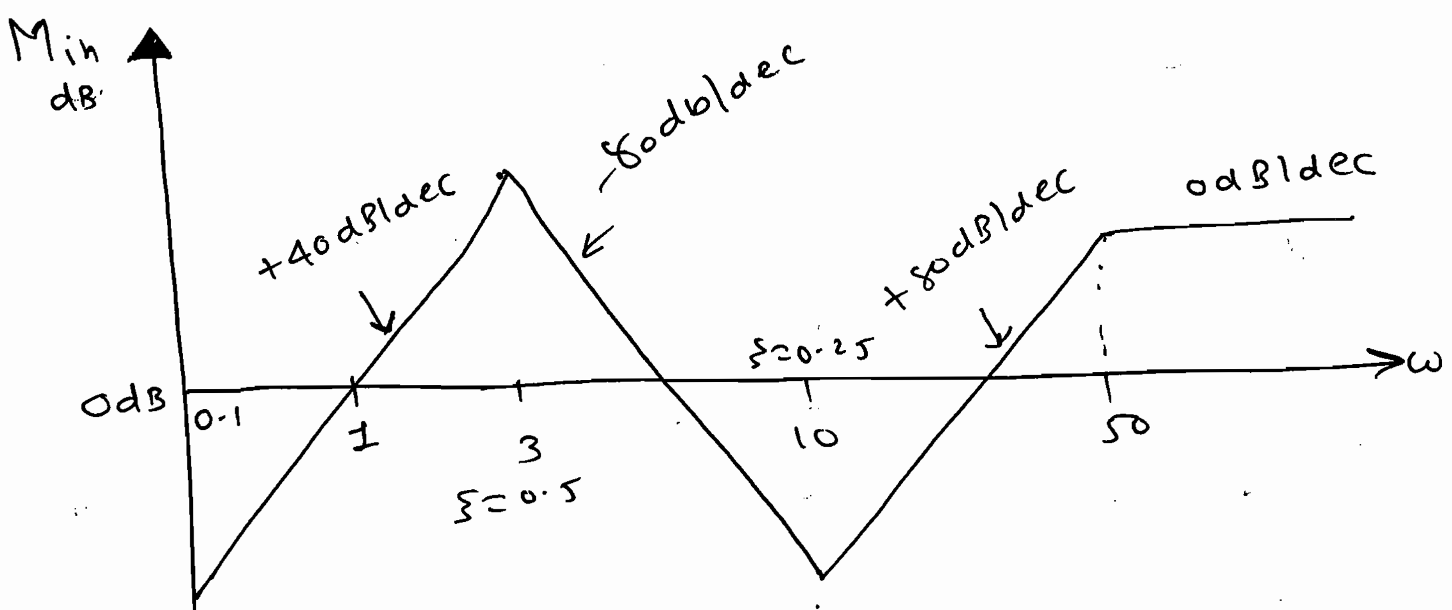
$\omega_{n_2}^2 = 100$

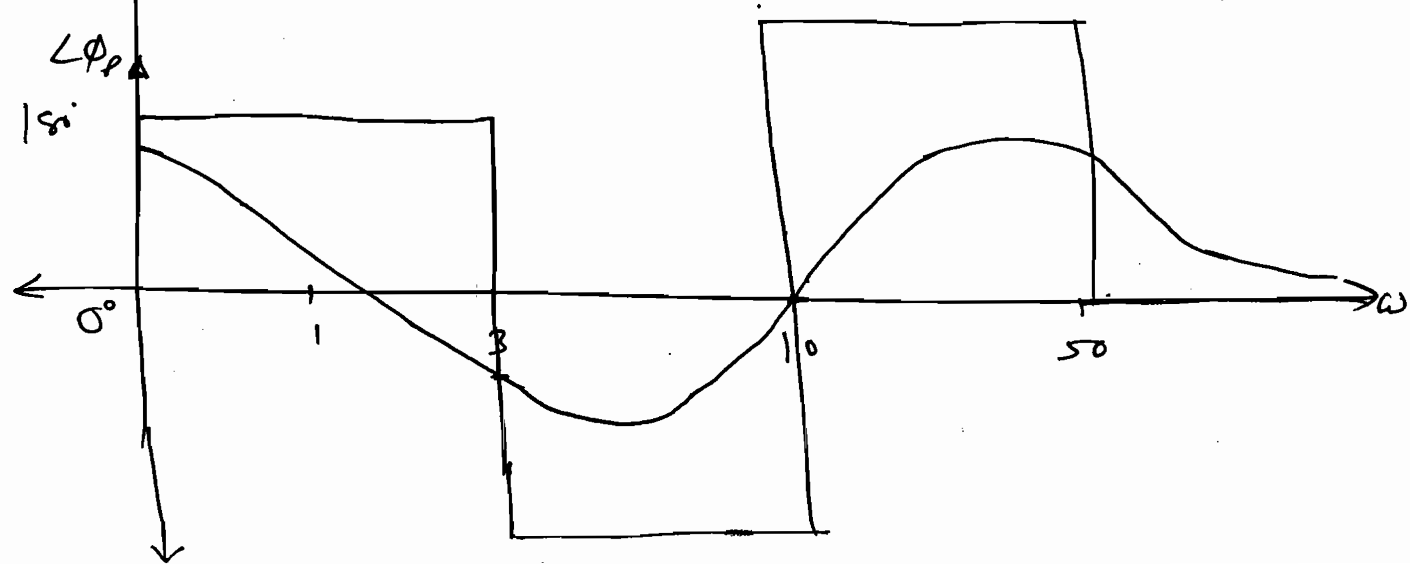
$\omega_{n_2} = 10$ rad/sec.

$\frac{2\xi_2}{\omega_n} = \frac{1}{20}$

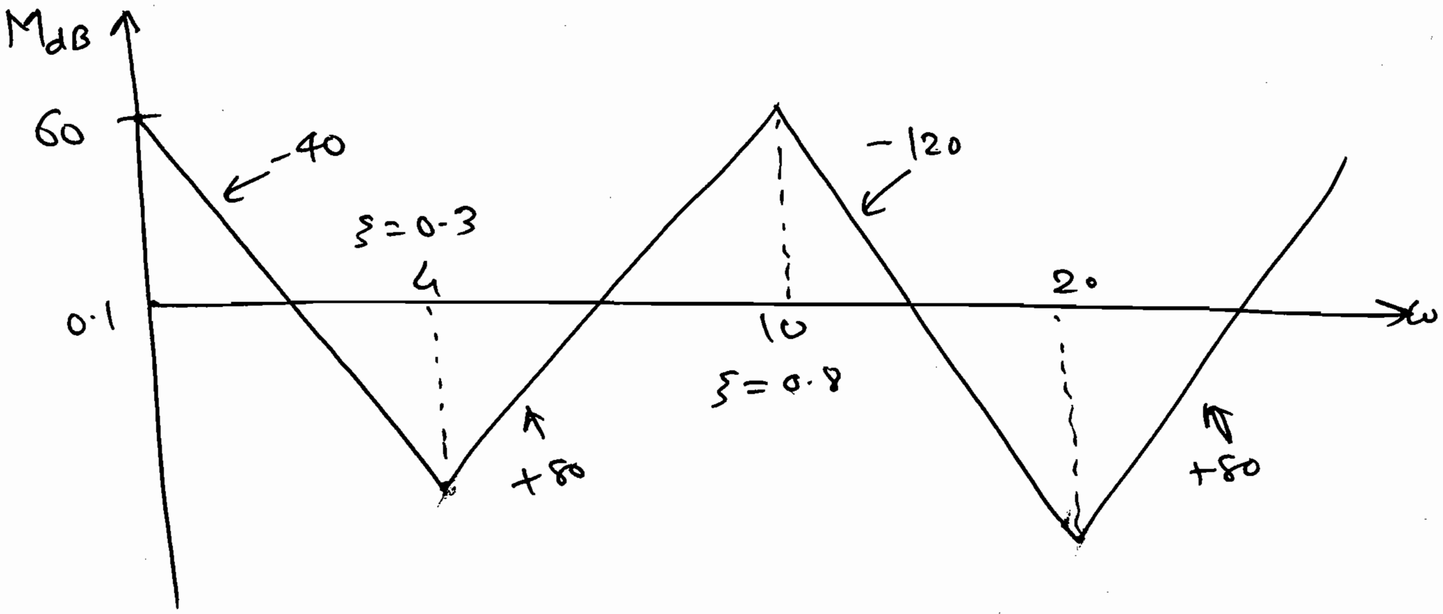
$\therefore \boxed{\xi_2 = 0.25}$

$\boxed{\omega_{n_3} = 50 \text{ rad/sec}}$





Q. Find the TF to the given asymptotic Mag. plot.



$$GH(s) = K \frac{\left(1 + \frac{2(0.3)s}{4} + \frac{s^2}{16}\right)^3 \left(1 + s/20\right)^{10}}{s^2 \left(\frac{s^2}{100} + \frac{2(0.8)s}{10} + 1\right)^5}$$

$$M_{dB}\Big|_{\omega = 0.1} = 60 \text{ dB.}$$

$$\Rightarrow \quad 60 \text{ dB} = 20 \log K - 40 \log (0.1).$$

$$20 = 20 \log K$$

$$\log = 1$$

$$\Rightarrow \quad \boxed{K = 10}$$

$$\Rightarrow \quad \boxed{TF = GH(s) = \frac{10 \left(\frac{s^2}{16} + \frac{0.6}{4} s + 1\right)^3 \left(1 + s/20\right)^{10}}{s^2 \left(\frac{s^2}{100} + 1.6s + 1\right)^5}}$$

☆ Polar Plots :-

* Purpose:

⇒ To draw the Freq. response of the OLTF.

⇒ To find the CL system stability.

⇒ To find the gain margin and phase Margin.

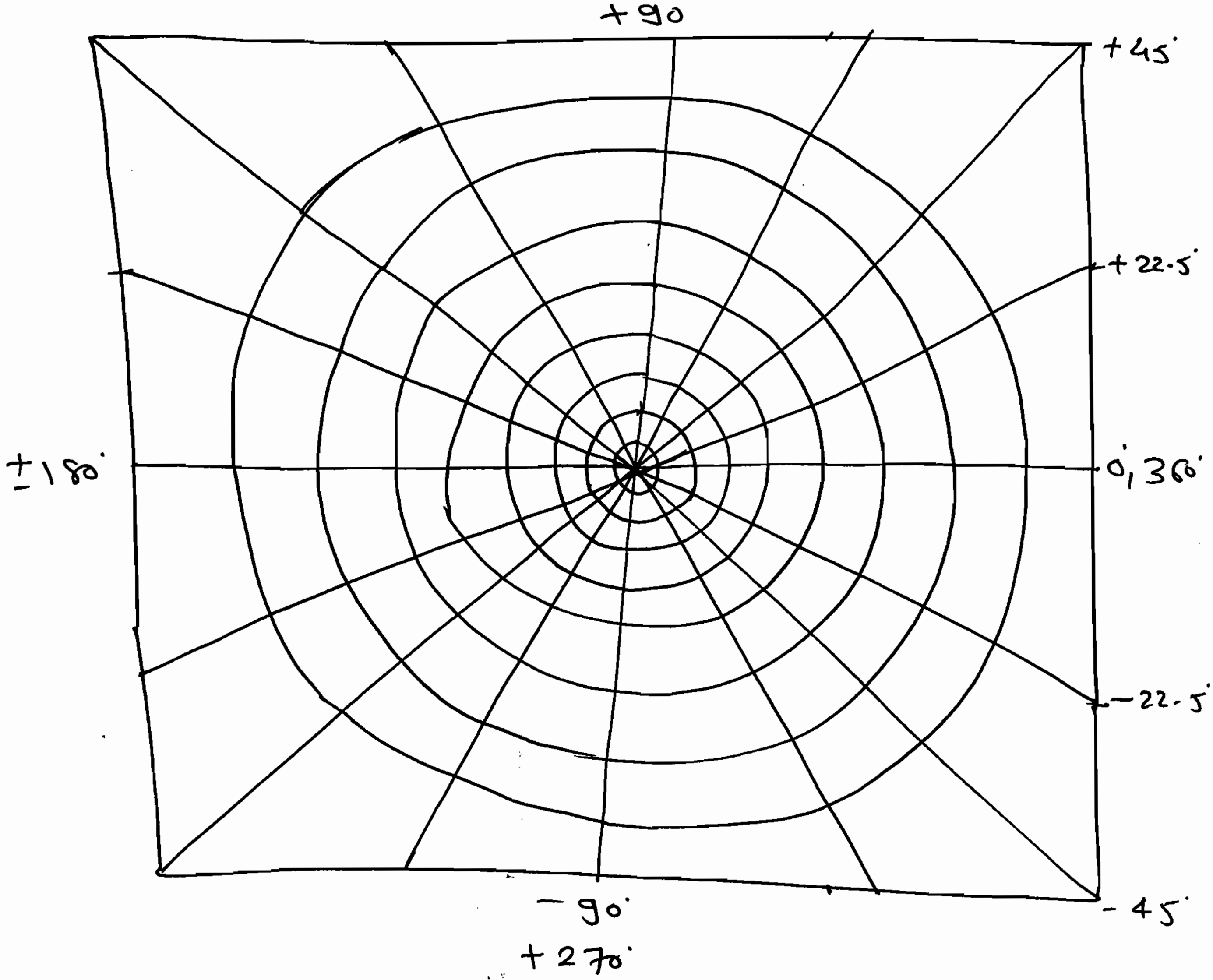
⇒ The Polar plots are used in the Nyquist plots.

⇒ The Polar plots is not a Complete freq. response plots. The Complete freq. response plots are Nyquist plots.

⇒ The freq. range for Polar plot is 0 to ∞ where as for Nyquist plot the freq. range is $-\infty$ to $+\infty$.

⇒ The Polar plot is nothing but the Mag. verses Phase plot.

$\Rightarrow$



$\boxed{Q}$ Draw the Polar plot for $GH(s) = 1/s$.

Soln:

$$GH(s) = \frac{1}{s}.$$

$$s \rightarrow j\omega$$

$$GH(j\omega) = \frac{1}{j\omega}.$$

$$M = |GH(j\omega)| = \frac{1}{\omega}. \quad (2).$$

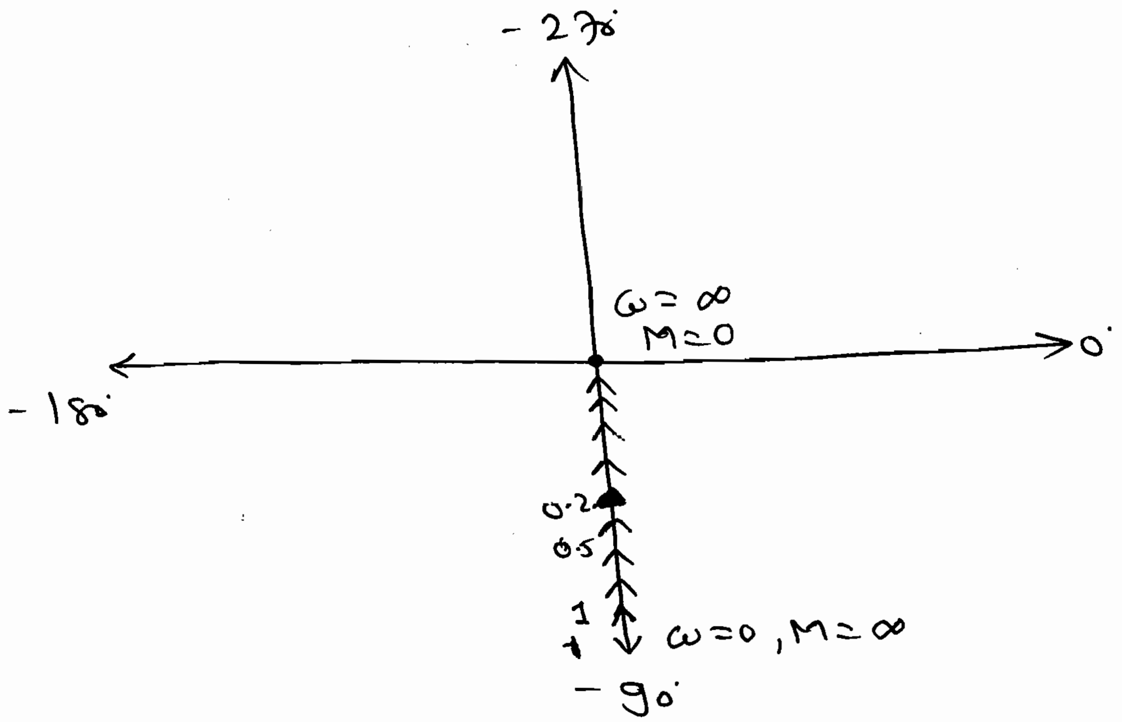
$$M_{in\,dB} = -20\log\omega.$$

$$\angle GH(j\omega) = -90°.$$

$\Rightarrow$

ω	M	ϕ
0	∞	$-90°$
1	1	$-90°$
2	0.5	$-90°$
5	0.2	$-90°$
10	0.1	$-90°$
$\vdots$	$\vdots$	$\vdots$
∞	0	$-90°$

$\Rightarrow$



$\boxed{e}$ $GH(s) = \dfrac{1}{(s\tau+1)}$.

Soln:

$$GH(j\omega) = \frac{1}{(j\omega\tau+1)}$$

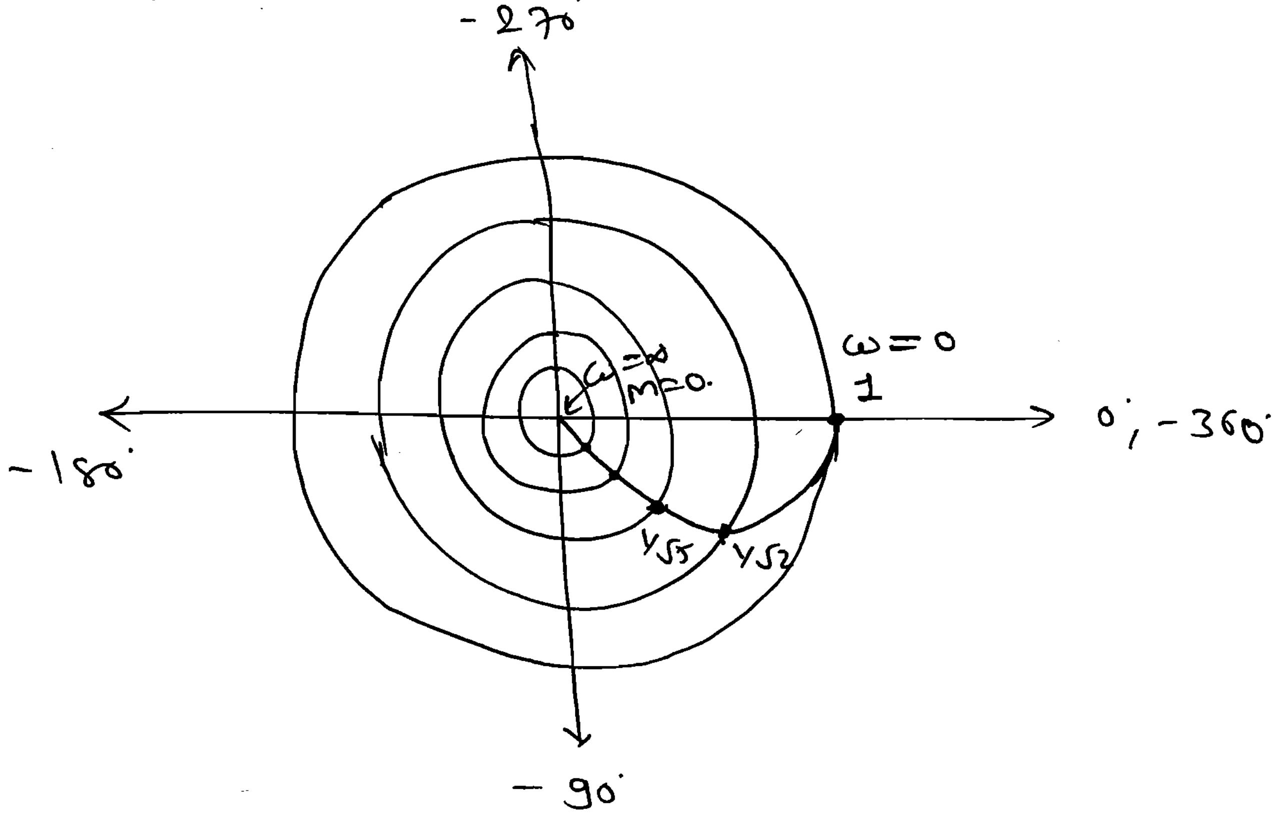
$$M = |GH(j\omega)| = \frac{1}{\sqrt{(\omega\tau)^2+1}}$$

$$\angle GH = \phi = -\tan^{-1}(\omega\tau).$$

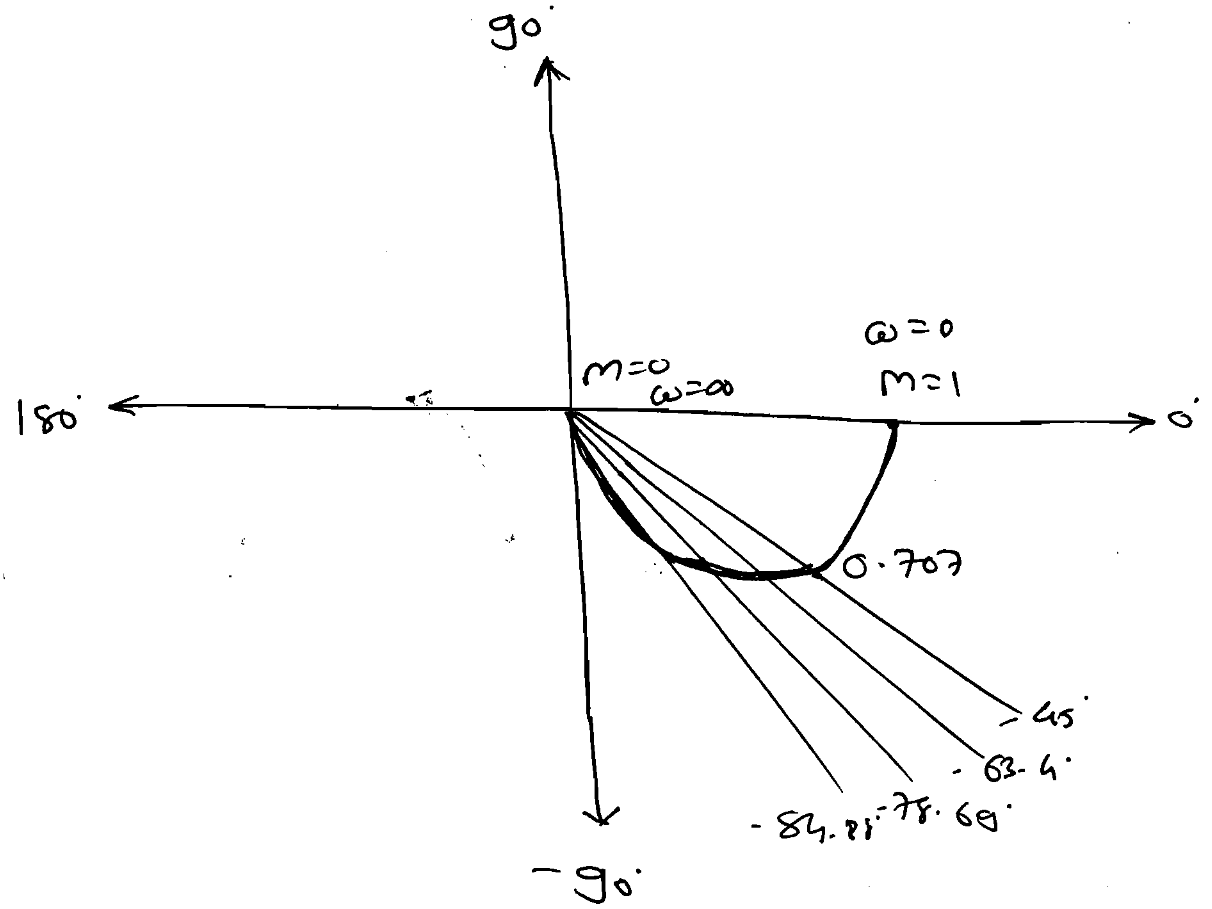
ω	M	ϕ
0	1	0°
$1/\tau$	$1/\sqrt{2}$	$135°$ $(-45°)$.
$2/\tau$	$1/\sqrt{5}$	-63.43

ω	M	ϕ
$5/\tau$	$1/\sqrt{26}$	-78.69°
$10/\tau$	$1/\sqrt{101}$	-84.28°
$\vdots$	$\vdots$	$\vdots$
∞	0	-90°

$\Rightarrow$



$\Rightarrow$



Q Draw the Polar plots:

① $GH(s) = \dfrac{(s+1)}{(s+10)}$ (HPF, Lead Comp.).

Sol^n:

$$GH(j\omega) = \frac{(1+j\omega)}{(10+j\omega)}$$

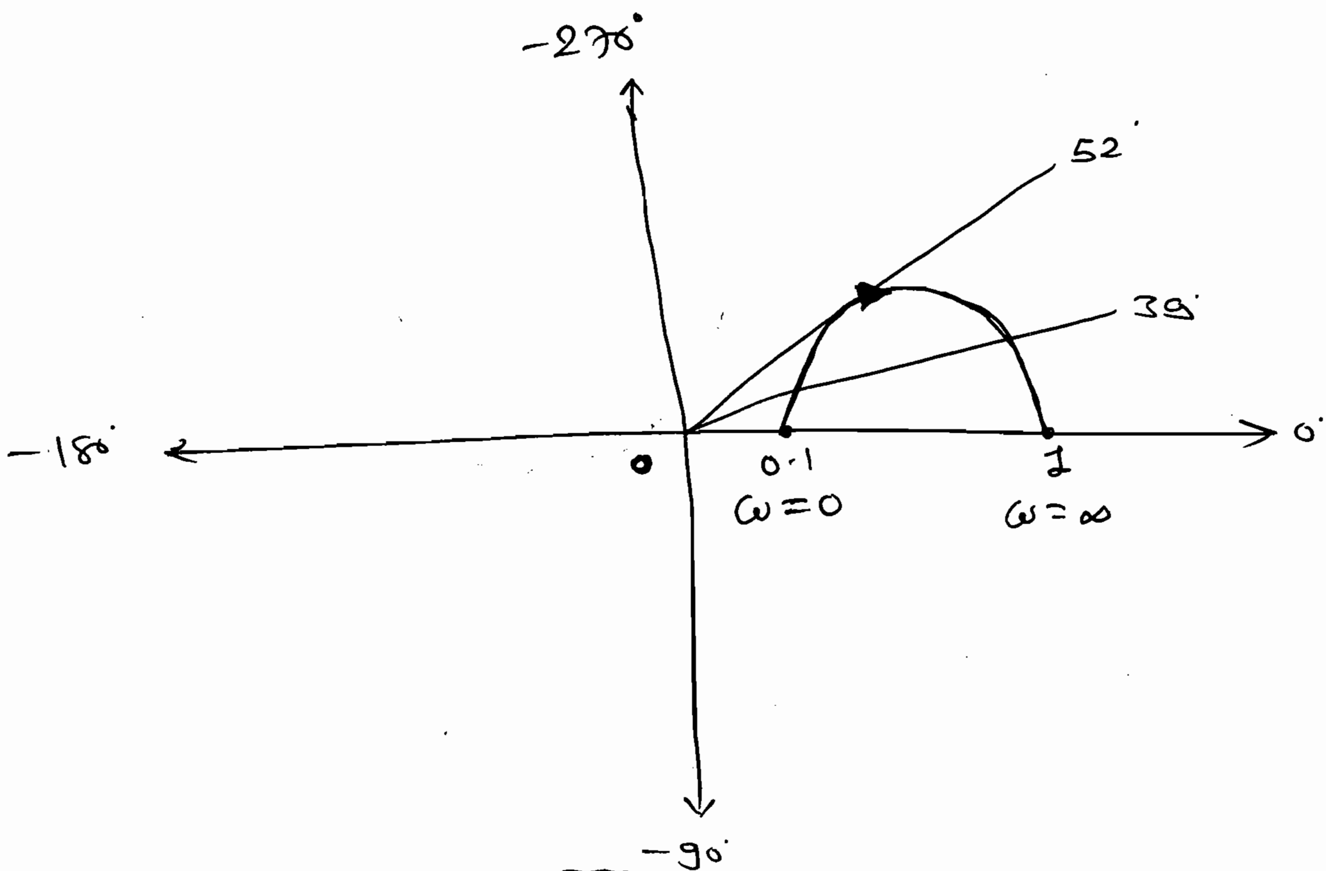
$$\Rightarrow \quad M = \frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}} \quad , \quad \phi = \tan^{-1}(\omega) - \tan^{-1}(\omega/10)$$

ω	M	ϕ
0	0.1	0°
1	0.141	39.29°
2	0.22	52.125°
5	0.456	52.125°
10	0.710	39.29°
⋮		
∞	1	0°

$$\lim_{\omega\to\infty} \frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}}$$

$$= \lim_{\omega\to\infty} \sqrt{\frac{1+1/\omega^2}{1+100/\omega^2}}$$

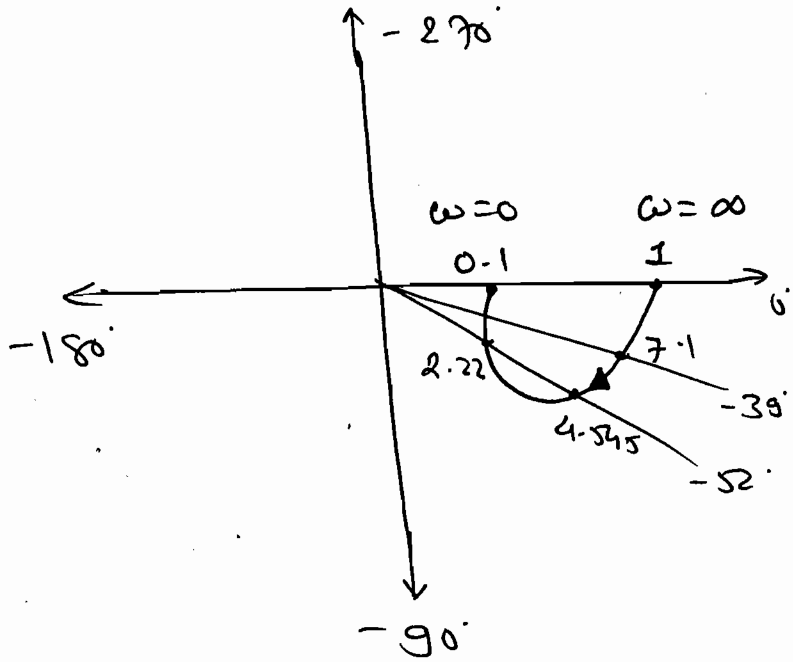
$$= \sqrt{\frac{1+0}{1+0}} = 1.$$



(a) $GH(s) = \dfrac{(s+10)}{(s+1)}$ (LPF Lag Cmp.).

Soln: $M = \dfrac{\sqrt{\omega^2+100}}{\sqrt{\omega^2+1}}$, $\phi = \tan^{-1}(\omega/10) - \tan^{-1}(\omega)$.

ω	M	ϕ
0	10	0°
1	7.1	-39°
2	4.545	-52°
5	2.22	-52°
10	0.14	-39°
⋮	⋮	
∞	1	0



* Procedure to draw the Polar plots:

Step-1:

⇒ Find the M_1 and ϕ_1 at $\omega=0$.

Step-2:

⇒ Find the M_2 and ϕ_2 at $\omega=\infty$.

Step-3:

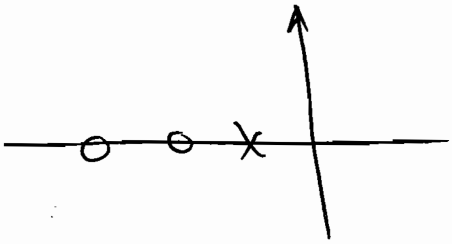
⇒ Ending direction

$\phi_1 - \phi_2$ = +ve → CW.
= -ve → ACW.

Step-4: Starting Direction:

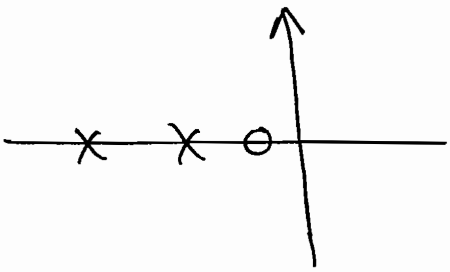
⇒ The starting direction is consider to the TF if it should have only +ve sign terms.

⇒ If the finite pole is near to the imaginary axis then the S.D. is Cω.



$f_p \rightarrow C\omega$.

⇒ If finite zero near to imaginary then the S.D. is ACω.



$f_z \rightarrow AC\omega$.

⇒ The above procedure is valid when M at ~~origin~~ i.e. $\omega = 0$ is greater or equal to M at $\omega = \infty$.

$$\boxed{M\Big|_{\omega=0} \geq M\Big|_{\omega=\infty}}$$

⇒ If the $M\big|_{\omega=0} < M\big|_{\omega=\infty}$ like HPF (when TF consist only zeros (or) zeros at origin and poles = zero) check the Mag.)

⇒ Like HPF draw the polar plot using standard procedure.

Q Draw the Polar Plots for $GH(s) = \frac{1}{s+1}$.

Soln:

$$GH(s) = \frac{1}{s+1}.$$

$s \to j\omega$

$$\Rightarrow GH(j\omega) = \frac{1}{j\omega+1}.$$

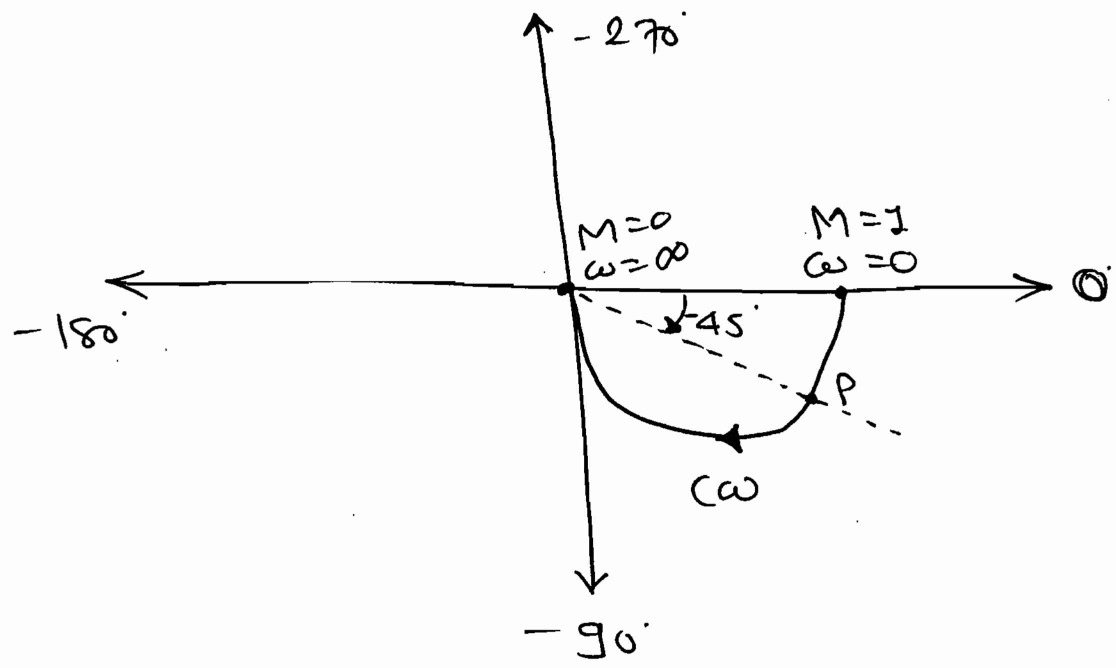
$$M = \frac{1}{\sqrt{\omega^2+1}}, \quad \angle\phi = -\tan^{-1}(\omega).$$

ω	M	ϕ
0	1	0°
∞	0	-90°

$\phi_1 - \phi_2 = 0 - (-90^\circ) = 90^\circ$

$= +ve \Rightarrow$ E.D. $\Rightarrow$ CW.

f-P $\Rightarrow$ S.D. $\Rightarrow$ CW.



$\Rightarrow$ given $\phi = -45^\circ$.

$\therefore -45^\circ = -\tan^{-1}(\omega)$

$\therefore$ $\boxed{\omega = 1 \text{ rad/sec}}$

Rectangular

$\Downarrow$

$\boxed{R\left(\frac{1}{2}, -\frac{1}{2}\right)}$

$$\Rightarrow M\Big|_{\omega=1} = \frac{1}{\sqrt{2}} = 0.707$$

$\Rightarrow$ I.P. $\boxed{P\left(\frac{1}{\sqrt{2}} \angle -45^\circ\right) \Rightarrow \text{polar}}$

$\boxed{a}$ $GH(s) = \dfrac{1}{(s+1)(s+2)}$.

Sol^n:

$s \to j\omega$

$$GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}.$$

$$M = |GH(j\omega)| = \frac{1}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}.$$

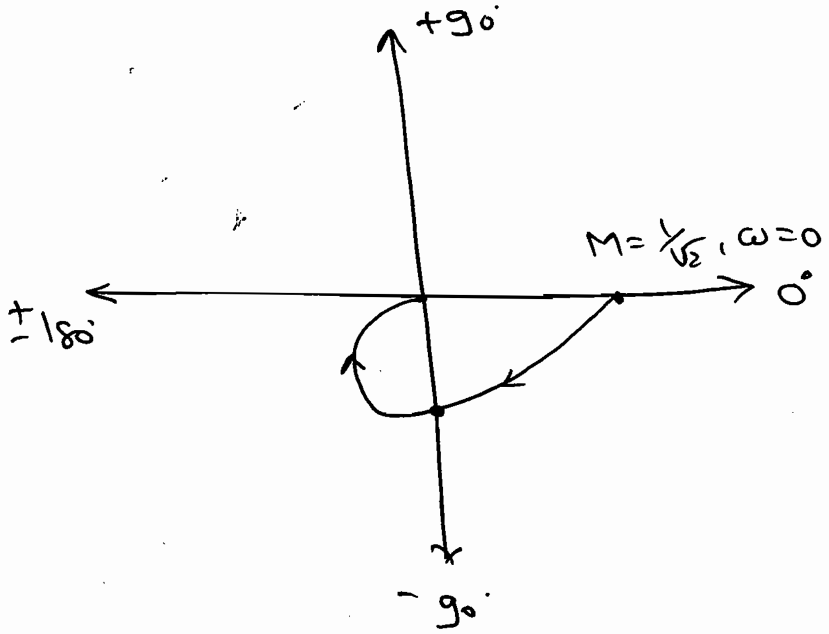
$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2).$$

$\omega = 0 \Rightarrow M = \frac{1}{2}$. & $\phi_1 = 0^\circ$.

$\omega = \infty \Rightarrow M = 0$ & $\phi_2 = -90^\circ - 90^\circ = -180^\circ$.

E.D. $\Rightarrow$ $\phi_1 - \phi_2 = 0 - (-180^\circ) = +180^\circ \Rightarrow$ CW.

S.D. $\Rightarrow$ FP $\Rightarrow$ CW.



$\Rightarrow$ Intersection Point with -90°.

$\therefore$ $\phi = -90^\circ$

$\Rightarrow -90^\circ = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\Rightarrow 90^\circ = \tan^{-1}(\omega) + \tan^{-1}(\omega/2)$.

$$\Rightarrow \quad 90^\circ = \tan^{-1}\left(\frac{\omega + \omega/2}{1 - \omega^2/2}\right).$$

$$\Rightarrow \quad \tan 90^\circ = \frac{\omega + \omega/2}{1 - \omega^2/2}.$$

$$\Rightarrow \quad \infty = \frac{3\omega}{2-\omega^2}.$$

$$\therefore \quad 2-\omega^2 = 0$$

$$\Rightarrow \quad \omega^2 = 2 \quad \Rightarrow \quad \boxed{\omega = \sqrt{2} \text{ rad/sec}}$$

$$M\Big|_{\omega=\sqrt{2}} = \frac{1}{\sqrt{2+1}\,\sqrt{4+2}} = \frac{1}{\sqrt{18}}.$$

$$\Rightarrow \quad \text{Polar} \quad \frac{1}{\sqrt{18}} \angle -90^\circ$$

$$\Rightarrow \quad \text{Rect} \quad \left(0,\ j\frac{1}{\sqrt{18}}\right).$$

$\boxed{a}$ $$GH = \frac{1}{(s+1)(s+2)(s+3)}.$$

Sol^n:
$$M = |GH(j\omega)| = \frac{1}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}.$$

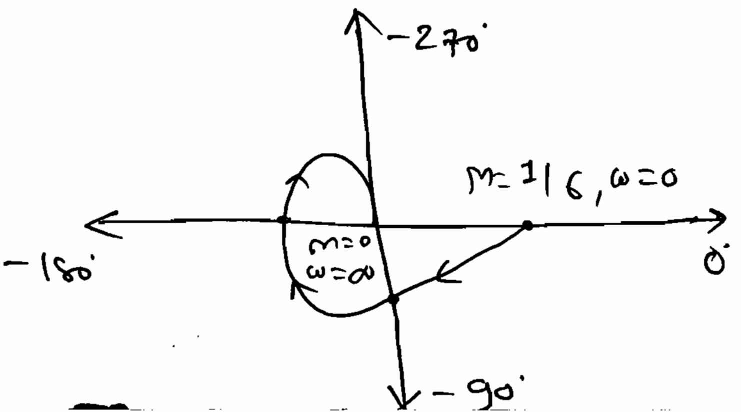
$$\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega/3).$$

$$\omega = 0 \Rightarrow \quad M = \frac{1}{\sqrt{36}} = 1/6., \quad \phi = 0^\circ$$

$$\omega = \infty \Rightarrow \quad M = 0 \quad \& \quad \phi = -90^\circ - 90^\circ - 90^\circ = -270^\circ.$$

E.D. $\Rightarrow$ $\phi_1 - \phi_2 = +270^\circ \Rightarrow +ve$
$= CW$

S.D. $\Rightarrow$ finite
pole $\Rightarrow$ CW



$\Rightarrow$ Intersection pt with $(-90°)$.

$$\phi = -90° \Rightarrow -90° = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$$

$$\Rightarrow 90° = \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\frac{\omega}{2} + \omega/3}{1 - \omega^2/6}\right).$$

$$\therefore 90° = \tan^{-1}(\omega) + \tan^{-1}\left(\frac{5\omega}{6-\omega^2}\right).$$

$$\therefore 90° = \tan^{-1}\left(\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \frac{5\omega^2}{6-\omega^2}}\right).$$

$$\therefore \infty = \frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - 5\omega^2/6-\omega^2}$$

$$\Rightarrow 1 - \frac{5\omega^2}{6-\omega^2} = 0 \Rightarrow 6-\omega^2 - 5\omega^2 = 0$$

$$\boxed{\omega = 1 \text{ rad/sec}}$$

$\Rightarrow$ I.P. with $-180°$

$$\Rightarrow -180° = -\tan^{-1}\left(\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \frac{5\omega^2}{6-\omega^2}}\right).$$

$$\Rightarrow \omega + \frac{5\omega}{6-\omega^2} = 0.$$

$$\therefore 6\omega - \omega^3 + 5\omega = 0$$

$$\therefore \omega^2 = 11$$

$$\boxed{\omega = \sqrt{11} \text{ rad/sec}}$$

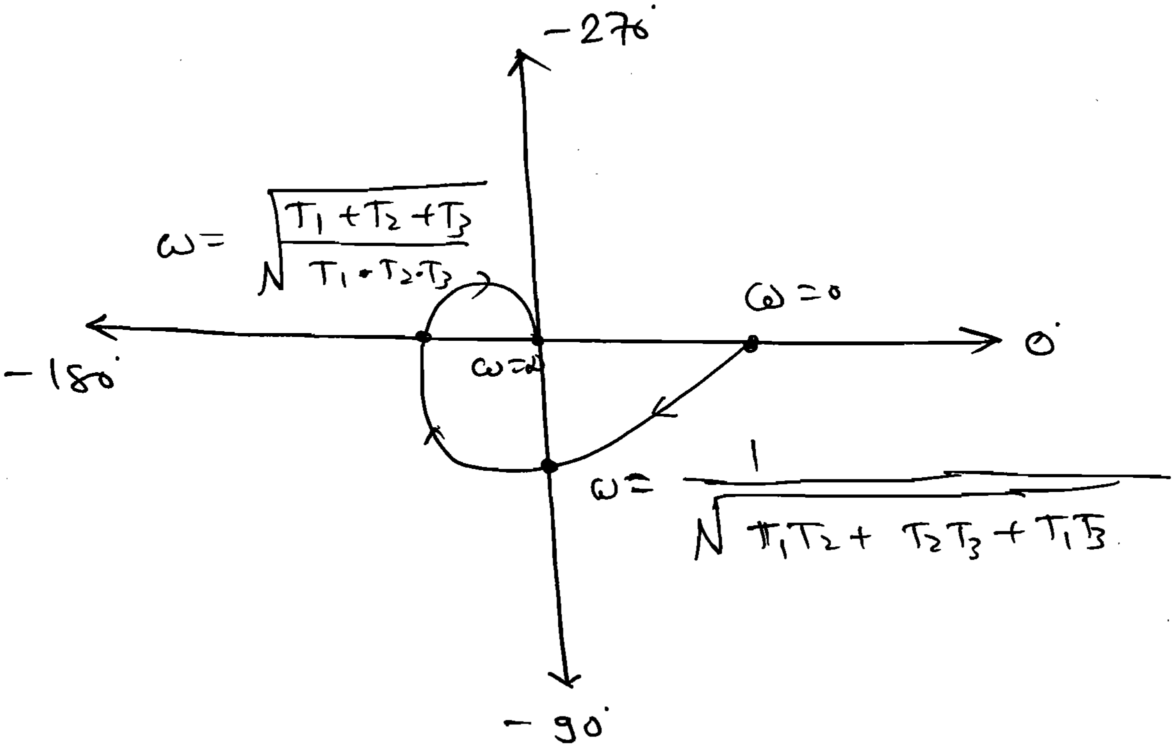
$$\Rightarrow M\Big|_{\omega=1} = \frac{1}{\sqrt{2} \times \sqrt{5}\sqrt{10}} = \frac{1}{\sqrt{100}} = 0.1$$

$\Rightarrow$ I.P. $(0, -j0.1)$.

