ॐ

OM

Sujal Patel (+918121564132).

ECE

PM1(B)

Communication System.

Sir: Murli Sir

All the best

## * Objective of Electronic Communication system:

→ objective of Electronic communication system is to transfer information from one place to another place using electrical signals.

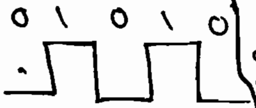
H.B →

Voice → 300 - 3.5 kHz e.g. - Telephone system

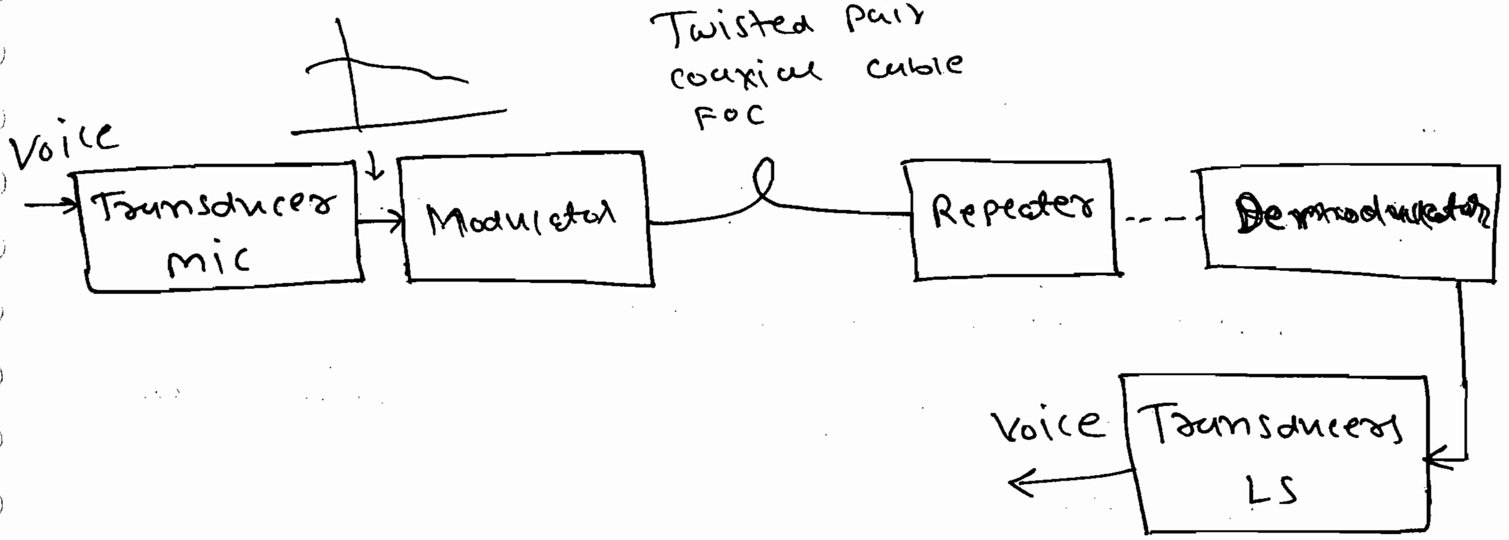
Audio → 20 - 20 KHz e.g. - Radio.

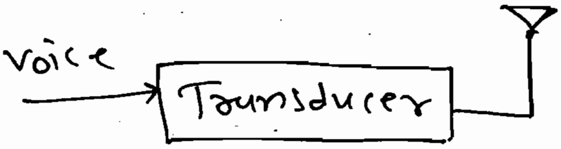
Video → 0 - 4.5 MHz e.g. - T.V.

Data → (Pulse width) e.g. - Internet



## * Block Diagram of Communication System:



⊛ 

$$\lambda = \frac{c}{f}$$

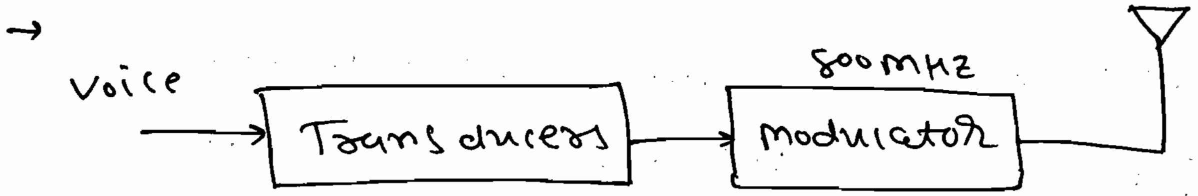
$$\lambda = \frac{3\times10^{8}}{300} = \text{very large}$$

→ Therefore, Antenna size may be very

Very large because antenna length ~~very~~ is to be define in terms of its wavelength (λ) which is practicully not possible.

→ Inorder to avoid this problem we should use modulator which convert ~~to~~ Low freq. signal into high freq. signal.

→



$$\therefore \quad \lambda = \frac{3\times10^{8}}{800\times10^{6}}$$

So, Antenna size reduced very much.

## * Advantages:

① To reduce the size of antenna.

② Multiplexing is possible.

③ To reduce the effect of Noise. and interface.

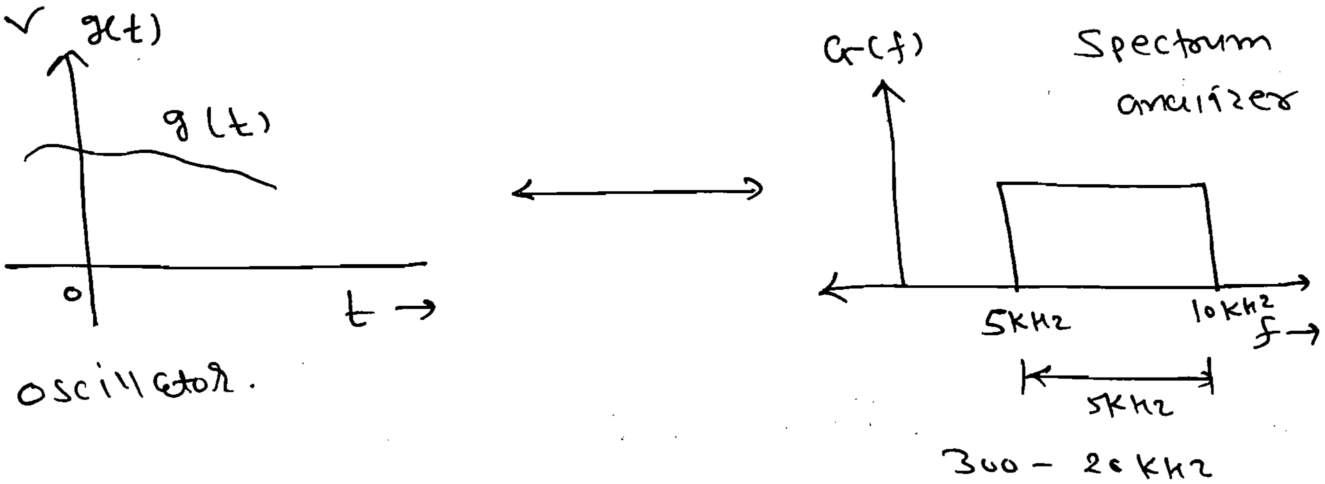
④ For narrow banding of signals.

## * Review of Fourier Transform:

→ Fourier transform concept is used to determine the frequency present in the signal.

→ Fourier transform converts time domain signals into freq. domain signal.

$$g(t) \longleftrightarrow G(f)$$



**NOTE:**

→ The oscillator will display time domain signal.

→ The Spectrum analyizer will display freq. domain spectrum.

## * Band width:

→ B Range of the frequiens occupied by the signal is called as the BW.

→ Mathamaticaly,

$$BW = f_H - f_L$$

$$BW = 10 - 5$$

$$BW = 5 \text{ KHz}$$

## * Spectrum:

→ It is a graphical representation of signal in freq. domain.

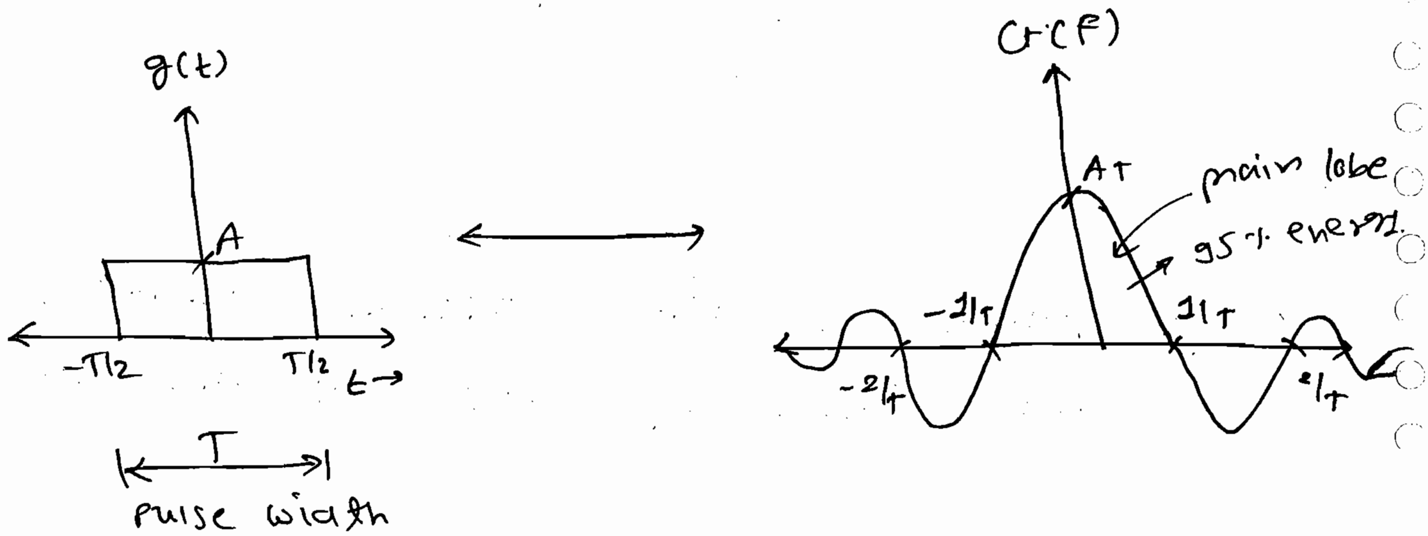
**NOTE:**

→ Practically the Bandwidth of signal should be low as possible and channel BW should be as high as possible.

$$\Rightarrow \quad g(t) \xleftrightarrow{FT} G(F).$$

$$\boxed{G(F) = \int_{-\infty}^{\infty} g(t) . e^{-j2\pi f t} . dt.}$$

⇒



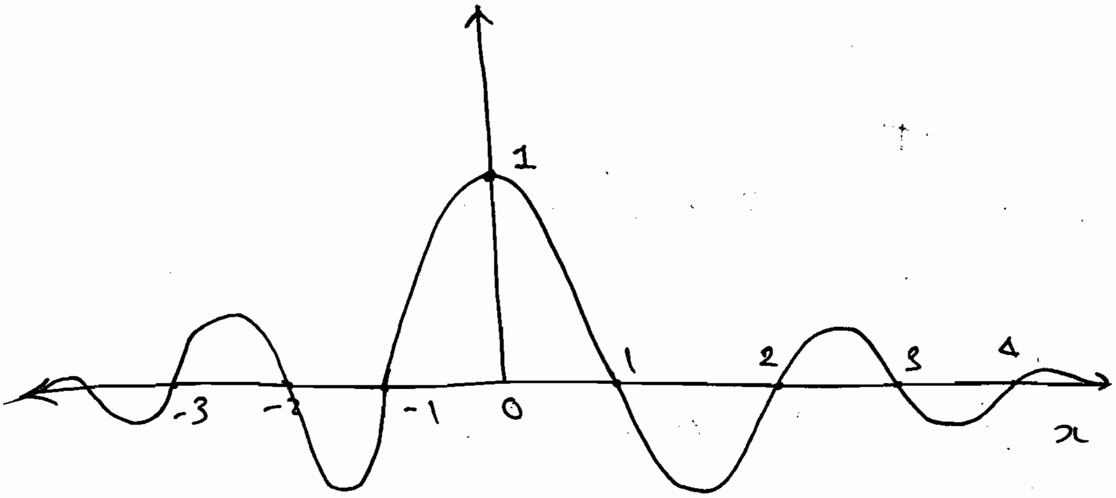
$$G(F) = \int_{-T/2}^{T/2} A \cdot e^{-j2\pi f t} \cdot dt.$$

$$\therefore G(f) = AT \frac{\sin(\pi f T)}{\pi f T}.$$

$$\therefore \boxed{G(f) = AT\, \text{sinc}(fT).}$$

$$* \quad \text{Sinc}(x) = \frac{\sin(\pi x)}{\pi x}.$$

$$= 0, \quad x = \pm 1, \pm 2, \pm 3, \pm 4, \ldots$$



$\Rightarrow \quad G(f) = AT\text{sinc}(fT).$

$\therefore \quad f = 0, \pm \frac{1}{T}, \pm \frac{2}{T}, \pm 3/T, \ldots$

Practically (-ve) freq. do not exist.

$\Rightarrow$ BW of a signal is:

$\Rightarrow \quad BW = f_H - f_L$

$= \infty - 0$

$\boxed{BW = \infty}$ Theoretical

⇒ Signal which is having finite duration is called Energy signal.

⇒ Signal which is having Infinite duration is called Power signal.

→ $$E = \int_{-\infty}^{\infty} g^2(t)\,dt = A^2T \text{ Joules.}$$

$$E = \int_{-\infty}^{\infty} |G(f)|^2\,df = A^2T \text{ Joules.}$$

$$\therefore \quad E = \int_{-1/T}^{1/T} |G(f)|^2\,df.$$

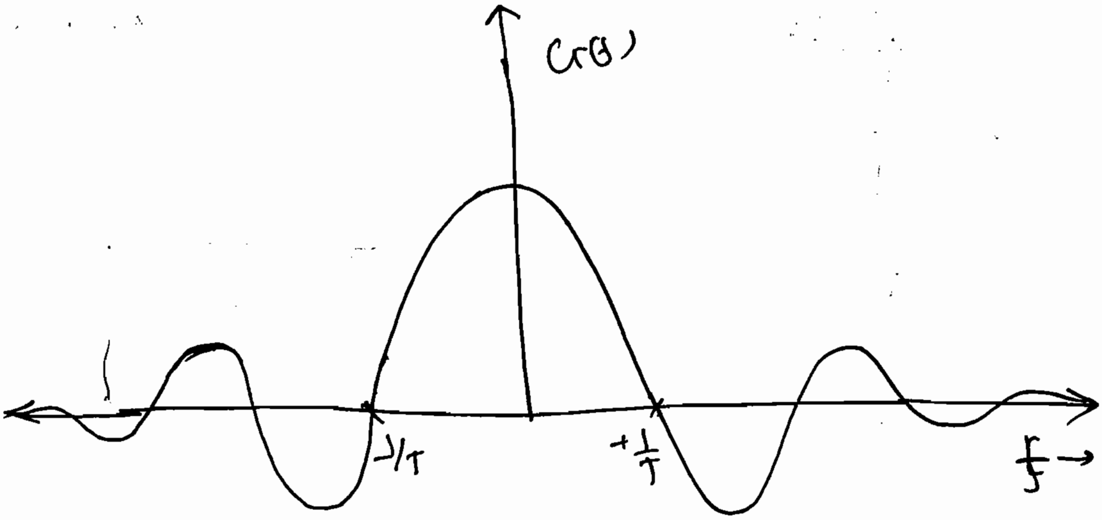
$$\therefore \quad \boxed{E = 0.95\,A^2T} \text{ J.}$$

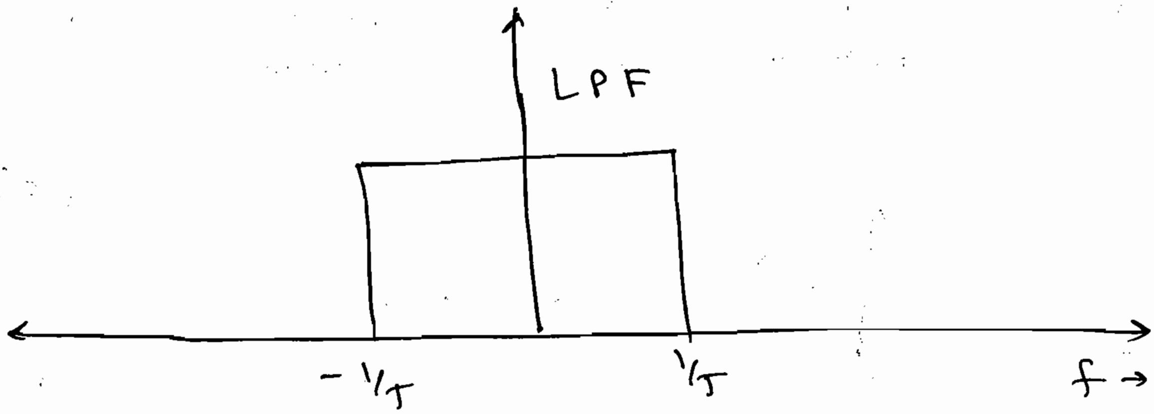
⇒ So, Energy present in main lobe,

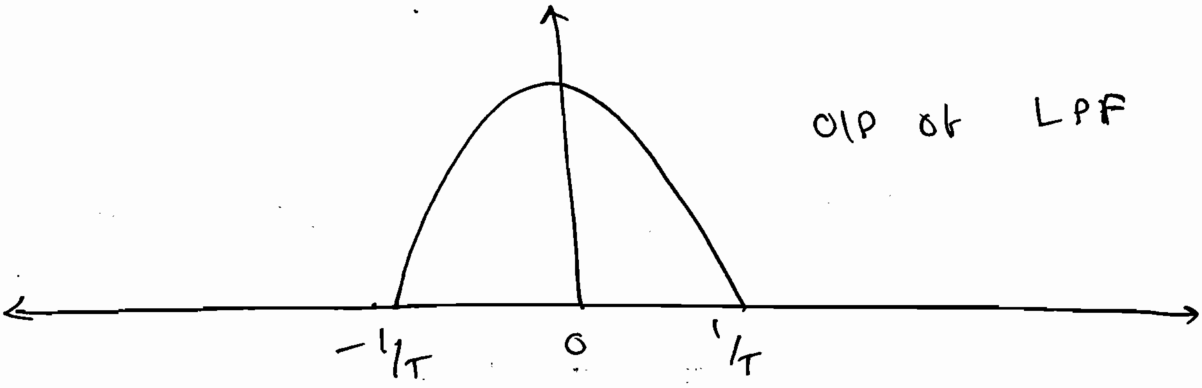
$$\boxed{E = 0.95\,A^2T \text{ J}}$$

⇒ 95 % of energy is Contained by only main lobe.

*







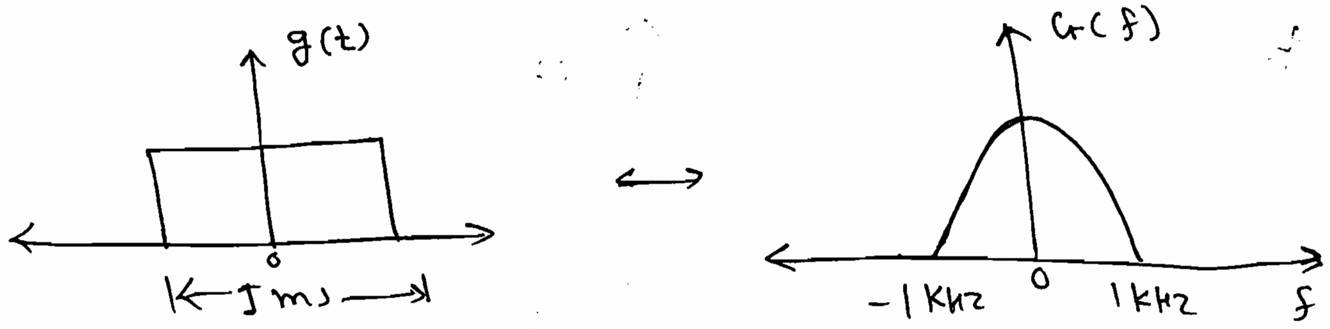
$\Rightarrow$ $BW = f_H - f_L$

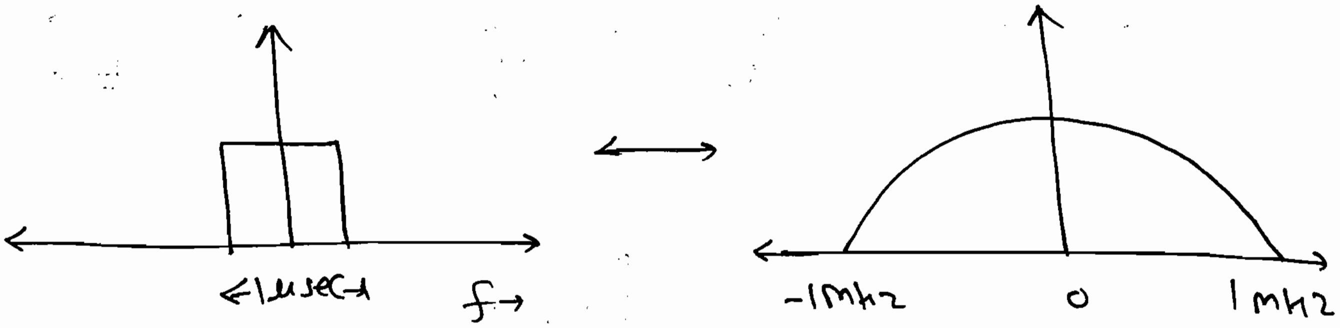
$= \frac{1}{T} - 0$

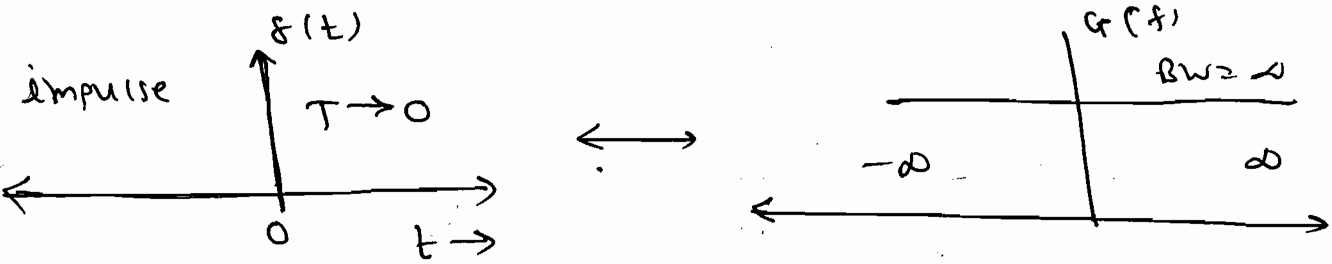
$\boxed{BW = 1/T}$ (Practical BW)

$\Rightarrow$ Practical BW of Rectangular pulse is inversely proportional to the pulse width.

*







⇒ BW of impulse is infinite.

* Freq. Shifting Property: [Modulation property]:

$$g(t) \longleftrightarrow G(f).$$

N.B.

$$g(t) . \cos 2\pi f_c t \longleftrightarrow \frac{G(f-f_c) + G(f+f_c)}{2}$$

Carrier

$f_c$: Carrier freq.

Mobile: 800 MHz

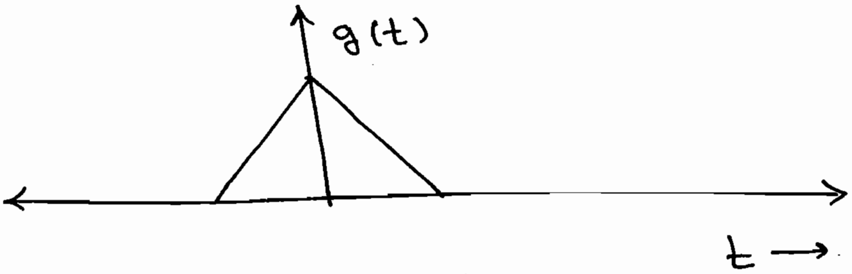
FM: 88 - 108 MHz

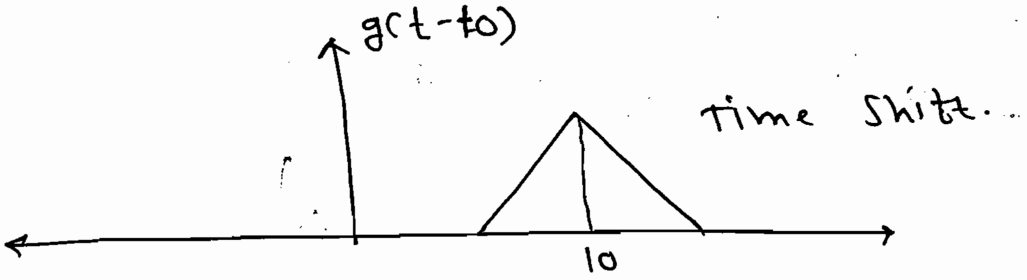
Radio: order of GHz

⇒ HB If a signal is Multiplied with carrier, the spectrum is shifted to the left and Right side, by $f_c$. The amplitude decreses by a factor of '2'.

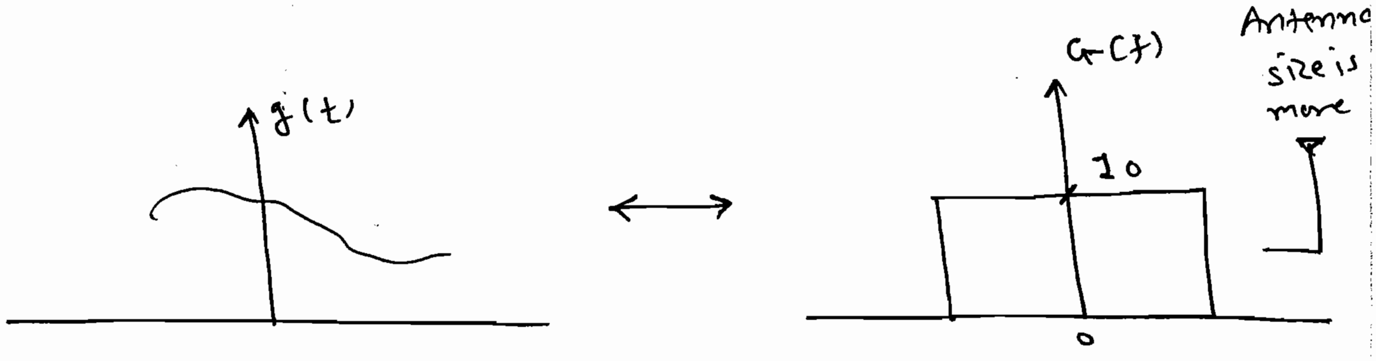
* Time Shift:

*

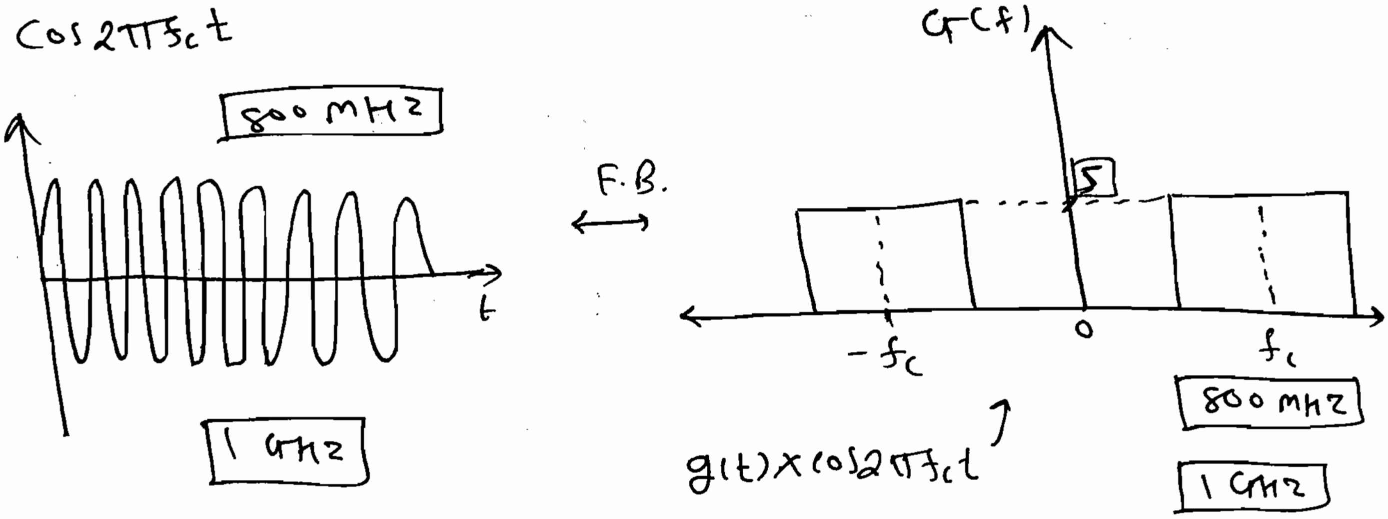




*



*

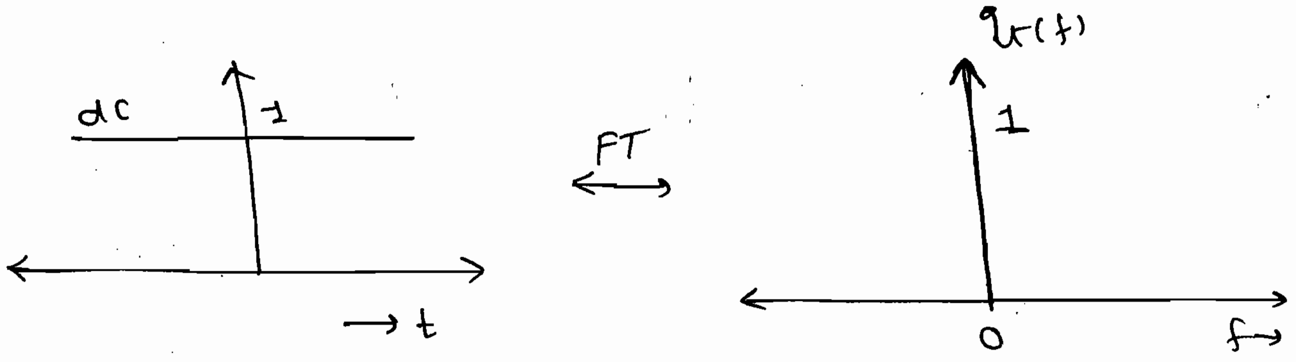


# * Modulation :-

→ Modulation is defined as a process of freq. translation by modulating a signal. The spectrum is shifted from low freq. region to the high freq. region. In order to modulate a signal it should be multiplied with a carrier. (high freq. sinusoidal signal).

$$\Rightarrow \quad \delta(t) \xleftrightarrow{FT} 1.$$

$$1 \xleftrightarrow{FT} \delta(F). \Rightarrow 2\pi\delta(\omega).$$



F=0 of Dc

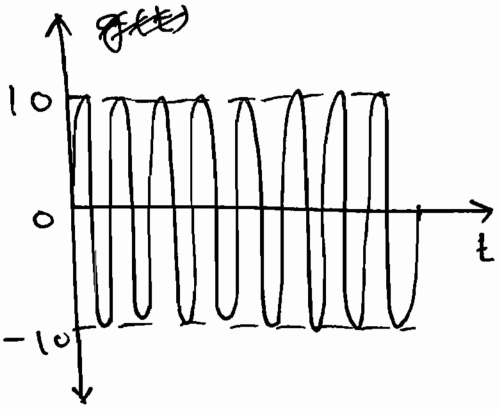
Impulse:

$$1.\ \cos 2\pi f_c t \xleftrightarrow{FT} \frac{\delta(f-f_c)+\delta(f+f_c)}{2}$$

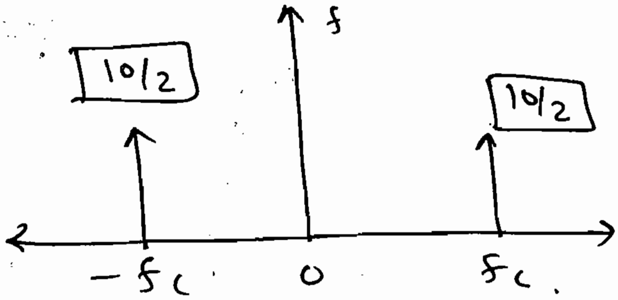
H.B

$$= 2\pi\left[\frac{\delta(\omega-\omega_c)+\delta(\omega+\omega_c)}{2}\right]$$

$$= \pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right].$$



$\longleftrightarrow$

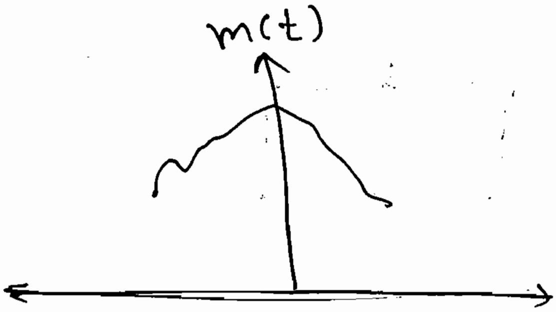


## * Concept of Modulation and Demodulation:

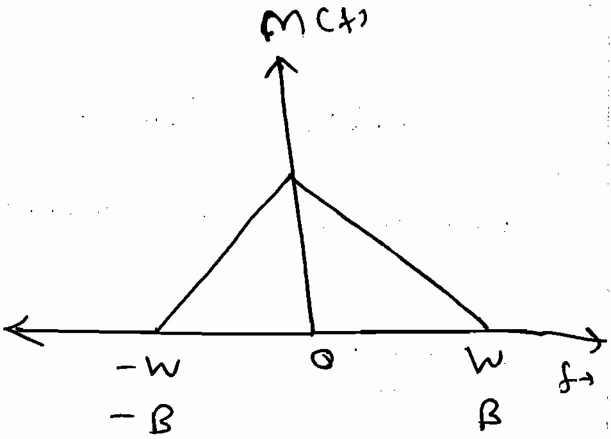
⇒ $m(t)$ = message signal.
= modulating signal
= Base-band signal.

→ A signal which is having significant low freq. is called base-band signal.

$$m(t) \xleftrightarrow{FT} M(f).$$



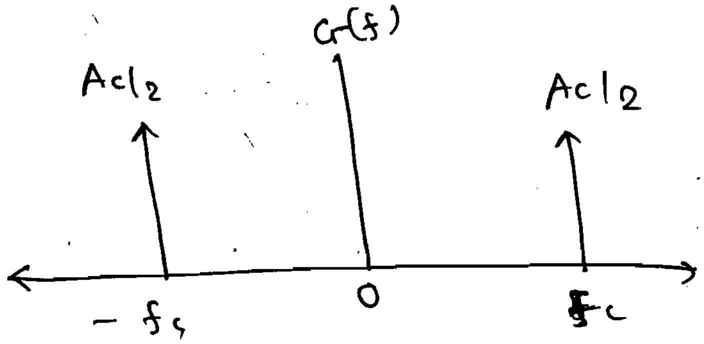
$\longleftrightarrow$



W = highest freq. of signal $m(t)$.

⇒ $c(t)$ = Carrier

$$= A_c \cos 2\pi f_c t \longleftrightarrow \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right].$$

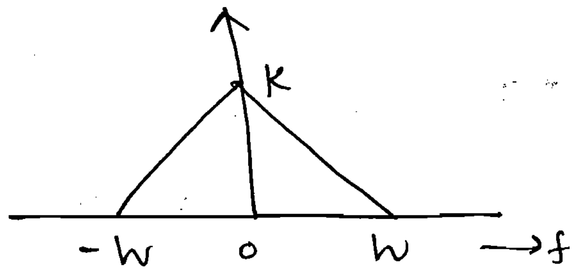


⇒ S(t) = modulated signal.

H.B.

∴ $\boxed{S(t) = m(t). c(t).}$

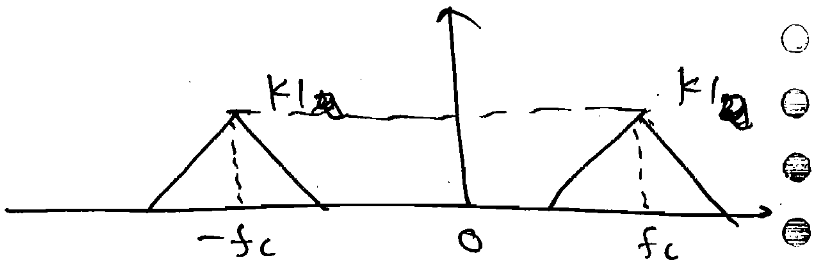
⇒ m(t) ⟷



Tx

Modulation $\downarrow$ X cos2πfct

$\boxed{S(t) = m(t). \cos 2\pi f_c t}$



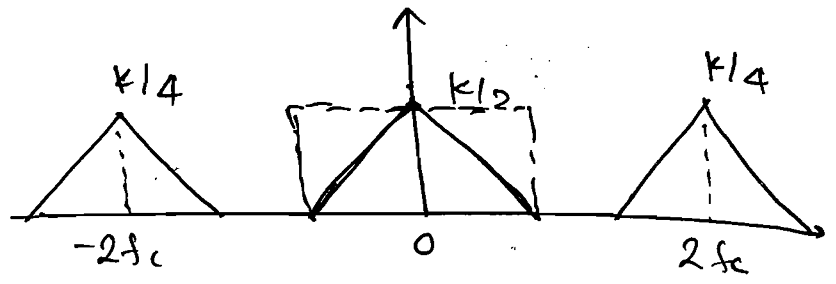
Rx

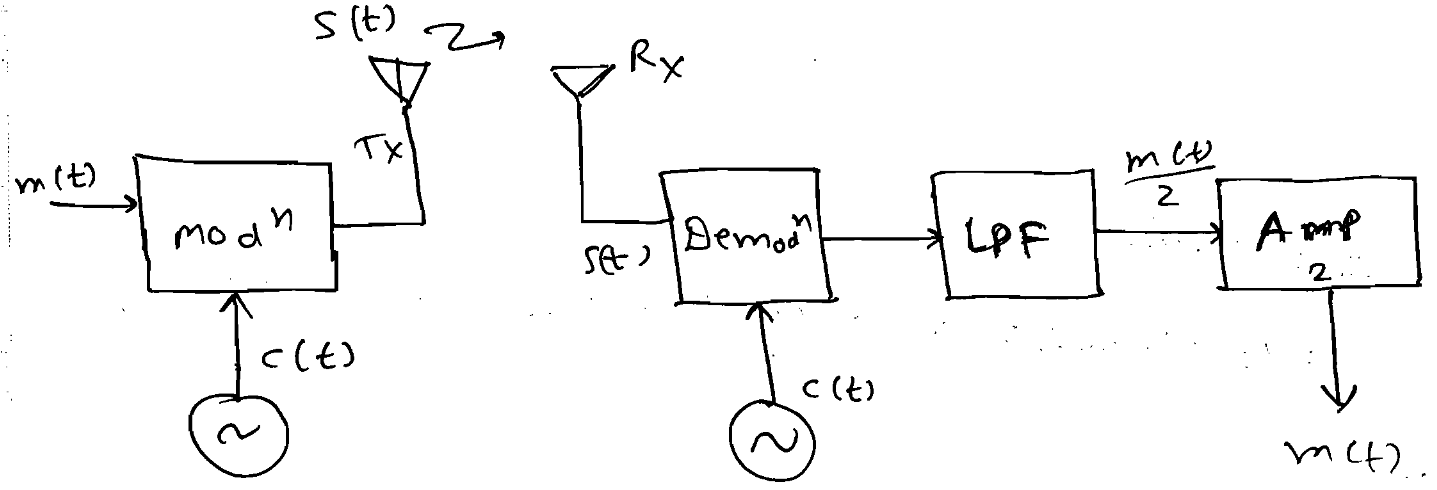
Demodulation $\downarrow$ X cos2πfct

$= m(t). \cos^2 2\pi f_c t$

$= \frac{m(t)}{2} + \frac{m(t) \cos 2\pi f_c 2t}{2}$

$= \frac{m(t)}{2}$

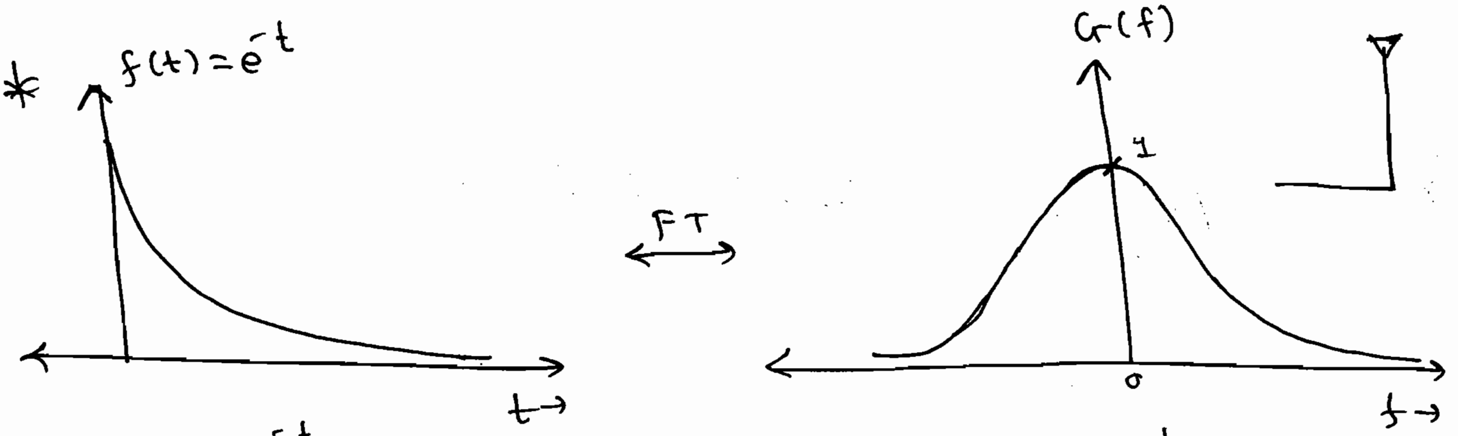




# Analog Communication

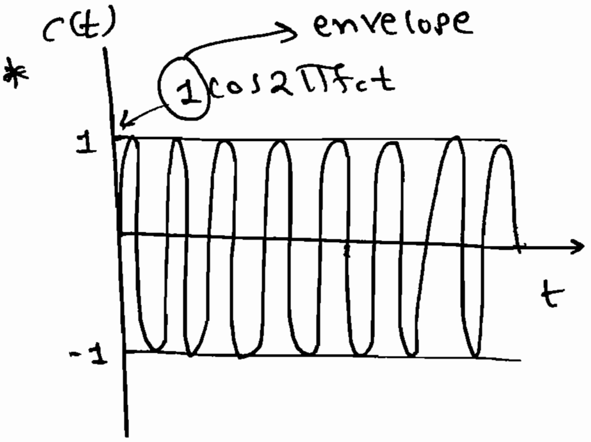
## * Modulation (Time domain):

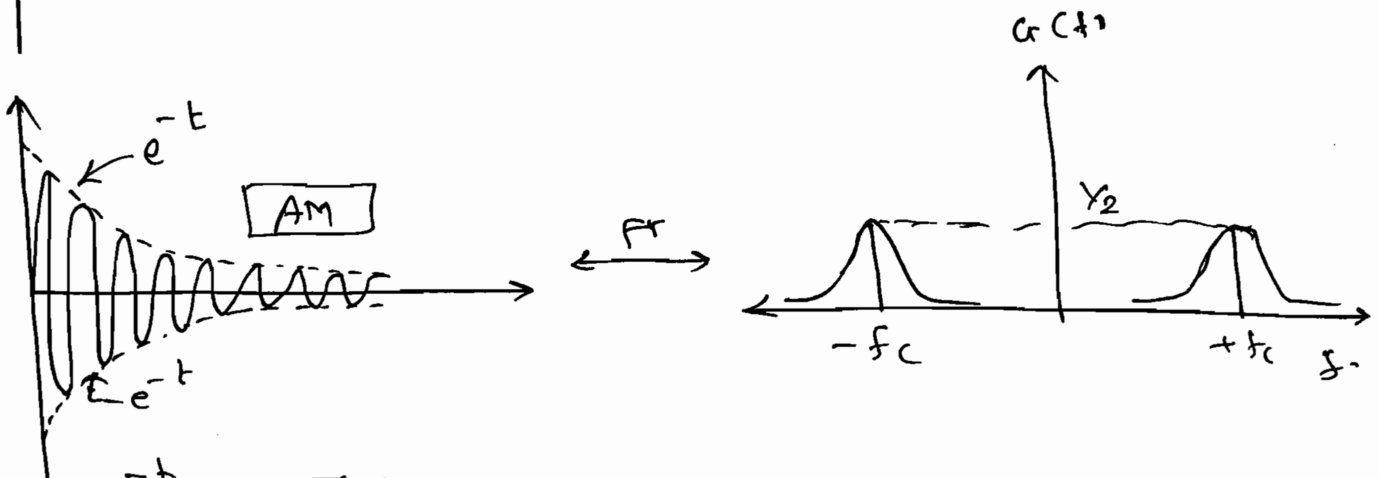
→ Modulation is defined as the process in which the peak amplitude of carrier is varied according to the message signal.



$f(t) = e^{-t}$

$F(f) = \dfrac{1}{1+j2\pi f}$





$e^{-t} \cdot \cos 2\pi f_c t$

⟹ Three <u>parameters</u> of the <u>Carrier</u> which can be varied according to the <u>message signal</u> are <u>Amplitude</u>, <u>Freq.</u> and <u>phase</u>.

→ So, three analog Communication techniques are, $\boxed{\text{AM, FM \& PM.}}$

* H.B

$$\underset{\underset{\text{Am}}{\downarrow}}{\underline{\underline{A_c}}} \cos\left(2\pi \underset{\underset{\text{Fm}}{\downarrow}}{\underline{\underline{f_c}}} t + \underset{\underset{\text{Pm}}{\downarrow}}{\underline{\underline{\phi}}}\right).$$

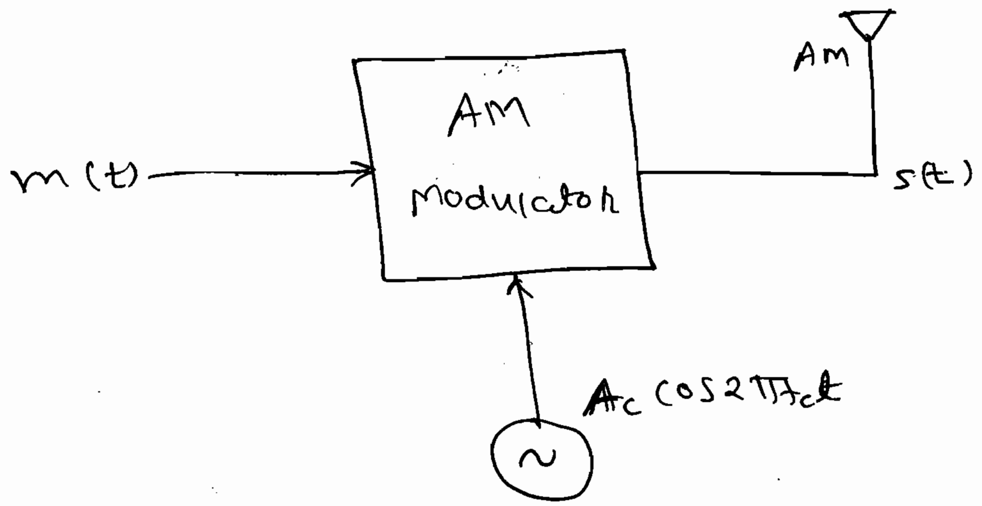
| | | |
|---|---|---|
| ① AM | ① NBFm | ① NBPm |
| ② DSB | ② WBFm | ② WBPm. |
| ③ SSB | | |
| ④ VSB | | |

## (1) Amplitude Modulation (AM) :

→ Amplitude Modulation is define as the Process in which amplitude of the carrier varying according to the message signal.

→ Assume that the message signal (m(t)) and carrier signal is applied to the AM modulator.

⇒



→ The time domain ean of the Am wave is

$$S(t) = A_c \cos 2\pi f_c t + K_a m(t) . A_c \cos 2\pi f_c t$$

$K_a$ = amplitude sensitivity of the modulator.

H.B.

$$\therefore \boxed{S(t) = A_c [1 + K_a m(t)] \cos 2\pi f_c t.}$$

This is the standard form.

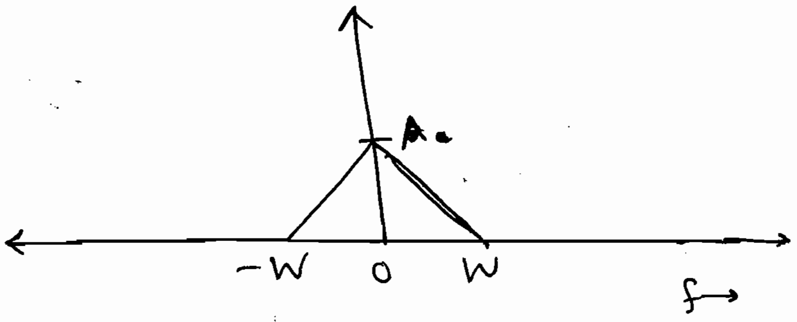
→ The peak amplitude of the carrier modulation is $A_c$ and after modulation $A_c[1 + K_a m(t)]$.

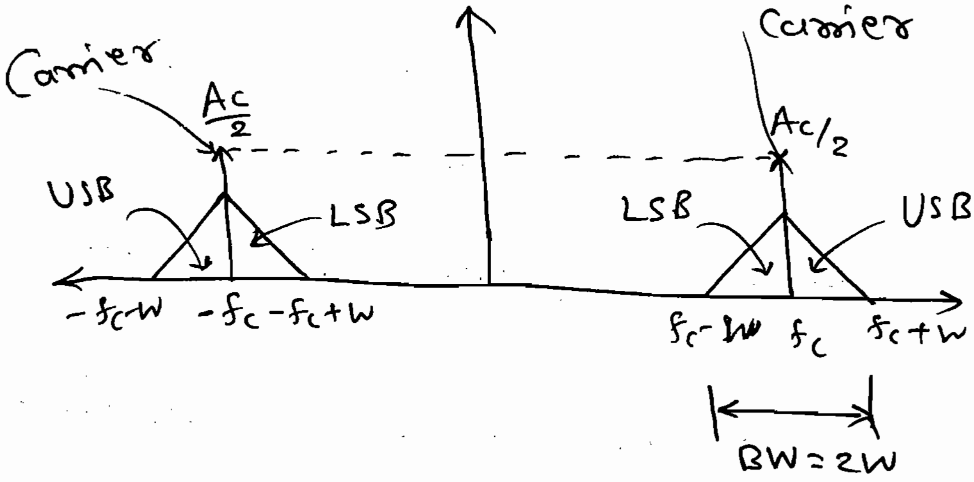
→ Now, Freq. domain eqn is,

H.B

$$\boxed{S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c K_a}{2}\left[M(f-f_c) + M(f+f_c)\right].}$$

→





→ $BW = f_H - f_L$

$BW = f_c + W - (f_c - W).$

$\therefore \boxed{BW = 2W.}$

∴ $\boxed{BW = 2 \times \text{Heignest freq. of } m(t).}$

→ AM spectrum consist of the following freqs.

① Carrier frequency '$f_c$'. ✓

② Upper sideband freq. '$f_c$' to '$f_c+W$'. ✓

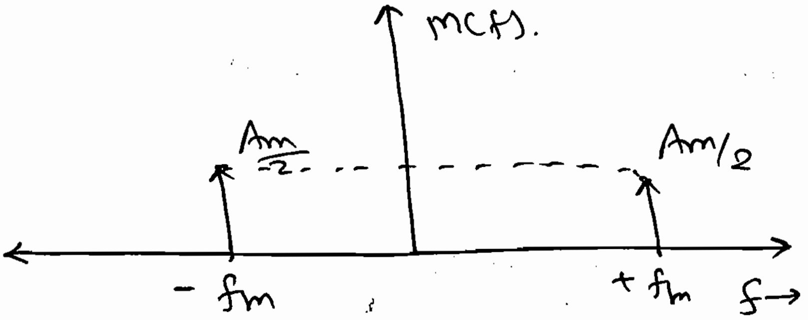
③ Lower sideband freq. '$f_c-W$' to '$f_c$'. ✓

## ✱ Single tone Modulation of AM :-

→ In the singletone Modulation message is the sinusoidal signal.

$$m(t) = A_m \cos 2\pi f_m t$$

$$M(f) = \frac{A_m}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right].$$



⇒ Time domain eqn of the Am signal for singletone modulation is.

H.B.

$$\boxed{S(t) = A_m \cos 2\pi f_c t + A_c \underline{\underline{K_a A_m}} \cos 2\pi f_m t \cdot \cos 2\pi f_c t}$$

$K_a A_m = \mu = m$

= modulation index

= depth of modulation.

⇒ The modulation index concept is required for the demodulation of Am.

→ Practically the modulation index lies betⁿ 0 to 1.

$$\boxed{0 < \mu, m \le 1.}$$

→ When $\mu = 0$ the carrier is not modulated.

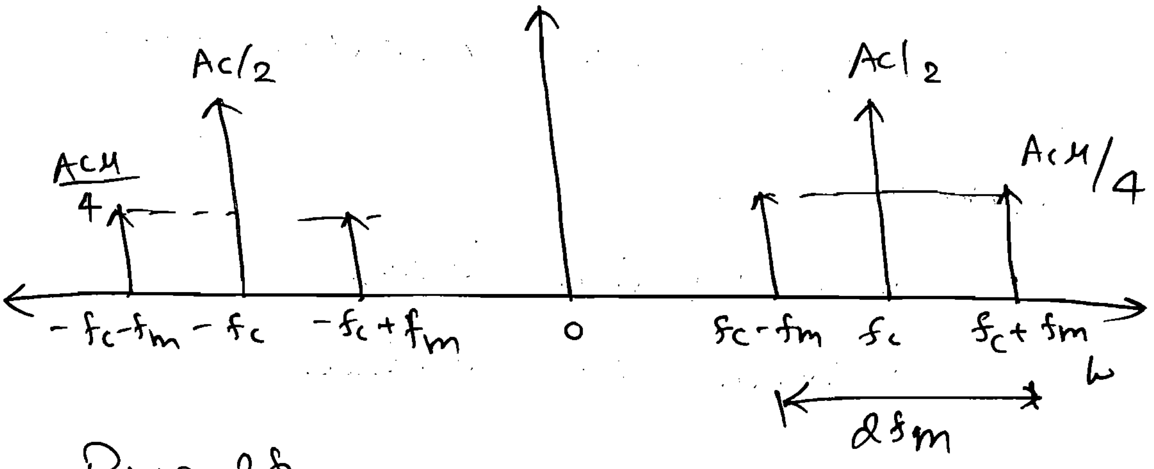
→ When $\mu > 1$ the ~~carrier~~ Am signal is said to be over modulated.

H.B

$$\therefore \boxed{s(t) = A_c\left[1 + \mu\cos 2\pi f_m t\right]\cos 2\pi f_c t.}$$

$$\therefore s(t) = \underset{\text{Carrier}}{A_c\cos 2\pi f_c t} + \underset{\text{USB}}{\frac{A_c\mu}{2}\cos 2\pi(f_c + f_m)t} + \underset{\text{LSB.}}{\frac{A_c\mu}{2}\cos 2\pi(f_c - f_m)t}$$

H.B.



Bw = 2fm.

## * Power Calculation:

→ $P_t = P_c + P_{USB} + P_{LSB}$. ← H.B.

→ $P_c = \frac{V_{rms}^2}{R} = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R}$.

$\therefore P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R}$

H.B. $\therefore$ $\boxed{P_c = \frac{A_c^2}{2R}}$.

R = Antenna Resistance (Ω) it not given then take R = 1Ω.

→ $P_{USB} = \frac{\left(\frac{A_c \mu}{2\sqrt{2}}\right)^2}{R}$

$\therefore P_{USB} = \frac{A_c^2 \mu^2}{8R}$ ← H.B.

$\therefore P_{LSB} = \frac{A_c^2 \mu^2}{8R}$ ← H.B.

$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$.

$\therefore P_t = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]$.

$\therefore$ $\boxed{P_t = P_c\left[1 + \frac{\mu^2}{2}\right].}$ ← H.B.

→ $P_t = \underbrace{P_c}_{\text{Carrier power}} + \underbrace{\frac{P_c \mu^2}{2}}_{\text{Side Band Power}}$.

→ Total Power $\boxed{P_t = P_c + P_{SB}.}$

→ Practically sideBand power is more important than the Carrier power because ~~stggn~~ signal to noise ratio depends only on the SideBand power.

→ To determine the sideband power w.r.t. to the total power modulation efficiency (or) power efficiency is used.

$$\eta = \frac{P_{SB}}{P_t} \leftarrow \underline{H.B.}$$

$$\therefore \eta = \frac{\frac{\cancel{P_c}\mu^2}{2}}{\cancel{P_c}\left(1+\frac{\mu^2}{2}\right)}$$

$$\therefore \boxed{\eta = \frac{\mu^2}{2+\mu^2}.} \leftarrow \underline{H.B.}$$

When $\mu = 1 \Rightarrow \eta = 1/3$ or $33.33\%$

→ $P_{SB} = \frac{1}{3} P_t.$

(or) $\boxed{P_{SB} = 33.33\% \, P_t.}$

→ $P_t = P_c + P_{SB}$.

$\mu = 1 \Rightarrow P_t = \frac{2}{3} P_t + \frac{1}{3} P_t$.

$P_t = 66.67\% P_t + 33.33\% P_t$.

$\mu = 0.707 \Rightarrow \mu = 1/\sqrt{2}$.

$\therefore P_t = 80\% P_t + 20\% P_t$.

→ In AM carrier takes more power than the sideband power. The maximum efficiency possible in the case of single tone modulation is 33.33 %.

→ The maximum efficiency possible in AM is 50 %. ( When a message is a square wave).

Ex-1 Consider an AM signal $s(t) = 20[1 + 0.9 \cos 2\pi 10^4 t] \cos 2\pi 10^6 t$. The signal is radiated into free space using an antenna having a resistance of 5 Ω. Sketch the spectrum and calculate the B.W., power and modulation efficiency.

Sol$^n$: Here, ~~A~~

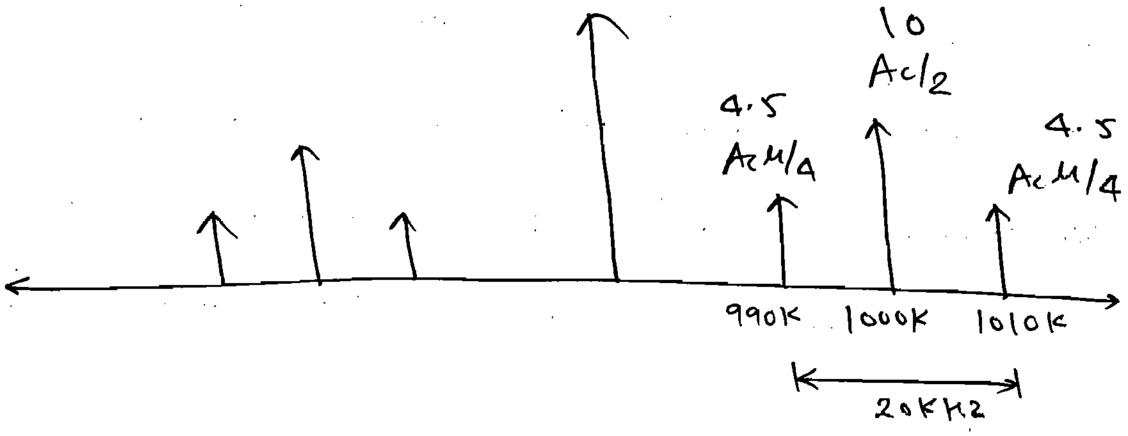
$$s(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$$

$$\therefore \quad s(t) = 20[1 + 0.9 \cos 2\pi 10^4 t] \cos 2\pi 10^6 t.$$

$\therefore \quad \mu = 0.9 \qquad A_c = 20.$

$f_m = 10$ KHz

$f_c = 1000$ KHz.



$\therefore \quad BW = 1010K - 990K$

$$\boxed{BW = 20\text{K Hz}}$$

(OR)

$BW = 2 f_m.$

$= 2 \times 10$

$$\boxed{BW = 20\text{ KHz}}$$

$P_c = \frac{A_c^2}{2R}.$

$\therefore \quad P_c = \frac{400}{2 \times 5}$

$$\boxed{P_c = 40 \text{ watts}}$$

$\therefore \quad P_t = P_c\left[1 + \frac{\mu^2}{2}\right].$

$\therefore \quad P_t = 40\left[1 + \frac{0.81}{2}\right]$

$$\boxed{P_t = 56.2. \text{ watts.}}$$

$P_t = P_c + P_{SB}.$

$\therefore \quad P_{SB} = 56.2 - 40$

$\therefore \quad P_{SB} = 16.2$ watts.

$\therefore \quad P_{USB} = P_{LSB} = 8.1$ watts.

$\therefore \quad \eta = \frac{P_{SB}}{P_t}$ (or) $\eta = \frac{\mu^2}{2+\mu^2}$.

$\therefore \quad \eta = \frac{16.2}{56.2}$ $\therefore \eta = \frac{0.81}{2.81}$

$\boxed{\eta = 28.8\%}$ $\therefore \boxed{\eta = 28.8\%}$

**Ex. 2** Consider an AM signal

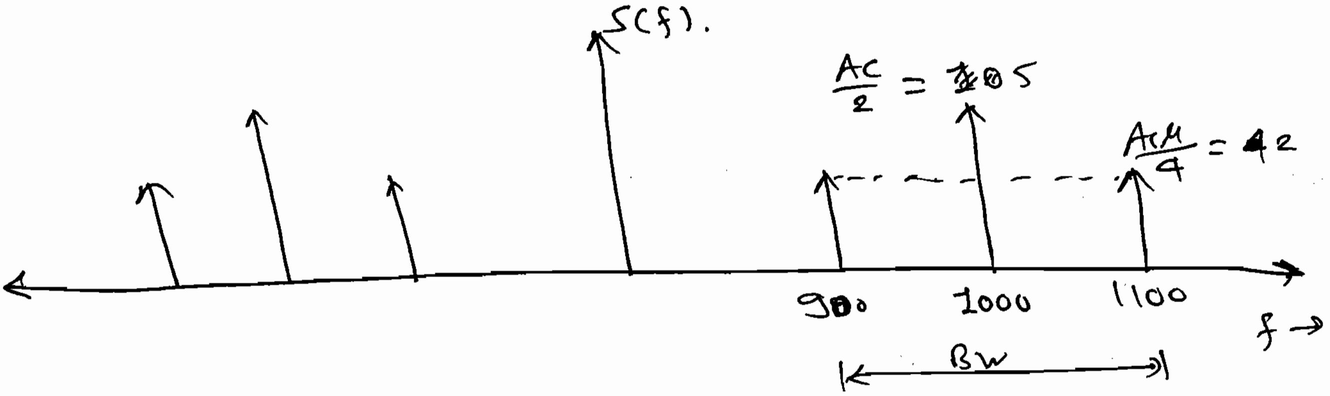
$s(t) = 4\cos 1800\pi t + 10\cos 2000\pi t + 4\cos 2200\pi t.$

Sketch the spectrum and calculate the B.W., power & Modulation efficiency.

**Sol^n:** $s(t) = A_c \cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c+f_m)$

$+ \frac{A_c\mu}{2}\cos 2\pi(f_c - f_m).$

$\rightarrow \quad s(t) = 10\cos 2000\pi t + 4\cos 1800\pi t + 4\cos 2200\pi t.$



→ $BW = 1100 - 900$

$$\boxed{BW = 200\text{Hz}}$$

→ ~~$\frac{A_c\mu}{2}$ [illegible]~~ $\times 0$

$$\boxed{A_c = 10}$$

$$\frac{A_c\mu}{2} = 4$$

$$\therefore \mu = \frac{4 \times 2}{10}$$

$$\boxed{\mu = 0.8}$$

$$P_c = \frac{A_c^2}{2R}$$

$R = 1\Omega$

$$\therefore P_c = \frac{100}{2}$$

$$\boxed{P_c = 50 \text{ watts}}$$

$$\Rightarrow \quad P_t = P_c\left[1 + \frac{\mu^2}{2}\right].$$

$$\therefore P_t = 50\left[1 + \frac{0.64}{2}\right].$$

$$\therefore \boxed{P_t = 66\text{W}}$$

$$\therefore \quad \eta = \frac{\mu^2}{2+\mu^2} = \frac{0.64}{2.64}$$

$$\boxed{\eta = 24.24\ \%.}$$

Ex-3 A Carrier signal $c(t) = 5\cos 2\pi 10^6 t$ is modulated by message signal $m(t) = 4\cos 8\pi \times 10^3 t$. to generate an Am signal. Calculate the B.W. and power.

Ans: $c(t) = 5\cos 2\pi \times 10^6 t.$

$A_c = 5\text{ V}$

$f_c = 1000\text{ KHz}$

$m(t) = 4 \cos 8\pi 10^3 t.$

$\therefore \quad A_m = 4, \quad f_m = 4 \text{ KHz}.$

$\therefore \quad BW = 2 f_m.$

$BW = 2 \times 4 K$

$$\boxed{BW = 8 K \text{ Hz}}$$

$\rightarrow \quad P_c = \frac{A_c^2}{2R} = \frac{25}{2 \times 1} = 12.5 \text{ W}.$

$$\therefore \boxed{P_c = 12.5 \text{ W}.}$$

* when $\mu$ is not given take

H.B

$K_a = \frac{1}{AC} \Rightarrow \boxed{\mu = \frac{A_m}{A_c}}.$

$\therefore \quad \mu = A_m / A_c$

$\therefore \quad \mu = \frac{4}{5} = 0.8.$

$\therefore \quad P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$

$\therefore \quad P_t = 12.5 \left[ 1 + \frac{0.64}{2} \right].$

$$\therefore \boxed{P_t = 16.5 \text{ watts.}}$$

$\eta = \frac{\mu^2}{2 + \mu^2}.$

$= \frac{0.64}{2.64}$

$$\therefore \boxed{\eta = 24.24 \%}$$

Ex-4 A carrier signal $c(t) = 5\cos 2\pi 10^6 t$ is modulated by a message signal $m(t) = \cos 4\pi 10^3 t$ to generate an AM signal with an modulation index of 0.5.