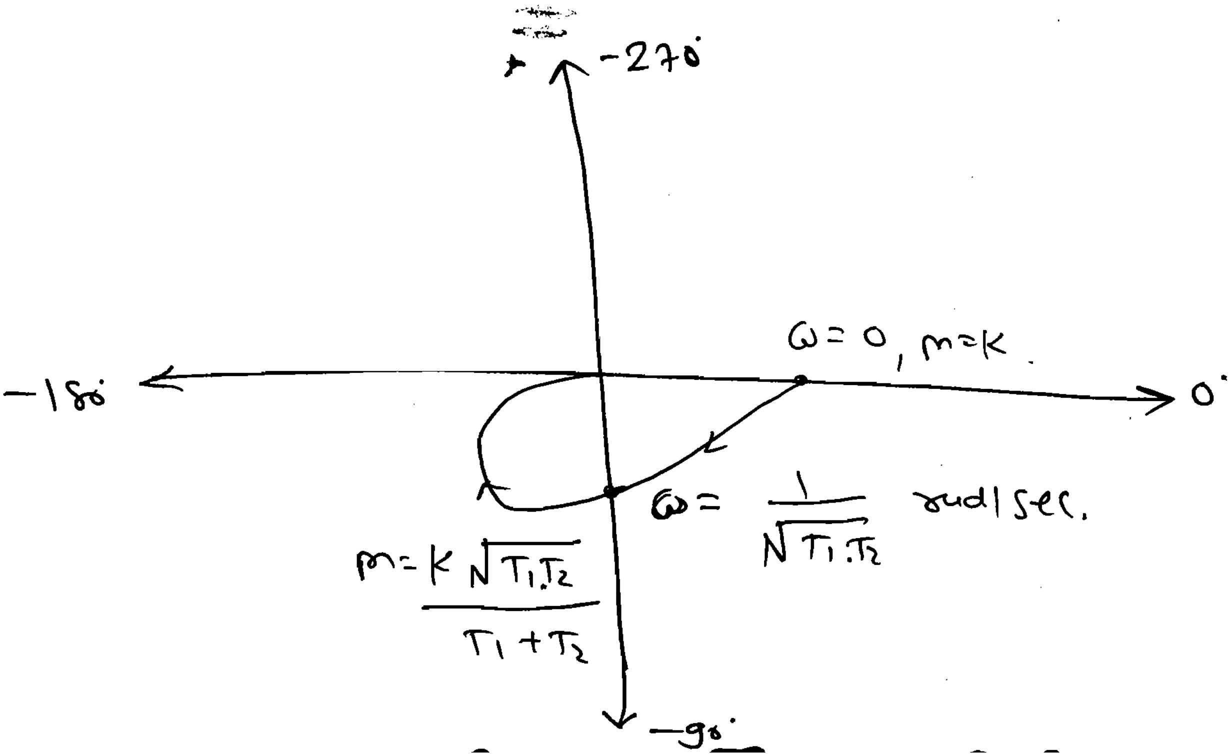
$$\Rightarrow \quad M\Big|_{\omega=\sqrt{11}} = \frac{1}{\sqrt{12}\times\sqrt{16}\times\sqrt{20}} = \frac{1}{60}.$$

I.P. $\left(-\frac{1}{60},\ -j0\right)$.

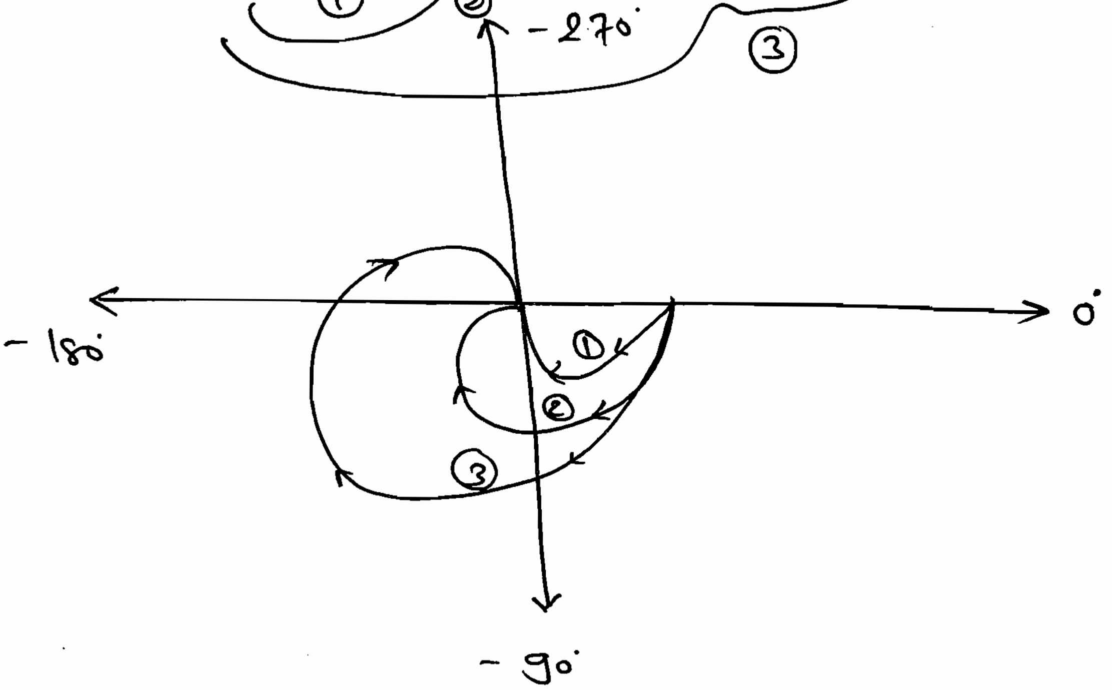
$$* \quad GH = \frac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$$



$$* \quad GH(s) = \frac{K}{(sT_1+1)(sT_2+1)}$$



* $GH(s) = \dfrac{K}{\underbrace{\underbrace{\underbrace{(sT_1+1)}_{①}(sT_2+1)}_{②}(sT_3+1)}_{③}}$



Note:

⇒ The addition of each finite pole in the left hand side shift Ending angle by $-90°$, in the clock-wise direction.

[a] $GH(s) = \dfrac{1}{s(s+1)}$.

Soln:

$|GH(\omega)| = M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$

⇒ $\phi = -90° - \tan^{-1}(\omega)$.

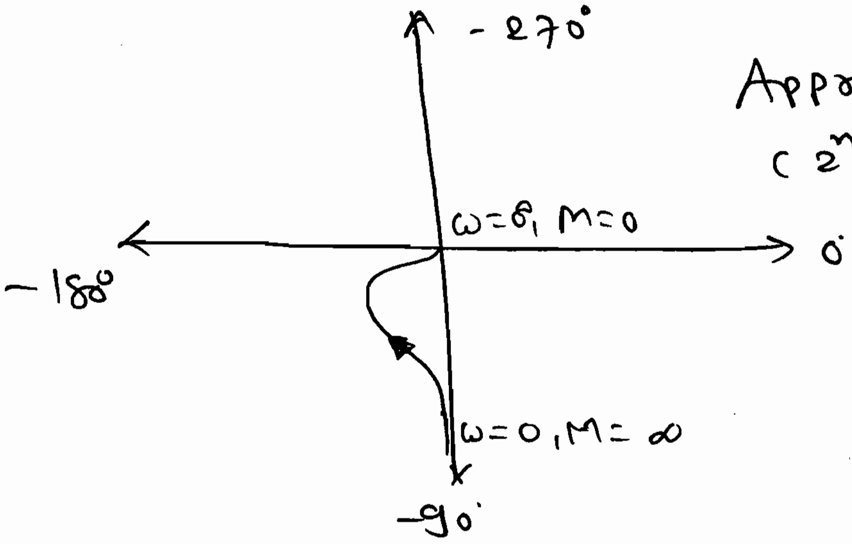
$\omega = 0 \Rightarrow M = \infty$ & $\phi = -90°$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -180°$

E.D. ⇒ $\phi_1 - \phi_2 = -90° + 180° = 90° = CW$

S.D. ⇒ b/p ⇒ CW.

⇒



Approx
(2nd priority).

⇒ $$GH = \frac{1}{j\omega(j\omega+1)} \times \frac{1-j\omega}{1-j\omega}$$

$$= \frac{1-j\omega}{j\omega(1+\omega^2)} = \frac{-j(1-j\omega)}{\omega(1+\omega^2)}$$

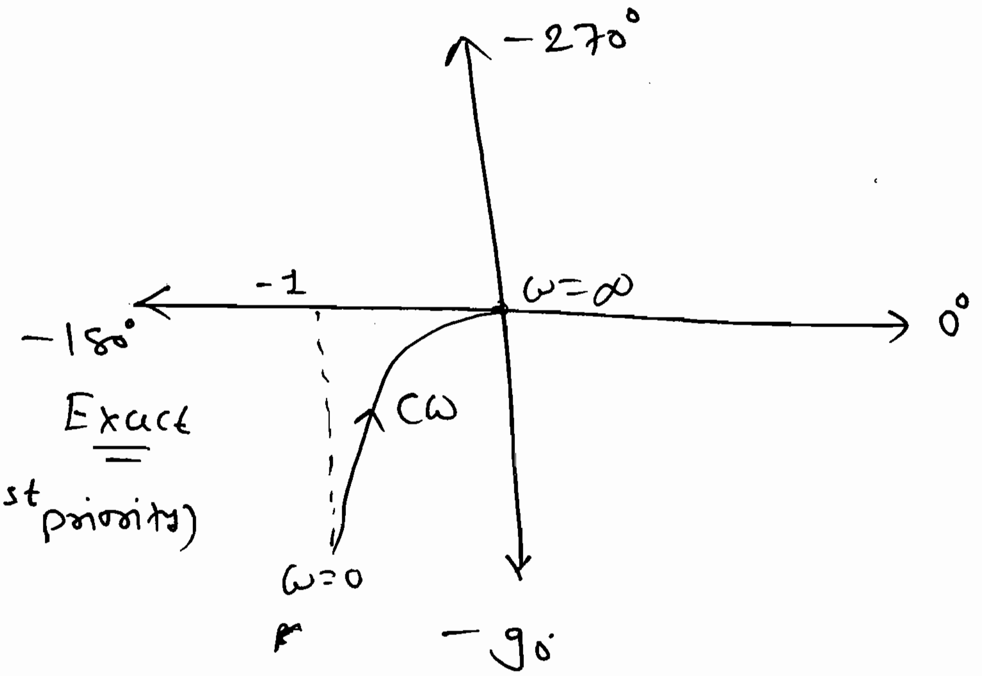
$$= \frac{-\omega - j(1)}{\omega(1+\omega^2)}$$

$$GH = \frac{-\cancel{\omega}}{\cancel{\omega}(1+\omega^2)} - \frac{j}{\omega(1+\omega^2)}$$

⇒ $\omega = 0$ $\quad GH = -1 - j(\infty)$.

$GH = -1 - j\infty$.

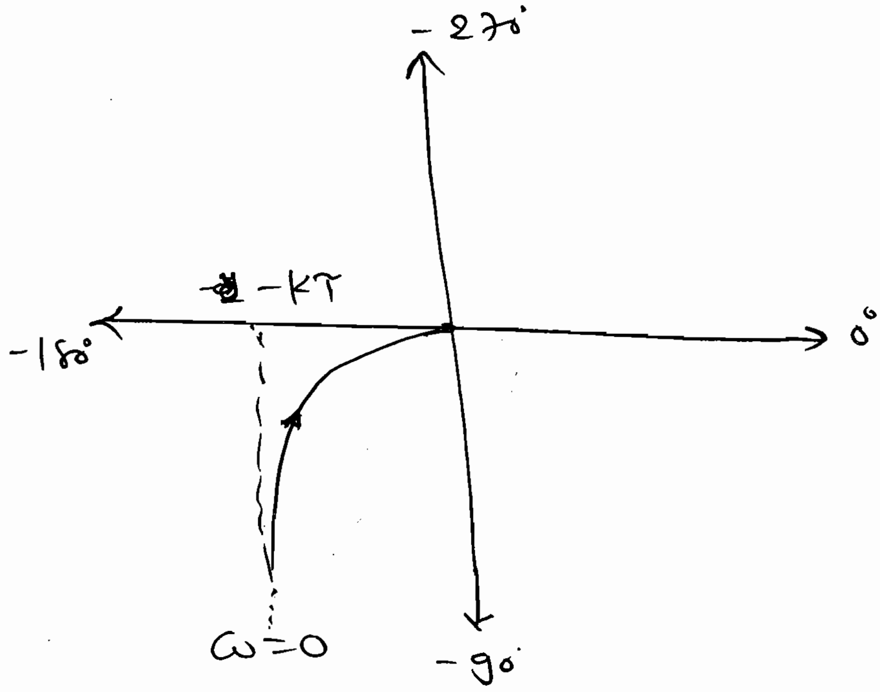
⇒



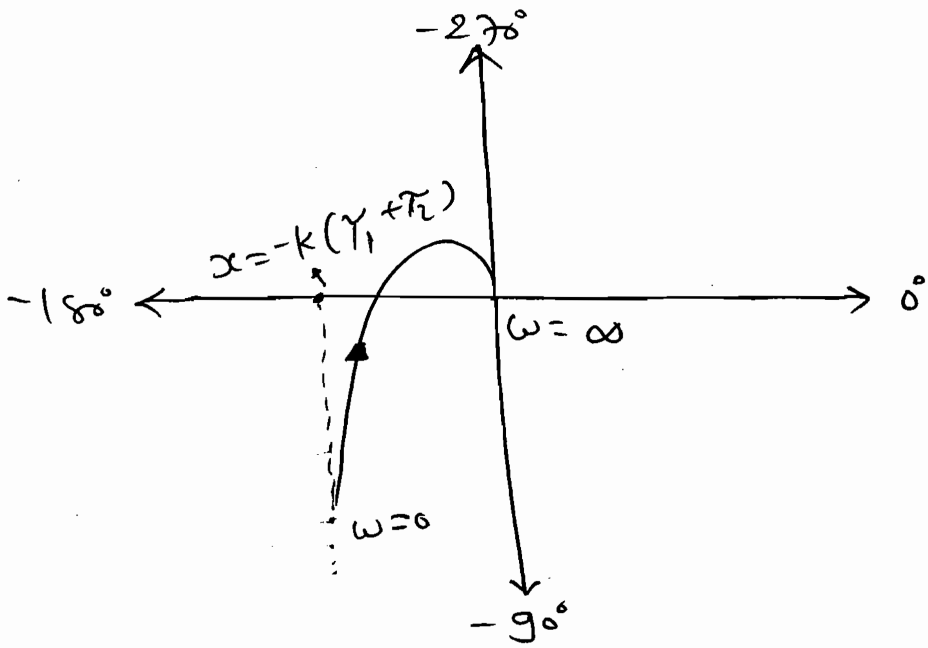
Exact

(1st priority)

* $GH(s) = \frac{K}{s(s\tau_1+1)}$.



* $GH(s) = \frac{K}{s(s\tau_1+1)(s\tau_2+1)}$.



$\boxed{Q}$ $GH = \frac{1}{s^2(s+1)}$.

Soln: $M = \frac{1}{\omega^2\sqrt{\omega^2+1}}$ & $\phi = -180° - \tan^{-1}(\omega)$.

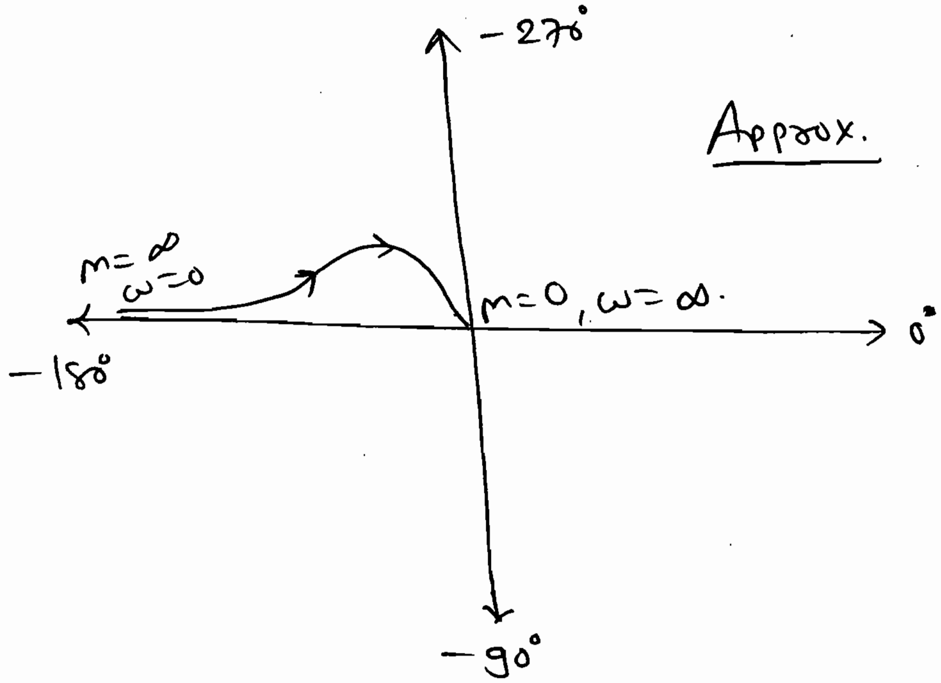
$\omega = 0 \Rightarrow M = \infty$ & $\phi = -180°$.

$\omega = \infty \Rightarrow M = 0$ & $\phi = -270°$.

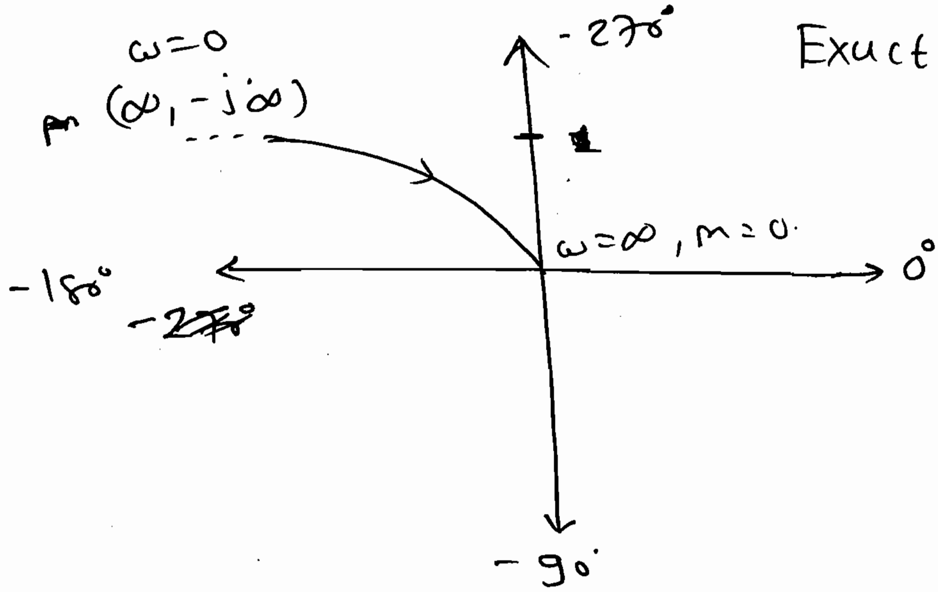
$\Rightarrow$ E.D. $\Rightarrow$ $\phi_1 - \phi_2 = -180^\circ - (-270^\circ) = +90^\circ$ $\Rightarrow$ +ve. $\Rightarrow$ CW

$\Rightarrow$ S.D. $\Rightarrow$ GP $\Rightarrow$ CW.

$\Rightarrow$



$\Rightarrow$



$$\Rightarrow GH(j\omega) = \frac{1}{-\omega^2 (j\omega+1)} \times \frac{1-j\omega}{1-j\omega}.$$

$$= \frac{1-j\omega}{-\omega^2(1+\omega^2)}.$$

$$GH(j\omega) = \frac{-1}{\omega^2(1+\omega^2)} - \frac{j}{\omega(1+\omega^2)}.$$

$$\Rightarrow GH(j\omega)\Big|_{\omega=0} = -\infty - j\infty = (-\infty, -j\infty).$$

Q $GH = \frac{1}{s^3(s+1)}$.

Sol^n:

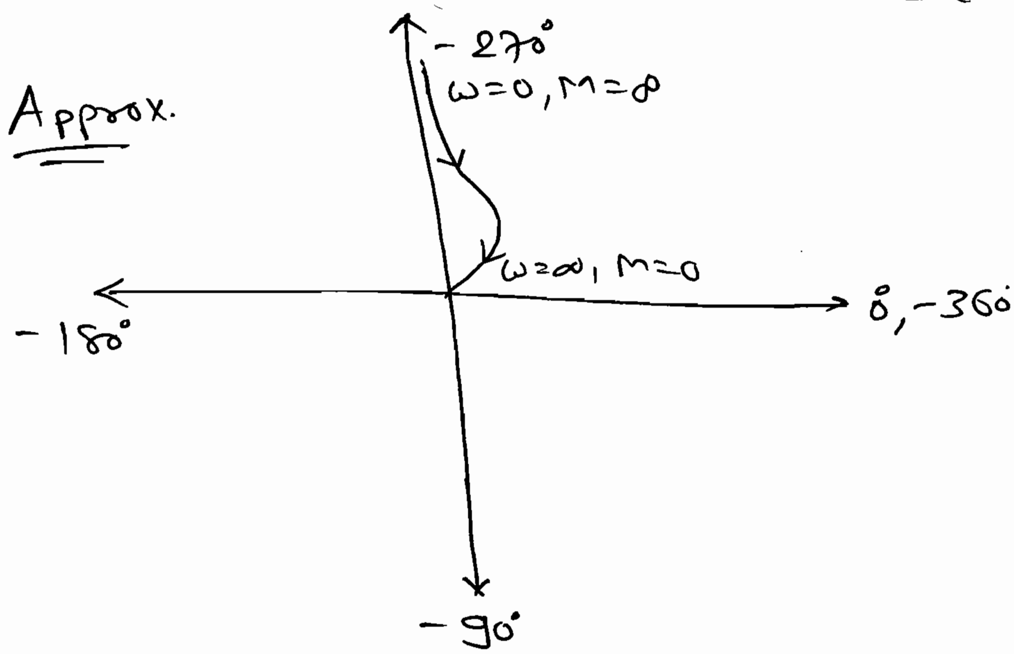
$M = \frac{1}{\omega^3\sqrt{\omega^2+1}}$ & $\phi = -270° - \tan^{-1}(\omega)$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -270°$. E.D. = +ve = CW

$\omega = \infty \Rightarrow M = 0$ & $\phi = -360°$. S.D = finite pole = CW.

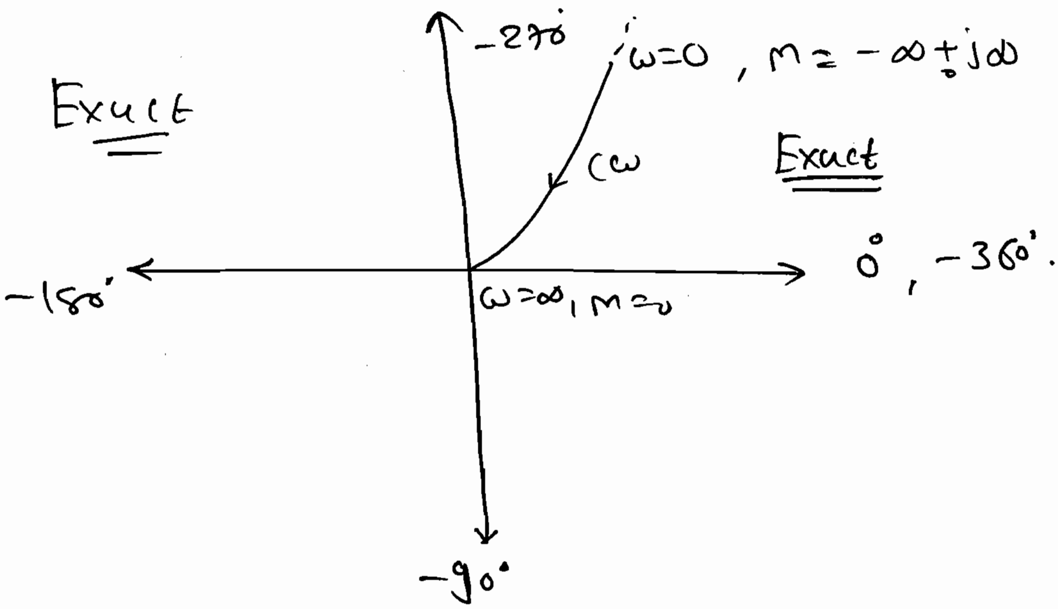
$\Rightarrow$

Approx.



$\Rightarrow$

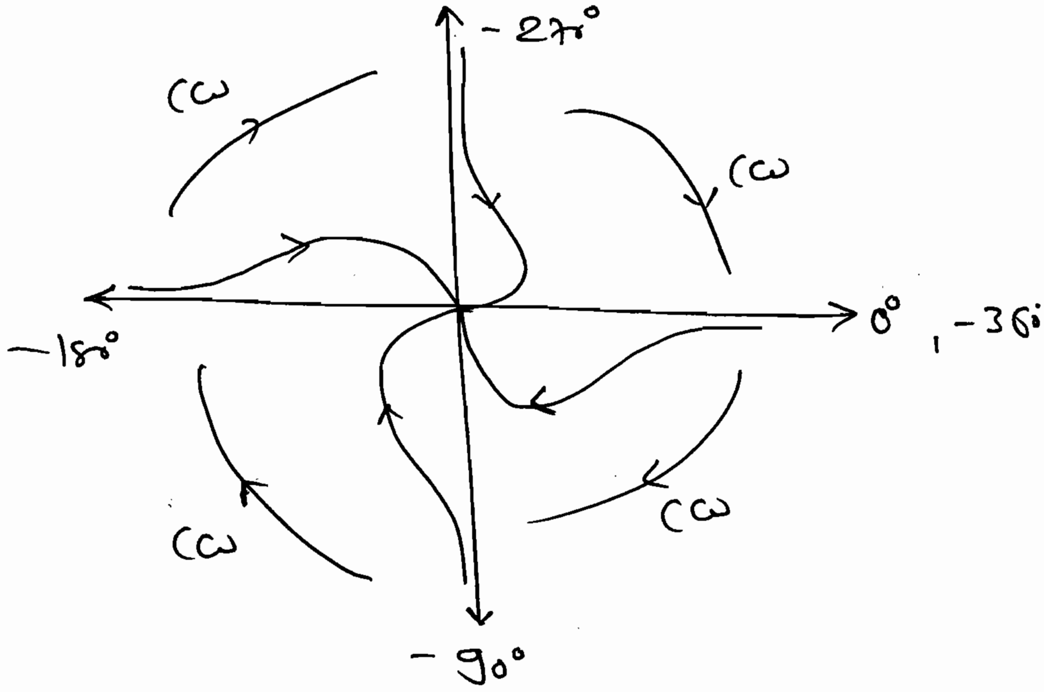
Exact



* **Note:**

$\Rightarrow$ The addition of each pole at origin shifted the total plot $-90°$ in the clock wise direction.

⇒



(a) $GH(s) = \frac{(s+1)}{s^3}$.

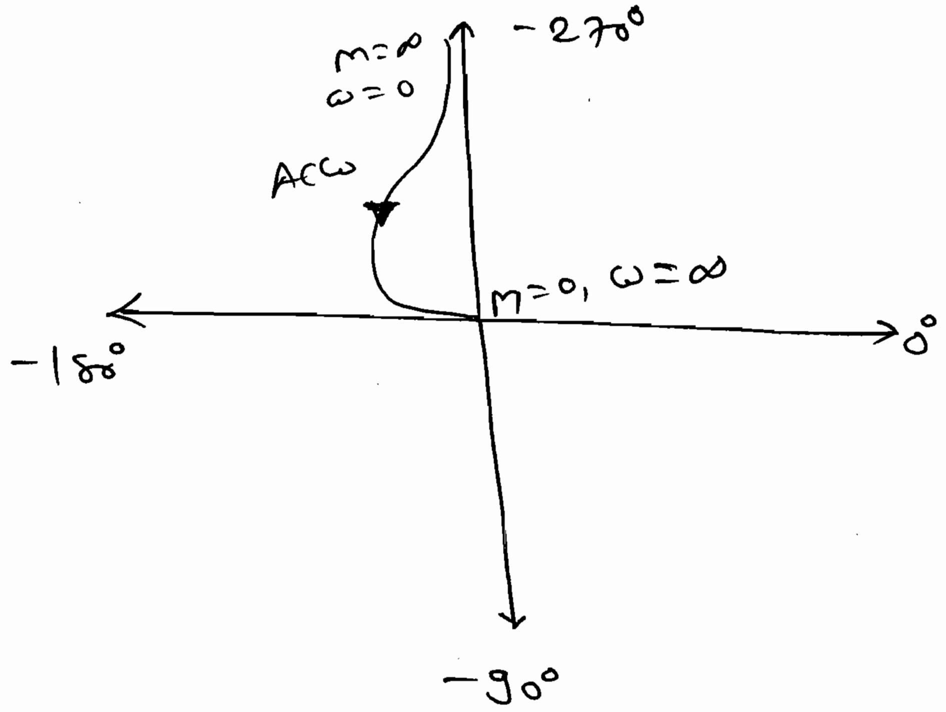
Sol^n: $M = \frac{\sqrt{\omega^2+1}}{\omega^3}$ & $\phi = -270° + \tan^{-1}\omega$.

⇒ $\omega = 0$ ⇒ $M = \infty$ & $\phi = -270°$.

⇒ $\omega = \infty$ ⇒ $M = 0$ & $\phi = \cancel{+360°}\ -180°$

⇒ E.D. ⇒ $\phi_1 - \phi_2 = -270° + 180° = -ve$ ⇒ ACW.

⇒ S.D. ⇒ PZ ⇒ ACW.



$\boxed{Q}$ $GH(s) = \dfrac{(s+1)(s+2)}{s^3}$.

$\Rightarrow$ $M = \dfrac{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}{\omega^3}$. &

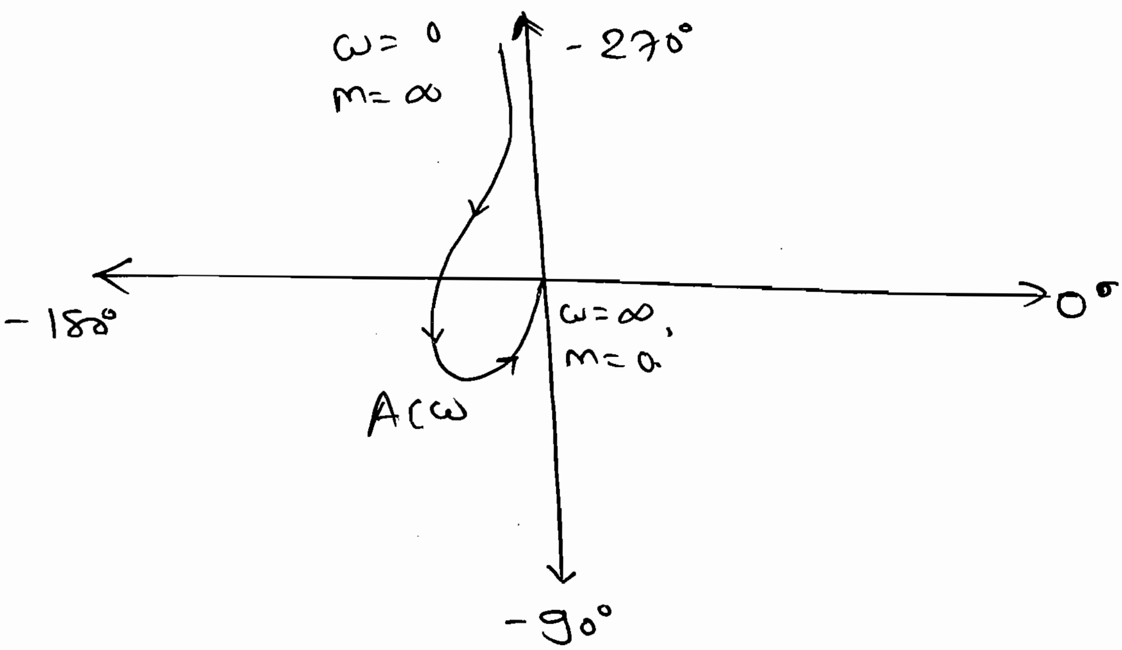
$\phi = -270° + \tan^{-1}\omega + \tan^{-1}(\omega/2)$.

$\Rightarrow$ $\omega = 0$ $\Rightarrow$ $M = \infty$ & $\phi = -270°$.

$\Rightarrow$ $\omega = \infty$ $\Rightarrow$ $M = 0$ & $\phi = -90°$.

$\Rightarrow$ E.P. $\Rightarrow$ $\phi_1 - \phi_2 = -270° + 90° = -ve$ $\Rightarrow$ A.C.W.

$\Rightarrow$ S.O. $\Rightarrow$ finite zero $\Rightarrow$ A.C.W.



$\boxed{Q}$ $GH(s) = \dfrac{(s+1)(s+2)(s+3)}{s^3}$

Sol^n: $M = \dfrac{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}{\omega^3}$.

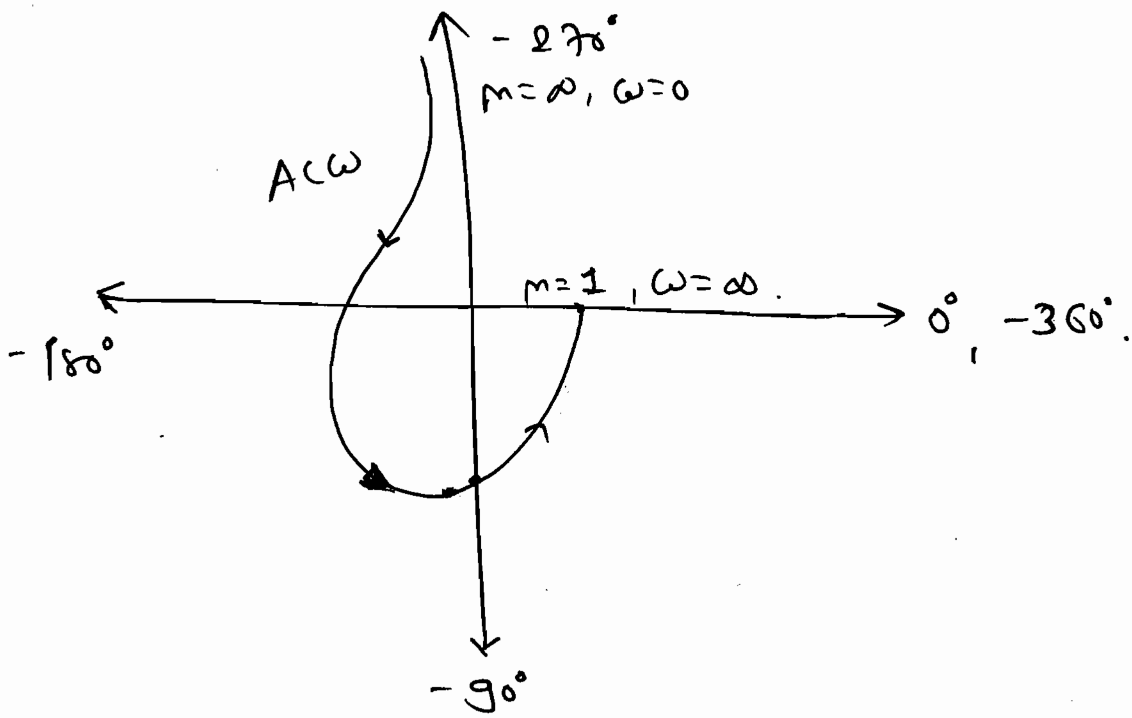
$\Rightarrow$ $\phi = -270° + \tan^{-1}(\omega) + \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3)$.

$\omega = 0 \Rightarrow$ $M = \infty$, & $\phi = -270°$.

$\omega = \infty \Rightarrow$ $\boxed{M = 1}$ & $\phi = 0°$.

$\Rightarrow$ E.D. $\Rightarrow$ $\phi_1 - \phi_2$ = -ve $\Rightarrow$ ACW

$\Rightarrow$ S.D. $\Rightarrow$ δz = +ve $\Rightarrow$ ACW



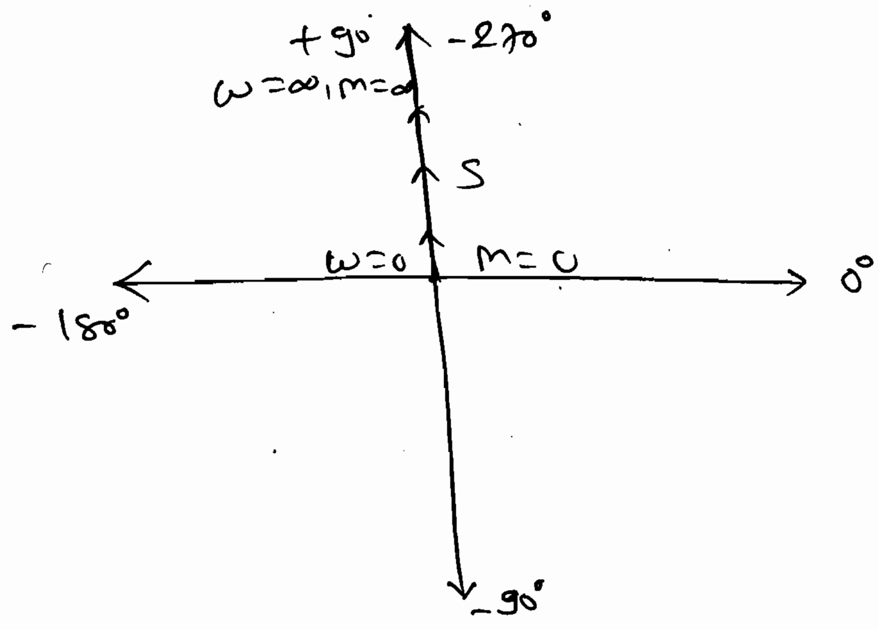
<u>Note:</u> The addition of each ~~time~~ finite zero shift the ending angle +90° in the Anticlockwise direction.

$\boxed{Q}$ $GH = S$.

<u>Soln:</u> $M = \omega$ & $\phi = 90°$

$\omega = 0 \Rightarrow M = 0$ & $\phi = 90°$

$\omega = \infty \Rightarrow M = \infty$ & $\phi = 90°$

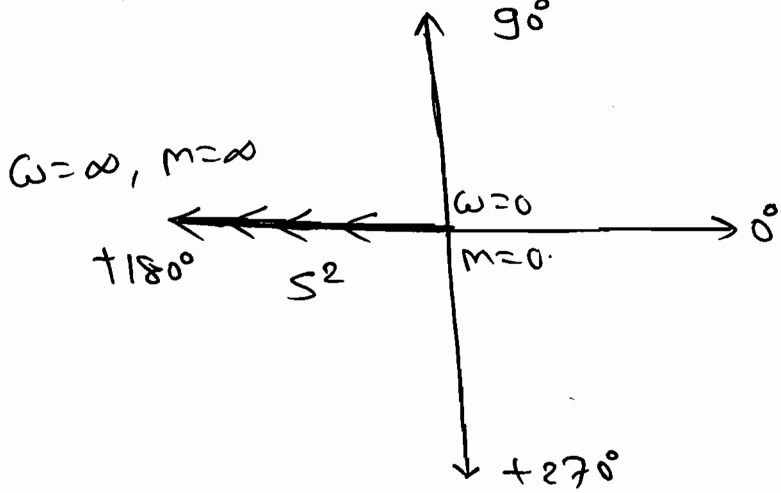


Note:

⇒ Whenever T.F. consist only poles (or) zeros at origin the polar plot is nothing but the angle line.

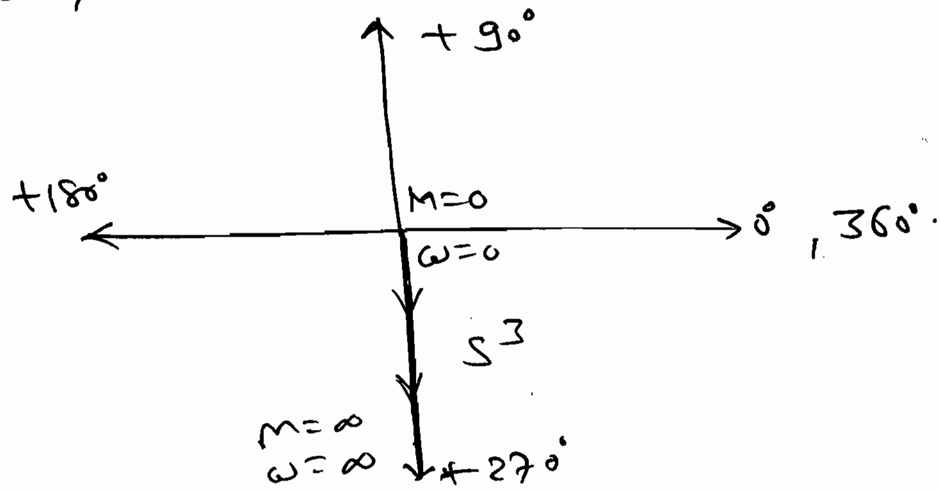
[a] $GH(s) = s^2$

Soln: $M = \omega^2$, $\phi = +180°$



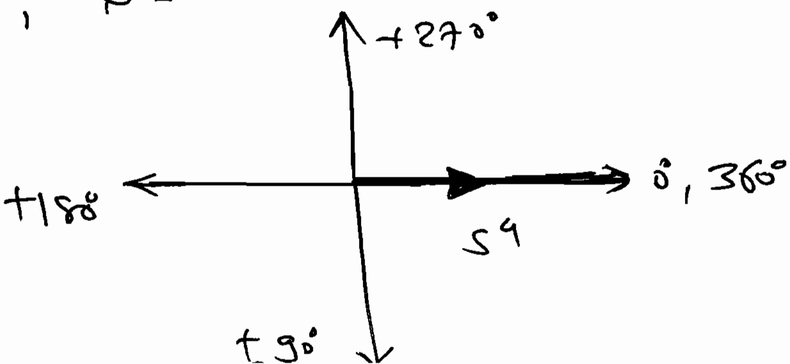
[a] $GH(s) = s^3$.

Soln: $M = \omega^3$, $\phi = +270°$.



[a] $GH(s) = s^4$

Soln: $M = \omega^4$, $\phi = +360°$ (or) $0°$



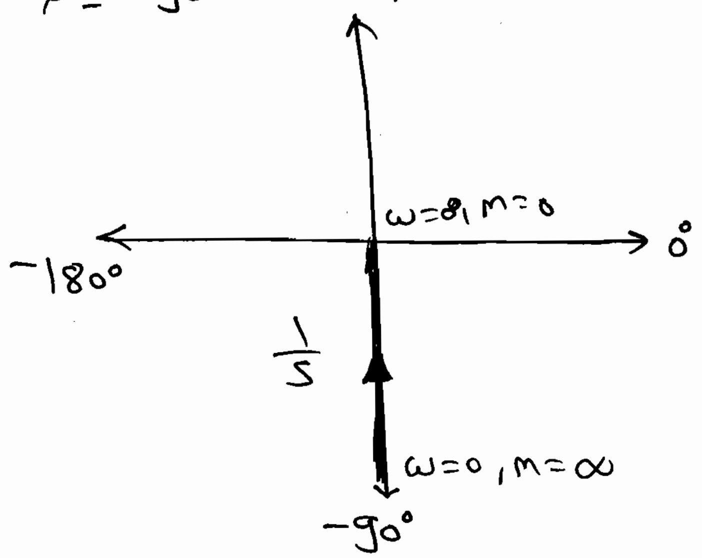
Note:

⇒ Whenever the zero at origin in added the total plot shifted $+90^\circ$ in the A.C.W direction.

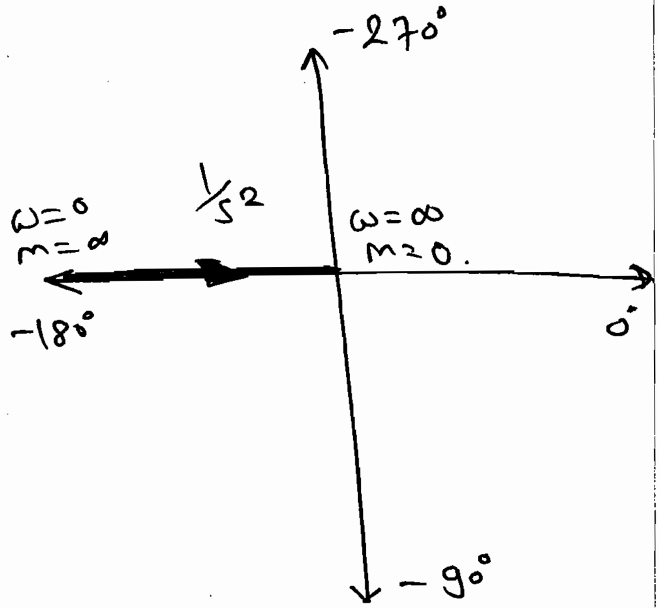
[a] $GH(s) = \frac{1}{s}$

Soln: $M = \frac{1}{\omega}$

$\phi = -90^\circ$

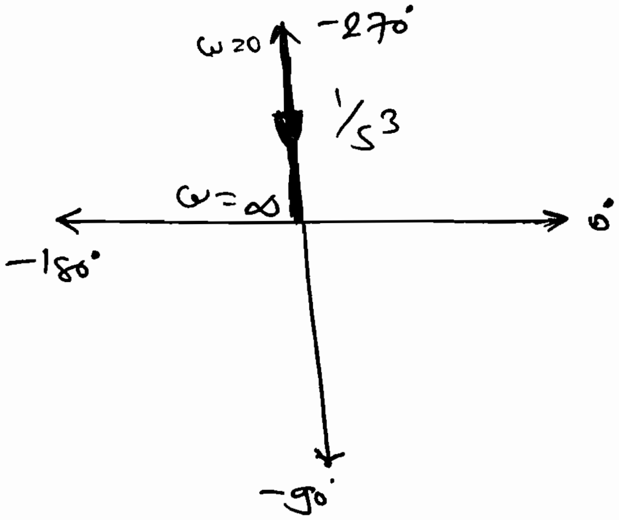


[a] $GH(s) = \frac{1}{s^2}$

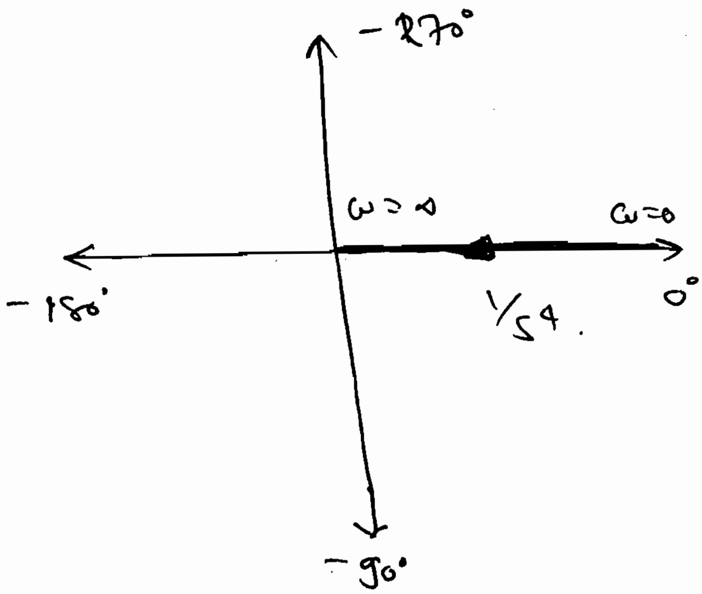


[a] $GH(s) = \frac{1}{s^3}$.

Soln: $M = \frac{1}{\omega^3}$, $\phi = -270^\circ$



[a] $GH(s) = \frac{1}{s^4}$.



Note: Whenever poles are added to the origin the total plot shifted $\mp 90^\circ$ in the the C.W. direction.

$\boxed{Q}$ $GH(s) = \dfrac{(s+1)}{s^3(s+2)}$.

Soln: $GH(j\omega) = \dfrac{(j\omega+1)}{-j\omega^3\,(j\omega+2)}$.

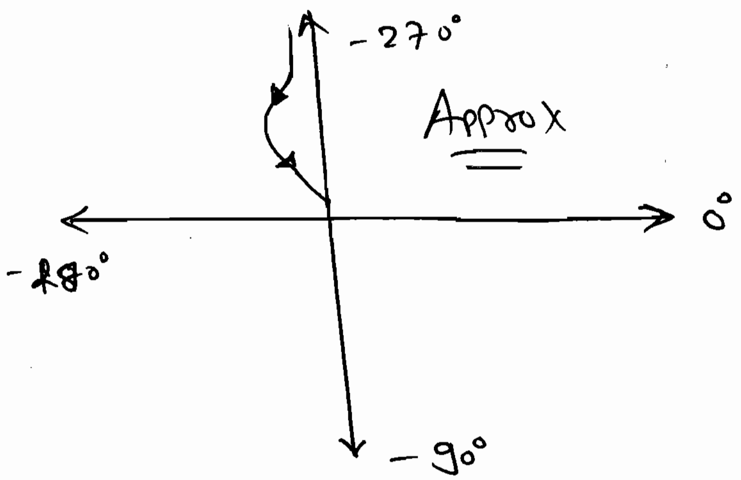
$M = \dfrac{\sqrt{\omega^2+1}}{\omega^3 \times \sqrt{\omega^2+4}}$. $\phi = -270^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -270^\circ$

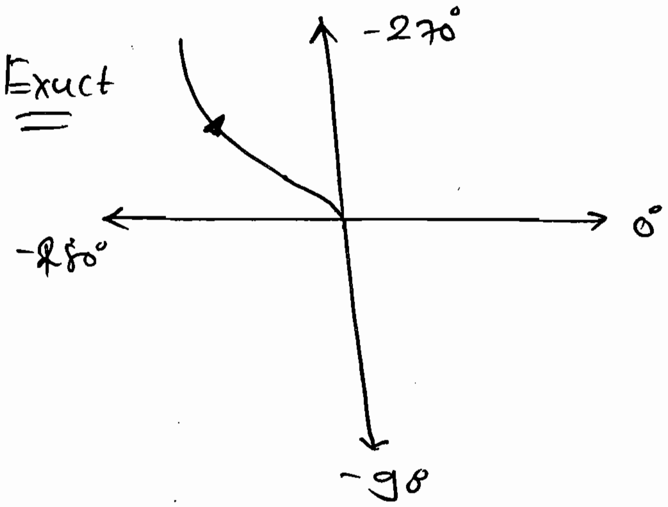
$\omega = \infty \Rightarrow M = 0$ & $\phi = -270^\circ$.

ED $\rightarrow$ X , SD $\rightarrow$ fz $-2\ -1$ $\Rightarrow$ ACW.

$\Rightarrow$



$\Rightarrow$



$\Rightarrow$ $\boxed{Q}$ $GH(s) = \dfrac{(s+2)}{s^3(s+1)}$.

Soln: $M = \dfrac{\sqrt{\omega^2+4}}{\omega^3\sqrt{\omega^2+1}}$.

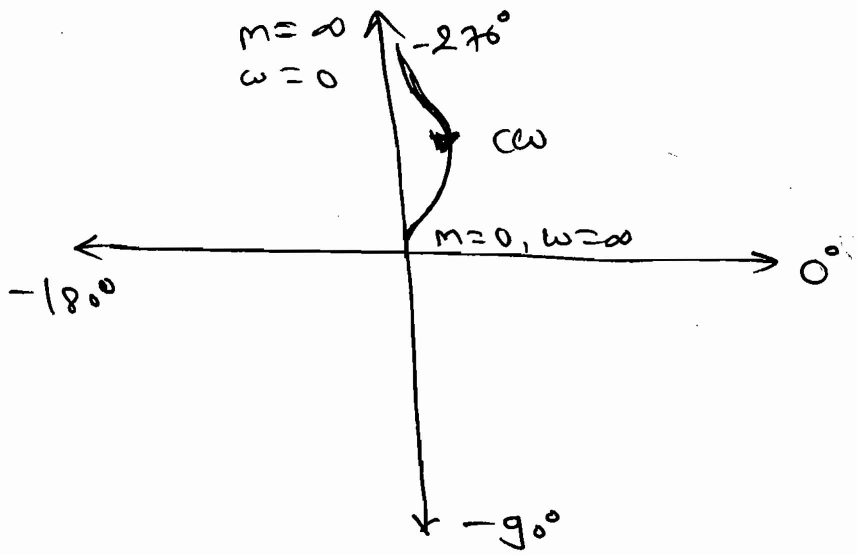
$\Rightarrow \quad \phi = -270^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2).$

$\omega = 0 \Rightarrow \quad M = \infty \quad \& \quad \phi = -270^\circ.$

$\omega = \infty \Rightarrow \quad M = 0 \quad \& \quad \phi = -270^\circ.$

$\Rightarrow$ S.D. $\Rightarrow$ fp $\Rightarrow$ CW.

E.D. $\Rightarrow$ X. $(\because \phi_1 - \phi_2 = 0^\circ)$



Q $\quad GH(s) = \dfrac{(s+1)}{s^3 (s+2)(s+3)}.$

Sol^n: $\quad M = \dfrac{\sqrt{\omega^2+1}}{\omega^3 \sqrt{\omega^2+4}\sqrt{\omega^2+9}}.$

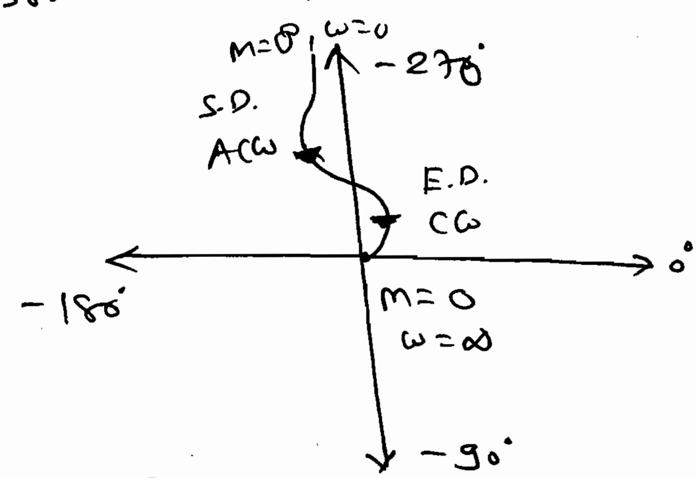
$\phi = -270^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$

$\omega = 0 \Rightarrow \quad M = \infty, \quad \phi = -270^\circ.$

$\omega = \infty \Rightarrow \quad M = 0, \quad \phi = -360^\circ$

E.D. $\Rightarrow \phi_1 - \phi_2 = -270^\circ + 360^\circ = +ve \Rightarrow$ CW.

S.D. $\Rightarrow$ fizero = ACW.



$\boxed{a}$ $GH(s) = \dfrac{(s+1)}{s^2(s+2)(s+3)}$.

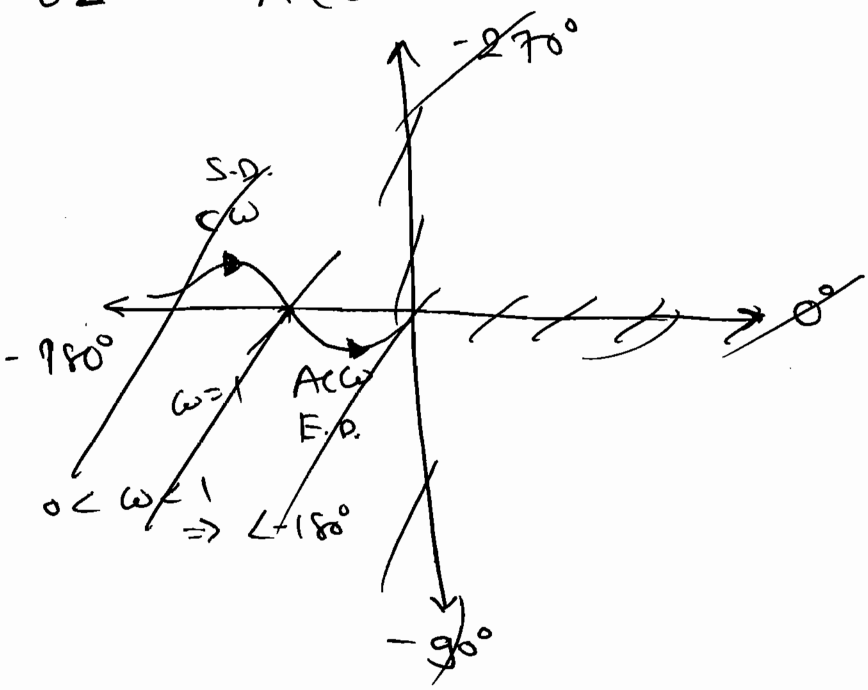
Sol^n: $M = \dfrac{\sqrt{\omega^2+1}}{\omega^2\sqrt{\omega^2+4}\,\sqrt{\omega^2+9}}$, $\phi = -180^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3)$.

$\omega = 0 \Rightarrow M = \infty, \quad \phi = -180^\circ$

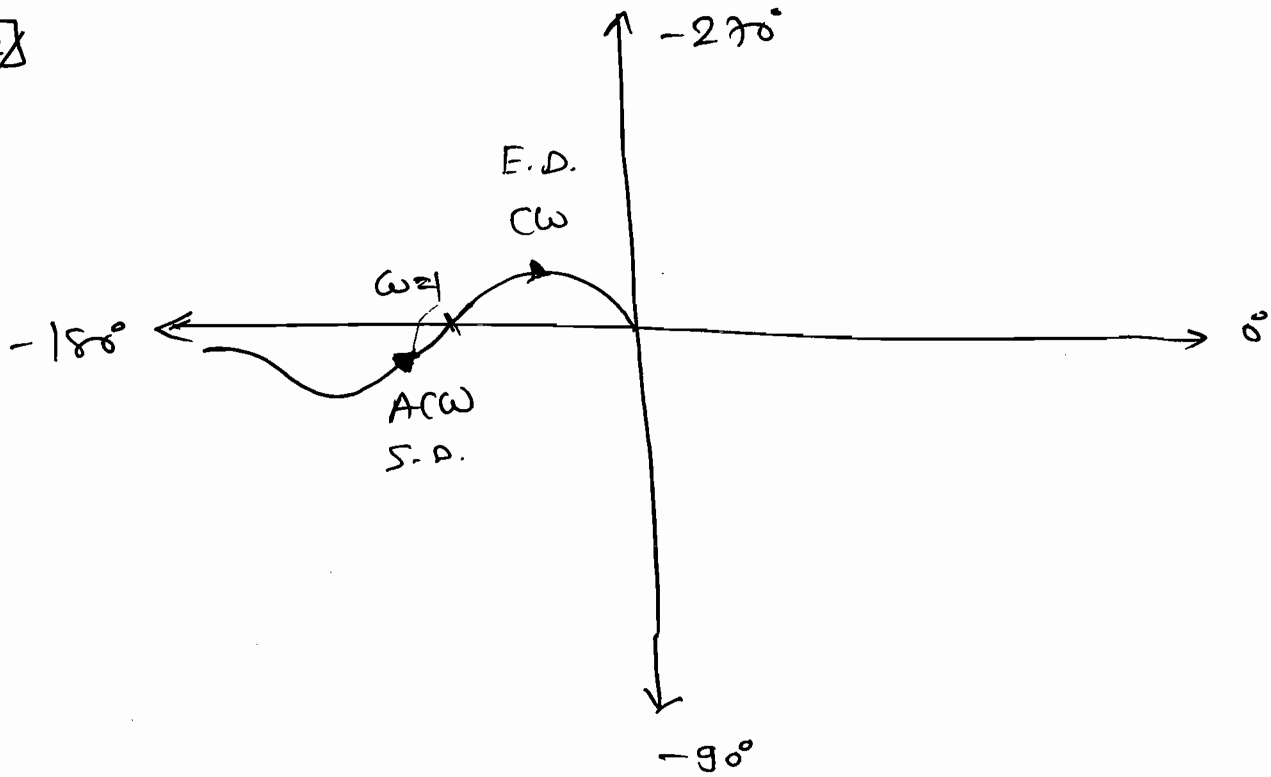
$\omega = \infty \Rightarrow M = 0, \quad \phi = -270^\circ$.

E.D. $\Rightarrow \phi_1 - \phi_2 = -180^\circ + 270^\circ = +ve \Rightarrow$ CW.

S.D. $\Rightarrow$ $\cancel{b}$2 $\Rightarrow$ ACW.



~~(b)~~



$\Rightarrow$ I.P. with $-180°$.

$\therefore \; \cancel{-180°} = \cancel{-180°} + \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$

$\therefore \; \tan^{-1}(\omega) = \tan^{-1}\left(\dfrac{\frac{\omega}{2} + \omega/3}{1 - \omega^2/6}\right).$

$\therefore \quad \omega = \dfrac{5\omega}{6 - \omega^2}.$

$\Rightarrow \quad 6\omega - \omega^3 = 5\omega.$

$6 - \omega^2 = 5.$

$\omega^2 = 1 \quad \Rightarrow \quad \boxed{\omega = 1 \text{ rad/sec}}$

$\boxed{a}$ $\quad GH(s) = \dfrac{(s+3)}{s^2(s+1)(s+2)}.$

Soln:

$$M = \frac{\sqrt{\omega^2+9}}{\omega^2\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}.$$

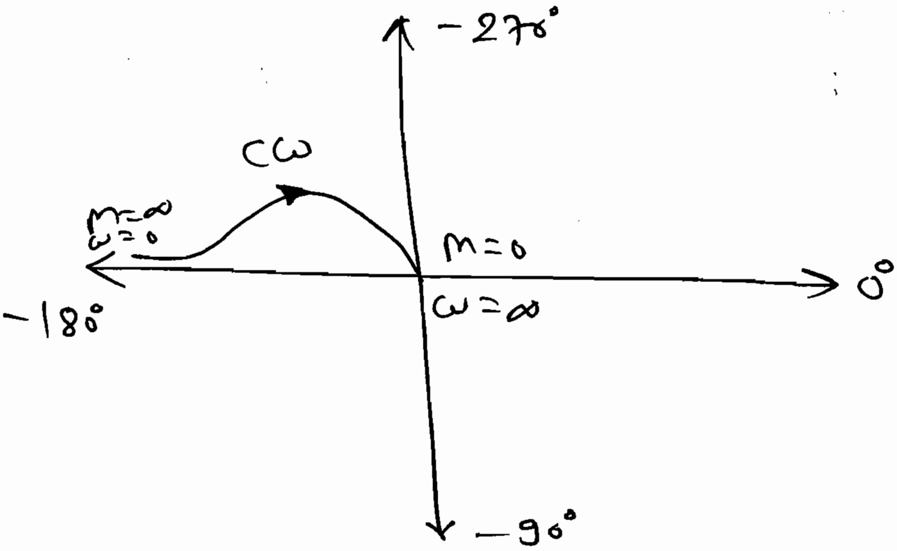
$\Rightarrow \quad \phi = -180° \cancel{+}\tan^{-1}\omega - \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3).$

$\omega = 0 \Rightarrow M = \infty \;\; \& \;\; \phi = -180°.$

$\omega = \infty \Rightarrow M = 0 \;\; \& \;\; \phi = -270°.$

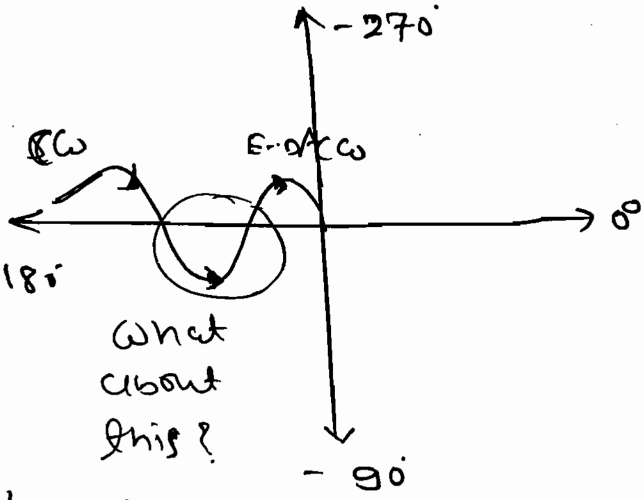
$\Rightarrow$ E.D. $\Rightarrow \phi_1 - \phi_2 = -180° + 270° \Rightarrow +ve \Rightarrow$ CW.

$\Rightarrow$ S.D. $\Rightarrow$ $\frac{1}{2}$P $\Rightarrow$ CW.



$\Rightarrow$ I.P. with -180°

$\angle GH = -180^\circ$



$\Rightarrow \cancel{-180^\circ} = \cancel{-180^\circ} - \tan^{-1}(\omega) - \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3)$

$\Rightarrow \tan^{-1}(\omega/3) = \tan^{-1}\left(\dfrac{\omega + \omega/2}{1 - \omega^2/2}\right)$.

$\Rightarrow \dfrac{\cancel{\omega}}{3} = \dfrac{3\cancel{\omega}}{2-\omega^2}$.

$2-\omega^2 = 9$

$\omega^2 = -7 \Rightarrow \omega = \pm j\sqrt{7}$ X Invaid point.

$\boxed{Q}$ $GH(s) = \dfrac{(s+1)(s+2)}{s^2(s+3)}$.

$\underline{\underline{\text{Sol}^n}}$: $M = \dfrac{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}{\omega^2 \cdot \times \sqrt{\omega^2+9}}$.

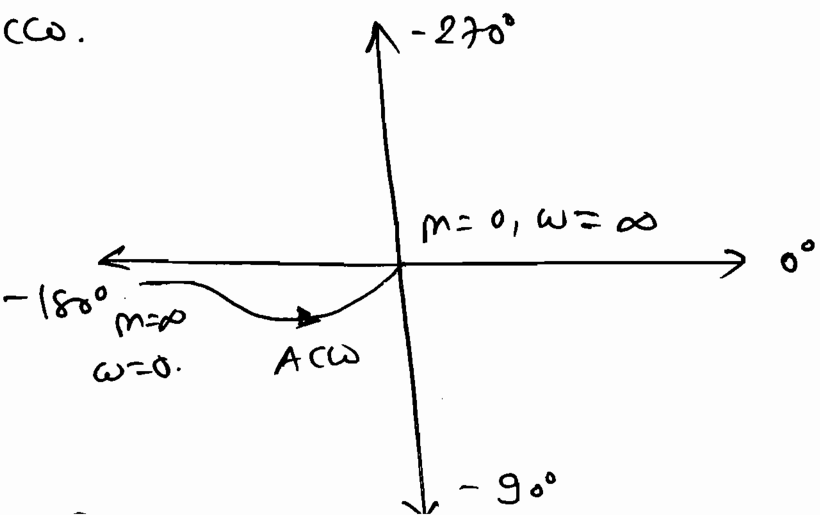
$\Rightarrow \phi = -180^\circ + \tan^{-1}(\omega) + \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\omega/3)$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -180^\circ$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -90^\circ$.

$\Rightarrow$ E.D. $\Rightarrow \phi_1 - \phi_2 = -ve \Rightarrow$ A.C.W.

$\Rightarrow$ S.D. $\Rightarrow$ ~~ϕ_2~~ $\Rightarrow$ ACW.



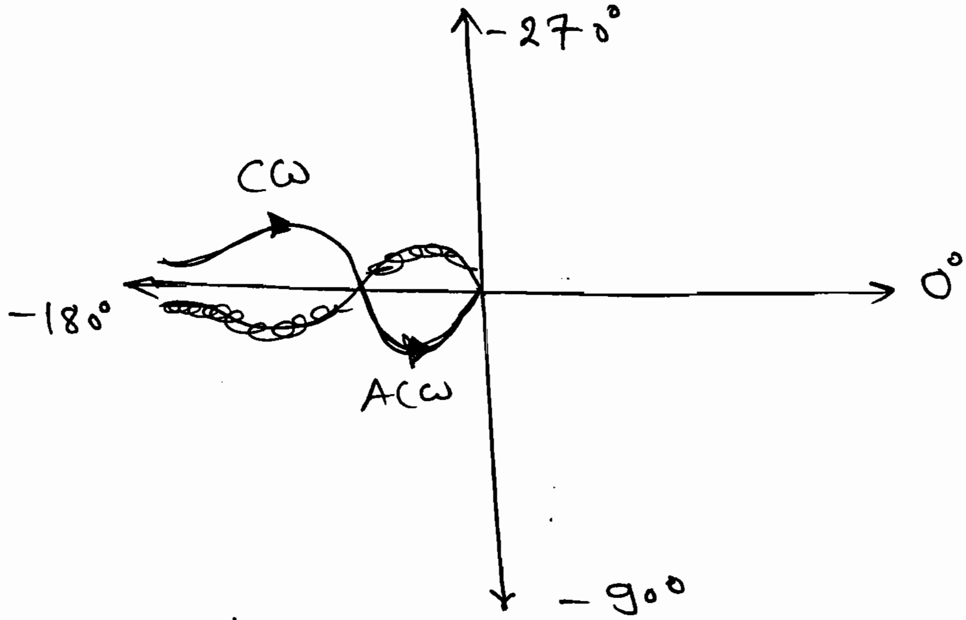
$\boxed{a}$ $GH(s) = \dfrac{(s+2)(s+3)}{s^2(s+1)}$.

Sol^n: $M = \dfrac{\sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}{\omega^2 \times \sqrt{\omega^2+1}}$.

$\phi = -180^\circ \cancel{\mp} \tan^{-1}(\omega) + \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3)$.

$\Rightarrow \omega = 0 \Rightarrow M = \infty, \quad \phi = -180^\circ.$ E.D. $\Rightarrow$ ACW

$\Rightarrow \omega = \infty \Rightarrow M = 0, \quad \phi = -90^\circ.$ S.D. $\Rightarrow$ ~~P.D.~~ $\Rightarrow$ CW.

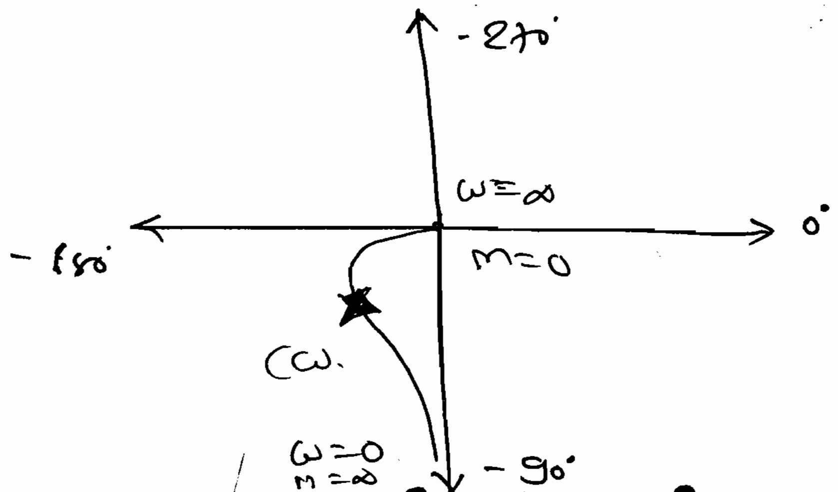


$\boxed{a}$ $GH(s) = \dfrac{1}{s(s+1)}$.

Sol^n: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$. & $\phi = -90^\circ - \tan^{-1}(\omega)$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -90^\circ$ E.D. $\Rightarrow$ CW

$\omega = \infty \Rightarrow M = 0$ & $\phi = -180^\circ$ S.D. $\Rightarrow$ CW.



$\boxed{Q}$ $GH = \dfrac{1}{s(s-1)}$.

Sol^n: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ & $\phi = -90^\circ - (180^\circ - \tan^{-1}(\omega))$

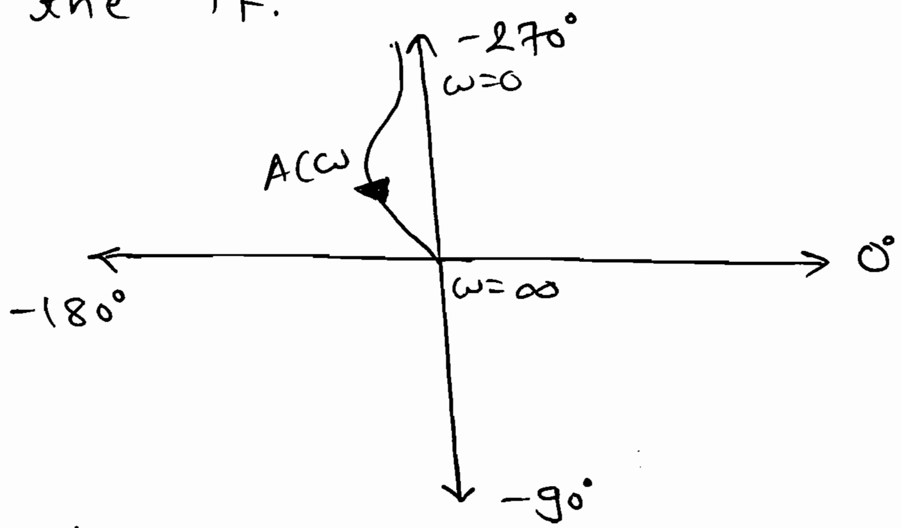
$\phi = -270^\circ + \tan^{-1}(\omega)$.

$\omega = 0 \Rightarrow \phi = -270^\circ$

$\omega = \infty \Rightarrow \phi = -180^\circ$

E.D. $\Rightarrow \phi_1 - \phi_2 = -270^\circ + 180^\circ = -ve \Rightarrow$ ACW.

$\boxed{S.D.}$ ✗ Not required because (-ve) sign in the TF.



$\boxed{Q}$ $GH = \dfrac{1}{s(-s-1)}$.

Sol^n: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ & $\phi = -90^\circ - (180^\circ + \tan^{-1}(\omega))$.

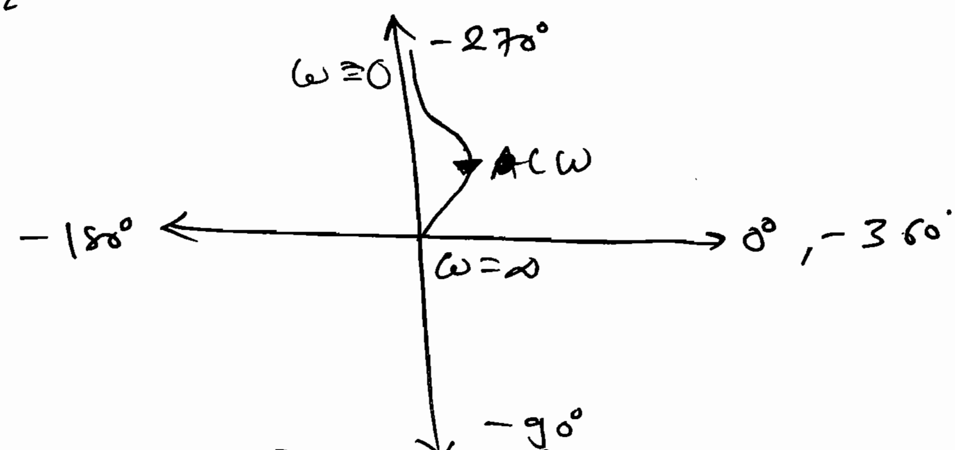
$= -270^\circ - \tan^{-1}\omega$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -270^\circ$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -360^\circ$.

E.D. $\Rightarrow \phi_1 - \phi_2 = -270^\circ + 360^\circ = +ve \Rightarrow$ CW.

S.D. ✗



$\boxed{a}$ $GH = \dfrac{1}{s(1-s)}$

<u>Soln:</u> $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ & $\phi = -90^\circ - (-\tan^{-1}(\omega))$

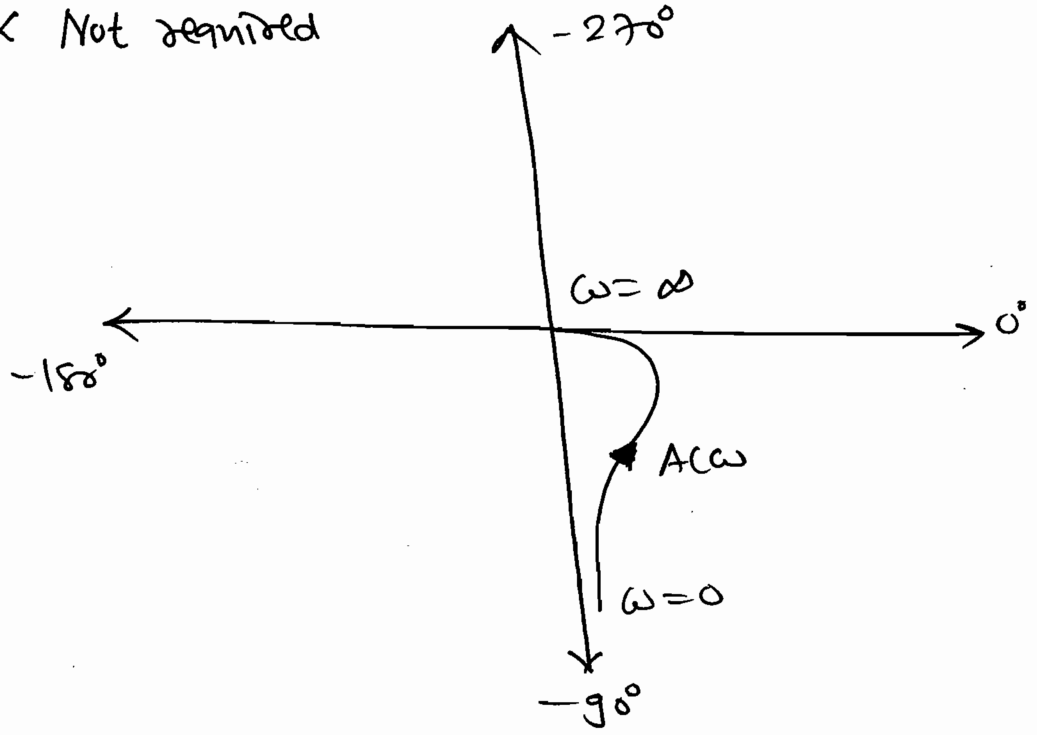
$= -90^\circ + \tan^{-1}\omega.$

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -90^\circ.$

$\omega = \infty \Rightarrow M = 0$ & $\phi = 0^\circ.$

<u>E.D.</u> $\Rightarrow \phi_1 - \phi_2 = -90^\circ = -ve \Rightarrow$ ACW.

<u>S.D.</u> X Not required



<u>Note:</u> The middle intersection pt that means (some freq ~~zone~~ zone C.ω & some freq. zone A.C.ω).

$\Rightarrow$ The middle intersection point possible when the phase angle having +ve & -ve term ($\tan^{-1}$) ~~of~~ provided that no of finite pole is not equal to no. of finite zero and these should be first two consecutive pole and zero (or) zero and pole otherwise no middle I.P. about that perticular point.

Note: E.D. $\phi_1 - \phi_2 = +ve \rightarrow CW$

$= -ve \rightarrow ACW$.

[finite p = finite z] $\Rightarrow$ E.D. X

$\Rightarrow$ S.D. fp $\rightarrow$ CW

fz $\rightarrow$ ACW.

if −ve sign in TF. then SD X.

$\boxed{Q}$ $GH = \dfrac{(S+2)}{(S+1).(S-1)}$.

Soln: $M = \dfrac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+1}}$.

$\phi = -\tan^{-1}(\omega) + \tan^{-1}(\omega/2) \mp (180^\circ - \tan^{-1}\omega)$.

$= \cancel{-\tan^{-1}(\omega)} + \tan^{-1}(\omega/2) \mp 180^\circ + \cancel{\tan^{-1}\omega}$

$\phi = -180^\circ + \tan^{-1}(\omega/2)$.

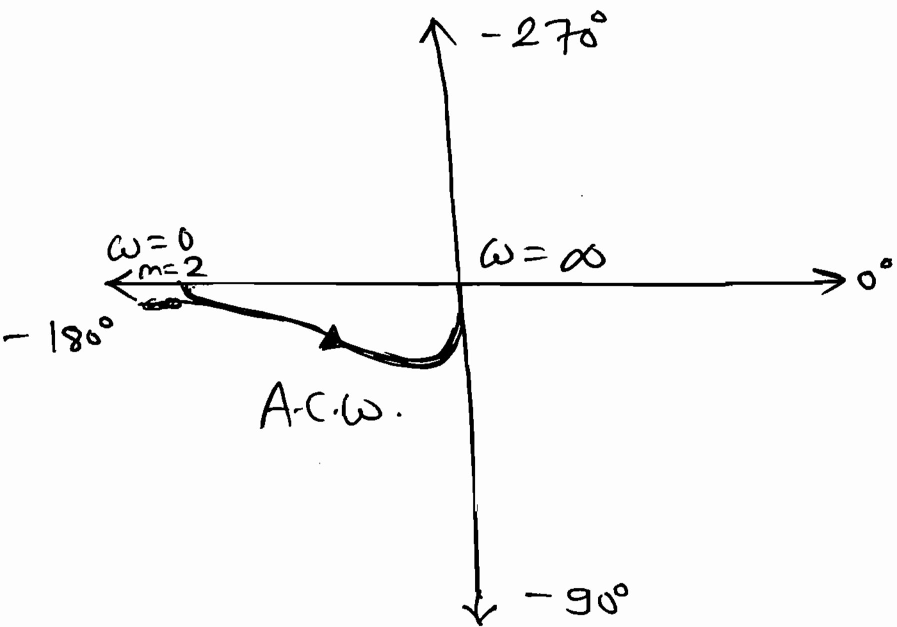
$\Rightarrow \omega = 0 \Rightarrow M = 2, \quad \phi = -180^\circ$.

$\Rightarrow \omega = \infty \Rightarrow M = 0, \quad \phi = -90^\circ$

E.D. $\phi_1 - \phi_2 = -180^\circ + 90^\circ = -90^\circ \Rightarrow ACW$

S.D X

$\Rightarrow$



$\boxed{Q}$ $GH = \dfrac{(s-3)}{s\,(s+1)}$

$\underline{\underline{sol^n}}:$ $M = \dfrac{\sqrt{\omega^2+9}}{\omega \times \sqrt{\omega^2+1}}$,

$\phi = -90^\circ - \tan^{-1}(\omega) + (180^\circ - \tan^{-1}(\omega/3))$.

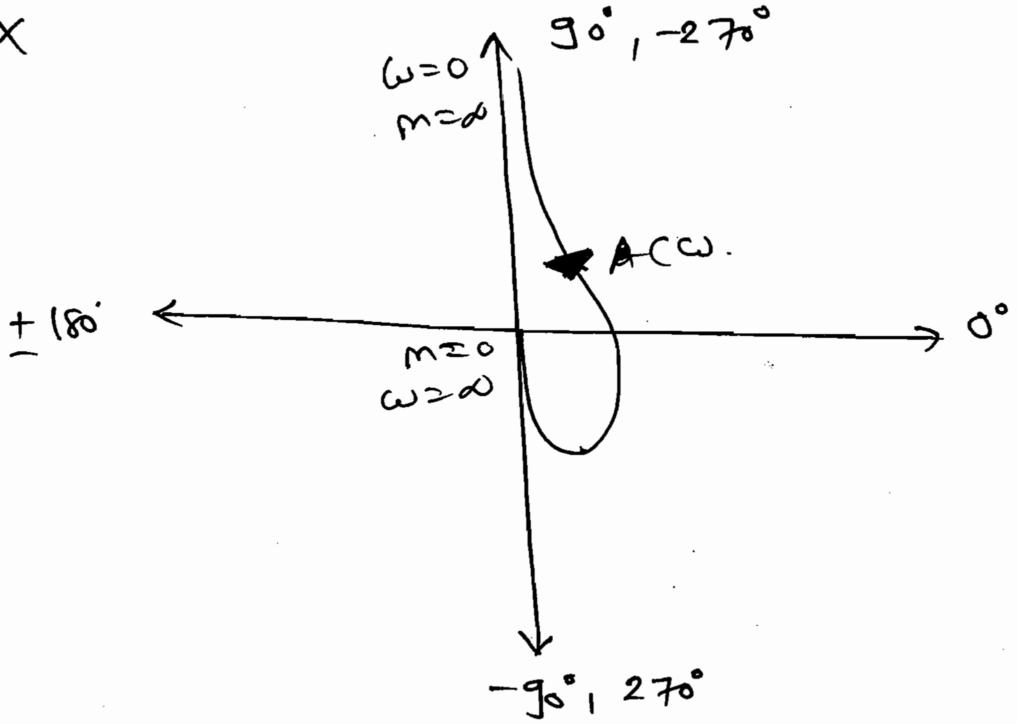
$\phi = +90^\circ - \tan^{-1}(\omega) - \tan^{-1}(\omega/3)$.

$\omega = 0 \Rightarrow M = \infty$, $\phi = +90^\circ$

$\omega = \infty \Rightarrow M = 0$, $\phi = -90^\circ$

$\underline{\underline{E.D.}} \Rightarrow \phi_1 - \phi_2 = +90^\circ - (-90^\circ) = +ve \Rightarrow$ cw.

$\underline{\underline{S.D}}$ X



$\boxed{Q}$ $GH(s) = \left(\dfrac{s+10}{s-10}\right)$.

$\underline{\underline{sol^n}}:$ $M = \sqrt{\dfrac{\omega^2+100}{\omega^2+100}} = 1.$

$\phi = \tan^{-1}(\omega/10) - 180^\circ + \tan^{-1}(\omega/10)$.

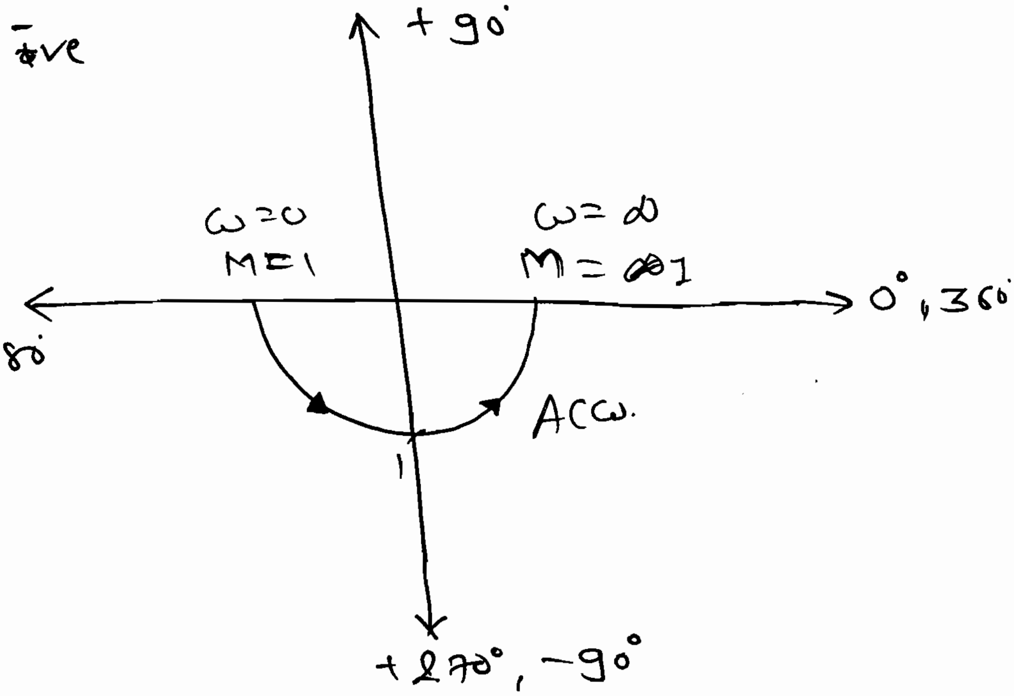
$\phi = -180^\circ + 2\tan^{-1}(\omega/10)$.

$\therefore \quad \omega = 0 \Rightarrow M = 1 \quad \& \quad \phi = -180^\circ$

$\omega = \infty \Rightarrow M = 1 \quad \& \quad \phi = 0^\circ \cancel{360^\circ}$

E.D. $\phi_1 - \phi_2 = -ve$
$\Rightarrow$ ACW

S.D X



$\boxed{Q}$ $GH = \dfrac{e^{-s}}{s(s+1)}$.

$GH = \dfrac{1-s}{s(s+1)}$ X Don't expand e^{-s} to $(1-s)$.

Soln: $s \to j\omega$

$GH(j\omega) = \dfrac{e^{-j\omega}}{j\omega\,(j\omega+1)}$.

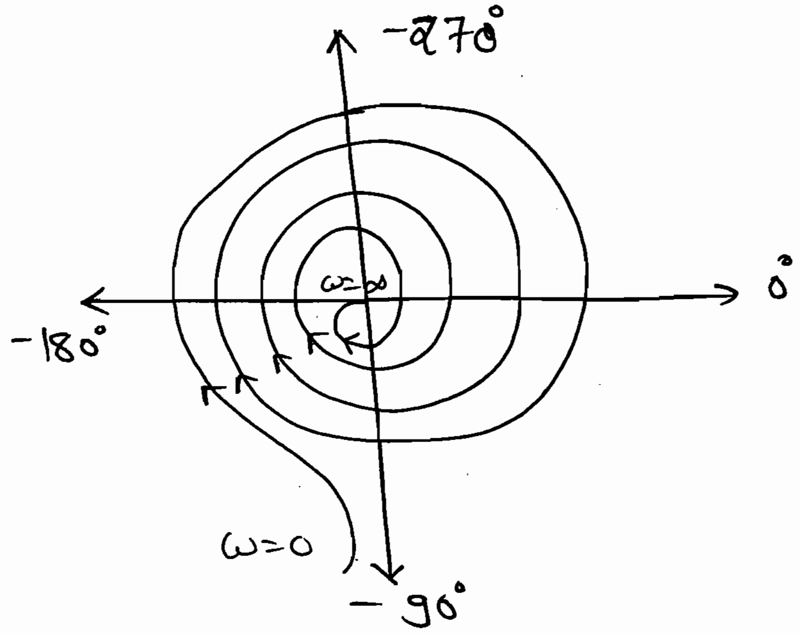
$\Rightarrow |GH(j\omega)| = M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$.

$\Rightarrow \phi = -90^\circ - \tan^{-1}\omega - \omega\left(\dfrac{180^\circ}{\pi}\right)$.

$$\boxed{\begin{aligned} \therefore\ e^{j\theta} &= \cos\theta + i\sin\theta \\ \angle\phi &= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta \\ \angle\phi &= \theta \end{aligned}}$$

$\Rightarrow \angle\phi = -90^\circ - \tan^{-1}\omega - 57.3^\circ\omega$.

$\Rightarrow$ $\omega\uparrow$	$M\downarrow$	$\angle\phi$
0	∞	-90°
1	0.707	-192°
2	0.22	-267°
3	0.04	-453°
10	0.01	-744°
∞	0	$-\infty$.

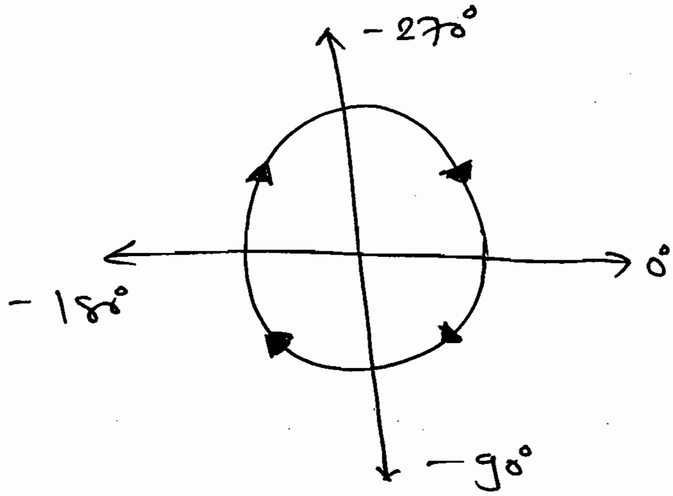


Q) $GH = \pi e^{-2s}$

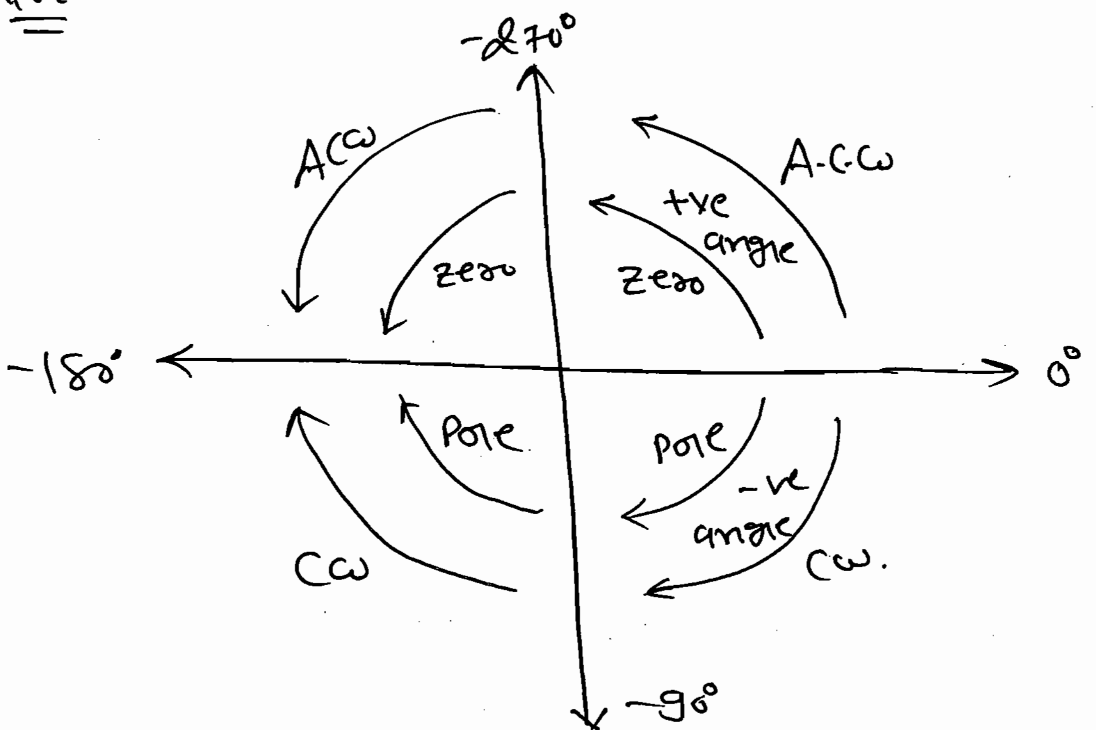
Solⁿ: $s \to j\omega \Rightarrow GH(j\omega) = \pi e^{-2j\omega}$

$M = \pi$ & $\phi = -2\omega \times \frac{180^\circ}{\pi}$

ω	M	$\angle\phi$
0	π	0°
$\pi/4$	π	-90°
$\pi/2$	π	-180°
$3\pi/4$	π	-270°
π	π	-360°



* Note:



☆ Nyquist Plot :-

* Purpose:

⇒ To draw the Complete freq. response of OLTF.