① Calculate B.W. and power.

② Determine the quantity $(P_{SB}/P_c)$

Gate : 2003

Ans:

→ $A_c = 5V$, $f_c = 1000\ kHz$

$A_m = 1V$, $f_m = 2\ kHz$.

$\mu = 0.5$.

$\therefore BW = 2 f_m$.

$\therefore \boxed{BW = 4\ kHz}$

$P_c = \frac{A_c^2}{2R}$

$\therefore P_c = \frac{25}{2}$

$\boxed{P_c = 12.5\ W}$

$\therefore P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]$.

$\therefore P_t = 12.5 \left[ 1 + \frac{0.25}{2} \right]$

$\therefore \boxed{P_t = 14.0625\ \text{watts}}$

$\therefore P_t = P_c + P_{SB}$.

$\therefore P_{SB} = 14.0625 - 12.5$

$\therefore P_{SB} = 1.5625$

$\therefore P_{SB}/P_c = \frac{1.5625}{12.5} = 0.125$.

$\therefore \boxed{P_{SB} = 12.5\ \%\ P_c}$

(OR)

$\longrightarrow$ quantity $\frac{P_{SB}}{P_C} = \frac{P_c \mu^2/2}{P_c}$.

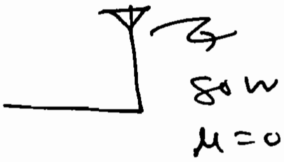
$\therefore \frac{P_{SB}}{P_c} = \frac{\mu^2}{2} = 0.125$.

$\therefore \boxed{\frac{P_{SB}}{P_c} = 12.5 \%}$

[ expressing SB Power w.r.t. Carrier Power].

Ex-5 An Am transmitter radiates 80 watts when the carrier is not modulated. Determine the total power radiated if the carrier is modulated and modulated index is 1.

Ans:



$P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]$.

when carrier is not modulated then $\mu = 0$. H.B

$\therefore P_t = P_c [1 + 0]$.

$\therefore \boxed{P_t = P_c}$

$\therefore P_t = P_c = 80$ Watts.

$\rightarrow$ when $\mu = 1$,

$\therefore P_t = 80 [1 + \frac{1}{2}]$.

$\therefore P_t = 80 \times 1.5 = 120$ W.

$$\therefore \quad P_t = 120W = 80W + 40W.$$

$$\therefore \quad P_t = P_c + (0.5 P_c)$$

$$P_t = P_c + (50\% \text{ of } P_c)$$

50 W → 75W

200 W → 300 W.

**NOTE:** H.B

→ when modulation index is changed from 0 to 1 the power increases by 50%.

$$\rightarrow \quad P_t = P_c\left[1 + \frac{\mu^2}{2}\right].$$

$$\left[\begin{array}{lcl} \mu = 0 & \rightarrow & P_t = P_c \\ \mu = 1 & \rightarrow & P_t = 1.5 P_c. \end{array}\right] \leftarrow \text{H.B}$$

Ex - 6 An AM transmitter radiates 50 watts when the carrier is modulated by sinusoidal signal and the modulation index 0.707.

① Determine the modulation efficiency, carrier power & sideband power.

② Determine the peak amplitude of the carrier before modulation and after modulation.

⟹  $P_t = 50W$ ; $\mu = 0.707$.

$$\therefore \eta = \frac{\mu^2}{2+\mu^2}.$$

$$\therefore \eta = \frac{1/2}{2+1/2} = 1/5.$$

$$\therefore \boxed{\eta = 20\%.} \qquad \eta = \frac{P_{SB}}{P_t}.$$

$$\therefore P_c = 80\% \cdot P_t$$

$$P_{SB} = 20\% \cdot P_t$$

$$\therefore P_c = 0.8 \times 50 = 40W.$$

$$P_t = 0.2 \times 50 = 10W.$$

→ Peak Amp. before modulation = $A_c$.

$$\therefore P_c = \frac{A_c^2}{2\times 1} = A_c^2.$$

$$\therefore A_c^2 = 80.$$

$$\therefore \boxed{A_c = 8.944\ V.}$$

→ Peak Amp. after modulation $= A_c[1+\mu\cos 2\pi f_m t]$

↙ HB

$$\therefore \boxed{\begin{array}{ll} V_{max} = A_c[1+\mu]. & (\theta = 0) \\ \hline V_{min} = A_c[1-\mu]. & (\theta = \pi]. \end{array}}$$

$$\Rightarrow V_{max} = 8.944\,[1+0.707].$$

$$\boxed{V_{max} = 15.27\ \cancel{W}.}$$

$$V_{min} = 8.944\,[1-0.707] \Rightarrow \boxed{V_{min} = 6.323V}$$

$$\Rightarrow \quad \frac{V_{max}}{V_{min}} = \frac{1+\mu}{1-\mu}.$$

$$\therefore V_{max}[1-\mu] = [1+\mu]V_{min}.$$

$$\therefore \boxed{\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}} \leftarrow \underline{\underline{H.B}}$$

↗ Formmula to determine $\mu$ practically.

$$\therefore V_{max} + V_{min} = 2A_c.$$

$$\therefore \boxed{A_c = \frac{V_{max} + V_{min}}{2}}$$

Ex-1 An amplitude of an Am signal varies from 5V to 15V. Determine the modulation index, carrier power, sidebund power & total power.

Ans: $V_{min} = 5V$, $V_{max} = 15V$.

$$\therefore \mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}. \qquad P_c = \frac{A_c^2}{2R}.$$

$$\therefore \mu = \frac{15-5}{20} \qquad A_c = \frac{V_{max} + V_{min}}{2}$$

$$\mu = \frac{1}{2}. \qquad A_c = \frac{20}{2}$$

$$\therefore \boxed{\mu = 0.5} \qquad \boxed{A_c = 10V}$$

$$\therefore P_c = \frac{100}{2}$$

$$\boxed{P_c = 50\ W}$$

$$\therefore P_{SB} = \frac{P_c \mu^2}{2}.$$

$$\therefore P_{SB} = \frac{50 \times 0.25}{2}$$

$$\therefore \boxed{P_{SB} = 6.25\ W}$$

$$\therefore P_t = P_c + P_{SB}$$

$$\therefore \boxed{P_t = 58.5\ W}$$

## * Antenna Current :-

→ Consider an antenna having a resistance of $R\ \Omega$ and '$I_t$' is the antenna current when AM signal is radiated into free space.

$$P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore I_t^2 . R = I_c^2 R \left[ 1 + \frac{\mu^2}{2} \right]$$

$$\therefore I_t^2 = I_c^2 \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore \boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}.} \leftarrow \underline{\underline{H.B.}}$$

→ $I_c$ is the antenna current before modulation & $I_t$ is the antenna current after modulation.

**Ex-2** The antenna current of an AM transmitter is 8A before modulation and 8.5 A after modulation. calculate the modulation index and modulation efficiency.

**Ans:** $I_c = 8A$, $I_t = 8.5A$.

$$\therefore \quad I_t^2 = I_c^2 \left[ \underrightarrow{1+\frac{\mu^2}{2}} \right].$$

$$\therefore \quad (8.5)^2 = (8)^2 \left[ 1+\frac{\mu^2}{2} \right].$$

$$\therefore \quad 1.128 = 1+\frac{\mu^2}{2}.$$

$$\therefore \quad \boxed{\mu = 0.5}.$$

$$\therefore \quad \eta = \frac{0.25}{2.25}.$$

$$\therefore \quad \boxed{\eta = 11.11\%.}$$

# * Multitone Modulation :-

=> Generalized Am signal is,

$$\rightarrow S(t) = A_c \cos 2\pi f_c t + A_c K_a \boxed{m(t)} \cos 2\pi f_c t.$$

=> m(t) is whatever sin or cosine => no change in B.W. & power.

Let, $m(t) = A_{m_1} \cos 2\pi f_{m_1} t \underset{=}{\overset{+}{-}} A_{m_2} \cos 2\pi f_{m_2} t$

$f_{m_2} > f_{m_1}$ with -ve sign no change in B.W. & power.

$$\Rightarrow S(t) = A_c \cos 2\pi f_c t + A_c K_a \left[ A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t \right] \cos 2\pi f_c t.$$

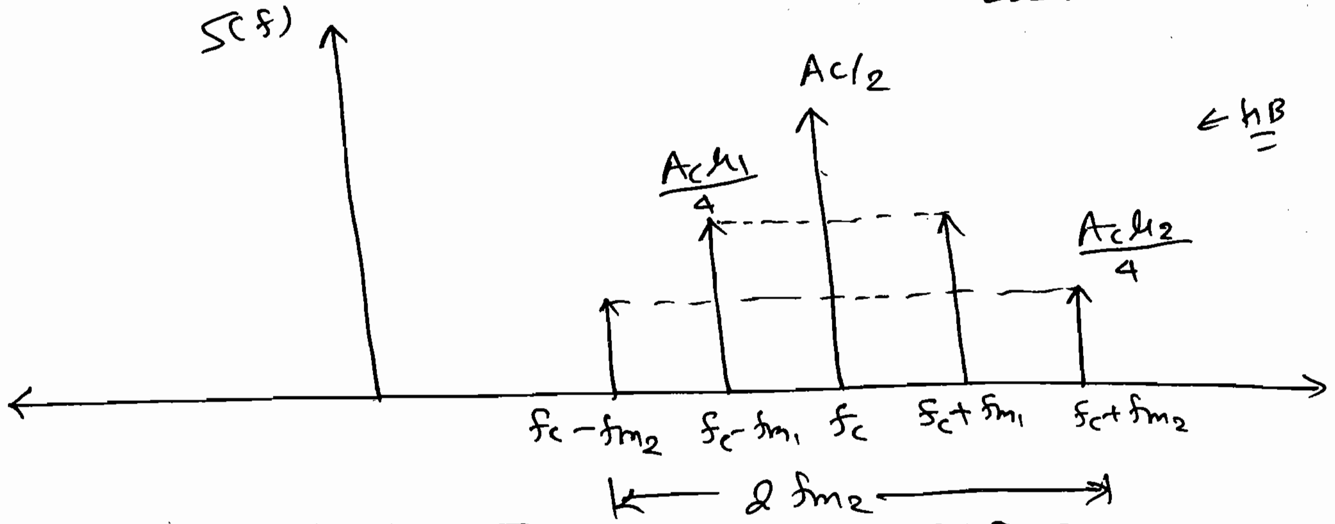
H.B

$$\therefore S(t) = \underset{\text{carrier}}{A_c \cos 2\pi f_c t} + \frac{A_c \mu_1}{2} \underset{\text{USB}}{\cos 2\pi (f_c + f_{m_1}) t}$$

H.B.

$$+ \frac{A_c \mu_1}{2} \underset{\text{LSB}}{\cos 2\pi (f_c - f_{m_1}) t} + \frac{A_c \mu_2}{2} \underset{\text{USB}}{\cos 2\pi (f_c + f_{m_2}) t}$$

$$+ \frac{A_c \mu_2}{2} \underset{\text{LSB.}}{\cos 2\pi (f_c - f_{m_2}) t}$$



B.W = 2 × Maximum Frequency.

$$P_t = P_c + P_{USB} + P_{LSB}.$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R}.$$

$$\therefore P_t = \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right].$$

$$\therefore \boxed{P_t = P_c\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right]}$$ ✓

let, $\mu_t^2 = \mu_1^2 + \mu_2^2 + \cdots$

$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \cdots}$ = Total modulation index.

$$\therefore \boxed{P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right].}$$ ← H.B.

$$\therefore P_t = P_c + \frac{P_c \mu_t^2}{2}.$$

$$\therefore \boxed{\eta = \frac{\mu_t^2}{2 + \mu_t^2}.}$$ ← H.B.

∴ **Note:**

⇒ The B.W. & Power is depends only on the magnitude spectrum. (not on phase spectrum).

Ex-1 A Carrier signal $c(t) = 20\cos 2\pi 10^6 t$ is modulated by a message signal having 3 frequencies 5 KHz, 10 KHz & 20 KHz. The Corresponding Modulation indices are 0.4, 0.5, 0.6. Sketch the and calculate the B.W., Power & Modulation efficiency.

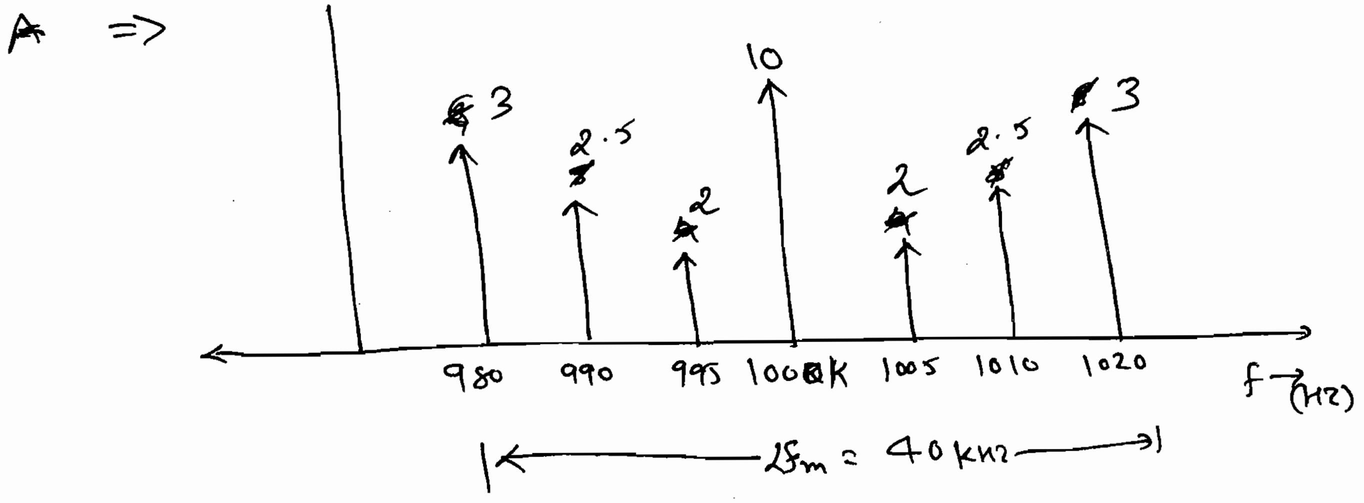
Soln:

$f_c = 1000$ KHz  $\mu_1 = 0.4$  $A_c = 20$.

$f_{m_1} = 5$ KHz  $\mu_2 = 0.5$.

$f_{m_2} = 10$ KHz  $\mu_3 = 0.6$.

$f_{m_3} = 20$ KHz



→ $B_w = 2 f_{m_3}$

$= 2 \times 20$

$B_w = 40$ KHz.

$\therefore \quad P_c = \frac{A_c^2}{2R}$.

$\therefore \quad P_c = \frac{400}{2(1)}$

$\therefore \quad \boxed{P_c = 200 \text{ watts}}$

$\therefore \quad P_t = P_c \left[ 1 + \frac{\mu_t^2}{2} \right]$.

$\therefore \quad \mu_t^2 = \mu_1^2 + \mu_2^2 + \mu_3^2$.

$\mu_t^2 = 0.16 + 0.25 + 0.36$

$\mu_t^2 = 0.77$.

$\therefore \quad P_t = 200 \left[ 1 + \frac{0.77}{2} \right]$

$\therefore \boxed{P_t = 277 W}$

$\therefore \quad \eta = \frac{\mu_t^2}{2 + \mu_t^2} = \frac{0.77}{2.77}$.

$\therefore \boxed{\eta = 27.8}$

Ex. 2 consider an Am signal
$s(t) = \left[10 + 4 \sin 2\pi 10^3 t - 8 \cos 2\pi 10^4 t\right] \times \cos 2\pi f_c t$.
Determine the B.W. & power.

Solⁿ:

$s(t) = 10\left[1 + 0.4 \sin 2\pi 10^3 t - 0.8 \cos 2\pi 10^4 t\right] \times \cos 2\pi f_c t$.

$\therefore \quad A_c = 10, \quad \mu_1 = 0.4, \quad \mu_2 = -0.8$.

$f_{m_1} = 1 \text{ kHz}, \quad f_{m_2} = 10 \text{ kHz}$.

∴ $BW = 2f_{m2}$

$\boxed{BW = 20\ KHz}$

$\mu_t^2 = 0.16 + 0.64$

$\mu_t^2 = 0.80$

→ $P_c = \frac{A_c^2}{2R}$

$P_c = \frac{100}{2}$

∴ $\boxed{P_c = 50\ W}$

$\boxed{\mu = |K_a A_m|}$

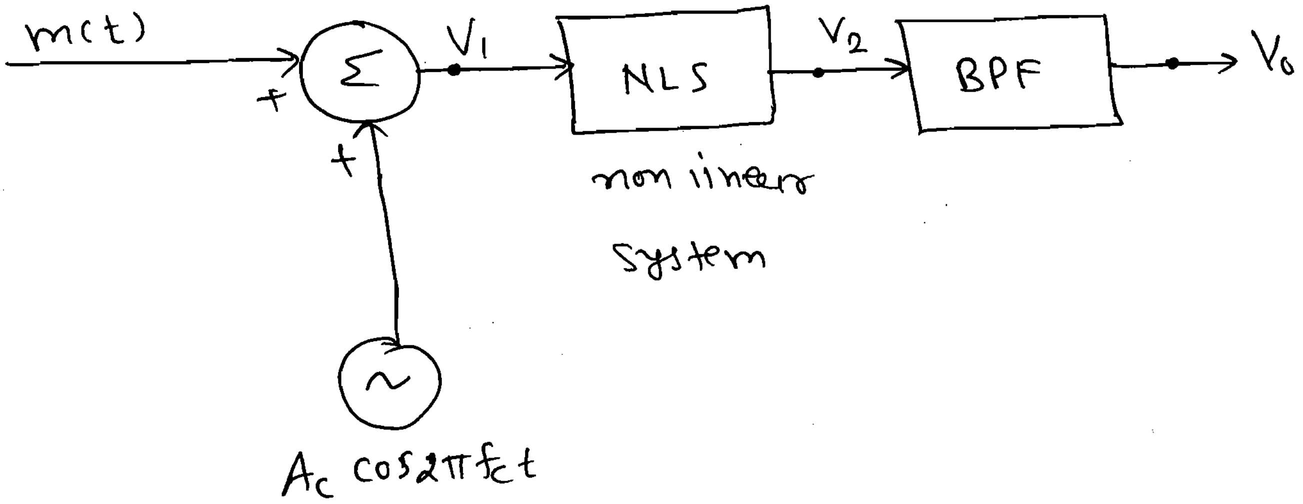
∴ $P_t = 50\left[1 + \frac{0.80}{2}\right]$

∴ $\boxed{P_t = 70W.}$

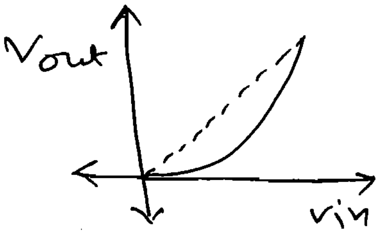
## * Generation of AM Signal :-

### ⊙ Square Law Modulator:

↙ H.B.



→ The generalized characteristic of NLS:



$\boxed{V_{out} = aV_{in} + bV_{in}^2}$ ↙ H.B.

$$\Rightarrow \quad V_1 = [m(t) + A_c \cos 2\pi f_c t].$$

$$\therefore \quad V_2 = aV_1 + bV_1^2.$$

$$\therefore \quad V_2 = am(t) + aA_c \cos 2\pi f_c t$$

$$+ bm^2(t) + 2bA_c m(t) . \cos 2\pi f_c t$$

$$+ bA_c^2 \cos^2 2\pi f_c t.$$

→ The bandpass Filter is used to select the freq. Component of Am signal.

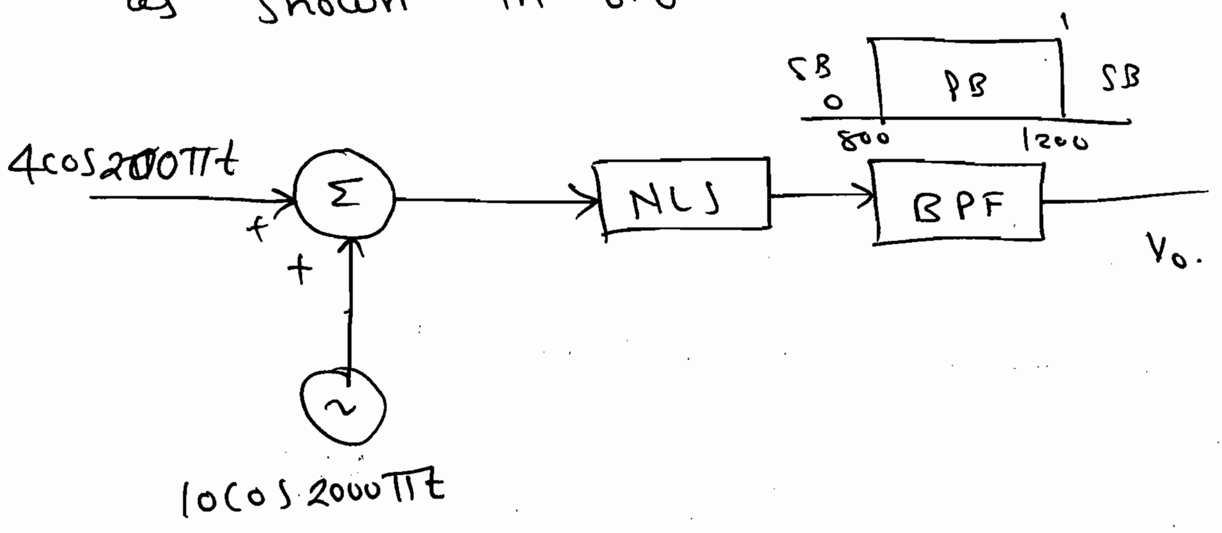
⇒ Output of bandpass filter.

$$\therefore \quad V_0 = aA_c \cos 2\pi f_c t + 2bA_c m(t) . \cos 2\pi f_c t$$

$$\therefore \boxed{V_0 = aA_c \left[ 1 + \left(\frac{2b}{a}\right) m(t) \right] \cos 2\pi f_c t.}$$

$$K_a = \frac{2b}{a}.$$

Ex-1 Consider a square law modulator as shown in fig.



The BPF is ideal filter having passband from 800 Hz to 1200 Hz. Det. modulation index, B.W. & power of Am signal.

$V_2 = V_1 + 0.1V_1^2$

Ans:

$V_1 = 4\cos 200\pi t + 10\cos 2000\pi t$.

∴ $V_2 = V_1 + 0.1V_1^2$.

∴ $V_2 = [4\cos 200\pi t + 10\cos 2000\pi t] + 0.1[4\cos 200\pi t + 10\cos 2000\pi t]^2$.

∴ $V_2 = \underset{\times}{4\cos 200\pi t} + 10\underset{\checkmark}{\cos 2000\pi t} + 0.16\cos^2 200\pi t \times$
$+ \underset{\checkmark}{8\cos 200\pi t \cdot \cos 2000\pi t} - \underset{\times}{1\cdot\cos^2 2060\pi t}$.

∴ $V_2 = 10\cos 2000\pi t + 8\cos 200\pi t \cdot \cos 2000t\pi$.

$V_2 = 10[1 + 0.8\cos 200\pi t]\cos 2000\pi t$.

∴ $V_2 = A_c[1 + \mu\cos 2\pi f_m t]\cos 2\pi f_c t$.

∴ $\boxed{A_c = 10V}$ $f_m = 100$ Hz

∴ $\mu = 0.8$ $f_c = 1000$ Hz.

∴ $B_W = 2\times100$

∴ $\boxed{BW = 200\text{ Hz}}$

$P_c = \frac{A_c^2}{2R}$

$P_c = \frac{100}{2} = 50$ watts

$P_t = P_c[1 + \frac{\mu^2}{2}]$.

∴ $P_t = 50[1 + \frac{0.64}{2}]$.

$\boxed{P_t = 66\text{ W}}$

## * Crute point of view :!

↙ H.B.

$\mu = K_a A_m$.

$\therefore \boxed{\mu = (2b/a)\, A_m.} = 0.2 \times 4 = 0.8.$

$\therefore \boxed{K_a = 2b/a.}$

$\therefore K_a = 2 \times 0.1 = 0.2$

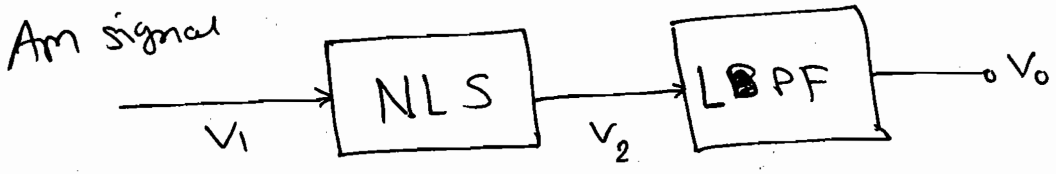
## * Demodulation of Am signal:

① Square law demodulator

② Envelope detector

③ Synchronous detector.

## ① Square Law demodulator:



$$V_1 = A_c \cos 2\pi f_c t + K_a m(t) A_c \cos 2\pi f_c t.$$

$$\therefore V_2 = a v_1 + b v_1^2$$

$$\therefore V_2 = a A_c \cancel{\cos 2\pi f_c t} + a K_a m(t) A_c \cancel{\cos 2\pi f_c t}$$

$$+ b A_c^2 \cancel{\cos^2} 2\pi f_c t + 2b A_c^2 K_a m(t).\ \cos^2 2\pi f_c t$$

$$\Downarrow$$

$$\frac{m(t)}{2} + \cancel{\frac{m(t) \cos 2\pi (2f_c)t}{2}}$$

$$+ b K_a^2 m^2(t) A_c^2 \cos^2 2\pi f_c t.$$

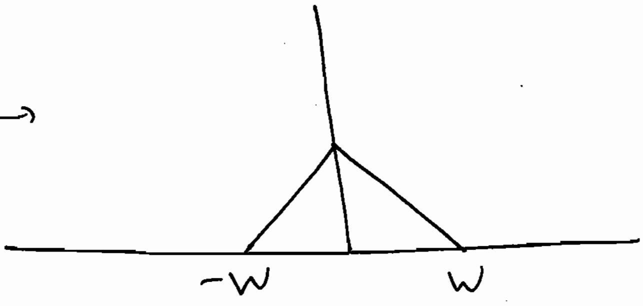
$$\Downarrow$$

$$\frac{m^2(t)}{2} + \cancel{\frac{m^2(t).\ \cos 2\pi f_c (2) t}{2}}$$

$$\therefore V_o = bA_c^2 m(t) K_a \cos . + \frac{bK_a^2 m^2(t)}{2} A_c^2 .$$

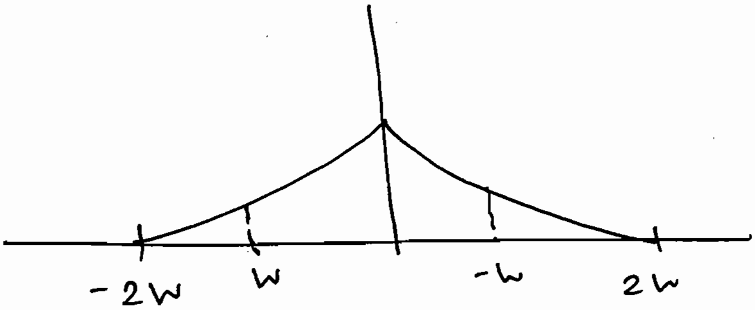
$$V_o = \underbrace{bA_c^2 k_a[m(t)]}_{\text{signal}} + \underbrace{\frac{bk_a^2 A_c^2}{2} [m^2(t)]}_{\text{noise}} .$$

$m(t) \longleftrightarrow$



$m^2(t) \longleftrightarrow$

$= M(f) * M(f).$



⇒ The output of demodulator consist of $m(t)$ and $m^2(t)$.

⇒ $m^2(t)$ is also having significant low frequencies so it is not eliminated.

$$\rightarrow \frac{\text{Signal Component}}{\text{Noise Component}} = \frac{2}{K_a m(t)} .$$

$$\therefore \boxed{\frac{S}{N} = \frac{2}{K_a A_m \cos 2\pi f_m t}} \leftarrow \underline{\text{H.B}}$$

$$\therefore \left(\frac{S}{N}\right)_{\min} = \frac{2}{K_a A_m}$$

$$= 2/\mu .$$

$$\left(\frac{S}{N}\right)_{min} = \frac{2}{\mu} = 0.2 \; ; \; \mu = 10$$

$$= 20 \; ; \; \mu = 0.1$$

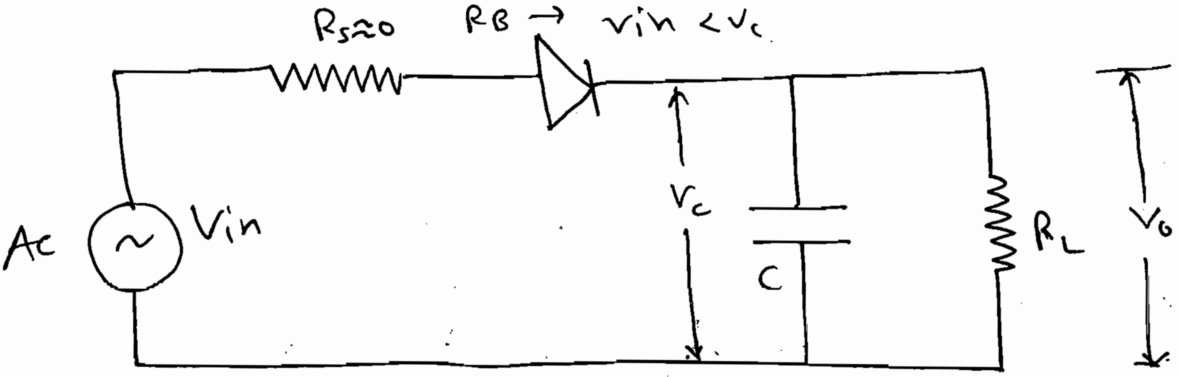
$$= 1 \; ; \; \mu = 2.$$

( $\mu$ should be $< 2$ in this case).

## ② Envelope Detector:

FB → $V_{in} > V_c$

RB → $V_{in} < V_c$

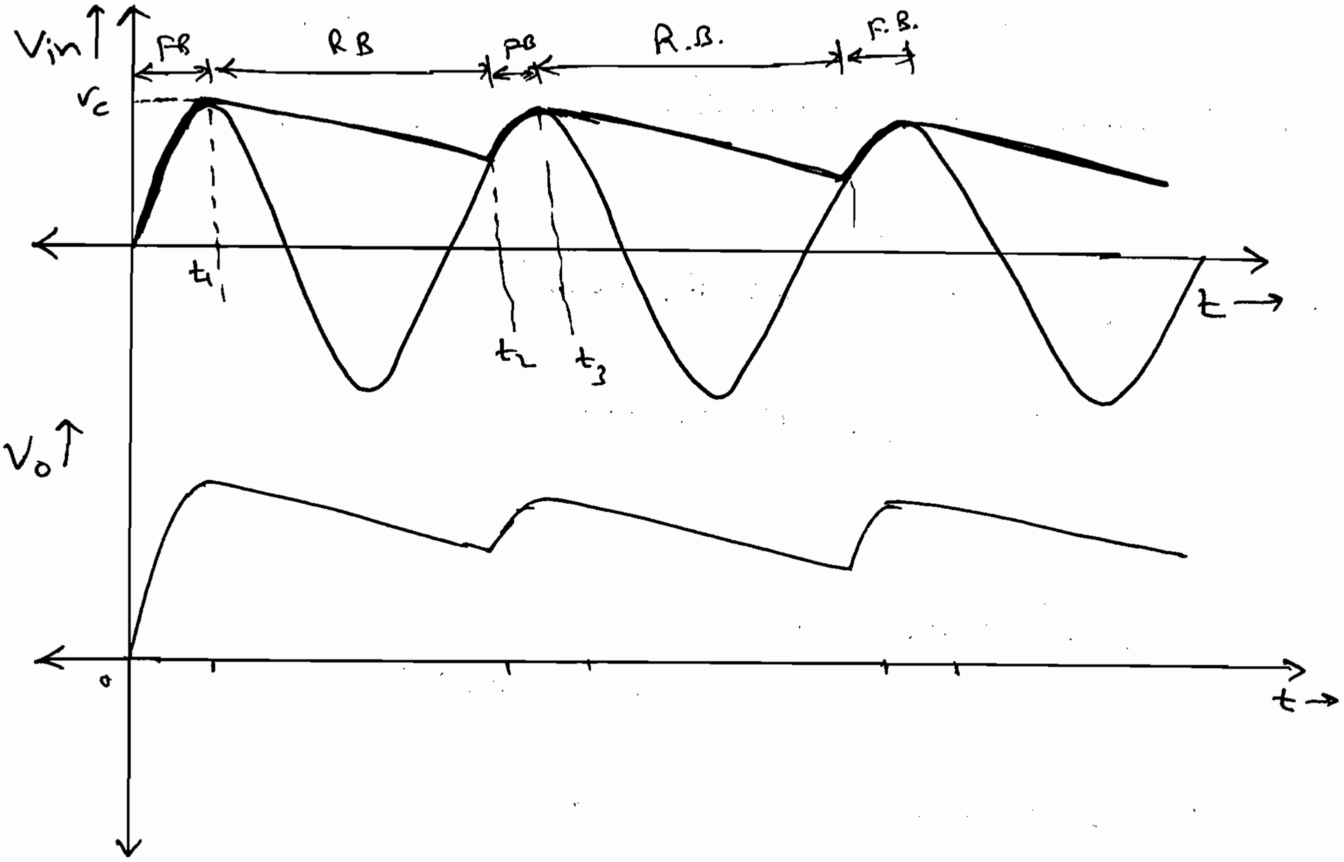


⇒ Hardware implementation is very simple. ($\mu \leq 1$).

→ In AM signal the peak amplitude of the carrier which is also called as envelope is varied according to the message signal. So, the envelope of the Am signal represent the message signal. Envelope detector is used to track the peak amplitude of an Am signal.

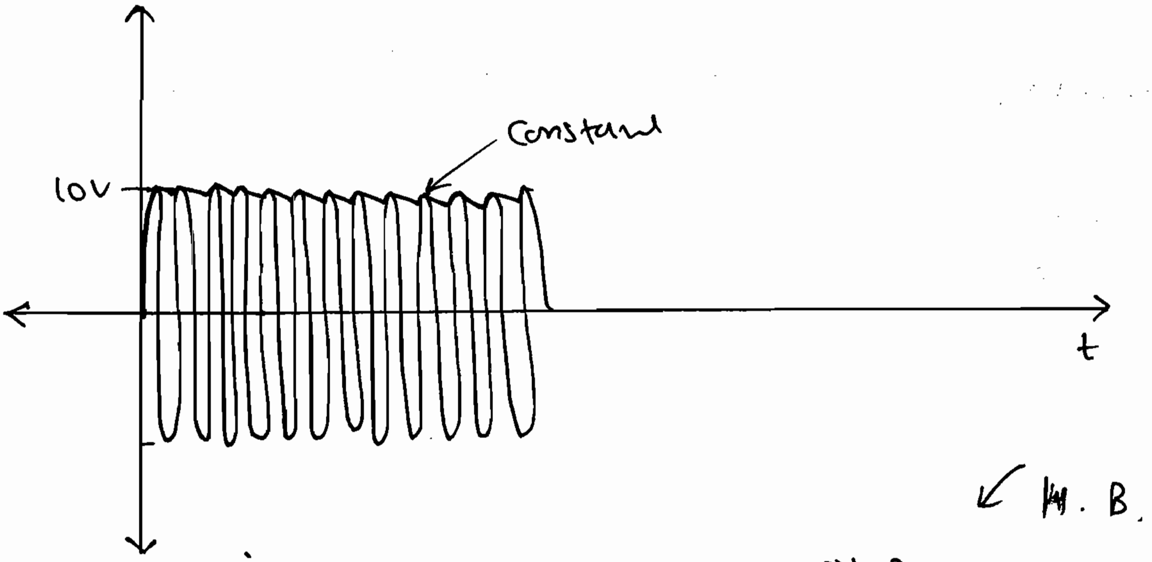
⇒ $R_sC$ → very low (charging time const^n)

$R_LC$ → very high (Discharging time con^n)



⇒ Am is very high freq. signal means time period is very very less. means Vo seen like as follow:

eg. if Am is $10\cos 2\pi f_c t$ then Vo is ~~[illegible]~~ 10V const.



$\boxed{\text{IMP}}$

| i/p | o/p |
|---|---|
| → $A_c[1+k_a m(t)]\cos 2\pi f_c t$ | $V_o = A_c[1+k_a m(t)]$. |
| → $A_c\cos 2\pi f_c t$ | $V_o = A_c$ |
| → $e^{-t}\cos 2\pi f_c t$ | $V_o = e^{-t}$. |

i/p                                                o/p

→ $A_c \cos 2\pi f_c t + B \sin 2\pi f_c t$          $V_0 = \sqrt{A_c^2 + B^2}$ . ✓

$$\Rightarrow \quad V_0 = A_c [1 + K_a m(t)].$$

$$V_0 = \underbrace{A_c}_{\text{DC}} + A_c K_a m(t)$$

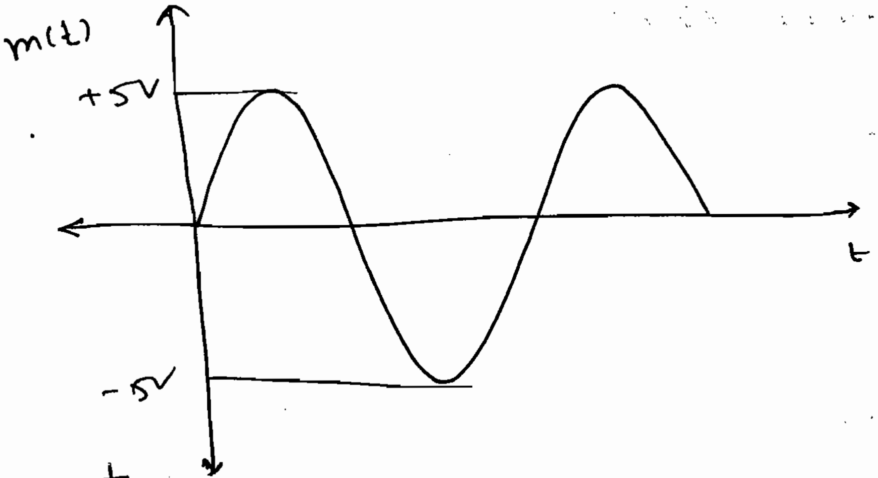
DC can be eliminated by filter.

$$V_0 = A_c K_a m(t) \qquad \text{if } \boxed{K_a = 1/A_c.}$$

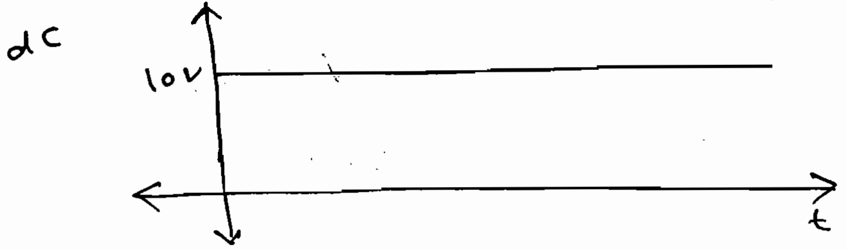
$$\therefore \boxed{V_0 = m(t).}$$

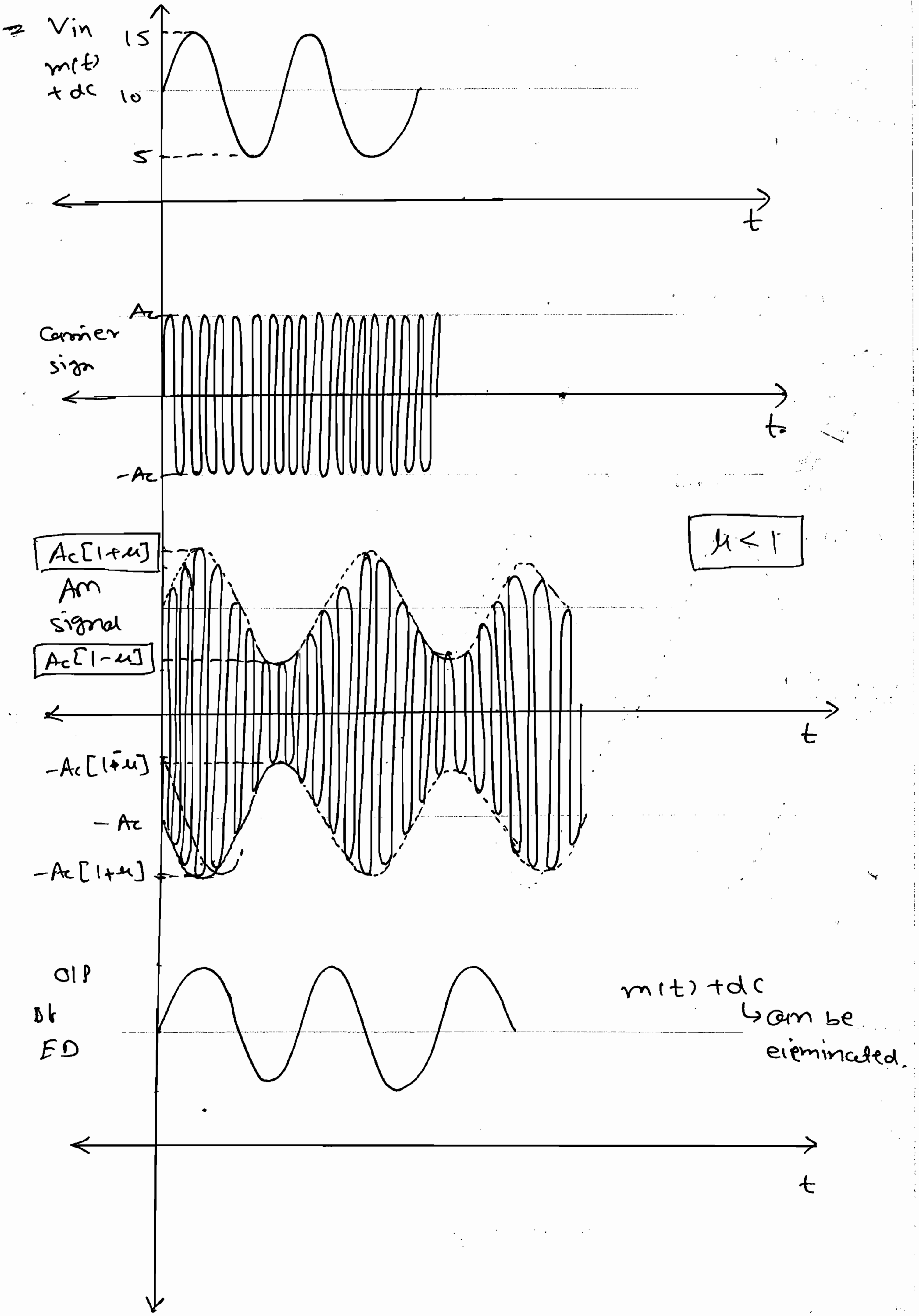
## * Limitation of envelope Detector:

→ Consider a sine signal whose amplitude Varies from +5 to -5. A dc Component of 10V is added to the message signal before multiplying with Carrier.



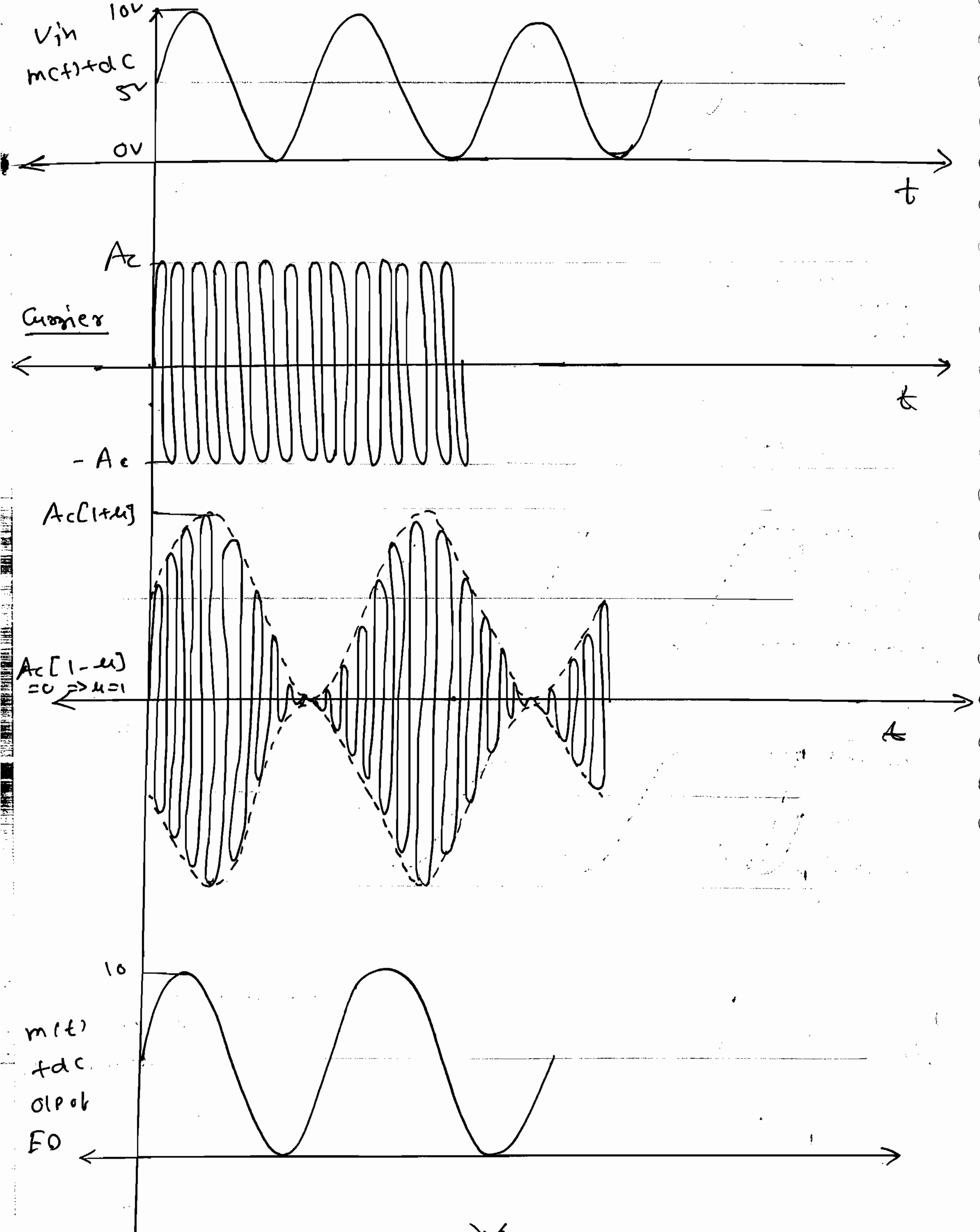
+





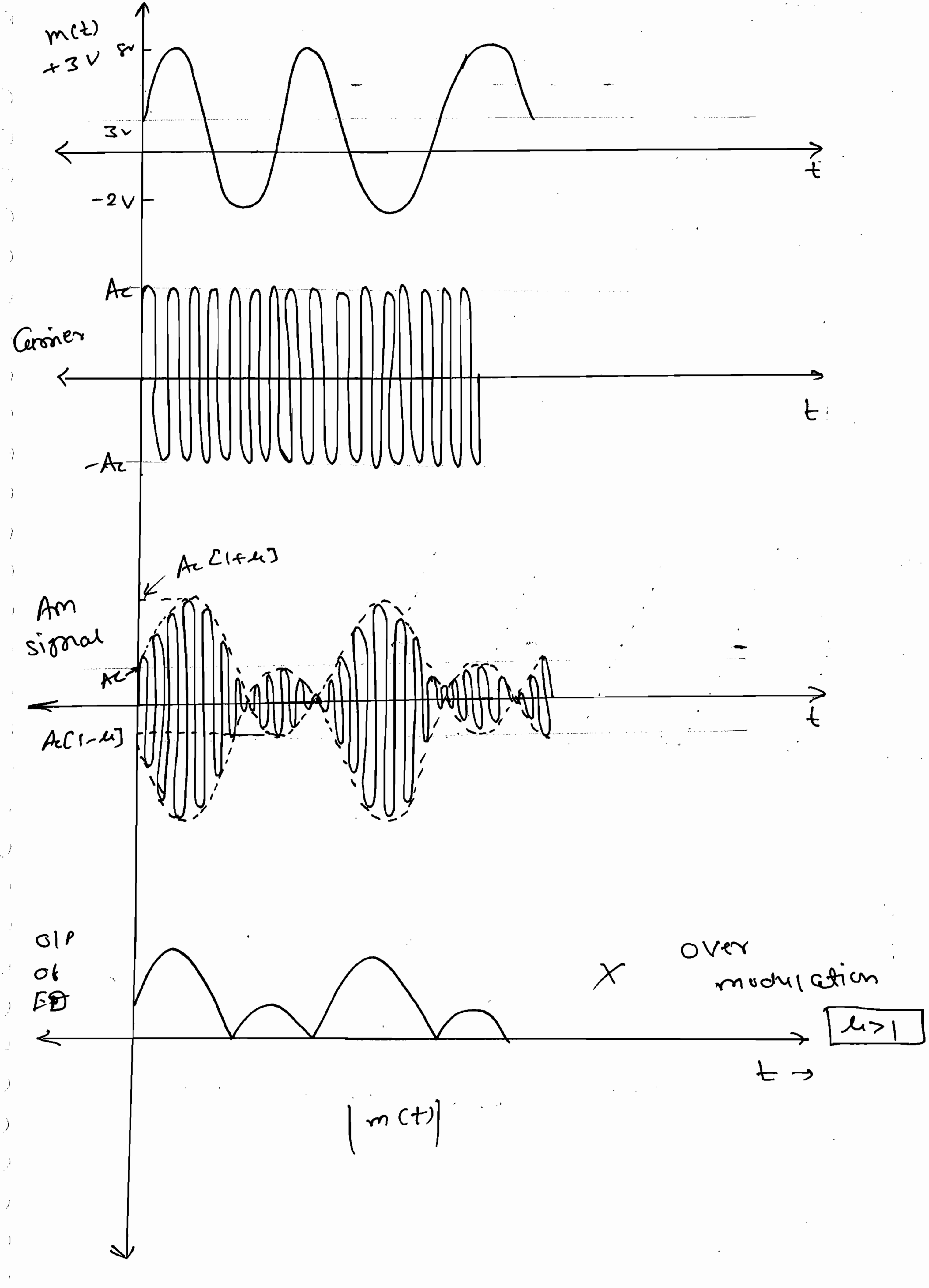
m(t) +dc
↳ can be eliminated.

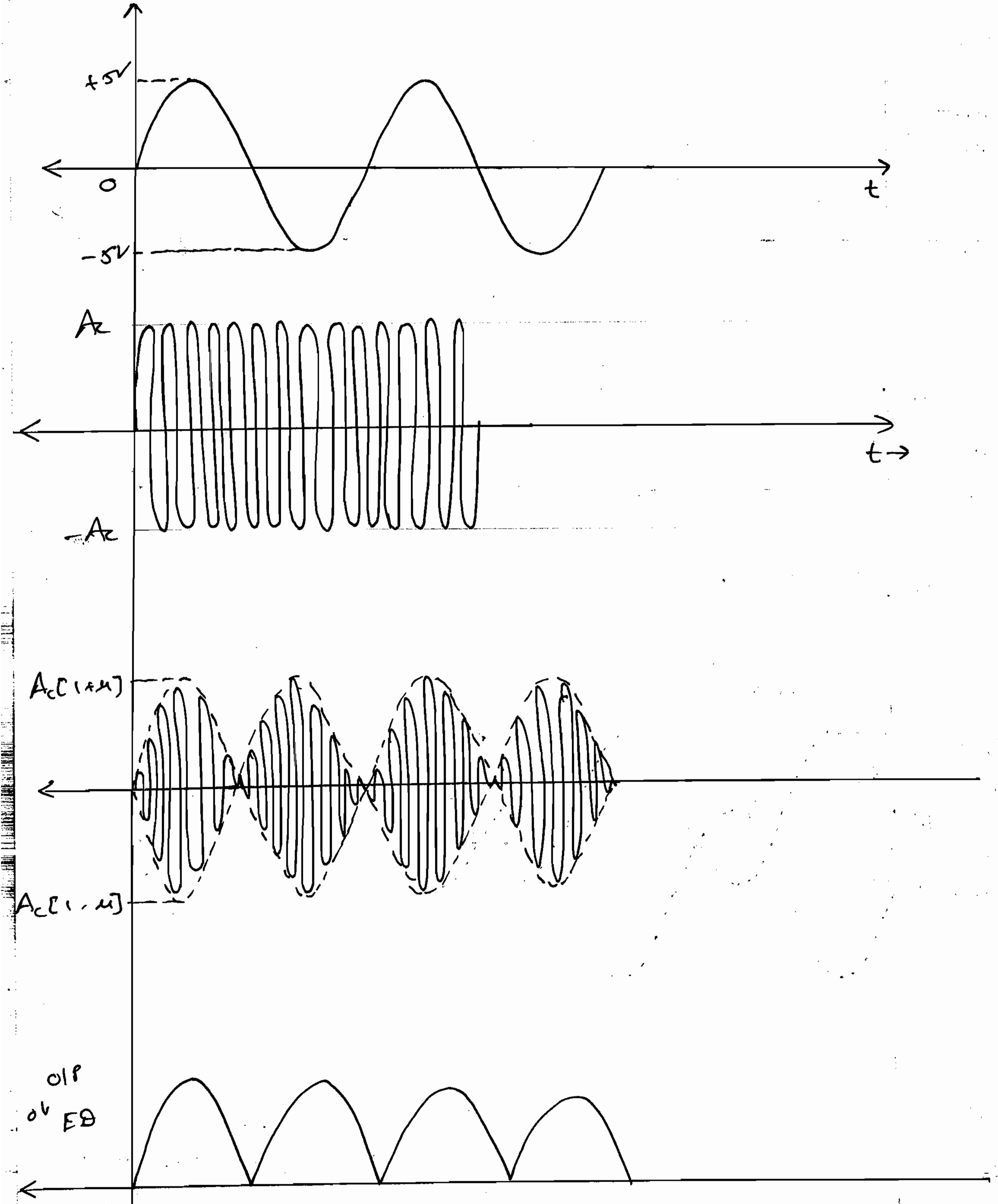
* $\boxed{\mu=1}$



m(t) + ~~dc~~

* $\boxed{\mu > 1}$


m(t)
+3V
8v
3v
-2V
t
Ac
Carrier
-Ac
t
Ac[1+μ]
Am signal
Ac
Ac[1-μ]
t
O/P of ED
X
over modulation
μ>1
t →
|m(t)|



|m(t)|

It is not an AM signal.

① $\mu \le 1 \longrightarrow$ $V_o = \overset{O/P}{A_c [1 + K_a m(t)]}$. ↙ H.B

② $\mu > 1 \longrightarrow$ $V_o = \left| A_c [1 + k_a m(t)] \right|$

③ $\rightarrow [dc + m(t)] \cos 2\pi f_c t$

$\rightarrow dc \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$.

③ A dc component is added to the message signal & multiplied with carrier to generate the Am signal but the demodulation is possible only when $\underline{\underline{m(t) + dc}}$ is positive.

④ $K_a$ is used in analysis to normalized the peak amplitude of the message signal so that the modulation index is $\le 1$.

$\mu = K_a A_m$.

When, $K_a = 1$ there is possibility of over modulation.

$\therefore \mu = A_m$ $m(t) = 2 \cos 2\pi \times 10^3 t$

$\therefore \mu = 2$

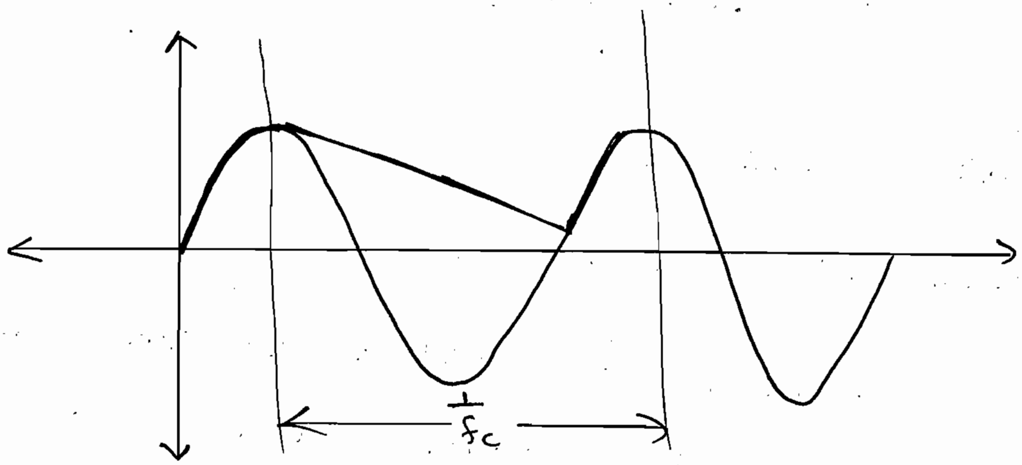
⑤ The time constant of envelope detector should satisfy following conditions.

① $R_L C >> \frac{1}{f_c}$.

② $R_L C << \frac{1}{f_m}$.

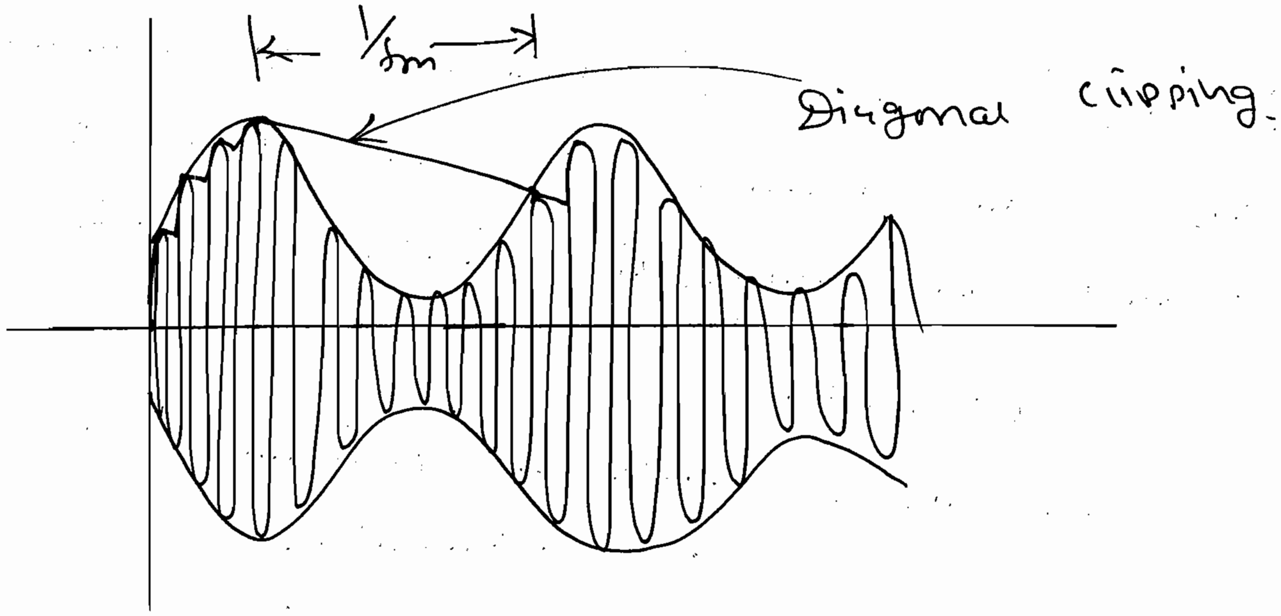
$$\therefore \boxed{\frac{1}{f_c} << R_L C << \frac{1}{f_m}} \leftarrow \underline{H.B.}$$

①



In this case we can not able to trace the output (or) envelope.

② *



Gate-2004

Ex-1 Am signal is demodulated using an envelop detector the carrier freq. is 1 MHz and the msg. freq. is 2 KHz. A suitable value for the time constant of the envelop detector is,

(A) 0.2 μs  (C) 20 μs ✓

(B) 1 μs  (D) 500 μs.

Ans: $\frac{1}{f_c} << R_L C << \frac{1}{f_m}$.

$\therefore \frac{1}{1 MHz} << R_L C << \frac{1}{2 kHz}$.

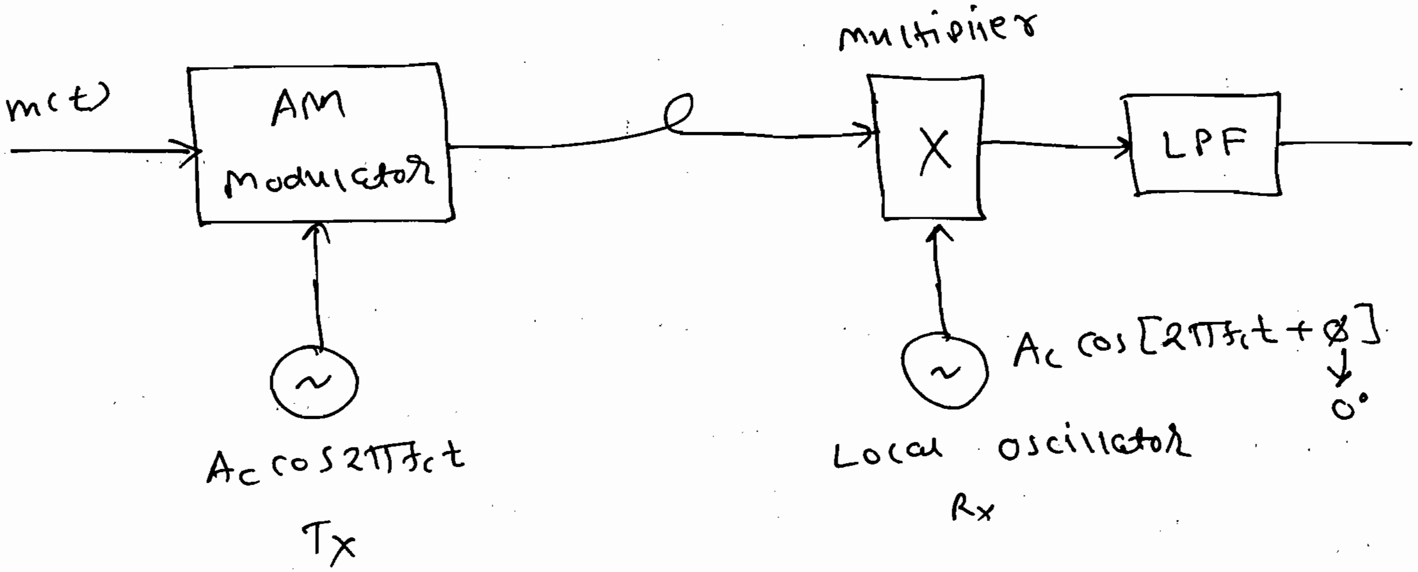
$\therefore \frac{1}{10^6} << R_L C << \frac{1}{2 \times 10^3}$.

$\therefore \boxed{1\mu s << R_L C << 500 \mu s.}$

So, valid ans is (c) 20 μsec.

## (3) Synchronous Detector:-

⇒ Synchronous detector is capable of demodulating the overmodulating $\overset{\text{AM}}{\downarrow}$ signal also



⇒ output of multiplier,

→ (Am) (LO).

$= [A_c \cos 2\pi f_c t + A_c m(t) \cos 2\pi f_c t] A_c \cos 2\pi f_c t.$

$= A_c^2 \cos^2 2\pi f_c t + A_c^2 m(t) \cos^2 2\pi f_c t.$

$= \frac{A_c^2}{2} + \frac{A_c^2 \cos 2\pi (2f_c) t}{2} + \frac{A_c^2}{2} m(t) + \frac{A_c^2 m(t)}{2} \cos 2\pi (2f_c) t.$

$\therefore$ o/p of LPF = $\frac{A_c^2}{2} m(t)$.

=> Assume that the phase shift is existing b/w carrier used at transmitter and carrier generated at receiver.

=> Then o/p of multiplier,

$$[A_c \cos 2\pi f_c t + A_c m(t) \cos 2\pi f_c t] \times A_c \cos[2\pi f_c t + \phi]$$

=> finally o/p of LPF is

$$\left[\frac{A_c^2}{2} m(t)\right] \cos \phi .$$

if $\phi = 0° \longrightarrow \frac{A_c^2}{2} m(t)$.