⇒ To find the range of K value for System stability.

⇒ To find the no. of close loop poles in the Right of S-plane.

⇒ To find the Gain Margin, Phase Margin, Gain cross over freq & Phase cross over freq.

⇒ To find the Relative stability by using Gain Margin & Phase Margin.

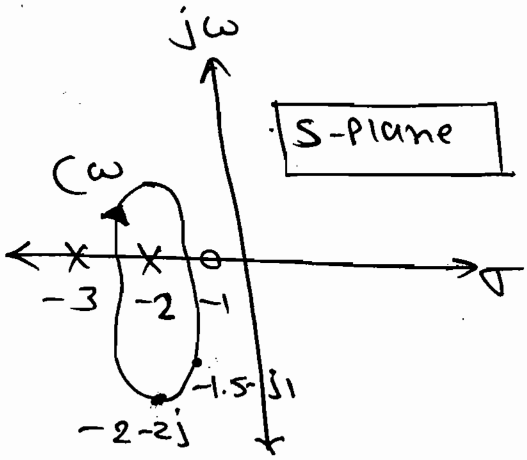
⇒ The Nyquist plot is developed by using a mathmatical principle called Principle of Arguments.

* Principle of Arguments:

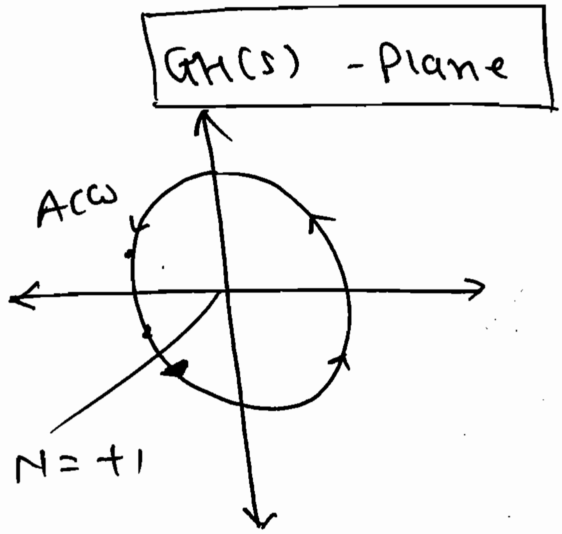
⇒ It states that if there are P poles and the Z zeros are enclosed by the random selected closed path than the corresponding $G(s) \cdot H(s)$ plane N-circles the origin with P-Z times i.e N = P-Z

$N = P - Z$.

e.g. $GH(s) = \dfrac{(s+1)}{(s+2)(s+3)}$.



⇒



$N = P - Z$

$\boxed{N = 1 - 0 = +1}$

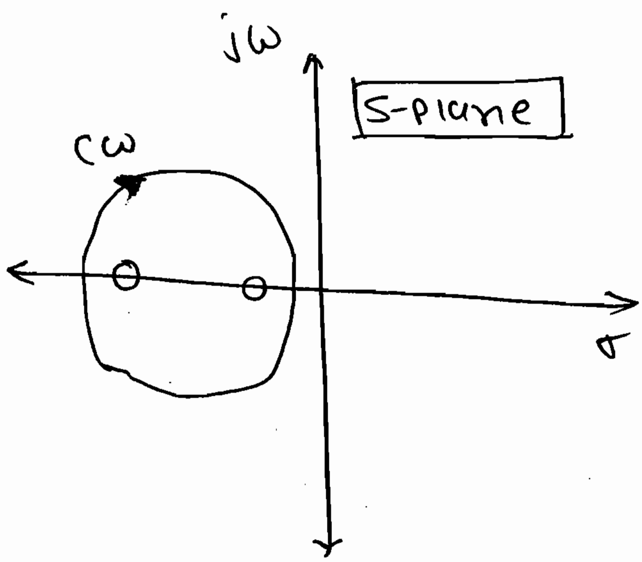
⇒

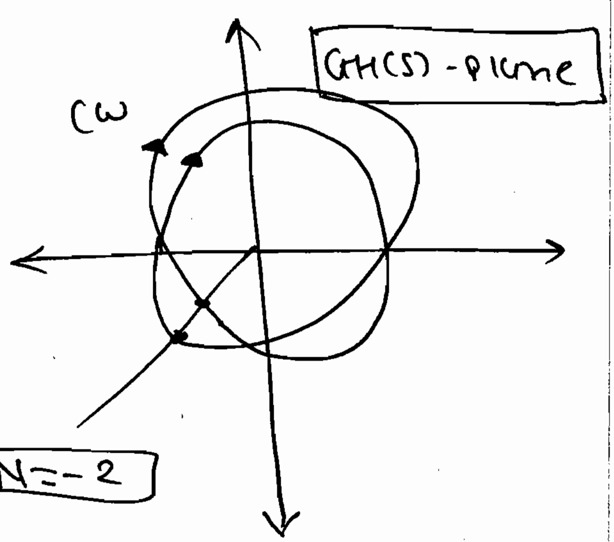
Pole →	Change in direction.
Zero →	No change in direction.

⇒

ACW →	+ve
CW →	-ve

⇒

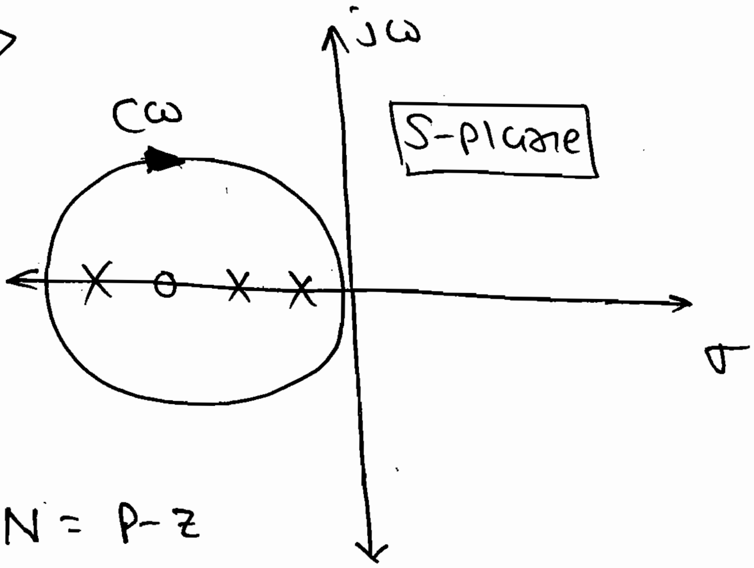




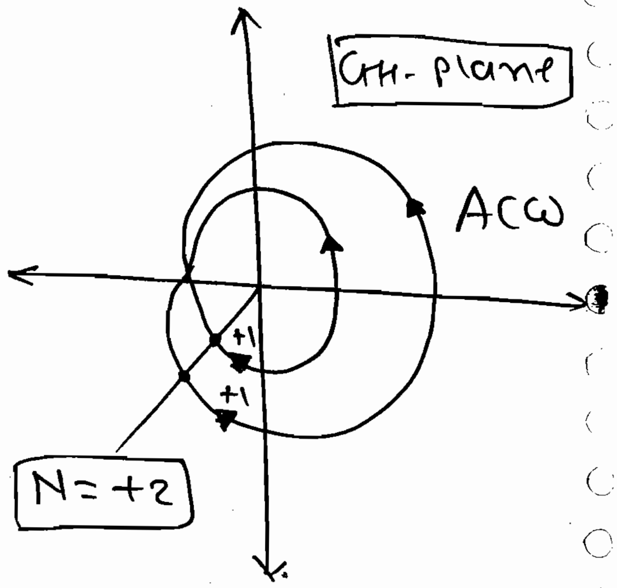
⇒ $N = P - Z$

$N = 0 - 2$ ⇒ $\boxed{N = -2}$

$\Rightarrow$



$\Rightarrow$

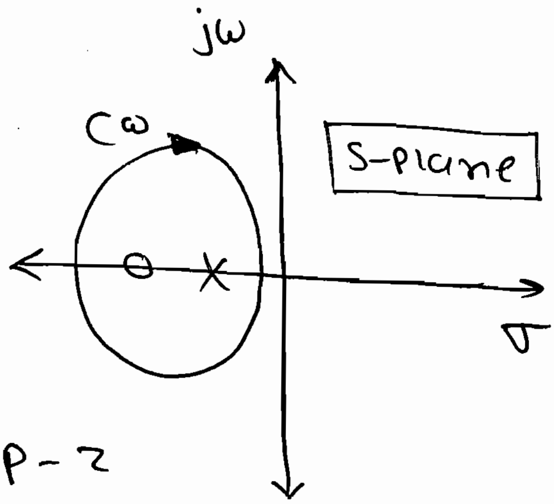


$N = P - Z$

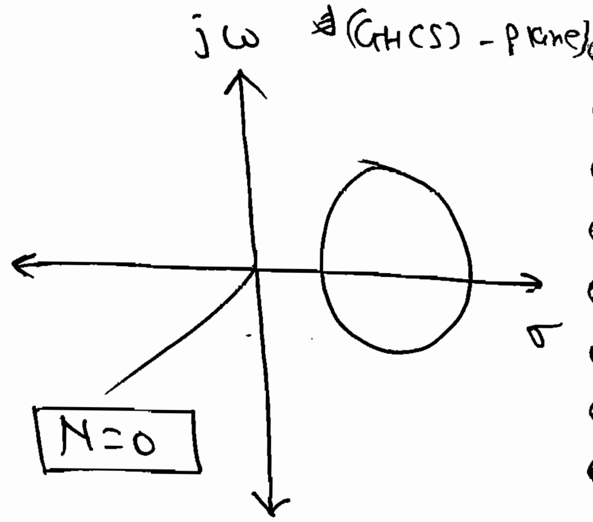
$N = 3 - 1$

$\boxed{N = +2}$

$\Rightarrow$



$\Rightarrow$

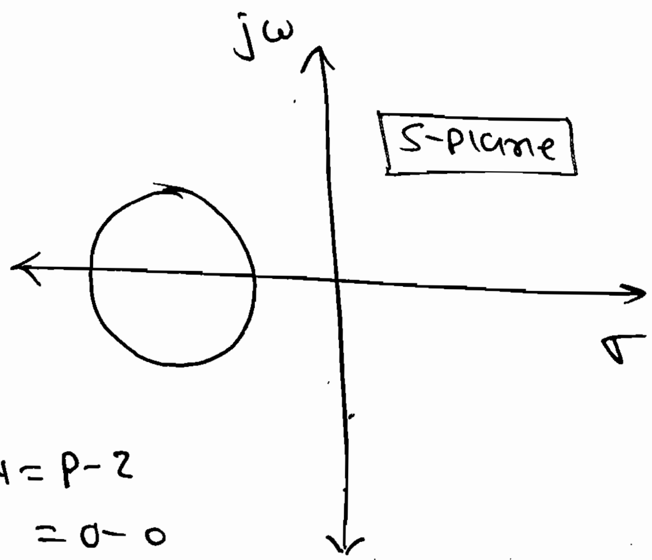


$N = P - Z$

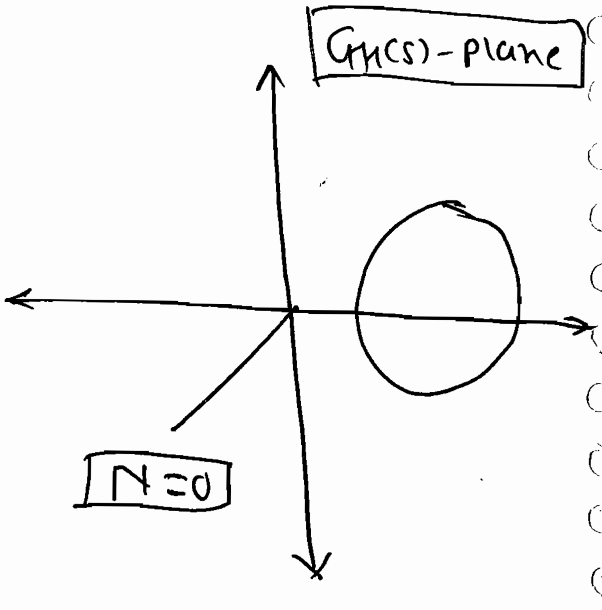
$= 1 - 1$

$N = 0$

$\Rightarrow$



$\Rightarrow$



$N = P - Z$

$= 0 - 0$

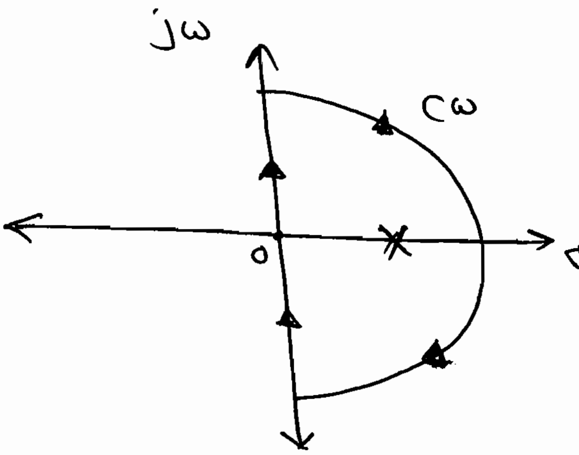
$\boxed{N = 0}$

⇒ The Random Selected closed path should not pass through any pole (or) zero.

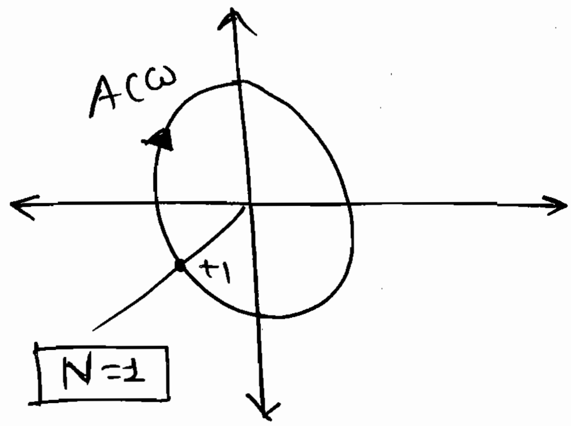
⇒ The Principle of Arg. concept is applied to the total Right half of S-plane with radius of ∞.

⇒ The Nyquist stability analysis is right of S-plane analysis.

⇒ $GH(s) = \dfrac{1}{(s-1)(s+2)(s+3)}$



⇒



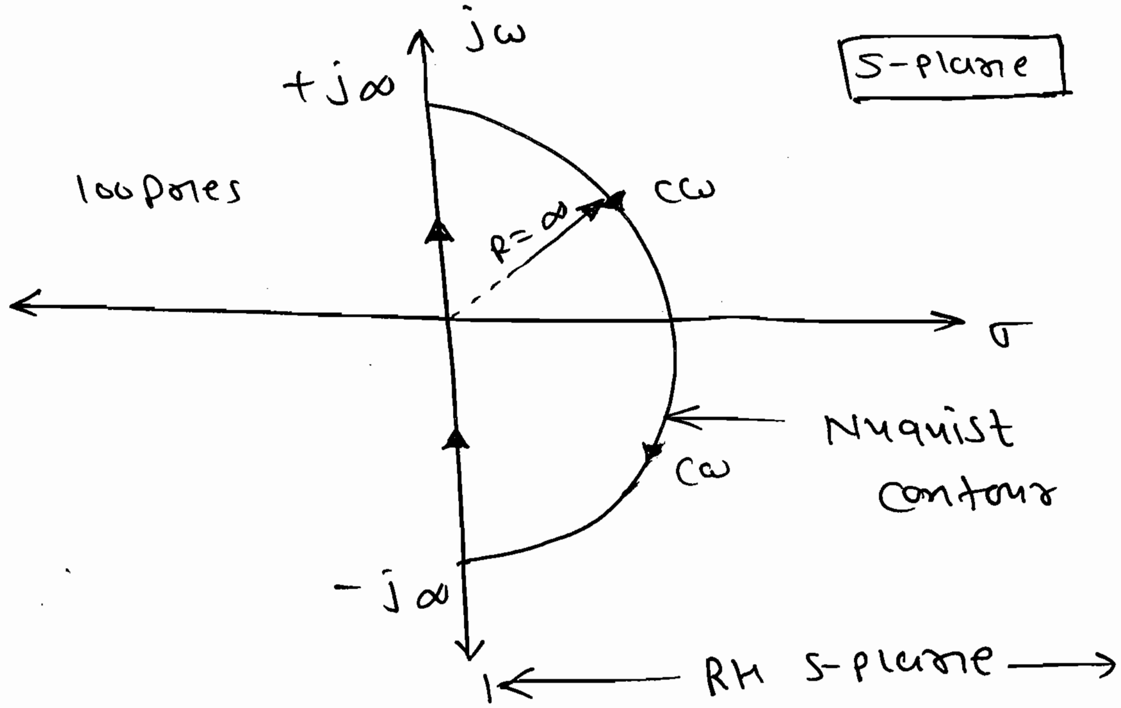
$N = P - Z$
$= 1 - 0 = +1 \Rightarrow \boxed{N = +1}$

⇒ OL sys. | OLTF

$N = P - Z$

- N ↓ No. of encirclements about origin
- P ↓ OL Pole RH
- Z ↓ OL zero RH.

$\Rightarrow$



$\Rightarrow$ To get about OLTF (or) OL sys.

Consider N as a No. of encirclements about origin.

$\Rightarrow$ P is no. of OL Poles on Right of the S-plane.

$\Rightarrow$ Z is no. of OL Zeros on Right of the S-plane.

* **Pole - Zero Configuration:-**

$\Rightarrow$ The open loop transfer function (OLTF) is given by.

$$\boxed{GH(s) = K\frac{N(s)}{D(s)}} \qquad \text{------} \quad (1)$$

$\Rightarrow$ The CLTF is given by.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}.$$

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + K\frac{N(s)}{D(s)}}.$$

$$\therefore \boxed{T.F. = \frac{G(s).D(s)}{D(s) + K N(s)}} \quad \text{———} \quad (2)$$

⇒ The Closed loop stability is given by Char. eqn.

$$q(s) = 1 + G(s).H(s)$$

$$= 1 + K\frac{N(s)}{D(s)}.$$

$$\therefore \boxed{q(s) = \frac{D(s) + K N(s)}{D(s)}} \quad \text{———} \quad (3).$$

⇒ Compare eqn - ① & ③.

$$\boxed{\text{Poles of CE = OLTF Poles.}}$$

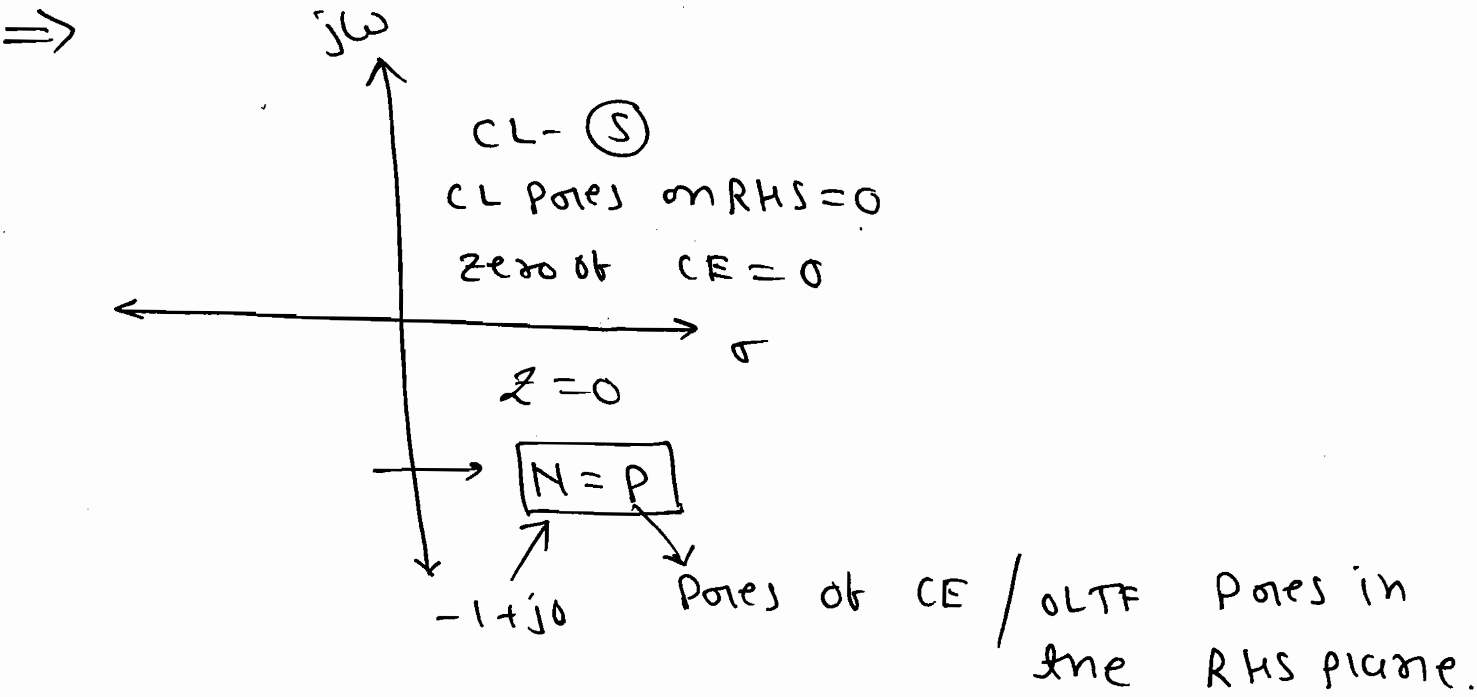
⇒ Compare eqn ② & ③.

$$\boxed{\text{Zeros of CE = CLTF Poles.}}$$ ***

⇒ $N = P - Z$

- N → No. of encirclements about critical point $(-1+j0)$
- P → Poles of CE in the RH s-plane → OLTF poles in the RH s-plane
- Z → Zeros of CE in the RHS plane → CLTF Poles in the RH s-plane

$\Rightarrow$



$\Rightarrow$ The Close-loop Pole is nothing but, zeros of CE which must be zero, in the right of s-plane that means $\boxed{Z=0}$ & $\boxed{N = P - Z = P}$

* **Nyquist Stability Criteria:-**

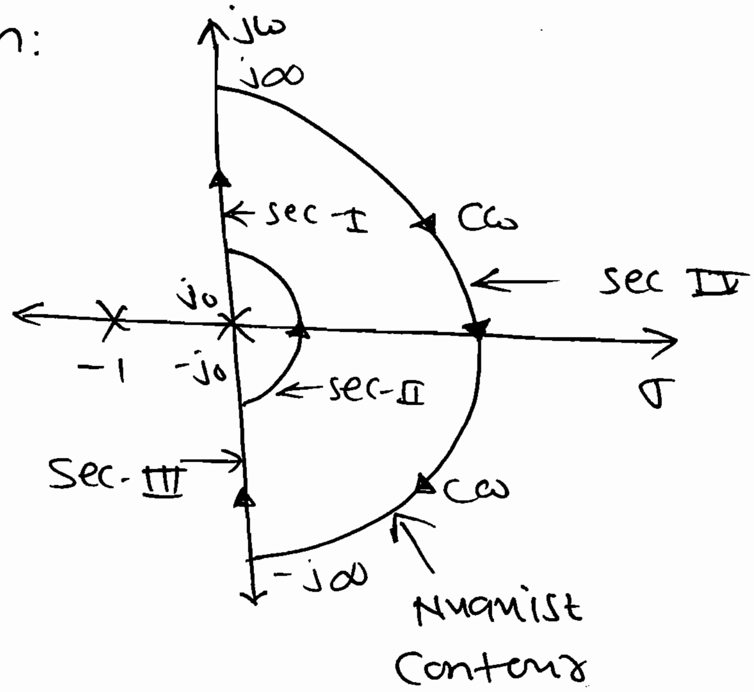
$\Rightarrow$ It states that the No. of ~~A~~-encirclements about the critical point must be equal to poles of chara. eqn which are nothing but OLTF Poles in the Right of s-plane,

i.e. $Z=0$, $N=P$.

Q Draw the Nyquist plot & find the sys. stability to the following sys.

$$G(s).H(s) = \frac{1}{s(s+1)}.$$

Soln:



$$M = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\phi = -90^\circ - \tan^{-1}(\omega).$$

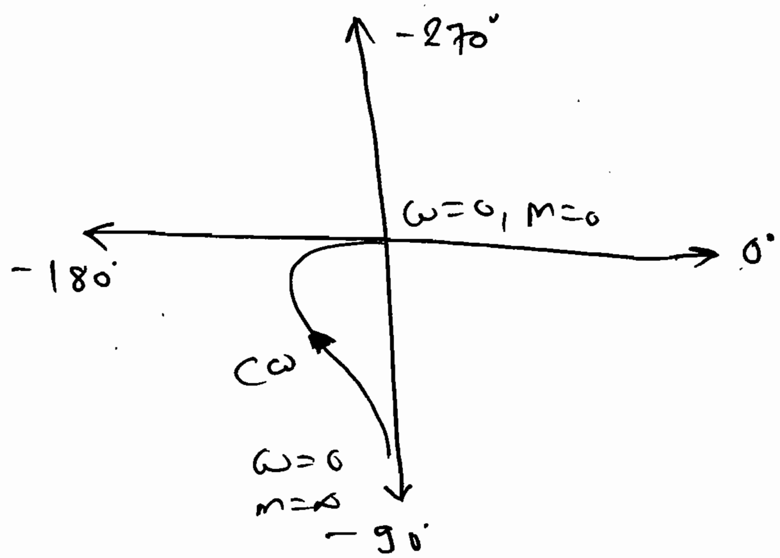
⇒ Sec-I:

→ $\omega = 0^+$ ⇒ $M = \infty$, $\phi = \angle -90^\circ$

$\omega = \infty^+$ ⇒ $M = 0$, $\phi = -180^\circ$

S.D. ⇒ fP ⇒ CW

E.D. ⇒ $\phi_1 - \phi_2 = +ve$ ⇒ CCW.



⇒ Sec-II:

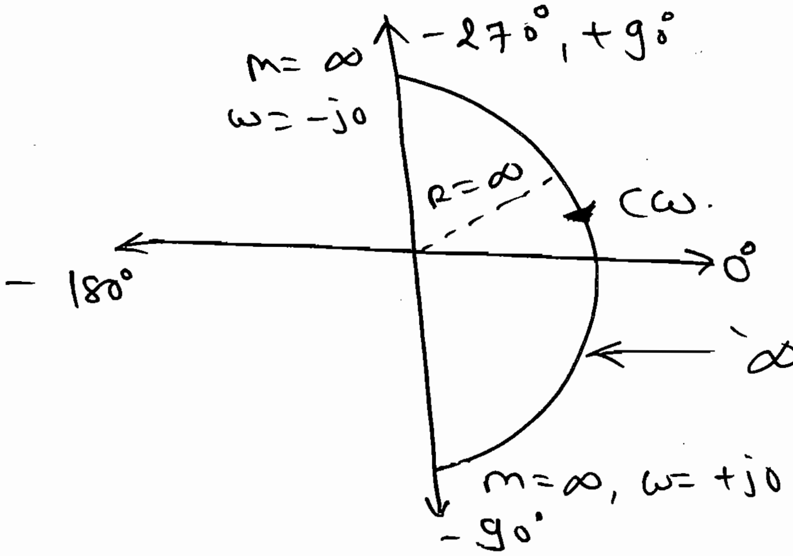
$\Rightarrow$ ~~ω=0~~ $\omega = -j0 \Rightarrow M = \infty \;, \quad \phi = -90^\circ + (180 - \tan^{-1}(0)).$

$$\phi = +90^\circ$$

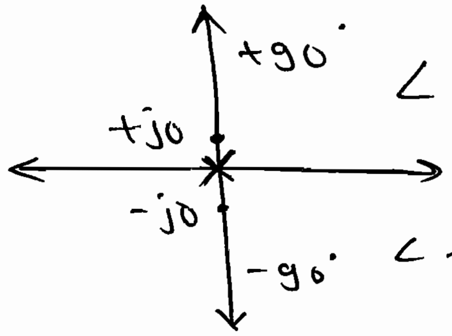
$\omega = +j0 \Rightarrow$ ~~ω=0~~ $, \quad M = \infty \;, \quad \phi = -90^\circ$

E.D. $\Rightarrow \quad \phi_1 - \phi_2 = +ve \Rightarrow$ C.W.

S.D. $\Rightarrow$ C.W.



$\Rightarrow$



$\angle \frac{1}{s}\Big|_{s=+j0} = \frac{\angle 1}{\angle +j0} = -90^\circ$

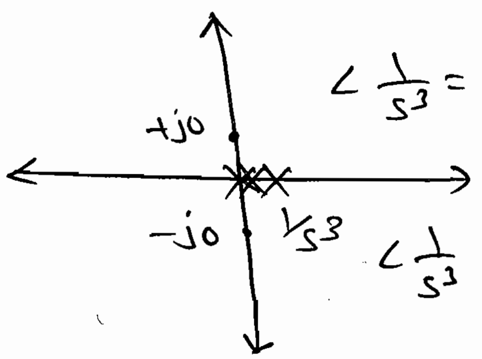
$\angle \frac{1}{s}\Big|_{s=-j0} = \frac{\angle 1}{\angle -j0} = +90^\circ$

$\boxed{180^\circ \times 1}$

$1/s$

1-pole at origin

$\Rightarrow$



$\angle \frac{1}{s^3} = \frac{\angle 1}{3\angle +j0} = -270^\circ$

$\angle \frac{1}{s^3} = \frac{\angle 1}{3\angle -j0} = +270^\circ$

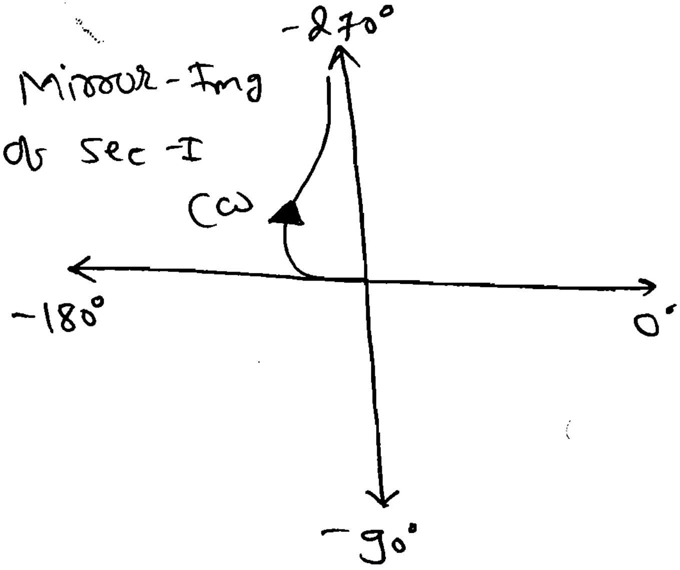
$540 \Rightarrow \boxed{180^\circ \times 3}$

$1/s^3$

3-pole at origin.

Note: No. of '∞' Radius half (180°) circles = No. of Poles at origin.

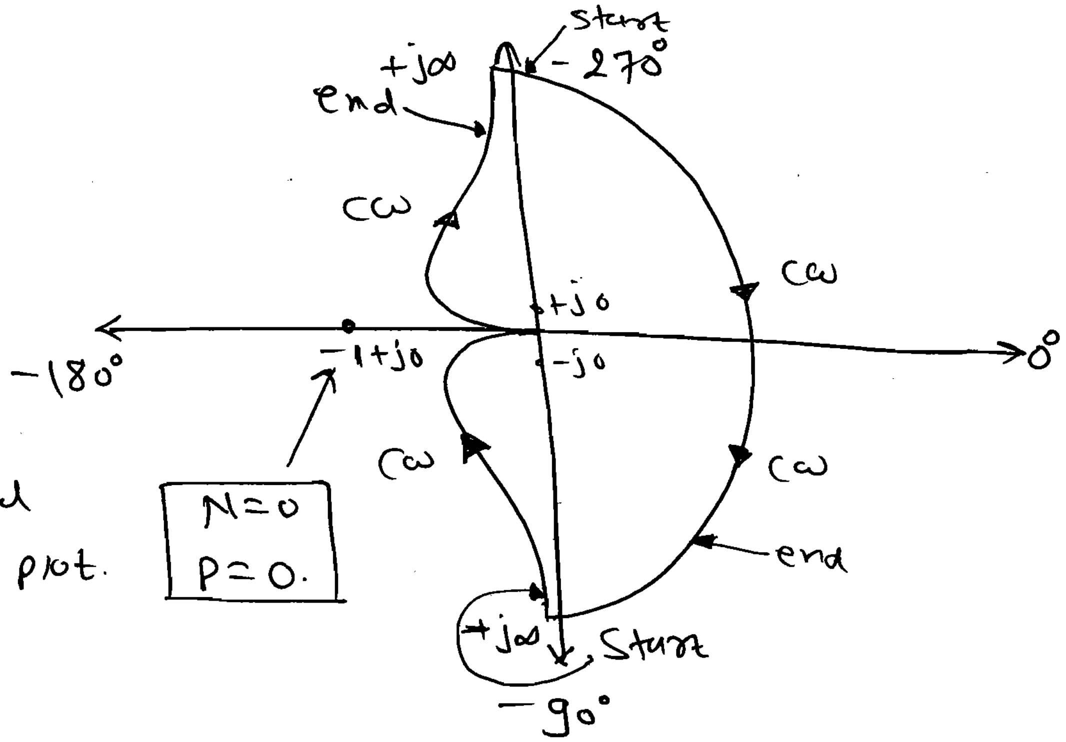
sec-III :-



⇒ sec-III is a Mirror-image of sec-I, about the Real axis but the direction is Continuous.

⇒ **Section-IV:**

⇒ The sec-IV gives the magnitude of at $\omega = \infty$ ⇒ $m = \pm 0$. that means it is a point at origin. neglect the sec-IV.



Actual polar plot.

Note:

⇒ The ∞ radius half circle should be start where the mirror img. end & the ∞ radius half circle end where the actual polar plot is start.

$\Rightarrow$ The ∞ radius half circle direction always CCW because it depends on nyquist contour direction.

$\boxed{Q}$ $GH(s) = \dfrac{10}{(s+1)}$.

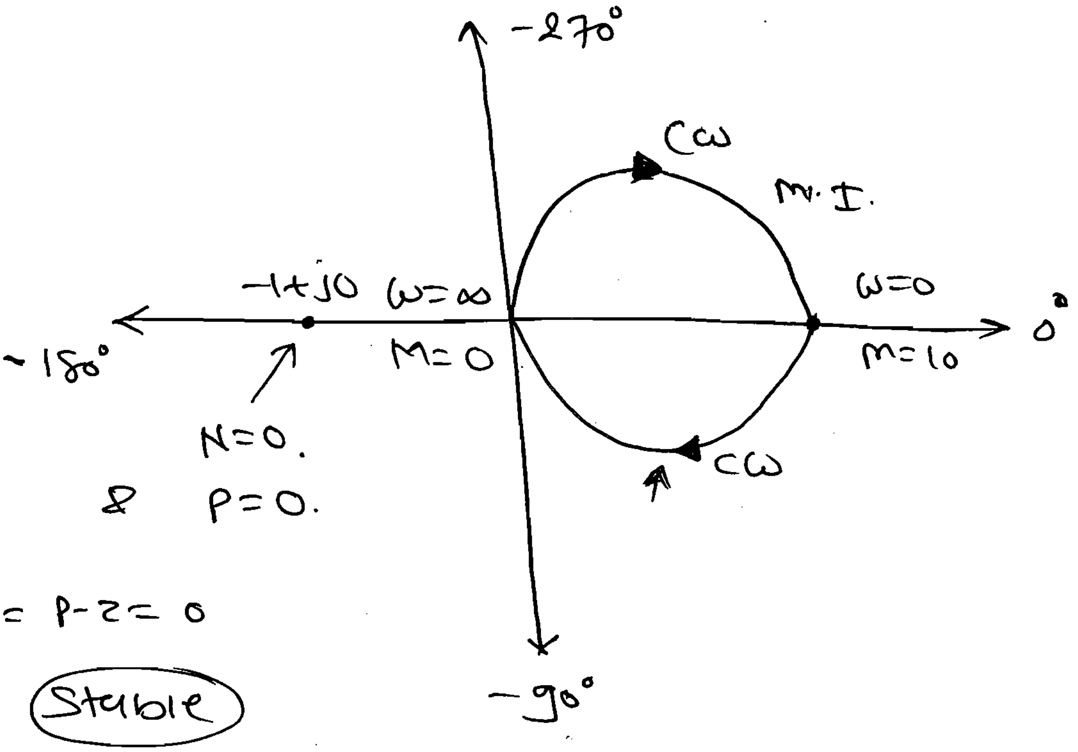
$\underline{\underline{\text{Soln:}}}$ $M = \dfrac{10}{\sqrt{\omega^2+1}}$. & $\phi = -\tan^{-1}(\omega)$.

$\omega = 0 \Rightarrow$ $M = 10$ & $\phi = 0°$.

$\omega = \infty \Rightarrow$ $M = 0$ & $\phi = -90°$

E.D. $\Rightarrow$ $\phi_1 - \phi_2 = +ve \Rightarrow CW$

S.D. $\Rightarrow$ $fp = CW$.



$N = P - Z = 0$

Stable

$\boxed{Q}$ $GH(s) = \dfrac{10}{(s+1)(s+2)}$.

$\underline{\underline{\text{Soln:}}}$ $M = \dfrac{10}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}$.

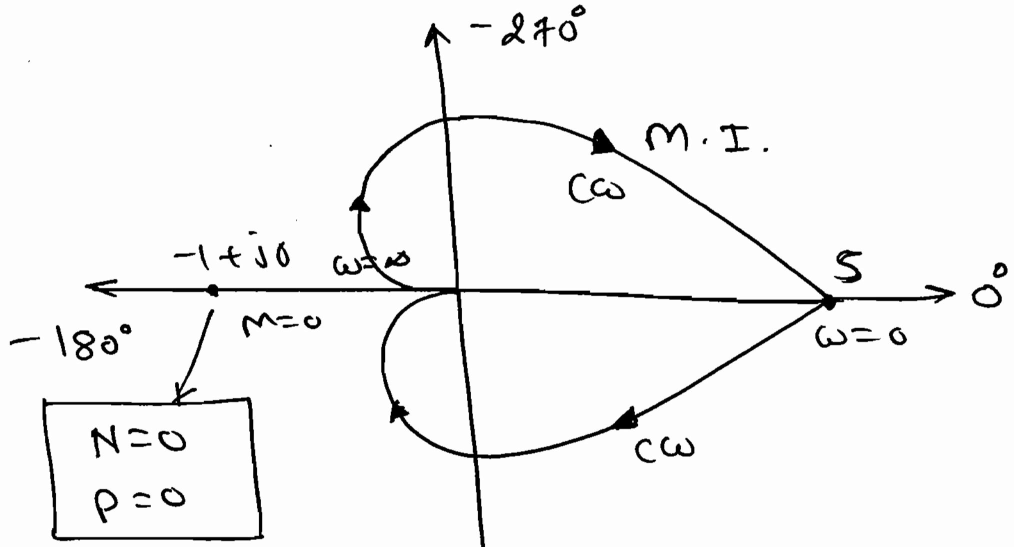
$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\Rightarrow \quad \omega = 0 \Rightarrow M = \frac{10}{2} = 5 \quad , \quad \phi_1 = 0^\circ$

$\omega = \infty \Rightarrow M = \infty \quad , \quad \phi_2 = -180^\circ .$

$\underline{\underline{E.D.}} \Rightarrow \phi_1 - \phi_2 = +ve = C\omega.$

$\underline{\underline{S.D.}} \Rightarrow GP = C\omega.$



$\boxed{N = P - Z = 0}$

CL (S) //

$\boxed{a}$ $GH(s) = \dfrac{10}{s^2 (s+1)(s+2)}$.

$\underline{\underline{\text{Sol}^n}}$: $M = \dfrac{10}{\omega^2 \sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}$.

$\phi = -180^\circ - \tan^{-1}(\omega) - \tan^{-1}(\omega/2).$

$\omega = 0 \Rightarrow M = \infty. , \phi_1 = -180^\circ$

$\omega = \infty \Rightarrow m = 0 , \phi_2 = -360^\circ$

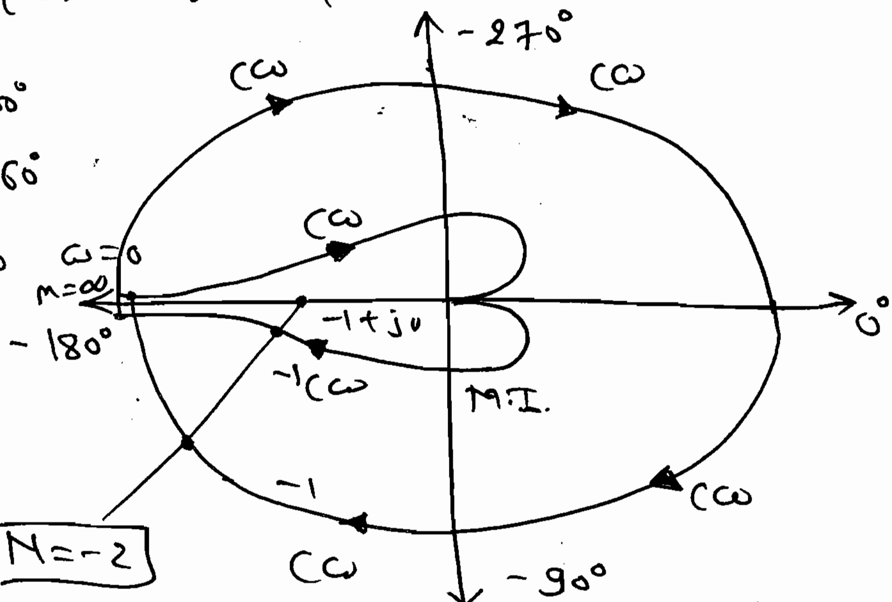
$\underline{\underline{E.D.}} \Rightarrow \phi_1 - \phi_2 = +ve = C\omega$

$\underline{\underline{S.D.}} \Rightarrow GP = C\omega.$

$P = 0$

$Z = 0$ so, $\boxed{N \neq P}$

$\Rightarrow$ (US)



$\Rightarrow$ Here, $N = -2$, but $P = 0$

so, $N \neq P \Rightarrow$ CL US.

$\Rightarrow N = P - Z$

$Z = P - N$

$Z = 0 - (-2)$

$\boxed{Z = 2} \Rightarrow$ 2 CL Pole in the Right of S-plane.

$\boxed{Q}$ $GH(s) = \dfrac{10}{s^3(s+10)}$.

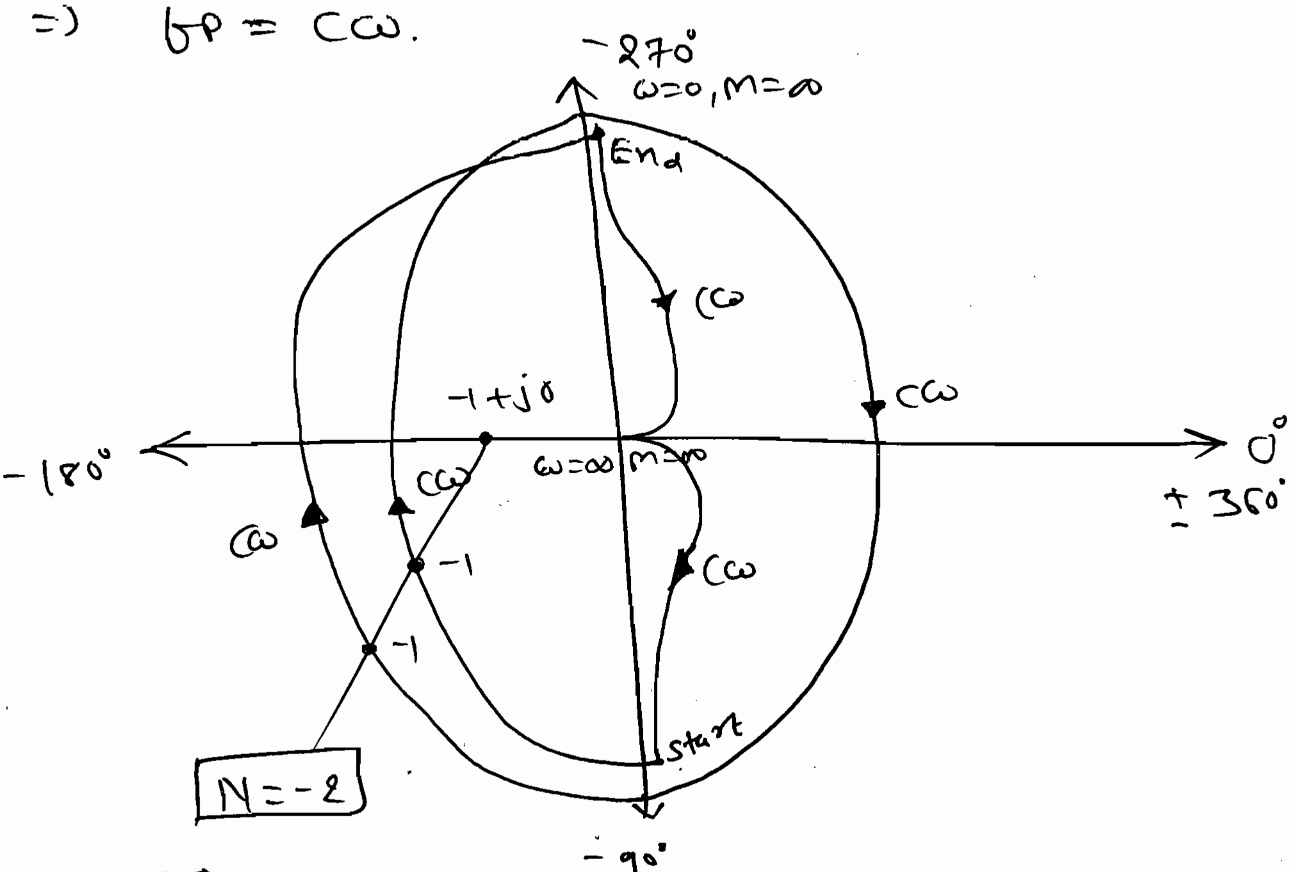
Sol^n: $M = \dfrac{10}{\omega^3\sqrt{\omega^2+100}}$, $\phi = -270^\circ - \tan^{-1}(\omega/10)$

$\Rightarrow \omega = 0 \Rightarrow M = \infty$, $\phi_1 = -270^\circ$.

$\Rightarrow \omega = \infty \Rightarrow M = 0$, $\phi_2 = -360^\circ$.