$\phi = 90° \longrightarrow 0 \rightarrow \boxed{\text{Quadrature null effect}}$

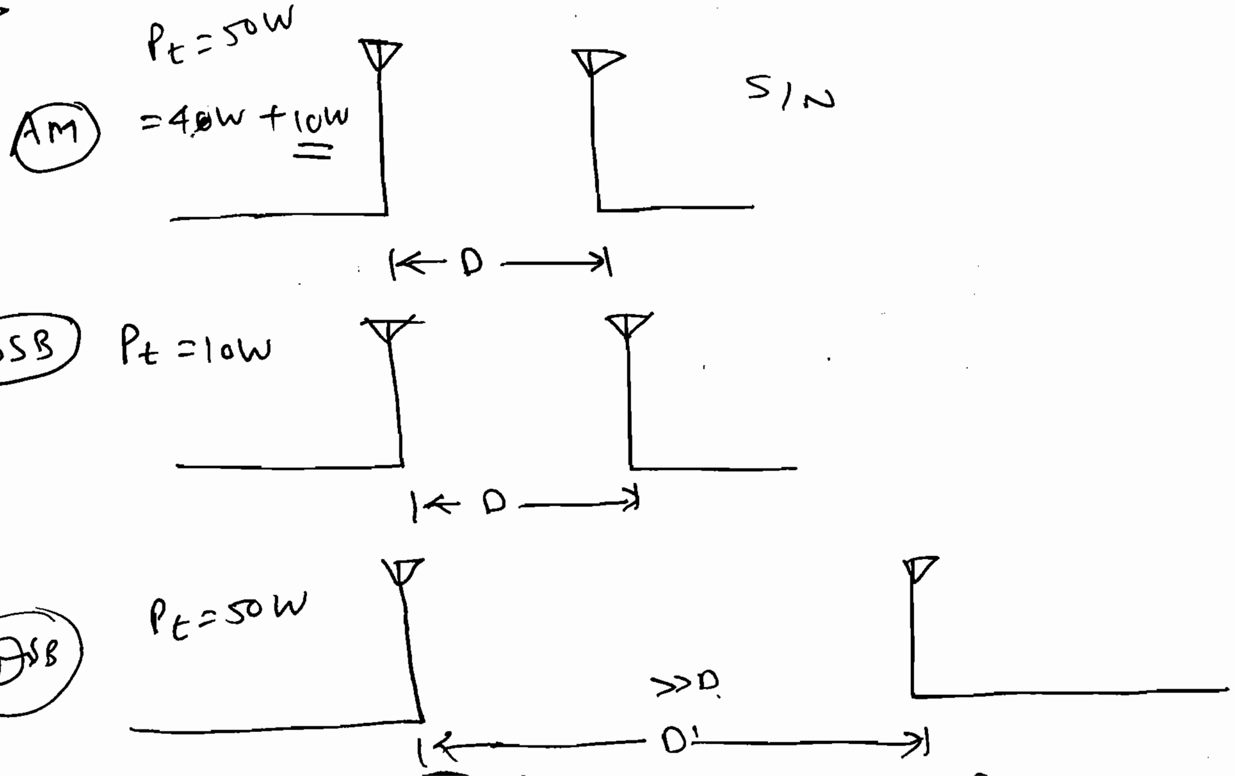
→ To overcome the Quadrature null effect Synchronization should be maintained b/w $T_x$ & $R_x$. So, Additional Hardware is required to maintain Synchronization. The Hardware Complexity of the synchronization detector is very high when compared with envelop detector. so envelope detector is used as Demodulator for AM.

## ② DSBSC Modulation:- ( Double side Band Suppressed Carrier)

⇒ In Am the maximum efficiency possible is 33.3%, in the case of singletone Modulation. So, the maximum sideband power is only 33.3% of the total power. When the Carrier is Suppressed the total power & sideband power are equal. So, modulation efficiency is 100%.

→ To cover the same distance in a wireless Communication system DSB Modulation require less power with compared with Am. If the Same power is used in DSB the distance B|w Tx & Rx is increased.

⇒



⇒ $s(t) = A_c \cancel{\cos 2\pi f_c t} + A_c k_a m(t) \cos 2\pi f_c t$.

$\boxed{k_a = 1}$

⇒ Time domain eqn of DSB.

$$\boxed{\begin{aligned} s(t) &= A_c m(t) \cos 2\pi f_c t \\ s(t) &= m(t) \cdot c(t) \end{aligned}}$$

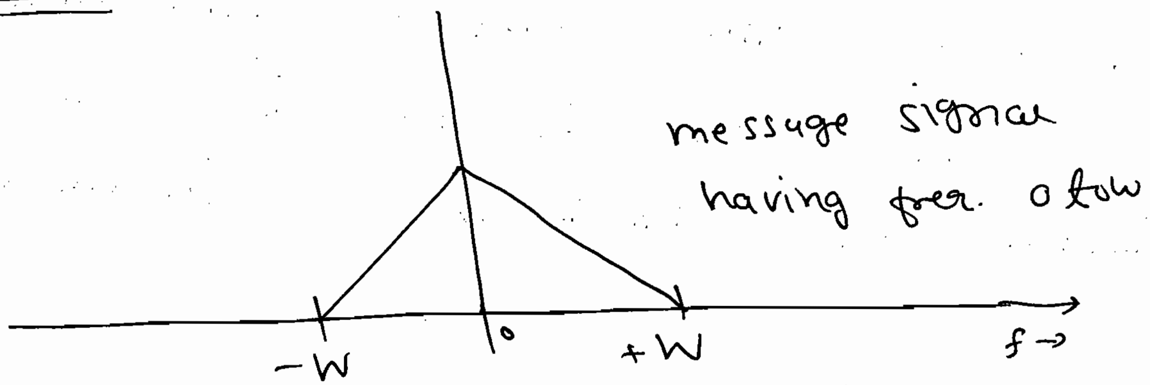
H.B.

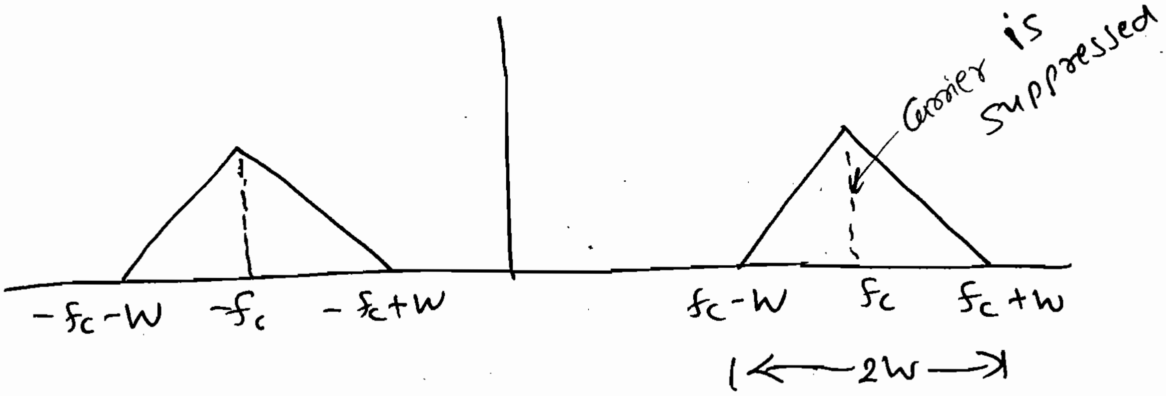
⇒ Freq. domain eqn of DSB,

$$\boxed{S(f) = \frac{A_c}{2}\left[ M(f - f_c) + M(f + f_c) \right]}$$

H.B.

⇒ Spectrum:





## * Singletone Modulation of DSB.

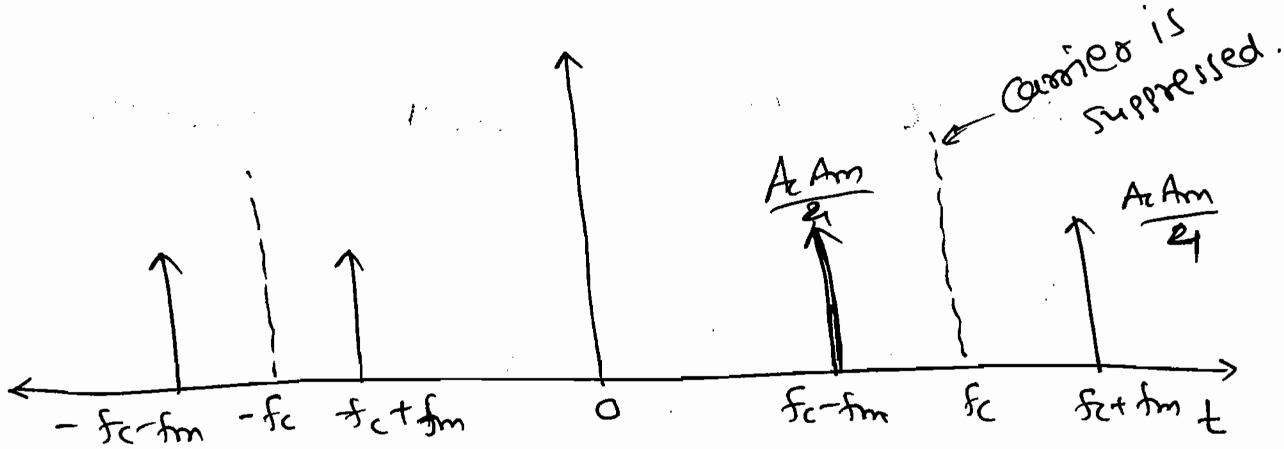
⇒ $s(t) = A_c\, m(t).\ \cos 2\pi f_c t$

$= A_c.A_m.\ \cos 2\pi f_m t.\ \cos 2\pi f_c t$

$= \frac{A_c.A_m}{2}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$

$$\therefore S(t) = \underbrace{\frac{A_cA_m}{2}\cos 2\pi(f_c+f_m)t}_{USB} + \underbrace{\frac{A_cA_m}{2}\cos 2\pi(f_c-f_m)t}_{LSB}.$$

H.B



## * Power Calculation:

⇒ $P_t = P_{USB} + P_{LSB}.$

$$\therefore = \frac{\left(\frac{A_cA_m}{2\sqrt2}\right)^2}{R} + \frac{\left(\frac{A_cA_m}{2\sqrt2}\right)^2}{R}.$$

$$P_t = \frac{A_c^2A_m^2}{8R} + \frac{A_c^2A_m^2}{8R}.$$

$$\therefore \boxed{P_t = \frac{A_c^2A_m^2}{4R}.}$$ H.B.

$$\boxed{P_t = \frac{A_c^2 A_m^2}{4R} \quad (W).}$$ $\underline{H.B}$

$\therefore \boxed{\eta = 100\%}$

$\Rightarrow$

$$P_t = \cancel{P_c} + \frac{P_c \mu^2}{2}.$$

$$\therefore P_t = \frac{P_c \mu^2}{2}.$$

$$\therefore P_t = \frac{A_c^2}{2R}, \quad K_a = 1 \Rightarrow A_m = \mu.$$

$$\therefore \boxed{P_t = \frac{A_c^2 A_m^2}{4R}.}$$ $\underline{H.B}$

Ex-1 A carrier signal $c(t) = 20\cos 2\pi 10^6 t$ is modulated by a message signal $m(t) = 5\cos 2\pi 10^4 t$ to generate a DSB signal. Sketch the spectrum & calculate the B.W., power & modulation efficiency.

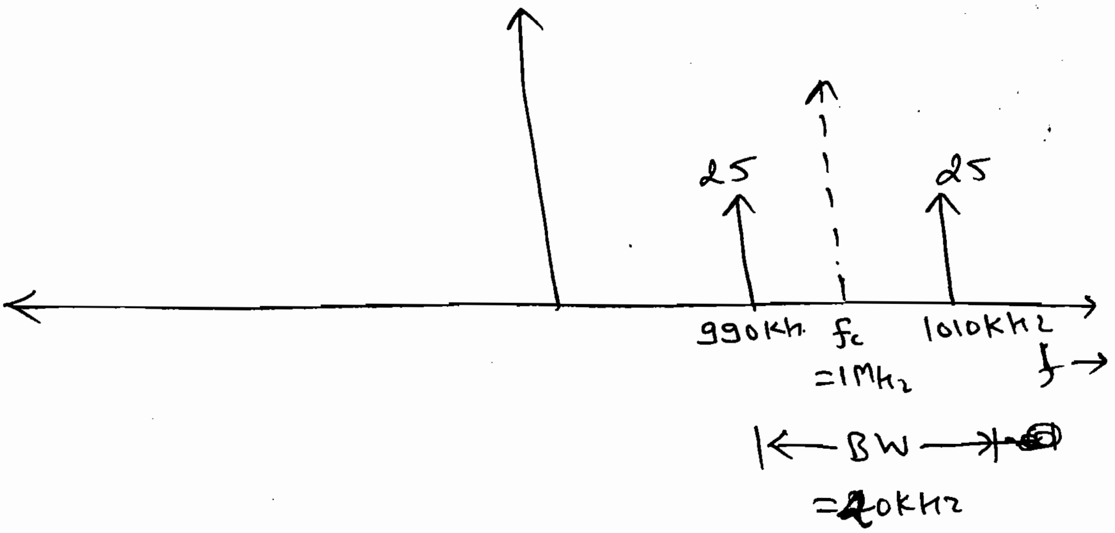
Ans: $c(t) = 20\cos 2\pi 10^6 t.$

$\Rightarrow A_c = 20.$

$f_c = 10^6\ Hz = 1\ MHz.$

$\Rightarrow \quad m(t) = 5 \cos 2\pi 10^4 t$

$\Rightarrow \quad f_m = 10 KHz.$ $\qquad A_m = 5$ ..



$\Rightarrow \quad BW = 2f_m = 2 \times 10 KHz$

$\boxed{BW = 20 KHz}$.

$\Rightarrow \quad P_t = \frac{A_c^2 A_m^2}{4R}.$ $\qquad \boxed{\eta = 100\%.}$

$\therefore P_t = \frac{\overset{100}{\cancel{400}} \times 25}{\cancel{4} \times R}$

$\therefore \boxed{P_t = 2.5 KW}$

Ex-2 Repeat the above Problem when the msg signal $m(t) = 5 \cos 2\pi 10^4 t + 2 \cos 8\pi 10^3 t.$

Ans: $f_{m1} = 10 KHz, \qquad f_{m2} = 4 KHz.$

$A_{m1} = 5 V, \qquad A_{m2} = 2 V.$

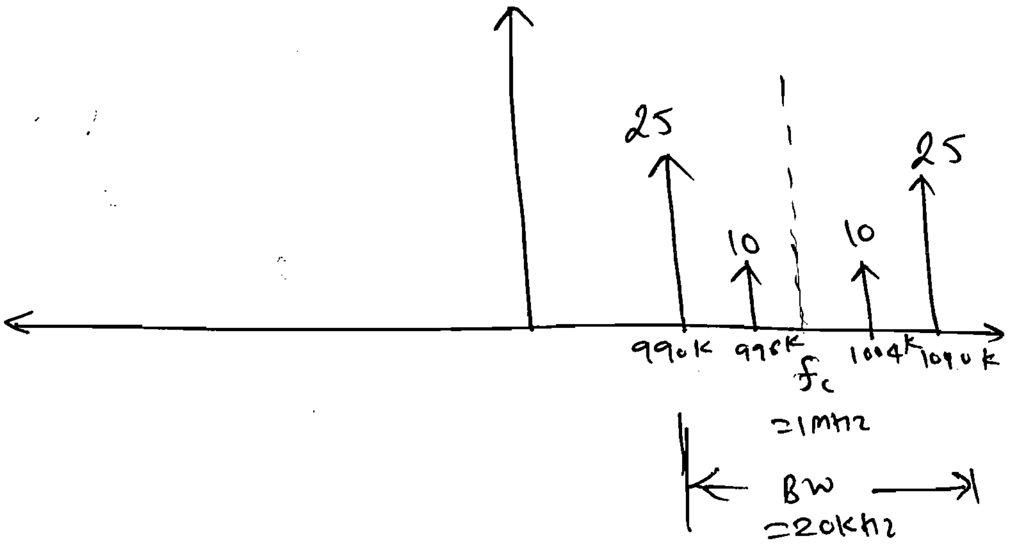
$\therefore P_t = \frac{A_c^2}{4R} [\mu_1^2 + \mu_2^2].$

$\therefore P_t = \frac{400}{4} [A_{m1}^2 + A_{m2}^2].$

$\therefore \quad P_t = 100 [25 + 4].$

$\therefore \quad \boxed{P_t = 2.9 \text{ KW}}$

$\boxed{\eta = 100\%.}$

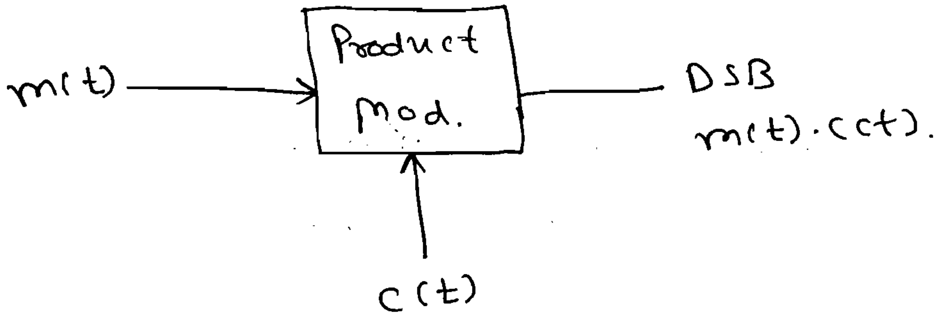


$\Rightarrow BW = 2 f_m.$

$\boxed{BW = 20 \text{ KHz}}$

## * Generation of DSB signal:

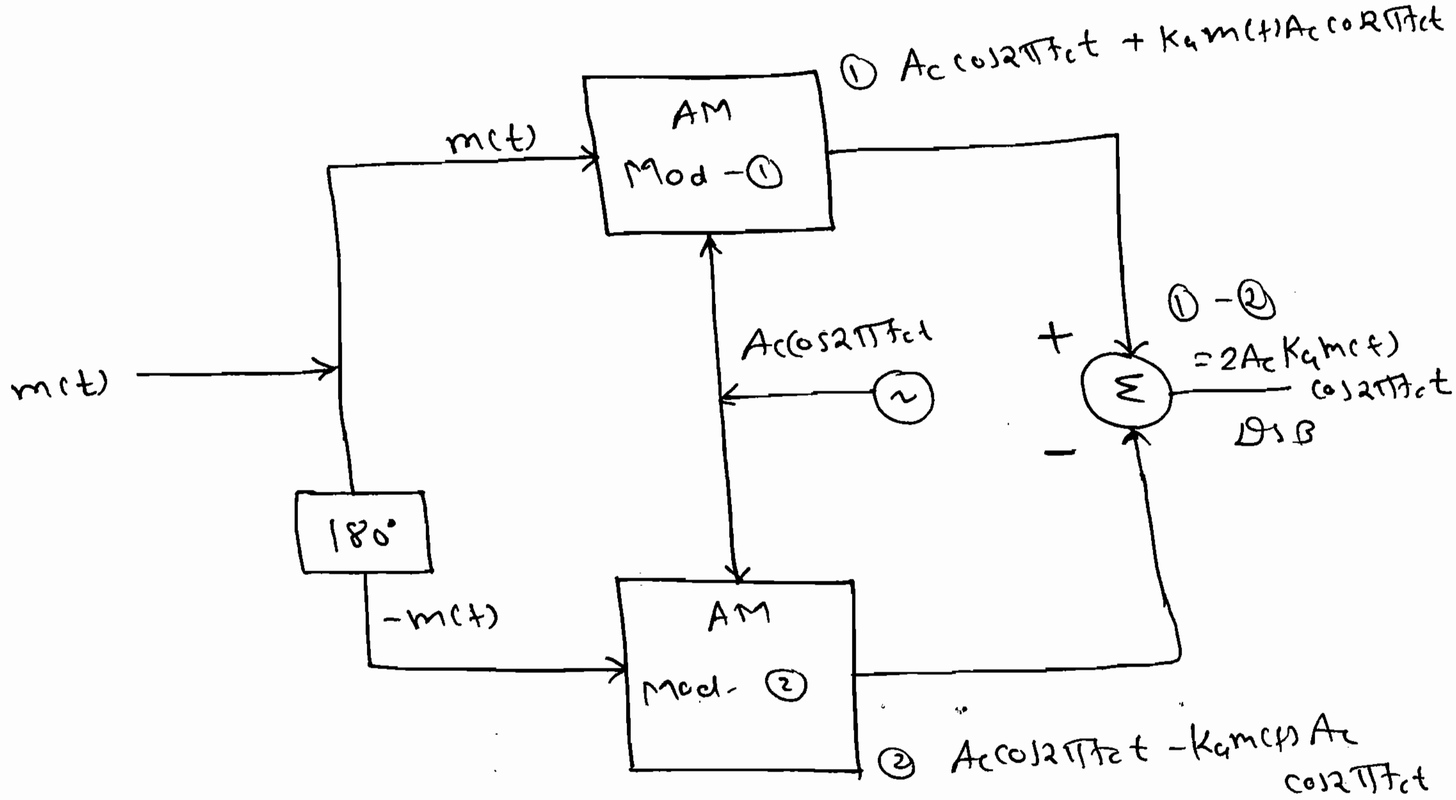
→ Any modulator which generate DSB signal it is also called a product modulator.



① Balanced Modulator.

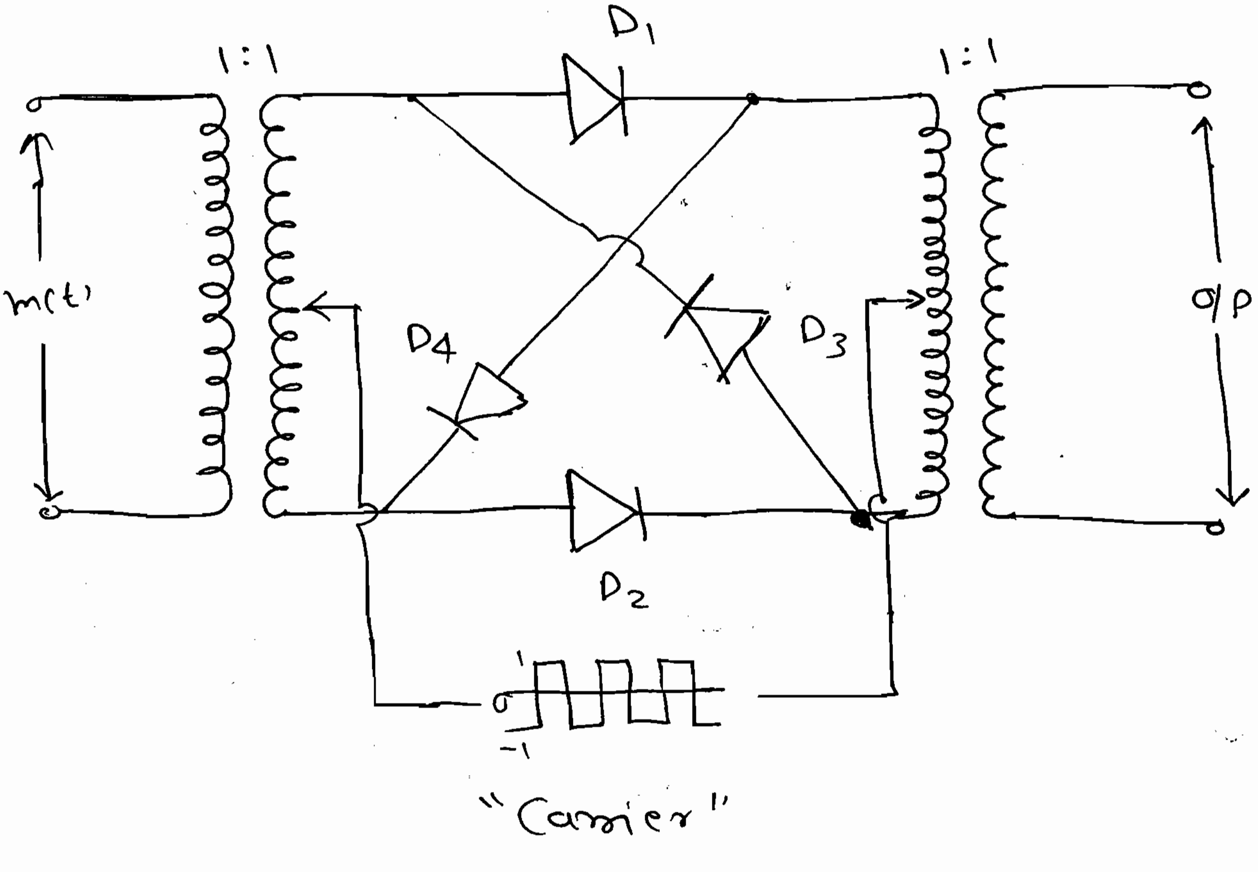
② Ring modulator.

## ① Balanced Modulator:



=> "Balanced modulator can be used as "Multiplier""

## ② Ring Modulator:

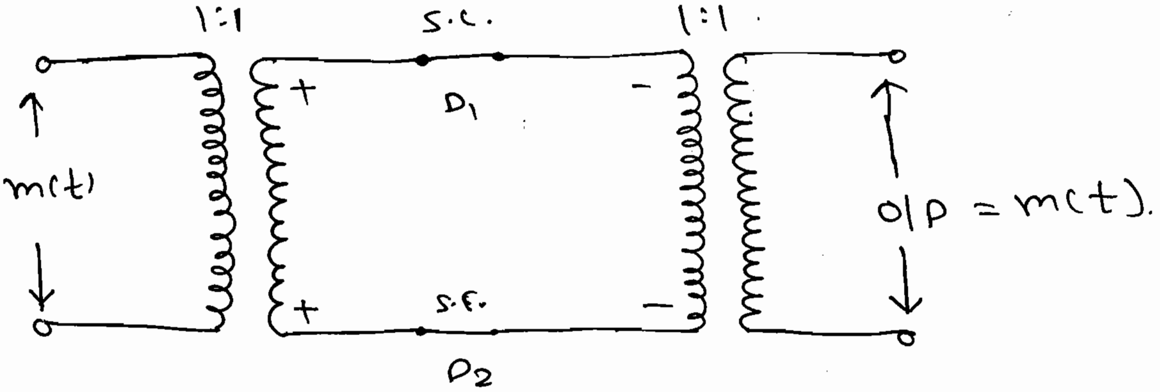


→ Carrier is taken as Square wave for convinience.

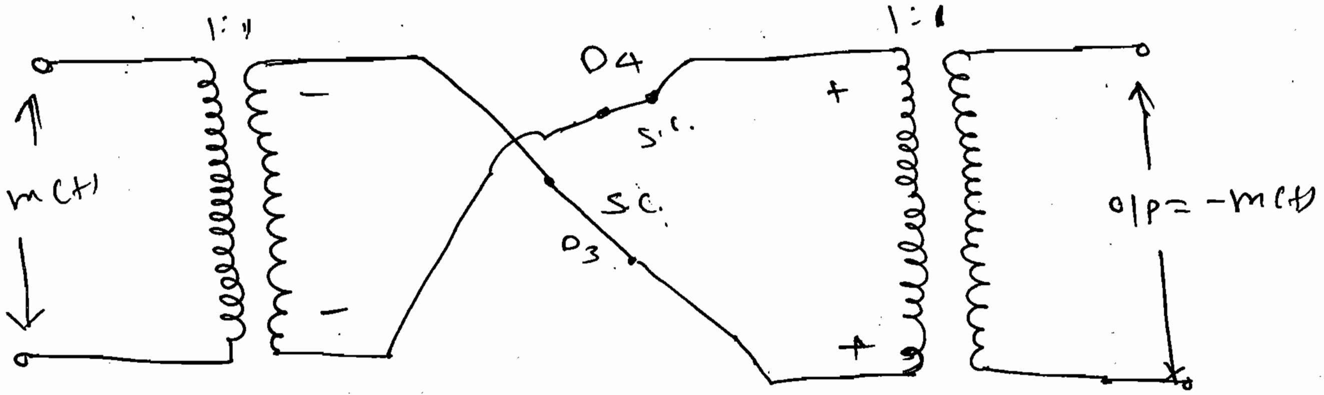
**NOTE:** => In all practical application we use sinusoidal signal because it contains only one freq '$f_c$'.

=> Other signal, according to the fourier series there will be infinite freq.

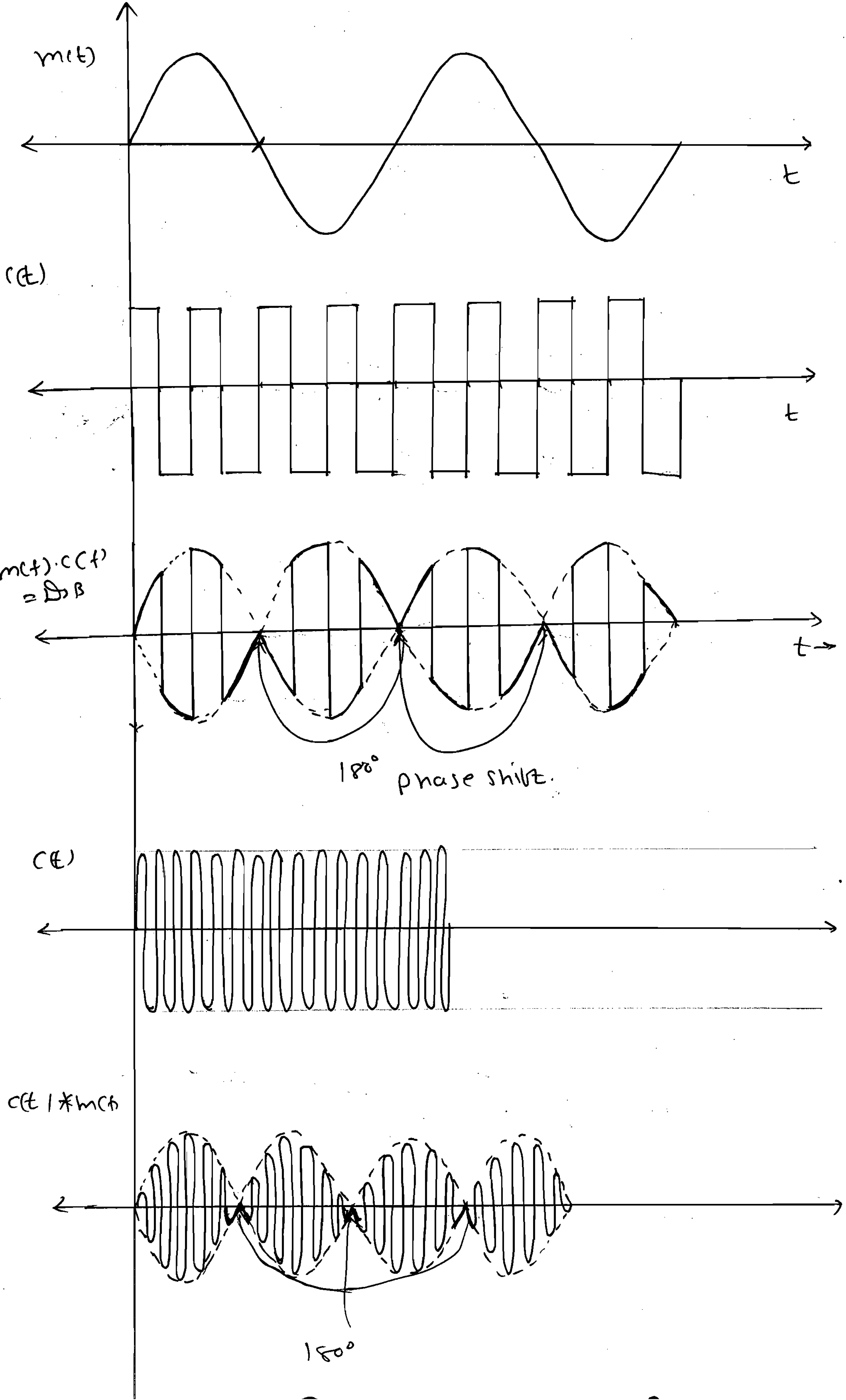
=> Carrier Polarity is '+Ve':



=> Carrier Polarity is '-Ve':



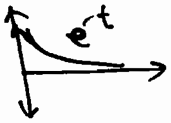
=> In DSB when signal changes its polarity (or) crosses 'zero' line then there will be 180° phase shift in DSB signal.

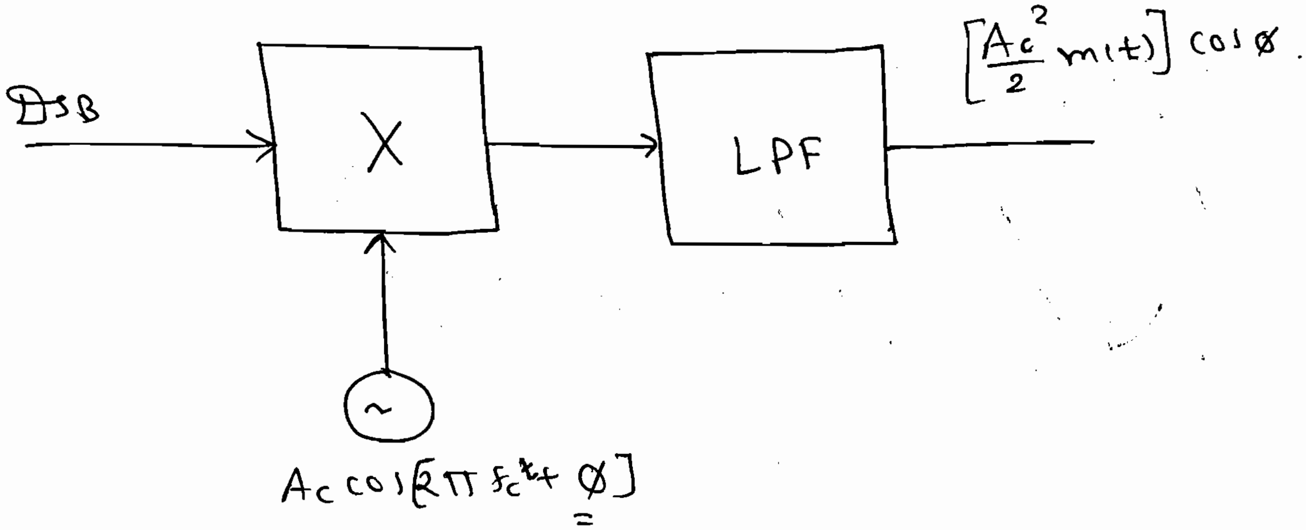

m(t)
t
c(t)
t
m(t)·c(t)
= DSB
t →
180° phase shift.
c(t)
c(t) * m(t)
180°

## * Demodulation of DSB signal:

→ When a DSB signal is passed through a Envelope detector, the o/p is "$|m(t)|$". So, Envelope detector is not used as demodulator. Therefore, Synchronous Detector is used as demodulator.

But, the signal which has amp. of +ve m(t) then it can be also demodulated by E.D for e.g. $e^{-t}$





$\Rightarrow$ o/p of Multiplier = (DSB)(Lo).

$$= [A_c m(t) \cos 2\pi f_c (t)]\, A_c \cos 2\pi f_c t$$

$$= A_c^2 m(t) \cos^2 2\pi f_c t$$

$$= \frac{A_c^2 m(t)}{2} + \frac{A_c^2}{2} m(t) \cancel{\cos 2\pi (2f_c) t}.$$

$\Rightarrow$ o/p of LPF = $\frac{A_c^2}{2} m(t)$.

$\Rightarrow$ Modulation concept used only in demodulation of AM by Envelope detector. It is not for synchronous demodulator for DSB.

$\Rightarrow$ If there are some phase shift $\phi$.

then, (DSB) (LO).

$$A_c m(t) . \cos(2\pi f_c t) . A_c \cos(2\pi f_c t + \phi).$$

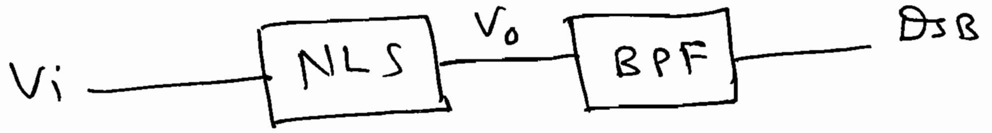
$$= \frac{A_c^2}{2} m(t) \cancel{\cos}(4\pi f_c t + \phi) + \frac{A_c^2}{2} m(t) . \cos\phi.$$

$\Rightarrow$ o/p of LPF $= \frac{A_c^2}{2} m(t) . \cos\phi.$

$\rightarrow$ The Hardware Complexity of DSB receiver is very high when compared with AM.

Ex-1 A DSB signal is generated using non-linear system having characteristic $V_o = aV_i + bV_i^3$, $V_i = [m(t) + \cos 2\pi f_1 t]$. The o/p of the non-linear system is passed through a bandpass filter to select the DSB signal. Determine the value of $f_1$ so that carrier freq. of DSB signal is 1 MHz.

Ⓐ 1 MHz Ⓑ 3 MHz Ⓒ 0.33 MHz Ⓓ 0.5 MHz.

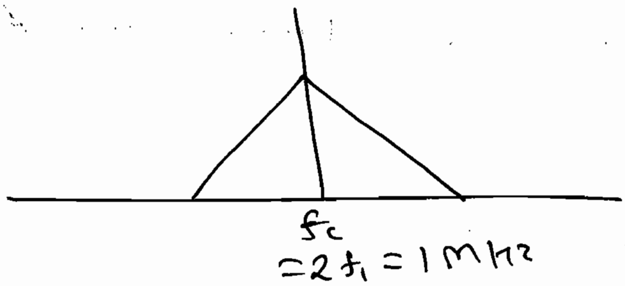


Ans: $V_o = aV_i + bV_i^3$.

$$\therefore V_o = a\cancel{m}(t) + a\cos \cancel{2\pi f_1 t} + b\cancel{m^3(t)} + b\cancel{\cos^3 2\pi f_1 t}$$

$$+ b3\cancel{m^2}(t) . \cos 2\pi f_1 t + 3bm(t) . \cos^2 2\pi f_1 t.$$

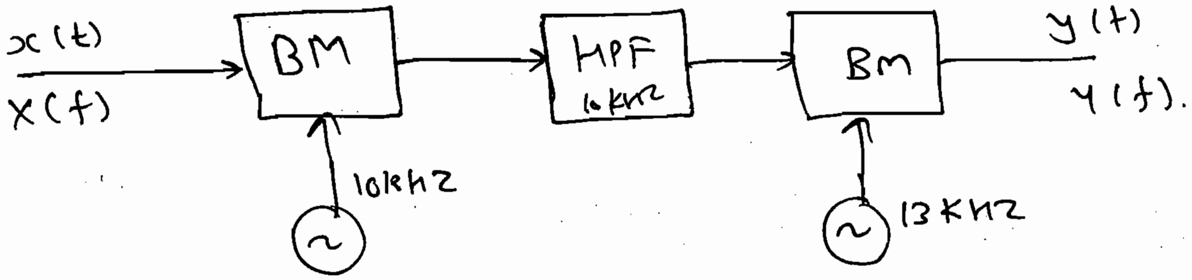
$$V_o = 3b\cancel{m}(t) + \frac{3bm(t)}{2} \cos 2\pi(2f_1)t$$



$\therefore \boxed{f_1 = 0.5\text{ MHz}.}$

GATE : 2004

**Ex-2** Consider the System Shown in fig. Determine the +ve freq. at which Y(f) is having spectral peaks.



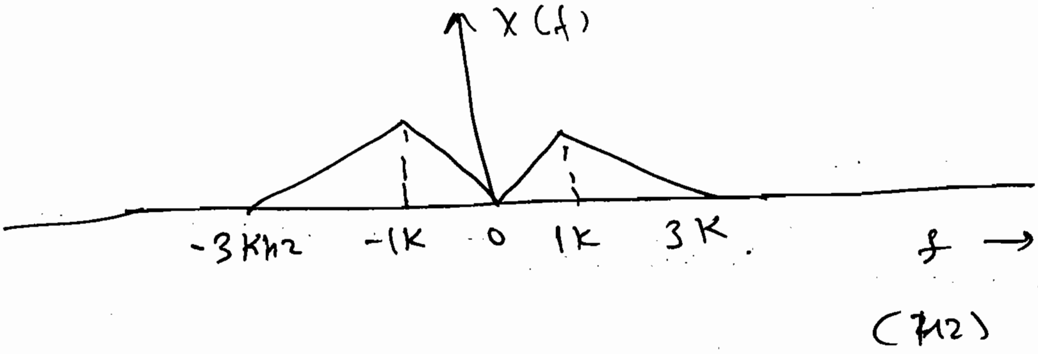
(A) 2 kHz & 14 kHz

(B) 2 kHz & 24 kHz

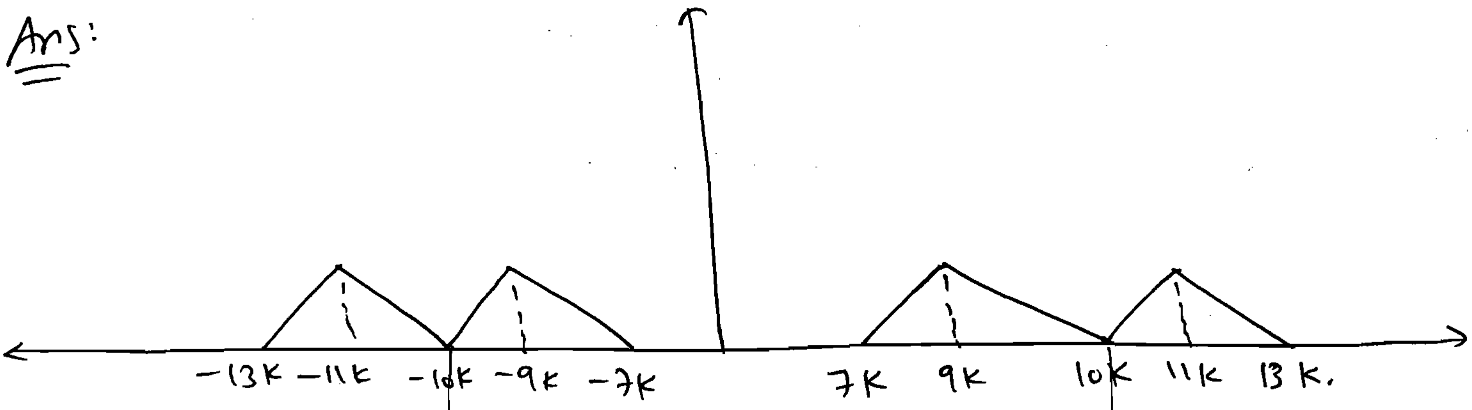
(c)' 1 kHz & 14 kHz

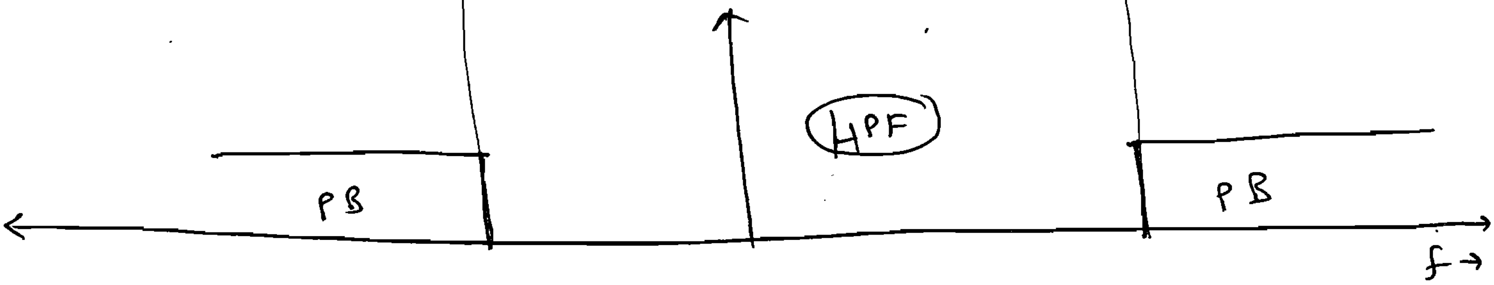
(D) 1 kHz & 24 kHz.

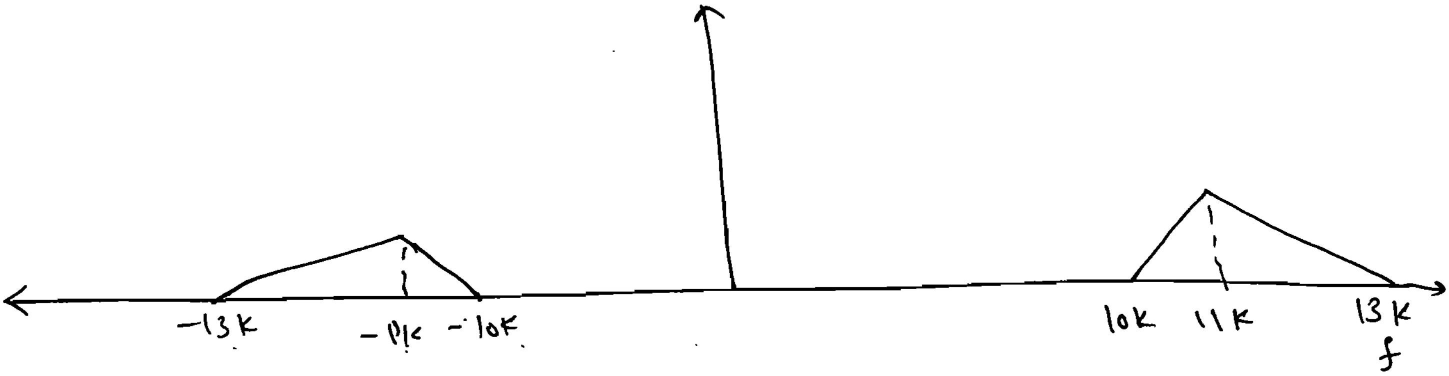
→ spectrum of i/p signal,

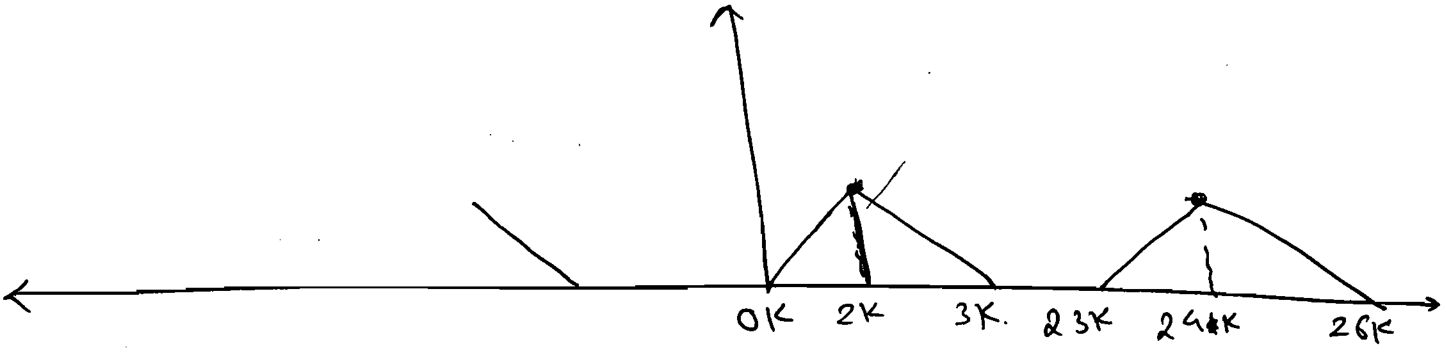


Ans:



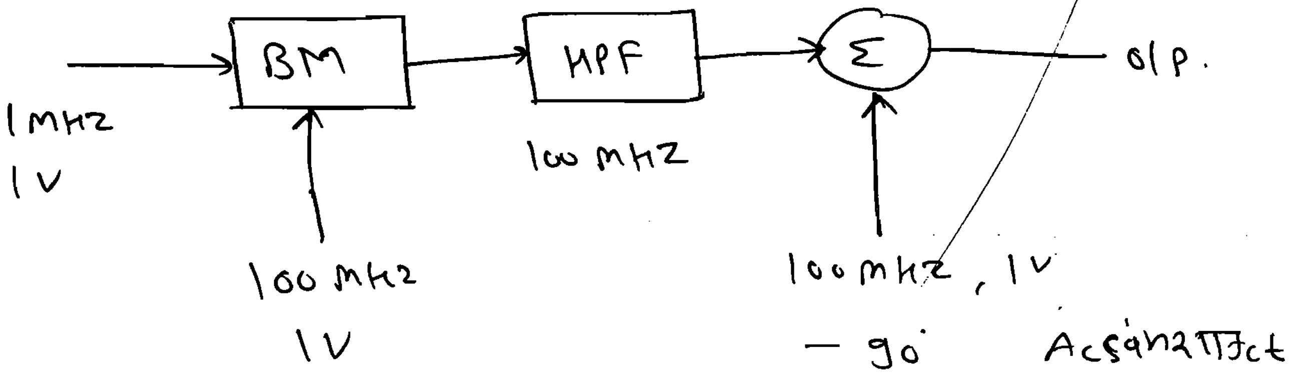






So, Ans: (B) 2 KHz & 24 KHz.

Ex-3 Consider a system as shown in fig.



determine the envelope of the o/p signal.

**Ans:**

$\rightarrow A_c \cos 2\pi f_c t \cdot A_m \cos 2\pi f_m t$

$$= \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m).$$

$$= \frac{1}{2} \cos 2\pi\, 101 m t + \frac{1}{2} \cos \cancel{2\pi}\, 99 M t.$$

$\rightarrow$ The o/p of HPF.

$$\Rightarrow \frac{1}{2} \cos 2\pi (f_c + f_m) t.$$

Now.

$$\Rightarrow o/p = \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t + A_c \sin 2\pi f_c t.$$

$$= \frac{A_c A_m}{2} \left[\cos 2\pi f_c t \cdot \cos 2\pi f_m t - \sin 2\pi f_c t \cdot \sin 2\pi f_m t\right] + A_c \sin 2\pi f_c t.$$

$$= \underbrace{\left[\frac{A_c A_m}{2} \cdot \cos 2\pi f_m t\right]}_{A} \cos 2\pi f_c t + \underbrace{\left[A_c - \frac{A_c A_m}{2} \sin 2\pi f_m t\right]}_{B} \sin 2\pi f_c t.$$

$$\therefore o/p = \sqrt{A^2 + B^2}$$

$$o/p = \sqrt{\frac{A_c^2 A_m^2}{4} \cos^2 2\pi f_m t + A_c^2 - \frac{\cancel{2} A_c^2 A_m}{\cancel{2}} \sin 2\pi f_m t + \frac{A_c^2 A_m^2}{4} \sin^2 2\pi f_m t}$$

$$= \sqrt{\frac{1}{4} + 1 - \sin 2\pi f_m t}$$

$$\boxed{o/p = \sqrt{5/4 - \sin 2\pi f_m t}}$$

## ③ SSB Moudulation: (single side Band Mod.)

→ The BW of the AM and DSB signal is Same. In order to multiplex more number of signal the BW of the signal should be as Low as possible. So, in this method only one side Band is transmitted through the Channel.

→ $A_c m(t). \cos 2\pi f_c t.$

$= A_c A_m \cos 2\pi f_c t . \cos 2\pi f_m t$

$= \underbrace{\frac{A_c.A_m}{2} \cos 2\pi (f_c + f_m) t}_{USB} + \underbrace{\frac{A_c.A_m}{2} \cos 2\pi (f_c - f_m) t}_{LSB}.$

⇒ Time domain signal of SSB is,

$\boxed{\frac{A_c A_m}{2} \cos 2\pi t (f_c \pm f_m)}$ ↙ H.B.

$+ \rightarrow$ USB

$- \rightarrow$ LSB.

⇒ $\frac{A_c A_m}{2} \cos 2\pi f_m t . \cos 2\pi f_c t \mp \frac{A_c A_m}{2} \sin 2\pi f_m t . \sin 2\pi f_c t.$

Now, $m(t) = A_m \cos 2\pi f_m t$

$\hat{m}(t) = A_m \sin 2\pi f_m t.$

= Hilbert transform.

→ $\boxed{\frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t}$

H.B ↗

− → USB

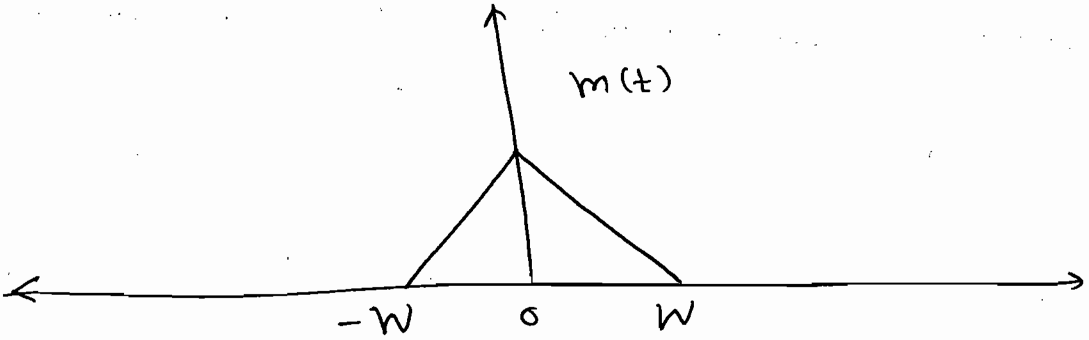
+ → LSB.

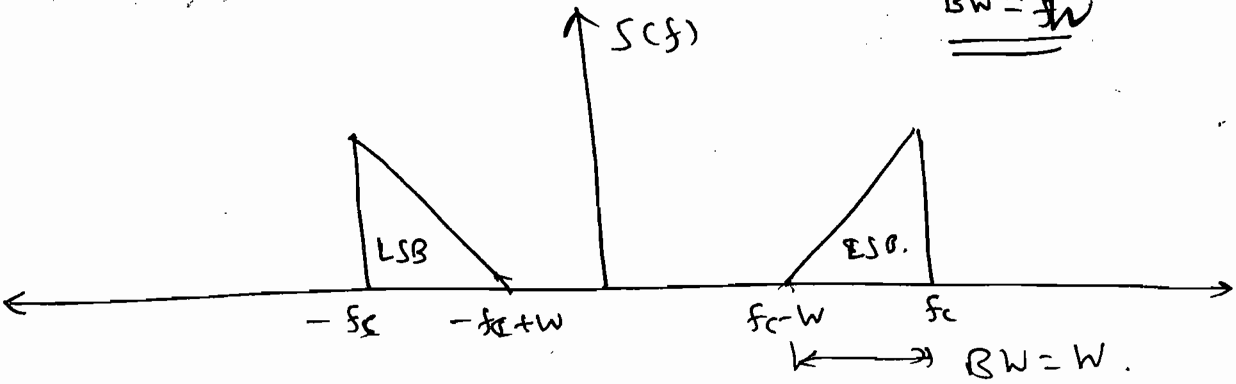
* Power Calculation:

⇒ $$P_t = \frac{A_c^2 A_m^2}{4R}.$$

$$P_t = \underbrace{\frac{A_c^2 A_m^2}{8R}}_{USB} + \underbrace{\cancel{\frac{A_c^2 A_m^2}{8R}}}_{LSB}$$

$$\therefore \boxed{P_t = \frac{A_c^2 A_m^2}{8R}.}$$ ↙ H.B.



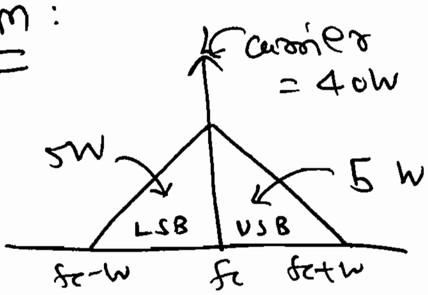


→ The BW & Power are decreasing by 50% when compared with the DSB signal.

⇒ When DSB modulation replace with SSB modulation then the power saving is 50%.

✱

① **Am :**



BW = 2fm

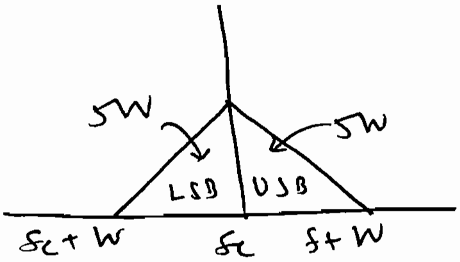
$P_t = 50W = 40W + 5W + 5W$

80% 10% 10%

$\mu = 0.707$

$\eta = 20\%$

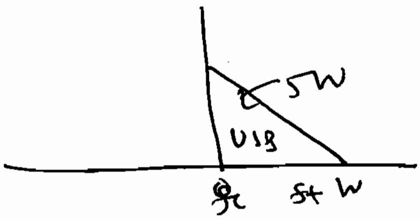
② **DSB :**



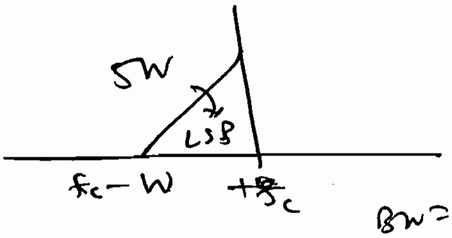
BW = 2fm

$P_t = 10W = 5W + 5W.$

③ **SSB :**



$P_t = 5W.$



BW = fm

## * Power Saving: ← H.B

**Case-(i): AM to DSB:**

$$\therefore \quad \text{Power Saving}(\%) = \frac{\text{Power saved}}{\text{Total power}} \times 100\%$$

$$= \frac{40}{50} \times 100\%$$

$$= \frac{P_c}{P_t} \times 100\%$$

$$= \frac{\cancel{P_c}}{\cancel{P_c}\left[1+\frac{\mu^2}{2}\right]} \times 100\%$$

$$\boxed{\text{Power saving}(\%) = \frac{2}{\mu^2+2} \times 100\%}$$

**case-(ii): Am to SSB.**

$$\therefore \quad \text{Power Saving}(\%) = \frac{\text{Power saved}}{\text{total Power}} \times 100\%$$

$$= \frac{40+5}{50} \times 100\%$$

$$= \frac{P_c + \frac{P_c\mu^2}{4}}{P_c\left[1+\frac{\mu^2}{2}\right]} \times 100\%$$

$$\boxed{\text{Power saving}(\%) = \left[\frac{1+\frac{\mu^2}{4}}{1+\frac{\mu^2}{2}}\right] \times 100\%}$$

**case-(iii): DSB to SSB %.**

$$\Rightarrow \quad \boxed{\text{Power saving} = \frac{5}{10} = 50\%}$$

# * Generation of SSB signal:

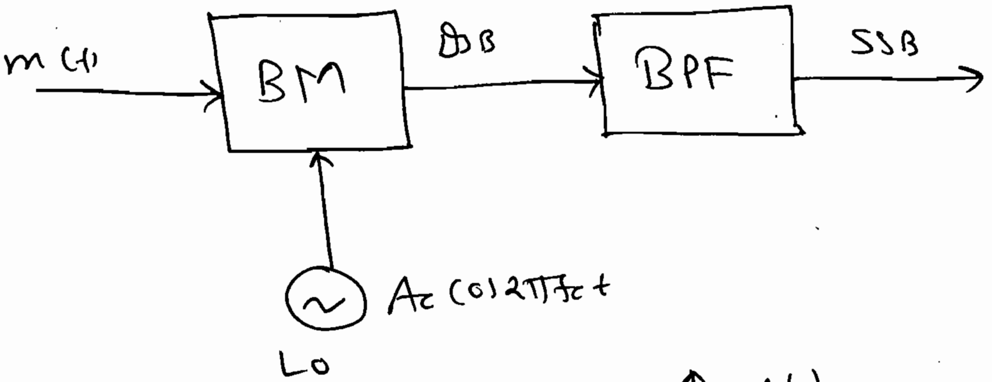
① Frequency discrimination method (or)

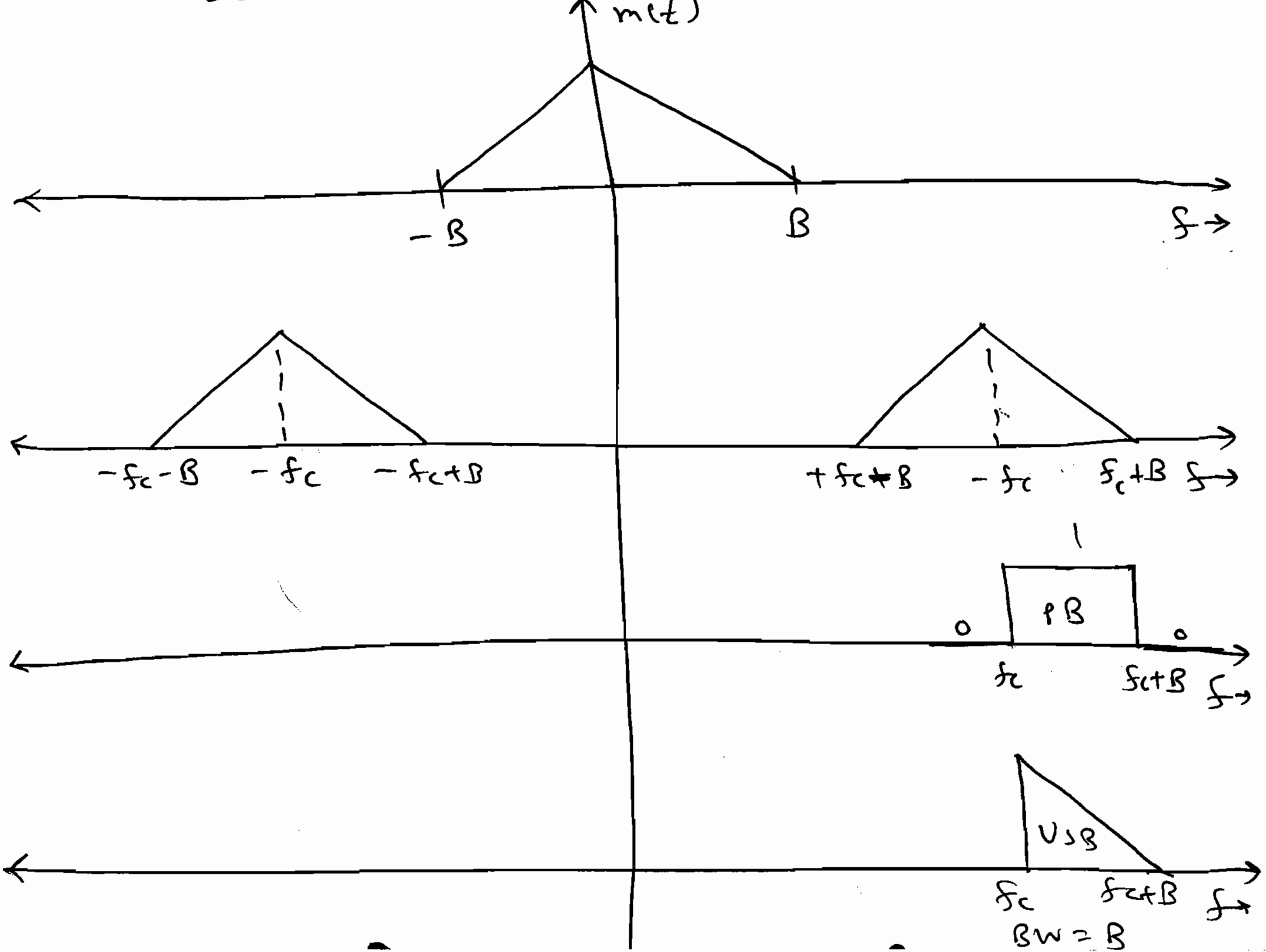
(Filtered method)

② Phase discrimination method:

H.B.

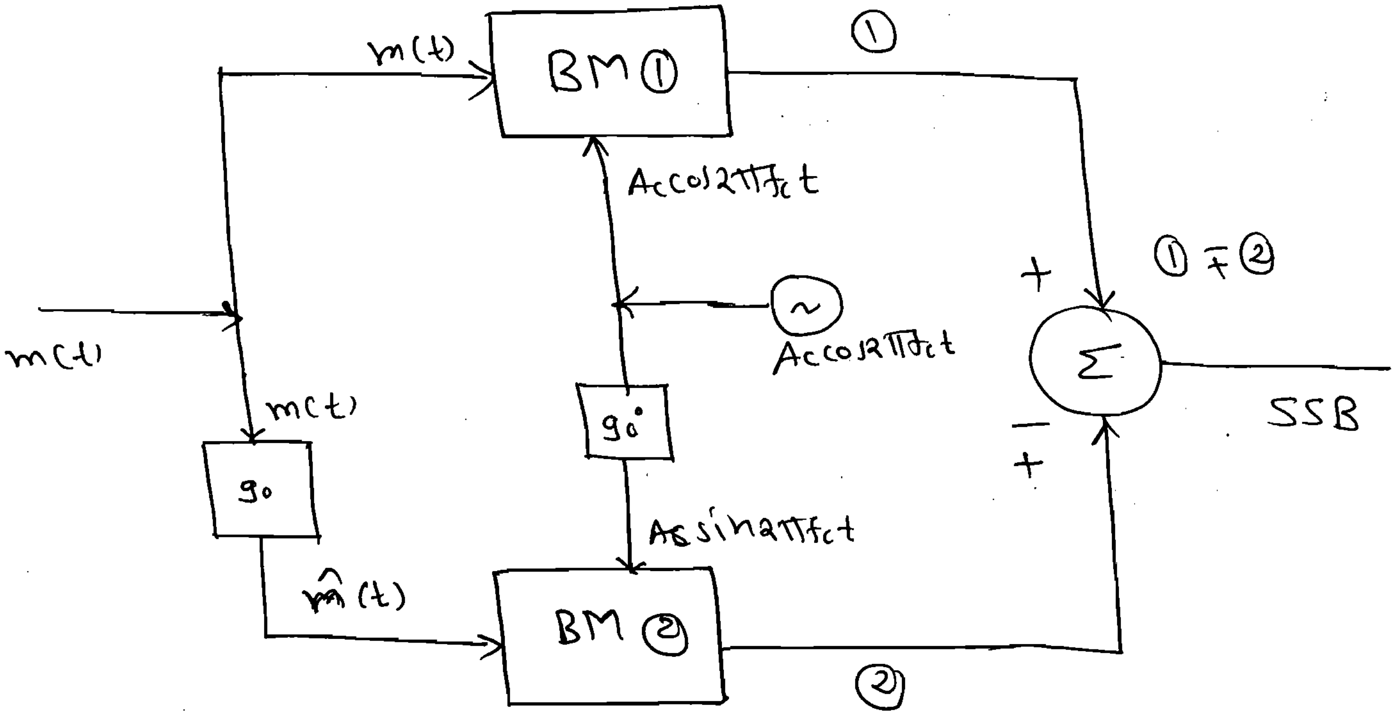
## ① Frequency discrimination method:





## ② Phase Discrimination Method :-

$$\Rightarrow S(t) = \frac{A_c}{2}.m(t)\cos 2\pi f_c t \mp \frac{A_c}{2}.\hat{m}(t).\sin 2\pi f_c t.$$



disadvantges

⇒ Hardware Complexity is very high.