E.D. $\Rightarrow \phi_1 - \phi_2 = +ve = $ CW.

S.D. $\Rightarrow$ SP = CW.



$P - Z = 0 - 0 = 0$

$\Rightarrow \boxed{N \neq P} \Rightarrow$ US

⇒ The no. of CL Poles on RHS plane

$N = P - Z$

$\therefore -2 = 0 - Z \Rightarrow \boxed{Z = 2} \rightarrow$ CL Poles on RH s-plane.

$\boxed{Q}$ $GH(s) = \frac{1}{s(1-s)}$.

Sol^n: $M = \frac{1}{\omega\sqrt{\omega^2+1}}$, $\phi = -90^\circ - (-\tan^{-1}\omega)$.

$\phi = -90^\circ + \tan^{-1}\omega$.

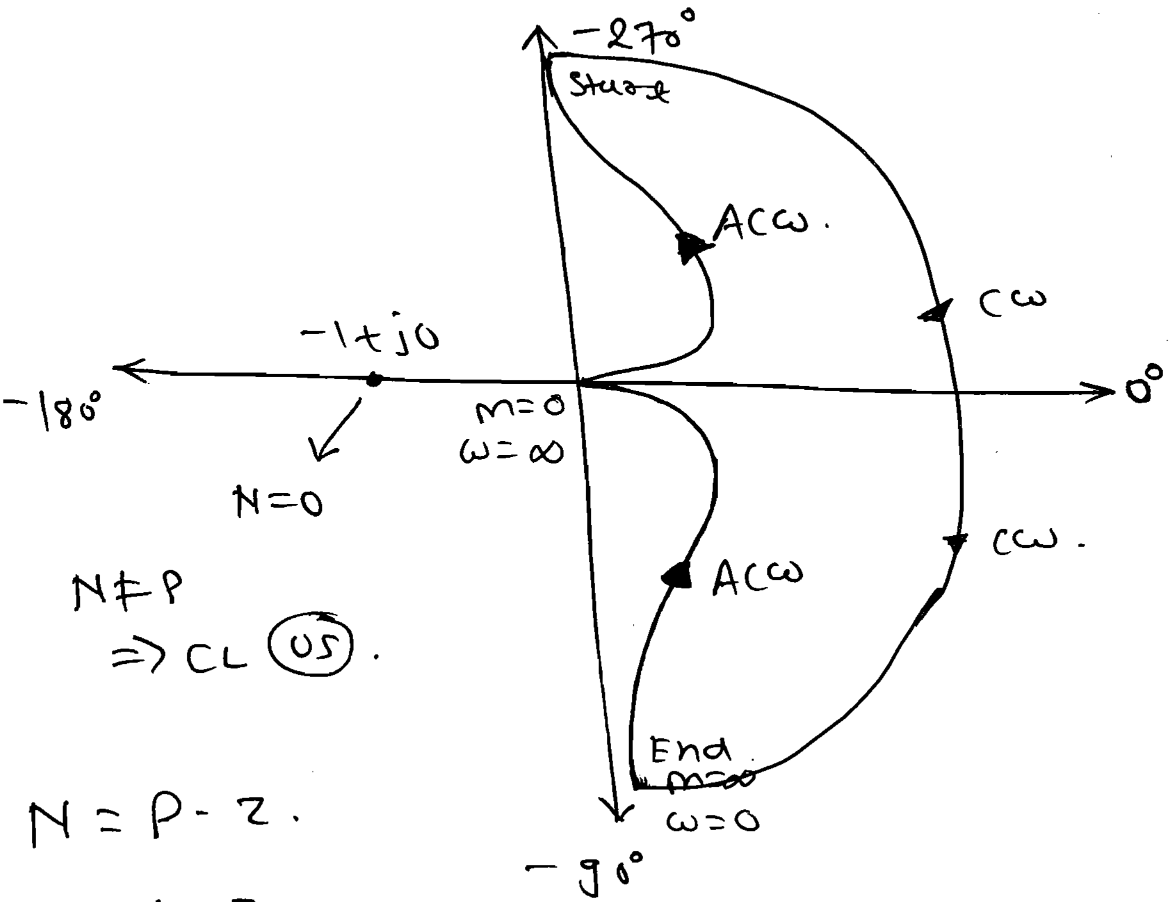
$\omega = 0 \Rightarrow M = \infty, \ \phi = -90^\circ$

$\omega = \infty \Rightarrow M = 0, \ \phi = 0^\circ$.

$\boxed{P = 1}$

E.D. ⇒ $\phi_1 - \phi_2 = -ve \Rightarrow$ ACω.

S.D. ✗ (∵ -ve sign in T.F.).



N≠P

⇒ CL US.

⇒ $N = P - Z$.

$\therefore 0 = 1 - Z$.

⇒ $\boxed{Z = 1}$ ⇒ 1 CL pole on RHS plane.

(OR) By RH- criterion

CE $\longrightarrow$ $1 + GH(s) = 0$

$\Rightarrow 1 + \frac{1}{s(1-s)} = 0$

$\Rightarrow s - s^2 + 1 = 0.$

$\Rightarrow -s^2 + s + 1 = 0.$

① sign change

s^2	-1	1
s^1	1	
s^0	1	

$\Rightarrow$ 1 sign chang

$\Rightarrow$ 1 CL pole on RHS plane!

Q Find the range of K value for close loop system stability by using Nyquist stability Analysis, to the following system.

$$G(s).H(s) = \frac{K}{(s+1)(s+2)(s+3)}.$$

Soln: $M = \frac{K}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}.$

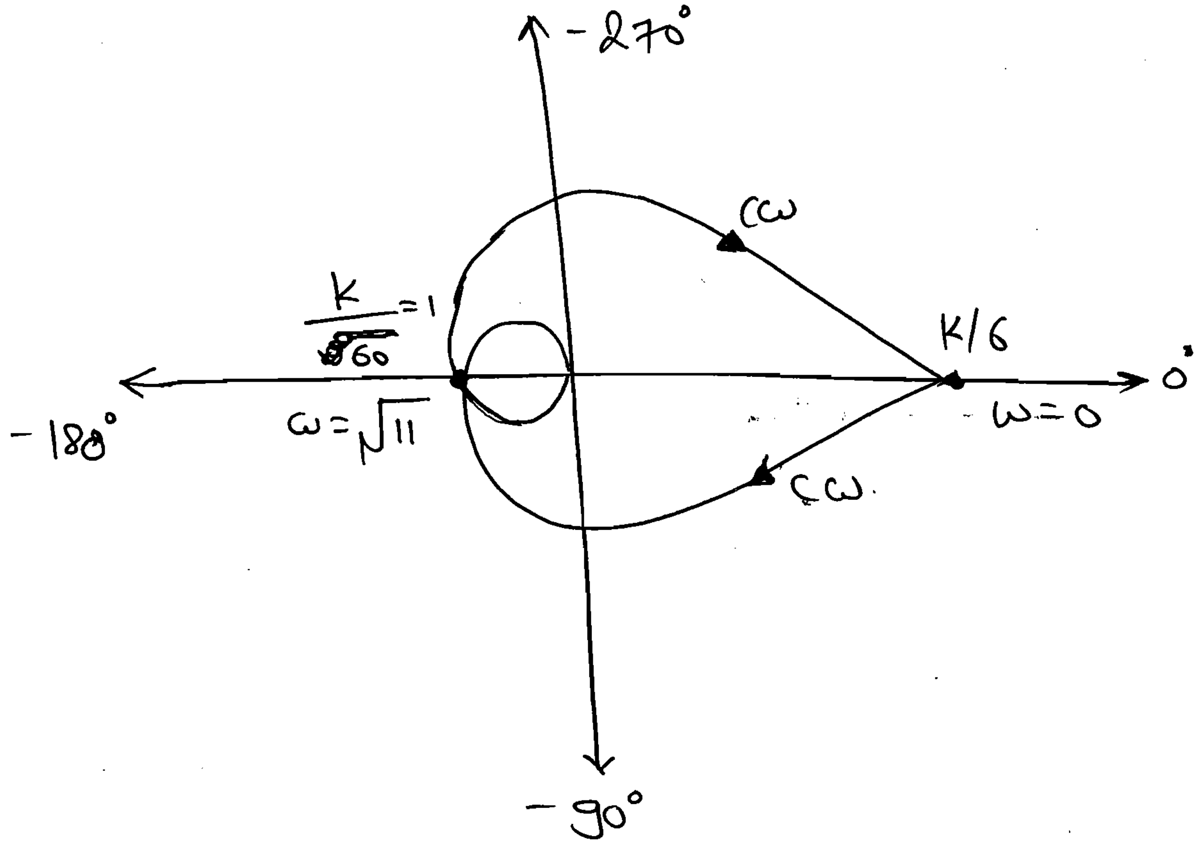
$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$

$\omega = 0 \Rightarrow M = \frac{K}{\sqrt{36}} = \frac{K}{6}, \quad \phi = 0°.$

$\omega = \infty \Rightarrow M = 0, \quad \phi = -270°.$

E.P $\Rightarrow$ $\phi_1 - \phi_2 = +ve = C.\omega.$

S.P $\Rightarrow$ FP = C.ω.



$\Rightarrow$ I.P. with -180° axis is $\frac{K}{60} = \frac{K}{60}$.

* **Procedure to find the range of k values:-**

$\boxed{S1}$: Assume that the I.P. with -180° must be equal to the critical point, that means the mag. of I.P. = mag of critical point

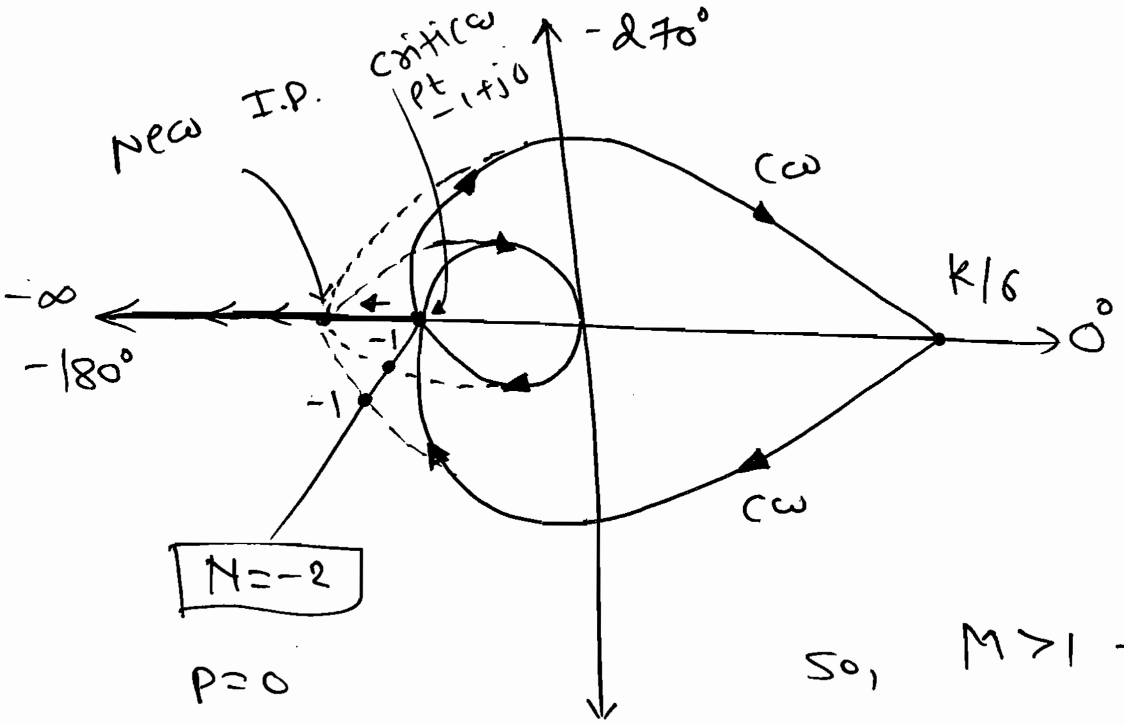
i.e. Mag. $\boxed{M=1}$

$\Rightarrow$ In the above case $\boxed{K/60=1.}$

$\boxed{S2}$: Shift the I.P. towards $-\infty$ by Considering $\boxed{M>1}$.

$\Rightarrow$ In this case, the critical point inside the loop. For this get the no. of encirclement & Condition for stability.

⇒



P=0

N ≠ P

CLS → US

So, M > 1 → US

⇒ $\frac{K}{60} > 1$

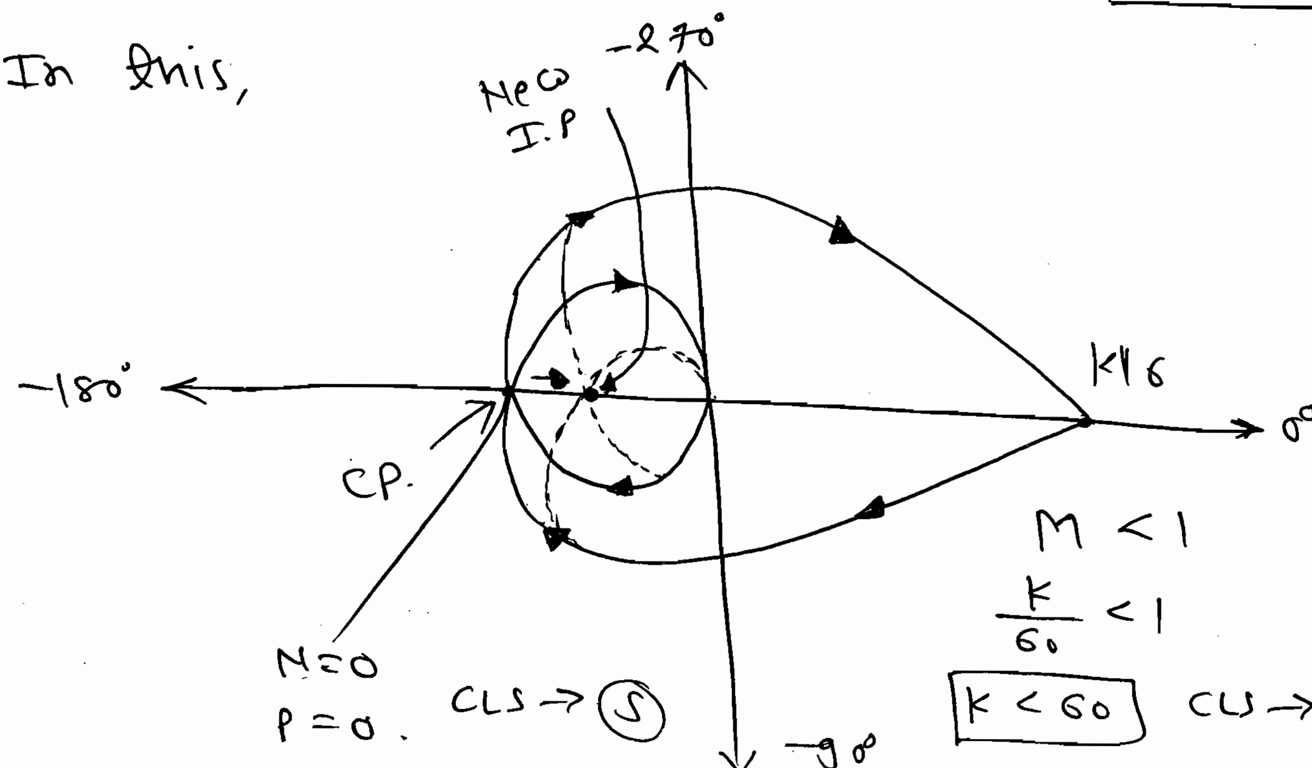
⇒ $\boxed{K > 60}$ ⇒ US.

N = P − Z ⇒ −2 = 0 − Z

⇒ $\boxed{Z = 2}$ → CL poles on RHS plane.

$\boxed{S3}$: Shift the intersection point towards the origin by considering $\boxed{M < 1.}$

⇒ In this,



M < 1

$\frac{K}{60} < 1$

$\boxed{K < 60}$ CLS → S.

P = 0. CLS → S

(54): Whenever the stability condition is less than certain value then the lower limit is desided by I.P. with $0°$.

$\Rightarrow$ The intersection point with $0°$ must be greater than -1.

$\Rightarrow$ In the above problem $\frac{K}{6} > -1$.

$$\Rightarrow \boxed{K > -6}$$

So, $\boxed{-6 < K < 60.}$ $\Rightarrow$ Stable system.

$\boxed{Q}$ $GH(s) = \dfrac{K(s+3)}{s(s-1)}$.

Soln: $M = \dfrac{K \times \sqrt{\omega^2+9}}{\omega \times \sqrt{\omega^2+1}}$.

$\Rightarrow \phi = -90° + \tan^{-1}(\omega/3) - 180° + \tan^{-1}(\omega)$.

$\phi = -270° + \tan^{-1}(\omega) + \tan^{-1}(\omega/3)$.

$\Rightarrow \omega = 0 \Rightarrow M = \infty, \quad \phi = -270°$.

$\Rightarrow \omega = \infty \Rightarrow M = 0, \quad \phi = -90°$.

E.D. $\Rightarrow \phi_1 - \phi_2 \Rightarrow -ve \Rightarrow$ ACW.

S.D X.

$\Rightarrow$ I.P. with $-180°$.

$\therefore -180° = -270° + \tan^{-1}\left(\dfrac{\omega + \omega/3}{1 - \omega^2/3}\right)$.

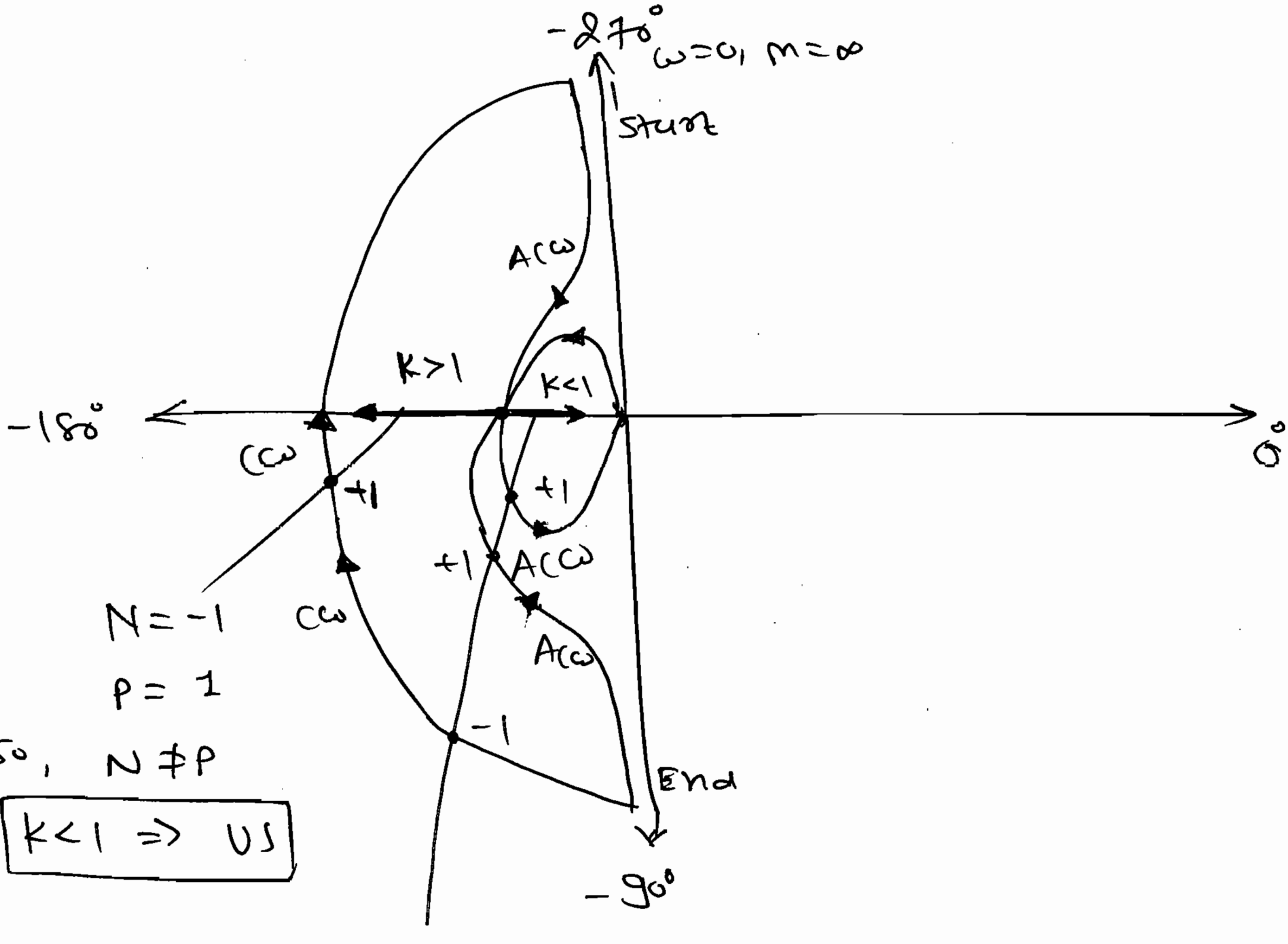
$\therefore 90° = \tan^{-1}\left(\dfrac{4\omega}{3-\omega^2}\right)$.

$\Rightarrow \quad \omega = \sqrt{3}$ rad/sec.

$$\Rightarrow \quad M\Big|_{\omega=\sqrt{3}} = \frac{K\sqrt{3+9}}{(\sqrt{3})^2 \times \sqrt{3+1}}$$

$$= \frac{K\sqrt{12}}{\sqrt{3}\times 2} = \frac{K \times \cancel{2\sqrt{3}}}{\cancel{2\sqrt{3}}} = K.$$

$$\therefore \quad \boxed{M\Big|_{\omega=\sqrt{3}} = K}$$



$N = -1$

$p = 1$

So, $N \neq P$

$\boxed{K<1 \Rightarrow US}$

$N = +1+1-1 = +1$

$N = +1, \quad P = +1.$

So, $\boxed{N=P} \Rightarrow$ CLS (S). $\boxed{K<1}$

No I.P. with 0°

So, $\boxed{0 < K < 1 \Rightarrow \text{CLS (S)}}$.

$\boxed{Q}$ $G(s)\cdot H(s) = \dfrac{K(s+2)}{(s+1)(s-1)} \Rightarrow \boxed{P=+1}$

Soln:

$$M = \frac{K\sqrt{\omega^2+4}}{\sqrt{\omega^2+1}\times\sqrt{\omega^2+1}}$$

$$\phi = -\cancel{\tan^{-1}\omega} + \tan^{-1}(\omega/2) - 180^\circ + \cancel{\tan^{-1}\omega}$$

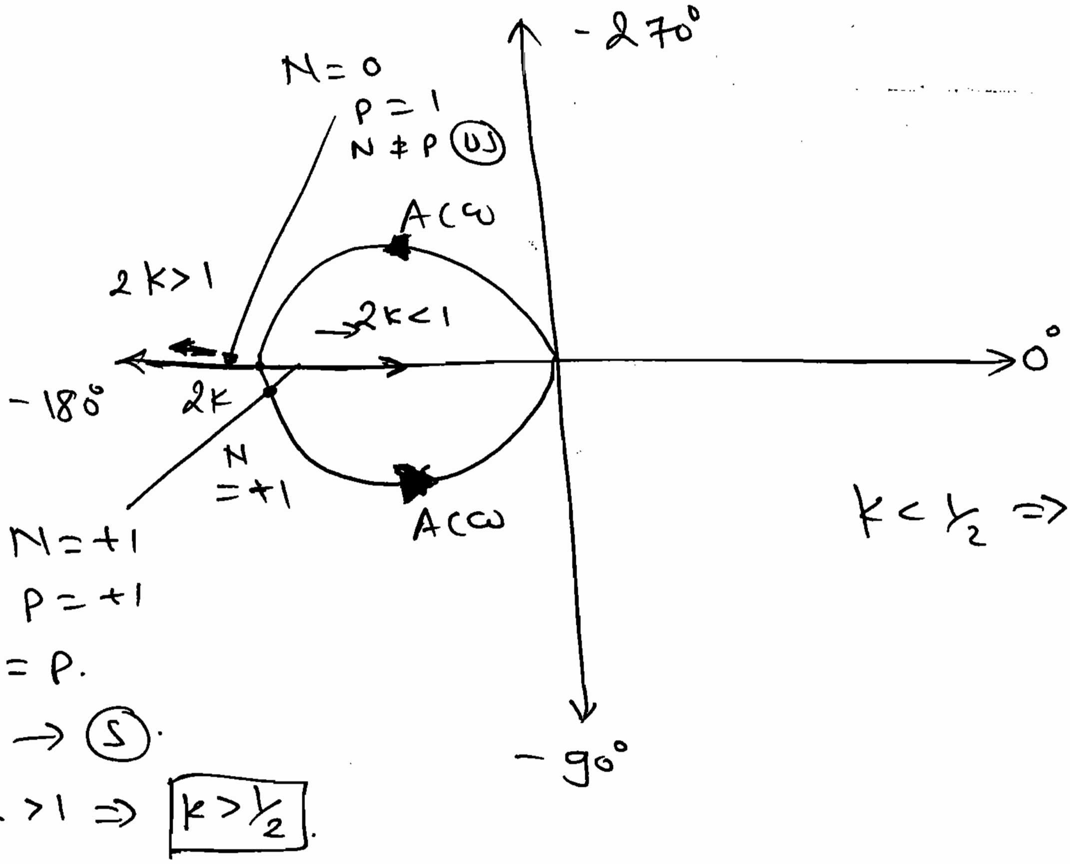
$$\phi = -180^\circ + \tan^{-1}(\omega/2).$$

$\omega = 0 \Rightarrow M = 2K, \quad \phi = -180^\circ.$

$\omega = \infty \Rightarrow M = 0, \quad \phi = -90^\circ.$

E.D. $\Rightarrow \phi_1 - \phi_2 = -ve \Rightarrow$ ACW

S.D. X.



$K < 1/2 \Rightarrow$ US

CLS → S.

$2K > 1 \Rightarrow \boxed{K > 1/2}$.

S

$\boxed{Q}$ $G(s)\cdot H(s) = \dfrac{K(s-2)}{(s+2)}$.

Soln:

$$M = \frac{K\sqrt{\cancel{\omega^2+4}}}{\sqrt{\cancel{\omega^2+4}}} = K.$$

$\Rightarrow \quad \phi = -\tan^{-1}(\omega/2) + 180^\circ - \tan^{-1}(\omega/2)$

$\phi = 180^\circ - 2\tan^{-1}(\omega/2).$

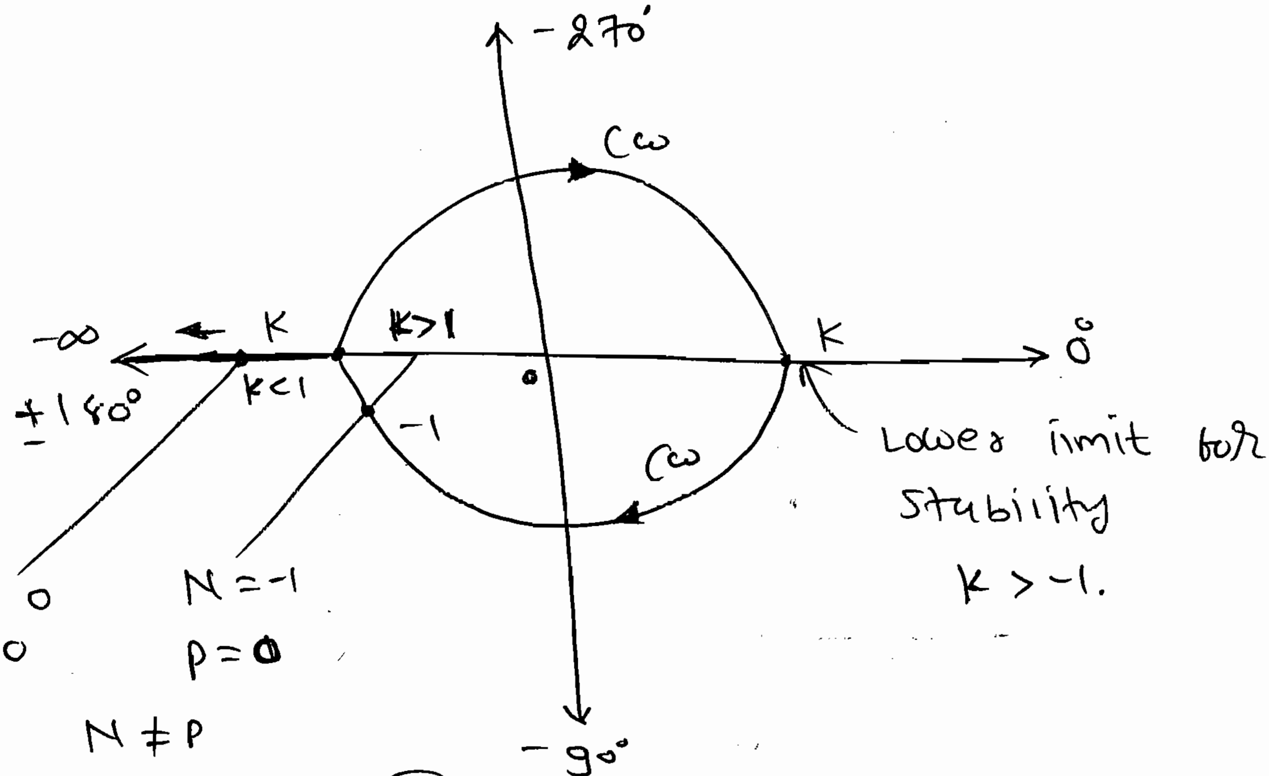
$\omega = 0 \Rightarrow M = K$, & $\phi_1 = +180^\circ$

$\omega = \infty \Rightarrow M = K$, & $\phi = 180^\circ - 2(90^\circ)$

$\phi_2 = 0^\circ$

$\Rightarrow$ P.D. $\Rightarrow \phi_1 - \phi_2 = 180^\circ - 0^\circ = +ve = \text{CCW}.$

S.D ✗.



CLS: S

$\boxed{K<1}$

$N \neq P$

So, $K>1 \Rightarrow$ US.

So, $\boxed{-1 < K < 1} \Rightarrow$ CLS $\rightarrow$ S.

* GM & PM:

$\Rightarrow \quad GM = \dfrac{1}{M|_{\omega = \omega_{pc}}}.$

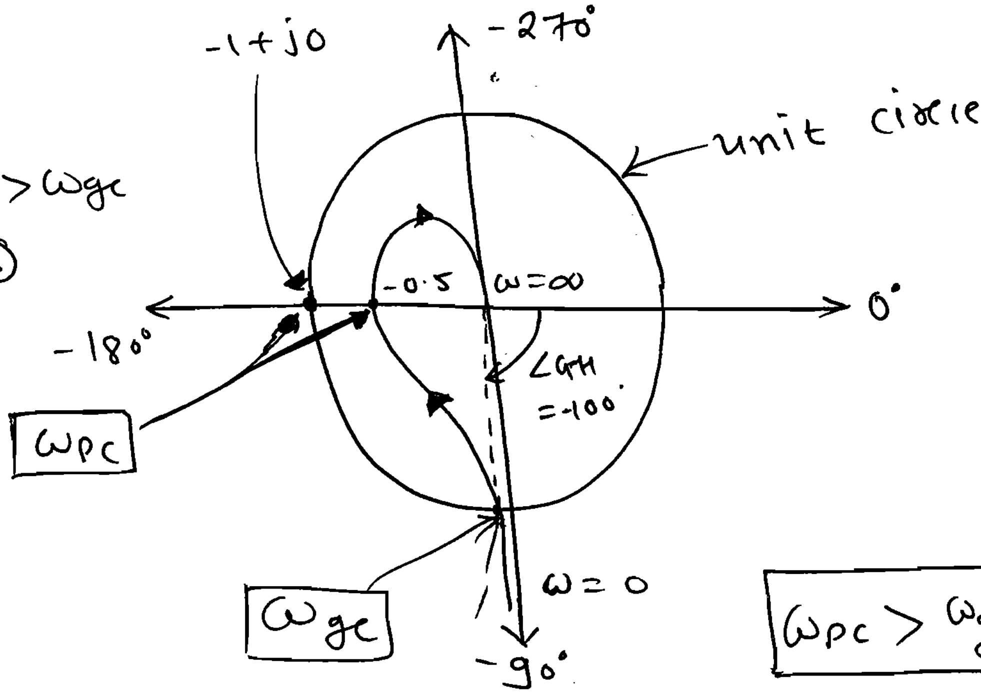
$\Rightarrow \quad PM = 180^\circ + \angle GH|_{\omega = \omega_{gc}}.$

Q Identify the Stability to the following Polar plots:

①

$\omega_{pc} > \omega_{gc}$

$\Rightarrow$ S



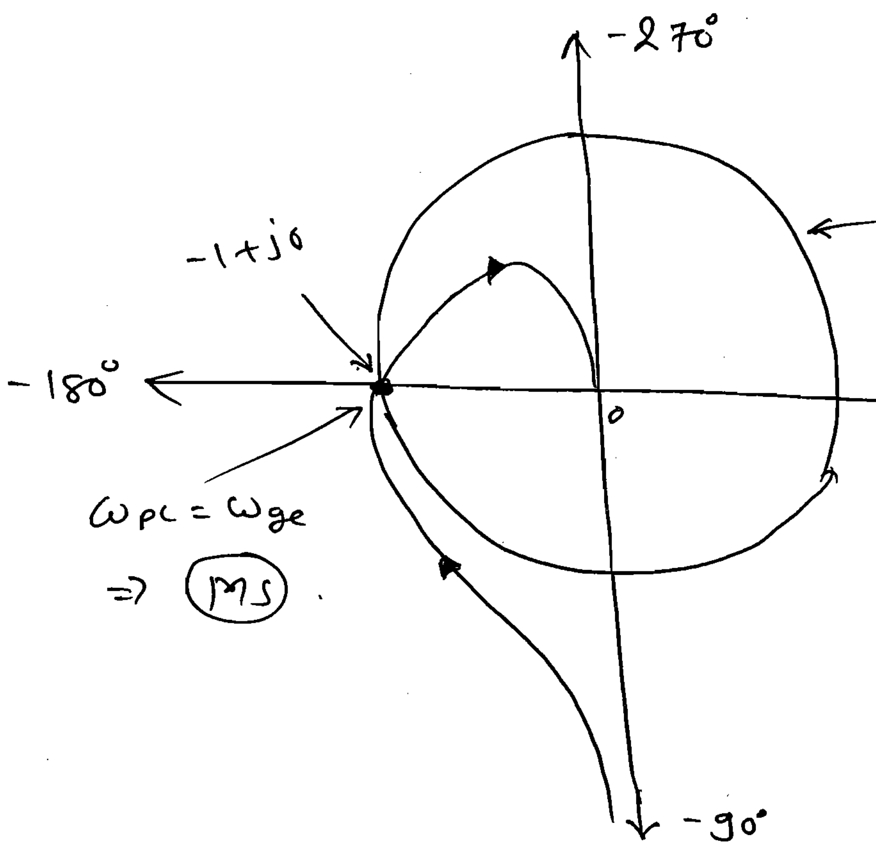
$\boxed{\omega_{pc} > \omega_{gc}} \rightarrow$ S

$$\Rightarrow GM = \frac{1}{M|_{\omega=\omega_{pc}}} = \frac{1}{0.5} = 2, > 1 \text{ (L)}$$

$$PM = 180° + \angle GH = 180° + (-100°) = +80° > 0$$

So, CL system $\rightarrow$ S.

②



$\omega_{pc} = \omega_{gc}$

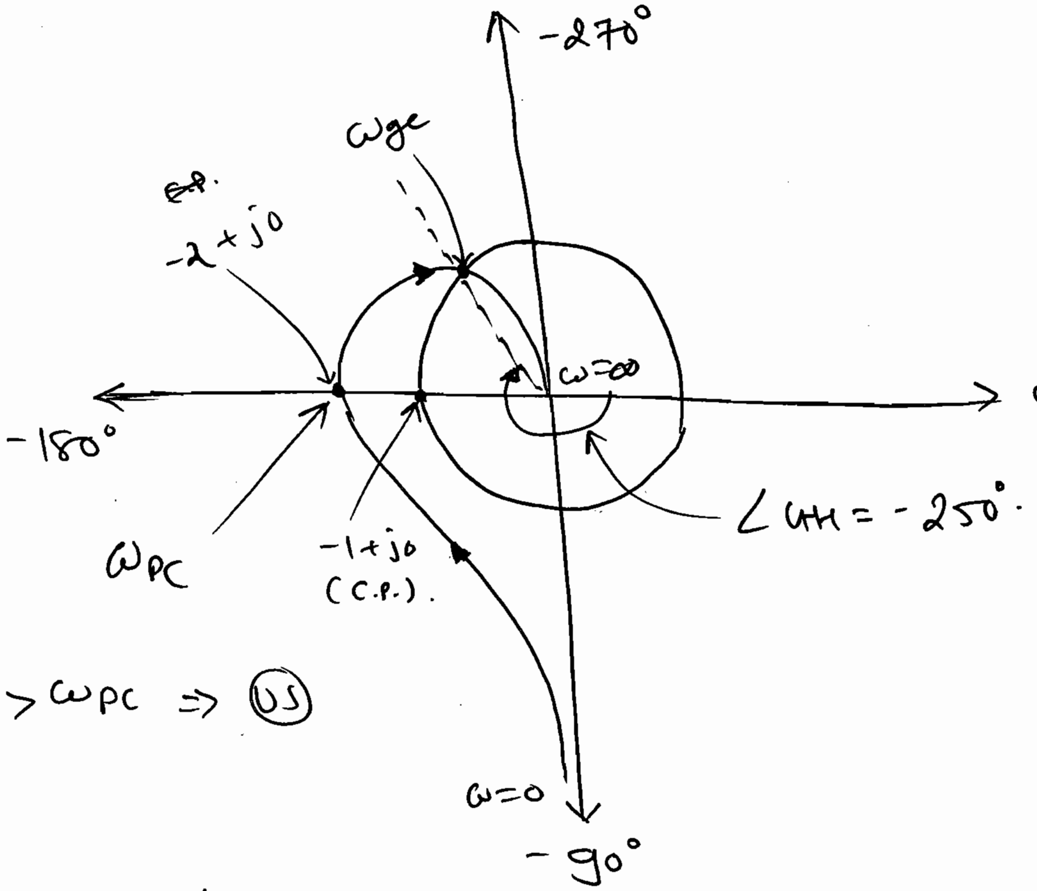
$\Rightarrow$ MS.

$$GM = \frac{1}{1} = 1.$$

$$PM = 180° - 180° = 0°$$

So, MS

(3)



$\omega_{gc} > \omega_{pc} \Rightarrow$ US

$$\Rightarrow GM = \frac{1}{M|_{\omega=\omega_{pc}}}$$

$$= \frac{1}{2} = 0.5 < 1 \quad \text{(L)}$$

US.

$$\Rightarrow PM = 180° + \angle GH = 180° - 250° = -70° < 0$$

So, CLS → US.

Note:

(i) Whenever the plot intersect −180° line with mag. less than 1 (i.e $\boxed{M<1}$), the sys. is $\boxed{\text{stable}}$ because here

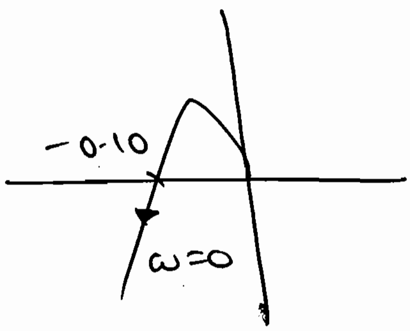
$\boxed{\omega_{pc} >> \omega_{gc}}$

(ii) Whenever the plot intersect −180° line with mag. $\boxed{M=1}$, then the sys. is $\boxed{\text{Marginal stable}}$ because here

$\boxed{\omega_{pc} = \omega_{gc}}$

(iii) Whenever the plot intersect $-180°$ line with mag. $\boxed{M>1}$ then the syst. is $\boxed{\text{unstable}}$ because here $\boxed{\omega_{gc} > \omega_{pc}}$

Q The Polar plot of $G(s) \cdot H(s)$ for $K=10$ is given below. The range of 'K' for sys. stability is ?

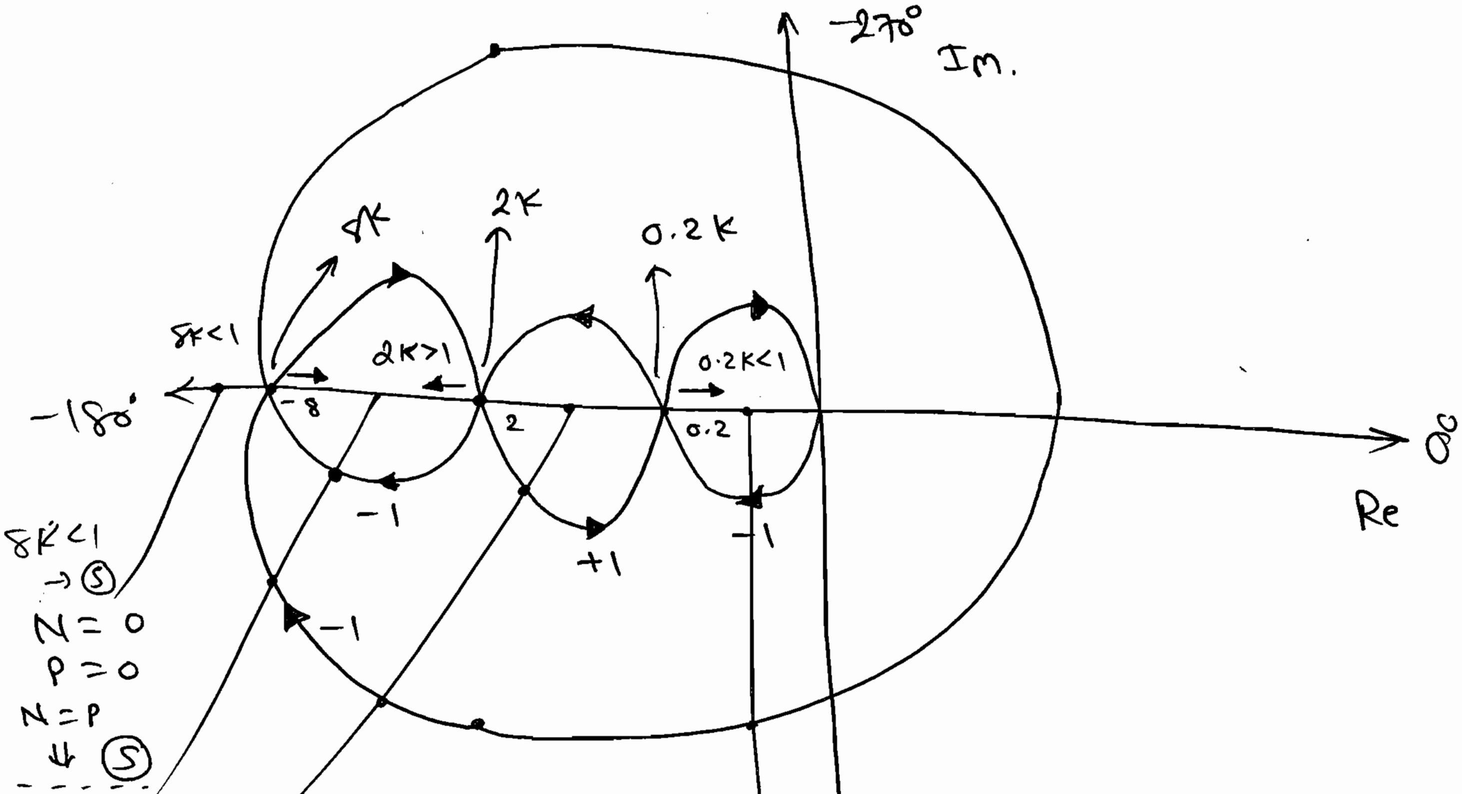


Soln: Note: To find the range of K value, product the K with given I.P. divided by given K value. (i.e here 10).

So, For CLS to be (S) $K \cdot \frac{0.10}{10} < 1.$

$\therefore \boxed{K < 100}$

Q The Polar diagram of a Conditionally Stable sys. for open loop gain $K=1$ is given is shown in the fig. The OLTF of the sys. is known to be stable. The CL system stable for ?



N = -2

N ≠ P

US

N = -1+1=0.

P = 0

N = P ⇒ S

2K>1 &

0.2K <1 → S

N=-1-1=-2

P=0

N ≠ P US

- -90°

(1) OL S

OL RH → $\boxed{P=0}$

$0.2K<1$ & $2K>1$

$\frac{K}{5}<1$

$\boxed{K<5}$

$\boxed{K>0.5}$

$\boxed{0.5<K<5}$ ⇒ S

(2) → $8K<1.$ ⇒ $\boxed{K<\frac{1}{8}}$ ⇒ S

So, Ans: $\boxed{K<1/8 \ \& \ 0.5<K<5 \Rightarrow S}$

$\boxed{*}$ Consider the following Nyquist plots of loop T.F. over $\omega = 0$ to $\omega = \infty$, which of the following plots represents stable closed loop sys.?

* Calculation of Gain Margin & Phase Margin:

Q Calculate the gain margin for

$$GH(s) = \frac{1}{s(s+1)(s+2)}.$$

Soln:

$$M = \frac{1}{\omega\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}.$$

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}(\omega/2).$$

Now, $$Gm = \frac{1}{M|_{\omega=\omega_{pc}}}$$

for ω_{pc}, $\phi|_{\omega=\omega_{pc}} = -180^\circ.$

$$\therefore -180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega_{pc} + \frac{\omega_{pc}}{2}}{1 - \frac{\omega_{pc}^2}{2}}\right).$$

$$\therefore \tan(90^\circ) = \frac{3\omega_{pc}}{2-\omega_{pc}^2}.$$

$$\therefore \boxed{\omega_{pc} = \sqrt{2} \text{ rad/sec.}}$$

Now, $$M|_{\omega=\omega_{pc}} = \frac{1}{\sqrt{2}\sqrt{3}\sqrt{6}} = 1/6.$$

So, $$Gm = \frac{1}{M|_{\omega=\omega_{pc}}} = 1/{1/6} = 6.$$

$$\therefore \boxed{Gm = 6}$$

$$Gm_{dB} = 20\log 6 = \boxed{15.56\,dB.}$$

* **Steps for finding GM:**

$\boxed{S1}$: find ω_{pc} by using $\angle GH = -180^\circ$.

$\boxed{S2}$: find $M|_{\omega=\omega_{pc}}$.

$\boxed{S3}$: $GM = \dfrac{1}{M|_{\omega=\omega_{pc}}}$.

$\boxed{Q}$ Calculate the PM for $GH(s) = \dfrac{1}{s(s+1)}$

Soln:

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}, \quad \phi = -90^\circ - \tan^{-1}(\omega).$$

* **Steps for finding PM:**

$\boxed{S1}$: find $\omega_{gc} \longrightarrow M = 1$.