### * Demodulation of SSB signals:

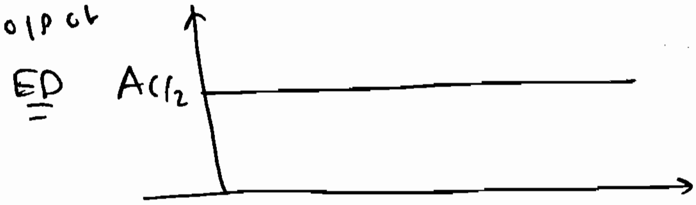
⇒

$$O/P = \sqrt{\left(\frac{A_c}{2}m(t)\right)^2 + \left(\frac{A_c\hat{m}(t)}{2}\right)^2}.$$

let, $m(t) = \cos 2\pi f_m t.$

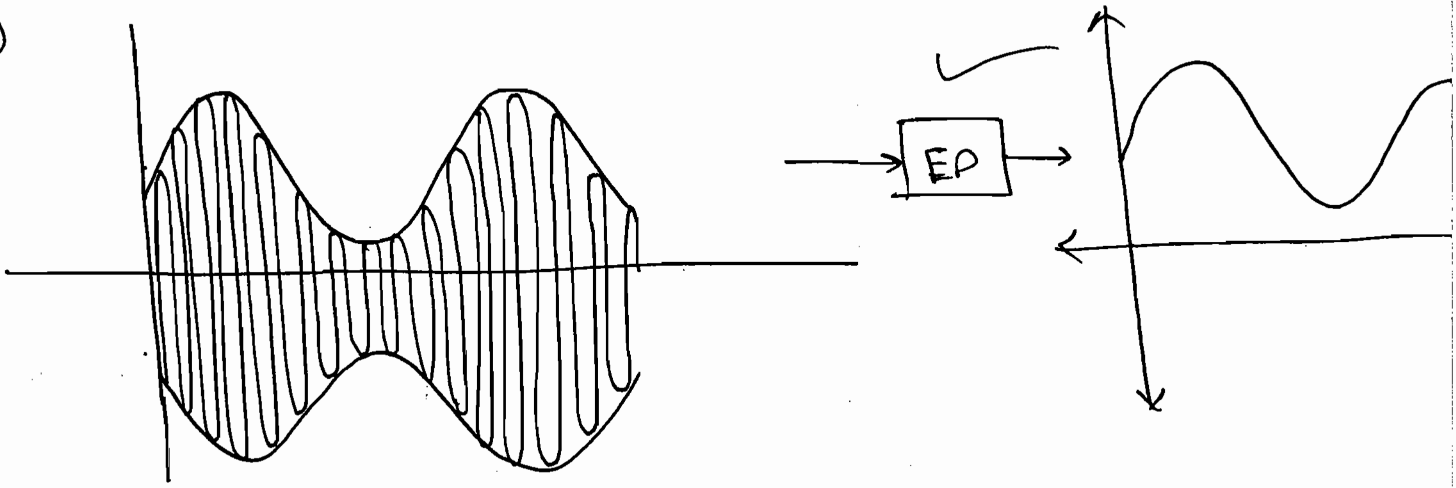
$$\therefore O/P = \left(\frac{A_c^2}{4}\cos^2 2\pi f_m t + \frac{A_c^2}{4}.\sin^2 2\pi f_m t\right)^{1/2}$$

$$O/P = \frac{A_c}{2}.$$

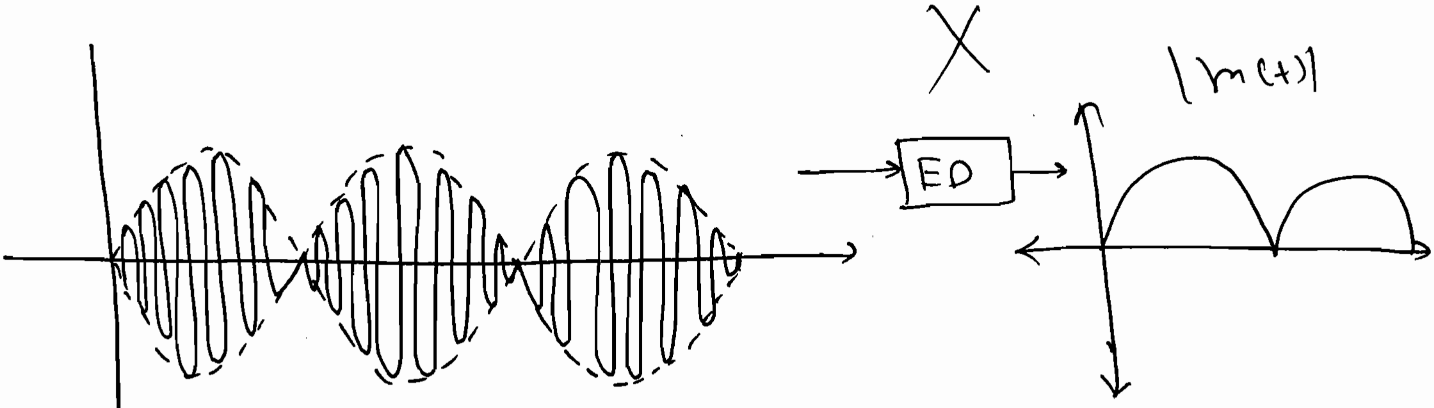


⇒ When the $m(t) = \cos 2\pi f_m t$. The o/p of a envelope detector is constant. So, ED is not used as demodulation for SSB.

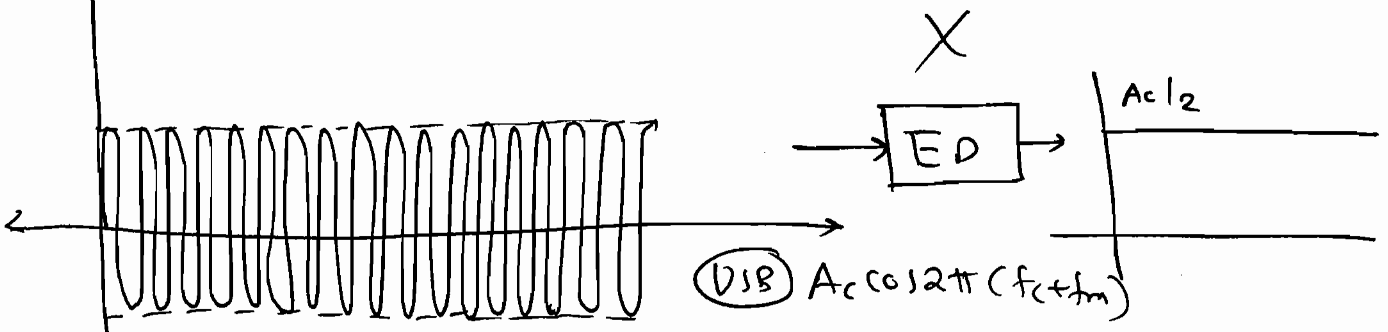
①



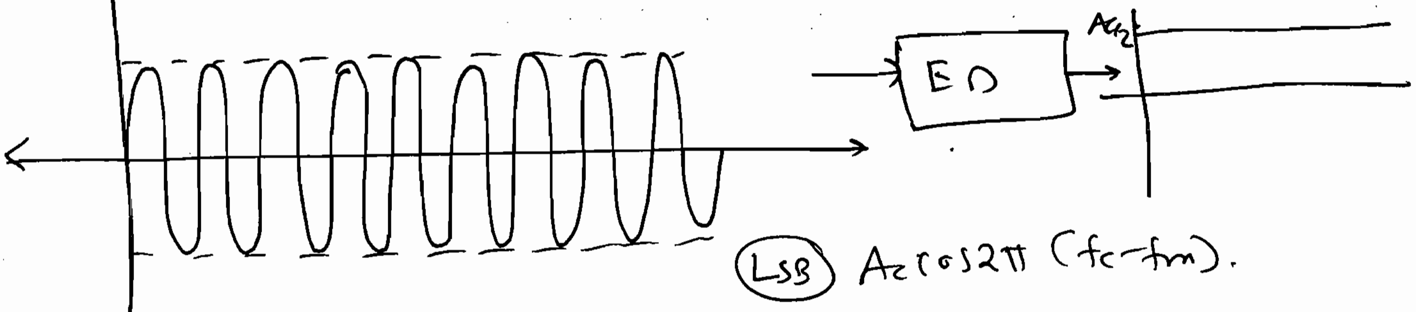
(2)



③

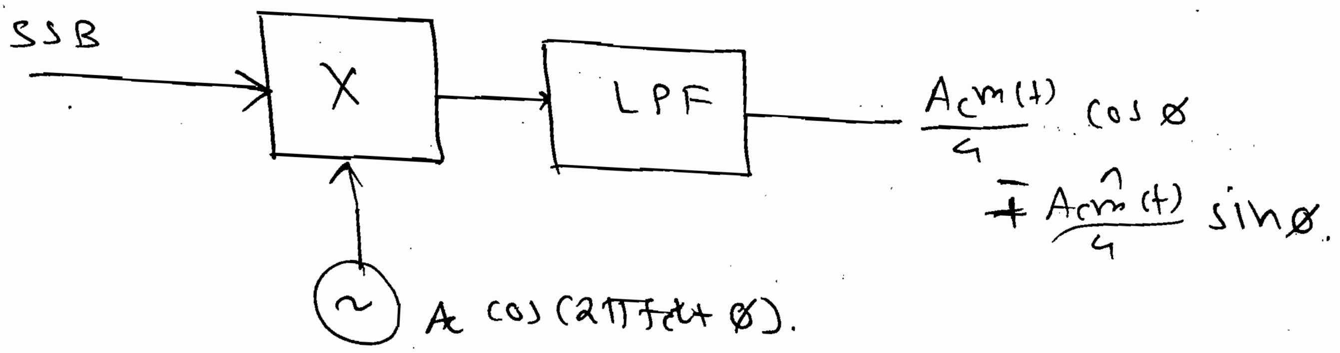


(4)



# * Synchrooneous Demodulator:

✓ H.B.



=> O/P of Multiplier:

$$\frac{A_c^2}{2} m(t)\cdot\cos^2 2\pi f_c t \mp \frac{A_c^2}{2}\hat{m}(t)\cdot\cos 2\pi f_c t\cdot\sin 2\pi f_c t$$

$$= \frac{A_c^2}{4} m(t) + \cancel{\frac{A_c^2 m(t)}{4}\cos 2\pi(4f_c)t} \mp \frac{A_c^2}{4}\cancel{\hat{m}(t)\sin 4\pi f_c t}$$

$$\mp \cancel{\frac{A_c^2}{4}\hat{m}(t)}.\ (0).$$

=> O/P of LPF

$$O/P = \frac{A_c^2}{4} m(t).$$

if there is phase shift of ø then

$$\boxed{O/P = \frac{A_c^2}{4} m(t)\cos\phi \mp \frac{A_c^2}{4}\hat{m}(t).\sin\phi.}$$

=> When $\phi = 0^\circ$ => $\frac{A_c^2}{4} m(t).$

$\phi = 90$ => $\frac{A_c^2}{4}\hat{m}(t).$

here, No Quadrature null effect.

=> When the phase shift is betn 0° and 90° the o/p of the demodulator are m(t) and $\hat{m}(t)$ and demodulation is not possible. So, synchronization required betn Tx and Rx.

## * Advantages of SSB over DSB. ✓ H.B

① BW is reduced by 50 %.

② Required power for same distance is reduced by 50 %.

③ No quadrature null effect. ✓✓

## * Disadvantages of SSB:

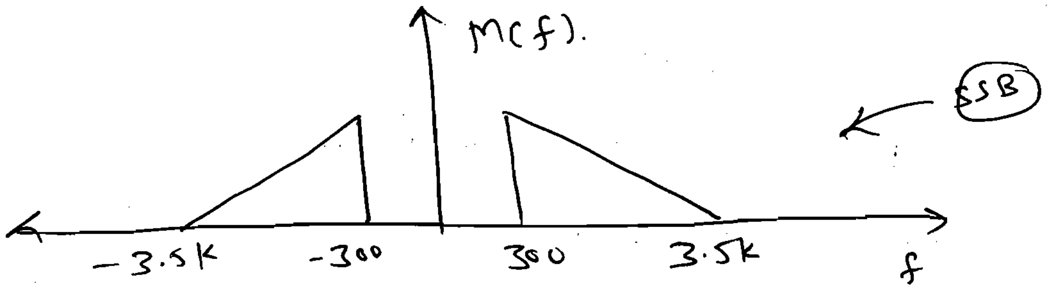
→ As the practical filter do not have sharp cut-off freq., It is not possible to suppress one side-band completely.

## * Application: (Most imp concept):

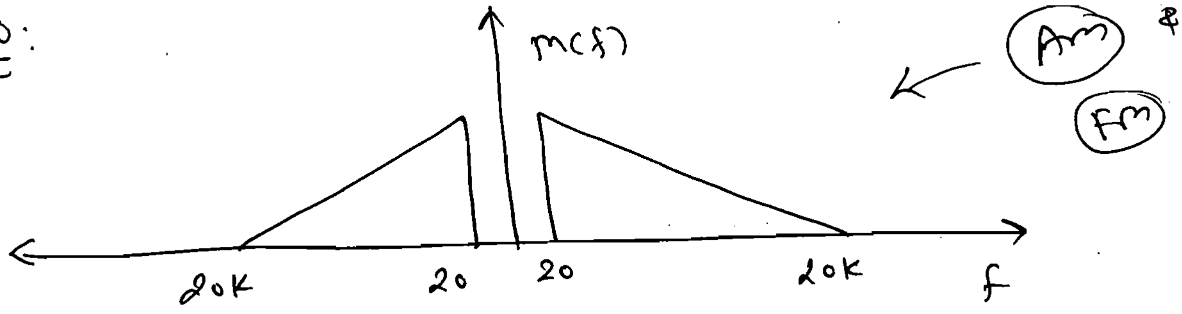
=> SSB modulation is suitable only for the transmission of voice signals only.

e.g. telephone system.

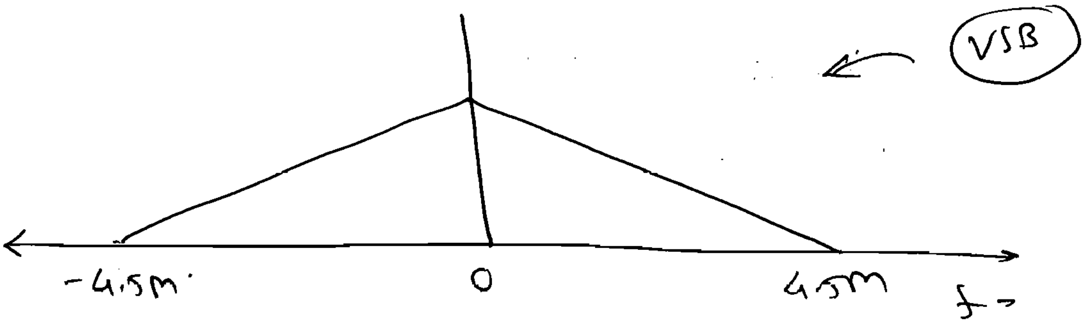
## ① Voice:



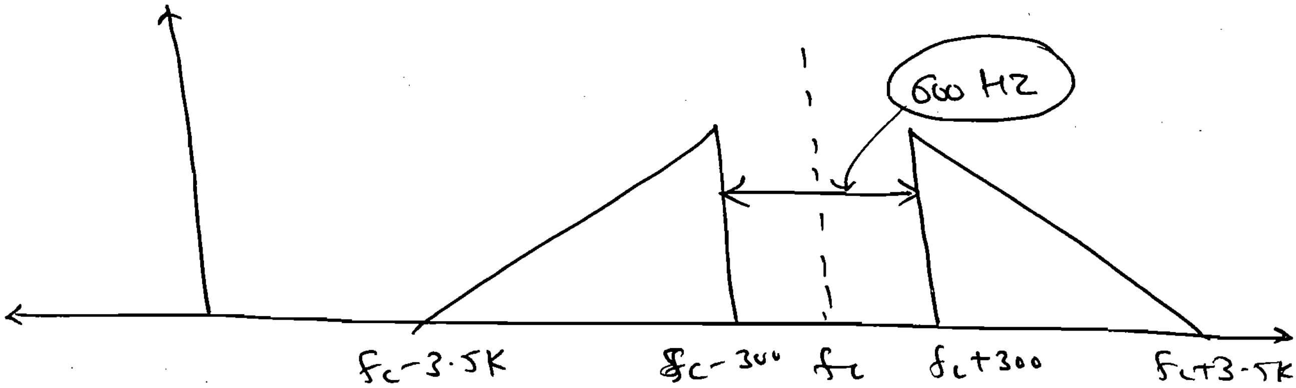
## ② Audio:

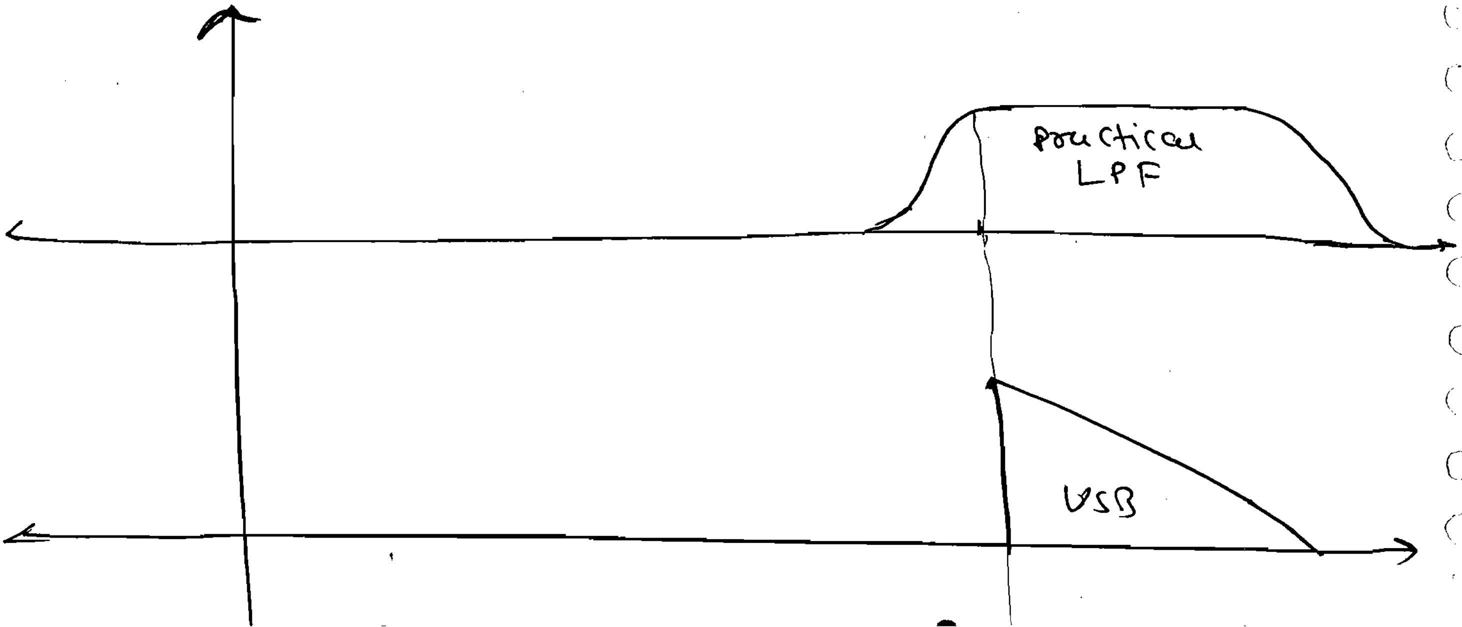


## ③ Vedio:



(✱)

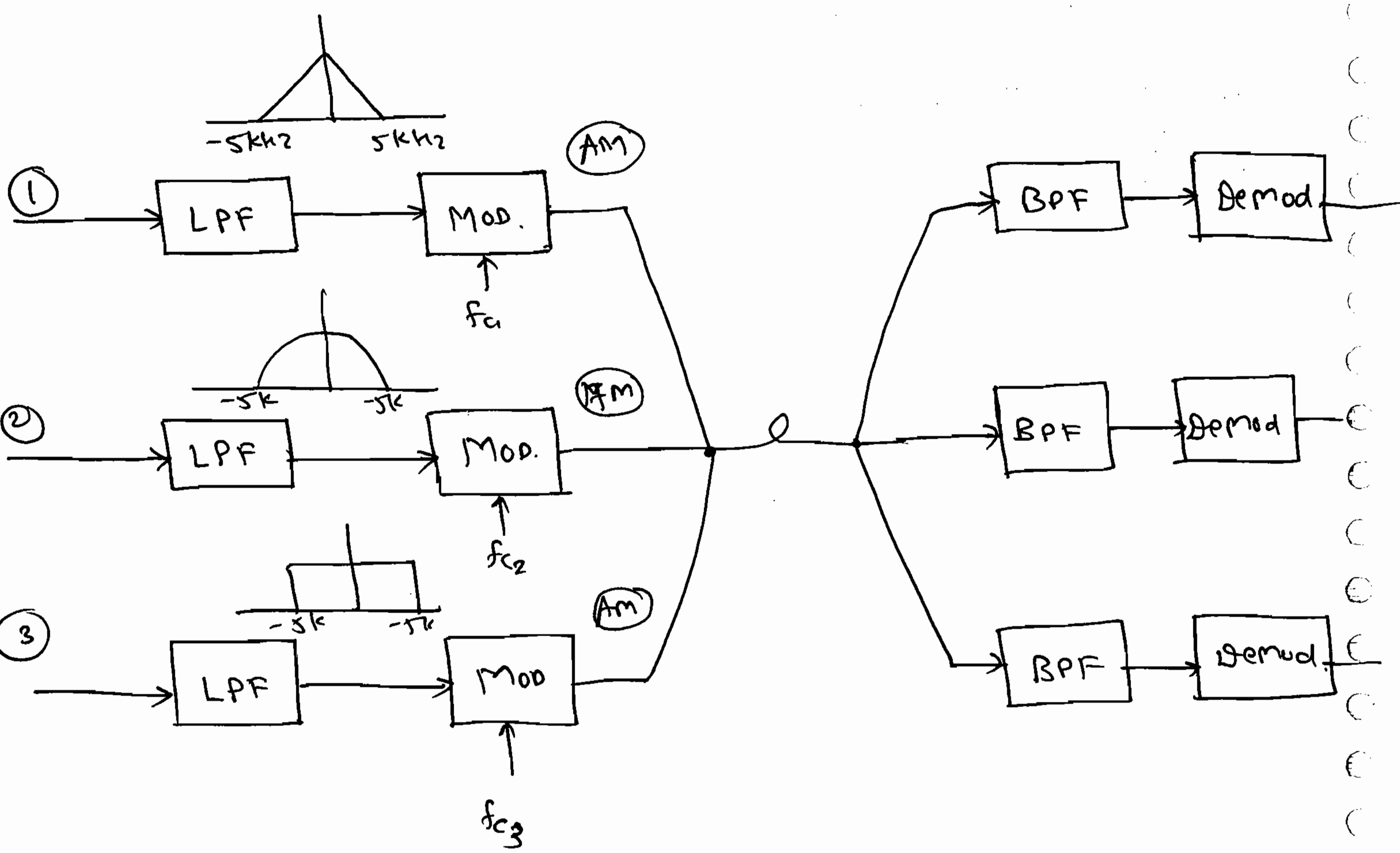




⇒ In the case of voice transmission a spectrum gape of 600 Hz is existing bet$^n$ USB and LSB. So, it is possible to eliminate one side band completely even though filter is not having sharp cut-off frequency.

# * Frequency Division Multiplexing (FDM):

⇒ Transmission of more than one signal through same ~~transmission~~ ~~line~~ Communication channel is called as Multiplexing.

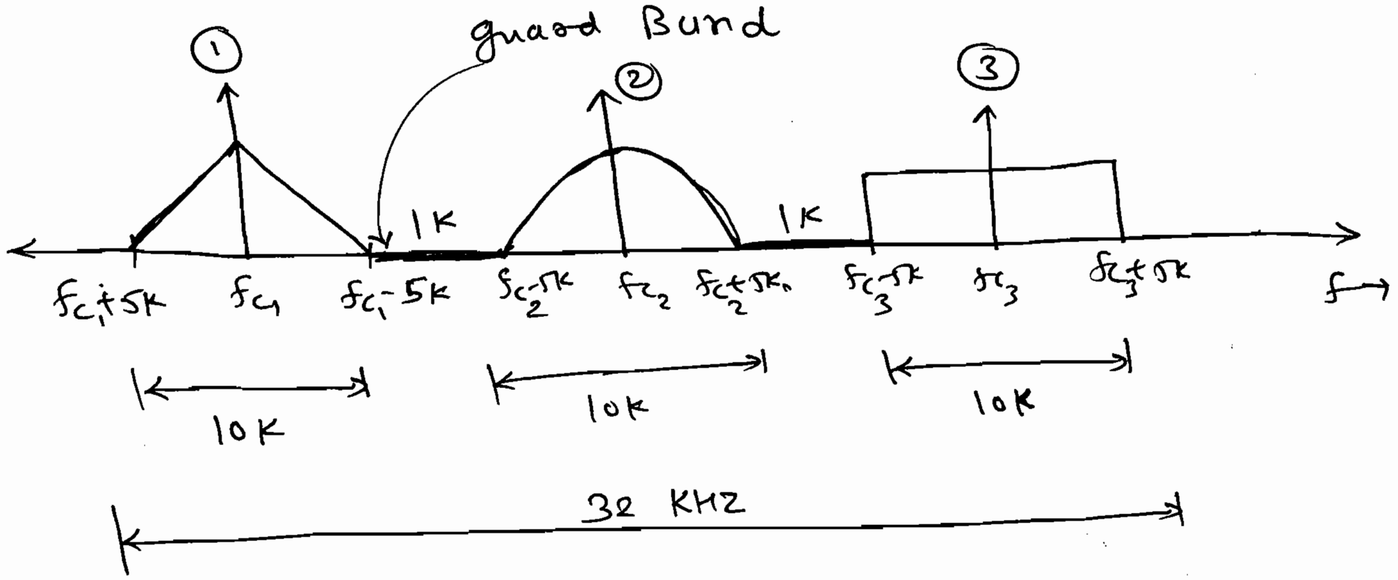


⇒ The LPFs are used to eliminate the insignificant high frequencies and cut-off freq. depends on the application.

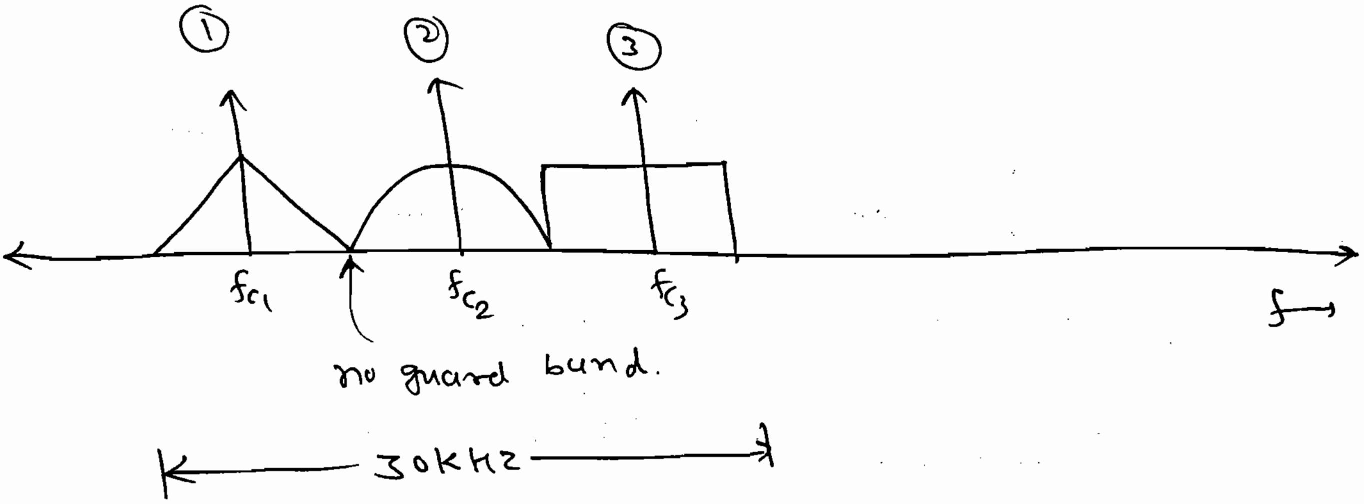
→ Assume that all the signals are band-limited to 5 KHz.

⇒ Assuming all modulators as Am, the spectrum of the multiplexed signal is as shown in fig.

=>



=>



=> Practically guard band is required. Because BPF will not sharp cut-off frequencies.

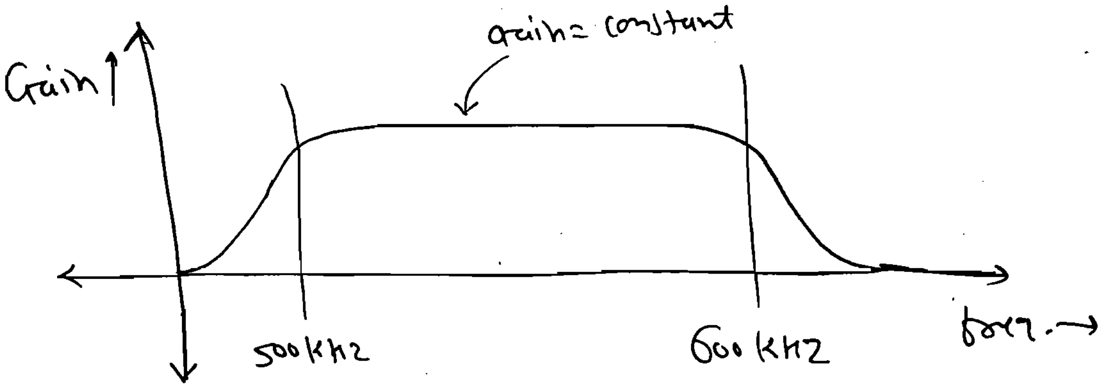
-> Bandpass filters are used at the receiver to select the required signal.

-> The no. of signals which can be multiplexed through a channel is depends on the channel B.W. & signal B.W.

=> Channel BW is defined as the range of freqs. that the channel is capable of transmitting without distortion.

⇒ In order to multiplexed more no. of signals; Channel BW should be as high as possible and signal BW should be as low as low possible.

⇒ For Channel BW.



Freq. response of Channel.

⇒ To transmitt a signal without any distortion the gain of the channel should be constant. For an ideal Channel the gain is constant from 0 to ∞.

⇒ In order to transmit a signal without any distortion BW of Channel should be greater than the BW of the signal.

⇒ Twisted Pair → 500 kHz ← Means the gain of Channel is remain constant through range of 500 kHz frequencies.

Coaxial Cable → 500 MHz.

FOC → GHz.

Ex-1: 10 signals are band limited to 5 KHz are transmitted through a channel after modulation using FDM, the guard band is 2 KHz. Determine the B.W. of the multiplexed signal.

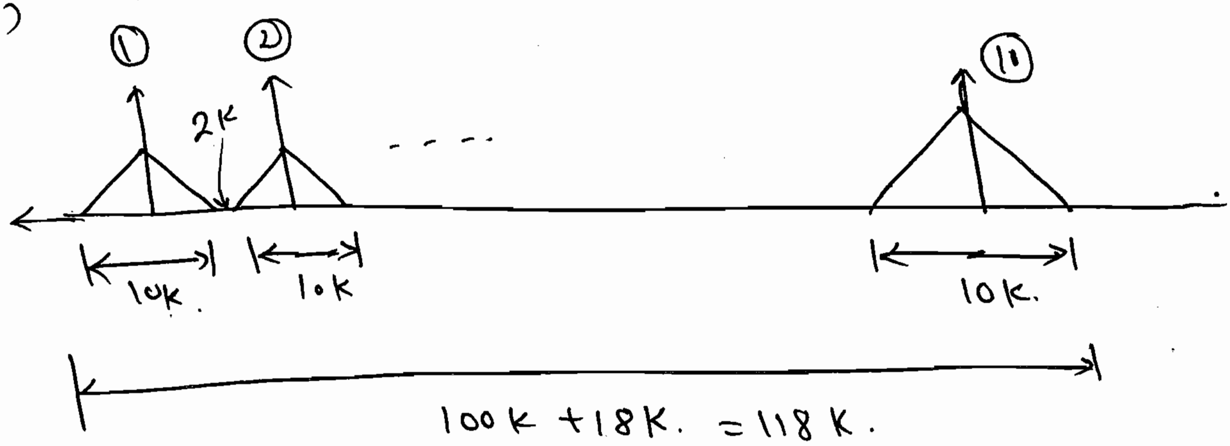
i) If all Modulators are AM.

ii) If all Modulators are DSB.

iii) If all Modulators are SSB.

Ans:

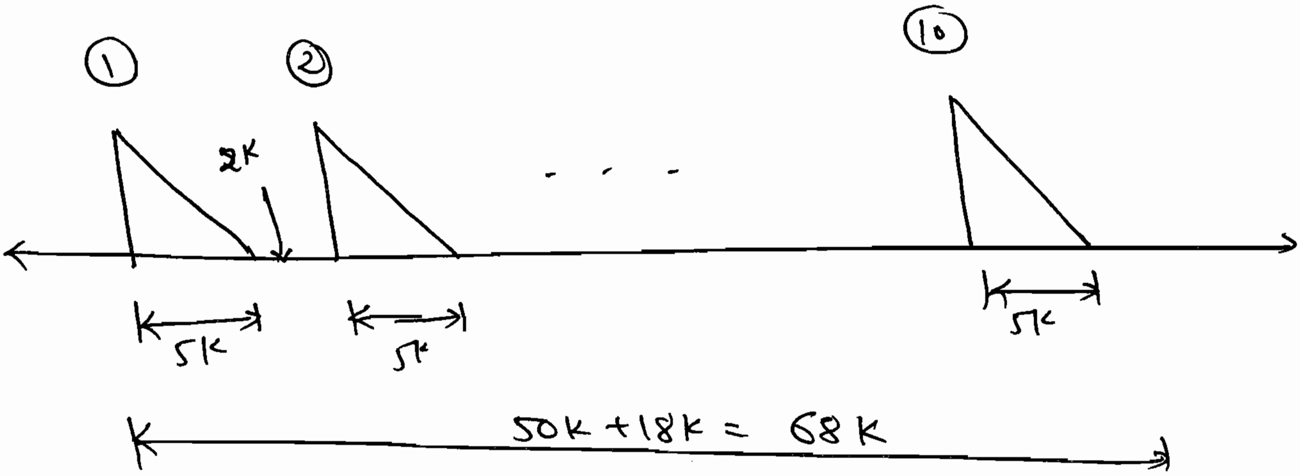
(i)



100K + 18K. = 118 K.

(ii) Same as above but there are no carrier.

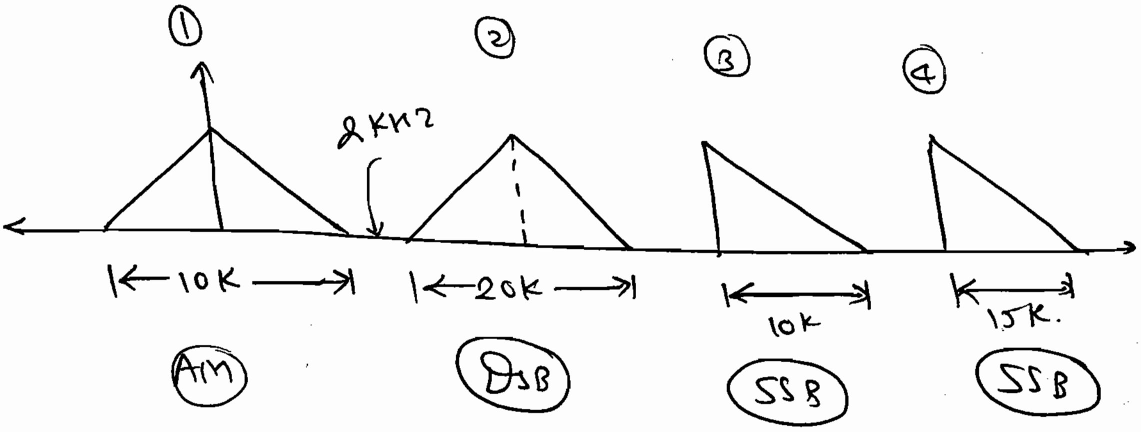
(iii)

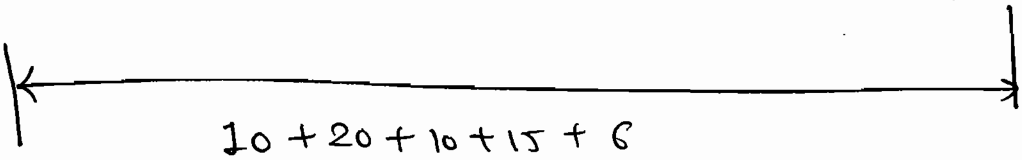


50K + 18K = 68 K

Q-2 4 signals each bandlimited to 5K, 10K, 10K, 15K, are transmitted through a channel after modulation using FDM. In modulators used are AM, DSB, SSB & SSB respectively. Assuming guard band of 2 KHz. Determine the minimum BW of channel required.

Ans:





$10 + 20 + 10 + 15 + 6$

$= 61 \text{ K HZ}.$

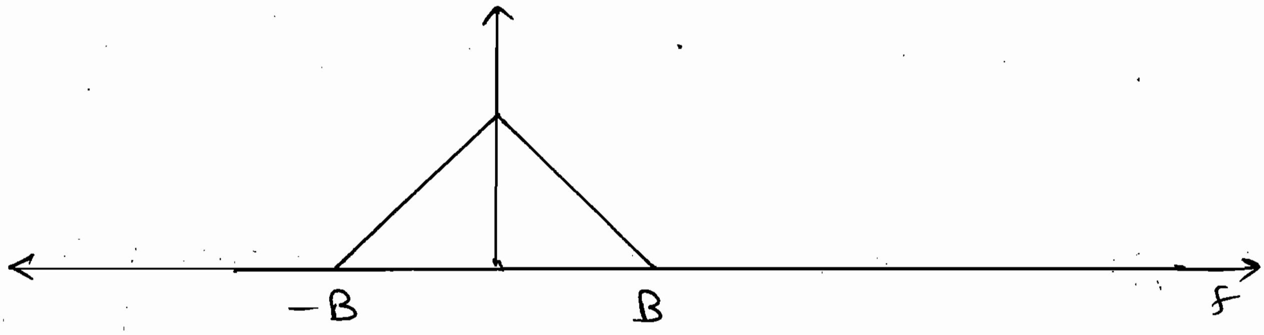
# ④ Vestigial Side Band Modulation: (VSB).

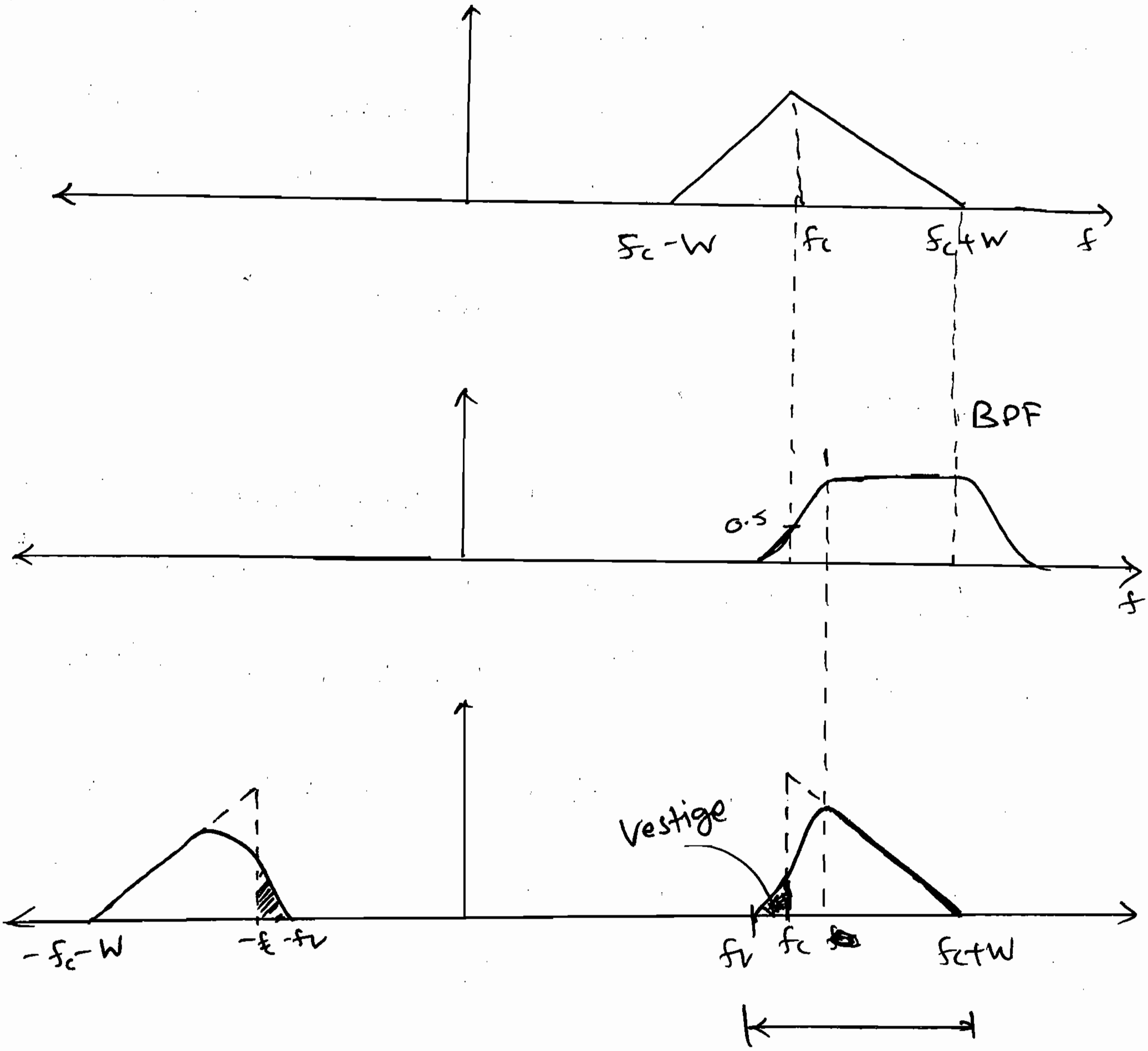
→ Video signal consist of significant freq. upto 4.5 MHz. If AM (or) DSB is used the BW is 9 MHz and it is not possible to transmit more no. of signals. So AM and DSB are not used for video signal transm.

→ SSB modulation is suitable for voice transmission only.

→ The Generation and Demodulation technique of VSB are same as SSB except few modification at the Band pass filter.

→ The Bandpass filter should be designed so that the gain is not constant in the entire upper side band.





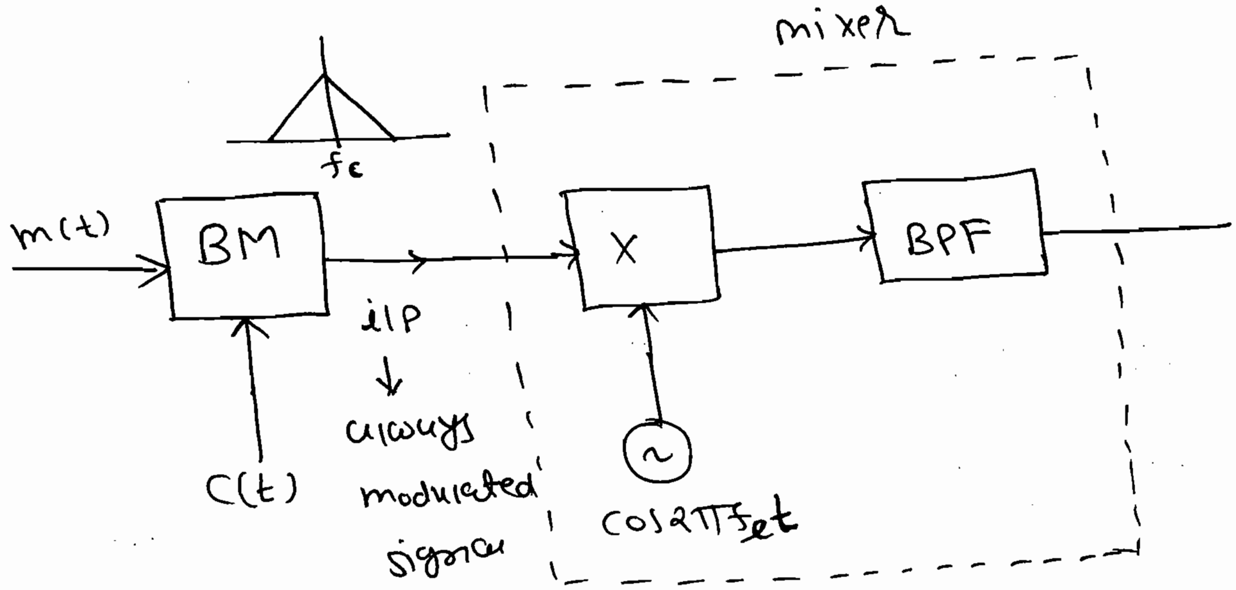
$BW = W + f_v.$

⇒ BW of Vestige depends on design of BPF.

# * Mixer: H.B.

→ Mixing (or) Heterodyning.

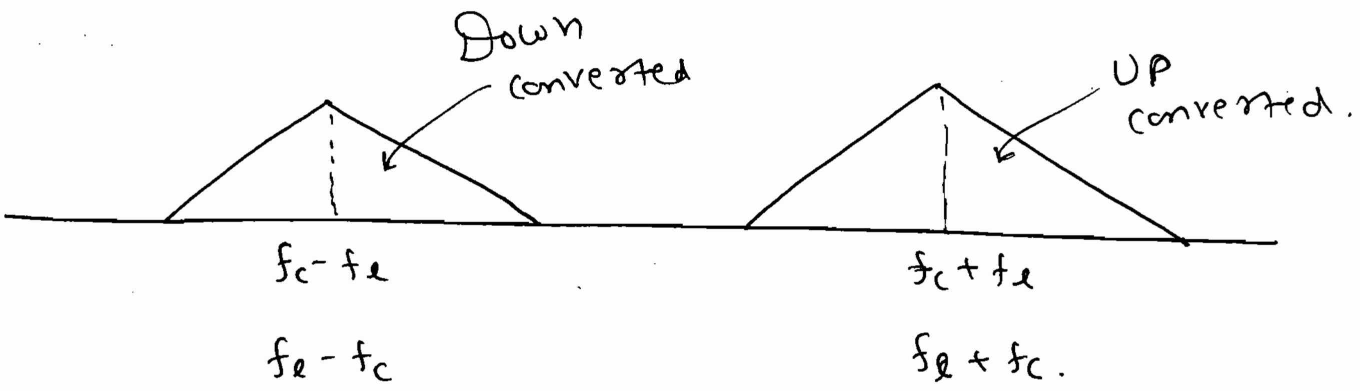
→ Mixer is used to change the carrier frequency of a modulated signal.



⇒ o/p of multiplier = $m(t) \cdot \cos 2\pi f_c t \cdot \cos 2\pi f_l t$.

① Condition - ⊛ :- $f_c > f_l$ ② Condition : $f_l > f_c$

$$\Rightarrow \frac{1}{2}\left[m(t) \cos 2\pi (f_c + f_l)t + m(t) \cos 2\pi (f_c - f_l)t\right].$$



⊛

$\boxed{f_l > f_c}$

~~$f$~~ $f_l = 8$ MHz, 12 MHz ✓

# ☆ Angle Modulation

⇒ The generalized eqn of the Carrier is

$$\overset{\overset{Am}{\downarrow}}{A_c} \cos\underbrace{(2\pi \overset{\overset{FM}{\downarrow}}{f_c} t + \overset{\overset{PM}{\downarrow}}{\phi})}_{\theta(t)}.$$

↙ H.B

$$\Rightarrow \quad A_c \cos(\theta(t)).$$

→ "$2\pi f_c t + \phi$" is called as the angle of the Carrier.

⇒ Whenever the frequency (or) the phase changes angle also changes. So FM & PM are also called as Angle modulation Technique.

## ✱ Frequency Modulation:-

**Definition:**

⇒ FM is defined as the process in which the frequency of the carrier is varied according to the message signal.

⇒ Freq. of Carrier before modulation is $f_c$ and after modulation $f_i$ where

$$\Rightarrow \quad \boxed{f_i = f_c + k_f m(t)}$$ ← H.B.

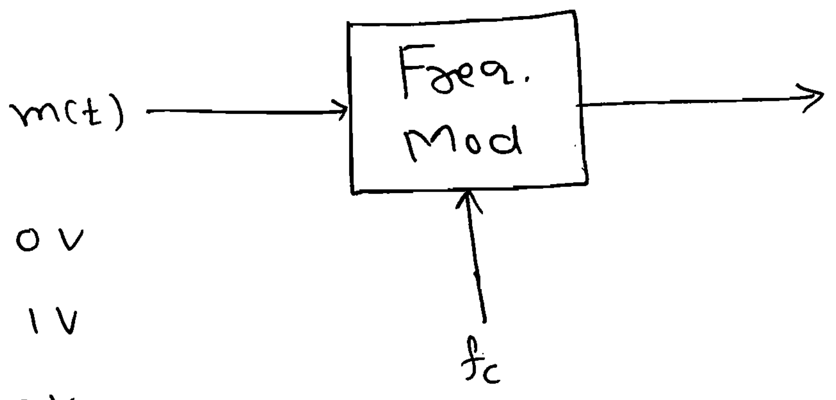
$K_f$ = Frequency Sensitivity of modulator $\left(\frac{Hz}{Volt}\right)$.

⇒ If the Carrier is not modulated $m(t)=0$

and $\boxed{f_i = f_c}$

→ '$K_f$' indicate the change in the freq. per 1V change of the message signal.

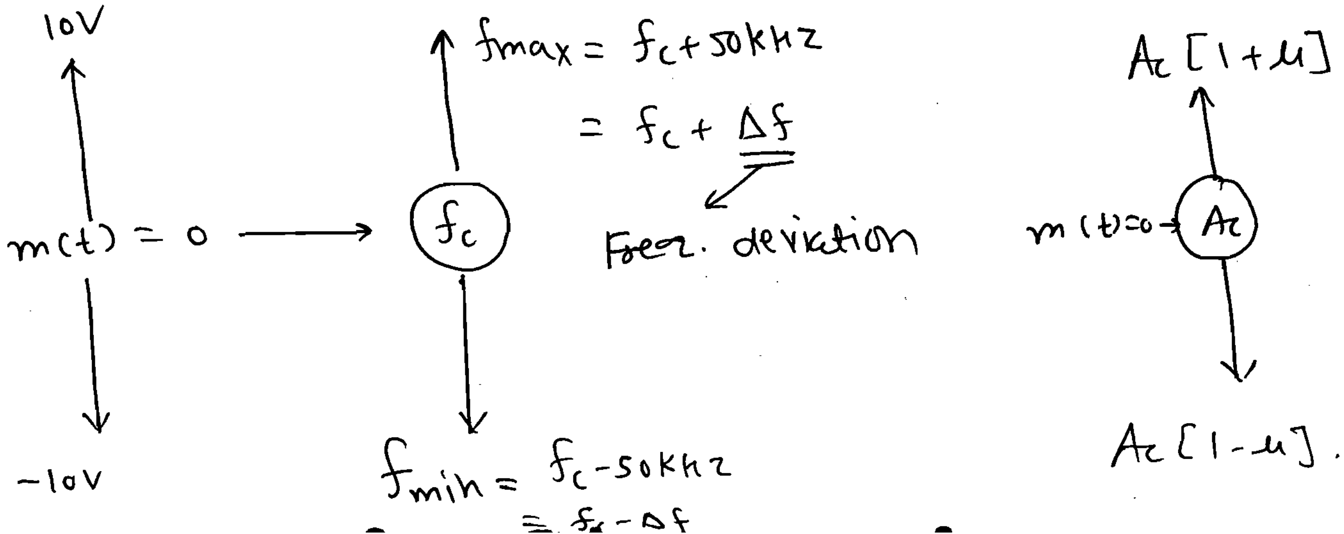
⇒ $K_f$ = 5 kHz/volt



$\rightarrow f_i = f_c + K_f m(t)$.

| | |
|---|---|
| = 0 V | $f_i = f_c$ |
| = 1 V | $f_i = f_c$ + 5kHz. |
| = 2 V | $f_i = f_c$ + 10 kHz |
| = 5 V | $f_i = f_c$ + 25kHz |
| = 10V | $f_i = f_c$ + 50 kHz |
| = -10V | $f_i = f_c$ - 50 kHz. |

→ FM modulator is a Voltage to Frequency converter.



⇒ In the case of single tode modulation,

$m(t) = A_m \cos 2\pi f_m t$.

→ $f_i = f_c + K_f m(t)$.

$f_i = f_c + K_f A_m \cos 2\pi f_m t$.

→ $f_{max} = f_c + K_f A_m. = f_c + \Delta f.$

→ $f_{min} = f_c - K_f A_m. = f_c - \Delta f.$

⇒ $\Delta f = K_f A_m.$ ✓

Generalized formula. ↙ H.B.

$$\therefore \boxed{\Delta f = K_f [m(t)]_{max}}$$

⇒ $A_c \cos(\theta(t))$.

→ $\theta(t) = \omega_i t$

$\therefore \theta(t) = 2\pi f_i t$.

$\therefore f_i \ \frac{d}{dt} \theta(t) = 2\pi f_i$

↙ H.B.

$$\therefore \boxed{f_i = \frac{1}{2\pi} \frac{d}{dt}(\theta(t)).}$$

⇒ $\theta(t) = 2\pi \int f_i \, dt$

$= 2\pi \int [f_c + K_f m(t)] \, dt$.

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi k_f \int m(t)\,dt.} \leftarrow \underline{H.B.}$$

→ The above eqn represent the angle in the case of FM.

→ So, Time domain eqn of FM for <u>multitone</u> Modulation is,

$\swarrow$ H.B.

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right].}$$

↑ multitone

→ for single tone modulation

let, $m(t) = A_m \cos 2\pi f_m t.$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int A_m \cos 2\pi f_m t\right].$$

$$= A_c \cos\left[2\pi f_c t + \frac{2\pi k_f \cdot A_m}{2\pi f_m} \cdot \sin 2\pi f_m t\right].$$

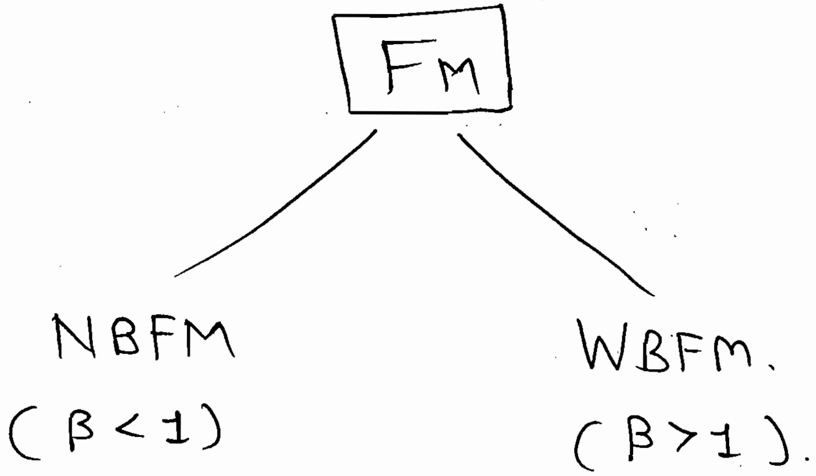
$$\therefore S(t) = A_c \cos\left[2\pi f_c t + \left(\frac{k_f A_m}{f_m}\right) \sin 2\pi f_m t\right].$$

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right].}$$

↑ single tone.

$$\Rightarrow \boxed{\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \text{Modulation index.} = \frac{\text{Freq. deviation}}{\text{message freq.}}}$$

⇛



→ In Am, modulation index Concept is used for the demodulation and when envelope Detector is used as demodulator.

→ In DSB, SSB and VSB, modulation index Concept is not used because the envelope detector is not used as demodulator.

→ In FM, Modulation index concept is required for anulyzing the Spectrum.

# ① Narrow Band FM (NBFM) : $\boxed{\beta < 1}$

→ $S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$.

$= A_c \cos 2\pi f_c t \cdot \cos[\beta \cdot \sin 2\pi f_m t]$

$- A_c \sin 2\pi f_c t \cdot \sin[\beta \sin 2\pi f_m t]$.

let, $\theta = \beta \sin 2\pi f_m t$.

$\therefore S(t) = A_m \cos 2\pi f_c t \cdot \cos\theta - A_c \sin 2\pi f_c t \cdot \sin\theta$.

$\theta < 1$.

$\cos\theta \simeq 1$

$\sin\theta \simeq \theta$.

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\Rightarrow \lim_{\theta \to 0} \sin\theta = \theta$

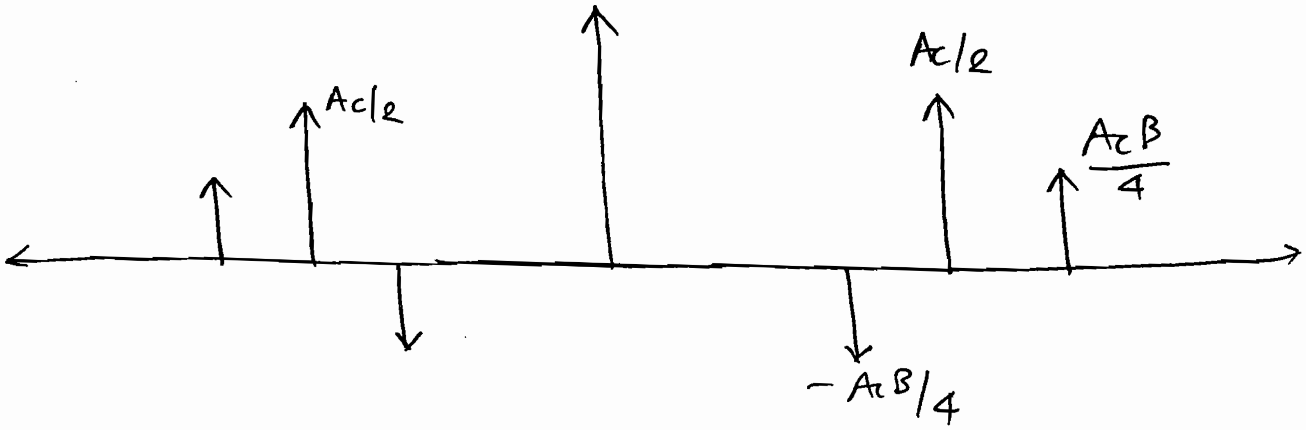
$\therefore S(t) = A_c \overset{\cos}{} 2\pi f_c t \cdot \underline{\underline{1}} - A_c \sin 2\pi f_c t \cdot \underline{\underline{\theta}}$.

H.B.

$\therefore \boxed{S(t) = A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t.}$

$\therefore \boxed{S(t) \simeq A_c \cos 2\pi f_c t + \frac{A_c\beta}{2} \cos 2\pi (f_c + f_m) t - \frac{A_c\beta}{2} \cos 2\pi (f_c - f_m) t.}$ ← H.B

⇒



→ The LSB in the spetrum is out of Phase by 180°.

Ex: (1) An Am signal and NBFm signal having same modulation index are added, the resultant signal is,

Ⓐ Am

Ⓑ DSB

Ⓒ SSB

Ⓓ SSB with Carrier. ✓

Ans: LSB = $\frac{A_c\mu}{4}$ , LSB = $-\frac{A_c\beta}{4}$

$\mu = \beta$ so, ans Ⓓ SSB with Carrier.

⇒ The magnitude spectrum of NBFm is same as Am. so, BW and power are also same as Am.

$$\rightarrow \boxed{P_t = P_c\left[1 + \frac{\beta^2}{2}\right].} \leftarrow$$ h.B

NOTE:

⇒ Practically NBFm is not used because of its similarity to Am.

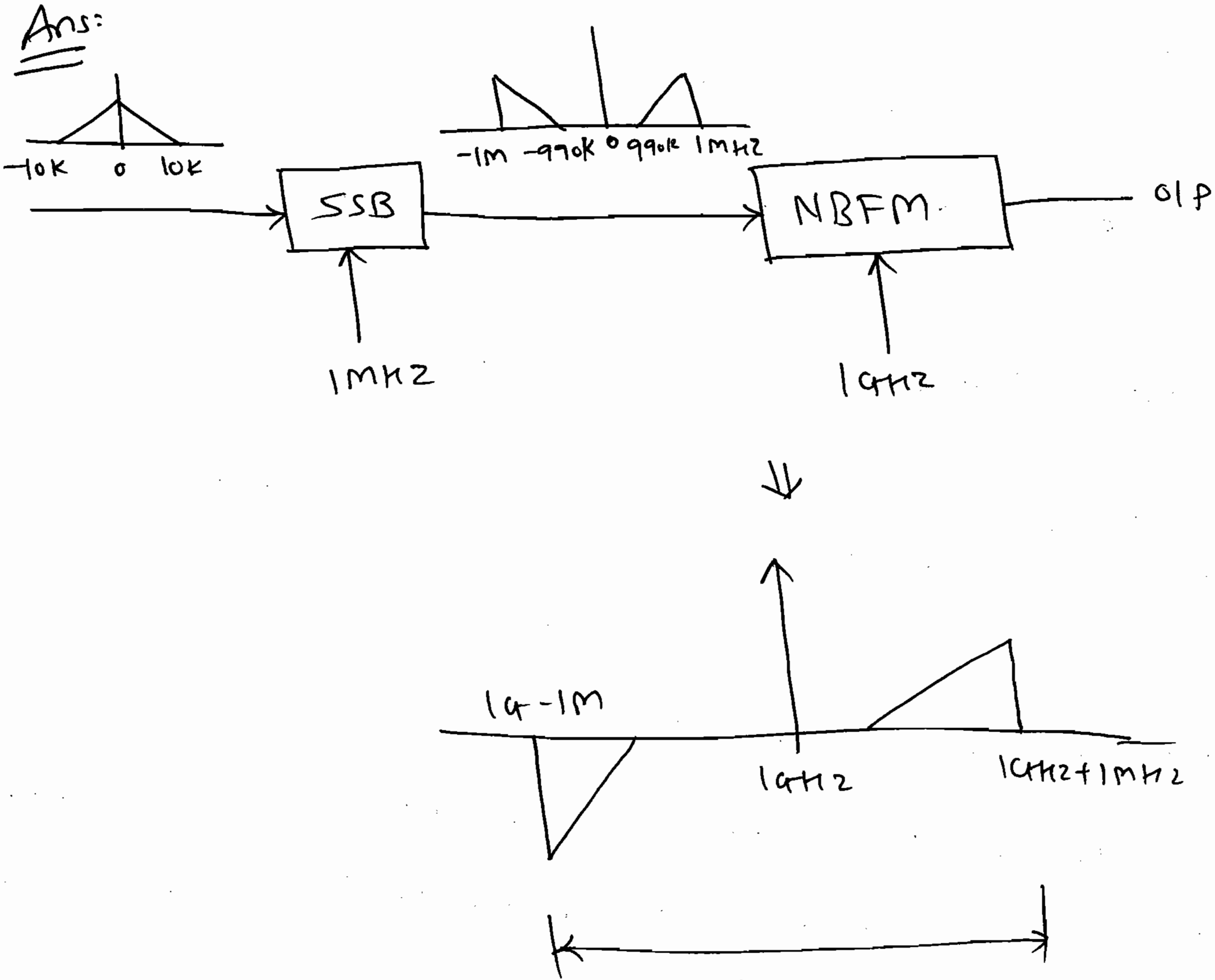
## Gate: 2006:

**Ex- 2** A message signal bandlimited to 10 KHz is lower sideband SSB modulated with a carrier frequency of 1 MHz. The resultant signal is again passed through NBFM modulator having a carrier freq. of 1 GHz. Determine the BW of signal at O/P.

Ⓐ 10 KHz Ⓑ 20 KHz

Ⓒ 1 MHz [✓ Ⓓ 2 MHz.]

**Ans:**



2 MHZ

= BW.

## ② Wide Band FM : (WBFM): ($\underline{\beta > 1}$)

⇒ Time domain eqn for the single tone modulation is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t].$$

## * Bessel Function:

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)} . d\theta$$

⇒ **Properties:**

① $J_n(x) = (-1)^n J_{-n}(x).$

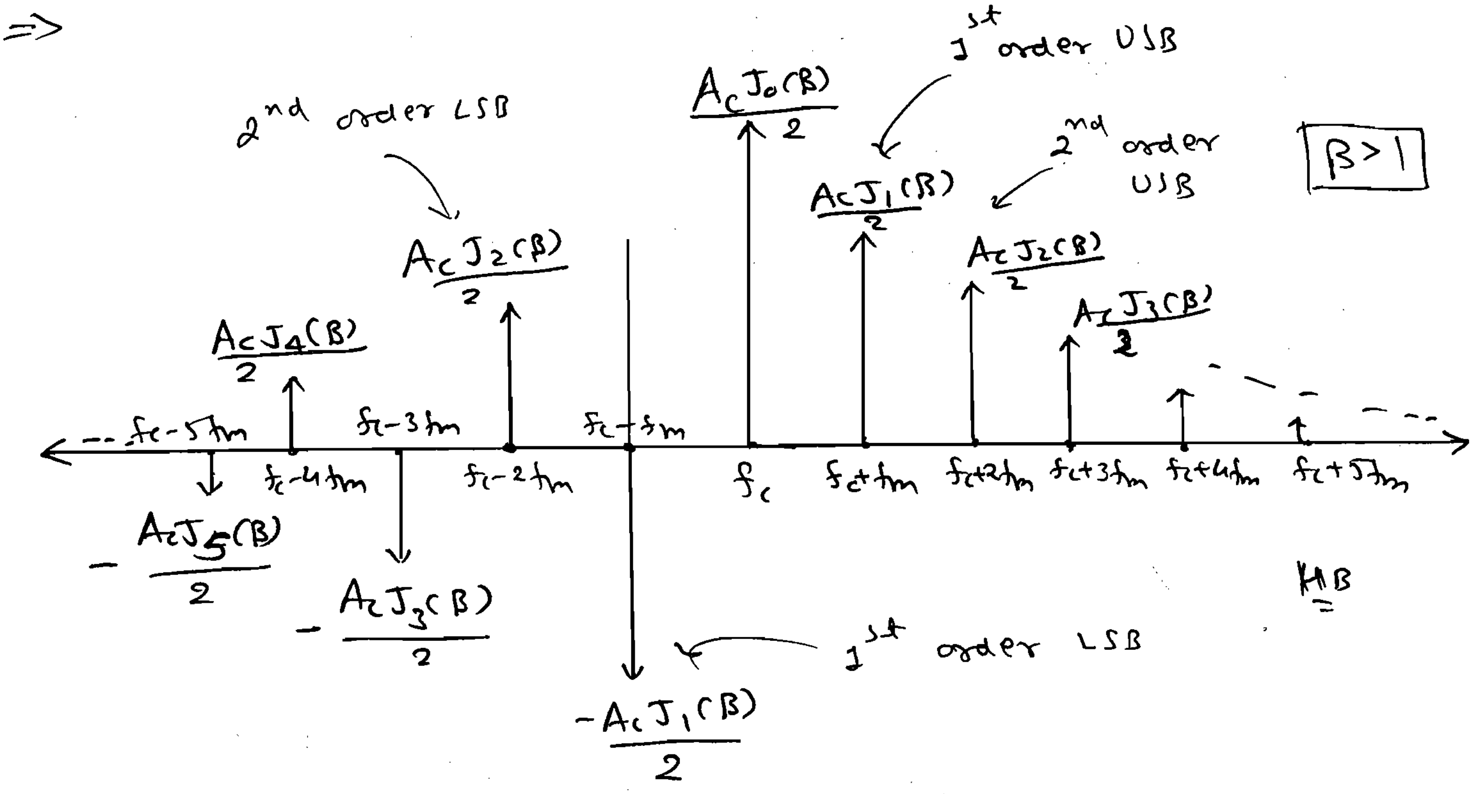
② $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1. \leftarrow \underline{H.B}$

⇒ Time domain eqn of WBFM is $\underline{H.B}$

$$\boxed{S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta).\cos[2\pi(f_c + n f_m)t]}$$

$$\begin{aligned}
\therefore s(t) = {} & A_c J_0(\beta).\cos 2\pi f_c t \\
& + A_c J_1(\beta).\cos 2\pi(f_c+f_m)t + A_c J_{-1}(\beta).\cos 2\pi(f_c-f_m)t \\
& + A_c J_2(\beta).\cos 2\pi(f_c+2f_m)t + A_c J_{-2}(\beta).\cos 2\pi(f_c-2f_m)t \\
& + A_c J_3(\beta).\cos 2\pi(f_c+3f_m)t + A_c J_{-3}(\beta).\cos 2\pi(f_c-3f_m)t \\
& + \ldots
\end{aligned}$$

=>



* Analysis of spectrum:

=> The spectrum consist of carrier and infinite no. of upper and lower sideband frequencies.

=> Theoretical BW is infinite.

=> The magnitude of the spectral component depends on the Bessel function coefficient. But Bessel function's values gradually decreases as $n$ increases. So, the magnitude of higher order of frequencies are negligible.

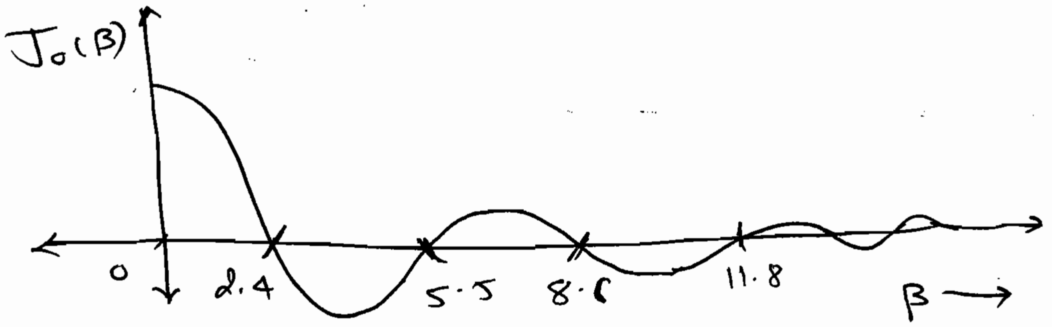
=> The carrier magnitude in the spectrum varies with modulation index.

=> The Bessel function coefficient $J_0(\beta)=0$ where $\beta = 2.4, 5.5, 8.6, \ldots$

⇒ The Bessel function coefficient

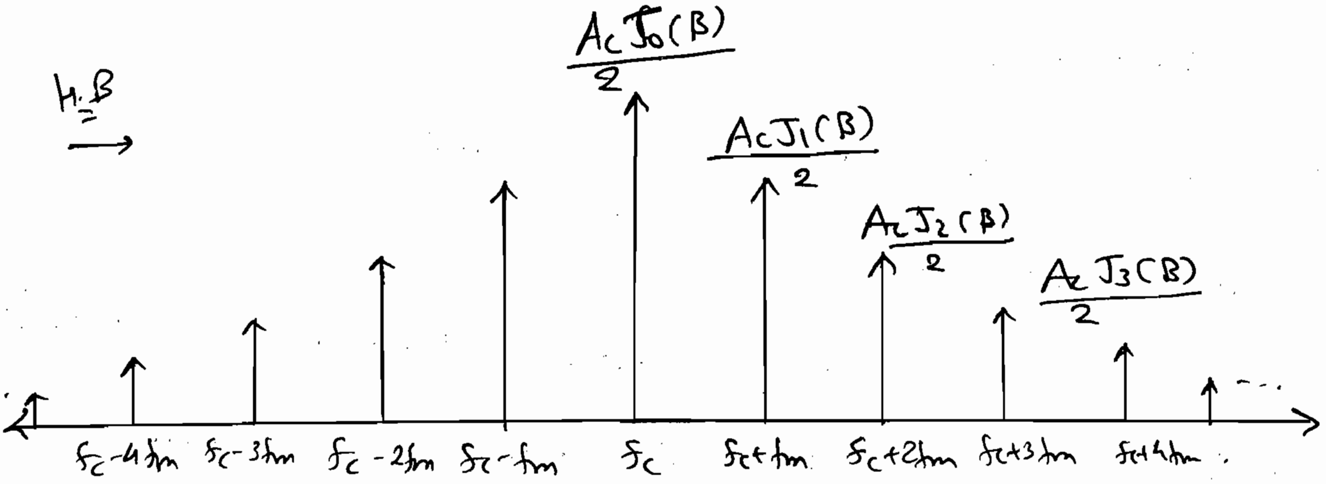
$\boxed{J_0(\beta) = 0.}$ ← H.B.

where $\beta = 2.4, 5.5, 8.6, \ldots$



⇒ For these values of β, the carrier will zero magnitude in the spectrum and modulation efficiency is 100%.

H.B →



[Magnitude Spectrum].

## * Power Calculation: I.B.

$\Rightarrow$

$$P = \frac{V_{rms}^2}{R}$$

$$\therefore P_{fc} = \frac{\left(\frac{A_c J_0(\beta)}{\sqrt{2}}\right)^2}{R}.$$

$$\therefore P_{fc} = \frac{A_c^2 J_0^2(\beta)}{2R}.$$

$$\left.\begin{aligned} \Rightarrow P_{fc+fm} &= \frac{A_c^2}{2R} J_1^2(\beta) \\ P_{fc-fm} &= \frac{A_c^2}{2R} J_1^2(\beta) \end{aligned}\right\} \text{first order SB Power} = \frac{A_c^2 J_1^2(\beta)}{R}$$

$$\left.\begin{aligned} \Rightarrow P_{fc+2fm} &= \frac{A_c^2}{2R} J_2^2(\beta) \\ P_{fc}-2fm &= \frac{A_c^2}{2R} J_2^2(\beta) \end{aligned}\right\} 2^{nd} \text{ order SB power} = \frac{A_c^2 J_2^2(\beta)}{R}.$$

$$\therefore P_t = \frac{A_c^2}{2R}\left[\sum_{n=-\infty}^{\infty} J_n^2(\beta)\right]$$

$$\therefore P_t = \frac{A_c^2}{2R} \cdot 1$$

$$\therefore \boxed{P_t = \frac{A_c^2}{2R}.} \rightarrow \text{same as Unmodulated carrier Power}$$

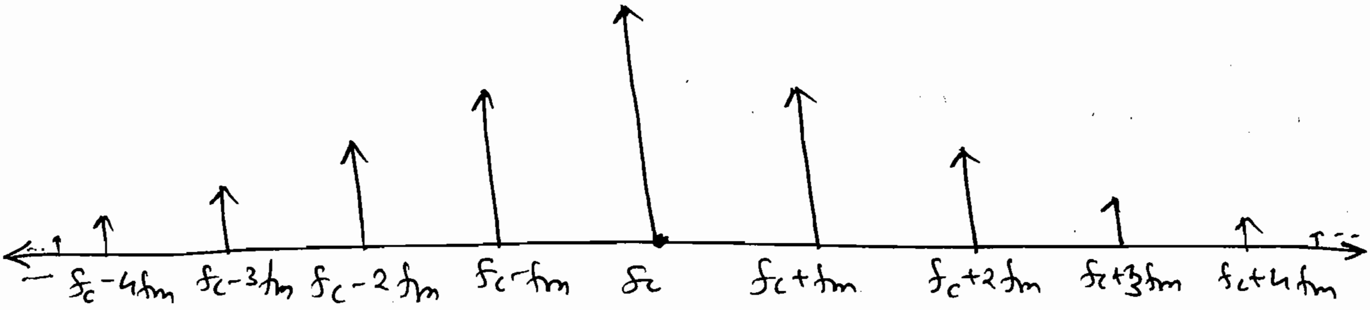
[$A_c \cos 2\pi f_c t$].

## * Transmission B.W. of FM signal using Carson's Rule:

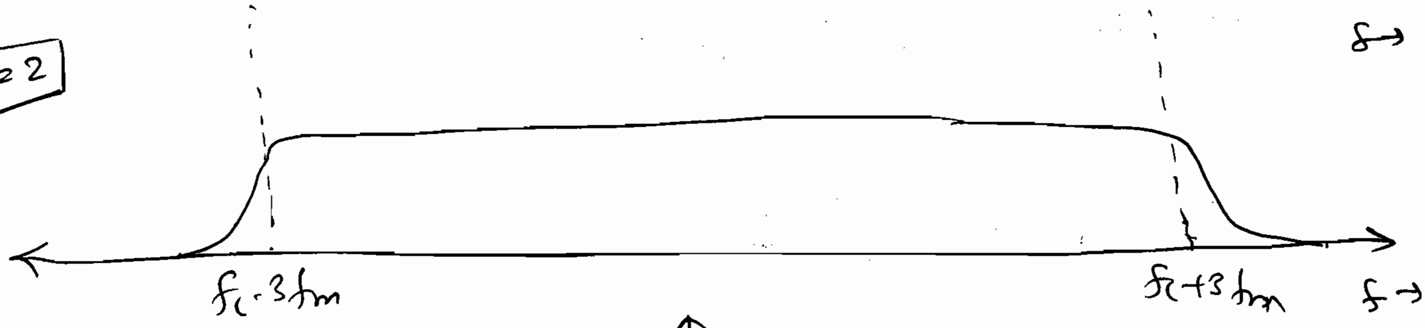
⇒ The theoretical B.W. of FM signal is ∞.

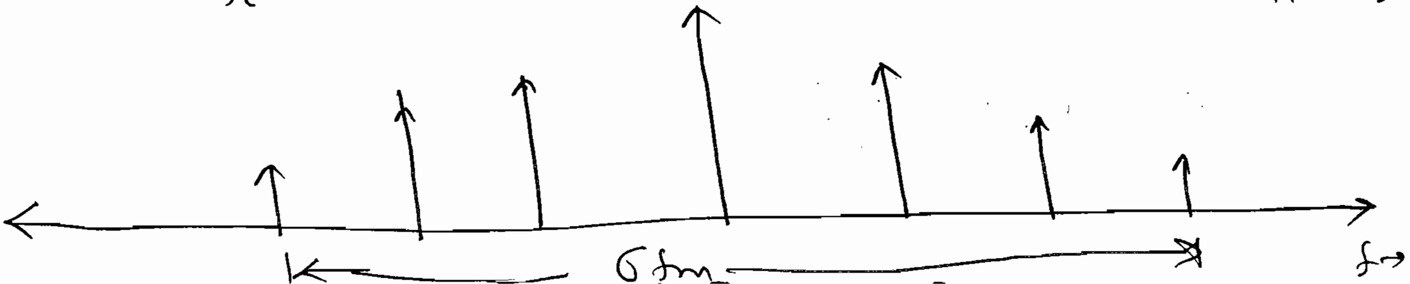
⇒ Practically BW of the signal should be as minimum as possible. So insignificant frequencies should be eliminated.

⇒ According to Carson's rule β+1 Upper and Lower sidebands will have significant magnitude and contains 99% of the total power. So, the FM signal is passed through a Band pass filter to eliminate the insignificant frequencies.



$\boxed{\beta=2}$





⇒ **Carson's rule:** H.B

$$\boxed{B.W. = 2(\beta+1) f_m} \quad — ①$$

$$= 2\left(\frac{\Delta f}{f_m} + 1\right) f_m$$

H.B.

$$\boxed{B.W. = 2\Delta f + 2 f_m} \quad — ②$$

**Ex-1** A carrier signal is frequency modulated by a sinusoidal signal of amplitude 20V and freq. 100 KHZ. The freq. sensitivity of modulator is 25 KHZ/V.

(i) Determine the freq. deviation, modulation index & BW.

(ii) Repeat the above calculation when the amplitude of the message signal is 40V.

**Ans:** $A_m = 20V$, $f_m = 100$ KHZ, $k_f = 25$ KHZ/V.

(i) $\Delta f = K_f \cdot A_m = 20 \times 25 = 500$ KHZ.

$$\therefore \beta = \frac{\Delta f}{f_m} = \frac{500 \text{ KHZ}}{100 \text{ KHZ}} = 5.$$

$\therefore \beta = 5.$

$BW = 2(\beta+1) f_m.$

$\therefore \quad BW = 2(6) \times 100\,kHz.$

$= 1200\,KHz.$

$\therefore \quad \boxed{BW = 1.2\,MHz}$

(ii) When $A_m = 40V$

$\Delta f = K_f . A_m = 25\,KHz/V \times 40V$

$\Delta f = 1000\,KHz$

$\therefore \beta = \frac{\Delta f}{f_m} = \frac{1000\,KHz}{100\,KHz} = 10.$

$\therefore \quad BW = 2(\beta+1) f_m$

$= 2 \times 11 \times 100\,KHz$

$\boxed{BW = 2.2\,MHz.}$

Ex-2 Consider the Fm signal

$s(t) = 10 \cos[2\pi 10^6 t + 8 \sin 4\pi 10^3 t]$

(i) Determine Modulation Index, Freq. deviation, BW and power.

(ii) Repeat the above calculation when the message freq. is doubled.

Ans: $A_c = 10$, $f_c = 10^6$, $\beta = 8$, $f_m = 2 \times 10^3$ Hz.

(i) $\boxed{\beta = 8}$

$\therefore \ \beta = \dfrac{\Delta f}{f_m}$

$\therefore \ \Delta f = \beta \cdot f_m$

$\Delta f = 8 \times 2 \text{KHz} = 16 \text{KHz}.$

$\therefore \ BW = 2(\beta+1) f_m = 2(9) \times 2 \text{KHz}$

$= ~~8{\cdot}8~~ \ 36 \text{ KHz}.$

$\therefore \ P_t = \dfrac{A_c^2}{2R} = \dfrac{100}{2 \times 1} = 50W.$

(ii)

~~$A_m' = 2A_m$.~~

$\therefore$

$\Rightarrow$ when, $f_m' = 2 f_m.$

$\therefore \ \beta = 4$ as $\beta = \dfrac{k_f \cdot A_m}{f_m}.$

So, $\Delta f = 4 \times 4 \text{ kHz} = 16 \text{ KHz}.$

$BW = 2(4+1) \, 4 \text{ KHz}$

$BW = 40 \text{ kHz}$

$\therefore \ \boxed{P_t = 50W.}$

**Ex-3** A Carrier signal is frequency modulated by a sinusoidal signal and the frequency deviation is 8 kHz. The message freq. is 2 kHz.

i) Determine the modulation index and BW.

ii) Repeat the above calculation when the amplitude of the message signal is increased by a factor of 3 and its freq. is reduced to 1 kHz.

**Ans:** $\Delta f = 8\text{ kHz}$, $f_m = 2\text{ kHz}$.

$\therefore$ (i) $\beta = \dfrac{\Delta f}{f_m} = \dfrac{8\text{ kHz}}{2\text{ khz}} = 4.$

$\therefore$ $BW = 2(\beta+1)f_m = 2(5) \times 2\text{ kHz}$

$= 20\text{ kHz}.$

(ii) When $A_m' = 3A_m$. and =>

$f_m' = f_m - 1\text{ kHz} = f_m/2 = 1\text{ kHz}.$

$\therefore$ $\Delta f = K_f . A_m'$

$\therefore$ $\Delta f = 3 \times 8 = 24\text{ kHz}.$

$\beta = \dfrac{24\text{ kHz}}{1\text{ kHz}} = 24.$

$BW = 2(\beta+1)f_m = 2(25) \times 1K = 50\text{ kHz}.$

Ex - 4 *** A carrier signal is freq. modulated by a sinusoidal signal and freq. deviation is 50 KHz. Determine the modulation index and BW when message freq. is (i) $f_m = 500$ KHz (ii) $f_m = 500$ Hz.

Ans:- $\Delta f = 50$ KHz, $f_m = 500$ KHz

(i) $\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ KHz}}{500 \text{ KHz}}$

$\boxed{\beta = 0.1}$ (NBFM)

So, $BW = 2 f_m$ (Same as AM)

$\boxed{BW = 1 \text{ MHz}}$

(ii) $\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ KHz}}{500} = 100 > 1$ (WBFM).

$\therefore BW = 2(\beta + 1) f_m$

$= 2 \times 101 \times 500$

$\therefore \boxed{BW = 101 \text{ KHz.}}$

Ex-5 A carrier signal is freq. Modulated by a sinusoidal signal and freq. deviation is 50 KHz. Det. the Modulation index and BW when the message freq. is

(i) $f_m = 500$ KHz (ii) $f_m = 500$ Hz.

Ex-8 ***A sinusoidal signal of 4 KHz is used as modulating signal for an Am and FM transmitters. Both of the transmitter uses the same carrier. The freq. deviation of the FM transmitter is 4 times the BW of the Am signal. The magnitudes of the freq. at $f_c + 4$ KHz are same in Am and FM. Determine the modulation index of Am & FM.

$J_1(2) = 0.577$ , $J_1(4) = 0.06$, $J_1(8) = 0.235$.

Ans: $f_m = 4$ KHz

$\Delta f = 4 \times BW_{Am}$

$\Delta f = 4 \times 2 f_m$

$\therefore \Delta f = 8 f_m$

$\therefore \beta = \frac{\Delta f}{f_m}$

$\therefore \boxed{\beta = 8}$

$$\frac{\cancel{A_c} J_1(\beta)}{2} = \frac{\cancel{A_c} \mu}{4}$$

$$J_1(8) = \frac{\mu}{2}$$

$\therefore \mu = 2 \times 0.235$

$\boxed{\mu = 0.47}$

Ex-7 A carrier signal is modulated by message signal $m(t) = A_m \cos 2\pi f_m t$. In a certain experiment conducted with $f_m = 1$ KHz increasing $A_m$ (starting from 0V) it was found that the carrier magnitude in a spectrum becomes zero for the first time when Am is 2V.

① Det. frequency sensitivity of Modulator.

② Det. the value of Am for which the carrier magnitude become zero for the second time.

Ans: (i) $\uparrow \beta = \frac{k_f \cdot A_m \uparrow}{f_m}$

$\beta = 2.4, 5.5, 8.6, \ldots$

$\therefore 2.4 = \frac{k_f \cdot 2V}{1\,kHz}$

$\therefore \boxed{k_f = 1.2\ kHz/volt.}$

(ii) $\beta = \frac{k_f \cdot A_m}{f_m}$.

$\therefore 5.5 = \frac{1.2 \times A_m}{1\,kHz.}$

$\therefore \boxed{A_m = 4.6V}$

***

Ex-8 FM transmitter radiates 100 W when the carrier is not modulated. the carrier is now modulated and the modulation index is adjusted so that the magnitude of the 1st order sidebands is zero in the spectrum under this condition ① ~~ex~~ calculate the power at carrier freq. ② calculate the power in all the remaining sideband. ③ calculate the power in the second order sideband.

$J_0(0) = 1$ $J_0(3.8) = -0.4$ $J_1(2.4) = 0.52$

$J_0(2.4) = 0$ $J_0(5.1) = -0.16$ $J_1(3.8) = 0.$

$J_1(5.1) = -0.33$

$J_2(2.4) = 0.43$

$J_2(3.8) = 0.41$

$J_2(5.1) = 0.$

Ans: $P_t = \frac{A_c^2}{2R} = 100W.$

$\therefore A_c^2 = 200.$

$A_c = 14.14\ V$

$\therefore$ MI is adjustead so that the magnitude of the 1st order SB. is zero

$\therefore$ so, $J_1(3.8) = 0 \Rightarrow J_1(\beta) = 0$

$\therefore \boxed{\beta = 3.8}$

① $P_{fc} = \frac{A_c^2}{2R} . J_0^2(\beta)$

$= \frac{A_c^2}{2R} . J_0^2(3.8)$

$= 100 \times (-0.4)^2$

$P_{fc} = 16\ W$

② $P_{SB} = P_t - P_{fc}$

$= 100 - 16$

$\therefore \boxed{P_{SB} = 84\ W}$

③ $$P_{2nd\ order} = \frac{A_c^2}{R} J_2^2(3.8)$$

$$= \frac{100}{2} \cdot (0.41)^2$$

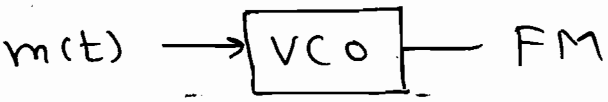
$$\therefore \boxed{P_{2nd\ order} = 33.3\ W}$$

## * Generation of WBFM signal.

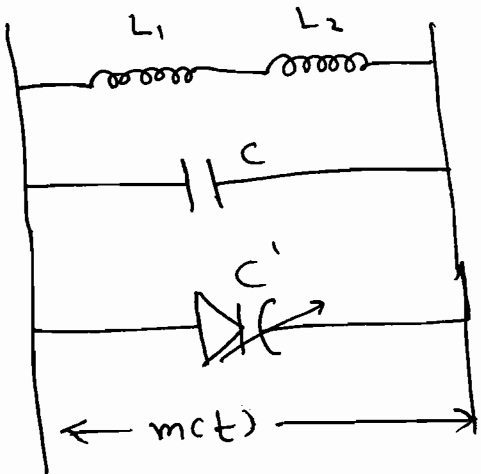
① Direct Method.

② Indirect method (OR) Arm Strong method (Practically it is not used).

### ① Direct Method:



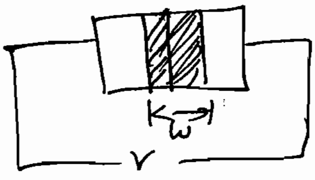
⇒ FM modulator is Voltage to frequency converter. So, VCO (voltage control oscillator) used to convert Voltage variation into freq. Variation.



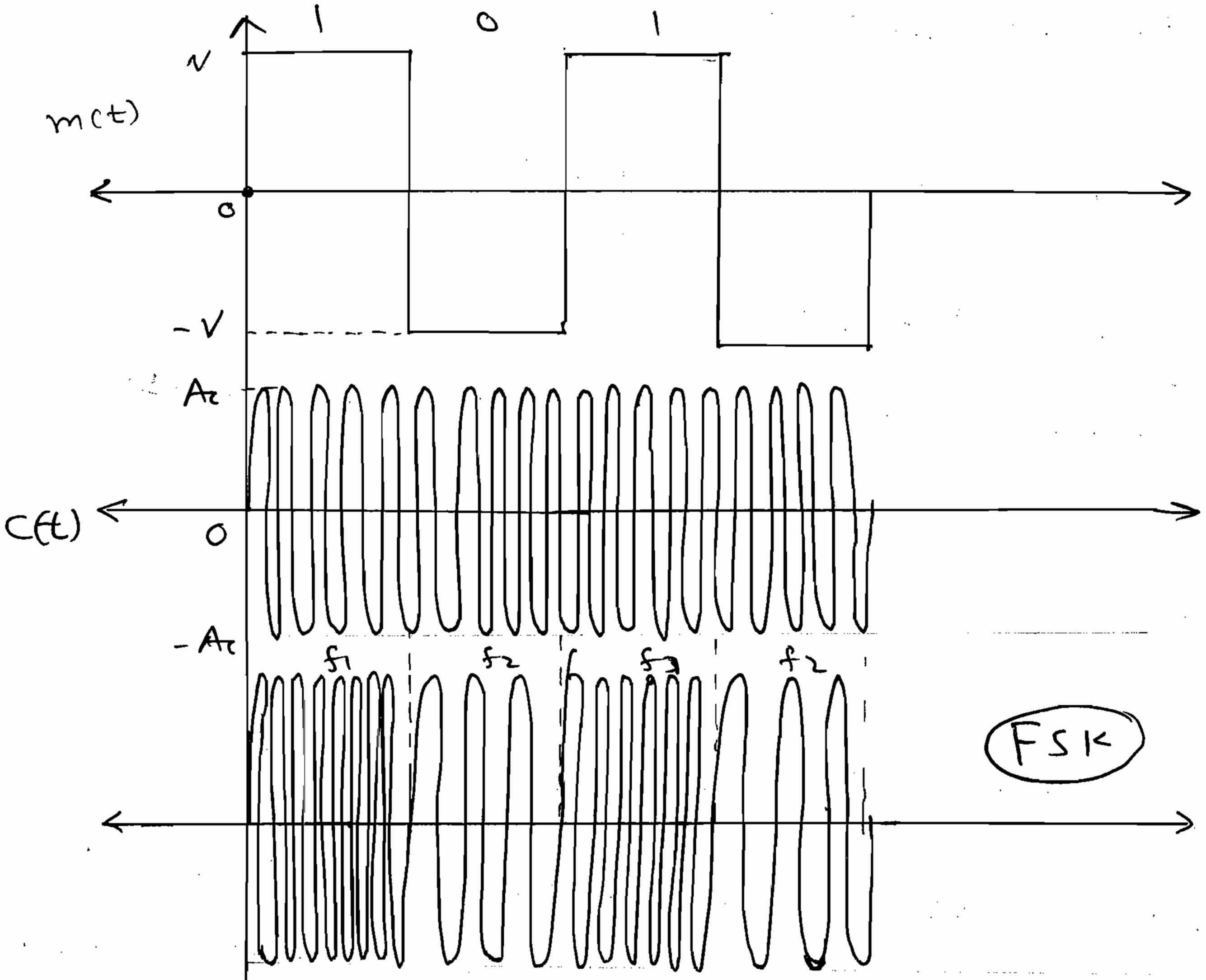
VCO

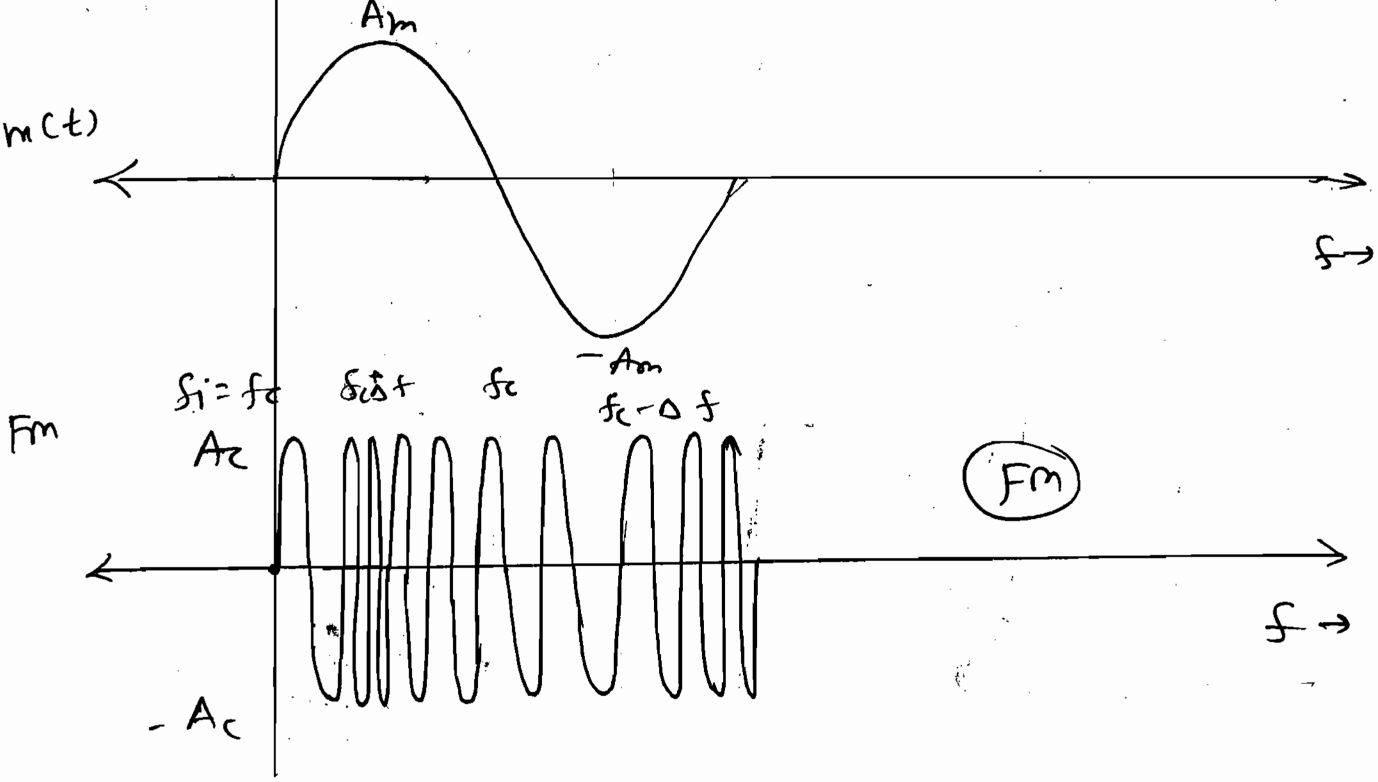
Varactor diode

$$C = \frac{\epsilon A}{W}$$



$$\therefore \boxed{f = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}}$$





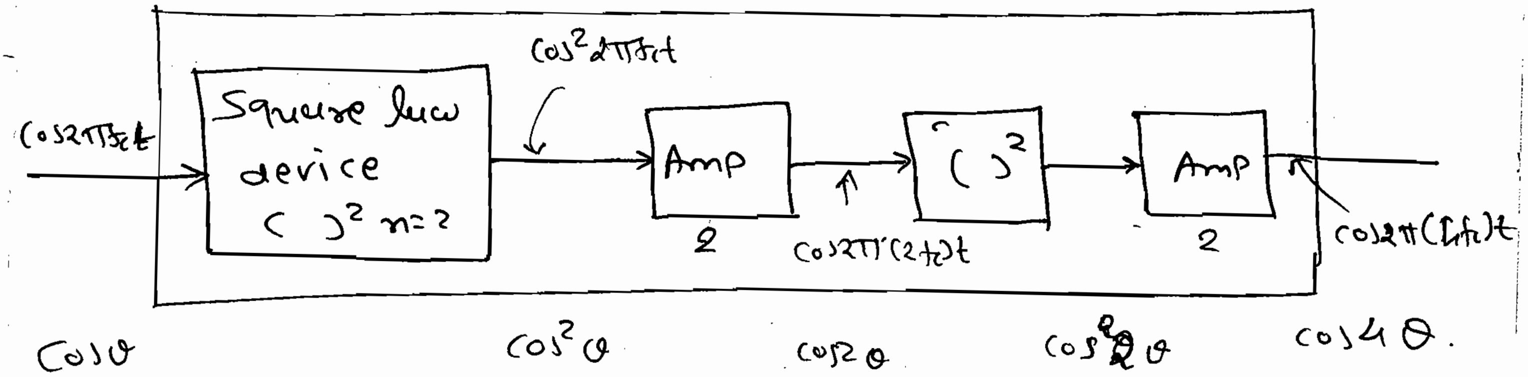
$$f_1 = f_c + k_f V = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C_1)}}$$

$$f_2 = f_c - k_f V = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C_2)}}.$$

## ② Armstrong Method:

⇒ Freq Multiplier 'n':

$\boxed{n=4}$



→ $\cos^2 2\pi f_c t = \frac{1}{2} + \frac{1}{2}\cos 2\pi(2f_c)t.$

→ Assume that the message signal & carrier signal are applied to NBFM modulator.

The o/p signal is:

$$A_c \cos[2\pi f_c t + \underbrace{\beta}_{\beta<1} \sin 2\pi f_m t]$$

⇒ If the above signal is passed through a 'freq. multiplier by n' then O/P is:

$$A_c \cos\left[n\left(2\pi f_c t + \beta \sin 2\pi f_m t\right)\right].$$

$$= A_c \cos\left[2\pi (n f_c) t + \underset{\swarrow W.B}{(n\beta)} \sin 2\pi f_m t\right].$$

⇒ In a freq. multiplier carrier freq. $f_c$ & $\beta$ is increased by a factor of $n$, but the message frequency is same.

i.e.

$$\rightarrow \text{NBFM} \rightarrow n=10 \rightarrow \text{WBFM} \longrightarrow \text{mixer} \longrightarrow \text{O/P}$$

| NBFM | WBFM | mixer |
|---|---|---|
| | | $f_l = 9$ MHz<br>$= 11$ MHz |
| $f_c = 1$ MHz | $f_c = 10$ MHz | $f_c = 1$ MHz |
| $\beta = 0.8$ | $\beta = 8$ | $\beta = 8$ |
| $f_m = 2$ kHz | $f_m = 2$ kHz | $f_m = 2$ kHz |
| $(\beta < 1)$ | $(\beta > 1)$ | O/P |

Ex-1 Consider a NBFM signal $s(t) = 10\cos\left[2\pi 10^6 t + 0.4 \sin 2\pi 10^3 t\right]$. The signal is passed through to freq. multiplier connected in cascade $n_1 = 2$ & $n_2 = 5$. Determine the carrier freq., modulation index, & BW at the O/P of first multiplier & second multiplier.

Ans: $A_c = 10V$, $f_c = 1 MHz$, $f_m = 1 KHz$, $\beta = 0.4$.

$f_c = 1 MHz$

$f_m = 1 kHz$

$\beta = 0.4$

$f_c = 2 MHz$

$f_m = 1 KHz$

$\beta = 0.8 < 1$

(NBFM)

$BW = 2 f_m$

$\boxed{BW = 2 KHz}$

$f_c = 10 MHz$

$f_m = 1 KHz$

$\beta = 4$

(WBFM)

$BW = 2(\beta+1) f_m$

$BW = 2 \times 5 \times 1 KHz$

$\boxed{BW = 10 KHz}$

⇒ We know that,

modulation index $\beta = \frac{\Delta f}{f_m}$ → No change.

↓H.B.

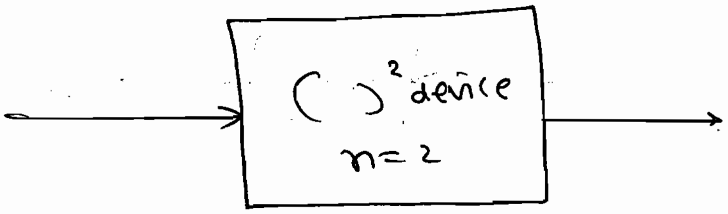
but $f_m$ is not change

∴ $\Delta f$ change, during conversion of NBFM to WBFM.

⇒ Hardware complexity of Armstrong Modulator is very high so it is not Practically used.

Ex-1 FM signal is having frequency deviation of 90 KHz is passed through a square law device the message freq. is 5 KHz. Determine β, BW, at the O/P.

Soln:



$\Delta f = 90$ KHz

$f_m = 5$ KHz

$\beta = 18$

$\Delta f = \beta f_m = 36 \times 5 = 180$ kHz

$\beta = 2 \times 18 = 36$

$BW = 2(\beta + 1) f_m$

$= 2 \times 36 \times 5$ kHz

$$\boxed{BW = 370 \text{ kHz}}$$

(OR)

$BW = 2\left(\frac{\Delta f}{f_m} + 1\right) f_m$

$= 2\Delta f + 2f_m$

$= 2(180) + 10$

$$\boxed{BW = 370 \text{ KHz}}$$

## * Demodulation of FM signal:

⇒ FM demodulator is a frequency to voltage converter.

① Frequency discrimination method.

② phase discrimination method.

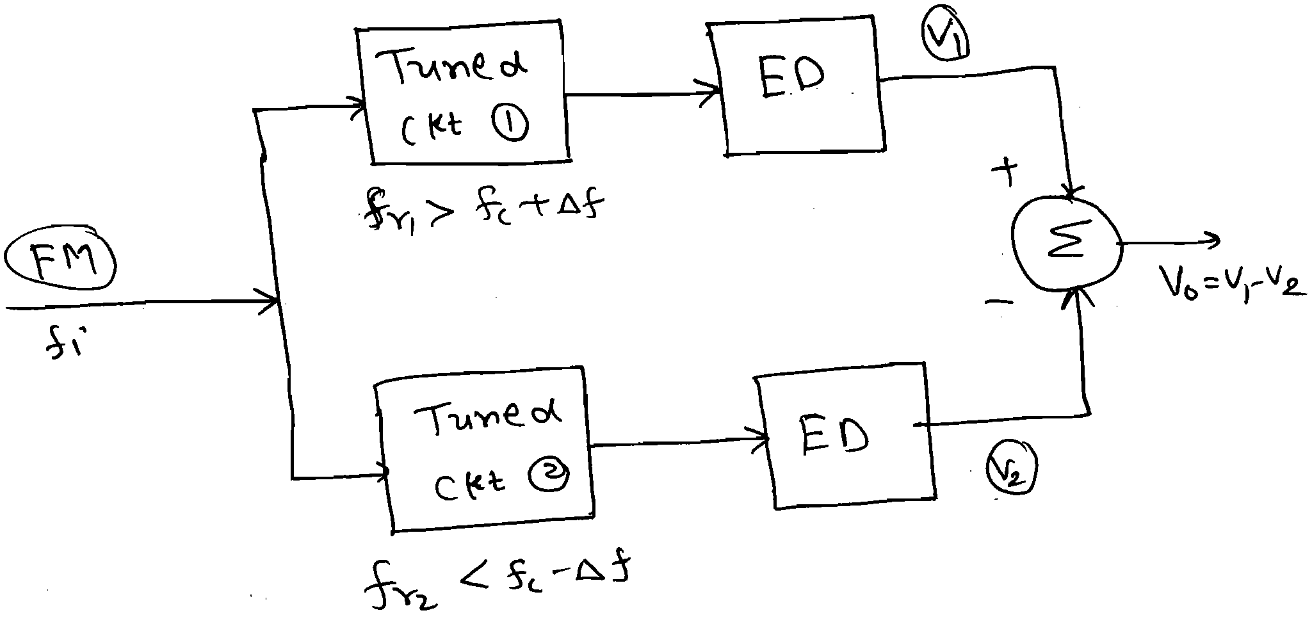
→ Foster seeley discrimination.

→ Ratio detector.

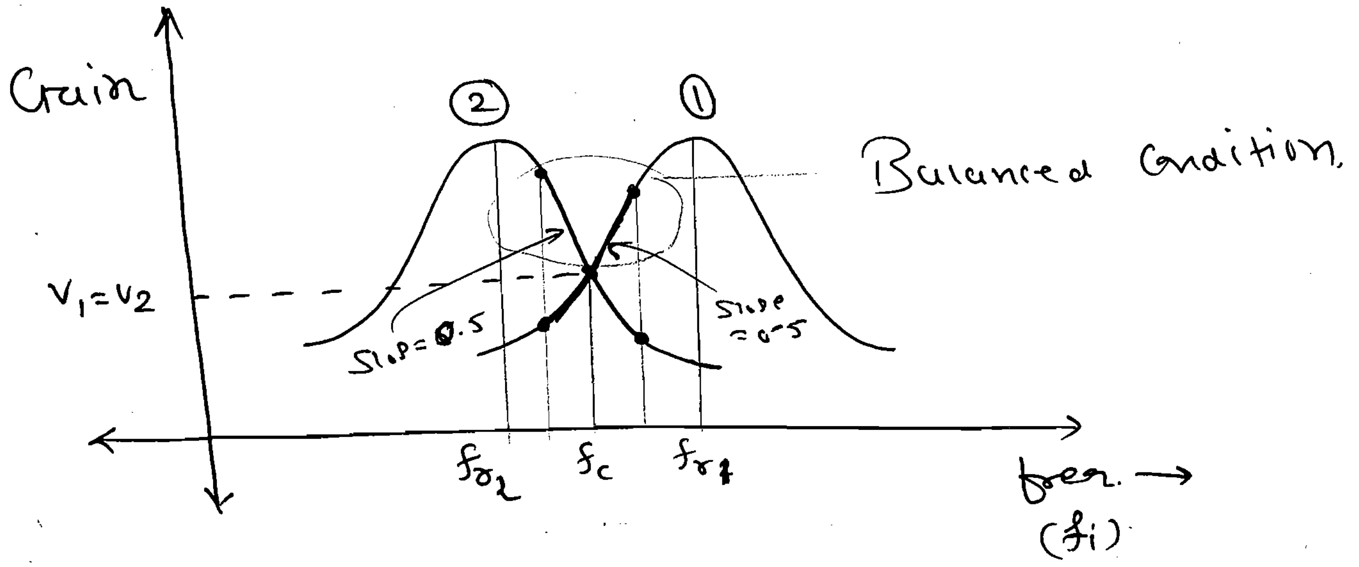
→ PLL (phase locked loop).

## ① Balanced Slop Detector :-

⇒



⇒ The Freq. Response of the Tuned ckt:



⇒ Condition:

① $f_i = f_c$ :

⇒ The gain of the 1st tuned ckt is same as the 2nd tuned ckt. So the o/p of the EDs $\boxed{V_1 = V_2}$. So, $V_0 = 0$

② $f_i > f_c$:

⇒ The gain of the 1st tuned ckt is larger than gain of the 2nd tuned ckt. So, $V_1 > V_2$ and so, the output voltage $V_o = V_1 - V_2 > 0$ (positive).

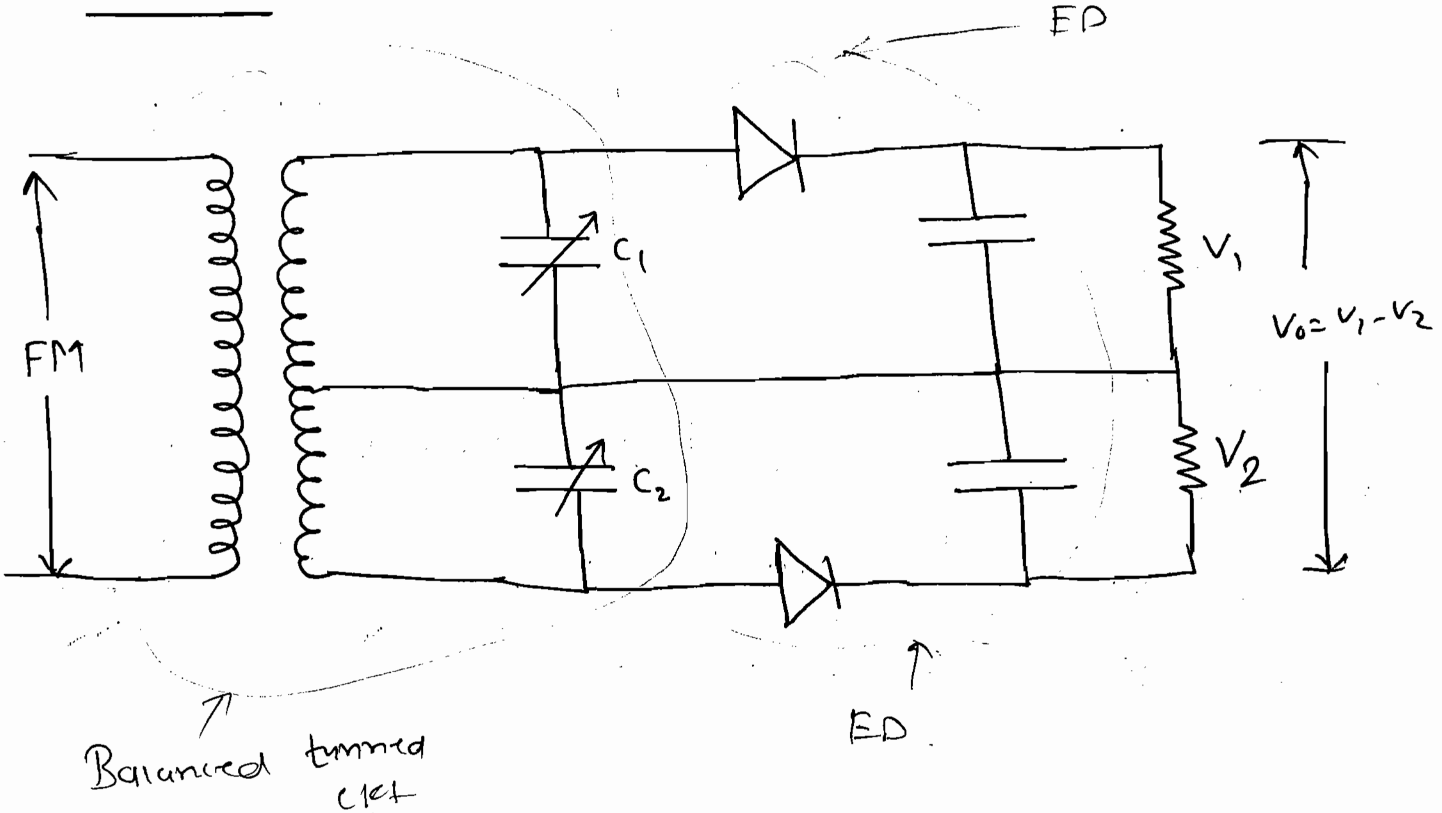
③ $f_i < f_c$:

⇒ The gain of the 1st tuned ckt is less than the 2nd tuned ckt.

So, $V_1 < V_2$.

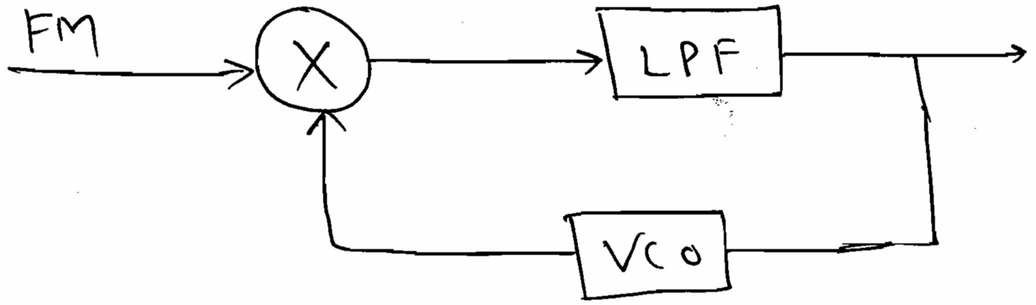
∴ o/p voltage, $V_o = V_1 - V_2$ = negative $< 0$.

* Hardware implementation of Balanced slope detector:



# * FM demodulation using PLL:

=>



first order PLL

=> The working principle of PLL is same of synchronous detector.

→ In AM, DSB & SSB the carrier frequency is constant. So, local oscillator is used to generate the same carrier.

→ In fm the carrier freq. is varied according to the message signal using VCO. So, the local oscillator is replaced with VCO to generate the same carrier.

→ If the input to the PLL is $\cos[2\pi f_c t + \phi]$ then o/p of PLL is:

then the o/p of PLL is:

$$\boxed{V_0 \propto \frac{d[\phi]}{dt}}$$

⇒ Time domain eqn of Multitone modln is:

$$A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\, dt\right].$$

∴ o/p $V_o \propto \frac{d}{dt}[\phi]$.

$$\therefore V_o \propto \frac{d}{dt}\left[2\pi K_f \int m(t)\, dt\right].$$

$$\therefore V_o \propto 2\pi K_f\, m(t).$$

$$\therefore V_o = \frac{\cancel{2\pi} K_f}{\cancel{2\pi} K_v} . m(t).$$

$$\therefore \boxed{V_o = \frac{K_f}{K_v} . m(t).}$$ ↙ I.B.

$K_f$ :→ freq. sensitivity of the Vco at transmitter.

$K_v$ :→ freq. sensitivity of the vco at Receiver.

# * Phase Modulation:

⇒ Time domain eqn of the PM signal is

$$S(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right]. \leftarrow \text{H.B.}$$

$$\therefore \boxed{\phi(t) = K_p m(t)} \text{ (rad).}$$

$K_p$ = phase sensitivity of the modulator (rad/volts).

for singletone modulation

$$m(t) = A_m \cos 2\pi f_m t.$$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + K_p A_m \cos 2\pi f_m t\right].$$

$$\text{H.B} \nearrow \boxed{\Delta\phi = K_p \cdot A_m} = \text{Phase deviation.}$$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + \Delta\phi \cos 2\pi f_m t\right].$$

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + \beta \cos 2\pi f_m t\right].} \leftarrow \text{H.B}$$

$$\Rightarrow \boxed{K_p m(t) = \Delta\phi = \beta = \text{modulation index.}} \leftarrow \text{H.B}$$

⇒ In phase modulation phase deviation and modulation index are same.

⇒ The magnitude Spectrum of the PM signal is Same as the FM signal except a phase shift of 90° at message freq. So, the magnitude Spectrum of the PM signal is Same as the FM signal.

**Ex-1** Consider an angle modulated signal $s(t) = 10\cos\left[2\pi f_c t + 5\sin 8\pi 10^3 t\right]$.

Determine the freq. deviation and phase deviation.

**Ans:** (i) FM → $\Delta f = \beta \cdot f_m = 5 \times 4\,KHz = 20\,KHz$.

(ii) PM → $\Delta\phi = \beta = 5$ rad.

**Ex-2** Consider an angle modulated ~~index~~ signal

$s(t) = 10\cos\left[2\pi 10^6 t + 8\sin 2\pi 10^3 t\right]$

(i) Assuming the given signal is FM determine the modulation index, freq. deviation, BW & power.

(ii) Repeat the above calculation when the message freq. is doubled.

Soln:

$A_c = 10$, $f_c = 1\,MHz$, $f_m = 1\,KHz$

(i) $\beta = 8$,

$\Delta f = \beta \cdot f_m = 8 \times 1\,KHz = 8\,KHz.$

$\therefore\ BW = 2(\beta+1) f_m = 2 \times 9 \times 1\,KHz = 18\,KHz.$

$P = \dfrac{A_c^2}{2R} = \dfrac{100}{2} = 50W.$

(ii) when $f_m' = 2 f_m = 2\,KHz.$

$\Delta f = 8\,KHz$

$\therefore\ \beta = \dfrac{\Delta f}{f_m} = \dfrac{8\,KHz}{2\,KHz} = 4.$

$\therefore\ BW = 2(\beta+1) f_m = 2(5) \times 2 = 20\,KHz.$

Ex-3 Repeat the above numerical problem assume that the given signal is PM.

Soln:

(i) $\beta = 8.$

Phase deviation $\Delta\phi = \beta = 8$ rad.

$BW = 2(\beta+1) f_m$

$= 2 \times 9 \times 10\,KHz$

$BW = 18\,KHz$

$P_t = \dfrac{A_c^2}{2R} = \dfrac{100}{2} = 50\,W$

(ii) when $f_m' = 2 f_m = 2 \times 1\cancel{0}\,KHz = 2\,KHz.$

$\beta = 8 = \Delta\phi.$

$P_t = 50\,W$

$BW = 2(\beta+1) f_m'$

$= 2(9) \times 2\,KHz. = 36\,KHz.$

## * Conclusion:

① In AM & PM, the BW of the modulated signal is double when the message freq. is double. But in FM, BW is not double.

② In PM, modulation index β is independent of modulating frequency.

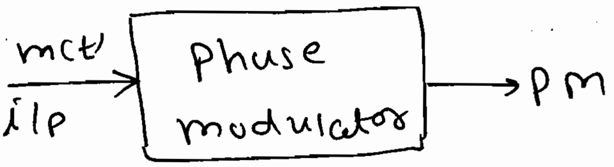
## * Time domain eqn for multitone modulation:

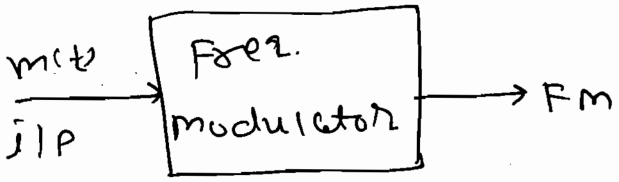
**PM**

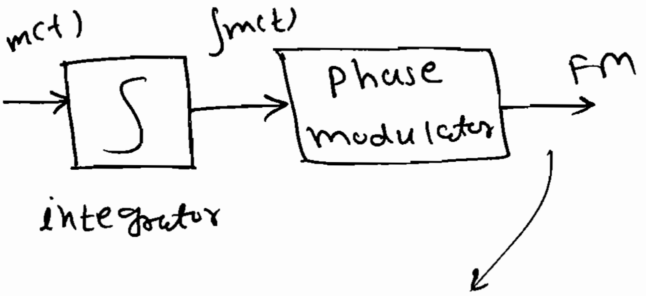
→ $S(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$

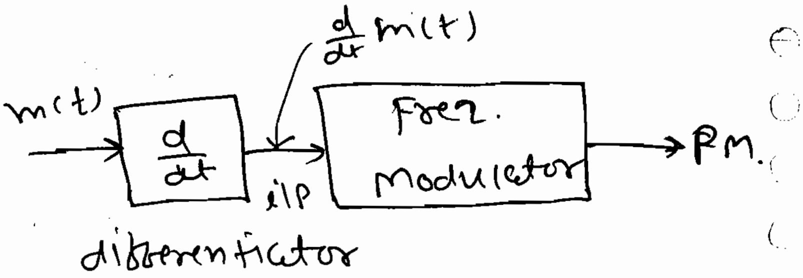
**FM**

→ $S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)dt\right]$







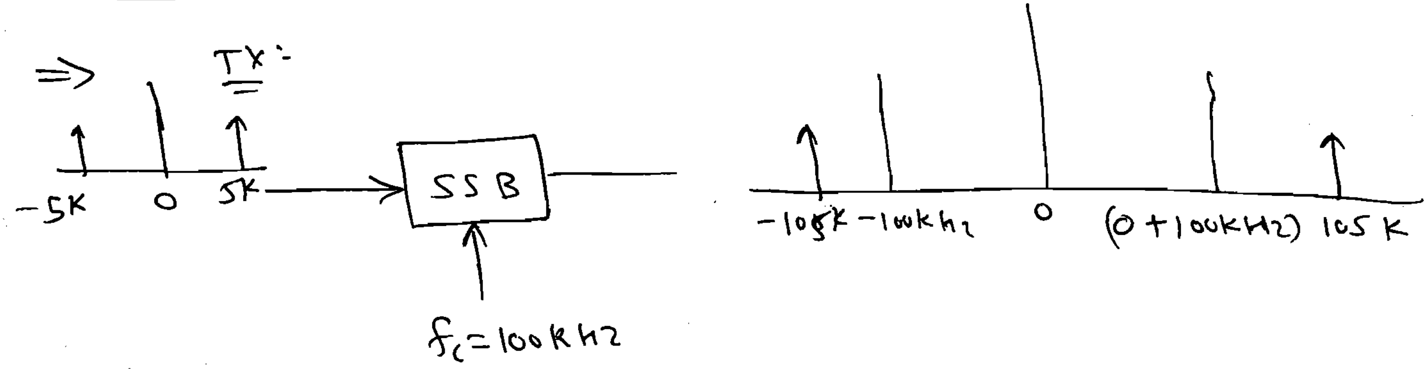


$A_c \cos\left[2\pi f_c t + K_p \int m(t)dt\right]$

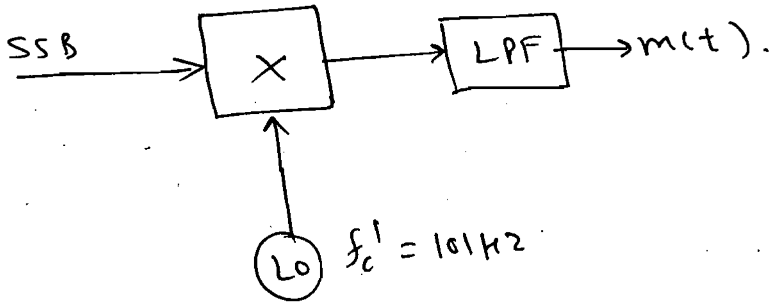
$K_p = 2\pi K_f$

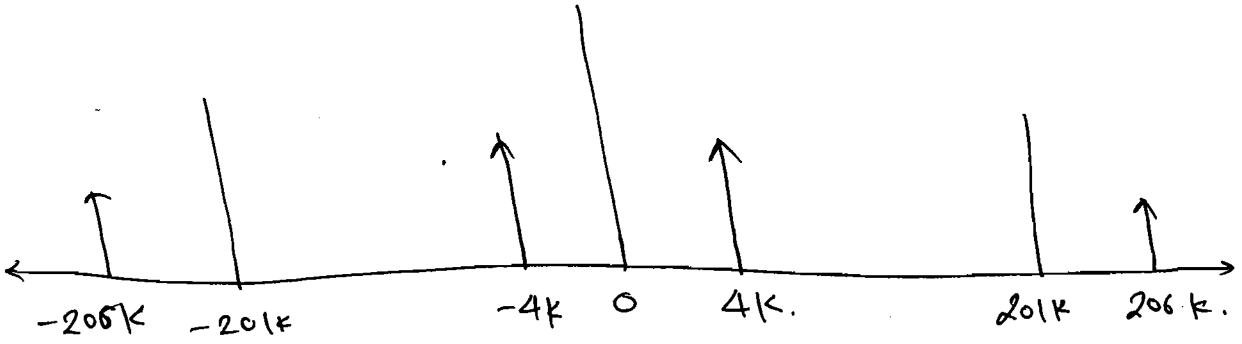
# * IMP Concept: ← N.B

=> Tx:



=> Reciever: Using coherent | synchronous detection.





Conclusion:

| | | | | |
|---|---|---|---|---|
| Tx: | $f_m$= 5 kHz | $f_c$ = 100kHz | USB → 105 kHz | LSB. |
| Rx: Lo → | 100kHz | → | o/p : $f_m$= 5 kHz. | 5kHz. |
| Lo → | 101 kHz | → | o/p : $f_m$= 4 kHz. | 6kHz |
| Lo → | 102 kHz | → | o/p : $f_m$= 3 kHz. | 7 kHz. |
| Lo → | 99 kHz | → | o/p : $f_m$= 6 kHz. | 4 kHz |
| Lo → | 98 kHz | → | o/p : $f_m$= 7 kHz. | 3 kHz |

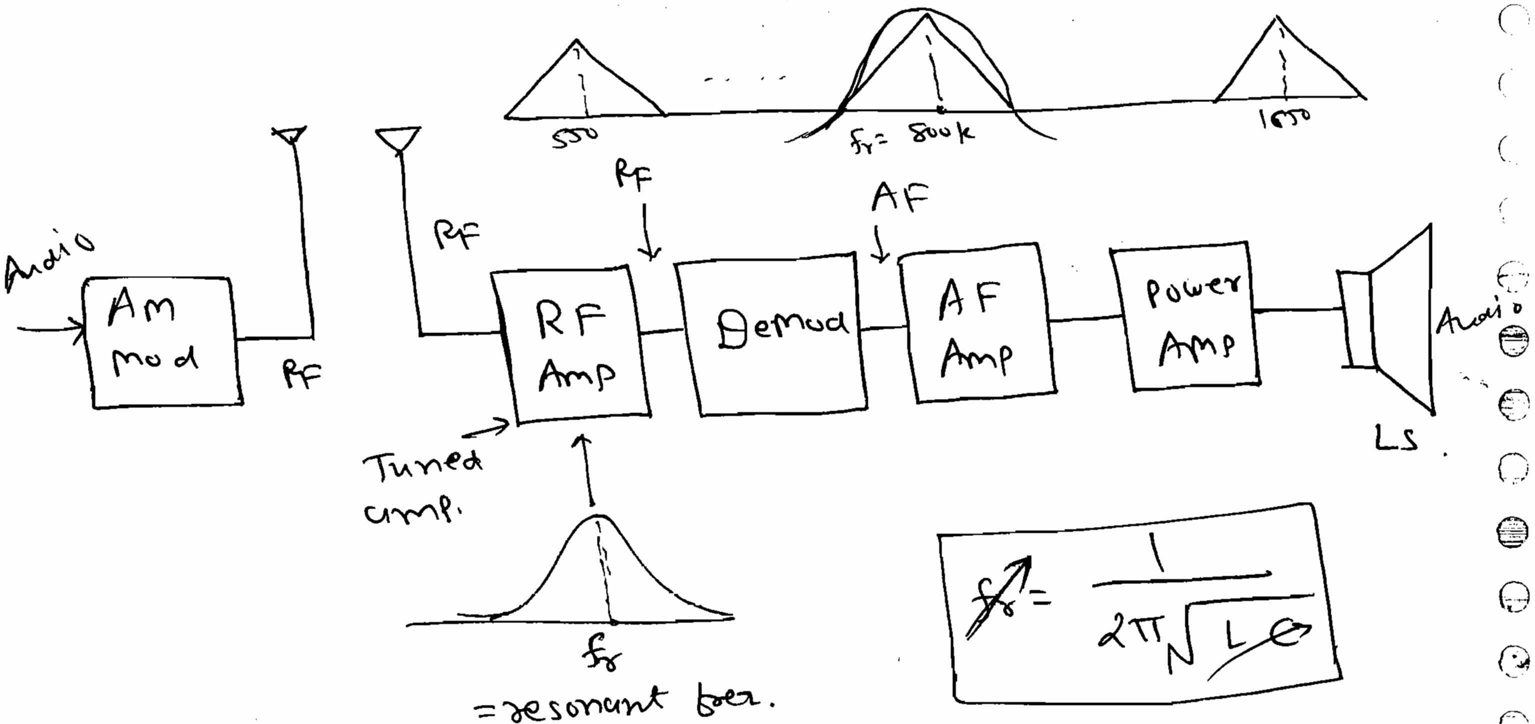
for LSB $f_m'$ = sub.

# ☆ Am Receivers :-

① Tuned Radio Freq. Receiver. (TRF Rx).

② Superhetrodyne Receiver.

## ① Tuned Radio Frequency Rx (TRF Rx) :-



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

FM : 88M - 108MHz
Am : 550 K - 1050 kHz.

← H.B.

⇒ In order to select the 800kHz signal, we have to be set resonant freq. at 800 kHz. by Changing capacitor value.

⇒ So, tunning in TRF Rx is nothing but the setting a $f_r$ in order to get desired signal.

⇒ International standard BW of signal = 10 KHz (including Guard Band)

⇒ After Receiving the signal from antenna RF Amp. is used to increase the signal strength. The RF amp. is also a tuned amp. which is used to select the required signal by adjusting the resonant freq.

⇒ Demodulator Converts RF signal into AF.

⇒ AF amp. and Power amp. is used to increase the signal strength to the required level.

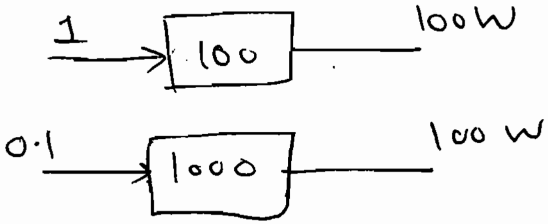
## * Characteristic Parameters of a Receiver:

① Sensitivity.
② Selectivity.
③ Fidelity.

## ① Sensitivity:

⇒ It is defined as the minimum signal strength that should be maintain at i/p of a receiver to get a standard o/p.

⇒ Sensitivity depends on the overall gain of the receiver.



## ② Selectivity:

⇒ Selectivity is defined as the ability of the receiver to select the required freqs only.

⇒ Assume that a receiver is tuned to 800 KHz the tuned ckt at the RF amplifier has to select the freq from 795 KHz to 805 KHz. This is possible only when the resonant freq. is adjusted to 800 KHz and Q should be 80. ($BW = \frac{fr}{Q}$) ⇒ $Q = 80$. But simultaneously variation of resonant freq. and Q is not possible in tuned ckt. ~~and Q is not~~

⇒ If the BW of the tuned ckt is greater than 10 KHz adjecent signal freq. are selected. If the BW is less than 10 KHz some of the required freqs are attenuated.

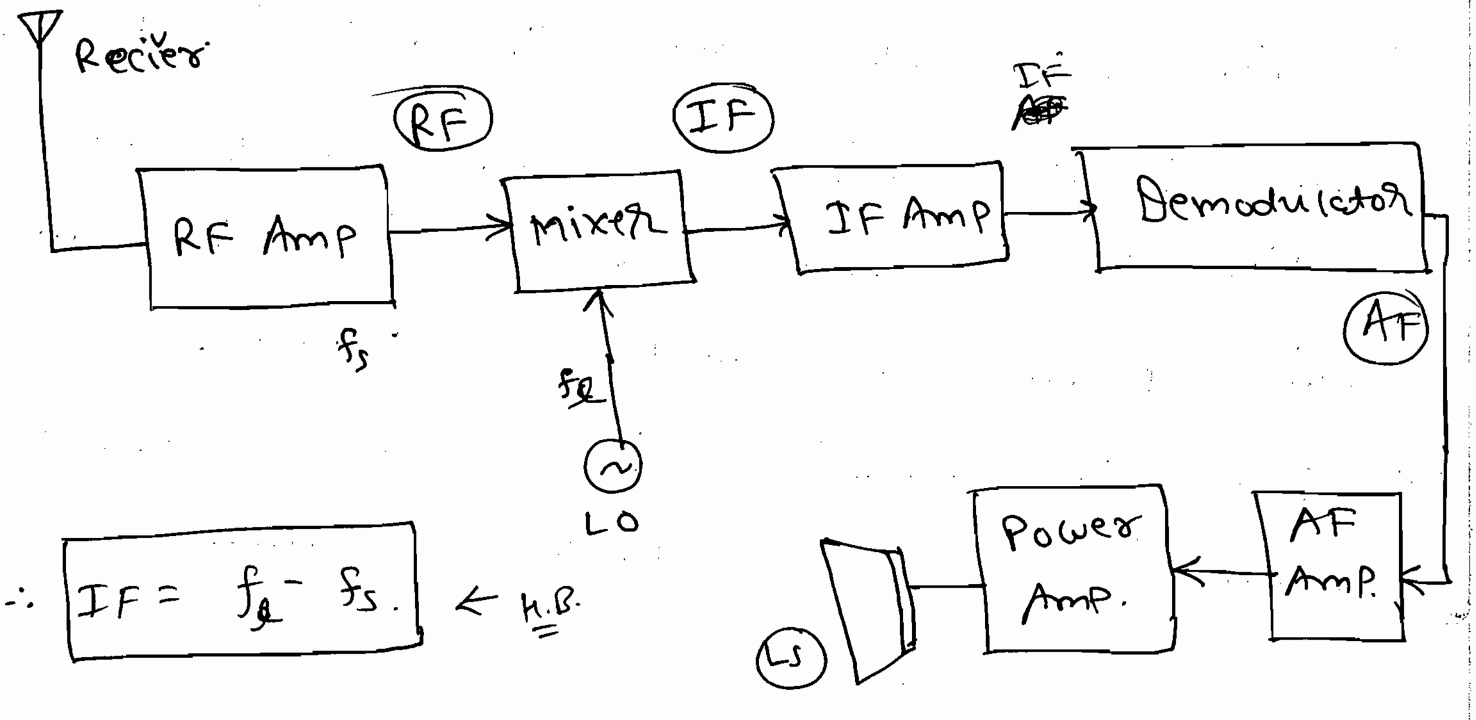
## ③ Fidelity:

⇒ Fidelity is defined as the ability of the receiver to reproduced all audio freq. at the o/p.