$\boxed{S2}$: $PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}}$.

$$\Rightarrow \quad M = 1 = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\Rightarrow \quad \omega^2(\omega^2+1) = 1$$

$$\omega^4 + \omega^2 - 1 = 0.$$

$$\therefore \omega^2 = 0.618 \checkmark, \quad \omega^2 = -1.618 \times$$

$$\therefore \boxed{\omega_{gc} = 0.786 \text{ rad/sec}}$$

$$\therefore \angle GH|_{\omega=\omega_{gc}} = -90 - \tan^{-1}(0.786).$$

$$\therefore \angle GH|_{\omega=\omega_{gc}} = -128.17^\circ.$$

$$\therefore PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}} = 180^\circ - 128.17^\circ$$

$$\boxed{PM = 51.83}$$

Q find the k value to get the PM $= 30°$, $G(s) \cdot H(s) = \frac{k}{s(s+1)}$.

Sol^n:

$$PM = 180° + \angle GH|_{\omega=\omega_{gc}}$$

$$\therefore 30° = 180° + (-90° - \tan^{-1}\omega).$$

$$\therefore -60° = -\tan^{-1}(\omega_{gc})$$

$$\omega = \tan 60°$$

$$\therefore \boxed{\omega = \sqrt{3} \text{ rad/s.}}$$

at $\omega = \omega_{gc}$, $M = 1$.

$$\therefore M|_{\omega=\omega_{gc}} = 1.$$

$$\therefore \frac{k}{\omega_{gc}\sqrt{\omega_{gc}^2+1}} = 1.$$

$$\therefore k = \sqrt{3} \times \sqrt{3+1} = 2\sqrt{3}.$$

$$\therefore \boxed{k = 2\sqrt{3}.}$$

Q (i) find the k value for the PM $= 60°$.

Sol^n: $G(s) \cdot H(s) = \frac{k}{s(s+2)(s+4)}$.

$$\Rightarrow M = \frac{k}{\omega\sqrt{\omega^2+4} \times \sqrt{\omega^2+16}}.$$

$$\phi = -90° - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/4).$$

$$\Rightarrow PM = 180° + \angle GH|_{\omega=\omega_{gc}}.$$

$$\therefore \quad 60^\circ = 180^\circ + \left(-90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega/4)\right).$$

$$\therefore \quad -30^\circ = -\tan^{-1}\left(\frac{\frac{\omega}{2} + \frac{\omega}{4}}{1 - \omega^2/8}\right).$$

$$\therefore \quad \frac{1}{\sqrt{3}} = \frac{6\omega}{8-\omega^2}.$$

$$8-\omega^2 = \cancel{6/\sqrt{3}\,\omega}. \quad 6\sqrt{3}\omega$$

$$\therefore \quad \cancel{-\omega^2 + \frac{6}{\sqrt{3}}\omega + 8 = 0}.$$

$$\therefore \quad -\omega^2 - 6\sqrt{3}\omega + 8 = 0$$

$$\Rightarrow \quad \boxed{\omega_{ge} = 0.72 \text{ rad/sec.}}$$

Now, $M\Big|_{\omega=\omega_{ge}} = 1.$

$$\therefore \quad \frac{K}{0.72\sqrt{(0.72)^2+4} \times \sqrt{(0.72)^2+16}} = 1.$$

$$\therefore \quad \boxed{K = 4.456}$$

(ii) find the K value to get GM = 20dB.

$\underline{\underline{\text{Soln:}}}$

$$GM = \frac{1}{M\big|_{\omega=\omega_{pc}}}$$

given GM = 20dB $\Rightarrow$ GM = 20 ~~20~~log(GM)

$$GM = 10.$$

$\therefore$ for ω_{pc} $\quad \phi = -180^\circ.$

$$\therefore \quad -180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{4}\right).$$

$\therefore \quad 90^\circ = \tan^{-1}\left(\dfrac{\frac{\omega}{2} + \frac{\omega}{4}}{1 - \omega^2/8}\right)$

$\therefore \quad 1 - \dfrac{\omega^2}{8} = 0$

$$\boxed{\omega_{pc} = \sqrt{8} \text{ rad/sec}}$$

$\Rightarrow \quad G_m = \dfrac{1}{M|_{\omega = \omega_{pc}}}$

$\therefore \quad 10 = \left(\dfrac{K}{\sqrt{8} \times \sqrt{8+4} \times \sqrt{8+16}}\right)^{-1}.$

$\therefore \quad 0.1 = \dfrac{K}{\sqrt{8} \times \sqrt{12} \times \sqrt{24}}.$

$\therefore \quad K = 48 \times 0.1.$

$\therefore \quad \boxed{K = 4.8}$

[a] The OLTF of unity f/b sys. is $G(s) = \left(\dfrac{as+1}{s^2}\right)$, the value of 'a' to get the PM = 45°.

Soln:

$M = \dfrac{\sqrt{(a\omega)^2 + 1}}{\omega^2}.$

$\phi = -180^\circ + \tan^{-1}(\omega a).$

$PM = 180^\circ + \angle GH|_{\omega = \omega_{gc}}.$

$\therefore \quad 45^\circ = \cancel{180} - \cancel{180} + \tan^{-1}(a\omega).$

$\therefore \quad 1 = a\omega \Rightarrow \boxed{\omega_{gc} = 1/a.}$

$$\Rightarrow \quad M\Big|_{\omega=\omega_{gc}} = 1.$$

$$\Rightarrow \quad \frac{\sqrt{1 + a^2 \times \frac{1}{a^2}}}{a^2} = 1.$$

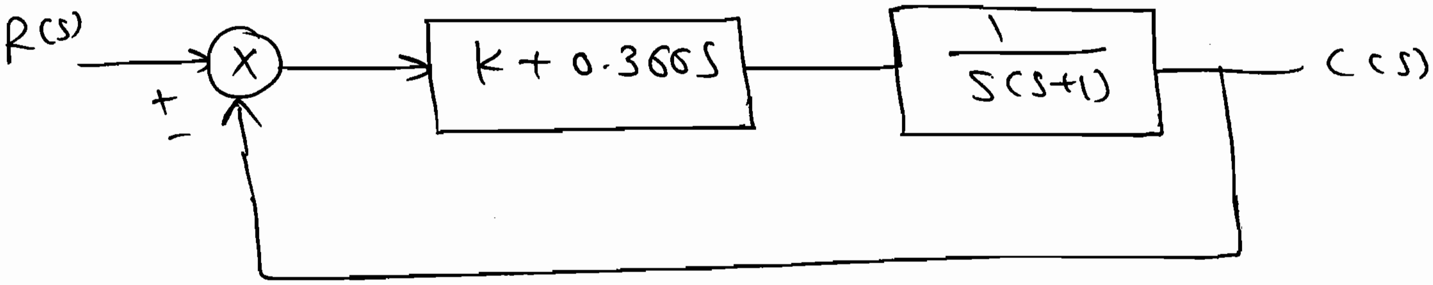
$$\therefore \quad a^2 = \frac{1}{\sqrt{2}}.$$

$$\therefore \quad a^4 = \frac{1}{2}.$$

$$a = (2)^{-1/4}.$$

$$\therefore \quad \boxed{a = 0.8408.}$$

$\boxed{a}$ If the component connected sys as shown in fig. has pm of 60° at a cross over freq. of 1 rad/sec, the value of K is ______.



Soln:

OLTF $\cancel{\frac{C(s)}{R(s)}}$ $G(s).H(s)$ $= \dfrac{(K + 0.366s)}{s(s+1)}.$

$$PM = 180^\circ + \angle GH\Big|_{\omega = 1\ \text{rad/sec}}.$$

$$\therefore \quad \cancel{45^\circ}\ 60^\circ = 180^\circ + \left(-90^\circ - \tan^{-1}(\omega_{gc}) + \tan^{-1}\left(\frac{0.366\omega}{K}\right)\right).$$

$\Rightarrow$ ~~ecosine~~

$\therefore \tan^{-1}(\omega) - \tan^{-1}\left(\frac{0.366\omega}{K}\right) = 30°$.

$\therefore \tan^{-1}(1) - \tan^{-1}\left(\frac{0.366(1)}{K}\right) = 30°$.

$\therefore 45° - \tan^{-1}\left(\frac{0.366}{K}\right) = 30°$.

$15° = \tan^{-1}\left(\frac{0.366}{K}\right)$.

$0.268 = \frac{0.366}{K}$.

$$\boxed{K = 1.366}$$

__Note:__ To Calculate GM & PM required OLTF of either unity (or) Non-unity f/b sys. i.e $G(s)$ (or) $G(s)\cdot H(s)$.

$\boxed{Q}$ The loop gain of a Nyquist plot

$GH(s) = \frac{\pi e^{-j0.25s}}{s}$ passes through the real axis at the point is ______.

__Soln:__ Passing through the -ve real axis means it is a I.P. with $-180°$. i.e. mag. at ω_{pc}.

$\therefore \angle GH(s) = -180° = -90° - 0.25\omega \times \frac{\cancel{\pi}\,180}{\cancel{180}\,\pi}$

$\therefore \cancel{0.25 \times \omega \times \frac{180}{180\pi}} = 90°$

$\therefore 0.25 \times \omega \times \frac{180 \cdot 2}{\pi} = 90$

$\therefore$ ~~$\omega_{pc}=$~~ $\omega_{pc} = \frac{\pi}{0.5} = 2\pi$

$\therefore \boxed{\omega_{pc} = 6.28 \text{ rad/sec}}$

$\therefore M \Big|_{\omega=\omega_{pc}} = \frac{\pi}{2\pi} = \frac{1}{2} = 0.5.$

$\therefore$ I. Pt. Rec $(-0.5, j0)$.

Polar $\Rightarrow$ $0.5 \angle 0°$.

Q Calculate the Gm & pm to the above sys.

Soln:

$$Gm = \frac{1}{M}\Big|_{\omega=\omega_{pc}}$$

$$= \frac{1}{0.5}$$

$$\Rightarrow \boxed{Gm = 2}$$

for ω_{gc} $M = 1$.

$$\therefore \frac{\pi}{\omega} = 1 \Rightarrow \boxed{\omega_{gc} = \pi}$$

$$\therefore Pm = 180° + \left(-90° - 0.25 \times \pi \times \frac{180°}{\pi}\right).$$

$$\boxed{Pm = 45°}$$

Q Calculate the Gm & pm for $GH(s) = \frac{e^{-s}}{s(s+1)}$.

Soln: $s \to j\omega$

$$GH(j\omega) = \frac{e^{-j\omega}}{j\omega(j\omega+1)}.$$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\phi = -90^\circ - \tan^{-1}(\omega) - \omega \times \frac{\cancel{\pi}}{\cancel{180}} \cdot \frac{180^\circ}{\pi}$$

$\angle GH = -180^\circ$ at $\omega = \omega_{pc}$.

$$\therefore -180^\circ = -90^\circ - \tan^{-1}(\omega) - \omega \times \frac{\pi}{180^\circ}$$

$$\therefore 90^\circ = \tan^{-1}(\omega) + \frac{\omega \times \pi}{180^\circ}$$

$$\therefore \tan^{-1}(\omega) + 57.3\omega - 90^\circ = 0.$$

$$\Rightarrow \boxed{\omega_{pc} = 0.86 \text{ rad/sec}}$$

In Calculator, write eqn i.e.
$\tan^{-1}(X) + 57.3(X) - 90$
then press shift + CALC.

$$G_m = \frac{1}{M|_{\omega=\omega_{pc}}}$$

$$= \frac{1}{\frac{1}{0.86\sqrt{(0.86^2+1)}}}$$

$$\boxed{G_m = 1.13}$$

$\Rightarrow$ Now, PM, $M|_{\omega=\omega_{gc}} = 1.$

$$\therefore \frac{1}{\omega_{gc}\sqrt{\omega_{gc}^2+1}} = 1.$$

$$\therefore \omega_{gc}^2(\omega_{gc}^2+1) - 1 = 0$$

$$\Rightarrow \boxed{\omega_{gc} = 0.786 \text{ rad/sec.}}$$

$$\therefore PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}}.$$

$$\therefore PM = 180^\circ + \left(-90^\circ - \tan^{-1}(0.786) - \left(0.786 \times \frac{180}{\pi}\right)\right)$$

$$\therefore \boxed{PM = 6.8^\circ}$$

$\boxed{Q}$ Calculate GM & PM $GH(s) = \frac{1}{(s+2)}$.

Sol^n: $M = \frac{1}{\sqrt{\omega^2+4}}$, $\phi = -\tan^{-1}(\omega/2)$.

$\Rightarrow$ $\cancel{-180° = -180° + \angle GH|_{\omega=\omega_{pc}}}$.

$\therefore$ $\cancel{-180° = 180° - \tan^{-1}(\omega_{pc}/2)}$.

$\therefore$ $\cancel{\tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = 360°}$.

$\therefore$ $\angle GH = -180°$ at $\omega = \omega_{pc}$.

$\therefore$ $-180° = -\tan^{-1}\left(\frac{\omega_{pc}}{2}\right)$.

$\therefore$ $\frac{\omega_{pc}}{2} = \tan(180°)$.

$\therefore$ $\frac{\omega_{pc}}{2} = 0$ X.

** one Pole can give max angle 90°

i.e. ω varies from 0 to ∞ angle will varies from 0 to 90°.

So, $\boxed{\omega_{pc} = \infty}$ rad/s.

$\Rightarrow$ $PM = 180° - \tan^{-1}(\omega/2)$.

$\therefore$ $M = \left.\frac{1}{\sqrt{\omega^2+4}}\right|_{\omega=\omega_{pc}}$

$= 1/\infty$

$\therefore \boxed{M = 0}$

$\Rightarrow$ $GM = \frac{1}{M|_{\omega=\omega_{pc}}} = 1/0 \Rightarrow \boxed{GM = \infty}$

$\Rightarrow \quad M\Big|_{\omega=\omega_{gc}} = 1.$

$\therefore \quad \dfrac{1}{\sqrt{\omega^2+4}} = 1. \quad \Rightarrow \quad \omega_{gc} = -3$

$\omega_{gc} = \pm\sqrt{3}$ X In valid.

		M
$\omega = 0$	$\longrightarrow$	0.5
$\omega = \infty$	$\longrightarrow$	0.

So, one pole give max mag. 0.5.

$\xrightarrow{M<1}$ ω_{gc} doesnot exist.

$\boxed{PM = \infty}$

Note:

$\Rightarrow$ whenever the plot (OR) T.F. gives less mag. than 1 (i.e) $M<1$ & -ve phase angle than -180° at all the freq. range then the $GM = PM = \infty$. (ω_{pc}, ω_{gc} does not exist).

$\boxed{e}$ $GH = 1/s$.

Soln: $M = 1/\omega$, $\phi = -90^\circ$.

$\xrightarrow{GM} \omega_{pc} \quad \angle GH = -180^\circ$

$-90^\circ = -180^\circ$ X

$\xrightarrow{\angle -180^\circ} \omega_{pc} = \infty$.

$m\Big|_{\omega=\omega_{pc}} = 0 \quad \Rightarrow \quad \boxed{GM = \infty}$

$\xrightarrow{PM} \omega_{gc} \Rightarrow$ $m = 1$

$\frac{1}{\omega_{gc}} = 1 \Rightarrow \omega_{gc} = 1$ rad/sec

$\therefore PM = 180° - 90°$

$\boxed{PM = 90°}$ $\boxed{\text{Stable}}$

Note:

$M < 1$, $\phi < -180°$, ω_{gc} & ω_{pc} = doesn't exist

$GM = PM = \infty$

options: ① $PM = \infty$ → 1^{st} priority.

② None → 2^{nd} priority.

③ $\omega_{gc} = 0$ & calculate PM → last priority.

$\boxed{Q}$ $GH = \frac{1}{s^2}$.

Soln: $M = \frac{1}{\omega^2}$, $\phi = -180°$.

$\therefore \angle GH = -180°$

$\angle GH|_{\omega=\omega_{pc}} = -180°$

$\underbrace{-180° = -180°}$

$\boxed{\omega_{pc} = \omega_{gc}}$

$m = 1$

$\frac{1}{\omega_{gc}^2} = 1$

$\boxed{\omega_{gc} = \omega_{pc} = 1 \text{ rad/sec}}$

$\Rightarrow M|_{\omega=\omega_{pc}} = 1/1 = 1.$ $\Rightarrow$ MS

$\boxed{GM = 1}$ ①

$GM_{in\,dB} = 0\,dB$

$PM = 180° + \angle GH|_{\omega=\omega_{gc}}$

$PM = 180° - 180°$

$\boxed{PM = 0°}$

$\boxed{Q}$ $GH(s) = \dfrac{1}{s^3}$.

Sol^n: $M = \dfrac{1}{\omega^3}$, $\angle GH = \phi = -270°$.

$\xrightarrow{Gm}$ $\angle GH|_{\omega=\omega_{pc}} = -180°$

$\therefore$ $-270° \neq 180°$

$> \xrightarrow{-180°} \omega_{pc} = 0.$

$$Gm = \frac{1}{M|_{\omega=\omega_{pc}}} = \frac{1}{\infty} = 0.$$

$\boxed{Gm = 0 < 1}$ US

$\xrightarrow{pm}$ ω_{gc} $M = 1.$,

$\Rightarrow$ $\omega_{gc} = 1.$ rad/sec.

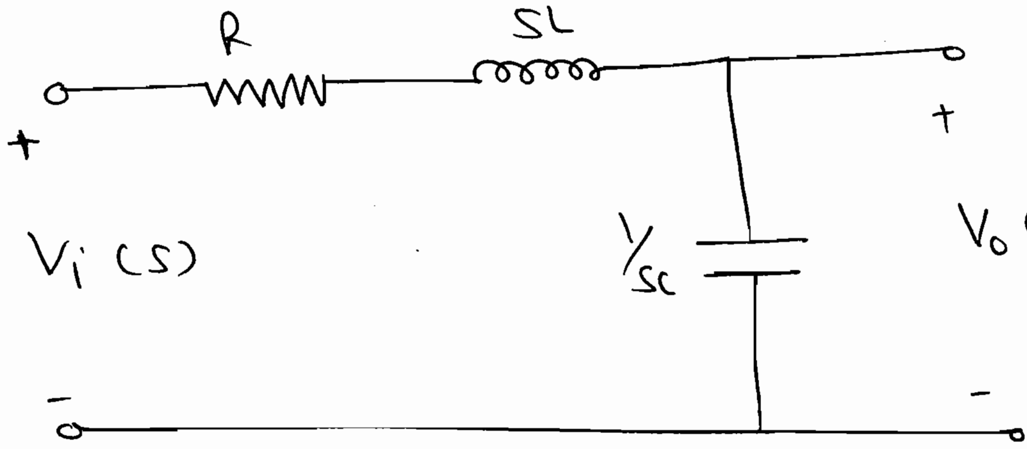
$pm = 180° - 270°$

$\boxed{pm = -90° < 0}$ $\Rightarrow$ U.S.

So, CLS US.

* Frequency Domain Specification:-

⇒ The general freq. response of RLC ckt is shown in fig.



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{R + SL + \frac{1}{SL}}.$$

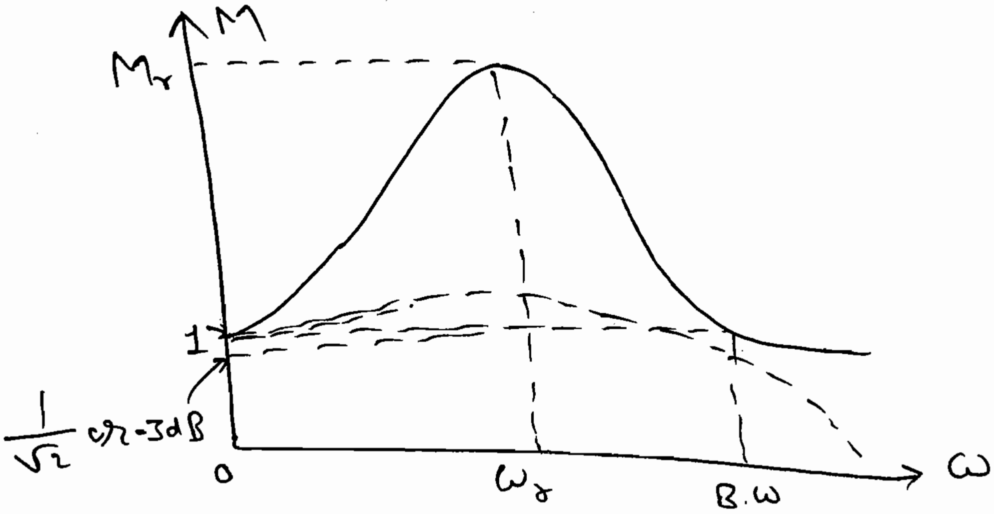
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$\therefore \boxed{\omega_n = \frac{1}{\sqrt{LC}} \text{ rad/sec.}}$$

$$2\xi\omega_n = R/L.$$

$$\therefore \boxed{\xi = \frac{R}{2} \times \sqrt{\frac{C}{L}}}$$

$$\boxed{Q = \frac{1}{2\xi} = \frac{1}{R} \times \sqrt{\frac{L}{C}}}$$



* **Resonant freq:-**

⇒ It is a freq at which max. magnitude occurs so,

$$\boxed{\omega_r = \omega_n \sqrt{1 - 2\xi^2} \quad \text{rad/sec}}$$

* **Resonant Peak:-**

⇒ It is a max. magnitude occurs at resonant freq.

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

⇒ $\boxed{\xi < \frac{1}{\sqrt{2}}}$ * then freq. domain specification is valid otherwise not valid.

⇒ when $\xi \geq 1/\sqrt{2}$, No resonant peak & No Resonant freq. exist.

* **Band-width:-**

⇒ It is the range of freq. at which the mag. dropped by -3 dB (or) $1/\sqrt{2}$ from the maximum value at the low freq.

⇒ Bw. for 1^{st} order

$$\boxed{Bw = \frac{1}{T}} \text{ Hz} \leftarrow \text{Bw for } 1^{st} \text{ order.}$$

⇒ Bw for 2^{nd} order.

$$\boxed{B.\omega. = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}} \text{ Hz}$$

☆ State Space Analysis :

⇒ State ⇒ future behaviour

PI (present i/p) | Past history

⇒ The State gives the future behaviour of the sys. based on the present I/p & past history of the system.

⇒ The past history (Initial condition) of the sys. described by the state variable.

⇒ The resistive ckt not having any state variable, because the o/p does not depends on the past history of the system.

⇒ The resistive ckt o/p depends on only i/p.

⇒ The resistive ckt cannot store any energy i.e. No past history, No state variables.

⇒ The resistive ckt is called memoryless system.

* No. of State Variable's:

⇒ If the RLC ckt given then the

$$\boxed{\text{no. of State Variable} = \text{Sum of Inductors \& Capacitors.}}$$

⇒ If the differential eqⁿ is given,

$$\boxed{\text{No. of State Variable} = \text{Order of diffⁿ eqⁿ.}}$$

* Standard form of State model:-

$$\underbrace{\dot{X}}_{\text{Diffⁿ State Vector}} = A\underbrace{X}_{\text{State Vector}} + B\underbrace{U}_{\text{I/P Vector}} \longrightarrow \text{State eqⁿ | Dynamic eqⁿ}$$

$$\underbrace{Y}_{\text{O/P vector}} = CX + DU \longrightarrow \text{O/p eqⁿ}$$

⇒

$A \longrightarrow$ State matrix.

$B \longrightarrow$ I/P matrix.

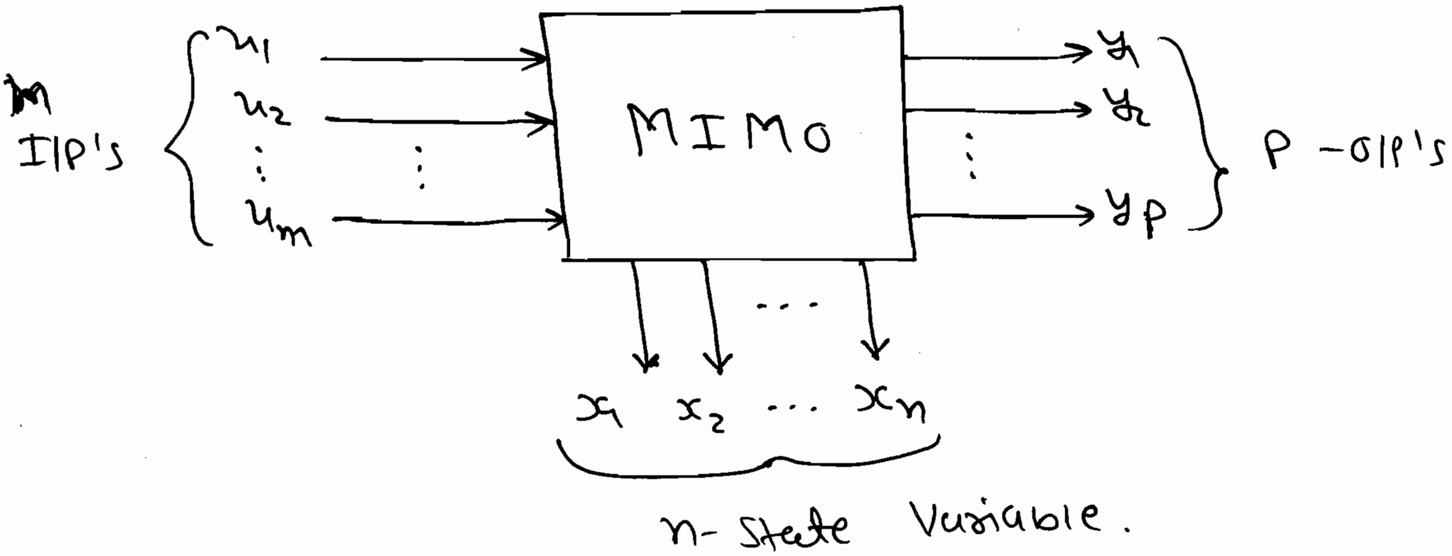
$C \longrightarrow$ O/P matrix.

$D \longrightarrow$ Transmission matrix.

* Order of Matrices:-

⇒ Consider the multi-I/P, multi O/P system as shown in fig.

$\Rightarrow$



$\Rightarrow$ I/P Vector $(U) = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1}$

$\Rightarrow$ O/P Vector $(Y) = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$

$\Rightarrow$ State Vector $(X) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$

$\Rightarrow$ $$\underbrace{\dot{X}}_{n\times 1} = \underbrace{A}_{n\times n}\,\underbrace{X}_{n\times 1} + \underbrace{B}_{n\times m}\,\underbrace{U}_{m\times 1}$$

⇒ The order of differential state vector must be equal to order of the state vector.

$$\underset{p\times 1}{Y} = \underset{p\times n}{C}\,\underset{n\times 1}{X} + \underset{p\times m}{D}\,\underset{m\times 1}{U}$$

* **State Model to Differential eqn:-**

Q) Write the state model to following systems:

① $\dddot{y} + 3\ddot{y} + 5\dot{y} + 7y = 10U.$

S.C. with opposite sign of co-efficient.

Soln: The No. of state variable required is 3, $\boxed{n=3}$

Let, $y = x_1$ —— ①

$\dot{x}_1 = \dot{y} = x_2$ — ②

$\dot{x}_2 = \ddot{y} = x_3$ —— ③

$\dot{x}_3 = \dddot{y} =$ —— ④.

⇒ To get the $\dot{x}_3$ interms of state variab le substitute all eqn in the given sys.

$\therefore \dot{x}_3 + 3x_3 + 5x_2 + 7x_1 = 10U.$

$\Rightarrow \dot{x}_3 = -7x_1 - 5x_2 - 3x_3 + 10U$ — ⑤

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} [U].$$

$$[y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

S.C.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \multicolumn{5}{l}{\text{coefficient from last to first with opposite sign}} \end{bmatrix}$$

Last Row

⇒ The above State model is Controllable Cannonical form. $\boxed{\text{CCF}}$

⇒ The State models are not unique, there are four types of state model.

1. Controllable Canonical form.
2. Observable Cannonical form.
3. Diagonalization (or) Normal form.
4. Jordon Cannoical form.

* Observable Canonical form:-

$$\Rightarrow A_{OCF} = (A_{CCF})^T = \begin{bmatrix} 0 & 0 & -7 \\ 1 & 0 & -5 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\Rightarrow \quad B_{CCF} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \begin{matrix} \text{end} \\ \uparrow \\ \uparrow \\ \text{start} \end{matrix} \quad \Rightarrow \quad B_{OCF} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} \begin{matrix} \text{start} \\ \downarrow \\ \downarrow \\ \text{end} \end{matrix}$$

$$\Rightarrow \quad C_{ccf} \quad \Rightarrow \quad B_{OCF} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}.$$

$$\Rightarrow \quad C_{CCF} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix},$$

$$\text{start} \longrightarrow \text{End}$$

$$C_{OCF} = \begin{bmatrix} c_3 & c_2 & c_1 & c_0 \end{bmatrix}$$

$$\text{end} \longleftarrow \text{start}.$$

$$C_{OCF} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

$\boxed{Q}$ $\quad \ddddot{y} + 2\dddot{y} + 4\ddot{y} + 6\dot{y} + 8y = 5U.$

$\underline{\underline{\text{soln:}}}$

$$y = x_1$$
$$\dot{x}_1 = \dot{y} = x_2$$
$$\dot{x}_2 = \ddot{y} = x_3$$
$$\dot{x}_3 = \dddot{y} = x_4.$$
$$\dot{x}_4 = \ddddot{y}$$

$$\dot{x}_4 + 2x_4 + 4x_3 + 6x_2 + 8x_1 = 5U.$$

$$\therefore \dot{x}_4 = -8x_1 - 6x_2 - 4x_3 - 2x_4 + 5U.$$

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -8 & -6 & -4 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

State model to the Transfer Function:

Q Write the State model to the given TF.

$$\frac{Y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}.$$

Soln:

$\dot{x}_1 = x_2$ $\quad$ x_1

$$\frac{Y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}.$$

$\dot{x}_2$ $\quad$ $\dot{x}_1 = x_2$ — ① $\quad$ x_1

$$\dddot{y} = \dot{x}_3 = s^3$$

so, $\boxed{s^n = \dot{x}_n}$

$\Rightarrow$ $U = \dot{x}_2 + 5x_2 + 6x_1$

$$\therefore \boxed{\dot{x}_2 = -6x_1 - 5x_2 + U.} \quad \text{— ②}.$$

$\Rightarrow$ $Y = 2\dot{x}_1 + 3x_1$

$$\boxed{Y = 2x_2 + 3x_1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [U].$$

$$[Y] = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Short -cuts:

$$\frac{Y(s)}{KU(s)} = \frac{K\,(2s+3)}{s^2+5s+6}$$

with same sign of co-eff. C matrix

A matrix with opposite sign of coeffs.

$\boxed{Q}$ $$\frac{Y(s)}{U(s)} = \frac{2s^3+4s^2+6}{s^5+3s^4+5s^3+7s^2+9s+10}$$

Soln:

$$\frac{Y(s)}{2U(s)} = \frac{2\,(s^3+2s^2+3)}{s^5+3s^4+5s^3+7s^2+9s+10}$$

$\dot{x}_5 \quad \dot{x}_4 \quad \dot{x}_3 \quad \dot{x}_2 \quad \dot{x}_1 \quad x_1$

$\therefore$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -10 & -9 & -7 & -5 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}.$$

$$C = \begin{bmatrix} 3 & 0 & 2 & 1 & 0 \end{bmatrix}.$$

* Diagonalization form:-

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+2)(s+3)}$$

$$= \frac{1/2}{s+1} - \frac{1}{(s+2)} + \frac{1/2}{(s+3)}.$$

$$\therefore \quad y = \frac{\frac{1}{2}U}{(S+1)} + \frac{-1U}{(S+2)} + \frac{\frac{1}{2}U}{(S+3)}.$$

$$y = x_1 + x_2 + x_3.$$

let, $\quad x_1 = \dfrac{\frac{1}{2}U}{(S+1)}, \quad x_2 = \dfrac{-1U}{(S+1)}, \quad x_3 = \dfrac{\frac{1}{2}U}{(S+3)}.$

$$\Rightarrow \quad Sx_1 + x_1 = \frac{1}{2}U$$

$$\therefore \quad \dot{x}_1 + x_1 = \frac{1}{2}U$$

$$\boxed{\dot{x}_1 = -x_1 + \frac{1}{2}U} \quad \text{—} \quad (1)$$

$$\Rightarrow \quad Sx_2 + x_2 = -U.$$

$$\therefore \quad \dot{x}_2 + x_2 = -U.$$

$$\Rightarrow \boxed{\dot{x}_2 = -x_2 - U.} \quad \text{—} \quad (2).$$

$$\Rightarrow \quad Sx_3 + 3x_3 = \frac{1}{2}U.$$

$$\therefore \quad \dot{x}_3 + 3x_3 = \frac{1}{2}U$$

$$\therefore \quad \boxed{\dot{x}_3 = -3x_3 + \frac{1}{2}U.} \quad \text{—} \quad (3).$$

Partial fraction

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} [U].$$

Poles (or) Eigen Values.

$$\Rightarrow [y] = \underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}_{\text{Always } 111\ldots} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow$ In the Diagonalization form B & C matrix are interchange.

* Jordan Canonical form:-

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{(s+2)^2 (s+3)}$$

$$= \frac{1}{(s+2)^2} + \frac{-1}{(s+2)} + \frac{1}{s+3}$$

$$\Rightarrow y = \frac{1U}{(s+2)^2} + \frac{-1U}{(s+2)} + \frac{1U}{(s+3)}$$

$$\Rightarrow y = x_1 - x_2 + x_3.$$

where, $x_1 = \dfrac{U}{(s+2)^2} = \dfrac{U}{(s+2)} \cdot \dfrac{1}{(s+2)}$

$$\therefore x_1 = \frac{x_2}{(s+2)}$$

$$\therefore s x_1 + 2x_1 = x_2.$$

$$\therefore \boxed{\dot{x}_1 = -2x_1 + x_2} \quad \text{—(1)}$$

$\Rightarrow x_2 = \dfrac{U}{(s+2)} \Rightarrow s x_2 + 2x_2 = U$

$$\Rightarrow \boxed{\dot{x}_2 = -2x_2 + U.} \quad \text{—(2)}$$

$$\Rightarrow \quad x_3 = \frac{U}{S+3}.$$

$$\therefore \quad Sx_3 + 3x_3 = U.$$

$$\therefore \quad \boxed{\dot{x}_3 = -3x_3 + U} \quad — ③.$$

→ Repeated Roots. i.e. Jordan block

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [U].$$

zero's

one's

$$[Y] = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

↓ Partical fraction

$\boxed{Q}$ $\quad \dfrac{Y}{U} = \dfrac{1}{(S+5)^3 (S+10)}.$

Soln:

$$A = \begin{bmatrix} -5 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

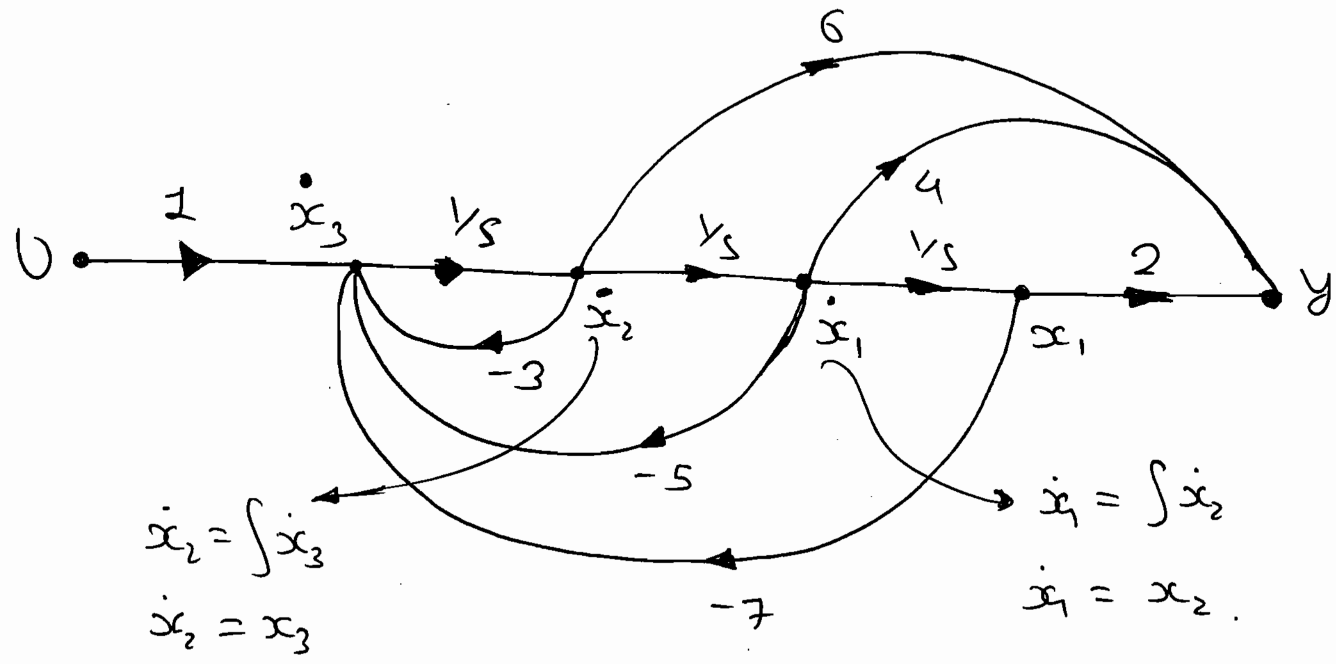
$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}.$$

$$\begin{matrix} k_1 & k_2 & k_3 & k_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ (S+5)^3 & (S+5)^2 & (S+5) & (S+10) \end{matrix}$$

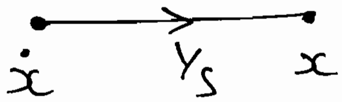
* State Model to the signal flow graph:

Q Write the state model to the following signal flow graph.

Soln:



⇒ To select the node as a state variable, the incoming branch to that particular node must be a integrator, like



$$\Rightarrow \quad \dot{x}_3 = 1.U - 3\dot{x}_2 - 5\dot{x}_1 - 7x_1.$$

$$\boxed{\dot{x}_3 = U - 3x_3 - 5x_2 - 7x_1}$$

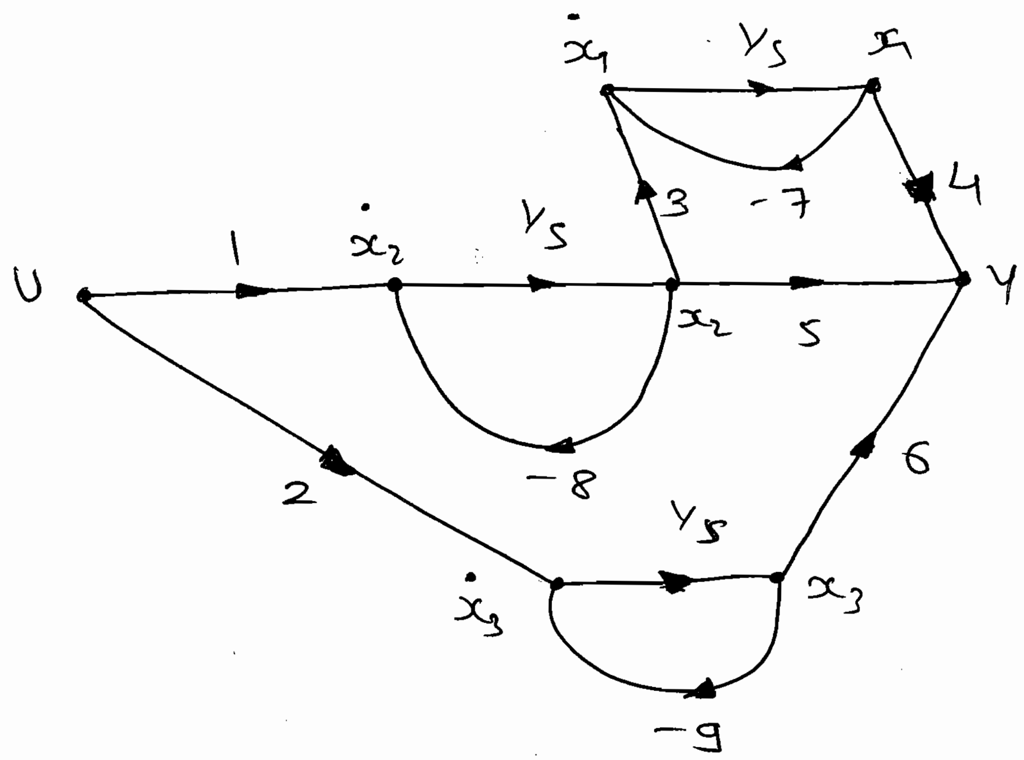
$$\therefore \quad y = 2x_1 + 4\dot{x}_1 + 6\dot{x}_2$$

$$\therefore \boxed{y = 2x_1 + 4x_2 + 6x_3.}$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u].$$

$$[y] = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

$\boxed{a}$



Sol^n:

$$\dot{x}_1 = -7x_1 + 3x_2.$$

$$\dot{x}_2 = -8x_2 + 1.U.$$

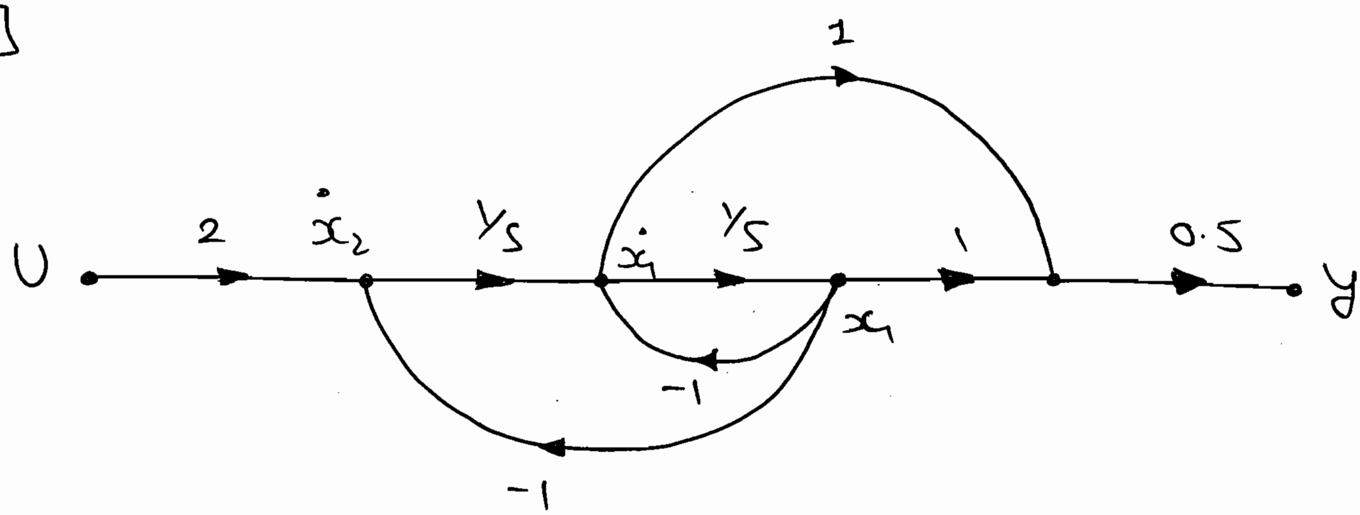
$$y = 4x_1 + 5x_2 + 6x_3$$

$$\dot{x}_3 = -9x_3 + 2.U.$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -7 & 3 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} [u].$$

$$\therefore [y] = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Q



Sol^n:

$$\dot{x}_1 = \int \dot{x}_2 - x_1 = x_2 - x_1$$

$$\dot{x}_2 = 2U - x_1.$$

$$\therefore y = (\dot{x}_1 + x_1)\,0.5$$

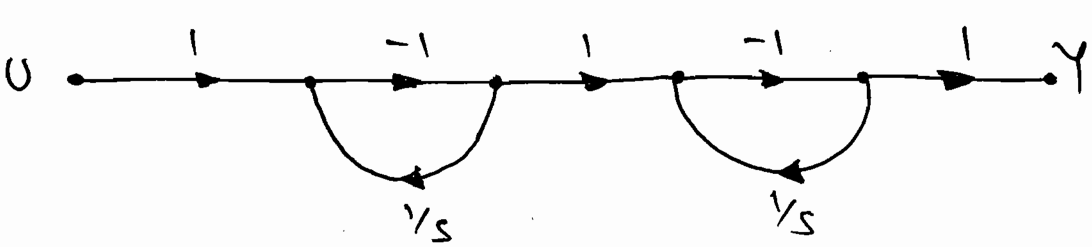
$$\therefore y = (x_2 - x_1 + x_1)\,0.5$$

$$\therefore \boxed{y = 0.5x_2}$$

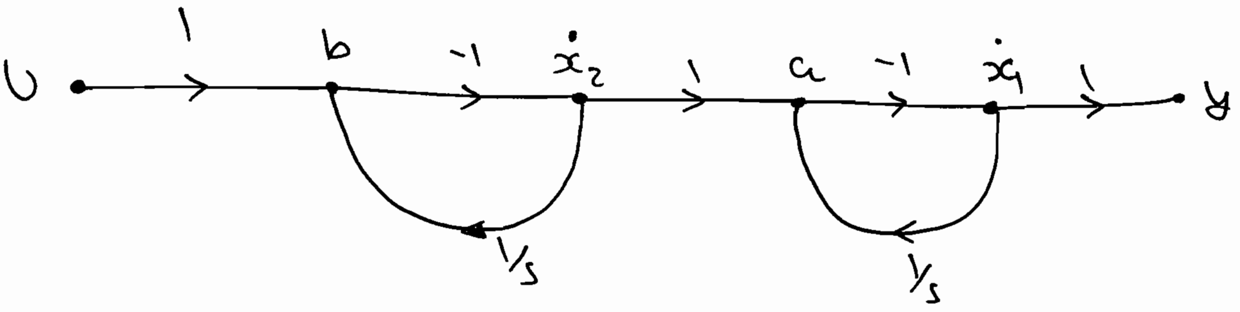
$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} [U].$$

$$\therefore [y] = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Q



Soln:



$$\Rightarrow \quad a = \dot{x}_2 + \frac{1}{s}\cdot\dot{x}_1$$

$$a = \dot{x}_2 + x_1$$

$$\Rightarrow \quad \cancel{\dot{x}_2} \quad \cancel{=} \quad b = 1.U + \dot{x}_2/s.$$

$$\therefore \quad b = U + x_2$$

$$\dot{x}_2 = -b \qquad\qquad \dot{x}_1 = -a.$$

$$\therefore \boxed{\dot{x}_2 = -U - x_2.} \qquad \therefore \dot{x}_1 = -x_1 - \dot{x}_2$$

$$\boxed{\dot{x}_1 = -x_1 + x_2 + U.}$$

$$\therefore \quad y = \dot{x}_1$$

$$\therefore \boxed{y = -x_1 + x_2 + U.}$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} [U].$$

$$\therefore \quad [y] = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1][U].$$

(*) **State model to the Electrical N/w:**

$\Rightarrow$ Select the state variables as Voltage across Capacitors, and Current through the ~~Conductor~~. Inductor.