⇒ In AM ~~radio~~ the audio signal is Band limited to 5 KHz as the Bandwidth alloted to each broadcasting station is 10KHz.

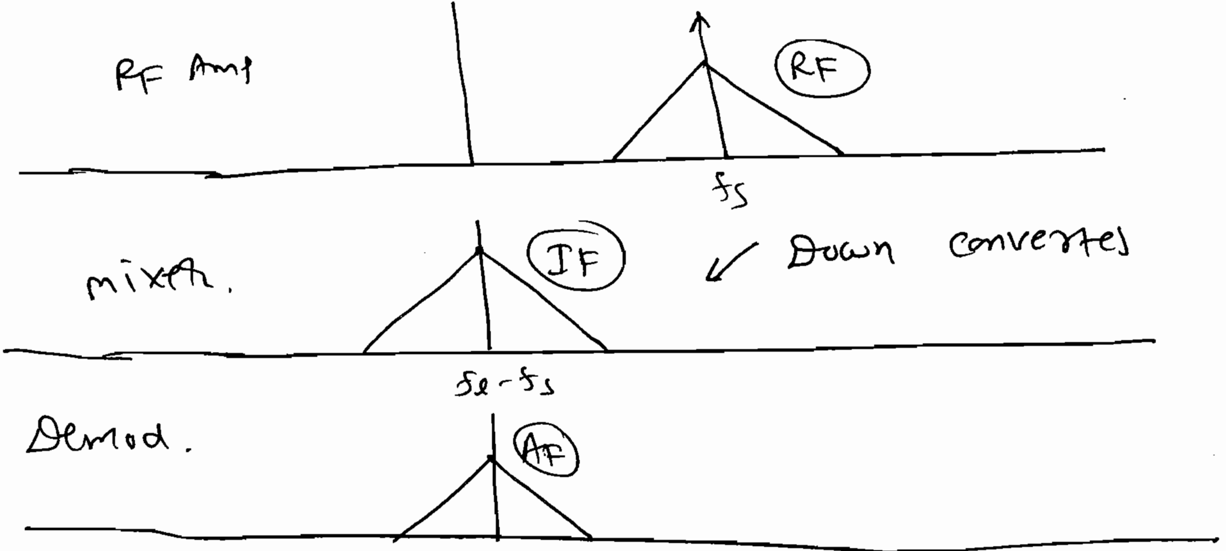
⇒ The highest audio freq. at o/p of the receiver is 5 KHz. So, the signal fidelity is very low in AM receivers.

## ② Superheterodyne Receiver:



$$\therefore \boxed{IF = f_l - f_s} \leftarrow \text{H.B.}$$

$f_s$ = Carrier freq. of the tuned station.



→ After receiving signal from the antenna RF Amp is used to increase the signal strength.

→ mixer is used to down convert the RF signal into IF.

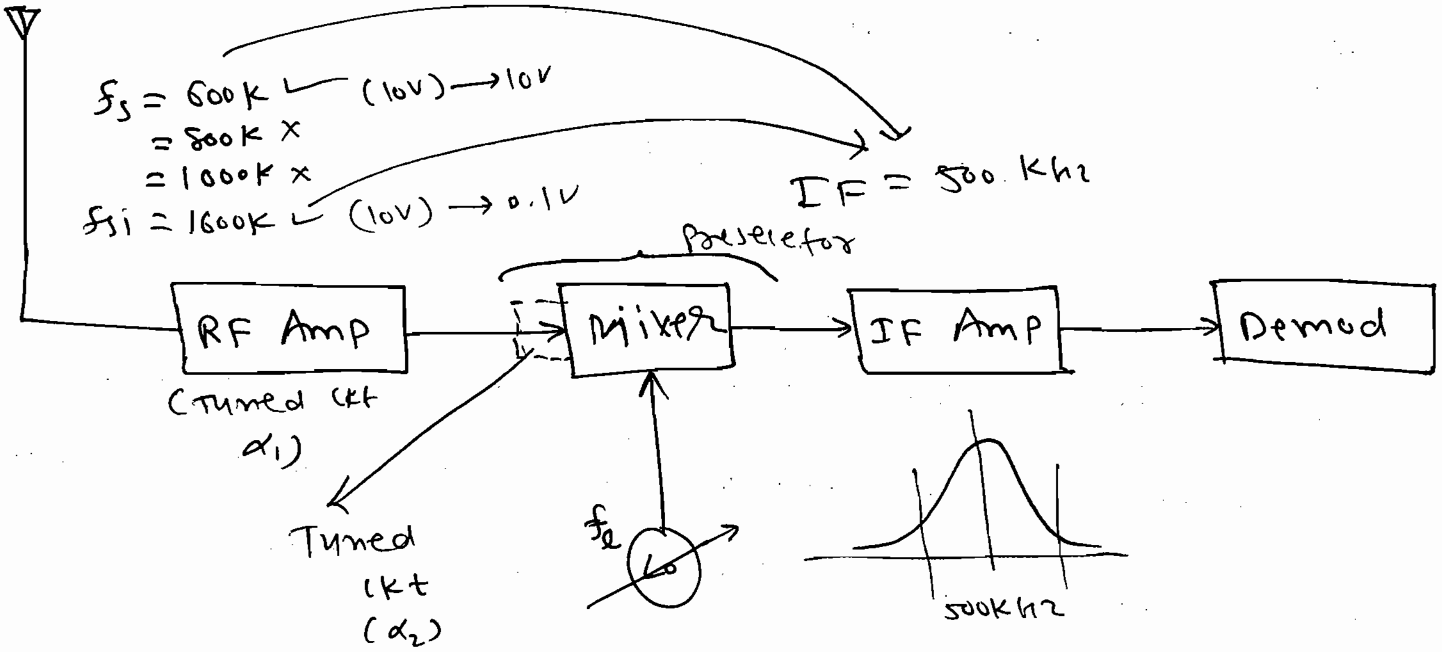
→ The IF is a constant value and depends on the Application.

→ In AM radio the IF used is 455 KHz

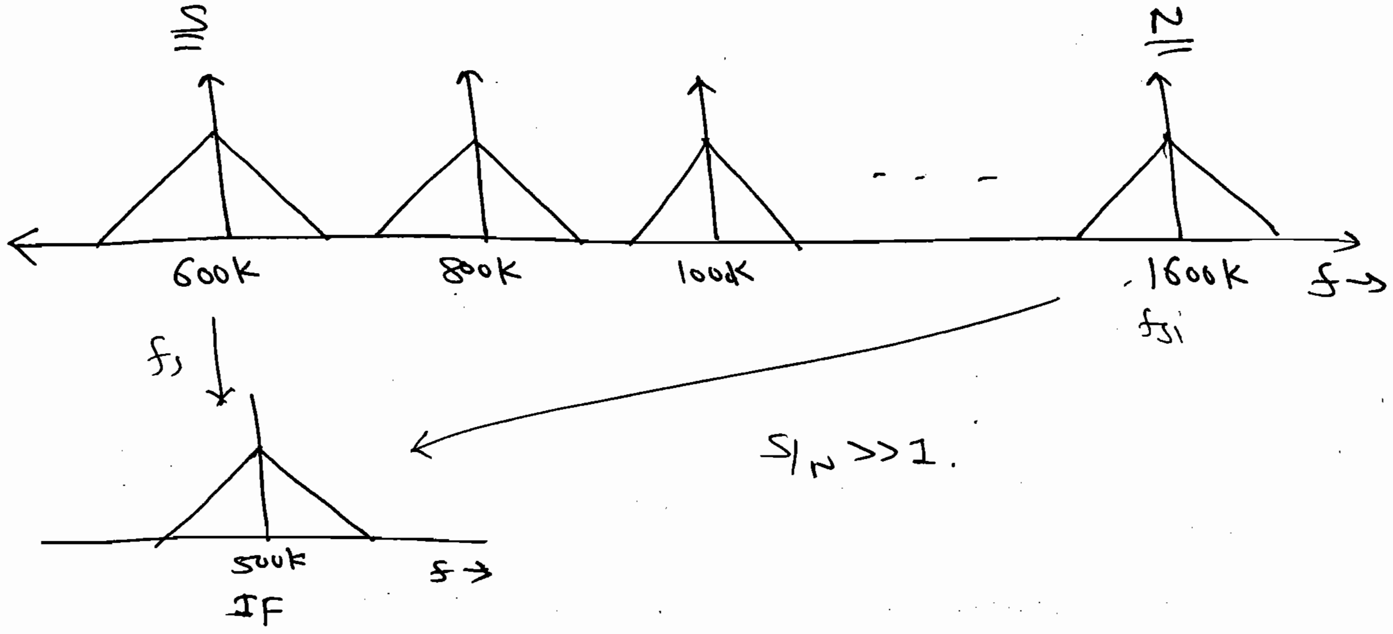
The local oscillator freq. should be adjusted so that $f_o - f_s$ is 455 KHz. This Process is called as tunning.

→ The IF amplifier is a tuned amplifier which is always tuned 455 KHz.

# * Image freq. and its Suppression:



=>

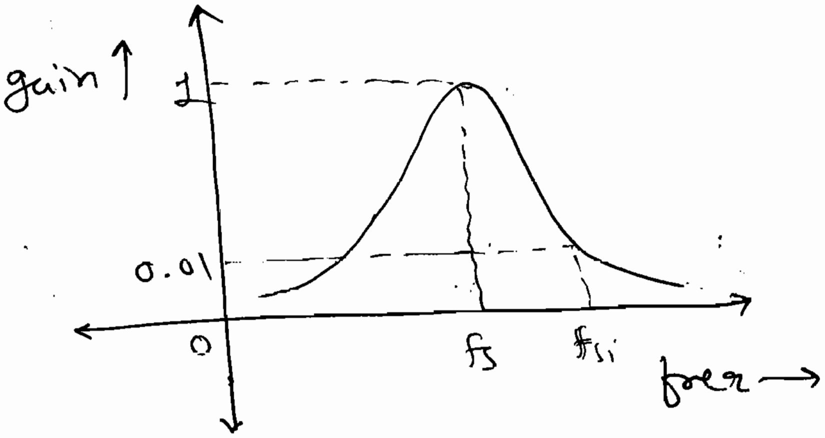


=> Consider a receiver with IF 500 KHz and assume that receiver is tuned at 600 KHz. The Local oscillator freq. is adjusted to 1100 KHz to down convert signal to 500KHz. Assume that another signal received from antenna having a carrier freq. of 1600 KHz. This signal is also down converted to 500KHz and causes interference to the required signal, the signal which is causing interference is called as the image freq.

$$\boxed{f_{si} = f_s + 2IF}$$ ← H.B

1600k = 600k + (2×500k)

→ To reduced the image frequency signal strength of a tuned ckt is used at the i/p of the mixer. the freq. response of tuned ckt as shown in fig.

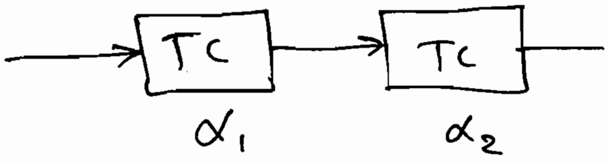


⇒ The gain of tuned ckt at fsi should be as minimum as possible.

⇒ To determine the suppression factor of the tuned ckt, Image Rejection Ratio is used.

$$\boxed{IRR = \alpha = \frac{G_{fs}}{G_{fsi}} = \sqrt{1 + Q^2\rho^2}}$$

$$\boxed{\text{where, } \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}}$$



$\alpha = \alpha_1 \cdot \alpha_2$.

**Q-1** A Superhetrodyne receiver having no RF Amp. is tuned to 555 kHz. The Local oscillator freq. is adjusted to 1010 kHz & Q is 50. Determine the image freq. and image rejection Ratio.

**Ans:**

$f_s = 555$ kHz $\qquad Q = 50$

$f_l = 1010$ kHz

$\therefore \ IF = f_l - f_s = 1010 - 555 = 455$ kHz.

$\therefore \ f_{si} = f_s + 2IF$

$\therefore \ f_{si} = 555 + (2 \times 455) = 555 + 910$

$\therefore \ \boxed{f_{si} = 1465 \text{ kHz}}$

$$\therefore \ \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}.$$

$$\therefore \ \rho = \frac{1465}{555} - \frac{555}{1465}.$$

$\therefore \ \rho = 2.26$.

$\therefore \ \alpha = \sqrt{1 + Q^2 \rho^2} = \sqrt{1 + (2500)^2 (2.26)^2}$

$\therefore \ \boxed{\alpha = 113.}$

**Ex-2** A Superhetrodyne receiver having RF Amp. is tuned to 1200 KHz, the IF 450 KHz. The Q of the tuned ckt at the RF Amp. and i/p of the mixer are same and is equal to 65. Calculate the image freq & IRR.

**Ans:**

$f_s = 1200$ KHz.

$IF = 450$ KHz.

$Q = Q_2 = 65$.

$\therefore \quad f_{si} = f_s + 2IF$

$= 1200 + 900$

$$\boxed{f_{si} = 2100 \text{ KHz}}$$

$$\therefore \quad \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$$= \frac{2100}{1200} - \frac{1200}{2100}$$

$$\therefore \boxed{\rho = 1.178}$$

$\therefore \quad \alpha = \alpha_1 \cdot \alpha_2$

$\alpha = \sqrt{1 + Q_1^2\rho^2} \cdot \sqrt{1 + Q_2^2\rho^2}$

$\therefore \quad \alpha = (1 + Q^2\rho^2) \quad (\because Q_1 = Q_2 = Q)$.

$\therefore \quad \alpha = 1 + (65)^2(1.178)^2$

$$\therefore \boxed{\alpha = 5870.}$$

Ex-3 A Superhetrodyne receiver having no RF Amp. is tuned to 1000 KHz the IF is 455 KHz & Q is 100. ① Determine the Image freq. & IRR. ② Repeat the above calculation when receiver is tuned to 25 MHz.

Ans: (i) $f_s = 1000$ KHz

$IF = 455$ KHz.

$Q = 100.$

$\therefore f_{si} = f_s + 2IF$

$\therefore f_{si} = 1000 + 910 = 1910$ KHz.

$\therefore \rho = \frac{1910}{1000} - \frac{1000}{1910}.$

$\therefore \rho = 1.3864.$

$\alpha = \sqrt{1 + Q^2 \rho^2} = \sqrt{1 + 1.38^2 . (100)^2}$

$\therefore \boxed{\alpha = 138.64}.$

(ii) $f_s = 25$ MHz.

$\therefore f_{si} = 25M + 2 \times 455K.$

$f_{si} = 25.910$

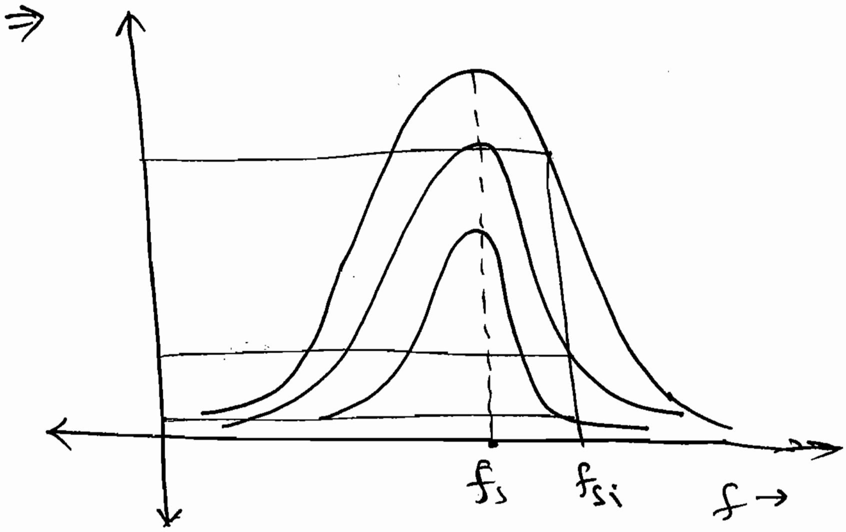
$\therefore \rho = \frac{25.910}{25} - \frac{25}{25.910}.$

$\therefore \rho = 0.036$

$\boxed{\alpha = 7.22}$

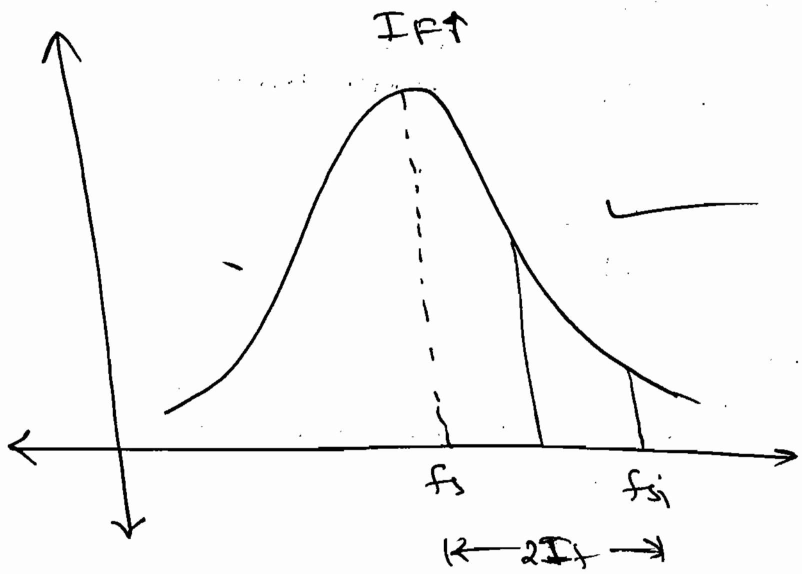
⇒ To improve the image Rejection Ratio, either Q (or) 'IF' should be increased.

⇒



Practically not possible because BW must 10kHz.

⇒



Ex-1 Determine the value of IF Required in the above numerical problem so that the IRR at 25 MHz is also equal to 138.6.

Ans: $f_s = 25$ MHz $\quad \alpha = \sqrt{1 + (100)^2 (1.386)^2}$

$Q = 100 \quad = 138.6.$

$\therefore \alpha = 138.6 = \sqrt{1 + (100)^2 \cdot \rho^2}$

$\Rightarrow \boxed{\rho = 1.38}$

$$\therefore \rho = 1.386 = \frac{25+2IF}{25} - \frac{25}{25+2IF}$$

$$\Rightarrow \boxed{IF = 11.4\ MHz}$$

* **In FM:**

88M - 108 M Hz

$\boxed{IF = 10.7\ MHz}$

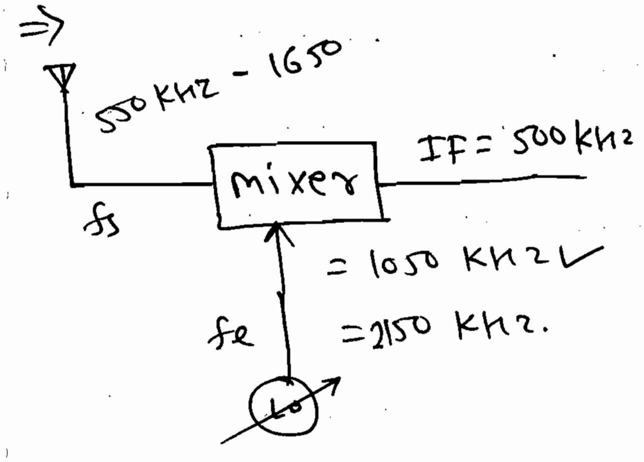
**In Am**

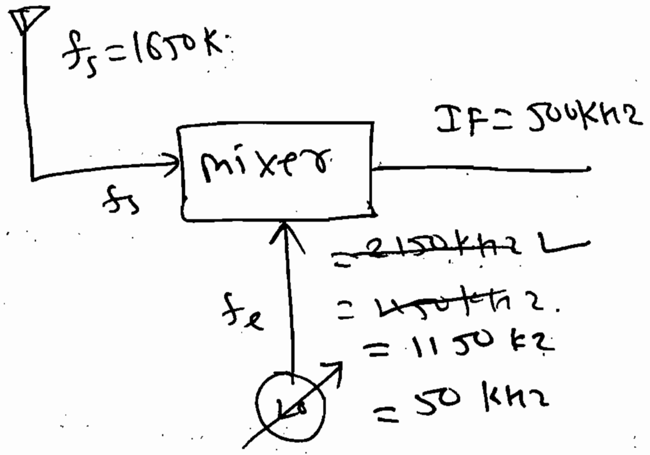
$\boxed{IF = 455\ KHz}$

**Imp:**

* Why should Local oscillator freq. should always larger than signal freq.?

⇒





$$\frac{1}{2\pi\sqrt{(L_1+L_2)\cancel{C}}}$$

① $f_e > f_s$

$$1050k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{max}}}$$

$$2150k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{min}}}$$

$$\therefore \frac{C_{max}}{C_{min}} = \left(\frac{2150}{1050}\right)^2$$

$$\simeq 4$$

② $f_e < f_s$

$$50k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{max}}}$$

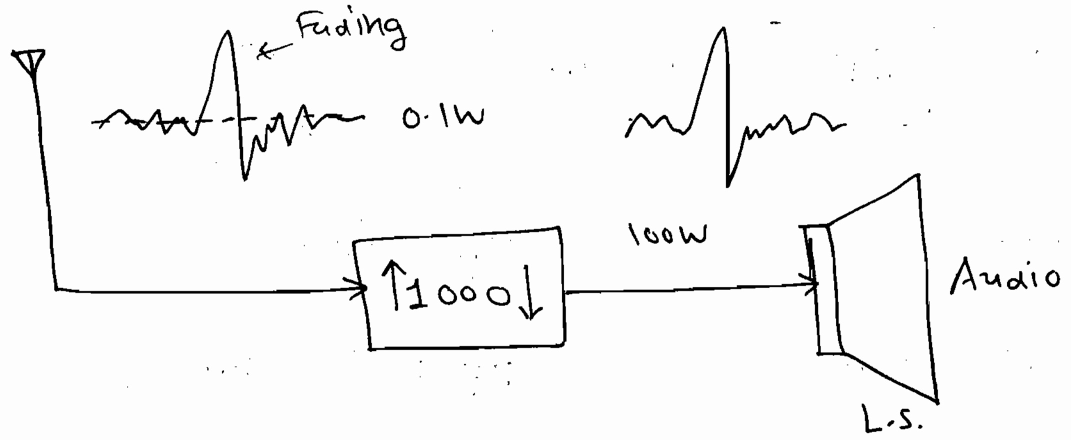
$$1150k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{min}}}$$

$$\therefore \frac{C_{max}}{C_{min}} = \left(\frac{1150}{50}\right)^2$$

$$= 529$$

It is impossible to get such a large ratio.

# ☆ AGC Circuit:



⇒ The signal strength from the antenna in any wireless communication system is not constant is called fading.

⇒ Signal fading occurs due to various propagation losses. If overall gain of the receiver is constant the o/p signal strength also changes.

⇒ AGC circuit is used to maintain a constant audio o/p irrespective of the variations at the i/p of the receivers

⇒ AGC circuit will control the overall gain of the receiver according to variation in the signal strength at the i/p.

# * FM Receiver :

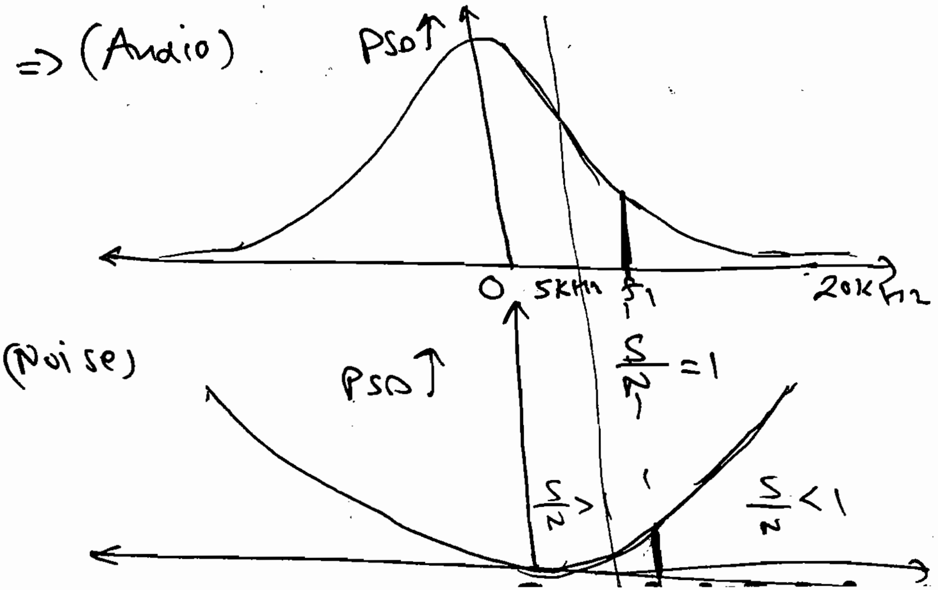
## * International Standards:

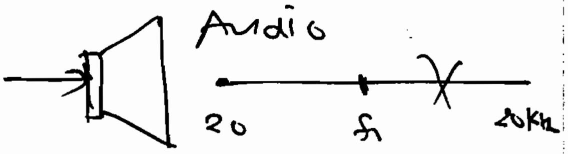
- 88 MHz = 108 MHz ← $f_c$
- IF = 10.7 MHz
- B.W. = 200 KHz = $2\Delta f + 2f_m$ = 150K + 30KHz + $\underbrace{20\text{KHz}}_{\text{Guard Band}}$
- $\Delta f$ = 75 KHz
- $f_m$ = 15 KHz
- $\beta = 5$

## * Preemphesis (Tx) and Deemphasis: (Rx)

=> These Concept are used in FM Radio not in AM Radio because Am signal is Band limited to 5 KHz and upto 5 KHz signal power is very more than the noise power and hence, $S/N >> 1$.

=> These techniques are used to improve the fideity of an Audio signals.

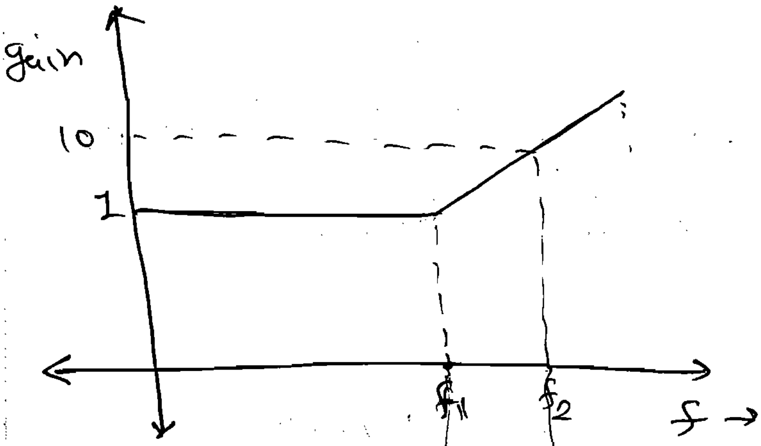




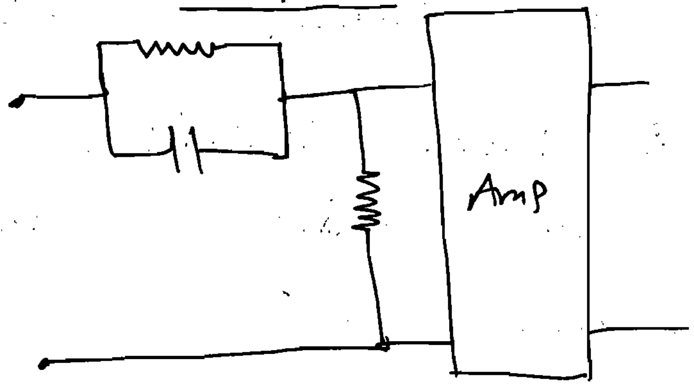
→ In Audio signal transmission $\frac{S}{N} > 1$ at low frequencies and $S/N < 1$ at high freq. So, high freq of the Audio signal will not be reproduced & signal Fidelity decreases.

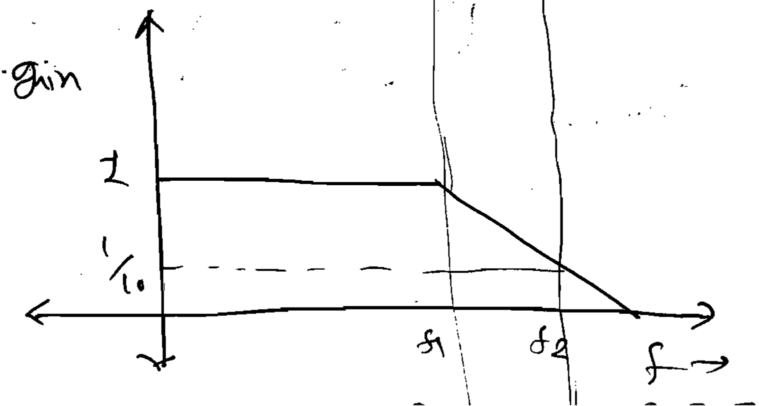
→ To improve the fidelity preemphasis technique is used ~~to~~ at Tx before modulator.

⇒ "Preemphesis is defined as the Process of boosting the high freqs of the Audio signal so that the signal to noise ratio is greater than '1' (SNR > 1) at all Audio frequencies."

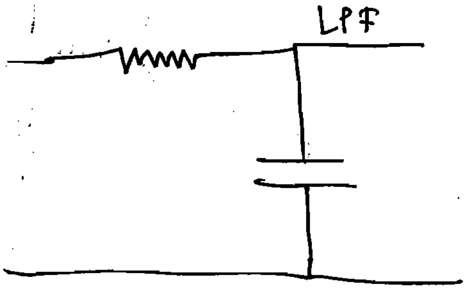


Preemphesis



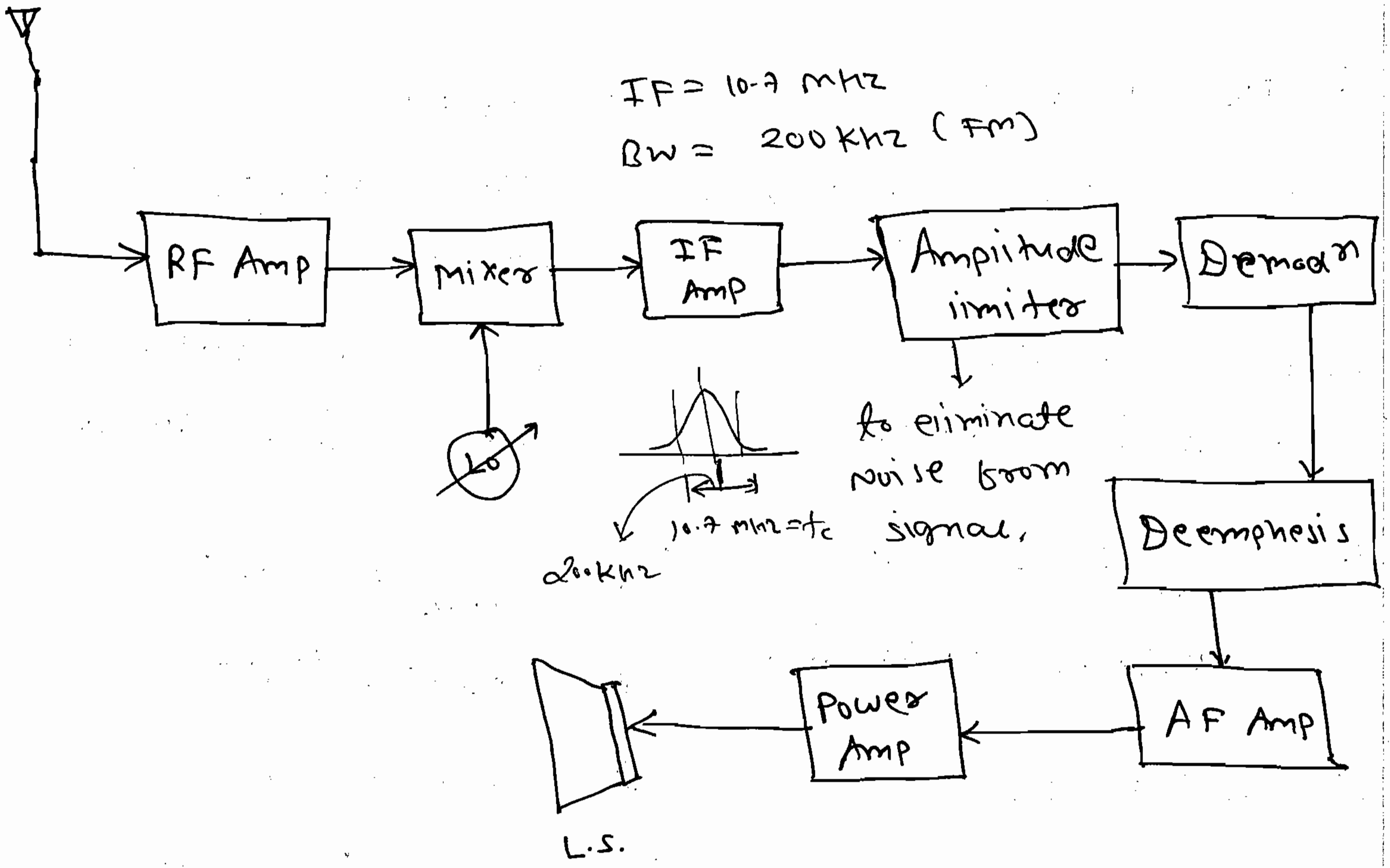


De emphesis



⇒ whenever Preemphesis and deemphesis used practically in music system, it is called as Dolby Noise Reduction System and Dolby is one of the laboratory of USA.

* Block Diagram of FM Radio :



⇒ FM demodulator is a freq. to voltage converter

→ Frequency variation will be converted back into voltage variation only if the envelope of the FM signal is constant.

→ Due to noise, Ampiitude distortion occurs and it is not possible to convert freq. into voltage. So Ampiitude limiter is used to eliminate the noise or to maintain a constant envelope.

→ In AM Amplitude limiter is not used as the information is in the form of amplitude.

* Major differences in AM & FM:

| AM | FM |
|---|---|
| ① $P_t = P_c + \frac{P_c\mu^2}{2}$ | ① $P_t = P_c$ |
| ② AM requires more power | ② FM requires less power. |
| ③ Power varies with modulation index. | ③ power is independent of modulation index. |
| ④ $\eta_{max} = 33.33\,\%$ (singletone modulation) | ④ $\eta_{max} = 100\%$ ; $\beta = 2.4, 5.5, 8.6, \ldots$ |
| ⑤ $BW = 2 f_m$ | ⑤ $2(\beta+1) f_m$ |
| ⑥ BW is very low. | ⑥ BW is ~~varies~~ very high. |
| ⑦ BW is independent of modulation index. | ⑦ B.W. varies with modulation index. |
| ⑧ AM receiver is less complex. | ⑧ FM receiver is more complex. |
| ⑨ The effect of Noise is more. | ⑨ The effect of noise is very less. |

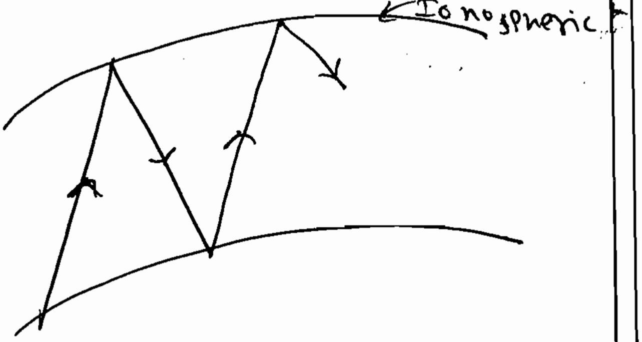
⑩ 550K – 1650 KHz

IF = 455KHz

B.W. = 10 KHz

$\mu = 1$

⑪ Ionospheric Propogation



⑫ → Area coverage is more

⑬ Freq. reuse is not possible.

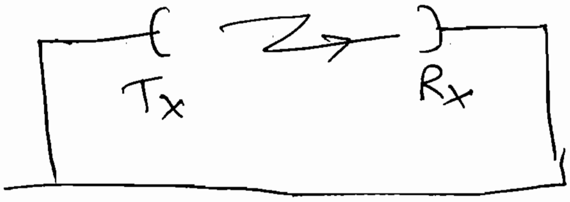
⑩ 88 M - 108 MHz

IF = 10.7 MHz

B.W. = 200 KHz

$\beta = 5$

⑪ Line of sight Propogation (LOS)



→ microwave freq. 'G' Hz

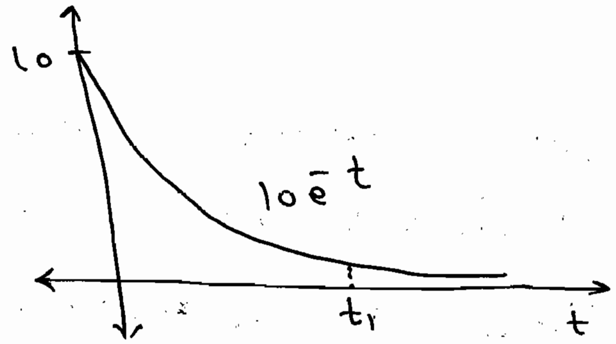
⑫ Area covered by antenna is resticated due to LOS.

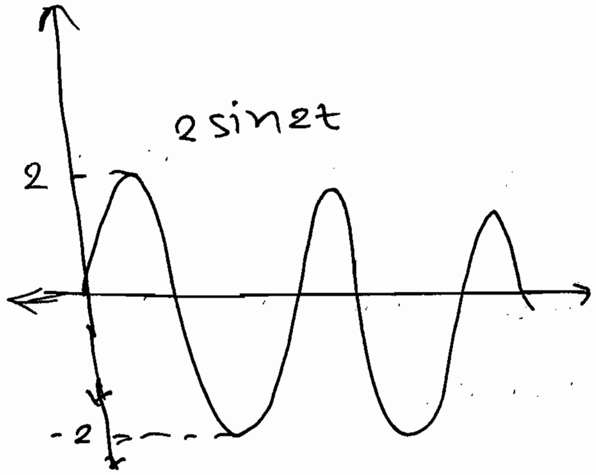
⑬ Freq. reuse is Possible.

# ★ Random Signal Theory

⇒ The i/p to the Communication receiver is signal & noise. The signal is completely deterministic but the noise voltage changes randomly w.r.t. time. So probability concepts are used to analyze a random signal.

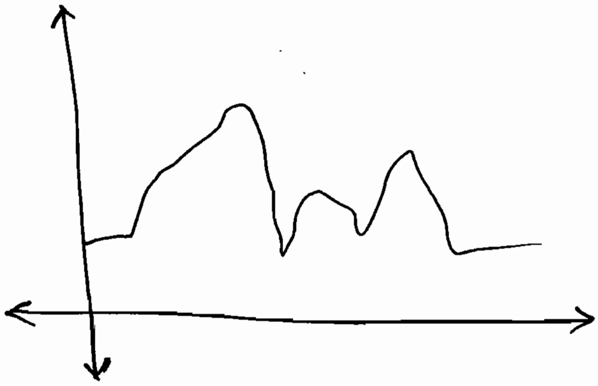
⇒ **Deterministic signals:**





→ At any instant of time it can be determine.

⇒ **Random signal:**



Impossible to represent Random ~~signal~~ Variations mathematically.

★ ~~Random~~ ~~Signal~~ ~~Theory~~:

* Probability Distribution Function (PDF)

(or) Cummulative Distribution Function (CDF).

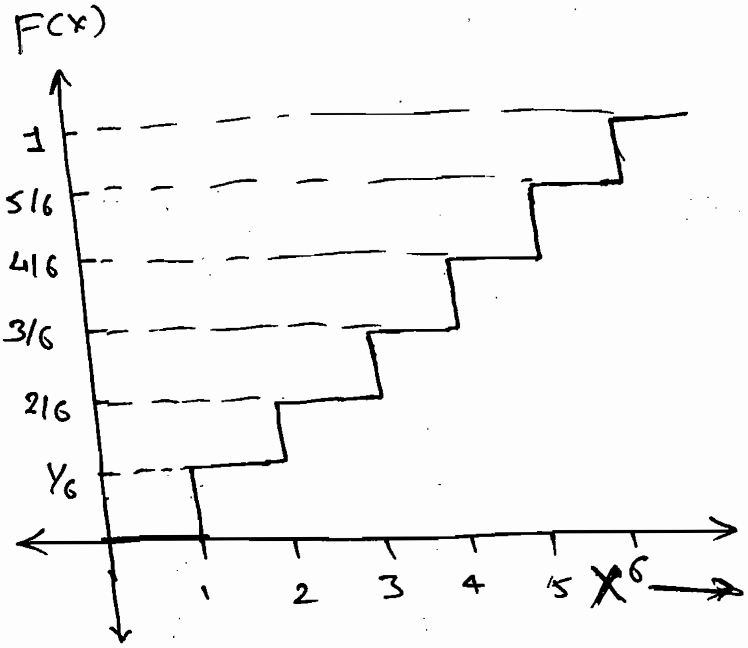
⟹ PDF mathmatically defined as,

$$F_X(x_1) = P[X \leq x_1]$$

$$F_X(5) = P[X \leq 5]$$

**Dice:**

| X | P[X=x] |
|---|---|
| 1 | 1/6 |
| 2 | 1/6 |
| 3 | 1/6 |
| 4 | 1/6 |
| 5 | 1/6 |
| 6 | 1/6 |



$$F(0.99) = P[x \leq 0.99] = 0$$

$$F(1) = P[x \leq 1] = 1/6.$$

$$F(1.8) = P[x \leq 1.8] = 1/6$$

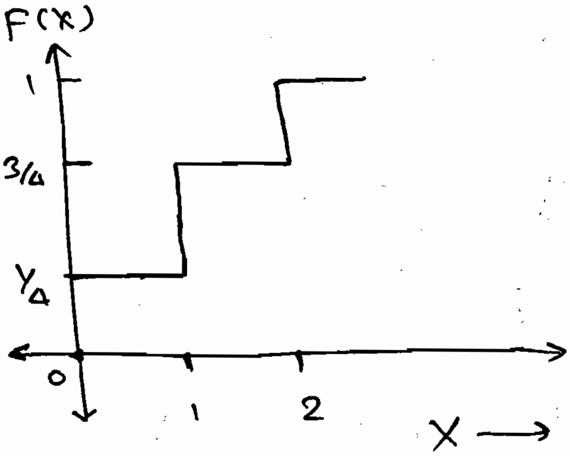
$$F[2] = P[x \leq 2] = 2/6$$

$\boxed{Q}$ A coin is tossed twice and the Random Variable X represent the number of Head. sketch the PDF.

Sol$^n$:

| | | X |
|---|---|---|
| T | T | 0 |
| T | H | 1 |
| H | T | 1 |
| H | H | 2 |

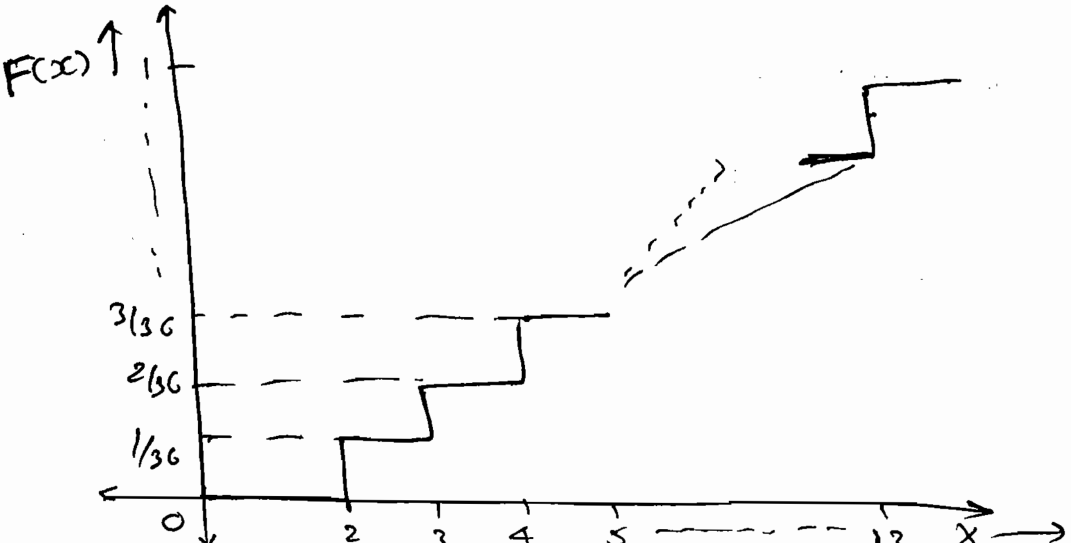
| X | P[X=x] |
|---|---|
| 0 | 1/4 |
| 1 | 1/2 |
| 2 | 1/4 |
| | 1 |



$\boxed{Q}$ Two dice are thrown simulteneously and Random Variable X represent the sum of the two outcomes. sketch the Probability Dis. $f^n$?

Ans:

| X | P[X=x] | |
|---|---|---|
| 2 | 1/36 | (1,1) |
| 3 | 2/36 | (1,2), (2,1) |
| 4 | 3/36 | (1,3), (3,1), (2,2) |
| 5 | 4/36 | |
| 6 | 5/36 | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | 1/36 | (6,6). |



## * Properties of PDF :

① $0 \leq F(x) \leq 1$

② $F(x)$ is a increasing function of $x$.

③ $F(\infty) = 1$ & $F[-\infty] = 0$.

$P[x \leq \infty] = 1$ & $P[x \leq -\infty] = 0$.

④ $P[x > x_1] = 1 - P[x \leq x_1] = 1 - F[x_1]$.

⑤ $P[x_1 < x \leq x_2] = F[x_2] - F[x_1]$

e.g. $P[3 < x \leq 5] = F[5] - F[3]$

$= 5/6 - 3/6$

$= 2/6 = 1/3$.

⑥ $P[x = x_1] = F[x_1] - F[\underset{\downarrow}{x_1^-}]$

Small decremental value

e.g. $P[x = 2] = F[2] - F[2^-]$

$= F[2] - F[1.99]$

$= 2/6 - 1/6$.

$= 1/6$.

***

$P[x = 3.4] = F[3.4] - F[3.4^-]$

$= F[3.4] - F[3.399]$

$= 3/6 - 3/6$

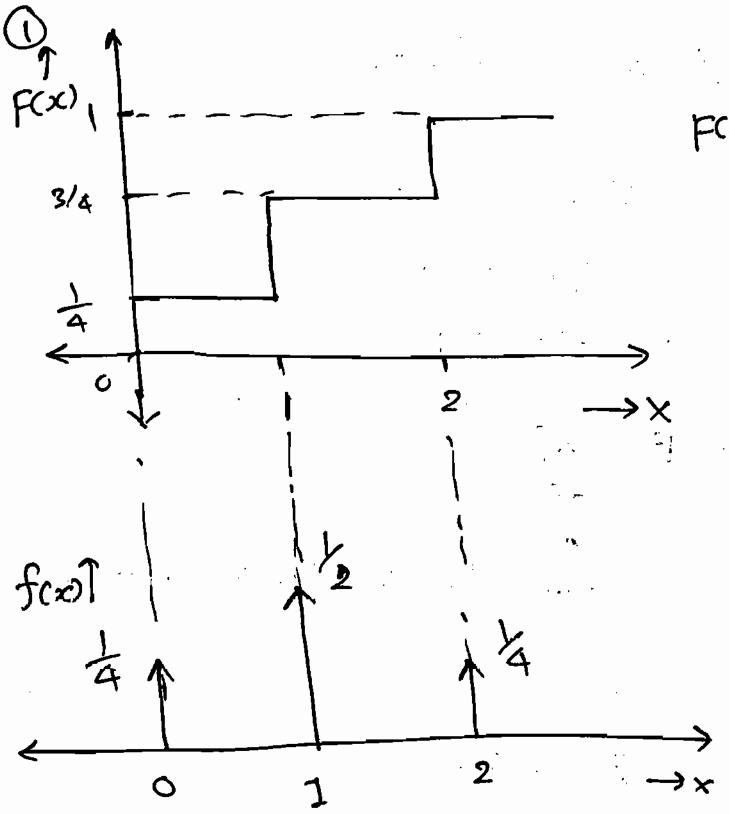
$= 0$.

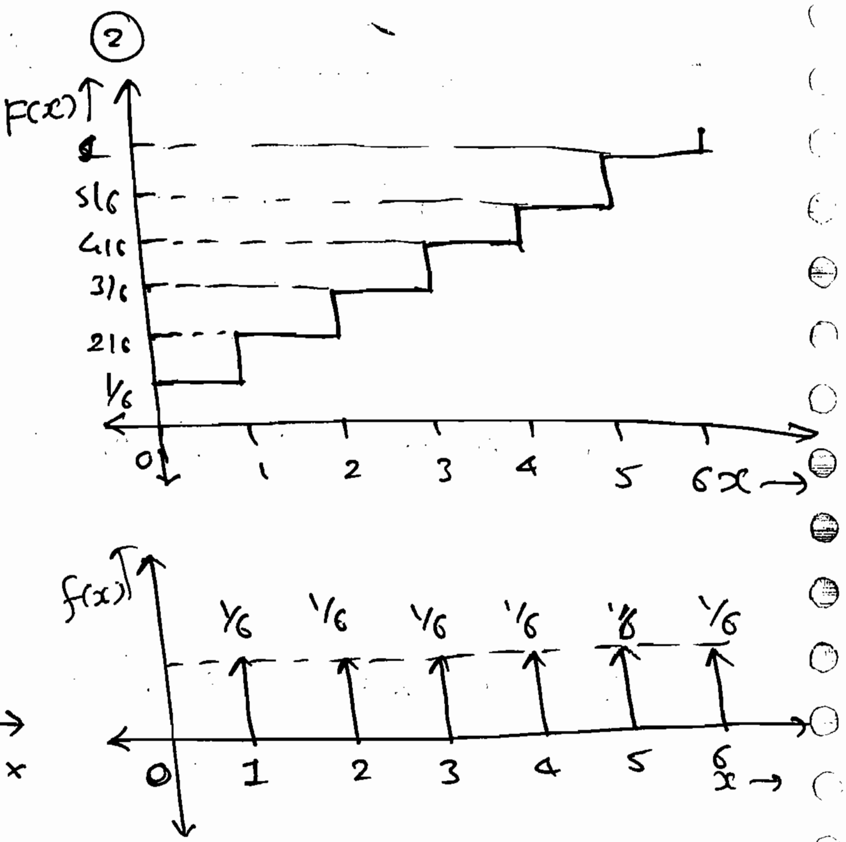
## * Probability density function ( pdf )

small letters

→ pdf indicates the distribution of total probability to various random variables.

$$\boxed{f(x) = \frac{d}{dx} F(x).}$$





## * Properties of pdf:

① $\int_{-\infty}^{\infty} f(x).\,dx = 1$ or Total area = 1 Imp.

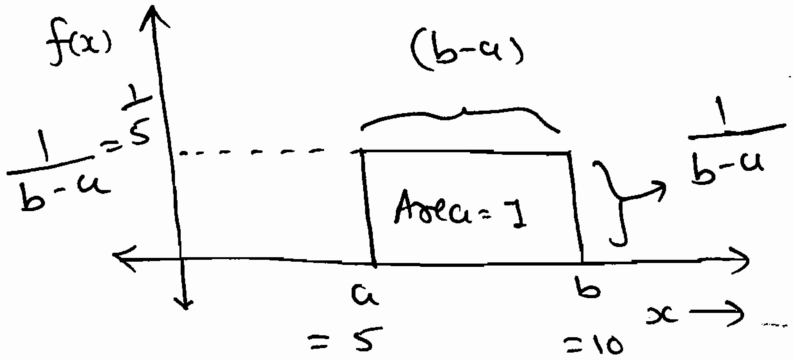
② $P[x \le x_1] = F(x_1) = \int_{-\infty}^{x_1} f(x)\,dx$

③ $P[x > x_1] = 1 - F(x_1) = \int_{x_1}^{\infty} f(x)\,dx.$

④ $P[x_1 < x \le x_2] = \int_{x_1}^{x_2} f(x)dx.$

## ① Uniform Density Function:

⇒ If pdf is constant within Specified range, then the Random Variable is said to be uniformly distributed.



Area

$A = (b-a) \times \frac{1}{(b-a)}$

$= 1$

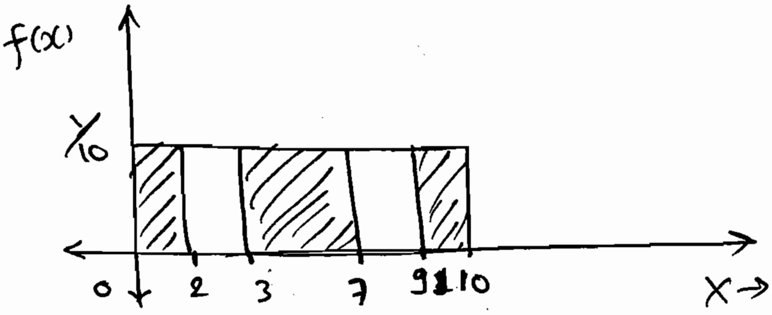
$\boxed{Q}$ A Continuous R.V. is uniformly distributed in the [0,10] ① Sketch the pdf & determine the p following probability.

i) $P[x \le 2]$ ② Sketch the PDF.

ii) $P[x > 9]$

iii) $P[3 < x \le 7]$.

Sol$^n$:



① $P[x \le 2] = \int_{-\infty}^{2} f(x).dx = \frac{1}{10} \times 2 = 0.2$

(ii) $P[x>9] = \int_{9}^{\infty} f(x).dx = (10-9) \times \frac{1}{10} = 0.1.$

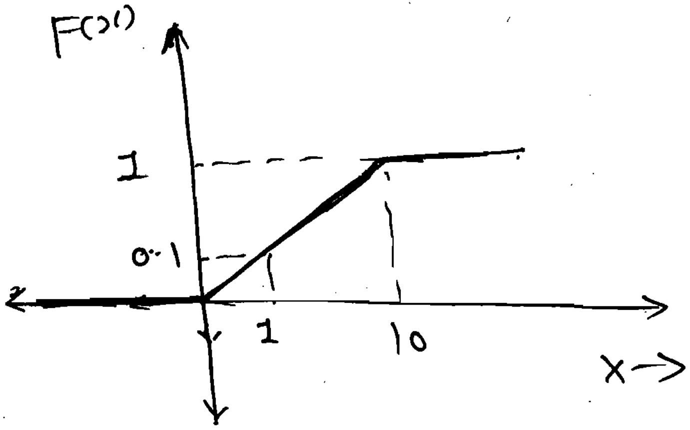
(iii) $P[3 < x \leq 7] = \int_{3}^{7} f(x).dx = \frac{1}{10} \times [7-3]$

$= 0.4.$

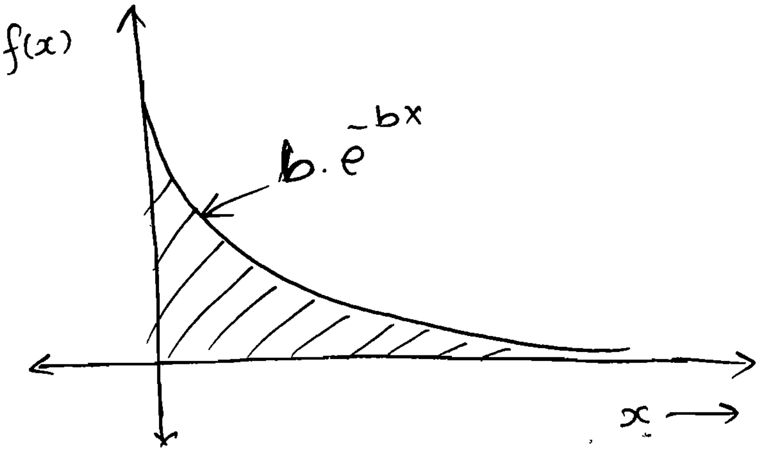
~~Q~~ ② $F(x) = \int_{-\infty}^{x} f(x).dx.$

$= \int_{0}^{\cancel{10}\,x} \frac{1}{10} . dx.$

$F(x) = \frac{x}{10}.$



## ② Exponential Density Function:



$f(x) = b.e^{-bx}, \quad x \geq 0$

$= 0, \quad x < 0.$

$\int_{0}^{\infty} a.e^{-bx}.dx = 1.$

$\therefore \frac{a}{b}.[0 - e^{-bx}]_{0}^{\infty} = 1.$

$\Rightarrow \frac{a}{b} = 1 \Rightarrow \boxed{a=b}$ Cond^n for valid Pdf.

* Identity Valid Pdf:

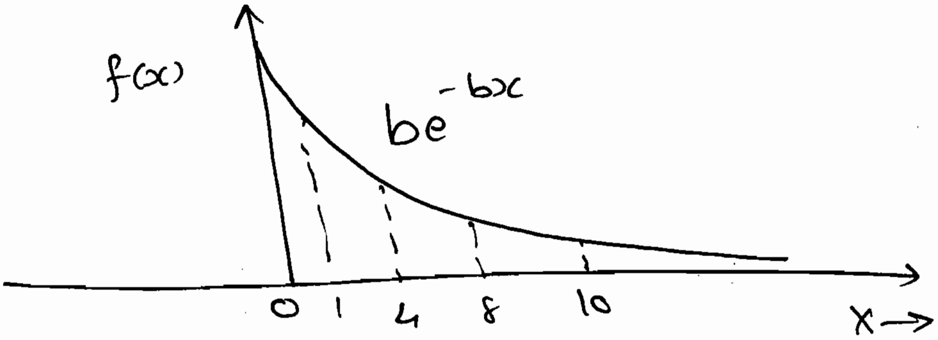
→ $f(x) = 10e^{-10x}, x \geqslant 0$ ✓ (valid). [∵ $a = b$]

→ $f(x) = 5e^{-10x}, x \geqslant 0$ ✗ ( ∵ $a \neq b \Rightarrow 10 \neq 5$].

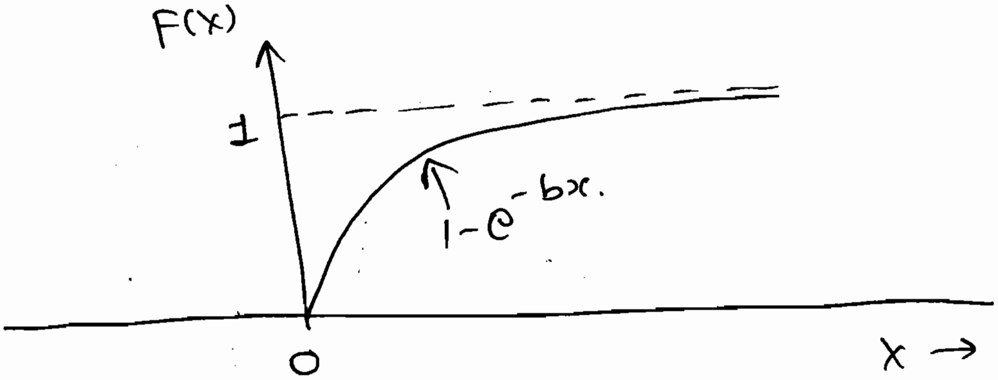
(Not valid)

⇒

pdf



PDF



$$\Rightarrow F(X) = \int_{-\infty}^{x} f(x).\,dx$$

$$= \int_{\cancel{-\infty}\,0}^{x} be^{-bx}.\, dx.$$

$$= -\frac{b}{b}\left[e^{-bx}\right]_0^x$$

$$\boxed{F(x) = [1 - e^{-bx}]}$$

# ③ Laplacian Density Function:

$$\rightarrow \quad f(x) = a \cdot e^{-b|x|}, \quad -\infty < x < \infty$$

$$= a \cdot e^{-bx}. \quad x > 0$$

$$= a.\ e^{bx}, \quad x < 0.$$

Cond^n for valid pdf:

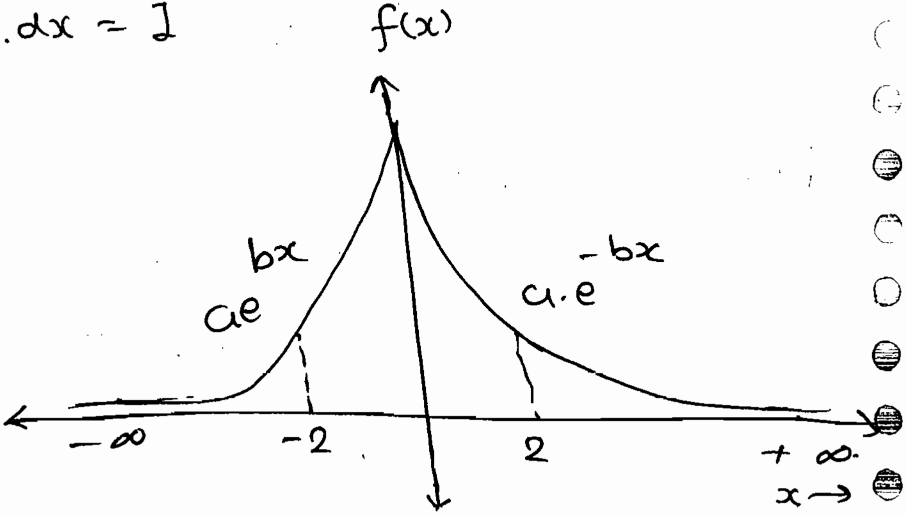
$$\Rightarrow \int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{-\infty}^{0} e^{-bx} .dx + \int_{0}^{\infty} e^{bx} .dx = 1$$

$$\therefore \quad \frac{a}{b} + \frac{a}{b} = 1$$

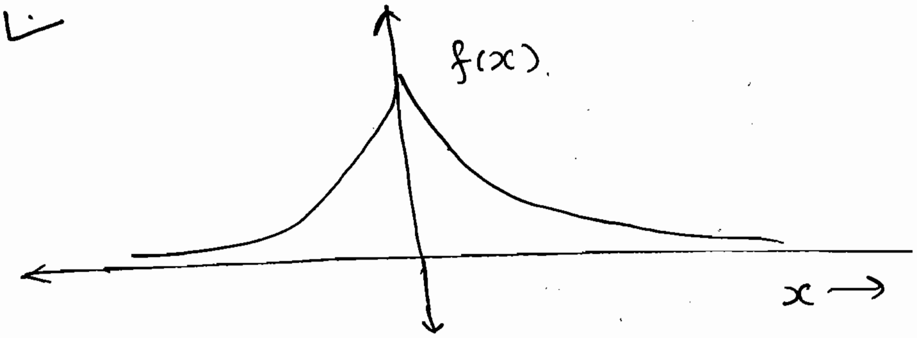
$$\frac{2a}{b} = 1.$$

$$\Rightarrow \boxed{a = b/2}$$

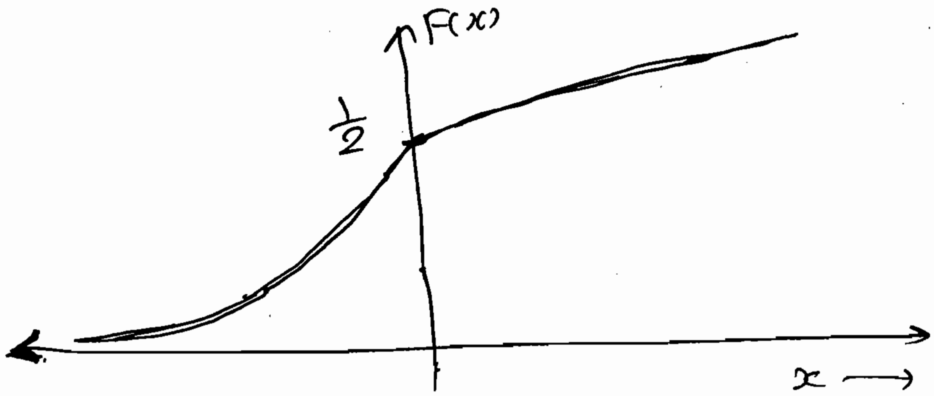


$\boxed{Q}$ Sketch F(x) from f(x). given.

$\boxed{Gate}$ L



Sol^n:



# * Statistical Averages of a Random Variable.

## ① Mean (or) Average Value:

$$\rightarrow \bar{X} = E[X] = m_1 = \int_{-\infty}^{\infty} x . f(x)\, dx \rightarrow \text{dc component only.}$$

## ② Mean Square Value:

$$\rightarrow \overline{X^2} = E[X^2] = m_2 = \int_{-\infty}^{\infty} x^2 . f(x)\, dx \rightarrow \text{Total Power.}$$

## ③ Variance:

$$\rightarrow \sigma^2 = E[(X-\bar{X})^2] = m_2 - m_1^2 = \text{ac Power}$$

= (mean square − square of mean).

## ④ Standard Deviation:

$$\rightarrow SD = \sqrt{\text{Variance}} = \sigma = \sqrt{m_2 - m_1^2}$$

$$= \sqrt{\text{ac power}}$$

= rms value of ac Component.

# * Properties of Mean:

① $E[K] = K$ , $K$ = constant

② $E[Kx] = K \cdot \bar{x}$

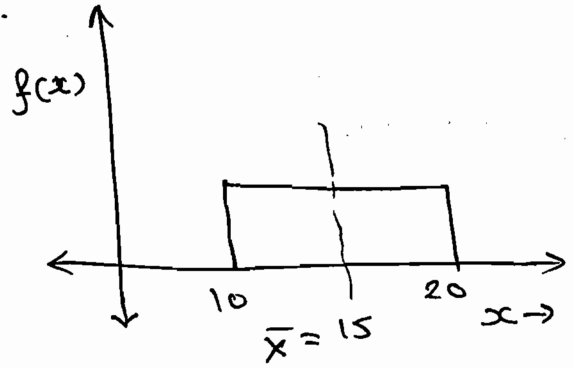
③ $E[x+K] = E[x] + E[K] = \bar{x} + K$

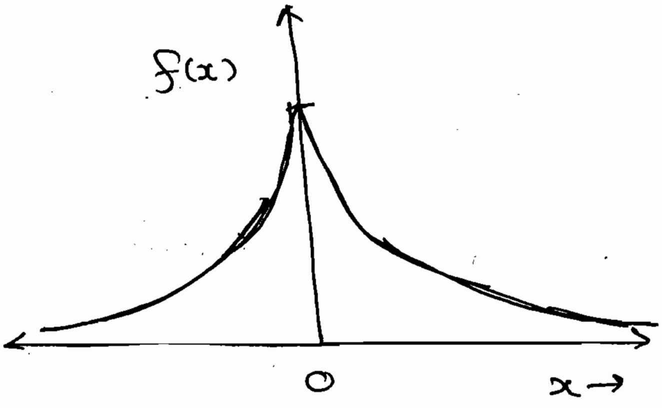
④ $E[x+y] = E[x] + E[y]$.

⑤ $E[xy] = E[x] \cdot E[y]$ provided that x & y are independent R.V.

⑥ If the Probability density function is Symmetrical w.r.t. a x-axis then the average value (mean) is equal to the symmetry point.

e.g.





$\bar{x} = 15$

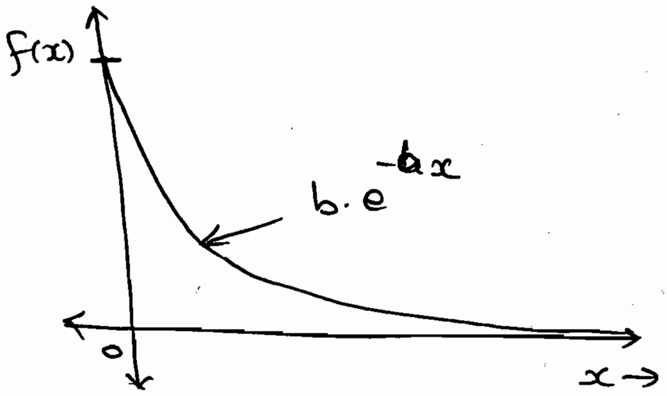
$\bar{x} = 0$.

$\boxed{Q-1}$ Determine the statistical avg. of exp. $f^n$.

Ans: $f(x) = b \cdot e^{-bx}$.

① mean

$$\bar{X} = E[X] = \int_0^\infty x . f(x) . dx$$



$$= \int_0^\infty x . b . e^{-bx} . dx.$$

$$= b. \left[ (x)\left(\frac{e^{-bx}}{-b}\right) - (1).\left(\frac{e^{-bx}}{b^2}\right)\right]_0^\infty$$

$$= b\left[ 0 - 0 + \frac{1}{b^2}\right]$$

$$\boxed{\bar{X} = \frac{1}{b}} = m_1$$

② Mean Sqaure,

$$m_2 = \overline{X^2} = E[x^2] = \int_0^\infty x^2 . b . e^{-bx} . dx.$$

$$= b\left[ (x^2).\left(\frac{e^{-bx}}{-b}\right) - (2x)\left(\frac{e^{-bx}}{b^2}\right) + (2)\left(-\frac{e^{-bx}}{b^3}\right)\right]_0^\infty$$

$$= b\left[ 0 + \frac{2}{b^3}\right]$$

$$\boxed{m_2 = \frac{2}{b^2}}$$

③ Variance = $\sigma^2 = m_2 - m_1^2$

$$\sigma^2 = \frac{2}{b^2} - \frac{1}{b^2} = \frac{1}{b^2}.$$

④ Standard deviation

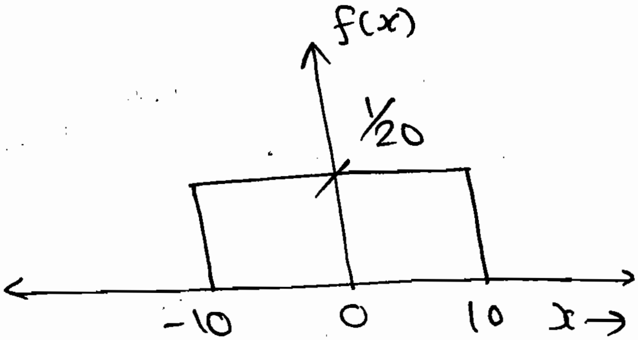
$$S.D. = \sqrt{m_2 - m_1^2}$$

$$\Rightarrow \boxed{S.D. = \frac{1}{b}}$$

$\boxed{Q-2}$ A continous R.V. is uniformly distn. in the [-10, 10]. Determine the statistical average.

Soln:

① $f(x) = \frac{1}{20}$.



① Mean

$$m_1 = \bar{X} = E[X] = \int_{-10}^{10} X \cdot \frac{1}{20} \cdot dx$$

$$\boxed{m_1 = 0}$$

② Mean square,

$$m_2 = \bar{X^2} = E[x^2] = \int_{-10}^{10} x^2 \cdot \frac{1}{20} \cdot dx.$$

$$m_2 = \frac{2}{20} \int_{0}^{10} x^2 \cdot dx = \frac{1}{10} \times \frac{1000}{3} = \frac{100}{3}.$$

$$\boxed{m_2 = \frac{100}{3}}$$

③ Variance,

$$\sigma^2 = \left(\sqrt{m_2 - m_1^2}\right)^2$$

$$\sigma^2 = m_2 - m_1^2$$

$$= \frac{100}{3} - 0$$

$$\boxed{\sigma^2 = \frac{100}{3}}$$

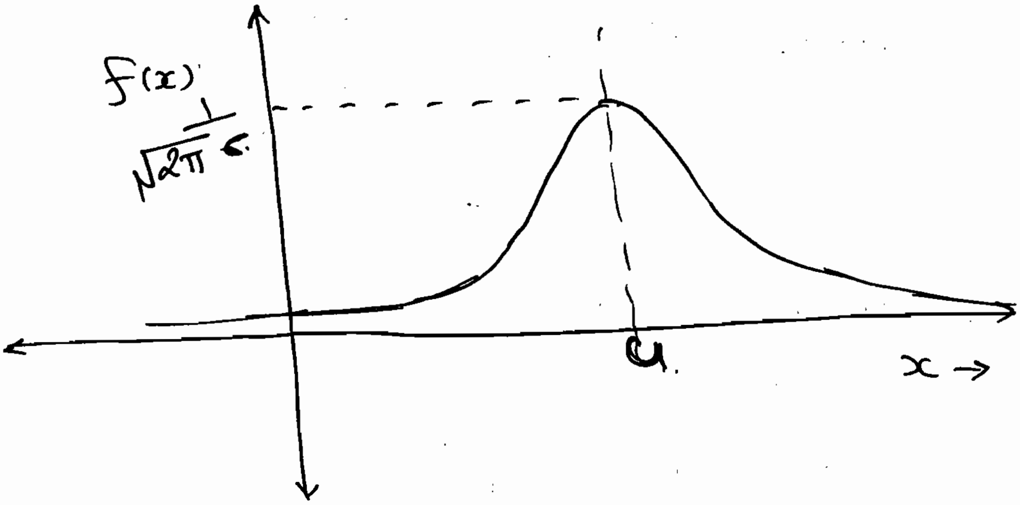
④ S.D. = $\sqrt{\text{variance}}$.

$$\boxed{S.D. = \frac{10}{\sqrt{3}}}$$

## ④ Gaussian Density Function:

⇒ Pdf 
$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}}}$$ 
$-\infty < x < \infty.$

①



⇒ ① Mean:

f(x) is symmetrical w.r.t. x axis at x=a.

So, mean = $m_1$ = E[X] = 'a'. ⇒ $\boxed{m_1 = a}$

② $$m_2 = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx = '\sigma^2 + a^2'$$

⇒ $\boxed{m_2 = \sigma^2 + a^2.}$

③ Variance $\sigma^2 = m_2 - m_1^2$

Variance = $\sigma^2 + a^2 - a^2 = \sigma^2$

∴ $\boxed{\text{Variance} = \sigma^2}$

④ $S.D. = \sigma = \sqrt{\text{Variance}}$

$$\boxed{S.D. = \sigma}$$

$\boxed{Q}$ find $m_2$ of $f(x) = \dfrac{1}{\sqrt{(\ \ )}} \cdot e^{-\frac{(x-2)^2}{100}}$.

Sol^n: Here, $a = 2$, $2\sigma^2 = 100$

$\sigma^2 = \frac{100}{2}$

$$\boxed{\sigma^2 = 50}$$

$S.D. = \sqrt{\sigma^2}$

$$\boxed{S.D. = 5\sqrt{2}}$$

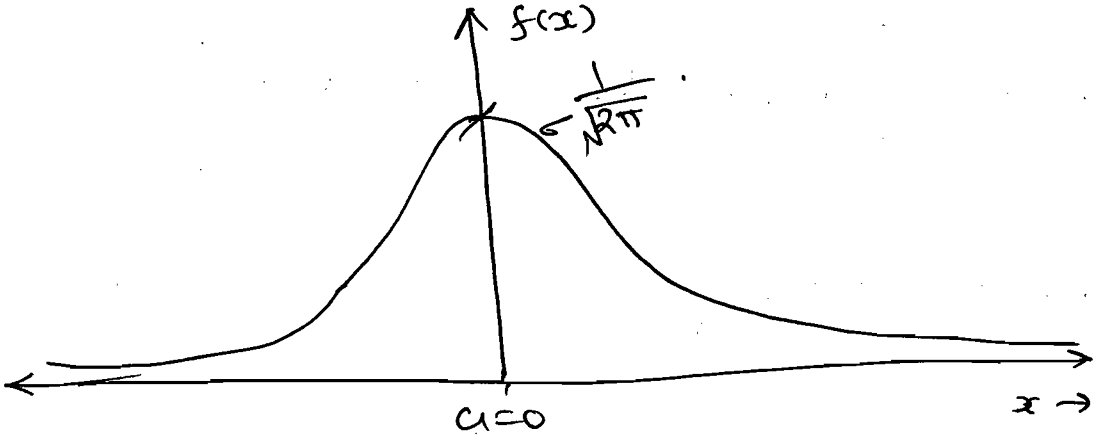
$\therefore m_2 = \sigma^2 + a^2$

$= 50 + 4$

$$\boxed{m_2 = 54} \qquad \boxed{m_1 = a = 2}$$

$\Rightarrow$ if $a=0$, then

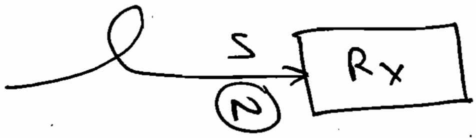
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



# ☆ Random Process:

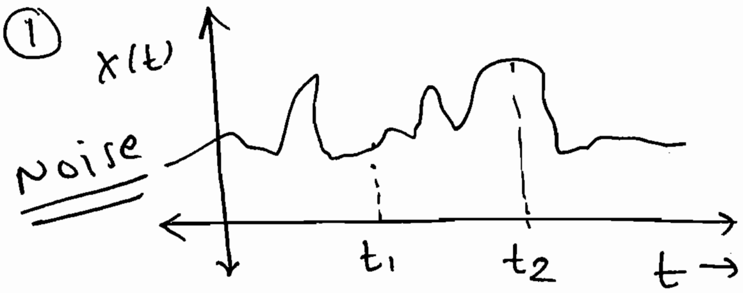
⇒ A random variable which is function of time is called as the Random Process.

⇒ Random Process $= X(t)$.

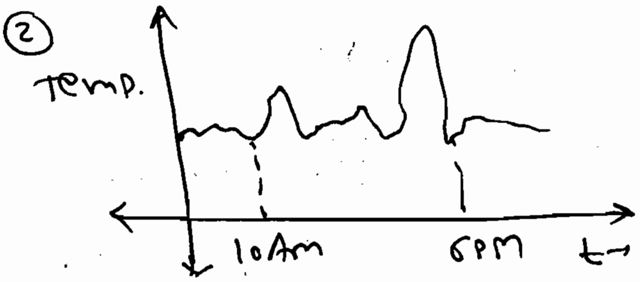


e.g.

①



②



## * PDF of a Random Process:

→ $X \rightarrow F[x_1] = P[X \leq x_1]$.

→ $X(t) \rightarrow F[x, t_1] = P[X(t) \leq x_1]$

→ 1st order PDF.

$$F[x, t_1] = P[x(t_1) \leq 2mV]$$

$$= P[X(2PM) \leq 25^{\circ}C].$$

$$= P[X[2Am) \leq 25^{\circ}C]$$

→ $F[x_1 x_2 t_1 t_2] = P[X(t_1) \leq x_1 \; \& \; X[t_2) \leq x_2]$.

→ 2nd order PDF.

## * Pdf of a Random Process:

→ $f(x) = \frac{d}{dx} F(x).$

→ $f(x_1 t_1) = \frac{\partial}{\partial x} F(x_1 t_1)$ → $1^{st}$ order.

→ $f(x_1 x_2 t_1 t_2) = \frac{\partial^2}{\partial x_1 \partial x_2} F[x_1 x_2 t_1 t_2]$

→ $2^{nd}$ order.

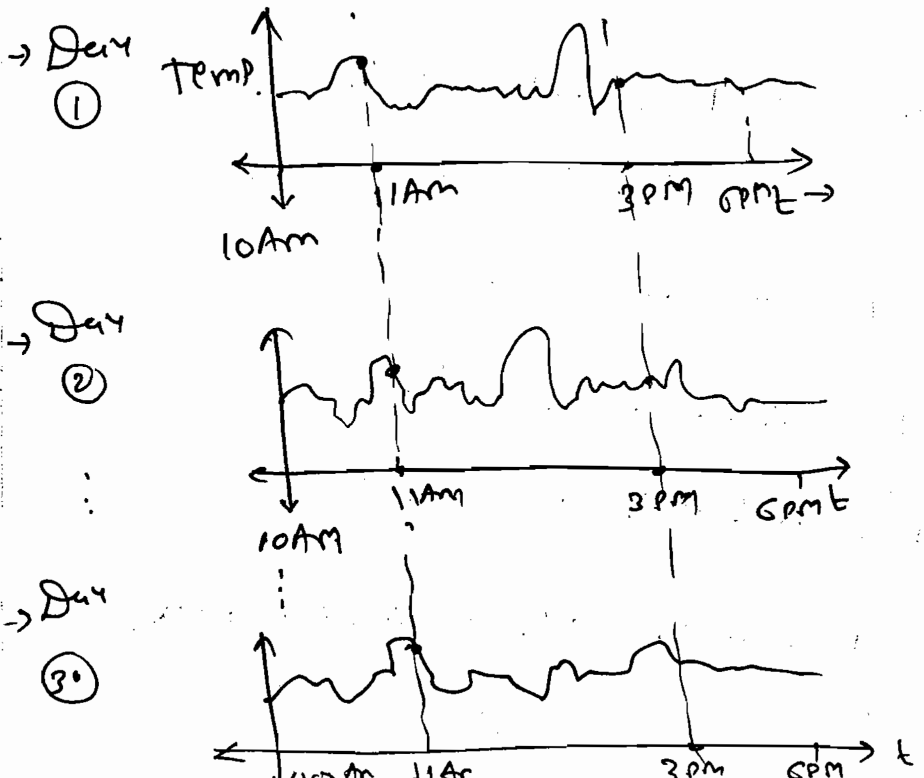
## * Statistical averages of a Random Process:

⇒ ① Ensemble averages.

② Time averages.

⇒ Ensemble means collection of data.

## ① Ensemble averages:



① mean,

$$m_1(t) = E[X(t)] = \int_{-\infty}^{\infty} X(t) \cdot f(xt) \cdot \underline{\underline{dx}}$$

② Mean Sauare,

$$m_2(t) = E[X^2(t)] = \int_{-\infty}^{\infty} X^2(t) \cdot f(xt) \; \underline{\underline{dx}}.$$

③ Variance,

$$= m_2(t) - m_1^2(t).$$

④ Standard Deviation,

$$S.D. = \sqrt{Variance}$$

$$S.D. = \sqrt{m_2(t) - m_1^2(t)}.$$

⑤ <u>Auto Corelation Function:</u> ✓

$$ACF = E[X(t_1) \cdot X(t_2)].$$

→ ACF is used to determine the simillarity of a Random Process at two instant of time,

$$\boxed{ACF = E[X(t) \cdot X(t+\tau)]}$$

$t_1 \to t.$

$t_2 \to t+\tau.$

## ② Time averages:

⟹ ① mean,

$$< x(t) > = m_1 = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} x(t)\, dt$$

② Mean square,

$$< x^2(t) > = m_2 = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} x^2(t)\, dt$$

③ Variance, $= m_2 - m_1^2$

④ $S.D. = \sqrt{m_2 - m_1^2}$.

⑤ $ACF = < x(t) \cdot x(t+\tau) >$ $\quad t_1 \to t$, $t_2 \to t+\tau$.

$$ACF = \frac{1}{\tau} \int_{t}^{t+\tau} x(t) \cdot x(t+\tau)\, dt.$$

⟹ If the Ensemble avgs and Time avgs are equal then the Random Process is called Ergodic Random process.

⟹ A random Process is said to be stationary of order one if the following condition is satisfied,

$$\Rightarrow \boxed{f(x_1, t_1) = f(x_1\ t_1+\Delta t).}$$

↑ stationary of order one.

$$\Rightarrow \boxed{f(x_1 x_2 t_1 t_2) = f(x_1 x_2\ t_1+\Delta t\ t_2+\Delta t).}$$

↑ stationary of order two.

* Wide Sense Stationary:

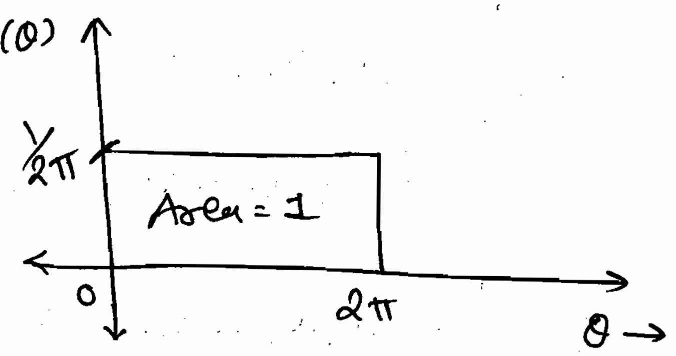
⇒ A random Process is said to be wide sense stationary if the following two conditions are satisfied:

$$\boxed{\begin{array}{l} ①\ E[X(t)] = \text{constant} \\ ②\ \text{The ACF should depends only on } t_2 - t_1 \text{ i.e. } \tau. \end{array}}$$

$\boxed{Q}$ Consider a Random Process $X[t] = A\cos[\omega_0 t + \theta]$, where $A$ & $\omega_0$ are constant and $\theta$ is a R.V. Uniformally distributed in $[0, 2\pi]$. Determine the mean & ACF. and show that Random Process is WSS.

Ans: $X(t) = A\cos[\omega_0 t + \theta]$.

$$\Rightarrow \quad f(\theta) = \frac{1}{2\pi}.$$



$$\rightarrow \quad E[x] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$E[x(t)] = \int_{0}^{2\pi} x(t) \cdot f(\theta) \cdot d\theta.$$

$$= \int_{0}^{2\pi} A\cos[\omega_0 t + \theta] \cdot \frac{1}{2\pi} \cdot d\theta.$$

$$\Rightarrow \quad \boxed{E[x(t)] = 0} \leftarrow \text{Ensemble mean.}$$

$$\Rightarrow \quad ACF = E[\,x(t_1) \cdot x(t_2)].$$

$$= E\left[A\cos[\,\omega_0 t_1 + \theta] \cdot A\cos[\omega_0 t_2 + \theta]\right]$$

$$= E\left[\frac{A^2}{2}\cos[\,\omega_0 (t_1 + t_2) + 2\theta + \frac{A^2}{2}\cos[\,\omega_0 (t_1 - t_2)]\,\right].$$

$$= E\left[\frac{A^2}{2} \cdot \cos[\,\omega_0 (t_2 + t_1) + 2\theta]\right]$$

$$+ E\left[\frac{A^2}{2} \cdot \cos[\,\omega_0 (t_2 - t_1)]\right].$$

$$= 0 + \frac{A^2}{2} \cdot \cos[\omega_0 (t_2 - t_1)].$$

⇒ which is independent of R.V. i.e here $\theta$.

So, ① $E[x(t)] = 0$ = constant.

② ACF is depends on $(t_2 - t_1)$.

So, Random Process in WSS.

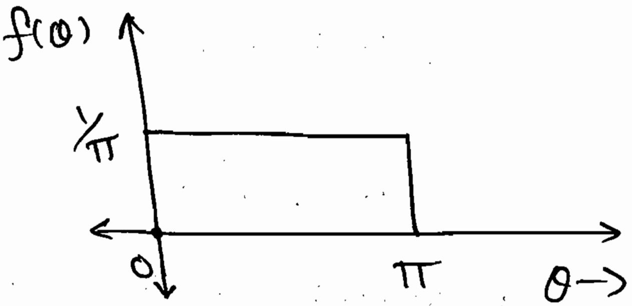
$$ACF = \frac{A^2}{2} \cos \omega_0 \tau$$

$$\boxed{ACF = \frac{A^2}{2} \cos 2\pi f_0 \tau. = R(\tau).}$$

$\boxed{Q-2}$ Consider a Random Process $x(t) = A\cos[\omega_0 t + \theta]$, $A$ & $\omega_0$ are constants and $\theta$ is a R.V. uniformally distributed in the $[0, \pi]$. check weither the R.P. is WSS or not?

Soln: $x(t) = A\cos[\omega_0 t + \theta]$.

→ $f(\theta) = \frac{1}{\pi}$.



① Ensemble mean,

$$E[x(t)] = \int_0^{\pi} A\cos[\omega_0 t + \theta] . \frac{1}{\pi} \, d\theta.$$

$$= \int_0^{\frac{\pi}{2}} A\cos(\omega_0 t + \theta) \, d\theta - \int_{\pi/2}^{\pi} A\cos(\omega_0 t + \theta) \, d\theta.$$

$$= A\left[\sin(\omega_0 t + \theta)\right]_0^{\frac{\pi}{2}} - A\left[\sin(\omega_0 t + \theta)\right]_{\pi/2}^{\pi}$$

$$= A\left[\cos\omega_0 t - \cancel{\sin\omega_0 t} + \cancel{\sin\omega_0 t} + \cos\omega_0 t\right]$$

$$= 2A\cos\omega_0 t$$

$$\boxed{ACF = \frac{A^2}{2}\cos 2\pi f_0 \tau}$$

$$\Rightarrow E[x(t)] = 2A\cos\omega_0 t$$

which is not constant.

So, Random Process is not WSS.

## * Properties of ACF:

$$\Rightarrow ACF = R(\tau) = E[x(t).\,x(t+\tau)].$$

① $R(0) = E[x^2(t)]$

$\Rightarrow ACF = R(0) =$ Mean square $=$ Power.

② The Fourier Transform of ACF is nothing but the Power spectral Density (PSD).

(W/Hz)

$$ACF \xleftrightarrow{FT} PSD$$

$$\Rightarrow \boxed{R(\tau) \xleftrightarrow{FT} S(f).}$$

$\Rightarrow$ PSD indicates how the total Power is distributed to various freq.

$$\Rightarrow \boxed{S(f) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-j2\pi f\tau} \cdot d\tau .}$$

$$\boxed{R(\tau) = \int_{-\infty}^{\infty} S(f) \cdot e^{j2\pi f\tau} \cdot df .} \leftarrow H.B.$$

$\omega = 2\pi f$

$f = \omega/2\pi$

IMP

$$\therefore \; R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) \cdot e^{j2\pi f\tau} \cdot d\omega .$$

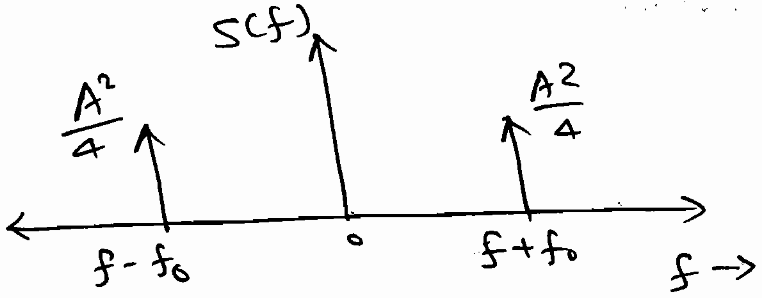
* $x(t) = A \cos(\omega_0 t + \theta).$

$\Rightarrow$ 8x ACF $= R(\tau) = \frac{A^2}{2} \cos 2\pi f_0 \tau .$

$\rightarrow S(f) = \frac{A^2}{4} [\delta(f-f_0) + \delta(f+f_0)].$

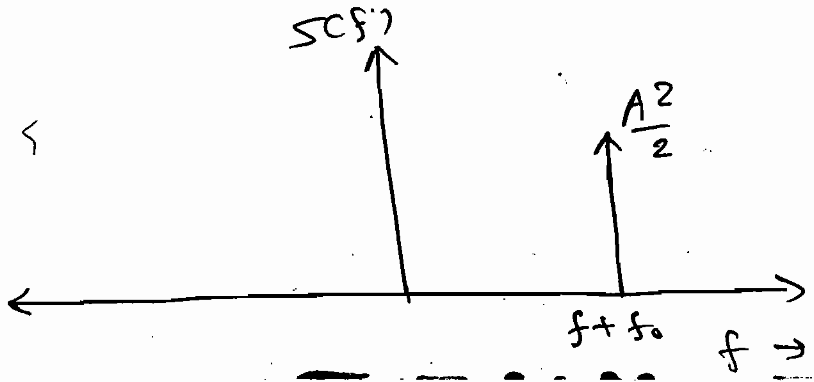
PSD

$\Rightarrow$



(a) Two sided PSD.

=)



(b) ~~Two~~ single sided PSD.

⇒ ③ $R(0) = \int_{-\infty}^{\infty} S(f).df$ = Power

→ Area under the PSD should gives Power.

→ Noise Power = Mean square

$= \int x^2 . f(x)\, dx.$

$= R(0).$

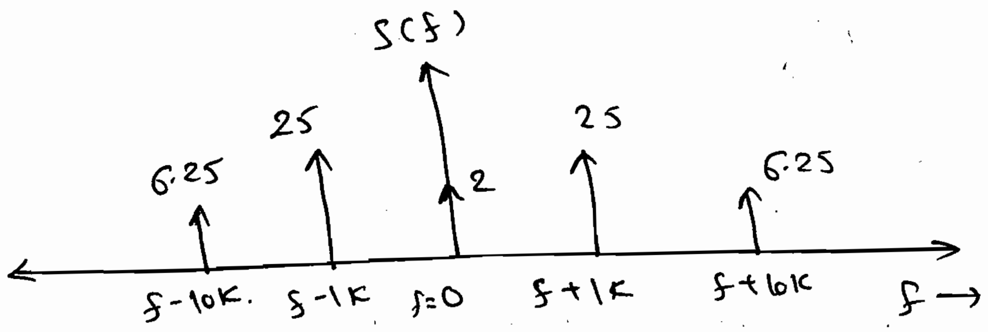
$= \int (PSD)\, \underline{\underline{df}}$

$= \frac{1}{2\pi} \int (PSD)\, \underline{\underline{d\omega}}.$

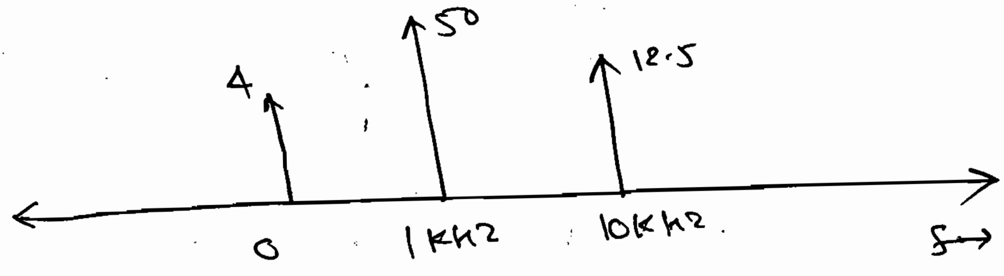
= Area under PSD.

* $2 + 10\cos 2\pi 10^3 t + 5\cos 2\pi 10^4 t$

$\underline{\underline{2}}$ ↙ DC.
f = 0Hz

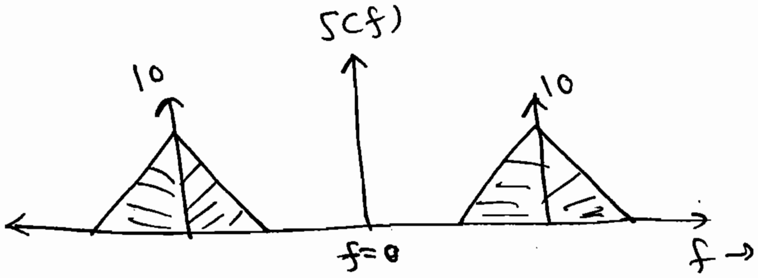


~~d~~. (Two sided PSD)



(single sided PSD)

⊛ IMP:



Mean = DC = impulse at f=0.

, otherwise mean = 0.

## ☆ Noise :-

* Generalized defⁿ:

⇒ Any unwanted signal interfering with the required signal is called as noise.

⇒ Noise are classified into two types:

① External Noise.

② Internal Noise.

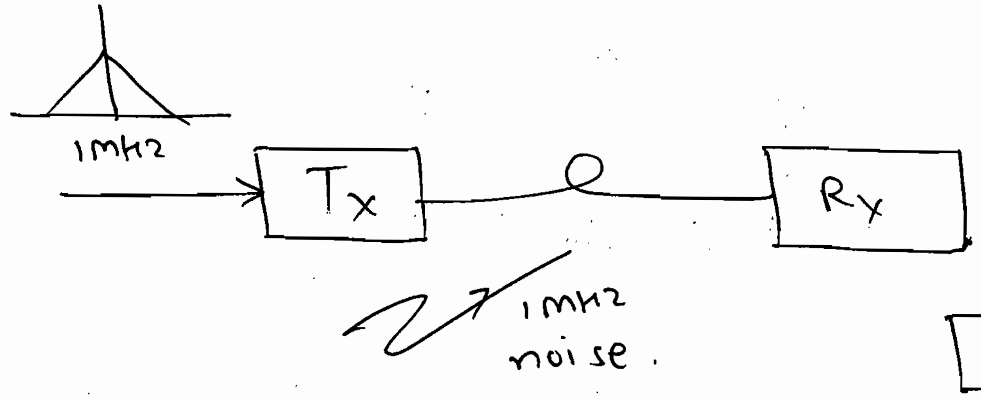
### ① External Noise:

⇒ Noise which is generated outside the system is called as the External Noise.

e.g. ① Atmospheric noise

→ Sudden electrical disturbutions.

=>



$\boxed{S/N >> 1}$

② Solar noise

→ radiation from SUN.

③ Industrial Noise.

## ② Internal Noise:

=> The Noise which is generated within the System is called Internal Noise.

e.g.: Radiations from electrical appliances.

=> Whenever the noise is transmitted intentionally to along with the signal then it disturbs the Communication system is called signal Jamming.

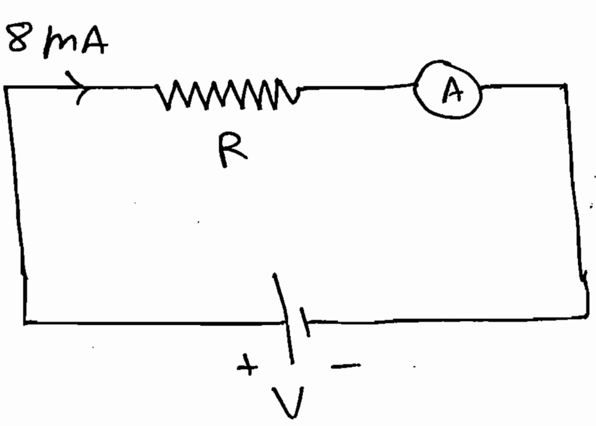
for e.g. Mobile Phone Jammer.

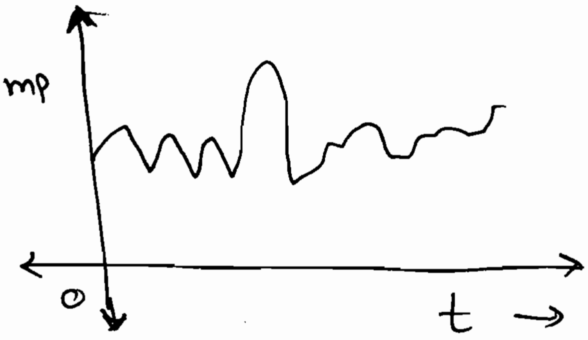
→ Transmit Noise signal at signal freq. having noise power greater than signal power. to Jam the desired signal.

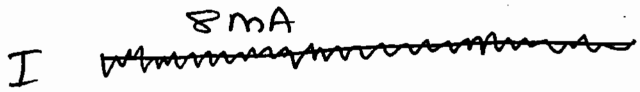
## ☆ Internal Noise:

### ① Thermal Noise (or) White noise:

⇒







⇒ Thermal Noise is given by following emperical formula.

$$\boxed{P = KTB \text{ watts.}}$$ ← H.B.

where, K = Boltzman Constant

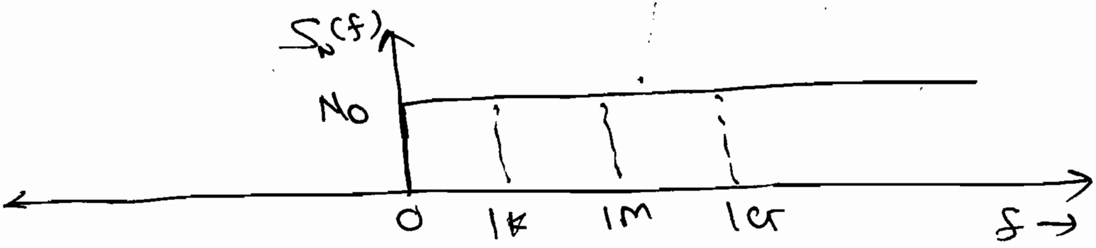
$= 1.38 \times 10^{-23}$ J/°K.

T = Temp.

B = Bandwidth.

$$\boxed{KT = \frac{\text{watts}}{\text{Hz}} = \text{PSD.}}$$ ← H.B.

→ $KT = N_o$ = PSD of Thermal Noise.

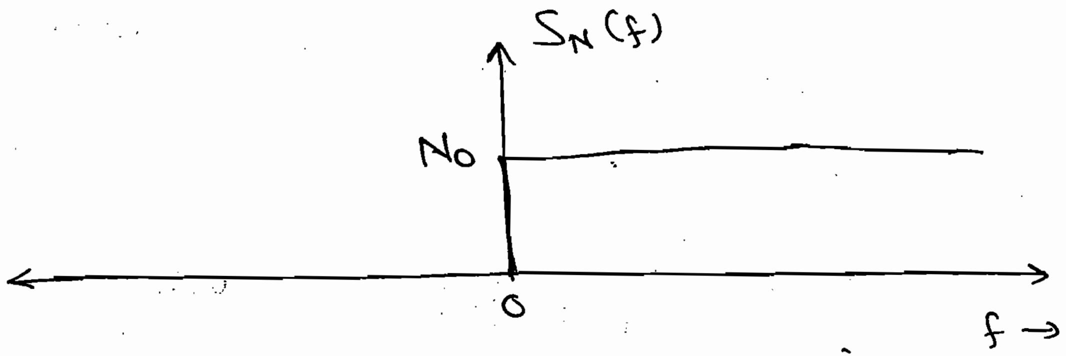


## * Coloured Noise:

⇒ When noise affecting only narrow band of freqs then it is called Coloured noise.

⇒ Noise is affecting all freqs. equally.
(∵ No is independent of freq.).

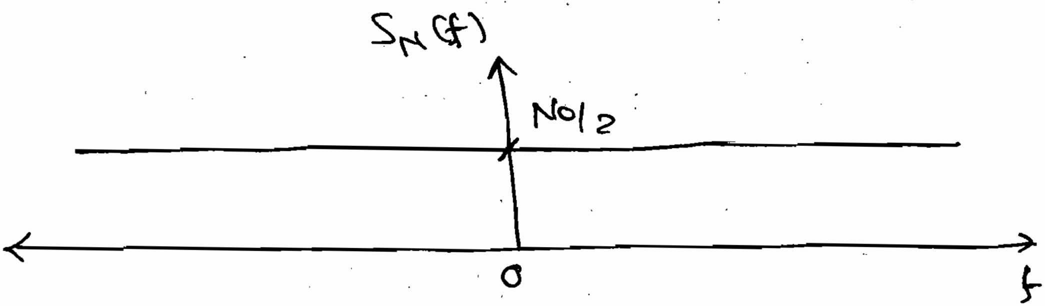
→



(singne side PSD of Noise)

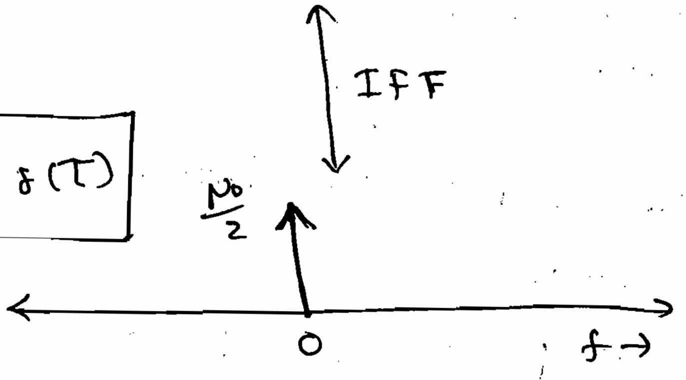
↙ H.B.

→

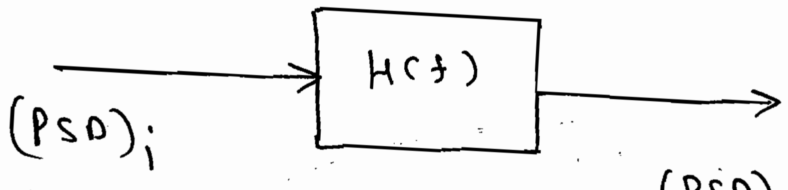


(Two Sided PSD of Noise)

H.B
↓
* * *
$$\boxed{R(\tau) = \frac{N_0}{2}\,\delta(\tau)}$$



Most
IMP
topic

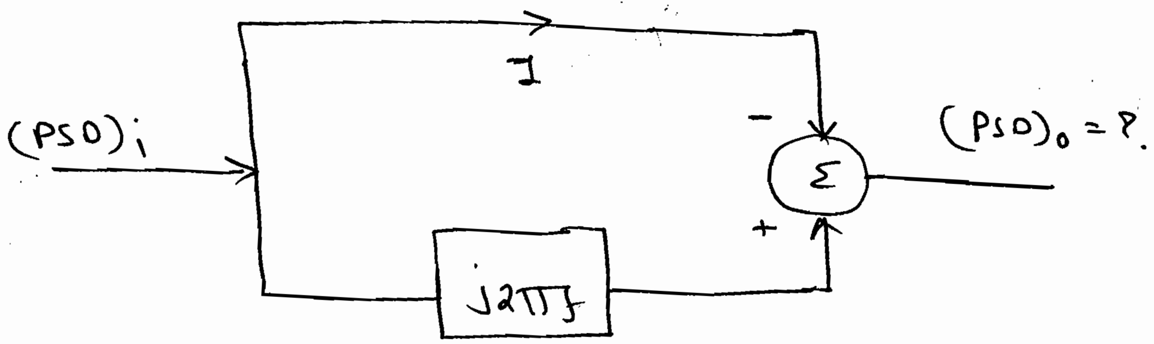


$$(PSD)_o = (PSD)_i \cdot |H(f)|^2.$$

$$\Rightarrow \boxed{PSD_o = (PSD)_i \cdot |H(f)|^2} \leftarrow \underline{\underline{H.B.}}$$

Gate:

Q



Sol$^n$:

$$H(f) = -1 + j2\pi f$$

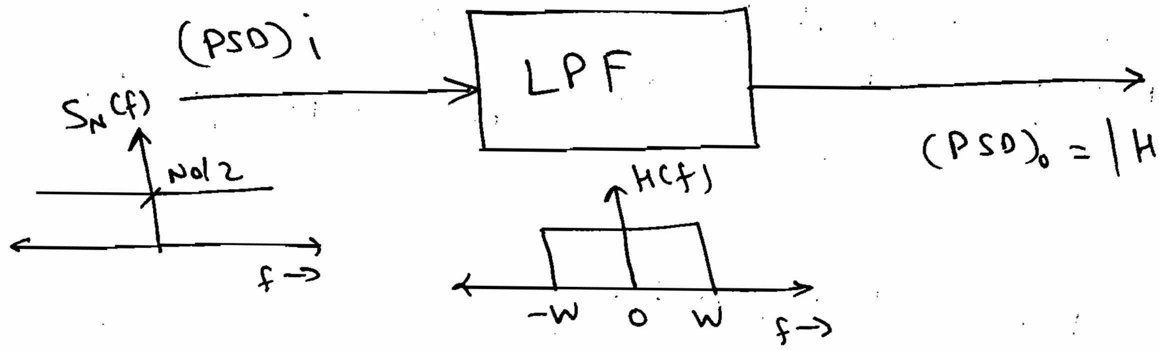
$$|H(f)| = \sqrt{1 + (2\pi f)^2}$$

$$|H(f)|^2 = 1 + (2\pi f)^2.$$

$$(PSD)_o = |H(f)|^2 . (PSD)_i$$

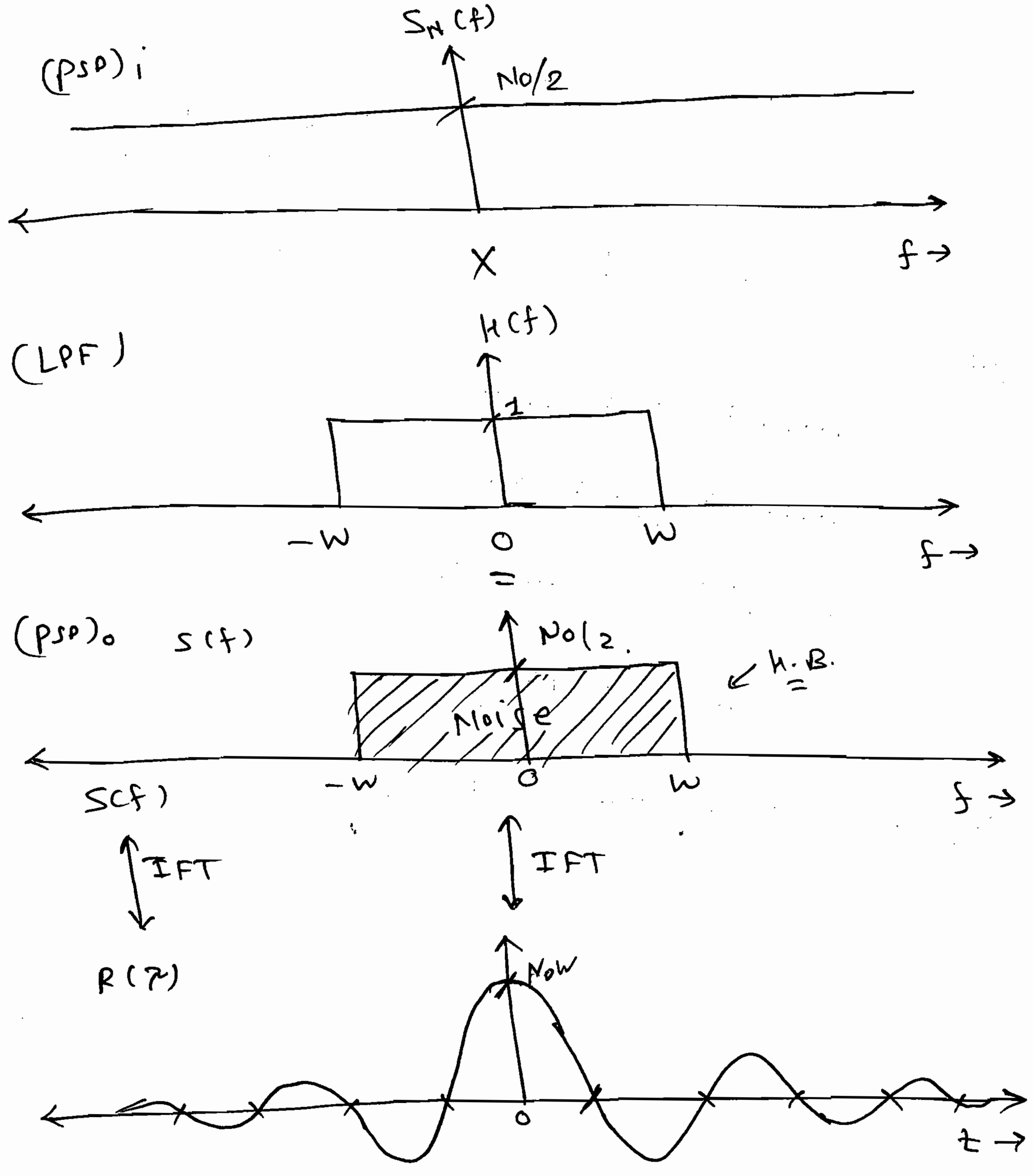
$$\therefore \boxed{(PSD)_o = (1 + 4\pi^2 f^2)(PSD)_i}$$

## ① When White Noise Passed through LPF :



$(PSD)_o = |H(f)|^2 . (PSD)_i$

$|H(f)| = 1 \Rightarrow |H(f)|^2 = 1.$



$$\Rightarrow \text{Noise Power} = \int_{-W}^{W} \frac{N_o}{2} \cdot df$$

$$\boxed{\text{Noise Power} = \text{'}N_o \cdot W\text{'} \text{ watts.}} \leftarrow \underline{\text{H.B.}}$$

$$\underline{\text{ACF}} \Rightarrow R(\tau) = \frac{N_o}{2} \times 2W \cdot \text{sinc}[2W\tau].$$

$$\boxed{R(\tau) = N_o W \cdot \text{sinc}(2W\tau).} \leftarrow \underline{\text{H.B}}$$

$$\Rightarrow R(0) = \text{Mean square} = \text{Power} = N_o W \text{ watts.}$$

Q White noise having a <u>two sided</u> spetral density of $4 \times 10^{-3}$ watts/Hz is passed through an ideal LPF having a cutoff freq 2 kHz. Determine the noise power and ACF at the o/p of the filter.

<u>Soln</u>:

$$\frac{N_o}{2} = 4 \times 10^{-3} \text{ watts/Hz}$$

$$\Rightarrow N_o = 8 \times 10^{-3} \text{ watts/Hz.}$$

$$W = 2\text{ kHz} = 2000 \text{ Hz.} = f$$

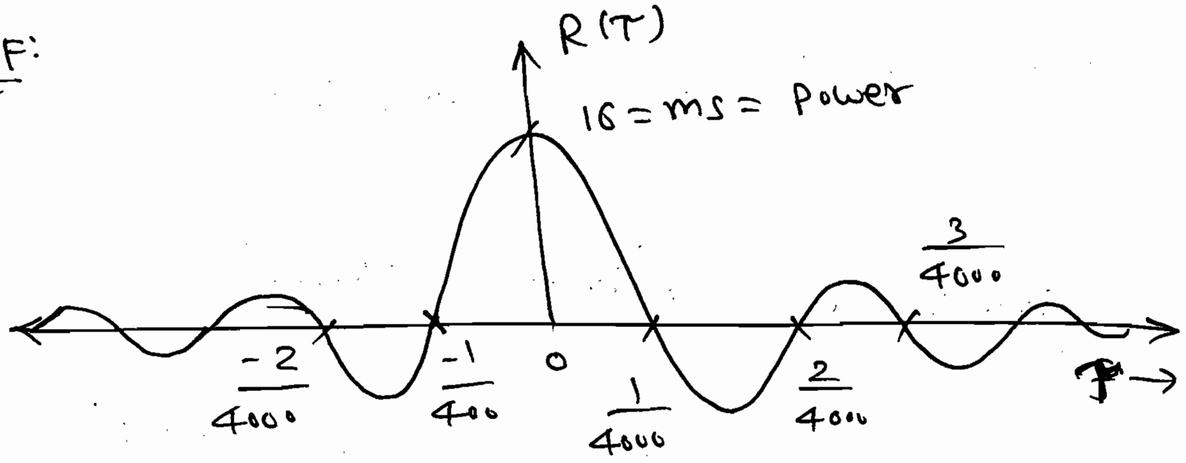
$$\text{Noise Power} = N_o W$$

$$= 8 \times 10^{-3} \times 2 \times 10^{3}$$

$$\boxed{\text{Noise Power} = 16 \text{ watts.}}$$

$$\Rightarrow \boxed{\text{ACF} = R(\tau) = 16\sin[4000\tau].}$$

$\Rightarrow$ ACF:



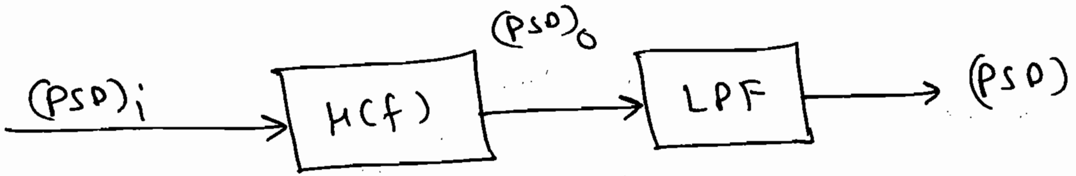
$\rightarrow$ Spacing is $\left(\frac{2}{4000} - \frac{1}{4000}\right) = \frac{1}{4000} = \frac{1}{2W}$.

$\rightarrow$ If the ACF is sampled at $\frac{1}{4000}, \frac{2}{4000}, \frac{3}{4000}$ at this, the samples are zero (or) this these samples are said to be uncorrelated.

$\boxed{Q}$ ** White Noise having a uniform spectrum density of $N_0$ watts/Hz is passed through a system having transfer function $H(f) = 2 \cdot e^{-j2\pi f t_0}$. The o/p of an ideal LPF having a cut-off freq of 'B' Hz. Determine Noise power at the o/p of the filter.

Gate-2005

Soln:



$H(f) = 2 \cdot e^{-j2\pi f t_0}$ $\quad$ $f_c = B$.

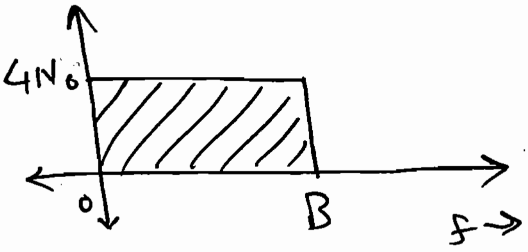
$\Rightarrow$ $|H(f)| = 2$

$\Rightarrow$ $|H(f)|^2 = 4$.

$\Rightarrow (PSD)_i = N_0$

$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$

$\therefore (PSD)_o = 4N_0$



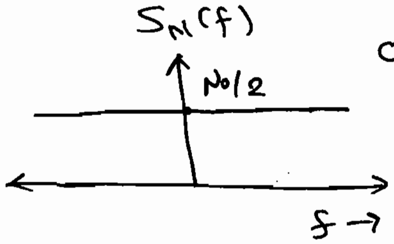
$\Rightarrow$ Noise power

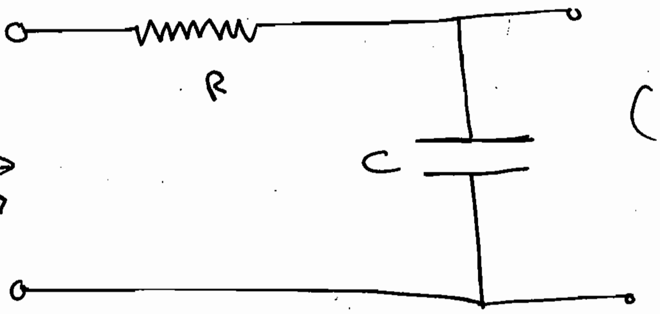
$$= \int_0^B 4N_0 \cdot df$$

$$\boxed{\text{Noise power} = 4N_0B \text{ watts.}}$$

* When white noise Passed through RC LPF.

$\Rightarrow$





$(PSD)_o = |H(f)|^2 (PSD)_i$

$$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \cdot V_{in}$$

$$\therefore H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + RsC}.$$

$s = j\omega$

$$\therefore H(j\omega) = \frac{1}{1 + j\omega CR}.$$

$\omega = 2\pi f$

$$\therefore H(f) = \frac{1}{1 + j2\pi fRC}.$$

$$\therefore \quad |H(f)|^2 = \frac{1}{1+(2\pi f RC)^2}$$

$$\therefore \quad (PSD)_0 = \frac{N_0}{2} \cdot \frac{1}{1+(2\pi f RC)^2}$$

$$\therefore \quad (PSD)_0 = \frac{\frac{N_0}{2}}{1+(\omega RC)^2} = \frac{\frac{N_0}{2(RC)^2}}{\omega^2+\left(\frac{1}{RC}\right)^2}$$

**ACF**

$$\underline{\underline{S(\omega)}} \qquad\qquad \underline{\underline{R(\tau)}}$$

$$\frac{2a}{\omega^2+a^2} \xleftrightarrow{FT} e^{-a|t|}$$

$$\Rightarrow \frac{N_0}{4RC} \cdot \frac{\frac{2}{RC}}{\omega^2+\left(\frac{1}{RC}\right)^2} \xleftrightarrow{FT} \frac{N_0}{4RC} \cdot e^{-\frac{|\tau|}{RC}}$$

$$\therefore \boxed{R(\tau) = \frac{N_0}{4RC} \cdot e^{-\frac{|\tau|}{RC}}} \leftarrow H.B.$$

Noise Power = mean square = R(0)

$$\boxed{\text{Noise Power} = \frac{N_0}{4RC}} \leftarrow \underline{H.B.}$$

**Q** The white noise having a Two sided PSD of $4\times10^{-3}$ W/Hz is passed through simple RC LPF ckt. having 3db cut off freq of ~~the o/p of~~ RC ckt ~~the~~ 2kHz. Determine the noise power at the o/p of RC ckt.

Sol$^n$:

$$\frac{N_o}{2} = 4 \times 10^{-3}$$

$\therefore \underline{N_o} = 8 \times 10^{-3}$ w/Hz.

$\therefore$ Now, $f_{3dB} = \dfrac{1}{2\pi RC}$ for RC LPF.

$$\therefore \quad 2000 = \frac{1}{2\pi RC}$$

$$\Rightarrow \frac{1}{RC} = 4\pi \times 1000$$

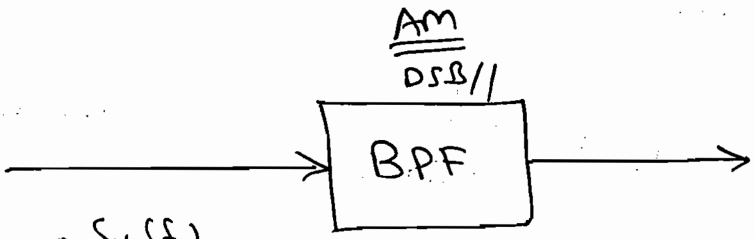
Now, Noise Power $= \dfrac{4 N_o}{4RC}$ watts

$$= \frac{8 \times 10^{-3}}{4} \times 4\pi \times 1000$$

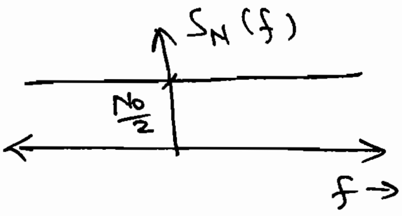
$= 25.12$ watts.

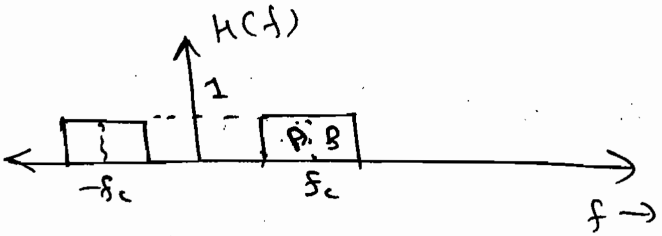
## ② When White noise Passed through BPF:

⇒



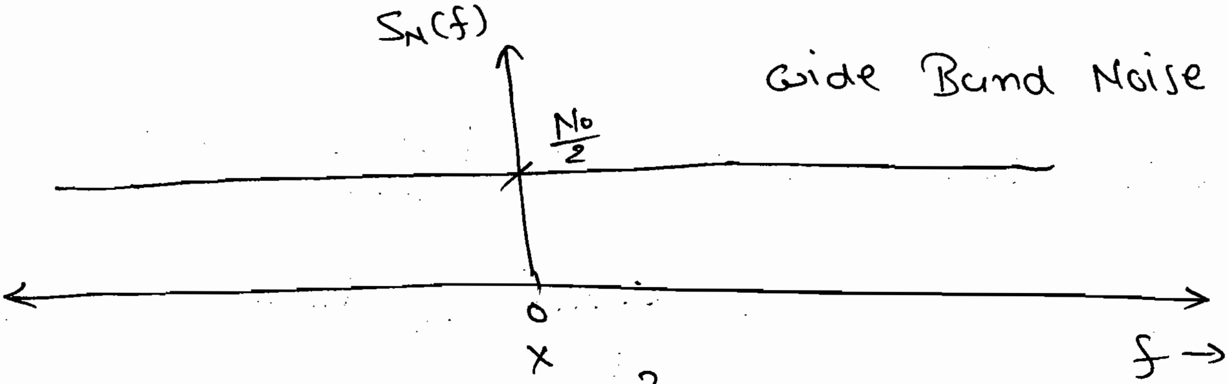
white noise



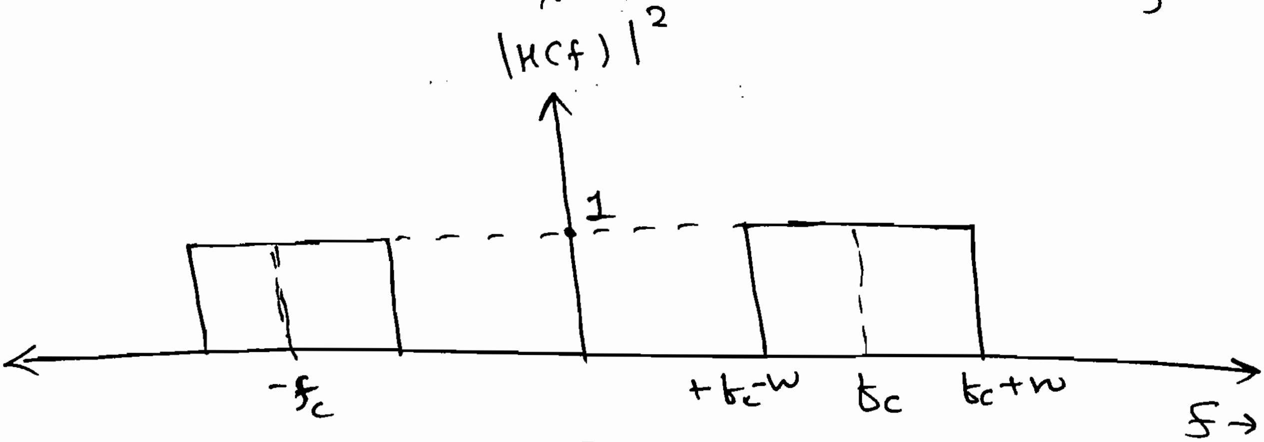


⇒

Wide Band Noise

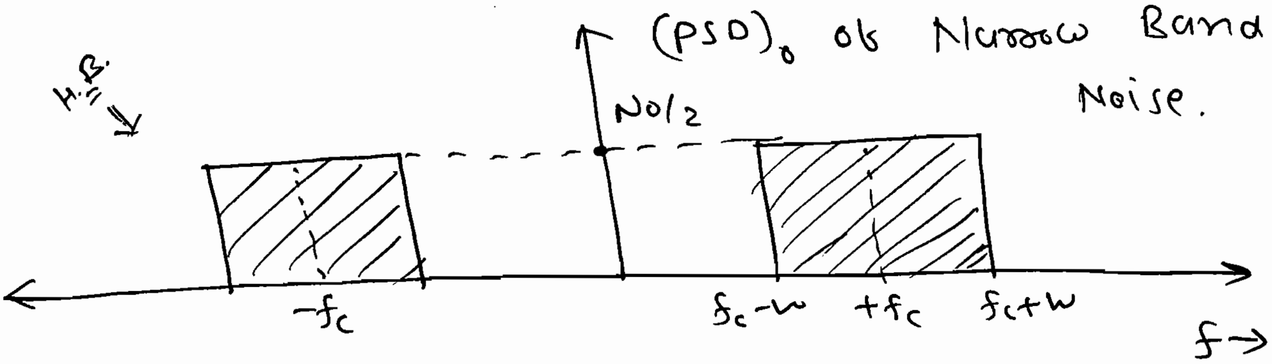


X



=

(PSD)$_0$ of Narrow Band Noise.



⇒ Noise Power = Area under (PSD)$_0$

$$= 2 \times \frac{N_0}{2} \times 2W$$

Noise Power = $2 N_0 W$

=> $\boxed{\text{Noise Power} = 2N_oW.}$ ← H.B

→ $\boxed{R(\tau) = 2N_ow \operatorname{sinc}(2w\tau) \cdot \cos 2\pi f_c t}$

=> $\cos 2\pi f_c t$ terms is come because (PSD)o shifted left as well as right.

✓ => If the white noise is passed through a BPF, then the resultant Noise at the o/p of the BPF is called as the <u>Narrow Band Noise.</u>

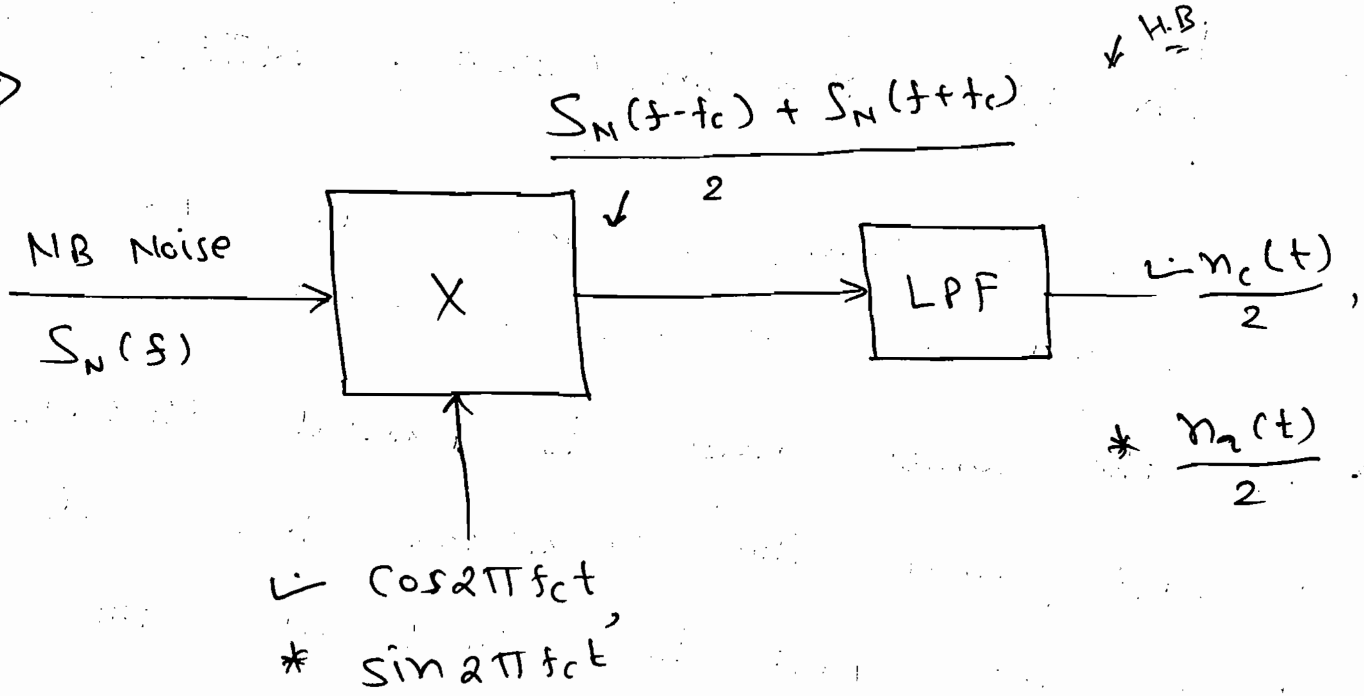
=> Narrow Band Noise represent mathematically as, ↙ H.B

$$\boxed{\underbrace{n_c(t)}_{\text{Inphase Component}} \cdot \cos 2\pi f_c t + \underbrace{n_q(t)}_{\text{Quadrature component}} \cdot \sin 2\pi f_c t.}$$

=> To determine the inphase (or) Quadrature Componenet, the following method is used.