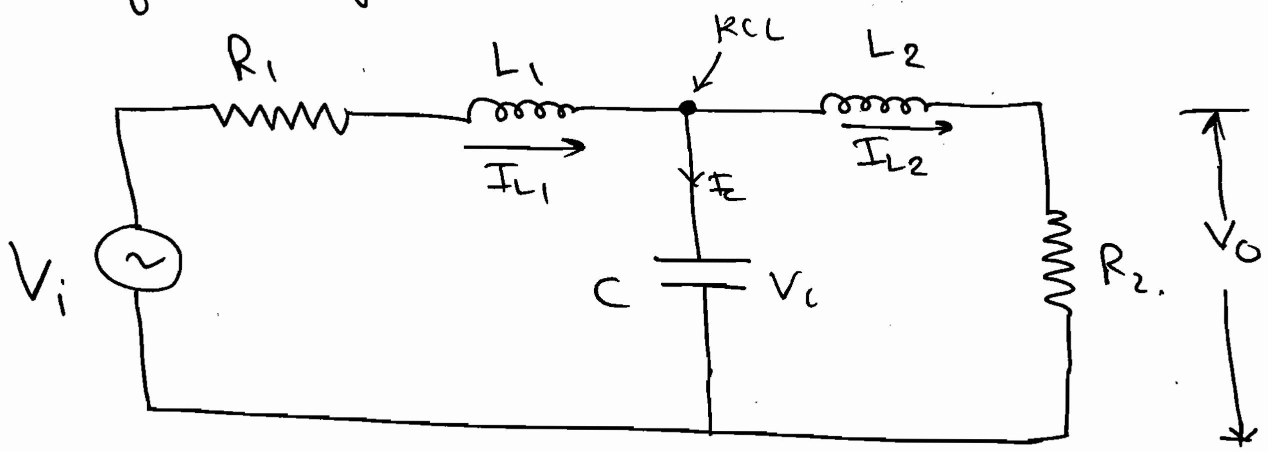
⇒ The no. of State Variables = sum of Inductors & Capacitors.

⇒ Write the independent KCL & KVL eqn.

⇒ At Capacitor jn apply KCL & apply KVL through the Inductor.

⇒ The resultant eqn should consist, State Variables, differential State Variables, I/P Variables & O/P Variables.

Q Write the State model to the following system:



Soln:

SV (State variable) = $\begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix}$.

⇒ KCL at Cap. jn

$$\therefore \quad -I_{L_1} + I_{L_2} + C\frac{dV_c}{dt} = 0.$$

$$\therefore \quad C\frac{dV_c}{dt} = I_{L_1} - I_{L_2}.$$

$$\therefore \boxed{\dot{V}_c = \frac{I_{L1}}{C} - \frac{I_{L2}}{C}} \quad \text{——} \; ①.$$

→ KVL₁:

$$V_i - I_{L1} \cdot R_1 - L_1 \frac{dI_{L1}}{dt} - V_c = 0.$$

$$\therefore L_1 \frac{dI_{L1}}{dt} = -I_{L1} R_1 - V_c + V_i.$$

$$\therefore \boxed{\dot{I}_{L1} = \frac{-R_1}{L_1} \cdot I_{L1} - \frac{V_c}{L_1} + \frac{V_i}{L_1}.} \quad \text{——} \; ②.$$

→ KVL₂:

$$V_c - L_2 \frac{dI_{L2}}{dt} - I_{L2} \cdot R_2 = 0.$$

$$\therefore L_2 \frac{dI_{L2}}{dt} = -I_{L2} \cdot R_2 + V_c.$$

$$\therefore \boxed{\frac{dI_{L2}}{dt} = -\frac{R_2}{L_2} \cdot I_{L2} + \frac{V_c}{L_2}.} \quad \text{——} \; ③.$$

$$\Rightarrow \begin{bmatrix} \dot{V}_c \\ \dot{I}_{L1} \\ \dot{I}_{L2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} V_c \\ I_{L1} \\ I_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ 0 \end{bmatrix} [V_i]$$

$$V_o = I_{L2} \cdot R_2.$$

$$\therefore [V_o] = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ I_{L1} \\ I_{L2} \end{bmatrix}.$$

* Transfer function from the State model:-

$\Rightarrow$

$$\boxed{T.F. = C\,[SI - A]^{-1} \cdot B + D.}$$

$$\boxed{T.F. = C \cdot \frac{adj\,[SI-A]}{|SI-A|} \cdot B + D.}$$

$\Rightarrow$ The det of $SI-A$ i.e. $|SI-A| = 0$ gives the Char. eq^n.

$\Rightarrow$ The roots of the CE is called Poles which are called eigen values.

Q Find the T.F. to the given State model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [U].$$

$$[y] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Sol^n: add s diagonally & change the sign of coefficient for to get $|SI-A|^{-1}$.

$$SI - A = \begin{bmatrix} s+2 & 3 \\ -4 & s-2 \end{bmatrix}.$$

$$\therefore (SI-A)^{-1} = \frac{adj\,(SI-A)}{|SI-A|} = \frac{\begin{bmatrix} s-2 & -3 \\ 4 & s+2 \end{bmatrix}}{s^2 - 4 + 12}.$$

$\therefore$ T.F = $C(SI-A)^{-1}.B. + D.$

$$= \frac{\begin{bmatrix} 1 & 1 \end{bmatrix}_{1\times2} \begin{bmatrix} s-2 & -3 \\ +4 & s+2 \end{bmatrix}_{2\times2} \begin{bmatrix} 3 \\ 5 \end{bmatrix}}{s^2+8}.$$

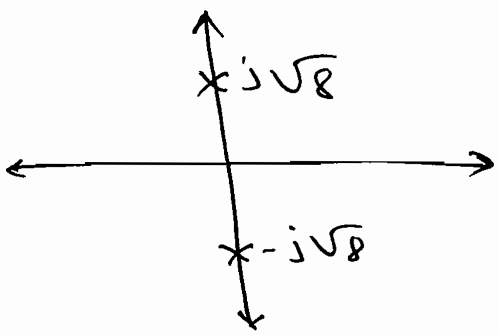
$$= \frac{\begin{bmatrix} s+2 & s-1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}}{s^2+8}$$

$$= \frac{3s+6+5s-5}{s^2+8}.$$

$$\therefore \boxed{TF = \frac{8s+1}{s^2+8}.}$$

CE $\rightarrow$ $s^2+8=0 \Rightarrow s = \pm j\sqrt{8}.$

Marginally Stable (or) Undamped sys.



$\omega_n = \sqrt{8}$ rad/sec

M.S

Q $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u].$

$[y] = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$

Sol^n:

$$SI - A = \begin{bmatrix} s & -3 \\ 2 & s+5 \end{bmatrix}.$$

$$(SI-A)^{-1} = \frac{adj(SI-A)}{|SI-A|}.$$

$$= \frac{\begin{bmatrix} s+5 & 3 \\ -2 & s \end{bmatrix}}{s^2+5s+6}.$$

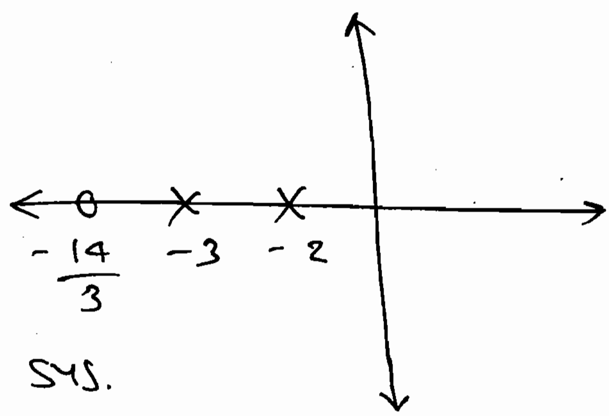
$$\therefore T.F. = C[SI-A]^{-1}B + D.$$

$$= \frac{\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} s+5 & 3 \\ -2 & s \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+5s+6}$$

$$= \frac{\begin{bmatrix} 2s+8 & 6+s \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+5s+6}$$

$$= \frac{2s+8+6+s}{s^2+5s+6}.$$

$$\boxed{T.F. = \frac{3s+14}{s^2+5s+6}}.$$



Stable / over-damped sys.

* Solution to the State eqⁿ :-

$\Rightarrow$ $\dot{X} = AX + BU$ $\rightarrow$ Non-Homogenous State eqⁿ.

M-I : Laplace Transform method.

$\Rightarrow$ $SX(S) - X(0) = AX(S) + BU(S).$

$\therefore$ $SX(S) - AX(S) = X(0) + BU(S).$

$\therefore$ $(SI - A)\, X(S) = X(0) + BU(S).$

$\therefore$ $X(S) = (SI - A)^{-1} X(0) + [SI - A]^{-1} . BU(S).$

$\Rightarrow$ Apply I.L.T.

$$\therefore \boxed{x(t) = \underbrace{L^{-1}\left\{ (SI - A)^{-1} X(0) \right\}}_{\text{Zero I/P Response due I.C.}} + \underbrace{L^{-1}\left\{ (SI - A)^{-1} BU(S) \right\}}_{\text{Zero State Response due to I/P.}}}$$

$\Rightarrow$ The zero I/P resp. (ZIR) is due to Initial Condition.

$\Rightarrow$ The zero State resp. (ZSR) is due to I/P.

M-II : Classical Method.

$$x(t) = \underbrace{e^{At}x(0)}_{ZIR} + \underbrace{\int_0^t \cancel{e^{A(t-\tau)}}\, \cdot B \cdot u(\tau)\, d\tau}_{ZSR} \quad \text{---(II)}$$

$\phi(t-\tau)$.

$\Rightarrow$ Compare ZIR terms, ***

$$\boxed{\phi(t) = e^{At} = L^{-1}[SI-A]^{-1}.}$$

STM : State Transmission matrix.

$$\Rightarrow \quad [SI-A]^{-1} = L[\phi(t)] = \phi(s).$$

$$\therefore \quad \boxed{\phi(s) = [SI-A]^{-1}}$$

$\Rightarrow$ Compare ZSR term:-

$$\boxed{\int_0^t \phi(t-\tau) B u(\tau)\, d\tau = L^{-1}[\phi(s) \cdot B \cdot U(s)].}$$

$$\Rightarrow \boxed{x(t) = e^{At} \cdot x(0) + L^{-1}[\phi(s) \cdot B \cdot U(s)]} \quad ***$$

* **Properties of STM:-**

$\Rightarrow$ STM : $\phi(t) = e^{At}$.

1. $\phi(0) = e^{0} = I$ (Identity matrix).
2. $\phi^{K}(t) = (e^{At})^{K} = e^{A(Kt)} = \phi(Kt)$.

e.g. $\phi^{-1}(t) = \phi(-t)$.

③ $\phi(t_1 + t_2) = \phi(t_1) \cdot \phi(t_2)$.

④ $\phi(t_2 - t_1) \cdot \phi(t_1 - t_0) = \phi(t_2 - t_0)$.

$\boxed{Q}$ Obtain the Complete sys. response of the system given below:

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad y = [1 \ -1]X.$$

Soln:

Homogenous State eqn:

$\dot{X} = AX \rightarrow$ is called as homogenous State eqn. $(U = 0)$.

$\rightarrow$ $x(t) = \phi(t) \cdot X(0) = Z.I.R. = e^{At} \cdot X(0)$.

$$\boxed{\begin{aligned} x(t) &= \phi(t) \cdot X(0) \\ \Rightarrow x(t) &= L^{-1}[(SI-A)^{-1} X(0)]. \\ x(t) &= L^{-1}[(SI-A)^{-1} \cdot X(0)] \end{aligned}}$$

$\Rightarrow$ The given State model is homogeneous

Hence the soln is

$$x(t) = Z.I.R. = e^{At} \cdot X(0) = \phi(t) \cdot X(0).$$

$\Rightarrow \xrightarrow{STM}$ $\phi(t) = e^{At} = L^{-1}[SI-A]^{-1}$.

$$\phi(t) = L^{-1}\left[(SI - A)^{-1}\right].$$

$$\Rightarrow (SI - A) = \begin{bmatrix} s & -1 \\ 2 & +s \end{bmatrix}$$

$$\Rightarrow (SI-A)^{-1} = \frac{\operatorname{adj}(SI-A)}{|SI-A|}.$$

$$(SI-A)^{-1} = \frac{\begin{bmatrix} s & +1 \\ -2 & s \end{bmatrix}}{s^2+2}.$$

$$\therefore \phi(t) = L^{-1}\begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}.$$

$$\therefore \phi(t) = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}.$$

$$\rightarrow x(t) = ZIR = \phi(t).\ X(0).$$

$$= \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}_{2\times 2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$x(t) = \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ \sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix} \quad \cancel{-\sqrt{2}\sin\sqrt{2}t} \quad \cancel{+\cos\sqrt{2}t}]$$

$\Rightarrow$ The Complete time Response is called $y(t)$.

→ Substitute x in y.

$$\therefore\ y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}.$$

$$\therefore\ y(t) = \cancel{\cos\sqrt{2}t} + \frac{1}{\sqrt{2}}\sin\sqrt{2}t + \sqrt{2}\sin\sqrt{2}t - \cancel{\cos\sqrt{2}t}.$$

$$\therefore\ \boxed{y(t) = \frac{3}{\sqrt{2}}\sin\sqrt{2}t.}$$

Q Obtain the time response for unit-step I/P for a sys. given by.

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \end{bmatrix} [0]$$

$$X[0] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} X.$$

Sol^n:

$$x(t) = \phi(t)\,X(0) + L^{-1}\left[\phi(s)\cdot B\,U(s)\right].$$

$$\Rightarrow\ \phi(t) = L^{-1}\left[(SI-A)^{-1}\right].$$

⇒ The given state model is non-homogeneous. Hence, sol^n is

$$x(t) = Z.I.R. + Z.S.R.$$

$$\Rightarrow\ \xrightarrow{Z.I.R.}\ e^{At}\cdot X(0) \Rightarrow \phi(t).X(0).$$

$$\Rightarrow \quad \phi(t) = L^{-1}\left[(sI-A)^{-1}\right].$$

$$\therefore \quad sI - A = \begin{bmatrix} s & -1 \\ +2 & s+3 \end{bmatrix}.$$

$$(sI-A)^{-1} = \frac{\operatorname{adj}(sI-A)}{|sI-A|}.$$

$$= \frac{\begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix}}{s^2+3s+2}.$$

$$\therefore \quad = \frac{\begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix}}{(s+2)(s+1)}.$$

$$\therefore \quad \phi(t) = L^{-1}\begin{bmatrix} \dfrac{(s+3)}{(s+2)(s+1)} & \dfrac{1}{(s+2)(s+1)} \\[2ex] \dfrac{-2}{(s+2)(s+1)} & \dfrac{s}{(s+2)(s+1)} \end{bmatrix}$$

$\therefore$ $\cancel{\phi(t)=}$

$$\phi(t) = L^{-1}\begin{bmatrix} \dfrac{2}{s+1} - \dfrac{1}{s+2} & \dfrac{1}{s+1} - \dfrac{1}{s+2} \\[2ex] \dfrac{-2}{(s+1)} + \dfrac{2}{s+2} & \dfrac{-1}{s+1} + \dfrac{2}{s+2} \end{bmatrix}$$

$$\Rightarrow \quad \phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\Rightarrow \text{Z.I.R.} = \phi(t) \cdot x(0).$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

$$\therefore \text{Z.I.R.} = \phi(t) \cdot x(0).$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}.$$

$$\Rightarrow \text{ZSR} = L^{-1}\left[\phi(s) \cdot B\, U(s)\right].$$

$$= L^{-1}\left[\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} \left[\frac{1}{s}\right]\right]$$

$$= L^{-1}\left[\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} 0 \\ 5/s \end{bmatrix}\right]$$

$$= L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{5}{(s+1)(s+2)} \end{bmatrix}$$

$$= L^{-1}\begin{bmatrix} \frac{5}{2S} - \frac{5}{S+1} + \frac{5}{2(S+2)} \\ \frac{5}{(S+1)} - \frac{5}{S+2} \end{bmatrix}$$

$$\underbrace{= \begin{bmatrix} \frac{5}{2} - 5e^{-t} + \frac{5}{2}.e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}.}_{ZSR}$$

$\rightarrow \quad x(t) = ZIR + ZSR.$

$$\Rightarrow \quad x(t) = \begin{bmatrix} \frac{5}{2} - 3e^{-t} + \frac{3}{2}e^{-2t} \\ 3e^{-t} \; \cancel{+} - 3e^{-2t} \end{bmatrix}.$$

$\Rightarrow y = [0 \quad 1]\, x(t).$

$$y[t] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{5}{2} - 3e^{-t} + \frac{3}{2}.e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix}.$$

$$\therefore \boxed{y(t) = 3e^{-t} - 3e^{-2t}}$$

* Controllability & Observability :-

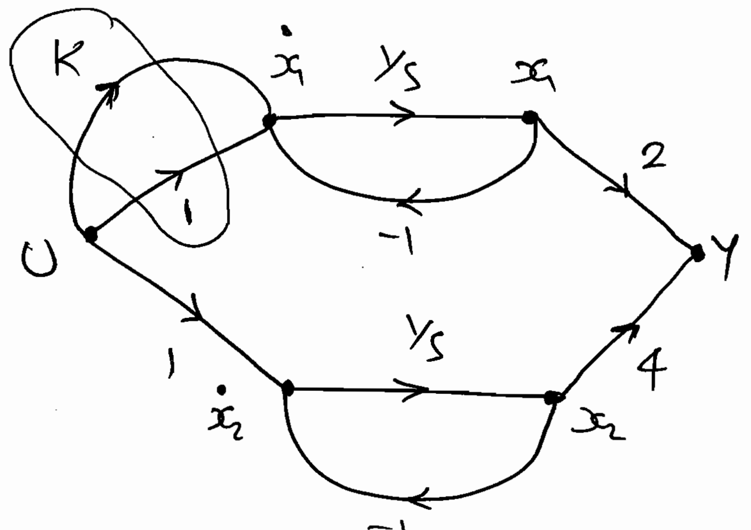
① Controllability :-

⇒ A sys. is ~~xxxxxx~~ said to be controllable if it is possible to transfer the initial states to the desired ~~state~~ state in a finite time interval by the controlled I/P.

⇒ If the SFG is given to check the controllability observe the continuous path from I/P to each & every state variable.

⇒ If the path is exist then it is called controllable.

[Q] Find the K value to become the system uncontrollable.



Solⁿ: To become the system uncontrollable no path exist betⁿ the u to $\dot{x}_i$

$$\longrightarrow K+1 = 0 \Rightarrow \boxed{K=-1}$$

* <u>Kalaman's test for Controllability</u> (Q_c):-

$$\Rightarrow \boxed{Q_c = [\, B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B \,].}$$

<u>Controllable</u>

$$\Rightarrow \boxed{\begin{array}{c}\text{Rank of } Q_c = \text{Rank of } A \\ |Q_c| \neq 0.\end{array}}$$

$\boxed{Q}$ Check the controllability to the given system.

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 4}.$$

Solⁿ:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\Rightarrow \quad Q_c = \begin{array}{c} \begin{array}{ccc} B & AB & A^2B \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & +1 \end{bmatrix} \end{array} \Rightarrow |Q_c| = 1(-1) \neq 0$$

So, Controllable.

(2) Observability:

⇒ A sys. is said to be observable if it is possible to determine the initial states of the sys. by observing the o/p in a finite time interval.

* Kalman's test for observability:-

⇒ $$Q_o = \left[C^T \quad A^T C^T \quad (A^T)^2 \cdot C^T \; \cdots \; (A^T)^{n-1} \cdot C^T \right].$$

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

⇒ Observability

$$\boxed{\begin{gathered}\text{Rank of } Q_o = \text{Rank of } A. \\ |Q_o| \neq 0\end{gathered}}$$

Q Check the controllability & observability for the following system.

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u.$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

Soln:

$$Q_c = \begin{bmatrix} B & AB \end{bmatrix}$$

$$\Rightarrow Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$|Q_c| = -2 + 2 = 0 \Rightarrow$ Not controllable.

$$\Rightarrow Q_o = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$\therefore |Q_o| = 1 - 1 = 0 \Rightarrow$ Not observable.

$\boxed{Q}$

$$\dot{x}_1 = -2x_1 + x_2 + u.$$

$$\dot{x}_2 = -x_2 + u.$$

$$y = x_1 + x_2.$$

Soln:

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

$$\rightarrow Q_c = \begin{array}{c} \begin{array}{cc} B & AB \end{array} \\ \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \end{array} \Rightarrow |Q_c| = 0$$

$\Rightarrow$ Not controllable.

$\Rightarrow \quad Q_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$

$\therefore |Q_o| = 2 \Rightarrow$ Observable.

$\Rightarrow$ The Pole-Zero Cancellation makes the system un-controllable & un-observable (or) Controllable & unobservable (or) uncontrollable & Observable.

* Gilbert test for Controllability & observability.

$\Rightarrow$ The Gilbert test is valid for only diagonalization form & Jordan Canonical form.

e.g.

Non-zero, controllable

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} u(t).$$

non-zero controllable $\rightarrow$ Controllable.

$$y(t) = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} x(t)$$

Non zero observable   Non-zero observable $\rightarrow$ Observable.

⇒ So, the given system is both observable and controllable.

Jordan block.

(e.g.)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} U.$$

non-zero

Controllable

$$[y] = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Non-zero ∴ observable

Non-zero observable.

Observable.

⇒ So, Sys. is controllable & observable.

☆ Controllers & Compunsators:-

* Purpose:-

⇒ If the System is unstable then Controllers and Compunsators are required to make it Stable & to achieve the required Performance.

⇒ If the System is stable then also required a compensator (or) controller to get the desired performance.

⇒ The Type-2 & Higher order sys. are usually un-stable. In this case it is essential to used lead componsator (or) PD Controller to make the sys. stable & to get the desired performance.

⇒ In Type-0 & Type-1 sys., the stable operation is achieved by adjusting the sys. gain.

⇒ In this case we can use any Componsator (or) Controller to get the required Specification.

Type-2:

$$G(s)\Big|_{w/c} = \frac{K}{s^2(s+2)(s+4)} \quad ; \quad H(s) = 1.$$

0 to ∞

$$\xrightarrow{CE} \quad s^4 + 6s^3 + \underbrace{8s^2 + K}_{s^1 \text{ missing}} = 0 \quad \text{—— (US)}$$

With P-D Controller $= (K_P + K_D s)$.

$$G(s)\Big|_{wc} = \frac{K(K_P + K_D s)}{s^2(s+2)(s+4)} \quad ; \quad H(s) = 1.$$

$$\xrightarrow{CE} \quad s^4 + 6s^3 + 8s^2 + KK_D s + KK_P = 0$$

$$\longrightarrow \text{(S)}.$$

Type-1:

$$G(s)\Big|_{w/c} = \frac{K}{s(s+2)(s+4)} \quad ; \quad H(s) = 1.$$

0 to 48.

$$\xrightarrow{CE} \quad s^3 + 6s^2 + 8s + K = 0 \quad \longrightarrow \text{(S)}.$$

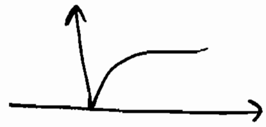
48

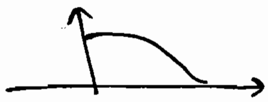
K

* Compensators:

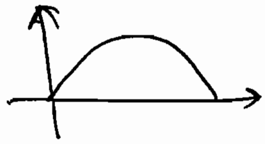
⇒ A Compensator is a electrical N/W which adds finite Poles & finite zeros to the System, so that the sys. performance is changed as per the requirement.

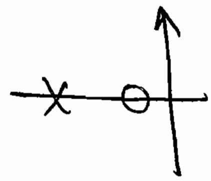
⇒ There are three types of Compensators.

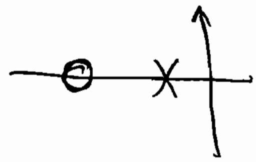
① Lead Compensator  → High pass filter.

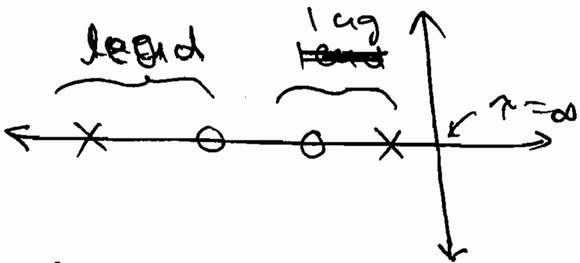
② Lag Compensator  → Low pass filter.

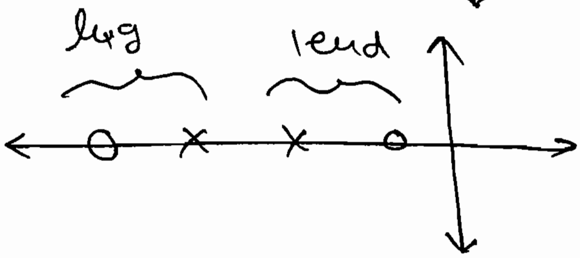
③ Lag- Lead Compensator  → Band Stop filter.

④ Lead - Lag Compensator  → Band pass filter.

* HPF → Lead Com. → +ve angle ⇒ zeros 

* LPF → Lag Comp. → −ve angle ⇒ Poles 

* BSF → $T_{lag} > T_{lead}$ 

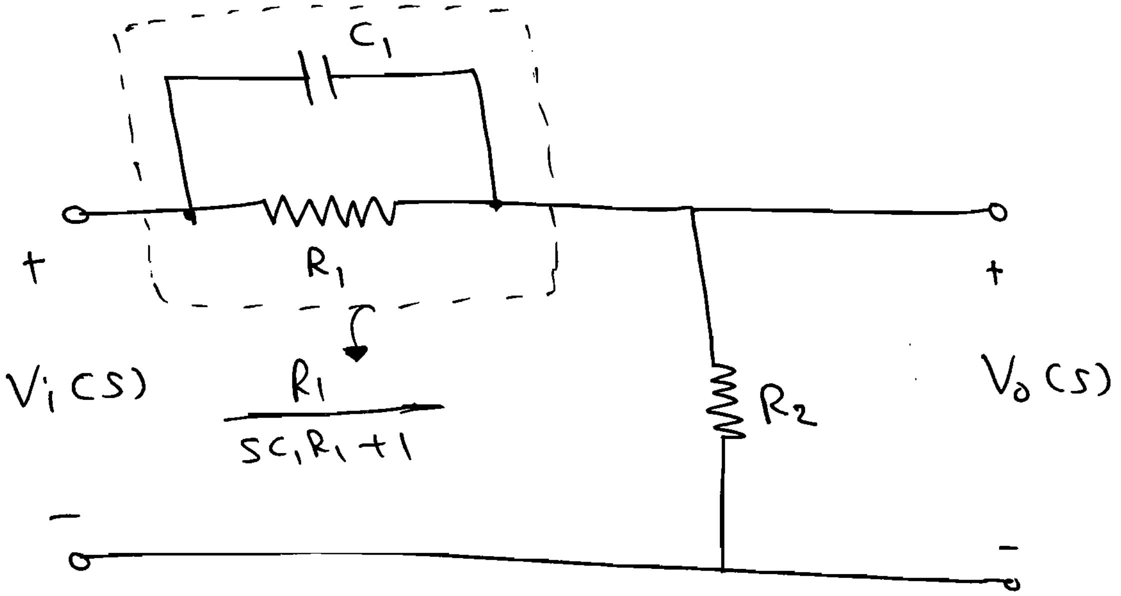
* BPF → $T_{lead} > T_{lag}$ 

① Lead Compensators :-

⇒ When sinusoidal I/P is applied to a n/w it produce a sinusoidal steady state o/p, having a phase lead with respect to I/P, then the n/w is called lead compensator.

⇒ The lead compensator improves the transient performance & also margin for the sys. stability.

⇒



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2}{\frac{R_1}{sC_1R_1+1} + R_2}.$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(sC_1R_1+1)}{R_1 + R_2(sC_1R_1+1)}.$$

⇒ S_1: T.F.

S_2: τ- const.

S_3: Poles & zeros → s-plane.

S_4: Bode plot.

S_5: Identify filters.

S_6: ω_m, ϕ_m, m/cm.

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2(1+sC_1R_1)}{R_1+R_2+sC_1R_1R_2}.$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2}{R_1+R_2}(1+sC_1R_1)}{\left[1+\frac{R_2}{R_1+R_2}sC_1R_1\right]}.$$

$\Rightarrow$ let, α is called lead Const. $= \frac{R_2}{R_1+R_2} < 1$

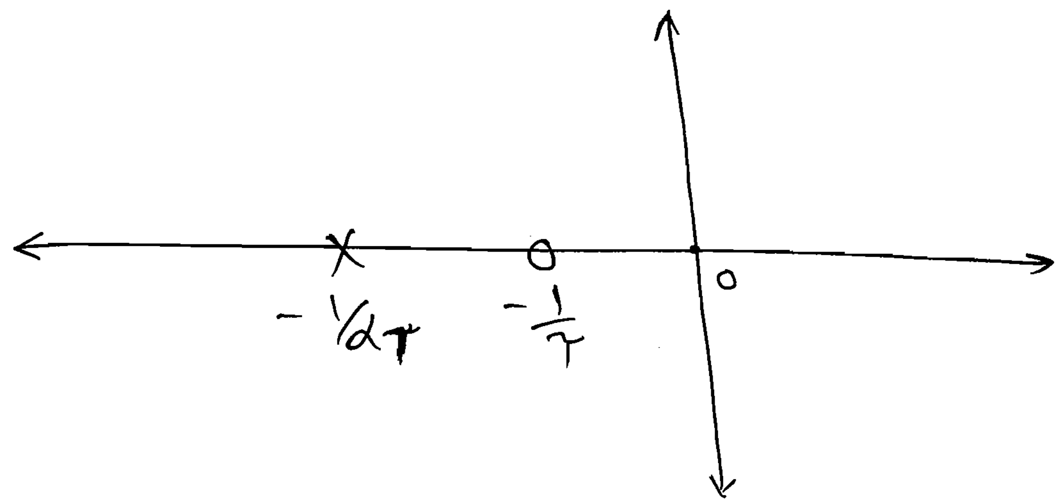
$(\alpha \nless 0.07)$.

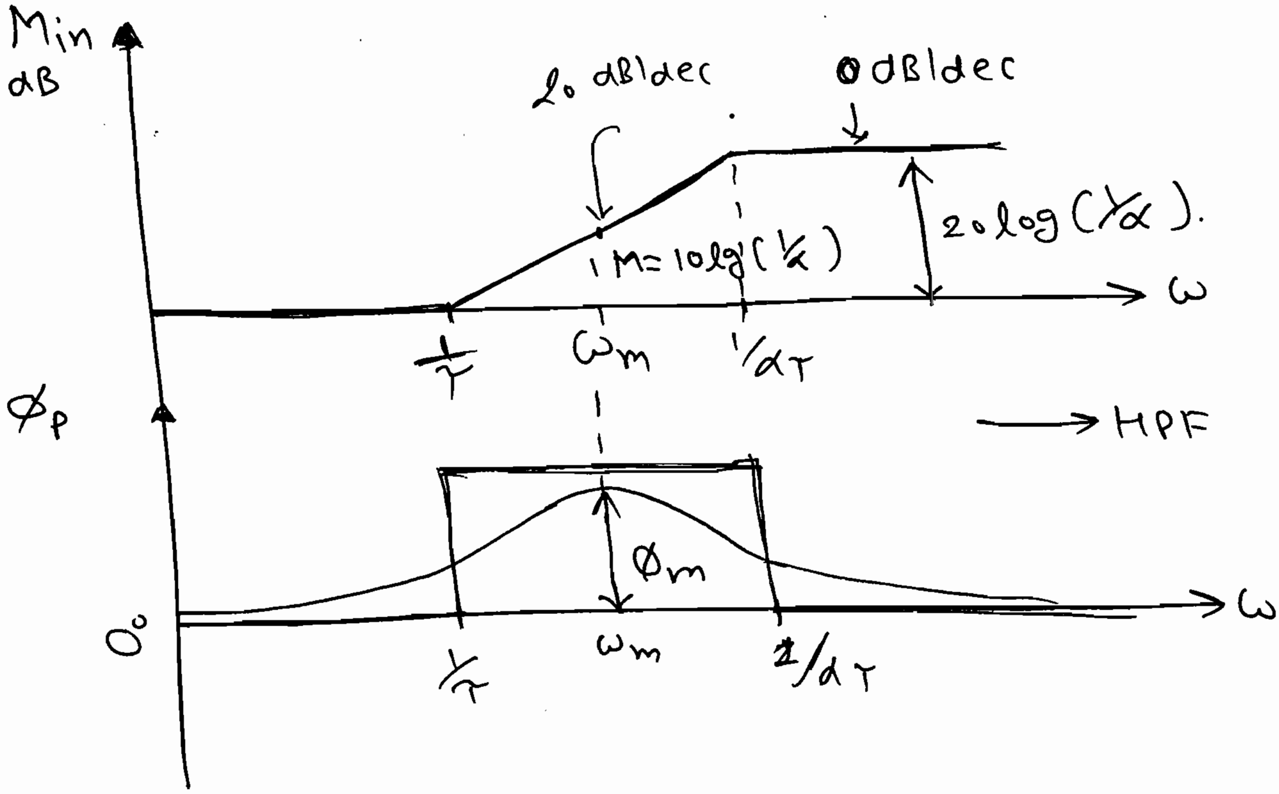
$\tau \rightarrow$ lead Time-Const. $= R_1C_1$.

$\alpha_{optimum} = 0.1$.

$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)}}.$$

$\Rightarrow$ The disadvantage of lead Componsator it is creates the attenuation in the sys. To eliminate attenuation we require to add amp. with the gain of $1/\alpha$, which add the cost & space to the system.



⇒ 

⇒ $\omega_m = \sqrt{\omega_{c1} \times \omega_{c2}}$, $\omega_m = \sqrt{\frac{1}{\tau} \times \frac{1}{\alpha\tau}}$

∴ $\omega_m = \frac{1}{\tau\sqrt{\alpha}}$ rad/sec.

∴ $\boxed{\phi_{max} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right).}$ ***

* Advantages:

⇒ A lead Compensator improves the transient performance.

⇒ The lead Compensator is a high pass filter hence the B.W. of the sys. improves.

⇒ As B.W. increases, the rise-time decreases the sys. gives very quick response.

⇒ The lead Compensator improves the

damping of the system. ($\xi \omega_n$). Hence, setteling time (t_s) decreases (↓).

⇒ The lead componsator improves the Gain margin & phase margin of the sys. Hence, relative stability improves.

⇒ The Lead compensator similar to P-D contoller.

* Disadvantages:-

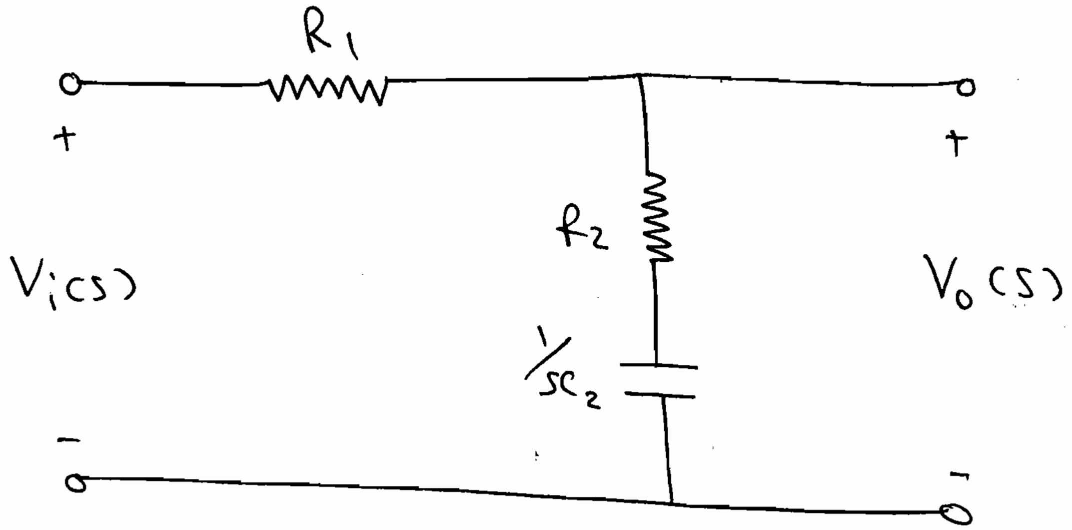
⇒ The lead compensator creates the attenuation in the sys. to eliminate the attenuation we required to add an ampifier with a gain of $1/\alpha$.

⇒ The lead comp. is a HPF. Hence noise power enter into the system. So, the SNR at output is poorer.

⇒ The max lead given by lead comp is 60°, if required more than 60° we required to use multi stage compensator.

② Lag Compensator:-

⇒



⇒ $$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_1 + R_2 + \frac{1}{sC_2}}.$$

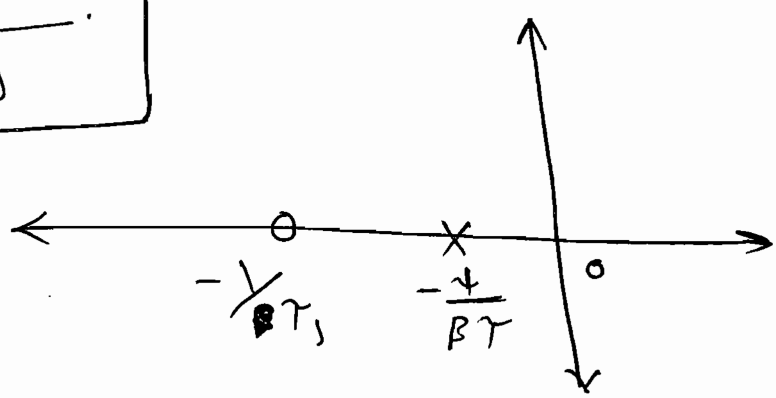
$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{sC_2R_2 + 1}{1 + sC_2(R_1 + R_2)}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{sC_2R_2 + 1}{1 + sC_2R_2\left(\frac{R_1 + R_2}{R_2}\right)}.$$

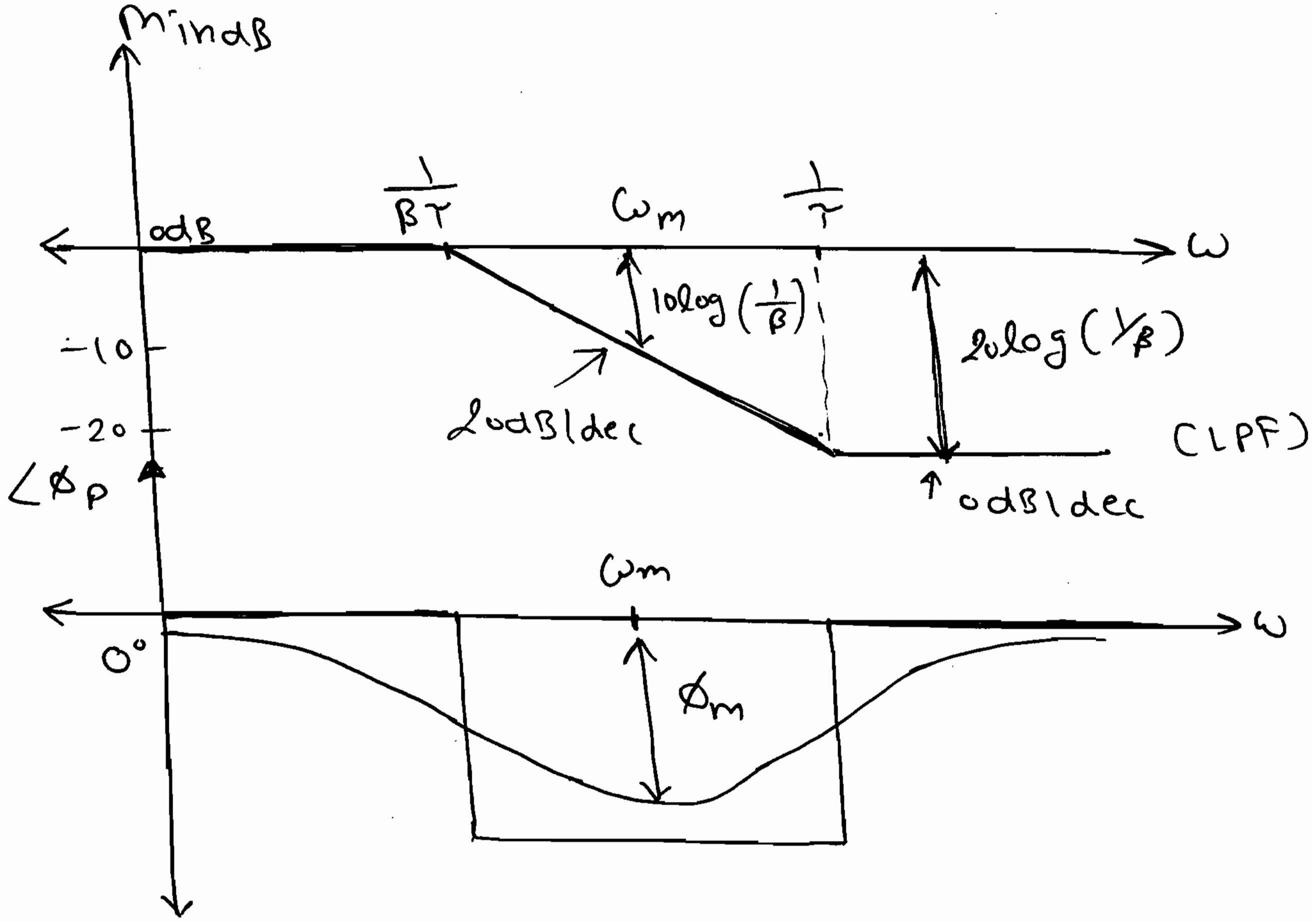
$\beta \Rightarrow$ lag Constant $= \frac{R_1 + R_2}{R_2} > 1$ $(\beta_{opt} = 10)$.

τ = lag time const. $= R_2C_2$.

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1 + \tau s}{1 + \beta\tau s}}.$$



⇒



$$\Rightarrow \quad \omega_m = \sqrt{\omega_{c1} \times \omega_{c2}} = \sqrt{\frac{1}{\beta\tau} \times \frac{1}{\tau}}.$$

$$\omega_m = \frac{1}{\sqrt{\beta}\cdot\tau}. \text{ rad/sec.}$$

$$\therefore \boxed{\phi_m = \sin^{-1}\left(\frac{\beta-1}{\beta+1}\right).} \quad ***$$

* Advantages:

⇒ The lag Compensator is a LPF, it improves the steady state Performance (steady state error ↓es, accurate o/p).

⇒ The lag Compensator is a LPF, it eliminate the noise in the system, hence SNR at the o/p is improved.

⇒ The main ~~advantage~~ purpose of lag compensator is to provide the sufficient phase margin to the system.

* <u>Disadvantages:-</u>

⇒ The lag compensator & decreases the BW; hence the rise time increases hence the system gives the slow response.

⇒ The lag compensator is similar to the PI controller. With lag comp. system becomes very sensitive with parameter variation.

③ <u>Lag-Lead Compensators:-</u>

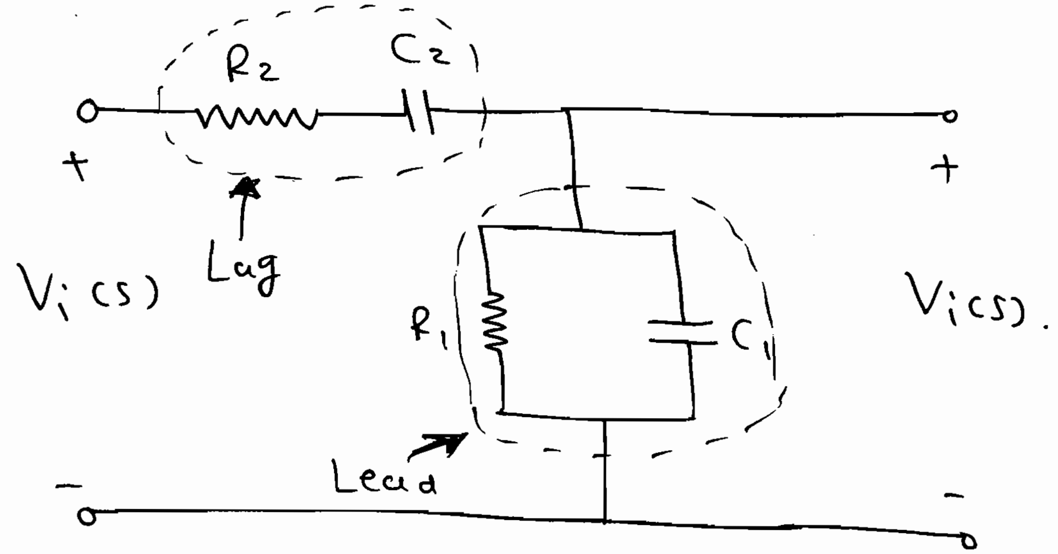
$(\tau_{lag} > \tau_{lead})$.

⇒ The Lag-Lead compensator is used to get the very quick response and good static accuracy.

(Rise time ↓ & e_{ss} ↓).

⇒ The ckt of lag-lead compensator is shown in fig.

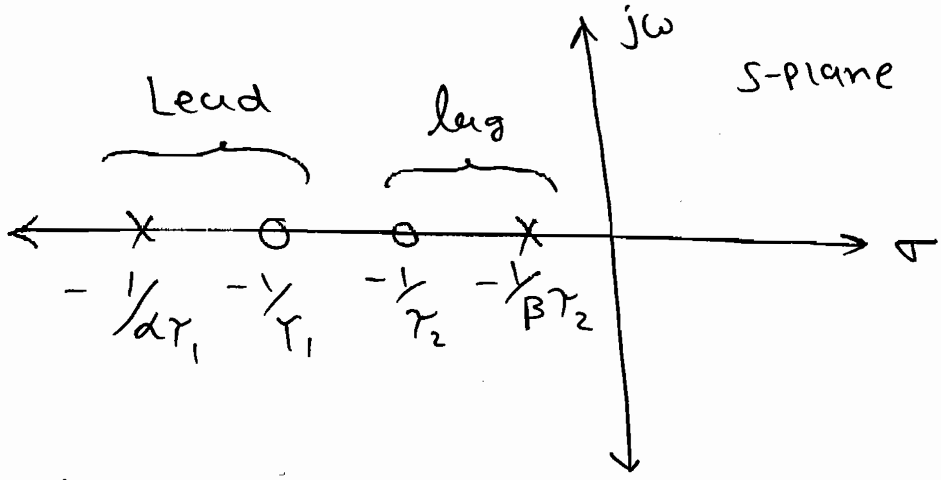
⇒



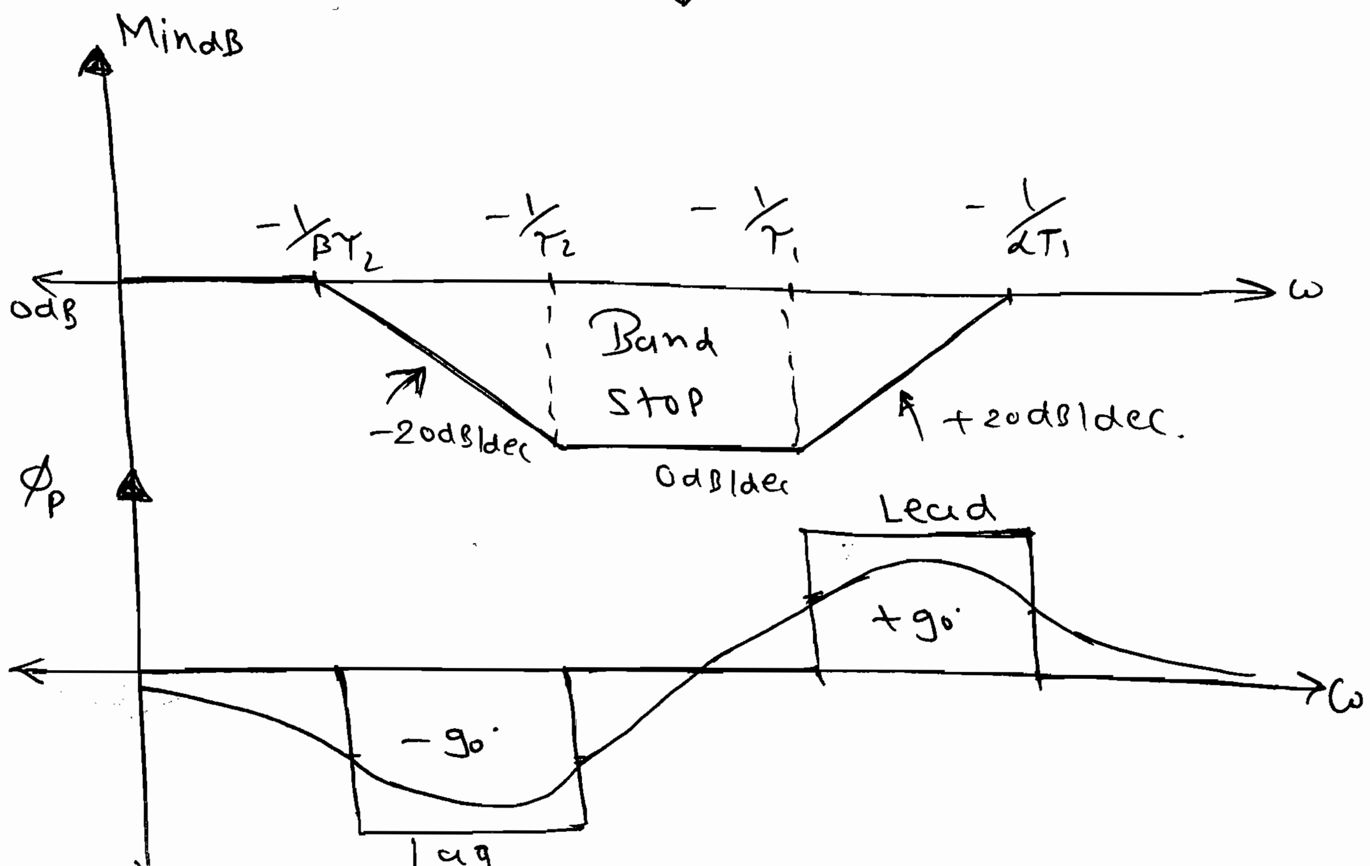
$$\frac{V_o(s)}{V_i(s)} = \left(\frac{1+\tau_1 s}{1+\alpha\tau_1 s}\right)\left(\frac{1+\tau_2 s}{1+\beta\tau_2 s}\right)$$

Lead Lag.

⇒

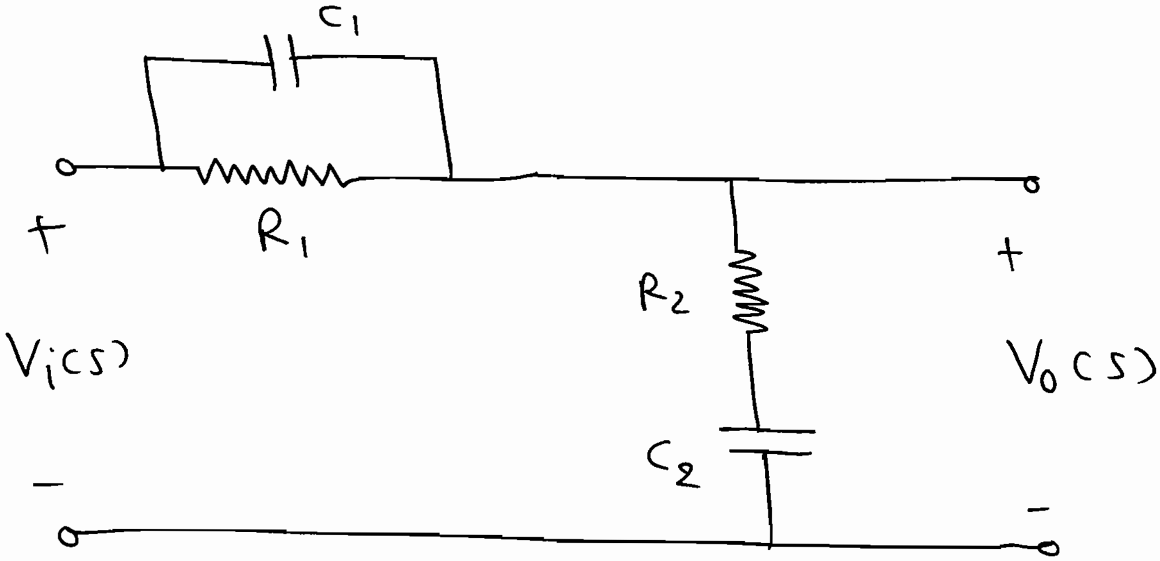


⇒



④ Lead- lag Compensators:

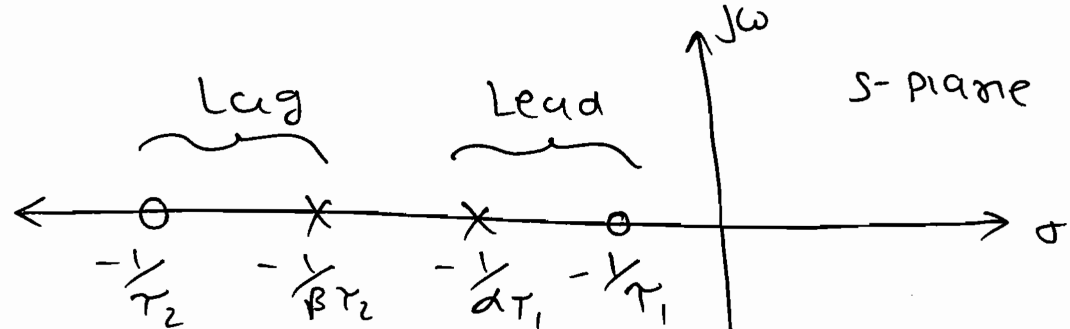
⇒



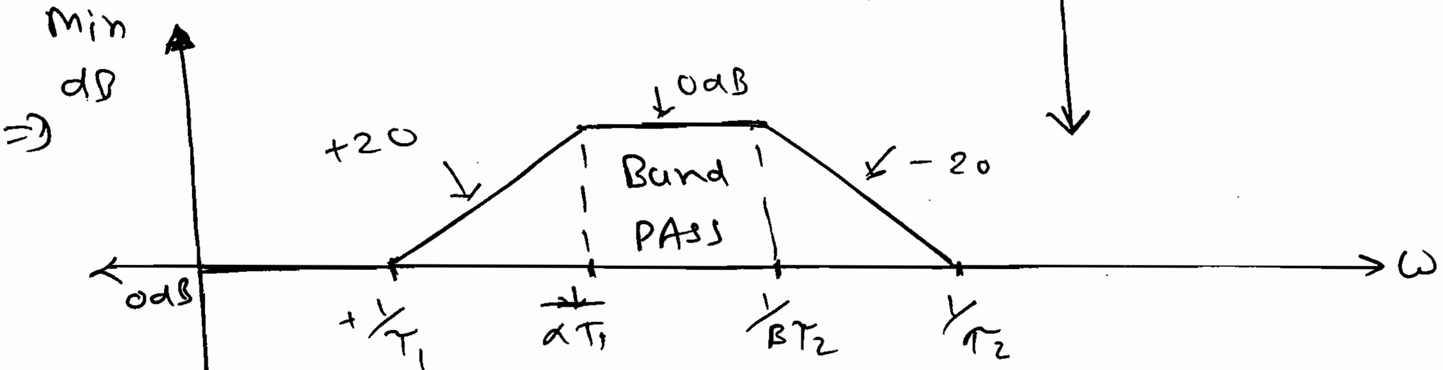
T.F. $$\frac{V_o(s)}{V_i(s)} = \left(\frac{1+\tau_1 s}{1+\alpha\tau_1 s}\right) \times \left(\frac{1+\tau_2 s}{1+\beta\tau_2 s}\right).$$

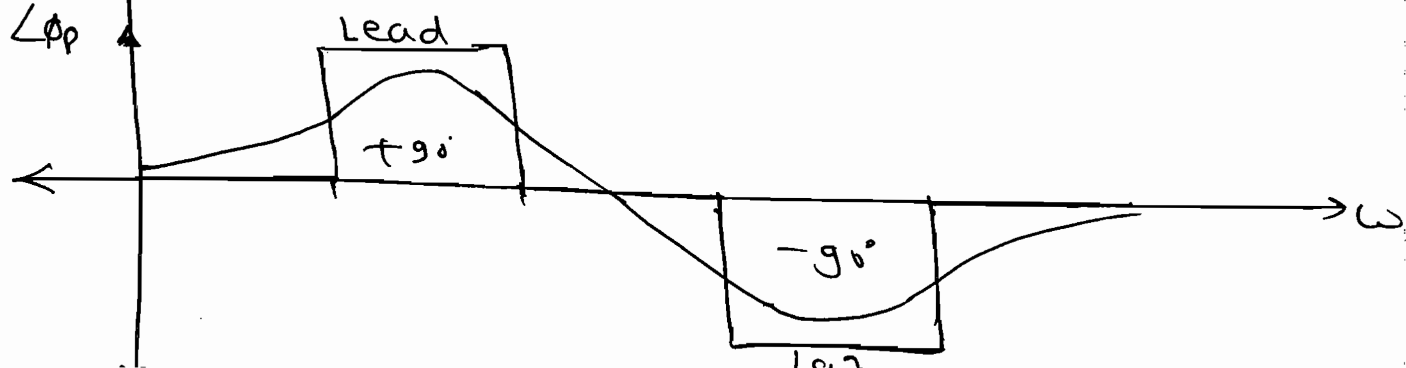
$$\boxed{\tau_{lead} > \tau_{lag}}$$

⇒



⇒





* Controllers :-

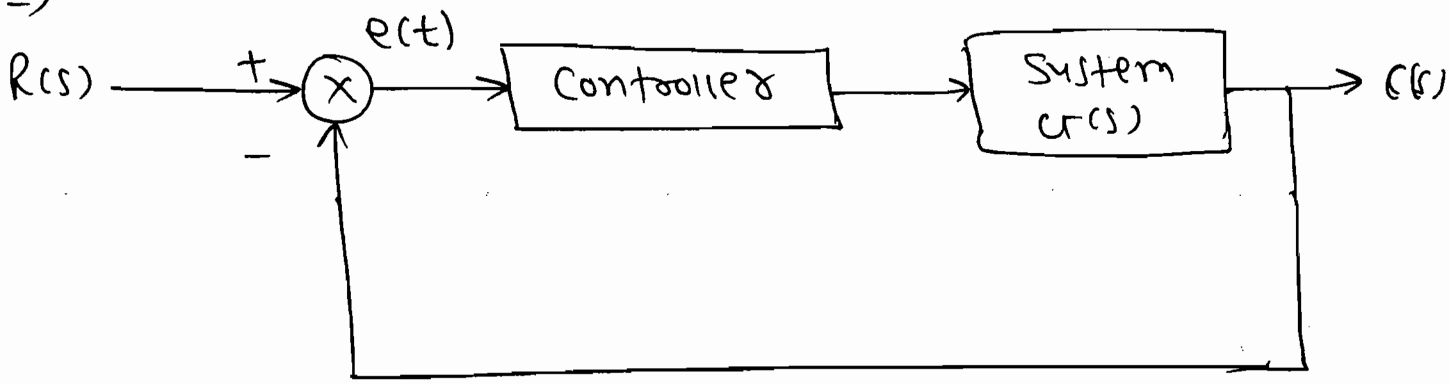
⇒ The Controller is a device which is used to controll the transient and steady state response as per requirement.

⇒ The best system demand smallest t_r, smallest t_s, smallest e_{ss}, smallest M_p.

⇒ To get above requirements we require to add a controller to the system.

⇒ The block diagram with the controller is shown in fig.

⇒



⇒ ① P Controller
② D Controller
③ I controller
④ PD controller.
⑤ PI Controller.
⑥ PID Controller.

① Proportional Controller :-

*** Purpose :-**

⇒ To change the transient response as per the requirement.

⇒ The T.F. of Proportional Controller is $\boxed{K_p}$

$$P \text{ controller} = K_p.$$

for (e.g) → $G(s)\Big|_{\substack{\text{without} \\ \text{Controller}}} = \dfrac{1}{s(s+10)}$.

⇒ $CLTF = \dfrac{1}{s^2+10s+1}$ ⇒ $\omega_n = 1$ rad/sec

$2\xi\omega_n = 10$

$\xi = 5 > 1$

⇒ overdamped system.

⇒ $G(s)\Big|_{\substack{\text{with} \\ \text{Controller}}} = \dfrac{K_p}{s(s+10)}$.

⟶ $CLTF = \dfrac{K_p}{s^2+10s+K_p}$.

→ let, $K_p = 100$ ⇒ $\omega_n = 10$ r/sec.

$2\xi\omega_n = 10$

$\boxed{\xi = 0.5}$ ⇒ Under damped sys.

→ let, $K_p = 25$, $\omega_n = 5$

& $\boxed{\xi = 1}$ ⇒ Critical damped sys.

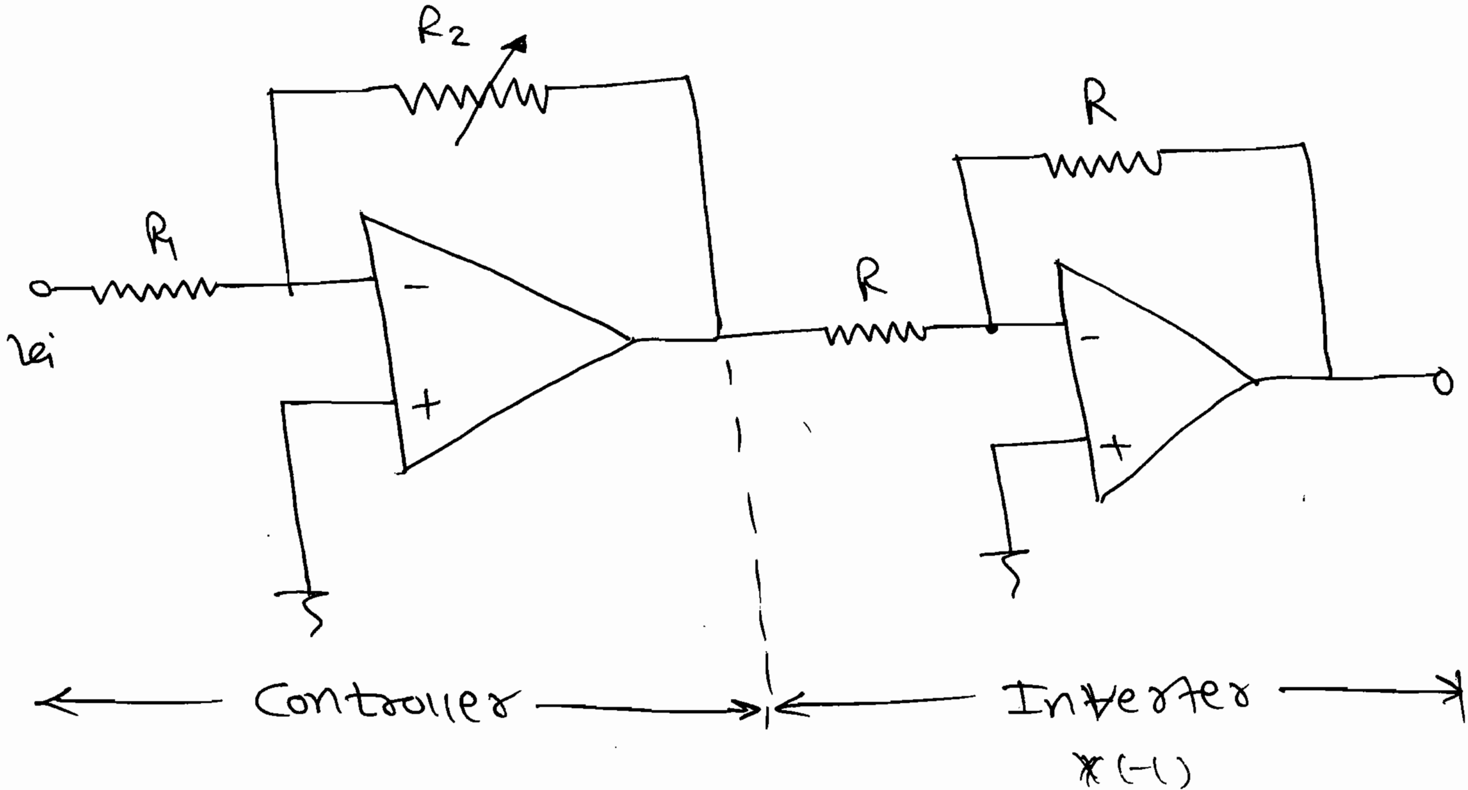
⇒ If selecting the proper value of K_p we can get the required transient response.

**⇒ $\boxed{K_p \uparrow \to \omega_n \uparrow \to \xi \downarrow \to \% M_p \uparrow \to \text{less RS \& more osc.}}$

⇒ Proportional controller can not eliminate error in the system.

⇒ If the $K_p \uparrow$, to get the better transient response $\xi \downarrow$. Hence the $\% M_p$ increases, the sys. become more oscillatory & ~~xxx~~ less Relative stable.

⇒ <u>Practical Proportional controller:-</u>



$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1} = K_p.}$$

② Integrated Controller (or) RESET Controller :-

* ~~Res.~~ Purpose :-

⇒ To decrease the steady state error (e_{ss}).

⇒ The T.F. of Integral Controller is $\frac{K_I}{s}$.

⇒ The integral Controller added the one Pole at origin Hence, Type is increases.

⇒ As the Type increases, the $e_{ss}\downarrow$ but the System Stability is affected.

e.g.:

→ $G(s)\Big|_{\text{without Controller}} = \frac{1}{s(s+10)}$, Type-1.

$\xrightarrow{CE}$ $s^2+10s+1=0$ → Stable.

→ $G(s)\Big|_{\text{with Controller}} = \frac{K_I}{s^2(s+10)}$, Type-2, ↑

$e_{ss}\downarrow$ (more accurate)

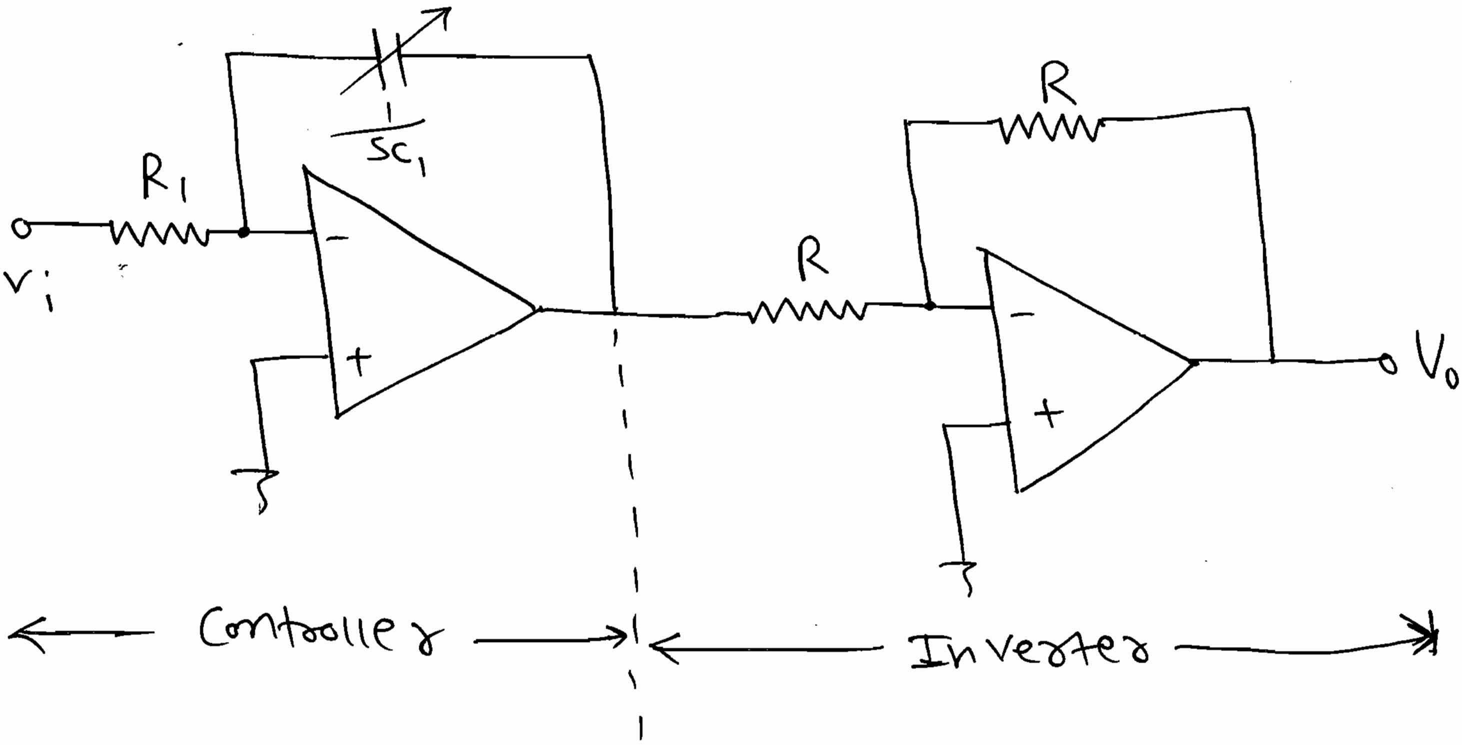
CE → $s^3 + 10s^2 + K_I = 0$ → Un-stable.

⇒ The integral Controller effect the sys. Stability. Hence, before using the integral Controller we required to verify the sys. Stability.

⇒ If the System Stability is affected the integral Controllers are not used.

* <u>Practical</u> <u>ckt</u> <u>of</u> <u>Integral</u> <u>Controller:-</u>

⇒



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R} = \frac{1}{sCR} = \frac{1}{\tau_I s} = \frac{K_I}{s}.$$

Where, $K_I = \dfrac{1}{\tau_I} = \dfrac{1}{RC}$.

③ Derivative Controller :- (RATE Controller)

* Purpose:-

⇒ To improve the Stability.

⇒ T.F. of Derivative controller is $K_D s$.

⇒ The Derivative Controller adds 1 zero at origin.

$$\text{T.F. of D Controller} = K_D s.$$

⇒ The best example of derivative Controller is Techo-meter.

→ With D Controller added one zero at origin. Hence the type is ↓.

→ As type ↓, the sys. stability improved but sys. become less accurate. (e_{ss} ↑).

e.g. → $G(s)\Big|_{\substack{\text{without} \\ \text{Controller}}} = \dfrac{1}{s^2(s+10)}$ Type-2.

$\xrightarrow{CE}$ $s^3 + 10s^2 + 1 = 0 \longrightarrow$ Unstable.

→ $G(s)\Big|_{\substack{\text{with} \\ \text{Controller}}} = \dfrac{K_D \cancel{s}}{s^{\cancel{2}}(s+10)} = \dfrac{K_D}{s(s+10)}$

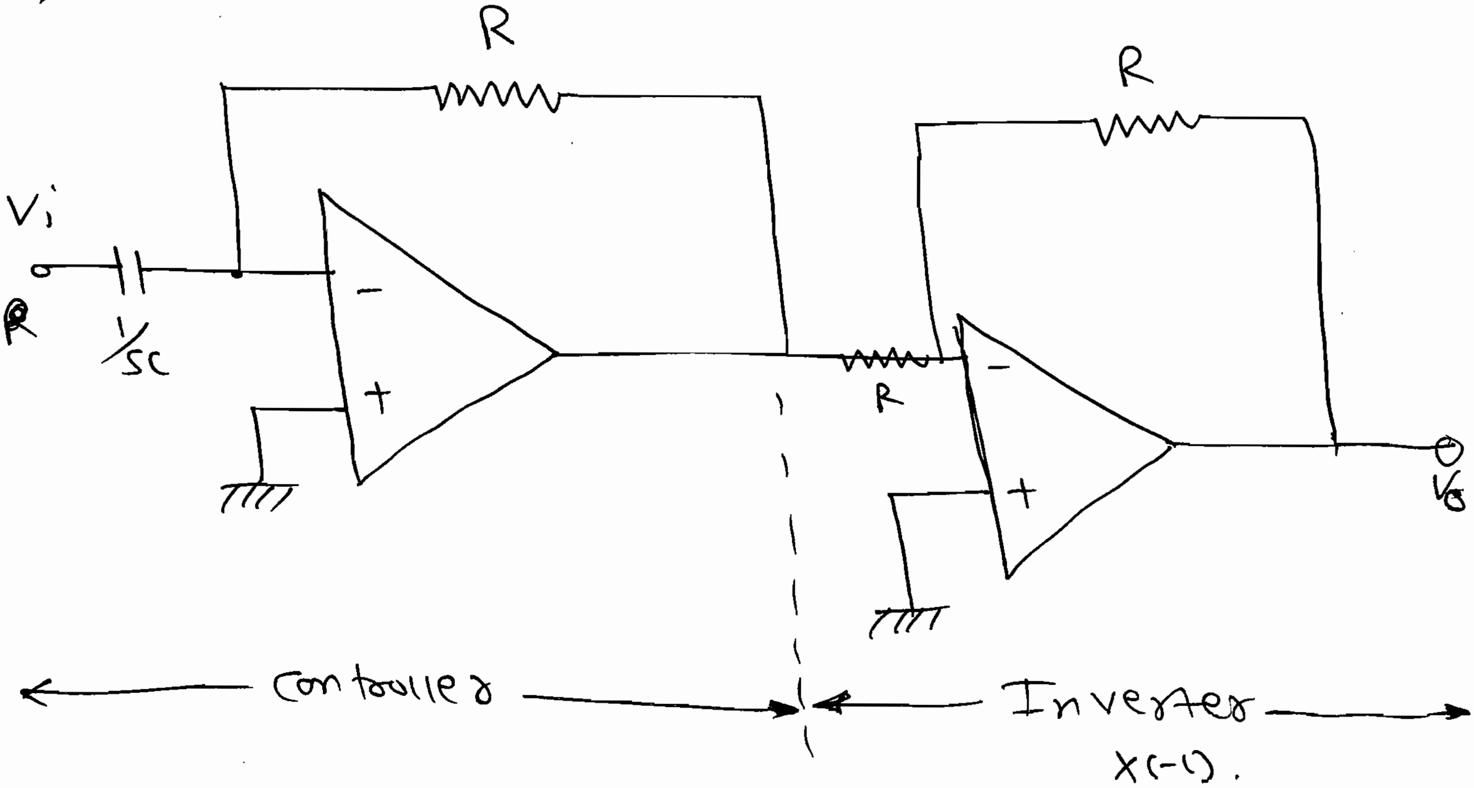
Type-1 ↓ e_{ss} ↑

less accurate

$CE \longrightarrow \quad s^2 + 10s + K_D = 0 \longrightarrow$ Stable.

* Practical ckt for Derivative Controller :-

$\Rightarrow$



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R}{\frac{1}{sC}} = sCR = \tau_d s = K_D s.$$

where, $K_D = \tau_D = RC.$

(4) PI Controller :-

* Purpose:-

$\Rightarrow$ To decrease the steady-state error without affecting the stability.

$\Rightarrow$ The ~~RI~~ ~~Controller~~ T.F. of the PI controller is

$$\boxed{T.F = K_P + \frac{K_I}{S}.}$$

$\Rightarrow$ T.F. $= \left(\dfrac{sK_P + K_I}{s}\right)$.

$\Rightarrow$ The P-I Controller added one Pole at origin which increases the Type of the system.

$\Rightarrow$ As type $\uparrow$, the $e_{ss} \downarrow$.

$\Rightarrow$ PI Controller added one finite zero in the left of the s-plane which avoid the effect on sys. Stability.

eg let, $G(s)\Big|_{\substack{\text{without}\\ \text{Controller}}} = \dfrac{1}{s(s+10)}$ Type-1

CE $\rightarrow$ $s^2 + 10s + 1 = 0$ $\longrightarrow$ Stable

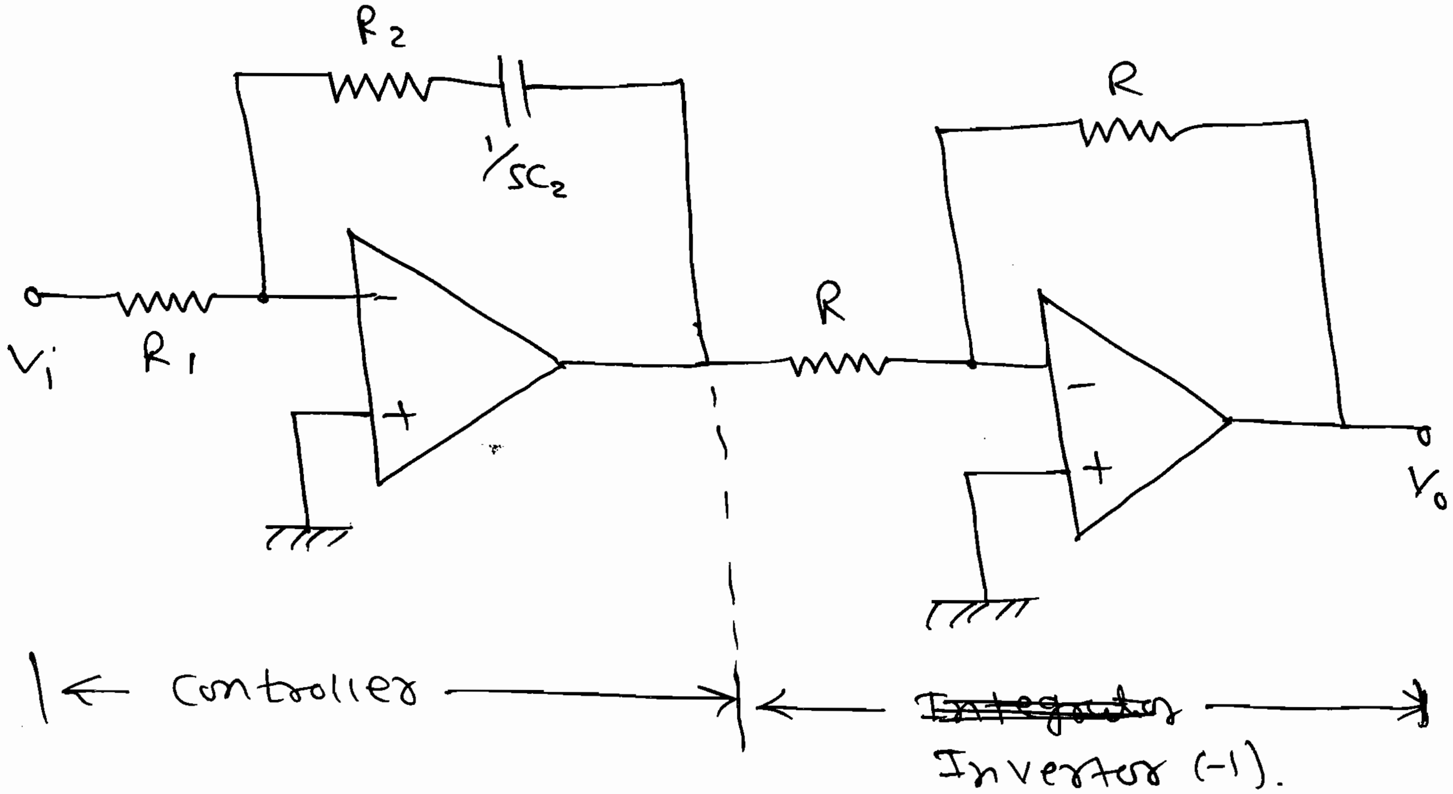
$\rightarrow$ $G(s)\Big|_{\substack{\text{with}\\ \text{Controller}}} = \dfrac{(sK_P + K_I)}{s^2(s+10)}$, Type-2 $\uparrow$, $e_{ss}\downarrow$ more accurate

CE $\rightarrow$ $s^3 + 10s^2 + sK_P + K_I = 0$ $\longrightarrow$ Stable

Stability is not affected.

* Practical ckt for P-I Controller:-

$\Rightarrow$ The Practical ckt is shown in fig.



$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_1 \cancel{+ R_2 + \frac{1}{sC_2}}}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1} + \frac{1}{sC_2R_1}.$$

$$T.F. = K_P + \frac{K_I}{s}.$$

Where, $K_P = \frac{R_2}{R_1}$, $K_I = \frac{1}{R_1C_2}$.

⑤ PD Controller :-

*** Purpose :-**

=> To improve the stability without affecting the steady state error (ess).

⇒ The T.F. of PD Controller is $(K_P + K_D s)$.

⇒ The P.D. Controller added one finite zero in the left hand side, which improves the sys. Stability.

⇒ PD Controller do not change the type, hence No effect on Steady state error.

⇒ The damping ratio with PD Controller is $\zeta_{PD} = \left[\zeta + \frac{\omega_n K_D}{2}\right]$.

⇒ $G(s)\Big|_{\text{without Controller}} = \dfrac{1}{s^2(s+10)}$; Type-2

$\xrightarrow{CE}$ $s^3 + 10s^2 + 1 = 0$ ⟶ Unstable.

No change in Type. Hence No change in e_{ss}

⇒ $G(s)\Big|_{\text{with Controller}} = \dfrac{(K_P + K_D s)}{s^2(s+10)}$; Type-2.

improved

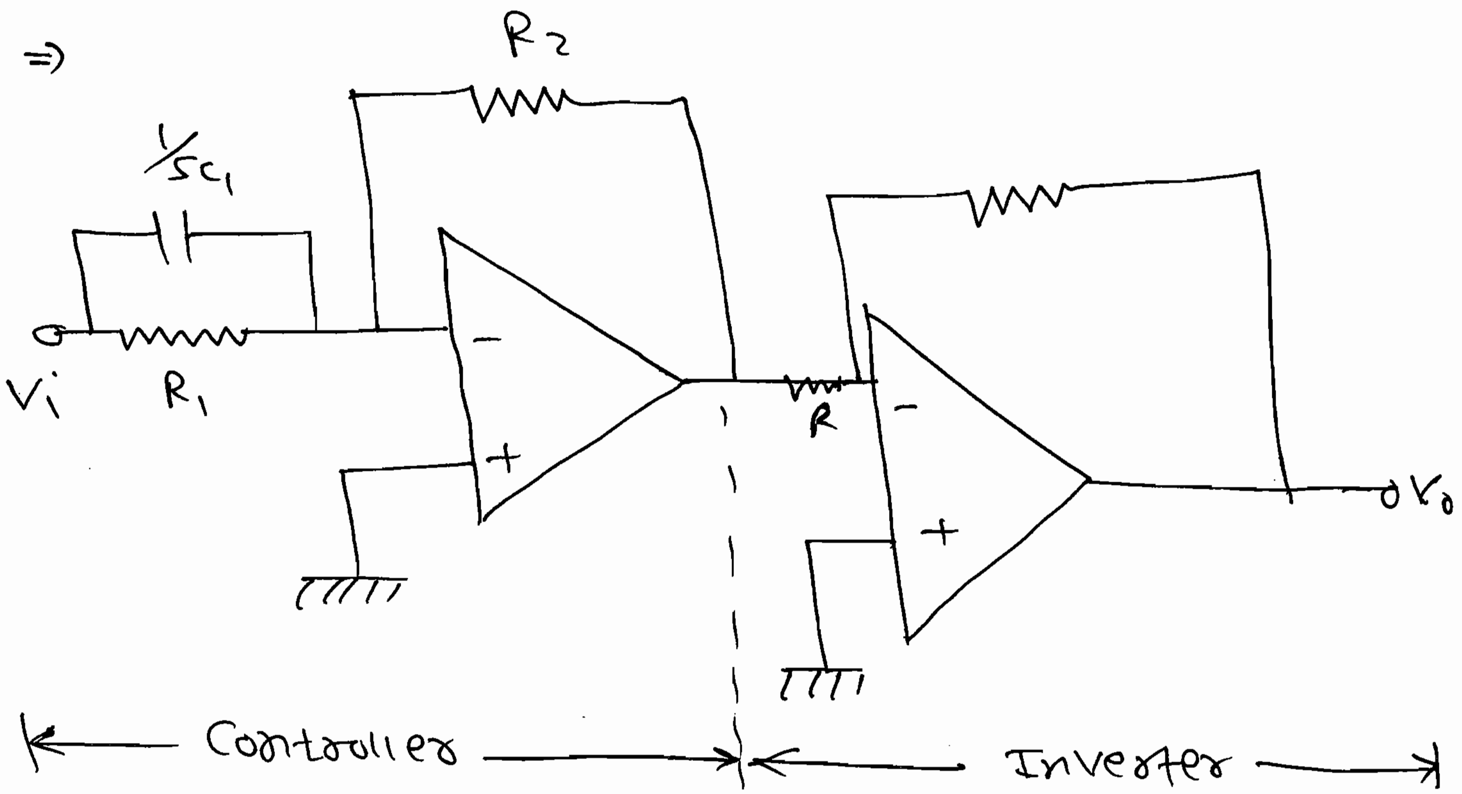
$\xrightarrow{CE}$ $s^3 + 10s^2 + K_D s + K_P = 0$ ⟶ Stable.

⇒ No Change in Type, Hence No Change in e_{ss}.

⇒ Stability improved.

* Practical ckt for PD Controller :-

⇒



$$\Rightarrow T.F. = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1 \parallel \frac{1}{sC_1}} = \frac{R_2}{\frac{R_1}{1+sC_1R_1}}$$

$$\therefore = \frac{R_2(1+sC_1R_1)}{R_1}$$

$$= \frac{R_2}{R_1} + sC_1R_2$$

$$= K_p + K_D s$$

Where, $K_p = \frac{R_2}{R_1}$, $K_D = C_1R_2$.

6 PID Controller :-

*** Purpose:-**

⇒ To decrease the steady state error (e_{ss}) & improve the stability.

⇒ The T.F. of the PID Controller:

$$T.F. = \left(K_P + \frac{K_I}{s} + K_D s\right).$$

$$T.F. = \left(\frac{K_D s^2 + K_P s + K_I}{s}\right).$$

⇒ The PID Controller added one pole at origin. Hence, the Type is ↑ Steady State error ↓.

⇒ The PID Controller added two finite zero in left-hand side.

⇒ one zero avoid the effect on System stability and other zero improves the system stability.

Eg:

$$G(s)\Big|_{\substack{\text{without}\\ \text{Controller}}} = \frac{1}{s^2(s+10)} \quad ; \text{ Type-2.}$$

$\xrightarrow{CE}$ $s^3 + 10s^2 + 1 = 0$ ⟶ Unstable. ↑

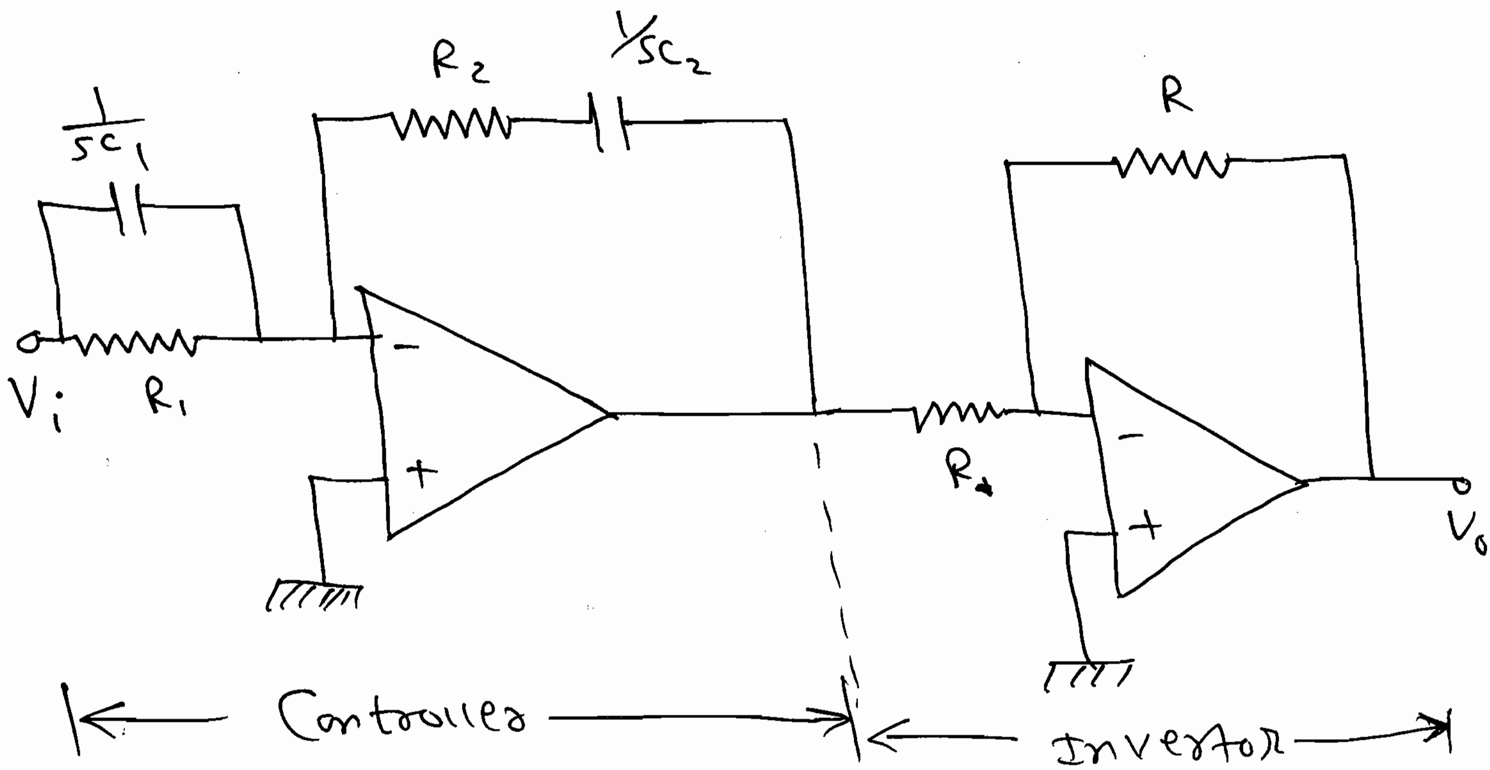
$$\Rightarrow G(s)\Big|_{\substack{\text{with}\\ \text{Controller}}} = \frac{K_D s^2 + K_P s + K_I}{s^3(s+10)} \quad ; \text{ Type-3. ↑}$$

e_{ss} ↓

improved

$\xrightarrow{CE}$ $s^4 + 10s^3 + K_D s^2 + K_P s + K_I = 0$ ⟶ Stable (more accurate)

* Practical ckt for PID Controller :-



$$\Rightarrow T.F. = \frac{V_o(s)}{V_i(s)}.$$

$$= \frac{R_2 + \frac{1}{sC_2}}{\frac{R_1}{sC_1R_1+1}}$$

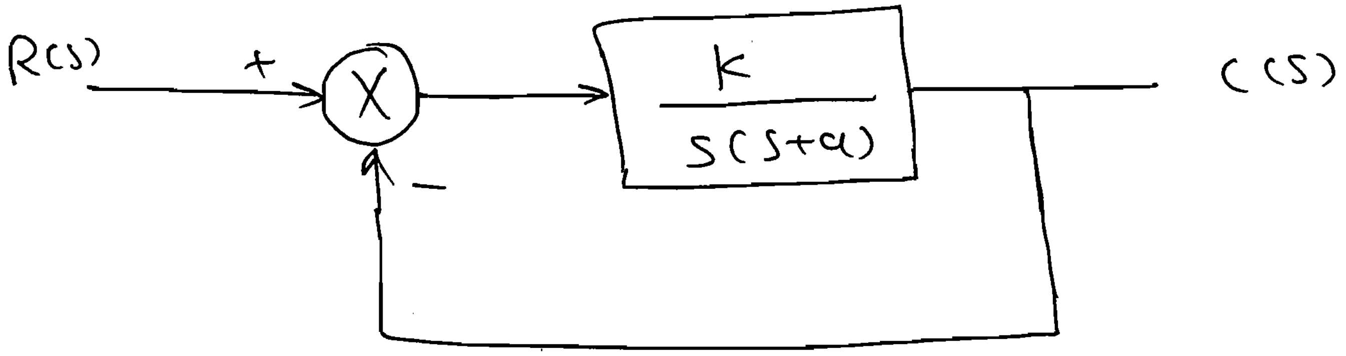
$$= \frac{(1+sC_2R_2)(1+sC_1R_1)}{sC_2R_1}$$

$$= \frac{1+s(C_2R_2+C_1R_1)+s^2C_2R_2C_1R_1}{sC_2R_1}$$

$$= \underbrace{\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)}_{K_P} + \underbrace{\left(\frac{1}{sC_2R_1}\right)}_{K_I/s} + \underbrace{\left(sC_1R_2\right)}_{K_D \cdot s}.$$

$$T.T. = K_P + \frac{K_I}{s} + K_D \cdot s$$

Q Find the steady state error of sensitivity to change in parameters (i) K (ii) a to the unit-ramp I/P to the following system.



Soln:

$$G(s) = \frac{K}{s(s+a)}.$$

$R(s) =$ unit ramp

$r(t) = t\,u(t)$. & Type-1

$$\therefore \quad e_{ss} = \frac{1}{K/a} = a/K.$$

(i) $$S_K^{e_{ss}} = \left(\frac{\partial e_{ss}}{\partial K}\right) \times \left(\frac{K}{e_{ss}}\right) = \frac{-\cancel{K}\,\cancel{a}}{\cancel{K^2}} \times \frac{\cancel{K}}{\cancel{a}/\cancel{K}}$$

$$\therefore \quad \boxed{S_K^{e_{ss}} = -1}$$

(ii) $$S_a^{e_{ss}} = \left(\frac{\partial e_{ss}}{\partial a}\right) \times \left(\frac{a}{e_{ss}}\right) = \frac{1}{\cancel{K}} \times \frac{\cancel{a}}{\cancel{a}/\cancel{K}}$$

$$\therefore \quad \boxed{S_K^{e_{ss}} = 1}$$