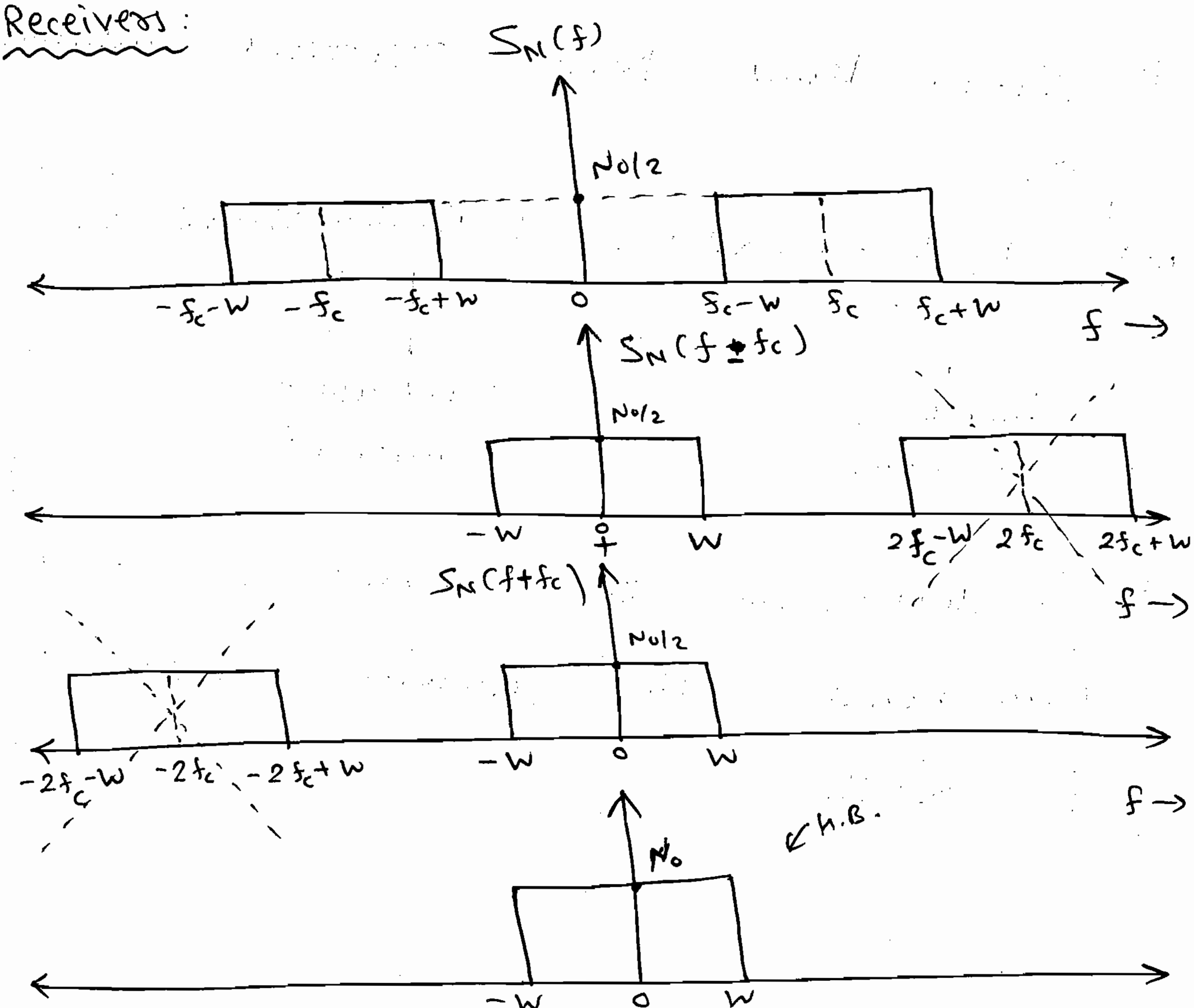
## => Method to determine $n_c(t)$ & $n_q(t)$:

=>



## * PSD of Inphase Component in Am & DSB Receivers:

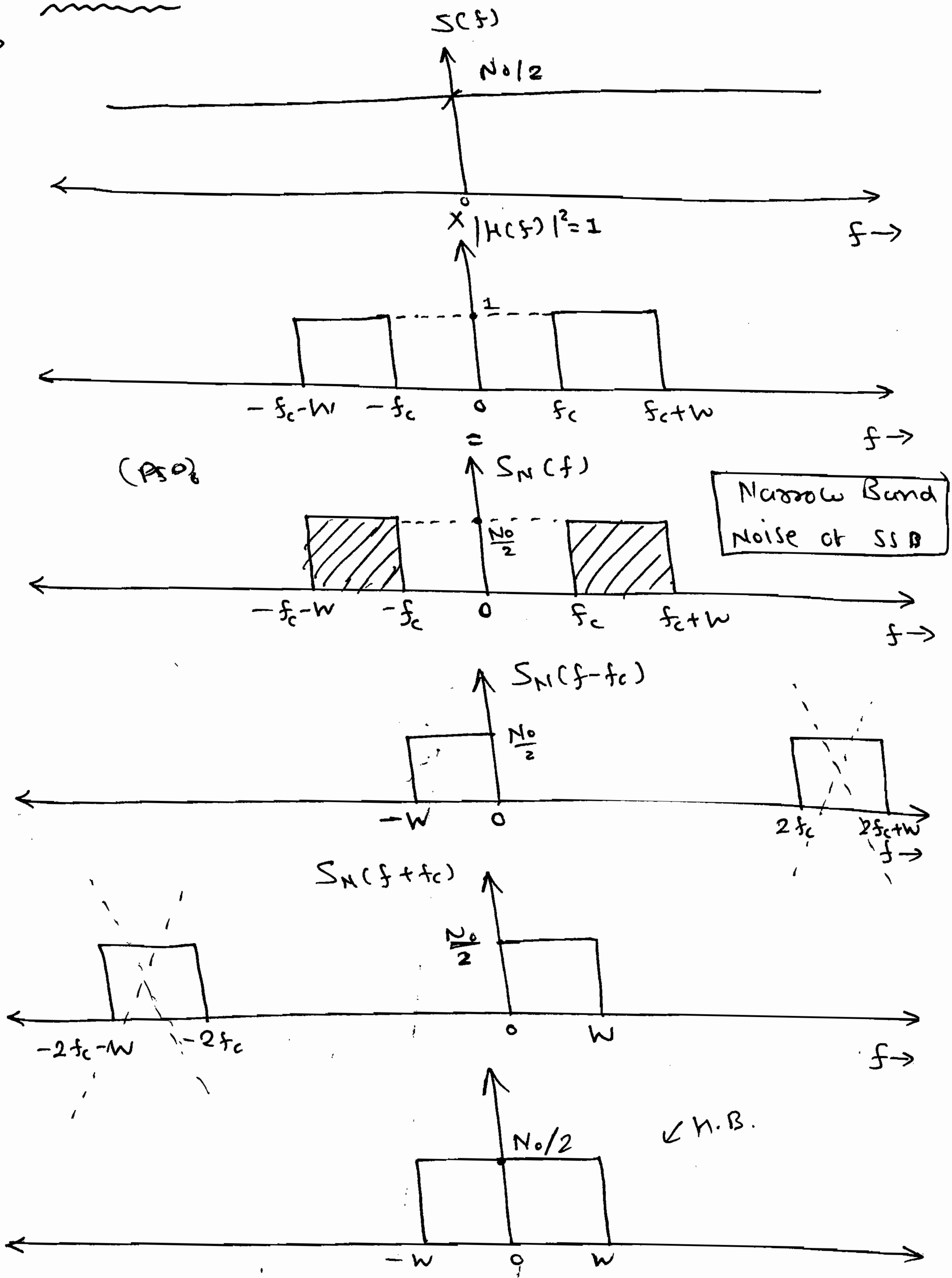
=>



PSD of $n_c(t)$ for Am & DSB

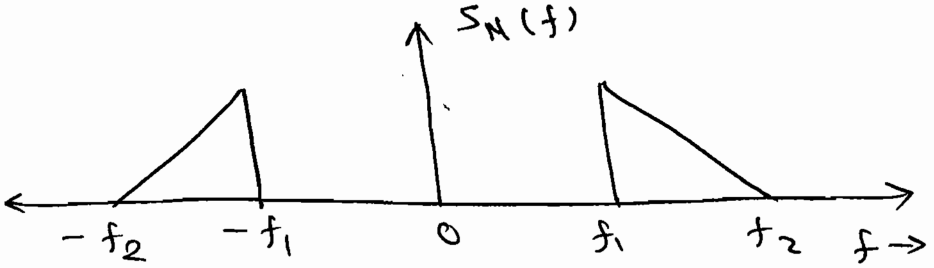
# * PSD of Inphase Component in SSB Receivers:

=>



PSD of $n_c(t)$ for SSB

$\boxed{Q}$ The Power spectral density of NB Noise is shown in the fig.

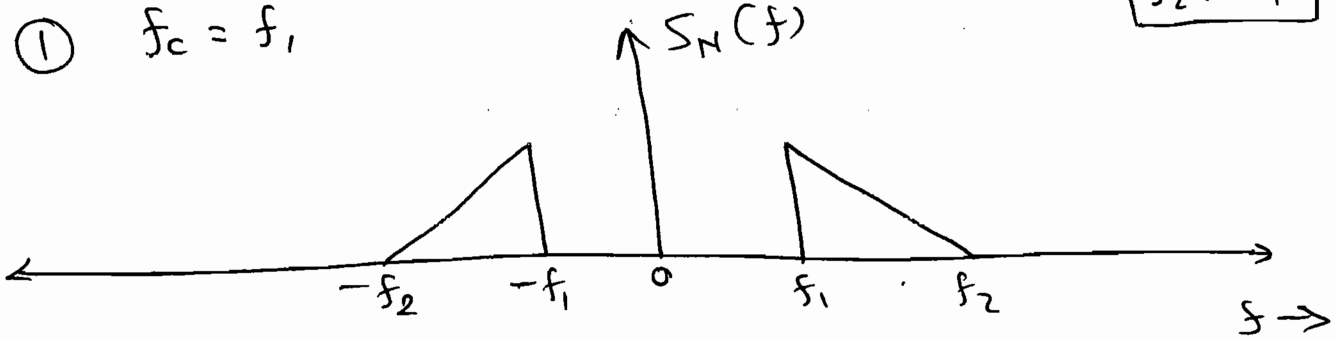


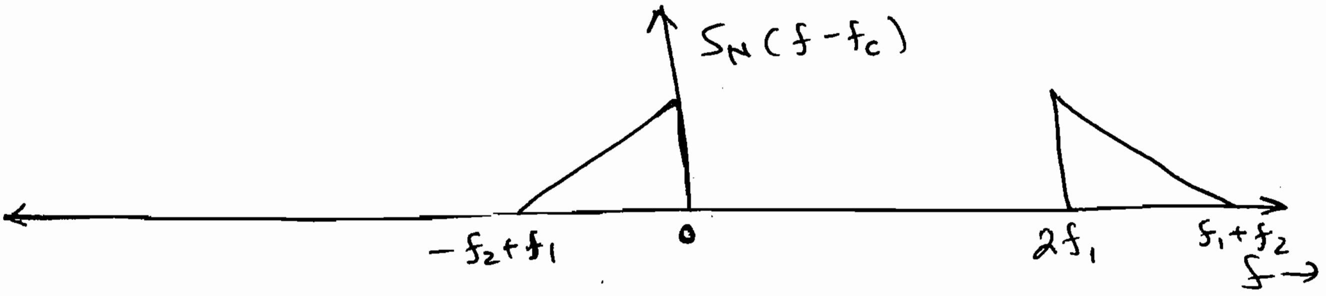
Sketch the PSD of inphase component when the carrier freq is

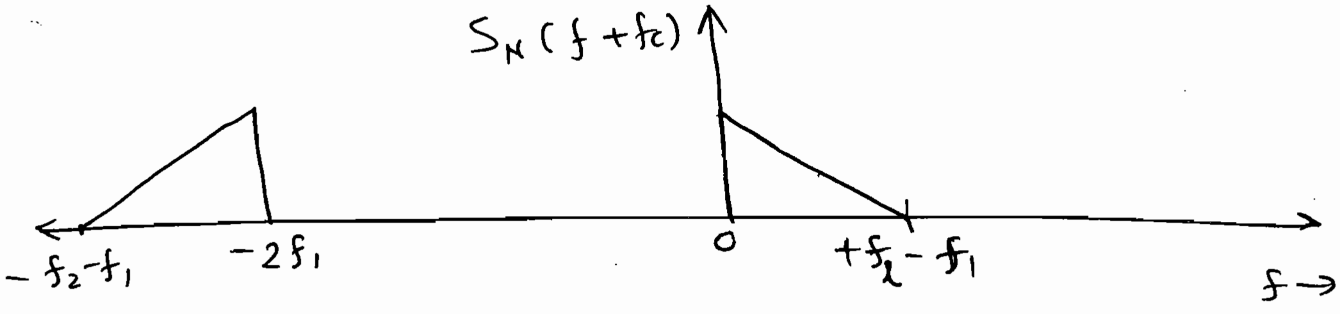
① $f_c = f_1$ ② $f_c = f_2$ ③ $f_c = \frac{f_1+f_2}{2}$.

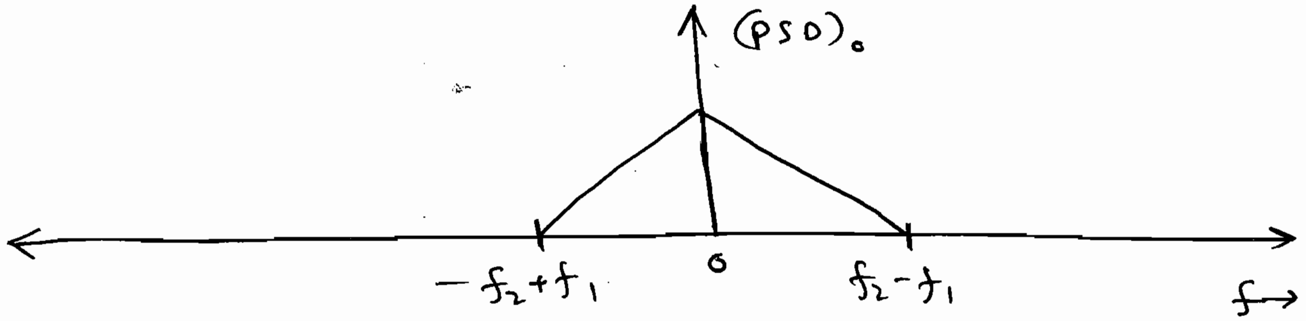
$\boxed{f_2 > f_1}$

Solⁿ: ① $f_c = f_1$

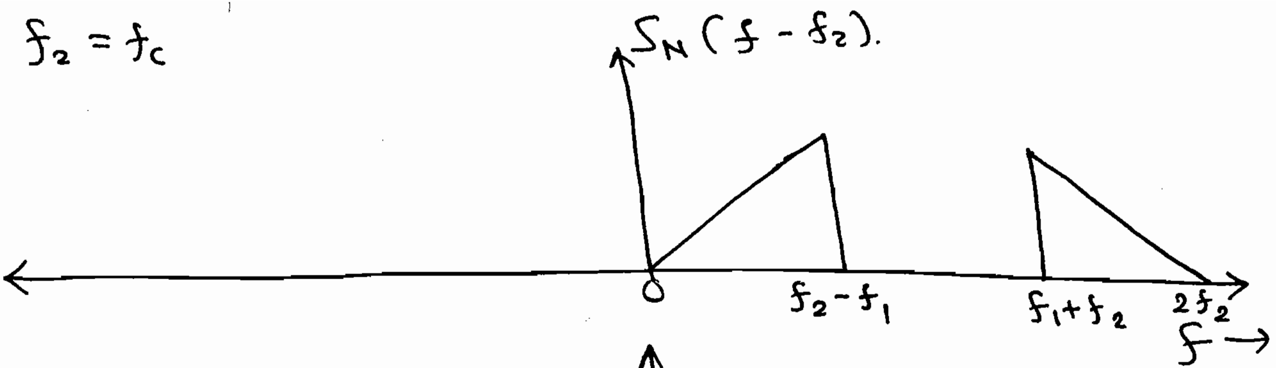


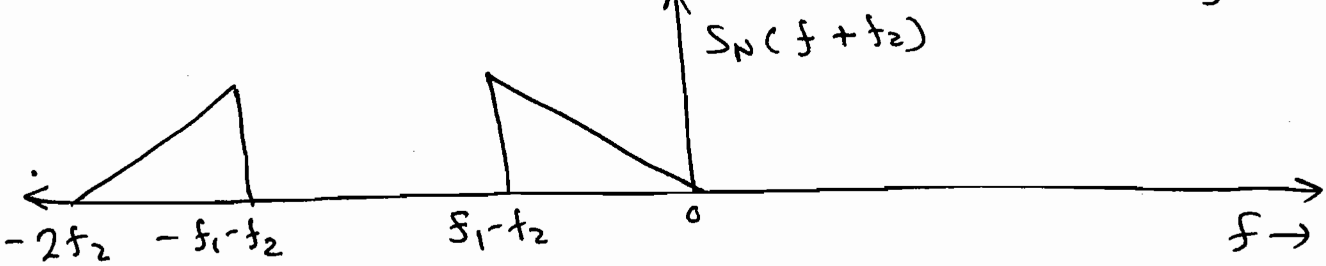


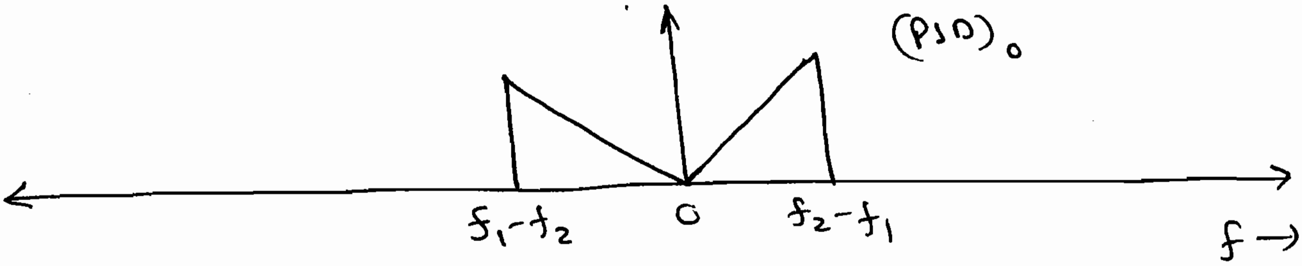




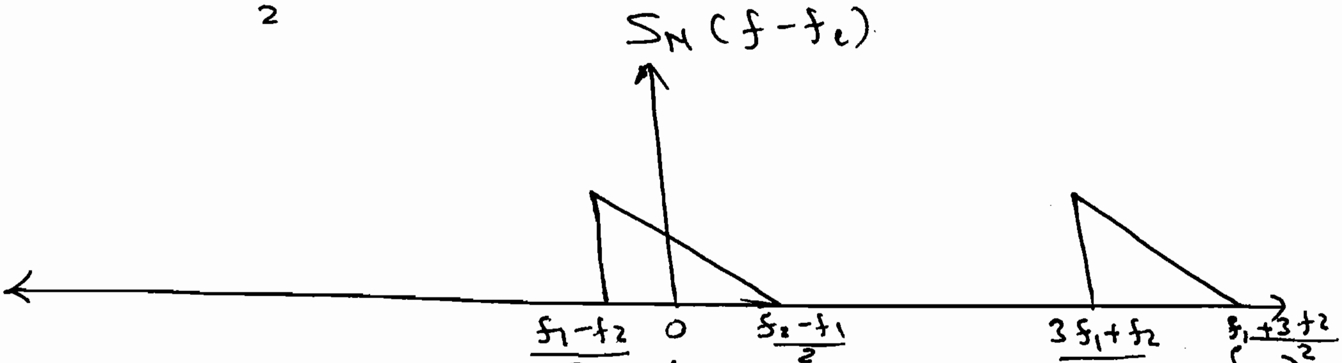
② $f_2 = f_c$

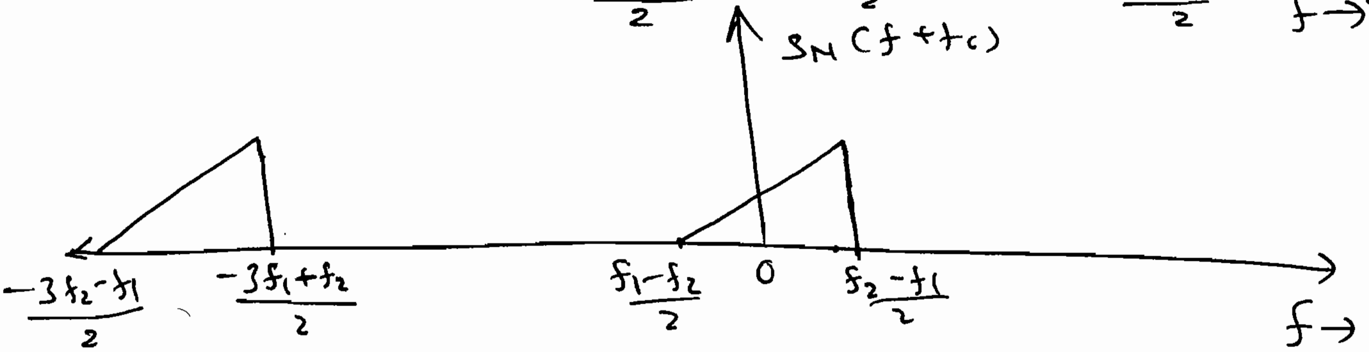


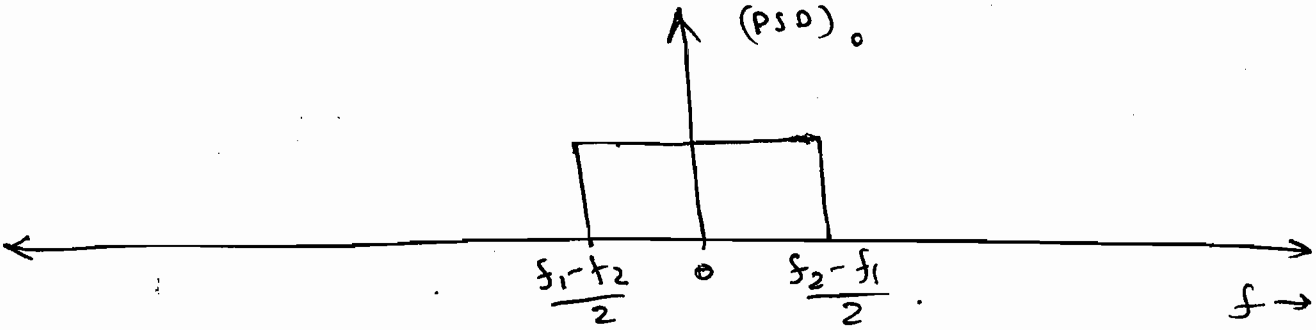




② $f_c = \frac{f_1 + f_2}{2}$







$\boxed{Q}$ PSD of noise is $10^{-8}\left[1-\frac{|f|}{10^8}\right]$ in freq. range $-10^8 \le f \le 10^8$ & '0' otherwise. The noise is passed through an ideal BPF having a centre freq. of 50 MHz. & a BW of 2 MHz. ① Sketch the PSD of at i/p & o/p at B.P.F.

② Find the noise power at o/p.

③ Sketch PSD of inphase component when $f_c$ = 50 MHz.

Solⁿ: Let, PSD of Noise $s(f) = y = 10^{-8}\left[1-\frac{|f|}{10^8}\right]$

Let, $s(f) = y$
$f = x.$

$$\therefore\ y = 10^{-8}\left[1-\frac{|x|}{10^8}\right].$$

$$\therefore\ y = 10^{-8} - 10^{-16}|x|.$$

$$\therefore\ 10^{-16}|x| + y = 10^{-8}$$

$$\therefore\ \frac{|x|}{10^8} + \frac{y}{10^{-8}} = 1.$$

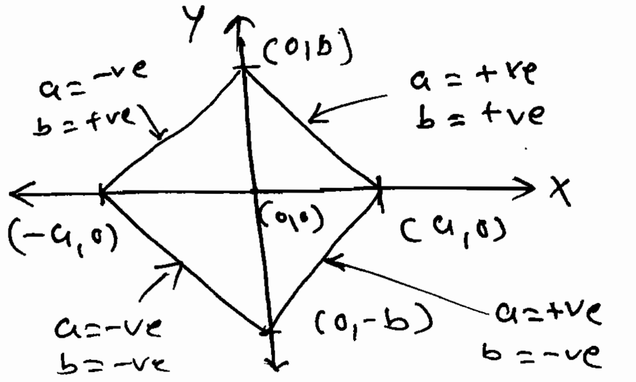
So, ① $x = +ve$.

$$\therefore\ \frac{x}{10^8} + \frac{y}{10^{-8}} = 1 \quad -①$$

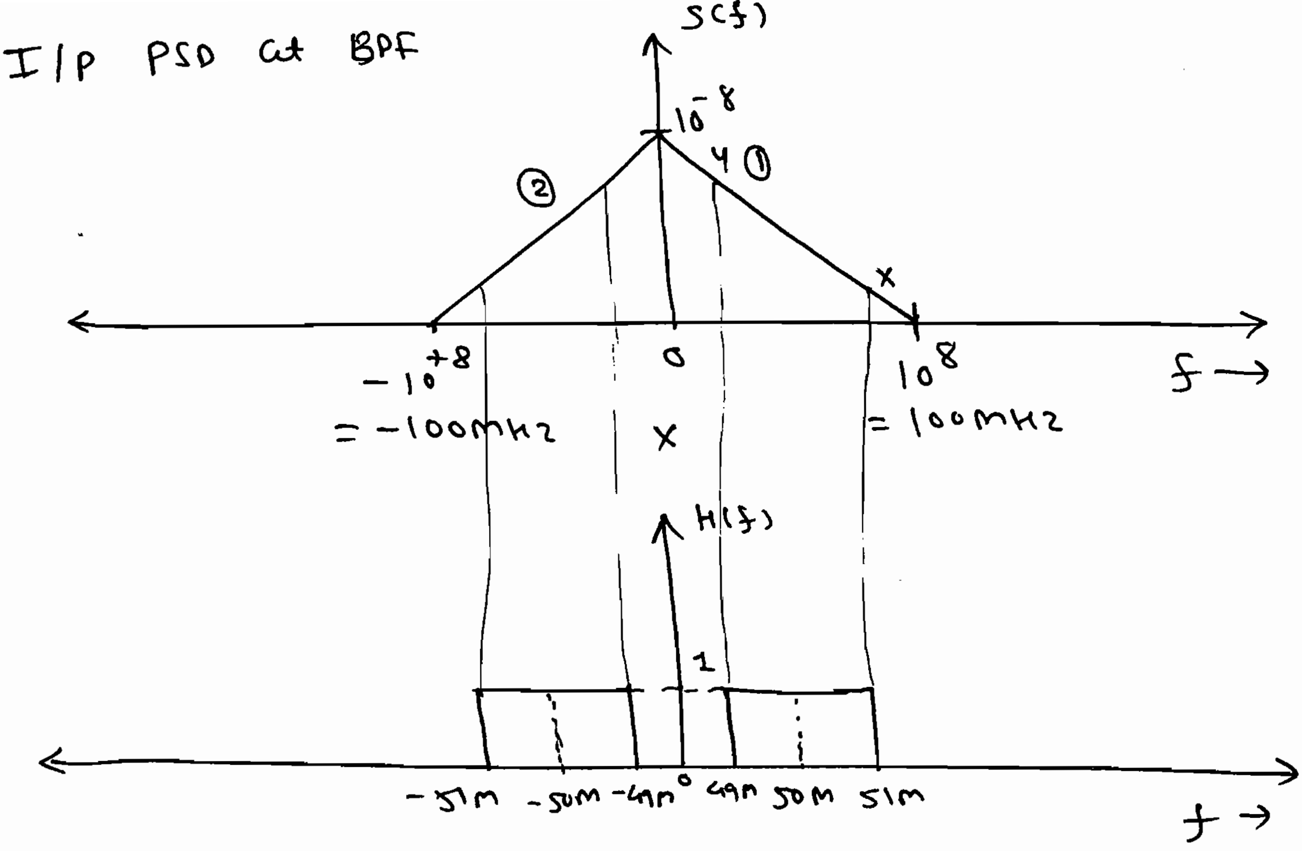
② $x = -ve$

$$\frac{-x}{10^8} + \frac{y}{10^{-8}} = 1 \quad -②$$

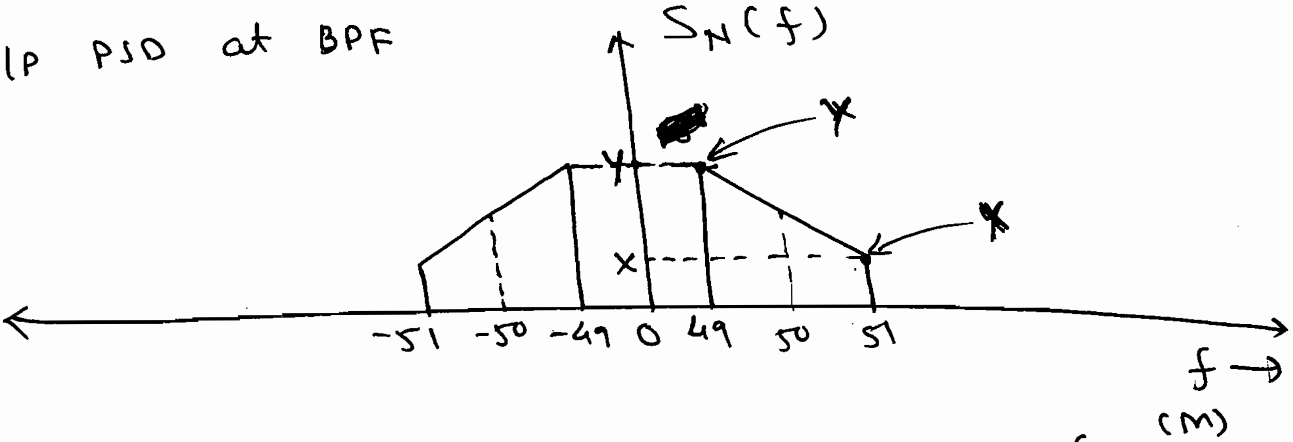
$$\pm\frac{x}{a} \pm \frac{y}{b} = 1.$$



① I/P PSD at BPF



O/P PSD at BPF



⇒ at f = 51M $S_N(f) = 10^{-8}\left[1 - \frac{51\times10^{6}}{10^{8}}\right]$

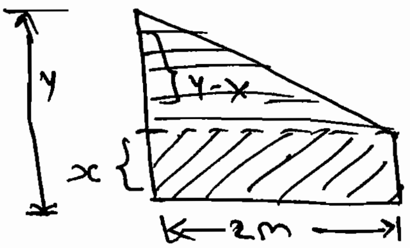
$x = S_N(f) = 0.49\times10^{-8}$ W/Hz.

⇒ at f = 49M $S_N(f) = 10^{-8}\left[1 - \frac{49}{100}\right]$

$y = S_N(f) = 0.51\times10^{-8}$ W/Hz.

⇒ Noise Power = Area under (PSD)o

Noise Power $= 2\left[(x\times 2M) + \frac{1}{2}(2M\times(y-x))\right]$



Noise power

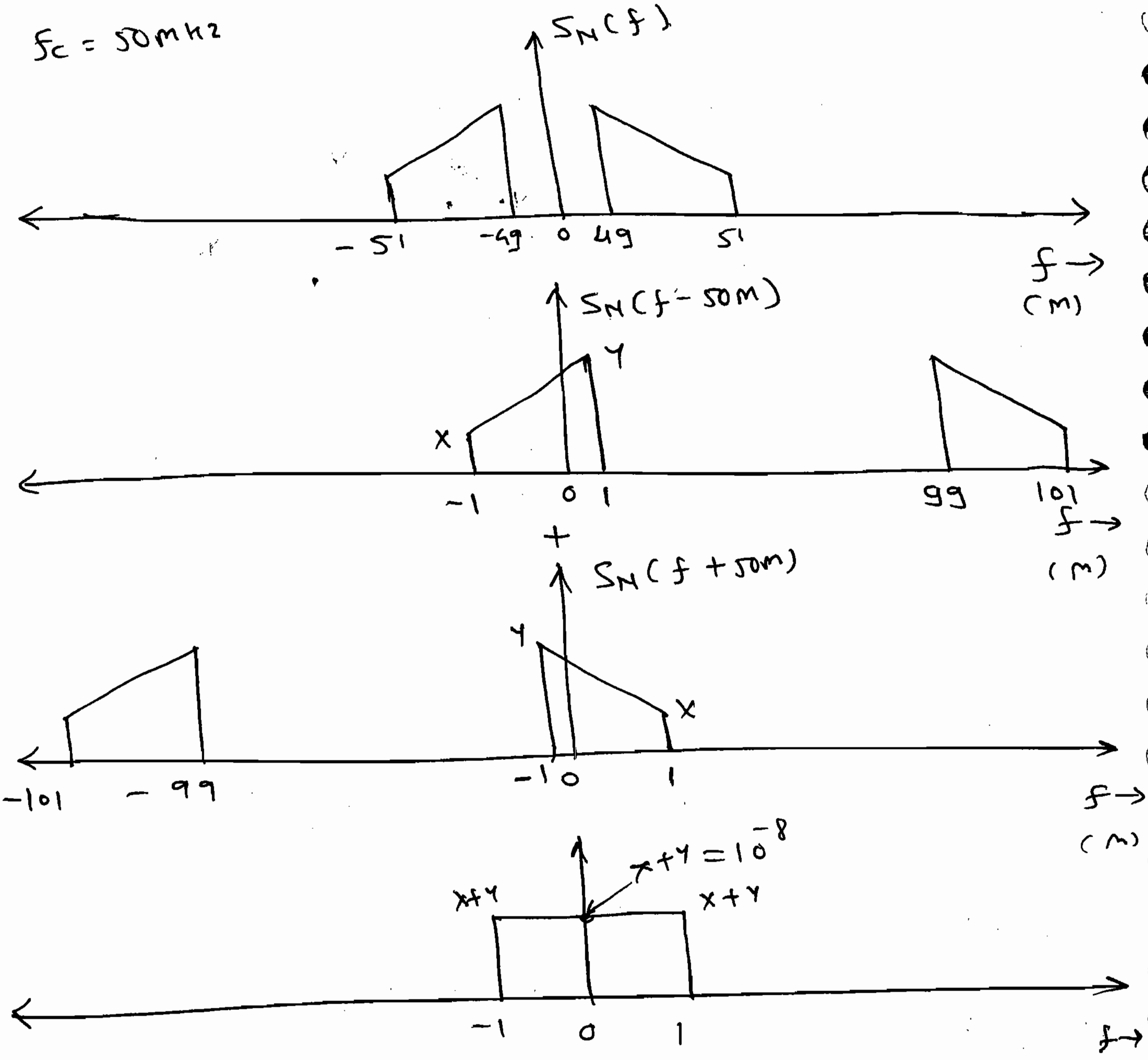
$$= 2\left[\left(2\times10^{+6} \times 0.49\times10^{-8}\right) + \left(\frac{1}{\cancel{2}} \times \cancel{2}\times10^{6} \times 2\times10^{-10}\right)\right]$$

$$= 2\left[9.8\times10^{-3} + 0.2\times10^{-3}\right]$$

$$= 2\times10\times10^{-3}$$

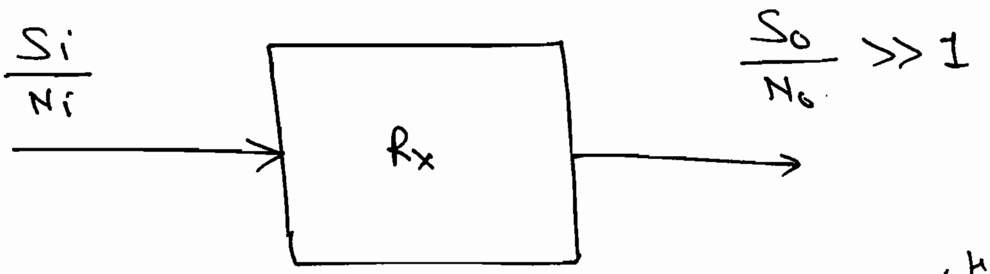
$\therefore$ $\boxed{\text{Noise Power} = 20\,\text{mW}}$

③ $f_c = 50\,\text{MHz}$



# ☆ Noise Analysis of Analog Communication:

⇒ To determine the Performance of the modulation technique, Figure of Merit (FoM) is used.

⇒ 

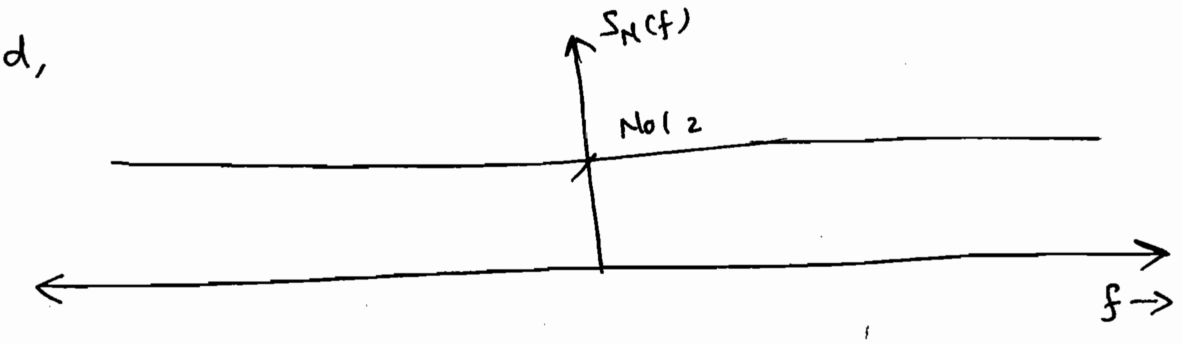
H.B.

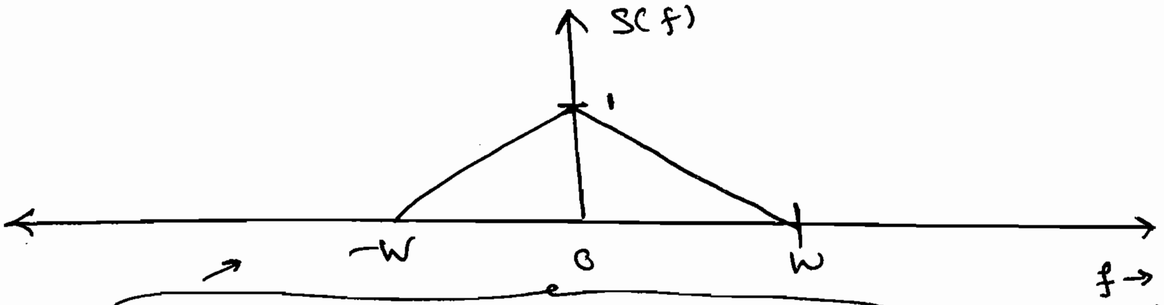
$$\Rightarrow \text{Figure of Merit} = \frac{S_o/N_o}{S_i/N_i} >> 1$$

H.B.

$$\Rightarrow \text{Noise Figure} = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} << 1.$$

⇒ To determine ni following Procedure is used,





AM DSB FM SSB

$n_i$ = noise Power in message BW

= area

$\therefore n_i = \left(\frac{N_0}{2} \times 2W\right)$

$\therefore \boxed{n_i = N_0 W \text{ watts.}}$ ↙ H.B.

## * Figure of Merit of a DSB Receivers:

⟹ Power

dc ⟶ $(dc)^2$

ac ⟶ $(rms)^2$

$10\cos 2\pi f_c t \longrightarrow \left(\frac{10}{\sqrt{2}}\right)^2 = 50W$

⇒ $\boxed{K(10\cos 2\pi f_c t) \longrightarrow K^2(50W).}$ ✓ H.B.

→ Time domain eqn of DSB is,

$s(t) = A_c m(t) . \cos 2\pi f_c t .$

$\therefore S_i = \left(\frac{A_c m(t)}{\sqrt{2}}\right)^2$

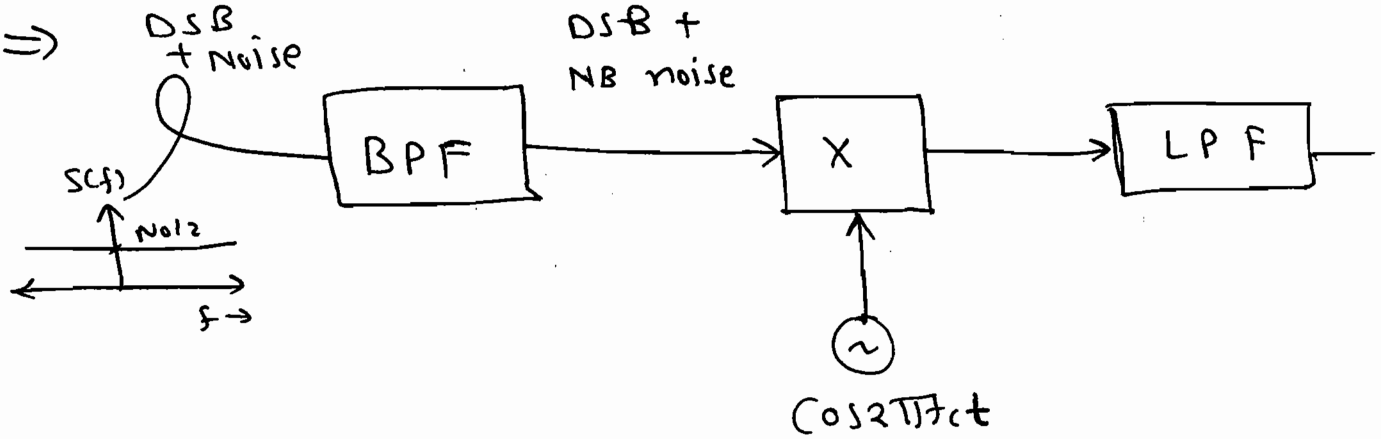
$S_i = \frac{A_c^2}{2} . m^2(t).$ , $n_i = N_0 W$ , Sir2

let, P = Power of message signal

$S_i = \frac{A_c^2 . P}{2}$

$\therefore \boxed{(SNR)_i = \frac{S_i}{N_i} = \frac{A_c^2 . P}{2 N_0 W}}$ ← H.B.

⇒



⇒ The o/p of multiplier is,

$[DSB + NB\ noise] \cos 2\pi f_c t$.

$$= \left[A_c \cdot m(t) \cdot \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_q(t) \cdot \sin 2\pi f_c t\right] \cos 2\pi f_c t$$

$$= A_c \cdot m(t) \cdot \cos^2 2\pi f_c t + n_c(t) \cdot \cos^2 2\pi f_c t + n_q(t) \cdot \sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

⇒ o/p of LPF is.

$m(t) \longrightarrow P$

$\frac{A_c}{2} \cdot m(t) \to \frac{A_c^2 \cdot P}{4}$

$$\underbrace{\frac{A_c \cdot m(t)}{2}}_{\text{signal}} + \underbrace{\frac{n_c(t)}{2}}_{\text{noise}}$$

$$\Rightarrow S_0 = \left(\frac{A_c \cdot m(t)}{2\sqrt{2}}\right)^2 = \frac{A_c^2 \cdot P}{2 \times 2} = \frac{A_c^2 \cdot P}{4}$$

$m(t) \longrightarrow P$

$\frac{A_c}{2} \cdot m(t) \to \frac{A_c^2}{4} \cdot P$

$n_0 =$ $\cancel{n_c(t)}$

Power

$\Rightarrow n_c(t) \longrightarrow (2 N_0 W)$.

$\therefore \frac{1}{2} n_c(t) \longrightarrow 2 N_0 W \times \frac{1}{4} = \frac{2 N_0 W}{4} = n_0$

so, $S_o = \frac{A_c^2 \cdot P}{4}$ , $n_o = \frac{2 N_o W}{4}$.

$$\therefore \boxed{SNR_o = \frac{S_o}{N_o} = \frac{A_c^2 \cdot P}{2 N_o W}} \leftarrow \underline{H.B.}$$

⇒ Figure of Merit $= \frac{(SNR)_o}{(SNR)_i}$

$$\underline{H.B.} \rightarrow \boxed{FoM = 1}$$ for DSB Receiver.

## * Figure of Merit of a SSB Receiver.

⇒ $(S_i)_{SSB} = 50\% \cdot (S_i)_{DSB}$.

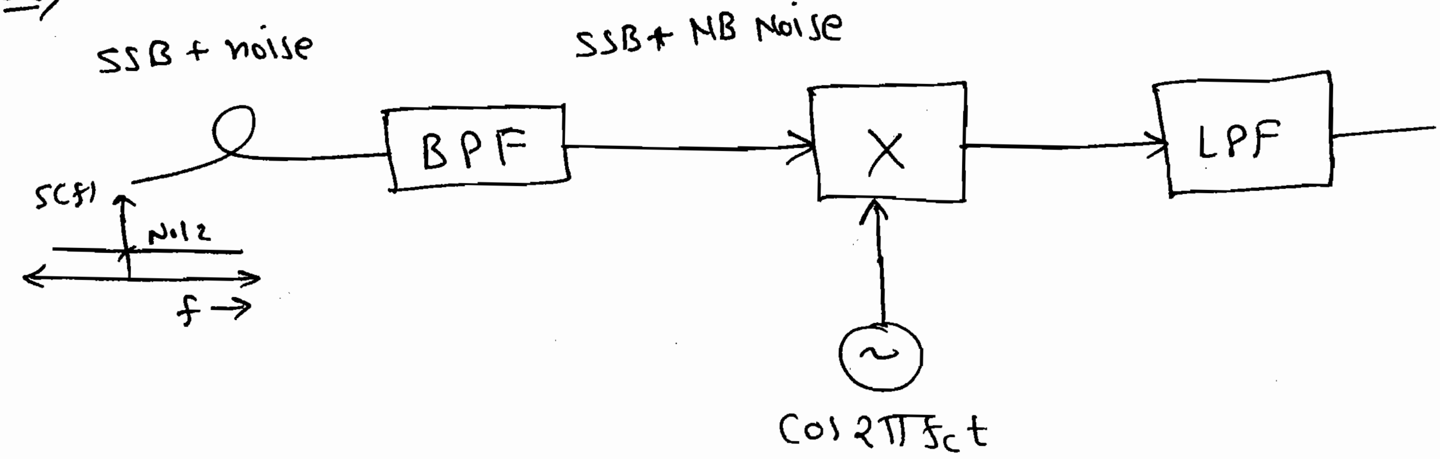
$\therefore (S_i)_{SSB} = \frac{1}{2} \cdot (S_i)_{DSB}$.

$\therefore (S_i)_{SSB} = \frac{A_c^2 \cdot P}{4}$.

$n_i = N_o \cdot W$.

$$\therefore \boxed{(SNR)_i = \frac{S_i}{n_i} = \frac{A_c^2 \cdot P}{4 N_o W}} \leftarrow H.B.$$

⇒



⇒ The o/p of multiplier is,

$(SSB + NB \text{ noise}) \cos 2\pi f_c t.$

$$= \left[\frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \hat{m}(t). \sin 2\pi f_c t + n_c(t). \cos 2\pi f_c t + n_q(t). \sin 2\pi f_c t\right] \cos 2\pi f_c t$$

$$= \frac{A_c}{2} m(t). \cos^2 2\pi f_c t \mp A_c \hat{m}(\cancel{t)}. \sin 2\pi f_c t. \cos 2\pi f_c t + n_c(t). \cos^2 2\pi f_c t + n_q(t). \cancel{\sin} 2\pi f_c t. \cos 2\pi f_c t.$$

⇒ o/p of the LPF is,

$$\therefore = \underbrace{\boxed{\frac{A_c}{4} m(t)}}_{\text{signal}} + \boxed{\frac{n_c(t)}{2}}. \longrightarrow \text{Noise},$$

$$\Rightarrow \quad m(t) \longrightarrow P$$

$$\rightarrow \frac{A_c}{4}.m(t) \longrightarrow \frac{A_c^2}{16}. P.$$

$$\rightarrow n_c(t) \longrightarrow N_0.W$$

$$\frac{1}{2} n_c(t) \longrightarrow \frac{1}{4}. N_0.W.$$

So, $S_o = \frac{A_c^2.P}{16}$, $n_o = \frac{N_0.W}{4}$.

$$\therefore (SNR)_o = \frac{S_o}{N_o} = \frac{\frac{A_c^2.P}{16}}{\frac{N_0.W}{4}}.$$

$$\therefore \boxed{(SNR)_o = \frac{A_c^2 \cdot P}{4 N_o W}} \leftarrow \underline{\underline{H.B.}}$$

$\Rightarrow$ Figure of Merit $= \dfrac{(SNR)_o}{(SNR)_o} = 1$

$\therefore$ $\boxed{F_oM = 1}$ for SSB Receiver.

## * Figure of Merit of AM:

$$\Rightarrow \quad s(t) = A_c \cos 2\pi f_c t + A_c K_a \cdot m(t) \cdot \cos 2\pi f_c t$$

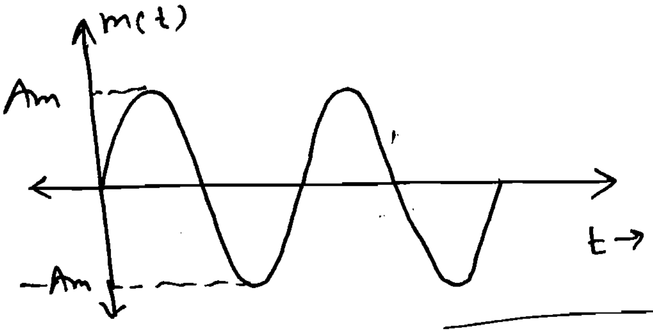
$$\therefore \quad S_i = \frac{A_c^2}{2} + \frac{A_c^2 \cdot K_a^2 \cdot m^2(t)}{2}$$

$$m(t) \xrightarrow{\text{Power}} P$$

$$\therefore \quad S_i = \frac{A_c^2}{2} + \frac{A_c^2 \cdot K_a^2 \cdot P}{2}$$

$$\therefore \boxed{S_i = \frac{A_c^2}{2} \cdot \left[1 + \underline{K_a^2 \cdot P}\right]} \leftarrow \underline{\underline{H.B.}}$$

$\Rightarrow$ if $m(t) = A_m \cos 2\pi f_m t$ = sinusoidal signal.



$$\text{rms} = V_m/\sqrt{2} = \frac{A_m}{\sqrt{2}}$$

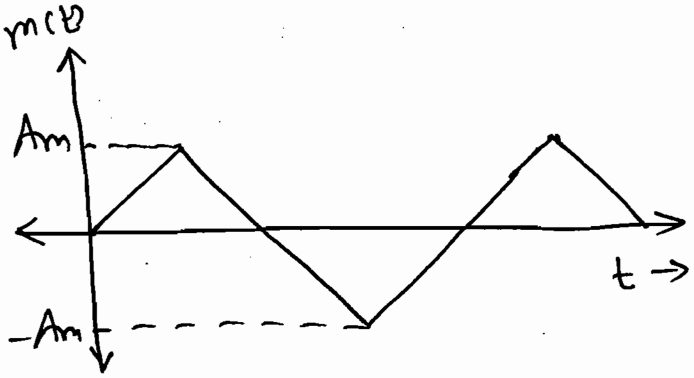
$$\therefore \quad P = \frac{(\text{rms})^2}{R} = \frac{A_m^2}{2R}$$

let, $R = 1\Omega$ $\quad S_i = \dfrac{A_c^2}{2}\left[1 + \dfrac{K_a^2 A_m^2}{2}\right] \boxed{= \dfrac{A_c^2}{2}\left[1 + \dfrac{\mu^2}{2}\right]}$ H.B

⇒ if m(t) = triangular wave.

⇒ rms = $\frac{Am}{\sqrt{3}}$.

∴ $P = \frac{Am^2}{3}$.



⇒ ∴ $S_i = \frac{Ac^2}{2}\left[1 + \frac{Ka^2 . Am^2}{3}\right]$

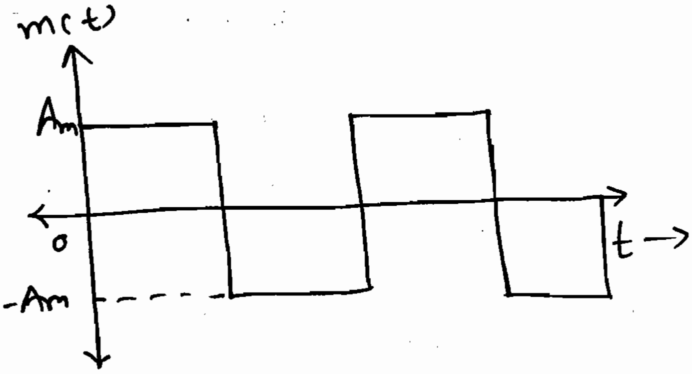
as $\mu = Ka . Am$.

∴ $\boxed{S_i = \frac{Ac^2}{2}\left[1 + \frac{\mu^2}{3}\right].}$ ← H.B.

⇒ if m(t) = Square wave.

∴ rms = Am

∴ $P = Am^2$.



∴ $\boxed{S_i = \frac{Ac^2}{2}\left[1 + \mu^2\right].}$ ← H.B.

⇒ $S_i = \underset{\text{Carrier Power}}{\frac{Ac^2}{2}} + \underset{\text{sideband power.}}{\frac{Ac^2 . Ka^2 P}{2}}$.

∴ $\eta = \frac{P_{SB}}{P_t}$.

$$\therefore \quad \eta = \frac{P_{SB}}{P_t}$$

$$\therefore \quad \eta = \frac{\frac{A_c^2 K_a^2 P}{2}}{\frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}}$$

$$\therefore \quad \boxed{\eta = \frac{K_a^2 P}{1 + K_a^2 P}} \leftarrow \underline{\underline{\text{H.B.}}}$$

$\therefore$ ① **sine** $\Rightarrow \quad P \rightarrow A_m^2/2.$

$$\therefore \quad \eta = \frac{K_a^2 A_m^2/2}{1 + K_a^2 \cdot A_m^2/2}$$

when $\underline{\underline{\mu = 1}}$

$$\boxed{\eta = 33.33\ \%}$$

$$\therefore \quad \eta = \frac{K_a^2 A_m^2}{2 + K_a^2 \cdot A_m^2}$$

$$\therefore \quad \boxed{\eta = \frac{\mu^2}{\mu^2 + 2}} \leftarrow \underline{\underline{\text{h.B}}}$$

② **Square wave** $\Rightarrow \quad P = A_m^2.$

$$\therefore \quad \eta = \frac{K_a^2 A_m^2}{1 + K_a^2 A_m^2}$$

when $\underline{\underline{\mu = 1}}$

$$\therefore \quad \boxed{\eta = \frac{\mu^2}{\mu^2 + 1}} \leftarrow \underline{\underline{\text{h.B}}}$$

$$\boxed{\eta = 50\ \%}$$

③ **Triangular wave** $\Rightarrow \quad P = A_m^2/3.$

$$\therefore \quad \eta = \frac{K_a^2 \cdot A_m^2/3}{1 + K_a^2 \cdot A_m^2/3}$$

when $\underline{\underline{\mu = 1}}$

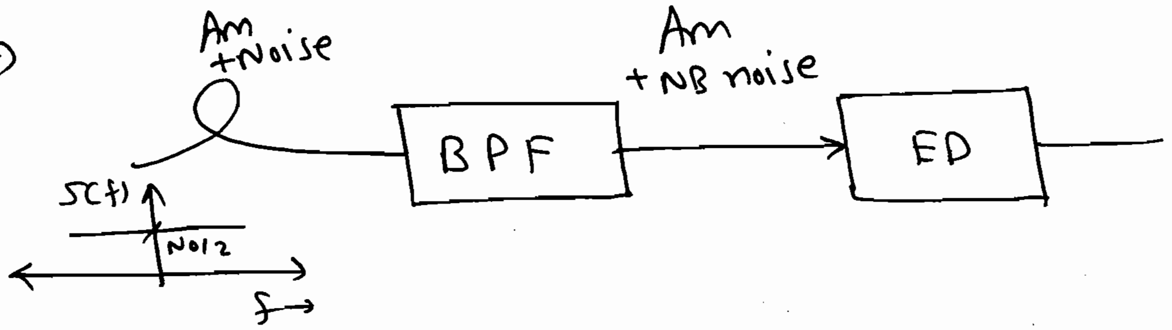
$$\therefore \quad \boxed{\eta = \frac{\mu^2}{\mu^2 + 3}} \leftarrow \underline{\underline{\text{h.B.}}}$$

$$\boxed{\eta = 25\ \%}$$

⇒ $(SNR)_i = \frac{S_i}{n_i}$

$$\therefore \boxed{(SNR)_i = \frac{A_c^2\,[1+K_a^2.m^2(t)]}{2N_oW}} \leftarrow \underline{H.B}$$

⇒



⇒ O/P of BPF is.

AM + NB noise.

$$= A_c[1+K_am(t)]\cos 2\pi f_ct + n_c(t).\cos 2\pi f_ct + n_q(t)\cdot\sin 2\pi f_ct$$

$$= [A_c[1+K_am(t)] + n_c(t)]\cos 2\pi f_ct + n_q(t)\sin 2\pi f_ct$$

Now, O/P of ED is

$$\sqrt{[A_c[1+K_am(t)]+n_c(t)]^2 + \underbrace{(n_q(t))^2}_{\rightarrow \text{very small.}}}$$

So, O/P of ED is

$$= A_c[1+K_am(t)]+n_c(t)$$

$$= \underbrace{\cancel{A_c}}_{dc} + \underbrace{A_cK_am(t)}_{signal} + \underbrace{n_c(t)}_{noise}.$$

$$\Rightarrow \quad m(t) \longrightarrow P$$

$$A_c K_a \, m(t) \longrightarrow A_c^2 . K_a^2 . P$$

$$\therefore \quad S_o = A_c^2 . K_a^2 . P .$$

$$n_c(t) \longrightarrow 2 N_o W .$$

$$\therefore \quad N_o = 2 N_o W .$$

H.B.

$$\therefore \quad \boxed{(SNR)_o = \frac{S_o}{N_o} = \frac{A_c^2 . K_a^2 . P}{2 N_o W}}$$

$$\therefore \quad \text{Figure of Merit} = \frac{(SNR)_o}{(SNR)_i}$$

$$= \frac{\dfrac{\cancel{A_c^2} . K_a^2 . P}{\cancel{2 N_o W}}}{\dfrac{\cancel{A_c^2} \left[ 1 + K_a^2 m^2(t) \right]}{\cancel{2 N_o W}}}$$

H.B

$$\therefore \quad \boxed{FoM = \frac{K_a^2 . P}{1 + K_a^2 . P} = \eta < 1 .}$$

for AM.

## * FoM of FM:

$$\Rightarrow \quad F.M = \frac{3 K_f^2 P}{W^2}.$$

$\therefore$ for single tone $P = \frac{A_m^2}{2}$.

$$\therefore \quad FoM = \frac{3}{2} \cdot \frac{K_f^2 \cdot A_m^2}{f_m^2}.$$

$$\therefore \quad FoM = \frac{3}{2} \cdot \left( \frac{K_f \cdot A_m}{f_m} \right)^2.$$

$$\beta = \frac{K_f \cdot A_m}{f_m}.$$

$$\therefore \quad \boxed{FoM = 3/2 \cdot \beta^2.} \leftarrow \underline{H.B}$$

for $\beta = 10$ $\quad FoM = 150$.

## * FoM of PM:

$$\Rightarrow \quad FoM = K_p^2 \cdot P.$$

for single tone modulation $\quad P = \frac{A_m^2}{2}$.

$$\therefore \quad FoM = \frac{K_p^2 \cdot A_m^2}{2}.$$

$$\therefore \quad \beta = \Delta\phi = K_p \cdot A_m.$$

$$\therefore \quad \boxed{FoM = \frac{\beta^2}{2} = \frac{\Delta\phi^2}{2}.} \leftarrow \underline{H.B}$$

*

$$\frac{S_i}{n_i} \longrightarrow \boxed{R_x} \longrightarrow \frac{S_o}{n_o}$$

$$\Rightarrow \text{For} \left.\begin{matrix} \text{DSB} \\ \text{SSB} \\ \text{VSB} \end{matrix}\right\} \longrightarrow \text{FoM} = 1 \Rightarrow (SNR)_o = (SNR)_i$$

$$\Rightarrow \text{For AM} \longrightarrow \text{FoM} = \frac{1}{3} \Rightarrow (SNR)_o = \frac{1}{3}(SNR)_i$$

when $[\mu = 1]$

$$\Rightarrow \text{For FM} \longrightarrow \text{FoM} = 150 \Rightarrow (SNR)_o = 150(SNR)_i$$

when $[\beta = 10]$

$$\Rightarrow \text{For PM} \longrightarrow \text{FoM} = 50 \Rightarrow (SNR)_o = 50\,(SNR)_i$$

$[\beta = 10]$

**Q** An Audio signal is Band limited to 15 KHz and transmitted through a channel after modulation. The signal is affected by white noise having a two sided PSD of $10^{-10}$ watts/Hz. The Power loss in the channel is 50 dB. The SNR required at the o/p of the receiver is 40 dB. Determine the transmitted 121 Power when the modulation used is, ① DSB

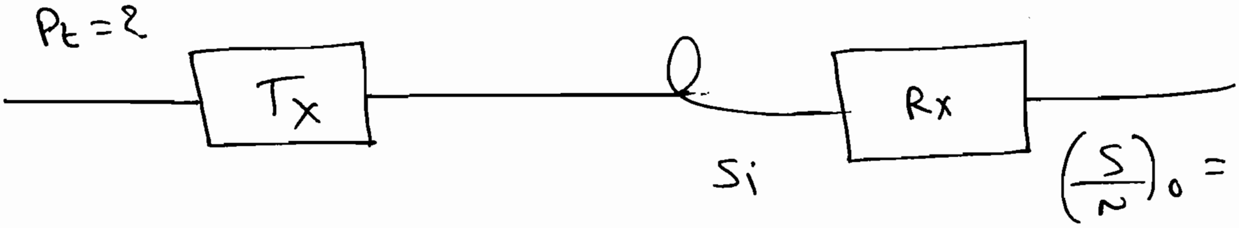
② Am with $\mu = 1$

③ FM $[\beta = 5]$.

Ans:

$P_L = 50\,dB$

$P_L = 10^5\,W$



$\left(\frac{S}{N}\right)_o = 40\,dB$

$\left(\frac{S}{N}\right)_o = 10^4\,W.$

① DSB:

For DSB FOM = 1.

$\frac{N_o}{2} = 10^{-10}$ watts/Hz

$\therefore \frac{S_o}{N_o} = \frac{S_i}{n_i}.$

$\therefore S_i = n_i \times 10^4.$

$n_i = N_o W = 2 \times 10^{-10} \times 15 \times 10^3$ watts/Hz.

$\therefore S_i = 2 \times 10^{-10} \times 15 \times 10^3 \times 10^4.$

$\therefore \boxed{S_i = 30\,m\,watts}$

$\therefore (S_i)_{dB} = (P_t)_{dB} - (P_L)_{dB}.$

$\therefore 20 \log (s_i)_{dB} = 20 \log (P_t)_{dB} - 20 \log (P_L)_{dB}.$

$\therefore \boxed{S_i = \frac{P_t}{P_c}.}$

$\Rightarrow \quad P_t = S_i \times P_L.$

$\therefore \quad P_t = 30 \times 10^{-3} \times 10^5$

$\boxed{P_t = 3\,kW}$

② AM with $\mu = 1.$

$\therefore \quad FoM = \dfrac{\mu^2}{\mu^2+2}$

$FoM = 1/3.$

$\therefore \quad \dfrac{S_o}{n_o} = \dfrac{1}{3} \cdot \dfrac{S_i}{n_i}.$

$\therefore \quad S_i = 3 \times n_i \times \dfrac{n_o}{S_o}.$

$= 3 \times 2 N_o \omega \times 10^4.$

$= 3 \times 2 \times 10^{-10} \times 15 \times 10^3 \times 10^4.$

$\therefore \quad \boxed{S_i = 90\, mW.}$

$\therefore \quad P_t = S_i \cdot P_L.$

$\therefore \quad P_t = 90 \times 10^{-3} \times 10^5$

$\boxed{P_t = 9\,W.}$

③ FM $[\beta = 5].$

$\therefore \quad FoM = 3/2 \cdot \beta^2 = \cancel{150} \cdot \frac{75}{2}.$

$\therefore \quad \left(\frac{S}{N}\right)_o = \frac{150}{4} \times \frac{S_i}{n_i}.$

611

$\therefore$ ~~$S_i = N_o W \times \frac{150}{4} \times 10^4$~~.

$\therefore$ ~~$S_i = 10^{-10} \times 2 \times 15 \times 10^3 \times \frac{75}{2} \times 10^4$~~.

$$\therefore \quad S_i = \frac{2}{75} \times 10^4 \times N_o \cdot W.$$

$$= \frac{2}{\cancel{75}_5} \times 10^4 \times 2 \times 10^{-10} \times \cancel{15} \times 10^3$$

$$S_i = \frac{4}{5} \times 10^{-3}$$

$$\therefore \quad S_i = 0.8 \text{ mW}.$$

$$\therefore \quad P_t = S_i \times P_L.$$

$$\therefore \quad P_t = 0.8 \times 10^{-3} \times 10^5.$$

$$\therefore \quad \boxed{P_t = 80 \text{ W}}$$

=> FM require very less Power Compared to other modulation technique. But BW is very large Compared to Am.

④ Pm $[\beta = 5]$.

$$\therefore \quad Fom = \beta^2/2 = \frac{25}{2}.$$

$$(Fom)_{pm} = \frac{1}{3} (Fom)_{Fm}.$$

$$\therefore \quad P_t = 3 \times 80$$

$$\therefore \quad \boxed{P_t = 240 \text{ W}}$$

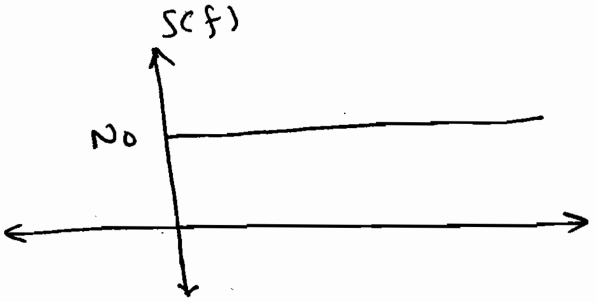
**Q-2** 1 mw Vedio signal Band limited to 100 MHz is transmitted through a channel the Power loss in the channel is 40 dB. The signal is affected by white noise having single sided power spectral density of $10^{-20}$ watts/Hz. Determine the $(SNR)_i$.

Ans:

$P_t = 1\ mw.$ , $W = f_m = 100 \times 10^6\ Hz$

$P_L = 40 dB$

$P_L = 10^4\ W.$



$\therefore n_i = N_o \cdot W$

$\therefore n_i = 10^8 \times 10^{-20}$

$\therefore n_i = 10^{-12}\ W.$

$\therefore S_i = P_t / P_L.$

$\therefore S_i = \frac{10^{-3}}{10^4} = 10^{-7}.$

$\therefore (SNR)_i = \frac{S_i}{n_i} = \frac{10^{-7}}{10^{-12}}.$

$\therefore (SNR)_i = 10^5$

$\therefore \boxed{(SNR)_{i\ dB} = 50\ dB.}$

ell